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DETERMINATION OF AERODYNAMIC FORCE AND HEAT TRANSFER PROPERTIES FOR A CONCAVE HEMISPHERICAL SURFACE IN FREE MOLECULAR FLOW

by CLARENCE RAY WIMBERLY
Aero-Astrodynamics Laboratory

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George C. Marshall
Space Flight Center,
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ABSTRACT

The methods and equations are presented for the aerodynamic force and heat transfer properties of a concave hemispherical surface at an arbitrary angle of attack in free molecule flow. The effect of multiple reflections is taken into account by assuming a cosine distribution after each collision. It is assumed that the emission of molecules from the surface is purely diffuse, and that the reflecting molecules are perfectly accommodated to the surface conditions. The resulting equations are not in closed form, but have been solved numerically for some typical values of the molecular speed ratio and surface-to-ambient-temperature ratio. It is found that under the above conditions the heat transfer characteristics are the same as those of a convex hemisphere. The drag is increased slightly over its counterpart, while the total lift is shown to be very small and would be zero except for interreflected molecules.

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ADVANCED VEHICLE AERODYNAMICS OFFICE
AERODYNAMICS DIVISION
AERO-ASTRODYNAMICS LABORATORY

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DEFINITION OF SYMBOLS

Symbol	Definition
A	surface area of hemisphere ($\pi D^2/2$)
A_r	reference area ($\pi D^2/4$)
a	thermal accommodation coefficient
C_D	aerodynamic drag coefficient
C_L	aerodynamic lift coefficient
C_p	specific heat at constant pressure
C_q	heat transfer coefficient
C_v	specific heat at constant volume
C_x	relative surface velocity in the x-direction
C_y	relative surface velocity in the y-direction
C_z	relative surface velocity in the z-direction
D	aerodynamic drag and surface diameter
d	distance between two differential surface elements
E_i	energy due to incident molecular flux
E_r	energy due to reflected molecular flux
E_w	energy due to molecules reflecting at the wall temperature
f	Maxwell's velocity distribution function
F_a	angle factor
k	Boltzmann's constant
m	molecular weight
N_i	total molecular incident flux
N_w	total reflected molecular flux at the wall temperature

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
n	molecular density
P	total pressure on a differential surface element
P_f	pressure on a surface element due to the freestream incident and reflected molecular flux
P_I	pressure in region I
P_{II}	pressure in region II
P_i	pressure due to incident freestream molecular flux
P_r	pressure due to reflected freestream molecular flux
P_w	pressure due to reflected freestream molecular flux at the wall temperature
q	local heat transfer to the surface per unit time and area
$\bar{Q}$	average heat transfer to the surface
R	gas constant
r	radius of hemisphere
S	ratio of the free stream velocity and the most probable molecular speed
S_w	molecular speed ratio at wall temperature
T	molecular temperature in freestream
T_w	wall temperature

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
U	surface freestream velocity
v	molecular velocity
α	surface angle of attack
β	angle of integration
γ	ratio of specific heats (C_p/C_v)
θ	element angle of attack
ν	angle of integration
π	constant (3.14159)
ρ	mass density
σ	reflection coefficient due to shear stresses
σ'	reflection coefficient due to pressures
τ_i	shear stress due to incident mass flux
τ_r	shear stress due to reflected mass flux
ϕ_1	direction angle for diffuse emission from element 1
ϕ_2	direction angle for diffuse emission from element 2

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SUMMARY

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An analysis of a concave hemispherical surface in free molecule flow is presented where the surface is oriented at an arbitrary angle of attack. General equations are formulated for the lift, drag, and heat transfer coefficients. The effect of multiple reflections is taken into account by assuming a completely diffuse reflection process where molecules are distributed equally in all directions after each reflection. The molecules are assumed to obey Maxwell's velocity distribution law before and after collisions, reflecting directionally in a cosine distribution. It is further assumed that the reflecting molecules are perfectly accommodated to the surface conditions. The resulting equations are not in closed form, but have been solved numerically for some typical values of the molecular speed ratio and surface-to-ambient-temperature ratio. It is found that under the above conditions the heat transfer characteristics are the same as those of a convex hemisphere. The drag is increased slightly over its counterpart, while the total lift is shown to be very small and would be zero except for interreflected molecules. Maximum drag and heat transfer occur at zero angle of attack, decreasing to zero at ninety degrees angle of attack. The lift is a function only of interreflected molecules and is zero at zero and ninety degrees angle of attack, the maximum occurring near an angle of attack of forty-five degrees.

I. INTRODUCTION

With the advent of orbital satellites and space vehicles, analyses of the aerodynamic force and heat transfer properties of these bodies and vehicles, in the high altitude, low density flow regime have become necessary. The complexity of the body shapes, and the peculiarities of the molecular interactions with surfaces in the flow have introduced interesting aspects to analyses in this flow regime by imposing the necessity of considering various contributions of the momentum and energy imparted to the body by the impinging and re-emitting molecules. Actually, much work has been done in determining the aerodynamic properties of flat and convex bodies in free molecule flow with experimental verification, but little has been done in determining these properties for concave bodies that create a molecular multi-reflection process. This paper is concerned with the determination of the aerodynamic properties of a concave hemispherical surface at various angles of attack in this high altitude, free molecule flow regime.

By definition, the free molecule flow regime is a gas region of extreme rarefaction where collisions between molecules are much less probable than are collisions between the molecules and a surface in the flow. Because of this molecular freedom, the gas does not behave as a continuous fluid, but rather performs in accordance with its complex molecular structure defined by the kinetic theory of ideal gases. Briefly, this theory is a statistical description of molecular motion, including the impact effect with other bodies in the flow. From this theory and Newton's laws of motion, values for the force and heat transfer properties imparted to a concave hemispherical surface submerged in the flow have been determined and are presented here. The equations were derived by treating the flows of incident and re-emitted molecules separately, where the incident flow consists of the freestream and reflected molecules. It is the inclusion of this additional incident flux created by the concavity of the surface that causes the interesting aspects of this investigation.

II. PREVIOUS WORK

The fundamental concept of the free molecule gas flow analysis performed here is based on textbooks by Jeans [1], Loeb [2], and Patterson [3], the paperback by Schaaf and Chambre [4], and the paper by Oppenheim [5]. The methods employed in the analysis of free molecule flow over non-convex surfaces are presented in the book by Jakob [6], and the papers by Chahine [7], Sparrow [8], and Pratt [9].

The fundamentals of particle dynamics and the kinetic theory of gases, including velocity distributions, pressures, heat conduction and interaction effects are fully discussed with derivations in the textbooks by Jeans [1] and Loeb [2]. Using these fundamentals, Patterson [3] and Schaaf and Chambre [4] derived the means for representing the force and heat transfer properties for a differential surface element and for a few flat and convex bodies. Patterson [3] limited his investigations to the drag and heat transfer properties of a cylinder in free molecule helium and discussed some effects of Knudsen number on heat transfer. Schaaf and Chambre [4] summarized the pertinent equations in deriving the aerodynamic force and heat transfer properties for a flat plate, cylinder, and sphere. Also included were data for the drag coefficient, lift-to-drag ratio, recovery factor and Stanton number for the flat plate at angles of attack, and for the drag coefficient, recovery factor and Stanton number for the cylinder, and sphere, each plotted as a function of the molecular speed ratio. Oppenheim [5] discusses in detail the derivation and application of the convective heat transfer equations to bodies in free molecule flow. He included specific equations and data for the heat transfer properties to flat plates, cylinders, semicylinders, spheres, and hemispheres.

Jakob [6], in Chapter 31, discusses diffuse radiation between two surfaces of arbitrary orientation. The results of his discussions and derivations produce an equation representing the fraction of energy emitted from one surface to another. Chahine [7], Sparrow [8], and

Pratt [9] used this in determining the force and heat transfer properties of concave surfaces. Chahine [7] determined the force and heat transfer properties for infinite concave cylinders and for concave spherical segments; however, he limited his investigations to the special case wherein the free-stream flow impinges directly on all parts of the concave surfaces. This is an especially strong restriction for hemispherical surfaces since, for this type of surface, the angle of attack must be zero. Sparrow [8] in his paper determined the local and average heat transfer rates, the adiabatic wall temperature, and the forces exerted on a concave cylindrical surface in free molecule flow at arbitrary angles of attack. Pratt [9] performed the same analysis as Chahine [7], obtaining slightly different results. He also restricted his investigation for hemispherical surface to zero angle of attack. The investigations by Chahine [7], Sparrow [8], and Pratt [9] all assumed that the surface was traveling at hypervelocities, thereby neglecting the molecular motion, and assumed that all molecular reflections were diffuse.

The analysis presented here is for the force and heat transfer properties of concave hemispherical surface moving at hypervelocities in free molecule flow including the effects of angle of attack from zero to ninety degrees. Perfect thermal accommodation of molecules at the surface is assumed where the molecules are diffusely reflected.

III. FUNDAMENTAL CONCEPTS

A. Free Molecule Flow

The free molecule flow regime may be defined mathematically as the gas region where the Knudsen number $K_n = \lambda/L \geqq 10$ [3]. This corresponds to a minimum altitude in the earth's atmosphere of about 75 miles for a characteristic length of one foot, based on the U. S. Standard Atmosphere, 1962 [10]. This is an arbitrary definition and expressed by some as not complete for defining this flow regime [4].

The Knudsen number, however, has been accepted by most as the best criterion to date for establishing the boundaries of free molecule flow and is defined as the ratio of the mean free path of the molecules (λ) and some characteristic length (L). The mean free path is the average distance a molecule travels before colliding with another molecule.

B. Maxwell's Velocity Distribution

Because of the character of free molecule flow the methods normally used in describing fluid motion in a continuum cannot be applied. Thus, in this region of extreme rarefaction, the molecular motion is necessarily described statistically because of the variation of molecular velocities in the flow. From the kinetic theory of gases [1,2], the number of molecules in a region of free molecule flow is a function not only of the volume of the region, but also of the velocity of each molecule in the region. This differential number of molecules is thus expressed as

$$dN = f(\bar{x},\bar{v},t)\ dx^3\ dv^3, \tag{1}$$

where the function $f(\bar{x},\bar{v},t)$ has the same characteristics as molecular density except in a six-dimensional phase space. By assuming that the gas region of investigation is homogeneous and steady, the function becomes independent of position and time. The derivation of the resulting function is presented in many books on gas kinetics such as those by Jeans [1] and Loeb [2]. This function is

$$f(\bar{v}) = n\left(\frac{m}{2\pi\ kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} \tag{2}$$

and is known as Maxwell's velocity distribution function [1,2,3,4]. As will be shown, this function is highly useful in determining the force and heat transfer properties of surfaces in free molecule flow.

