NASA TECHNICAL MEMORANDUM

NASA TM X-53272

MAY 26, 1965

NASA TM X-53272

THE EFFECT OF TRUNCATION ON TESTS OF HYPOTHESES FOR NORMAL POPULATIONS

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THE EFFECT OF TRUNCATION ON TESTS OF HYPOTHESES FOR NORMAL POPULATIONS[1]

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ABSTRACT

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The loss of power involved in using the usual tests of hypotheses for complete normal populations when, in fact, the population is a truncated normal one is examined. The technique employed is to obtain an asymptotic expansion of the distribution of sums of samples of size n drawn from a truncated normal population. An electronic computer is then employed to consider enough terms of this expansion to obtain the desired accuracy for the loss function, which is tabulated as a function of selected truncation points and sample sizes. Author

[1] The research reported in this paper was submitted as a Ph. D dissertation in statistics at the University of Georgia, Athens, Georgia. This research was performed under NASA Contract NAS8-11175 with the Aerospace Environment Office, Aero-Astrodynamics Laboratory, Marshall Space Flight Center, Huntsville, Alabama. Mr. O. E. Smith and Mr. J. D. Lifsey are the NASA contract monitors. The author is currently with the RCA Service Company, Missile Test Project, Patrick AFB, Florida.

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

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By

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AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

FOREWORD

This report presents results of an investigation performed by the Department of Statistics, University of Georgia, Athens, Georgia as part of NASA Contract NAS8-11175 with the Aerospace Environment Office, Aero-Astrodynamics Laboratory, NASA-George C. Marshall Space Flight Center, Huntsville, Alabama. This research was performed by Mr. Britain J. Williams under the supervision of Dr. A. C. Cohen, Jr., the contract principal investigator, and was submitted in June 1964 as a Ph. D. dissertation in statistics. The NASA contract monitors are Mr. O. E. Smith and Mr. J. D. Lifsey.

The results of this investigation represent a contribution in the area of statistical tests of hypotheses for samples of size n drawn from truncated normal populations.

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THE EFFECT OF TRUNCATION ON TESTS OF HYPOTHESES FOR NORMAL POPULATIONS

SUMMARY

When performing tests of hypotheses on the means of samples drawn from a normal population, the experimenter usually assumes that the sample was drawn from a complete normal population. Because of the physical limitations imposed by the act of recording measurements, this is never the case. When the recording devices used in sampling permit all variables to be measured within three standard deviations of the mean, the assumption that the sample was drawn from a complete normal population does not seriously affect the power of the tests. However, situations arise where difficulty of measurement or the expense of obtaining data restrict the sampling interval to less than three standard deviations of the mean.

Aggarwal and Guttman [1, 2] have examined the loss of power when using tests based on the assumption that the variable being sampled has a complete normal distribution when, in fact, the distribution is a symmetrically truncated normal distribution. They derived the distribution of means based on samples of size $n \geq 4$ using convolutions of

$$f(x) = \begin{cases} (C/\sqrt{2\pi}) \exp(-x^2/2), & |x| < a \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where C is given by

$$\frac{1}{C} = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} e^{-\frac{1}{2}t^2} \, dt . \tag{2}$$

In this paper, the work of Aggarwal and Guttman [1] is extended to non-symmetric truncation and arbitrary sample size. An asymptotic series for the distribution of sums of samples of size n drawn from a truncated normal population is developed, and tables which show the loss of power as a function of selected truncation points and sample sizes are presented.

I. INTRODUCTION

Francis [12] examined the situation of a normal population whose members had values greater (or less) than a given rejected value. Cohen [6, 7, 8, 9] has extensively examined the question of estimating the mean and variance of a normally distributed population from truncated samples. The problem of selecting a truncation point for a sample from a normal population with known parameters in order to meet requirements on the means was investigated by Clark [5].

Birnbaum and Andrews [4] have pointed out that $n\bar{x}$ has a limiting normal distribution. Thus, for large n, one can obtain an approximate cumulative distribution of $\bar{x}$. For arbitrary n, however, no general formula giving the distribution of means (or sums) of samples of size n drawn from a truncated normal population is available.

II. ASYMPTOTIC EXPANSION FOR THE DISTRIBUTION OF SUMS OF A TRUNCATED VARIATE

The distribution function and density function of a standard normal variate will be denoted by

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-t^2/2)\ dt \tag{3}$$

and

$$\phi(x) = \Phi'(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2), \quad -\infty \le x \le \infty, \tag{4}$$

respectively. If x is distributed as a normal variate with mean μ and standard deviation σ, its distribution function and density function will be denoted by $F(x)$ and $f(x) = F'(x)$, respectively. When the population is truncated on the left at $\mu + \sigma a$ and on the right at $\mu + \sigma b$, the density function will be denoted by

$$f(x; a, b) = \frac{C}{\sigma\sqrt{2\pi}} \exp[(x-\mu)^2/2\sigma^2], \quad \mu + \sigma a \le x \le \mu + \sigma b, \tag{5}$$

where C is given by

$$\frac{1}{C} = \frac{1}{\sqrt{2\pi}} \int_a^b \exp(-t^2/2)\, dt = F(b) - F(a) . \tag{6}$$

Let x be a standard normal variate with distribution function given by (3). Then

$$\int_{-\infty}^{\infty} \exp(itx)\, d\Phi(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(itx - x^2/2)\, dx = \exp(-t^2/2) . \tag{7}$$

Repeated partial differentiation of (7) gives

$$\int_{-\infty}^{\infty} \exp(itx)\, d\Phi^{[n]}(x) = (-it)^n \exp(-t^2/2) . \tag{8}$$

If $\phi(x)$ is given by (4), then

$$\int_{-\infty}^{\infty} \exp(itx)\, \phi^{[n]}(x)\, dx = (-it)^n \exp(-t^2/2) . \tag{9}$$

Let $X_1, X_2, \ldots, X_n$ be n independent random variables with the density function of X_i, $i = 1, 2, \ldots, n$, given by (5) and consider the variable

$$X = X_1 + X_2 + \ldots + X_n . \tag{10}$$

If we denote the mean and variance of X_i by μ_1 and σ_1^2, respectively, then the mean and variance of X is given by $\mu = n\mu_1$ and $\sigma^2 = n\sigma_1^2$. Let $\psi(Y)$ denote the characteristic function of $Y = (X - \mu)/\sigma$ and $\psi_1(t)$ denote the characteristic function of $t = X_1 - \mu_1$. Then

$$\psi(Y) = \left[\psi_1\left(\frac{t}{\sigma_1\sqrt{n}}\right)\right]^n .$$

For $v = 1, 2, \ldots$ let χ_v denote the cumulants of $X_1 - \mu$ and χ'_v denote the cumulants of $X_1 - \mu_1$, and put

$$\lambda_v = \frac{\chi_v}{\sigma^v} \quad \text{and} \quad \lambda'_v = \frac{\chi'_v}{\sigma_1^v}.$$

From (A-10) (see Appendix A) we obtain

$$\psi_1(t) = \exp\left[\sum_{v=1}^{\infty} \frac{\lambda'_v}{v!} (it)^v\right],$$

and thus

$$\psi(t) = \left[\psi_1\left(\frac{t}{\sigma_1\sqrt{n}}\right)\right]^n = \exp\left[n \sum_{v=1}^{\infty} \frac{\lambda'_v}{v!} \left(\frac{it}{\sqrt{n}}\right)^v\right].$$

Since $X_1 - \sigma_1$ has mean zero and standard deviation σ_1, it follows that $\chi'_1 = 0$ and $\chi'_2 = \sigma_1^2$, so that $\lambda'_1 = 0$ and $\lambda'_2 = 1$. Hence.

$$e^{t^2/2}\,\psi(t) = \exp\left[n \sum_{v=3}^{\infty} \frac{\lambda'_v}{v!}\left(\frac{it}{\sqrt{n}}\right)^v\right]. \tag{11}$$

$$= \exp\left[(it)^2 \sum_{v=1}^{\infty} \frac{\lambda'_{v+2}}{(v+2)!}\left(\frac{it}{\sqrt{n}}\right)^v\right]$$

$$= \sum_{h=0}^{\infty} \frac{(it)^{2h}}{h!}\left[\sum_{v=1}^{\infty} \frac{\lambda'_{v+2}}{(v+2)!}\left(\frac{it}{\sqrt{n}}\right)^v\right]^h.$$

Expanding the right-hand side of (11) in powers of $n^{-\frac{1}{2}}$ and collecting terms gives

$$\psi(t) = e^{-t^2/2} + \sum_{v=1}^{\infty} \frac{b_{v,v+2}(it)^{v+2} + \dots + b_{v,3v}(it)^{3v}}{n^{v/2}} e^{-t^2/2}, \qquad (12)$$

where $b_{v,v+2h}$ is a polynomial in $\lambda'_3, \lambda'_4, \dots, \lambda'_{v-h+3}$ which is independent of n.

Applying the integral relation (9) to (12) yields the following form of the density function of Y:

$$f_n(Y) = \phi(Y) + \sum_{v=1}^{\infty} (-1)^v \frac{b_{v,v+2}\phi^{[v+2]}(Y) + \dots + b_{v,3v}\phi^{[3v]}(Y)}{n^{v/2}} \qquad (13)$$

This series is due to Edgeworth [11]. A corresponding expansion for $F_n(Y)$ is obtained by replacing $\phi(Y)$ in (13) by $\Phi(Y)$. Cramér [10] has shown that this series is asymptotic in powers of $n^{-\frac{1}{2}}$ and that the remainder term is of the same order as the first term neglected.

III. TESTS OF HYPOTHESES

Consider a sample of size n drawn from a normal population with mean μ and variance σ^2. We may assume without loss of generality that $\sigma^2 = 1$. If the parent population is truncated on the left at $\mu + \sigma a$ and on the right at $\mu + \sigma b$, the density function of the truncated variable will be denoted by $f(X; \mu, \sigma, a, b)$.

