January to July 1962
SUMMARY REPORT

POWER RELAY DESIGN

Analyze, Study and Establish an Optimum Power Relay Design for Application in Saturn Launch Vehicle Systems

Contract No. NAS 8-2552

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Prepared for:

George C. Marshall Space Flight Center
Huntsville, Alabama

SCHOOL OF ELECTRICAL ENGINEERING
OKLAHOMA STATE UNIVERSITY
Stillwater

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Prepared for

N. A. S. A.

GEORGE C. MARSHALL SPACE FLIGHT CENTER
HUNTSVILLE, ALABAMA

by

C. F. Cameron, Project Director

SCHOOL OF ELECTRICAL ENGINEERING
OKLAHOMA STATE UNIVERSITY
STILLWATER, OKLAHOMA

Report Period 1 January to 30 June 1962

July 1, 1962

OKLAHOMA STATE UNIVERSITY

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FOREWORD

This summary report contains the material that was developed during the period 1 January to 30 June, 1962 on the research contract.

This material has been organized into four major topics. These topics are: contactor characteristics, vibration, contact study and contactor design.

The report itself has been divided into two divisions: the first containing the abstracts and conclusions of each of the technical sections, the scope of work defined in the contract and a summary of the engineering and service time spent on the project. The second division of this report contains the technical material developed in more detail and the results obtained during the period of time involved. A table of contents at the beginning of each part should be helpful in locating each section.

The information contained in each of the technical parts is compiled from the three interim report sections and consequently contains the section numbering used in that particular interim report. In order to maintain continuity of presentation the interim report sections used to make up each part of this report may not be in chronological order. In case the chronological order is desired the tabs identifying the sections in a given interim report are assigned a particular color. The interim report numbers for the time interval of this six months report are the 1st, 2nd and 3rd. The tab color associated with the interim report is as follows: 1st - rose, 2nd - blue and 3rd - yellow.

The various sections of the interim reports are written by different project personnel. An effort has been made to make the different sections conform to a consistent pattern of presentation and format but inevitably some differences exist.

The project technical personnel consist of graduate research assistants who are actively pursuing a M.S. or Ph.D. degree and some of the faculty of the College of Engineering. It is through their efforts and those of the technical supervisors at the George C. Marshall Space Flight Center that this report is possible.

ABSTRACTS

PART A Contactor Characteristics

TRANSIENT COIL CURRENT OF A CONTACTOR - Section I - 1st

Two contactors were used to find the transient coil current characteristics for operate and release. These oscillograms were obtained for the 25 ampere contactor and for the 200 ampere contactor. The voltage across the contacts was recorded for the main NO contacts and the auxiliary NO and NC contacts. Since the transient coil current and the voltage across the contacts were simultaneous traces on the oscillograms, the relative time could be observed when these operations took place.

CONTACTOR TRANSIENT CHARACTERISTICS - Section IV - 1st

The transient characteristics are shown by the oscillograms which are given in the first four figures of this section. The trace of the transient coil current and the trace of the armature displacement give the dynamic characteristics during this period.

A series of traces of the transient coil current were obtained with different values of voltage. It seemed desirable to find some voltage at which the contactor would function and a double or triple cusp would not appear in the coil current during the transient period.

PART B Vibration

VIBRATION TEST - Section IV - 2nd

This section outlines the attempt to obtain enough data concerning the vibrational failure of the relays such that the design can be corrected. The procedure has been to select possible causes and check each possibility individually until a condition is found that noticably affects the performance of the relay. It is hoped that this analysis will point out some design criteria which may be applied to relays in general.

VIBRATION TEST CONTINUED - Section II - 3rd

The problem of failure of the relay, for the purpose of this discussion shall be defined as a separating of the contacts when the coil is energized. The contact system was considered and five possible causes of failure defined. Of these five one had been investigated previously, one was discarded as unlikely, and one was investigated in some detail. This report is concerned with the motion of the movable contact bar with respect to the armature shaft. The spring tension was varied and the effects noted.

PART C <u>Contact Study</u>

PRELIMINARY INVESTIGATION AND PROPOSAL OF RELAY CONTACT DESIGN - Section III - 1st

In this preliminary study, three areas are discussed, which relate to design. Design terminology as applied to devices in general with some definitions is given. The second part deals with the electrical contact system in particular. Whereas, the third part is concerned with an attempt to work out a scheme which can be applied to an electric contact system with given load requirements.

CONTACT RATING - Section II - 2nd

This discussion is an attempt to furnish a partial answer to the question, "What are the actual load conditions to which a contactor is subjected?" An outline is made of one analysis of the problem. No doubt, this study should be extended.

THEORETICAL INVESTIGATION AND SOME EXPERIMENTAL DATA FOR ELECTRICAL CONTACT FAILURE CAUSED BY ELECTRICAL LOADING - Section III - 2nd

This section is a preliminary attempt to find analytical relationships with which to predict the life of a contactor contact system with respect to electrical load with a given degree of certainty. The degree of certainty is expressed as a probability for the number of contactors of interest which are expected to meet the predicted life. The life is expressed in terms of number of operations based on a given electrical load condition. This was obtained from more basic considerations involving the two relationships;

probability for failure vs mass transfer, and mass transfer vs arc energy. The final relationship used, relates number of operations (N), to arc energy (A), through arc energy per cycle, (A_c) for a given load condition.

FURTHER DISCUSSION OF CONTACT FAILURE DUE TO ELECTRICAL LOADING - Section III - 3rd

A discussion of the determination of the constants of an equation of mass transfer caused by arc energy is given. Test are suggested for obtaining data which may be used to evaluate the constants of the relationship between mass transfer and arc energy.

PART D <u>Contactor Design</u>

VERIFICATION OF THE FORM OF CONTACTOR DESIGN EQUATIONS - Section II - 1st

In previous work several design equations have been developed for electromagnetic relays. Before some of these equations should be used in a modification of a contactor, it is best to verify that the same assumptions are justified for a contactor as well as an electromagnetic relay.

The sum of the pick-up time and the transit time is equal to the total seating time. It is, therefore, necessary to verify the equations for pick-up time and transit time.

AN APPLICATION OF THE THEORY OF DESIGN - Section V - 2nd

A design modification is the same type of a problem as a new design. The first question to be answered is, "Will the desired modification yield a device which can be made?" Some of the same limitations which are encountered in the original design must be observed.

PRELIMINARY CONTACTOR REDESIGN - Section I - 2nd

Preliminary vibration testing of the contactors in the de-energized state indicated that the plunger was moving when the contactor was vibrated along its axis of operation. In order to hold the plunger stationary, the initial back tension on the plunger must be increased. Increasing the back tension requires that the other contactor parameters be changed. Two possible combinations of fixed parameters were selected and the other parameters computed. The procedure used to take the parameters specified and list them on the design matrix is given. Since the numerical data about the values of the parameters existing on the given contactor were not known, the changes are given in terms of percent.

CONTINUATION OF PRELIMINARY CONTACTOR REDESIGN - Section I - 3rd

It appears that some combination of increased coil power and coil length might be the most feasible in the redesign of the contactor. Additional calculations are given in this section to show the result of increasing the back tension by a combination of coil power and coil length. Several parameters are plotted against coil power.

CONCLUSIONS

PART A Contactor Characteristics

TRANSIENT COIL CURRENT OF A CONTACTOR - Section I - 1st

The two contactors which were studied by means of obtaining the transient coil current and voltage across the contacts characteristics showed one common trait. There was a double hump immediately after the first cusp of the current build-up trace. The conclusion was that an obstruction such as the picking-up of an additional spring caused this hesitation in the motion of the armature.

CONTACTOR TRANSIENT CHARACTERISTICS - Section IV - 1st

It was assumed that the transient characteristics of contactors would be similar to the transient characteristics of relays. The oscillograms which were obtained for this section demonstrate that this assumption is correct. The first four oscillograms show that the transient current trace has irregularities in it which correspond to the trace of the instanteous position of the armature and that there was a hesitation of the armature during its travel.

The last two oscillograms prove that the armature hesitation may be suppressed by increasing the impressed voltage on the coil. It is believed that the armature hesitation causes unsatisfactory functioning of the contactor.

PART B Vibration

VIBRATION TEST - Section IV - 2nd

The investigation to date has dealt with the armature and the contact mountings. The armature, although appearing to have some type of motion relative to the coil, does not seem to have much effect on the failure of the contacts when the relay is energized. The mountings of the stationary contacts have some effect, although the conclusions to this part of the test are not yet complete. The mountings of the movable contacts have a much greater effect on the possibility of failure than any other factor yet considered. The investigation of these mountings is still underway.

A permanent failure was detected in the NC auxiliary set of contacts on the 50 amp relay. The failure was the breaking of one of the contacts during the vibrational test.

VIBRATION TEST CONTINUED - Section II - 3rd

It was found that be selection of the proper spring tension on the moving contact bar, the failure of the contacts (that was found to exist in all relays tested) could be eliminated. It was also found that the extremes of adjustment (i.e. very little or very great tension) made the relay fail under much less extreme conditions.

It is believed that although the spring system is so non-linear as to make analytical studies very difficult, it would be desirable to study this type of contact arrangement in much more detail.

PART C <u>Contact Study</u>

PRELIMINARY INVESTIGATION AND PROPOSAL OF RELAY CONTACT DESIGN - Section III - 1st

The type of electric load on the contacts of a relay seems to be more significant than the numerical value of the current. Specifications which are more realistic for electric contactors would, no doubt, be of great value.

Intermittent opening and closing of the contacts, such as that which takes place during contact chatter, and with highly inductive loads will seriously overheat the device with less than rated current through the contacts.

CONTACT RATING - Section II - 2nd

Some rational scheme or logical method should be devised whereby contact specifications may be obtained.

A partial list of one set of requirements is outlined. No attempt has been made to make this outline all inclusive. It would seem desirable to have test data upon which to base valid conclusions.

THEORETICAL INVESTIGATION AND SOME EXPERIMENTAL DATA FOR AN ELECTRICAL CONTACT FAILURE CAUSED BY ELECTRICAL LOADING - Section III - 2nd

The investigations to date have primarily dealt with constructing a mathematical model with which to test for validity. The only experimental work in this area to date has been an investigation of mass transfer versus arc energy. Based on the evidence of the experiments to date, it appears likely that a not too complicated form can be obtained relating the two parameters. However this form also appears to have at least two parameters which are functions of several variables.

The correlation between the proposed theory and the observed experiments to date, are quite encouraging as to the possibilities for obtaining workable expressions with which to rate contact life for a given relay and load-duty cycle condition.

FURTHER DISCUSSION OF CONTACT FAILURE DUE TO ELECTRICAL LOADING - Section III - 3rd

Information from tests should allow some predictions to be made concerning number of operations for a given load and application for a pair of contacts. When these tests have been completed the validity of the proposed scheme may be determined.

PART D <u>Contactor Design</u>

VERIFICATION OF THE FORM OF CONTACTOR DESIGN EQUATIONS - Section II - 1st

Two design equations, one used to predict the pick-up time and the other used to predict the plunger transit time, were checked to determine if the form was accurate for contactor design. These equations were checked in regard to the influence of the supply voltage E and the total circuit resistance R_t. Before the other parameters involved in the equations can be changed the contactors must be unsealed which will be done at a later time. The form of the equations for pick-up time t_p, plunger transit time k as a function of the per unit pick-up current h are:

$$t_p = A \ln \frac{1}{1 - h} \quad \text{when E is variable}$$

$$t_p = \frac{B}{h} \ln \frac{1}{1 - h} \quad \text{when } R_t \text{ is variable}$$

$$k = C \left(\frac{h}{1 - h}\right)^{\frac{1}{3}} \quad \text{when E is variable}$$

$$k = D \left(\frac{1}{1 - h}\right)^{\frac{1}{3}} \quad \text{when } R_t \text{ is variable}$$

$$h = i_p/i_{ss} = G\, R_t/E$$

i_p = pick-up coil current

i_{ss} = steady state coil current

E = supply emf in the Thevenin's theorem sense

R_t = total circuit resistance

The letters A, B, C, D and G are constants as far as the voltage E and resistance R_t is concerned. These constants are functions of other contactor design parameters which are listed but these have not been completely checked at this time. Since these design equations were developed for a different electromechanical device all the parameters involved should be verified for the contactor. This verification will be continued.

AN APPLICATION OF THE THEORY OF DESIGN - Section V - 2nd

The examples given in this section illustrate that the number of items of the specification is fixed when there is a fixed number of relationships and parameters. As mentioned, arbitrary specifications may result in conflicting requirements. A modification of a design is really a new design problem and a logical method of procedure should be used.

PRELIMINARY CONTACTOR REDESIGN - Section I - 2nd

The results of the preliminary redesign points out the fact that only a certain number of parameters can be fixed or changed. If a given mechanical arrangement of the elements is to be used then the parameters which determine this must be fixed. These fixed parameters along with the ones being changed are limited to 8 in number.

When the coil dimensions are fixed, among other things, the coil power must increase in order to increase the back tension. In this case the coil power required increased directly with the back tension. However, this depends upon the parameters that are selected. Since heat dissipation was not known, the redesign resulting in an increase in coil power may be undesirable. A second computation was made with the coil power fixed and the coil length variable. The results of this computation indicated that the coil length must increase directly with the increase in back tension. These results show that an increase in the mechanical work performed by the contactor must be accompanied by an increase in coil power or an increase in coil volume or a combination of both.

CONTINUATION OF PRELIMINARY CONTACTOR REDESIGN - Section I - 3rd

Increasing the back tension, P_o, on the plunger in order to raise the G level requires certain changes in the other parameters. For the set of specified parameters used, which contain P, η , E, M, R_s, x_o and μA, it is shown that the product of the coil power P and the coil length ℓ is directly proportional to P_o. The influence on the unspecified parameters of changing the coil power is shown by a set of curves for various values of the factor β . The factor β is the ratio of the core diameter to the outside coil diameter. A value of β which will minimize the coil length for a given value of coil power is obtained. In addition, the influence of the coil bobbin insulation is presented by comparison of the curves in the figures. For the contactor considered, an increase in coil efficiency of approximately 50% can be obtained by changing the core diameter and the bobbin insulation thickness.

SCOPE OF WORK

The work will consist of the following:

(a) Review several contactor designs presently employed for space vehicle applications and select the most promising designs for further analysis.

(b) Analyze in detail the design to determine the parameters which are not consistent with the requirements.

(c) Propose a modified design which would more nearly satisfy the required performance.

(d) The design performance of the contactor is as follows:

   (1) Withstand 20g or more vibration with a frequency range of 10 to 2000 cps.

   (2) That the contactor have a minimum life of 10,000 operations at rated load.

   (3) Temperature limits - 65° to + 125°F.

   (4) Contactor shall be contained in a hermetically sealed package.

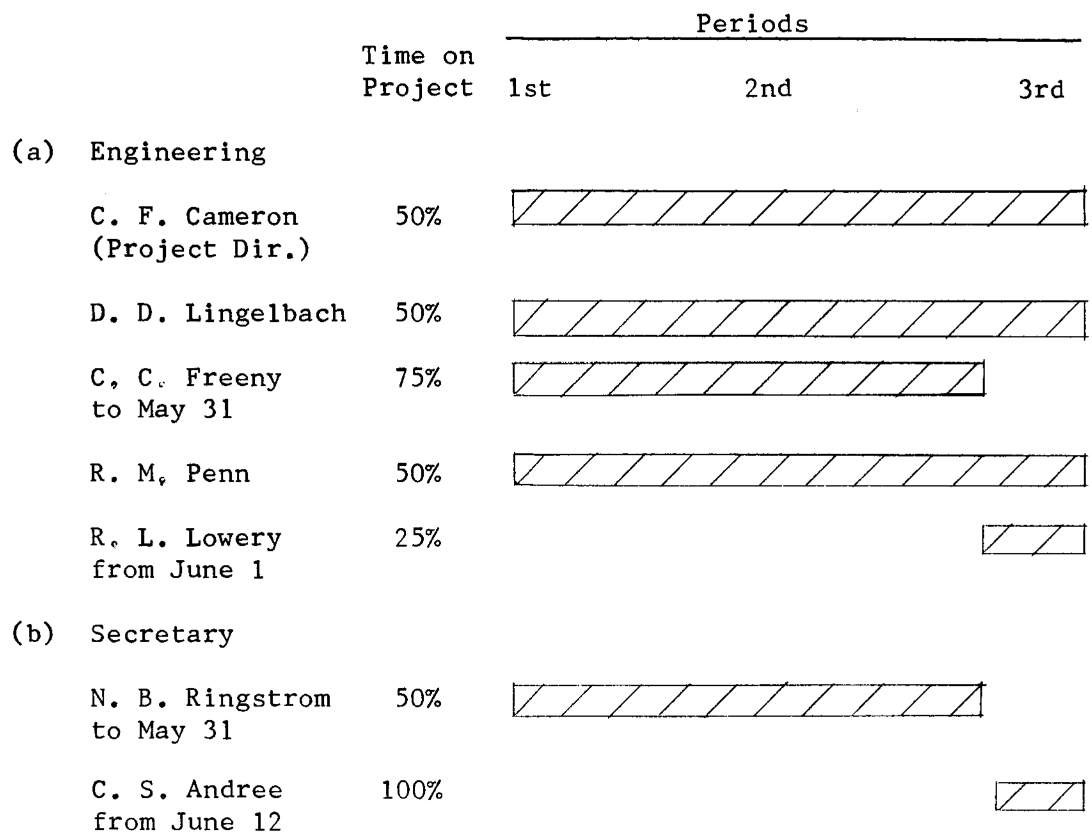
(e) Evaluate modified design unit.

SUMMARY OF MAN HOURS

Summary Report: 1 January to 30 June, 1962

Interim Report Periods:

1st	1 January to 28 February, 1962
2nd	1 March to 30 April, 1962
3rd	1 May to 30 June, 1962



The academic staff of the Oklahoma State University is appointed for nine months plus a vacation of one month for a given salary. When a person works two months during the summer, this pay is at the rate of ten per cent of his pay for the academic year per month. Working hours are 40 hours per week except where vacation periods are established by the University. Research personnel are assigned a given percentage of their total time to a project, and relieved of other duties for the corresponding time assigned to the project.

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Contactor Characteristics

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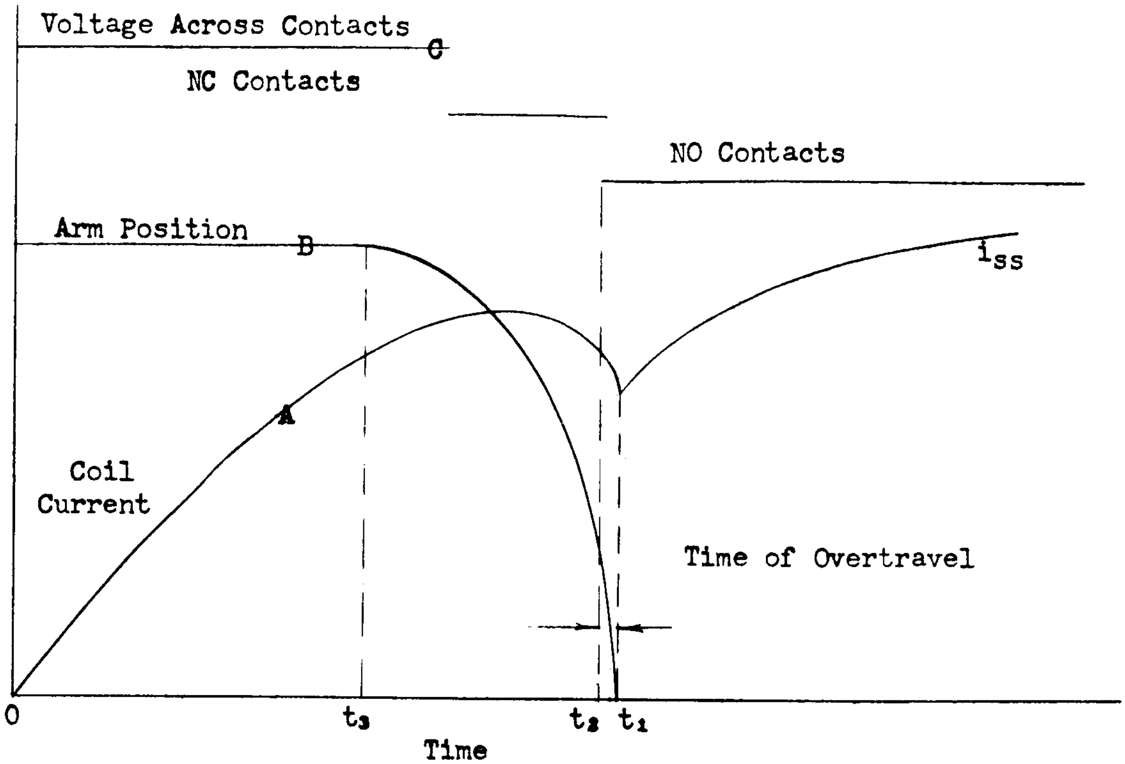
SECTION I

TRANSIENT COIL CURRENT OF A CONTACTOR

Much may be learned about the behavior of an electrical contactor or a heavy duty relay by observing the transient coil current and the voltage across the contacts. These traces may be recorded by a camera attached to a dual beam oscilloscope. Since the two beams of the oscilloscope give a record of events which have taken place simultaneously, this scheme may be used to analyze the sequence of events in a device such as a contactor.

The figure which is included herewith shows a typical set of transient characteristics for a relay. In this study, relay and contactor will be used interchangeably. It might be said that a contactor is a heavy duty relay. In the figure the time scale is on the horizontal axis and the vertical axis may be used to represent current, voltage, position or some other quantity. Two of these quantities may be recorded simultaneously as a function of time. Since the time is the same for each trace at some particular point on the horizontal axis, these oscillograms are an excellent means of explaining the happenings in such a device as a contactor.

The trace which is labeled "A" in the illustrative diagram shows the instantaneous current for a time interval of zero time to steady-state current conditions, which may be fifty or one hundred milliseconds later. It is to be noted that there is a very pronounced cusp in this current trace. The sharp tip of the cusp indicates the time at which the armature has completed its travel. Curve "B" is a trace which indicates the instantaneous position of the armature. In this diagram, the coil is energized at zero time and the armature was in the open position. The armature has closed at time (t_1) which coincides with the sharp point on the current cusp. Numerous oscillograms have proved the validity of this statement.



TRANSIENT CHARACTERISTICS

At time (t_3) the armature starts to move and a short time later the NC contacts have opened which is indicated by the three horizontal lines marked "C". The short horizontal lines show that during transfer, the NC contacts are open and NO contacts are open, after which the NO contacts close, which is indicated as time (t_2). The time of overtravel of the NO contacts is shown as the distance from t_2 to t_1. This drawing was made for a transfer switch or Form C contacts.

Much significant information may be obtained from oscillograms of this nature. Contact chatter or bounce may be recorded. Hesitation of the armature during its travel may be indicated. When the armature strikes the core and rebounds, the current trace after time (t_1) is not a smooth curve. The height of the current trace before the cusp compared to the steady-state value of current gives an idea of the stability of the device. The steady-state current is indicated as i_{ss}. Operate time is the time from zero to t_1.

Under release conditions, the transient current may be recorded as well as the voltage across the contacts. These release traces also have certain general characteristics. These curves or traces for operate and release may be regarded as the transient characteristics.

Each relay design type will exhibit certain peculiarities which are common to that particular design type. Any variation in these characteristics indicates that some abnormal situation has arisen.

The oscillograms shown in Figures 1 through 12 were made in order to have a record of the transient characteristic of each of the contactors received. If during testing of the contactors any changes occur, a comparison can be made by recording the transient characteristics after testing and comparing them with the original oscillograms. These oscillograms are recorded at some particular voltage, usually the rated voltage. However, additional information can be obtained by recording the transients at different values of voltage.

Figure 1 shows simultaneously the coil current build-up and the contact voltage across the power contacts L_1-T_1 of the 25 ampere contactor #1. Since the power contacts are a NO pair, the contactor voltage trace has only two levels. Comparison of the coil current trace and the contact voltage trace shows that the power contacts function at the first cusp. Or in other words, the functioning of the power contacts in this case seems to cause the first cusp.

Figure 2 shows the contact voltage across the NO contacts of the auxillary set and the coil current build-up. The breaks in the contact voltage trace (a) indicates contact bounce which continues for some little time. For inductive loads this could be a very unsatisfactory situation.

The oscillogram of Figure 3 gives the transient coil current and the voltage across the NC auxiliary contacts. In all of the traces for the current build-up in the first three oscillograms, the current shows three different cusps, however the last one is rather minor.

The transients during the release period are shown in Figures 4, 5 and 6. The Figure 4 shows the decay of the coil current and the opening of the NO contacts for the 25 ampere contactor. The hump on the current decay trace has a saddle. This seems to be a characteristic of this particular contactor. At the moment, no opinion has been formed as to why this particular shape exists. Figures 5 and 6 are somewhat similar to Figure 4.

Oscillograms which are given in Figures 7, 8, 9, 10, 11 and 12 are those obtained on the 200 ampere contactor. The transient current trace has a double hump but the decay trace is somewhat different than that of the 25 ampere contactor.

The voltage across the auxiliary contacts of Figure 8 shows some contact bounce. The other figures do not give much evidence of bounce. Figures 7, 8 and 9 are the transients for operate conditions and Figures 10, 11, and 12 are for release conditions.

These oscillograms give some ideas about the functioning of the contactor. Below each oscillogram is given the various conditions which were imposed on that device.

It is evident that much more will have to be learned about these contactors before specific recommendations can be made for improvement. It seems evident, however, that the armature hesitation for operation conditions will bear further investigation. It is planned to continue with this idea in an attempt to cause the armature to move directly from the open position to the closed position when the coil is energized.

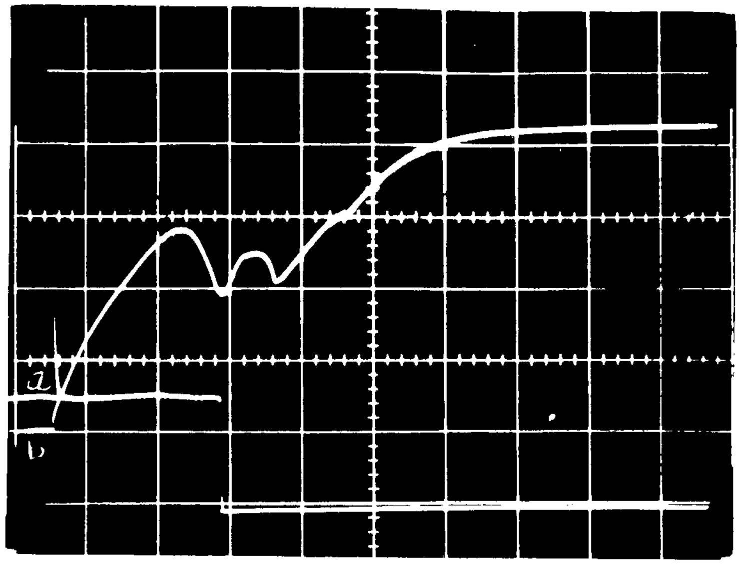


Figure 1

Traces:

(a) Contact Voltage

(b) Coil Current Build-up

Oscillogram Data:

Relay - 25 amp

Contacts - NO (main set)

Coil Voltage - 28 volts

Coil Current - 430 ma

Time Scale - 10 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

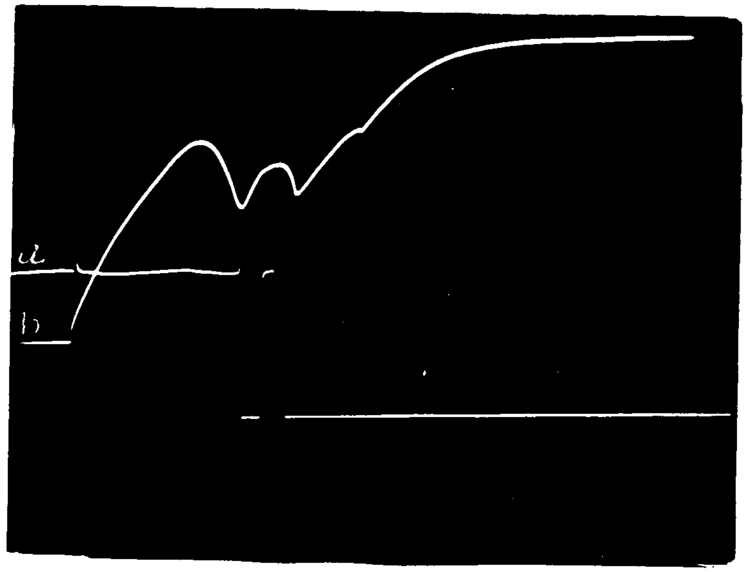


Figure 2

Traces:

(a) Contact Voltage

(b) Coil Current

Oscillogram Data:

Relay - 25 amp

Contacts - NO (auxiliary)

Coil Voltage - 28 volts

Coil Current - 430 ma

Time Scale - 10 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

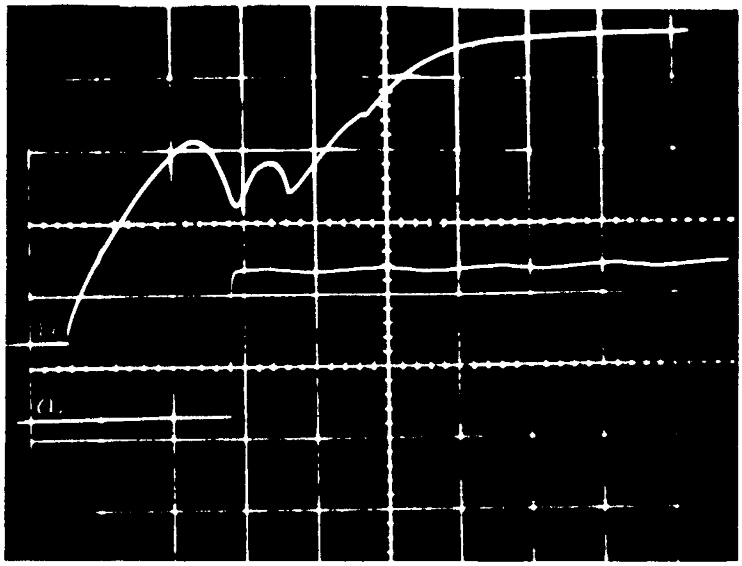


Figure 3

Traces:

(a) Contact Voltage

(b) Coil Current Build-up

Oscillogram Data:

Relay - 25 amp

Contacts - NC (auxiliary)

Coil Voltage - 28 volts

Coil Current - 430 ma

Time Scale - 10 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

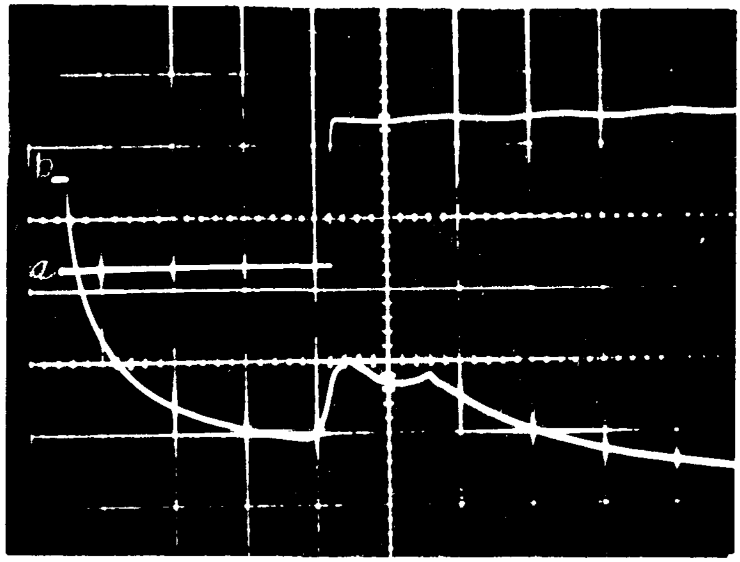


Figure 4

Traces:

(a) Contact Voltage

(b) Coil Current Decay

Oscillogram Data:

Relay - 25 amp

Contacts - NO (main set)

Coil Voltage - 28 volts

Coil Current - 430 ma

Time Scale - 10 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

Coil Discharge Path - diode

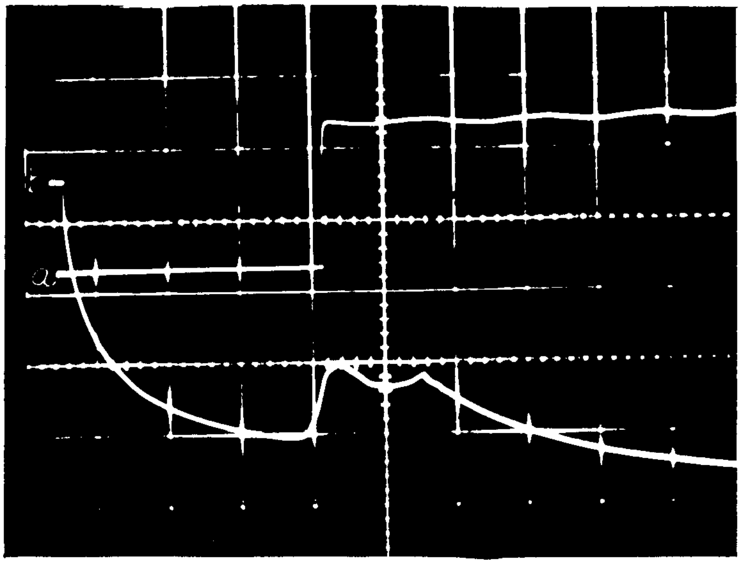


Figure 5

Traces:

(a) Contact Voltage

(b) Coil Current Decay

Oscillogram Data:

Relay - 25 amp

Contacts - NO (auxiliary)

