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Attrition of Graduate Engineers

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In order to make projections of the future supply of graduate engineers it is necessary to project the number of engineering graduates and attrition from the existing stock of engineers. The attrition depends on the population exposed to attrition and the rate of attrition. It is well known that occupational mobility and mortality rates vary with age. To estimate attrition we should ideally prefer to know the detailed composition by age of the engineering population and the attrition rates of each of the age groups. It is possible, however, to estimate attrition rates that apply to the entire population. This is the objective of this paper. We shall discuss several methods of estimating the overall attrition rate. The same methods can be applied to the various engineering specialties.

Attrition among engineering graduates tends to be high during the first years after graduation. Of June, 1951 engineering graduates, only 61 percent were working as engineers one year later, or 79 percent of those engineering graduates that were working in civilian jobs (Table 1). Of June, 1958 engineering graduates 60 percent were working as engineers two years later, or 80 percent of engineering graduates that were working in civilian jobs. The percentage of graduates in the various specialties working in the same specialty varied considerably with rapidly growing specialties such as aeronautical and electrical engineering showing the lowest rates of attrition. There appears to be a high concordance of rank between the one and two year attrition rates

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Table 1

One Year, Two Years, and Overall Attrition Rates, by Engineering Specialty

Engineering specialty	Percent of graduates not working as engineers in same specialty: One year after graduation (1952)	Two years after graduation (1960)	Estimated overall attrition rate
All engineers	39.2	34.7	3.0
Aeronautical	29.8	----	-5.1
Chemical	40.8	58.0	5.8
Civil	37.4	38.3	3.8
Electrical	26.0	29.0	3.3
Mechanical	41.9	45.5	6.4
Mining	----	63.1	----
Industrial	----	71.1	-1.2

Sources: Hugh Folk, "The Supply of Engineers and Scientists," Department of Economics, Washington University, Working Paper 6506, June, 1965, derived from National Science Foundation data and Table 2 below.

and the overall attrition rate (which are estimated below). The pattern of attrition shown by one-year, two-year, and overall attrition rates is one of attrition declining with increasing age.

The pattern of attrition is the result of voluntary movement of engineering graduates primarily into business occupations with relatively few entering science (see Folk [2]). The movement is termed voluntary because during the entire period of 1950-1960 the market for engineers as a whole had numerous job vacancies(Folk, [1]). Very few indeed were the graduates that wished to work as an engineer that could not find jobs as engineers. The attrition rate is highest among mechanical engineering graduates, many of whom work as aeronautical or industrial engineering which are in many applications considered to be subspecialties of mechanical engineering. The next highest attrition rate is that of chemical engineering. The attrition rates of aeronautical and industrial engineering are actually negative, i.e., there is an inflow of graduates from other specialties that outnumbers those aeronautical and industrial engineers that leave the occupations by death or mobility. This is to say that aeronautical and industrial engineering make a large net claim on other academic specialties (some of which are nonengineers, especially in the case of industrial engineers.)

The estimates of attrition presented in this paper exceed by a considerable amount the estimates mentioned, and presumably used, by the Bureau of Labor Statistics for the National Science Foundation [5] and [6]. The BLS estimated attrition owing to death and retirement from Wolfbein's "Tables of Working Life" [7] for white urban males which implied a decade separation rate of 15.7 percent. Attrition among new B.S. graduates was not allowed for, since 85.6 percent of bachelor's degrees during period, 86.1 percent of the masters degrees, and all of the doctor's degrees were considered net additions during the period. No allowance was made for transfer from the occupation to nontechnical jobs, but, according to the BLS, "losses from this source are believed to be minor relative to total scientific and engineering personnel needs" ([6,] p. 31).

