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AURORAL PARTICLES FROM THE GEOMAGNETIC TAIL, I

BY
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GODDARD SPACE FLIGHT CENTER
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ABSTRACT

An analytical study is made of particle trajectories about a neutral sheet. Such a magnetically neutral sheet has been observed in the earth's magnetic tail by Ness.

A simple model is constructed where the magnetic field varies linearly, reversing across the neutral sheet, and the electric field is everywhere homogenous, perpendicular to the magnetic field and parallel to the neutral sheet. The important results in this case are that either protons or electrons never come out of the neutral sheet, and gain an unlimited amount of energy.

Another model is constructed by adding a small component of the magnetic field perpendicular to the neutral sheet. Such a component could be furnished by the earth's dipole field in the tail. This component not only serves to bring the particles out of the neutral sheet, but turns both electrons and protons 90° from the accelerating electric field, in toward the earth. Furthermore, all particles that are injected into the neutral sheet far enough from the earth emerge with small pitch angles to the magnetic lines of force. Particles injected at about 50 R_e (earth-radii) emerge in a cone of about 1/2° about a line of force, the angle of the cone varying inversely as the cube of the distance from the earth. Protons injected at 50 R_e emerge with an energy of about 13 kev, and electrons have only about 6 eV. In order to have emergent electrons with energies of the order of auroral electron energies, the electrons must be injected at about 150 R_e. Protons injected at this distance back in the tail will emerge with energies somewhere around 50 to 100 kev.

When a magnetospheric magnetic field model is considered, consisting of a dipole field added to a tail field like the simple model, the above results imply that electron auroras will occur at higher latitudes than proton auroras and that proton auroras will occur before electron auroras in time.

These results, showing that particles are accelerated in and then ejected from neutral sheets may also be important in situations such as solar flares, the day-side neutral sheet and neutral sheets occurring in interplanetary space.

ACKNOWLEDGMENTS

Interesting discussions were had with Drs. J. W. Dungey, M. P. Nakada, W. N. Hess and many other people in the Theoretical Division of the Goddard Space Flight Center. The research was done while the author was a National Academy of Sciences, National Research Council Resident Research Associate at Goddard.

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INTRODUCTION

Ness (1964) has reported the discovery of a magnetically neutral sheet in the earth's geomagnetic tail, with measurements taken on board the IMP-1 satellite. A somewhat uniform magnetic field of about $20\,\gamma$ ($1\,\gamma = 10^{-5}$ gauss $= 10^{-9}$ weber/m^2) is found from about $10\,R_e$ (earth-radii) to at least $30\,R_e$ in the anti-solar direction. This field is predominantly in the solar direction above a plane (roughly identified as the magnetic equatorial plane), and in the anti-solar direction below this plane. The magnetic field reverses across a sheet of thickness about $0.1\ R_e$ and goes to zero within this neutral sheet.

It is of interest to look at charged particle trajectories about such a sheet, to see if the sheet can be a source of auroral particles, the particles originally coming from the solar wind. Adiabatic theory, which is good throughout the rest of the magnetosphere, cannot be used across such a neutral sheet, because the magnetic field changes significantly in distances much less than a gyroradius. The charged particle equations of motion must therefore be either solved analytically, or computed numerically.

Speiser (1964(b), 1965) has made numerical calculations, using a computer, of proton trajectories about a model neutral or current sheet. Using Liouville's theorem, he finds that the largest intensity of particles emergent from the sheet directly along magnetic lines of force, occurs in a thin output sheet. Such a model is thus capable of producing thin sheets of incoming particles for auroras. The mapping of these sheets onto the earth is near the auroral zones, and will be discussed in a future publication.

A SIMPLE LINEAR MODEL

The simplest model for the fields about a neutral sheet which can be discussed analytically is:

$$\mathbf{B} = -\,b\,\frac{x}{d}\;\hat{e}_y, \tag{1}$$

$$\mathbf{E} = - a\, \hat{e}_z \qquad (2)$$

where b is the strength of the magnetic field when x = d, the sheet half-thickness, and a is the strength of the electric field. The electric field is added because it is believed that such a field exists across the magnetospheric tail. This belief is based on measurements of current systems and electric fields over the polar caps, and the assumption that such a field would be transmitted across the tail if the magnetic lines of force are equipotentials.