A uniformly most powerful test of the null hypothesis

H_0: $\mu = 0$

against the alternative hypothesis

H_a: $\mu > 0$

is given by

$$\left.\begin{array}{l}\text{reject } H_o \text{ if } \overline{X} > Z_\alpha/\sqrt{n} \\ \text{accept } H_o \text{ otherwise}\end{array}\right\}, \qquad (14)$$

where Z_α is the point exceeded with probability α using the distribution of the standard normal variable, $Z \sim N(0,1)$. A usual test of size α for a complete normal population becomes a test of size

$$\alpha' = \Pr(Z_t > Z_\alpha/\sqrt{n}),$$

where Z_t is a truncated normal variate with density function $f(Z_t;\ 0,\ 1,\ a,\ b)$.

If $X \sim N(\mu,\ 1)$, then $\overline{X} \sim N(\mu,\ 1/n)$ and the usual power function, the probability of accepting H_o when, in fact, H_a is true, is given by

$$\begin{aligned} P_u(\mu) &= \Pr(X > Z_\alpha/\sqrt{n} \mid X \sim N(\mu,\ 1/n)) \\ &= \Pr(Z > Z_\alpha - \mu\sqrt{n} \mid Z \sim N(0,\ 1)). \end{aligned} \qquad (15)$$

If X is a truncated normal variate with density function $f(X;\ \mu,\ 1,\ a,\ b)$, the actual power of the test is given by

$$P_a(\mu) = \Pr(X > Z_\alpha/\sqrt{n} \mid X \sim f_n(X;\ \mu,\ 1,\ a,\ b)) \qquad (16)$$

$$= \Pr(Z_t > (Z_\alpha - \mu\sqrt{n})/\sigma_1 \mid Z_t \sim f_n(Z_t;\ \mu,\ 1,\ a - \mu_1,\ b - \mu_1)),$$

where μ_1 and σ_1^2 are the mean and variance of $f(X;\ \mu,\ 1,\ a,\ b)$ and $f_n(X;\ \mu,\ 1,\ a,\ b)$ denotes the density function of means based on samples of size n drawn from $f(X;\ \mu,\ 1,\ a,\ b)$.

Let $L(\mu)$ denote the loss of power when using the usual tests of the hypothesis (14) when, in fact, the population is truncated. Then,

$$L(\mu) = P_u(\mu) = P_a(\mu).$$

Tables 1 through 16 express $L(\mu)$ as a percentage of $P_u(\mu)$ for the one sided test (14) with $\alpha = 0.05$ for various sample sizes and truncation points. The units used are standard deviations of the parent population, and a minus zero in the body of the tables indicates accuracy to within 10^{-3} units while an unsigned zero denotes accuracy to within 10^{-38} units.

IV. CONCLUSIONS

Examination of the tables reveals that there is very little loss of power when the sample is of size 10 or larger and the true value of the mean is more than 0.5 standard deviations away from the value specified in the null hypothesis. Also, as Aggarwal and Guttman [1] have pointed out, there is a change in sign from positive to negative in the loss of power as soon as μ exceeds $Z_\alpha/\sqrt{n}$.

When the value of the true mean of the population is equal to the value specified in the null hypothesis, the size of the test and the power function at that point assume the same value. Thus, from column 1 of the tables, the actual size, α', of the test is easily computed. From Table 1 we observe that, for $n = 5$, the usual test of size $\alpha = 0.05$ is actually of size $\alpha' = 0.001$ when the population is symmetrically truncated within one standard deviation of the mean. Tables 2 and 3 reveal that the actual size increases to 0.014 and 0.031 when the population is symmetrically truncated at 1.5 and 2.0 standard deviations from the mean. Tables 4, 5 and 6 indicate that there is no appreciable loss in power or decrease in the size of the test if the population is symmetrically truncated more than 2.5 standard deviations from the mean.

Let $X \sim N(\mu, 1)$ and consider the variable $X_t \sim f(X; \mu, 1, -\infty, b)$ resulting from single truncation on the right at the point $\mu + b$. Denoting the mean and variance of X_t by μ_1 and σ_1^2, respectively, the mean and variance of $\overline{X}_t \sim f_n(X; \mu, 1, -\infty, b)$ are given by μ_1 and σ_1^2/n, respectively, for samples of size n. Since $\mu_1 < \mu$ we may put $\mu - \mu_1 = \epsilon > 0$, and it follows from the weak law of large numbers that for any constant δ satisfying $0 < \delta < 1$,

$$\Pr(|\overline{X}_t - \mu_1| < \epsilon) > 1 - \delta,$$

where $\overline{X}_t$ is based on a sample of size $n > \sigma_1^2/\delta\epsilon^2$.

Thus, if the usual test (14) is employed when, in fact, the distribution is singly truncated on the right, the actual size of the test will tend to zero as n increases. Similarly, if (14) is employed when, in fact, the population is truncated on the left, the actual size of the test tends to one as n increases.

Examination of Table 7 reveals that when the population is singly truncated on the right at 1.5 standard deviations from the mean, the usual test of size $\alpha = 0.05$ is actually of size $\alpha' = 0.032$ when $n = 5$ and decreases rapidly as n increases. For example, when $n = 15$ the actual size of the test is reduced to $\alpha' = 0.005$. Similarly, from Table 12, when the population is singly truncated on the left at 1.5 standard deviations from the mean, the usual test of size $\alpha = 0.05$ yields an actual test of size $\alpha' = 0.075$ for $n = 5$ and increases with n.

Thus, for singly truncated populations, there is no appreciable loss of power involved in using the usual test when truncation is beyond 1.5 standard deviations from the mean and the value of the population mean is more than 0.5 standard deviations from the value specified in the null hypothesis. However, there is considerable change in the size of the test for large samples. Hence, when using the usual test on means based on large samples from populations singly truncated within 2.5 standard deviations of the population mean, an improvement in the size of the test can be realized by further truncation to obtain a symmetrically truncated population.

APPENDIX A

SOME BASIC THEORY

Moments of Truncated Normal Variates

Let X be a standard normal variate truncated on the left at a and on the right at b. The density function of X is

$$\phi(X; a, b) = \frac{C}{\sqrt{2\pi}} \exp(-X^2/2), \quad a \le X \le b, \tag{A-1}$$

where C is given by

$$\frac{1}{C} = \frac{1}{\sqrt{2\pi}} \int_a^b \exp(-t^2/2)\,dt.$$

The r^{th} moment about the origin of X is defined to be

$$\mu_r^* = \int_a^b X^r \phi(X; a, b)\,dX, \quad r = 1, 2, \ldots . \tag{A-2}$$

Substituting (A-2) in (A-1) and integrating by parts we obtain, for m = 1, 2, ...,

$$\mu^*_{2m} = \frac{C}{\sqrt{2\pi}} \left[e^{-a^2/2} \sum_{i=0}^{m-1} a^{2(m-i)-1} K_i - e^{-b^2/2} \sum_{i=0}^{m-1} b^{2(m-i)-1} \right] \tag{A-3}$$

$$+ \frac{(2m)!}{m!\,2^m}$$

where $K_i = \frac{(2m)!\,(m-i)!}{m!\,[2(m-i)]\,2^i}$ and, for m = 0, 1, 2, ...,

$$\mu^*_{2m+1} = \frac{C}{\sqrt{2\pi}} \left[e^{-a^2/2} \sum_{i=0}^{m} a^{2(m-i)} K_i' - e^{-b^2/2} \sum_{i=0}^{m} b^{2(m-i)} K_i' \right], \tag{A-4}$$

where $K_i' = \dfrac{m!\ 2^i}{(m-i)!}$.

If the density function of the random variable X is given by (5), then

$$\mu_r' = \int_{\mu+\sigma b}^{\mu+\sigma a} X^r f(X; a, b)\, dX = \frac{C}{\sqrt{2\pi}} \int_a^b (\sigma y + \mu)^r e^{-y^2/2}\, dy. \qquad \text{(A-5)}$$

Expanding $(\sigma y + \mu)^r$ by the binomial theorem, we obtain

$$\mu_r' = \sum_{i=0}^{r} \binom{r}{i} \sigma^i \mu^{m-i} \mu_i^*. \qquad \text{(A-6)}$$

Characteristic Function and Cumulants

If X is a random variable with distribution function F(X), the characteristic function of X is defined by

$$\psi(t) = E(e^{itX}) = \int_{-\infty}^{\infty} e^{itX}\, dF(X), \qquad \text{(A-7)}$$

where E( ) denotes the expected value of the quantity in parentheses. Differentiating (A-7) with respect to t and evaluating the v^{th} derivative at $t = 0$, we obtain

$$\psi^{[v]}(0) = i^v \int_{-\infty}^{\infty} X^v\, dF(X) = i^v \mu_v' .$$

Thus, in a neighborhood of $t = 0$, we have the MacLaurin's series expansion

$$\psi(t) = 1 + \sum_{v=1}^{\infty} \frac{\mu_v'}{v!} (it)^v . \qquad \text{(A-8)}$$

Taking the logarithm of both sides of (A-8) and expanding the right-hand side by

$$\log (1 + z) = \sum_{k=1}^{\infty} \frac{z^k}{k} (-1)^{k-1} ,$$

we obtain

$$\log \psi(t) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \left[\sum_{v=1}^{\infty} \frac{\mu'_v}{v!} (it)^v \right]^k . \tag{A-9}$$

Expanding the right-hand side of (A-9) and collecting powers of (it) yields

$$\log \psi(t) = \sum_{v=1}^{\infty} \frac{\lambda'_v}{v!} (it)^v . \tag{A-10}$$

The coefficients λ'_v were introduced by Thiele [13] and are called the cumulants or semi-invariants of the distribution.

Replacing $\psi(t)$ in the left-hand side of (A-10) by (A-8) and expanding by $\log (1 + z)$, we obtain

$$\sum_{v=1}^{\infty} \frac{\lambda'_v}{v!} = \sum_{\rho=1}^{\infty} \frac{(-1)^{\rho-1}}{\rho} \left[\sum_{k=1}^{\infty} \frac{\mu'_k}{k!} (it)^k \right]^{\rho} . \tag{A-11}$$

Collecting powers of (it) yields the following relation between the cumulants and moments:

$$\lambda'_r = r! \sum_{m=1}^{r} \sum \left(\frac{\mu'_{p_1}}{p_1!}\right)^{n_1} \left(\frac{\mu'_{p_2}}{p_2!}\right)^{n_2} \cdots \left(\frac{\mu'_{p_m}}{p_m!}\right)^{n_m} \frac{(-1)^p (p-1)!}{n_1!\, n_2! \ldots n_m!} , \tag{A-12}$$

where the second summand extends over all positive integers $n_1, n_2, \ldots, n_m$ and ρ satisfying $n_1 + n_2 + \ldots + n_m = \rho$ and $p_1 n_1 + p_2 n_2 + \ldots + p_m n_m = r$.