II. Estimating Attrition Rates

The method adopted for estimating the attrition rates assumes that the stock of employed engineers in 1950, Eo, grows at a gross compound rate γ and is subject to attrition at a compound rate α over the period 1950-60, thus,

$$E_{60} = E_{50} (1 + \gamma)^{10} (1 - \alpha)^{10} \quad (1)$$

The net rate of change, ρ, is

$$E_{60} = E_{50} (1 + \rho)^{\tau} \quad (2)$$

So that

$$(1 + \rho) = (1 + \gamma - \alpha - \gamma\alpha) \quad (3)$$

Thus if there was no attrition in the period, ρ would equal γ . If there was no growth over the period, ρ would equal α .

It is possible to compute ρ by the formula

$$\rho = \frac{1}{\tau} \text{ antilog of } (\log E_{60} - \log E_{50})$$

To find γ and α an additional condition relating γ to α must be found. We will examine first "exact" methods, which are time consuming, and then some approximate methods. 1/

First consider that, in general, employment in one period plus the growth increment in that period minus the attrition decrement in that period is equal to employment in the next period.

$$E_t + G_t - A_t = E_{t + 1} \quad (5)$$

1/ For other examples, see Folk and Yett [3].

Most of the growth increment of graduate engineers, the graduating class (G_t), enter the labor market in June. Although there will be some stragglers that finish work for their degrees each semester and quarter. If we can estimate engineering employment as of, or shortly before, the June graduation date, we can compute a gross growth rate for the period, γ_t, on the base E_t, such that

$$E_t (1 + \gamma_1) = E_t + G_t \tag{6}$$

During the following year, the number of engineers ($E_t + G_t$) is subject to attrition at rate α_t, such that

$$(E_t + G_t) \; (1 - \alpha_t) = E_{t=1} \tag{7}$$

By specifying that the growth increment is subject to attrition, but that the attrition decrement does not reduce the employment on which the growth process operates during year t, we make a descriptively accurate and computationally convenient specification.[2/]

Since we have employment figures for graduate engineers as of **April 1**, in 1950 and 1960 and the number of graduates in classes of June, 1950 through June, 1959, we can compute the average attrition rate as follows:

[2/] It could alternatively be assumed that (1) neither growth increments were subject to attrition or attrition decrements subject to growth; (2) attrition occurs over the year and growth at the end of the year; or (3) growth and attrition occur simultaneously. Our specification describe the engineering growth process better than any of these alternative assumptions.

$$E_{50} + G_{50} - A_{50} = E_{51} \tag{8}$$

By (7),

$$(E_{50} + G_{50})(1-\alpha_o) = E_{51} \tag{9}$$

E_{52} can be derived in the same way

$$\{(E_{50} + G_{50})\ (1-\alpha_0) + G_{51}\}(1-\alpha_1) = E_{52} \tag{10}$$

and so on, until

$$(E_{50} + G_{50})\ \left(\prod_{t=50}^{59} (1-\alpha_t)\right) + \sum_{t=51}^{59} G_t \prod_{k=t}^{59} (1-\alpha_k)) = E_{60} \tag{11}$$

Let us assume that the expected value of the attrition rate α_t is constant in the long run with constant variance then the maximum likelihood estimator of $(1-\alpha)$ is

$$\delta = (1-\hat{\alpha}) = \sqrt[10]{\prod_{t=50}^{59} (1-\alpha_t)} \tag{12}$$

Substituting δ for the $(1-\alpha_t)$ in (11),

we obtain a 10th degree polynomial in δ

$$(E_{50} + G_{50})\ \delta^{10} + G_{51}\delta^{9} + \ldots G_{58}\delta^{2} + G_{59}\delta - E_{60} = 0 \tag{13}$$

By Descartes' Rule of Signs we know that (13) has at most one positive real root. This is readily approximated by numerical methods either with a desk calculator or by a high-speed digital computor. Because all coefficients but one are positive, the positive real root may be obtained by a simple trial-and-error method that converges quickly:

(1) Let δ be the first trial root, calculate δ, δ^2, ... δ^{10}. In doing this it is necessary to carry each multiplication to the greatest degree of accuracy possible.