Coil Voltage - 28 volts

Coil Current - 430 ma

Time Scale - 10 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

Coil Discharge Path - diode

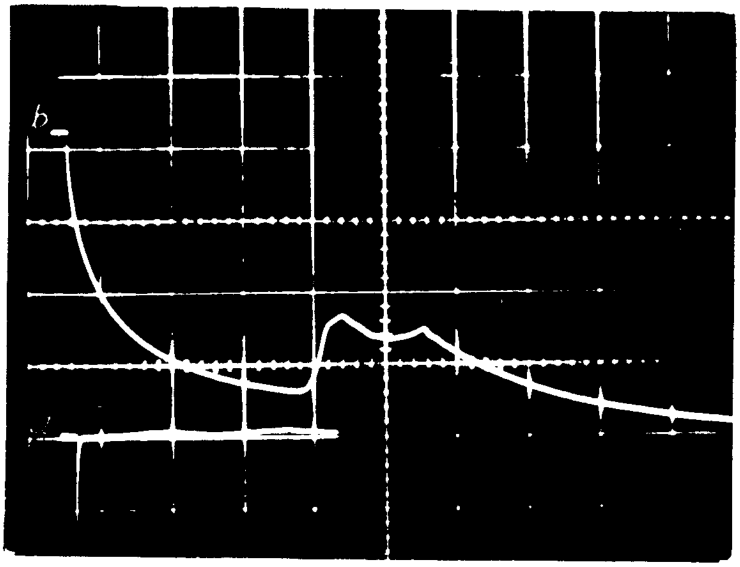


Figure 6

Traces:

(a) Contact Voltage

(b) Coil Current Decay

Oscillogram Data:

Relay - 25 amp

Contacts - NC (auxiliary)

Coil Voltage - 28 volts

Coil Current - 430 ma

Time Scale - 10 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

Coil Discharge Path - diode

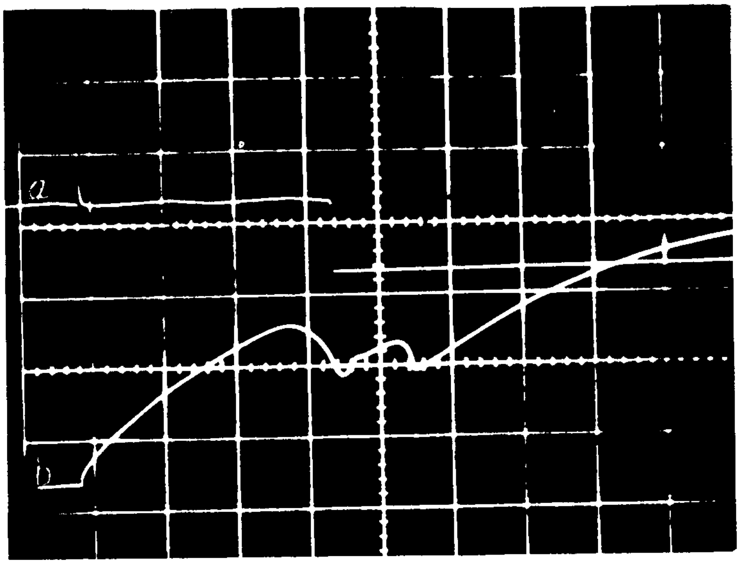


Figure 7

Traces:

(a) Contact Voltage

(b) Coil Current Build-up

Oscillogram Data:

Relay - 200 amp

Contacts - NO (main set)

Coil Voltage - 28 volts

Coil Current - 350 ma

Time Scale - 10 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

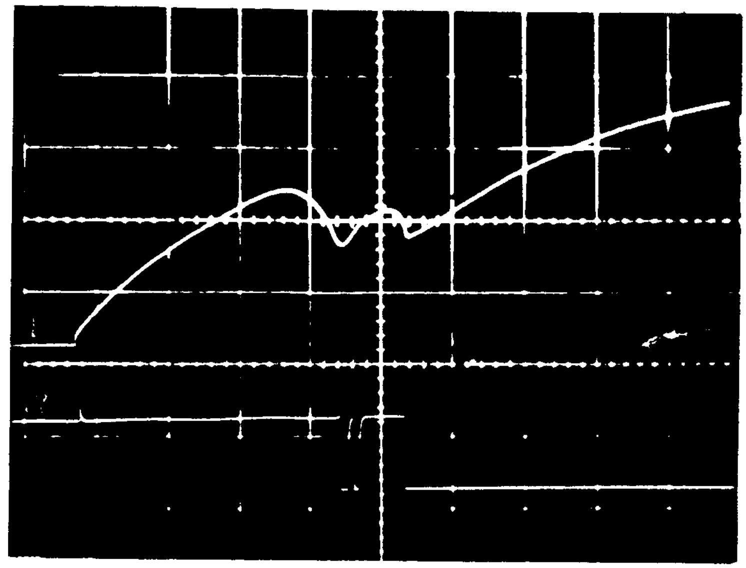


Figure 8

Traces:

(a) Contact Voltage

(b) Coil Current Build-up

Oscillogram Data

Relay - 200 amp

Contacts - NO (auxiliary)

Coil Voltage - 28 volts

Coil Current - 350 ma

Time Scale - 10 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

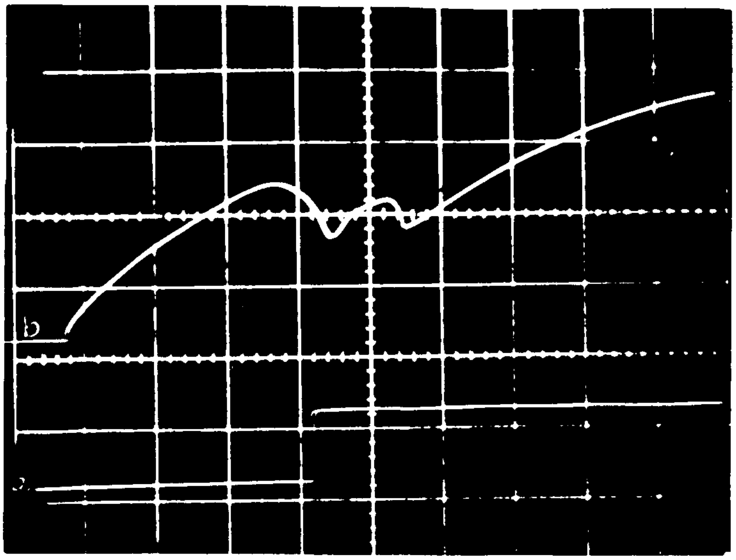


Figure 9

Traces:

(a) Contact Voltage

(b) Coil Current Build-up

Oscillogram Data:

Relay - 200 amp

Contacts - NC (auxiliary)

Coil Voltage - 28 volts

Coil Current - 350 ma

Time Scale - 10 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

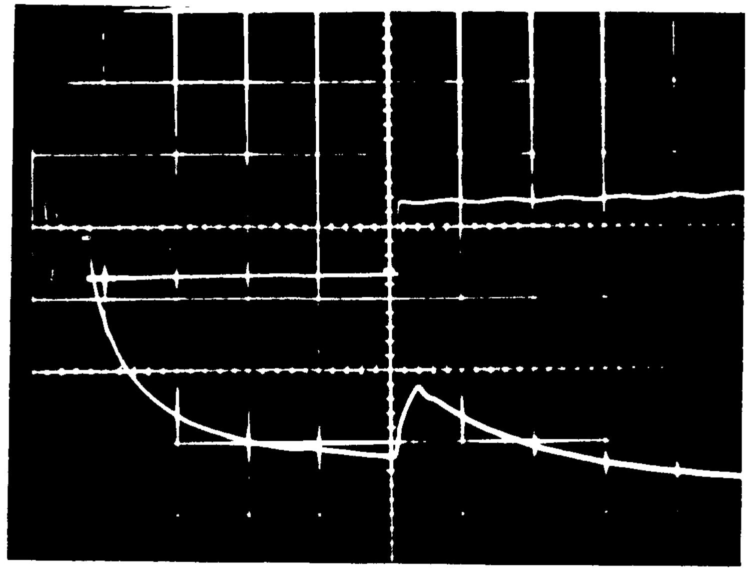


Figure 10

Traces:

(a) Contact Voltage

(b) Coil Current Decay

Oscillogram Data:

Relay - 200 amp

Contacts - NO (main set)

Coil Voltage - 28 volts

Coil Current - 350 ma

Time Scale - 20 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

Coil Discharge Path - diode

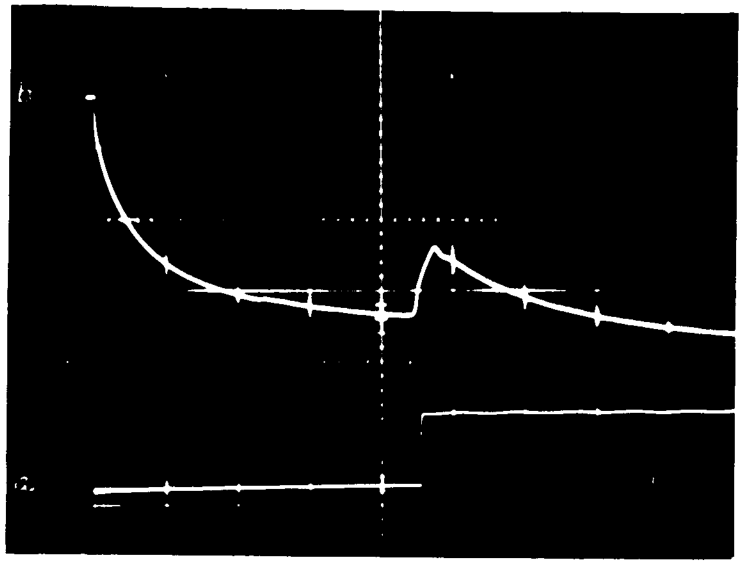


Figure 11

Traces:

(a) Contact Voltage

(b) Coil Current Decay

Oscillogram Data:

Relay - 200 amp

Contacts - NO (auxiliary)

Coil Voltage - 28 volts

Coil Current - 350 ma

Time Scale - 20 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

Coil Discharge Paths - diode

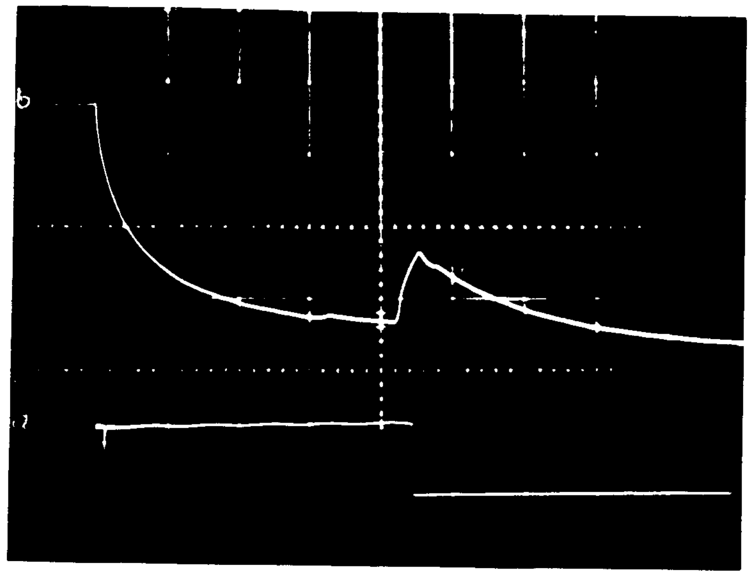


Figure 12

Traces:

(a) Contact Voltage

(b) Coil Current Decay

Oscillogram Data:

Relay - 200 amp

Contacts - NC (auxiliary)

Coil Voltage - 28 volts

Coil Current - 350 ma

Time Scale - 20 ms per cm

Current Scale - 100 ma per cm

Contact Voltage - 20 volts

Coil Discharge Path - diode

SECTION IV

CONTACTOR TRANSIENT CHARACTERISTICS

In the design of relays it is sometimes desirable to mount the movable contact (of a normally open set of contacts) such that it will touch the fixed contact before the armature has completed its travel. It is possible that this design, under extreme operating conditions, could lead to a premature failure of the relay.

Failure of the type relay being discussed in this report is defined to be an opening of the contacts, the open time exceeding 10^{-4} seconds, during the period of time when they are intended to be closed.

It is desired that the relay carry the rated current and undergo vibrations up to 20 times the force of gravity at frequencies of 10 to 2000 cps. In the steady state operated condition, the contacts are held together by a force which for the purpose of this discussion we will define as the maximum force. When this force exists on the contacts, the contact surfaces will be termed, "under maximum pressure."

A direct cause of failure could be the opening of the contacts due to the forces induced by vibrations. To minimize the probability of this type of failure, it is obvious that maximum pressure is required at all times when the contacts are closed.

An indirect cause of failure could be the deterioration of the contacts themselves caused by overheating and arcing. Neglecting the arc energy, the temperature of the contacts is, among other things, a function of the I^2R loss in the contacts. The contact resistance is a function of the pressure on the contact surfaces, an increase in pressure results in a decrease in resistance.

To minimize the undesirable effects of heat on the contact surfaces, and therefore reduce the probability of failure, maximum pressure is desired at all times when there is current flowing between the contacts.

This discussion will deal with the transient characteristics of the relay, and it will be shown that there exists a time interval during operation, such that during the interval the contacts are carrying current but not under maximum pressure. It will also be shown that if such a condition exists, it may be minimized by increasing the coil voltage a sufficient amount. (It should also be noted that other parameters could be changed with the same result.)

Consider a relay with one or more sets of normally open contacts, such that the contact surfaces touch before the armature seats. The controlling circuit of such a relay is essentially an R-L circuit, and the current in the coil can be expressed as

$$(1) \qquad i = \frac{1}{R}\left(E - N\frac{d\phi}{dt}\right)$$

where
i = circuit current
R = circuit resistance
E = applied voltage
N = turns linked by flux
X = distance (of armature travel)
t = time
ϕ = magnetic flux

During the transient period, the flux is related to time through the changing air gap and coil current. Therefore the term $\frac{d\phi}{dt}$ could be more properly written as $\frac{\partial\phi}{\partial t}\frac{di}{dt} + \frac{\partial\phi}{\partial x}\frac{dx}{dt}$. However, for the purpose of this discussion it will be sufficient to use the expression for current in the form of equation (1).

After the voltage is applied to the coil, the current must attain a certain value (called the pick-up current) such that the magnetic force produced is sufficient to overcome the back tension and cause the armature to move. During this time, (termed the pick-up time) the current will follow a curve similar to an exponential rise as determined by the value of R and L in the circuit. This is as expected because of the relatively small change in inductance during this period. This rise in current is seen by referring to Figure 1 which shows the coil current and armature displacement of a relay of the type under discussion.

As the armature begins to move, (as indicated by the droping of trace a in Figure 1) the changing air gap produces a very noticable effect upon the inductance of the circuit. As the armature velocity increases, the term $\frac{d\phi}{dt}$, originally a decreasing term, begins to increase as the energy stored in the air gap is put back into the circuit. This in turn changes the current from an increasing function to a decreasing function. The current continues to decrease until the first set of contacts touch. This is shown by the vertex of the first cusp in the coil current trace. At this point, the force produced by the coil current is not sufficient to overcome the added resistance of the first set of contacts. The current must again build up to a new "pick-up value" before the armature will continue its motion. Note that the current was at one time at a level which would have allowed the armature to push past the first set of contacts, but was reduced by decreasing air gap. As the current reaches the required value, the armature again starts to move. The same sequence of events occur at the time of making of the second set of contacts (the second cusp on trace b). The system then has a third pick-up time to allow the current to rise again. After the armature is seated (the third cusp) the current rises to its steady state value.

During the time between the make of the first set of contacts and the seating of the armature, the contacts are not under maximum pressure. Therefore, the probability of failure is greater at this time than it would be under steady state operation.

Figure 2 shows the same relay operated at a slightly higher voltage (5.8 volts). The time of make without maximum pressure for the first set of contacts has been reduced from 72 to 46 milliseconds.

Figure 3 shows the operation at a much higher voltage (12.6 volts). With this applied voltage the current rise is such that when the first contacts make, the magnetic force is great enough to continue the movement of the armature. Note that the time between make of the contacts (indicated by the interruption in the trace) and the seating of the armature (indicated by the vertex of the cusp) has been reduced to a value so small as to be undetectable at the trace speed shown.

Figure 4 shows operation of the relay at 26 volts. At this voltage the operating time is so short that there should be no problem concerning a less than maximum pressure on the contacts.

Figures 5 and 6 show the effect of applied voltage on the operation of the relay being discussed. Trace (1) shows the first set of contacts start to carry current (at the first cusp) a full 34 milliseconds before maximum pressure is applied (at the last cusp). By increasing the voltage one volt, (trace 2) the time is reduced to 22 milliseconds. At an increase of three volts, (trace 4) the time is reduced to 10 milliseconds.

This time is continually reduced by application of higher voltages until it becomes unmeasurable as in trace (10).

From this discussion it can be concluded that the probability of failure is increased by operation of a relay below a certain desired applied voltage.

Referring again to Figure 1 of this section, trace (a) is the instantaneous position of the armature and trace (b) is the transient coil current. These two traces were obtained simultaneously by means of a dual beam oscilloscope. It is interesting to note that the armature does not move directly from an open position to a closed position but that its progress is interrupted several times during the transit period. These interruptions are reflected, so to speak, into the transient coil current. In fact, it has been demonstrated that when the transient coil current has several cusps then the armature has not had an uninterrupted travel during the transit period.

Unless otherwise stated, the horizontal axis of the oscillograms are time scales. Usually, the milliseconds per centimeter for the time scale are indicated on the sheet. The vertical axis may be current, armature position or voltage and when required the calibration is indicated.

The oscillograms of Figures 1, 2, 3 and 4 were made to illustrate the hesitation of the armature during its travel. As previously mentioned, these oscillograms show that the irregularities of the current trace are the result of the interruptions of the travel of the armature. When the armature moves directly from the open position to the closed position with no interruption, the current trace is smooth. This is shown in Figures 3 and 4.

It is believed that unsatisfactory functioning of a contactor may result from armature hesitating during its travel. This is particularly true during the release condition, when an arc may form across the contacts. An arc which may take place with an inductive load should be broken rapidly, if not the arc could permanently damage the contact structure.

The cause of the interruption of the travel of the armature or plunger may be the restoring spring, the auxiliary contacts and the main contacts. By increasing the voltage impressed on the coil of the contactor, this armature hesitation is greatly reduced or eliminated entirely. The oscillograms of Figures 5 and 6 illustrate the validity of this statement.

Several factors may be involved. An increase in temperature will cause an increase in resistance which, in turn, will cause a decrease in current and therefore the ampere-turns. A reduction in the magnetic pull will be the result. Another situation could cause the same undesirable condition, that is, the power supply could have a voltage drop which would not allow the proper value of current for satisfactory functioning of the contactor.

Before final judgement is passed, it is proposed to investigate all of these details thoroughly in an attempt to explore all of the possibilities.

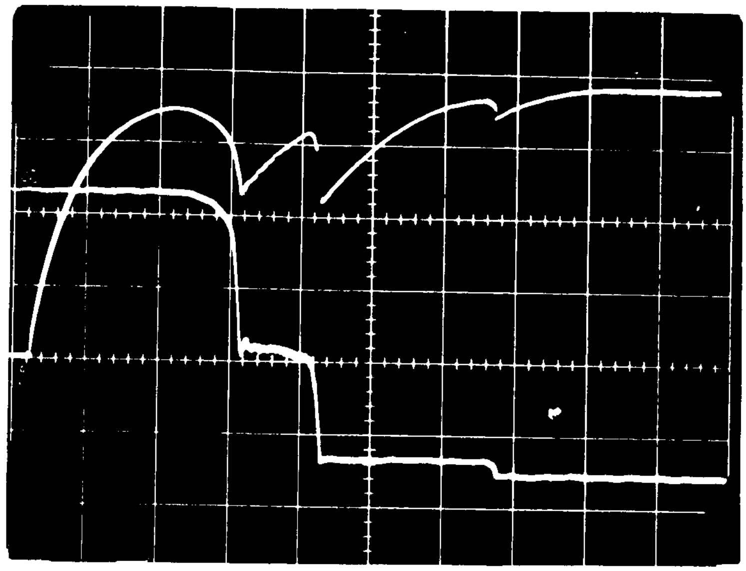


Figure 1

Traces:

(a) Armature displacement

(b) Coil current build-up

Oscillogram Data:

Time Scale: 20 milliseconds per centimeter

Current Scale: 96.6 milliamperes per centimeter

Coil Voltage: 5.4 volts dc.

Steady State Coil Current: 357 milliamperes

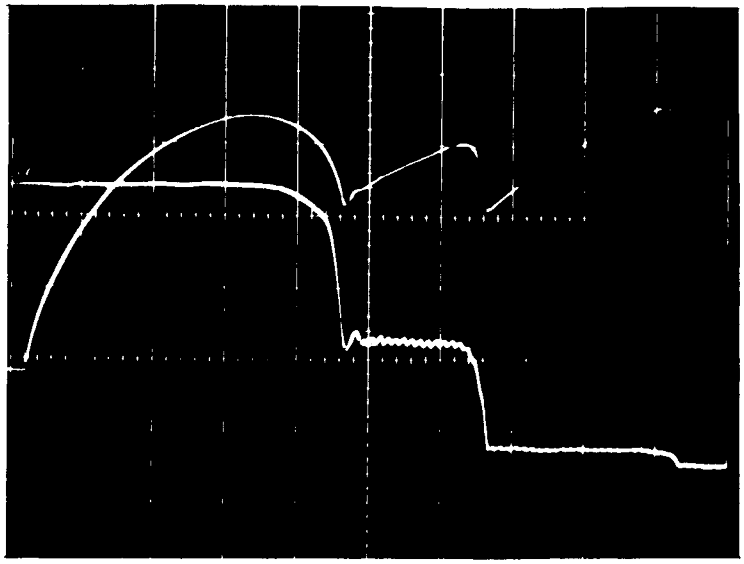


Figure 2

Traces:

(a) Armature displacement

(b) Coil current build-up

Oscillogram Data:

Time Scale: 10 milliseconds per centimeter

Current Scale: 96.6 milliamperes per centimeter

Coil Voltage: 5.8 volts dc.

Steady State Coil Current: 384 milliamperes

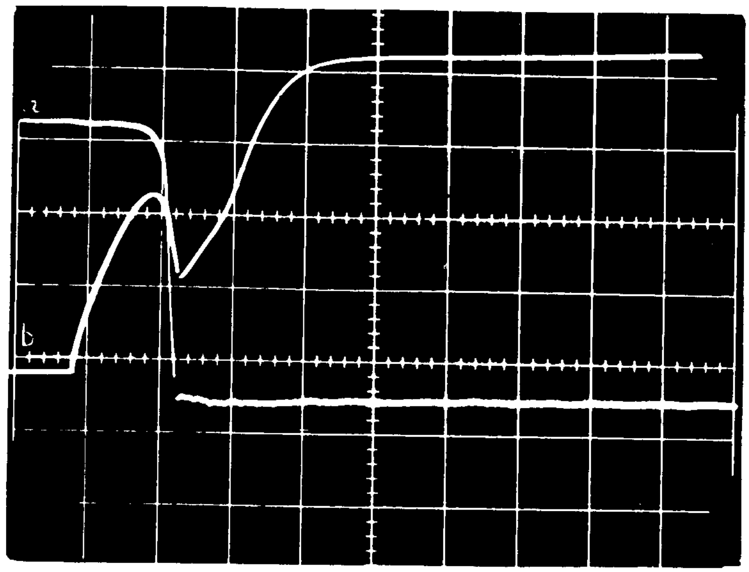


Figure 3

Traces:

(a) Armature displacement

(b) Coil current build-up

Oscillogram Data:

Time Scale: 10 milliseconds per centimeter

Current Scale: 190 Milliamperes per centimenter

Coil Voltage: 12.6 volts dc.

Steady State Coil Current: 835 milliamperes

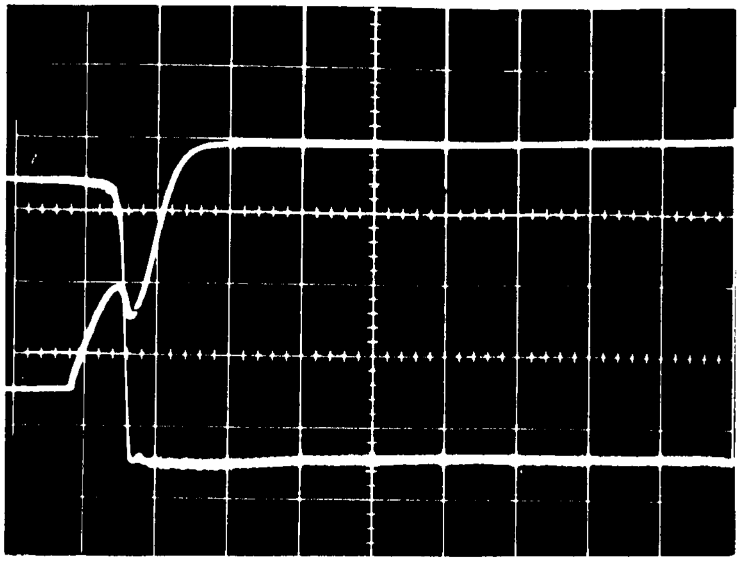


Figure 4

Traces:

(a) Armature displacement

(b) Coil current build-up

Oscillogram Data:

Time Scale: 10 milliseconds per centimeter

Current Scale: 490 milliamperes per centimeter

Coil Voltage: 26 volts dc.

Steady State Coil Current: 1720 milliamperes

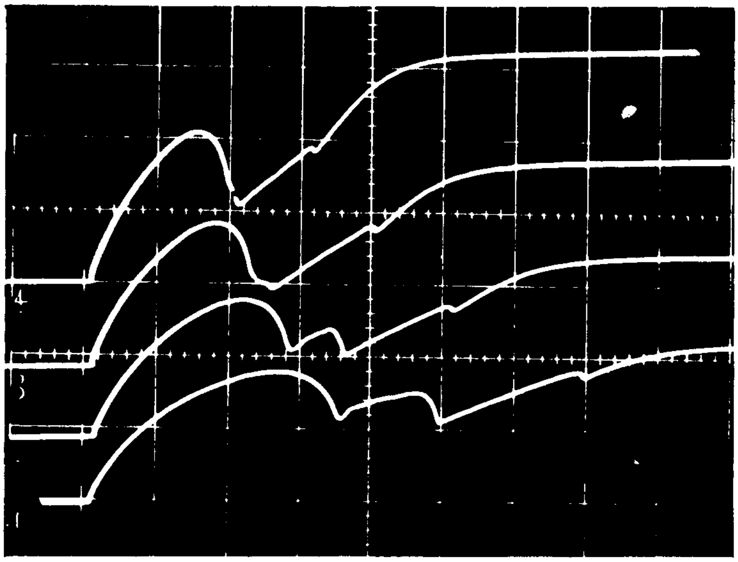


Figure 5

Coil Current Build-up

Traces:

(1) Coil Voltage: 6.4 volts dc.
Steady State Coil Current: 424 milliamperes

(2) Coil Voltage: 7.4 volts dc.
Steady State Coil Current: 490 milliamperes

(3) Coil Voltage: 8.4 volts dc.
Steady State Coil Current: 556 milliamperes

(4) Coil Voltage: 9.4 volts dc.
Steady State Coil Current: 623 milliamperes

Oscillogram Data:

Time Scale: 10 milliseconds per centimeter

Current Scale: 189 milliamperes per centimeter

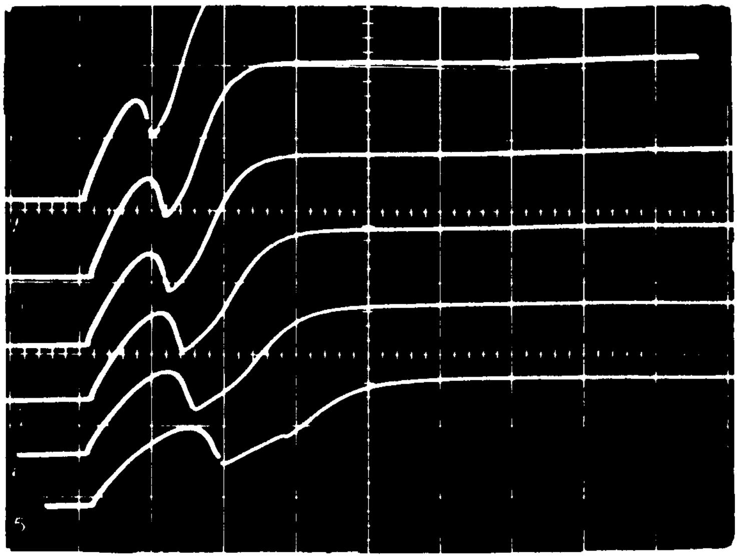


Figure 6

Coil Current Build-up

Traces:

(5) Coil Voltage: 10.5 volts dc.
Steady State Coil Current: 695 milliamperes

(6) Coil Voltage: 12.6 volts dc.
Steady State Coil Current: 834 milliamperes

(7) Coil Voltage: 14.8 volts dc.
Steady State Coil Current: 980 milliamperes

(8) Coil Voltage: 17.4 volts dc.
Steady State Coil Current: 1150 milliamperes

(9) Coil Voltage: 19.4 volts dc.
Steady State Coil Current: 1282 milliamperes

(10) Coil Voltage: 23 volts dc.
Steady State Coil Current: 1520 milliamperes

Oscillogram Data:

Time Scale: 10 milliseconds per centimeter

Current Scale: 427 milliamperes per centimeter

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Part B

Vibration

Tab Color Code

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Blue	2nd Interim	1 March - 30 April, 1962
Yellow	3rd Interim	1 May - 30 June, 1962

SECTION IV

VIBRATION TEST

Failure of a relay under severe vibration is a common problem. This investigation is being conducted with the following two goals in mind. First, a particular group of relays shall be tested and an attempt made to determine and correct the cause of failure for each individual relay. Second, it is hoped that the study of these relays will produce some design criteria (concerning vibration problems) for the class of relays in general.

The group of relays tested consisted of the following types:

(a) 25 amp, three sets of NO main contacts, one set NO and one set NC auxiliary contacts

(b) 50 amp, one set of NO main contacts, one set NO and one set NC auxiliary contacts

(c) 100 amp, one set of NO main contacts, one set NO and one set NC auxiliary contacts

(d) 200 amp, one set of NO main contacts, one set NO and one set NC auxiliary contacts.

Each type of relay was attached to the vibration table and checked for contact failure over the frequency range of 10 to 2000 cps. (The relays were energized at the rated coil voltage.) The following failures were noted:

(a) 25 amp relay - At a frequency of 390 cps, the center set of main contacts failed at 14 g, the outer sets failed at 40 g. No failure of the auxiliary contacts was noted at this frequency.

(b) 50 amp relay - At 1300 cps, the main contacts failed at 10 g. No failure of the auxiliary contacts was noted at this frequency.

(c) 100 amp relay - At 680 cps, the main contacts failed at 17.7 g. No failure of the auxiliary contacts was noted at this frequency.

(d) 200 amp relay - At 960 cps, the main contacts failed at 14 g. No failure of the auxiliary contacts was noted at this frequency.

It should be noted that relays of the same type were found to correspond as to the frequency at which failure occurred and varied only slightly in the level of acceleration required.

In view of the results of the first test it was decided to check on the possibility of armature motion while energized and its relation, if any, to the failures.

The 100 amp relay was chosen for the study of armature motion. Photographs were taken of the coil current and the contact voltage to obtain a permanent record of results.

Figure 1 shows the coil current and contact voltage of the 100 amp relay undergoing 17.7 g's at 680 cps. The upper trace is the contact voltage, the lower trace is the coil current. The coil voltage is 10 volts. The two traces indicate that opening of the contacts corresponds to the motion of the armature. Note that the contacts stay open longer every other time and this corresponds to a more extreme armature displacement.

Figure 2 shows the same relay under the same conditions, except that the coil voltage is increased to 28 volts. The coil current indicates less armature motion, but the contacts continue to open.

In Figure 3, the coil voltage has been raised to 50 volts. This has noticeably reduced the armature motion but seems to have little effect on the contact failure.

Another possible cause of contact failure is the flexing of the stationary contact mounts which pass through the case of the relay. In order to investigate this possibility, the mounting studs for the contacts were braced to the upper part of the relay case. The results are shown in Figures 4, 5 and 6.

In Figure 4, with 10 volts applied to the coil, the contacts are seen to open at a higher frequency (760 cps.) The armature motion is noticeably less than in Figure 1, which was without the braced mountings. The contacts no longer fail at 680 cps. as they did without the brace.

The same pattern of failure occurs in Figures 5 and 6 with the armature motion becoming less as the coil voltage is increased.

The result of bracing the contacts then seems to be a reduction of armature motion and a change in the frequency at which failure occurred.

A second 100 amp relay was tested with the contacts braced, with the result shown in Figure 7. With 28 volts applied to the coil, there seems to be very little armature motion, although the contacts are opening.

Figure 8 shows the same relay with the brace removed and 10 volts applied to the coil. The coil current indicates a much greater motion of the armature. The failure frequency has returned to the 680 cps. as was the case in Figure 1.

The result of increasing the coil voltage to 50 volts is shown in Figure 9. The armature motion is reduced with no apparent affect on the contacts.

A 25 amp relay was tested with a blocked armature. The effect of blocking the armature was only to change the frequency at which the contacts opened. This seems to indicate that the problem is not the armature but with the contacts themselves. A series of test to investigate the contacts and their mountings is now underway. Only one permanent failure was noted in these tests. This took place on the 50 ampere relay during the vibration test. The NC auxiliary contacts broke loose from the mounting which was detected after the vibration test was completed.