The co-ordinate system being used and its relationship to the earth is sketched in Figure 1. The above magnetic and electric fields are also indicated in Figure 1.

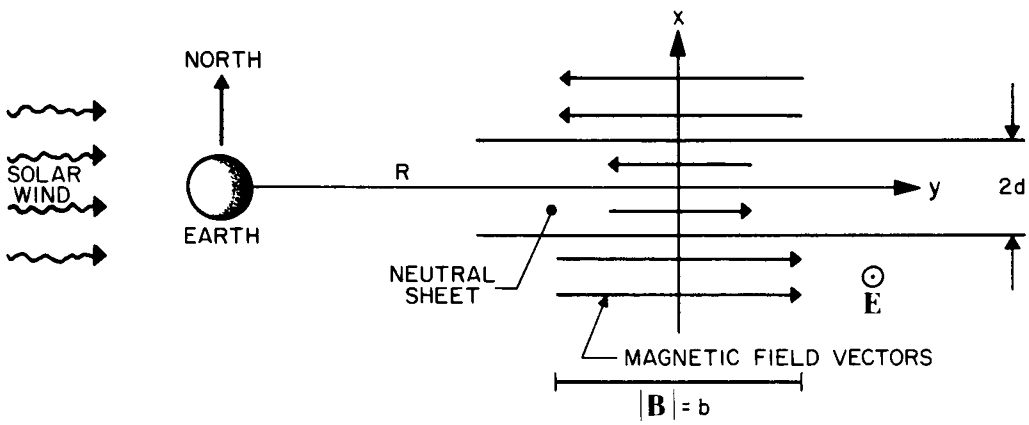

Figure 1—Cartesian co-ordinate system, $\hat{e}_y$ is anti-solar, along the earth-sun line, $\hat{e}_x$ points North, and $\hat{e}_z$ is into the paper. The magnetic field reverses linearly across a neutral sheet of thickness 2d, the electric field is uniform and everywhere points out of the paper. R is the distance from the earth to the origin of the co-ordinate system.

The equations of motion for a particle in the neutral sheet, using these fields are:

$$\ddot{x} = C_1 \dot{z}\, x \qquad (3)$$

$$\ddot{y} = 0 \qquad (4)$$

$$\ddot{z} = - C_3 - C_1\, x \dot{x} \qquad (5)$$

where $C_1 = \frac{q}{m}\frac{b}{d}$, and $C_3 = \frac{q}{m}\, a$. Mks units are used.

Speiser (1964, (a),(b)) has given the solution to these equations for large time. (See Appendix A.) The result is that the particle executes a damped oscillation about $x = 0$ (the amplitude going as $1/t^{1/4}$), while accelerating in the $+\hat{e}_z$ direction for electrons and the $-\hat{e}_z$ direction for protons. This can be understood as follows. The electric field in equation (5) accelerates a proton in the $-\hat{e}_z$ direction. $\dot{z}$ then becomes proportional to $-t$ ignoring the term $-C_1 x \dot{x}$ for the moment. Equation (3) then becomes

$$\ddot{x} = -kx \tag{6}$$

where $k = C_1 |\dot{z}|$, or $k = \left(\frac{q}{m}\right)^2 \frac{ab}{d} \; t$.

Equation (6) is just the equation for oscillatory motion, with spring constant k. k gets larger with time, however, implying that the spring gets stiffer with time, thus the amplitude of oscillation decays.

The term $-C_1 x \dot{x}$ in equation (5) is thus oscillatory and approaches zero, justifying its neglect. k is proportional to q^2, so particles of either sign will oscillate.

The oscillation in $x(t)$ is due to the $\mathbf{V} \times \mathbf{B}$ force which is always toward $x = 0$ for x either positive or negative, because of the reversal of the magnetic field.

The net result of this simple model is that charged particles of either sign never come out of the neutral sheet, and their energy increases without bound. See Figure 2 for a sketch of the results of this simple model.

