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THERMAL STRAIN ANALYSIS OF ADVANCED MANNED SPACECRAFT HEAT SHIELDS

Final Report

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ABSTRACT

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Numerical methods and computer programs are presented for the analysis of heat shields. The finite element technique is used to determine stresses and displacements developed in composite axisymmetric solids of arbitrary geometry subjected to axisymmetric thermal or mechanical loads. This technique is then applied to the development of an automated computer program for the analysis of axisymmetric heat shields subjected to axisymmetric thermal and pressure loadings. Finally, the numerical technique is extended to the analysis of heat shields subjected to non-axisymmetric thermal loading.

Several examples are presented to illustrate the application of the method and to demonstrate its validity. FORTRAN II card listings and descriptions of the use of the above programs are given in the appendices.

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LIST OF SYMBOLS

a_j, b_j, a_k, b_k = Element Dimensions

E = Modulus of Elasticity

u = Displacement in the r Direction

v = Displacement in the θ Direction

w = Displacement in the z Direction

α = Thermal Coefficient of Expansion

β = Over-Relaxation Factor

γ = Shear Strain

$\epsilon_{r,\theta,z}$ = Strain in the Radial, Circumferential and Longitudinal Direction

ν = Poisson's Ratio

$\sigma_{r,\theta,z}$ = Stress in the Radial, Circumferential and Longitudinal Direction

τ = Shearing Stress

$[\ ]^T$ = Matrix Transpose

$[a]$ = Displacement Transformation Matrix

$[C]$ = Matrix of Elastic Coefficients

$[G_n]$ = Displacement Transformation Matrix - Harmonic n

$[S]$ = Matrix of Element Corner Forces

$[u]$ = Matrix of Element Corner Displacements

$[k]$ = Element Stiffness Matrix

$[\bar{r}]$ = Nodal Point Displacements

$[R]$ = Nodal Point Loads

$[K]$ = Stiffness Matrix for Complete Structure

INTRODUCTION

The purpose of this investigation is the development of methods of analysis and digital computer programs to aid in establishing the structural integrity of manned spacecraft heat shields. The results of the analysis which are presented in this report indicate only the capabilities of the computer programs and do not necessarily represent the behavior of a specific heat shield. The final evaluation of the structural capability of a heat shield must be based on a certain amount of engineering judgement, in connection with the use of the computer programs.

In this investigation the finite element method is used to determine stresses and displacements developed in solids of revolution. First, a numerical procedure and a digital computer program are developed for the analysis of composite axisymmetric solids of arbitrary geometry subjected to axisymmetric thermal or mechanical loads. Second, this program is specialized to the analysis of axisymmetric heat shields. Finally, the same numerical technique is extended to the analysis of heat shields subjected to non-axisymmetric thermal loading. A description of the method of analysis and the use of the above computer programs is presented. In addition, FORTRAN II listings of the above programs are incorporated in this report.

During the initial phases of this contract, finite difference techniques were used to solve the governing differential equations for displacements of the system. However, considerable difficulty was encountered in the solution of the resulting set of linear equations. An iterative approach, coupled with over-relaxation techniques, resulted in inadequately convergent displacements. The direct solution technique gave a matrix for the set of simultaneous equations which was ill-conditioned. An additional difficulty of the finite difference technique was encountered in satisfying the boundary conditions at the edge of the heat shield. The finite element approach proved more practical and more versatile; therefore, the finite difference method was discontinued.

PART I: METHOD OF ANALYSIS

A. INTRODUCTION

The "finite element method" is a general method of structural analysis in which a continuous structure is replaced by a finite number of elements interconnected at a finite number of nodal points -- (such an idealization is inherent in the conventional analysis of frames and trusses). In this investigation the finite element method is applied to the determination of stresses and displacements developed in axisymmetric elastic structures of arbitrary geometry and material properties which are subjected to thermal and mechanical loads.

An assemblage of different types of axisymmetric elements is used to represent the continuous structure. Approximations are made on the displacements within each element of the system. Based on these approximations, equilibrium equations are developed for all elements. From "direct stiffness techniques", the equilibrium equations, in terms of unknown nodal point displacements, are developed at each nodal point. A solution of this set of equations constitutes a solution to the finite element system.

B. EQUILIBRIUM EQUATIONS FOR AN ARBITRARY FINITE ELEMENT

1. Strain-Displacement Relationship

The first step in the determination of the stiffness (corner forces in terms of corner displacements and temperature changes), of a finite element is to assume a solution for the displacement field within the element. It is desirable that this assumed displacement field satisfies compatibility between other elements in the system. Based on this solution for the displacements within the element, it is possible to develop an expression for the strains at any point within the element in terms of the nodal points (corner) displacements. This expression in matrix form is

$$\left[\epsilon\right] = \left[a\right]\left[u\right] \tag{1.1}$$

where $\left[\epsilon\right]$ is a column matrix of the M components of strain

$\left[u\right]$ is a column matrix of the N nodal point displacements

$\left[a\right]$ is an M x N strain-displacement transformation matrix - this matrix may be a function of space

2. Stress-Strain Relationship

For an elastic material, the stresses at any point within the element are expressed in terms of the corresponding strains by the elastic stress-strain relationship. Or, in matrix form

$$\left[\sigma\right] = \left[C\right]\left[\epsilon\right] + \left[\tau\right] \tag{1.2}$$

where $\left[\sigma\right]$ is a column matrix of the M components of stress

$\left[\epsilon\right]$ is a column matrix of the M components of strain

$\left[\tau\right]$ is a column matrix of the M components of thermal stress

$\left[C\right]$ is an MxM matrix of material property coefficients

The size (M) of these matrices will depend on the type of element being considered. The coefficients of matrices $\left[C\right]$ and $\left[\tau\right]$ will depend on material properties. Since $\left[C\right]$ is completely arbitrary, anisotropic materials can be handled. Also, each element in the system may have different properties; therefore, composite structures are readily represented by the finite element idealization.

3. Internal Work

The internal work, or strain energy, which is associated with an infinitesimal volume element dV within the finite element is given by

$$dW_I = \frac{1}{2} (\epsilon_1 \sigma_1 + \epsilon_2 \sigma_2 \cdots\cdots + \epsilon_M \sigma_M) \, dV$$

or in matrix form

$$dW_I = \frac{1}{2} \left[\epsilon\right]^T \left[\sigma\right] dV \tag{1.3}$$

The substitution of Equation (1.2) into Equation (1.3) yields

$$dW_I = \frac{1}{2} \left[\epsilon\right]^T \left[C\right] \left[\epsilon\right] dV + \frac{1}{2} \left[\epsilon\right]^T \left[\tau\right] dV \tag{1.4}$$

Equation (1.1) may be written in transposed form as

$$[\epsilon]^T = [u]^T [a]^T \tag{1.5}$$

After Equations (1.1) and (1.5) are substituted into Equation (1.4), the internal work is given by

$$dW_I = \frac{1}{2} [u]^T [a]^T [C] [a] [u] \, dV + \frac{1}{2} [u]^T [a]^T [\tau] \, dV \tag{1.6}$$

The total strain energy stored within the element is found by integrating Equation (1.6) over the volume of the finite element. Or

$$W_I = \frac{1}{2} [u]^T \int [a]^T [C] [a] [u] \, dV + \frac{1}{2} [u]^T \int [a]^T [\tau] \, dV \tag{1.7}$$

4. External Work

The work supplied externally at the nodal points of the finite element is given by

$$W_E = \frac{1}{2} U_1 S_1 + \frac{1}{2} U_2 S_2 \dots\dots\dots\dots + \frac{1}{2} U_N S_N$$

or in matrix form

$$W_E = \frac{1}{2} [u]^T [S] \tag{1.8}$$

where $[u]^T$ is a row matrix of the N nodal point displacements

$[S]$ is a column matrix of the N corresponding nodal point forces

5. <u>External Work = Internal Work</u>

The external work, Equation (1.8), is equated to the internal work, Equation (1.7), yielding

$$\left[u\right]^T\left[S\right] = \left[u\right]^T\left[k\right]\left[u\right] + \left[u\right]^T\left[L\right] \tag{1.9}$$

where the element stiffness matrix

$$\left[k\right] = \int\left[a\right]^T\left[C\right]\left[a\right] dV \tag{1.10}$$

and the thermal load matrix

$$\left[L\right] = \int\left[a\right]^T\left[\tau\right] dV \tag{1.11}$$

Equation (1.9) represents an energy balance (scalar equation) for a single nodal point displacement pattern. If the final displacements $\left[u_i\right]$ are assumed to be composed of N separate displacement patterns, $\left[\bar{u}_{ij}\right]$ j = 1,...N, and if the final forces $\left[S_i\right]$ are assumed to be composed of N corresponding sets of forces, $\left[\bar{S}_{ij}\right]$ j = 1, ... N, Equation (1.9) may be written as

$$\left[\bar{u}\right]^T\left[\bar{S}\right] = \left[\bar{u}\right]^T\left[k\right]\left[\bar{u}\right] + \left[\bar{u}\right]^T\left[L\right] \tag{1.12}$$

To eliminate the term $\left[\bar{u}\right]^T$, the displacement patterns must be selected in such a manner as to assure an inverse of $\left[\bar{u}\right]^T$. An acceptable matrix is a diagonal matrix of the final displacement, or

$$\left[\bar{u}\right]^T = \left[\bar{u}\right] = \left[\diagdown u \diagdown\right] \tag{1.13}$$

Equation (1.12) is now premultiplied by $\left[\diagdown u \diagdown\right]^{-1}$ yielding

$$\left[I\right]\left[\bar{S}\right] = \left[I\right]\left[k\right]\left[\ u\ \right] + \left[I\right]\left[L\right] \tag{1.14}$$

where $\left[I\right]$ is a diagonal unit matrix.

Since only linear systems are considered, the N displacement patterns may be superimposed. Or

$$\left[S\right] = \left[k\right]\left[u\right] + \left[L\right] \tag{1.15}$$

Since

$$S_i = \sum_{j=1,\ldots N} \bar{S}_{ij} \quad \text{and} \quad u_i = \bar{u}_{ii}$$

Equation (1.15) expresses nodal point forces in terms of nodal point displacements and temperature changes within the element.

C. EQUILIBRIUM EQUATIONS FOR A SYSTEM OF FINITE ELEMENTS

The first step in the procedure is to express <u>all</u> element forces in terms of external nodal point displacements for each element in the system. This is accomplished by expanding Equation (1.14) in terms of the N possible nodal point displacements; this will yield M matrix equations of the form

$$\left[S^m\right] = \left[k^m\right]\left[r\right] + \left[L^m\right] \qquad m = 1, \ldots. M \tag{1.16}$$

where M is the total number of elements in the system.

The matrix $\left[k^m\right]$ is termed the complete stiffness of element m and involves only terms which are associated with the displacements of the connecting nodal points. Consequently, the majority of the coefficients of this matrix equation are zero. The matrix $\left[r\right]$ contains all possible nodal point displacements of the complete finite element system. The matrix $\left[S^m\right]$ is a column matrix containing the forces acting on element m in the direction of the nodal point displacements $\left[r\right]$. The thermal load matrix $\left[L^m\right]$ and the element stiffness matrix $\left[k^m\right]$ are given by Equations (1.11) and (1.10); however, the order (size) of these matrices has now been expanded to correspond with the total number of nodal point displacements.

In order to satisfy equilibrium of all nodal points, the sum of the internal element forces must be equal to the external nodal point loads. Or

$$\left[P\right] = \sum_{m=1,\dots M} \left[S^m\right] \tag{1.17}$$

where $\left[P\right]$ is the externally applied nodal point loads. The substitution of Equation (1.16) into Equation (1.17) yields

$$\left[P\right] = \sum_{m=1,\dots M} \left[k^m\right]\left[r\right] + \sum_{m=1,\dots M} \left[L^m\right] \tag{1.18}$$

or rewritten in the following form:

$$\left[R\right] = \left[K\right]\left[r\right] \tag{1.19}$$

where

$$\left[R\right] = \left[P\right] - \sum_{m=1,\ldots M} \left[L^m\right] \tag{1.20}$$

$$\left[K\right] = \sum_{m=1,\ldots M} \left[k^m\right] \tag{1.21}$$

Equation (1.19), which is an equilibrium relationship between external loads and internal forces, represents a system of N linear equations in terms of N unknown displacements.

D. SOLUTION OF EQUILIBRIUM EQUATIONS

Equation (1.19) represents the relationship between all nodal point forces and all nodal point displacements. Mixed boundary conditions are considered by rewriting Equation (1.19) in the following partitioned form:

$$\begin{bmatrix} R_a \\ \hline R_b \end{bmatrix} = \left[\begin{array}{c|c} K_{aa} & K_{ab} \\ \hline K_{ba} & K_{bb} \end{array}\right] \begin{bmatrix} r_a \\ \hline r_b \end{bmatrix} \tag{1.22}$$

where $\left[R_a\right]$ = the specified nodal point forces

$\left[R_b\right]$ = the unknown nodal point forces

$\left[r_a\right]$ = the unknown nodal point displacements

$\left[r_b\right]$ = the specified nodal point displacements

Equation (1.22) may be expressed in terms of two separate equations, or

$$\left[R_a\right] = \left[K_{aa}\right]\left[r_a\right] + \left[K_{ab}\right]\left[r_b\right] \tag{1.23}$$

$$\left[R_b\right] = \left[K_{ba}\right]\left[r_a\right] + \left[K_{bb}\right]\left[r_b\right] \tag{1.24}$$

Equation (1.23) is rewritten in the following reduced form:

$$\left[K_{aa}\right]\left[r_a\right] = \left[\bar{\bar{R}}_a\right] \tag{1.25}$$

where the modified load vector, $\left[\bar{\bar{R}}_a\right]$ is given by

$$\left[\bar{\bar{R}}_a\right] = \left[R_a\right] - \left[K_{ab}\right]\left[r_b\right] \tag{1.26}$$

In Part II of this report, the Gauss-Seidel iterative technique is used to solve Equation (1.25) for the unknown nodal point displacements $\left[r_a\right]$. Appendix A gives a direct solution approach which is used in the automated computer programs for the thermal stress analysis of heat shields, Parts III and IV of this report.

E. ELEMENT STRESSES

After the nodal point displacements have been determined, the strains within any element in the system are evaluated by the direct application of Equation (1.1). The corresponding stresses are calculated from the stress-strain relationship, Equation (1.2).

PART II GENERAL COMPUTER PROGRAM FOR THE ANALYSIS OF ARBITRARY AXISYMMETRIC STRUCTURES

A. INTRODUCTION

The stress analysis of an axisymmetric structure of arbitrary shape, subjected to thermal and mechanical loads is of considerable practical interest. Although the governing differential equations have been known for many years, closed form solutions have been obtained for only a limited number of structures. Thus, the investigator must often rely on experimental or numerical procedures to solve this problem.

Experimental methods, such as Photoelasticity, have proven to be versatile tools in the analysis of many axisymmetric structures. However, for structures composed of several different materials or structures with thermal loading, this approach is limited.

The finite difference method, which involves the replacement of the derivatives in the differential equations and boundary conditions with difference equations, has been the most popular of the numerical techniques. However, for structures of composite materials and of arbitrary geometry, the procedure is difficult to apply.

In this section the finite element method is used to determine the stresses and displacements developed within arbitrary, elastic solids of revolution subjected to thermal or mechanical axisymmetric loads. The finite element approach replaces the continuous structure with a system of triangular rings interconnected at a finite number of nodal points (joints). Loads acting on the structure are replaced by statically equivalent concentrated forces acting at the nodal points of the finite element system. Figure 2.1 illustrates a finite element idealization of a typical axisymmetric solid.

B. STIFFNESS OF TRIANGULAR RING

1. Strain-Displacement Relationship

Continuity between elements of the system is maintained by requiring that within each element "lines initially straight remain straight in their displaced position". This linear displacement field, which is illustrated in Figure 2.2, is defined in terms of u(r,z) and

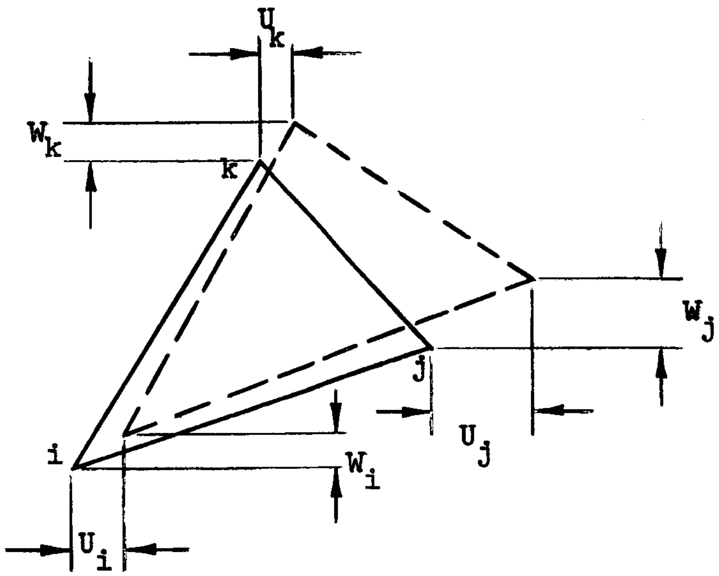
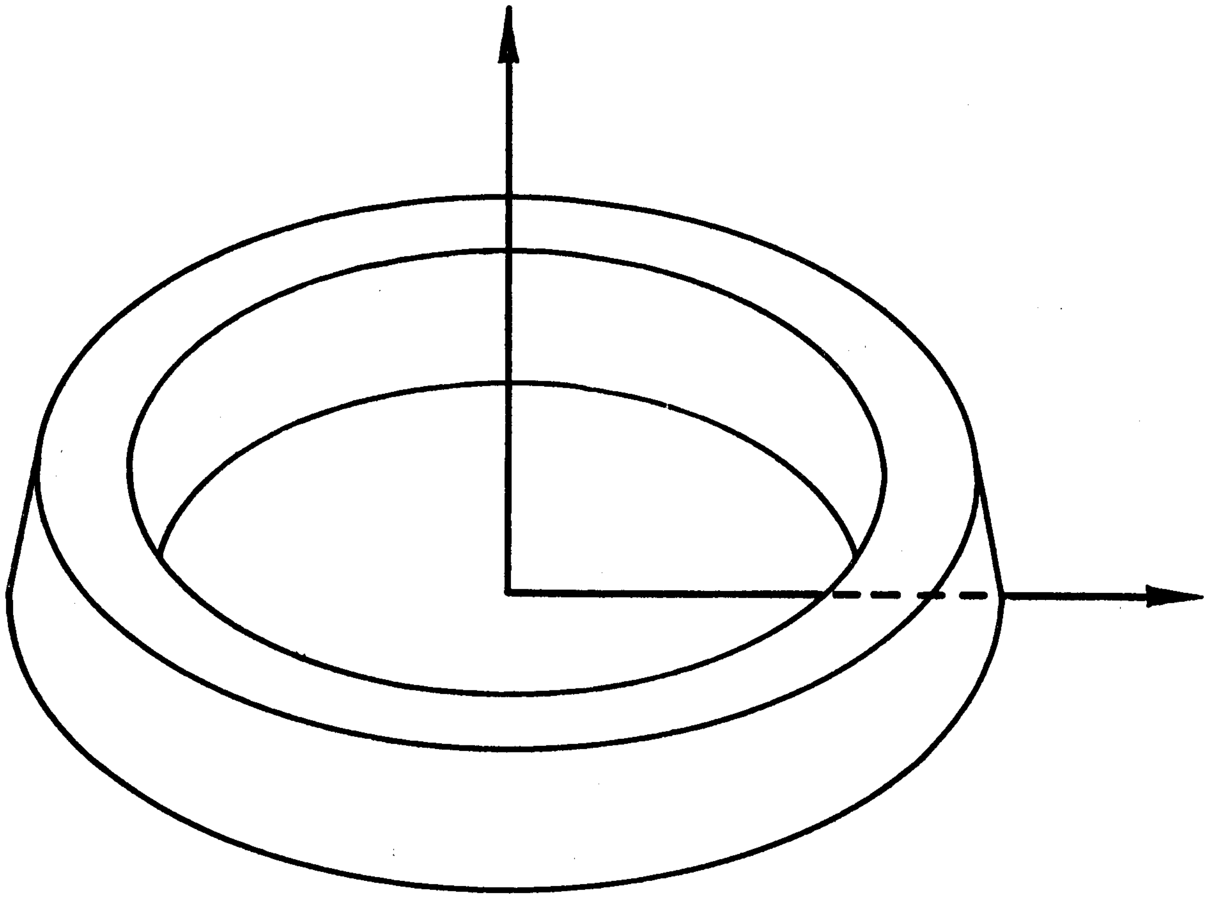
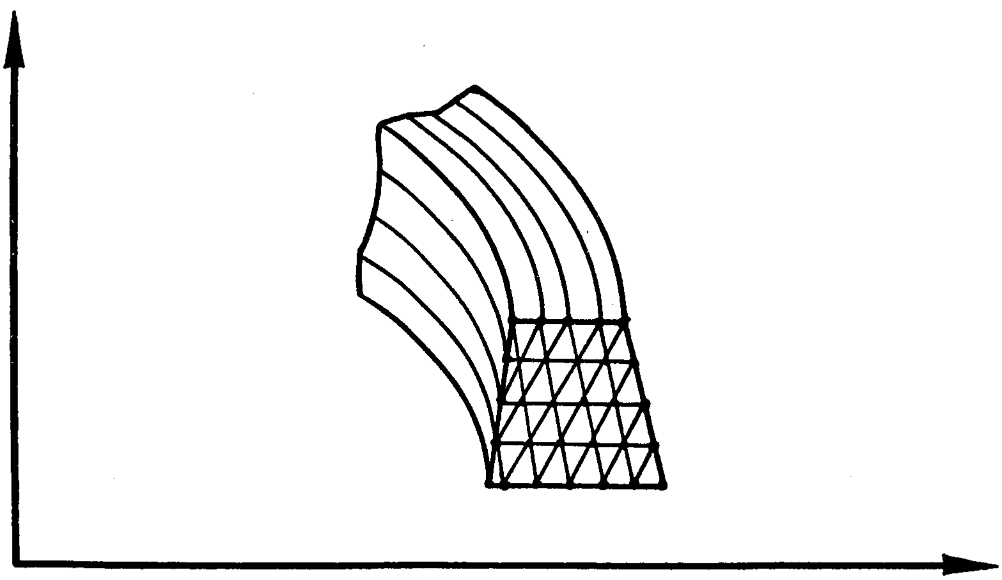


FIG. 2.2 ASSUMED DISPLACEMENT PATTERN



a. ACTUAL RING



b. TRIANGULAR ELEMENT APPROXIMATION

FIG. 2.1 THE FINITE ELEMENT IDEALIZATION

$w(r,z)$ by equations of the following form:

$$u(r,z) = C_1 + C_2 r + C_3 z \qquad (2.1a)$$

$$w(r,z) = C_4 + C_5 r + C_6 z \qquad (2.1b)$$

If Equations (2.1a) and (2.1b) are evaluated at the three corners i, j, k of the triangle, the following set of equations is obtained:

$$\begin{bmatrix} u_i \\ w_i \\ u_j \\ w_j \\ u_k \\ w_k \end{bmatrix} = \begin{bmatrix} 1 & r_i & z_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_i & z_i \\ 1 & r_j & z_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_j & z_j \\ 1 & r_k & z_k & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_k & z_k \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} \qquad (2.2)$$

By solving the system of Equations (2.2) for the constants $C_1, \ldots, C_6$, they are expressed in terms of corner displacements. The strains in the rz-plane are obtained from the assumed displacement field by considering the basic definition of strain.

$$\bar{\epsilon}_r = \frac{\partial u}{\partial r} = C_2 \tag{2.3a}$$

$$\bar{\epsilon}_z = \frac{\partial w}{\partial z} = C_6 \tag{2.3b}$$

$$\bar{\gamma}_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = C_3 + C_5 \tag{2.3c}$$

At any point within the element the tangential strain $\bar{\epsilon}_\theta$ is

$$\bar{\epsilon}_\theta(r,z) = \frac{u(r,z)}{r}$$

The average tangential strain is found by averaging the strains at the vertices of the triangle, or

$$\epsilon_\theta = \frac{1}{3}\left(\frac{u_i}{r_i} + \frac{u_j}{r_j} + \frac{u_k}{r_k}\right) \tag{2.3d}$$

After eliminating the constants C_n between Equations (2.2) and (2.3), the average element strains are expressed in terms of corner displacements by the following matrix equation:

$$
\begin{bmatrix} \epsilon_r \\ \epsilon_z \\ \epsilon_\theta \\ \gamma_{rz} \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} b_j - b_k & 0 & b_k & 0 & -b_j & 0 \\ 0 & a_k - a_j & 0 & -a_k & 0 & a_j \\ \frac{\lambda}{3r_i} & 0 & \frac{\lambda}{3r_j} & 0 & \frac{\lambda}{3r_k} & 0 \\ a_k - a_j & b_j - b_k & -a_k & b_k & a_j & -b_j \end{bmatrix} \begin{bmatrix} u_i \\ w_i \\ u_j \\ w_j \\ u_k \\ w_k \end{bmatrix} \quad (2.4a)
$$

or in symbolic form

$$
[\epsilon] = [a][u] \quad (2.4b)
$$

where

$$
\begin{aligned}
a_j &= r_j - r_i \\
a_k &= r_k - r_i \\
b_j &= z_j - z_i \\
b_k &= z_k - z_i \\
\lambda &= a_j b_k - a_k b_j
\end{aligned}
$$

The geometry of a typical triangle is illustrated in Figure 2.3

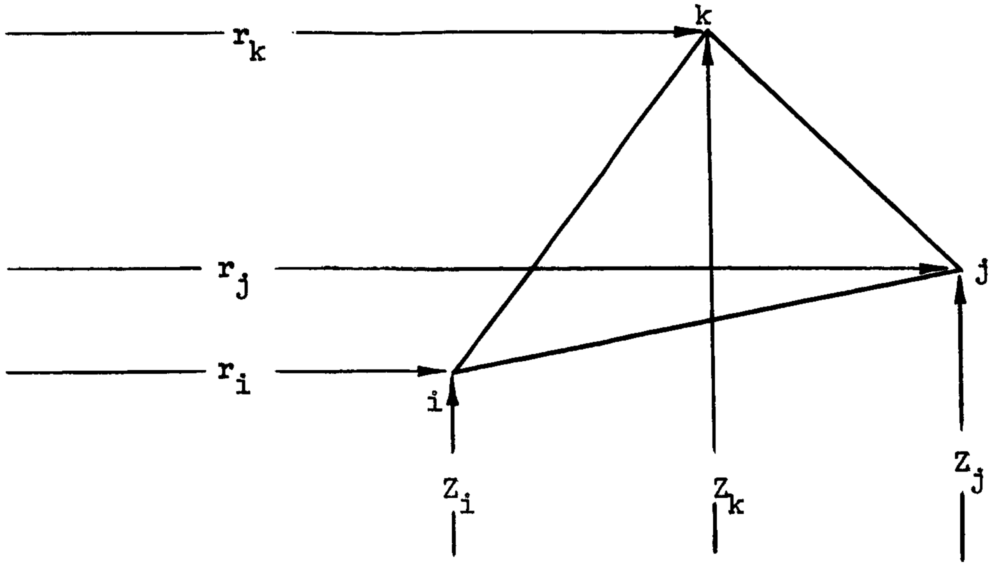


FIG. 2.3 ELEMENT DIMENSIONS

2. Stress-Strain Relationship

One important advantage of the finite element approach is that structures with anisotropic materials can be treated. In general, the stress-strain relationship is of the form

$$\begin{bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \sigma_{rz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & \epsilon_r \\ C_{21} & C_{22} & C_{23} & C_{24} & \epsilon_z \\ C_{31} & C_{32} & C_{33} & C_{34} & \epsilon_\theta \\ C_{41} & C_{42} & C_{43} & C_{44} & \gamma_{rz} \end{bmatrix} + \begin{bmatrix} \tau_r \\ \tau_z \\ \tau_\theta \\ \tau_{rz} \end{bmatrix} \tag{2.5a}$$

or symbolically $[\sigma] = [C]\,[\epsilon] + [\tau]$ (2.5b) where $\left[\tau\right]$ is the matrix of thermal stresses for a given temperature change. For example, the stress-strain relationship for an isotropic material is given by

$$\begin{bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \sigma_{rz} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_r \\ \epsilon_z \\ \epsilon_\theta \\ \gamma_{rz} \end{bmatrix} + \begin{bmatrix} \tau \\ \tau \\ \tau \\ 0 \end{bmatrix} \qquad (2.6)$$

$$\text{where } \tau = \frac{E\alpha}{(1-2\nu)} \Delta T \qquad (2.7)$$

3. Element Stiffness

The stiffness of a typical triangular ring, which is an expression for corner forces in terms of corner displacements, is given by Eq. (1.10) as

$$\left[k\right] = \int \left[a\right]^T \left[c\right] \left[a\right] dV$$

And the thermal load matrix is given by Eq. (1.11) as

$$\left[L\right] = \int \left[a\right]^T \left[\tau\right] dV$$

Since the coefficients in matrices $\left[a\right]$ and $\left[C\right]$ are assumed not to be a function of space the above equations reduce to

$$[k] = V [a]^T [C] [a] \tag{2.8}$$

$$[L] = V [a]^T [\tau] \tag{2.9}$$

If a one-radian segment is considered, an approximate expression for the volume of a triangular ring segment is

$$V = \bar{r} A \tag{2.10}$$

where $\bar{r}$ is the average radius given by

$$\bar{r} = (r_i + r_j + r_k)/3 \tag{2.11}$$

and A is the cross-sectional area given by

$$A = (a_j b_k - a_k b_j)/2 \tag{2.12}$$

From Eq. (1.15) the six corner forces acting at the vertices of a one-radian triangular segment is given in terms of the six corner displacements and the temperature change within the element by

$$[S] = [k] [u] + [L] \tag{2.13}$$

C. EQUILIBRIUM EQUATIONS FOR COMPLETE STRUCTURE

The equilibrium of the complete system of triangular rings, which is an expression for nodal point loads in terms of nodal point displacements, is given by the following matrix equation:

$$[R] = [K] [r] \tag{2.14}$$

The stiffness matrix $\left[K\right]$ and the load matrix $\left[R\right]$ are determined by "direct stiffness" techniques as indicated in the previous section, Eqs. (1.20) and (1.21). In addition to the thermal loads, the $\left[R\right]$ matrix is composed of concentrated external forces acting at the nodal points of the system. Hence, pressures acting on the boundary of a segment of the structure are replaced by statically equivalent forces acting at the nodal points.

Mixed boundary conditions are considered by a simple transformation of Eq. (2.14); Eqs. (1.22) to (1.26) give the details of this modification.

D. DETERMINATION OF DISPLACEMENT AND STRESSES

Equation (2.14) is solved for the unknown nodal point displacements by the application of the well-known Gauss-Seidel iterative procedure. This involves the repeated calculation of new displacements from the equation

$$r_n^{(s)} = K_{nn}^{-1} \left[R_n - \sum_{i=1,..n-1} K_{ni} r_i^{(s)} - \sum_{i=n+1,..n} K_{ni} r_i^{(s-1)} \right. \qquad (2.15)$$

where n is the number of the unknown and s is the cycle of iteration.

The only modification of the procedure introduced in this analysis is the simultaneous application of Equation (2.15) to both components of displacements at each nodal point. Therefore, r_n and R_n become vectors with r and z components.

The rate of convergence of the Gauss-Seidel procedure can be greatly increased by the use of an over-relaxation factor. This factor is applied by first calculating the change in displacement $\Delta r_n^{(s)}$ of nodal point n and then determining the new displacement from the following equation:

$$r_n^{(s)} = r_n^{(s-1)} + \beta \Delta r_n^{(s)} \qquad (2.16)$$

where β is the over-relaxation factor.

The solution of an over-relaxation factor, which gives the best convergence, depends on the characteristics of the particular problem. However, experience has indicated that for most structures, the optimum over-relaxation factor is between 1.8 and 1.95.

Since only the non-zero terms in Equation (2.14) are developed and stored by the computer program, a solution of several hundred equations is possible, thereby, making it possible to solve large finite element systems.

For each element the average strains are calculated directly from the nodal point displacements by the application of Equation (2.4). The average element stresses are then determined from the stress-strain relationship for the element, Equation (2.5). In addition, at each nodal point, stresses are computed by averaging the stresses in all elements attached to the point.

E. COMPUTER PROGRAM

The complete analysis of an axisymmetric solid by the finite element method involves three separate phases. First, the structure must be idealized by a system of triangular rings. Second, this system is solved for displacements and stresses from given nodal point forces. Third, the displacements and stresses are presented graphically for further evaluation and utilization.

The selection of the system of finite elements for a particular problem is completely arbitrary; therefore, axisymmetric structures, composed of many interacting components, of practically any shape may be handled. By numbering all elements and nodal points, in a convenient manner, the system can be defined in the form of three numerical arrays - nodal point array, element array and boundary point array. The nodal point array contains the coordinates and the loads or displacements that are associated with each nodal point of the system. The element array contains, for each element in the system, the location of the element (the three nodal point numbers defining the corners of the element and other possible parameters which are associated with the element (i.e., elastic constants, density and temperature changes). The boundary array indicates the type of restraint that exists at boundary nodal points.

These three arrays, along with some basic control information, constitute the numerical input for the digital computer program. The program itself performs three major tasks in the analysis of the finite element system of triangular rings. First, the equilibrium equations for the system are formed from the basic numerical description of the system. Second, this set of equations is solved for the nodal point displacements. Third, the internal stresses are determined from these displacements.