APPENDIX B

COMPUTER PROCEDURE

The computational work involved in obtaining the various tables of Section IV was performed on the University's IBM 7094 computer. For computing purposes, formula (13) was expressed as

$$F_n(Y) = \sum_{h=0}^{N} \sum \frac{1}{r_1!\; r_2! \ldots r_N!} \left(\frac{\lambda_3'}{3!}\right)^{r_1} \cdots \left(\frac{\lambda_{N+2}'}{(N+2)!}\right)^{r_N} \frac{\phi^{[p+2h]}(Y)}{n^{p/2}} \qquad \text{(B-1)}$$

$$+ R_n(N, Y),$$

where the second summand extends over all non-negative integral values of $r_1, r_2, \ldots, r_N$, p and h satisfying

$$r_1 + r_2 + \ldots + r_N = h \quad \text{and} \quad r_1 + 2r_2 + \ldots + Nr_N = p.$$

For $N = 13$, it was found that $R_n(N, Y)$ is of the order 0.0001 for $n = 4$. The accuracy increases with n.

The moments U(I), I = 1, 2, ..., 20, of $f_n(X; A, B)$ are computed by the subroutine MOMEN. The first two moments are computed directly and then the recursive formula

$$\mu'_m = \frac{C}{2\pi}\left[A^{m-1} e^{-A^2/2} - B^{m-1} e^{-B^2/2}\right] + (m-1)\,\mu_{m-2}$$

is used to obtain the remaining moments. The subroutine CUMUL accepts the moments as inputs and computes the cumulants. When control returns to the calling program, the array C(I) contains $\lambda_i/i!$. This array is accepted as input by the subroutine DENSI which evaluates $F_n(nX)$, where $F_n(X)$ is given by (B-1).

The value $\phi^{v}(X)$, for given values of X and natural numbers v, is computed by the two formulas

$$\phi^{[v]}(X) = (-1)^{v} H_{v}(X)\ \phi(X),$$

$$\phi^{[v]}(X) = \phi^{[v-1]}(X),$$

where $\phi(X)$ is given by (4) and $H_v(X)$ is the vth Hermite polynomial. The subroutine HCOEF computes the coefficients of the Hermite polynomials and, for given values of X and v, HERMIT computes $H_v(X)$.

The equation $\Phi(X) = P$, where Φ is defined by (3), is solved by the subroutines NORMX and NORMP. The subroutine NORMX computes P for a given value of X and NORMP solves for X for a given value of P in the range of Φ. A detailed description of these two subroutines is given in [3].

The source statement listing of these subroutines is found on pages 14-19. These programs are all coded in IBM FORTRAN IV and have been successfully executed on an IBM 7094 computer.

```
C      THIS PROGRAM COMPUTES THE TABLES OF LOSS OF POWER
C      FOR SYMMETRIC TRUNCATION.

     1 FORMAT(15X,40H                         PERCENT LOSS OF POWER)
     2 FORMAT(15X,60H    WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =
      10 IF IN )
     3 FORMAT(15X,60H FACT U = C AND THE POPULATION IS SYMMETRICALLY TRUN
      1CATED AT)
     4 FORMAT(15X,24H THE TERMINUS POINT A = ,F6.2,///)
     5 FORMAT(15X,60H      C      0.0       0.5       1.0       1.5       2.0
      1    2.5 )
     6 FORMAT(15X,5H    N)
     7 FORMAT(15X,1H ,I4,6F9.2)
     8 FORMAT(1H1,///)
   777 FORMAT(15X,30H                        TABLE,I3,//)
       ITAB=1
       DIMENSION FACT(20),COEF(100,50),U(20),C(20),HOLD(6)
       CALL NFACT(FACT)
       CALL HCOEF(COEF)
    10 B=1.
   110 A=-B
       CALL MOMEN(A,B,U)
       CALL CUMUL(U,C,FACT)
       WRITE (7,8)
       WRITE (6,8)
       WRITE (7,777) ITAB
       WRITE (6,777) ITAB
       ITAB = ITAB + 1
       WRITE (7,1)
       WRITE (6,1)
       WRITE (7,2)
       WRITE (6,2)
       WRITE (7,3)
       WRITE (6,3)
       WRITE (7,4) B
       WRITE (6,4) B
       WRITE (7,5)
       WRITE (6,5)
       WRITE (7,6)
       WRITE (6,6)
       ICK = 1
       DO 121 N=5,30
       GO TO 14
    11 CONTINUE
   121 CONTINUE
       ICK = 2
       DO 16 N = 35,50,5
       GO TO 14
    15 CONTINUE
    16 CONTINUE
       ICK = 3
       DO 18 N = 60,100,10
       GO TO 14
    17 CONTINUE
    18 CONTINUE
       ICK = 4
       DO 20 N = 200,500,100
       GO TO 14
    21 CONTINUE
```

```
   20 CONTINUE
      ICK = 5
      N = 1000
      GO TO 14
   22 CONTINUE
      B = B + 0.5
      IF(B - 3.7)110,19,19
   19 CALL PUNCH
      STOP
   14 SD = (C(2)/FLOAT(N))**0.5
      XBAR = 0.0
      J = 1
   12 CONTINUE
C           THE FOLLOWING COMPUTES PERCENT LOSS OF POWER
C           ALPHA IS THE SIZE OF THE TEST DESIRED
      ALPHA = 0.05
      P = 1.0 - ALPHA
      CALL NORMP(P,X,Z)
      ZALPHA = X
      Y = ZALPHA - (XBAR*(FLOAT(N)**0.5))
      CALL NORMX(Y,P,Z)
C          PUSUAL IS THE POWER OF THE USUAL TEST
      PUSUAL = 1.0 - P
      Y = ZALPHA/(FLOAT(N)**0.5) - XBAR
      Y = (Y - C(1))/SD
      CALL DENSI(FNX,N,Y,C,FACT,COEF)
C          PACTUL IS THE ACTUAL POWER OF THE TEST
      PACTUL = 1.0 - FNX
C           PCTLOS IS THE PERCENT LOSS OF POWER
      PCTLOS = ((PUSUAL - PACTUL)/PUSUAL)*100.0
      XBAR = XBAR + 0.5
      HOLD(J) = PCTLOS
      J = J+1
      IF(XBAR - 2.7) 12,13,13
   13 WRITE (7,7) N, (HOLD(J), J = 1,6)
      WRITE (6,7) N, (HOLD(J), J = 1,6)
      GO TO (11,15,17,21,22), ICK
      END


C     THIS SUBROUTINE COMPUTES N FACTORIAL FOR N = 1,2,...,20.

      SUBROUTINE NFACT(FACT)
      DIMENSION FACT(20)
      FACT(1) = 1.
      DO 1 I = 2,20
      X = I
    1 FACT(I) = X*FACT(I-1)
      RETURN
      END


C     THIS IS A SUB-ROUTINE, CODED IN FORTRAN IV, WHICH COMPUTES
C     THE COEFFICIENTS OF THE HERMITE POLYNOMIALS , H(N), TO N = 100.
C     THESE COEFFICIENTS ARE STORED IN THE ARRAY COEF(N,M), WHERE
C     COEF(I,J) IS THE COEFFICIENT OF X TO THE (N - 2(J-1)) POWER.

      SUBROUTINE HCOEF(COEF)
```

```
      DIMENSION COEF(100,50)
      DO 1 I = 1,100
      COEF(I,1) = 1.0
      DO 1 J = 2,50
    1 COEF(I,J) = 0.0
      COEF(2,2) = -1.0
      DO 2 I = 3,100
      DO 2 J = 2,50
      XN = I - 1
    2 COEF(I,J) = COEF(I-1,J) - (XN*COEF(I-2,J-1))
      RETURN
      END

C     THIS IS A SUBROUTINE TO EVALUATE THE NTH HERMITE POLYNOMIAL
C     AT THE POINT X. HNX IS THE OBTAINED VALUE AND COEF IS THE
C     COEFFICIENT MATRIX.

      SUBROUTINE  HERMIT(N,X,HNX,COEF)
      DIMENSION COEF(100,50)
      IF(N)1,2,4
    1 HNX = 0.0
      GO TO 3
    2 HNX = 1.0
    3 CONTINUE
      RETURN
    4 HNX = 0.0
      DO 5 J = 1,50
      ICK = N - 2*(J-1)
      IF(ICK)3,7,6
    6 CONTINUE
      HNX = HNX + (COEF(N,J)*(X**(N - 2*(J-1))))
      GO TO 5
    7 CONTINUE
      HNX = HNX + COEF(N,J)
    5 CONTINUE
      GO TO 3
      END



      SUBROUTINE MOMEN(A,B,U)
      DIMENSION U(20)
      DO 1 I = 1,20
    1 U(I) = 0.
      X = B
      CALL NORMX(X,P,Z)
      C = P
      X = A
      CALL NORMX(X,P,Z)
      C1 = C - P
      C = 1.0/C1
      RTP = 2.506628274
      ETA = EXP(-(A*A)/2.)
      ETB = EXP(-(B*B)/2.)
      U(1) = (C/RTP)*(ETA - ETB)
      U(2) = (C/RTP)*(A*ETA - B*ETB) + 1.0
      DO 2 M = 3,20
      XM = M
      U(M) = (C/RTP)*(A**(M-1)*ETA - B**(M-1)*ETB)+((XM-1.)*U(M-2))
```

```
    2 CONTINUE
      RETURN
      END

C     THIS PROGRAM ACCEPTS THE MOMENTS, U(I), I = 1,2,...,20,
C     AS INPUT AND COMPUTES THE CUMULANTS.
C     THIS PROGRAM ACCEPTS THE MOMENTS ABOUT THE ORIGIN AS
C     INPUT AND COMPUTES THE CUMULANTS (UP TO 12).

      SUBROUTINE CUMUL(U,C,FACT)
      DIMENSION CC(20,20),U(20),C(20),FACT(20)
      SAVE = U(1)
      IF(U(1))20,24,20
   20 DO 23 KK = 1,20
      KR = 21 - KK
      U(KR) = U(KR) + (-1.0*U(1))**KR
      KRM1 = KR - 1
      IF(KRM1)24,24,25
   25 DO 23 IR = 1,KRM1
      KRMIR = KR - IR
      U(KR) = U(KR) +(FACT(KR)/(FACT(KRMIR)*FACT(IR)))*((-1.0*U(1))
     1 ** KRMIR)*U(IR)
   23 CONTINUE
   24 CONTINUE
      DO 1 I = 1,20
      C(I) = 0.
      DO 1 J = 1,20
      CC(I,J) = 0.
    1 CONTINUE
      DO 2 J = 1,20
      CC(1,J) = U(J)/FACT(J)
    2 CONTINUE
      DO 3 I = 2,20
      DO 3 J = 2,20
      J1 = J - 1
      DO 3 K = 1,J1
      K1 = J - K
      CC(I,J) = CC(I,J) + CC(1,K)*CC(I-1,K1)
    3 CONTINUE
      DO 4 I = 2,20
      DO 4 K = 2,20
      CC(I,K) = CC(I,K)*(-1.**(I-1))/FACT(I)
    4 CONTINUE
      DO 6 J =1,20
      DO 5 I = 1,20
      C(J) = C(J) + CC(I,J)
    5 CONTINUE
      C(J) = FACT(J)*C(J)
    6 CONTINUE
      C(1) = SAVE
      RETURN
      END

C     THIS PROGRAM COMPUTES THE AREA UNDER THE STANDARD
C     NORMAL CURVE FROM MINUS INFINITY TO X.

      SUBROUTINE NORMX(X,P,Z)
      G = 1.12837917*EXP(-(X*X/2.))
```

```
      Z = G/2.82842712
      XA = ABS(X)
      IF(XA - 2.5) 105,106,106
  106 U = 1./(XA+1./(XA+2./(XA+3./(XA+4./(XA+5./(XA+6./(XA+7./(XA+8./(XA
     1+9./(XA+10./(XA+11./(XA+12./XA))))))))))))
      IF(X) 107,108,108
  107 P = U*Z
      GO TO 101
  108 P = 1. - U*Z
      GO TO 101
  105 ET = 1.41421356/(1.41421356+0.3275911*ABS(X))
      U = G * ((((0.94064607*ET-1.28782245)*ET+1.25969513)*ET-0.25212866
     18)*ET+0.225836846)*ET
      IF(X) 102,103,103
  102 P = U/2.
      GO TO 101
  103 P = 1. - U/2.
  101 CONTINUE
      RETURN
      END

C       THIS PROGRAM COMPUTES THE POINT X EXCEEDED WITH PROBABILITY 1 - P

      SUBROUTINE NORMP(P,X,Z)
      I = 1
      J = 1
      IF(P) 213,214,215
  215 IF(P-1.) 216,217,213
  216 IF(P-0.5) 202,203,203
  202 Q = P
      GO TO 204
  203 Q = 1. - P
  204 ET = SQRT(-2.*ALOG(Q))
      NCYCL = 0
      XN=ET-((0.010328*ET+0.802853)*ET+2.515517)/(((0.001308*ET+0.189269
     1)*ET+1.432788)*ET+1.)
      IF(P-0.5) 205,256,256
  205 XN = -XN
  256 XA = ABS(XN)
      G = 1.1283917*EXP(-XN*XN/2.)
      Z = G/2.82842712
      IF(XA - 2.5) 206,251,251
  251 U = 1./(XA+1./(XA+2./(XA+3./(XA+4./(XA+5./(XA+6./(XA+7./(XA+8./(XA
     1+9./(XA+10./(XA+11./(XA+12./XA))))))))))))
      IF(XN) 107,107,108
  107 PN = U*Z
      GO TO 209
  108 PN = 1. - U*Z
      GO TO 209
  206 ETA = 1.41421356/(1.41421356+0.3275911*ABS(XN))
      U=G*((((0.94064607*ETA-1.28782245)*ETA+1.25969513)*ETA-0.252128668
     1)*ETA+0.225836846)*ETA
      IF(XN) 207,208,208
  207 PN = U/2.
      GO TO 209
  208 PN = 1.- U/2.
  209 ER = (PN - P)
      IF(ABS(ER) - P*1.E-7) 210,210,211
  211 NCYCL = NCYCL + 1
```

```
      IF(NCYCL - 10) 1211,1211,269
 269  GO TO 210
1211  XN = XN - ER/Z
      GO TO 256
 210  X = XN
      GO TO 201
 214  X = -.99999999E38
      Z = 0.
      GO TO 201
 217  X = .99999999E38
      Z = 0.
 201  CONTINUE
      GO TO 240
 213  PRINT 230, I,J,P
 230  FORMAT(41H0ERRONEOUS PROBABILITY INPUT IN ELEMENT (I4,1H,I4,7H) VA
     1LUE F16.8)
      GO TO 201
 240  RETURN
      END


      SUBROUTINE DENSI(FNX,N,Y,C,FACT,COEF)
      DIMENSION CC(14,14),C(20), COEF(100,50), FACT(20)
      DO 1 I = 1,14
      DO 1 J = 1,14
      CC(I,J) = 0.
    1 CONTINUE
      XN = N
      X = Y
      ROOTN = XN**0.5
      SD = C(2)**0.5
      DO 2 J = 1,14
      CC(1,J)=(C(J+2)/SD**(J+2))/(FACT(J+2)*ROOTN**J)
    2 CONTINUE
      DO 4 I = 2,14
      DO 4 J = 2,14
      JM1 = J - 1
      DO 3 K = 1,JM1
      JMK = J - K
      CC(I,J) = CC(I,J) + CC(1,K)*CC(I-1,JMK)
    3 CONTINUE
      CC(I,J) = (-1.**J)*CC(I,J) /FACT(I)
    4 CONTINUE
      FNX = 0.
      DO 5 I = 1,13
      DO 5 J = 1,13
      IV = 2*I + J - 1
      CALL HERMIT(IV,X,HNX,COEF)
      FNX = FNX + CC(I,J)*(-1.**IV)*HNX*EXP(-(X*X)/2.)*0.39894228
    5 CONTINUE
      CALL NORMX(X,P,Z)
      FNX = FNX + P
      RETURN
      END
```