THE LINEAR MODEL WITH SMALL PERPENDICULAR FIELD ADDED

Consider now the addition of a small magnetic field component perpendicular to the neutral sheet, i.e., in the $+\hat{e}_x$ direction referring to the co-ordinate system of Figure 1. Such a component could be furnished by the addition of the earth's dipole field. (See Figure 3). The magnetic field of equation (1) now becomes:

$$\mathbf{B} = b\left(\eta\, \hat{e}_x - \frac{x}{d}\, \hat{e}_y\right) \tag{7}$$

where η will be assumed small.

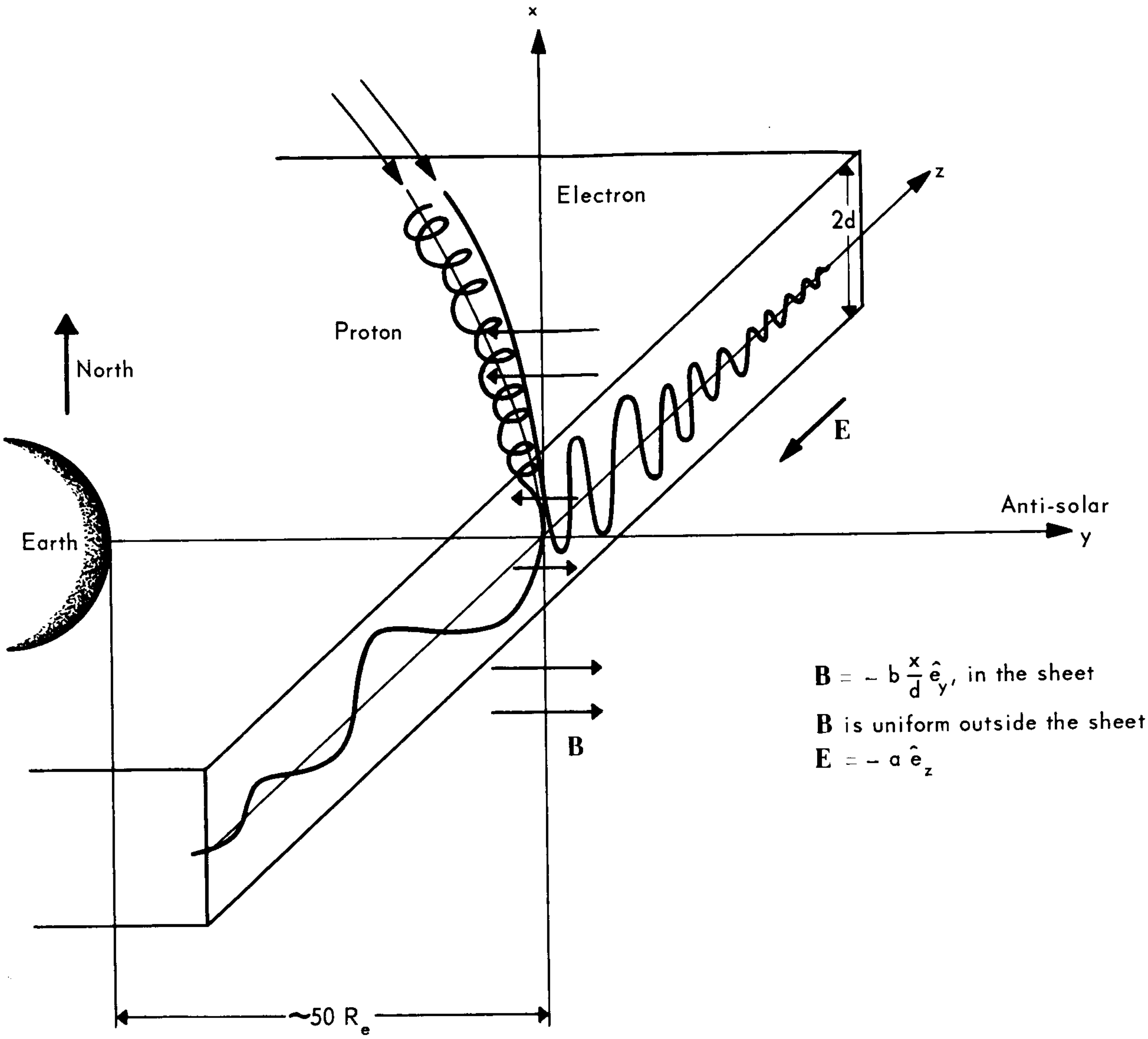

Figure 2–Sketch of particle trajectories using the fields of the simple model (linear reversal of the magnetic field across the neutral sheet, no magnetic field component perpendicular to the sheet, and a uniform electric field in the $-\hat{e}_z$ direction). Electrons are accelerated in the $+\hat{e}_z$ direction, protons in the $-\hat{e}_z$ direction. The amplitude of oscillation decays, so particles never come out and their energy goes to infinity.