C. Reflection and Accommodation Coefficients

Specification of molecule-surface interactions is necessary to determine the momentum and energy transfer to the surface. Fortunately, this can be done by simply knowing certain average parameters which characterize the interaction phenomena. These parameters are the well known reflection and accommodation coefficients [3,4]. Two parameters are used in defining the reflection process to take into account both the tangential and normal force components. These reflection coefficients are.

$$\sigma = \frac{\tau_i - \tau_r}{\tau_i} \tag{3}$$

$$\sigma' = \frac{P_i - P_r}{P_i - P_w} . \tag{4}$$

For a purely specular reflection process, $\sigma = \sigma' = 0$, while for a completely diffuse reflection process, $\sigma = \sigma' = 1$. The thermal accommodation coefficient is defined by

$$a = \frac{dE_i - dE_r}{dE_i - dE_w} \tag{5}$$

and is used in defining energy exchanges with the surface. From this definition, it is easily seen that for perfect thermal accommodation at the surface, $a = 1$, while for the case of no energy exchange, $a = 0$. Experimentally determined values of σ, σ' and a for various typical materials and air-surface combinations are available [3,4].

D. Pressure and Shear Stress Calculations

The determination of the pressure and shear stress on a differential surface element (Figure 1) caused by the incident freestream and reflected molecules, where the macroscopic velocity is in the xy-plane, can now be carried out by considering the time rate of change of momentum normal and parallel to the surface element. By use of equations (1) and (2), the pressure and shear stress caused by the incident freestream are obtained as follows:

$$P_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} m\, C_x^2\, f(\bar{v})\, dC_x\, dC_y\, dC_z$$

$$= \frac{\rho U^2}{2\sqrt{\pi}\, S^2}\left\{\sqrt{\pi}\,[\tfrac{1}{2} + \eta^2][1 + \text{erf}(\eta)] + \eta e^{-\eta^2}\right\} \qquad (6)$$

and

$$\tau_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} m\, C_x C_y\, f(\bar{v})\, dC_x\, dC_y\, dC_z$$

$$= -\frac{\rho U^2 \cos\theta}{2\sqrt{\pi}\, S}\left\{e^{-\eta^2} + \sqrt{\pi}\,\eta[1 + \text{erf}(\eta)]\right\}, \qquad (7)$$

where

$\eta = S \sin\theta$

$S = U/\sqrt{2RT}$

$$\operatorname{erf}(\eta) = 2/\sqrt{\pi} \int_0^{\eta} e^{-x^2} dx$$

$$C_x = U_x + v_x$$

$$C_y = U_y - v_y$$

$$C_z = v_z .$$

The limits of integration include all possible magnitudes of the molecular velocities. The lower limit of dC_x is zero since no negative velocities in the x-direction will strike the front side of the surface element. The function $f(\bar{v})$ is given by equation (2). The angle θ is the element angle of attack. The net pressure and shear stress can be determined by using the definitions of the reflection coefficients, equations (3) and (4):

$$P_f = P_i + P_r = (2 - \sigma') P_i + \sigma' P_w \tag{8}$$

$$\tau_f = \tau_i - \tau_r = \sigma\tau_i , \tag{9}$$

where P_w is the pressure exerted on the wall due to molecules reflecting at the wall temperature with no macroscopic velocity contribution, i.e., $S = 0$ and $T = T_w$. With the help of equations (1) and (6), the equation for P_w becomes [4]

$$P_w = \frac{1}{2} m \sqrt{2\pi R T_w} \; N_i , \tag{10}$$

where

$$N_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} C_x\, f(\bar{v})\, dC_x\, dC_y\, dC_z\, dA$$

$$= \frac{\rho}{m} \sqrt{RT/2\pi} \left\{ e^{-\eta^2} + \sqrt{\pi}\, \eta[1 + \mathrm{erf}(\eta)] \right\} dA. \tag{11}$$

By combining equations (6), (7), (8), (9), (10) and (11), the net pressure and shear stress from the freestream incident flux of molecules on the front side of the surface element dA (Figure 1), including the contribution of reflecting molecules, become

$$P_f = \frac{\rho U^2}{2S^2} \left\{ \left(\frac{2 - \sigma'}{\sqrt{\pi}} \eta + \frac{\sigma'}{2} \sqrt{\frac{T_w}{T}} \right) e^{-\eta^2} + \left[(2 - \sigma')(\eta^2 + \frac{1}{2}) \right. \right.$$

$$\left. \left. + \frac{\sigma'}{2} \sqrt{\frac{\pi T_w}{T}}\, \eta \right] \left[1 + \mathrm{erf}(\eta) \right] \right\} \tag{12}$$

$$\tau_f = - \frac{\sigma \rho U^2 \cos\theta}{2\sqrt{\pi}\, S} \left\{ e^{-\eta^2} + \sqrt{\pi}\, \eta\, [1 + \mathrm{erf}(\eta)] \right\}. \tag{13}$$

E. Heat Transfer Calculations

The fundamental concepts of heat transfer to a differential surface element in free molecule flow is carried out in detail by Patterson [3], Schaaf and Chambre [4], and Oppenheim [5], where the contributions of energy imparted to the surface are the translational kinetic energy and internal molecular energy. The total kinetic energy caused by translational motion of molecules that strike the front side of dA per unit time and per unit area is

$$dE_{i,t} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} \frac{1}{2} mC^2 f(\bar{v}) C_x \, dC_x \, dC_y \, dC_z$$

$$= \rho RT \sqrt{RT/2\pi} \left\{ (S^2 + 2)e^{-\eta^2} + \sqrt{\pi} (S^2 + \frac{5}{2}) \eta[1 + \text{erf}(\eta)] \right\}. \qquad (14)$$

From the kinetic theory of gases [1,2] and the principle of equipartition of energy, the internal energy per molecule is jkT/2, where j is the internal degrees of freedom. The flux of internal energy per unit time and per unit area is given by

$$dE_{i,i} = j \frac{kT}{2} N_i, \qquad (15)$$

where

$$j = \frac{5 - 3\gamma}{\gamma - 1}$$

and N_i is defined by equation (11). From the definition of the accommodation coefficient, equation (5), the heat transfer to the surface element becomes

$$q_f = dE_i - dE_r = a(dE_i - dE_w), \qquad (16)$$

where

$$dE_i = dE_{i,t} + dE_{i,i} \tag{17}$$

and dE_w is the energy transfer from the surface caused by molecules reflecting at the wall temperature with no macroscopic velocity contribution, i.e., $S = 0$ and $T = T_W$. Inspection of equations (11) and (14), while considering the contribution of internal energy, reveals the equation

$$dE_w = \frac{\gamma + 1}{2(\gamma - 1)} mRT_w N_w . \tag{18}$$

Inserting equations (14), (15), and (18) into equation (16) gives the heat transfer to the surface element (Figure 1) caused by freestream incident and reflected molecules on the front side of the surface per unit time and per unit area

$$q_f = a\rho RT \sqrt{\frac{RT}{2\pi}} \left\{ \left(S^2 + \frac{\gamma}{\gamma - 1} - \frac{\gamma + 1}{2(\gamma - 1)} \frac{T_w}{T} \right) \left[e^{-\eta^2} + \sqrt{\pi}\, \eta [1 + \operatorname{erf}(\eta)] \right] - \frac{1}{2} e^{-\eta^2} \right\} . \tag{19}$$

IV. EFFECTS OF CONCAVITY

To determine the effect of the concavity of the surface, it is necessary to represent the fraction of reflecting molecules that re-impinge on the surface and the effect on the mass and energy transfer to the surface. This may be done by assuming a diffuse reflection process which is based on Lambert's cosine law of diffusion and is defined simply as a cosine distribution of radiation or mass flux to all directions of space [6]. More precisely, from Figure 2, it can be seen that from the surface element dA_1, mass is diffusely emitted in the direction OM proportional to the cosine of the angle $\emptyset$ between OM and ON, the normal to dA_1. For concave surfaces, a fraction of molecules reflect or reemit, either diffusely or partially diffusely, from portions of the surface and impinge on other parts of the surface, thereby creating additional contributions to the forces and heat transfer properties of the body. This molecular fraction of reimpinging molecules will be referred to throughout this paper as the angle factor. The angle factors for a concave hemispherical surface at an arbitrary angle of attack can be determined by considering two differential surface elements as shown in Figure 3. Since the diffuse interchange of thermal radiation also applies to the diffuse interchange of mass [6,8], the general relationship for the angle factor derived by Jakob [6] for mass directly incident (normal) to the surface element can be used

$$F_a = \int_{A_2} \frac{\cos \emptyset_1 \cos \emptyset_2}{\pi d^2} dA_2. \tag{20}$$

For concave hemispherical surfaces, it can be easily seen from Figures 4 and 5 that

$$\cos \emptyset_1 = \cos \emptyset_2 = \frac{d}{2r}$$

and

$$dA_2 = r^2 \sin \beta \, d\beta \, d\nu_2.$$

Thus, for one reflection,

$$F_a = \frac{1}{4\pi} \int_0^{\pi} \sin \beta \quad d\beta \int_0^{\nu_o} d\nu_2 = \frac{\nu_o}{2\pi} . \tag{21}$$

As may be shown from Figure 4, $\nu_o = \pi - 2\alpha$. The angle factor for one reflection can thus be expressed as

$$F_a = \frac{1}{2} - \frac{\alpha}{\pi}, \tag{22}$$

where the surface angle of attack, α, is measured in radians. This quantity represents the fraction of energy caused by incident molecules from one reflection that re-impinge on a differential surface element at any point on the hemispherical surface. For a greater number of collisions the angle factor becomes a fraction of the molecules that was previously reflected. The general expression for the angle factors for multi-reflections can thus be written

$$F_a = \left(\frac{1}{2} - \frac{\alpha}{\pi}\right) (1/2)^{j-1} \tag{23}$$

where j = 1,2,3, ... , representing the number of interreflections (one less than the number of collisions).

These angle factors defined by equation (23) represent only the component of energy caused by molecules striking normal to the surface element. The tangential component is not given since, by definition of diffuse emission, reflecting molecules are distributed equally and in opposite directions, thereby cancelling any effect they may have.

V. ANALYSIS OF A CONCAVE HEMISPHERICAL SURFACE

Based on the fundamental concepts as presented in Section III and the concavity effects of Section IV, the force and heat transfer can be determined for a concave hemispherical surface at an arbitrary angle of attack. A completely diffuse reflection process is assumed with perfect thermal accommodation; thus, $\sigma = \sigma' = 1$, which is considered justified by experimental results [4].