1. <u>Input Information</u>

To define the system of finite elements, all nodal points and elements are numbered as illustrated in Figure 2.4. Based on this numbering system, the following sequence of punched cards constitutes the input to the computer program.

a. TITLE CARD (72H)

Columns 2 to 72 of this card contain information to be printed with results

b. CONTROL CARD (6I4, 2E12.5)

Columns	1 - 4	Number of elements
	5 - 8	Number of nodal points
	9 - 12	Number of restrained boundary points
	13 - 16	Cycle interval for print of unbalanced forces

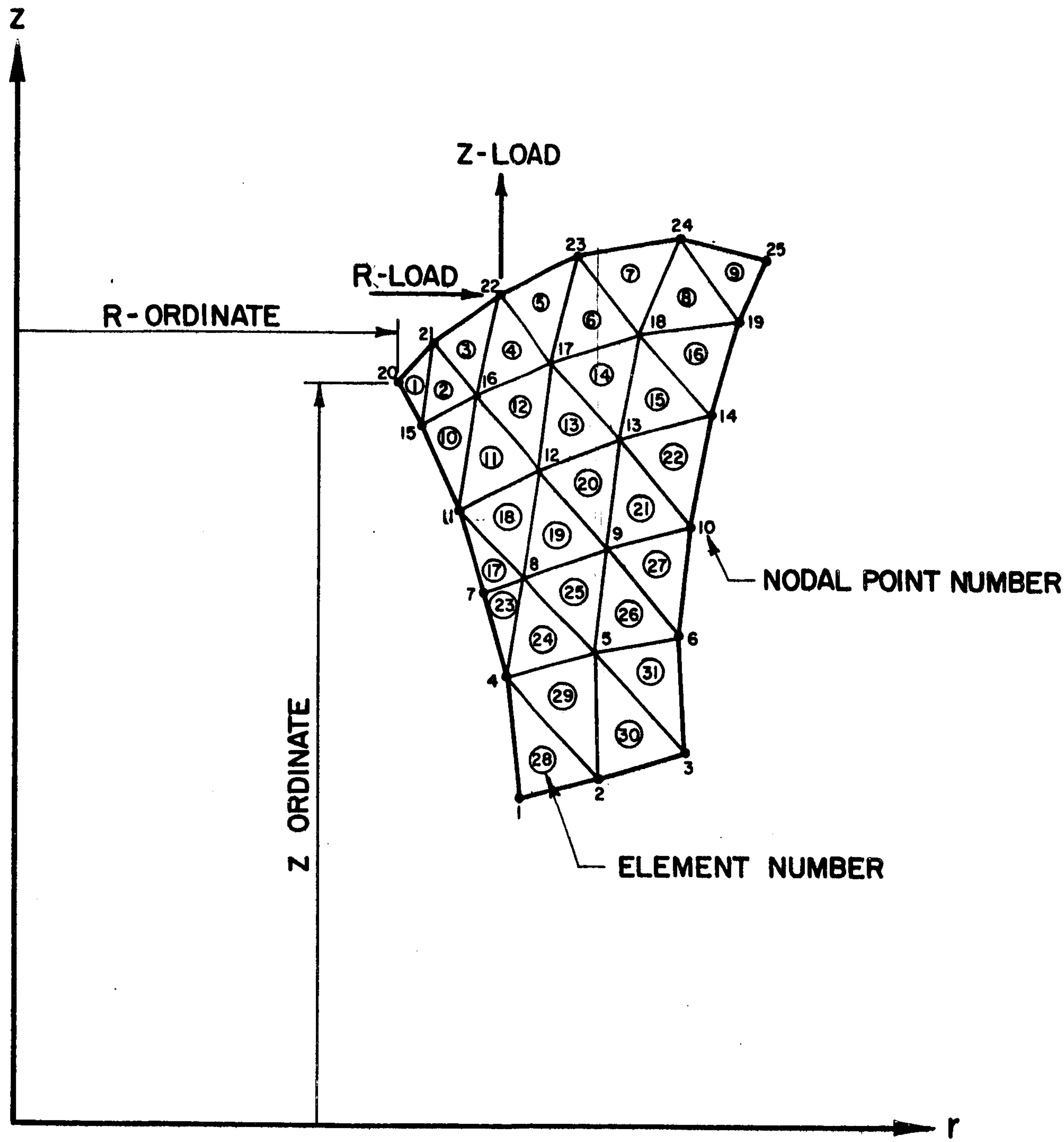


FIG. 2.4 NUMBERING SYSTEM FOR ELEMENTS AND NODAL POINTS

17-20	Cycle interval for print of results
21-24	Maximum number of cylces problem may run
25-36	Convergence limit for unbalanced forces
37-48	Over-relaxation factor

c. ELEMENT ARRAY - 1 card per element (4I4, 4E12.4, F8.4)

Column	1-4	Element Number	
	5-8	Nodal point number i	
	9-12	Nodal point number j	in counter-clockwise order
	13-16	Nodal point number k	
	17-28	Modulus of elasticity E	
	29-40	Density of element ρ	
	41-52	Poisson's ratio ν	
	53-64	Coefficient of thermal expansion α	
	65-72	Temperature change within element ΔT	

d. NODAL POINT ARRAY - 1 card per nodal point (1I4, 4F8.1, 2F12.8)

Column	1-4	Nodal point number	
	5-12	R-ordinate	
	13-20	Z-ordinate	
	21-38	R-load	Total force acting on a one radian segment.
	29-36	Z-load	
	37-48	R-displacement	
	49-60	Z-displacement	

On free nodal points, the displacements are initial guesses for the iterative solution. On restrained nodal points, the input displacements are the specified final displacements of the nodal point.

e. BOUNDARY POINT ARRAY - 1 card per restrained nodal point (2I4, IF6)

Columns	1-4	Nodal point numbers
	5-8	0 if point is fixed in both directions
		1 if point is fixed in the R-direction
		2 if point is free to move along a line of slope S
	9-16	Slope S (type 2 points only)

2. Output Information

The following information is generated and printed by the computer program:

a. Input Data
b. Nodal Point Displacement
c. Average Element Stresses
d. Average Nodal Point Stresses

3. Timing

For the IBM 7094 the computational time required by the program is approximately 0.004 x n x m seconds, where n equals the number of nodal points and m equals the number of cycles of iteration. Depending on the desired degree of convergence, it may be necessary to extend the iteration process.

4. Program Listing

A card listing of the FORTRAN II source deck for the general axisymmetric program is included in Appendix D of this report. This program is compiled for a maximum size of 550 elements or 340 nodal points.

F. EXAMPLE

An infinite cylinder subjected to steady state temperature distribution, for which an exact solution is known, is selected as a means of verifying the finite element analysis. A finite element idealization of a section of the cylinder is shown in Figure 2.5a. The temperature distribution which is assumed constant within each element, is plotted in Figure 2.5b. The hoop stresses are compared with the exact solution in Figure 2.5c. Considering the coarse mesh, agreement with the exact solution is very good except at the two boundary points. This discrepancy is due to the fact that nodal point stresses are calculated by averaging the stresses in the attached elements. Therefore, the boundary nodal point stress reflects the average stress in the elements near the boundary.

In general, good boundary stresses are obtained by plotting the interior stresses and extrapolating to the boundary. This type of engineering judgement is always necessary in evaluating results from a finite element analysis.

G. DISCUSSION

This section demonstrates the application of the finite element technique to the stress analysis of structures of revolution. The approach reduces the stress analysis to a simple procedure. In order to use the

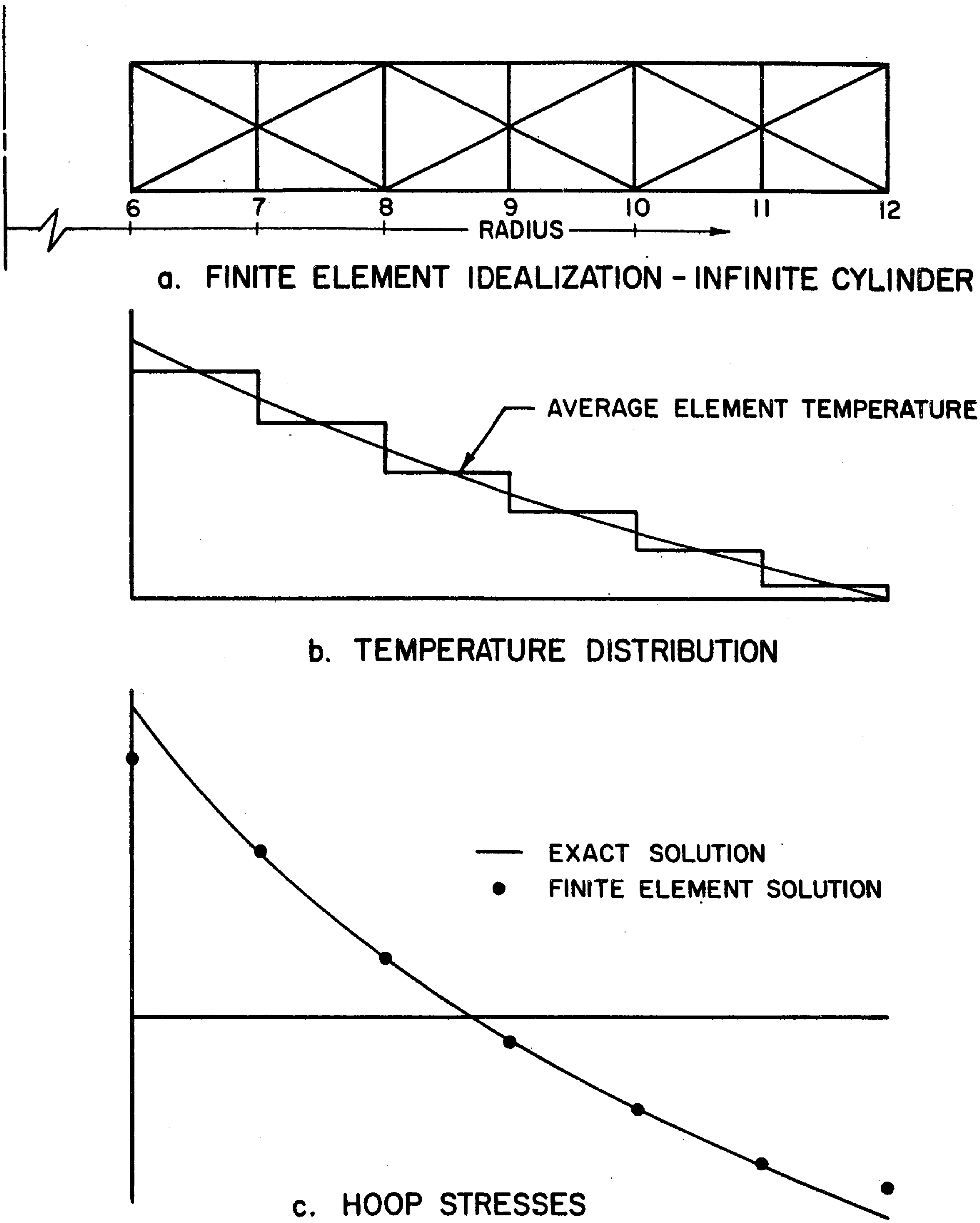


FIG. 2.5 ANALYSIS OF INFINITE CYLINDER

program, it is only necessary to select an element idealization of the structure and to supply the computer program with data that numerically define the system of elements. Therefore, the program may be used as a tool in design since changes in materials and geometry of the structures may involve only minor changes in the input data.

In addition, the program may be extended to include the effects of anisotropic materials. For this case, the input to the program must be expanded to include the general elastic constants defined by Eq. (2.5).

For the analysis of a specific type of structure, this program can be further automated by incorporating a mesh generator and the calculation of temperature-dependent material properties. In the next section of this report the program is specialized to the thermal stress analysis of axisymmetric heat shields for manned spacecraft.

PART III AUTOMATED PROGRAM FOR AXISYMMETRIC HEAT SHIELDS SUBJECTED TO AXISYMMETRIC LOADS

A. INTRODUCTION

The general computer program for the analysis of arbitrary axisymmetric structure, as given in the previous section, can be applied to the thermal stress analysis of heat shields. However, the use of this program for such a complex structure involves a large amount of detail work to select the finite element idealization and to prepare the computer input. In addition, the convergence of the Gauss-Seidel iteration procedure is slow for this type of structure and a solution may require an excessive amount of computer time.

By restricting the general computer program to the analysis of heat shields and by automating the input, a considerably more efficient program can be developed. The geometry of the heat shield is supplied to the computer program in the form of R and Z coordinates and ablator thickness at various points along the bond line. The required triangular mesh and the temperature at the grid points are generated within the program. Material properties at various temperatures are supplied in tabular form and the program automatically develops analytical expressions for the material properties by least square techniques. The flanges of the sandwich shell are idealized by special conical shell elements and the honeycomb core material is treated as a separate material. This approach eliminates the need for the establishing of a pseudo-thickness for the composite sandwich shell. The solution of the equilibrium equations, which was previously obtained by an iterative approach is accomplished by a direct solution procedure. Because of their significance, stresses within the sandwich plates and at the bond line are included in the computer output.

B. MESH GENERATION

A typical finite element idealization of the cross-section of a heat shield is shown in Figure 3.1. The basic element in this system is a quadrilateral ring, which in turn is composed of two triangular rings (Part II). In this particular case, the sandwich shell is represented by the first two rows of elements and the ablator is idealized by four rows of elements; there are 30 points in the meridional direction. The specific mesh configuration is a variable which is supplied to the computer program.

In general, the geometry of the shell is given by the R-Z coordinates of the points at the bond line between the sandwich shell and the ablator. The points on lines perpendicular to the bond line inside the variable thickness ablator and inside the constant thickness sandwich shell are generated automatically within the program. Thin shell cone elements are used to represent the face plates of the sandwich shell.

From a given temperature distribution at the bond line, the grid point temperatures are assumed to be constant within the shell and are assumed to vary parabolically within the ablator.

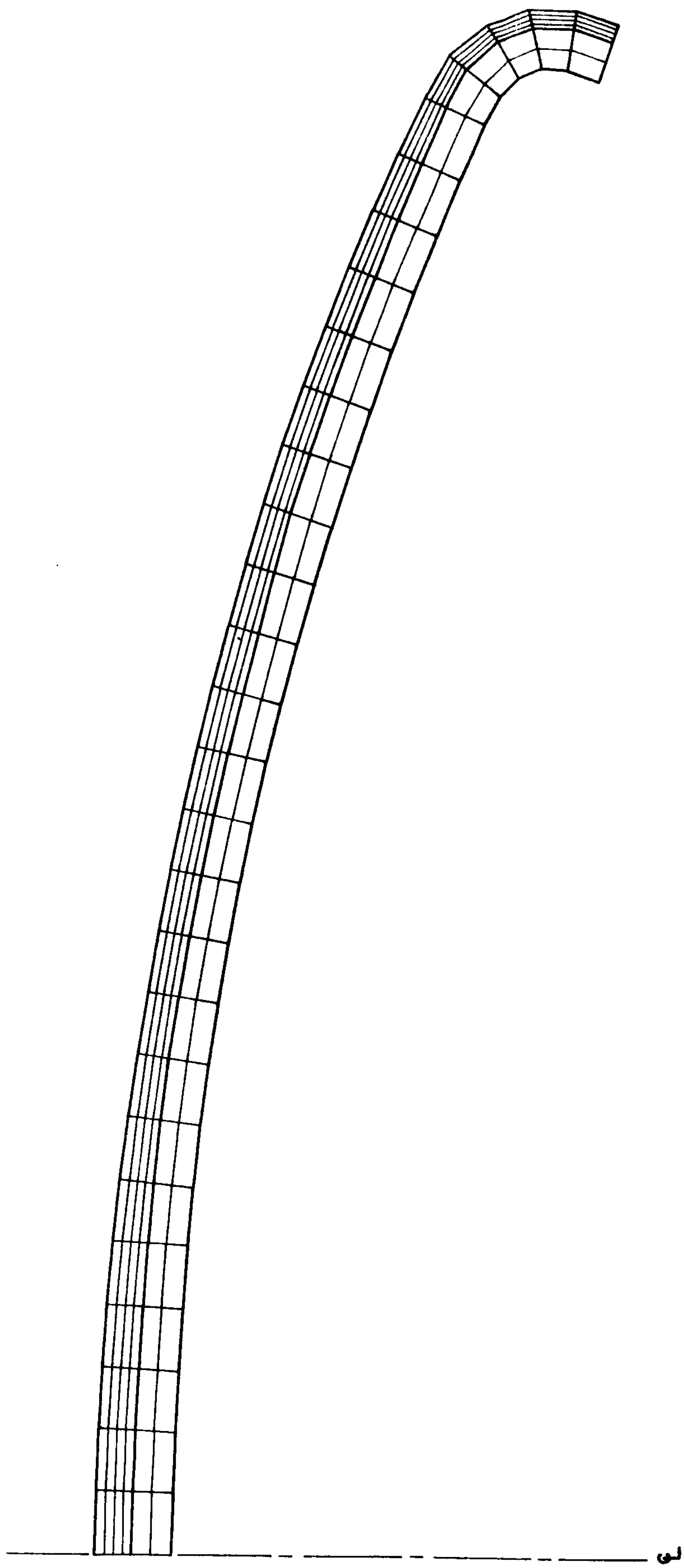


Fig. 3.1 Typical Finite Element Idealization of Heat Shield

C. DETERMINATION OF DISPLACEMENTS AND STRESSES

Based on temperature dependent material properties, the equilibrium relationship for each quadralateral ring is developed and then combined to form the equilibrium equations of the complete system of rings. Similarly the stiffness properties of the face plates of the sandwich shell are incorporated into the equilibrium of the system. The axisymmetric behavior of a typical conical element is a special case of the non-axisymmetric behavior which is given in Appendix C. The unknowns in this set of equations are the vertical and radial displacements at each grid point in the system. The satisfying of possible displacement boundary conditions requires that these equations be modified as indicated by Equation (1.25). Because of the physical characteristics of the heat shield, the resulting set of equations is in band form. Appendix A indicates the necessary modification to restrict the standard Gaussian elimination procedure to the solution of symmetrical band systems. This approach results in a definite increase in capacity and speed over the iterative technique and it eliminates the problem of convergence.

After the equilibrium equations are solved for the unknown grid point displacements, the average stresses within each triangular ring are calculated as indicated in Part II of this report. Based on the stresses in the two triangular rings, average quadrilateral stresses are calculated for each quadrilateral ring in the system.

D. COMPUTER PROGRAM

The first step in the stress analysis of an axisymmetric heat shield is to select points at the bond line at regular intervals along the meridian of the shield. A quadrilateral mesh is automatically developed by the program from the R and Z coordinates. The material properties vs. temperature for the ablator and bond are supplied to the computer in tabular form and the computer program automatically determines analytical expressions for the properties by a least square procedure (see Appendix B for details of method). Finally, the grid points which are to be restrained and the external loads which act at grid points are specified.

1. Input Information

The following sequence of punched cards numerically defines the heat shield to be analyzed.

a. FIRST CARD - (72H)

Columns 1 to 72 of this card contains information to be printed with results

b. SECOND CARD - (6I5, 2F10.2)

Columns	1 - 5	Number of points along the shield - NMAX
	6 - 10	Number of points thru the thickness-MMAX
	11 - 15	Location of bond line - MB
	16 - 20	Number of material property cards - NP

21 - 25 Number of points with radial and axial loads - NL

26 - 30 Number of additional boundary conditions - NB

31 - 40 Surface temperature of ablator

41 - 50 Zero stress temperature

c. THIRD CARD - Properties of Sandwich Core (4F10.2)

Columns 1 - 10 Modulus of elasticity

11 - 20 Poisson's ratio

21 - 30 Coefficient of thermal expansion

31 - 40 Thickness of core

d. FOURTH CARD - Properties of Sandwich Face Plates (4F12.2)

Columns 1 - 10 Modulus of elasticity

11 - 20 Poisson's ratio

21 - 30 Coefficient of thermal expansion

31 - 40 Thickness of single face plate

e. GEOMETRY CARDS - (4F10.2)

One card per point along shield, in order from axis of symmetry to edge (NMAX cards).

Columns 1 - 10 R-ordinate at bond line

11 - 20 Z-ordinate at bond line

21 - 30 Temperature at bond line

31 - 40 Normal thickness of ablator

f. MATERIAL PROPERTY CARDS - (4F10.2)

One card for each temperature (NP cards)

Columns 1 - 10 Temperature

11 - 20 Modulus of elasticity of ablative material

21 - 30 Modulus of elasticity of bond material

31 - 41 Coefficient of thermal expansion for ablator and bond materials

g. LOAD CARDS - (2I5, 2F10.2)

One card for each point which is loaded externally (NL cards). N and M specify the grid location of the point.

Columns 1 - 5 N(Meridional direction)

6 - 10 M (Thickness direction)

11 - 20 Radial Load } Total load acting on one radian segment

21 - 30 Axial Load } Total load acting on one radian segment

h. BOUNDARY CONDITION CARDS - (3I5)

One card for each point which is restraint (NB cards). N and M specify the grid location of the point.

Columns 1 - 5 N (Meridional direction)

6 - 10 M (Thickness direction)

11 - 15 Boundary Code

Code = 1 point fixed in R-direction

Code = 2 point fixed in Z-direction

Code = 3 point fixed in both the R and Z-directions

2. <u>Output Information</u>

The following information is generated and printed by the computer program.

a. Input data

b. Least squares evaluation of the temperature dependent material property data

c. Coordinates and temperatures of all grid points

d. R and Z displacement at all grid points

e. Average stresses in quadrilateral rings

f. Stresses in sandwich face plates

g. Stresses in bond layer

3. <u>Timing</u>

The computer time required by this program for an axisymmetric analysis of a heat shield is approximately

$$\text{time} = A + B \cdot (\text{NMAX}) \cdot (\text{MMAX})^2 \text{ (Seconds)}$$

The constants A and B depend on the specific computer system which is employed. For the IBM 7094 A=20 and B=0.02, and the time required for a 30 x 7 mesh is 50 seconds.

4. <u>Program Listing</u>

A card listing of the FORTRAN II source deck for the automated computer program for the axisymmetric stress analysis of heat shields is given in Appendix E. The program is compiled for a maximum grid size of 40 points in the meridional direction and 10 points through the thickness. Material properties can be specified by a maximum of 50 cards.

E. EXAMPLES

Several axisymmetric analyses of a heat shield were conducted to evaluate the significance of the various structural parameters. A typical finite element idealization of the heat shield is shown in Figure 3.1.

1. Effect of Mesh Size

The first example was selected to illustrate the effect of mesh size on the accuracy of the displacements and stresses developed within the heat shield. For a structure fixed at the edge, typical results of two analyses with different meshes are shown in Figure 3.2. This example illustrates that two layers of elements in the sandwich shell are adequate for the purposes of predicting stresses. It is of interest to note that the stress distribution varies linearly within the sandwich shell, thereby confirming the assumption made in thin shell theory. The displacements for these two analyses differed by less than one percent.

2. Effect of Ablator Thickness on Stress Distribution

Figure 3.3 shows typical results of three analyses of heat shields with different ablator thicknesses. In general, the magnitude of

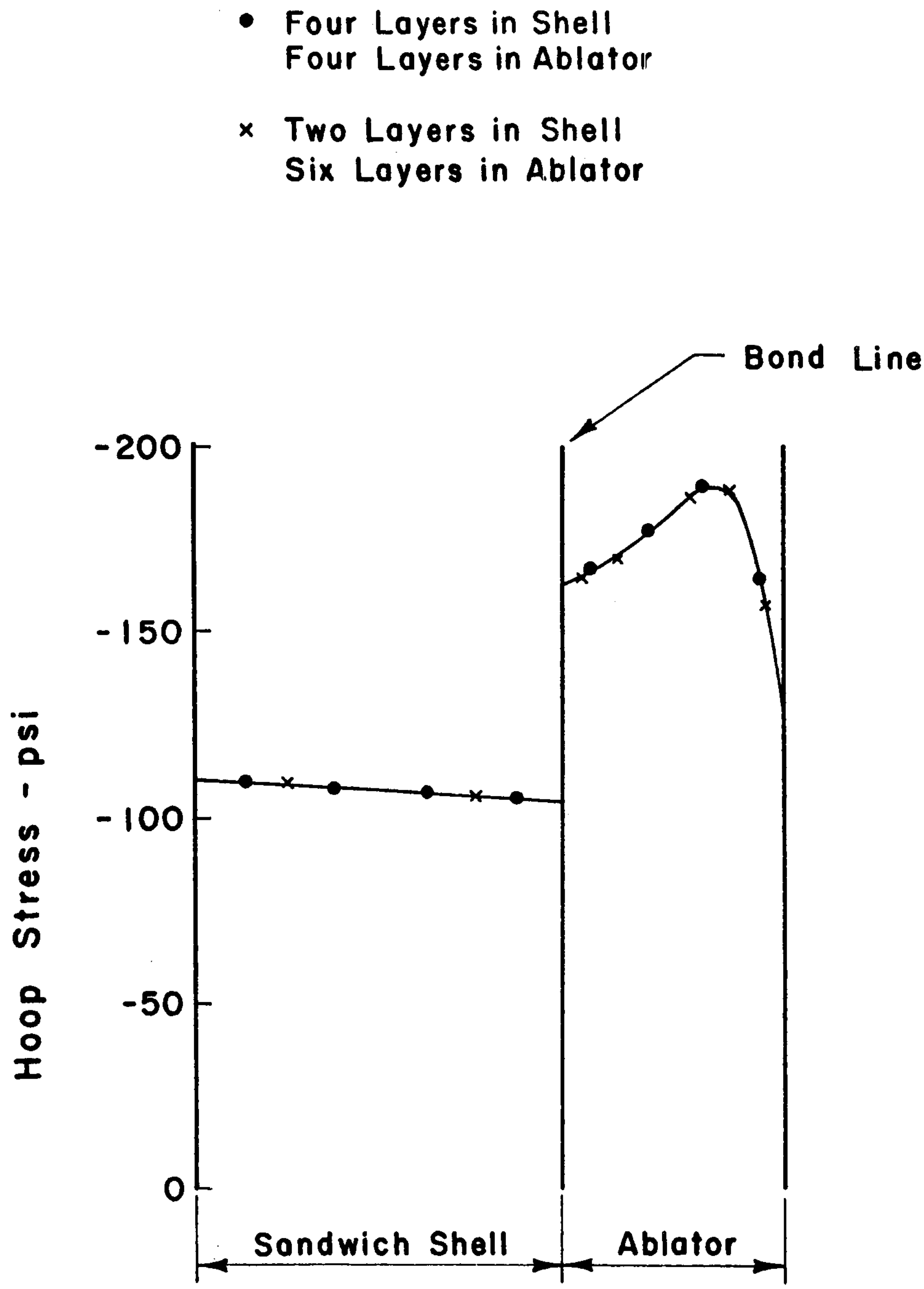


Fig. 3.2 Effect of Mesh Size on Stress Distribution

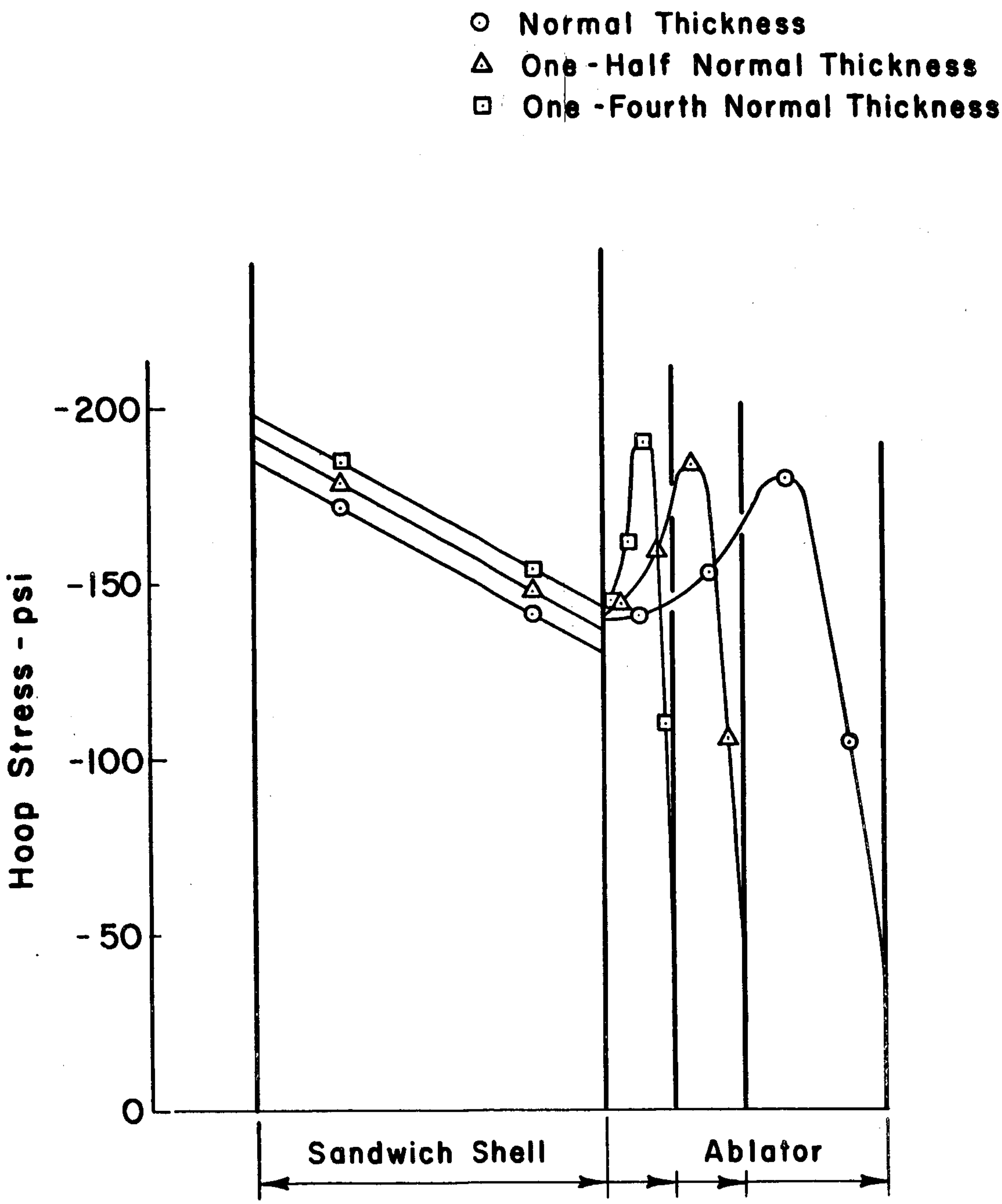


Fig. 3.3 Effect of Thickness of Ablator on Stress Distribution

the maximum stresses in the ablator were in good agreement. This example illustrates that the thickness of the ablator is not an important structural parameter at the temperature of re-entry.

3. Effect of Boundary Conditions on the Behavior of the Heat Shield

The support condition which is imposed on the heat shield is an extremely important parameter. Figure 3.4 illustrates the deflected position of the bond line for two different support conditions. The resulting stresses differ significantly. Therefore, it is important that the boundary condition which is imposed on the finite element system is a realistic approximation of the physical support condition which exists in the actual heat shield.

F. DISCUSSION

The automated computer program presented in this section reduces the analysis of an arbitrary heat shield subjected to axisymmetric thermal or mechanical loads to a simple procedure. The program automatically generates the finite element grid, evaluates temperature-dependent material properties, solves the equilibrium equations for the grid point displacements and calculates stresses within elements, sandwich shell face plates and at the bond layer. Arbitrary boundary conditions can be imposed since any of the grid points may be restrained in either the R or Z directions.

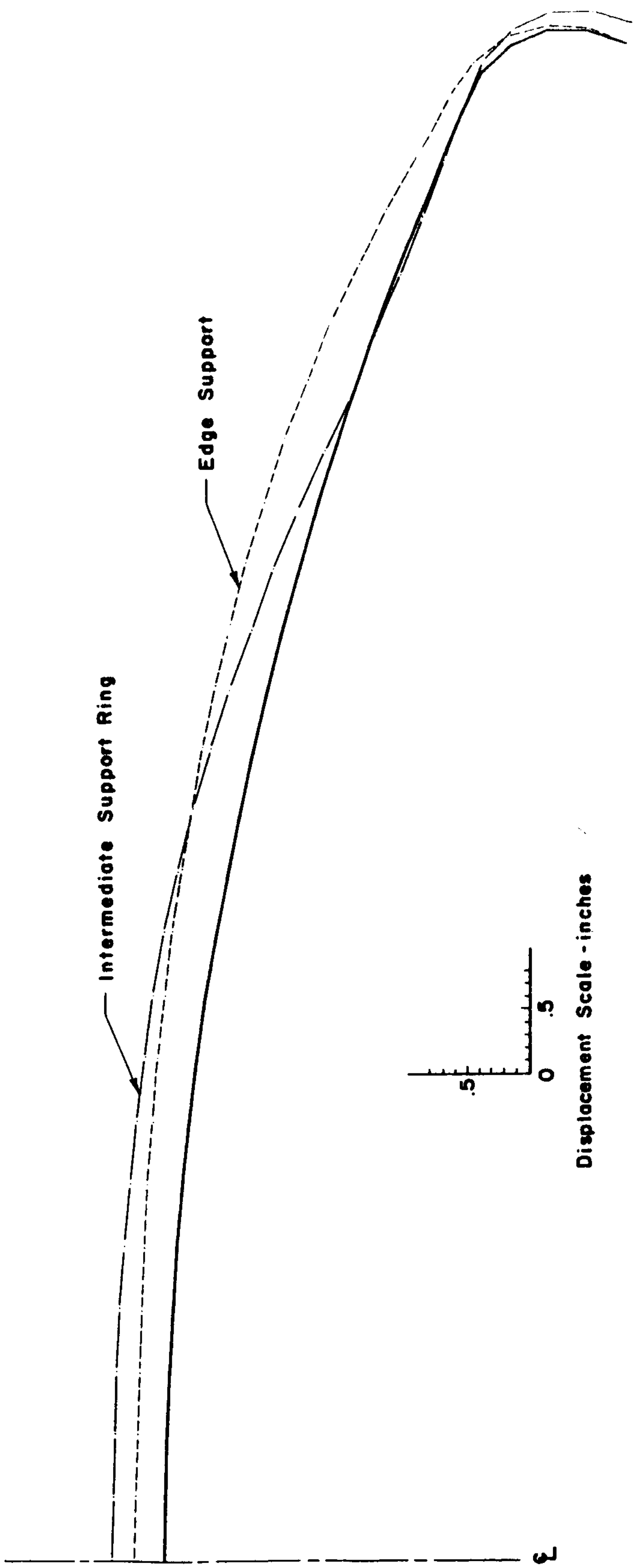


Fig. 3.4 Effect of Boundary Conditions on Deflected Shape (Bond Line)

PART IV: AUTOMATED PROGRAM FOR AXISYMMETRIC HEAT SHIELDS SUBJECTED TO NON-AXISYMMETRIC LOADS

A. INTRODUCTION

In general, the heat shield of a manned spacecraft is composed of a constant thickness sandwich shell and an ablator which varies in thickness in both the meridional and circumferential directions. The temperature distribution experienced by the heat shield is also non-axisymmetric. Because the ablator, at high temperatures, is not a major structural element, contributing to the overall behavior of the heat shield, an approximation of its properties in the circumferential direction is justified. The approximation, that it is axisymmetric, reduces the stress analysis of a non-axisymmetric heat shield to the stress analysis of an axisymmetric structure subjected to non-axisymmetric thermal loads. This involves the expansion of the temperature distribution and the final displacements of the system in Fourier series. By making use of the orthogonality properties of the harmonic functions the three-dimensional analysis is divided into a series of uncoupled two-dimensional analyses.