From information of tests conducted at NASA, 3 out of 10 relays tested failed in the same manner. This seems to indicate that the auxiliary contacts need more bracing.

The results of the test performed to date are inconclusive, but it is hoped that with the results of additional tests, a clear picture of the cause of contact failure on these relays can be established.

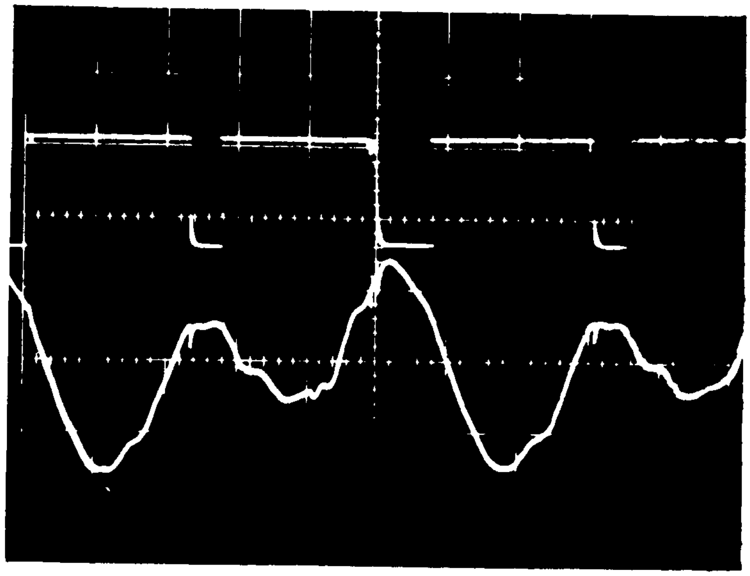

Figure 1

Traces:

Top trace: Contact Voltage

Lower trace: Coil Current

Oscillogram Data:

Relay - 100 amp #1

Contacts - NO (main)

Coil Voltage - 10 volts

Time Scale - .5 ms per cm

Current Scale - 1 ma per cm

Contact Voltage - 20 volts

Frequency - 680 cps

Acceleration - 17.7 g (rms)

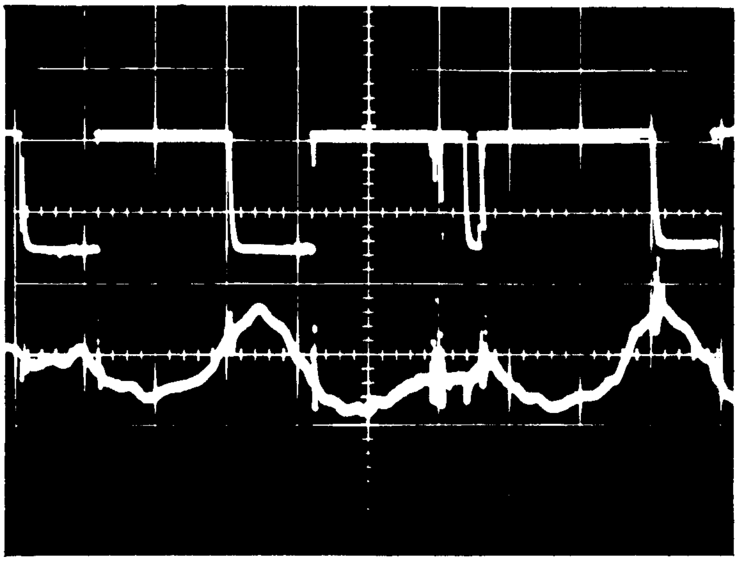

Figure 2

Traces:

Top Trace: Contact Voltage

Lower Trace: Coil Current

Oscillogram Data:

Relay - 100 amp #1

Contacts - NO (main)

Coil Voltage - 28 volts

Time Scale - .5 ms per cm

Current Scale - 1 ma per cm

Contact Voltage - 20 volts

Frequency - 680 cps

Acceleration - 17.7 g (rms)

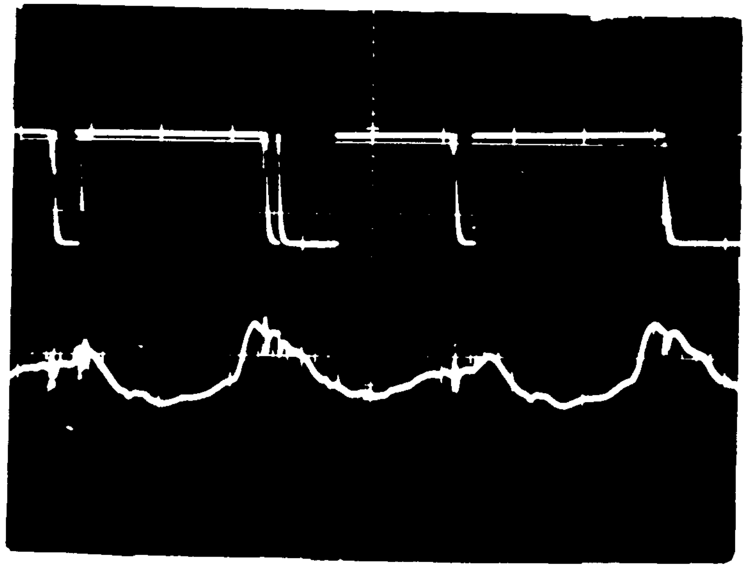

Figure 3

Traces:

Top Trace: Contact Voltage

Lower Trace: Coil Current

Oscillogram Data:

Relay - 100 amp #1

Contacts - NO (main)

Coil Voltage - 50 volts

Time Scale - .5 ms per cm

Current Scale - 1 ma per cm

Contact Voltage - 20 volts

Frequency - 680 cps

Acceleration - 19.8 g (rms)

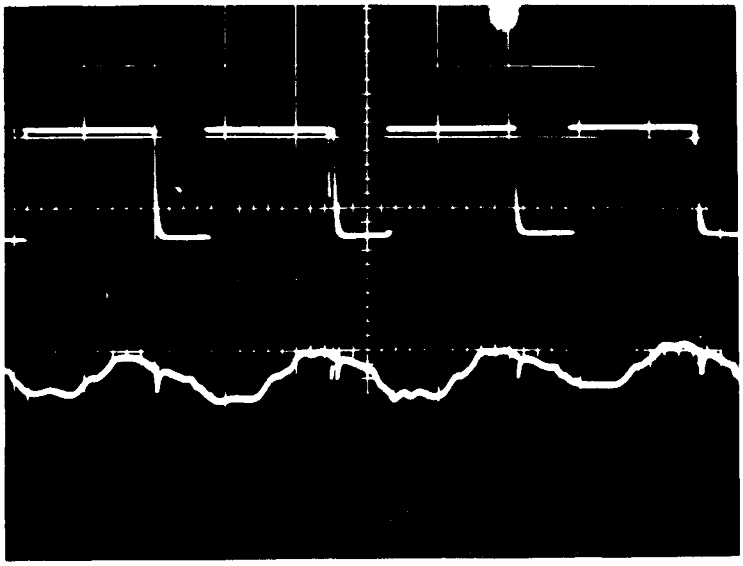

Figure 4

Traces:

Top Trace: Contact Voltage

Lower Trace: Coil Current

Oscillogram Data:

Relay - 100 amp #1

Contacts - NC (main)

Coil Voltage - 10 volts

Time Scale - .5 ms per cm

Current Scale - 1 ma per cm

Contact Voltage - 20 volts

Frequency - 760 cps

Acceleration - 17.7 g (rms)

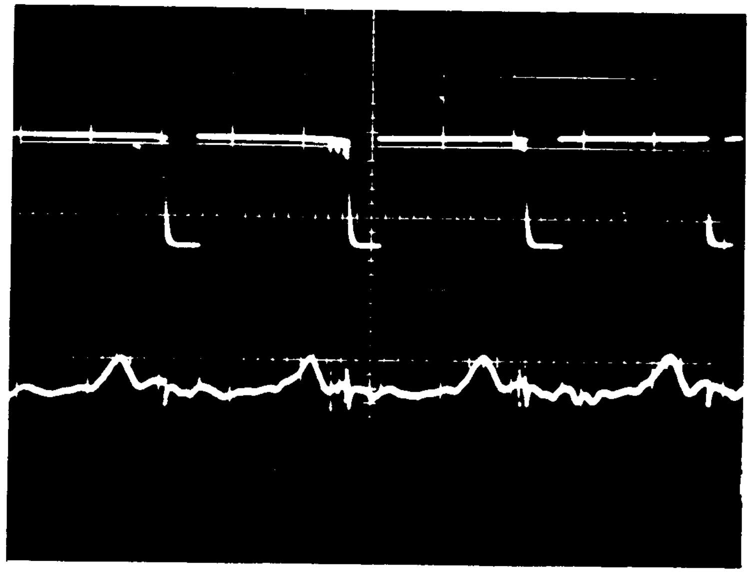

Figure 5

Traces:

Top Trace: Contact Voltage

Lower Trace: Coil Current

Oscillogram Data:

Relay - 100 map #1

Contacts - NO (main)

Coil Voltage - 28 volts

Time Scale - .5 ms per cm

Current Scale - 1 ma per cm

Contact Voltage - 20 volts

Frequency - 760 cps

Acceleration - 17.7 g (rms)

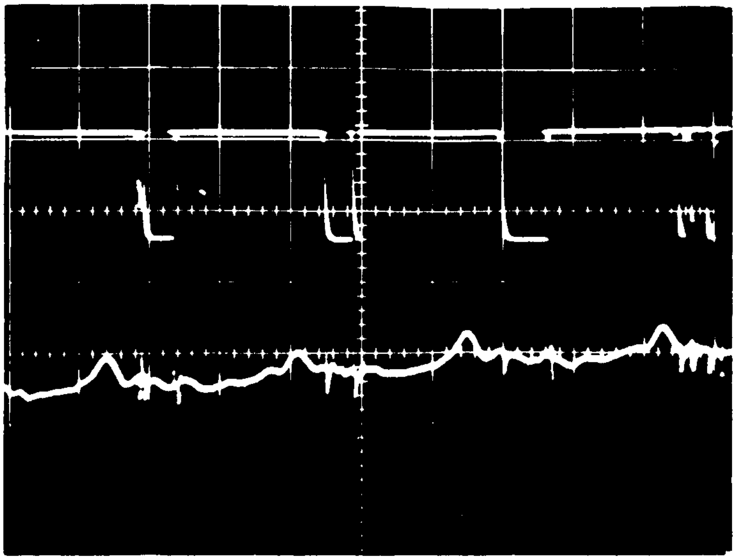

Figure 6

Traces:

Top Trace: Contact Voltage

Lower Trace: Coil Current

Oscillogram Data:

Relay - 100 amp #1

Contacts - NO (main)

Coil Voltage - 50 volts

Time Scale - .5 ms per cm

Current Scale - 1 ma per cm

Contact Voltage - 20 volts

Frequency - 760 cps

Acceleration - 17.7 g (rms)

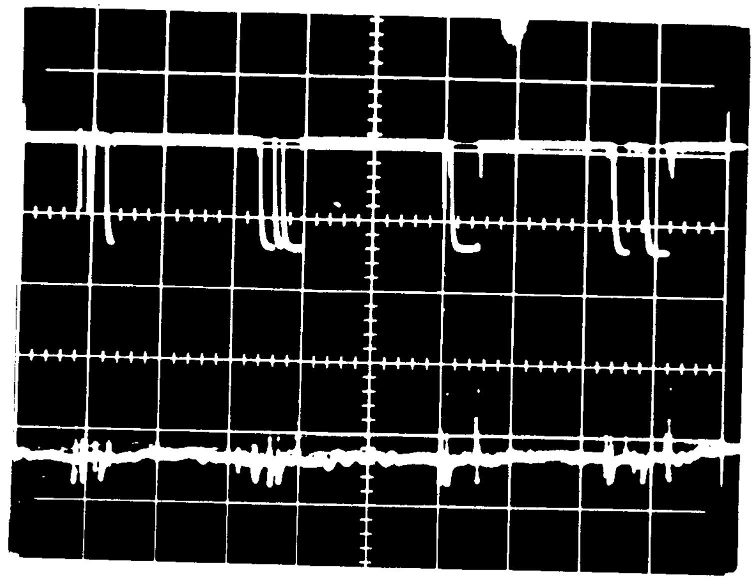

Figure 7

Traces:

Top Trace: Contact Voltage

Lower Trace: Coil Current

Oscillogram Data:

Relay - 100 amp #2

Contacts - NO (main)

Coil Voltage - 28 volts

Time Scale - .5 ms per cm

Current Scale - 1 ma per cm

Contact Voltage - 20 volts

Frequency - 750 cps

Acceleration - 17.7 g (rms)

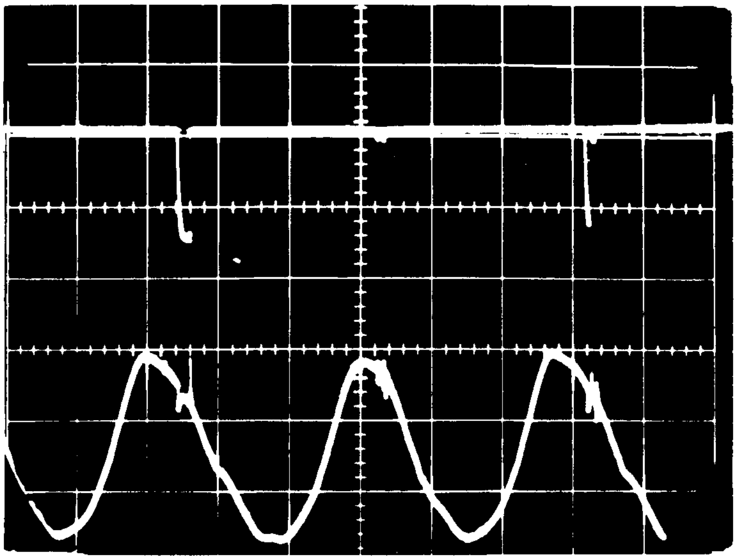

Figure 8

Traces:

Top Trace: Contact Voltage

Lower Trace: Coil Current

Oscillogram Data:

Relay - 100 amp #2

Contacts - NO (main)

Coil Voltage - 10 volts

Time Scale - .5 ms per cm

Current Scale - 1 ma per cm

Contact Voltage - 20 volts

Frequency - 680 cps

Acceleration - 14.1 g (rms)

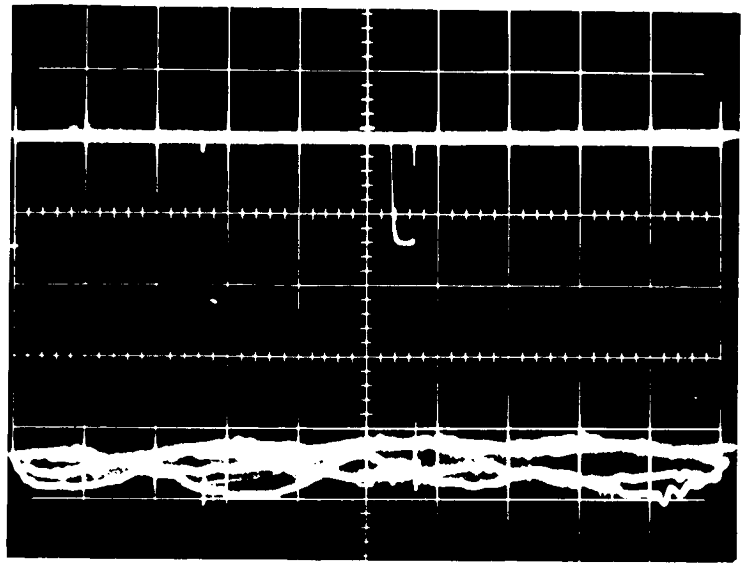

Figure 9

Traces:

Top Trace: Contact Voltage

Lower Trace: Coil Current

Oscillogram Data:

Relay - 100 amp #2

Contacts - NO (main)

Coil Voltage - 50 volts

Time Scale - .5 ms per cm

Current Scale - 1 ma per cm

Contact Voltage - 20 volts

Frequency - 680 cps

Acceleration - 14.1 g (rms)

VIBRATION TESTING

In order to have a more logical procedure to follow in the search for the cause of separation of contacts when the relays under consideration are subjected to extreme vibration, the contact system was examined to determine all the possible causes of separation. The system under consideration is shown in Figure 1.

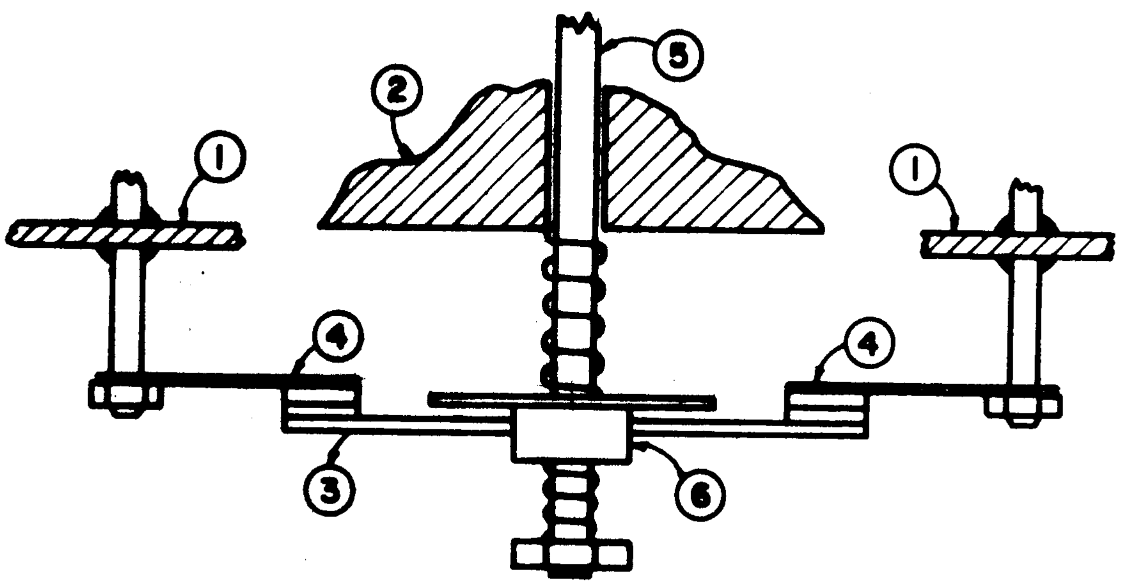

Figure 1. Relay Contact System

The possible causes of separation of the contacts are listed below:

1) Motion of point 1 with respect to point 2

2) Motion of point 5 with respect to point 2

3) Motion of point 6 with respect to point 5

4) Motion of point 4 with respect to point 1

5) Motion of point 3 with respect to point 6

Consider cause number one. Any movement of point 1 with respect to 2 would be a result of flexing the case enclosing the relay. This is definitely a possibility on the relays tested. If experimental evidence does show the case to be flexing to a harmful degree, a relocation of the mounting bracket to the center of the case would be a possible solution to the problem.

If the armature were to move with respect to the coil (cause number two), the contacts could easily open. This possibility has previously been investigated on several relays and the evidence obtained to date seems to justify the elimination of this cause from consideration for the present.

The movable contacts are mounted on a bar which is allowed to move on the armature shaft in order to provide some armature overtravel. This bar is restrained by two springs. It seems very likely that this arrangement could produce a separation of the contacts at the resonant frequency of the spring and mass system. This cause will be discussed at greater length later in the report.

The stationary contacts, being mounted as long cantilevers, are very susceptible to vibrations. Any extreme motion caused by flexing of the mounting or in the bar itself could possibly open the contacts. This cause is also considered worthy of some investigation.

A flexing of the movable contact bar itself is considered unlikely because of the rigidity of this particular part.

The list of possible causes has now been reduced to three. Of these three, the most likely cause is believed to be number three; therefore, this was the next topic to be investigated. It should be pointed out that the failure is not necessarily due to one condition alone but could be a result of several conditions.

Investigation of overtravel spring system

It was felt that the one characteristic that would have the greatest effect on the contact failure was the motion of the movable contact bar with respect to the armature shaft. In order to check on this possibility, two relays were opened by sawing a small round hole in the base such that the adjusting nut on the end of the armature shaft could be reached. Both relays were then checked for failure at several spring tension adjustments. The results are as follows:

100 amp relay #1

With the original manufactures adjustment of the spring system the main contacts failed at a frequency of 830 cycles per second. The required R.M.S. acceleration level was 15.5G. This was the only frequency at which any failure was noted. Figure 2 shows the opening of the contacts (top trace) and the exciting current of the vibration table (lower trace). The picture was steady on the oscilloscope as it appears in the figure.

The spring was loosened approximately four turns of the adjusting nut. The results are shown in figures 3 and 4. The failure is continuous over the entire frequency range of 20 to 2000 cps. Very low values (3 to 9 G) of acceleration were required.

Increasing the spring tension by about one turn yielded the failure shown in figure 5. Note that 21G is required to open the contacts and the frequency has shifted about eighty cycles. The change in frequency is attributed to the change in the spring constants as the loading is increased.

Another increase in the spring tension (one turn) resulted in reducing the failure to zero. A second increase did not change the result. In other words, the increase in spring tension stopped the opening of the contacts up to at least 20G. The "at least" is used because this was the maximum acceleration available at that frequency.

50 amp relay #1:

With the original spring adjustment the relay was observed to fail at 810 cps (6G) and at 1250 cps (14G). This is shown in figures 6 and 7. With the spring tension nut tightened one turn, failure was noted at 890 cps (8G) and 1200 cps (30G). (See figures 8 and 9). Note that the frequency of both failures was changed but the required G level was increased only for the 1200 cps failure. The 800 cps failure was an intermittent failure and could be started or stopped by tapping the case with a pencil.

Decreasing the tension (2 turns) lowers the required acceleration level required to 8.5 and 6G. (Figures 10 and 11). The frequencies are again changed because of the non-linearity of the system.

100 amp relay #2

This relay was not opened but was tested to show that the failure frequency corresponded to the other 100 amp relay. In figure 12 it can be seen that the contacts are separating at 800 cps at an acceleration level of 10.5G. This should correspond to the fundamental frequency of the contact system. Figure 13 shows the contact voltage at 1600 cps corresponding to the second harmonic. Note that a higher G level is required as would be expected. The other 100 amp relay tested did not fail at 1600 cps, however the G level required at 800 cps was much higher; and it is assumed that the equipment was not capable of producing the acceleration required at 1600 cps to separate the contacts.

The results of these tests seem to indicate that the present problem of failure is the result of an improper adjustment of the overtravel and back tension springs. The next planned study will be to verify more completely the results of this test, and then to proceed with the formulation of the necessary relationships in order that the design of this system of contacts can be incorporated into the already existing relay design procedure.

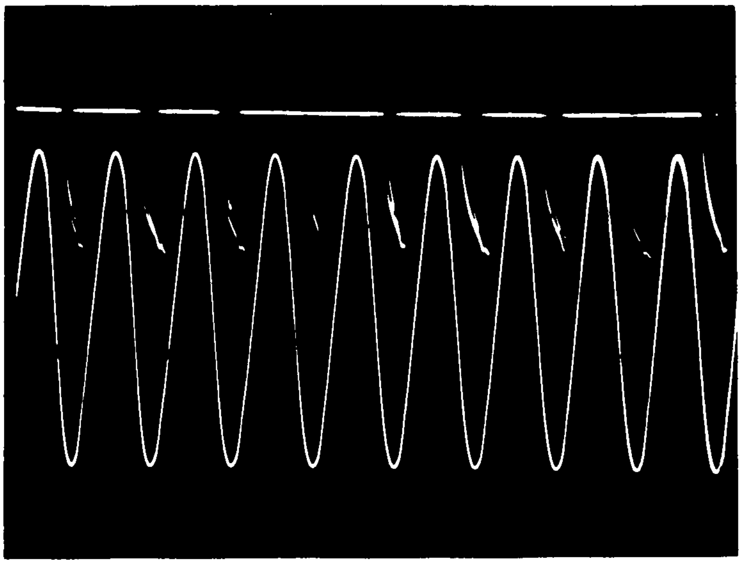

Figure 2

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 100 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 830 cps

Acceleration - 15.5 G (rms)

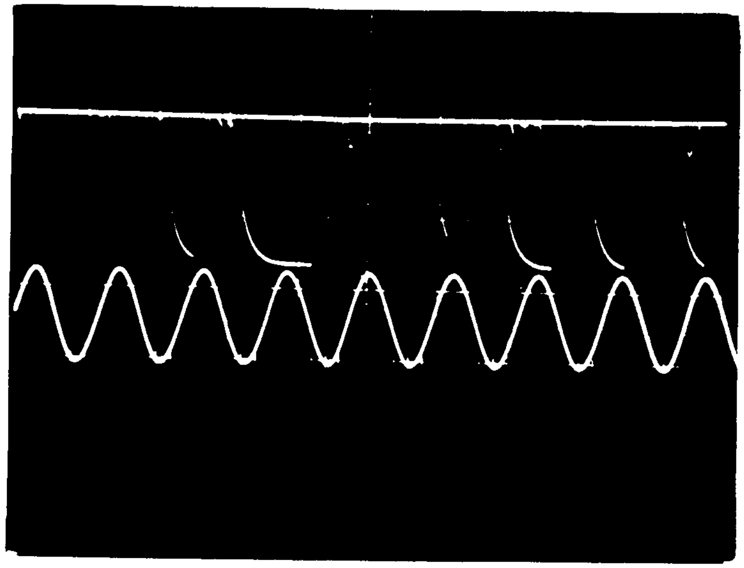

Figure 3

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 100 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 795 cps

Acceleration - 8.5 G (rms)

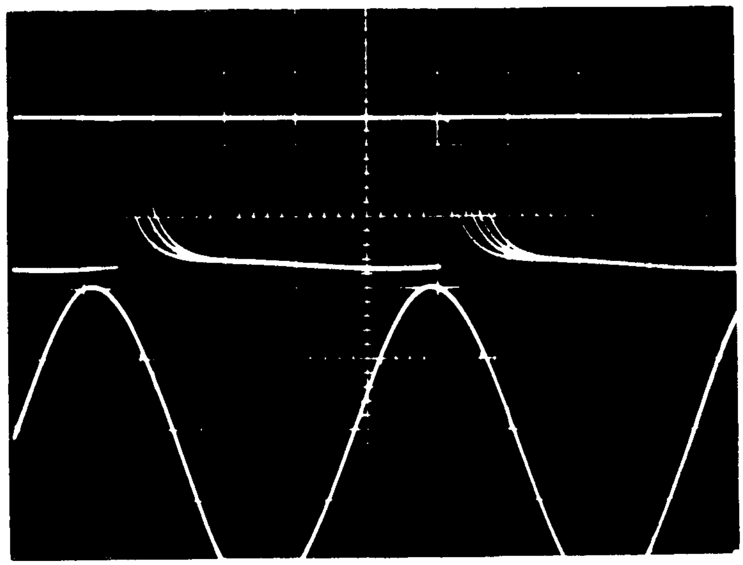

Figure 4

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 100 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 385 cps

Acceleration - 4.5 G (rms)

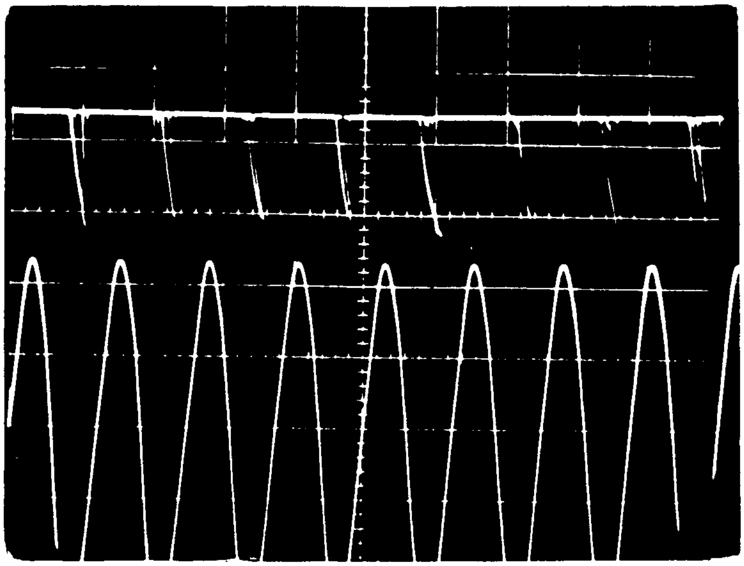

Figure 5

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 100 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 750 cps

Acceleration - 21 G (rms)

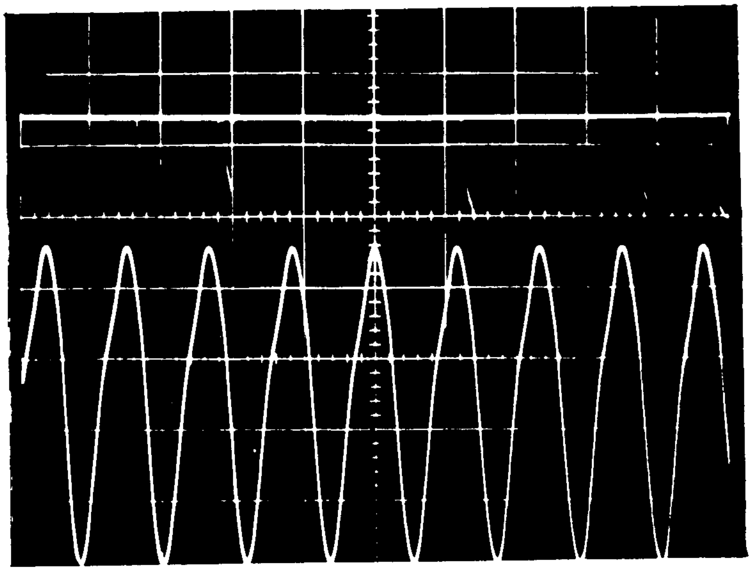

Figure 6

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 50 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 810 cps

Acceleration - 6.4 G (rms)

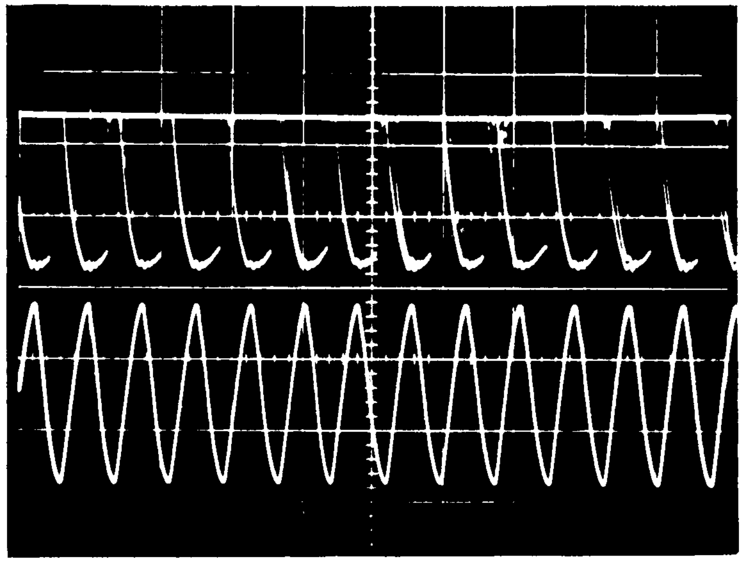

Figure 7

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 50 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 1250 cps

Acceleration - 14 G (rms)

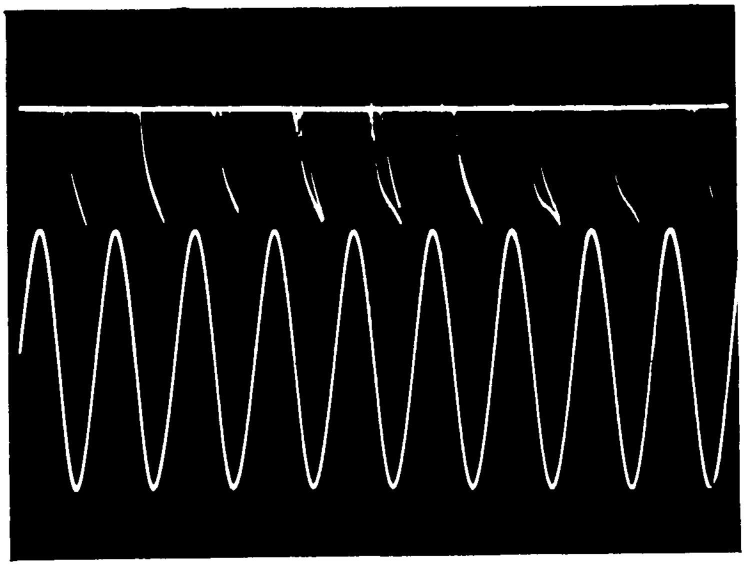

Figure 8

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 50 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 890 cps

Acceleration - 5.5 G (rms)

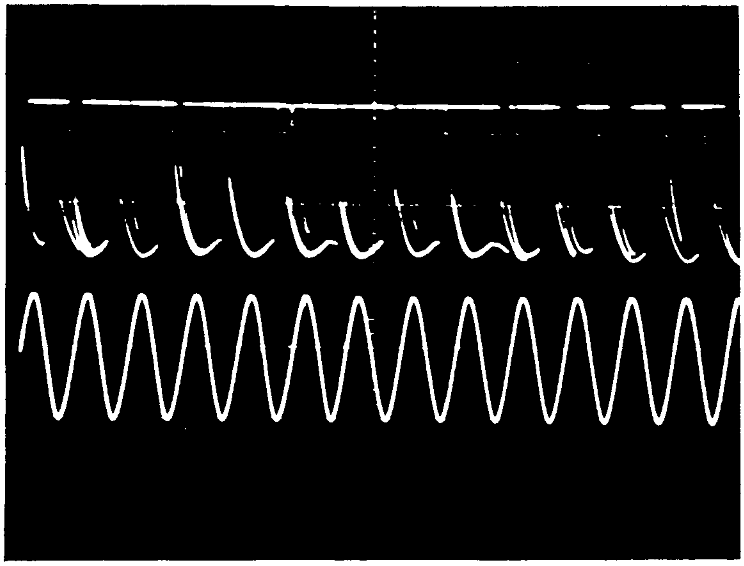

Figure 9

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 50 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 1200 cps