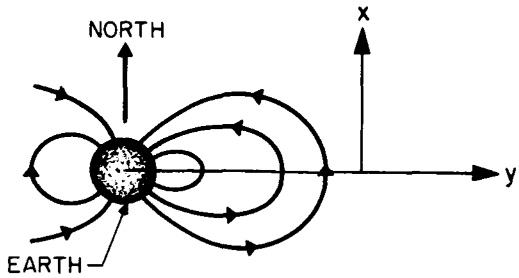

Figure 3–Sketch of earth's dipole field lines. Near the geomagnetic equatorial plane ($x = 0$), the dipole field lines are predominantly in the $+\hat{e}_x$ direction, and the magnitude falls off with the cube of the distance.

The equations of motion now become:

$$\ddot{x} = C_1 \, x \, \dot{z} \tag{8}$$

$$\ddot{y} = C_2 \, \eta \, \dot{z} \tag{9}$$

$$\ddot{z} = - C_3 - C_1 \, x \, \dot{x} - C_2 \, \eta \, \dot{y} \tag{10}$$

where $C_1 = (q/m)\,(b/d)$, $C_2 = (q/m)\,b$, $C_3 = (q/m)a$, C_1 and C_3 being the same as for the simple model. When $\eta = 0$, the simple model results.

Even without solving these equations, the particle motion can be understood qualitatively as was done for the simple model.

Consider a proton (the arguments also hold for electrons with appropriate changes in sign) incident on this neutral sheet with small velocity. (Strictly speaking, the sheet is not now a neutral sheet.) The proton will be accelerated initially in the negative z direction (see Equation (10)), gaining a velocity, $\dot{z}$, proportional to $-t$ as for the simple model. From Equation (9), there will then be an acceleration in the $-y$ direction proportional to $\dot{z}$ or $-t$, and thus $\dot{y}$ will be proportional to $-t^2$. (Note that $\dot{y} \, \alpha \, C_2 \, C_3 \, \alpha \, (q/m)^2$, so either protons or electrons are turned toward the $-\hat{e}_y$ direction.)

As long as $\dot{z}$ is negative, Equation (8) will imply oscillatory motion in $x(t)$ by the same arguments in the previous section on the simple model. Thus, as long as $\dot{z}$ is negative, the term $-C_1 x\dot{x}$ in Equation (10) is oscillatory and will be assumed small as a first order approximation. The oscillations may now be either damped or growing depending on whether $\dot{z}$ increases or decreases with time. The last term in Equation (10) grows as $+t^2$ so that $\dot{z}$ will grow negatively until $\ddot{z}$ goes to zero, and $\dot{z}$ will then diminish in absolute value going to zero and even becoming positive after some time. Thus $x(t)$ will execute damped oscillatory motion until $\ddot{z} = 0$ (the spring gets stiffer). After $\ddot{z}$ becomes positive and until $\dot{z} = 0$, $x(t)$ will execute growing oscillatory motion. After $\dot{z}$ goes positive, however, $x(t)$ will no longer oscillate but will increase exponentially, thus ejecting the particle from the neutral sheet.

Summarizing, the addition of a small magnetic field component, $+B_x$, to a linear neutral sheet model, serves to bring the particles out of the sheet and also turns charged particles of either sign in toward the earth (the $-\hat{e}_y$ direction in Figure 1). Such a component $+B_x$ is consistent with the direction of the earth's field near the geomagnetic equatorial plane. Therefore, particles which have gained energy from the electric field are turned toward the earth, 90° from that field, and just as the energy gained is changed into motion toward the earth, the particle is ejected from the neutral sheet. It is shown in Appendix B that both protons and electrons emerge from the sheet with the same velocity. See Figure 4 for a sketch of the particle trajectories in this model.