T A B L E S

TABLE 1

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SYMMETRICALLY TRUNCATED AT THE TERMINUS POINT A = 1.00

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	99.51	43.90	-18.97	-4.59	-0.24	-0.00
6	99.01	34.79	-17.90	-2.18	-0.06	-0.00
7	98.76	25.99	-15.01	-1.02	-0.01	-0.00
8	98.61	17.94	-11.84	-0.47	-0.00	-0.00
9	98.49	10.76	-9.00	-0.22	-0.00	-0.00
10	98.41	4.49	-6.69	-0.10	-0.00	-0.00
11	98.34	-0.89	-4.89	-0.04	-0.00	-0.00
12	98.28	-5.43	-3.55	-0.02	-0.00	-0.00
13	98.23	-9.19	-2.56	-0.01	-0.00	-0.00
14	98.19	-12.22	-1.83	-0.00	-0.00	-0.00
15	98.16	-14.62	-1.31	-0.00	-0.00	-0.00
16	98.13	-16.43	-0.93	-0.00	-0.00	0.
17	98.10	-17.75	-0.66	-0.00	-0.00	0.
18	98.08	-18.63	-0.47	-0.00	-0.00	0.
19	98.06	-19.14	-0.33	-0.00	-0.00	0.
20	98.04	-19.34	-0.24	-0.00	-0.00	0.
21	98.02	-19.28	-0.17	-0.00	-0.00	0.
22	98.01	-19.01	-0.12	-0.00	-0.00	0.
23	97.99	-18.58	-0.08	-0.00	-0.00	0.
24	97.98	-18.02	-0.06	-0.00	-0.00	0.
25	97.97	-17.36	-0.04	-0.00	0.	0.
26	97.96	-16.63	-0.03	-0.00	0.	0.
27	97.95	-15.86	-0.02	-0.00	0.	0.
28	97.94	-15.05	-0.01	-0.00	0.	0.
29	97.93	-14.24	-0.01	-0.00	0.	0.
30	97.92	-13.42	-0.01	-0.00	0.	0.
35	97.89	-9.64	-0.00	-0.00	0.	0.
40	97.87	-6.65	-0.00	-0.00	0.	0.
45	97.85	-4.50	-0.00	0.	0.	0.
50	97.83	-3.00	-0.00	0.	0.	0.
60	97.81	-1.31	-0.00	0.	0.	0.
70	97.79	-0.56	-0.00	0.	0.	0.
80	97.78	-0.24	-0.00	0.	0.	0.
90	97.77	-0.10	-0.00	0.	0.	0.
100	97.77	-0.04	0.	0.	0.	0.
200	97.73	-0.00	0.	0.	0.	0.
300	97.72	-0.00	0.	0.	0.	0.
400	97.72	0.	0.	0.	0.	0.
500	97.71	0.	0.	0.	0.	0.
1000	97.71	0.	0.	0.	0.	0.