A. Force Coefficients

The force coefficients can be determined by integrating the pressure contributions of a surface element over the hemispherical surface. This pressure includes that caused by the freestream molecular flux as defined by equation (12), plus contributions of reflecting molecules. For clarity, the pressure distribution over the surface can be divided into two regimes (see Figure 4). The area of direct incidence is defined by $0 \leqq \nu \leqq \nu_o$ and is termed as region I, while the area receiving only reflected molecules is defined by $\nu_o \leqq \nu \leqq \pi$ and is termed as region II. The dividing line is a semi-circle and can be easily shown to be at the angle

$$\nu_o = \pi - 2\alpha, \tag{24}$$

where the surface angle of attack, α, appears only in the xy-plane as shown in Figure 1. No generality is lost in this assumption since the surface is axisymmetric. The pressure at any point in region I can be written as

$$P_I = P_f + 2\left(\frac{1}{2} - \frac{\alpha}{\pi}\right) P_r \sum_{j=1}^{\infty} (1/2)^{j-1}, \tag{25}$$

where $P_r = P_w$, since $\sigma' = 1$. The value of P_w is defined by equation (10). It can be shown that the infinite series in equation (25) converges, and that the sum of the series converges to

$$\sum_{j=1}^{\infty} (1/2)^{j-1} = 2.$$

Equation (25) can thus be written as

$$P_I = P_f + 2\left(\frac{1}{2} - \frac{\alpha}{\pi}\right) m\sqrt{2\pi RT_w}\, N_i, \tag{26}$$

where N_i and P_f are defined by equations (11) and (12), respectively. Similarly, the pressure at any point in region II can be determined to be the same as equation (26) minus the freestream contribution. The pressure on the surface in region II can thus be written

$$P_{II} = 2\left(\frac{1}{2} - \frac{\alpha}{\pi}\right) m\sqrt{2\pi RT_w}\, N_i. \tag{27}$$

The differential force on the surface in the direction of flow measured by the angle of attack α can now be determined. The derivation is facilitated by reference to Figure 4. This force includes the contributions of both pressure and shear stresses plus the shading effect due to angle of attack.

$$dD = (P_I \sin\theta\, dA_I - \tau_f \cos\theta\, dA_I + P_{II} \sin\theta\, dA_{II}) \sin\beta \tag{28}$$

where P_I and τ_f are defined by equations (26) and (13), respectively, and where P_{II} is defined by equation (27). Also required are the differential surface areas, which are shown to be

$$dA_I = dA_{II} = r^2 d\beta \sin\beta\, d\nu. \tag{29}$$

The element angle of attack is defined as $\theta = \pi - \alpha - \nu$. The total drag, or force, in the direction of flow can be determined by integrating equation (28) over the hemispherical surface.

$$D = \int_0^{\pi} \int_0^{\nu_o} \left[P_f + 2\left(\frac{1}{2} - \frac{\alpha}{\pi}\right) m \sqrt{2\pi R T_w}\, N_i \right] \sin\theta\, r^2 \sin^2\beta\, d\nu d\beta$$

$$- \int_0^{\pi} \int_0^{\nu_o} \tau_f \cos\theta\, r^2 \sin^2\beta\, d\nu d\beta$$

$$+ \int_0^{\pi} \int_{\nu_o}^{\pi} P_{II} \sin\theta\, r^2 \sin^2\beta\, d\nu d\beta \tag{30}$$

As previously defined the values of P_f, N_i, τ_f and P_{II} are shown to be a function of the element angle of attack, θ. The appropriate substitution into equation (30) can thus be made for ν, changing the limits of integration. Furthermore, by inserting equations (11), (12), (13), and (27) into the above equation, where $\sigma' = \sigma = 1$, the drag equation becomes

$$D = \frac{\rho R T \pi r^2}{2} \int_{\alpha}^{\pi-\alpha} [g(\theta) + h(\theta)] \sin\theta\, d\theta + \frac{\rho R T \pi r^2}{2} \int_{\alpha}^{\pi-\alpha} f(\theta) \cos^2\theta\, d\theta$$

$$+ \frac{\rho R T \pi r^2}{2} \left(\frac{1}{2} - \frac{\alpha}{\pi}\right) \sqrt{T_w/T} \int_{-\alpha}^{\pi-\alpha} \sin\theta\, d\theta, \tag{31}$$

where

$$g(\theta) = \left[\frac{1}{\sqrt{\pi}} S \sin\theta + \frac{1}{2}\sqrt{T_w/T}\right] e^{-S^2 \sin^2\theta} \tag{32}$$

$$h(\theta) = [1 + \mathrm{erf}(S \sin\theta)]\left[\frac{1}{2} + S^2 \sin^2\theta + \frac{S \sin\theta}{2}\sqrt{\pi T_w/T}\right] \tag{33}$$

$$f(\theta) = e^{-S^2\sin^2\theta} + \sqrt{\pi}\, S \sin\theta\, [1 + \mathrm{erf}(S \sin\theta)]. \tag{34}$$

Similarly, the lift, or force normal to the direction of flow can be determined. Utilizing Figure 4, the differential lift can be determined as

$$dL = (P_I \cos\theta\, dA_I + \tau_f \sin\theta\, dA_I + P_{II} \cos\theta\, dA_{II}) \sin\beta \tag{35}$$

where P_I, τ_f, and P_{II} retain the same definitions. A similar analogue can be applied for the lift as was applied for the drag, yielding the following equation

$$L = \int_0^{\pi}\int_0^{\nu_0} \tau_f \sin\theta\, r^2 \sin^2\beta\, d\nu d\beta$$

$$+ \int_0^{\pi}\int_0^{\nu_0} \left[P_f + 2\left(\frac{1}{2} - \frac{\alpha}{\pi}\right) m \sqrt{2\pi R T_w}\, N_i\right] \cos\theta\, r^2 \sin^2\beta\, d\nu d\beta$$

$$+ \int_0^{\pi}\int_{\nu_0}^{\pi} P_{II} \cos\theta\, r^2 \sin^2\beta\, d\nu d\beta. \tag{36}$$

By making the same substitutions as was previously done for the drag equation the resulting lift equation becomes

$$L = \frac{\rho RT\pi r^2}{2}\int_{\alpha}^{\pi-\alpha}[g(\theta)+h(\theta)]\cos\theta\, d\theta - \frac{\rho RT\pi r^2}{2}\int_{\alpha}^{\pi-\alpha} f(\theta)\sin\theta\cos\theta\, d\theta$$

$$+ \frac{\rho RT\pi r^2}{2}\left(\frac{1}{2}-\frac{\alpha}{\pi}\right)\sqrt{T_w/T}\int_{-\alpha}^{\pi-\alpha} f(\theta)\cos\theta\, d\theta. \qquad (37)$$

The force coefficients C_D and C_L for the hemisphere can thus be written

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A_r} = \frac{1}{2S^2}\int_{\alpha}^{\pi-\alpha}[g(\theta)+h(\theta)]\sin\theta\, d\theta$$

$$+ \frac{1}{\sqrt{\pi}S}\int_{\alpha}^{\pi-\alpha} f(\theta)\cos^2\theta\, d\theta + \frac{1}{S^2}\left(\frac{1}{2}-\frac{\alpha}{\pi}\right)\sqrt{T_w/T}\int_{-\alpha}^{\pi-\alpha}\sin\theta\, d\theta$$

$$(38)$$

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A_r} = \frac{1}{2S^2}\int_{\alpha}^{\pi-\alpha}[g(\theta)+h(\theta)]\cos\theta\, d\theta$$

$$- \frac{1}{\sqrt{\pi}S}\int_{\alpha}^{\pi-\alpha} f(\theta)\sin\theta\cos\theta\, d\theta - \frac{1}{S^2}\left(\frac{1}{2}-\frac{\alpha}{\pi}\right)\sqrt{T_w/T}\int_{-\alpha}^{\pi-\alpha}\cos\theta\, d\theta$$

$$(39)$$

B. Heat Transfer Coefficients

The heat transfer properties considered here for the concave hemispherical surface is based on the general theory of freestream convective heat transfer in free molecule flow presented in the paper by Oppenheim [6], plus the contributions due to reflected molecules. The heat transfer to a differential surface element due to the incident flux is defined by equation (19). The heat transfer due to reflected molecules is determined by using the angle factors of Section IV where the energy impinging on a differential element is equal to molecular energy emitted from the rest of the surface times the appropriate angle factor. For example, if dE_r leaves element dA_1, then the incident amount that is transferred to dA_2 is $F_a dE_r$. The initial heat transfer caused by the incident freestream and reflected molecules is defined by equation (16) and may be rewritten here as

$$q_f = dE_i - dE_r = a[dE_i - dE_w].$$

The contribution of heat transfer to the surface by the molecules incident after one collision may be expressed as

$$q_1 = \left(\frac{1}{2} - \frac{\alpha}{\pi}\right)(dE_r - dE'_r) = \left(\frac{1}{2} - \frac{\alpha}{\pi}\right)(dE_r - dE_w)a. \tag{40}$$

Successive contributions of the heat transfer for multi-collisions greater than one are

$$q_j = \left(\frac{1}{2} - \frac{\alpha}{\pi}\right)(1/2)^{j-1}(dE_r^{j-1} - dE_r^j) = a\left(\frac{1}{2} - \frac{\alpha}{\pi}\right)(1/2)^{j-1}(dE_r^{j-1} - dE_w) \tag{41}$$

where $j = 2,3,4, \ldots, j_o$, and represents the number of interreflections. The superscript numbers are used to define the energy carried away by the molecules caused by the j^{th} interreflection. Desired is the total heat transfer to a differential surface element caused by an infinite number of interreflections, or a finite number that fully represent the effects of concavity as a function of dE_i and dE_w. By assuming a finite number of interreflections, the resulting simultaneous equations can be solved, producing the heat transfer equation for the incident and each interreflected flux of molecules. A summation of these contributions produce the total heat transfer to the surface element, and may be expressed as

$$q = q_f + \sum_{j=1}^{j_o} q_j \tag{42}$$

where q_f is defined by equation (19), and q_1 is obtained from equations (16) and (43).

$$q_1 = q_f(1 - a)\left(\frac{1}{2} - \frac{\alpha}{\pi}\right). \tag{43}$$

The additional contributions of the heat transfer for integer values of $j \geqq 2$ is similarly obtained and may be written

$$q_j = q_f(1 - a)^j \, (1/2)^{j-1} \left(\frac{1}{2} - \frac{\alpha}{\pi}\right)$$

where $j = 2, 3, 4, \ldots, j_o$. The total heat transfer to a differential surface element in region I is obtained by inserting equations (43) and (44) into equation (42)

$$q_I = q_f \left[1 + \left(\frac{1}{2} - \frac{\alpha}{\pi}\right) \sum_{j=1}^{j_o} (1 - a)^j \, (1/2)^{j-1} \right]. \tag{45}$$