RESULTS OF THE APPROXIMATE THEORY

The detailed calculations are contained in Appendix B, to which the following alphabetic equation indices refer. First integrals of the equations of motion (Equations (8), (9) and (10)) are obtained exactly (Equations (i), (j) and (n)). A first approximation is used to obtain the time of ejection, τ, which turns out to be inversely proportional to $\eta b(q/m)$. (Equation (s)) Electrons are therefore ejected from the neutral sheet much sooner than protons. The velocity toward the earth at the time of ejection is found to be independent of (q/m), (Equation (w)) and the maximum pitch angle of the emergent particles (Equation (z)) is proportional to η, and to $|\dot{x}_0/u - c|$, where u is the bulk flow velocity exterior to the neutral sheet, and c is a number which is not very different from 1. This result implies that $\alpha = 0$ for $\eta = 0$, which means that the particles would never come out of the neutral sheet if the small perpendicular field were not added. This agrees with the result of the simple model, and implies that α will be small if η is small. Since α is also proportional to $|\dot{x}_0/u - c|$, particles incident on the current sheet with $\dot{x}_0 \approx u$ will all emerge with pitch angles close to zero.

APPLICATIONS

Choose $R = 50 R_e$, $b = 20\gamma$, $a = 0.3$ volts/km. B_x for the earth's dipole field at $R = 50 R_e$ is $B_x \approx 0.3\gamma$. Therefore $\eta = 1.5 \times 10^{-2}$. Using Equation (v) for