TABLE 2

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SYMMETRICALLY TRUNCATED AT THE TERMINUS POINT A = 1.50

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	71.64	19.85	-8.68	-3.37	-0.25	-0.00
6	72.68	15.16	-8.82	-1.88	-0.06	-0.00
7	73.10	10.99	-8.13	-0.97	-0.01	-0.00
8	73.32	7.43	-7.04	-0.47	-0.00	-0.00
9	73.43	4.40	-5.85	-0.22	-0.00	-0.00
10	73.50	1.82	-4.71	-0.10	-0.00	-0.00
11	73.54	-0.36	-3.70	-0.04	-0.00	-0.00
12	73.56	-2.21	-2.85	-0.02	-0.00	-0.00
13	73.58	-3.75	-2.16	-0.01	-0.00	-0.00
14	73.58	-5.03	-1.62	-0.00	-0.00	-0.00
15	73.58	-6.09	-1.19	-0.00	-0.00	-0.00
16	73.58	-6.94	-0.87	-0.00	-0.00	0.
17	73.58	-7.61	-0.63	-0.00	-0.00	0.
18	73.57	-8.13	-0.46	-0.00	-0.00	0.
19	73.57	-8.52	-0.33	-0.00	-0.00	0.
20	73.56	-8.78	-0.23	-0.00	-0.00	0.
21	73.55	-8.94	-0.16	-0.00	-0.00	0.
22	73.55	-9.02	-0.12	-0.00	-0.00	0.
23	73.54	-9.01	-0.08	-0.00	-0.00	0.
24	73.53	-8.95	-0.06	-0.00	-0.00	0.
25	73.52	-8.83	-0.04	-0.00	0.	0.
26	73.52	-8.66	-0.03	-0.00	0.	0.
27	73.51	-8.46	-0.02	-0.00	0.	0.
28	73.51	-8.23	-0.01	-0.00	0.	0.
29	73.50	-7.97	-0.01	-0.00	0.	0.
30	73.49	-7.70	-0.01	-0.00	0.	0.
35	73.47	-6.19	-0.00	-0.00	0.	0.
40	73.44	-4.72	-0.00	-0.00	0.	0.
45	73.43	-3.46	-0.00	0.	0.	0.
50	73.41	-2.47	-0.00	0.	0.	0.
60	73.38	-1.18	-0.00	0.	0.	0.
70	73.36	-0.53	-0.00	0.	0.	0.
80	73.35	-0.23	-0.00	0.	0.	0.
90	73.34	-0.10	-0.00	0.	0.	0.
100	73.33	-0.04	0.	0.	0.	0.
200	73.28	-0.00	0.	0.	0.	0.
300	73.26	-0.00	0.	0.	0.	0.
400	73.26	0.	0.	0.	0.	0.
500	73.25	0.	0.	0.	0.	0.
1000	73.24	0.	0.	0.	0.	0.

TABLE 3

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SYMMETRICALLY TRUNCATED AT THE TERMINUS POINT A = 2.00

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	37.03	8.39	-3.69	-1.75	-0.24	-0.01
6	37.15	6.29	-3.80	-1.04	-0.07	-0.00
7	37.39	4.49	-3.60	-0.61	-0.02	-0.00
8	37.61	3.00	-3.22	-0.34	-0.00	-0.00
9	37.78	1.76	-2.79	-0.18	-0.00	-0.00
10	37.92	0.75	-2.34	-0.09	-0.00	-0.00
11	38.03	-0.14	-1.93	-0.04	-0.00	-0.00
12	38.12	-0.87	-1.56	-0.02	-0.00	-0.00
13	38.19	-1.49	-1.24	-0.01	-0.00	-0.00
14	38.24	-2.00	-0.97	-0.00	-0.00	-0.00
15	38.29	-2.43	-0.75	-0.00	-0.00	-0.00
16	38.32	-2.78	-0.58	-0.00	-0.00	0.
17	38.35	-3.06	-0.44	-0.00	-0.00	0.
18	38.38	-3.29	-0.33	-0.00	-0.00	0.
19	38.40	-3.47	-0.24	-0.00	-0.00	0.
20	38.42	-3.60	-0.18	-0.00	-0.00	0.
21	38.43	-3.69	-0.13	-0.00	-0.00	0.
22	38.44	-3.76	-0.09	-0.00	-0.00	0.
23	38.46	-3.79	-0.07	-0.00	-0.00	0.
24	38.47	-3.79	-0.05	-0.00	-0.00	0.
25	38.47	-3.78	-0.03	-0.00	0.	0.
26	38.48	-3.74	-0.02	-0.00	0.	0.
27	38.49	-3.69	-0.02	-0.00	0.	0.
28	38.49	-3.63	-0.01	-0.00	0.	0.
29	38.50	-3.55	-0.01	-0.00	0.	0.
30	38.50	-3.47	-0.01	-0.00	0.	0.
35	38.52	-2.95	-0.00	-0.00	0.	0.
40	38.53	-2.38	-0.00	-0.00	0.	0.
45	38.53	-1.85	-0.00	0.	0.	0.
50	38.53	-1.40	-0.00	0.	0.	0.
60	38.53	-0.74	-0.00	0.	0.	0.
70	38.53	-0.37	-0.00	0.	0.	0.
80	38.53	-0.17	-0.00	0.	0.	0.
90	38.53	-0.08	-0.00	0.	0.	0.
100	38.53	-0.03	0.	0.	0.	0.
200	38.52	-0.00	0.	0.	0.	0.
300	38.52	0.	0.	0.	0.	0.
400	38.52	0.	0.	0.	0.	0.
500	38.52	0.	0.	0.	0.	0.
1000	38.51	0.	0.	0.	0.	0.

TABLE 4

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SYMMETRICALLY TRUNCATED AT THE TERMINUS POINT A = 2.50

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	13.45	2.96	-1.35	-0.60	-0.14	-0.01
6	13.66	2.35	-1.48	-0.35	-0.05	-0.00
7	13.87	1.69	-1.38	-0.23	-0.02	-0.00
8	14.06	1.12	-1.21	-0.15	-0.01	-0.00
9	14.22	0.65	-1.04	-0.09	-0.00	-0.00
10	14.35	0.27	-0.88	-0.05	-0.00	-0.00
11	14.45	-0.05	-0.74	-0.03	-0.00	-0.00
12	14.54	-0.32	-0.61	-0.02	-0.00	-0.00
13	14.61	-0.53	-0.49	-0.01	-0.00	-0.00
14	14.67	-0.71	-0.40	-0.00	-0.00	-0.00
15	14.72	-0.86	-0.32	-0.00	-0.00	-0.00
16	14.77	-0.99	-0.25	-0.00	-0.00	0.
17	14.80	-1.09	-0.20	-0.00	-0.00	0.
18	14.83	-1.17	-0.15	-0.00	-0.00	0.
19	14.86	-1.23	-0.12	-0.00	-0.00	0.
20	14.88	-1.28	-0.09	-0.00	-0.00	0.
21	14.90	-1.31	-0.07	-0.00	-0.00	0.
22	14.92	-1.34	-0.05	-0.00	-0.00	0.
23	14.94	-1.35	-0.04	-0.00	-0.00	0.
24	14.95	-1.36	-0.03	-0.00	-0.00	0.
25	14.96	-1.36	-0.02	-0.00	0.	0.
26	14.98	-1.35	-0.02	-0.00	0.	0.
27	14.99	-1.33	-0.01	-0.00	0.	0.
28	14.99	-1.32	-0.01	-0.00	0.	0.
29	15.00	-1.29	-0.01	-0.00	0.	0.
30	15.01	-1.27	-0.00	-0.00	0.	0.
35	15.04	-1.10	-0.00	-0.00	0.	0.
40	15.06	-0.92	-0.00	-0.00	0.	0.
45	15.07	-0.73	-0.00	0.	0.	0.
50	15.08	-0.57	-0.00	0.	0.	0.
60	15.10	-0.32	-0.00	0.	0.	0.
70	15.10	-0.17	-0.00	0.	0.	0.
80	15.11	-0.08	-0.00	0.	0.	0.
90	15.11	-0.04	-0.00	0.	0.	0.
100	15.11	-0.02	0.	0.	0.	0.
200	15.12	-0.00	0.	0.	0.	0.
300	15.13	0.	0.	0.	0.	0.
400	15.13	0.	0.	0.	0.	0.
500	15.13	0.	0.	0.	0.	0.
1000	15.13	0.	0.	0.	0.	0.

TABLE 5

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SYMMETRICALLY TRUNCATED AT THE TERMINUS POINT A = 3.00

C	0.0	0.5	1.0	1.5	2.0	2.5
5	2.72	0.49	-0.32	-0.05	-0.08	-0.01
6	3.20	0.74	-0.61	0.01	-0.03	-0.01
7	3.45	0.61	-0.53	-0.02	-0.01	-0.00
8	3.62	0.41	-0.41	-0.03	-0.01	-0.00
9	3.75	0.24	-0.32	-0.03	-0.00	-0.00
10	3.85	0.09	-0.25	-0.02	-0.00	-0.00
11	3.94	-0.02	-0.20	-0.01	-0.00	-0.00
12	4.01	-0.11	-0.16	-0.01	-0.00	-0.00
13	4.07	-0.18	-0.13	-0.01	-0.00	-0.00
14	4.12	-0.23	-0.11	-0.00	-0.00	-0.00
15	4.16	-0.28	-0.09	-0.00	-0.00	-0.00
16	4.19	-0.31	-0.07	-0.00	-0.00	0.
17	4.23	-0.34	-0.06	-0.00	-0.00	0.
18	4.25	-0.36	-0.05	-0.00	-0.00	0.
19	4.28	-0.38	-0.04	-0.00	-0.00	0.
20	4.30	-0.39	-0.03	-0.00	-0.00	0.
21	4.32	-0.40	-0.02	-0.00	-0.00	0.
22	4.33	-0.40	-0.02	-0.00	-0.00	0.
23	4.35	-0.40	-0.01	-0.00	-0.00	0.
24	4.36	-0.40	-0.01	-0.00	-0.00	0.
25	4.37	-0.40	-0.01	-0.00	0.	0.
26	4.38	-0.40	-0.01	-0.00	0.	0.
27	4.39	-0.39	-0.00	-0.00	0.	0.
28	4.40	-0.39	-0.00	-0.00	0.	0.
29	4.41	-0.38	-0.00	-0.00	0.	0.
30	4.41	-0.37	-0.00	-0.00	0.	0.
35	4.44	-0.32	-0.00	-0.00	0.	0.
40	4.46	-0.27	-0.00	-0.00	0.	0.
45	4.47	-0.22	-0.00	0.	0.	0.
50	4.48	-0.17	-0.00	0.	0.	0.
60	4.50	-0.10	-0.00	0.	0.	0.
70	4.50	-0.05	-0.00	0.	0.	0.
80	4.51	-0.03	-0.00	0.	0.	0.
90	4.51	-0.01	-0.00	0.	0.	0.
100	4.52	-0.01	0.	0.	0.	0.
200	4.53	-0.00	0.	0.	0.	0.
300	4.53	0.	0.	0.	0.	0.
400	4.53	0.	0.	0.	0.	0.
500	4.53	0.	0.	0.	0.	0.
1000	4.53	0.	0.	0.	0.	0.