B. THEORY FOR THE ANALYSIS OF AN AXISYMMETRIC BODY SUBJECTED TO NON-AXISYMMETRIC LOADS

A theory is presented for the analysis of solids of revolution subjected to non-axisymmetric loads which are symmetric about a plane containing the axis of revolution. Figure 4.1, a view of a plane perpendicular to the axis of revolution, shows the trace of the plane of symmetry. Anisotropic material properties, which are constant along any circumferential line, are included in this formulation.

The structure is idealized as a series of rings with triangular cross-sections; the rings are interconnected at their nodal circles, i.e., at the circles containing the vertices of the triangles, (Figure 4.2). Loads acting on the structure are replaced by statically equivalent concentrated forces acting along the nodal circles.

1. Strain-Displacement Relationship

By noting the axisymmetry of the geometry and the material properties of the body and the plane of symmetry for deformations, the displacements in r, θ, z coordinates may be written in the following form:

$$u_r = \sum u_{rn}(r,z) \cos n\theta \qquad (4.1a)$$

$$u_\theta = \sum u_{zn}(r,z) \sin n\theta \qquad (4.1b)$$

$$u_z = \sum u_{\theta n}(r,z) \cos n\theta \qquad (4.1c)$$

Within each ring element the r and z variation of the Fourier coefficients of the displacements are assumed to be linear, i.e.,

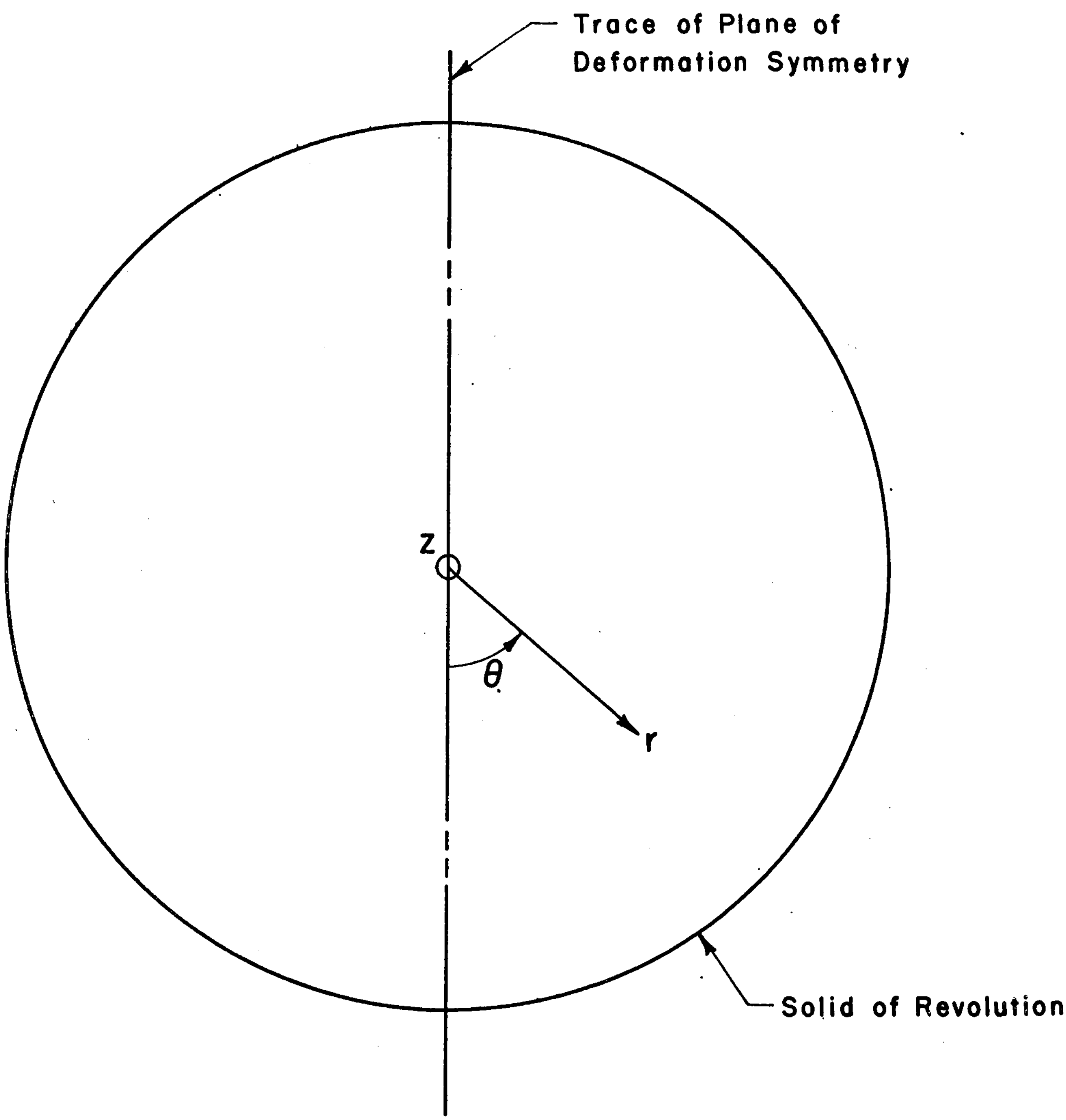


Fig. 4.1 Cylindrical Coordinate System Embedded in a Solid of Revolution

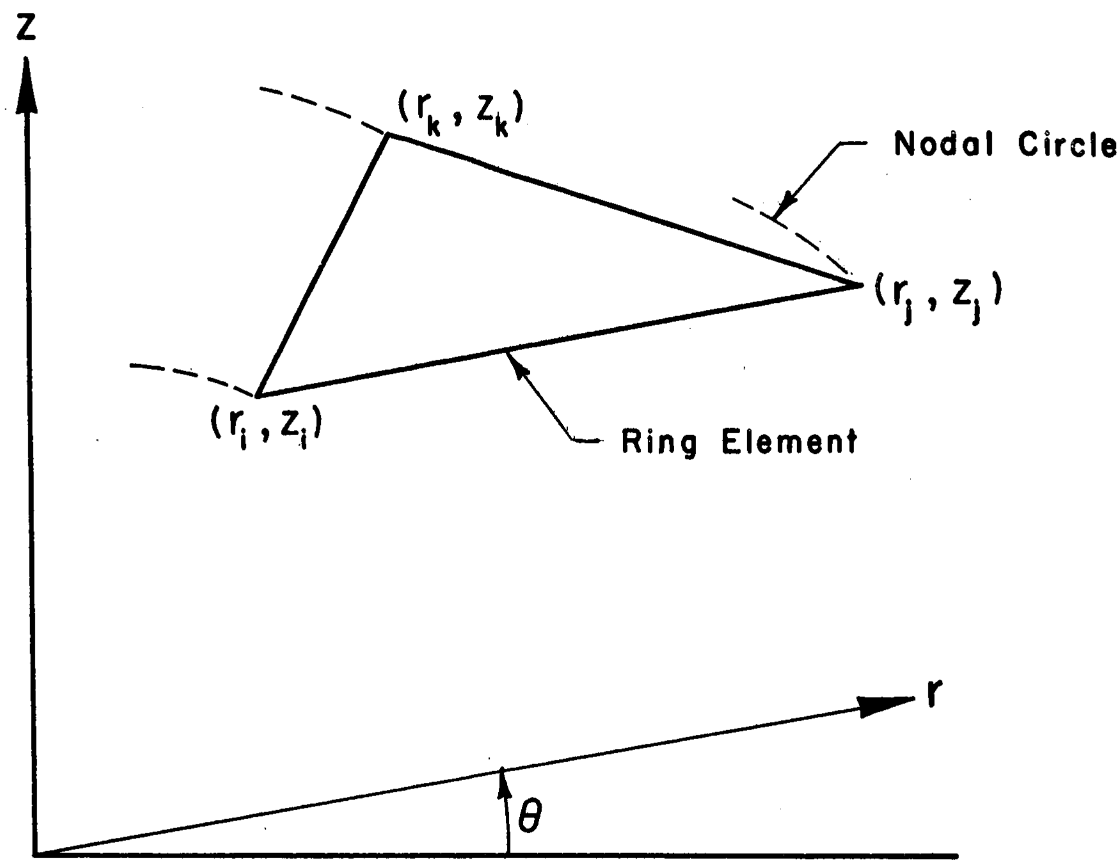


Fig. 4.2 Cross Section of a Ring Element

$$u_{rn} \approx k_{1n} + k_{2n}\, r + k_{3n} z \tag{4.2a}$$

$$u_{\theta n} \approx k_{4n} + k_{5n}\, r + k_{6n} z \tag{4.2b}$$

$$u_{zn} \approx k_{7n} + k_{8n}\, r + k_{9n} z \tag{4.2c}$$

Now expressing the constants k_{mn} in terms of the corner values of the Fourier coefficients of the displacements, i.e., in terms of

u^i_{rn}, $u^i_{\theta n}$, u^i_{zn}, u^j_{rn}, $u^j_{\theta n}$, u^j_{zn}, u^k_{rn}, $u^k_{\theta n}$ and u^k_{zn}

$$\begin{bmatrix} k_{1n} & k_{4n} & k_{7n} \\ k_{2n} & k_{5n} & k_{8n} \\ k_{3n} & k_{6n} & k_{9n} \end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} u^i_{rn} & u^i_{\theta n} & u^i_{zn} \\ u^j_{rn} & u^j_{\theta n} & u^j_{zn} \\ u^k_{rn} & u^k_{\theta n} & u^k_{zn} \end{bmatrix} \tag{4.3a}$$

with

$$\begin{bmatrix} T \end{bmatrix} = \frac{1}{D} \begin{bmatrix} r_j z_k - z_j r_k & r_k z_i - r_i z_k & r_i z_j - r_j z_i \\ z_j - z_k & z_k - z_i & z_i - z_j \\ r_k - r_j & r_i - r_k & r_j - r_i \end{bmatrix} \tag{4.3b}$$

and $$D = r_j(z_k - z_i) + r_i(z_j - z_k) + r_k(z_i - z_j) \tag{4.3c}$$

Combining Equation (4.1) with the strain-displacement relationships, the following expressions for the strains are found:

$$\epsilon_r = \frac{\partial u_r}{\partial r} = \sum \epsilon_{rn} \cos n\theta \tag{4.4a}$$

$$\epsilon_\theta = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} = \sum \epsilon_{\theta n} \cos n\theta \tag{4.4b}$$

$$\epsilon_z = \frac{\partial u_z}{\partial z} = \sum \epsilon_{zn} \cos n\theta \tag{4.4c}$$

$$\gamma_{r\theta} = \left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) = \sum \gamma_{r\theta n} \sin n\theta \tag{4.4d}$$

$$\gamma_{rz} = \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) = \sum \gamma_{rzn} \cos n\theta \tag{4.4e}$$

$$\gamma_{\theta z} = \left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) = \sum \gamma_{\theta zn} \sin n\theta \tag{4.4f}$$

where

$$\epsilon_{rn} = \frac{\partial u_{rn}}{\partial r} \tag{4.5a}$$

$$\epsilon_{\theta n} = \frac{1}{r}(nu_{\theta n} + u_r) \tag{4.5b}$$

$$\epsilon_{zn} = \frac{\partial u_{zn}}{\partial z} \tag{4.5c}$$

$$\gamma_{r\theta n} = \frac{\partial u_{\theta n}}{\partial r} - \frac{u_{\theta n}}{r} - \frac{nu_{rn}}{r} \tag{4.5d}$$

$$\gamma_{rzn} = \frac{\partial u_{zn}}{\partial r} + \frac{\partial u_{rn}}{\partial z} \tag{4.5e}$$

$$\gamma_{\theta zn} = \frac{\partial u_{\theta n}}{\partial z} - n\frac{u_{zn}}{r} \tag{4.5f}$$

Within a given ring the approximate values of strain for the harmonic n are calculated by combining Equations (4.2), (4.3) and (4.5). The hoop strain is assumed to be constant within the ring and $\frac{u_{rn}}{r}$ is approximated by

$$\frac{u^i_{rn}}{3\bar{r}} + \frac{u^j_{rn}}{3\bar{r}} + \frac{u^k_{rn}}{3\bar{r}}, \quad \text{with} \quad \bar{r} = \frac{1}{3}(r_i + r_j + r_k)$$

Thus, for the harmonic n the six components of strain within the element are given in terms of the nine corner displacements by the following matrix equation:

$$\left[\epsilon_n\right] = \left[G_n\right]\left[u_n\right] \tag{4.6a}$$

The strain-displacement transformation matrix (for convenience it is written in its transposed form) is defined on the next page, Eq. (4.6b).

2. Stress-Strain Relationship

The stress-strain relationship for the harmonic n is written in the following symbollic form:

$$\left[G_n\right] = \left[C\right]\left[\epsilon_n\right] + \left[\tau_n\right] \tag{4.7a}$$

For an isotropic material this becomes

$$\begin{bmatrix} \sigma_{rn} \\ \sigma_{\theta n} \\ \sigma_{zn} \\ \sigma_{r\theta n} \\ \sigma_{rzn} \\ \sigma_{\theta zn} \end{bmatrix} = \begin{bmatrix} \bar{\alpha} & \bar{\beta} & \bar{\beta} & 0 & 0 & 0 \\ \bar{\beta} & \bar{\alpha} & \bar{\beta} & 0 & 0 & 0 \\ \bar{\beta} & \bar{\beta} & \bar{\alpha} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \epsilon_{rn} \\ \epsilon_{\theta n} \\ \epsilon_{zn} \\ \epsilon_{r\theta n} \\ \epsilon_{rzn} \\ \epsilon_{\theta zn} \end{bmatrix} + \begin{bmatrix} \tau_n \\ \tau_n \\ \tau_n \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.7b}$$

$$
\left[G_n\right]^T = \begin{bmatrix}
\frac{z_j - z_k}{D} & \frac{1}{3\bar{r}} & 0 & -\frac{n}{3\bar{r}} & \frac{r_k - r_j}{D} & 0 \\
\frac{z_k - z_i}{D} & \frac{1}{3\bar{r}} & 0 & -\frac{n}{3\bar{r}} & \frac{r_i - r_k}{D} & 0 \\
\frac{z_i - z_j}{D} & \frac{1}{3\bar{r}} & 0 & -\frac{n}{3\bar{r}} & \frac{r_j - r_i}{D} & 0 \\
0 & \frac{1}{3\bar{r}} & 0 & \left(\frac{z_j - z_k}{D} - \frac{1}{3\bar{r}}\right) & 0 & \frac{r_k - r_j}{D} \\
0 & \frac{n}{3\bar{r}} & 0 & \left(\frac{z_j - z_k}{D} - \frac{1}{3\bar{r}}\right) & 0 & \frac{r_k - r_j}{D} \\
0 & \frac{n}{3\bar{r}} & 0 & \left(\frac{z_k - z_i}{D} - \frac{1}{3\bar{r}}\right) & 0 & \frac{r_i - r_k}{D} \\
0 & 0 & \frac{r_k - r_j}{D} & 0 & \frac{z_j - z_k}{D} & -\frac{n}{3\bar{r}} \\
0 & 0 & \frac{r_i - r_k}{D} & 0 & \frac{z_k - z_i}{D} & -\frac{n}{3\bar{r}} \\
0 & 0 & \frac{r_j - r_i}{D} & 0 & \frac{z_i - z_j}{D} & -\frac{n}{3\bar{r}}
\end{bmatrix} \tag{4.6b}
$$

where
$$\bar{\alpha} = \frac{(1-\nu)\, E}{(1+\nu)(1-2\nu)} \tag{4.8a}$$

$$\bar{\beta} = \frac{\nu \quad E}{(1+\nu)(1-2\nu)} \tag{4.8b}$$

$$\mu = \frac{E}{2(1+\nu)} \tag{4.8c}$$

$$\tau_n = - \frac{E\alpha}{(1-2\nu)} \; T_n \tag{4.8d}$$

T_n is the Fourier coefficient for the expansion of the average temperature change within the ring

$$T = \sum T_n \cos n\theta \tag{4.9}$$

3. <u>Equilibrium Equation for Harmonic n</u>

By recognizing the othogonality of the harmonic functions the same procedure which was used in Part I may be used to develop the equilibrium equations for an element subjected to harmonic loading. Therefore, Equation (1.15) is rewritten as

$$\left[S_n\right] = \left[k_n\right]\left[u_n\right] + \left[L_n\right] \tag{4.10}$$

where
$$\left[k_n\right] = \int \left[G_n\right]^T\left[C\right]\left[G_n\right] dV \tag{4.11}$$

$$\left[L_n\right] = \int \left[G_n\right]^T\left[\tau_n\right] dV \tag{4.12}$$

Within a ring the matrices $\left[G_n\right]$ and $\left[C\right]$ are not a function of space; therefore, Equations (4.11) and (4.12) reduce to

$$\left[k_n\right] = V \cdot \left[G_n\right]^T \left[C\right] \left[G_n\right] \qquad (4.13)$$

$$\left[L_n\right] = V \cdot \left[G_n\right]^T \left[\tau_n\right] \qquad (4.14)$$

where the volume V is given by Equation (2.10). Equilibrium of the overall structure requires that the sum of the nodal circle forces for all rings with a common nodal circle must equal the applied nodal force. This results on an equation of the following form for each harmonic:

$$\left[K_n\right] \left[r_n\right] = \left[R_n\right] \qquad (4.15)$$

where the displacement vector $\left[r_n\right]$ contains all the displacement amplitudes u_{rn}, $u_{\theta n}$ and u_{zn} for all nodal circles in the system.

The equilibrium of the face plates is incorporated by a similar procedure. Appendix C gives the details of this development.

4. Determination of Displacements and Stresses

The number of harmonics required to represent the three-dimensional temperature distribution indicates the number of two-dimensional problems which must be solved. For each harmonic Equation (4.15) must be solved for the unknown displacement amplitudes. The corresponding strain amplitudes are calculated for each finite element by Equation (4.6) and then stress amplitudes are found by the application of Equation (4.7). The final displacements of the system for any angle are calculated from Equation (4.1). The final stresses are determined from the stress amplitudes by the following equations:

$$\sigma_r(r,z,\theta) = \sum \sigma_{rn} \cos n\theta \quad (4.16a)$$

$$\sigma_z(r,z,\theta) = \sum \sigma_{zn} \cos n\theta \quad (4.16b)$$

$$\sigma_\theta(r,z,\theta) = \sum \sigma_{\theta n} \sin n\theta \quad (4.16c)$$

$$\sigma_{rz}(r,z,\theta) = \sum \sigma_{rzn} \cos n\theta \quad (4.16d)$$

$$\sigma_{r\theta}(r,z,\theta) = \sum \sigma_{r\theta n} \sin n\theta \quad (4.16e)$$

$$\sigma_{\theta z}(r,z,\theta) = \sum \sigma_{\theta zn} \sin n\theta \quad (4.16f)$$

C. COMPUTER PROGRAM

The use of the non-axisymmetric heat shield program is similar to the axisymmetric program (Part III). The only additional input required is the three-dimensional temperature distribution. The computer program automatically develops the necessary Fourier coefficients for the temperature distribution and sums the series of two-dimensional analyses to produce the final displacements and stresses in the system.

1. Input Information

The following sequence of punched cards numerically defines the heat shield to be analyzed:

a. FIRST CARD - (72H)

Columns 1 to 72 of this card contains information to be pointed with results

b. SECOND CARD - (6I5, 2F10.2)

Columns	1 - 5	Number of points along meridian of shield - NMAX
	6 - 10	Number of points through thickness - MMAX
	11 - 15	Location of bond line - MB
	16 - 20	Number of material property cards - NP
	21 - 25	Number of harmonic to be used in analysis - NL
	26 - 30	Number of boundary condition cards - NB
	31 - 40	Surface temperature of ablator
	41 - 50	Temperature of zero stress

c. THIRD CARD - Properties of Sandwich Core (4F10.2)

Columns	1 - 10	Modulus of elasticity
	11 - 20	Poisson's ratio
	21 - 30	Coefficient of thermal expansion
	31 - 40	Thickness of core

d. FOURTH CARD - Properties of Sandwich Fact Plates (4F10.2)

Columns	1 - 10	Modulus of elasticity
	11 - 20	Poisson's ratio
	21 - 30	Coefficient of thermal expansion
	31 - 40	Thickness of single face plate

e. GEOMETRY CARDS - (4F10.2)

One card per point along shield in order from axis of symmetry to edge (NMAX cards).

Columns 1 - 10 R-ordinate at bond line

11 - 20 Z-ordinate at bond line

21 - 30 Temperature at bond line

31 - 40 Normal thickness of ablator

The temperature information is used by the program to determine the axisymmetric temperature-dependent material properties.

f. MATERIAL PROPERTY CARDS - (4F10.2)

One card for each temperature (NP cards)

Columns 1 - 10 Temperature

11 - 20 Modulus of elasticity of ablative material

21 - 30 Modulus of elasticity of bond material

31 - 41 Coefficient of thermal expansion for ablative and bond materials

g. THREE-DIMENSIONAL TEMPERATURE DISTRIBUTION CARDS

A table of bond line temperature values at 10 degree increments along 9 circumferential lines is punched in the following form:

<u>1st. card - (9F8.0)</u>

R-ordinates of 9 circumferential points on bond line

<u>2nd card - (9F8.0)</u>

Z-ordinates of 9 circumferential points on bond line

<u>3rd to 21st card - (9F8.0)</u>

One card for each 10 degree increment (0 to 180^{o}). Each card contains the 9 temperatures which correspond to the above R and Z-ordinates.

h. BOUNDARY CONDITION CARDS - (3I5)

One card per restrained nodal circle (NB cards)

Column 1 - 5 N } mesh point N, M
6 - 10 M

11 - 15 = 1 restrained in R-direction
2 restrained in θ-direction
3 restrained in Z-direction
4 restrained in R and θ-directions
5 restrained in R and Z-directions
6 restrained in θ and Z-directions
7 restrained in R, θ and Z directions

i. PRINT ANGLE CARDS - (1F5.0)

Column 1 - 5 angle θ

For each "print angle card" a complete set of displacement and stresses are printed for angle θ.

2. Output Information

The following information is generated and printed by the computer program:

a. Input data
b. Least squares evaluation of the temperature-dependent material property data
c. Coordinates and temperature of all grid points
d. Two-dimensional Fourier temperature coefficients
e. For each print angle
 (1) R, Z, and θ displacement at all grid points
 (2) Average stresses in quadrilatoral rings
 (3) Stresses in sandwich face plates

3. Timing

The computer time required by this program for the non-axisymmetric analysis of a heat shield is approximately

$$\text{time} = A + B \cdot (\text{NMAX}) \cdot (\text{MMAX})^2 \cdot \text{NL} \quad \text{(seconds)}$$

For the IBM 7094 computer A=20 and B=0.05, and the time required for a 30 x 7 mesh with 4 harmonics is approximately 5 minutes.

4. Program Listing

A card listing of the FORTRAN II source deck for the computer program for the non-axisymmetric analysis of heat shields is given in Appendix E. The program is compiled for a maximum grid size of 30 points in the meridional direction and 8 points through the thickness. A maximum of 10 harmonics may be considered. Material properties can be specified by a maximum of 50 cards.

Standard input tape 5 and output tape 6 are used by the program. Tape 20 is used for temporary storage within the program; it may be necessary to change this tape unit to conform with local computer center policy.

D. EXAMPLES

Two analyses of axisymmetric heat shields subjected to non-axisymmetric temperature distribution were conducted to illustrate the application of the program. In both cases, the axisymmetric finite element mesh was similar to the mesh given by Figure 3.1. For the purpose of reference the station layout along the bond line is shown in Figure 4.3. The three-dimensional bond layer temperature and ablator thickness distribution for angles $\theta = 0, 90^\circ, 180^\circ$ are plotted in Figure 4.4.

In Analysis A the axisymmetric properties of the ablator are assumed to be equal to the properties of the actual ablator at $\theta = 0$. In Analysis B the ablator properties at $\theta = 180^\circ$ are used. For both analyses the surface temperature of the ablator is 1000 F and the temperature at the bond surface is given by Figure 4.5. Station 20 (Figure 4.3) is restrained at the inside surface of the shield to simulate the effect of an intermediate support ring.

The computer output for a non-axisymmetric analysis contains displacements and stresses at many points in the heat shield; however, only the typical results are presented. For Analysis A the deflected shape of the bond line at three sections is plotted in Figure 4.6a. The non-axisymmetric behavior is significant. The displacements from Analysis B are shown in Figure 4.6b; they are essentially the same as those found by Analysis A. This again indicates that the ablator's thickness and property variations in the circumferential direction are not of major importance at these temperatures. Hoop stresses at station 15 are