The heat transfer to a differential surface element in region II is obtained similarly by excluding the freestream contribution.

$$q_{II} = \left(\frac{1}{2} - \frac{\alpha}{\pi}\right) \sum_{j=1}^{j_o} (1 - a)^j \, (1/2)^{j-1}. \tag{46}$$

The average heat transfer to the surface is thus obtained by integrating equations (45) and (46) over the surface

$$\bar{Q} = \frac{1}{A} \int_I q_I \, dA_I + \frac{1}{A} \int_{II} q_{II} \, dA_{II}. \tag{47}$$

Values of the accommodation coefficient determined experimentally for air on various surfaces are shown to be between 0.88 and 0.94 [4]. It is therefore considered justified to assume that a = 1. For this assumption, the average heat transfer equation reduces to

$$\bar{Q} = \frac{1}{A} \int_I q_f \, dA_I \tag{48}$$

where A is the surface area, and q_f is defined by equation (19). By inserting these relationships into equation (48), the average heat transfer equation becomes

$$\bar{Q} = \frac{\rho RT}{\pi}\sqrt{RT/2\pi}\left[S^2 + \frac{\gamma}{\gamma - 1} - \frac{\gamma + 1}{2(\gamma - 1)}\frac{T_w}{T}\right]\int_{\alpha}^{\pi-\alpha} f(\theta)\, d\theta$$

$$+ \frac{\rho RT}{2\pi}\sqrt{RT/2\pi}\int_{\alpha}^{\pi-\alpha} e^{-S^2\sin^2\theta}\, d\theta \qquad (49)$$

where γ is the ratio of specific heats and a function of the type of gas (for air, $\gamma = 7/5$), and $f(\theta)$ is defined by equation (34). For convenience, equation (49) may be made dimensionless by dividing by $\rho(2RT)^{3/2}$. The first equation for the heat transfer coefficient may then be written

$$C_q = \frac{1}{4\pi^{3/2}}\left[S^2 + \frac{\gamma}{\gamma - 1} - \frac{\gamma + 1}{2(\gamma - 1)} \cdot \frac{T_w}{T}\right]\int_{\alpha}^{\pi-\alpha} f(\theta)\, d\theta$$

$$+ \frac{1}{8\pi^{3/2}}\int_{\alpha}^{\pi-\alpha} e^{-S^2\sin^2\theta}\, d\theta. \qquad (50)$$

VI. CONCLUSIONS

The analysis performed here can be used to determine the aerodynamic forces and average heat transfer to a concave hemispherical surface in free molecule flow. The resulting equations, although not in closed form, can be integrated numerically using standard computer techniques. As shown in the analysis, the drag and lift coefficients are good for an infinite number of collisions where the average heat transfer equation is independent of the number of interreflections.

The resulting equations are general in that they are a function of surface velocity, ambient and wall temperatures, type of gas, and angle of attack. The equations are limited to high molecular speed ratios where molecules impinge only on the concave surface.

The drag, lift, and heat transfer coefficients have been obtained by solving the resulting equations numerically for some typical values of the molecular speed ratio and surface-to-ambient-temperature ratio and plotted as a function of angle of attack (figures 6 through 14). As shown, the average heat transfer coefficient is independent of the concavity. Maximum values of the heat transfer and drag occur at zero angle of attack, decreasing with increasing angle of attack to values of zero at ninety degrees angle of attack. By comparison, the drag for the concave hemisphere is shown to be slightly higher (about one per cent) than the drag for a convex hemisphere. The total lift is shown to be very small and would be zero except for interreflected molecules.

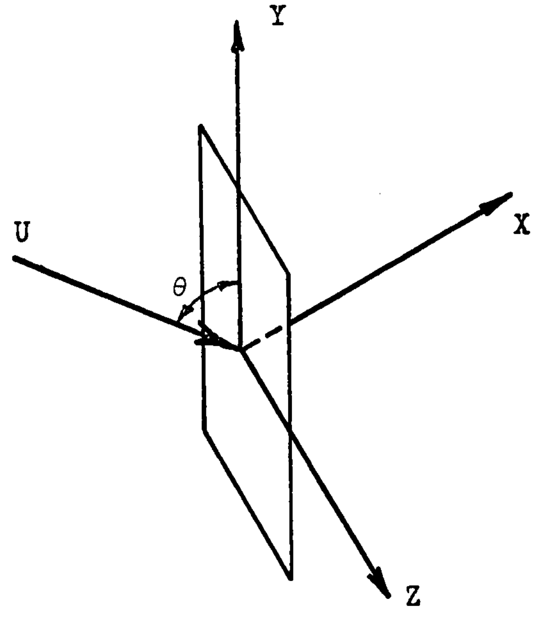

Figure 1 - Cartesian Coordinates of a Differential Surface Element

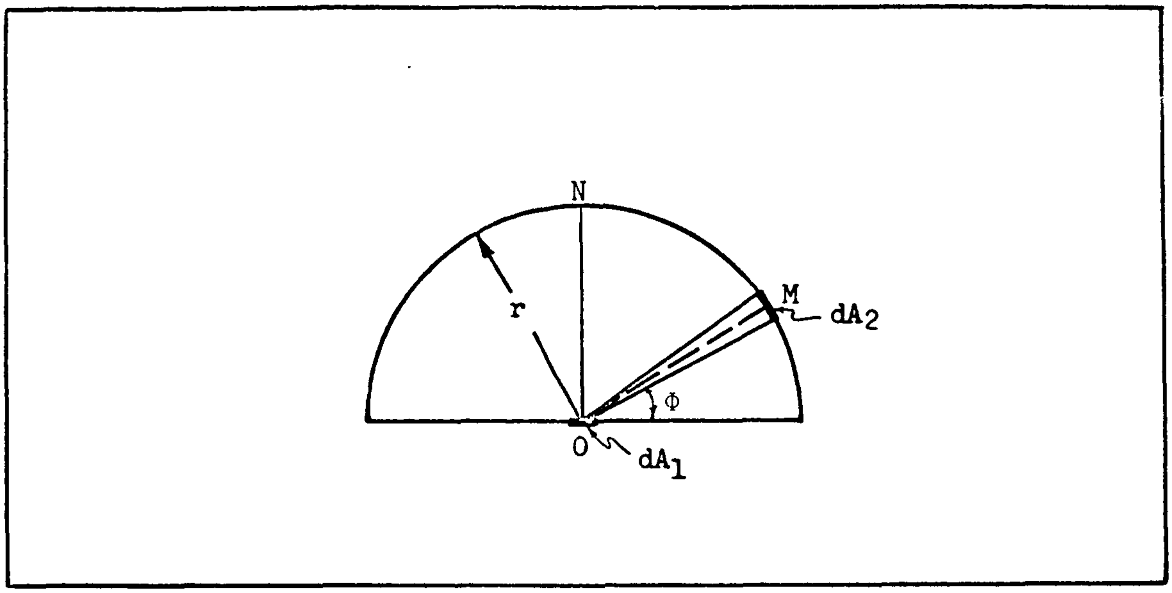

Figure 2 - Illustration of Lambert's Cosine Law of Diffusion

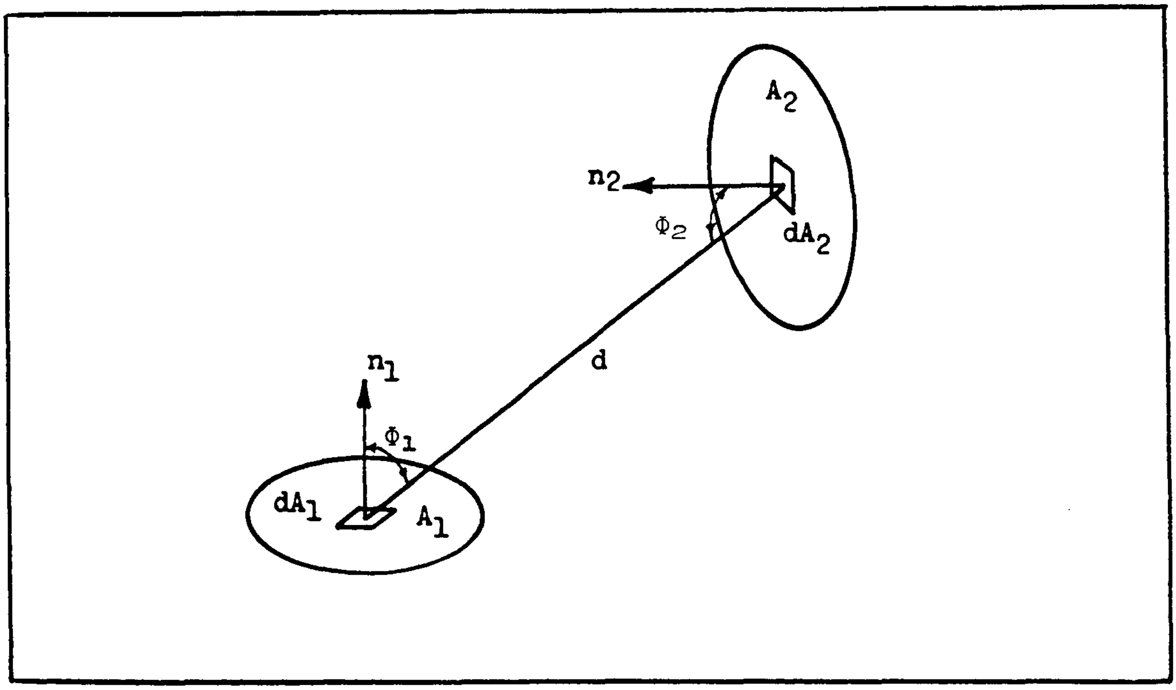

Figure 3 - Diffuse Emission Between Two Surface Elements of Arbitrary Orientation