Acceleration - 21 G (rms)

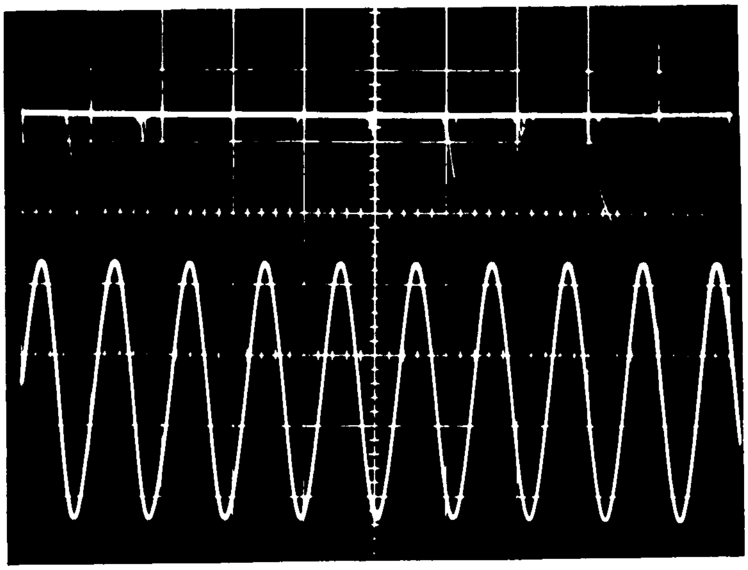

Figure 10

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 50 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 890 cps

Acceleration - 8.5 G (rms)

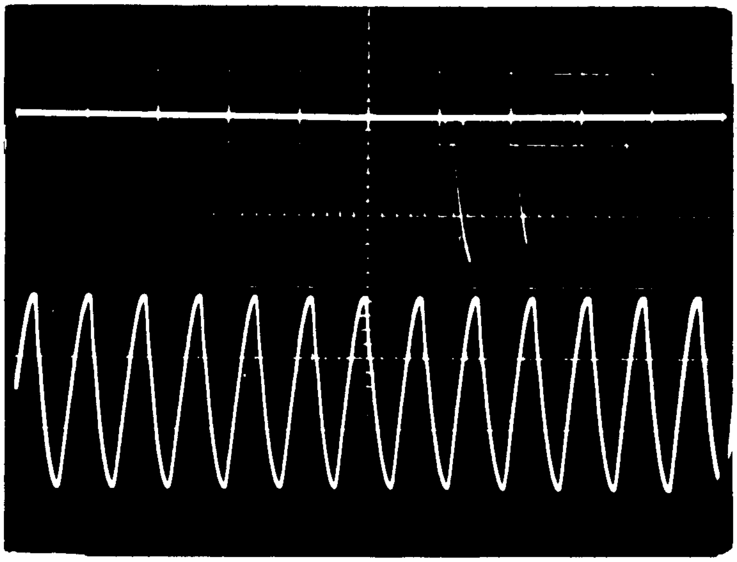

Figure 11

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 50 amp #1

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 1200 cps

Acceleration - 5.5 G (rms)

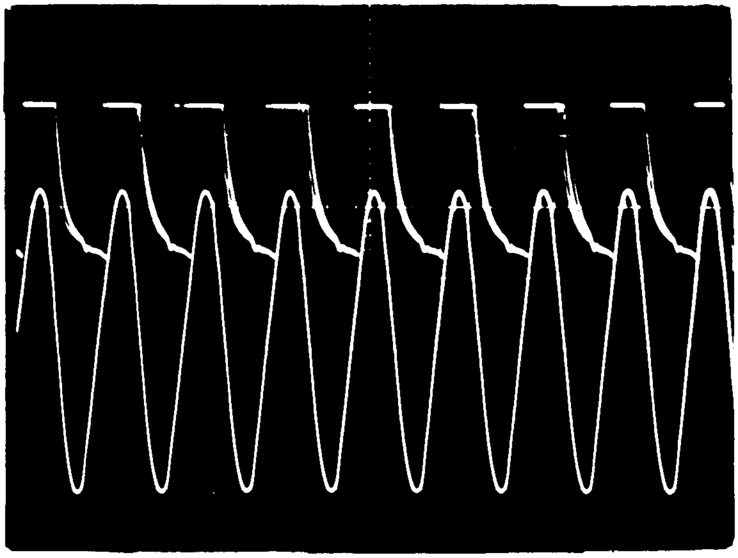

Figure 12

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 100 amp #2

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 800 cps

Acceleration - 10.5 G (rms)

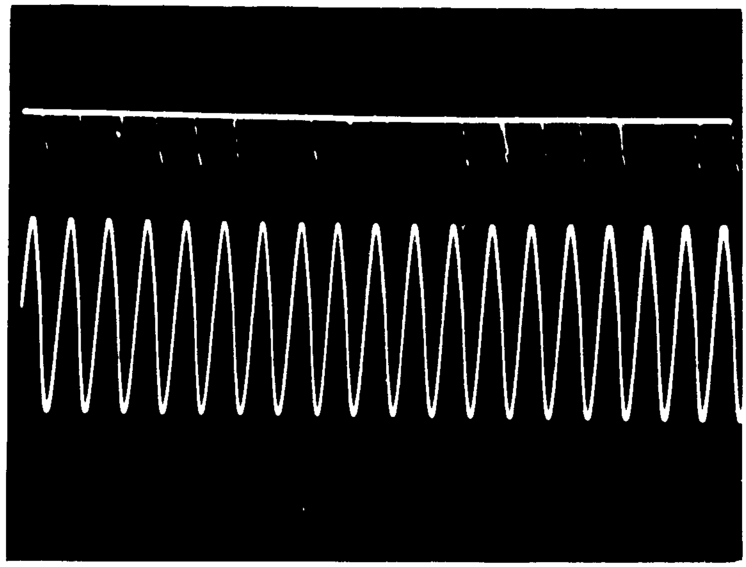

Figure 13

Traces:

Upper Trace: Contact Voltage

Lower Trace: Exciter Current

Oscillogram Data:

Relay - 100 amp #2

Contacts - NO (Main)

Coil Voltage - 28 volts

Time Scale - 1 ms per cm

Contact Voltage - 10 volts

Frequency - 1600 cps

Acceleration - 19 G (rms)

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Contact Study

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SECTION III

PRELIMINARY INVESTIGATION AND PROPOSAL OF RELAY CONTACT DESIGN

This section is concerned with some problems dealing with the design of a relay contact system, given the specifications. In particular, the type of contact systems of immediate interest are of the heavy duty (current) type. However, in order to arrive at a design procedure for these types, a more general discussion is needed at this time due to the lack of information concerning contact design.

The first portion of this section, Part I, is a discussion of some design terminology which is frequently used but seldom defined. Some definitions are given with the intent of adding clarity to discussions in subsequent reports. Also, some problems related to these definitions are discussed.

The second phase of this section, Part II, deals with the particular type of relay, to be evaluated and re-designed under the present research contract. The discussion is limited to the contact system and the specifications which will govern their design. The requirements for the mechanical design and electrical design are separated, and the preliminary investigation of these factors is given. Some oscillograms of particular electrical loads are given at the end of this section in connection with this initial evaluation report.

The final topic to be presented, Part III, is a proposal directed at the problem of designing contacts to satisfy the electrical load requirements. Two basic assumptions are presented with the intent of obtaining a single parameter with which to relate duty cycle, type current load, relay discharge time and obtain the probable number of operations to failure due to electrical properties.

PART I

The contact design problem is difficult for many reasons. One of these reasons is because of the lack of methods and communication for the design process itself. The following discussion is intended to give more concrete definition to some of the basic concepts used in design. The following ideas are defined in terms of the quantities, system, criteria, parameter, relationship and restricted. Design: The construction of a system based on criteria will be called design. (This will be denoted by the design of (S) when referring to a particular system.)

Set of Specifications: A collection of criteria (denoted by $[c_i]$. and a collection of parameters (denoted by $[P_J]$) is said to form a set of specifications (denoted by $[S_r]$) if:

(i) For each criteria $[c_i]$ there is a relationship (denoted by f_i) such that, $f_i([P_J])$ restricts a subset $[P_J]$. (If this restricted set is denoted by $[S_P]_i$ then $f_i([P_J]) \longrightarrow [S_P]_i$ can be used to denote (i), where $\longrightarrow$ stands for implies.) The set $[S_r]$ is the totality of the restricted parameters.

The above definition emphasizes the complexity involved, of taking a requirement for design and obtaining a set of specifications. The undefined quantities: parameter and restricted, are usually well understood for any particular case. For example, physical quantities, (voltage (E), time (t), temperature (T), etc.), are very commonly used as parameters. Restricted, for many cases is defined as; assigning a value or range of values to a parameter. The more difficult problem is that of selecting the set of parameters, and the relationships, from the given criteria, which in turn restrict the parameters. Many examples could be given in which this can be easily done, but for the most part this is a difficult problem due to the nature of the set of criteria. Part II is an illustration of the problems involved in this type criteria.

Design Process: A process which uses a fixed set of criteria and yields a design for (S) will be called, a design process.

Best Design: A best design is any system (S) which has the following properties.

(i) The design of (s) was a design process

(ii) The criteria for the design process forms a set of specifications

(iii) The system has a set of parameters, a subset of these being the same as the set of specifications in (ii).

This definition is nothing more than a formal statement of the common conception usually associated with this idea. That is, a best design produces a system which has all the properties which initiated the design. Note, however, that this definition disallows variable criteria when discussing the best design, and parameters belonging to the system which do not belong to the set formed by the fixed criteria. Also, note that a best design is not necessarily unique. Although a best design in each design problem would be the ultimate, this does not appear to be the actual situation. For this reason the following definition appears to be more useful when evaluating designs which have no evaluating criteria given.

Better Design: Let two designs, say d_1 and d_2, be such that the same criteria is used in the design of d_1 and d_2, and for any set of specifications $[S_r]$ formed by the criteria for d_1 and d_2 are not both best designs. Then d_1 is said to be a better design and d_2 if the parameters of d_2 (denoted by $[P_2]$) and d_1 (denoted by $[P_1]$) have the following property. $[P_2] \cap [S_r] \subset [P_1] \cap [S_r]$ where $A \cap B$ denotes, parameters common between A and B, and $A \subset B$ denotes that B has all the parameters of A plus some more.

This definition allows two designs to be compared assuming that not both are best designs. The comparison is a matter of seeing which design comes closest to a best design relative to a common basis. The common basis is most important since without this property the comparison of two designs becomes arbitrary when no comparison criteria is included in the criteria for design.

Before leaving Part I it is mentioned again that the preceeding discussion is only intended to point out some of the main problems concerned with initiating and terminating a design. The discussion of best and better designs indicates reasons for the many different opinions relating to a good design. Although these opinions many times have a good motivation seldom can they be used by the designer until a system has been designed. Even though the definitions were given in a general form it is felt that to have a basis for ideas involved in a problem is very useful. Also, since converting from a general case to a particular case is much easier than the converse problem it is hoped that the satisfactory solution to the problem at hand will be enhanced by the above structure.

PART II

As mentioned in Part I it is usually a difficult task to take a set of criteria and form a set of specifications. Also, except for a few cases a design process which has well defined steps for producing a system is available. Although the objective of this study is to obtain a better design for a particular system the above problems enter into the realization of this objective. This is the case since no design process is available with which to handle the given criteria, and the set of specifications formed by the given criteria has not been determined.

In this section the particular system for design is the contact system for a relay contactor. The given criteria can be stated as follows:

1. The contacts are to function properly (electrically) with RMS acceleration up to 20 g's from 10-2000 cps having chatter time less than 10^{-4} sec. along the three major axis.
2. The electrical load can be handled for a given life (number of operations), duty cycle, and ambient temperature range.
3. The following parameters have an upper limit (i), volume, (ii) weight.
4. The parameter, dielectric strength, has a lower bound.
5. The order of the above criteria is to be used in comparing any two designs which are not best designs.

A problem which is an immediate consequence of the above criteria is that of finding a set of parameters with which to form a set of specifications using the above criteria. The first criteria (1) likely implies that; the masses (M_j) of the contact arms, plunger, and armature; the back tensions (P_o) of the N.C. contacts and armature; the spring constants (K_i); the magnetic pull (F) on operate; and the geometry of the contact system are a sufficient set of parameters which if restricted properly in a contactor will satisfy criteria (1). This set then will be used to try and determine a proper restriction relation. The investigation of this relation to date has not yielded a satisfactory expression with which to predicate failure caused by chatter knowing the above parameters. From experimental tests however, one design which is being observed failed to meet specification (1). This was reported in "Special Test Data for the #6042H32, 25 Amp Relay" conducted by M-ASTR-EC. Therefore a redesign of the above relay to meet (1) keeping all other properties the same would be a better design.

Specification (2) is not as easily broken into a set of identifiable parameters as was (1). Also a procedure for relating the parameters involved in this set of specifications into some usable relationships has not been developed at the present time. In this particular design investigation the problem is compounded by the variable nature of the electrical load and duty cycle. The electrical load has been tentatively divided into the following specifications.

(a) Maximum inrush current $\leq$ twice rated current with a specified time interval.

(b) For inductive load the time constant must be specified along with rated current.

(c) The type of circuit voltage (AC, DC) and desired value should be specified for all loads.

(d) For resistive load the rated current should be specified.

Although no definite relationships have been found with which to handle the specifications of (2) in an analytical manner the following investigations have been initiated.

A. In order to handle the stady state (or rated) value of contact current, assume an ambient and maximum temperature distribution for the contact structure, and develop a set of relationships which can predict this distribution. The following parameters are desirable to belong to these relationships.

(1) I_c = current through contacts

(2) $R_c = \frac{V_c}{I_c}$ = loaded contact resistance

(3) Contact dimensions

(4) Ambient temperature and maximum allowable temperature distribution

(5) Contact material (mass desity, specific heat)

B. In order to design for inductive loads and related arcing problems a relationship involving arc time, arc energy temperature, and some of the parameters of (A) are being investigated.

C. The contact life is being studied from a statistical approach with the intent of obtaining some correlation among the following factors.

(1) number of operations

(2) arc energy

(3) contact volume

(4) duty cycle

(5) contact electrical and mechanical properties.

In the above indicated investigations, one important factor which influences the results has been omitted. This factor is that of, contact failure which is caused by electrical loading and mechanical wear. Unless contact failure is defined, any relationship describing contact characteristics would be hard to apply in designing for satisfactory operation. This problem is discussed in Part III. Before leaving the discussion of the particular re-design problem under investigation the experimental work on electrical loading for a particular design is presented.

The oscillograms of figures (1) to (8) give the voltage current characteristics of a particular set of contacts undergoing controlled loading. The contacts used were rated at 25 amperes. The oscillograms of figures (1) to (4) are characteristics of contact voltage and current during make and break using a resistive load and approximately one half rated current and 24 vdc. (Note that the apparent negative current for these traces was due to the high gain needed when using a small series resistance to obtain the current trace. This problem was corrected for the remaining characteristics shown.) The observed characteristics for these contacts under resistive load using the duty cycle 11 sec. on - 4 sec. off, after 4000 operations as compared to the original characteristics, was essentially unchanged, if not improved from the standpoint of arcing on release. The temperature increase of the contact terminals over that produced by the coil was not measurable at room temperature. The contact resistance under load also had no apparent change. After this period of testing the load was changed to an inductive load keeping the duty cycle as before. Oscillogram (5) gives the contact voltage and current on release when the relay was cold (ten operations) at approximately one half rated current and 24 v dc. The blow-out of the arc is clearly marked by the impulse of voltage which accompanies this process in inductive circuits. Note, however, that the current does not extinguish until the current decays to near zero. This causes the arc time to be of the order 200-300 times greater than for the resistive case. The arc energy calculated as

$$A_E = \int_0^{T_o} V_c I_c \, dt$$

can be compared for these two cycles and different type loads using the information given in oscillograms (3) and (5). This was done using V_c = 12 volts, I_c = 4 amps, T_o = (.06)(.25)ms for the resistive case. For the inductive case increments of 2 ms were used and the integral computed numerically using mean current and voltage during each interval.

A_E for resistive load = $(7.2)10^{-3}$ watt-sec.

A_E for inductive load = $(1451)10^{-3}$ watt-sec.

Due to the large differences in the above arc energies the relay was cycled 250 and 750 operations using the duty cycle mentioned earlier. Oscillogram (7) and (8) give the characteristics at the end of the above periods respectively. The operate characteristics are also given in these last two figures. The one noticable feature between figure (6) and (7) or (8) is the arc time has changed by 10 ms which is $\approx 75\%$ of that in (6). This checks close to value of % increase in resistance due to heating which is $\approx 80\%$ between figure (6) and (7) or (8). This suggests a representation is possible involving E, Rt, and L which could be used to compute the arc energy for an inductive load knowing the above parameters. This proposal is discussed further in Part III. Although no apparent changes in the contact characteristics are evident with the oscillograms shown, the temperature of the contact terminals was markedly increased ($\approx 240^{\circ}F$) over the resistive load. The contact resistance had no noticable increase with the above tests.

In order to obtain an upper limit on duty cycle at rated load, the inductive load was used but the duty cycle was changed to 5 cps with 120 ms on - 80 ms off. The contact characteristics were monitored on the oscilloscope and at $\approx (5\text{-}6)10^3$ operations the arc became intermittently unstable (failed to extinguish each time). At $\approx 10^4$ operations the arc would not extinguish when the contacts were open. This test was repeated on a different set of contacts with similar results. The contact terminals heated to a temperature of $\approx 400^{\circ}F$ in 10^3 operations under this load-duty cycle combination. Also, once the contacts were subjected to this amount of arcing, the arcing characteristics appeared to be permanently changed. That is, when allowed to cool to normal room temperature the test was repeated and continuous arcing was experienced at random during the first 10^2 operations.

These results have led to investigating the ideas presented in Part III.

PART III

Many definitions could be used for contact failure, but one which could be applied usefully to all design problems is, indeed, difficult. The following discussion is directed at obtaining a single definition for contact failure which could be used for a wide class of relays independent of relay duty. If this could be done satisfactorily, then the remaining problem would be that of relating types of contact duty to the failure condition.

At the present time many types of contact failures are defined, some of the more common definitions being related to:

(1) contact resistance

(2) welding together

(3) melting away

(4) voltage breakdown

Failures defined in terms of (1) are usually given in order to prevent the positive type failures of (2) and (3). Very seldom is the actual contact resistance of importance in the contact circuit aside from its influence on the reliability of controlling a circuit in a predetermined manner. However, since contact resistance is easily measured, and for any load a value can be computed for which the contact resistance must stay below, then a definition in terms of contact resistance is practical from the users point of view. Assuming then, that value of contact resistance for failure can be specified, how can the designer use this information without extensive testing in order to design the contacts? The following proposal is made with the hope of being able to obtain a contact design procedure which will help answer the above question.

Assume that for a chosen contact material, contact pressure and contact volume, the P_r[contact resistance $> R_o$] = f[total average arc energy Y_A].

That is, the probability that the contact resistance is less than some specified amount is a function of the total average arc energy. Average arc energy will be defined as:

$$A = \frac{1}{T_o} \int_o^{T_o} V_c I_c dt$$

where: V_c = voltage during arcing

I_c = current during arcing

T_o = time of arcing.

Then total average arc energy is given by:

$$A_T = \Sigma A_j \qquad j = 1, 2, \text{---} h = \text{number of arcing periods.}$$

Also, for a given contact circuit voltage, initial air gap, gap media, contact volume, and contact material, the P_r[break down voltage $< V_o$] = g [total average arc energy]. That is, the probability that the break down voltage is less than some specified amount is a function of the total average arc energy.

If the above proposals can be justified and the relationships found to relate these quantities the contact design problem will be greatly simplified. One of the main advantages in having the above type relationships is that of having a common basis by which to compute the number of probable operations until failure, as a function of load. That is, if arc energy can be used to predict failure then expressions for most loads (resistive, inductive, capacitive) can be obtained which give this energy. This in turn would allow the user to calculate the probability of failure for a given relay design knowing his load and duty cycle.

In concluding this section the evaluation of the contact system for the relay under test is as follows.

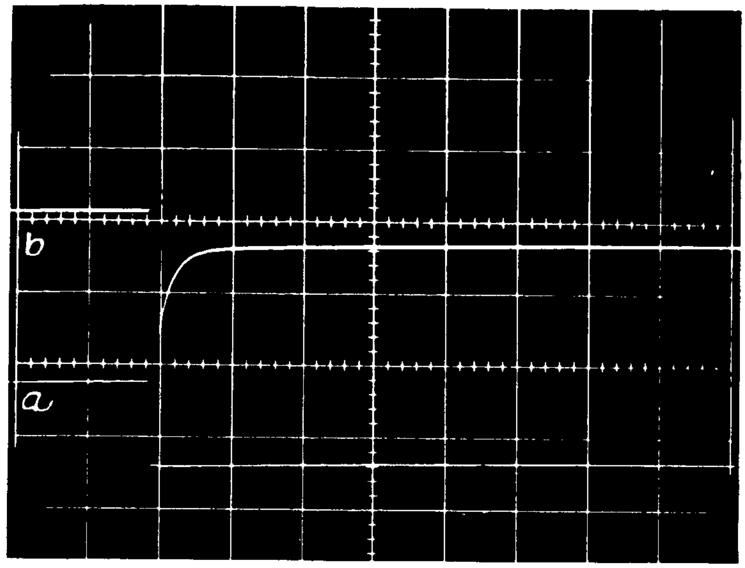
1. The vibration criteria is not satisfactory (this was not tested at OSU but at NASA as mentioned earlier).

2. The electrical load capabilities of the contact system can not be judged except that one load-duty cycle combination was found which produced continuous arcing. The resistive load-duty cycle operation produced no noticable harmful effects. The problem of rating the contacts satisfactorily hinges on finding failure relationships independent of the load condition.

Principle investigations being carried on in connection with the above observations are:

1. Investigating the mechanical system dynamically (lumped and distributed approach) in order to correlate vibration failure to the spring, mass, force, and geometry of the contact and armature system.

2. Investigating the arc energy relationships for various load conditions. Investigating the influence of arcing on the contact materials as to material deformation is desirable.



Oscillogram #1

Contact Voltage on Operate After 2000 Operations
Using Resistive Load for 25 amp Contactor

Traces:

(a) Voltage

(b) Current

Duty Cycle - 11 seconds on, 4 seconds off

Oscillogram Data:

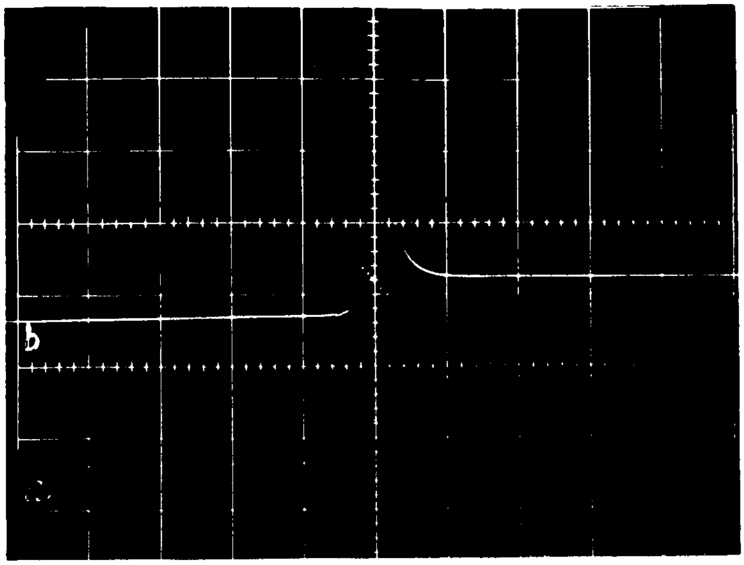
Time scale: .25 ms/cm

Voltage Scale: 20 v/cm

Current Scale: 50 mv/cm

Steady State Contact Current = 13 amps

Relay Operated at $\approx$ 28 volts



Oscillogram #2

Contact Voltage and Current on Release After 2000 Operations Using Resistive Load for 25 amp Contactor

Traces:

(a) Voltage

(b) Current

Duty Cycle - 11 seconds on, 4 seconds off

Oscillogram Data:

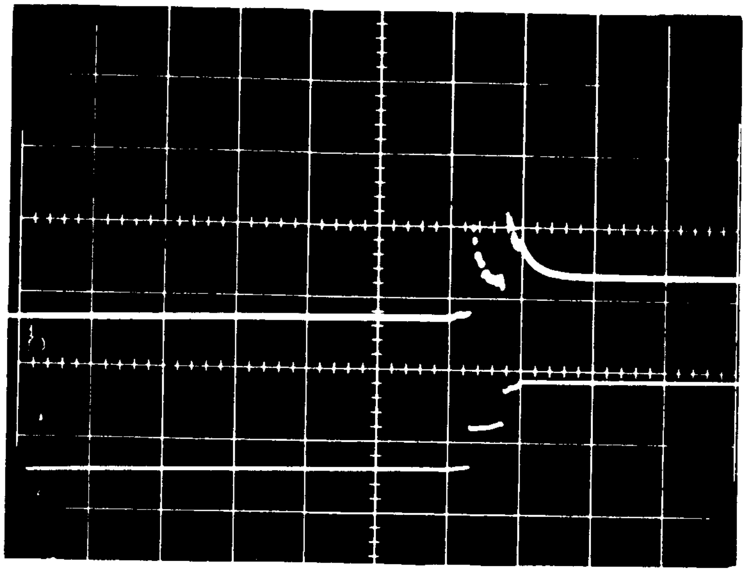
Time Scale: .25 ms/cm

Voltage Scale: 20 v/cm

Current Scale: 50 mv/cm

Steady State Contact Current = 13 amps

Relay Operated at ≈28 volts



Oscillogram #3

Contact Voltage and Current on Release After 3400 Operations Using Resistive Load for 25 amp Contactor

Traces:

(a) Voltage

(b) Current

Duty Cycle - 11 seconds on, 4 seconds off

Oscillogram Data:

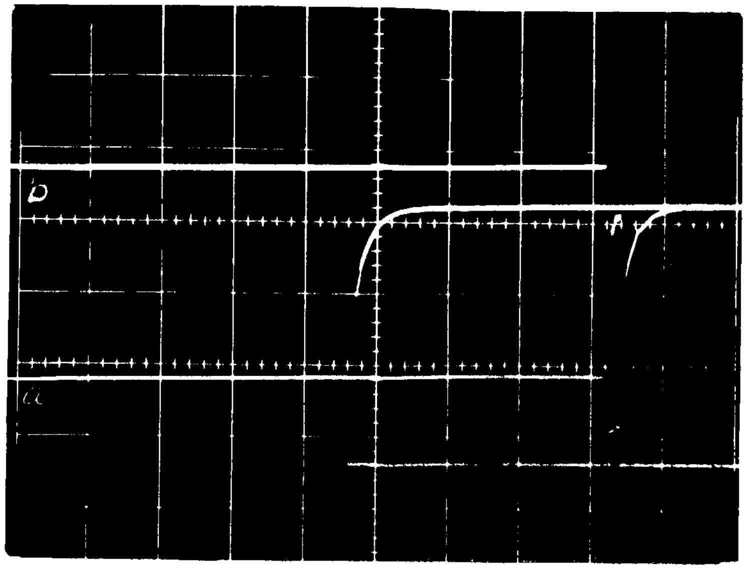
Time Scale: .25 ms/cm

Voltage Scale: 20 v/cm

Current Scale: 50 mv/cm

Steady State Contact Current = 13 amps

Relay Operated at $\approx$28 volts



Oscillogram #4

Contact Voltage and Current on Operate after 3400 Operations Using Resistive Load for 25 amp Contactor

Traces:

(a) Two sets of voltage on release

(b) Two sets of current on release

Duty Cycle - 11 seconds on, 4 seconds off

Oscillogram Data:

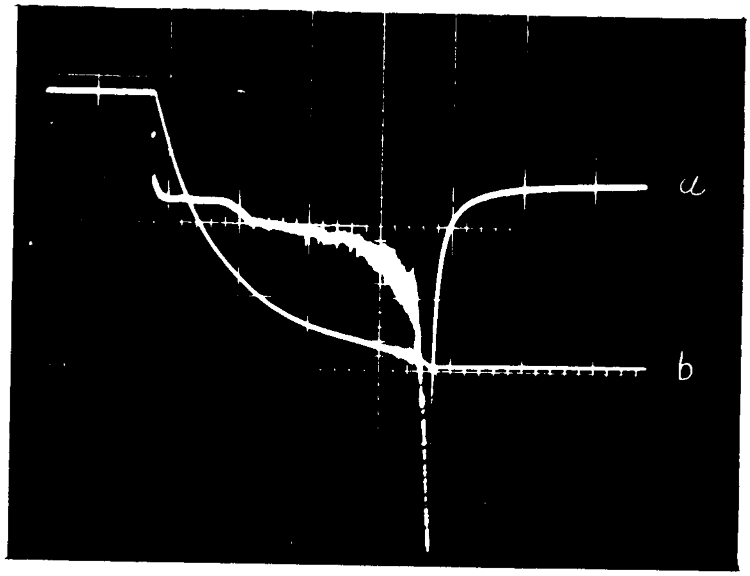
Time Scale: .25 ms/cm

Voltage Scale: 20 v/cm

Current Scale: 50 mv/cm

Steady State Contact Current = 13 amps

Relay Operated at ≈28 volts



Oscillogram #5

Contact Voltage and Current on Release After Ten Operations Using Inductive Load for 25 amp Contactor

Traces:

(a) Voltage

(b) Current

Oscillogram Data:

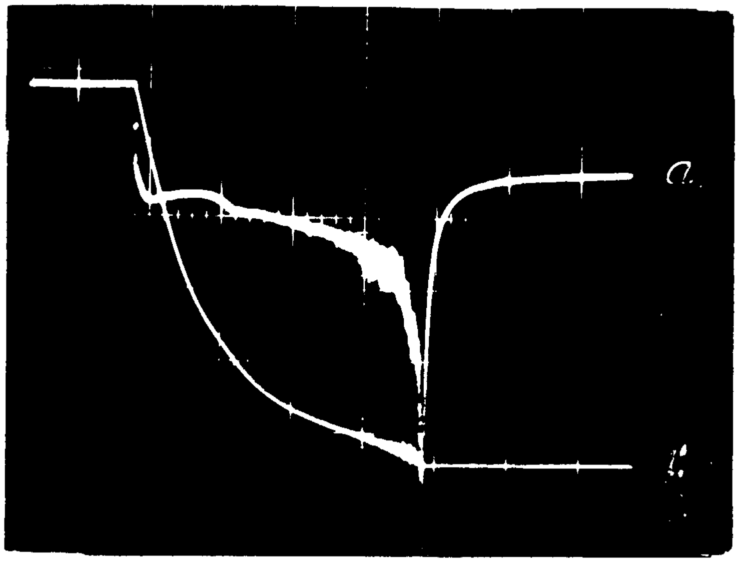
Time Scale: 10 ms/cm

Voltage Scale: 20 v/cm

Current Scale: 2 v/cm

Current Shunt $\approx$.8 Ω cold

Relay Operated at $\approx$28 volts.