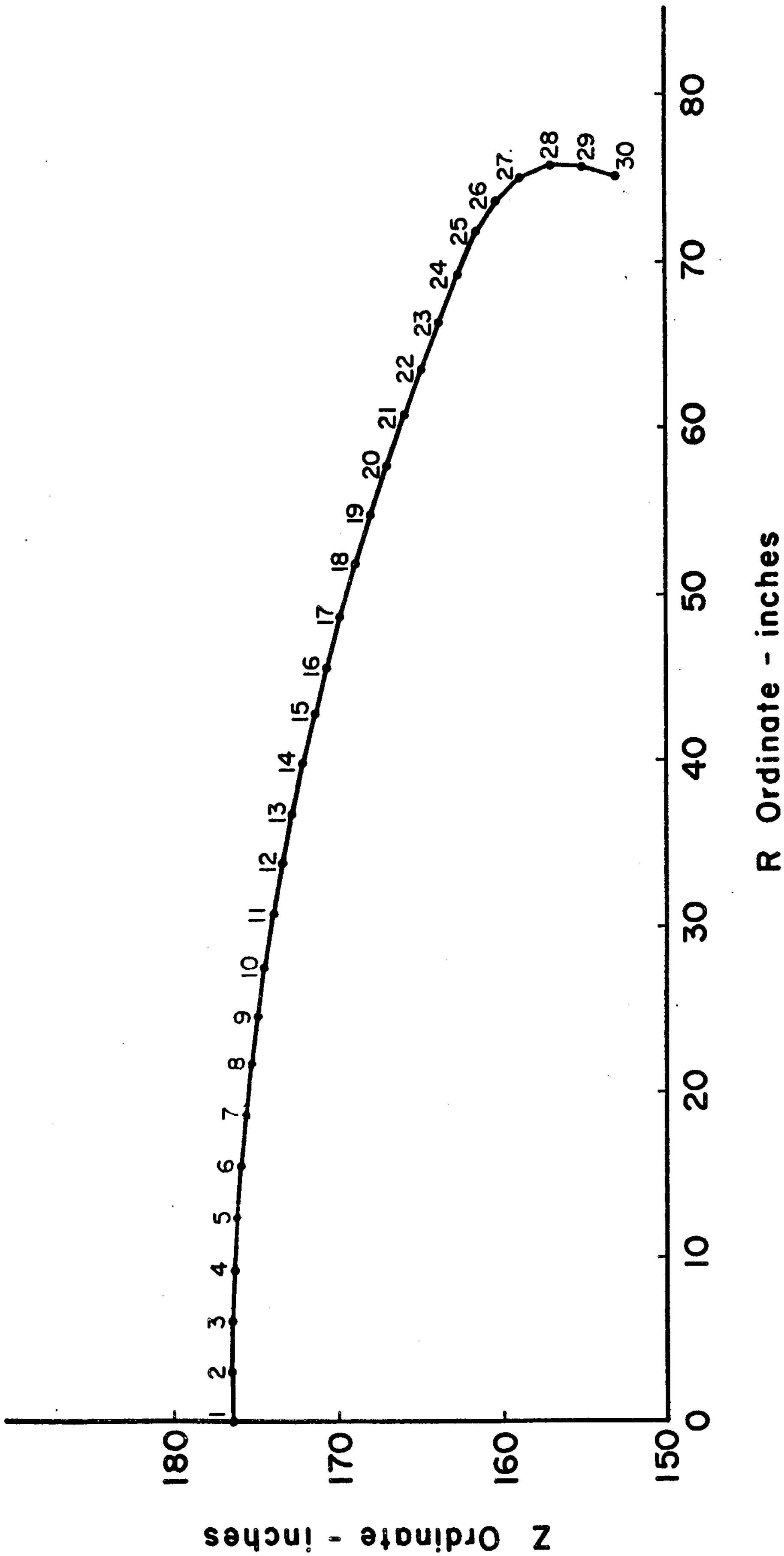


Fig. 4.3 Station along Bond Line

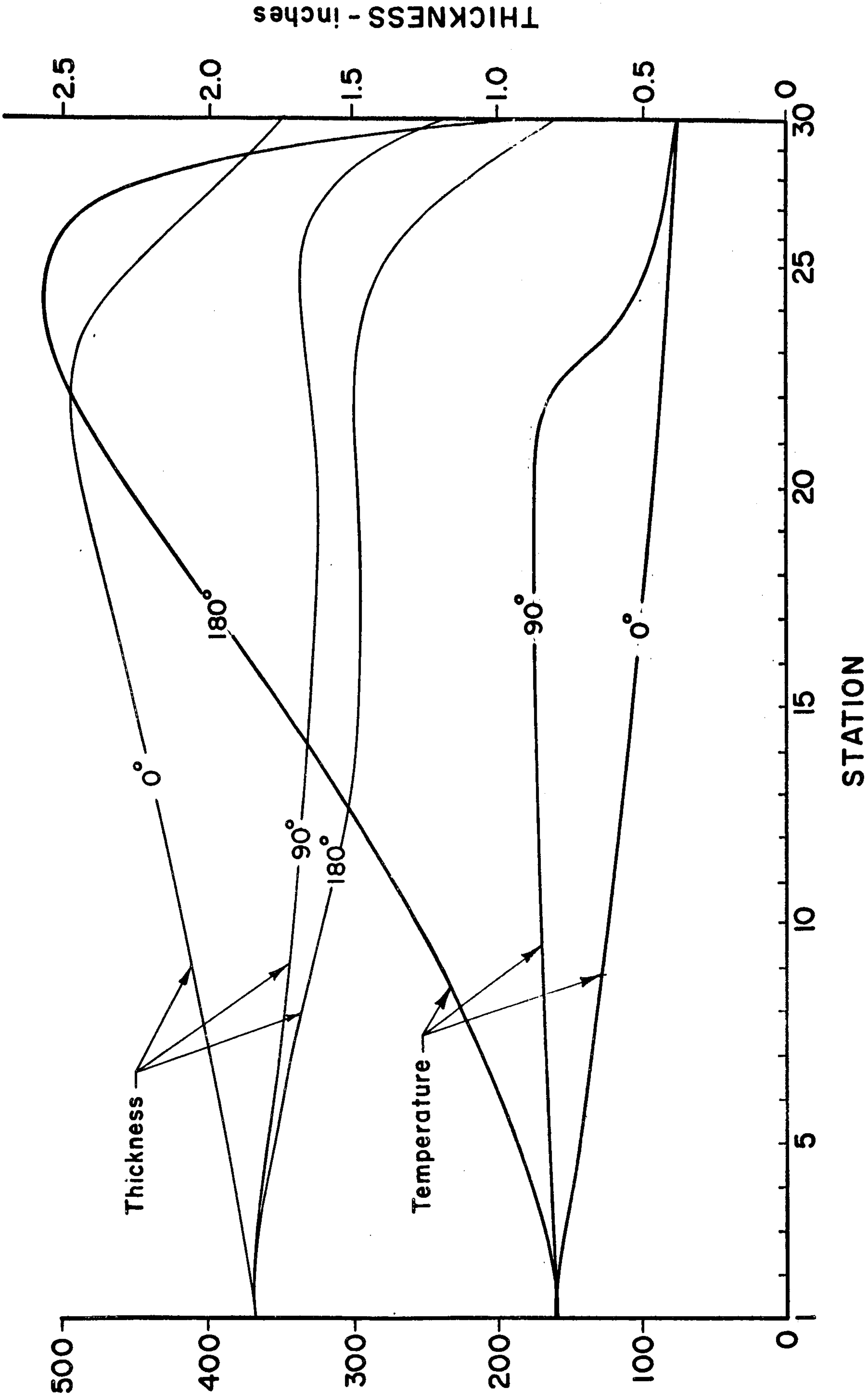


Fig. 4.4 Bond Layer Temperature and Ablator Thickness Distribution

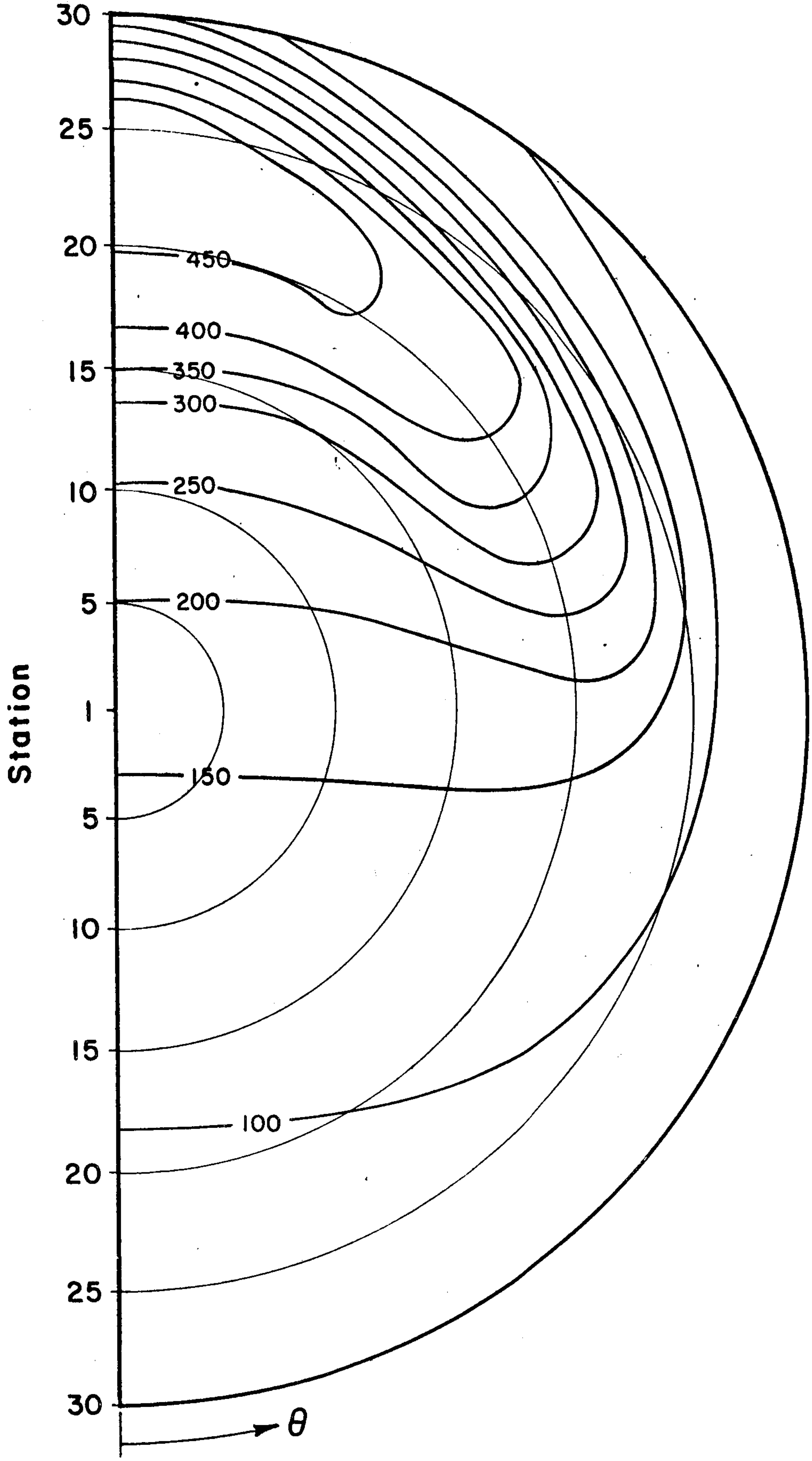


Fig. 4.5 Temperature at Bond Surface

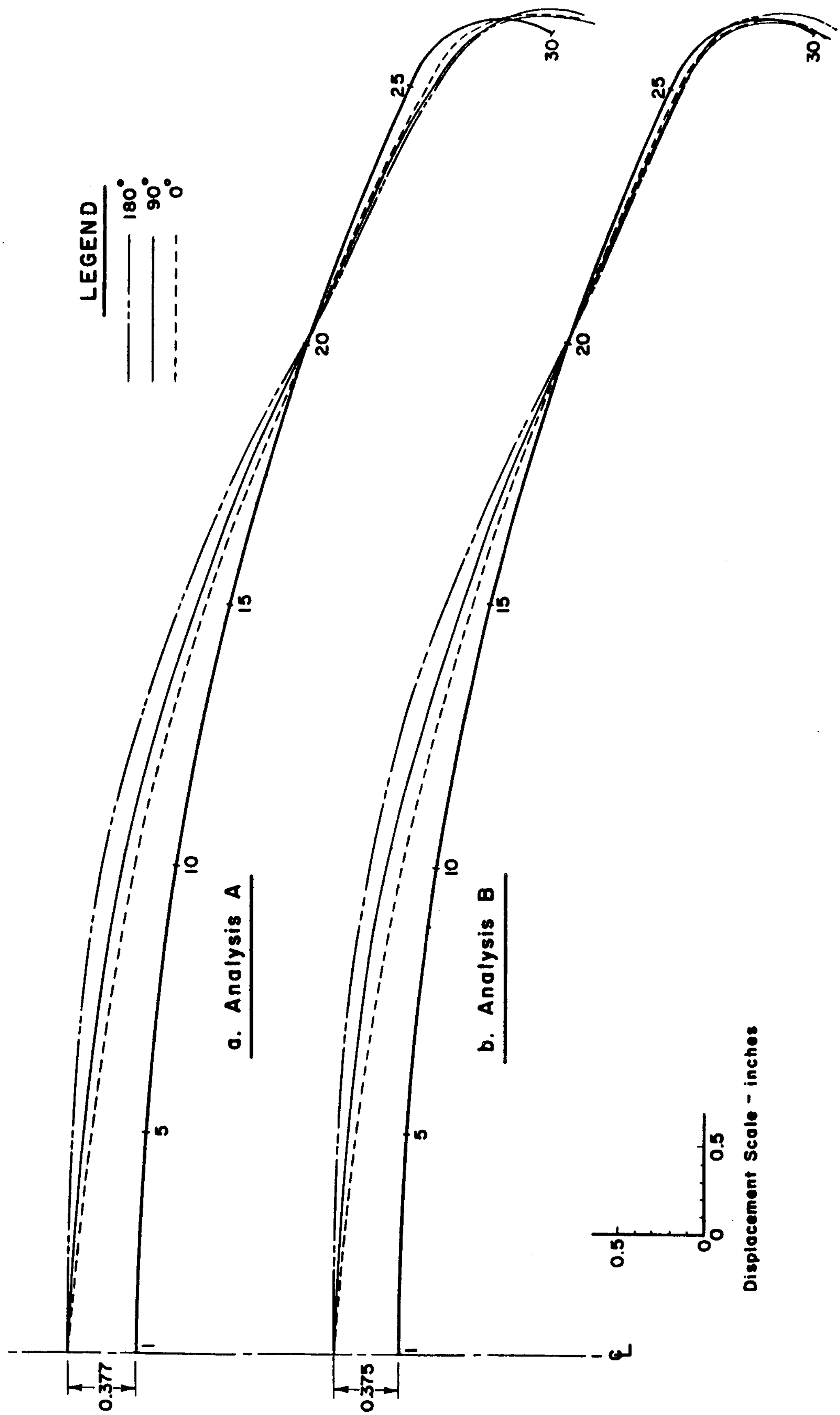
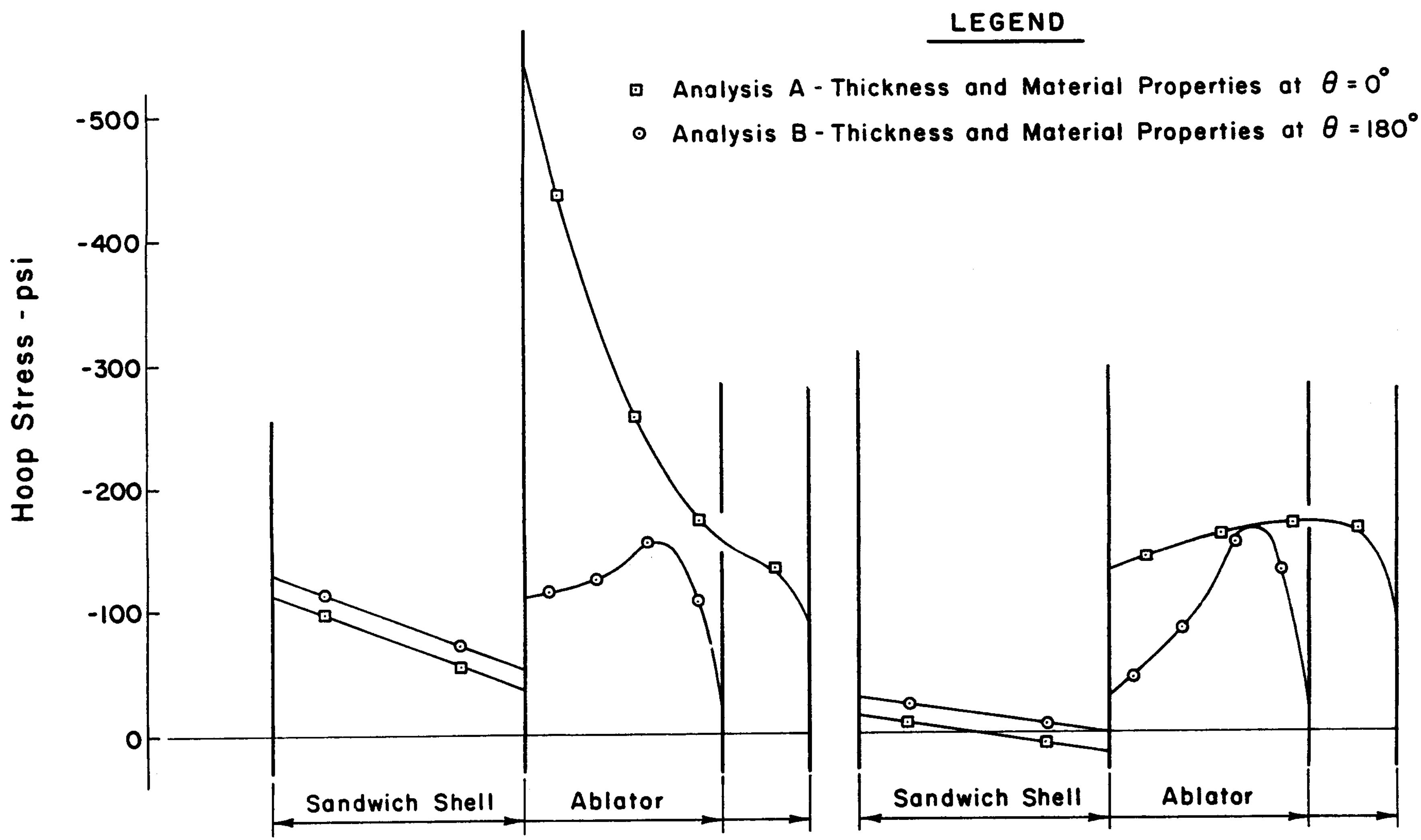


Fig. 4.6 Deflected Shape at Bond Surface

plotted in Figure 4.7 for two values of θ. In both analyses the stresses within the sandwich shell are in fair agreement since the material properties do not change at these temperatures. However, within the ablator, where the material properties are strongly temperature-dependent, the stresses differ significantly. Of course, the particular solution is most reasonable if the assumed material properties correspond with those at the position of the desired stress. Hence, for $\theta = 0$, Analysis A is considered the best approximation and similarly for $\theta = 180^{\circ}$, Analysis B is the most reasonable.

E. DISCUSSION

In this section a method and the resulting computer program are presented for the analysis of axisymmetric heat shields subjected to non-axisymmetric thermal loads. The program may be used to analyze an approximate solution to a non-axisymmetric heat shield if a number of solutions are obtained and then are judicially evaluated. At the section where the assumed axisymmetric ablator properties (temperature and thickness) correspond to the local properties, the stresses will be a good approximation of the actual state of stress in the non-axisymmetric heat shield. Therefore, for each angle θ for which stresses are desired a separate structure must be evaluated.



a. Hoop Stress - Station at $\theta = 180°$

b. Hoop Stress - Station at $\theta = 0°$

Fig. 4.7 Typical Results of Non-Axisymmetric Heat Shield Analysis

It should be pointed out that the computer program can be extended to include non-axisymmetric pressure loading, displacement boundary conditions and the effects of elastic supports. However, this additional investigation was beyond the scope of the present effort.

APPENDIX A

SOLUTION OF EQUILIBRIUM EQUATIONS

The equilibrium equations for a system of finite elements may be written in the following form:

$$A_{11} X_1 + A_{12} X_2 + A_{13} X_3 \text{-----------------} + A_{1N} X_N = B_1 \qquad \text{(A1a)}$$

$$A_{21} X_1 + A_{22} X_2 + A_{23} X_3 \text{-----------------} + A_{2N} X_N = B_2 \qquad \text{(A1b)}$$

$$A_{31} X_1 + A_{32} X_2 + A_{33} X_3 \text{-----------------} + A_{3N} X_N = B_3 \qquad \text{(A1c)}$$

$$A_{N1} X_1 + A_{N2} X_2 + A_{N3} X_3 \text{-----------------} + A_{NN} X_N = B_N \qquad (-)$$

or symbolically

$$\left[A\right]\left[X\right] = \left[B\right] \qquad \text{(A1)}$$

where $\left[A\right]$ = the stiffness matrix

$\left[X\right]$ = the unknown displacements

$\left[B\right]$ = the applied loads

Gaussian Elimination

The first step in the solution of the above set of equations is to solve Equation (A1a) for X_1, Or

$$X_1 = B_1/A_{11} - (A_{12}/A_{11})\, X_2 - (A_{13}/A_{11})\, X_3 \text{ -------------- } (A_{1N}/A_{11})\, X_N \qquad \text{(A2)}$$

If Equation (A2) is substituted into Equations (A1b, c, ..., N) a modified set of N-1 equations is determined.

$$A^1_{22}\, X_2 + A^1_{23}\, X_3 \text{ -------------- } + A^1_{2N}\, X_N = B^1_1 \qquad \text{(A3a)}$$

$$A^1_{32}\, X_2 + A^1_{33}\, X_3 \text{ -------------- } + A^1_{3N}\, X_N = B^1_2 \qquad \text{(A3b)}$$

$$A^1_{N2}\, X_2 + A^1_{N3}\, X_3 \text{ -------------- } + A^1_{NN}\, X_N = B^1_N$$

where

$$A^1_{ij} = A_{ij} - A_{i1}\, A_{1j}/A_{11} \quad i, j = 2, \ldots, N \qquad \text{(A4a)}$$

$$B^1_i = B_i - A_{i1}\, B_1/A_{11} \quad i = 2, \ldots, N \qquad \text{(A4b)}$$

A similar procedure is used to eliminate X_2 from Equation (A3), etc.

A general algorithm for the elimination of X_n may be written as

$$X_n = (B_n^{n-1}/A_{nn}^{n-1}) - \sum (A_{nj}^{n-1}/A_{nn}^{n-1})\, X_j \qquad j = n+1, \ldots, N \tag{A5}$$

$$A_{ij}^{n} = A_{ij}^{n-1} - A_{in}^{n-1}\,(A_{nj}^{n-1}/A_{nn}^{n-1}) \qquad i, j = n+1, \ldots, N \tag{A6}$$

$$B_i^{n} = B_i^{n-1} - A_{in}^{n-1}\,(B_n^{n-1}/A_{nn}^{n-1}) \qquad i = n+1, \ldots, N \tag{A7}$$

Equations A5, A6 and A7 may be rewritten in compact form:

$$X_n = D_n - \sum C_{nj}\, X_j \qquad j = n+1, \ldots, N \tag{A8}$$

$$A_{ij}^{n} = A_{ij}^{n-1} - A_{in}^{n-1}\, C_{nj} \qquad i, j = n+1, \ldots, N \tag{A9}$$

$$B_i^{n} = B_i^{n-1} - A_{in}^{n-1}\, D_n \qquad i = n+1, \ldots, N \tag{A10}$$

where

$$D_n = B_n^{n-1}/A_{nn}^{n-1}$$

$$C_{nj} = A_{nj}^{n-1}/A_{nn}^{n-1}$$

After the above procedure is applied N-1 times the original set of equations is reduced to the following single equation

$$A_{NN}^{N-1}\, X_N = B_N^{N-1}$$

which is solved directly for X_N

$$X_N = B_N^{N-1}/A_{NN}^{N-1}$$

In terms of the previous notation, this is

$$X_N = D_N \tag{A11}$$

The remaining unknowns are determined in reverse order by the repeated application of Equation (A8).

Simplification for Band Matrices

For many finite element systems it is possible to place the stiffness matrix in a "band" form which results in the concentration of the elements of the stiffness matrix along the main diagonal. Therefore, the following simplifications in the general algorithm (Equations A8, A9 and A10) are possible:

$$X_n = D_n - \sum C_{nj} X_j \qquad j = n+1, \ldots, n+M-1 \tag{A12}$$

$$A_{ij}^n = A_{ij}^{n-1} - A_{in}^{n-1} C_{nj} \qquad i, j = n+1, \ldots, n+M-1 \tag{A13}$$

$$B_i^n = B_i^{n-1} - A_{in}^{n-1} D_n \qquad i = n+1, \ldots, n+M-1 \tag{A14}$$

where M is the band width of the matrix.

The number of numerical operations can further be reduced by recognizing that the reduced matrix at any stage of procedure is symmetric. Accordingly, Equation (A13) may be replaced by the following equation:

$$A_{ij}^{n} = A_{ij}^{n-1} - A_{in}^{n-1} C_{nj} \qquad \begin{aligned} i &= n+1, \ldots., n+M-1 \\ j &= i, \ldots., n+M-1 \end{aligned} \qquad \text{(A15)}$$

since

$$A_{ji}^{n} = A_{ij}^{n}$$

The number of numerical operations required for the solution of a band matrix is proportional to NM^2 as compared to N^3 which is required for the solution of a full matrix. Also, the computer storage required by the band matrix procedure is NM as compared to N^2 required by a set of N arbitrary equations.

This technique has been used in the automated axisymmetric program for the analysis of a typical heat shield idealized by a 10 x 40 mesh of quadrilateral elements. A solution to 800 simultaneous equations was necessary, which required less than two minutes of computing time on the IBM 7094.

APPENDIX B

MATRIX FORMULATION OF THE LEAST SQUARE CURVE-FIT PROCEDURE

Consider the problem of selecting the "best" polynomial of the form $y = C_1 + C_2X + C_3X^2 + C_4X^3, \ldots, C_NX^{N-1}$ to represent the following set of data points:

$$X_1, Y_1; X_2; Y_2; \text{-------------------}; X_M, Y_M$$

If the above polynomial is evaluated at points X_1 to X_M, M equation of the following form are found:

$$C_1 + C_2\,X_m + C_2\,X_m^2 + \text{-------------} C_N\,X_m^{N-1} = Y_m \quad m = 1, \ldots, M$$

or in matrix form

$$\begin{bmatrix} 1 & X_1 & X_1^2 & \text{-----} & X_1^{N-1} \\ 1 & X_2 & X_2^2 & \text{-----} & X_2^{N-1} \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ 1 & X_m & X_m^2 & \text{------} & X_m^{N-1} \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ 1 & X_M & X_M^2 & \text{------} & X_M^{N-1} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ - \\ - \\ - \\ C_N \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \text{---} \\ Y_m \\ \text{---} \\ Y_M \end{bmatrix} \qquad \text{(B1a)}$$

or symbolically

$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} C \end{bmatrix} = \begin{bmatrix} Y \end{bmatrix} \tag{B1b}$$

where

$$\begin{bmatrix} A \end{bmatrix} = \text{a } M \times N \text{ matrix}$$

$$\begin{bmatrix} C \end{bmatrix} = \text{a } N \times 1 \text{ matrix}$$

$$\begin{bmatrix} Y \end{bmatrix} = \text{a } M \times 1 \text{ matrix}$$

If Equation (B1) is premultiplied by $\begin{bmatrix} A \end{bmatrix}^T$, a set of N linear equations in N unknowns is created. Consequently,

$$\begin{bmatrix} B \end{bmatrix} \begin{bmatrix} C \end{bmatrix} = \begin{bmatrix} D \end{bmatrix} \tag{B2}$$

where

$$\begin{bmatrix} B \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}^T \begin{bmatrix} A \end{bmatrix} = \text{a } N \times N \text{ matrix}$$

$$\begin{bmatrix} D \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}^T \begin{bmatrix} Y \end{bmatrix} = \text{a } N \times 1 \text{ matrix}$$

Equation (B2) can now be solved directly for the unknown coefficients $\begin{bmatrix} C \end{bmatrix}$.

This procedure is numerically equivalent to the standard least square procedure. However, it is presented here in a form which is readily programmed for the digital computer The technique is not restricted to polynomials.

Figure B1 illustrates the application of the method in the evaluation of the elastic modulus for temperature dependent materials. A fourth order polynomial was used.

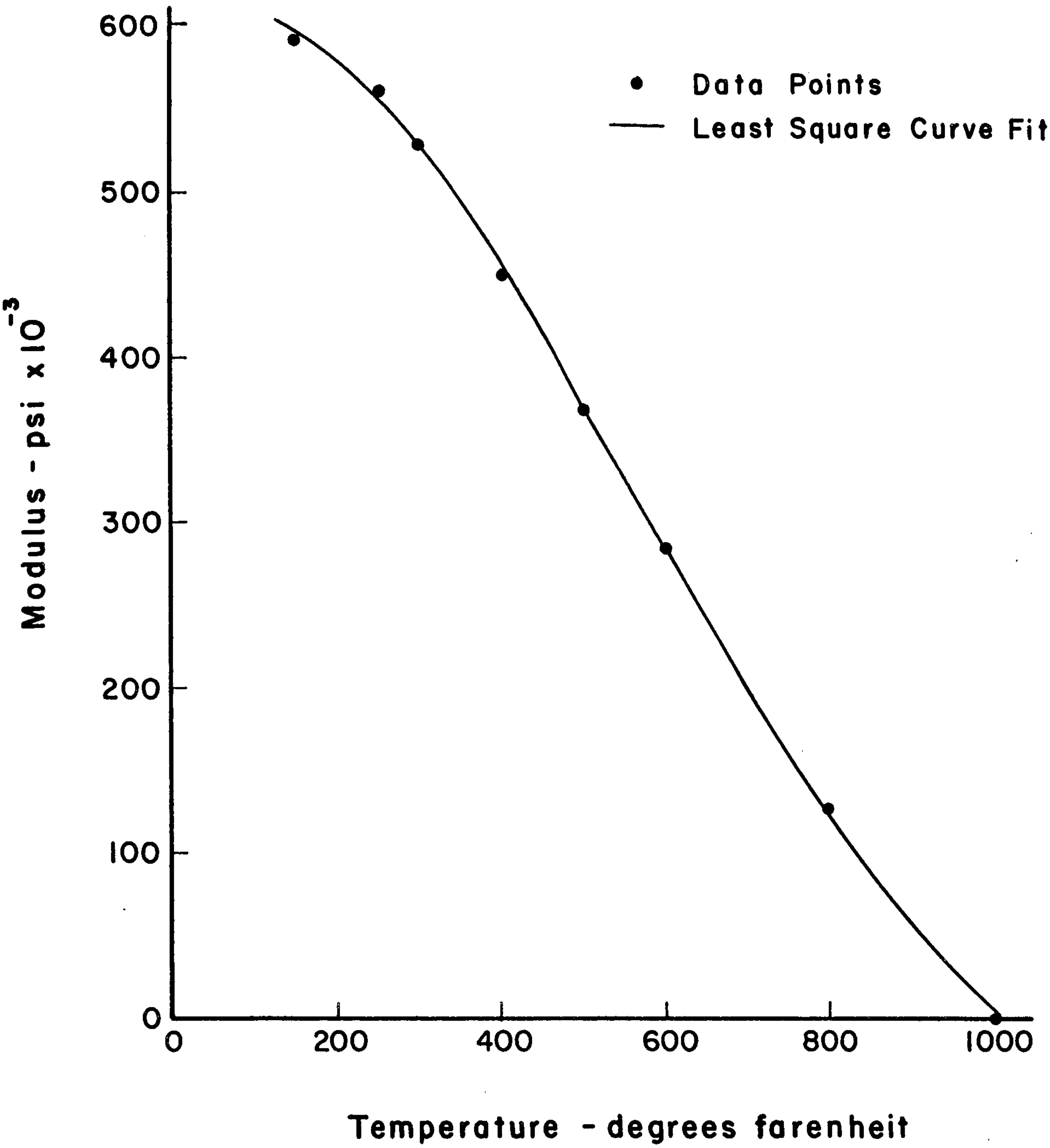


Fig. B1 Example of Least Square Curve Fit

APPENDIX C - MATHEMATICAL MODEL OF SANDWICH SHELL

The sandwich shell substructure is composed of a honeycomb core material and two steel face plates. The orthotropic core material is readily represented by solid triangular rings as indicated in Part IV of this report. However, for the description of the behavior of the face plates, a shell type element is used. The appropriate theory is given in this appendix.

It is assumed that the face plates are idealized by series of truncated cone elements which are connected at mesh points of the finite element system. The cross section of a typical truncated cone element is shown in Figure C1.

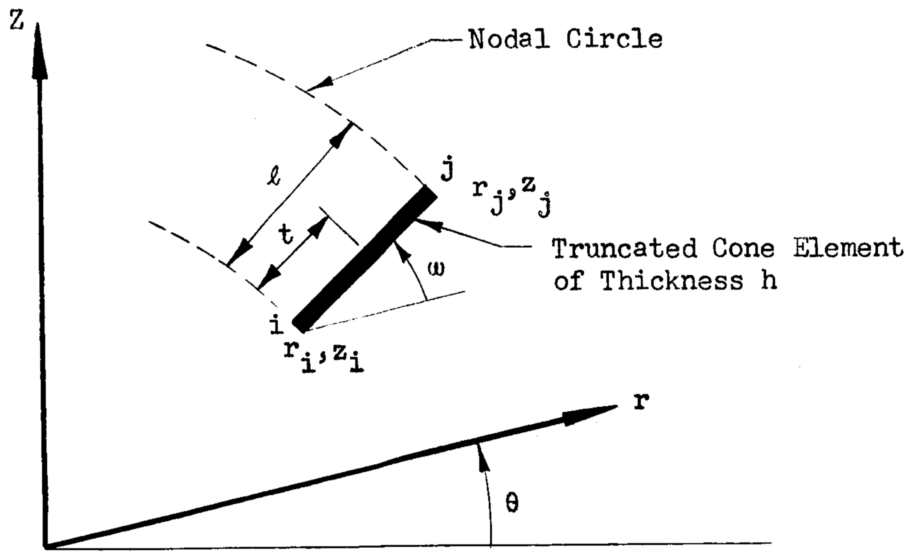


FIGURE C1 - CROSS SECTION OF TRUNCATED CONE ELEMENT

The displacements of the system in the r, θ, z coordinate system are written in the following form:

$$u_r = \sum u_{rn} (r,z) \cos n\theta$$

$$u_z = \sum u_{zn} (r,z) \cos n\theta$$

$$u_\theta = \sum u_{\theta n} (r,z) \sin n\theta$$

where $u_{rn}(r,z)$, $u_{zn}(r,z)$ and $u_{\theta n}(r,z)$ are the two-dimensional displacement functions (Fourier coefficients) associated with the harmonic n.

Within each truncated cone element, the two-dimensional displacement functions are assumed to vary linearly. The displacement at some point t within the element are given in terms of the nodal circle displacements by

$$u_r(t,\theta) = \sum \left[\frac{\ell-t}{\ell} u^i_{rn} + \frac{t}{\ell} u^j_{rn} \right] \cos n\theta \qquad \text{(C1)}$$

$$u_z(t,\theta) = \sum \left[\frac{\ell-t}{\ell} u^i_{zn} + \frac{t}{\ell} u^j_{zn} \right] \cos n\theta \qquad \text{(C2)}$$

$$u_\theta(t,\theta) = \sum \left[\frac{\ell-t}{\ell} u^i_{\theta n} + \frac{t}{\ell} u^j_{\theta n} \right] \sin n\theta \qquad \text{(C3)}$$

Accordingly, the displacement in the t-direction is

$$u_t(t,\theta) = u_r(t,\theta)\cos\omega + u_z(t,\theta)\sin\omega \tag{C4}$$

where
$$\cos\omega = \frac{a}{\ell}$$
$$\sin\omega = \frac{b}{\ell}$$

The inplane strains within the truncated cone element are

$$\epsilon_t = \frac{\partial u_t}{\partial t} = \sum \epsilon_{tn}\cos n\theta \tag{C5}$$

$$\epsilon_\theta = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} = \sum \epsilon_{\theta n}\cos n\theta \tag{C6}$$

$$\gamma_{\theta t} = \frac{1}{r}\frac{\partial u_t}{\partial \theta} + \frac{\partial u_\theta}{\partial t} = \sum \gamma_{\theta tn}\sin n\theta \tag{C7}$$

By combining Equations (C1) through (C7), the inplane strains for harmonic n are expressed in terms of nodal circle displacements by the following matrix equations:

$$\begin{bmatrix} \epsilon_{tn} \\ \epsilon_{\theta n} \\ \gamma_{\theta tn} \end{bmatrix} = \begin{bmatrix} -\frac{a}{\ell^2} & -\frac{b}{\ell^2} & 0 & \frac{a}{\ell^2} & \frac{b}{\ell^2} & 0 \\ \frac{\ell-t}{r\ell} & 0 & \frac{n(\ell-t)}{r\ell} & \frac{t}{r\ell} & 0 & \frac{nt}{r\ell} \\ -\frac{na}{r\ell^2}(\ell-t) & -\frac{nb}{r\ell^2}(\ell-t) & -\frac{1}{\ell} & -\frac{nat}{r\ell^2} & -\frac{nbt}{r\ell^2} & \frac{1}{\ell} \end{bmatrix} \begin{bmatrix} u_{rn}^i \\ u_{zn}^i \\ u_{\theta n}^i \\ u_{rn}^j \\ u_{tn}^j \\ u_{\theta n}^j \end{bmatrix} \tag{C8}$$

If Equation (C8) is evaluated at the center of the element, the average element strains are given by

$$\begin{bmatrix} \bar{\epsilon}_{tn} \\ \\ \bar{\epsilon}_{\theta n} \\ \\ \bar{\gamma}_{\theta tn} \end{bmatrix} = \begin{bmatrix} -\frac{a}{\ell^2} & -\frac{b}{\ell^2} & 0 & \frac{a}{\ell^2} & \frac{b}{\ell^2} & 0 \\ \frac{1}{2\bar{r}} & 0 & \frac{n}{2\bar{r}} & \frac{1}{2\bar{r}} & 0 & \frac{n}{2\bar{r}} \\ -\frac{na}{2\bar{r}} & -\frac{nb}{2\bar{r}\ell} & -\frac{1}{\ell} & -\frac{na}{2\bar{r}\ell} & -\frac{nb}{2\bar{r}\ell} & \frac{1}{\ell} \end{bmatrix} \begin{bmatrix} u^i_{rn} \\ u^i_{zn} \\ u^i_{\theta n} \\ u^j_{rn} \\ u^j_{zn} \\ u^j_{\theta n} \end{bmatrix} \quad \text{(C9a)}$$

where $\bar{r} = (r_i + r_j)/2$

or Equation (C9a) written in symbolic form

$$\left[\epsilon_n\right] = \left[G_n\right]\left[u_n\right] \quad \text{(C9b)}$$

From Hooke's law, the stresses within the element are given by

$$\begin{bmatrix} \sigma_{tn} \\ \sigma_{\theta n} \\ \tau_{\theta tn} \end{bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_{tn} \\ \epsilon_{\theta n} \\ \nu_{\theta tn} \end{bmatrix} - \begin{bmatrix} \tau_1 \\ \tau_2 \\ 0 \end{bmatrix} \quad \text{(10a)}$$

where $\tau_1 = \tau_2 = \frac{1 + \nu}{1 - \nu^2} E\alpha_t T_n$

Equation (10a) expressed in symbolic form is

$$\left[\sigma_n\right] = \left[C\right]\left[\epsilon_n\right] + \left[\tau\right] \tag{10b}$$

From Equation (1.15), the nodal circle forces in terms of the nodal circle displacement for the harmonic n are given by

$$\left[S_n\right] = \left[k_n\right]\left[U_n\right] + \left[L_n\right] T_n \tag{C11}$$

where

$$\left[K_n\right] = \bar{r}A \left[G_n\right]^T \left[C\right]\left[G_n\right]$$

$$\left[L_n\right] = \bar{r}A \left[G_n\right]^T \left[\tau\right]$$

For the truncated cone element

$$A = h \cdot \ell$$

These forces are included into the overall equilibrium of the system for the harmonic n by the same technique used for the system of triangular rings. When n equals zero, these equations reduce to the axisymmetric case.