TABLE 6

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SYMMETRICALLY TRUNCATED AT THE TERMINUS POINT A = 3.50

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	-0.94	-0.51	0.07	0.15	-0.06	-0.01
6	-0.21	0.18	-0.35	0.14	-0.01	-0.01
7	0.08	0.26	-0.28	0.06	-0.01	-0.00
8	0.24	0.19	-0.17	0.01	-0.01	-0.00
9	0.36	0.11	-0.09	-0.00	-0.00	-0.00
10	0.45	0.04	-0.05	-0.01	-0.00	-0.00
11	0.52	-0.01	-0.02	-0.01	-0.00	-0.00
12	0.58	-0.05	-0.01	-0.01	-0.00	-0.00
13	0.63	-0.07	-0.01	-0.00	-0.00	-0.00
14	0.67	-0.09	-0.01	-0.00	-0.00	-0.00
15	0.71	-0.10	-0.01	-0.00	-0.00	-0.00
16	0.74	-0.11	-0.01	-0.00	-0.00	0.
17	0.77	-0.11	-0.01	-0.00	-0.00	0.
18	0.79	-0.12	-0.01	-0.00	-0.00	0.
19	0.81	-0.12	-0.01	-0.00	-0.00	0.
20	0.83	-0.12	-0.01	-0.00	-0.00	0.
21	0.85	-0.12	-0.01	-0.00	-0.00	0.
22	0.86	-0.11	-0.01	-0.00	-0.00	0.
23	0.87	-0.11	-0.00	-0.00	-0.00	0.
24	0.89	-0.11	-0.00	-0.00	-0.00	0.
25	0.90	-0.11	-0.00	-0.00	0.	0.
26	0.91	-0.10	-0.00	-0.00	0.	0.
27	0.91	-0.10	-0.00	-0.00	0.	0.
28	0.92	-0.10	-0.00	-0.00	0.	0.
29	0.93	-0.09	-0.00	-0.00	0.	0.
30	0.93	-0.09	-0.00	-0.00	0.	0.
35	0.96	-0.07	-0.00	-0.00	0.	0.
40	0.98	-0.06	-0.00	-0.00	0.	0.
45	0.99	-0.05	-0.00	0.	0.	0.
50	1.00	-0.04	-0.00	0.	0.	0.
60	1.01	-0.02	-0.00	0.	0.	0.
70	1.01	-0.01	-0.00	0.	0.	0.
80	1.02	-0.01	-0.00	0.	0.	0.
90	1.02	-0.00	-0.00	0.	0.	0.
100	1.03	-0.00	0.	0.	0.	0.
200	1.03	0.	0.	0.	0.	0.
300	1.04	0.	0.	0.	0.	0.
400	1.04	0.	0.	0.	0.	0.
500	1.04	0.	0.	0.	0.	0.
1000	1.04	0.	0.	0.	0.	0.

TABLE 7

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE RIGHT AT THE POINT A = 1.50

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	35.98	46.29	15.07	1.92	0.45	0.14
6	70.43	40.80	9.50	1.01	-0.24	-0.01
7	79.88	40.41	8.08	0.80	-0.05	-0.00
8	83.43	40.19	7.11	0.44	-0.01	0.00
9	85.26	39.75	6.12	0.17	-0.00	-0.00
10	86.47	39.16	5.16	0.04	-0.00	-0.00
11	87.42	38.48	4.29	-0.00	-0.00	-0.00
12	88.21	37.77	3.52	-0.01	-0.00	-0.00
13	88.92	37.04	2.85	-0.01	-0.00	-0.00
14	89.55	36.30	2.29	-0.01	-0.00	-0.00
15	90.13	35.56	1.82	-0.00	-0.00	-0.00
16	90.67	34.81	1.42	-0.00	-0.00	0.
17	91.16	34.06	1.11	-0.00	-0.00	0.
18	91.62	33.31	0.85	-0.00	-0.00	0.
19	92.05	32.55	0.65	-0.00	-0.00	0.
20	92.44	31.80	0.49	-0.00	-0.00	0.
21	92.81	31.05	0.37	-0.00	-0.00	0.
22	93.16	30.31	0.27	-0.00	-0.00	0.
23	93.48	29.56	0.20	-0.00	-0.00	0.
24	93.79	28.82	0.15	-0.00	-0.00	0.
25	94.08	28.08	0.11	-0.00	0.	0.
26	94.35	27.34	0.08	-0.00	0.	0.
2	94.60	26.62	0.06	-0.00	0.	0.
28	94.84	25.89	0.04	-0.00	0.	0.
29	95.07	25.18	0.03	-0.00	0.	0.
30	95.28	24.47	0.02	-0.00	0.	0.
35	96.19	21.06	0.00	-0.00	0.	0.
40	96.90	17.90	0.00	-0.00	0.	0.
45	97.45	15.05	0.00	0.	0.	0.
50	97.89	12.52	0.00	0.	0.	0.
60	98.53	8.42	0.	0.	0.	0.
70	98.96	5.47	-0.00	0.	0.	0.
80	99.25	3.47	-0.00	0.	0.	0.
90	99.45	2.14	-0.00	0.	0.	0.
100	99.60	1.30	0.	0.	0.	0.
200	99.97	0.00	0.	0.	0.	0.
300	100.00	0.00	0.	0.	0.	0.
400	100.00	0.00	0.	0.	0.	0.
500	100.00	0.00	0.	0.	0.	0.
1000	100.00	0.	0.	0.	0.	0.

TABLE 8

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE RIGHT AT THE POINT A = 2.00

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	28.54	17.99	5.15	0.43	-0.10	0.01
6	40.90	16.60	3.27	0.61	-0.05	-0.01
7	45.18	16.16	2.57	0.32	-0.02	-0.00
8	47.37	15.74	2.17	0.14	-0.01	-0.00
9	48.87	15.29	1.85	0.05	-0.00	-0.00
10	50.07	14.83	1.55	0.01	-0.00	-0.00
11	51.12	14.37	1.28	-0.00	-0.00	-0.00
12	52.08	13.93	1.03	-0.01	-0.00	-0.00
13	52.98	13.49	0.82	-0.00	-0.00	-0.00
14	53.81	13.07	0.64	-0.00	-0.00	-0.00
15	54.61	12.65	0.50	-0.00	-0.00	-0.00
16	55.37	12.25	0.38	-0.00	-0.00	0.
17	56.09	11.85	0.29	-0.00	-0.00	0.
18	56.78	11.46	0.21	-0.00	-0.00	0.
19	57.45	11.07	0.16	-0.00	-0.00	0.
20	58.09	10.69	0.12	-0.00	-0.00	0.
21	58.71	10.32	0.08	-0.00	-0.00	0.
22	59.31	9.95	0.06	-0.00	-0.00	0.
23	59.88	9.59	0.04	-0.00	-0.00	0.
24	60.44	9.24	0.03	-0.00	-0.00	0.
25	60.98	8.89	0.02	-0.00	0.	0.
26	61.51	8.56	0.01	-0.00	0.	0.
27	62.02	8.23	0.01	-0.00	0.	0.
28	62.51	7.90	0.01	-0.00	0.	0.
29	63.00	7.59	0.00	-0.00	0.	0.
30	63.47	7.28	0.00	-0.00	0.	0.
35	65.64	5.87	0.00	-0.00	0.	0.
40	67.57	4.65	0.00	-0.00	0.	0.
45	69.30	3.64	0.00	0.	0.	0.
50	70.87	2.80	0.00	0.	0.	0.
60	73.62	1.60	0.	0.	0.	0.
70	75.95	0.88	0.	0.	0.	0.
80	77.96	0.46	-0.00	0.	0.	0.
90	79.72	0.24	-0.00	0.	0.	0.
100	81.27	0.12	0.	0.	0.	0.
200	90.47	0.00	0.	0.	0.	0.
300	94.53	0.	0.	0.	0.	0.
400	96.67	0.00	0.	0.	0.	0.
500	97.89	0.	0.	0.	0.	0.
1000	99.70	0.	0.	0.	0.	0.

TABLE 9

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE RIGHT AT THE POINT A = 2.50

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	9.71	5.62	1.62	0.15	-0.06	-0.00
6	15.05	5.29	0.75	0.30	-0.02	-0.01
7	16.95	5.12	0.56	0.15	-0.01	-0.00
8	17.97	4.91	0.51	0.06	-0.01	-0.00
9	18.69	4.70	0.47	0.02	-0.00	-0.00
10	19.28	4.49	0.42	-0.00	-0.00	-0.00
11	19.80	4.30	0.36	-0.01	-0.00	-0.00
12	20.27	4.13	0.29	-0.01	-0.00	-0.00
13	20.72	3.97	0.24	-0.00	-0.00	-0.00
14	21.14	3.82	0.18	-0.00	-0.00	-0.00
15	21.54	3.67	0.14	-0.00	-0.00	-0.00
16	21.92	3.53	0.11	-0.00	-0.00	0.
17	22.28	3.40	0.08	-0.00	-0.00	0.
18	22.64	3.27	0.06	-0.00	-0.00	0.
19	22.98	3.14	0.04	-0.00	-0.00	0.
20	23.31	3.02	0.03	-0.00	-0.00	0.
21	23.63	2.90	0.02	-0.00	-0.00	0.
22	23.94	2.78	0.01	-0.00	-0.00	0.
23	24.24	2.67	0.01	-0.00	-0.00	0.
24	24.54	2.56	0.01	-0.00	-0.00	0.
25	24.83	2.45	0.00	-0.00	0.	0.
26	25.11	2.34	0.00	-0.00	0.	0.
27	25.38	2.24	0.00	-0.00	0.	0.
28	25.65	2.14	0.00	-0.00	0.	0.
29	25.92	2.05	0.00	-0.00	0.	0.
30	26.18	1.96	0.00	-0.00	0.	0.
35	27.40	1.53	-0.00	-0.00	0.	0.
40	28.53	1.18	-0.00	-0.00	0.	0.
45	29.58	0.90	-0.00	0.	0.	0.
50	30.56	0.67	-0.00	0.	0.	0.
60	32.36	0.36	0.	0.	0.	0.
70	33.99	0.19	-0.00	0.	0.	0.
80	35.48	0.09	-0.00	0.	0.	0.
90	36.85	0.04	-0.00	0.	0.	0.
100	38.13	0.02	0.	0.	0.	0.
200	47.76	0.00	0.	0.	0.	0.
300	54.30	0.	0.	0.	0.	0.
400	59.27	0.00	0.	0.	0.	0.
500	63.26	0.	0.	0.	0.	0.
1000	75.89	0.	0.	0.	0.	0.