(2) Calculate $(E_o + 60)\ \delta^{10}$. Obviously if $(E_{50} + G_{50})\ \delta^{10} > E_{60}$, δ_1 is too large and it is not necessary to compute the partial sum $(E_{50} + G_{50})\delta_1^{10}$ $+ G_{51}\ \delta_1^{9} + \ldots + G_{50}\ \delta^{2} + G_{59}\ \delta = S_1$, if $(E_{50} + G_{50})\ \delta^{10} < E_{60}$ compute S_1 and then $S_1 - E_{60} = \Delta_1$. If $\Delta_1 > 0$, select $\delta_2 < \delta_1$, if $\Delta_1 < 0$, select $\delta_2 > \delta_1$ . .

The difference between δ_1 and δ_2 should depend on the size of Δ_1.

Now compute $(E_{50} + G_{50})\ \delta_2^{10}$. If $(S_1 - (E_{50} + G_{50})\ \delta_2^{10}) < \Delta_1$ then choose $\delta_3 < \delta_2$ and you need not compute δ_2. Now compute S_3. If $S_3 - E_{60} = \Delta_3 > 0$, S_3 is too large, and so forth.

Repeat steps (2) and (3) as necessary. The computations converge quite rapidly, and it is possible to obtain δ to four or five decimal places in 10 or 12 iterations. Increasing experience is useful in judging by how much trial roots should be increased or decreased.

Trial roots may be readily chosen from the results of the approximate methods given below.

<u>Approximate methods</u>. While extraction of the root of our special 10th degree polynomial is not very difficult, it is time consuming. Several approximate methods of estimating attrition have been proposed.

(1) Meyer [4] proposes simultaneous solution of the equations:

$$E_0\ (1 + g - a)^{\tau} = E_{\tau} \tag{14}$$

$$\frac{g}{a} = \frac{\sum_{t=0}^{\tau} G_t + E_0}{E_0 + \sum_{t=0}^{\tau} G_t - E_{\tau}} \tag{15}$$

Equation (14) is a simplification of our equation (1). It assumes that there is no interaction of growth (g) and attrition (a) during a single period.

Equation (15) asserts that the ratio of the gross growth rate to the attrition rate is equal to the ratio of the gross gain (i.e. graduations) over the period to the loss over the period ($E_0 + \Sigma G_t$ is the total number in the occupation in the whole period.) Underlying (15) is the assertion that $\tau g E_0 = \sum_{t=1}^{\tau} G_t$ and $\tau\, a\, E_0 = E_0 + \sum_{t=1}^{\tau} G_t - E_\tau$. Both of these expressions ignore compounding by use of simple interest τg and τa (and thereby bias both rates estimated upward), ignore attrition during the period (thereby biasing g downward since the base is too large during the period), and ignore growth during the period thereby biasing the attrition rate upward since the base is consistently too large during the period. $g > \gamma$ and $a > \alpha$, but it cannot be said if $\frac{g}{a} \gtrless \frac{\gamma}{\alpha}$. The use of equation (14) instead on (1) also biases the difference of the rates downward.

(2) Another method is to use

$$E_0\,(1 + g - a - ga)^\tau = E_\tau \tag{16}$$

$$E_0\,(1 + g)^\tau = E_0 + \sum_{t=0}^{\tau} G_t \tag{17}$$

Here the gross growth rate g is based on the **attrition for less assumption** took place over the period. If no attrition had taken place, then E_τ would have equalled $E_0 + \sum_{t=1}^{\tau} G_t$. It can readily be shown that the growth rate g is too low, or has negative bias. The rate g is the geometric mean of the annual rates g_t obtained as follows:

$$\begin{aligned} E_0\ (1 + g_0) &= E_0 + G_1 = \hat{E}_1 \\ \hat{E}_1\ (1 + g_1) &= E_0 + G_1 + G_2 = \hat{E}_2 \\ E_{\tau-1}\ (1 + g_{\tau-1}) &= E_0 + \sum_{t=0}^{\tau} G_t = \hat{E}_\tau \end{aligned} \tag{18}$$

or

$$E_0 \prod_{t=0}^{\tau-2} (1 - g_t) = E_0\,(1 + g_0)\,(1 + g_1)\,\ldots\,(1 + g_{\tau-1}) = E_0 + \sum_{t=0}^{\tau} G_t \tag{19}$$