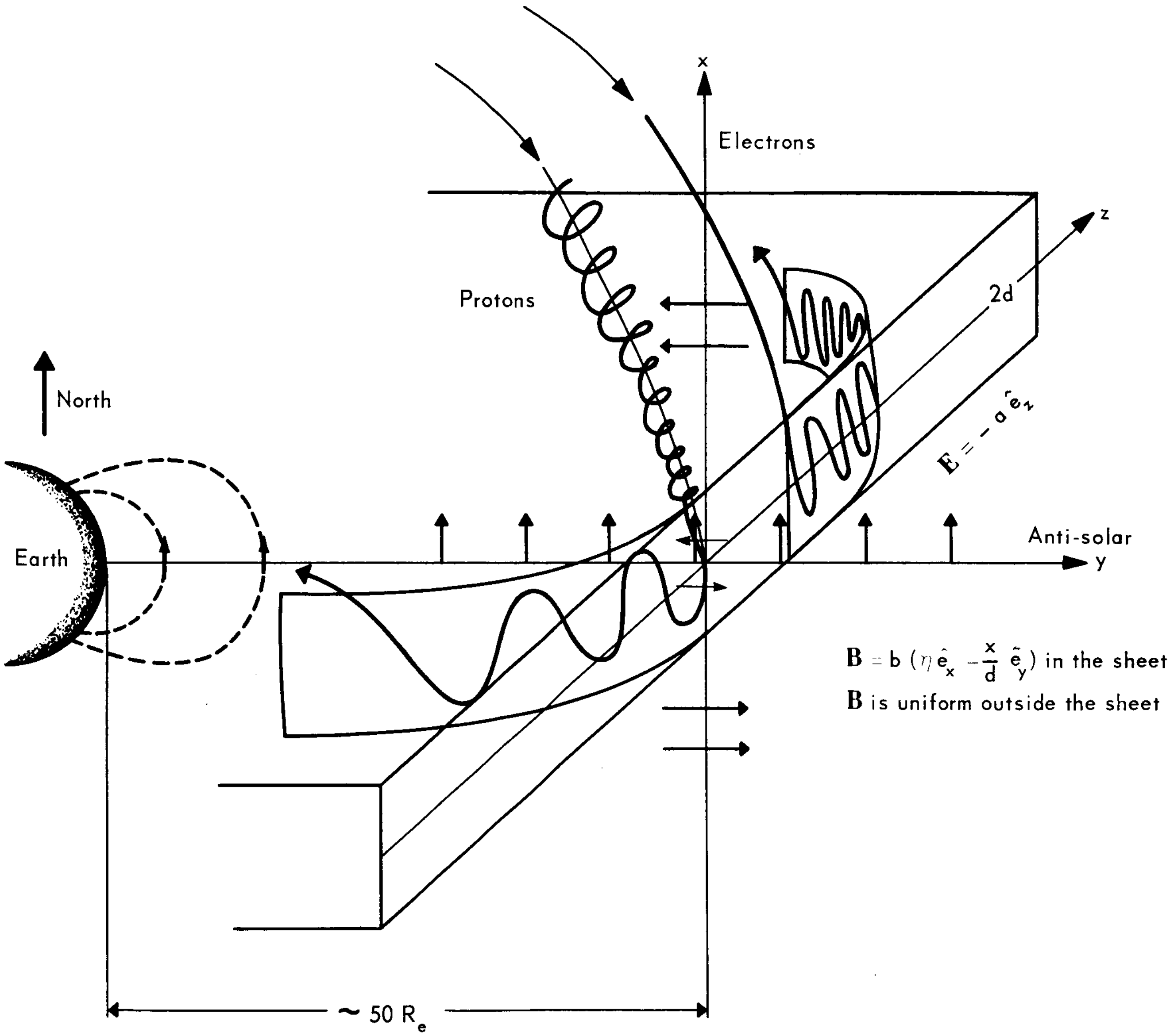

Figure 4—Sketch of particle trajectories using the fields of the simple model plus a small component perpendicular to the neutral sheet. Such a component could be furnished by the earth's dipole field as indicated. Both protons and electrons oscillate about the sheet accelerating in opposite directions, and are turned in the same direction (toward the earth) by the small magnetic field component perpendicular to the sheet. When the particles are turned 90°, they are ejected from the neutral sheet. Electrons come out much sooner than protons, hence gain less energy. The dimensions shown are illustrative and not to scale.

the distance the particle has moved toward the earth at the time of ejection, we obtain $y(\tau) \approx -14\ R_e$ for protons and ≈ -40 km for electrons. The emergent velocity $\dot{y}\ (\tau)$ (Equation (w)) is about 1500 km/sec, giving an energy for protons of about 13 kev and about 6 eV for electrons. If the quantity $|\dot{x}_0/u - c|$ in Equation (z) is assumed to be of the order of 1, then in this example, the maximum pitch angle is of the order of 0.3°. (All particles will emerge within a cone having α as the half-angle.)

The right order of magnitude for auroral protons is 13 kev, but 6 eV is too small for auroral electrons. However, the output velocity is proportional to $1/\eta$, thus the output energy is proportional to $1/\eta^2$. For the earth, η is proportional to $1/R^3$. So to get 6 kev electrons (which is of the order of auroral electron energies), we need only to have them injected in the neutral sheet at about 150 R_e, assuming the neutral sheet extends that far. At that distance, the electrons still only stay in the sheet for about 6 R_e before they are ejected.

Injecting protons at 150 R_e back in the tail would not increase their energy 1000 fold as for the electron, because $y(\tau)$ (Equation (v)) would need to be about -14,000 R_e, so the protons would follow the neutral sheet in toward the earth until the stronger B_x (larger η) would bring them out. The proton energy would thus not get above 50 to 100 kev.

If one adds the earth's dipole field to a tail field such as indicated in Figure 1, then field lines from high latitudes are dragged back into the tail. A field line crossing the neutral sheet at 150 R_e therefore comes from a higher latitude than a field line crossing at 40 R_e. Therefore, the above results would imply electron auroras at higher latitudes than proton auroras, which is usually observed. Also, if some change in the characteristics of the solar wind (velocity, particle density, magnetic field direction) triggers this neutral sheet acceleration process, then the above results would predict proton auroras before electron auroras, since it will take time for the change in the solar wind to move from about 50 R_e to 150 R_e in the tail. The electrons emergent at 150 R_e will all be within a cone of about .02° (Equation (z)). This implies that more electrons than protons should be able to get down to auroral altitudes.