TABLE 10

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE RIGHT AT THE POINT A = 3.00

C	0.0	0.5	1.0	1.5	2.0	2.5
5	0.50	1.14	0.60	0.13	-0.06	-0.00
6	3.18	1.37	-0.02	0.21	-0.02	-0.01
7	4.09	1.37	-0.03	0.10	-0.01	-0.00
8	4.55	1.29	0.03	0.04	-0.01	-0.00
9	4.85	1.19	0.08	0.01	-0.00	-0.00
10	5.09	1.10	0.10	-0.00	-0.00	-0.00
11	5.29	1.02	0.10	-0.01	-0.00	-0.00
12	5.47	0.96	0.09	-0.00	-0.00	-0.00
13	5.63	0.91	0.08	-0.00	-0.00	-0.00
14	5.78	0.87	0.06	-0.00	-0.00	-0.00
15	5.92	0.83	0.05	-0.00	-0.00	-0.00
16	6.05	0.79	0.03	-0.00	-0.00	0.
17	6.17	0.76	0.02	-0.00	-0.00	0.
18	6.29	0.73	0.02	-0.00	-0.00	0.
19	6.40	0.70	0.01	-0.00	-0.00	0.
20	6.50	0.67	0.01	-0.00	-0.00	0.
21	6.61	0.65	0.00	-0.00	-0.00	0.
22	6.71	0.62	0.00	-0.00	-0.00	0.
23	6.80	0.60	0.00	-0.00	-0.00	0.
24	6.90	0.57	0.00	-0.00	-0.00	0.
25	6.99	0.55	-0.00	-0.00	0.	0.
26	7.07	0.53	-0.00	-0.00	0.	0.
27	7.16	0.50	-0.00	-0.00	0.	0.
28	7.24	0.48	-0.00	-0.00	0.	0.
29	7.33	0.46	-0.00	-0.00	0.	0.
30	7.41	0.44	-0.00	-0.00	0.	0.
35	7.78	0.35	-0.00	-0.00	0.	0.
40	8.13	0.27	-0.00	-0.00	0.	0.
45	8.45	0.20	-0.00	0.	0.	0.
50	8.76	0.15	-0.00	0.	0.	0.
60	9.32	0.08	0.	0.	0.	0.
70	9.84	0.04	-0.00	0.	0.	0.
80	10.32	0.02	-0.00	0.	0.	0.
90	10.76	0.01	-0.00	0.	0.	0.
100	11.19	0.00	0.	0.	0.	0.
200	14.56	0.00	0.	0.	0.	0.
300	17.08	0.	0.	0.	0.	0.
400	19.17	0.	0.	0.	0.	0.
500	20.98	0.	0.	0.	0.	0.
1000	27.77	0.	0.	0.	0.	0.

TABLE 11

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE RIGHT AT THE POINT A = 3.50

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	-1.88	-0.37	0.32	0.18	-0.07	-0.00
6	-0.37	0.28	-0.22	0.19	-0.01	-0.01
7	0.14	0.39	-0.18	0.06	-0.01	-0.00
8	0.39	0.35	-0.08	0.03	-0.01	-0.00
9	0.55	0.29	-0.01	0.01	-0.00	-0.00
10	0.67	0.23	0.03	-0.00	-0.00	-0.00
11	0.77	0.19	0.04	-0.01	-0.00	-0.00
12	0.85	0.16	0.04	-0.00	-0.00	-0.00
13	0.92	0.14	0.04	-0.00	-0.00	-0.00
14	0.98	0.12	0.03	-0.00	-0.00	-0.00
15	1.04	0.11	0.02	-0.00	-0.00	-0.00
16	1.09	0.10	0.02	-0.00	-0.00	0.
17	1.13	0.10	0.01	-0.00	-0.00	0.
18	1.17	0.10	0.01	-0.00	-0.00	0.
19	1.21	0.09	0.00	-0.00	-0.00	0.
20	1.24	0.09	0.00	-0.00	-0.00	0.
21	1.28	0.09	0.00	-0.00	-0.00	0.
22	1.31	0.09	-0.00	-0.00	-0.00	0.
23	1.34	0.09	-0.00	-0.00	-0.00	0.
24	1.36	0.08	-0.00	-0.00	-0.00	0.
25	1.39	0.08	-0.00	-0.00	0.	0.
26	1.41	0.08	-0.00	-0.00	0.	0.
27	1.44	0.08	-0.00	-0.00	0.	0.
28	1.46	0.08	-0.00	-0.00	0.	0.
29	1.48	0.08	-0.00	-0.00	0.	0.
30	1.50	0.07	-0.00	-0.00	0.	0.
35	1.60	0.06	-0.00	-0.00	0.	0.
40	1.68	0.05	-0.00	-0.00	0.	0.
45	1.76	0.04	-0.00	0.	0.	0.
50	1.82	0.03	-0.00	0.	0.	0.
60	1.95	0.02	0.	0.	0.	0.
70	2.06	0.01	-0.00	0.	0.	0.
80	2.16	0.00	-0.00	0.	0.	0.
90	2.26	0.00	-0.00	0.	0.	0.
100	2.35	0.00	0.	0.	0.	0.
200	3.07	0.	0.	0.	0.	0.
300	3.62	0.	0.	0.	0.	0.
400	4.09	0.	0.	0.	0.	0.
500	4.50	0.	0.	0.	0.	0.
1000	6.09	0.	0.	0.	0.	0.

TABLE 12

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE LEFT AT THE POINT A = -1.50

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	-49.31	-41.45	-21.17	-2.36	0.37	-0.01
6	-54.70	-37.87	-17.09	-2.54	-0.08	0.00
7	-59.72	-37.50	-12.57	-1.06	-0.02	-0.00
8	-65.61	-38.11	-9.64	-0.45	-0.00	-0.00
9	-72.03	-38.65	-7.49	-0.21	-0.00	0.
10	-78.71	-38.80	-5.77	-0.10	-0.00	0.
11	-85.51	-38.53	-4.37	-0.05	-0.00	0.
12	-92.36	-37.90	-3.26	-0.02	-0.00	0.
13	-99.24	-36.97	-2.40	-0.01	-0.00	0.
14	-106.11	-35.83	-1.76	-0.00	0.	0.
15	-112.99	-34.53	-1.27	-0.00	0.	0.
16	-119.85	-33.11	-0.92	-0.00	0.	0.
17	-126.71	-31.62	-0.66	-0.00	0.	0.
18	-133.55	-30.10	-0.47	-0.00	0.	0.
19	-140.38	-28.56	-0.33	-0.00	0.	0.
20	-147.21	-27.02	-0.24	-0.00	0.	0.
21	-154.02	-25.50	-0.17	-0.00	0.	0.
22	-160.82	-24.02	-0.12	-0.00	0.	0.
23	-167.61	-22.58	-0.08	-0.00	-0.00	0.
24	-174.39	-21.19	-0.06	-0.00	-0.00	0.
25	-181.16	-19.85	-0.04	0.	0.	0.
26	-187.93	-18.57	-0.03	0.	0.	0.
27	-194.69	-17.35	-0.02	0.	0.	0.
28	-201.44	-16.19	-0.01	0.	0.	0.
29	-208.18	-15.10	-0.01	0.	0.	0.
30	-214.92	-14.06	-0.01	0.	0.	0.
35	-248.49	-9.73	-0.00	0.	0.	0.
40	-281.92	-6.63	-0.00	-0.00	0.	0.
45	-315.18	-4.47	-0.00	0.	0.	0.
50	-348.27	-2.99	-0.00	0.	0.	0.
60	-413.88	-1.31	-0.00	0.	0.	0.
70	-478.58	-0.56	-0.00	0.	0.	0.
80	-542.20	-0.24	-0.00	0.	0.	0.
90	-604.57	-0.10	-0.00	0.	0.	0.
100	-665.55	-0.04	0.	0.	0.	0.
200	-1179.44	-0.00	0.	0.	0.	0.
300	-1511.63	-0.00	0.	0.	0.	0.
400	-1702.51	0.	0.	0.	0.	0.
500	-1803.99	0.	0.	0.	0.	0.
1000	-1898.34	0.	0.	0.	0.	0.

TABLE 13

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE LEFT AT THE POINT A = -2.00

C N	0.0	0.5	1.0	1.5	2.0	2.5
5	-5.81	-13.02	-7.86	-1.36	-0.16	-0.04
6	-13.80	-11.81	-7.06	-1.22	-0.04	-0.00
7	-17.26	-12.46	-5.89	-0.71	-0.02	0.00
8	-19.64	-13.23	-4.88	-0.36	-0.01	-0.00
9	-21.69	-13.84	-4.00	-0.18	-0.00	-0.00
10	-23.60	-14.25	-3.23	-0.09	-0.00	-0.00
11	-25.46	-14.48	-2.57	-0.04	-0.00	-0.00
12	-27.27	-14.57	-2.02	-0.02	-0.00	0.
13	-29.05	-14.55	-1.56	-0.01	-0.00	0.
14	-30.80	-14.42	-1.20	-0.00	-0.00	0.
15	-32.53	-14.22	-0.91	-0.00	-0.00	0.
16	-34.24	-13.95	-0.68	-0.00	-0.00	0.00
17	-35.93	-13.64	-0.51	-0.00	-0.00	0.00
18	-37.60	-13.28	-0.37	-0.00	-0.00	0.
19	-39.24	-12.89	-0.27	-0.00	-0.00	0.
20	-40.87	-12.47	-0.20	-0.00	-0.00	0.
21	-42.49	-12.04	-0.14	-0.00	-0.00	0.
22	-44.08	-11.59	-0.10	-0.00	0.	0.
23	-45.66	-11.13	-0.07	-0.00	0.	0.
24	-47.23	-10.68	-0.05	-0.00	0.	0.
25	-48.78	-10.22	-0.04	-0.00	0.	0.
26	-50.32	-9.76	-0.03	-0.00	0.	0.
27	-51.85	-9.31	-0.02	-0.00	0.	0.
28	-53.36	-8.86	-0.01	-0.00	0.	0.
29	-54.86	-8.43	-0.01	-0.00	0.	0.
30	-56.35	-8.00	-0.01	-0.00	0.	0.
35	-63:65	-6.06	-0.00	-0.00	0.	0.
40	-70.73	-4.47	-0.00	-0.00	0.	0.
45	-77.62	-3.23	-0.00	0.	0.	0.
50	-84.36	-2.29	-0.00	0.	0.	0.
60	-97.46	-1.09	-0.00	0.	0.	0.
70	-110.16	-0.50	-0.00	0.	0.	0.
80	-122.54	-0.22	-0.00	0.	0.	0.
90	-134.66	-0.09	-0.00	0.	0.	0.
100	-146.58	-0.04	0.	0.	0.	0.
200	-258.89	-0.00	0.	0.	0.	0.
300	-364.41	-0.00	0.	0.	0.	0.
400	-465.54	0.	0.	0.	0.	0.
500	-562.68	0.	0.	0.	0.	0.
1000	-985.56	0.	0.	0.	0.	0.