If attrition is positive but less than growth in every period, it can be shown that the path of $\hat{E}_t$ will lie above the path of E_t in every period except the first. In fig. 1 we have assumed that graduations grow at the rate g sufficient to make $\hat{E}_t$ grow at the same rate. Since $\hat{E}_t$ grows at a more rapid rate than E_t a given graduating class G_t is always added to an $\hat{E}_t$ larger than the corresponding E_t. This

$$\frac{G_t}{\hat{E}_t} < \frac{G_t}{E_t} \text{ implies } g_t < \quad t \tag{20}$$

for all periods but the first when $g_0 = \quad 0$. Therefore

$$\sqrt[\tau]{\prod_{t=0}^{\tau-1}(1 + g_t)} < \gamma \tag{21}$$

From (16) we can see that (22)

$$a < \alpha$$

Hence, this method is biased with certainty toward low growth rates and low attrition rates.

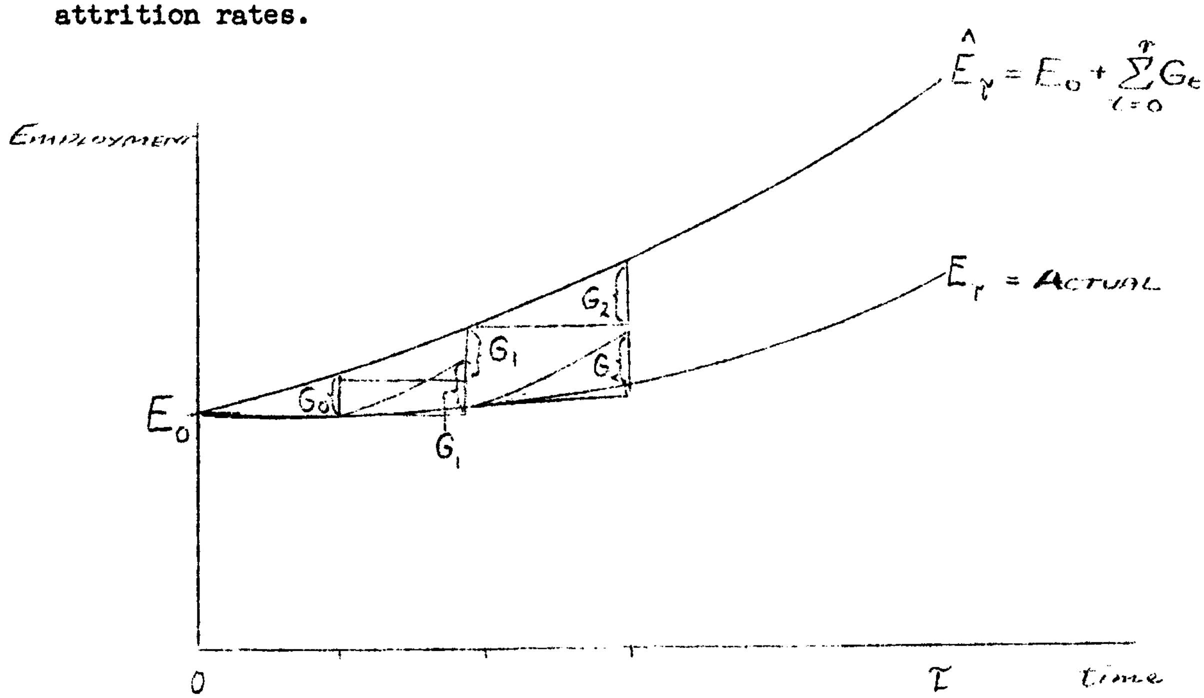

fig. 1.