Oscillogram #6

Contact Voltage and Current on **Release After Ten** Operations Using Inductive Load for 25 amp Contactor

Traces:

(a) Voltage

(b) Current

Oscillogram Data:

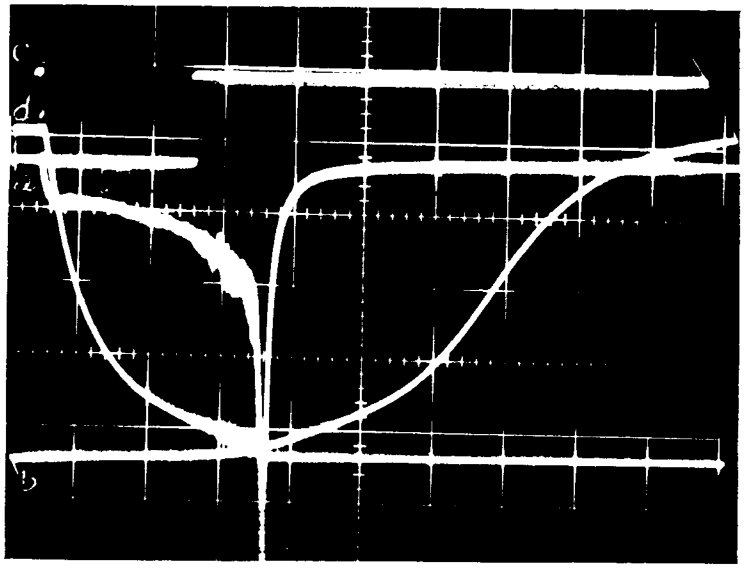
Time Scale: 10 ms/cm

Voltage Scale: 20 v/cm

Current Scale: 1 v/cm

Current Shunt $\approx$ .25 Ω cold

Relay Operated at $\approx$ 28 volts



Oscillogram #7

Contact Current and Voltage on Operate and Release After 250 Operations Using Inductive Load for 25 amp Contactor

Traces:

(a) Voltage Operate

(b) Current Operate

(c) Voltage Release

(d) Current Release

Duty Cycle - 11 Seconds on, 4 Seconds off

Oscillogram Data:

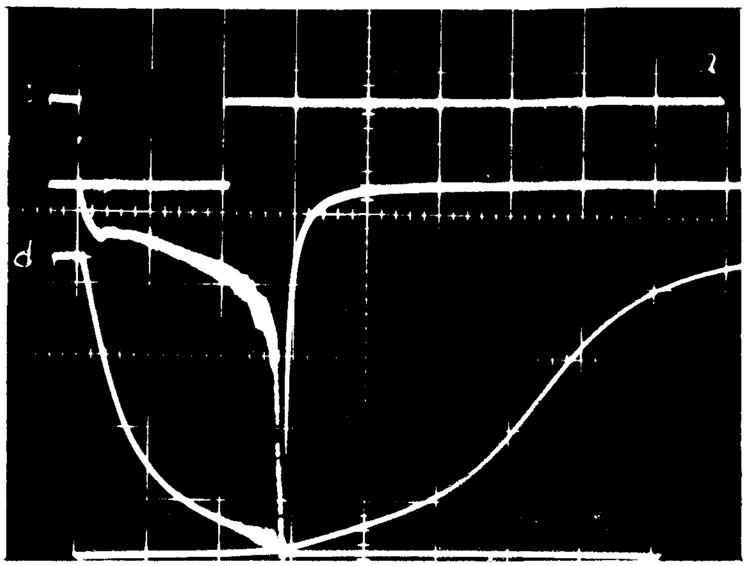
Time Scale: 10 ms/cm

Voltage Scale: 20 v/cm

Current Scale: 1 v/cm

Current Shunt $\approx$.25 Ω cold

Relay Operated at $\approx$ 28 volts



Oscillogram #8

Contact Current and Voltage on Operate and Release After 750 Operations Using Inductive Load for 25 amp Contactor

Traces:

(a) Voltage Operate

(b) Current Operate

(c) Voltage Release

(d) Current Release

Duty Cycle - 11 Seconds on, 4 Seconds off

Oscillogram Data:

Time Scale: 10 ms/cm

Voltage Scale: 20 v/cm

Current Scale: 1 v/cm

Current Shunt $\approx 25\,\Omega$ cold

Relay Operated at $\approx$ 28 volts

SECTION II

CONTACT RATING

As mentioned previously, a test was conducted on a contactor whose contacts were rated at 25 amperes. With a resistive load, no damage was evident to the contactor. Since it was known that a cantilever spring which held one contact of a pair had been welded under service conditions, an evident question was, "What are the actual load conditions to which a contactor is subjected?"

An answer to this question was that a contactor with a set of contacts rated at 25 amperes, 50 amperes, 100 amperes or 200 amperes, could be subjected to practically every type of a load which was possible. In other words, the current of the contact circuit could be caused by a resistance, an inductance or a capacitance. On the contact load circuit, any or all of these conditions could exist simultaneously and for varying times.

The contactor was expected to open and close circuits for all of the varying conditions and in addition, open and close circuits for motors, lamps, etc., when the foregoing situation existed. It was found, however, that the mating contacts would only function for a relatively few cycles for a rated current which was obtained from a highly inductive load. The terminals to the contactor became excessively hot and under some conditions, the arc across the contacts would continue to exist for some time after the contacts were separated. As would be expected the contacts were damaged excessively.

The question then is, what is an adequate set of specifications for electrical contacts? It is not expected to answer this question immediately, however, some satisfactory solution must be found if contacts in electrical contactors and in electromechanical relays are to preform under the conditions enumerated. It is quite probable that the description of the duty of the contacts is inclusive to the extent that this could serve as a final specification but it does point to a glaring lack of information in this area.

In an attempt to outline the problem, the following outline was prepared. While this is a preliminary outline, it does indicate that an intelligent application of contacts must be made as well as an improvement in the scheme used to rate contacts.

REQUIRED CONTACT SPECIFICATIONS

1. Stability
2. No chatter, 10 to 2000 cps, 20 g
3. Load on contacts
   (a) Capacitive, inrush of current shall be no more than twice rated current; time constant no greater than ________.
   (b) Inductive load with a time constant of less than ________.
   (c) State rated voltage, and whether it is DC or AC.
   (d) Steady state current is equal to rated current.
4. Life -- duty cycle -- number of operations, contact resistance -- rated load -- temperature.
5. Temperature
6. Coil voltage - range - nominal - power supply internal impedance.
7. Sealed - leakage 10^{-8} cc/sec.

- -

Upper Bound	Lower Bound
Operate time	Insulation resistance
Release time	Dielectric strength
Coil power	
Weight	
Volume	

Some rational scheme to arrive at specifications for contacts must be used if contactors and relays are to be utilized satisfactorily. Of course, the specifications alone cannot replace the proper application of these devices. In other words, after proper contact specification has bee determined, the proper contactor must be used for a given job. Not much has been accomplished in this direction but the proper usage of equipment and devices is a prime prerequisite if satisfactory functioning is to be expected.

It is quite probable that a special contact rating will be required for those places where a highly inductive current is to be interrupted. At the moment, not much information is available for the closing of contacts for incadescent lamp loads and motor loads. The meager data on hand seems to indicate that the breaking of circuit carrying a highly inductive load is the most severe case. If this is correct then, the problem resolves into the specification and design of contacts for this case.

SECTION III

THEORETICAL INVESTIGATION AND SOME EXPERIMENTAL DATA FOR ELECTRICAL CONTACT FAILURE CAUSED BY ELECTRICAL LOADING

The interim report covering the period 1-1962 to 3-1962 indicated the possibility of relating electrical load contact failures to arc energy. This section presents the theoretical development of several relationships based on ideas discussed in the preceding report. A brief review of these earlier ideas and the modifications used in this development are given before proceeding with the detailed development.

The primary assumption discussed previously was that the probability for "failure" due to electrical load is related to arc energy. This assumption is qualified in the ensuing work by restricting the study to medium and large current carrying contacts, designed for cycle duty, (i.e., not one shot relays.) For these type contact systems, the following electrical contact system conditions are said to constitute a failure under current load $\leq$ rated load.

(1) Prolonged arcing in the open position on break.

(2) Contact lead connections becoming faulty due to electrical load heat generated by contacts.

(3) Contacts welded closed.

(4) Failure to establish electrical continuity on make.

(5) Electrical discontinuity occurs during steady state load $\leq$ rated load, due to heating by I^2R of the contact system.

(6) Contact system discontinuity failures induced by electrical contact heat, not covered by (1) through (5), (i.e., springs failing due to excessive heat, etc.).

Most of the above conditions are common electrical failures associated with cyclic contactors. An additional condition is now defined which is somewhat unusual but is believed to have merit, along with usefulness in the following analytical development. This condition is called Condition A.

Condition A:

If all of the mass from either set of a contact pair is reduced by the original amount then either (i) a condition of the type (1) through (5) has occurred or (ii) the expected value of the probability for continued satisfactory operation is zero. Also, if no mass from either contact is removed the probability of failure is zero.

"Condition A", although open to criticism, allows a starting point from which analytical relationships can be derived. If these relationships prove useful in failure estimation and contact design, then criticism of the hypothesis can be of a constructive nature.

Using the essence of "Condition A", the following concise assumption is given.

For the probability of failure less than λ, ($Pr\,[failure] < \lambda$), there is a distribution of mass transfer, $f(M_T)$ such that:

(a) $f(M_T)$ is a continuous probability density function with the properties given in (1)

(b) M_T = mass transfer due to arc energy for the contact pair which monotonically loses mass under a given load condition, such that the steady state current is $\leq$ rated load current.

(c) M_V = original mass of the above contact.

$$(1)\quad \int_0^{M_{T_o}} f(M_T)dM_T = \lambda_o > \Pr\,[\text{failure}] \quad \text{if } M_T \text{ is in the range } \left[0, M_{T_o}\right]$$

$$\int_0^{M_V} f(M_T)dM_T = 1 = F(M_V)$$

$$0 = F(M_T) \quad \text{if } M_T < 0$$

$$1 = F(M_T) \quad \text{if } M_T \geq M_V$$

The above hypothesis leads to the investigation of the failure, as defined earlier, as a function of the mass transfer. The exact nature of this distribution can only be found by an infinite number of tests which is an obvious impossibility. However, on the basis that the distribution is not symmetrical, but is skewed in the increasing M_T direction and is a function of only one parameter other than M_V, the following density appears to have possibilities.

$$(2)\qquad g(M_T;M_V,\rho_s) = C(1-\epsilon^{M_T/\rho_s M_V}) \qquad 0 \leq M_T \leq M_V$$

$$= 0 \quad \text{otherwise}$$

$$\text{where}\quad C = \frac{1}{\int_0^{M_V} g(M_T;M_V,\rho_s)\,dM_T} = \frac{1}{M_V\left[1+\rho_s-\rho_s\epsilon^{1/\rho_s}\right]}$$

$$\text{which gives}\quad g(M_T;M_V,\rho_s) = \frac{1-\epsilon^{M_T/M_V\rho_s}}{M_V\left[1+\rho_s-\rho_s\epsilon^{1/\rho_s}\right]} \qquad 0 \leq M_T \leq M_V$$

Also the above distribution has the monotonic property that if $\lambda_1 > \lambda_2$, then $M_{T_1} > M_{T_2}$, which certainly appears reasonable.

The investigation to date on the correlation between mass transfer and arc energy indicates that M_T might be approximated by the following function. The data of Figures (1) and (2) indicates some results of the investigation to date, and is discussed in more detail in connection with using the results of the present development.

$$M_T = C_1 A(1 + C_2 A) \quad \text{where } A = \text{arc energy} = \sum_{i=1}^{N} \int_{o}^{T_{o_i}} E_i I_i d_T = \sum_{i=1}^{N} A_{c_i}$$

A_{c_i} = the arc energy of the ith arc period.

T_{o_i} = ith arc period.

E_i = arc voltage during the ith arc period.

I_i = arc current during the ith arc period.

C_1 and C_2 are constants which are presumed to be functions of the following factors.

(a) Average contact temperature during operation

(b) Type of contact material

(c) Contact pressure during closure

(d) Impact pressures

(e) Contact surface area

(f) Sealed or unsealed and dielectric media along with media pressure

(g) Gravity and electrical polarity.

To obtain the probability for failure in terms of arc energy A, the following density transformation can be made.*

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* "Introduction to the Theory of Statistics" Mood, McGraw-Hill

$$H(A;M_V,\rho_s,\ C_1,\ C_2) = g(M_T(A))\ \left|\frac{dM_T}{dA}\right|;A(0)< A < A(M_V)$$

where $A(0)$ = the value of A when $M_T = 0$

$A(M_V)$ = the value of A when $M_T = M_V$

since $\left|\frac{dM_T}{dA}\right| = C_1(1+2C_2A)$ the above becomes

$$(3)\quad H(A;M_V,\rho_s,C_1,C_2) = \frac{C_1(1+2C_2A)\left[1-\epsilon^{C_1A(1+C_2A)/\rho_s M_V}\right]}{M_V\left[1+\rho_s-\rho_s\epsilon^{1/\rho_s}\right]}$$

$$= 0 \text{ otherwise}$$

where A has the limits

$$0 < A < \frac{1}{2C_2}\left[\sqrt{1+\frac{4M_VC_2}{C_1}}-1\right]$$

Also if the load cycle of a contactor is repetitive in nature, the arc energy can be written as:

$$A = NA_c$$

where N = number of operations

A_c = arc energy per operation.

Transforming the H(A) density to an h(N) density (N, the number of operations) gives, using the same procedure as before:

$$(4)\quad h(N;\rho_s,C_1,C_2,A_c) = \frac{C_1(1+2C_2A_cN)\left[1-\epsilon^{A_cC_1N(1+AC_2N)/M_V\rho_s}\right]A_c}{M_V\left[1+\rho_s-\rho_s\epsilon^{1/\rho_s}\right]}$$

where N is in the interval $0 < N < \frac{1}{2A_cC_2}\left[\sqrt{1+\frac{4M_VC_2}{C_1}}-1\right]$

This implies the probability of failure is directly related to the number of operations permitted.

In order to demonstrate the use of equation (4) consider the following illustration.

Knowing the type of load for which a contactor is to be used and assuming that it is repetitive, then A_c can be found. Also, for a choice of relay the constants M_V, C_1 and C_2 can presumably be found. The problem then is to determine the number of operations, N_o, such that the probability of failure is less than λ_o. (Note that ρ_s is as yet not known but is presumably a constant once it has been determined. A method for finding ρ_s will be discussed later.) The above problem can be stated as: Find N_o such that

$$\int_0^{N_o} h(N;\rho_s,C_1,\ C_2,A_c)d_N = \lambda_o > \Pr\ [\text{failure}]$$

carrying out the integration yields the following equality which must be satisfied by N_o.

$$\lambda_o = \frac{A_cC_1/M_VN_o\left[1+C_2A_cN_o\right] + \rho_s\left[1-\epsilon^{A_cC_1N_o(1+A_cC_2N_o)/M_V\rho_s}\right]}{1+\rho_s-\rho_s\epsilon^{1/\rho_s}}$$

letting $u_o = C_1N_o\left[1+C_2A_cN_o\right]A_c/M_V\rho_s$

$$J = \frac{\lambda_o}{\rho_s}\left[1+\rho_s-\rho_s\epsilon^{1/\rho_s}\right]$$

(5) gives $J = u_o + 1 - \epsilon^{u_o}$ $\qquad o \gtrless u_o \gtrless 1/\rho_s$

$$o \gtrless J \gtrless \left[1+\rho_s-\rho_s\epsilon^{1/\rho_s}\right]\rho_s$$

Expression (5) does not have an explicit form for u_o. However, the following development yields an explicit form which is of practical interest: noting that

$$\epsilon^{u_o} = 1 + u_o + \frac{u_o^2}{2!} + ----$$

gives $$J = -\frac{u_o^2}{2}\left[1+\theta(u_o)\right]$$

where $$\theta(u_o) = \frac{2}{3!} u_o + \frac{2}{4!} u_o^2 + ----$$

Also $\theta(u_o)$ is a monotonically increasing function as is u_o^2 over $0 \lesseqgtr u_o \lesseqgtr 1/\rho_s$.

Therefore $\theta(u_o) \leq \theta(1/\rho_s) = - \left[1 + 2\rho_s \left(1 + \rho_s - \rho_s \epsilon^{1/\rho_s}\right)\right]$

and $\theta(o) = 0.$

Using these values to give upper and lower bounds on u_o implies that u_o belongs to the interval

$$\left[\sqrt{\frac{\lambda_o}{\rho_s^2}}, \quad \sqrt{-2J}\right].$$

A conservative value of u_o, and hence N_o, is obtained by letting

$u_o = \sqrt{\frac{\lambda_o}{\rho_s^2}}$. If $\rho_s \geq 3$, the result is within 4% of the true value of u_o. To illustrate the actual use of this expression for some possible values of C_1, C_2, A_c, the data presented in Figures (1) and (2) is used.

The data of Figures (1) and (2) was obtained from unsealed relays using an inductive load. The circuit diagram for each contact pair is shown in Figures (A) and (B) below in order to show the electrical polarity and gravity sense.

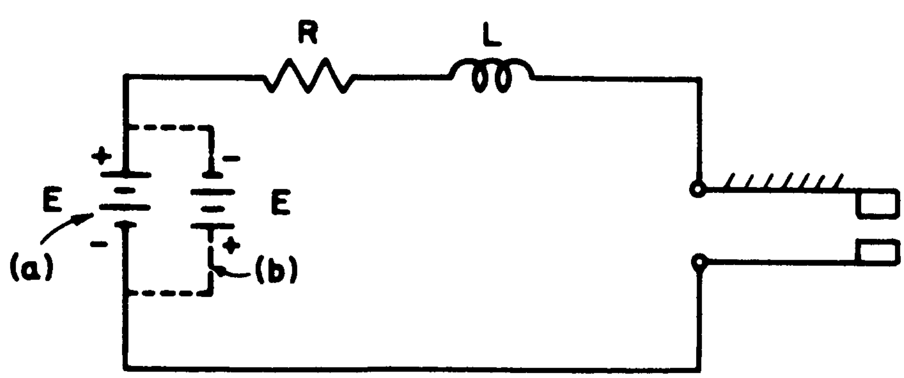

Fig. A

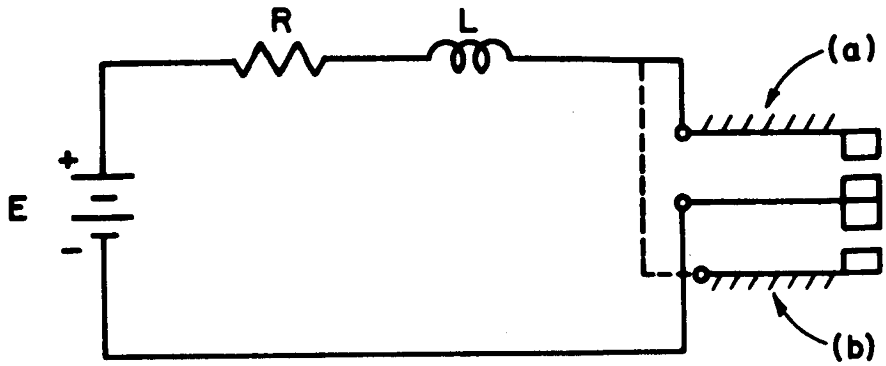

Fig. B

The operating data is listed below

Figure 1 (a), (b)	Duty cycle 350 ms on 150 ms off Resistive load = 1.45 Ω Supply voltage = 25 volts Inductance $\approx$ 1 henry
Figure 2 (a), (b)	Duty cycle 40 ms on 25 ms off Resistive load = 1.45 Ω Supply voltage = 25 volts Inductance $\approx$ 1 henry

The arc energy per cycle was computed, from the E-I characteristics during break, by the method used in the preceding report of approximating the integral by a summation of intervals. The value so obtained was, $A_c = 3.702$ watt-sec/cycle. This value represents the area under the curve of Figure 3, which is a plot of average arc power vs time increments for a break condition of the tested relays.

Figure (1) shows marked differences in the mass transferred for the two contact pairs of Figure A. The only test difference between

(a) and (b) of Figure A was the electrical polarity. Mass transfer was considerably greater when the contact was an anode than when it was a cathode.

The two contact pairs of Figure (2) show similar mass transfer characteristics although one is mass gained and the other is mass lost. The main difference between the test conditions for Figures (1) and (2) was the duty cycle and hence the average contact temperature. This suggests that the main contributor to the mass transfer vs arc energy relationship is temperature, for a given contact material. (The fact that temperature would influence the relationship is of course a physical fact already known.) From Figure (1) the data suggests that below a temperature region variables such as polarity and gravity can be a noticable factor.

Assuming that the dotted lines through the data of Figures (1) and (2) is representative of the M_T vs A relationship, and can be fitted by the second order polynomial suggested earlier over the range of interest, the constants C_1 and C_2 will be evaluated using the formulas below.

$$C_1 = \frac{M_{T_2} A_1^2 - M_{T_1} A_2^2}{A_1 A_2 [A_1 - A_2]}$$

M_{T_1} = mass at arc energy value A_1 which has been transferred

M_{T_2} = mass at arc energy value A_2 which has been transferred

$$C_2 = \frac{M_{T_1} A_2 - M_{T_2} A_1}{M_{T_2} A_1^2 - M_{T_1} A_2^2}$$

Some care has to be exercised in selecting A_1 and A_2 in order to obtain a reasonable fit to the data, when the curve is other than a straight line. The constants were evaluated for curves (a) and (b) of Figure (1) and (b) of Figure (2) using the circled points for A_1 and A_2. The constants are listed below.

Figure 1(a)	$C_1 = (1.31)10^{-4}$	gms/watt-sec.
	$C_2 = -(.9)10^{-6}$	per watt-sec.
Figure 1(b)	$C_1 = (1.555)10^{-5}$	gms/watt-sec.
	$C_2 = -(.895)10^{-6}$	per watt-sec.
Figure 2(b)	$C_1 = (1.13)10^{-3}$	gms/watt-sec.
	$C_2 = -(1.57)10^{-6}$	per watt-sec.

Using the constants from Figure 1(b) and Figure 2(b) along with the value for M_V of .5 gms the following examples are given using the value of arc energy per cycle mentioned previously.

For $\lambda_o = 10^{-3} >$ Pr [failure] and using the conservative estimate of u_o gives

$$u_o = \sqrt{\frac{10^{-3}}{\rho_s^2}}$$

using the definition of u_o in terms of N_o gives

$$N_o = -\frac{1}{2C_2A_c}\left[1 - \sqrt{1 + \frac{4M_V\rho_s u_o C_2}{C_1}}\right]$$

which is difficult to evaluate when $\frac{4M_V\rho_s u_o C_2}{C_1} \ll 1$

However, by using the binomial expression and neglecting higher terms than the first gives a good approximation. This technique will be used for the data of Figure 2(b) and N_o becomes:

$$N_o \approx \frac{M_V\rho_s u_o}{A_c C_1}$$

(Note: there is another value of N_o at $\approx -\frac{1}{+C_2}$ when $C_2 < 0$ but this occurs after M_T supposedly has passed through M_V)

or $$N_o \approx \frac{(.5)\sqrt{10^{-3}}}{(3.7)\ (1.13)10^{-3}} = 38 \text{ operations}$$

which says that for the load-duty cycle conditions imposed on the relay tested in order for not more than 1 in 10^3 relays to fail the relay should not be operated under these conditions for more than 38 operations. (This might seem like a severe restriction but it should be kept in mind that the load on the relay contact pair of Figure 2(b) was roughly 100 watts. Also both relays failed, due to burn out of the original contact-to-terminal connections.) This occurred at about 4500 operations. Also in order for the temperature to stabilize with a duty cycle of 900 cycle/min., it is apparent that the transient time for stabilization is a contributing factor to the number of operations until failure. In this light the above calculation implies that this relay would not reliably operate with this load condition.

For the data of Figure 1(a) and (b), the values of N_o for $\lambda_o = 10^{-3}$ are:

Figure 1 (a) = 326

Figure 1 (b) = 2730

These values indicate that for the duty cycle-load condition used, (the load was the same as in Figure (2)) continuous operation should be limited to 326 operations for the polarity of (a) and 2730 for the polarity of (b). In a practical sense this number of operations still suggests that the relays under test are not capable of reliable operation using the load-duty cycle of the test conditions. This too was varified by failure of connector straps after $\approx$ 300 min. of operation or $\approx$ 36,000 operations for 2 of 4 relays. Using $\lambda_o = 1/2$ in the above formula to predict the number of operations for Pr [Failure] < 1/2 gives

$N_o \approx 7400$ for relay used in (a)

$N_o \approx 60{,}000$ for relay used in (b)

Although the above illustrations were based on limited data and the proposed incomplete theory given earlier, the results suggest that some useful practical expressions are possible by further development of the proposed approach. Before discussing some of the possible modifications to the proposed theory, a method for finding the "best" estimator for the skew factor ρ_s is given. The method is based on the statistical principle of "Maximum Likelihood Estimator".* The procedure is as follows:

Let $(M_{T_1}, M_{V_1}), (M_{T_2}, M_{V_2})$----$(M_{T_m}, M_{V_n})$ represent the original mass and the mass transferred at failure for a random set of contactors under random load conditions. (Steady state current must be ≤ rated current.)

From the likelihood function

$$L = g_1(M_{T_1}; \rho_s, M_{V_1})\, g_2(M_{T_2}; \rho_s, M_{V_2}) \text{----} g_n(M_{T_m}; \rho_s, M_{V_n})$$

maximize L with respect to the skew factor ρ_s in terms of the measured values of M_{T_i} and M_{V_i}. This gives the following expression which must be satisfied by ρ_s.

$$\sum_{i=1}^{n} \left[\frac{X_i\, \epsilon^{X_i/\rho_s}}{1 - \epsilon^{X_i/\rho_s}} \right] = n\rho_s \left[\frac{\rho_s + \epsilon^{1/\rho_s} - \rho_s\, \epsilon^{1/\rho_s}}{1 + \rho_s - \rho_s\, \epsilon^{1/\rho_s}} \right]$$

where $X_i = M_{T_i} / M_{V_i}$

n = number of relays tested

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* For reference see "Introduction to Theory of Statistics", Mood, McGraw Hill.

Although this expression is not simple, it is felt that due to the possible results which could be obtained by having a good estimator for ρ_s, effort should be given to solving the above. Also in order to check the validity of the failure-mass transfer relation being independent of the type of load, the above relationship could be evaluated twice using two sets of data from two different types of load. To see the influence of the skew factor, ρ_s, on the probability distributions and the probability λ, these are plotted in Figures 4 and 5 for several values of ρ_s.

As mentioned earlier this report is a first attempt at finding a useful theoretical set of relationships which can be verified by experimental data. In this area it goes without saying that this has not been done to date. Although much work has been done in the physics area of contacts, this work has not been integrated into analytical relationships of the type being sought after by contact designers and relay customers.[1,2] This is the goal of this particular investigation. The first report gave only the basic point of view with which to attack this problem. It is felt that the more concrete investigations (experimental and theoretical) presented in this interim report indicate a direction, and will be helpful in finding well founded expressions with which to work.

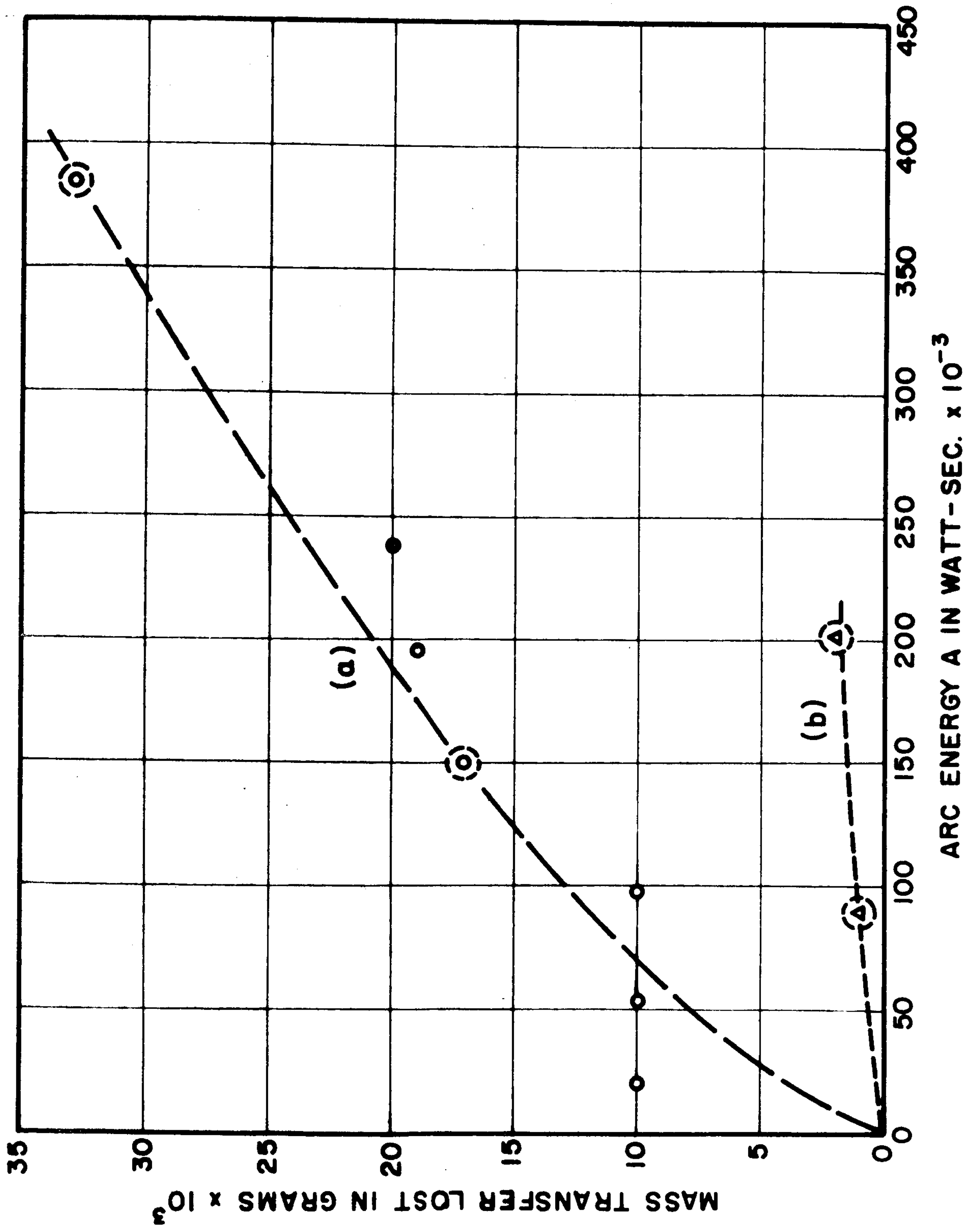
Some of the possible modifications to the proposed theory are discussed below and will be investigated further if future experimental data so dictates.

---

1. "Electric Contacts" Ragnar Holms
2. "The Physics of Electrical Contacts" Llewellyn Jones

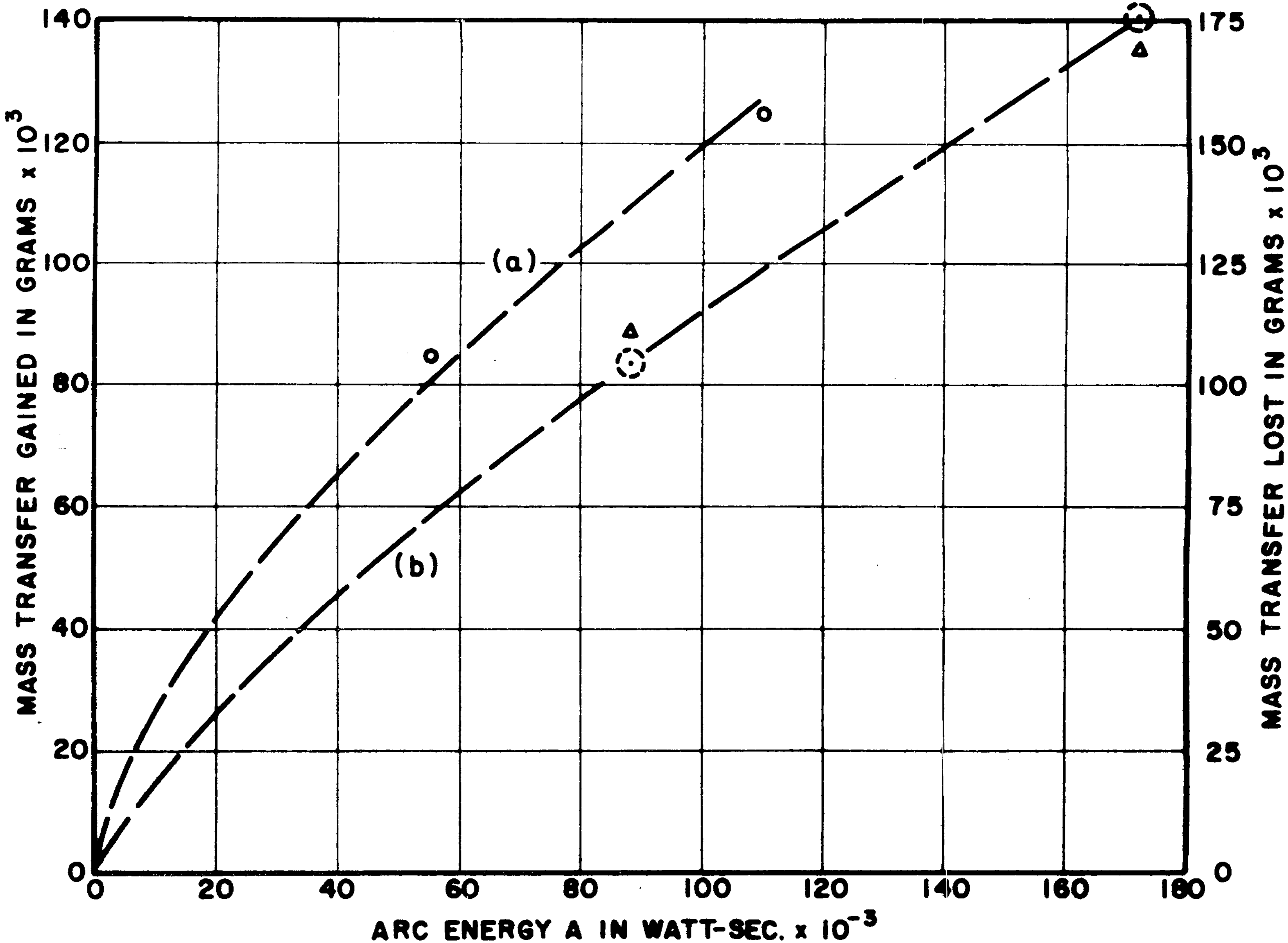
I. The types of failure listed in (1) through (6) be reclassified into one or more groups. This can be checked by the method indicated earlier for finding ρ_s.