TABLE 14

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE LEFT AT THE POINT A = -2.50

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	-0.65	-4.17	-2.57	-0.41	-0.14	-0.02
6	-3.86	-3.21	-2.52	-0.41	-0.03	-0.00
7	-4.99	-3.40	-2.13	-0.28	-0.02	-0.00
8	-5.64	-3.72	-1.78	-0.17	-0.01	-0.00
9	-6.15	-4.00	-1.48	-0.10	-0.00	-0.00
10	-6.61	-4.21	-1.22	-0.05	-0.00	-0.00
11	-7.05	-4.36	-1.00	-0.03	-0.00	-0.00
12	-7.49	-4.45	-0.81	-0.02	-0.00	-0.00
13	-7.91	-4.51	-0.65	-0.01	-0.00	-0.00
14	-8.34	-4.52	-0.51	-0.00	-0.00	-0.00
15	-8.75	-4.51	-0.40	-0.00	-0.00	-0.00
16	-9.17	-4.47	-0.31	-0.00	-0.00	0.
17	-9.58	-4.42	-0.24	-0.00	-0.00	0.
18	-9.98	-4.35	-0.18	-0.00	-0.00	0.
19	-10.38	-4.27	-0.14	-0.00	-0.00	0.
20	-10.77	-4.17	-0.10	-0.00	0.00	0.
21	-11.16	-4.07	-0.08	-0.00	0.00	0.
22	-11.55	-3.96	-0.06	-0.00	-0.00	0.
23	-11.93	-3.84	-0.04	-0.00	-0.00	0.
24	-12.30	-3.72	-0.03	-0.00	-0.00	0.
25	-12.67	-3.60	-0.02	-0.00	0.	0.
26	-13.04	-3.47	-0.02	-0.00	0.	0.
27	-13.40	-3.35	-0.01	-0.00	0.	0.
28	-13.76	-3.22	-0.01	-0.00	0.	0.
29	-14.11	-3.09	-0.01	-0.00	0.	0.
30	-14.46	-2.97	-0.00	-0.00	0.	0.
35	-16.16	-2.37	-0.00	-0.00	0.	0.
40	-17.78	-1.84	-0.00	-0.00	0.	0.
45	-19.33	-1.40	-0.00	0.	0.	0.
50	-20.82	-1.04	-0.00	0.	0.	0.
60	-23.66	-0.55	-0.00	0.	0.	0.
70	-26.33	-0.27	-0.00	0.	0.	0.
80	-28.87	-0.13	-0.00	0.	0.	0.
90	-31.29	-0.06	-0.00	0.	0.	0.
100	-33.63	-0.03	0.	0.	0.	0.
200	-53.82	-0.00	0.	0.	0.	0.
300	-70.93	0.	0.	0.	0.	0.
400	-86.46	0.	0.	0.	0.	0.
500	-101.01	0.	0.	0.	0.	0.
1000	-166.17	0.	0.	0.	0.	0.

TABLE 15

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE LEFT AT THE POINT A = -3.00

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	-0.22	-1.67	-0.67	0.06	-0.08	-0.01
6	-1.34	-0.68	-0.89	-0.01	-0.02	-0.01
7	-1.64	-0.63	-0.71	-0.04	-0.01	-0.00
8	-1.74	-0.75	-0.55	-0.04	-0.01	-0.00
9	-1.80	-0.88	-0.42	-0.03	-0.00	-0.00
10	-1.85	-0.98	-0.33	-0.02	-0.00	-0.00
11	-1.91	-1.05	-0.27	-0.01	-0.00	-0.00
12	-1.97	-1.10	-0.21	-0.01	-0.00	-0.00
13	-2.03	-1.13	-0.17	-0.01	-0.00	-0.00
14	-2.09	-1.15	-0.14	-0.00	-0.00	-0.00
15	-2.16	-1.15	-0.11	-0.00	-0.00	-0.00
16	-2.23	-1.15	-0.09	-0.00	-0.00	0.
17	-2.31	-1.15	-0.07	-0.00	-0.00	0.
18	-2.38	-1.13	-0.06	-0.00	-0.00	0.
19	-2.46	-1.11	-0.04	-0.00	-0.00	0.
20	-2.53	-1.09	-0.03	-0.00	-0.00	0.
21	-2.61	-1.07	-0.03	-0.00	-0.00	0.
22	-2.69	-1.04	-0.02	-0.00	-0.00	0.
23	-2.76	-1.02	-0.02	-0.00	-0.00	0.
24	-2.84	-0.99	-0.01	-0.00	-0.00	0.
25	-2.91	-0.96	-0.01	-0.00	0.	0.
26	-2.99	-0.93	-0.01	-0.00	0.	0.
27	-3.07	-0.90	-0.01	-0.00	0.	0.
28	-3.14	-0.87	-0.00	-0.00	0.	0.
29	-3.21	-0.84	-0.00	-0.00	0.	0.
30	-3.29	-0.80	-0.00	-0.00	0.	0.
35	-3.65	-0.65	-0.00	-0.00	0.	0.
40	-4.00	-0.52	-0.00	-0.00	0.	0.
45	-4.33	-0.40	-0.00	0.	0.	0.
50	-4.65	-0.31	-0.00	0.	0.	0.
60	-5.25	-0.17	-0.00	0.	0.	0.
70	-5.82	-0.09	-0.00	0.	0.	0.
80	-6.36	-0.04	-0.00	0.	0.	0.
90	-6.87	-0.02	-0.00	0.	0.	0.
100	-7.35	-0.01	0.	0.	0.	0.
200	-11.37	-0.00	0.	0.	0.	0.
300	-14.56	0.	0.	0.	0.	0.
400	-17.32	0.	0.	0.	0.	0.
500	-19.79	0.	0.	0.	0.	0.
1000	-29.93	0.	0.	0.	0.	0.

TABLE 16

PERCENT LOSS OF POWER
WHEN USING ONE SIDED TESTS OF THE HYPOTHESIS U =0 IF IN FACT U = C AND THE POPULATION IS SINGLY TRUNCATED ON THE LEFT AT THE POINT A = -3.50

C / N	0.0	0.5	1.0	1.5	2.0	2.5
5	-1.26	-1.08	-0.02	0.19	-0.06	-0.01
6	-1.10	-0.12	-0.42	0.14	-0.01	-0.01
7	-0.98	0.02	-0.31	0.05	-0.01	-0.00
8	-0.89	-0.03	-0.19	0.01	-0.01	-0.00
9	-0.82	-0.10	-0.11	-0.00	-0.00	-0.00
10	-0.77	-0.16	-0.06	-0.01	-0.00	-0.00
11	-0.72	-0.20	-0.04	-0.01	-0.00	-0.00
12	-0.69	-0.23	-0.02	-0.01	-0.00	-0.00
13	-0.66	-0.25	-0.01	-0.00	-0.00	-0.00
14	-0.64	-0.26	-0.01	-0.00	-0.00	-0.00
15	-0.63	-0.27	-0.01	-0.00	-0.00	-0.00
16	-0.62	-0.27	-0.01	-0.00	-0.00	0.
17	-0.61	-0.27	-0.01	-0.00	-0.00	0.
18	-0.61	-0.26	-0.01	-0.00	-0.00	0.
19	-0.61	-0.26	-0.01	-0.00	-0.00	0.
20	-0.61	-0.25	-0.01	-0.00	-0.00	0.
21	-0.61	-0.24	-0.01	-0.00	-0.00	0.
22	-0.61	-0.23	-0.01	-0.00	-0.00	0.
23	-0.62	-0.23	-0.00	-0.00	-0.00	0.
24	-0.62	-0.22	-0.00	-0.00	-0.00	0.
25	-0.63	-0.21	-0.00	-0.00	0.	0.
26	-0.64	-0.20	-0.00	-0.00	0.	0.
27	-0.64	-0.19	-0.00	-0.00	0.	0.
28	-0.65	-0.19	-0.00	-0.00	0.	0.
29	-0.66	-0.18	-0.00	-0.00	0.	0.
30	-0.67	-0.17	-0.00	-0.00	0.	0.
35	-0.72	-0.14	-0.00	-0.00	0.	0.
40	-0.77	-0.11	-0.00	-0.00	0.	0.
45	-0.82	-0.08	-0.00	0.	0.	0.
50	-0.88	-0.06	-0.00	0.	0.	0.
60	-0.98	-0.04	-0.00	0.	0.	0.
70	-1.08	-0.02	-0.00	0.	0.	0.
80	-1.18	-0.01	-0.00	0.	0.	0.
90	-1.27	-0.00	-0.00	0.	0.	0.
100	-1.36	-0.00	0.	0.	0.	0.
200	-2.09	0.	0.	0.	0.	0.
300	-2.67	0.	0.	0.	0.	0.
400	-3.15	0.	0.	0.	0.	0.
500	-3.59	0.	0.	0.	0.	0.
1000	-5.31	0.	0.	0.	0.	0.

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NASA TM X-53272 APPROVAL May 26, 1965

THE EFFECT OF TRUNCATION ON TESTS OF HYPOTHESES FOR NORMAL POPULATIONS

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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