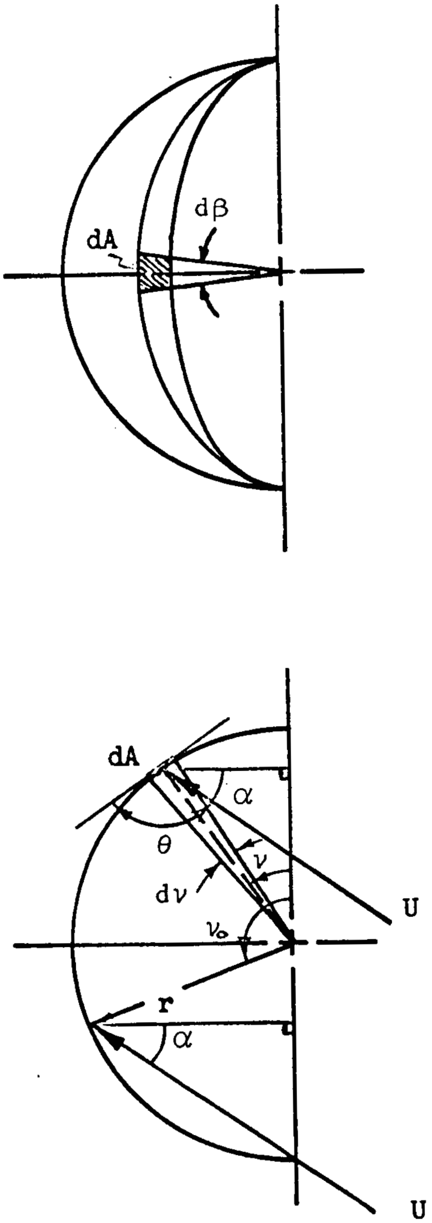

Figure 4 - Angular and Regional Definitions of Hemisphere

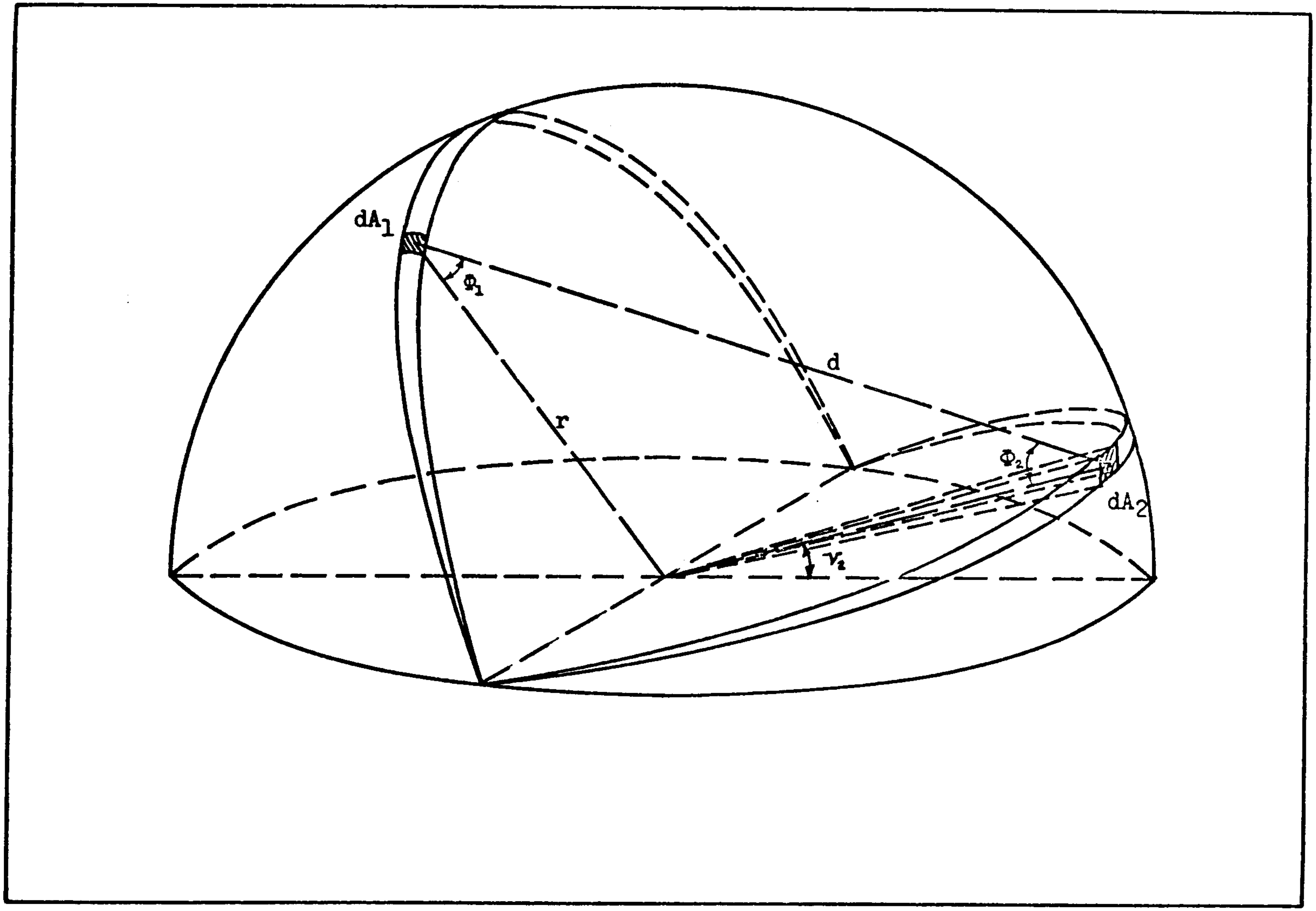

Figure 5 - Diffuse Emission Inside Hemisphere

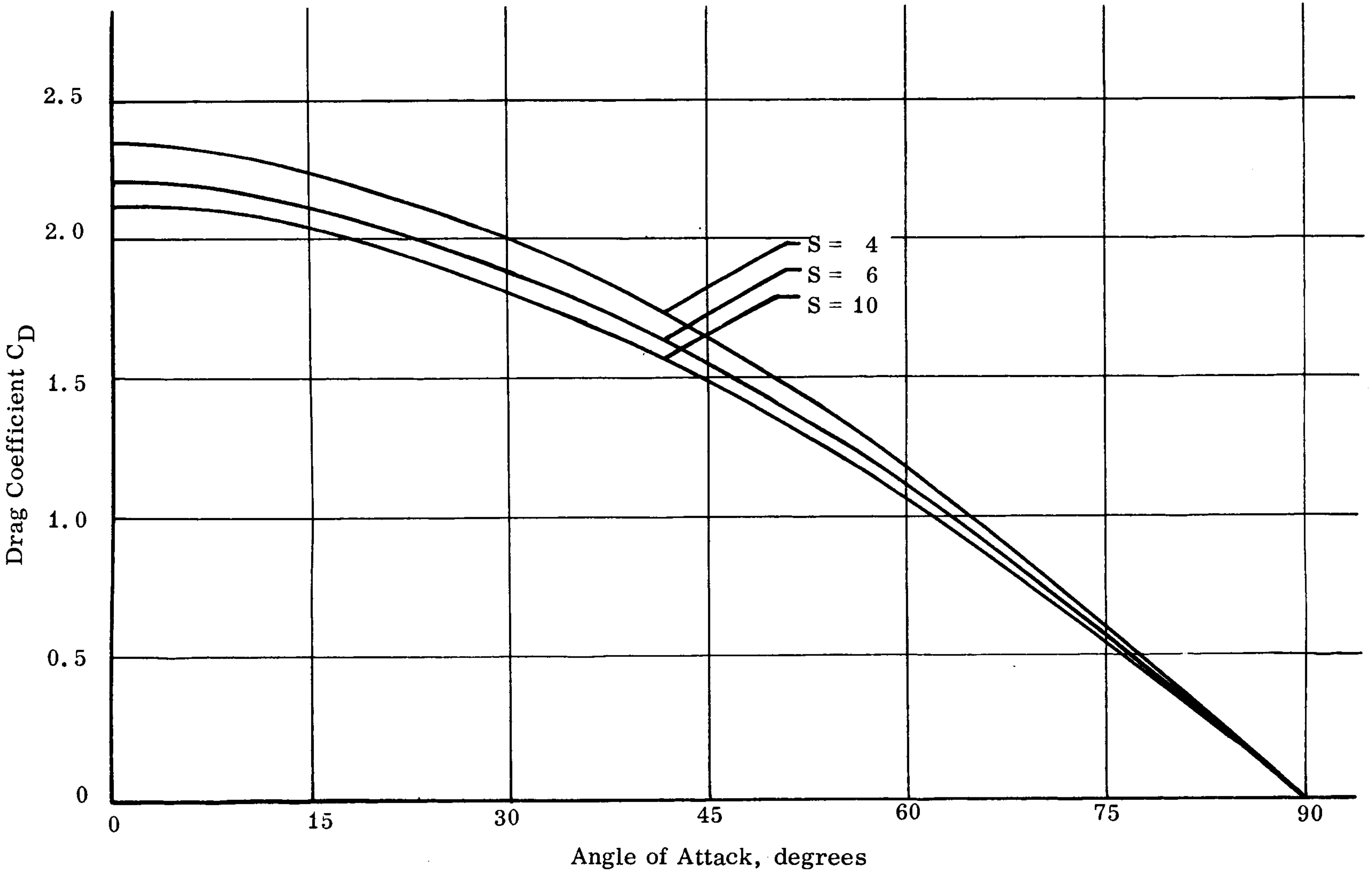

Figure 6. Drag Coefficient, $T_W/T = 0.4$

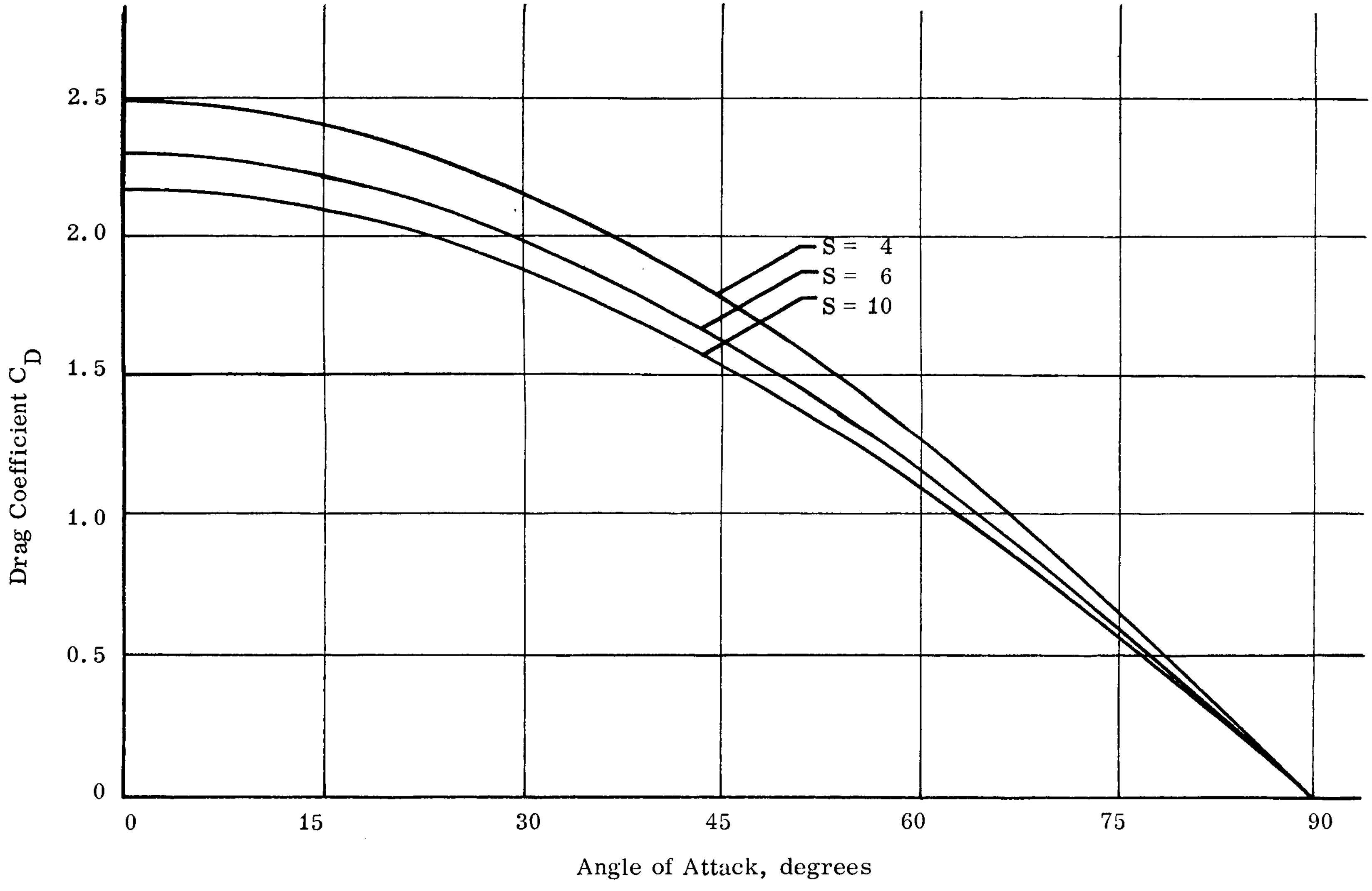

Figure 7. Drag Coefficient, $T_W/T = 1.0$

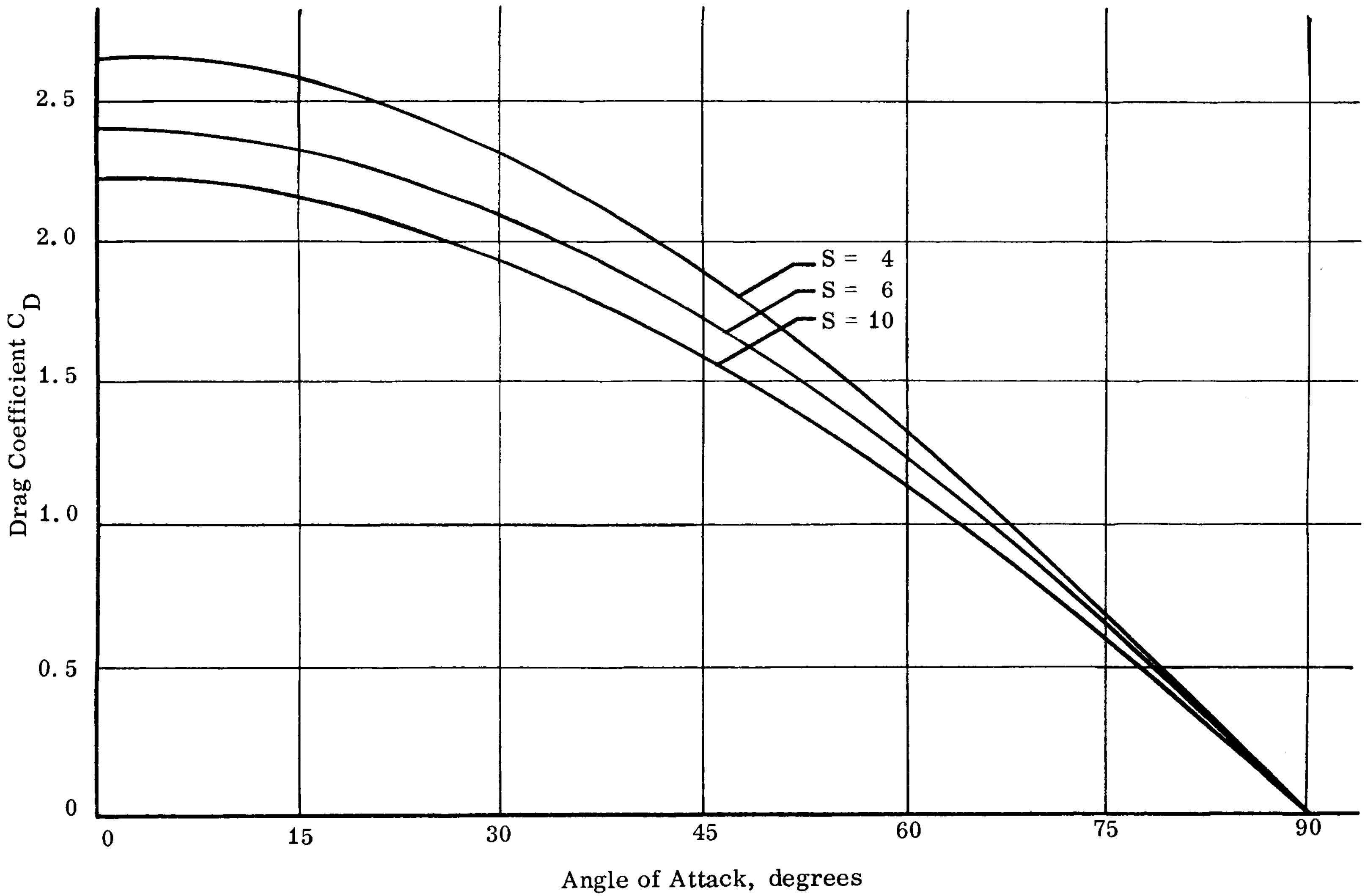

Figure 8. Drag Coefficient, $T_w/T = 2.0$

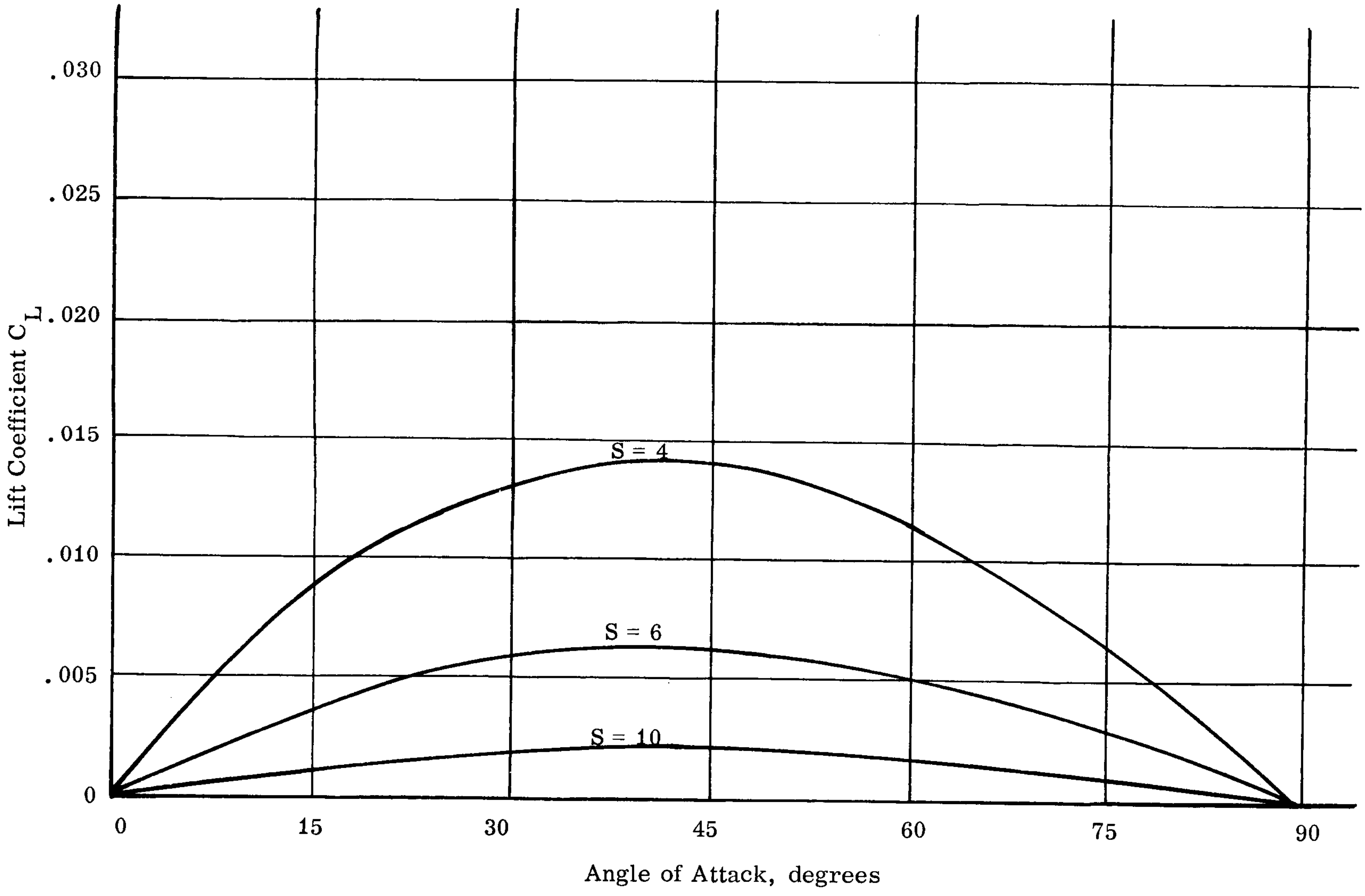

Figure 9. Lift Coefficient, $T_w/T = 0.4$

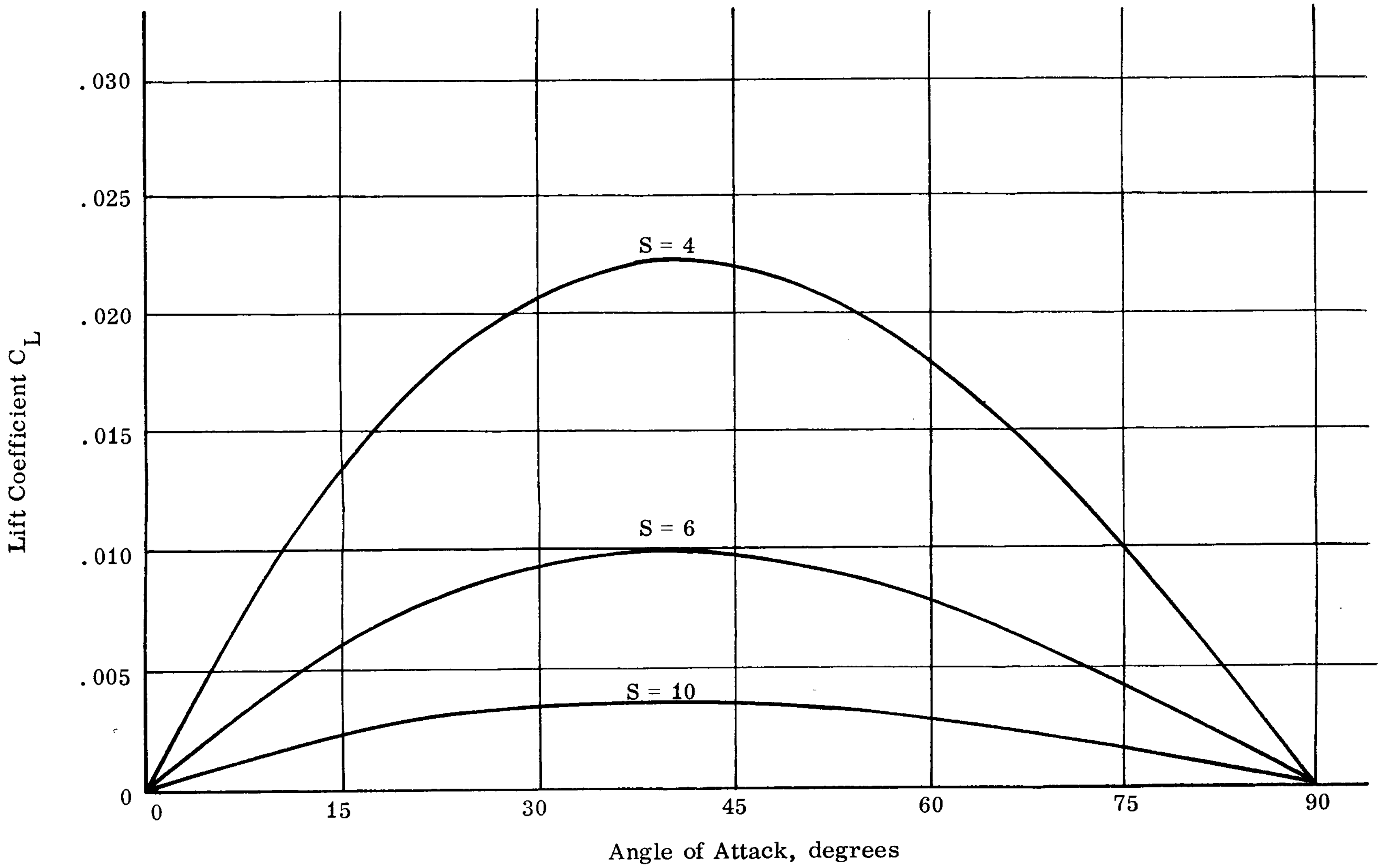

Figure 10. Lift Coefficient, $T_w/T = 1.0$

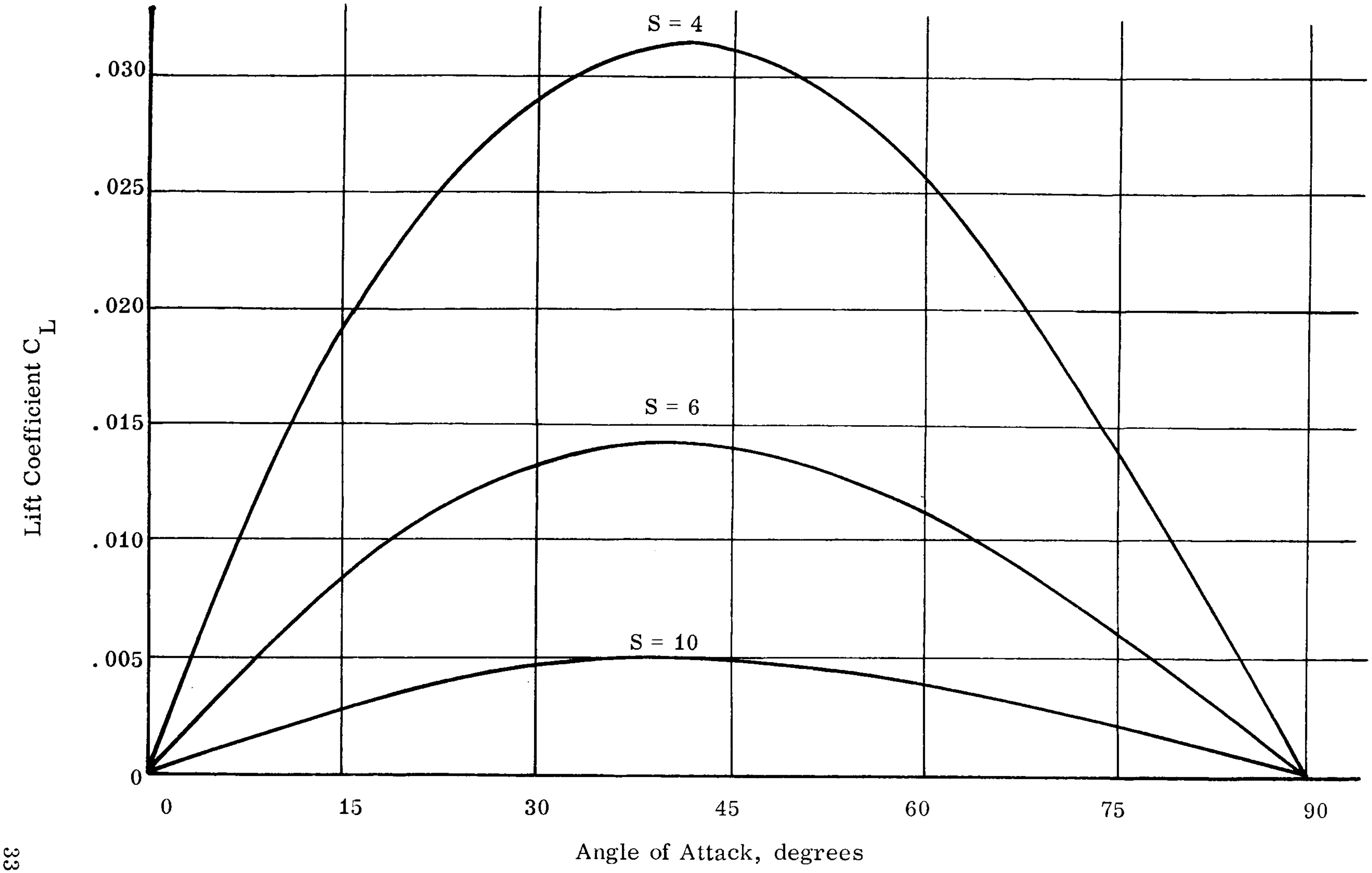

Figure 11. Lift Coefficient, $T_w/T = 2.0$

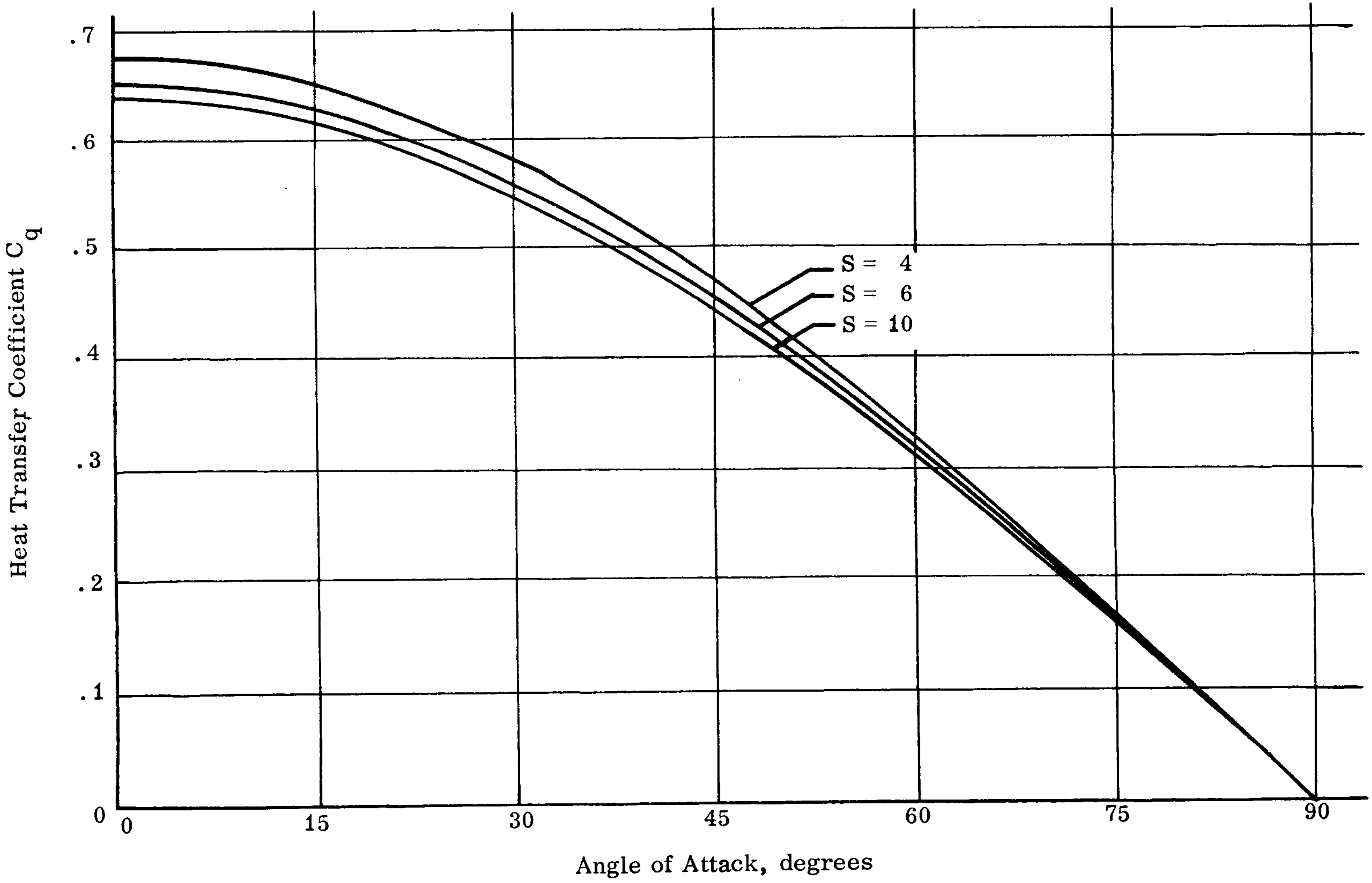

Figure 12. Average Heat Transfer Coefficient, $T_w/T = 0.4$