APPENDIX D

PROGRAM LISTING - ARBITRARY AXISYMMETRIC STRUCTURES

```
CAXISY    STRESS ANALYSIS OF AXISYMMETRIC SOLIDS                         AXISY001
C                                                                        AXISY002
C      DIMENSION AND COMMON STATEMENTS                                   AXISY003
C                                                                        AXISY004
       DIMENSION THDIS(6),THFRC(6)                                       AXISY005
       DIMENSION NPNUM(340),XORD(340),YORD(340),                         AXISY006
      1DSX(340),DSY(340),XLOAD(340),YLOAD(340),NP(340,10),SXX(340,9),    AXISY007
      2SXY(340,9),SYX(340,9),SYY(340,9),NAP(340)                         AXISY008
       DIMENSION NUME(550),NPI(550),NPJ(550),NPK(550),ET(550),XU(550),   AXISY009
      1RO(550),COED(550),DT(550),THERM(550),AJ(550),BJ(550),AK(550),     AXISY010
      2BK(550),SIGXX(550),SIGYY(550),SIGXY(550),SLOPE(340)               AXISY011
       DIMENSION NPB(340),NFIX(340),LM(3),A(6,6),B(6,6),S(6,6)           AXISY012
       COMMON SXX,SXY,SYX,SYY                                            AXISY013
       EQUIVALENCE (SIGXX,RO,NPB), (SIGYY,COED,NFIX), (SIGXY,DT,SLOPE)   AXISY014
C                                                                        AXISY015
C      READ AND PRINT OF DATA                                            AXISY016
C                                                                        AXISY017
       KINN=5                                                            AXISY018
       KOUT=6                                                            AXISY019
  150  READ INPUT TAPE KINN,100                                          AXISY020
       WRITE OUTPUT TAPE KOUT,99                                         AXISY021
       WRITE OUTPUT TAPE KOUT,100                                        AXISY022
       READ INPUT TAPE KINN,1,NUMEL,NUMNP,NUMBC,NCPIN,NOPIN,NCYCM,TOLER, AXISY023
      1XFAC,T1                                                           AXISY024
       WRITE OUTPUT TAPE KOUT,101,NUMEL                                  AXISY025
       WRITE OUTPUT TAPE KOUT,102,NUMNP                                  AXISY026
       WRITE OUTPUT TAPE KOUT,103,NUMBC                                  AXISY027
       WRITE OUTPUT TAPE KOUT,104,NCPIN                                  AXISY028
       WRITE OUTPUT TAPE KOUT,105,NOPIN                                  AXISY029
       WRITE OUTPUT TAPE KOUT,106,NCYCM                                  AXISY030
       WRITE OUTPUT TAPE KOUT,107,TOLER                                  AXISY031
       WRITE OUTPUT TAPE KOUT,108,XFAC                                   AXISY032
       READ INPUT TAPE KINN,2,(NUME(N),NPI(N),NPJ(N),NPK(N),ET(N),RO(N), AXISY033
      1XU(N),COED(N),DT(N),N=1,NUMEL)                                    AXISY034
       READ INPUT TAPE KINN,3,(NPNUM(M),XORD(M),YORD(M),XLOAD(M),YLOAD(M)AXISY035
      1,DSX(M),DSY(M),M=1,NUMNP)                                         AXISY036
       IF (T1) 160,155,160                                               AXISY037
  155  WRITE OUTPUT TAPE KOUT,110                                        AXISY038
       WRITE OUTPUT TAPE KOUT,2,(NUME(N),NPI(N),NPJ(N),NPK(N),ET(N),RO(N)AXISY039
      1,XU(N),COED(N),DT(N),N=1,NUMEL)                                   AXISY040
       WRITE OUTPUT TAPE KOUT,111                                        AXISY041
       WRITE OUTPUT TAPE KOUT,109,(NPNUM(M),XORD(M),YORD(M),XLOAD(M),    AXISY042
      1YLOAD(M),DSX(M),DSY(M),M=1,NUMNP)                                 AXISY043
C                                                                        AXISY044
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C     INITIALIZATION                                                    AXISY045
C                                                                       AXISY046
  160 NCYCLE=0                                                          AXISY047
      NUMPT=NCPIN                                                       AXISY048
      NUMOPT=NOPIN                                                      AXISY049
      DO 175 L=1,NUMNP                                                  AXISY050
      DO 170 M=1,9                                                      AXISY051
      SXX(L,M)=0.0                                                      AXISY052
      SXY(L,M)=0.0                                                      AXISY053
      SYX(L,M)=0.0                                                      AXISY054
      SYY(L,M)=0.0                                                      AXISY055
  170 NP(L,M)=0                                                         AXISY056
      NP(L,10)=0                                                        AXISY057
  175 NP(L,1)=L                                                         AXISY058
C                                                                       AXISY059
C     FOR EACH ELEMENT OF THE SYSTEM                                    AXISY060
C                                                                       AXISY061
      NTAG=0                                                            AXISY062
      DO 2000 N=1,NUMEL                                                 AXISY063
C                                                                       AXISY064
C      1. INITIALIZATION                                                AXISY065
C                                                                       AXISY066
      I=NPI(N)                                                          AXISY067
      J=NPJ(N)                                                          AXISY068
      K=NPK(N)                                                          AXISY069
      THICK=(XORD(I)+XORD(J)+XORD(K))/3.                                AXISY070
      AJ(N)=XORD(J)-XORD(I)                                             AXISY071
      AK(N)=XORD(K)-XORD(I)                                             AXISY072
      BJ(N)=YORD(J)-YORD(I)                                             AXISY073
      BK(N)=YORD(K)-YORD(I)                                             AXISY074
  176 AREA=(AJ(N)*BK(N)-BJ(N)*AK(N))/2.                                 AXISY075
      IF (AREA) 701,701,177                                             AXISY076
  701 WRITE OUTPUT TAPE KOUT,711,(N)                                    AXISY077
      NTAG=1                                                            AXISY078
  177 COMM=(1.+XU(N))*(1.-2.*XU(N))                                     AXISY079
      THERM(N)=-ET(N)*COED(N)*DT(N)/(1.-2.XU(N))                        AXISY080
C                                                                       AXISY081
C      2. COMPUTE ELEMENT STIFFNESS MATRIX                              AXISY082
C                                                                       AXISY083
C        A.  FORM MATRIX (A)                                            AXISY084
C                                                                       AXISY085
      A(1,1)=BJ(N)-BK(N)                                                AXISY086
      A(1,2)=0.0                                                        AXISY087
      A(1,3)=BK(N)                                                      AXISY088
```

```
      A(1,4)=0.0                                                        AXISY089
      A(1,5)=-BJ(N)                                                     AXISY090
      A(1,6)=0.0                                                        AXISY091
      A(2,1)=0.0                                                        AXISY092
      A(2,2)=AK(N)-AJ(N)                                                AXISY093
      A(2,3)=0.0                                                        AXISY094
      A(2,4)=-AK(N)                                                     AXISY095
      A(2,5)=0.0                                                        AXISY096
      A(2,6)=AJ(N)                                                      AXISY097
      A(3,1)=AK(N)-AJ(N)                                                AXISY098
      A(3,2)=BJ(N)-BK(N)                                                AXISY099
      A(3,3)=-AK(N)                                                     AXISY100
      A(3,4)=BK(N)                                                      AXISY101
      A(3,5)=AJ(N)                                                      AXISY102
      A(3,6)=-BJ(N)                                                     AXISY103
      C=.66666667*AREA                                                  AXISY104
      A(4,1)=C/XORD(I)                                                  AXISY105
      A(4,2)=0.0                                                        AXISY106
      A(4,3)=C/XORD(J)                                                  AXISY107
      A(4,4)=0.0                                                        AXISY108
      A(4,5)=C/XORD(K)                                                  AXISY109
      A(4,6)=0.0                                                        AXISY110
C                                                                       AXISY111
C        B.  FORM MATRIX (B)                                            AXISY112
C                                                                       AXISY113
      COMM=.25*ET(N)*THICK/(COMM*AREA)                                  AXISY114
      E=XU(N)*COMM                                                      AXISY115
      D=(1.-XU(N))*COMM                                                 AXISY116
      C=(.5-XU(N))*COMM                                                 AXISY117
      B(1,1)=D                                                          AXISY118
      B(1,2)=E                                                          AXISY119
      B(1,3)=0.0                                                        AXISY120
      B(1,4)=E                                                          AXISY121
      B(2,1)=E                                                          AXISY122
      B(2,2)=D                                                          AXISY123
      B(2,3)=0.0                                                        AXISY124
      B(2,4)=E                                                          AXISY125
      B(3,1)=0.0                                                        AXISY126
      B(3,2)=0.0                                                        AXISY127
      B(3,3)=C                                                          AXISY128
      B(3,4)=0.0                                                        AXISY129
      B(4,1)=E                                                          AXISY130
      B(4,2)=E                                                          AXISY131
      B(4,3)=0.0                                                        AXISY132
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      B(4,4)=D                                                          AXISY133
C                                                                       AXISY134
C        C.  FORM ELEMENT STIFFNESS MATRIX (A)T*(B)*(A)                 AXISY135
C                                                                       AXISY136
      DO 182 J=1,6                                                      AXISY137
      DO 182 I=1,4                                                      AXISY138
      S(I,J)=0.0                                                        AXISY139
      DO 182 K=1,4                                                      AXISY140
  182 S(I,J)=S(I,J)+B(I,K)*A(K,J)                                       AXISY141
      DO 183 J=1,6                                                      AXISY142
      DO 183 I=1,4                                                      AXISY143
  183 B(J,I)=S(I,J)                                                     AXISY144
      DO 184 J=1,6                                                      AXISY145
      DO 184 I=1,6                                                      AXISY146
      S(I,J)=0.0                                                        AXISY147
      DO 184 K=1,4                                                      AXISY148
  184 S(I,J)=S(I,J)+B(I,K)*A(K,J)                                       AXISY149
C                                                                       AXISY150
C      3. ADD ELEMENT STIFFNESS TO TOTAL STIFFNESS                      AXISY151
C                                                                       AXISY152
      LM(1)=NPI(N)                                                      AXISY153
      LM(2)=NPJ(N)                                                      AXISY154
      LM(3)=NPK(N)                                                      AXISY155
      DO 200 L=1,3                                                      AXISY156
      DO 200 M=1,3                                                      AXISY157
      LX=LM(L)                                                          AXISY158
      MX=0                                                              AXISY159
  185 MX=MX+1                                                           AXISY160
      IF(NP(LX,MX)-LM(M)) 190,195,190                                   AXISY161
  190 IF(NP(LX,MX)) 185,195,185                                         AXISY162
  195 NP(LX,MX)=LM(M)                                                   AXISY163
      IF (MX-10) 196,702,702                                            AXISY164
  702 WRITE OUTPUT TAPE KOUT,712,(LX)                                   AXISY165
      NTAG=1                                                            AXISY166
  196 SXX(LX,MX)=SXX(LX,MX)+S(2*L-1,2*M-1)                              AXISY167
      SXY(LX,MX)=SXY(LX,MX)+S(2*L-1,2*M)                                AXISY168
      SYX(LX,MX)=SYX(LX,MX)+S(2*L,2*M-1)                                AXISY169
  200 SYY(LX,MX)=SYY(LX,MX)+S(2*L,2*M)                                  AXISY170
C                                                                       AXISY171
C      5. COMPUTE BODY FORCES                                           AXISY172
C                                                                       AXISY173
      DL=AREA*THICK*RO(N)/3.                                            AXISY174
      I=NPI(N)                                                          AXISY175
      J=NPJ(N)                                                          AXISY176
```

```
      K=NPK(N)                                                          AXISY177
      C=COED(N)*DT(N)                                                   AXISY178
      THDIS(1)=C*XORD(I)                                                AXISY179
      THDIS(2)=0.0                                                      AXISY180
      THDIS(3)=C*XORD(J)                                                AXISY181
      THDIS(4)=C*BJ(N)                                                  AXISY182
      THDIS(5)=C*XORD(K)                                                AXISY183
      THDIS(6)=C*BK(N)                                                  AXISY184
      DO 204 L=1,6                                                      AXISY185
      THFRC(L)=0.0                                                      AXISY186
      DO 204 M=1,6                                                      AXISY187
  204 THFRC(L)=THFRC(L)+S(L,M)*THDIS(M)                                 AXISY188
      XLOAD(I)=XLOAD(I)+THFRC(1)                                        AXISY189
      YLOAD(I)=YLOAD(I)+THFRC(2)-DL                                     AXISY190
      XLOAD(J)=XLOAD(J)+THFRC(3)                                        AXISY191
      YLOAD(J)=YLOAD(J)+THFRC(4)-DL                                     AXISY192
      XLOAD(K)=XLOAD(K)+THFRC(5)                                        AXISY193
      YLOAD(K)=YLOAD(K)+THFRC(6)-DL                                     AXISY194
 2000 CONTINUE                                                          AXISY195
C                                                                       AXISY196
C     COUNT OF ADJACENT NODAL POINTS                                    AXISY197
C                                                                       AXISY198
      DO 206 M=1,NUMNP                                                  AXISY199
      MX =1                                                             AXISY200
  205 MX=MX+1                                                           AXISY201
      IF (NP(M,MX)) 206,206,205                                         AXISY202
  206 NAP(M)=MX-1                                                       AXISY203
C                                                                       AXISY204
C     INVERSION OF NODAL POINT STIFFNESS                                AXISY205
C                                                                       AXISY206
      DO 210 M=1,NUMNP                                                  AXISY207
      COMM=SXX(M,1)*SYY(M,1)-SXY(M,1)*SYX(M,1)                          AXISY208
      TEMP=SYY(M,1)/COMM                                                AXISY209
      SYY(M,1)=SXX(M,1)/COMM                                            AXISY210
      SXX(M,1)=TEMP                                                     AXISY211
      SXY(M,1)=-SXY(M,1)/COMM                                           AXISY212
  210 SYX(M,1)=-SYX(M,1)/COMM                                           AXISY213
C                                                                       AXISY214
C     MODIFICATION OF BOUNDARY FLEXIBILITIES                            AXISY215
C                                                                       AXISY216
      WRITE OUTPUT TAPE KOUT,112                                        AXISY217
      READ INPUT TAPE KINN,4,(NPB(L),NFIX(L),SLOPE(L),L=1,NUMBC)        AXISY218
      WRITE OUTPUT TAPE KOUT,4,(NPB(L),NFIX(L),SLOPE(L),L=1,NUMBC)      AXISY219
      DO 240 L=1,NUMBC                                                  AXISY220
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      WRITE OUTPUT TAPE KOUT,120,NCYCLE,SUM                             AXISY265
  305 IF (SUM-TOLER) 400,400,310                                        AXISY266
  310 IF(NCYCM-NCYCLE) 400,400,315                                      AXISY267
  315 IF (NCYCLE-NUMOPT) 244,320,320                                    AXISY268
  320 NUMOPT=NUMOPT+NOPIN                                               AXISY269
C                                                                       AXISY270
C     PRINT OF NODAL POINT DISPLACEMENTS                                AXISY271
C                                                                       AXISY272
  400 WRITE OUTPUT TAPE KOUT,99                                         AXISY273
      WRITE OUTPUT TAPE KOUT,100                                        AXISY274
      WRITE OUTPUT TAPE KOUT,121                                        AXISY275
      WRITE OUTPUT TAPE KOUT,122,(NPNUM(M),DSX(M),DSY(M),M=1,NUMNP)     AXISY276
C                                                                       AXISY277
C     CALCULATION AND PRINT OF ELEMENT STRESSES                         AXISY278
C                                                                       AXISY279
      WRITE OUTPUT TAPE KOUT,123                                        AXISY280
      DO 420 N=1,NUMEL                                                  AXISY281
      I=NPI(N)                                                          AXISY282
      J=NPJ(N)                                                          AXISY283
      K=NPK(N)                                                          AXISY284
C                                                                       AXISY285
C      1. COMPUTE ELEMENT STRAINS                                       AXISY286
C                                                                       AXISY287
      C=AJ(N)*BK(N)-BJ(N)*AK(N)                                         AXISY288
      EPX=((BJ(N)-BK(N))*DSX(I)+BK(N)*DSX(J)-BJ(N)*DSX(K))/C            AXISY289
      EPY=((AK(N)-AJ(N))*DSY(I)-AK(N)*DSY(J)+AJ(N)*DSY(K))/C            AXISY290
      EPZ=(DSX(I)+DSX(J)+DSX(K))/(XORD(I)+ XORD(J)+XORD(K))             AXISY291
      GAM=( (AK(N)-AJ(N))*DSX(I)-AK(N)*DSX(J)+AJ(N)*DSX(K)              AXISY292
     1   +(BJ(N)-BK(N))*DSY(I)+BK(N)*DSY(J)-BJ(N)*DSY(K) )/C            AXISY293
C                                                                       AXISY294
C      2. COMPUTE ELEMENT STRESSES                                      AXISY295
C                                                                       AXISY296
      E=ET(N)*XU(N)/((1.+XU(N))*(1.-2.*XU(N)))                          AXISY297
      D=E*(1.-XU(N))/XU(N)                                              AXISY298
      C=E*(1.-2.*XU(N))/(2.*XU(N))                                      AXISY299
      X= D*EPX + E*EPY + E*EPZ +THERM(N)                                AXISY300
      Y= E*EPX + D*EPY + E*EPZ +THERM(N)                                AXISY301
      Z= E*EPX + E*EPY + D*EPZ +THERM(N)                                AXISY302
      XY=C*GAM                                                          AXISY303
      SIGXX(N)=X                                                        AXISY304
      SIGYY(N)=Y                                                        AXISY305
      SIGXY(N)=XY                                                       AXISY306
C                                                                       AXISY307
C      3.COMPUTE PRINCIPAL STRESSES                                     AXISY308
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C                                                                       AXISY309
      C=(X+Y)/2.0                                                       AXISY310
      R=SQRTF(((Y-X)/2.0)**2+XY**2)                                     AXISY311
      XMAX=C+R                                                          AXISY312
      XMIN=C-R                                                          AXISY313
      PA=0.5*57.29578*ATANF ( 2.* XY/(Y-X))                             AXISY314
      IF (2.*X-XMAX-XMIN) 405,420,420                                   AXISY315
  405 IF (PA) 410,420,415                                               AXISY316
  410 PA=PA+90.0                                                        AXISY317
      GO TO 420                                                         AXISY318
  415 PA=PA-90.0                                                        AXISY319
C                                                                       AXISY320
C       4.PRINT OF ELEMENT STRESSES                                     AXISY321
C                                                                       AXISY322
  420 WRITE OUTPUT TAPE KOUT,125,(NUME(N),X,Y,Z,XY,XMAX,XMIN,PA)        AXISY323
C                                                                       AXISY324
C     CALCULATION AND PRINT OF NODAL POINT STRESSES                     AXISY325
C                                                                       AXISY326
      WRITE OUTPUT TAPE KOUT,823                                        AXISY327
      DO 900  M=1,NUMNP                                                 AXISY328
       X=0.0                                                            AXISY329
       Y=0.0                                                            AXISY330
      XY=0.0                                                            AXISY331
      SRX=0.0                                                           AXISY332
      SRY=0.0                                                           AXISY333
      E=0.0                                                             AXISY334
      XX=0.0                                                            AXISY335
      TEMP=0.0                                                          AXISY336
      R=0.0                                                             AXISY337
      DO 860 N=1,NUMEL                                                  AXISY338
      I=NPI(N)                                                          AXISY339
      J=NPJ(N)                                                          AXISY340
      K=NPK(N)                                                          AXISY341
      IF (M-I) 830,850,830                                              AXISY342
  830 IF (M-J) 835,845,835                                              AXISY343
  835 IF (M-K) 860,840,860                                              AXISY344
  840 I=NPK(N)                                                          AXISY345
      K=NPI(N)                                                          AXISY346
      GO TO 850                                                         AXISY347
  845 I=NPJ(N)                                                          AXISY348
      J=NPI(N)                                                          AXISY349
  850 A=ABSF(XORD(J)+XORD(K)-2.*XORD(I))                                AXISY350
      B=ABSF(YORD(J)+YORD(K)-2.*YORD(I))                                AXISY351
      RY=B/(A+B)                                                        AXISY352
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      M=NPB(L)                                                          AXISY221
      NP(M,1)=0                                                         AXISY222
      IF(NFIX(L)-1) 225,220,215                                         AXISY223
  215 C=(SXX(M,1)*SLOPE(L)-SXY(M,1))/(SYX(M,1)*SLOPE(L)-SYY(M,1))       AXISY224
      R=1.-C*SLOPE(L)                                                   AXISY225
      SXX(M,1)=(SXX(M,1)-C*SYX(M,1))/R                                  AXISY226
      SXY(M,1)=(SXY(M,1)-C*SYY(M,1))/R                                  AXISY227
      SYX(M,1)=SXX(M,1)*SLOPE(L)                                        AXISY228
       SYY(M,1)=SXY(M,1)*SLOPE(L)                                       AXISY229
      GO TO 240                                                         AXISY230
  220 SYY(M,1)=SYY(M,1)-SYX(M,1)*SXY(M,1)/SXX(M,1)                      AXISY231
      GO TO 230                                                         AXISY232
  225 SYY(M,1)=0.0                                                      AXISY233
  230 SXX(M,1)=0.0                                                      AXISY234
  235 SXY(M,1)=0.0                                                      AXISY235
      SYX(M,1)=0.0                                                      AXISY236
  240 CONTINUE                                                          AXISY237
C                                                                       AXISY238
C      ITERATION ON NODAL POINT DISPLACEMENTS                           AXISY239
C                                                                       AXISY240
      IF (NTAG) 150,243,150                                             AXISY241
  243 WRITE OUTPUT TAPE KOUT,119                                        AXISY242
  244 SUM=0.0                                                           AXISY243
      DO 290 M=1,NUMNP                                                  AXISY244
      NUM=NAP(M)                                                        AXISY245
      IF (SXX(M,1)+SYY(M,1)) 275,290,275                                AXISY246
  275 FRX=XLOAD(M)                                                      AXISY247
      FRY=YLOAD(M)                                                      AXISY248
      DO 280 L=2,NUM                                                    AXISY249
      N=NP(M,L)                                                         AXISY250
      FRX=FRX-SXX(M,L)*DSX(N)-SXY(M,L)*DSY(N)                           AXISY251
  280 FRY=FRY-SYX(M,L)*DSX(N)-SYY(M,L)*DSY(N)                           AXISY252
      DX=SXX(M,1)*FRX+SXY(M,1)*FRY-DSX(M)                               AXISY253
      DY=SYX(M,1)*FRX+SYY(M,1)*FRY-DSY(M)                               AXISY254
      DSX(M)=DSX(M)+XFAC*DX                                             AXISY255
      DSY(M)=DSY(M)+XFAC*DY                                             AXISY256
  285 SUM=SUM+(ABSF(DX)+ABSF(DY))/(SXX(M,1)+SYY(M,1))                   AXISY257
  290 CONTINUE                                                          AXISY258
C                                                                       AXISY259
C     CYCLE COUNT AND PRINT CHECK                                       AXISY260
C                                                                       AXISY261
      NCYCLE=NCYCLE +1                                                  AXISY262
      IF (NCYCLE-NUMPT) 305,300,300                                     AXISY263
  300 NUMPT=NUMPT+NCPIN                                                 AXISY264
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      SRY=SRY+RY                                                        AXISY353
      Y=Y+SIGYY(N)*RY                                                   AXISY354
      RX=A/(A+B)                                                        AXISY355
      SRX=SRX+RX                                                        AXISY356
      X=X+SIGXX(N)*RX                                                   AXISY357
      R=R+1.0                                                           AXISY358
      XY=XY+SIGXY(N)                                                    AXISY359
      E=E+ET(N)                                                         AXISY360
      XX=XX+XU(N)                                                       AXISY361
      TEMP=TEMP+THERM(N)*(1.-2.XU(N))                                   AXISY362
  860 CONTINUE                                                          AXISY363
      X=X/SRX                                                           AXISY364
      Y=Y/SRY                                                           AXISY365
      XY=XY/R                                                           AXISY366
      E=E/R                                                             AXISY367
      XX=XX/R                                                           AXISY368
      TEMP=TEMP/R                                                       AXISY369
      Z=E*DSX(M)/XORD(M)+XX*(X+Y)+TEMP                                  AXISY370
      C=(X+Y)/2.0                                                       AXISY371
      R=SQRTF(((Y-X)/2.0)**2+XY**2)                                     AXISY372
      XMAX=C+R                                                          AXISY373
      XMIN=C-R                                                          AXISY374
      PA=0.5*57.29578*ATANF ( 2.* XY/(Y-X))                             AXISY375
      IF (2.*X-XMAX-XMIN) 805,820,820                                   AXISY376
  805 IF (PA) 810,820,815                                               AXISY377
  810 PA=PA+90.0                                                        AXISY378
      GO TO 820                                                         AXISY379
  815 PA=PA-90.0                                                        AXISY380
  820 WRITE OUTPUT TAPE KOUT,125, (M,X,Y,Z,XY,XMAX,XMIN,PA)             AXISY381
  900 CONTINUE                                                          AXISY382
C                                                                       AXISY383
      IF (SUM-TOLER) 440,440,430                                        AXISY384
  430 IF (NCYCM-NCYCLE) 440,440,243                                     AXISY385
C                                                                       AXISY386
  440 GO TO 150                                                         AXISY387
C                                                                       AXISY388
C        FORMAT STATEMENTS                                              AXISY389
C                                                                       AXISY390
    1 FORMAT (6I4,2E12.5,4I1)                                           AXISY391
    2 FORMAT (4I4,4E12.4,F8.4)                                          AXISY392
    3 FORMAT (1I4,4F8.0,2F12.8)                                         AXISY393
    4 FORMAT (2I4,1F8.3)                                                AXISY394
    5 FORMAT (3E15.8)                                                   AXISY395
   99 FORMAT (1H1)                                                      AXISY396
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100 FORMAT (72H  BCD INFORMATION                                                      AXISY397
   1                     )                                                            AXISY398
101 FORMAT(29HONUMBER OF ELEMENTS          =1I4/)                                     AXISY399
102 FORMAT(29H NUMBER OF NODAL POINTS      =1I4/)                                     AXISY400
103 FORMAT(29H NUMBER OF BOUNDARY POINTS   =1I4/)                                     AXISY401
104 FORMAT(29H CYCLE PRINT INTERVAL        =1I4/)                                     AXISY402
105 FORMAT(29H OUTPUT INTERVAL OF RESULTS  =1I4/)                                     AXISY403
106 FORMAT(29H CYCLE LIMIT                 =1I4/)                                     AXISY404
107 FORMAT(29H TOLERANCE LIMIT             =1E12.4/)                                  AXISY405
108 FORMAT(29H OVER RELAXATION FACTOR      =1F6.3)                                    AXISY406
109 FORMAT (1I8,4F12.1,2F12.8)                                                        AXISY407
110 FORMAT (74H1EL.   I   J   K       E        DENSITY      POISSON                   AXISY408
   1  ALPHA      DELTA T)                                                             AXISY409
111 FORMAT (80H1      NP       X-ORD       Y-ORD       X-LOAD      Y-LOAAXISY410
   1D       X-DISP      Y-DISP)                                                       AXISY411
112 FORMAT (20H BOUNDARY CONDITIONS)                                                  AXISY412
119 FORMAT(34H0          CYCLE      FORCE UNBALANCE)                                  AXISY413
120 FORMAT (1I12,1E20.6)                                                              AXISY414
121 FORMAT (42H0NODAL POINT X-DISPLACEMENT Y-DISPLACEMENT)                            AXISY415
122 FORMAT (1I12,2E15.6)                                                              AXISY416
123 FORMAT(120H1  ELEMENT     X-STRESS        Y-STRESS        Z-STREAXISY417
   1SS        XY-STRESS      MAX-STRESS     MIN-STRESS      DIRECTION)AXISY418
124 FORMAT (1I10,3F20.4,5X,3F15.2)                                                    AXISY419
125 FORMAT (1I6,4F17.3,1X,3F15.2)                                                     AXISY420
711 FORMAT (32H0ZERO OR NEGATIVE AREA, EL. NO.=1I4)                                   AXISY421
712 FORMAT (33H0OVER 8 N.P. ADJACENT TO N.P. NO.1I4)                                  AXISY422
823 FORMAT(120H1  N-POINT     X-STRESS        Y-STRESS        Z-STREAXISY423
   1SS        XY-STRESS      MAX-STRESS     MIN-STRESS      DIRECTION)AXISY424
C                                                                                     AXISY425
    END                                                                               AXIS0426
```