II. Using a different expression to represent mass transfer in terms of arc energy.



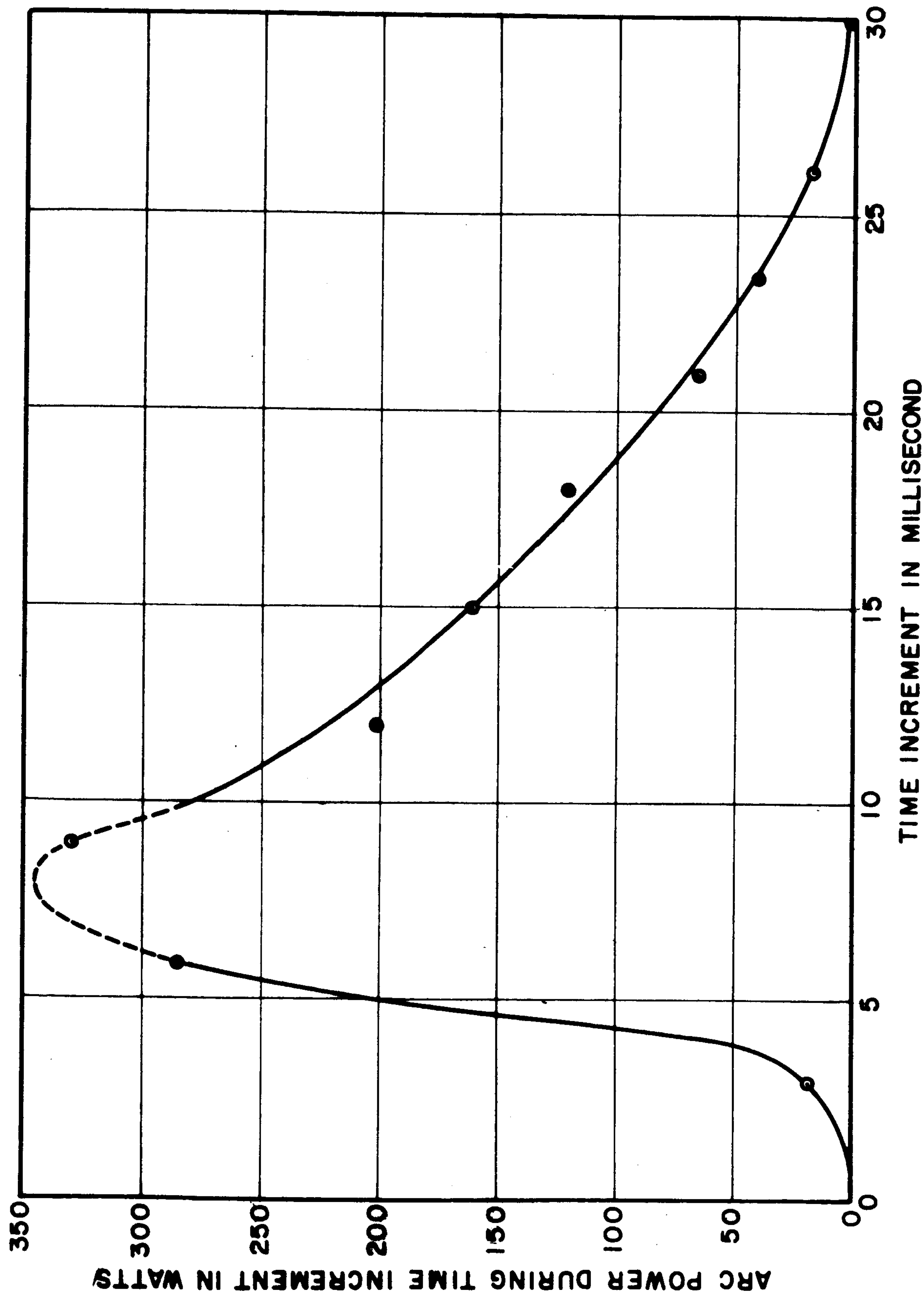
ARC ENERGY A IN WATT-SEC. x 10^-3
MASS TRANSFER LOST IN GRAMS x 10^3
0
50
100
150
200
250
300
350
400
450
5
10
15
20
25
30
35
(a)
(b)

Figure 1



MASS TRANSFER GAINED IN GRAMS x 10^3
MASS TRANSFER LOST IN GRAMS x 10^3
ARC ENERGY A IN WATT-SEC. x 10^{-3}
(a)
(b)
0
20
40
60
80
100
120
140
160
180
0
25
50
75
100
125
150
175

Figure 2



ARC POWER DURING TIME INCREMENT IN WATTS
TIME INCREMENT IN MILLISECOND
0
50
100
150
200
250
300
350
5
10
15
20
25
30

Figure 3

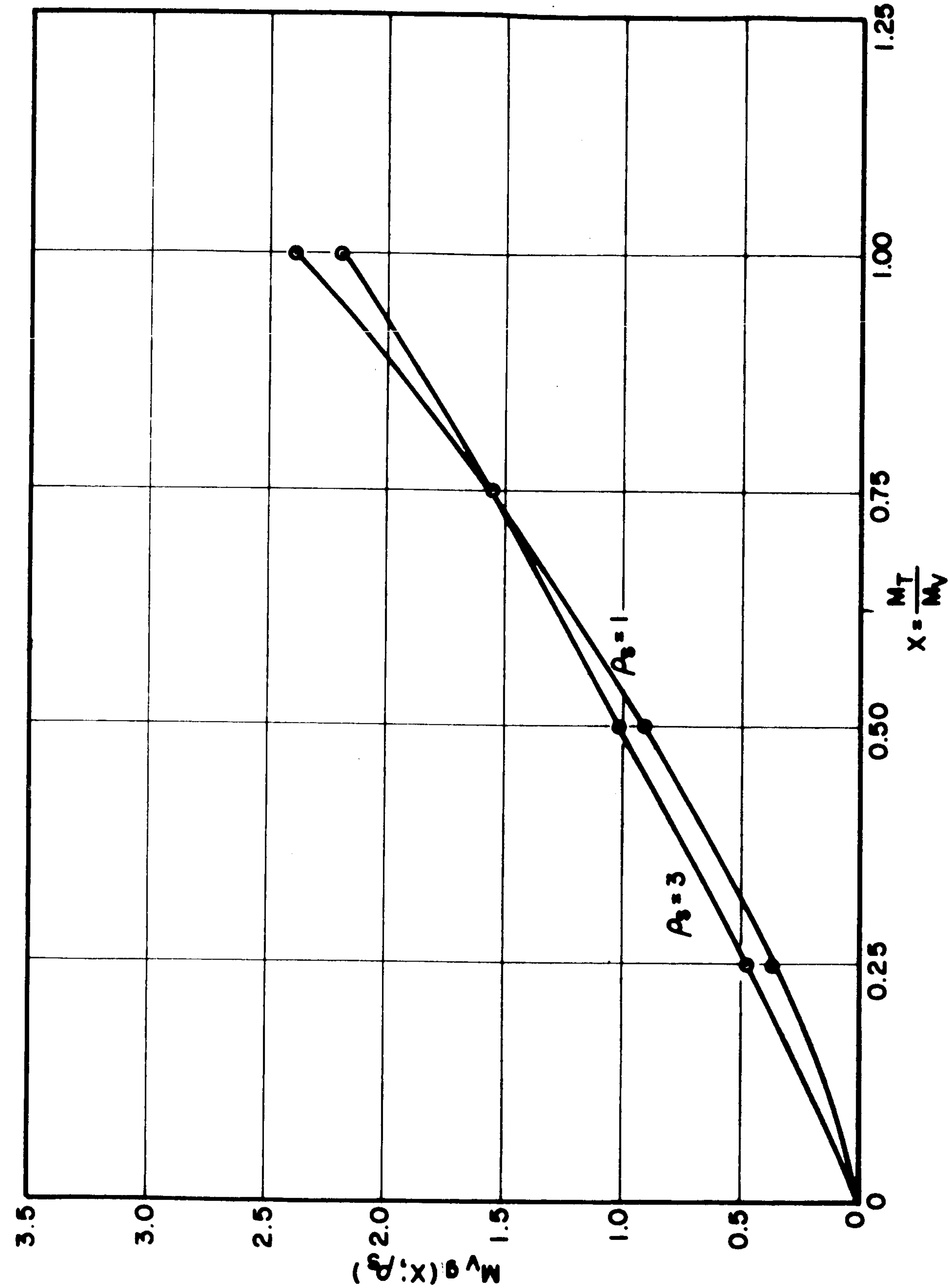
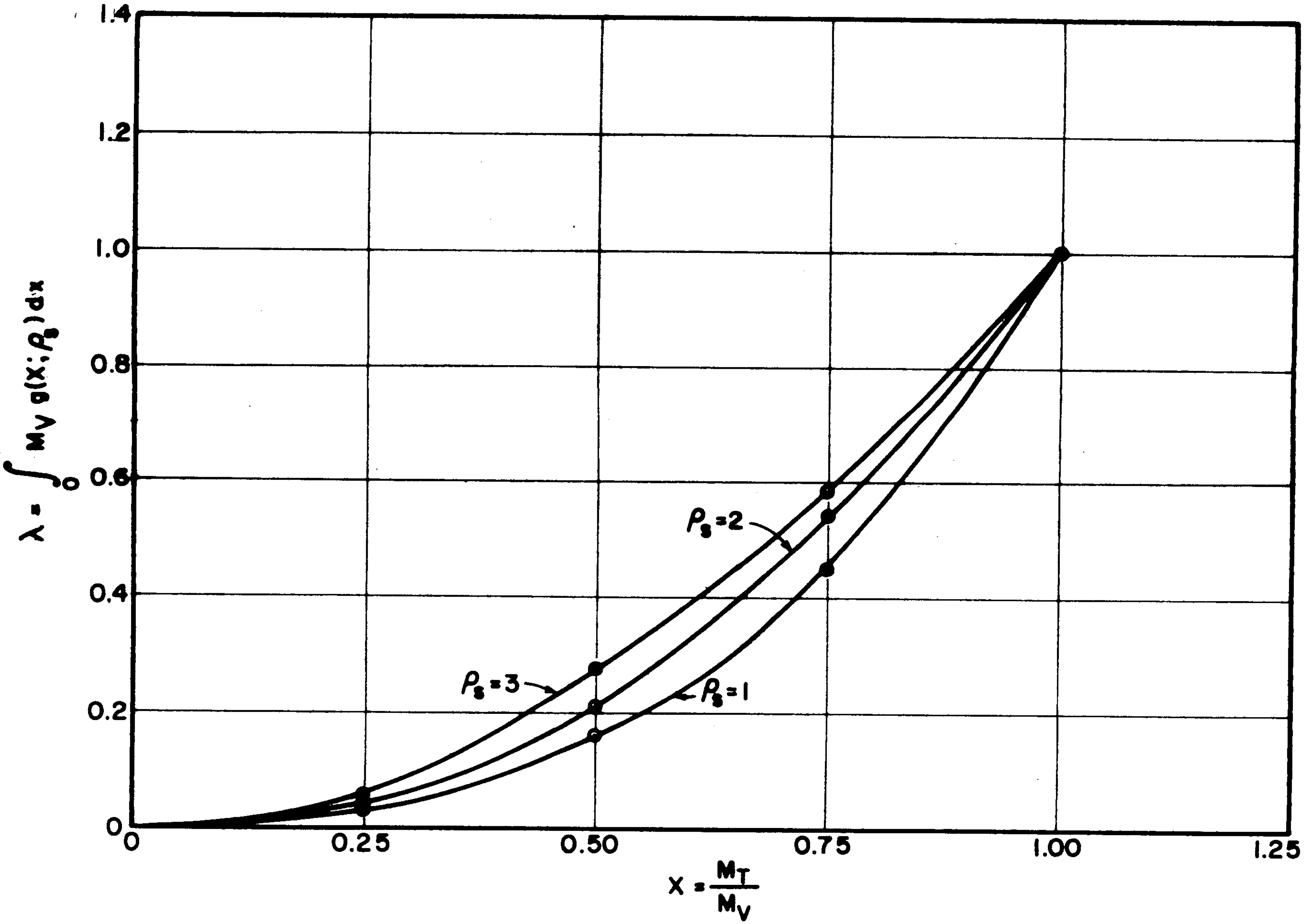

3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
0.25
0.50
0.75
1.00
1.25
$M_V\, g(X; \rho_s)$
$X = \frac{M_T}{M_V}$
$\rho_s = 1$
$\rho_s = 3$

Figure 4



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
$\lambda = \int_0^x M_V\, g(x;\rho_s)\, dx$
0
0.25
0.50
0.75
1.00
1.25
$X = \frac{M_T}{M_V}$
$\rho_s = 3$
$\rho_s = 2$
$\rho_s = 1$

Figure 5

FURTHER DISCUSSION OF CONTACT FAILURE DUE TO ELECTRICAL LOADING
Report No. 3 Section No. III

In the report covering the period 1 March 1962 to 30 April 1962, a method for finding contactor failures caused by electrical loading was proposed. The basic assumptions related contact failures to mass transfer caused by electrical load conditions. Mass transfer was then related to arc energy which was in turn related to the number of operations for a given duty cycle. This report discusses several aspects which were omitted or slighted in the preceding report, along with a more detailed testing scheme with which to test the proposed theory.

One of the important steps in being able to predict the number of operations for a given probability of failure was that of relating mass transfer to arc energy. Based on the results of several tests it was assumed that these were related as:

$$M_T = c_1 A(1 + c_2 A)$$

M_T = mass transfer due to arc energy for the contact pair which monotonically loses mass under a given load condition such that the steady state current is less than or equal to the rated current.

$$A = \text{Total arc energy} = \sum_{i=1}^{N} \int_{0}^{T_{oi}} E_i \, I_i d_T = \sum_{i=1}^{N} Ac_i$$

Ac_i = arc energy of ith arc period

T_{oi} = ith arc period

E_i = arc voltage during ith arc period

I_i = arc current during ith arc period.

The constants c_1 and c_2 were presumed to be functions of:

(a) Average contact temperature during operation

(b) Type of contact material

(c) Contact pressure during closure

(d) Impact pressures

(e) Contact surface area

(f) Sealed or unsealed, dielectric media and media pressure

(g) Gravity and electrical polarity.

By lumping all of the above factors into the constants c_1 and c_2, the analysis of failure was considerably simplified. However, by attacking the problem in this manner, a considerable amount of testing would be needed for a given relay, duty-cycle, environmental conditions, and electrical load, before the constants c_1 and c_2 could be obtained with which to predict failure. This is an undesirable situation when many different applications are to be considered.

Two possible solutions to this problem based on experimental data are presented in this report. Although both of these methods would require a considerable number of tests it could be small compared to the amount of testing required for finding c_1 and c_2 for each different application.

The first method for evaluating c_1 and c_2 is as follows:

(1) choose sets of compatible primary values for the variables; temperature, contact material (denoted in terms of constituents), media (denoted in terms of constituents), media pressure, contact surface area, sealed or unsealed and contact pressures. (Compatible is meant to be the values of the above parameters commonly used together in the range of application.)

(2) Vary the parameters gravitational force and electrical polarity between their extreme values.

This process would result in a set of discrete ranges for the parameters c_1 and c_2 which could be used to evaluate all applications which fell in one of these sets of ranges. Although this procedure would limit the versatility and accuracy of the failure estimate relationship, it does have the advantage of simplicity.

The second method for evaluating c_1 and c_2 is that of curve fitting. This would require considerably more tests than the above method. However, it offers a method for having c_1 and c_2 as analytic functions of the important parameters mentioned earlier.

Since the evaluation of c_1 and c_2 as a function of their parameters would only be considered subject to the validity of the failure theory, further discussion of methods to accomplish this task will be postponed until the validity is decided. These methods are discussed however, since the first method mentioned, could be obtained when testing for the validity of the mass transfer relationship.

The previous discussions assumed that the mass transfer could be related to arc energy by two constants c_1 and c_2. The experimental evidence to date has only indicated that this might be the case. Also, the test data to date indicates that in actuality, the data is far from smooth. This presents the problem of calculating c_1 and c_2 from any set of test data and interpreting the results. The following procedure is suggested based on obtaining conservative values of c_1 and c_2. That is, obtaining values for c_1 and c_2 which gives a conservative value for the number of operations for a given probability of failure. Also this method offers a way to verify if the mass transfer arc energy relationship can be approximated by the type relationship mentioned earlier, using limited information.

In order for the probability distribution in terms of arc energy to hold, the value of arc energy A_0 when the mass transfer $M_T = M_V$ is given by:

$$A_o = \frac{1}{2c_2}\left[\sqrt{1 + \frac{4M_T c_2}{c_1}} - 1\right] , \tag{1}$$

which gives a criterion for testing the possibility of using limited information to calculate c_1 and c_2. For example, if c_1 and c_2 were calculated from test data and if in addition $\frac{4M_T c_2}{c_1} < -1$, then this implies A_o is imaginary which is obviously not true. Also if A_o is less than some experimental value, this, too, is an indication that $M_T \neq c_1 A(1 + c_2 A)$.

In order to obtain conservative values of c_1 and c_2 when the test data is non-smooth as in Figure 1, the following procedure is suggested. Use two sets of actual data, $[A_1, M_{T1}]$ and $[A_2, M_{T2}]$ which gives a curve such that $M_{T\,calculated} \geq M_{T\,actual}$ over the range for which the data was obtained. In Figure 1, this would be the sets $[(50)10^4, (7.5)10^{-3}]$ and $[(300)10^4, (25)10^{-3}]$.

Another method of looking at the calculated values of c_1 and c_2 which is independent of the method of calculation is normality. Assuming normality for c_1 and c_2 with the variance being independent of the parameters mentioned earlier, gives a check on the repeatability of the mass transfer arc energy relationship. The assumption that c_1 and c_2 are normally distributed is certainly reasonable due to the number of small random variables associated with relay contact designs and the limited ability to control the test parameters associated with c_1 and c_2. To find the variance of c_1 and c_2, under the above assumption with an "identical" contact system, run the M_T versus A characteristics and compute c_1 and c_2. (These can be computed by the method suggested earlier, or some other scheme, but the same method should be used each time.) Then since c_1 and c_2 are distributed as:

$$\eta(c_1, u_1, \sigma_1^2) = \frac{1}{\sigma_1\sqrt{2\pi}}\, \mathcal{E}^{-1/2\left[\frac{c_1 - u_1}{\sigma_1}\right]^2} \tag{2}$$

$$\eta(c_2,u_2,\sigma_2^2) = \frac{1}{\sigma_2\sqrt{2\pi}}\ \varepsilon^{-1/2\left[\frac{c_2-u_2}{\sigma_2}\right]^2} \tag{3}$$

where u_1 = mean of c_1 and is a function of the parameters of c_1

σ_1^2 = variance of c_1 and is independent of the parameters of c_1

u_2 = mean of c_2 and is a function of the parameters of c_2

σ_2^2 = variance of c_2 and is independent of the parameters of c_2.

The best estimators of u_1, σ_1^2, u_2 and σ_2^2 are given as:

$$\bar{u}_1 = \frac{1}{N}\sum_{i=1}^{N} c_{1i} \tag{4}$$

$$\bar{\sigma}_1^2 = \frac{1}{N}\sum_{i=1}^{N} (c_{1i} - \bar{u}_1)^2 \tag{5}$$

$$\bar{u}_2 = \frac{1}{N}\sum_{i=1}^{N} c_{2i} \tag{6}$$

$$\bar{\sigma}_2^2 = \frac{1}{N}\sum_{i=1}^{N} (c_{2i} - \bar{u}_2)^2 \tag{7}$$

where N = number of times the test was run.

The standard deviation of c_1 and c_2 is then given by $\bar{\sigma}_1$ and $\bar{\sigma}_2$ respectively. That is, to the calculated values of c_1 and c_2 should be added $\pm\bar{\sigma}_1$ and $\pm\bar{\sigma}_2$. The size of $\bar{\sigma}_1$ and $\bar{\sigma}_2$ relative to the mean at any operating condition is a measure of the accuracy of the failure relationship for that range of operation.

To summarize the above discussion, the following tests are needed with which to check the mass transfer arc energy relationship for; (a) basic form and (b) repeatability.

I. To test the basic form several different types of relay contactors should be selected and operated under widely separated load and duty cycle conditions recording mass transfer versus arc energy. These tests should be run until failure or the mass transfered is equal to M_V. (Note that this type of test is not feasible with sealed relays. However the form of M_T versus A is not presumed a function of this parameter.)

II. To test for repeatability, several "identical" relay contactors should be operated under "identical" conditions recording mass transfered and arc energy.

The information from the above tests along with the relationships (1), (5) and (7) developed earlier in this report should give a reasonable verification or negation of the M_T versus A relationship and repeatability. Also using the information obtained in I and the relationship:*

$$\sum_{i=1}^{N}\left[\frac{X_i \epsilon^{X_i/\rho_s}}{1-\epsilon^{X_i/\rho_s}}\right] = n\rho_s\left[\frac{\rho_s + \epsilon^{1/\rho_s} - \rho_s \epsilon^{1/\rho_s}}{1 + \rho_s - \rho_s \epsilon^{1/\rho_s}}\right]$$

where $X_i = \dfrac{M_{T_i}}{M_{V_i}}$

n = number of relays tested

ρ_s = skew factor

M_{T_i} = mass transfered at failure for ith relay tested

M_{V_i} = original mass of ith test relay

the skew factor ρ_s can be estimated.

Although the above testing is outlined for unsealed relays the results should yield sufficient evidence to indicate the soundness of the proposed theory. Also by using unsealed relays, the problems associated with measuring M_V and M_T are easily overcome, contrary to the sealed relay situation.

*This relationship developed on page 12-III of the Interim Report for 1 March to 30 April 1962.

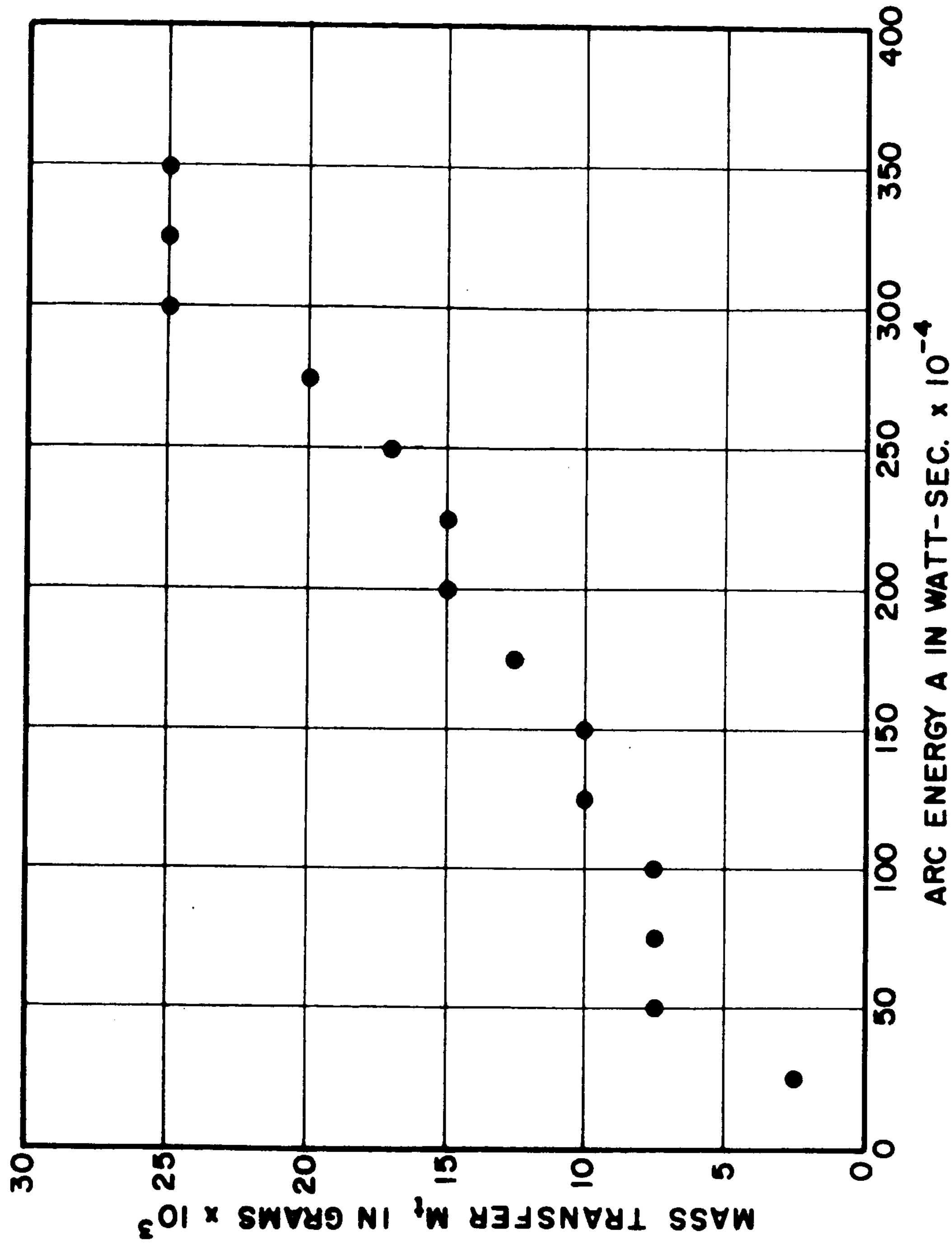

ARC ENERGY A IN WATT-SEC. x 10^{-4}
MASS TRANSFER M_t IN GRAMS x 10^3
0
50
100
150
200
250
300
350
400
0
5
10
15
20
25
30

Figure 1

TABLE OF CONTENTS

Part D

Contactor Design

Tab Color Code

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Blue	2nd Interim	1 March - 30 April, 1962
Yellow	3rd Interim	1 May - 30 June, 1962

SECTION II

VERIFICATION OF THE FORM OF CONTACTOR DESIGN EQUATIONS

Several of the design equations developed in previous work were based upon certain assumptions that were justified because of the nature of the electromechanical device. Before these can be used in the design modification of an electrical contactor, the accuracy of these must be determined.

Of the group of design relationships involved, only two or three have to be verified, however, each one involves several variables. Some of the variables involved are easily varied such as the supply voltage E and the series resistance R. Other variables are difficult to vary and require that the contactor be open or unsealed to do this.

Two of the equations must be modified in order to predict accurately the total seating time of the plunger. This exists because of the nature of these contactors is such that compound spring action occurs during the plunger closure. Because of the great change in the spring constant at the point the power contacts make, the plunger essentially stops and waits until additional magnetic pull is obtained before continuing its travel. At room temperature and coil voltage slightly below rated this additional time caused by plunger hesitation is significant enough to have to be considered on existing designs.

Since the manner in which different variables influence the functioning time of a contactor varies, it is necessary to define a variable that will place the changes on a common reference. In addition this variable should have some other desirable properties. One of these properties is that, as this new variable approaches some limit, the functioning time of the contactor should approach infinity (non-operate condition). In addition it must contain the variables which influence the functioning time. The most convenient variable found to date is defined as the ratio of the coil pick-up current and the coil steady state current. The coil pick-up current i_p is determined by the same variables that determine the magnetic flux and the initial back tension on the plunger. These variables are the magnetic circuit reluctance, the coil turns and the initial plunger back tension. These may be represented in various ways but let this ratio of pick-up current to steady state current be a symbol h and called per unit pick-up current. Then the following relation may be given.

$$h = \frac{i_p}{i_{ss}} = \frac{R_t i_p}{E} = \frac{R_t}{E} \frac{(x_o + \alpha)\sqrt{2P_c}}{N\sqrt{\mu A}}, \qquad (1)$$

Where: R_t = total resistance presented to the supply emf E (ohms)

E = supply emf in the Thevenins theorem sense (volts)

N = total turns on the coil

P_o = effective back tension on the plunger (newtons)

x_o = effective travel of the plunger of the magnetic circuit (meters)

α = effective non-working length of the magnetic circuit in equivalent length of air (meters)

A = effective cross sectional area of the working air gap (square meters)

μ = permeability of free space ($4\pi \times 10^{-7}$ webers/amp - turn meter)

An examination of equation (1) shows that as the steady state current (i_{ss}) approaches the magnitude of the pick-up current (i_p) then the value of h approaches one. From this definition when h approaches the value one, then the functioning time approaches infinity.

The plunger pick-up time (t_p) is defined as the time interval from the instant the coil is energized until the magnetic pull on the plunger equals the plunger back tension. At this time plunger motion commences. In terms of the previous variables the pick-up time t_p is given as

$$t_p = \frac{N^2 \mu A}{(x_o + \alpha) R_t} \ln \frac{1}{1-h} \qquad (2)$$

Since the supply voltage E only occurs in the variable h then when E is varied t_p should have the form

$$t_p = C \ln \frac{1}{1-h} \qquad (3)$$

From the definition of the per unit pick-up current (h) can only have useful values between 0 and 1. Figure 7, curve a, shows the form of equation 3. As h approaches 0 by varying E (E must approach infinity) the t_p approaches zero.

E Variable

To check the form of equation 3 requires that data be obtained of the transient coil current build-up as a function of time with E as a variable. These data are shown by the traces in Figures 1, 2 and 3. Trace a, in Figure 1 is for the highest voltage or smallest h value. The influence of increasing h or decreasing E is shown by the next two traces b and c. Traces a, b and c of Figure 2 show a continuation of decreasing E or increasing h as well as those of traces a, b and c of Figure 3. Commencing with trace a, of Figure 1, and progressing through from a to c on the others and ending with trace c of Figure 3, covers the following values of h; 0.418, 0.460, 0.500, 0.535, 0.657, 0.767, 0.822, 0.920 and 0.99.

Examination of the traces shows several things taking place as h is increased by decreasing the supply voltage E. For the highest value of E, which is the smallest value of h, the coil current essentially has one smooth cusp. A small second cusp is evident. It is this second cusp that suggests that the plunger is hesitating or stopping in its travel. As h is increased by decreasing E this second cusp and its corresponding build-up becomes longer in time. For some values of h this second build-up and cusp show some additional humps. This suggested some additional plunger or contact rebound. This assumption is further verified by examining the contact voltage trace which is shown simultaneously along with the coil current. In those cases where additional changes occur in the second hump, definite contact chatter is recorded. This chatter is shown in the contact voltage trace as the short breaks in the horizontal traces. The contact voltage trace also shows that the first cusp is definitely determined by the closure of the power contacts.

Since the time involved in the second build-up and cusp is a significant portion of the total plunger seating time, it is desirable to account for this in the design equations. The shape of the second build-up and cusp suggests that this time interval consists of a second pick-up and transit time. Since it is not experimentally possible to determine the second pick-up current in these sealed contactors this must be determined by solving two equations simultaneously. One of these equations is that given by equation 3 and the other must be the equation of the plunger transit time. From previous work the form of the armature transit time equation is given as

$$k = \left\{ \frac{18\, M x_o^2 R_t}{E^2 h(1-h)\left[1-\left(\frac{\alpha}{x_o+\alpha}\right)^2\left(1+\frac{K x_o}{P_o}\right)\right]} \right\}^{1/3} \tag{4}$$

where:

M = effective mass of the plunger (kilograms)

x_o = effective travel of the plunger of the magnetic circuit (meters)

R_t = total resistance presented to the supply emf E (ohms)

E = supply emf in the Thevenin's theorem sense (volts)

h = per unit pick-up current (see equation 1)

α = effective non-working length of the magnetic circuit in equivalent length of air (meters)

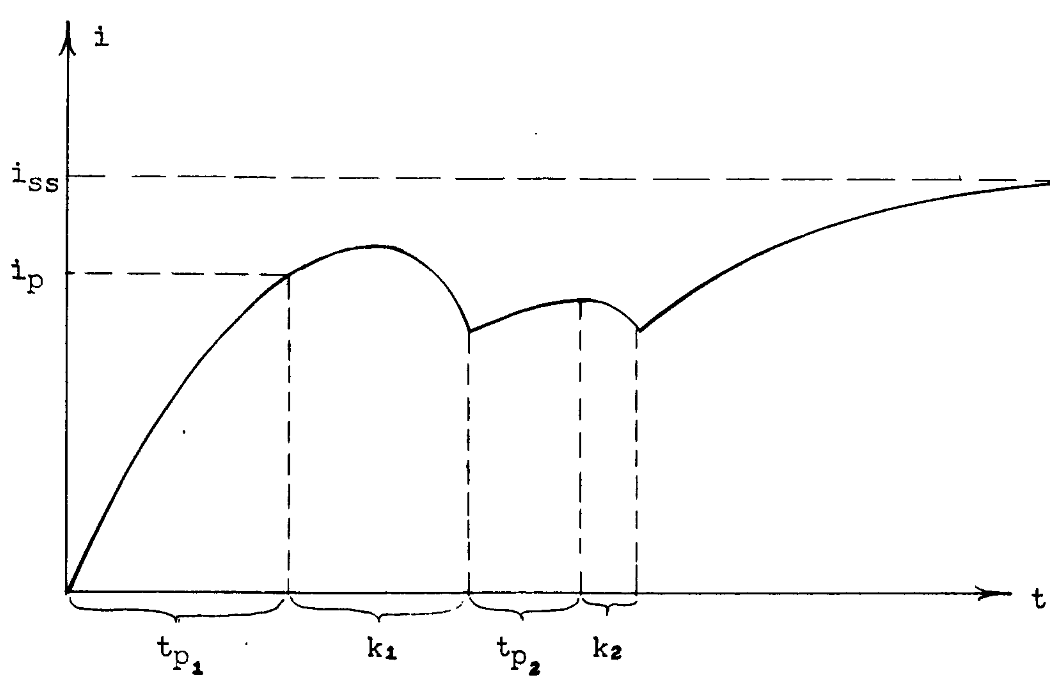
K = effective spring constant of the restoring spring acting through the distance x_o (newtons/meter)

P_o = effective restoring force on the plunger existing at the air gap x_o (coil unenergized) (newtons).

Since h is a function of E then when h is changed by varying E, the form of equation 4 is

$$k = D\left[\frac{h}{1-h}\right]^{1/3} \tag{5}$$

Curve a, in Figure 8, shows the form of k as h is changed by varying E. Comparison of curve a, in Figure 7, and curve a, in Figure 8, shows that the pick-up time t_p and the transit time k have some similiar points at h = 0 and h = 1 but in between the variations are somewhat different. To verify the form of equations 3 and 5 requires that experimental data of t_p and k as a function of h be obtained. This is obtained from the Figures 1, 2 and 3 in the following manner. The actual value of pick-up current is recorded, then knowing the current scale used with the oscillogram trace the time at which the current reaches the value is the pick-up time. This is shown graphically in the following sketch.