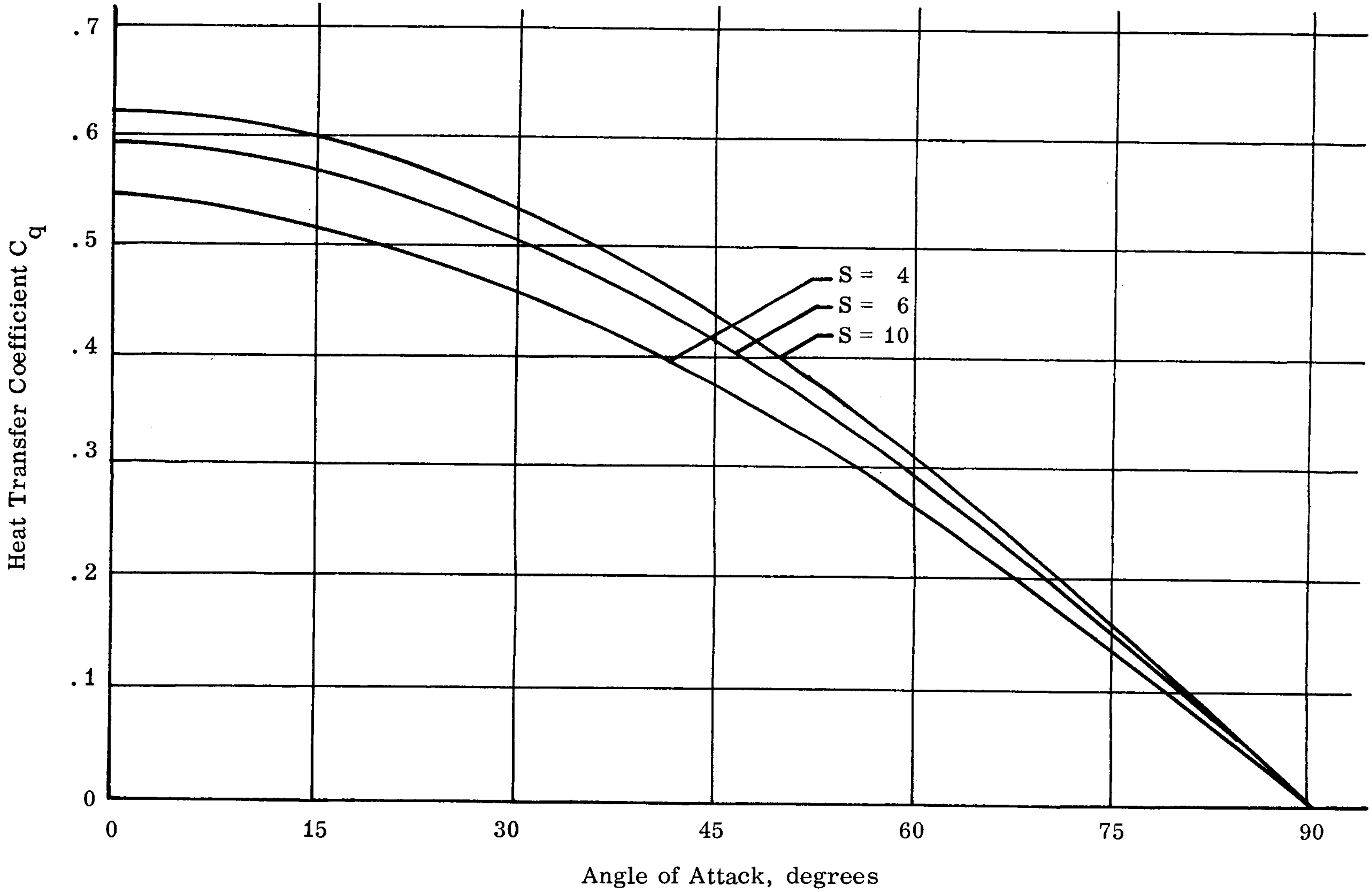

Figure 13. Average Heat Transfer Coefficient, $T_w/T = 1.0$

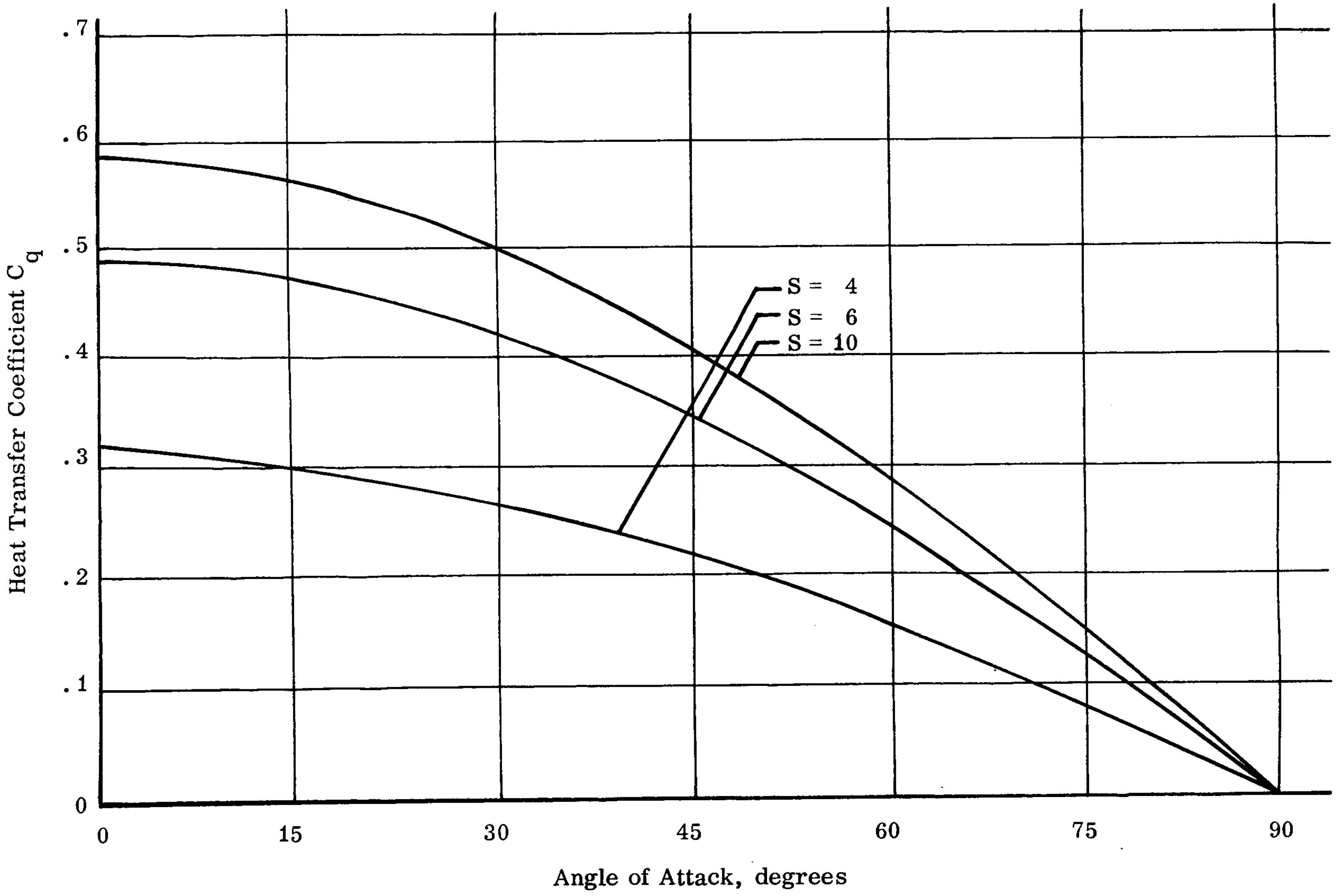

Figure 14. Average Heat Transfer Coefficient, $T_w/T = 2.0$

BIBLIOGRAPHY

1. Jeans, James, An Introduction to the Kinetic Theory of Gases, Cambridge University Press, 1962.

2. Loeb, L. B., The Kinetic Theory of Gases, Dover Publications, Inc., New York, Third Edition, 1961.

3. Patterson, G. N., The Molecular Flow of Gases, John Wiley and Sons, New York, 1956.

4. Schaaf, S. A., and P. L. Chambre, "Flow of Rarefied Gases," in Fundamentals of Gas Dynamics (H. W. Emmons, Ed.), Sect. H, pp. 687-739, Princeton University Press, Princeton, New Jersey, 1958.

5. Oppenheim, A. K., "Generalized Theory of Convective Heat Transfer in a Free-Molecule Flow," University of California, Berkely, Calif., 1952.