APPENDIX E

PROGRAM LISTING - AXISYMMETRIC HEAT SHIELDS

```
CASHS           ANALYSIS OF AXISYMMETRIC HEAT SHIELDS                           ASHS0001
C                                                                               ASHS0002
      DIMENSION                                                                 ASHS0003
     1 XX(5),YY(5),S(10,10),P(10),X(40,10),Y(40,10),U(40,10),V(40,10),          ASHS0004
     1 SS(800,24),RR(800),KODE(40,10),HED(12),T(40,10),CC(5,3)                  ASHS0005
     4 ,A(4,4),B(4,4)                                                           ASHS0006
C                                                                               ASHS0007
      COMMON KINN,KOUT,NMAX,MMAX,MB,NMT,NUMLD,NUMBC,TE,TR,                      ASHS0008
     1 ES,XS,AS,TS,ESH,XSH,ASH,TSH,                                             ASHS0009
     1 HED,X,Y,T,U,V,CC,XX,YY,S,P,KODE,RR,SS                                    ASHS0010
C                                                                               ASHS0011
C     READ AND PRINT OF DATA                                                    ASHS0012
C                                                                               ASHS0013
   50 KINN=5                                                                    ASHS0014
      KOUT=6                                                                    ASHS0015
C                                                                               ASHS0016
      CALL INPUT                                                                ASHS0017
C                                                                               ASHS0018
      MX=2*MMAX                                                                 ASHS0019
      NS=NMAX*MMAX                                                              ASHS0020
      MM=MMAX-1                                                                 ASHS0021
      NN=NMAX-1                                                                 ASHS0022
      MBAND=2*(MMAX+2)                                                          ASHS0023
      NEQ=2*NS                                                                  ASHS0024
C                                                                               ASHS0025
C     FORMATION OF STIFFNESS ARRAY                                              ASHS0026
C                                                                               ASHS0027
      DO 175 I=1,NEQ                                                            ASHS0028
      DO 170 J=1,MBAND                                                          ASHS0029
  170 SS(I,J)=0.0                                                               ASHS0030
  175 RR(I)=0.0                                                                 ASHS0031
C                                                                               ASHS0032
      DO 200 N=1,NN                                                             ASHS0033
      DO 200 M=1,MM                                                             ASHS0034
      DO 180  I=1,10                                                            ASHS0035
      P(I)=0.0                                                                  ASHS0036
      DO 180  J=1,10                                                            ASHS0037
  180 S(I,J)=0.0                                                                ASHS0038
      XX(1)=X(N,M)                                                              ASHS0039
      XX(2)=X(N,M+1)                                                            ASHS0040
      XX(3)=X(N+1,M)                                                            ASHS0041
      XX(4)=X(N+1,M+1)                                                          ASHS0042
```

```
      YY(1)=Y(N,M)                                                      ASHS0043
      YY(2)=Y(N,M+1)                                                    ASHS0044
      YY(3)=Y(N+1,M)                                                    ASHS0045
      YY(4)=Y(N+1,M+1)                                                  ASHS0046
      NNN=MB-M                                                          ASHS0047
      TT=.25*(T(N,M)+T(N,M+1)+T(N+1,M)+T(N+1,M+1))                      ASHS0048
      CALL TRIST(1,4,2,TT,NNN)                                          ASHS0049
      CALL TRIST(1,3,4,TT,NNN)                                          ASHS0050
C                                                                       ASHS0051
      DO 200 I=1,4                                                      ASHS0052
      II=((N-1)*MMAX+M-1)*2+I                                           ASHS0053
      KK=II+MX                                                          ASHS0054
      RR(II)=RR(II)+P(I)                                                ASHS0055
      RR(KK)=RR(KK)+P(I+4)                                              ASHS0056
      DO 190 J=I,4                                                      ASHS0057
      JJ=J-I+1                                                          ASHS0058
      SS(II,JJ)=SS(II,JJ)+S(I,J)                                        ASHS0059
  190 SS(KK,JJ)=SS(KK,JJ)+S(I+4,J+4)                                    ASHS0060
      DO 200 J=1,4                                                      ASHS0061
      JJ=J-I+1+MX                                                       ASHS0062
  200 SS(II,JJ)=SS(II,JJ)+S(I,J+4)                                      ASHS0063
C                                                                       ASHS0064
C     STIFFNESS OF PLATE ELEMENTS                                       ASHS0065
C                                                                       ASHS0066
      M=MB                                                              ASHS0067
      DO 500 N=1,NN                                                     ASHS0068
C                                                                       ASHS0069
      RI=X(N,M)                                                         ASHS0070
      ZI=Y(N,M)                                                         ASHS0071
      RJ=X(N+1,M)                                                       ASHS0072
      ZJ=Y(N+1,M)                                                       ASHS0073
      AA=ZJ-ZI                                                          ASHS0074
      BB=RJ-RI                                                          ASHS0075
      XL2=AA**2+BB**2                                                   ASHS0076
      XL=SQRTF(XL2)                                                     ASHS0077
C                                                                       ASHS0078
      A(1,1)=1.0/(RI+RJ)                                                ASHS0079
      A(1,2)=0.0                                                        ASHS0080
      A(1,3)=A(1,1)                                                     ASHS0081
      A(1,4)=0.0                                                        ASHS0082
      A(2,1)=-BB/XL2                                                    ASHS0083
      A(2,2)=-AA/XL2                                                    ASHS0084
```

```
      A(2,3)=-A(2,1)                                                    ASHS0085
      A(2,4)=-A(2,2)                                                    ASHS0086
C                                                                       ASHS0087
      B(1,1)=(ESH/(1.-XSH**2))*TSH*XL*(RI+RJ)/2.                        ASHS0088
      B(1,2)=B(1,1)*XSH                                                 ASHS0089
      B(2,1)=B(1,2)                                                     ASHS0090
      B(2,2)=B(1,1)                                                     ASHS0091
C                                                                       ASHS0092
      DO 470 I=1,2                                                      ASHS0093
      DO 470 J=1,4                                                      ASHS0094
      S(I,J)=0.0                                                        ASHS0095
      DO 470 K=1,2                                                      ASHS0096
  470 S(I,J)=S(I,J)+B(I,K)*A(K,J)                                       ASHS0097
C                                                                       ASHS0098
      DO 480 I=1,4                                                      ASHS0099
      DO 480 J=1,4                                                      ASHS0100
      B(I,J)=0.0                                                        ASHS0101
      DO 480 K=1,2                                                      ASHS0102
  480 B(I,J)=B(I,J)+A(K,I)*S(K,J)                                       ASHS0103
C                                                                       ASHS0104
      AT=ASH*(T(N,M)+T(N+1,M))/2.                                       ASHS0105
      A(1)=AT*RI                                                        ASHS0106
      A(2)=AT*ZI                                                        ASHS0107
      A(3)=AT*RJ                                                        ASHS0108
      A(4)=AT*ZJ                                                        ASHS0109
C                                                                       ASHS0110
      DO 485 I=1,4                                                      ASHS0111
      P(I)=0.0                                                          ASHS0112
      DO 485 K=1,4                                                      ASHS0113
  485 P(I)=P(I)+B(I,K)*A(K)                                             ASHS0114
C                                                                       ASHS0115
      DO 495 I=1,2                                                      ASHS0116
      II=((N-1)*MMAX+MB-1)*2+I                                          ASHS0117
      KK=II+MX                                                          ASHS0118
      RR(II)=RR(II)+P(I)                                                ASHS0119
      RR(KK)=RR(KK)+P(I+2)                                              ASHS0120
      DO 490 J=I,2                                                      ASHS0121
      JJ=J-I+1                                                          ASHS0122
      SS(II,JJ)=SS(II,JJ)+B(I,J)                                        ASHS0123
  490 SS(KK,JJ)=SS(KK,JJ)+B(I+2,J+2)                                    ASHS0124
      DO 495 J=1,2                                                      ASHS0125
      JJ=J-I+1+MX                                                       ASHS0126
```

```
  495 SS(II,JJ)=SS(II,JJ)+B(I,J+2)                                 ASHS0127
C                                                                  ASHS0128
  500 CONTINUE                                                     ASHS0129
C                                                                  ASHS0130
C     READ IN SPECIFIED RADIAL AND AXIAL LOADS                     ASHS0131
C                                                                  ASHS0132
      IF(NUMLD) 505, 202, 505                                      ASHS0133
C                                                                  ASHS0134
  505 WRITE OUTPUT TAPE KOUT, 2000                                 ASHS0135
C                                                                  ASHS0136
      DO 550 I=1,NUMLD                                             ASHS0137
      READ INPUT TAPE KINN, 1001,N,M,PR,PZ                         ASHS0138
      WRITE OUTPUT TAPE KOUT, 2001, N,M,PR,PZ                      ASHS0139
      K = 2*( (N-1)*MMAX+M ) -1                                    ASHS0140
      RR(K) = RR(K) + PR                                           ASHS0141
  550 RR(K+1) = RR(K+1) + PZ                                       ASHS0142
C                                                                  ASHS0143
C     BOUNDARY CONDITIONS                                          ASHS0144
C                                                                  ASHS0145
  202 DO 60 N=1,NMAX                                               ASHS0146
      DO 60 M=1,MMAX                                               ASHS0147
   60 KODE(N,M)=0                                                  ASHS0148
C                                                                  ASHS0149
C       FIX EDGE                                                   ASHS0150
C                                                                  ASHS0151
      DO 205 M=1,MMAX                                              ASHS0152
  205 KODE(1,M)=1                                                  ASHS0153
C                                                                  ASHS0154
C     READ IN ADDITIONAL BOUNDARY CONDITIONS                       ASHS0155
C                                                                  ASHS0156
      WRITE OUTPUT TAPE KOUT, 2002                                 ASHS0157
C                                                                  ASHS0158
      DO 208 I=1,NUMBC                                             ASHS0159
      READ INPUT TAPE KINN,1000,NNN,MMM,KKK                        ASHS0160
      WRITE OUTPUT TAPE KOUT, 1000, NNN,MMM,KKK                    ASHS0161
  208 KODE(NNN,MMM)=KKK                                            ASHS0162
C                                                                  ASHS0163
      KK=0                                                         ASHS0164
      DO 250 N=1,NMAX                                              ASHS0165
      DO 250 M=1,MMAX                                              ASHS0166
      IF (KODE(N,M)) 210,250,210                                   ASHS0167
  210 LL=1                                                         ASHS0168
```

```
      LH=2                                                              ASHS0169
      IF (KODE(N,M)-2) 211,212,213                                      ASHS0170
  211 LH=1                                                              ASHS0171
      GO TO 213                                                         ASHS0172
  212 LL=2                                                              ASHS0173
  213 DO 245 L=LL,LH                                                    ASHS0174
      II=KK+L                                                           ASHS0175
      DO 240 J=1,MBAND                                                  ASHS0176
      SS(II,J)=0.0                                                      ASHS0177
      III=II+1-J                                                        ASHS0178
      IF (III) 240,240,230                                              ASHS0179
  230 SS(III,J)=0.0                                                     ASHS0180
  240 CONTINUE                                                          ASHS0181
      SS(II,1)=1.                                                       ASHS0182
  245 RR(II)=0.0                                                        ASHS0183
  250 KK=KK+2                                                           ASHS0184
C                                                                       ASHS0185
C                                                                       ASHS0186
C     SOLVE FOR DISPLACEMENTS                                           ASHS0187
C                                                                       ASHS0188
      CALL SYMSOL(SS,RR,NEQ,MBAND)                                      ASHS0189
C                                                                       ASHS0190
      K=1                                                               ASHS0191
      DO 400 N=1,NMAX                                                   ASHS0192
      DO 400 M=1,MMAX                                                   ASHS0193
      U(N,M)=RR(K)                                                      ASHS0194
      V(N,M)=RR(K+1)                                                    ASHS0195
  400 K=K+2                                                             ASHS0196
C                                                                       ASHS0197
      WRITE OUTPUT TAPE KOUT,2003                                       ASHS0198
      CALL PRINTM(U,NMAX,MMAX,40)                                       ASHS0199
      WRITE OUTPUT TAPE KOUT,2004                                       ASHS0200
      CALL PRINTM(V,NMAX,MMAX,40)                                       ASHS0201
C                                                                       ASHS0202
C     CALCULATE AND PRINT STRESSES                                      ASHS0203
C                                                                       ASHS0204
      CALL STRESS                                                       ASHS0205
C                                                                       ASHS0206
      GO TO 50                                                          ASHS0207
C                                                                       ASHS0208
C     FORMAT STATEMENTS                                                 ASHS0209
C                                                                       ASHS0210
```

```
 1000 FORMAT (3I5)                                                    ASHS0211
 1001 FORMAT (2I5, 2F10.3)                                            ASHS0212
 2000 FORMAT (46H1 POINTS WITH SPECIFIED RADIAL AND AXIAL LOADS //    ASHS0213
     113X 26HN    M    R-LOAD    Z-LOAD)                              ASHS0214
 2001 FORMAT (9X 2I5, 2F10.3)                                         ASHS0215
 2002 FORMAT (44H1 POINTS WITH ADDITIONAL BOUNDARY CONDITIONS //      ASHS0216
     116H    N    M  CODE)                                            ASHS0217
 2003 FORMAT (16H1R-DISPLACEMENTS)                                    ASHS0218
 2004 FORMAT (16H1Z-DISPLACEMENTS)                                    ASHS0219
      END                                                             ASHS0220
```

```
      SUBROUTINE INPUT                                                  ASHS0221
C                                                                       ASHS0222
C     COMMON AND DIMENSION STATEMENTS                                   ASHS0223
C                                                                       ASHS0224
      DIMENSION HED(12),R(40,10),Z(40,10),T(40,10),CC(5,3),TA(40),      ASHS0225
     1 TT(50),A(50,5),B(5,5),D(5,3),EE(50,3),U(40,10),V(40,10)          ASHS0226
     2,KR(5)                                                            ASHS0227
C                                                                       ASHS0228
      COMMON KINN,KOUT,NMAX,MMAX,MB,NMT,NUMLD,NUMBC,TE,TR,              ASHS0229
     1 ES,XS,AS,TS,ESH,XSH,ASH,TSH,                                     ASHS0230
     1 HED,R,Z,T,U,V,CC,TA,TT,A,B,D,EE                                  ASHS0231
C                                                                       ASHS0232
C     READ AND PRINT INPUT DATA                                         ASHS0233
C                                                                       ASHS0234
      READ INPUT TAPE KINN,1000,                                        ASHS0235
     1HED,NMAX,MMAX,MB,NMT,NUMLD,NUMBC,TE,TR,ES,XS,AS,TS,ESH,XSH,ASH,TSHASHS0236
      WRITE OUTPUT TAPE KOUT,2000,                                      ASHS0237
     1HED,NMAX,MMAX,MB,NMT,NUMLD,NUMBC,TE,TR                            ASHS0238
C                                                                       ASHS0239
C     CHECK THE SIZE OF NMAX, MMAX, NMT AND LOCATION OF MB.  CHECK FOR  ASHS0240
C     ADDITIONAL BOUNDARY CONDITIONS.  IF ANY INPUT ERRORS EXIT.        ASHS0241
C                                                                       ASHS0242
      DO 1 I=1,5                                                        ASHS0243
    1 KR(I) = 0                                                         ASHS0244
C                                                                       ASHS0245
      IF (40-NMAX) 5, 10, 10                                            ASHS0246
    5 KR(1) = 1                                                         ASHS0247
   10 IF (10-MMAX) 15, 20, 20                                           ASHS0248
   15 KR(2) = 2                                                         ASHS0249
   20 IF (MMAX-MB) 25, 25, 30                                           ASHS0250
   25 KR(3) = 3                                                         ASHS0251
   30 IF (50-NMT) 35, 40, 40                                            ASHS0252
   35 KR(4) = 4                                                         ASHS0253
   40 IF (NUMBC) 45, 45, 50                                             ASHS0254
   45 KR(5) = 6                                                         ASHS0255
C                                                                       ASHS0256
   50 DO 65 I=1,5                                                       ASHS0257
      IF(KR(I)) 60, 65, 60                                              ASHS0258
   60 WRITE OUTPUT TAPE KOUT, 2011, KR(I)                               ASHS0259
      KR = 1                                                            ASHS0260
   65 CONTINUE                                                          ASHS0261
C                                                                       ASHS0262
      IF(KR) 70, 75, 70                                                 ASHS0263
   70 CALL EXIT                                                         ASHS0264
C                                                                       ASHS0265
   75 WRITE OUTPUT TAPE KOUT, 2006                                      ASHS0266
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      WRITE OUTPUT TAPE KOUT,2008, ES,XS,AS,TS                          ASHS0267
      WRITE OUTPUT TAPE KOUT,2007                                       ASHS0268
      WRITE OUTPUT TAPE KOUT,2008, ESH,XSH,ASH,TSH                      ASHS0269
      READ INPUT TAPE KINN,1001,                                        ASHS0270
     1 (R(N,MB),Z(N,MB),T(N,MB),TA(N),N=1,NMAX)                         ASHS0271
      WRITE OUTPUT TAPE KOUT,2001,                                      ASHS0272
     1 (R(N,MB),Z(N,MB),T(N,MB),TA(N),N=1,NMAX)                         ASHS0273
C                                                                       ASHS0274
C     LEAST SQUARE EVALUATION OF MATERIAL PROPERTIES                    ASHS0275
C                                                                       ASHS0276
      READ INPUT TAPE KINN,1002,                                        ASHS0277
     1 (TT(I),EE(I,1),EE(I,2),EE(I,3),I=1,NMT)                          ASHS0278
      WRITE OUTPUT TAPE KOUT, 2009                                      ASHS0279
      WRITE OUTPUT TAPE KOUT,2002,                                      ASHS0280
     1 (TT(I),EE(I,1),EE(I,2),EE(I,3),I=1,NMT)                          ASHS0281
C                                                                       ASHS0282
      DO 100 I=1,NMT                                                    ASHS0283
      A(I,1)=1.0                                                        ASHS0284
      A(I,2)=TT(I)                                                      ASHS0285
      A(I,3)=TT(I)*TT(I)                                                ASHS0286
      A(I,4)=A(I,3)*TT(I)                                               ASHS0287
  100 A(I,5)=A(I,4)*TT(I)                                               ASHS0288
C                                                                       ASHS0289
      CALL LEAST(A,EE,B,D,CC,NMT,5,3)                                   ASHS0290
C                                                                       ASHS0291
      WRITE OUTPUT TAPE KOUT, 2010                                      ASHS0292
      WRITE OUTPUT TAPE KOUT,2002,                                      ASHS0293
     1 (TT(I),EE(I,1),EE(I,2),EE(I,3),I=1,NMT)                          ASHS0294
C                                                                       ASHS0295
C     GENERATE MESH                                                     ASHS0296
C                                                                       ASHS0297
      MC = MB+1                                                         ASHS0298
      MBB = MB-1                                                        ASHS0299
      SHL = FLOATF(MBB)                                                 ASHS0300
      ABL = FLOATF(MMAX-MB)                                             ASHS0301
      DO 200 N=1,NMAX                                                   ASHS0302
C                                                                       ASHS0303
C     CHECK FOR END POINTS                                              ASHS0304
C                                                                       ASHS0305
      NL = N-1                                                          ASHS0306
       NH=N+1                                                           ASHS0307
      IF(NL)160,150,160                                                 ASHS0308
  150 SS = 0.                                                           ASHS0309
      CX = 1.                                                           ASHS0310
      GO TO 185                                                         ASHS0311
  160 IF(N-NMAX)180,170,180                                             ASHS0312
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```
  170  NH=N                                                         ASHS0313
C                                                                   ASHS0314
C      COMPUTE SIN AND COS AT EACH POINT ON BOND LINE               ASHS0315
C                                                                   ASHS0316
  180 XX = R(  NH,MB) - R(NL,MB)                                    ASHS0317
      YY = Z( NH ,MB) - Z(NL,MB)                                    ASHS0318
      ZZ = SQRTF(XX**2 + YY**2)                                     ASHS0319
      SS = YY/ZZ                                                    ASHS0320
      CX = XX/ZZ                                                    ASHS0321
C                                                                   ASHS0322
C      COMPUTE COORDINATES OF POINTS IN CONSTANT THICKNESS SHELL    ASHS0323
C                                                                   ASHS0324
  185 DO 190 M=1,MBB                                                ASHS0325
      TT = TS*FLOATF(MB-M)/SHL                                      ASHS0326
      T(N,M) = T(N,MB)                                              ASHS0327
      R(N,M) = R(N,MB) + TT*SS                                      ASHS0328
  190 Z(N,M) = Z(N,MB) - TT*CX                                      ASHS0329
C      COMPUTE COORDINATES OF POINTS IN VARIABLE THICKNESS ABLATER  ASHS0330
C                                                                   ASHS0331
      DO 200 M=MC,MMAX                                              ASHS0332
      TT = TA(N)*FLOATF(M-MB)/ABL                                   ASHS0333
      T(N,M) = T(N,MB) + (TE - T(N,MB)) * (TT/TA(N))**2             ASHS0334
      R(N,M) = R(N,MB) - TT*SS                                      ASHS0335
  200 Z(N,M) = Z(N,MB) + TT*CX                                      ASHS0336
C                                                                   ASHS0337
C      DUMP OUT R-ORDINATE TABLE AND Z-ORDINATE TABLE               ASHS0338
C                                                                   ASHS0339
      WRITE OUTPUT TAPE KOUT,2003                                   ASHS0340
      CALL PRINTM (R,NMAX,MMAX,40)                                  ASHS0341
      WRITE OUTPUT TAPE KOUT,2004                                   ASHS0342
      CALL PRINTM (Z,NMAX,MMAX,40)                                  ASHS0343
      WRITE OUTPUT TAPE KOUT,2005                                   ASHS0344
      CALL PRINTM (T,NMAX,MMAX,40)                                  ASHS0345
      RETURN                                                        ASHS0346
C                                                                   ASHS0347
C      FORMAT STATEMENTS                                            ASHS0348
C                                                                   ASHS0349
 1000 FORMAT (12A6/6I5,2F10.2/4F10.2/4F10.2)                        ASHS0350
 1001 FORMAT (4F10.2)                                               ASHS0351
 1002 FORMAT (4F10.2)                                               ASHS0352
 2000 FORMAT ( 1H1 12A6/                                            ASHS0353
     1 33HONUMBER OF POINTS ALONG LENGTH--- I3/                     ASHS0354
     2 33H NUMBER OF POINTS THRU THICKNESS- I3/                     ASHS0355
     3 33H LOCATION OF BOND LINE----------- I3/                     ASHS0356
     4 33H NUMBER OF PROPERTY CARDS-------- I3/                     ASHS0357
     5 33H NUMBER OF POINTS WITH R,Z LOADS- I3/                     ASHS0358
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     6 33H ADDITIONAL BOUNDARY CONDITIONS-- I3/                      ASHS0359
     7 33H SURFACE TEMPERATURE OF ABLATOR-- F6.0/                    ASHS0360
     8 33H ZERO STRESS TEMPERATURE--------- F6.0)                    ASHS0361
 2001 FORMAT (15H1    R-ORDINATE 5X 10HZ-ORDINATE 5X 10HBOND TEMP. 3X ASHS0362
     1 17HABLATOR THICKNESS / (3F15.3,1F20.4))                       ASHS0363
 2002 FORMAT (15H0  TEMPERATURE 5X 10HMODULUS A 5X 9HMODULUS B   1X  ASHS0364
     120H COEFF. OF EXPANSION /(3F15.0,1E20.5))                      ASHS0365
 2003 FORMAT (14H1  R-ORDINATES )                                    ASHS0366
 2004 FORMAT (14H1  Z-ORDINATES )                                    ASHS0367
 2005 FORMAT (14H1  TEMPERATURE )                                    ASHS0368
 2006 FORMAT (37H0PROPERTIES OF SANDWICH CORE MATERIAL )             ASHS0369
 2007 FORMAT (30H0PROPERTIES OF SANDWICH PLATES )                    ASHS0370
 2008 FORMAT                                                         ASHS0371
     1(28H  MODULUS OF ELASTICITY----- F10.0/                        ASHS0372
     2 28H  POISSONS RATIO------------ F10.4/                        ASHS0373
     3 28H  COEFFICIENT OF EXPANSION-- F10.8/                        ASHS0374
     4 28H  THICKNESS----------------- F10.4)                        ASHS0375
 2009 FORMAT ( 25H1  MATERIAL PROPERTY DATA )                        ASHS0376
 2010 FORMAT ( 1H1 5X 55HLEAST SQUARE EVALUATION OF ABOVE MATERIAL PROPEASHS0377
     1RTY DATA )                                                     ASHS0378
 2011 FORMAT ( 49H0 ERROR IN ABOVE INPUT DATA.  CHECK VALUE ON LINE I3 )ASHS0379
      END                                                            ASHS0380
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      SUBROUTINE TRIST(IX,JX,KX,TT,NNN)                                 ASHS0381
C                                                                       ASHS0382
      DIMENSION XX(5),YY(5),S(10,10),P(10),A(6,6),B(6,6),C(6,6),LM(3)   ASHS0383
     1 , HED(12),X(40,10),Y(40,10),T(40,10),CC(5,3),EE(6),U(40,10)      ASHS0384
     3 ,V(40,10)                                                        ASHS0385
C                                                                       ASHS0386
      COMMON KINN,KOUT,NMAX,MMAX,MB,NMT,NUMLD,NUMBC,TE,TR,              ASHS0387
     1 ES,XS,AS,TS,ESH,XSH,ASH,TSH,                                     ASHS0388
     1 HED,X,Y,T,U,V,CC,XX,YY,S,P,C                                     ASHS0389
C                                                                       ASHS0390
C     INITIALIZATION                                                    ASHS0391
C                                                                       ASHS0392
      AJ=XX(JX)-XX(IX)                                                  ASHS0393
      AK=XX(KX)-XX(IX)                                                  ASHS0394
      BJ=YY(JX)-YY(IX)                                                  ASHS0395
      BK=YY(KX)-YY(IX)                                                  ASHS0396
      TEMP=AJ*BK-BJ*AK                                                  ASHS0397
      AREA=TEMP/2.                                                      ASHS0398
      THICK=(XX(IX)+XX(JX)+XX(KX))/3.                                   ASHS0399
      DO 50 I=1,36                                                      ASHS0400
      A(I)=0.0                                                          ASHS0401
   50 B(I)=0.0                                                          ASHS0402
C                                                                       ASHS0403
C     CALCULATE MATERIAL PROPERTIES                                     ASHS0404
C                                                                       ASHS0405
      DO 60 I=1,3                                                       ASHS0406
   60 EE(I)=CC(1,I)+TT*(CC(2,I)+TT*(CC(3,I)+TT*(CC(4,I)+TT*CC(5,I))))   ASHS0407
      IF (NNN) 70,70,80                                                 ASHS0408
   70 EP=EE(1)                                                          ASHS0409
      ALP=EE(3)                                                         ASHS0410
      XNU=0.3155+0.000174*TT                                            ASHS0411
      GO TO 90                                                          ASHS0412
   80 EP=ES                                                             ASHS0413
      ALP=AS                                                            ASHS0414
      XNU=XS                                                            ASHS0415
   90 AT=ALP*(TT-TR)                                                    ASHS0416
      COMM=EP/(1.-2.*XNU)                                               ASHS0417
      TE=COMM*AT                                                        ASHS0418
      COMM=COMM/(1.+XNU)                                                ASHS0419
C                                                                       ASHS0420
C     FORM MATRIX (B)                                                   ASHS0421
C                                                                       ASHS0422
```

```
      E=XNU*COMM                                                        ASHS0423
      D=(1.-XNU)*COMM                                                   ASHS0424
      C=(.5-XNU)*COMM                                                   ASHS0425
      B(1,1)=D                                                          ASHS0426
      B(1,2)=E                                                          ASHS0427
      B(1,4)=E                                                          ASHS0428
      B(2,1)=E                                                          ASHS0429
      B(2,2)=D                                                          ASHS0430
      B(2,4)=E                                                          ASHS0431
      B(3,3)=C                                                          ASHS0432
      B(4,1)=E                                                          ASHS0433
      B(4,2)=E                                                          ASHS0434
      B(4,4)=D                                                          ASHS0435
C                                                                       ASHS0436
C     FORM MATRIX (A)                                                   ASHS0437
C                                                                       ASHS0438
      A(1,1)=BJ-BK                                                      ASHS0439
      A(1,3)=BK                                                         ASHS0440
      A(1,5)=-BJ                                                        ASHS0441
      A(2,2)=AK-AJ                                                      ASHS0442
      A(2,4)=-AK                                                        ASHS0443
      A(2,6)=AJ                                                         ASHS0444
      A(3,1)=AK-AJ                                                      ASHS0445
      A(3,2)=BJ-BK                                                      ASHS0446
      A(3,3)=-AK                                                        ASHS0447
      A(3,4)=BK                                                         ASHS0448
      A(3,5)=AJ                                                         ASHS0449
      A(3,6)=-BJ                                                        ASHS0450
      C=.333333*TEMP/THICK                                              ASHS0451
      A(4,1)=C                                                          ASHS0452
      A(4,3)=C                                                          ASHS0453
      A(4,5)=C                                                          ASHS0454
      DO 95 I=1,36                                                      ASHS0455
   95 A(I)=A(I)/TEMP                                                    ASHS0456
C                                                                       ASHS0457
C     FORM TRIANGULAR ELEMENT STIFFNESS MATRIX (A)T(B)(A)               ASHS0458
C                                                                       ASHS0459
      COMM=THICK*AREA                                                   ASHS0460
      DO 100 L=1,4                                                      ASHS0461
      DO 100 M=1,6                                                      ASHS0462
      C(L,M)=0.0                                                        ASHS0463
      DO 100 N=1,4                                                      ASHS0464
```

```
  100 C(L,M)=C(L,M)+B(L,N)*A(N,M)                                    ASHS0465
      DO 210 L=1,6                                                    ASHS0466
      DO 210 M=1,6                                                    ASHS0467
      B(L,M)=0.0                                                      ASHS0468
      DO 200 N=1,4                                                    ASHS0469
  200 B(L,M)=B(L,M)+A(N,L)*C(N,M)                                    ASHS0470
  210 B(L,M)=B(L,M)*COMM                                              ASHS0471
C                                                                     ASHS0472
C     COMPUTE TEMPERATURE FORCES                                      ASHS0473
C                                                                     ASHS0474
      A(1)=AT*XX(IX)                                                  ASHS0475
      A(2)=0.0                                                        ASHS0476
      A(3)=AT*XX(JX)                                                  ASHS0477
      A(4)=AT*BJ                                                      ASHS0478
      A(5)=AT*XX(KX)                                                  ASHS0479
      A(6)=AT*BK                                                      ASHS0480
C                                                                     ASHS0481
      DO 220 I=1,6                                                    ASHS0482
      EE(I)=0.0                                                       ASHS0483
      DO 220 K=1,6                                                    ASHS0484
  220 EE(I)=EE(I)+B(I,K)*A(K)                                         ASHS0485
C                                                                     ASHS0486
C     ADD ELEMENT STIFFNESS TO QUADRILATERAL STIFFNESS                ASHS0487
C                                                                     ASHS0488
      LM(1)=2*IX-2                                                    ASHS0489
      LM(2)=2*JX-2                                                    ASHS0490
      LM(3)=2*KX-2                                                    ASHS0491
      DO 250 I=1,3                                                    ASHS0492
      DO 250 K=1,2                                                    ASHS0493
      II=LM(I)+K                                                      ASHS0494
      KK=2*I-2+K                                                      ASHS0495
      P(II)=P(II)+EE(KK)                                              ASHS0496
      DO 250 J=1,3                                                    ASHS0497
      DO 250 L=1,2                                                    ASHS0498
      JJ=LM(J)+L                                                      ASHS0499
      LL=2*J-2+L                                                      ASHS0500
  250 S(II,JJ)=S(II,JJ)+B(KK,LL)                                      ASHS0501
C                                                                     ASHS0502
      RETURN                                                          ASHS0503
C                                                                     ASHS0504
      END                                                             ASHS0505
```

```
      SUBROUTINE STRESS                                                 ASHS0506
C                                                                       ASHS0507
C                                                                       ASHS0508
      DIMENSION T(40,10),EE(3),CC(5,3),HED(12),                         ASHS0509
     1 XX(5),YY(5),S(10,10),P(10),X(40,10),Y(40,10),U(40,10),V(40,10)   ASHS0510
     2 ,A(9,9),B(9,2),C(6,6),D(9,9),E(9,2),SIGTT(40,10),SIGXM(40,10)    ASHS0511
     3 ,SIGXX(40,10),SIGYY(40,10),SIGXY(40,10),SIGNN(40,10),SIGSS(40,10)ASHS0512
     4 ,SIGNS(40,10),BSIGS(40),BSIGT(40),SIG(4)                         ASHS0513
C                                                                       ASHS0514
      COMMON KINN,KOUT,NMAX,MMAX,MB,NMT,NUMLD,NUMBC,TE,TR,              ASHS0515
     1 ES,XS,AS,TS,ESH,XSH,ASH,TSH,                                     ASHS0516
     1 HED,X,Y,T,U,V,CC,XX,YY,S,P,C                                     ASHS0517
     1 ,A,B,D,E,NN,MM,SIGXX,SIGYY,SIGXY,SIGMX,SIGMN,SIGTT,BSIGS,BSIGT   ASHS0518
     1  ,EE,SIGNN,SIGNS,SIGSS,SIGXM,SIG                                 ASHS0519
C                                                                       ASHS0520
C                                                                       ASHS0521
      NN=NMAX-1                                                         ASHS0522
      MM=MMAX-1                                                         ASHS0523
      DO 200 N=1,NN                                                     ASHS0524
      DO 200 M=1,MM                                                     ASHS0525
C                                                                       ASHS0526
      XX(1)=X(N,M)                                                      ASHS0527
      XX(2)=X(N,M+1)                                                    ASHS0528
      XX(3)=X(N+1,M)                                                    ASHS0529
      XX(4)=X(N+1,M+1)                                                  ASHS0530
      YY(1)=Y(N,M)                                                      ASHS0531
      YY(2)=Y(N,M+1)                                                    ASHS0532
      YY(3)=Y(N+1,M)                                                    ASHS0533
      YY(4)=Y(N+1,M+1)                                                  ASHS0534
      NNN=MB-M                                                          ASHS0535
      TT=.25*(T(N,M)+T(N,M+1)+T(N+1,M)+T(N+1,M+1))                      ASHS0536
C                                                                       ASHS0537
      CALL TRIST(1,4,2,TT,NNN)                                          ASHS0538
C                                                                       ASHS0539
      P(1)=U(N,M)                                                       ASHS0540
      P(2)=V(N,M)                                                       ASHS0541
      P(3)=U(N+1,M+1)                                                   ASHS0542
      P(4)=V(N+1,M+1)                                                   ASHS0543
      P(5)=U(N,M+1)                                                     ASHS0544
      P(6)=V(N,M+1)                                                     ASHS0545
C                                                                       ASHS0546
      DO 150 I=1,4                                                      ASHS0547
```

```
      SIG(I)=0.0                                                        ASHS0548
      DO 150 K=1,6                                                      ASHS0549
  150 SIG(I)=SIG(I)+C(I,K)*P(K)                                         ASHS0550
C                                                                       ASHS0551
      CALL TRIST(1,3,4,TT,NNN)                                          ASHS0552
C                                                                       ASHS0553
      P(1)=U(N,M)                                                       ASHS0554
      P(2)=V(N,M)                                                       ASHS0555
      P(3)=U(N+1,M)                                                     ASHS0556
      P(4)=V(N+1,M)                                                     ASHS0557
      P(5)=U(N+1,M+1)                                                   ASHS0558
      P(6)=V(N+1,M+1)                                                   ASHS0559
C                                                                       ASHS0560
      DO 170 I=1,4                                                      ASHS0561
      DO 160 K=1,6                                                      ASHS0562
  160 SIG(I)=SIG(I)+C(I,K)*P(K)                                         ASHS0563
  170 SIG(I)=SIG(I)/2.0                                                 ASHS0564
C                                                                       ASHS0565
      SIGXX(N,M)=SIG(1)-TE                                              ASHS0566
      SIGYY(N,M)=SIG(2)-TE                                              ASHS0567
      SIGXY(N,M)=SIG(3)                                                 ASHS0568
      SIGTT(N,M)=SIG(4)-TE                                              ASHS0569
C                                                                       ASHS0570
      COST = .5*( X(N,MB) + X(N+1,MB) - X(N,1) - X(N+1,1) ) / TS        ASHS0571
      SINT = .5*( Y(N,MB) + Y(N+1,MB) - Y(N,1) - Y(N+1,1) ) / TS        ASHS0572
      COS2=COST**2                                                      ASHS0573
      SIN2=SINT**2                                                      ASHS0574
      TT=2.0*SIGXY(N,M)*SINT*COST                                       ASHS0575
      SIGNN(N,M)=SIGXX(N,M)*COS2+SIGYY(N,M)*SIN2+TT                     ASHS0576
      SIGSS(N,M)=SIGXX(N,M)*SIN2+SIGYY(N,M)*COS2-TT                     ASHS0577
      SIGNS(N,M)=(SIGYY(N,M)-SIGXX(N,M))*SINT*COST+SIGXY(N,M)*          ASHS0578
     1 (COS2-SIN2)                                                      ASHS0579
  200 CONTINUE                                                          ASHS0580
C                                                                       ASHS0581
C     PRINT STRESSES                                                    ASHS0582
C                                                                       ASHS0583
      WRITE OUTPUT TAPE KOUT,1006                                       ASHS0584
      CALL PRINTM(SIGTT,NN,MM,40)                                       ASHS0585
      WRITE OUTPUT TAPE KOUT,1007                                       ASHS0586
      CALL PRINTM(SIGXX,NN,MM,40)                                       ASHS0587
      WRITE OUTPUT TAPE KOUT,1008                                       ASHS0588
      CALL PRINTM(SIGYY,NN,MM,40)                                       ASHS0589
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      WRITE OUTPUT TAPE KOUT,1009                                       ASHS0590
      CALL PRINTM(SIGXY,NN,MM,40)                                       ASHS0591
      WRITE OUTPUT TAPE KOUT,1011                                       ASHS0592
      CALL PRINTM(SIGSS,NN,MM,40)                                       ASHS0593
C                                                                       ASHS0594
C     CALCULATION OF STRESSES IN SANDWICH PLATES                        ASHS0595
C                                                                       ASHS0596
      ED=ESH/(1.-XSH**2)                                                ASHS0597
      EE=XSH*ED                                                         ASHS0598
      MM=MB-1                                                           ASHS0599
C                                                                       ASHS0600
      DO 225 N=1,NN                                                     ASHS0601
      DO 225 M=1,MB,MM                                                  ASHS0602
      RI=X(N,M)                                                         ASHS0603
      ZI=Y(N,M)                                                         ASHS0604
      RJ=X(N+1,M)                                                       ASHS0605
      ZJ=Y(N+1,M)                                                       ASHS0606
      AA=ZJ-ZI                                                          ASHS0607
      BB=RJ-RI                                                          ASHS0608
      XL2=AA**2+BB**2                                                   ASHS0609
      AT =.5*ASH*(T(N,M)+T(N+1,M))                                      ASHS0610
      EPT=(U(N,M)+U(N+1,M))/(RI+RJ)-AT                                  ASHS0611
      EPS=(BB*(U(N+1,M)-U(N,M))+AA*(V(N+1,M)-V(N,M)))/XL2 -AT           ASHS0612
      SIGTT(N,M)=ED*EPT+EE*EPS                                          ASHS0613
  225 SIGSS(N,M)=EE*EPT+ED*EPS                                          ASHS0614
C                                                                       ASHS0615
      WRITE OUTPUT TAPE KOUT, 1013,                                     ASHS0616
     1( N, SIGTT(N,1), SIGSS(N,1), SIGTT(N,MB), SIGSS(N,MB), N=1,NN )   ASHS0617
C                                                                       ASHS0618
C     CALCULATION OF STRESSES IN BOND LAYER                             ASHS0619
C                                                                       ASHS0620
      DO 300 N=1,NN                                                     ASHS0621
      TT = 0.50*( T(N,MB)+T(N+1,MB) )                                   ASHS0622
      DO 260 I=1,3                                                      ASHS0623
  260 EE(I)=CC(1,I)+TT*(CC(2,I)+TT*(CC(3,I)+TT*(CC(4,I)+TT*CC(5,I))))   ASHS0624
      XNU = 0.3155+0.000174*TT                                          ASHS0625
      AA = Y(N+1,MB)-Y(N,MB)                                            ASHS0626
      BB = X(N+1,MB)-X(N,MB)                                            ASHS0627
      XL2 = AA**2+BB**2                                                 ASHS0628
      EEE = XNU*SIGNN(N,MB)/EE(2) - EE(3)*TT                            ASHS0629
      EPT = ( U(N,MB)+U(N+1,MB) )/( X(N+1,MB)+X(N,MB) ) + EEE           ASHS0630
      EPS = (BB*(U(N+1,MB)-U(N,MB))+AA*(V(N+1,MB)-V(N,MB)))/XL2+EEE     ASHS0631
```

```
      COMM = EE(2)/(1.-XNU**2)                                          ASHS0632
      BSIGS(N) = COMM*(EPS+XNU*EPT)                                     ASHS0633
  300 BSIGT(N) = COMM*(EPT+XNU*EPS)                                     ASHS0634
      WRITE OUTPUT TAPE KOUT, 1014, (N, BSIGS(N), BSIGT(N), SIGNN(N,MB),ASHS0635
     1 SIGNS(N,MB), N=1,NN )                                            ASHS0636
C                                                                       ASHS0637
  250 RETURN                                                            ASHS0638
C                                                                       ASHS0639
C     FORMAT STATEMENTS                                                 ASHS0640
C                                                                       ASHS0641
 1006 FORMAT (10H1 T-STRESS)                                            ASHS0642
 1007 FORMAT (10H1 R-STRESS)                                            ASHS0643
 1008 FORMAT (10H1 Z-STRESS)                                            ASHS0644
 1009 FORMAT (10H1RZ-STRESS)                                            ASHS0645
 1011 FORMAT (10H1 S-STRESS)                                            ASHS0646
 1013 FORMAT ( 1H1 42X 27HSTRESSES IN SANDWICH PLATES /                 ASHS0647
     1         27X 11HLOWER PLATE 39X 11HUPPER PLATE /                  ASHS0648
     2         20X 5HMERID 15X 4HHOOP 25X 5HMERID 15X 4HHOOP /          ASHS0649
     3  4X 1HN 14X 6HSTRESS 14X 6HSTRESS 24X 6HSTRESS 14X 6HSTRESS //   ASHS0650
     4     ( I5, 2F20.0, 10X 2F20.0) )                                  ASHS0651
 1014 FORMAT (1H1 11X 22HSTRESSES IN BOND LAYER //                      ASHS0652
     145H    N      MERID       HOOP    NORMAL     SHEAR //             ASHS0653
     2(I5, 4F10.1) )                                                    ASHS0654
      END                                                               ASHS0655
```

```
      SUBROUTINE LEAST(A,B,C,D,E,N,M,L)                                 ASHS0656
C                                                                       ASHS0657
      DIMENSION A(50,5),B(50,3),C(5,5),D(5,3),E(5,3)                    ASHS0658
C                                                                       ASHS0659
C     (C)=(A)T*(A)   AND (D)=(A)T*(B)                                   ASHS0660
C                                                                       ASHS0661
      DO 200 I=1,M                                                      ASHS0662
      DO 100 J=1,M                                                      ASHS0663
      C(I,J)=0.0                                                        ASHS0664
      DO 100 K=1,N                                                      ASHS0665
  100 C(I,J)=C(I,J)+A(K,I)*A(K,J)                                       ASHS0666
      DO 200 J=1,L                                                      ASHS0667
      D(I,J)=0.0                                                        ASHS0668
      DO 200 K=1,N                                                      ASHS0669
  200 D(I,J)=D(I,J)+A(K,I)*B(K,J)                                       ASHS0670
C                                                                       ASHS0671
C     INVERT (C)                                                        ASHS0672
C                                                                       ASHS0673
      CALL INVERT(C,M,5,B,E)                                            ASHS0674
C                                                                       ASHS0675
C      (E)=(C)*(D)                                                      ASHS0676
C                                                                       ASHS0677
      DO 300 I=1,M                                                      ASHS0678
      DO 300 J=1,L                                                      ASHS0679
      E(I,J)=0.0                                                        ASHS0680
      DO 300 K=1,M                                                      ASHS0681
  300 E(I,J)=E(I,J)+C(I,K)*D(K,J)                                       ASHS0682
C                                                                       ASHS0683
C     CHECK RESULTS  (B)=(A)*(E)                                        ASHS0684
C                                                                       ASHS0685
      DO 400 I=1,N                                                      ASHS0686
      DO 400 J=1,L                                                      ASHS0687
      B(I,J)=0.0                                                        ASHS0688
      DO 400 K=1,M                                                      ASHS0689
  400 B(I,J)=B(I,J)+A(I,K)*E(K,J)                                       ASHS0690
C                                                                       ASHS0691
      RETURN                                                            ASHS0692
C                                                                       ASHS0693
      END                                                               ASHS0694
```

```
      SUBROUTINE SYMSOL (A,B,NN,MM)
C
      DIMENSION A(800,24),B(800),C(24)
C
      N = 0
  100 N = N+1
C
C     REDUCE N TH EQUATION
C
C        1. DIVIDE RIGHT SIDE BY DIAGONAL ELEMENT
C
      B(N) = B(N) / A(N,1)
C
C        2. CHECK FOR LAST EQUATION
C
      IF(N-NN) 150,300,150
C
C        3. DIVIDE N TH EQUATION BY DIAGONAL ELEMENT
C
  150 DO 200 K=2,MM
      C(K) = A(N,K)
  200 A(N,K) = A(N,K) / A(N,1)
C
C        4. REDUCE REMAINING EQUATIONS
C
      DO 260 L=2,MM
      I = N+L-1
      IF(NN-I) 260,240,240
  240 J=0
      DO 250 K=L,MM
      J=J+1
  250 A(I,J) = A(I,J) - C(L) * A(N,K)
      B(I) = B(I) - C(L) * B(N)
  260 CONTINUE
      GO TO 100
C
C     BACK SUBSTITUTION
C
  300 N = N-1
C
C        1. CHECK FOR FIRST EQUATION
C
```

```
      IF(N) 350,500,350                                                 ASHS0825
C                                                                       ASHS0826
C        2. CALCULATE UNKNOWN B(N)                                      ASHS0827
C                                                                       ASHS0828
  350 DO 400 K=2,MM                                                     ASHS0829
      L = N+K-1                                                         ASHS0830
      IF(NN-L) 400,370,370                                              ASHS0831
  370 B(N) = B(N) - A(N,K) * B(L)                                       ASHS0832
  400 CONTINUE                                                          ASHS0833
      GO TO 300                                                         ASHS0834
C                                                                       ASHS0835
  500 RETURN                                                            ASHS0836
C                                                                       ASHS0837
      END                                                               ASHS0838
```

```
      SUBROUTINE INVERT(A,NN,N,M,C)                                     ASHS0695
C                                                                       ASHS0696
C          GENERAL MATRIX INVERSION SUBROUTINE                          ASHS0697
C                                                                       ASHS0698
      DIMENSION A(1),M(1),C(1)                                          ASHS0699
C                                                                       ASHS0700
      DO 90 I=1,NN                                                      ASHS0701
   90 M(I)=-I                                                           ASHS0702
C                                                                       ASHS0703
      DO 140 I=1,NN                                                     ASHS0704
C                                                                       ASHS0705
C     LOCATE LARGEST ELEMENT                                            ASHS0706
C                                                                       ASHS0707
      D=0.0                                                             ASHS0708
      DO 112 L=1,NN                                                     ASHS0709
      IF (M(L)) 100,100,112                                             ASHS0710
  100 J=L                                                               ASHS0711
      DO 110 K=1,NN                                                     ASHS0712
      IF (M(K)) 103,103,108                                             ASHS0713
  103 IF (ABSF(D)-ABSF(A(J))) 105,105,108                               ASHS0714
  105 LD=L                                                              ASHS0715
      KD=K                                                              ASHS0716
      D=A(J)                                                            ASHS0717
  108 J=J+N                                                             ASHS0718
  110 CONTINUE                                                          ASHS0719
  112 CONTINUE                                                          ASHS0720
C                                                                       ASHS0721
C     INTERCHANGE ROWS                                                  ASHS0722
C                                                                       ASHS0723
      TEMP=-M(LD)                                                       ASHS0724
      M(LD)=M(KD)                                                       ASHS0725
      M(KD)=TEMP                                                        ASHS0726
      L=LD                                                              ASHS0727
      K=KD                                                              ASHS0728
      DO 114 J=1,NN                                                     ASHS0729
      C(J)=A(L)                                                         ASHS0730
      A(L)=A(K)                                                         ASHS0731
      A(K)=C(J)                                                         ASHS0732
      L=L+N                                                             ASHS0733
  114 K=K+N                                                             ASHS0734
C                                                                       ASHS0735
C     DIVIDE COLUNM BY LARGEST ELEMENT                                  ASHS0736
```

```
C                                                                       ASHS0737
      NR=(KD-1)*N+1                                                     ASHS0738
      NH=NR+N-1                                                         ASHS0739
      DO 115 K=NR,NH                                                    ASHS0740
  115 A(K)=A(K)/D                                                       ASHS0741
C                                                                       ASHS0742
C     REDUCE REMAINING ROWS AND COLUMNS                                 ASHS0743
C                                                                       ASHS0744
      L=1                                                               ASHS0745
      DO 135 J=1,NN                                                     ASHS0746
      IF (J-KD) 130,125,130                                             ASHS0747
  125 L=L+N                                                             ASHS0748
      GO TO 135                                                         ASHS0749
  130 DO 134 K=NR,NH                                                    ASHS0750
      A(L)=A(L)-C(J)*A(K)                                               ASHS0751
  134 L=L+1                                                             ASHS0752
  135 CONTINUE                                                          ASHS0753
C                                                                       ASHS0754
C     REDUCE ROW                                                        ASHS0755
C                                                                       ASHS0756
      C(KD)=-1.0                                                        ASHS0757
      J=KD                                                              ASHS0758
      DO 140 K=1,NN                                                     ASHS0759
      A(J)=-C(K)/D                                                      ASHS0760
  140 J=J+N                                                             ASHS0761
C                                                                       ASHS0762
C     INTERCHANGE COLUMNS                                               ASHS0763
C                                                                       ASHS0764
      DO 200 I=1,NN                                                     ASHS0765
      L=0                                                               ASHS0766
  150 L=L+1                                                             ASHS0767
      IF(M(L)-I) 150,160,150                                            ASHS0768
  160 K=(L-1)*N+1                                                       ASHS0769
      J=(I-1)*N+1                                                       ASHS0770
      M(L)=M(I)                                                         ASHS0771
      M(I)=I                                                            ASHS0772
      DO 200 L=1,NN                                                     ASHS0773
      TEMP=A(K)                                                         ASHS0774
      A(K)=A(J)                                                         ASHS0775
      A(J)=TEMP                                                         ASHS0776
      J=J+1                                                             ASHS0777
  200 K=K+1                                                             ASHS0778
```

```
C                                                                       ASHS0779
      RETURN                                                            ASHS0780
C                                                                       ASHS0781
      END                                                               ASHS0782
```

```
      SUBROUTINE PRINTM (A,NR,NC,MAXR)                                  ASHS0839
C     SUBROUTINE TO PRINT ANY ARRAY                                     ASHS0840
      DIMENSION A(1),NHED(10)                                           ASHS0841
      COMMON KINN,KOUT                                                  ASHS0842
C                                                                       ASHS0843
      DO 50 I=1,NC,10                                                   ASHS0844
      II=NC-I+1                                                         ASHS0845
      IF (II-10) 20,20,10                                               ASHS0846
   10 II=10                                                             ASHS0847
   20 DO 30 J=1,II                                                      ASHS0848
   30 NHED(J)=I+J-1                                                     ASHS0849
C                                                                       ASHS0850
      WRITE OUTPUT TAPE KOUT,120,(NHED(J),J=1,II)                       ASHS0851
C                                                                       ASHS0852
      DO 50 J=1,NR                                                      ASHS0853
      KL=J+(I-1)*MAXR                                                   ASHS0854
      KH=KL+(II-1)*MAXR                                                 ASHS0855
   50 WRITE OUTPUT TAPE KOUT,130,(J,(A(K),K=KL,KH,MAXR))                ASHS0856
C                                                                       ASHS0857
      RETURN                                                            ASHS0858
C                                                                       ASHS0859
  120 FORMAT (8H0   N/M  10I11)                                         ASHS0860
  130 FORMAT (I5,3X,10F11.3)                                            ASHS0861
      END                                                               ASHS0862
```