If only one build-up and cusp exist then the subscript one will not be needed or used. The pick-up current i_p associated with the first pick-up time t_{p1} is the only pick-up current that can be measured with these contactors when they are sealed. If the contactor is open then it might be possible to determine the second value of the pick-up current by setting the plunger to a position corresponding to that of the second pick-up point.

Table I shows the values of the pick-up and transit times as functions of the variable h as E is varied. These values are obtained from measurements made from the traces in Figures 1, 2 and 3.

Table I Variable E

Figure-Trace	h	t_{p1}	k_1	$t_{p2} + k_2$
		ms	ms	ms
1 - a	0.418	9.5	10	3.5
1 - b	0.460	10.5	11.5	5.5
1 - c	0.500	13.0	11.0	8.7
2 - a	0.535	14.0	11.5	10.0
2 - b	0.657	21.0	13.0	16.0
2 - c	0.767	29.0	17.5	25.0
3 - a	0.822	34.0	19.0	30.0
3 - b	0.920	49.5	29.5	42.0
3 - c	0.99	$t_{p1}+k_1=215$		65.0

The experimental values of t_{p1} and k_1 are plotted in Figure 9 as solid lines. The dashed lines are the results of using equations 3 and 5 with the value at h = 0.657 the same as the experimental. This is equivalent of determining a value of C and D for each of the equations by using the times measured at h = 0.657.

Only the sum of t_{p2} and k_2 can be measured directly from the traces since the second pick-up current can not be directly determined. If the sum of t_{p2} and k_2 is plotted against h_1 in Figure 9, a smooth curve is obtained that has an apparent h axis intercept of $h_1 = 0.333$. This implies that the h variable associated with the second pick-up and transit must be different from h_1. Since as h_1 approches 1 both the first and second cusps approach infinity then this suggests that h_2 approaches 1 as h_1 approaches 1. The h_2 variable then can be related to the h_1 variable as

$$h_2 = 1.5h_1 - 0.5. \tag{6}$$

The sum of t_{p_2} and k_2 is plotted against h_2 in Figure 10. A spot check of the shape of curve a in Figure 7 with this curve shows that the sum of $t_{p_2} + k_2$ is not a pick-up time function alone. Also a check of curve a in Figure 8 shows that the sum of t_{p_2} and k_2 is not a transit time function alone. This implies that the function being sought consists of two time intervals as the sum suggests. Let this sum of t_{p_2} and k_2 be called the second seating time t_{s_2} as

$$t_{s_2} = t_{p_2} + k_2. \tag{7}$$

From equations 3 and 5 the form of equation 7 is

$$t_{s_2} = C_2 \ln \frac{1}{1-h_2} + D_2 \left(\frac{h_2}{1-h_2}\right)^{1/3} \tag{8}$$

Since this equation involves two unknows at least two sets of values of t_{s2} and h_2 must be used. Using the pair $t_{s2} = 6.3$, $h_2 = 0.2$ and the pair $t_{s2} = 34$, $h_2 = 0.8$ gives $C_2 = 17.32$ and $D_2 = 3.87$. Using these values of C_2 and D_2 the second pick-up time t_{p2} and the second transit time k_2 can be calculated as functions of h_2. The computed values of t_{p2} and k_2 are plotted in Figure 10. The computed value of t_{s2} is plotted as the dashed line in Figure 10 and follow the experimental curve (solid line) fairly close. Since the transit time seems to be, in general, a small percentage of the total seating time, the form of this seating time could be approximated by equation 3 with $h_1 = h_2$.

From the previous explanation the total seating time t_s of the plunger as a function of h, when h is changed by varying E, can be represented as:

$$t_s = C_1 \ln\frac{1}{1-h_1} + C_2 \ln\frac{1}{1-h_2} + D_1\left[\frac{h_1}{1-h_1}\right]^{1/3} + D_2\left[\frac{h_2}{1-h_2}\right]^{1/3} \qquad (10)$$

where: h_1 = the ratio of the first pick-up current to the steady state coil current. The first pick-up current is measured by determining the smallest value of coil current which will result in complete closure of the plunger.

h_2 = the ratio of the second pick-up current to the steady state coil current. The second pick-up current in most cases can not be measured directly for a sealed device. If the device is open then the second pick-up current is the coil current that will seat the plunger when the plunger back stop is set so that the NO power contacts are just touching. For a sealed device the value of h_1 at which the second cusp just vanishes is used in the following equations as h_1'.

$$h_2 = ah_1 + b \qquad \text{where:} \quad a = 1/(1-h_1') \text{ and } b = -h_1'/(1-h_1')$$

to obtain h_2 as a function of h_1. Since h_1 is directly measurable the h_2 can be determined in terms of h_1 as $h_2 = ah_1 + b$.

Equation 10 then gives the form for the total seating time t_s as a function of the variable h_1 when h_1 is changed by varying the supply voltage E. If only one cusp exists then the equation for the total seating time, when h_1 is varied by changing E, is simplified to

$$t_{s1} = C_1 \ln\frac{1}{1-h_1} + D_1\left[\frac{h_1}{1-n_1}\right]^{1/3}$$

R Variable

Equation 1 shows that h may be changed by varying the total series resistance R_t. Equation 2 shows that the pick-up time t_p is a function of both h and R_t. If equation 2 is rearranged so that R_t is written as a function of h, then the form of the pick-up time equation becomes

$$t_p = \frac{G}{h} \ln \frac{1}{1-h} \qquad (12)$$

when h is varied by changing R_t. Curve b of Figure 7 shows a plot of the form of equation 12. This shows that the pick-up time can not approach zero by changing R to zero. To verify the form of equation 12, traces of coil current build-up with variable R_t were recorded. These traces are shown by Figures 4, 5 and 6. The top trace in each oscillogram is for the smallest value of h and increases for the next trace down. Starting with the top traces of Figure 4 and progressing down in each oscillogram through Figure 5 and 6 the values of h are approximately 0.4, 0.5, 0.6, 0.7, 0.8, 0.83, 0.9, 0.95 and 0.99. At the lowest value of h, which is also the smallest R_t, the coil current build-up consists of essentially one smooth cusp. As h is increased a second hump and cusp appear and the time involved with this second hump increases with increasing R_t. A direct comparison of the variation of this second hump and cusp with those of the traces in Figures 1, 2 and 3 is not possible since a different 25 ampere contactor had to be used to obtain the influence of R_t on the functioning times of the contactor. This change was necessary because the characteristics of the first 25 ampere contactor were changed during a contact load run when the contacts were over-heated. It appears that the spring used to hold the moving contact on the plunger was annealed during the particular contact load run. The contactor involved has not been unsealed yet because other tests are to be run on it before breaking the seal. Final evaluation of this overheating will be made after the contactor is unsealed.

The data obtained from Figures 4, 5 and 6 are plotted as curves in Figure 11 and are shown in Table II. The two solid lines are plots of the pick-up time t_{p_1} and the plunger transit time k_1 as functions of the per unit pick-up current h_1. When R_t is changed to vary h the form of the equation being verified as indicated by equation (4) is

$$k = H \left(\frac{1}{1-h}\right)^{1/3} \quad . \tag{13}$$

TABLE II VARIABLE R

Figure-Trace	h	t_{p_1}	k_1	$t_{p_2} + k_2$
		ms	ms	ms
4 - a	0.40	8.7	10.0	1
4 - b	0.50	10.0	10.3	1.8
4 - c	0.60	11.3	11.3	4.0
5 - a	0.70	14.0	11.5	8.0
5 - b	0.80	17.0	13.0	11.5
5 - c	0.83	18.5	14.0	13.0
6 - a	0.90	25.0	15.0	17.0
6 - b	0.95	35.0	16.0	20.0
6 - c	0.99	62.0	50.0	24.0

The form of equation (13) is shown by curve b in Figure 8 which gives the transit time k as a function of h when h is varied by changing R_t. To check whether the contactor behaves in the manner given by equation 13 as h is varied by changing R_t, the form of equation 13 is plotted as a dashed line in Figure 11. The dashed lines are the results of using equations 12 and 13 with the values at h = .7 the same as the experimental. Fairly close comparison exists between the experimental data and the theoretical data.

The total time involved with the second hump and cusp is called the total second seating time t_{s_2}. Values of t_{s_2} obtained from the oscillograms are plotted in Figures 11 and 12. In Figure 11, t_{s_2} is plotted against h_1 and in Figure 12 it is plotted against h_2 where h_2 is computed as explained in the notation of equation 10. This gives h_2 as

$$h_2 = 1.5h_1 - .5. \tag{14}$$

The curve of t_{s2} versus h_2 in Figure 12 shows that the form of the relationship describing this can not be the form shown by adding equations 12 and 13 since neither of these go to zero as $h \rightarrow 0$. This suggests that maybe the form of equation 8 could be used. Assume the form of t_{s2} is

$$t_{s2} = G_2 \ln\frac{1}{1-h_2} + H_2\left(\frac{h_2}{1-h_2}\right)^{1/3} \qquad (15)$$

then the value of G_2 and H_2 can be obtained by selecting two pairs of values of t_{s2} and h_2. If the two pairs of t_{s2} and h_2 are selected in this case the value of H_2 comes out to essential zero. This probably is due to the fact that there is some doubt that h_2 goes to zero when h_1 goes to zero. However, in this case the best fit using the form shown by equation 15 is $G_2 = 8.73$ and $H_2 = 0$. This gives

$$t_{s2} = 8.73 \ln \frac{1}{1-h_2} \qquad (16)$$

This curve is plotted as the dotted line in Figure 12 and shows close comparison with the experimental curve up to an h_2 value of 0.9. The experimental curve in Figure 12 appears to intercept the ordinate at about 25 for $h_2 = 1$. This fact suggests that the constant term in equation 14 may be a little small, since at $h_2 = 1$, the time should be infinite. Additional data would have to be obtained to check this point.

The results and discussion of this section indicates that the form of equation 2 and 4 which gives the pick-up time t_p and the plunger transit time k as functions of the variables E and R_t is accurate enough to determine the initial functioning times. In regard to the second build-up and cusp of the coil current the form of equation 2 and 4 is correct for E. When h_2 is changed by changing R_t it appears that best form of the equation for predicting this time is:

$$t_{s2} = G_1 \ln \frac{EN\sqrt{\mu A}}{EN\sqrt{\mu A} - (x_o+\alpha)(\sqrt{2P_o})R_t}$$

where: E = supply emf in the Thevenin's theorem sense (volts)

N = total turns on the coil

μ = permeability of free space ($4\pi \times 10^{-7}$ weber/ampere-turn meter)

A = effective cross sectional area of the working air gap (square meters)

x_o = effective plunger travel existing when power contacts are just touching (meters)

α = effective non-working length of the magnetic circuit in equivalent length of air (meters)

P_o = effective back tension on the plunger when power contact are just touching (newtons)

R_t = total resistance presented to the supply E (ohms)

G_2 = a constant determined at present by $G_2 = t_{s_2}/\ln(1/1-h_2)$ where t_{s_2} and h_2 are a measured pair of values.

If the second build-up and cusp seem to be a desirable characteristic of the contactor operation then additional development would need to be undertaken along the line of predicting the pick-up time when the coil is carrying a bias current. At present the second build-up and cusp is not considered necessary or desirable.

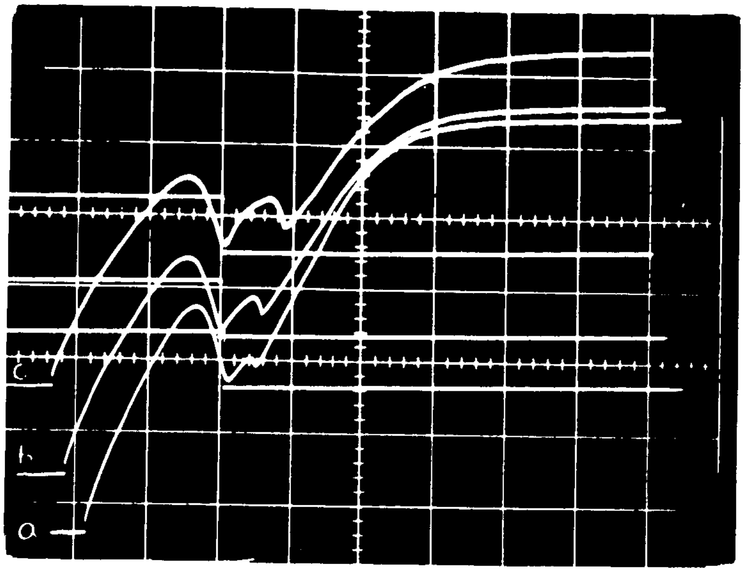


Figure 1

Coil Current Build-up and Contact Voltage on T_1-L_1 Contacts of 25 Ampere Contactor #1

Traces: Coil Current Build-up

(a) h = 0.418, E = 31 v dc

(b) h = 0.460, E = 28.8 v dc

(c) h = 0.500, E = 27.3 v dc

Oscillogram Data:

Time Scale: 10 ms/cm

Current Scale: 100 ma/cm

Contact Voltage 20 v dc

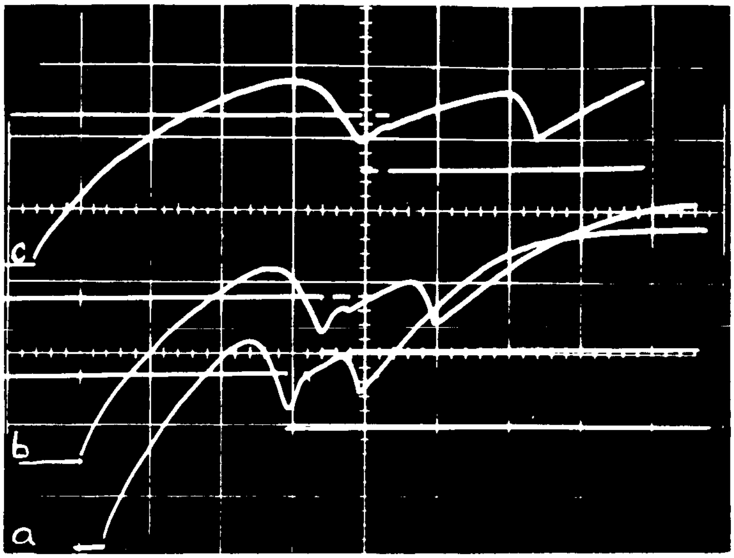


Figure 2

Coil Current Build-up and Contact Voltage on T_1-L_1 Contacts of 25 Ampere Contactor #1

Traces: Coil Current Build-up

(a) h = 0.535, E = 25 v dc

(b) h = 0.657, E = 20 v dc

(c) h = 0.767, E = 17.5 v dc

Oscillogram Data:

Time Scale: 10 ms/cm

Current Scale: 100 ma/cm

Contact Voltage 20 v dc

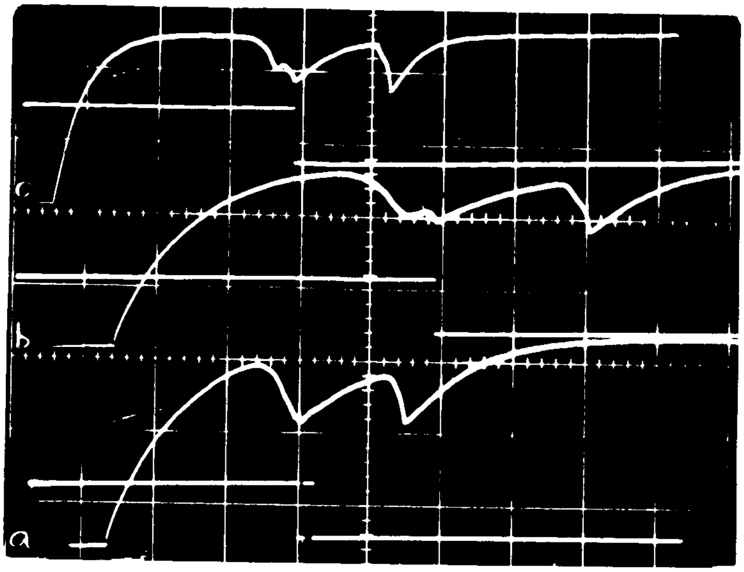


Figure 3

Coil Current Build-up and Contact Voltage on T_1-L_1 Contacts of 25 Ampere Contactor #1

Traces: Coil Current Build-up

(a) h = 0.822, E = 16 v dc

(b) h = 0.920, E = 14.1 v dc

(c) h = 0.99, E = 13.2 v dc

Oscillogram Data:

Time Scale: Traces a & b, 20 ms/cm

Trace c, 50 ms/cm

Current Scale: 100 ma/cm

Contact Voltage 20 v dc

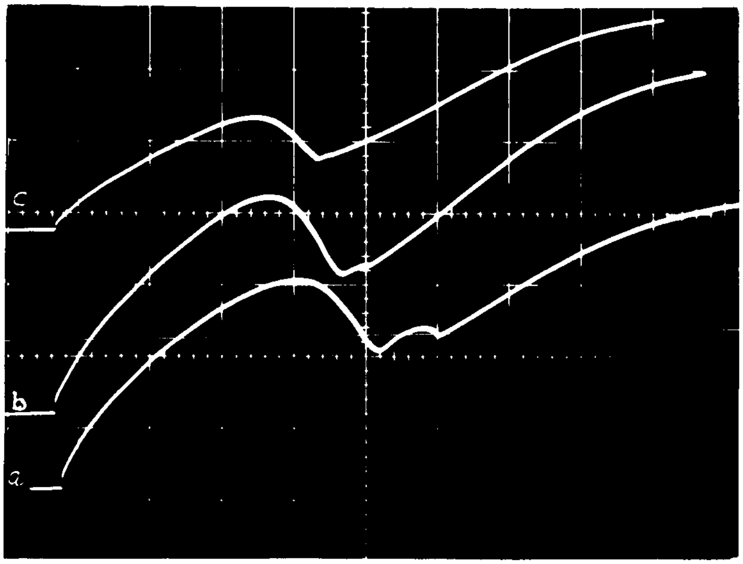


Figure 4

Coil Current Build-up for 25 Ampere Contactor
#2

Traces: Coil Current Build-up

(a) $h = 0.60$, $R_t = 87.5\Omega$

(b) $h = 0.50$, $R_t = 72.8\Omega$

(c) $h = 0.40$, $R_t = 57.7\Omega$

Oscillogram Data:

Time Scale: 5 ms/cm

Current Scale: Traces a & b, 95 ma/cm

Trace c, 195 ma/cm

$E = 33.5$ v dc

$i_p = 231$ ma

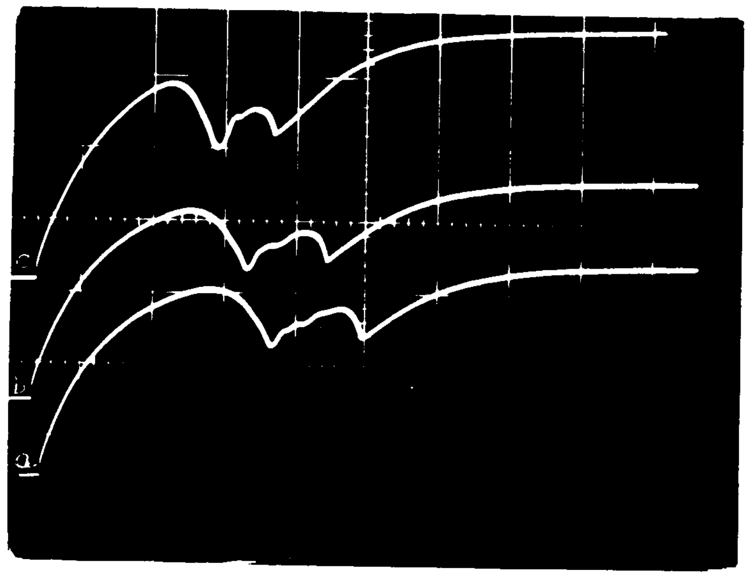


Figure 5

Coil Current Build-up for 25 Ampere Contactor #2

Traces: Coil Current Build-up

(a) $h = .83$, $R_t = 121\Omega$

(b) $h = .80$, $R_t = 116.5\Omega$

(c) $h = .70$, $R_t = 102\Omega$

Oscillogram Data:

Time Scale: 10 ms/cm

Current Scale: 95 ma/cm

$E = 33.5$ v dc

$i_p = 231$ ma

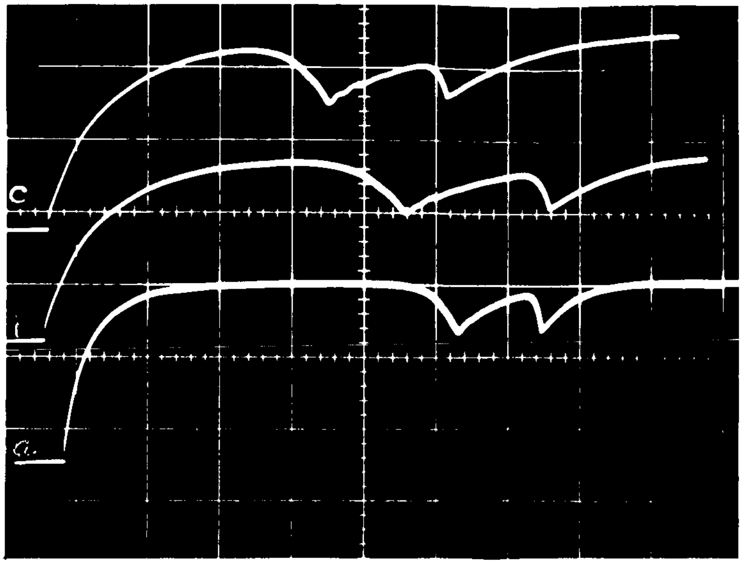


Figure 6

Coil Current Build-up for 25 Ampere Contactor #2

Traces: Coil Current Build-up

(a) $h = .99$, $R_t = 144.5\Omega$

(b) $h = .95$, $R_t = 138.5\Omega$

(c) $h = .90$, $R_t = 131.0\Omega$

Oscillogram Data:

Time Scale: Trace a, 20 ms/cm

Traces b & c, 10 ms/cm

Current Scale: 95 ma/cm

$E = 33.5$ v dc

$i_p = 231$ ma

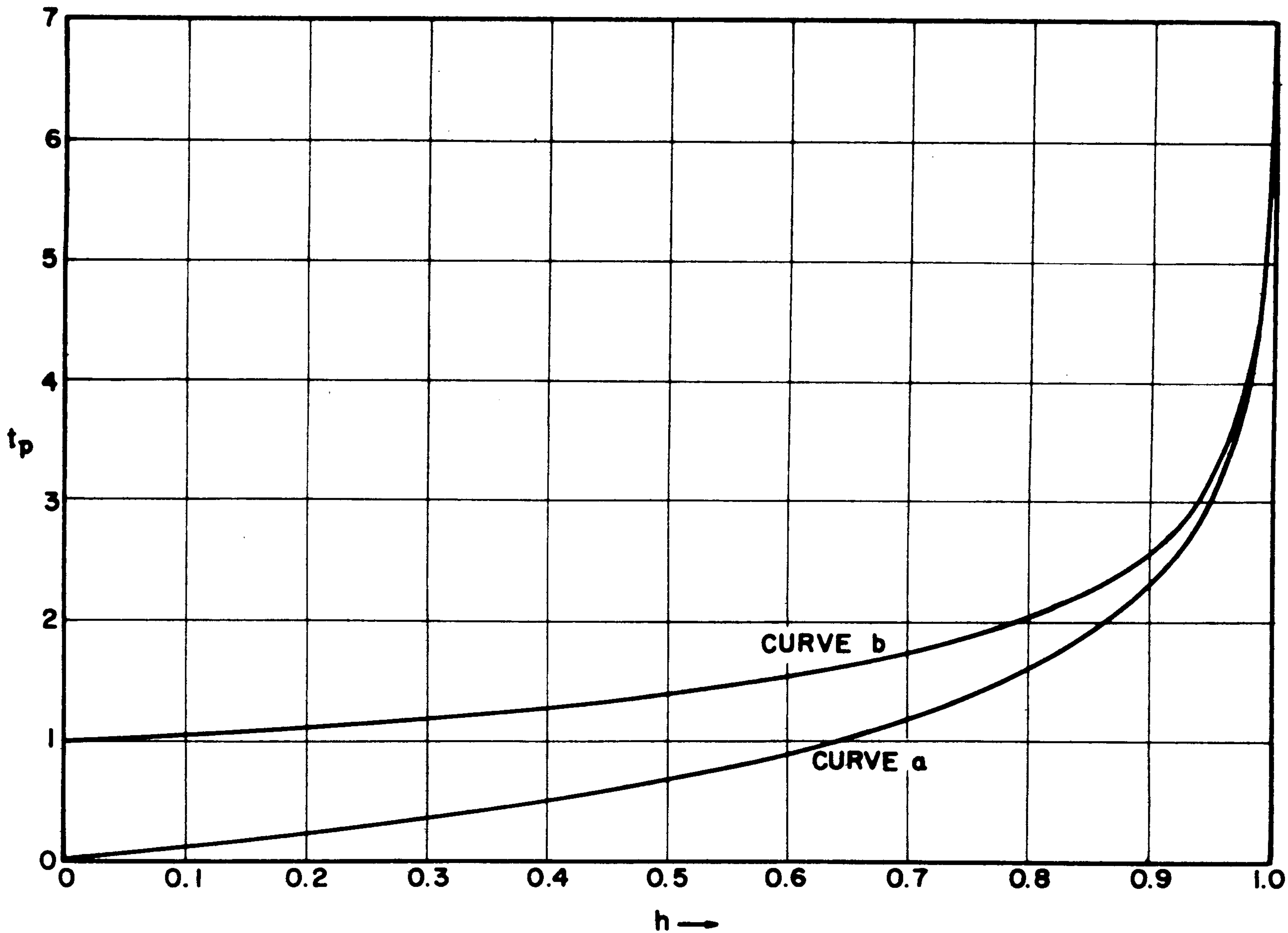


Figure 7 Normalized Pick-up Time (t_p) as a Function of the Per Unit Pick-up Current (h)

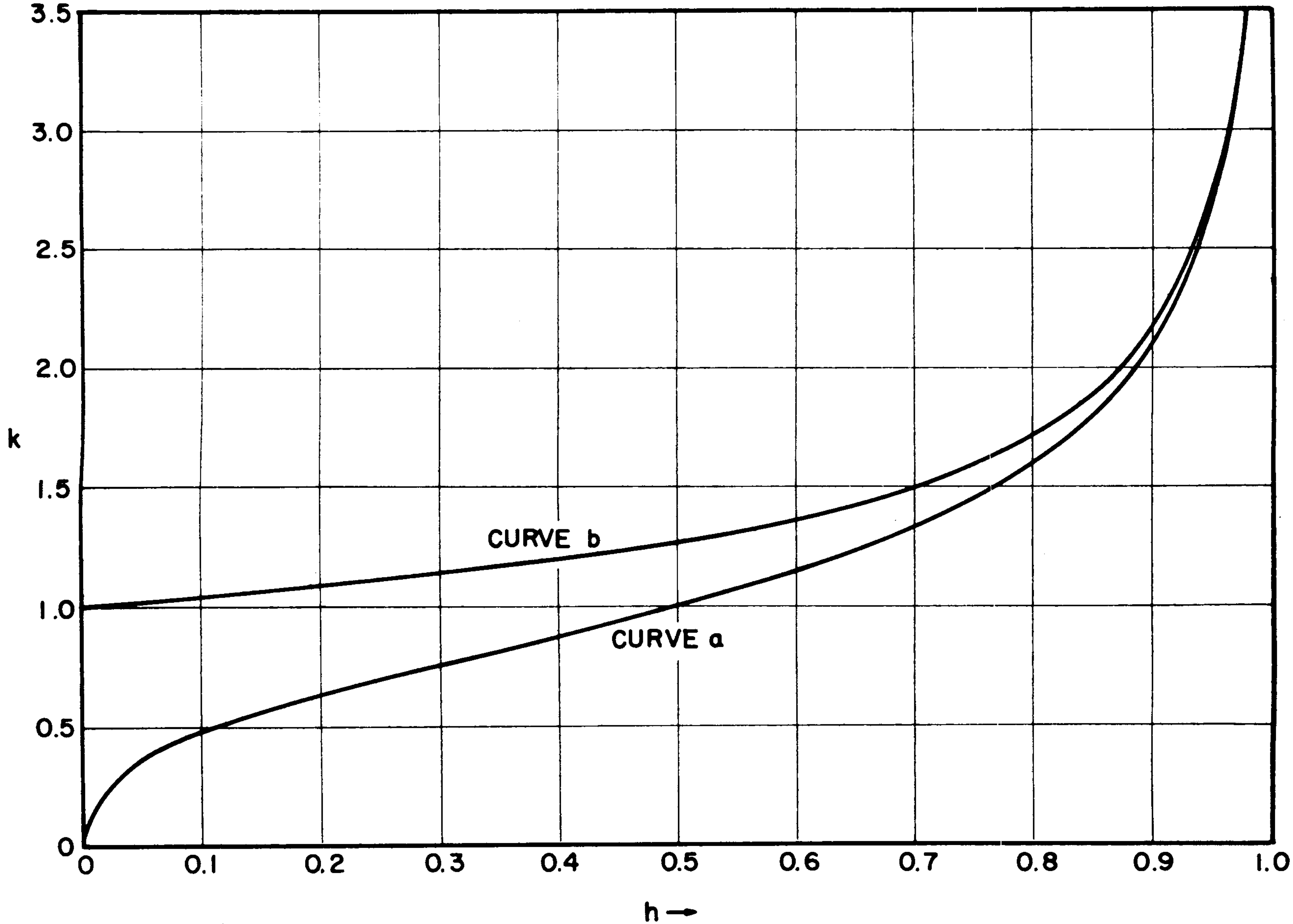


Figure 8 Normalized Plunger Transit Time (k) as a Function of the Per Unit Pick-up Current (h)

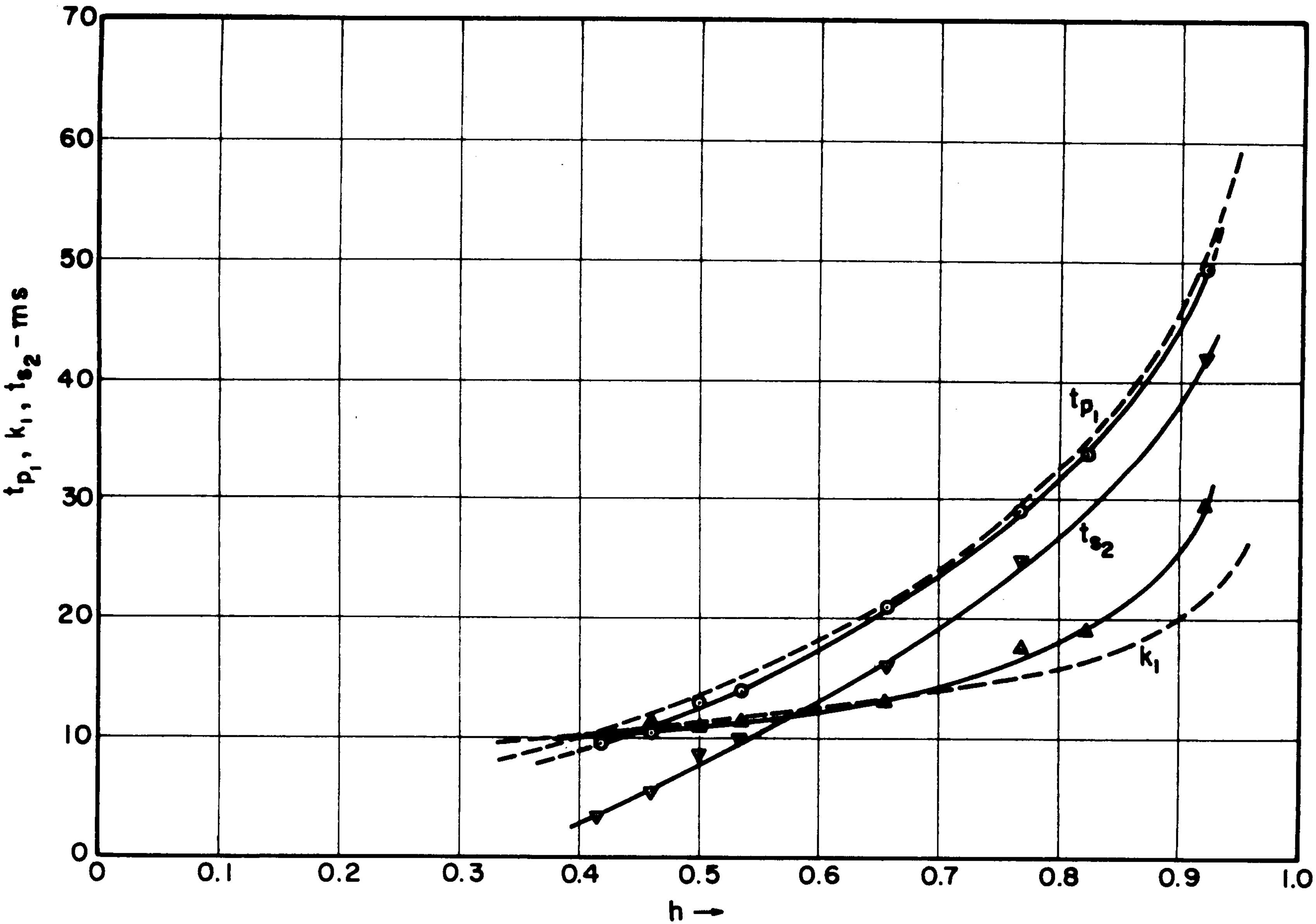


Figure 9 Experimental Values of t_{p1}, k_1 and t_{s2} as a Function of h_1 for 25 ampere Contactor #1 when E is Varied