6. Jakob, Max, Heat Transfer, Volume II, John Wiley and Sons, Inc., 1949.

7. Chahine, Monstafa T., "Free Molecule Flow over Non-convex Surfaces," Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif.

8. Sparrow, E. M., V. K. Jansson, T. S. Lundgren, and T. S. Chen, "Heat Transfer and Forces for Free-Molecule Flow on a Concave Cylindrical Surface," Journal of Heat Transfer, February, 1964.

9. Pratt, M. J., "The Free-Molecule Flow Characteristics of Concave Surfaces," The College of Aeronautics, Cranfield, Note No. 155, June, 1963.

10. U. S. Standard Atmosphere, 1962, prepared under sponsorship of the National Aeronautics and Space Administration, United States Air Force, and the United States Weather Bureau, Washington, D. C., December 1962.

APPROVAL

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DETERMINATION OF AERODYNAMIC FORCE AND HEAT TRANSFER PROPERTIES FOR A CONCAVE HEMISPHERICAL SURFACE IN FREE MOLECULE FLOW

by Clarence Ray Wimberly

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E. L. Linsley

Edward L. Linsley
Chief, Advanced Vehicle Aerodynamics Office

W Dahm

Werner K. Dahm
Chief, Aerodynamics Division

E. D. Geissler
Director, Aero-Astrodynamics Laboratory

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Houston, Texas 77058

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Advanced Spacecraft & Technology Div.
Manned Spacecraft Center
Houston, Texas 77058

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P. O. Box 273

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Director, Jet Propulsion Laboratory
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