(3) Another method is to combine (16) with

$$E_0 (1-a)\gamma = E_t = \sum_{t=0}^{\tau} G_t \qquad (23)$$

It can readily be shown that ignoring the growth that takes place leads to too small a base for the attrition rate which as a result is too large since the base is always too small.

(4) Another method can be obtained by combining equation (17) and (23) as follows:

$$\frac{(1 + g)}{(1 - a)} = \sqrt[\tau]{\frac{E_0 + \sum_{t=0}^{\tau} G_t}{E_t - \sum_{t=0}^{\tau} G_t}} \qquad (24)$$

and solving (24) simultaneously with equation (16). Since $g < \gamma$ and $a > \alpha$, the negative bias of $(1 + g)$ may be compensated for or over-compensated for.

The results of these approximate methods together with the polynomial results are given in Table 2. The biases of methods (2) and (3) are clearly shown. Method (4) is extremely unstable, giving rather absurd answers for mechanical and chemical engineers. This occurs because the term $E_t - \sum_{t=0}^{\tau} G_t$ is quite small and $E_0 - \sum_{t=0}^{\tau} G_t$ is quite large. Method (1) appears to be positively biased also, but performs somewhat better than the other methods.

In an application of the methods to nursing attrition, however, method (4) performed slightly better than method (1), so there is no a priori reason to choose one method over the other (Folk and Yett [3]). Method (1) should certainly be preferred whenever $\sum_{t=0}^{\tau} G_t$ is large relative to E_0 and E_t.

Table 2

Gross Growth and Attrition Rates of Engineering Specialties by Four Methods

	Net growth rate	Gross growth rate				Attrition rate			
	$(1+r)$	$\frac{x}{y}$	$\left(\frac{1+x}{1-y}\right)^{10}$	$(1+x)^{10}$	Polynomial	$\frac{x}{y}$	$\left(\frac{1+x}{1-y}\right)^{10}$	$(1+x)^{16}$	Polynomial
All engineers	5.54	8.94	9.88	7.97	8.77	3.41	3.96	2.26	2.97
Civil	2.85	7.01	7.84	6.04	6.86	4.16	4.63	3.01	3.75
Electrical	6.30	10.07	11.42	8.90	9.78	3.77	4.59	2.38	3.27
Mechanical	4.64	12.29	24.11	9.69	11.82	7.66	15.69	4.60	6.42
Chemical	3.08	9.77	13.46	7.82	9.47	6.69	9.15	4.40	5.84
Aeronautical	11.65	5.97	5.79	7.33	6.22	-5.69	-5.54	-4.02	-5.11
Industrial	8.84	7.43	7.26	7.78	7.53	-1.41	-1.44	- .96	-1.22

Source: Derived from data in Appendix Table 1 and 2.

Appendix Table 1

Engineering Bachelor's Degrees, by Specialty

	Total[a]	Aero-nautical	Chemical	Civil	Electrical	Industrial	Mechanical
1949-50	52,732	1,553	4,529	7,772	13,270	3,369	14,332
1950-51	41,893	1,271	3,756	7,060	9,385	2,583	10,752
1951-52	30,286	816	2,859	5,354	6,373	1,823	7,606
1952-53	24,164	645	2,276	4,400	4,905	1,525	5,917
1953-54	22,236	666	3,955	3,955	4,486	1,342	5,419
1954-55	22,589	686	2,027	3,868	4,860	1,495	5,829
1955-56	26,306	895	2,466	4,227	6,222	1,651	6,728
1956-57	31,211	1,109	2,818	4,683	8,108	1,926	7,907
1957-58	35,332	1,337	3,008	5,134	9,567	2,108	9,060
1958-59	38,134	1,551	3,131	5,394	10,786	2,297	9,592
Total 1949-59	324,883	10,529	28,912	51,847	77,962	20,119	83,142

a. Includes only those curricula accredited by the Engineer's Council for Professional Development at some level in at least one institution.

Source: U. S. Department of Health, Education, and Welfare, Office of Education, "Degrees in the Biological and Physical Sciences, Mathematics, and Engineering," OE-54029.

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