APPENDIX A

The first integrals of Equation (3) and (5) are:

$$\dot{z} = \dot{z}_0 - C_3 t - \frac{1}{2} C_1 (x^2 - x_0^2) \tag{a}$$

and

$$\frac{1}{2}(\dot{x}^2 + \dot{z}^2) + C_3 z = \frac{1}{2}(\dot{x}_0^2 + \dot{z}_0^2) + C_3 z_0 \tag{b}$$

Equation (b) is just the equation of conservation of energy. The zero subscripted values refer to initial values. Equation (3) becomes, using (a):

$$\ddot{x} = - C_1 x \left(-\dot{z}_0 + \frac{1}{2} C_1 (x^2 - x_0^2) + C_3 t \right) \tag{c}$$

For large enough time, the quantity in parenthesis on the right hand side of Equation (c) will be positive and monotonically increasing implying oscillatory, bounded motion in $x(t)$. Thus for large time Equation (c) is approximately:

$$\ddot{x} \approx - C_1 C_3 x t = - \left(\frac{q}{m}\right)^2 \frac{a b}{d} x t \tag{d}$$

The solution to Equation (d) is given by Jahnke and Emde (1945, pp 147) as

$$x = \sqrt{t'} \, Z_{1/3} \left(\frac{2}{3} t'^{3/2} \right) \tag{e}$$

where

$$t' = \left(\frac{b a}{d}\right)^{1/3} \left(\frac{q}{m}\right)^{2/3} t,$$

and $Z_{1/3}$ is a linear combination of Bessel Functions of the First and Second Kinds, of order one-third. Approximating (e) for large time, (Jahnke and Emde, 1945, pp. 138),

$$x \approx \frac{t^{-1/4}}{\left(\frac{ba}{d}\right)^{1/12}\left(\frac{q}{m}\right)^{1/6}}\left[A\cos\left(\frac{2}{3}\,t^{3/2}\left(\frac{ba}{d}\right)^{1/2}\left(\frac{q}{m}\right)\right)+B\sin\left(\frac{2}{3}\,t^{3/2}\left(\frac{ba}{d}\right)^{1/2}\left(\frac{q}{m}\right)\right)\right] \quad \text{(f)}$$

A and B are constants depending on the initial values.

$z(t)$ may be obtained as a function of time by integrating Equation (a):

$$z(t) \approx -\left(\frac{q}{m}\right)\frac{at^2}{2}+\left(\dot{z}_0+\left(\frac{q}{m}\right)\left(\frac{b}{d}\right)\frac{x_0^2}{2}\right)t+z_0 \quad \text{(g)}$$

+ smaller oscillatory terms

From (f), after large time the amplitude of oscillation decays as $1/t^{1/4}$, and from (g) and (b) it is seen that the kinetic energy increases as t^2.

APPENDIX B

First integrals of Equations (8), (9), and (10) are easily obtained. They are:

$$\frac{1}{2}(\dot{x}^2 - \dot{x}_0^2) = - C_3(z - z_0) - \frac{1}{2}(\dot{z}^2 - \dot{z}_0^2) - C_2\eta \int_0^t \dot{y}\,\dot{z}\,dt \tag{h}$$

$$\dot{y} - \dot{y}_0 = C_2\,\eta\,(z - z_0) \tag{i}$$

$$(\dot{z} - \dot{z}_0) = - C_3\,t - \frac{C_1}{2}(x^2 - x_0^2) - C_2\,\eta\,(y - y_0) \tag{j}$$

Equation (j) becomes, on another integration:

$$(z - z_0) = - \frac{C_3 t^2}{2} - \frac{C_1}{2}\int_0^t x^2 dt - C_2\eta \int_0^t y dt + C_4 t \tag{k}$$

where

$$C_4 = \dot{z}_0 + \frac{C_1 x_0^2}{2} + C_2\eta y_0$$

Using Equation (k) with Equation (i), we have:

$$\dot{y} = \dot{y}_0 + \eta\left(- C_5 t^2 + C_6 t - C_7 \int_0^t x^2 dt - C_2^2 \eta \int_0^t y dt\right) \tag{l}$$

where

$$C_5 = \frac{C_2 C_3}{2}, \quad C_6 = C_2 C_4, \quad C_7 = \frac{C_1 C_2}{2}.$$

From Equation (9) we have:

$$C_2 \eta \int_0^t \dot{y}\, \dot{z}\, dt = \frac{1}{2} (\dot{y}^2 - \dot{y}_0^2) \tag{m}$$