APPENDIX F

PROGRAM LISTING - NON-AXISYMMETRIC HEAT SHIELDS

```
CNAHS     NON-AXISYMMETRIC ANALYSIS HEAT SHIELDS
C
      DIMENSION R(30,8),Z(30,8),T(30,8),TC(30,8),E(10,10),CC(10,10),
     1 XX(5),YY(5),S(15,15),P(15),KODE(30,10),
     2 SS(720,30),RR(720),NTAG(3),C(9,9),B(9,9),G(9,9)
      COMMON KINN,KOUT,N,M,L,NMAX,MMAX,MB,NMT,NH,NB,TE,TTT,ES,XS,AS,TS,
     1 ESH,XSH,ASH,TSH,R,Z,T,TC,E,CC,XX,YY,S,P,C,B,G,KODE,RR,SS
C
C
   50 KINN=5
      KOUT=6
      REWIND 20
C
      CALL INPUT
C
      MX=3*MMAX
      NS=NMAX*MMAX
      MM=MMAX-1
      NN=NMAX-1
      MBAND=3*(MMAX+2)
      NEQ=3*NS
      MBB=MB-1
      MCC=MB+1
C
C     READ IN ADDITIONAL BOUNDARY CONDITIONS
C
      DO 60 N=1,NMAX
      DO 60 M=1,MMAX
   60 KODE(N,M)=0
      WRITE OUTPUT TAPE KOUT,2000
      DO 70 K=1,NB
      READ INPUT TAPE KINN, 1000, N,M,KK
      KODE(N,M)=KK
      WRITE OUTPUT TAPE KOUT,2001, N,M,KODE(N,M)
   70 CONTINUE
C
C     SOLVE STRUCTURE FOR EACH HARMONIC
C
      DO 500 L=1,NH
      WRITE OUTPUT TAPE KOUT,2006, L
C
C     COMPUTE FOURIER COEFFICIENTS FOR HARMONIC L
C
      XL=0.0
```

```
      TT=0.0                                                            NAHS0045
      IF(L-1) 102,101,102                                               NAHS0046
  101 TT=TE                                                             NAHS0047
  102 DO 105 N=1,NMAX                                                   NAHS0048
      TC(N,MMAX)=TT                                                     NAHS0049
      TC(N,MB)=E(1,L)+XL*(E(2,L)+XL*(E(3,L)+XL*(E(4,L)+XL*(E(5,L)       NAHS0050
     1 +XL*E(6,L)))))                                                   NAHS0051
      XL=XL+SQRTF((R(N+1,MB)-R(N,MB))**2+(Z(N+1,MB)-Z(N,MB))**2)        NAHS0052
      DO 104 M=1,MBB                                                    NAHS0053
  104 TC(N,M)=TC(N,MB)                                                  NAHS0054
      TXX=(TC(N,MMAX)-TC(N,MB))/FLOATF(MMAX-MB)**2                      NAHS0055
      DO 105 M=MCC,MMAX                                                 NAHS0056
  105 TC(N,M)=TC(N,MB)+TXX*FLOATF(M-MB)**2                              NAHS0057
C                                                                       NAHS0058
      IF(L-1) 106,106,108                                               NAHS0059
  106 DO 107 N=1,NMAX                                                   NAHS0060
      DO 107 M=1,MMAX                                                   NAHS0061
  107 TC(N,M)=TC(N,M)-TTT                                               NAHS0062
  108 CALL PRINTM(TC,NMAX,MMAX,30)                                      NAHS0063
C                                                                       NAHS0064
C     FORMATION OF STIFFNESS ARRAY                                      NAHS0065
C                                                                       NAHS0066
      DO 175 I=1,NEQ                                                    NAHS0067
      DO 170 J=1,MBAND                                                  NAHS0068
  170 SS(I,J)=0.0                                                       NAHS0069
  175 RR(I)=0.0                                                         NAHS0070
C                                                                       NAHS0071
      DO 200 N=1,NN                                                     NAHS0072
      DO 200 M=1,MM                                                     NAHS0073
C                                                                       NAHS0074
      DO 180 I=1,15                                                     NAHS0075
      P(I)=0.0                                                          NAHS0076
      DO 180 J=1,15                                                     NAHS0077
  180 S(I,J)=0.0                                                        NAHS0078
C                                                                       NAHS0079
      XX(1)=R(N,M)                                                      NAHS0080
      XX(2)=R(N,M+1)                                                    NAHS0081
      XX(3)=R(N+1,M)                                                    NAHS0082
      XX(4)=R(N+1,M+1)                                                  NAHS0083
      YY(1)=Z(N,M)                                                      NAHS0084
      YY(2)=Z(N,M+1)                                                    NAHS0085
      YY(3)=Z(N+1,M)                                                    NAHS0086
      YY(4)=Z(N+1,M+1)                                                  NAHS0087
C                                                                       NAHS0088
```

```
      CALL STIFFT(1,4,2)                                                NAHS0089
      CALL STIFFT(1,3,4)                                                NAHS0090
C                                                                       NAHS0091
C       ADD QUADRILATERAL STIFFNESS TO TOTAL STIFFNESS                  NAHS0092
C                                                                       NAHS0093
      DO 200 I=1,6                                                      NAHS0094
      II=((N-1)*MMAX+M-1)*3+I                                           NAHS0095
      KK=II+MX                                                          NAHS0096
      RR(II)=RR(II)+P(I)                                                NAHS0097
      RR(KK)=RR(KK)+P(I+6)                                              NAHS0098
      DO 190 J=I,6                                                      NAHS0099
      JJ=J-I+1                                                          NAHS0100
      SS(II,JJ)=SS(II,JJ)+S(I,J)                                        NAHS0101
  190 SS(KK,JJ)=SS(KK,JJ)+S(I+6,J+6)                                    NAHS0102
      DO 200 J=1,6                                                      NAHS0103
      JJ=J-I+1+MX                                                       NAHS0104
  200 SS(II,JJ)=SS(II,JJ)+S(I,J+6)                                      NAHS0105
C                                                                       NAHS0106
C     STIFFNESS OF PLATE ELEMENTS                                       NAHS0107
C                                                                       NAHS0108
      MBB=MB-1                                                          NAHS0109
      DO 600 N=1,NN                                                     NAHS0110
      DO 600 M=1,MB,MBB                                                 NAHS0111
C                                                                       NAHS0112
      CALL STIFFP                                                       NAHS0113
C                                                                       NAHS0114
      DO 600 I=1,3                                                      NAHS0115
      II=((N-1)*MMAX+M-1)*3+I                                           NAHS0116
      KK=II+MX                                                          NAHS0117
      RR(II)=RR(II)+P(I)                                                NAHS0118
      RR(KK)=RR(KK)+P(I+3)                                              NAHS0119
      DO 590 J=I,3                                                      NAHS0120
      JJ=J-I+1                                                          NAHS0121
      SS(II,JJ)=SS(II,JJ)+S(I,J)                                        NAHS0122
  590 SS(KK,JJ)=SS(KK,JJ)+S(I+3,J+3)                                    NAHS0123
      DO 600 J=1,3                                                      NAHS0124
      JJ=J-I+1+MX                                                       NAHS0125
  600 SS(II,JJ)=SS(II,JJ)+S(I,J+3)                                      NAHS0126
C                                                                       NAHS0127
C     SET BOUNDARY CONDITIONS ALONG AXIS FOR HARMONIC L                 NAHS0128
C                                                                       NAHS0129
      DO 208 M=1,MMAX                                                   NAHS0130
      IF(L-2) 205,206,207                                               NAHS0131
  205 KODE(1,M)=4                                                       NAHS0132
```

```
      GO TO 208                                                         NAHS0133
  206 KODE(1,M)=3                                                       NAHS0134
      GO TO 208                                                         NAHS0135
  207 KODE(1,M)=7                                                       NAHS0136
  208 CONTINUE                                                          NAHS0137
C                                                                       NAHS0138
C        MODIFY STIFFNESS MATRIX                                        NAHS0139
C                                                                       NAHS0140
      K=1                                                               NAHS0141
      DO 300 N=1,NMAX                                                   NAHS0142
      DO 300 M=1,MMAX                                                   NAHS0143
      NTAG(1)=0                                                         NAHS0144
      NTAG(2)=0                                                         NAHS0145
      NTAG(3)=0                                                         NAHS0146
      KK=KODE(N,M)                                                      NAHS0147
      IF(KK) 250,250,210                                                NAHS0148
  210 GO TO (230,220,245,215,225,240,235),KK                            NAHS0149
  215 NTAG(1)=K                                                         NAHS0150
  220 NTAG(2)=K+1                                                       NAHS0151
      GO TO 250                                                         NAHS0152
  225 NTAG(3)=K+2                                                       NAHS0153
  230 NTAG(1)=K                                                         NAHS0154
      GO TO 250                                                         NAHS0155
  235 NTAG(1)=K                                                         NAHS0156
  240 NTAG(2)=K+1                                                       NAHS0157
  245 NTAG(3)=K+2                                                       NAHS0158
  250 K=K+3                                                             NAHS0159
      DO 300 LL=1,3                                                     NAHS0160
      II=NTAG(LL)                                                       NAHS0161
      IF(II)300,300,260                                                 NAHS0162
  260 DO 270 J=1,MBAND                                                  NAHS0163
      SS(II,J)=0.0                                                      NAHS0164
      III=II+1-J                                                        NAHS0165
      IF(III) 270,270,265                                               NAHS0166
  265 SS(III,J)=0.0                                                     NAHS0167
  270 CONTINUE                                                          NAHS0168
      SS(II,1)=1.0                                                      NAHS0169
      RR(II)=0.0                                                        NAHS0170
  300 CONTINUE                                                          NAHS0171
C                                                                       NAHS0172
C     SOLVE FOR DISPLACEMENTS                                           NAHS0173
C                                                                       NAHS0174
C                                                                       NAHS0175
      CALL SYMSOL(SS,RR,NEQ,MBAND)                                      NAHS0176
```

```
C                                                                   NAHS0177
      CALL STRESS                                                   NAHS0178
C                                                                   NAHS0179
  500 CONTINUE                                                      NAHS0180
C                                                                   NAHS0181
      CALL OUTPUT                                                   NAHS0182
C                                                                   NAHS0183
C     FORMAT STATEMENTS                                             NAHS0184
C                                                                   NAHS0185
 1000 FORMAT (3I5)                                                  NAHS0186
 2000 FORMAT (20H1BOUNDARY CONDITIONS // 18H     N     M  CODE )    NAHS0187
 2001 FORMAT (3I6)                                                  NAHS0188
 2006 FORMAT (3H1L=I3)                                              NAHS0189
C                                                                   NAHS0190
      END                                                           NAHS0191
```

```
      SUBROUTINE INPUT
C
C     COMMON AND DIMENSION STATEMENTS
C
      DIMENSION R(30,8),Z(30,8),T(30,8),TC(30,8),E(10,10),CC(10,10),
     1 RR(10),ZZ(10),TA(30),TT(50),A(50,10),B(10,10),D(10,10),
     2 EE(50,10),HED(12)
C
      COMMON KINN,KOUT,N,M,L,NMAX,MMAX,MB,NMT,NH,NB,TE,TTT,ES,XS,AS,TS,
     1 ESH,XSH,ASH,TSH,R,Z,T,TC,E,CC,RR,ZZ,TA,TT,A,B,D,EE,HED
C
C     READ AND PRINT INPUT DATA
C
      READ INPUT TAPE KINN,1000,
     1 HED,NMAX,MMAX,MB,NMT,NH,NB,TE,TTT,ES,XS,AS,TS,ESH,XSH,ASH,TSH
      WRITE OUTPUT TAPE KOUT,2000, HED,NMAX,MMAX,MB,NMT,NH,NB,TE,TTT
      WRITE OUTPUT TAPE KOUT,2006
      WRITE OUTPUT TAPE KOUT,2008, ES,XS,AS,TS
      WRITE OUTPUT TAPE KOUT,2007
      WRITE OUTPUT TAPE KOUT,2008, ESH,XSH,ASH,TSH
      READ INPUT TAPE KINN,1001,
     1 (R(N,MB),Z(N,MB),T(N,MB),TA(N),N=1,NMAX)
      WRITE OUTPUT TAPE KOUT,2001,
     1 (R(N,MB),Z(N,MB),T(N,MB),TA(N),N=1,NMAX)
C
C     LEAST SQUARE EVALUATION OF MATERIAL PROPERTIES
C
      READ INPUT TAPE KINN,1002,
     1 (TT(I),EE(I,1),EE(I,2),EE(I,3),I=1,NMT)
      WRITE OUTPUT TAPE KOUT, 2011
      WRITE OUTPUT TAPE KOUT,2002,
     1 (TT(I),EE(I,1),EE(I,2),EE(I,3),I=1,NMT)
C
      DO 50 I=1,NMT
      A(I,1)=1.0
      A(I,2)=TT(I)
      A(I,3)=TT(I)*TT(I)
      A(I,4)=A(I,3)*TT(I)
   50 A(I,5)=A(I,4)*TT(I)
C
      CALL LEAST(A,EE,B,D,CC,NMT,5,3)
C
      WRITE OUTPUT TAPE KOUT, 2012
      WRITE OUTPUT TAPE KOUT,2002,
```

```
     1 (TT(I),EE(I,1),EE(I,2),EE(I,3),I=1,NMT)                        NAHS0236
C                                                                       NAHS0237
C     GENERATE MESH                                                     NAHS0238
C                                                                       NAHS0239
      MC = MB+1                                                         NAHS0240
      MBB = MB-1                                                        NAHS0241
      SHL = FLOATF(MBB)                                                 NAHS0242
      ABL = FLOATF(MMAX-MB)                                             NAHS0243
      DO 200 N=1,NMAX                                                   NAHS0244
C                                                                       NAHS0245
C     CHECK FOR END POINTS                                              NAHS0246
C                                                                       NAHS0247
      NL = N-1                                                          NAHS0248
      NNH=N+1                                                           NAHS0249
      IF(NL)160,150,160                                                 NAHS0250
  150 SS = 0.                                                           NAHS0251
      CX = 1.                                                           NAHS0252
      GO TO 185                                                         NAHS0253
  160 IF(N-NMAX)180,170,180                                             NAHS0254
  170 NNH=N                                                             NAHS0255
C                                                                       NAHS0256
C     COMPUTE SIN AND COS AT EACH POINT ON BOND LINE                    NAHS0257
C                                                                       NAHS0258
  180 XX = R( NNH,MB) - R(NL,MB)                                        NAHS0259
      YY = Z(NNH ,MB) - Z(NL,MB)                                        NAHS0260
      ZZ = SQRTF(XX**2 + YY**2)                                         NAHS0261
      SS = YY/ZZ                                                        NAHS0262
      CX = XX/ZZ                                                        NAHS0263
C                                                                       NAHS0264
C     COMPUTE COORDINATES OF POINTS IN CONSTANT THICKNESS SHELL         NAHS0265
C                                                                       NAHS0266
  185 DO 190 M=1,MBB                                                    NAHS0267
      TT = TS*FLOATF(MB-M)/SHL                                          NAHS0268
      T(N,M) = T(N,MB)                                                  NAHS0269
      R(N,M) = R(N,MB) + TT*SS                                          NAHS0270
  190 Z(N,M) = Z(N,MB) - TT*CX                                          NAHS0271
C                                                                       NAHS0272
C     COMPUTE COORDINATES OF POINTS IN VARIABLE THICKNESS ABLATER       NAHS0273
C                                                                       NAHS0274
      DO 200 M=MC,MMAX                                                  NAHS0275
      TT = TA(N)*FLOATF(M-MB)/ABL                                       NAHS0276
      T(N,M) = T(N,MB) + (TE - T(N,MB)) * (TT/TA(N))**2                 NAHS0277
      R(N,M) = R(N,MB) - TT*SS                                          NAHS0278
  200 Z(N,M) = Z(N,MB) + TT*CX                                          NAHS0279
```

```
C                                                                       NAHS0280
C     DUMP OUT R-ORDINATE TABLE AND Z-ORDINATE TABLE                    NAHS0281
C                                                                       NAHS0282
      WRITE OUTPUT TAPE KOUT,2003                                       NAHS0283
      CALL PRINTM (R,NMAX,MMAX,30)                                      NAHS0284
      WRITE OUTPUT TAPE KOUT,2004                                       NAHS0285
      CALL PRINTM (Z,NMAX,MMAX,30)                                      NAHS0286
      WRITE OUTPUT TAPE KOUT,2005                                       NAHS0287
      CALL PRINTM (T,NMAX,MMAX,30)                                      NAHS0288
C                                                                       NAHS0289
C     READ IN THREE-DIMENSIONAL TEMPERATURE DISTRIBUTION                NAHS0290
C                                                                       NAHS0291
      TT(1)=0.0                                                         NAHS0292
      DO 240 I=2,19                                                     NAHS0293
  240 TT(I)=TT(I-1)+10.                                                 NAHS0294
      READ INPUT TAPE KINN, 1003,((RR(J),J=1,9),(ZZ(J),J=1,9))          NAHS0295
      WRITE OUTPUT TAPE KOUT,2009,((RR(J),J=1,9),(ZZ(J),J=1,9))         NAHS0296
      READ INPUT TAPE KINN,  1003,((EE(I,J),J=1,9),I=1,19)              NAHS0297
      WRITE OUTPUT TAPE KOUT, 2013                                      NAHS0298
      WRITE OUTPUT TAPE KOUT,2010,(TT(I),(EE(I,J),J=1,9),I=1,19)        NAHS0299
C                                                                       NAHS0300
      DO 250 I=1,19                                                     NAHS0301
      XX=0.17453*FLOATF(I-1)                                            NAHS0302
      DO 250 J=1,NH                                                     NAHS0303
      YY=XX*FLOATF(J-1)                                                 NAHS0304
  250 A(I,J)=COSF(YY)                                                   NAHS0305
C                                                                       NAHS0306
      DO 270 I=1,9                                                      NAHS0307
      DO 260 J=1,NH                                                     NAHS0308
      E(J,I)=.5*(EE(1,I)*A(1,J)+EE(19,I)*A(19,J))                       NAHS0309
      DO 255 K=2,18                                                     NAHS0310
  255 E(J,I)=E(J,I)+EE(K,I)*A(K,J)                                      NAHS0311
  260 E(J,I)=E(J,I)/9.                                                  NAHS0312
  270 E(1,I)=E(1,I)/2.                                                  NAHS0313
C                                                                       NAHS0314
      DO 280 I=1,19                                                     NAHS0315
      DO 280 J=1,9                                                      NAHS0316
      EE(I,J)=0.0                                                       NAHS0317
      DO 280 K=1,NH                                                     NAHS0318
  280 EE(I,J)=EE(I,J)+A(I,K)*E(K,J)                                     NAHS0319
C                                                                       NAHS0320
      WRITE OUTPUT TAPE KOUT, 2014                                      NAHS0321
      WRITE OUTPUT TAPE KOUT,2010,(TT(I),(EE(I,J),J=1,9),I=1,19)        NAHS0322
C                                                                       NAHS0323
```

```
C      DETERMINE FOURIER COEFFICIENTS AS A FUNCTION OF SPACE                 NAHS0324
C                                                                            NAHS0325
       XL=0.0                                                                NAHS0326
       DO 300 I=1,9                                                          NAHS0327
       A(I,1)=1.0                                                            NAHS0328
       A(I,2)=XL                                                             NAHS0329
       A(I,3)=XL**2                                                          NAHS0330
       A(I,4)=A(I,3)*XL                                                      NAHS0331
       A(I,5)=A(I,4)*XL                                                      NAHS0332
       A(I,6)=A(I,5)*XL                                                      NAHS0333
       XL=XL+SQRTF((RR(I+1)-RR(I))**2+(ZZ(I+1)-ZZ(I))**2)                    NAHS0334
       DO 300 J=1,NH                                                         NAHS0335
   300 EE(I,J)=E(J,I)                                                        NAHS0336
C                                                                            NAHS0337
       CALL LEAST(A,EE,B,D,E,9,6,NH)                                         NAHS0338
C                                                                            NAHS0339
       RETURN                                                                NAHS0340
C                                                                            NAHS0341
C      FORMAT STATEMENTS                                                     NAHS0342
C                                                                            NAHS0343
  1000 FORMAT (12A6/ 6I5,2F10.2/ 4F10.2/4F10.2)                              NAHS0344
  1001 FORMAT (4F10.2)                                                       NAHS0345
  1002 FORMAT (4F10.2)                                                       NAHS0346
  1003 FORMAT (9F8.0)                                                        NAHS0347
  2000 FORMAT ( 1H1 12A6/                                                    NAHS0348
      1 33HONUMBER OF POINTS ALONG LENGTH--- I3/                             NAHS0349
      2 33H NUMBER OF POINTS THRU THICKNESS- I3/                             NAHS0350
      3 33H LOCATION OF BOND LINE----------- I3/                             NAHS0351
      4 33H NUMBER OF PROPERTY CARDS-------- I3/                             NAHS0352
      5 33H NUMBER OF HARMONICS------------- I4/                             NAHS0353
      6 33H NUMBER OF BOUNDARY CONDITIONS--- I4/                             NAHS0354
      7 33H SURFACE TEMPERATURE OF ABLATOR-- F6.0/                           NAHS0355
      8 33H ZERO STRESS TEMPERATURE--------- F6.0)                           NAHS0356
  2001 FORMAT (15H1    R-ORDINATE 5X 10HZ-ORDINATE 5X 10HBOND TEMP. 3X       NAHS0357
      1 17HABLATOR THICKNESS / (3F15.3,1F20.4))                              NAHS0358
  2002 FORMAT (15H0  TEMPERATURE 5X 10HMODULUS A 5X 9HMODULUS B   1X         NAHS0359
      120H COEFF. OF EXPANSION /(3F15.0,1E20.5))                             NAHS0360
  2003 FORMAT (14H1  R-ORDINATES )                                           NAHS0361
  2004 FORMAT (14H1  Z-ORDINATES )                                           NAHS0362
  2005 FORMAT (14H1  TEMPERATURE )                                           NAHS0363
  2006 FORMAT (37H0PROPERTIES OF SANDWICH CORE MATERIAL )                    NAHS0364
  2007 FORMAT (30H0PROPERTIES OF SANDWICH PLATES )                           NAHS0365
  2008 FORMAT                                                                NAHS0366
      1(28H  MODULUS OF ELASTICITY----- F10.0/                               NAHS0367
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    2 28H  POISSONS RATIO------------- F10.4/                          NAHS0368
    3 28H  COEFFICIENT OF EXPANSION-- F10.8/                           NAHS0369
    4 28H  THICKNESS----------------- F10.4)                           NAHS0370
 2009 FORMAT ( 5H1  R=9F12.4/5H   Z=9F12.4)                            NAHS0371
 2010 FORMAT (/(F5.0,9F12.1))                                          NAHS0372
 2011 FORMAT (20H1MATERIAL PROPERTIES)                                 NAHS0373
 2012 FORMAT (38H1LEAST SQUARE EVALUATION OF ABOVE DATA )              NAHS0374
 2013 FORMAT (36H0ANGLE        BOND LINE TEMPERATURES )                NAHS0375
 2014 FORMAT (1H010X,47H FOURIER SERIES EXPANSION OF ABOVE TEMPERATURES)NAHS0376
      END                                                              NAHS0377
```

```
      SUBROUTINE STIFFT(IX,JX,KX)                                       NAHS0378
C                                                                       NAHS0379
      DIMENSION R(30,8),Z(30,8),T(30,8),TC(30,8),E(10,10),CC(10,10),    NAHS0380
     1 XX(5),YY(5),S(15,15),P(15),G(9,9),B(9,9),C(9,9),LM(3),EE(9)      NAHS0381
      COMMON KINN,KOUT,N,M,L,NMAX,MMAX,MB,NMT,NH,NB,TE,TTT,ES,XS,AS,TS, NAHS0382
     1 ESH,XSH,ASH,TSH,R,Z,T,TC,E,CC,XX,YY,S,P,C,B,G                    NAHS0383
C                                                                       NAHS0384
C     INITIALIZATION                                                    NAHS0385
C                                                                       NAHS0386
      II=IX                                                             NAHS0387
      JJ=JX                                                             NAHS0388
      KK=KX                                                             NAHS0389
      DO 50 I=1,81                                                      NAHS0390
      G(I)=0.0                                                          NAHS0391
   50 B(I)=0.0                                                          NAHS0392
      AJ=XX(JJ)-XX(II)                                                  NAHS0393
      AK=XX(KK)-XX(II)                                                  NAHS0394
      BJ=YY(JJ)-YY(II)                                                  NAHS0395
      BK=YY(KK)-YY(II)                                                  NAHS0396
      AREA=(AJ*BK-BJ*AK)/2.                                             NAHS0397
      RBAR=(XX(II)+XX(JJ)+XX(KK))/3.                                    NAHS0398
C                                                                       NAHS0399
C     CALCULATE MATERIAL PROPERTIES                                     NAHS0400
C                                                                       NAHS0401
      TT=(T(N,M)+T(N,M+1)+T(N+1,M)+T(N+1,M+1))/4.                       NAHS0402
      DO 60 I=1,3                                                       NAHS0403
   60 EE(I)=CC(1,I)+TT*(CC(2,I)+TT*(CC(3,I)+TT*(CC(4,I)+TT*CC(5,I))))   NAHS0404
      IF(MB-M) 70,70,80                                                 NAHS0405
   70 EP=EE(1)                                                          NAHS0406
      ALP=EE(3)                                                         NAHS0407
      XNU=0.3155+0.000174*TT                                            NAHS0408
      GO TO 90                                                          NAHS0409
   80 EP=ES                                                             NAHS0410
      ALP=AS                                                            NAHS0411
      XNU=XS                                                            NAHS0412
   90 CONTINUE                                                          NAHS0413
C                                                                       NAHS0414
C     FORM MATRIX (B)                                                   NAHS0415
C                                                                       NAHS0416
      COMM=EP/((1.+XNU)*(1.-2.*XNU))                                    NAHS0417
      CX=(.5-XNU)*COMM                                                  NAHS0418
      DX=(1.-XNU)*COMM                                                  NAHS0419
      EX=XNU*COMM                                                       NAHS0420
      B(1,1)=DX                                                         NAHS0421
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```
      B(1,2)=EX                                                         NAHS0422
      B(1,3)=EX                                                         NAHS0423
      B(2,1)=EX                                                         NAHS0424
      B(2,2)=DX                                                         NAHS0425
      B(2,3)=EX                                                         NAHS0426
      B(3,1)=EX                                                         NAHS0427
      B(3,2)=EX                                                         NAHS0428
      B(3,3)=DX                                                         NAHS0429
      B(4,4)=CX                                                         NAHS0430
      B(5,5)=CX                                                         NAHS0431
      B(6,6)=CX                                                         NAHS0432
C                                                                       NAHS0433
C      FORM  G MATRIX                                                   NAHS0434
C                                                                       NAHS0435
      D=XX(JJ)*(YY(KK)-YY(II))+XX(II)*(YY(JJ)-YY(KK))                   NAHS0436
     1 +XX(KK)*(YY(II)-YY(JJ))                                          NAHS0437
      XN=L-1                                                            NAHS0438
C                                                                       NAHS0439
      G(1,1)=(YY(JJ)-YY(KK))/D                                          NAHS0440
      G(1,4)=(YY(KK)-YY(II))/D                                          NAHS0441
      G(1,7)=(YY(II)-YY(JJ))/D                                          NAHS0442
      G(2,1)=1./(XX(II)+XX(JJ)+XX(KK))                                  NAHS0443
      G(2,4)=G(2,1)                                                     NAHS0444
      G(2,7)=G(2,1)                                                     NAHS0445
      G(2,2)=XN*G(2,1)                                                  NAHS0446
      G(2,5)=XN*G(2,4)                                                  NAHS0447
      G(2,8)=XN*G(2,7)                                                  NAHS0448
      G(3,3)=(XX(KK)-XX(JJ))/D                                          NAHS0449
      G(3,6)=(XX(II)-XX(KK))/D                                          NAHS0450
      G(3,9)=(XX(JJ)-XX(II))/D                                          NAHS0451
      G(4,1)=-G(2,2)                                                    NAHS0452
      G(4,4)=-G(2,5)                                                    NAHS0453
      G(4,7)=-G(2,8)                                                    NAHS0454
      G(4,2)=G(1,1)-G(2,1)                                              NAHS0455
      G(4,5)=G(1,4)-G(2,4)                                              NAHS0456
      G(4,8)=G(1,7)-G(2,7)                                              NAHS0457
      G(5,1)=G(3,3)                                                     NAHS0458
      G(5,4)=G(3,6)                                                     NAHS0459
      G(5,7)=G(3,9)                                                     NAHS0460
      G(5,3)=G(1,1)                                                     NAHS0461
      G(5,6)=G(1,4)                                                     NAHS0462
      G(5,9)=G(1,7)                                                     NAHS0463
      G(6,2)=G(3,3)                                                     NAHS0464
      G(6,5)=G(3,6)                                                     NAHS0465
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```
      G(6,8)=G(3,9)                                                     NAHS0466
      G(6,3)=G(4,1)                                                     NAHS0467
      G(6,6)=G(4,4)                                                     NAHS0468
      G(6,9)=G(4,7)                                                     NAHS0469
C                                                                       NAHS0470
C     FORM RING STIFFNESS MATRIX (A)T(B)(A)                             NAHS0471
C                                                                       NAHS0472
      DO 150 I=1,6                                                      NAHS0473
      DO 150 J=1,9                                                      NAHS0474
      C(I,J)=0.0                                                        NAHS0475
      DO 150 K=1,6                                                      NAHS0476
  150 C(I,J)=C(I,J)+B(I,K)*G(K,J)                                       NAHS0477
C                                                                       NAHS0478
      COMM=RBAR*AREA                                                    NAHS0479
      DO 165 I=1,9                                                      NAHS0480
      DO 165 J=1,9                                                      NAHS0481
      B(I,J)=0.0                                                        NAHS0482
      DO 160 K=1,6                                                      NAHS0483
  160 B(I,J)=B(I,J)+G(K,I)*C(K,J)                                       NAHS0484
  165 B(I,J)=B(I,J)*COMM                                                NAHS0485
C                                                                       NAHS0486
C     COMPUTE TEMPERATURE LOADS                                         NAHS0487
C                                                                       NAHS0488
      TEM=(TC(N,M)+TC(N,M+1)+TC(N+1,M)+TC(N+1,M+1))/4.                  NAHS0489
      TTT=TEM*EP*ALP/(1.-2.*XNU)                                        NAHS0490
      TEM=TTT*RBAR*AREA                                                 NAHS0491
      DO 170 I=1,9                                                      NAHS0492
  170 EE(I)=(G(1,I)+G(2,I)+G(3,I))*TEM                                  NAHS0493
C                                                                       NAHS0494
C     ADD TRIANGULAR RING STIFFNESS TO QUADRILATERAL RING STIFFNESS     NAHS0495
C                                                                       NAHS0496
      LM(1)=3*II-3                                                      NAHS0497
      LM(2)=3*JJ-3                                                      NAHS0498
      LM(3)=3*KK-3                                                      NAHS0499
      DO 250 I=1,3                                                      NAHS0500
      DO 250 K=1,3                                                      NAHS0501
      II=LM(I)+K                                                        NAHS0502
      KK=3*I-3+K                                                        NAHS0503
      P(II)=P(II)+EE(KK)                                                NAHS0504
      DO 250 J=1,3                                                      NAHS0505
      DO 250 LX=1,3                                                     NAHS0506
      JJ=LM(J)+LX                                                       NAHS0507