So that Equation (h) becomes the energy integral:

$$\frac{1}{2} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + C_3 z = \frac{1}{2} (\dot{x}_0^2 + \dot{y}_0^2 + \dot{z}_0^2) + C_3 z_0 = \frac{E_0}{m} \tag{n}$$

Using Equation (j), Equation (8) becomes:

$$\ddot{x} = - k(t, x, y, \eta)\, x \tag{o}$$

where

$$k(t, x, y, \eta) = C_3 t + \frac{C_1 x^2}{2} + C_2 \eta y + C_8 \tag{p}$$

and

$$C_8 = - \frac{C_1 x_0^2}{2} - C_2 \eta y_0 - \dot{z}_0 .$$

k goes to zero when

$$t = -\frac{1}{C_3}\left(\frac{C_1 x^2}{2} + C_2\,\eta y + C_8\right) = \tau \qquad \text{(q)}$$

τ is therefore the critical time when k goes to zero and then becomes negative ejecting the particle. ($\dot{z}$ becomes positive.)

APPROXIMATIONS

Integrating Equation (l), we obtain:

$$y = y_0 + \dot{y}_0 t + \frac{C_6 t^2}{2} - \frac{\eta\, C_5 t^3}{3} \qquad \text{(r)}$$

Where the integral over x^2 in (l) is assumed small since $x(t)$ is oscillating, and the integral over y in (l) is multiplied by η^2, so will also be neglected in this first approximation.

Without loss of generality, but to facilitate finding the critical time of ejection τ, initial conditions are chosen such that $y_0 = \dot{y}_0 = 0$ and $\dot{z}_0 = -C_1 x_0^2/2$, which implies that $C_4 = C_6 = C_8 = 0$. Using Equations (q) and (r) with these initial conditions, τ becomes:

$$\tau = \frac{\sqrt{6}}{\left(\eta\left(\frac{q}{m}\right) b\right)} \qquad \text{(s)}$$

From Equation (r)

$$y(t) = -\frac{\eta C_5 t^3}{3}, \qquad \text{(t)}$$

and

$$\dot{y}(t) = -\eta C_5 t^2 \tag{u}$$

At $t = \tau$, Equations (t) and (u) become:

$$y(\tau) = -\frac{\sqrt{6}\, a}{(\eta b)^2 \left(\frac{q}{m}\right)} \tag{v}$$

and

$$\dot{y}(\tau) = -\frac{3a}{\eta b} \tag{w}$$

Thus, in this first approximation, the ejection velocity is independent of (q/m) because electrons are ejected much sooner than protons. (As the next approximation, if $y(t)$ from Equation (t) is used in the integral over y from Equation (k) it is found that τ is increased by about 30%, $\dot{y}(\tau)$ is decreased by about 50%, and the ejection velocity is still independent of (q/m). The first approximation should therefore give the right order of magnitude.)

PITCH ANGLE DISTRIBUTIONS OF EMERGENT PARTICLES

In order to calculate the pitch angle of an emergent particle, we must estimate the maximum value $\dot{x}$ can have at the time of ejection. Since $x(t)$ oscillates until $t = \tau$, it is reasonable to assume that $\dot{x}_{max} \approx \dot{x}_0$. Using this assumption and Equation (w) with Equation (7) for the magnetic field, with $x = d$, we have:

$$\mathbf{V} = \dot{x}_0\, \hat{e}_x - \frac{3a}{\eta b}\, \hat{e}_y \tag{x}$$

and the cosine of the pitch angle α, becomes:

$$\cos\alpha = \frac{(\mathbf{B}\cdot\mathbf{V})}{BV} = 1 - \frac{2\eta^2\left(\frac{\dot{x}_0}{u} - 3\right)^2}{36} \tag{y}$$

where u = a/b, the bulk flow velocity exterior to the neutral sheet, and only terms of order η^2 are kept. For small α, $\cos\alpha \approx 1 - \alpha^2/2$, so

$$\alpha \approx \frac{\eta\left|\frac{\dot{x}_0}{u} - 3\right|}{3} \tag{z}$$

The number 3 in Equation (z) comes from the 3 in Equation (w). This would be decreased to 3/2 by the second approximation mentioned above.

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