      LL=3*J-3+LX                                                       NAHS0508
  250 S(II,JJ)=S(II,JJ)+B(KK,LL)                                        NAHS0509
```

```
C                                                                 NAHS0510
      RETURN                                                      NAHS0511
C                                                                 NAHS0512
      END                                                         NAHS0513
```

```
      SUBROUTINE STIFFP                                                 NAHS0514
C                                                                       NAHS0515
      DIMENSION R(30,8),Z(30,8),T(30,8),TC(30,8),E(10,10),CC(10,10),    NAHS0516
     1 XX(5),YY(5),S(15,15),P(15),G(9,9),B(9,9),C(9,9)                  NAHS0517
      COMMON KINN,KOUT,N,M,L,NMAX,MMAX,MB,NMT,NH,NB,TE,TTT,ES,XS,AS,TS, NAHS0518
     1 ESH,XSH,ASH,TSH,R,Z,T,TC,E,CC,XX,YY,S,P,C,B,G                    NAHS0519
C                                                                       NAHS0520
C     FORMATION OF G MATRIX                                             NAHS0521
C                                                                       NAHS0522
      XN=L-1                                                            NAHS0523
      A=R(N+1,M)-R(N,M)                                                 NAHS0524
      B=Z(N+1,M)-Z(N,M)                                                 NAHS0525
      XL2=A**2+B**2                                                     NAHS0526
      XL=SQRTF(XL2)                                                     NAHS0527
      RBAR=R(N+1,M)+R(N,M)                                              NAHS0528
C                                                                       NAHS0529
      G(1,1)=-A/XL2                                                     NAHS0530
      G(1,2)=0.0                                                        NAHS0531
      G(1,3)=-B/XL2                                                     NAHS0532
      G(1,4)=-G(1,1)                                                    NAHS0533
      G(1,5)=0.0                                                        NAHS0534
      G(1,6)=-G(1,3)                                                    NAHS0535
      G(2,1)=1./RBAR                                                    NAHS0536
      G(2,2)=XN*G(2,1)                                                  NAHS0537
      G(2,3)=0.0                                                        NAHS0538
      G(2,4)=G(2,1)                                                     NAHS0539
      G(2,5)=G(2,2)                                                     NAHS0540
      G(2,6)=0.0                                                        NAHS0541
      G(3,1)=-XN*A/RBAR/XL                                              NAHS0542
      G(3,2)=1./XL                                                      NAHS0543
      G(3,3)=-XN*B/RBAR/XL                                              NAHS0544
      G(3,4)=G(3,1)                                                     NAHS0545
      G(3,5)=-G(3,2)                                                    NAHS0546
      G(3,6)=G(3,3)                                                     NAHS0547
C                                                                       NAHS0548
C     FORMATION OF B MATRIX                                             NAHS0549
C                                                                       NAHS0550
      B(1,1)=ESH/(1.-XSH**2)                                            NAHS0551
      B(1,2)=XSH*B(1,1)                                                 NAHS0552
      B(1,3)=0.0                                                        NAHS0553
      B(2,1)=B(1,2)                                                     NAHS0554
      B(2,2)=B(1,1)                                                     NAHS0555
      B(2,3)=0.0                                                        NAHS0556
      B(3,1)=0.0                                                        NAHS0557
```

```
      B(3,2)=0.0
      B(3,3)=.5*ESH/(1.+XSH)
C
C     FORMATION OF PLATE STIFFNESS MATRIX (G)T*(B)*(G)
C
      DO 150 I=1,3
      DO 150 J=1,6
      C(I,J)=0.0
      DO 150 K=1,3
  150 C(I,J)=C(I,J)+B(I,K)*G(K,J)
C
      COMM=RBAR*XL*TSH/2.
      DO 165 I=1,6
      DO 165 J=1,6
      S(I,J)=0.0
      DO 160 K=1,3
  160 S(I,J)=S(I,J)+G(K,I)*C(K,J)
  165 S(I,J)=S(I,J)*COMM
C
C     COMPUTE TEMPERATURE LOADS
C
      TEM=(TC(N,M)+TC(N+1,M))/2.
      TTT=TEM*ESH*ASH*(1.+XSH)/(1.-XSH**2)
      TEM=TTT*COMM
      DO 170 I=1,6
  170 P(I)=(G(1,I)+G(2,I))*TEM
C
      RETURN
C
      END
```

```
      SUBROUTINE STRESS                                                 NAHS0588
      DIMENSION R(30,8),Z(30,8),T(30,8),TC(30,8),E(10,10),CC(10,10),   NAHS0589
     1 XX(5),YY(5),S(15,15),P(15),U(30,8),V(30,8),W(30,8),KODE(30,10), NAHS0590
     2 C(9,9),RR(720),SIG(6,30,8),PSIG(3,30,2),B(9,9),G(9,9)           NAHS0591
      COMMON KINN,KOUT,N,M,L,NMAX,MMAX,MB,NMT,NH,NB,TE,TTT,ES,XS,AS,TS, NAHS0592
     1 ESH,XSH,ASH,TSH,R,Z,T,TC,E,CC,XX,YY,S,P,C,B,G,KODE,RR,U,V,W,SIG, NAHS0593
     3 PSIG                                                             NAHS0594
C                                                                       NAHS0595
C     SEPARATE DISPLACEMENTS FOR HARMONIC L                             NAHS0596
C                                                                       NAHS0597
      K=1                                                               NAHS0598
      DO 400 N=1,NMAX                                                   NAHS0599
      DO 400 M=1,MMAX                                                   NAHS0600
      U(N,M)=RR(K)                                                      NAHS0601
      V(N,M)=RR(K+1)                                                    NAHS0602
      W(N,M)=RR(K+2)                                                    NAHS0603
  400 K=K+3                                                             NAHS0604
C                                                                       NAHS0605
C     CALCULATION OF STRESSES FOR HARMONIC L                            NAHS0606
C                                                                       NAHS0607
      NN=NMAX-1                                                         NAHS0608
      MM=MMAX-1                                                         NAHS0609
      DO 100 N=1,NN                                                     NAHS0610
      N=N                                                               NAHS0611
      DO 100 M=1,MM                                                     NAHS0612
      M=M                                                               NAHS0613
      XX(1)=R(N,M)                                                      NAHS0614
      XX(2)=R(N,M+1)                                                    NAHS0615
      XX(3)=R(N+1,M)                                                    NAHS0616
      XX(4)=R(N+1,M+1)                                                  NAHS0617
      YY(1)=Z(N,M)                                                      NAHS0618
      YY(2)=Z(N,M+1)                                                    NAHS0619
      YY(3)=Z(N+1,M)                                                    NAHS0620
      YY(4)=Z(N+1,M+1)                                                  NAHS0621
C                                                                       NAHS0622
      CALL STIFFT(1,4,2)                                                NAHS0623
C                                                                       NAHS0624
      P(1)=U(N,M)                                                       NAHS0625
      P(2)=V(N,M)                                                       NAHS0626
      P(3)=W(N,M)                                                       NAHS0627
      P(4)=U(N+1,M+1)                                                   NAHS0628
      P(5)=V(N+1,M+1)                                                   NAHS0629
      P(6)=W(N+1,M+1)                                                   NAHS0630
      P(7)=U(N,M+1)                                                     NAHS0631
```

```
      P(8)=V(N,M+1)                                                     NAHS0632
      P(9)=W(N,M+1)                                                     NAHS0633
C                                                                       NAHS0634
      DO 80 I=1,6                                                       NAHS0635
      SIG(I,N,M)=0.0                                                    NAHS0636
      DO 80 K=1,9                                                       NAHS0637
   80 SIG(I,N,M)=SIG(I,N,M)+C(I,K)*P(K)                                 NAHS0638
C                                                                       NAHS0639
      CALL STIFFT(1,3,4)                                                NAHS0640
C                                                                       NAHS0641
      P(1)=U(N,M)                                                       NAHS0642
      P(2)=V(N,M)                                                       NAHS0643
      P(3)=W(N,M)                                                       NAHS0644
      P(4)=U(N+1,M)                                                     NAHS0645
      P(5)=V(N+1,M)                                                     NAHS0646
      P(6)=W(N+1,M)                                                     NAHS0647
      P(7)=U(N+1,M+1)                                                   NAHS0648
      P(8)=V(N+1,M+1)                                                   NAHS0649
      P(9)=W(N+1,M+1)                                                   NAHS0650
C                                                                       NAHS0651
      DO 95 I=1,6                                                       NAHS0652
      DO 90 K=1,9                                                       NAHS0653
   90 SIG(I,N,M)=SIG(I,N,M)+C(I,K)*P(K)                                 NAHS0654
   95 SIG(I,N,M)=SIG(I,N,M)/2.                                          NAHS0655
      SIG(1,N,M)=SIG(1,N,M)-TTT                                         NAHS0656
      SIG(2,N,M)=SIG(2,N,M)-TTT                                         NAHS0657
  100 SIG(3,N,M)=SIG(3,N,M)-TTT                                         NAHS0658
C                                                                       NAHS0659
      WRITE TAPE 20,((U(N,M),W(N,M),V(N,M),(SIG(I,N,M),I=1,6),          NAHS0660
     1 N=1,NMAX),M=1,MMAX)                                              NAHS0661
C                                                                       NAHS0662
C     COMPUTE STRESSES IN FACE PLATES                                   NAHS0663
C                                                                       NAHS0664
      DO 210 N=1,NN                                                     NAHS0665
      N=N                                                               NAHS0666
      DO 210 K=1,2                                                      NAHS0667
      M=(K-1)*(MB-1)+1                                                  NAHS0668
      CALL STIFFP                                                       NAHS0669
      P(1)=U(N,M)                                                       NAHS0670
      P(2)=V(N,M)                                                       NAHS0671
      P(3)=W(N,M)                                                       NAHS0672
      P(4)=U(N+1,M)                                                     NAHS0673
      P(5)=V(N+1,M)                                                     NAHS0674
      P(6)=W(N+1,M)                                                     NAHS0675
```

```
      DO 200 I=1,3                                                      NAHS0676
      PSIG(I,N,K)=0.0                                                   NAHS0677
      DO 200 J=1,6                                                      NAHS0678
  200 PSIG(I,N,K)=PSIG(I,N,K)+C(I,J)*P(J)                               NAHS0679
      PSIG(1,N,K)=PSIG(1,N,K)-TTT                                       NAHS0680
  210 PSIG(2,N,K)=PSIG(2,N,K)-TTT                                       NAHS0681
C                                                                       NAHS0682
      WRITE TAPE 20, ((( PSIG(I,N,K),I=1,3),K=1,2),N=1,NN)              NAHS0683
C                                                                       NAHS0684
      RETURN                                                            NAHS0685
C                                                                       NAHS0686
      END                                                               NAHS0687
```

```
      SUBROUTINE OUTPUT                                                 NAHS0688
C                                                                       NAHS0689
      DIMENSION U(30,8),V(30,8),W(30,8),XU(30,8),XV(30,8),XW(30,8),     NAHS0690
     1 SIG(6,30,8),XSIG(6,30,8),PSIG(3,30,2),XPSIG(3,30,2)              NAHS0691
      COMMON KINN,KOUT,N,M,L,NMAX,MMAX,MB,NMT,NH,NB,TE,TTT,ES,XS,AS,TS, NAHS0692
     1 U,V,W,XU,XV,XW,SIG,XSIG,PSIG,XPSIG                               NAHS0693
C                                                                       NAHS0694
      NN=NMAX-1                                                         NAHS0695
      MM=MMAX-1                                                         NAHS0696
C                                                                       NAHS0697
   50 READ INPUT TAPE KINN,1000,THETA                                   NAHS0698
C                                                                       NAHS0699
      DO 110 N=1,NMAX                                                   NAHS0700
      DO 100 M=1,MMAX                                                   NAHS0701
      U(N,M)=0.0                                                        NAHS0702
      V(N,M)=0.0                                                        NAHS0703
      W(N,M)=0.0                                                        NAHS0704
      DO 100 I=1,6                                                      NAHS0705
  100 SIG(I,N,M)=0.0                                                    NAHS0706
      DO 110 K=1,2                                                      NAHS0707
      DO 110 I=1,3                                                      NAHS0708
  110 PSIG(I,N,K)=0.0                                                   NAHS0709
C                                                                       NAHS0710
      REWIND 20                                                         NAHS0711
      DO 500 LL=1,NH                                                    NAHS0712
      TT=FLOATF(LL-1)*THETA*.017453289                                  NAHS0713
      COSINE=COSF(TT)                                                   NAHS0714
      SINE=SINF(TT)                                                     NAHS0715
C                                                                       NAHS0716
      READ  TAPE 20,((XU(N,M),XW(N,M),XV(N,M),(XSIG(I,N,M),I=1,6),      NAHS0717
     1 N=1,NMAX),M=1,MMAX)                                              NAHS0718
C                                                                       NAHS0719
      DO 200 N=1,NMAX                                                   NAHS0720
      DO 200 M=1,MMAX                                                   NAHS0721
      U(N,M)=U(N,M)+XU(N,M)*COSINE                                      NAHS0722
      W(N,M)=W(N,M)+XW(N,M)*COSINE                                      NAHS0723
      V(N,M)=V(N,M)+XV(N,M)*SINE                                        NAHS0724
      DO 150 I=1,3                                                      NAHS0725
  150 SIG(I,N,M)=SIG(I,N,M)+XSIG(I,N,M)*COSINE                          NAHS0726
      SIG(4,N,M)=SIG(4,N,M)+XSIG(4,N,M)*SINE                            NAHS0727
      SIG(5,N,M)=SIG(5,N,M)+XSIG(5,N,M)*COSINE                          NAHS0728
  200 SIG(6,N,M)=SIG(6,N,M)+XSIG(6,N,M)*SINE                            NAHS0729
C                                                                       NAHS0730
      READ TAPE 20,  (((XPSIG(I,N,K),I=1,3),K=1,2),N=1,NN)              NAHS0731
```

```
C                                                                       NAHS0732
      DO 300 N=1,NN                                                     NAHS0733
      DO 300 K=1,2                                                      NAHS0734
      PSIG(1,N,K)=PSIG(1,N,K)+XPSIG(1,N,K)*COSINE                       NAHS0735
      PSIG(2,N,K)=PSIG(2,N,K)+XPSIG(2,N,K)*COSINE                       NAHS0736
  300 PSIG(3,N,K)=PSIG(3,N,K)+XPSIG(3,N,K)*SINE                         NAHS0737
C                                                                       NAHS0738
  500 CONTINUE                                                          NAHS0739
C                                                                       NAHS0740
C     OUTPUT DISPLACEMENTS AND STRESSES                                 NAHS0741
C                                                                       NAHS0742
      WRITE OUTPUT TAPE KOUT,2003, THETA                                NAHS0743
      CALL PRINTM(U,NMAX,MMAX,30)                                       NAHS0744
      WRITE OUTPUT TAPE KOUT,2004, THETA                                NAHS0745
      CALL PRINTM(W,NMAX,MMAX,30)                                       NAHS0746
      WRITE OUTPUT TAPE KOUT,2005, THETA                                NAHS0747
      CALL PRINTM(V,NMAX,MMAX,30)                                       NAHS0748
C                                                                       NAHS0749
      DO 600 M=1,MM                                                     NAHS0750
      WRITE OUTPUT TAPE KOUT, 2007, THETA                               NAHS0751
  600 WRITE OUTPUT TAPE KOUT,2000,  (N,M,(SIG(I,N,M),I=1,6),N=1,NN)     NAHS0752
C                                                                       NAHS0753
      WRITE OUTPUT TAPE KOUT, 2008, THETA                               NAHS0754
      WRITE OUTPUT TAPE KOUT,2006,(N,((PSIG(I,N,K),I=1,3),K=1,2),N=1,NN)NAHS0755
C                                                                       NAHS0756
      GO TO 50                                                          NAHS0757
C                                                                       NAHS0758
 1000 FORMAT (F5.0)                                                     NAHS0759
 2000 FORMAT (12H0    N      M 10X,2HRR 10X,2HTT 10X,2HZZ 10X,2HRT      NAHS0760
     1 10X,2HRZ 10X,2HTZ / (2I6,6F12.2))                                NAHS0761
 2003 FORMAT (20H1R-DISPLACEMENTS AT  F4.0,8H DEGREES)                  NAHS0762
 2004 FORMAT (20H1Z-DISPLACEMENTS AT  F4.0,8H DEGREES)                  NAHS0763
 2005 FORMAT (20H1T-DISPLACEMENTS AT  F4.0,8H DEGREES)                  NAHS0764
 2006 FORMAT (I5,6F18.1)                                                NAHS0765
 2007 FORMAT  (28H1AVERAGE ELEMENT STRESSES AT F5.0,8H DEGREES)         NAHS0766
 2008 FORMAT (44H1AVERAGE STRESSES IN SANDWICH FACE PLATES ATF5.0,8H DEGNAHS0767
     1REES//22X, 12HBOTTOM PLATE 44X, 9HTOP PLATE /5H    N 2(54H     MERNAHS0768
     1ID. STRESS       HOOP STRESS    IN-PLANE SHEAR ))                 NAHS0769
C                                                                       NAHS0770
      END                                                               NAHS0771
```

```
      SUBROUTINE LEAST(A,B,C,D,E,N,M,L)                                 NAHS0772
C                                                                       NAHS0773
      DIMENSION A(50,10),B(50,10),C(10,10),D(10,10),E(10,10)            NAHS0774
C                                                                       NAHS0775
C     (C)=(A)T*(A)   AND (D)=(A)T*(B)                                   NAHS0776
C                                                                       NAHS0777
      DO 200 I=1,M                                                      NAHS0778
      DO 100 J=1,M                                                      NAHS0779
      C(I,J)=0.0                                                        NAHS0780
      DO 100 K=1,N                                                      NAHS0781
  100 C(I,J)=C(I,J)+A(K,I)*A(K,J)                                       NAHS0782
      DO 200 J=1,L                                                      NAHS0783
      D(I,J)=0.0                                                        NAHS0784
      DO 200 K=1,N                                                      NAHS0785
  200 D(I,J)=D(I,J)+A(K,I)*B(K,J)                                       NAHS0786
C                                                                       NAHS0787
C     INVERT (C)                                                        NAHS0788
C                                                                       NAHS0789
      CALL INVERT(C,M,10,B,E)                                           NAHS0790
C                                                                       NAHS0791
C      (E)=(C)*(D)                                                      NAHS0792
C                                                                       NAHS0793
      DO 300 I=1,M                                                      NAHS0794
      DO 300 J=1,L                                                      NAHS0795
      E(I,J)=0.0                                                        NAHS0796
      DO 300 K=1,M                                                      NAHS0797
  300 E(I,J)=E(I,J)+C(I,K)*D(K,J)                                       NAHS0798
C                                                                       NAHS0799
C     CHECK RESULTS  (B)=(A)*(E)                                        NAHS0800
C                                                                       NAHS0801
      DO 400 I=1,N                                                      NAHS0802
      DO 400 J=1,L                                                      NAHS0803
      B(I,J)=0.0                                                        NAHS0804
      DO 400 K=1,M                                                      NAHS0805
  400 B(I,J)=B(I,J)+A(I,K)*E(K,J)                                       NAHS0806
C                                                                       NAHS0807
      RETURN                                                            NAHS0808
C                                                                       NAHS0809
      END                                                               NAHS0810
```

```
      SUBROUTINE INVERT(A,NN,N,M,C)                                     NAHS0811
C                                                                       NAHS0812
C          GENERAL MATRIX INVERSION SUBROUTINE                          NAHS0813
C                                                                       NAHS0814
      DIMENSION A(1),M(1),C(1)                                          NAHS0815
C                                                                       NAHS0816
C     CHECK FOR 1 X 1 MATRIX                                            NAHS0817
C                                                                       NAHS0818
      IF(NN-1) 300,70,80                                                NAHS0819
   70 A(1)=1./A(1)                                                      NAHS0820
      GO TO 300                                                         NAHS0821
C                                                                       NAHS0822
   80 DO 90 I=1,NN                                                      NAHS0823
   90 M(I)=-I                                                           NAHS0824
C                                                                       NAHS0825
      DO 140 I=1,NN                                                     NAHS0826
C                                                                       NAHS0827
C     LOCATE LARGEST ELEMENT                                            NAHS0828
C                                                                       NAHS0829
      D=0.0                                                             NAHS0830
      DO 112 L=1,NN                                                     NAHS0831
      IF (M(L)) 100,100,112                                             NAHS0832
  100 J=L                                                               NAHS0833
      DO 110 K=1,NN                                                     NAHS0834
      IF (M(K)) 103,103,108                                             NAHS0835
  103 IF (ABSF(D)-ABSF(A(J))) 105,105,108                               NAHS0836
  105 LD=L                                                              NAHS0837
      KD=K                                                              NAHS0838
      D=A(J)                                                            NAHS0839
  108 J=J+N                                                             NAHS0840
  110 CONTINUE                                                          NAHS0841
  112 CONTINUE                                                          NAHS0842
C                                                                       NAHS0843
C     INTERCHANGE ROWS                                                  NAHS0844
C                                                                       NAHS0845
      TEMP=-M(LD)                                                       NAHS0846
      M(LD)=M(KD)                                                       NAHS0847
      M(KD)=TEMP                                                        NAHS0848
      L=LD                                                              NAHS0849
      K=KD                                                              NAHS0850
      DO 114 J=1,NN                                                     NAHS0851
      C(J)=A(L)                                                         NAHS0852
      A(L)=A(K)                                                         NAHS0853
      A(K)=C(J)                                                         NAHS0854
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```
      L=L+N                                                             NAHS0855
  114 K=K+N                                                             NAHS0856
C                                                                       NAHS0857
C     DIVIDE COLUNM BY LARGEST ELEMENT                                  NAHS0858
C                                                                       NAHS0859
      NR=(KD-1)*N+1                                                     NAHS0860
      NH=NR+N-1                                                         NAHS0861
      DO 115 K=NR,NH                                                    NAHS0862
  115 A(K)=A(K)/D                                                       NAHS0863
C                                                                       NAHS0864
C     REDUCE REMAINING ROWS AND COLUMNS                                 NAHS0865
C                                                                       NAHS0866
      L=1                                                               NAHS0867
      DO 135 J=1,NN                                                     NAHS0868
      IF (J-KD) 130,125,130                                             NAHS0869
  125 L=L+N                                                             NAHS0870
      GO TO 135                                                         NAHS0871
  130 DO 134 K=NR,NH                                                    NAHS0872
      A(L)=A(L)-C(J)*A(K)                                               NAHS0873
  134 L=L+1                                                             NAHS0874
  135 CONTINUE                                                          NAHS0875
C                                                                       NAHS0876
C     REDUCE ROW                                                        NAHS0877
C                                                                       NAHS0878
      C(KD)=-1.0                                                        NAHS0879
      J=KD                                                              NAHS0880
      DO 140 K=1,NN                                                     NAHS0881
      A(J)=-C(K)/D                                                      NAHS0882
  140 J=J+N                                                             NAHS0883
C                                                                       NAHS0884
C     INTERCHANGE COLUMNS                                               NAHS0885
C                                                                       NAHS0886
      DO 200 I=1,NN                                                     NAHS0887
      L=0                                                               NAHS0888
  150 L=L+1                                                             NAHS0889
      IF(M(L)-I) 150,160,150                                            NAHS0890
  160 K=(L-1)*N+1                                                       NAHS0891
      J=(I-1)*N+1                                                       NAHS0892
      M(L)=M(I)                                                         NAHS0893
      M(I)=I                                                            NAHS0894
      DO 200 L=1,NN                                                     NAHS0895
      TEMP=A(K)                                                         NAHS0896
      A(K)=A(J)                                                         NAHS0897
      A(J)=TEMP                                                         NAHS0898
```

```
      J=J+1                                                             NAHS0899
  200 K=K+1                                                             NAHS0900
C                                                                       NAHS0901
  300 RETURN                                                            NAHS0902
C                                                                       NAHS0903
      END                                                               NAHS0904
```

```
      SUBROUTINE SYMSOL (A,B,NN,MM)                                     NAHS0905
C                                                                       NAHS0906
      DIMENSION A(720,30),B(720),C(30)                                  NAHS0907
C                                                                       NAHS0908
      N = 0                                                             NAHS0909
  100 N = N+1                                                           NAHS0910
C                                                                       NAHS0911
C     REDUCE N TH EQUATION                                              NAHS0912
C                                                                       NAHS0913
C        1. DIVIDE RIGHT SIDE BY DIAGONAL ELEMENT                       NAHS0914
C                                                                       NAHS0915
      B(N) = B(N) / A(N,1)                                              NAHS0916
C                                                                       NAHS0917
C        2. CHECK FOR LAST EQUATION                                     NAHS0918
C                                                                       NAHS0919
      IF(N-NN) 150,300,150                                              NAHS0920
C                                                                       NAHS0921
C        3. DIVIDE N TH EQUATION BY DIAGONAL ELEMENT                    NAHS0922
C                                                                       NAHS0923
  150 DO 200 K=2,MM                                                     NAHS0924
      C(K) = A(N,K)                                                     NAHS0925
  200 A(N,K) = A(N,K) / A(N,1)                                          NAHS0926
C                                                                       NAHS0927
C        4. REDUCE REMAINING EQUATIONS                                  NAHS0928
C                                                                       NAHS0929
      DO 260 L=2,MM                                                     NAHS0930
      I = N+L-1                                                         NAHS0931
      IF(NN-I) 260,240,240                                              NAHS0932
  240 J=0                                                               NAHS0933
      DO 250 K=L,MM                                                     NAHS0934
      J=J+1                                                             NAHS0935
  250 A(I,J) = A(I,J) - C(L) * A(N,K)                                   NAHS0936
      B(I) = B(I) - C(L) * B(N)                                         NAHS0937
  260 CONTINUE                                                          NAHS0938
      GO TO 100                                                         NAHS0939
C                                                                       NAHS0940
C     BACK SUBSTITUTION                                                 NAHS0941
C                                                                       NAHS0942
  300 N = N-1                                                           NAHS0943
C                                                                       NAHS0944
C        1. CHECK FOR FIRST EQUATION                                    NAHS0945
C                                                                       NAHS0946
      IF(N) 350,500,350                                                 NAHS0947
C                                                                       NAHS0948
```

```
C          2. CALCULATE UNKNOWN B(N)                                    NAHS0949
C                                                                       NAHS0950
  350 DO 400 K=2,MM                                                     NAHS0951
      L = N+K-1                                                         NAHS0952
      IF(NN-L) 400,370,370                                              NAHS0953
  370 B(N) = B(N) - A(N,K) * B(L)                                       NAHS0954
  400 CONTINUE                                                          NAHS0955
      GO TO 300                                                         NAHS0956
C                                                                       NAHS0957
  500 RETURN                                                            NAHS0958
C                                                                       NAHS0959
      END                                                               NAHS0960
```

```
      SUBROUTINE PRINTM (A,NR,NC,MAXR)                                  NAHS0961
      DIMENSION A(1),NHED(10)                                           NAHS0962
      COMMON KINN,KOUT                                                  NAHS0963
C                                                                       NAHS0964
      DO 50 I=1,NC,10                                                   NAHS0965
      II=NC-I+1                                                         NAHS0966
      IF (II-10) 20,20,10                                               NAHS0967
   10 II=10                                                             NAHS0968
   20 DO 30 J=1,II                                                      NAHS0969
   30 NHED(J)=I+J-1                                                     NAHS0970
C                                                                       NAHS0971
      WRITE OUTPUT TAPE KOUT,120,(NHED(J),J=1,II)                       NAHS0972
C                                                                       NAHS0973
      DO 50 J=1,NR                                                      NAHS0974
      KL=J+(I-1)*MAXR                                                   NAHS0975
      KH=KL+(II-1)*MAXR                                                 NAHS0976
   50 WRITE OUTPUT TAPE KOUT,130,(J,(A(K),K=KL,KH,MAXR))                NAHS0977
C                                                                       NAHS0978
      RETURN                                                            NAHS0979
C                                                                       NAHS0980
  120 FORMAT (8H0   N/M  10I11)                                         NAHS0981
  130 FORMAT (I5,3X,10F11.3)                                            NAHS0982
      END                                                               NAHS0983
```