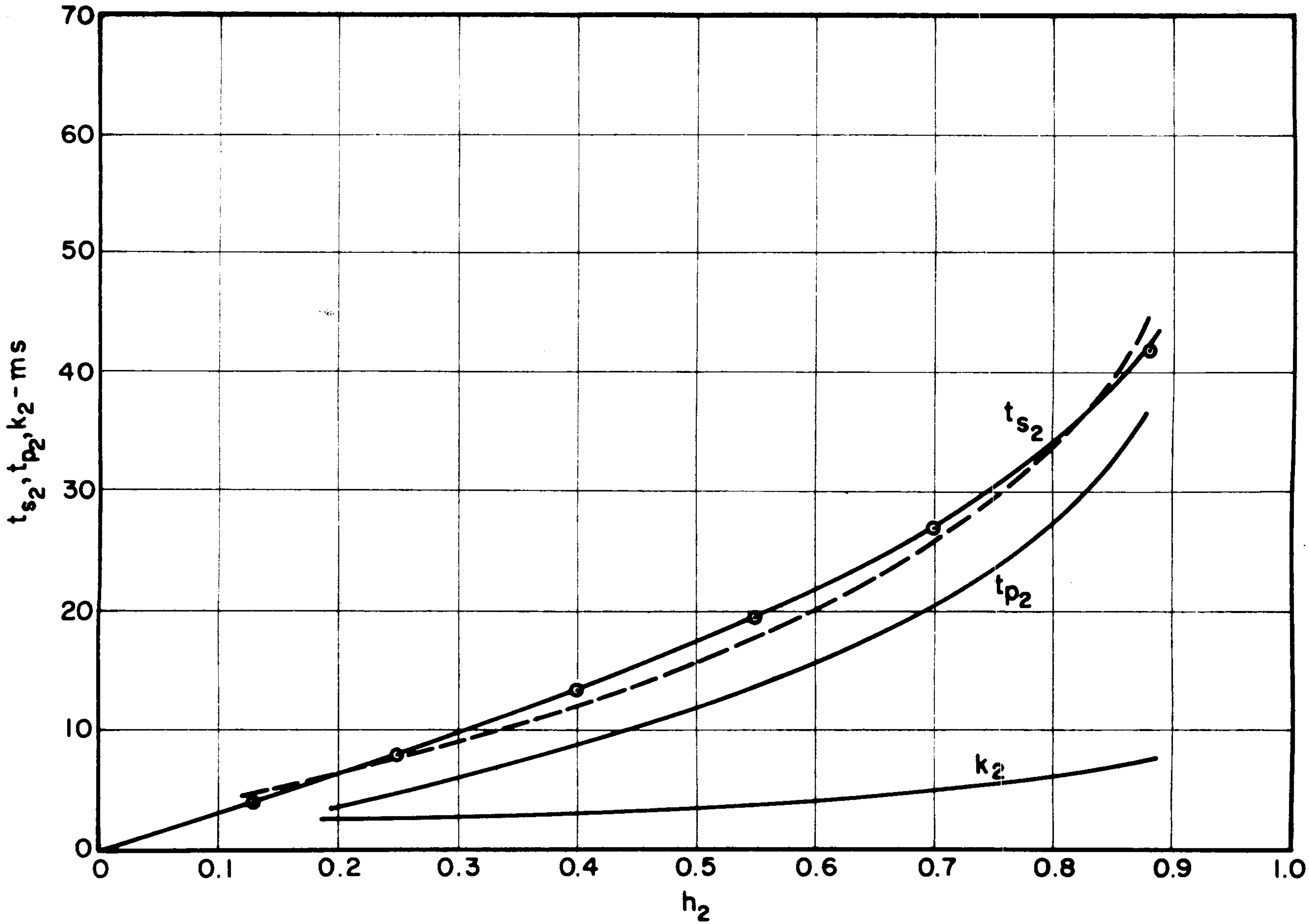


Figure 10 Measured Values of t_s and Computed Values of t_{p2}, k_2 and t_{s2} as a Function of h_2 When E is Varied for 25 ampere Contactor #1

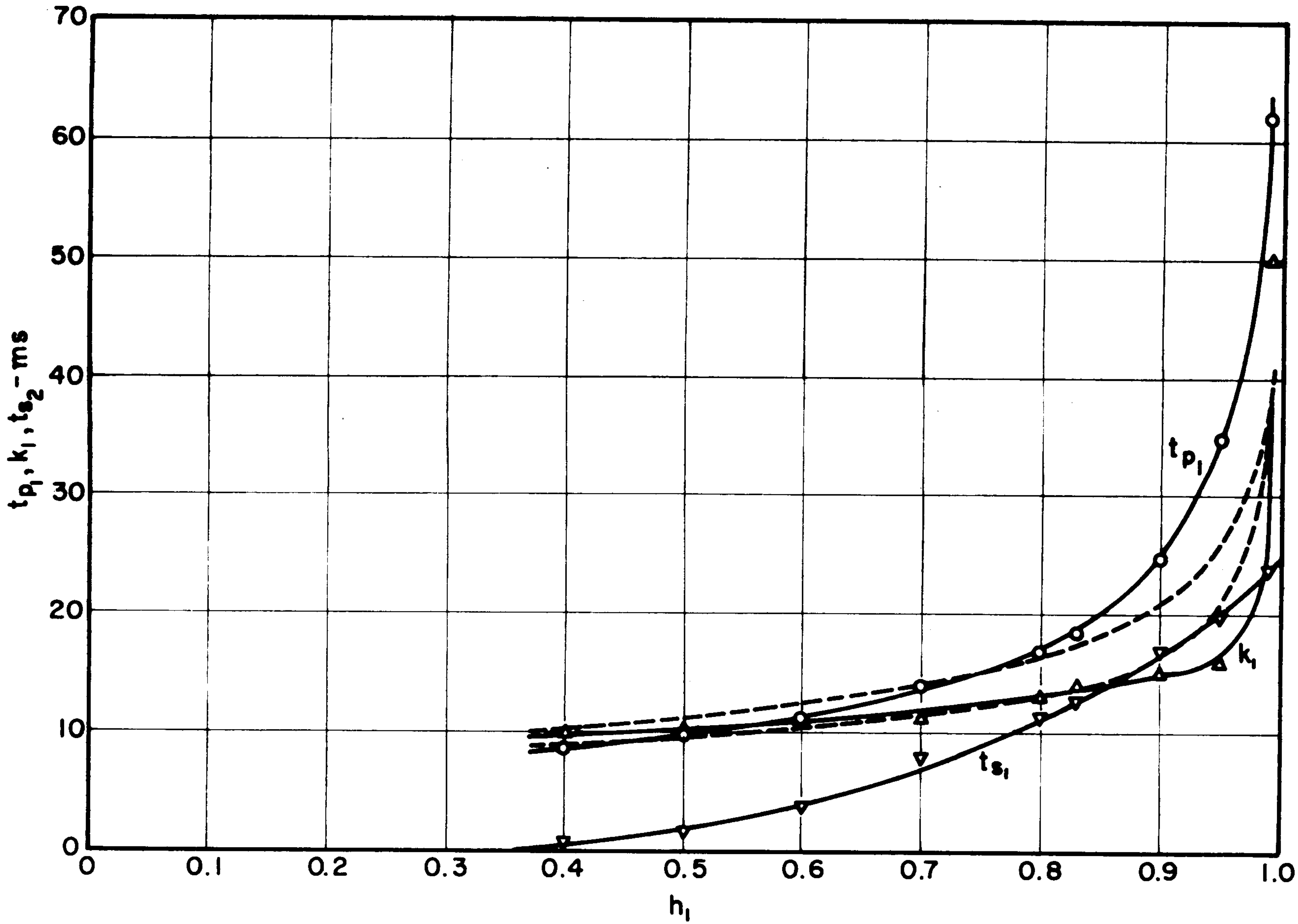


Figure 11 Measured Values of t_{p1}, k_1, t_{s2} and Computed Values of t_{p1} and k_1 as a Function of h_1 When R_t is Varied for 25 ampere Contactor #2

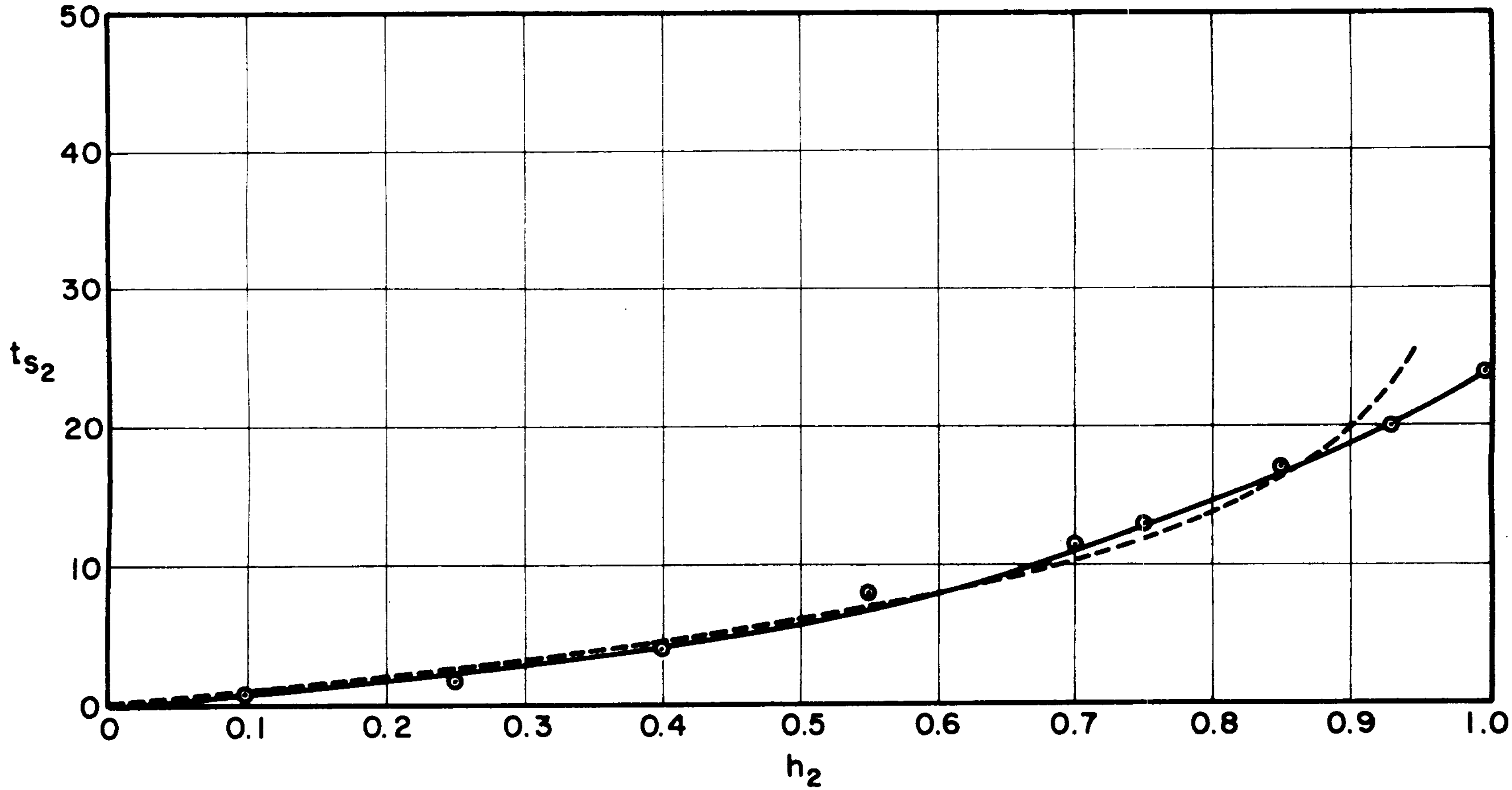


Figure 12 Measured and Computed Value of t_{s2} as a Function of h_2 when R_t is Varied for the 25 Ampere Contactor #2

AN APPLICATION OF THE THEORY OF DESIGN

BY

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SCHOOL OF ELECTRICAL ENGINEERING

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INTRODUCTION

A concept is necessary before a design can be started. It is usual to conceive of a model which has the desired characteristics. Most often the physical device which is constructed is somewhat different than the conception of the mental model. There may be many concepts which are impossible to translate into physical reality. To assist in the process of translation from a mental model to a physical object, the process known as design is utilized.

Synthesis is the combining of separate elements of thought into a whole or a combination of elements into a completed unit. Analysis of a device or unit may be accomplished after the device is finished. It is seen that analysis is the opposite of synthesis. Design implies that synthesis follows some logical procedure according to a plan and in some instances the design involves the formulation of a logical plan which may be followed in the building up of the elements which compose the whole or device.

Many schemes have been developed whereby physical devices may be studied by analytical means. A knowledge of the physical laws which govern a given device makes it possible to analyze the interrelation of electrical, mechanical, thermal and other characteristics which a device exhibits under certain conditions.

The realm of synthesis and design have not been explored sufficiently to have logical procedures which may be used in the building up process. Intuition seems to have been the process most often used to transform the concept into physical reality. Very frequently, the translation process has been accompanied by much trial and error or cut and try.

It is the purpose of this paper to investigate a scheme which may be used in the design of a device after some indication has been given of parameters which must be fixed. The indication of the fixed parameters is intertwined with synthesis and design. The fixing of a set of parameters is called fixing a set of specifications. Calling for a given set of specifications implies that the specifier knows what he needs which may not always be the case.

SOME DESIGN PROBLEMS AND SOLUTIONS RESULTING FROM DEPENDENT SPECIFICATIONS

Before considering a specific system with which to illustrate the points of interest, a brief discussion of the definitions and ideas used is given in order to clarify the procedures and examples.

The criteria with which to design a system as adopted in this work can be grouped in two classes. The first class, called the primary specifications, is that set of parameters of the system which are not specified by the designer. The second class is that set of specifications chosen by the designer. There is one relationship between these two classes which must not be violated. This relationship may be stated as follows; any specification chosen by the designer must not contradict any primary specification. The total specifications for design are given by the union of the above two classes. This total set will be denoted by (S) called the specification set.

As given in the paper "Theory of Design" the types of relationships and parameters having the following properties are the only ones being considered in the design procedures. 1

Given a set of N parameters (denoted by (P_N)) and V relationships (denoted by (f_V)) they form a system if the following conditions hold:

(i) $0 < V \leq N$
(ii) (P_N) = union of parameters belonging to each relationship.
(iii) There does not exist a proper subset of the V relationships such that all the parameters belonging to this subset is different than all the parameters belonging to the remaining relationships.

The problems to be discussed will be limited to the type of primary specifications which refer to a particular system. (In this paper the particular system will be a D.C. relay.) That is, all problems will be based on the assumption that the specification parameters (S) are contained in the system parameters (P_N). Two solutions are defined for the design problem as restricted to the above limitations. The first type solution will be called a general solution. The other type will be called a particular solution. Using the following symbolism, these type solutions are defined:

Let the parameters belong to each relationship be denoted by $(P)_i$, where i denotes f_i.
Given (S) and some $(P)_i$ such that the parameters of $(P)_i$ are common to (S) except for exactly one parameter of $(P)_i$, denote this parameter by $(\overline{P})_i = (P)_i - (S) \bigcap (P)_i$.
(The parameter $(\overline{P})_i$ is said to be specified by f_i.)

A general solution for a set of specifications (S) is said to exist if each relationship f_i and the parameters (S) and $((\overline{P})_J), J \neq i$ determines $(\overline{P})_i$. A set of specifications which yields a general solution is said to be an independent set.

A particular solution for a set of specifications (S) exists if the following properties are satisfied:

(i) Each specification of (S) is a range of values for a parameter of the system.

(ii) The system parameters common to (S) are in the range of values specified by (S).

To illustrate the difference between a particular solution and a general solution the following example is given for a relay system. The system is represented by the matrix in Figure (1) where the N parameters are listed vertically and the V relationships horizontally. The identification of the parameters and relationships is given below:

δ = diameter of bare wire
η = design stability point
$\mathcal{R}_p$ = reluctance of magnetic circuit presented to the coil with the armature in the pick-up position
E = open circuit supply voltage (D.C.)
k = armature transit or travel time
K = effective spring constant of spring system
ℓ = coil length
M = effective armature mass
N = coil turns
P = total power (steady state) supplied to relay coil circuit by voltage source
P_o = back tension at pick-up
R_c = coil resistance
R_t = total resistance of relay coil circuit presented to voltage source
$R_s = R_t - R_c$
s = outside coil diameter
t_p = armature pick-up time
t_s = armature seating time
x_o = armature air gap
μA = (permeability of free space)(effective cross sectional area of working air gap)
f_1 = stability equation
f_2 = pick-up time equation
f_3 = transit time equation
f_4 = power equation
f_5 = circuit resistance equation
f_6 = coil resistance equation
f_7 = coil turns equation
f_8 = stability inequality
f_9 = magnetic circuit equation
f_{10} = pole face equation
f_{11} = total time equation

In addition to the above N parameters and V relationships the following quantities are involved when using the above relationships, however, they are not considered parameters because of their restricted range of values.

β = ratio of core diameter to outside coil diameter
ρ = resistivity of conductor material
α = air equivalent of non-working part of the magnetic circuit when using a series representation

σ = ratio of twice the coil bobbin wall thickness to the outside coil diameter
g = winding space factor
a = ratio of the pole face diameter (d') to the core diameter (d)

Using this system as an example, assume that the following specifications were to be satisfied by this system:

design stability	$\eta \leq .5$
bare wire size	$\delta = (4)10^{-3}$
coil length	$\ell \leq 2$ inches
outside coil diameter	$s \leq 1$ inch
armature mass	$M \leq 20$ gms mass
external circuit resistance	$R_s = 0$
supply voltage	$E = 28$ v dc
back tension	$P_o \geq 200$ gms force

Applying the set of specifications (η, δ, ℓ, s, M, R_s, E, P_o) to the matrix in Figure 1 shows that this set of specifications has a general solution. This is indicated on the matrix by the fact that each f_i column has exactly one parameter denoted by $\square$ indicating $(\bar{P})_i$. Since there is a general solution then there is a particular solution obtained by substituting the above values in the design equations, using the sequence of the general solution. When this is done the remaining parameters of the system are as follows:

$$\mu A = .606 \ \frac{\text{Maxwell-inches}}{\text{amp-turn}}$$
$$N = (1.9)10^4 \text{ turns}$$
$$R_c = 2600\ \Omega$$
$$R_t = 2600\ \Omega$$
$$P = .3 \text{ watts}$$
$$x_o = .0126 \text{ inches}$$

$$t_p = 23 \text{ ms}$$
$$K \leq (1.59)10^4 \text{ gm force/inch}$$
$$\mathcal{R}_p = .0415 \text{ amp-turn/Maxwell}$$
$$k \leq 9.9 \text{ ms}$$
$$t_s \leq 32.9 \text{ ms}$$

These calculations were carried out using the following restricted parameter values:

$\beta + \sigma = .6$ (which yields maximum pull per watt)
$\rho = (.679)10^{-6}$ ohm-inches
$\alpha = x_o$
$\sigma = .1$
$g = .6$
$a = 1$

This example illustrates the ease with which particular solutions can be found for a set of specifications when they exist if a general solution exists for that same set of specifications. The only problems which arise are those of physical realizability which depend upon the particular values specified.

The more complicated design problem arises when, for a given set of specifications, no general solution exists. This arises when the number of specifications exceeds the number N-V which for this system is 8. Also, in many cases this situation arises for specifications in number ≤ 8. Specifically a dependent set of specifications (S) has the following properties:

For a system, (f_i), which has specifications, (S) then either,
(1) some particular f_i specifies $(\bar{P})_i$ which belongs to (S), or
(2) there is an f_i and $f_J (f_J \neq f_i)$, each of which specify the same $(\bar{P})$.

When (1) or (2) occur then a general solution does not exist. However, depending upon the values of a particular set of specifications not having a general solution, it is possible that a particular solution exists. However, this is usually not the case unless very loose numerical boundaries are placed on the specifications.

The following example illustrates this point along with a methodical scheme of investigating these situations. Assume that for the relay system as described earlier, the following specifications were used:

$$\begin{aligned} \eta &\leq .5 \\ E &= 28 \text{ v dc} \\ P &\leq .5 \text{ watts} \\ R_s &\leq 100\ \Omega \\ \ell &\leq 1 \text{ inch} \\ s &\leq .5 \text{ inch} \\ x_o &\geq 15\ 10^{-3} \text{ inches} \\ P_o &\geq 50 \text{ gm} \end{aligned}$$

Using the system matrix of figure 2, it is immediately evident that there is no general solution for $(\eta, E, P, R_s, \ell, s, x_o, P_o)$ since f_1 and f_7 both specify the number of turns. To check for a particular solution the following procedure is used:

> Using the equality sign in the above specifications, compute down to N_1 from f_1 and N_7 from f_7. In this case, $N_1 = (13.5)10^4$ and $N_7 = (10.2)10^4$ turns, but in f_1 there is no way to decrease N_1 using the inequalities above. Therefore, unless N_7 determined by f_7 can be increased to a value of $(13.5)10^4$ turns, using the inequalities above, there is no particular solution. Inspection of f_6 and f_7 implies that using the specifications above $N_7 \leq (10.9)10^4$ turns. Therefore, there is no way to meet the above specifications except possibly by using different values for the restricted parameters α, β, σ, g and a.

This example illustrates the easy way with which an infinite number of relay specifications can be shown to be impossible using a methodical logic process. Also, it points out the strong implication for no particular solution when there is no general solution. The implication becomes weaker as the range of values for the specifications becomes larger.

DESIGN OF UNCOMMON CLASSES OF RELAYS

To further illustrate the advantage of a logical design procedure, consider a request for a relay with the following specifications:

(1) must preform without failure under k_1 g's
(2) must switch K_2 volts at K_3 amperes
(3) must fit into a space $K_4 \times K_5 \times K_6$ inches.

○	δ						δ	δ									
○	η	η	η	η													
□	$\mathcal{R}_p$									$\boxed{\mathcal{R}_p}$							
○	E	E		E	E												
□	k			$\boxed{k}$								k					
□	K								$\boxed{K}$								
○	ℓ						ℓ	ℓ									
○	M			M													
□	N	N	N					$\boxed{N}$									
□	P				$\boxed{P}$												
○	P_o	P_o							P_o								
□	R_c					R_c	$\boxed{R_c}$										
□	R_t	R_t	R_t	R_t	R_t	$\boxed{R_t}$											
○	R_s					R_s											
○	s						s	s			s						
□	t_p		$\boxed{t_p}$									t_p					
□	t_s											$\boxed{t_s}$					
□	x_o	$\boxed{x_o}$	x_o	x_o					x_o	x_o							
□	μA	μA	μA							μA	$\boxed{\mu A}$						
		F	G	J	E	D	C	B	H	I	A	K					
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 1

□	δ						$\boxed{\delta}$	δ									
○	η	η	η	η													
□	R_p									$\boxed{R_p}$							
○	E	E		E	E												
	k			k								k					
□	K								$\boxed{K}$								
○	ℓ						ℓ	ℓ									
	M			M													
	N	$\boxed{N}$	N					$\boxed{N}$									
○	P				P												
○	P_o	P_o							P_o								
□	R_c					$\boxed{R_c}$	R_c										
□	R_t	R_t	R_t	R_t	$\boxed{R_t}$	R_t											
○	R_s					R_s											
○	s						s	s			s						
□	t_p		$\boxed{t_p}$									t_p					
	t_s											t_s					
○	x_o	x_o	x_o	x_o					x_o	x_o							
□	μA	μA	μA							μA	$\boxed{\mu A}$						
		A*	I		D	E	F	G*	C	B	A						
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 2

Any relay which satisfies these three requirements is an acceptable design. This set is referred to as the primary set of specifications. There also exists a secondary set of specifications chosen by the designer as mentioned earlier. In other words, if the primary specifications do not completely determine N-V parameters of the relay design matrix, the designer then selects some additional design matrix parameters at his discretion. Many times economic factors influence this selection.

The design process, as explained earlier, requires that the specifications for the device be stated in terms of known parameters. Comparing the specifications listed above with the parameters listed on the design matrix it is easily seen that there is a gap between the specifications and the beginning of the logical design process. This gap may be filled by another design, say a contact design matrix, relating the parameters determined by the primary specifications to the parameters in the actuator design matrix. From the contact design matrix the primary specifications applicable to the actuator design matrix are determined. Investigation of a suitable contact design matrix is under development at the present time.

Ideally, all significant parameters of a relay system could be listed on a design matrix, allowing the engineer to proceed in a logical manner, directly from the specifications to the final design. In other words, the present design matrix is sub-matrix of a larger, more complete relay design matrix.

As an example of this overall procedure, consider the set of specifications listed above. The first condition is related to shock. The second condition refers to the design of the contacts (switching capacity.) Assume that a contact design matrix is used to obtain a set of contacts satisfying this requirement. The parameters specified by this contact design matrix would be x_o, P_o and M.

Considering the third condition, the parameters specified are ℓ and s. These plus the parameters specified by conditions 1 and 2 specify P_o, x_o, M, ℓ, and s. Since it is possible to fix eight parameters and obtain a design with this design matrix, the designer now has an opportunity to select three other parameters of his choice.

The choices are δ, (in order to use standard size wire), η (for good stability), and R_s (the external series resistance.) Combining the primary specifications and the chosen parameter, the parameters η, δ, R_s, ℓ, s, x_o, M, and P_o are fixed. Inherent restrictions are on β, σ, g', a and α.

In order to illustrate the result of selecting a set of parameters which are not usually selected, the following example using numerical values is given.

$\eta = .5$	$P_o = 70$ gm	$\beta = .4$
$\delta = 5 \times 10^{-3}$"	$x_o = 20 \times 10^{-3}$"	$\sigma = .1$
$\ell = 1$"	$M = 8$ gm	$\delta' = .47$
$s = .75$"	$R_s = 0$	$a = 1$
		$\alpha = x_o$

Applying the set of parameters η, δ, R_s, ℓ, s, x_o, M and P_o to the actuator design matrix, shown in Figure 3 shows that a general solution exists. The order of solution of the equations is shown by the alphabetical listing near the bottom of the matrix. A circle is used to indicate the parameters in the original set and squares are used to indicate the parameters that are specified or fixed by the relationships. A square in the relationship column indicated the parameter specified by that relationship.

Calculation of the numerical values of the fixed parameters are listed as:

$\mu A = 0.225$	$E = 17.55$ volts	$R_p = 0.1775$
$N = 4480$ turns	$P = 1.24$ watts	$k = 2.9$ ms
$R_c = 253$ ohms	$t_p = 3.1$ ms	$t_s = 6.0$ ms
$R_t = 253$ ohms	$K = 3500$ grams/in	

Examination of the specifications will show that neither coil voltage, coil resistance or coil power was specified. The quantities specified were that this device was to stand certain shock requirement, switch a certain load and occupy a certain space. It will be observed by selecting normal values for P_o, x_o, ℓ, s, R_s, M, δ and η that somewhat normal values result. In other words, the coil resistance, coil power, and coil voltage obtained are the ordinary or normal values encountered in relays. This illustrates the advantage of having a versatile design procedure which does not force the designer to always start with the same set of specifications. This allows the designer to take the customers specifications instead of the ones needed for his particular design routine.

DESIGN PROBLEMS RESULTING FROM MODIFICATION

A design modification involves the same limitations in regard to dependency as any other set of specifications. This means that once the mathematical model of the device has been developed then only N-V of the parameters can be selected in order to obtain a general solution; where N is the number of parameters and V the number of relationships. In addition, these N-V parameters must be checked for dependency before it is known whether a general solution exists or not.

For the model involved in this discussion only 8 parameters may be selected to check for dependency. In a design modification this condition is not usually appreciated since these N-V, or 8 parameters in this case, must include the parameters desiring to be modified and those that are to be held constant. This means that one trying to retain certain parameters of a device having desirable values and trying to change certain other parameters having undesirable values find usually that this number is far greater than N-V. An example of a particular design modification will help illustrate some of these points.

Consider the case of modifying a certain relay to carry additional contact springs. This implies that it is desired to use the same relay frame, armature, core and individual contact springs in addition to the same coil voltage. Using the relay frame implies that the same overall coil space is available at least as far as coil height is concerned. Translating these desires into a set of parameters could result in more than 8 parameters being involved. In fact if the coil diameter and wire size are listed as being fixed then the number of parameters is easily more than 8. Since coil diameter had only an upper bound on it and wire sizes are only restricted to certain

○	δ						δ	δ						2			
○	η	η	η	η										1			
□	$\mathcal{R}_p$									$\boxed{\mathcal{R}_p}$				11			
□	E	$\boxed{E}$		E	E									16			
□	k			$\boxed{k}$								k		17			
□	K								$\boxed{K}$					10			
○	ℓ						ℓ	ℓ						3			
○	M			M										12			
□	N	N	N					$\boxed{N}$						6			
□	P				$\boxed{P}$									18			
○	P_o	P_o							P_o					8			
□	R_c					R_c	$\boxed{R_c}$							7			
□	R_t	R_t	R_t	R_t	R_t	$\boxed{R_t}$								14			
○	R_s					R_s								13			
○	s						s	s			s			4			
□	t_p		$\boxed{t_p}$									t_p		15			
□	t_s											$\boxed{t_s}$		19			
○	x_o	x_o	x_o	x_o					x_o	x_o				9			
□	μA	μA	μA							μA	$\boxed{\mu A}$			5			
		H	G	I	J	F	C	B	D	E	A	K					
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 3

numbers a particular solution to this design modification was possible. A general solution is not possible because more than 8 parameters would have been selected if s and δ were included.

The eight parameters selected were P_o, η, E, M, ℓ, μA, x_o and R_s. The parameters P_o, η, and E were selected because these were to be changed. The parameters M, ℓ, μA, x_o, and R_s were selected in order to use the same relay frame, coil bobbin and armature as existed on the original relay. Actually the spring constant K would also change as the number of contact springs were changed but this is related to the P_o and since this was a modification the ratio of P_o to K was assumed the same. Since the original relay was satisfactory with the ratio of P_o to K existing, so will be the modification as far as the ratio of P_o to K is concerned. Applying the parameters (P_o, η, E, M, ℓ, μA, x_o and R_s) to the matrix in Figure 4 shows that only the relationships f_8, f_9 and f_{10} give direct solutions to a parameter in each of them. In order to distinguish between the original 8 parameters and the other parameters which are determined by the 8 via the relationships when using the design matrix in Figure 4, two sets of symbols are used. In column one a circle is used in the rows involving the original 8 parameters. When a parameter is determined by a relationship, such as s in f_{10}, then a square is used in the f_{10} column and the first column for that row. Figure 4 shows the 8 parameters marked on the design matrix and that three of the other parameters were specified by the use of the relationships. This means that a general solution, as defined previously, can not be determined by simple procedure as used in the previous examples. In fact a general solution might not exist. However, the set has not been shown to be dependent so that a general solution is still possible at this point of the procedure.

Careful examination of the so far unused relationships show that f_1, f_5, f_6, and f_7 collectively have only four unspecified parameters. These parameters are δ, N, R_c and R_t. Since there are four relationships and four unspecified parameters involved with this group, then there exists a possibility of solving these relationships simultaneously for the four unspecified parameters. At this stage of the procedure the particular form of the relationship determines whether a solution is possible. With the particular equations used in this design, it was possible to solve the four equations f_1, f_5, f_6 and f_7 simultaneously for δ, N, R_c and R_t. Therefore, these variables are specified and are so marked on the design matrix by Δ's in Figure 5. Figure 5 is a continuation of Figure 4. Figure 5 shows the original 8 parameters plus the parameters specified by the use of f_8, f_9, f_{10} directly and f_1, f_5, f_6 and f_7 simultaneously. Inspection of the design matrix in Figure 5 shows that t_p is now specified by relationship f_2, k is specified by f_4 and lastly t_s is specified by f_{11} since t_p and k are fixed. The order of selection or specification is indicated by the numbers in the 15th column. The next to the bottom row shows the order of using the relationships to specify the other parameters. When solving relationships simultaneously the order is not definite since all equations are used collectively.

Figure 6 gives the completed design matrix showing that a general solution exists since all the V or 11 remaining parameters have been specified by the V or 11 relationships. Now that it has been shown a general solution exists for the design modification described earlier, numerical values for the parameters will be used to check for physical realizability, and determine the remaining parameters.

Starting with the contact system each form C contact set has a contact pressure of 68 grams and a spring constant of 2287 grams/inch when referred to the armature pull center. Therefore for an 8 form C contact system P_o = 544 grams and K = 18,300 grams/inch. The parameters associated with the relay frame, armature and coil bobbin were M, ℓ, μA and x_o. These were measured and the values were M = 10 grams, ℓ = 0.94 inches, μA = .582 and x_o = 0.017 inches. The two other parameters selected were E = 24 volts dc and h = 0.56. The last parameter h is the stability factor and was selected as some where in the range of .5 or so. The value of 0.56 was used in order to utilize wire of a standard size. Values for the restricted set of variables β, ρ, α, σ, a and g were calculated from the existing relay where applicable. The values for these were α = 0.036 inches, a = 2.105, g_r = 0.692 and g_n = 0.639 and ρ = resistivity of copper wire. The values of β and σ were not selected since the main restriction was an upper bound on the outside coil diameter s. The value of s was to be less than 0.75 inches. The values of the 8 parameters and the restricted variables along with the results obtained from using them are tabulated below. In addition a set of measured values from the modified relay are shown in Table I.

TABLE I

Values of given design parameters and variables	Values obtained from modified relay
P_o = 544 grams	503
K = 18,300 grams/inch	n.m.*
M = 10 grams	n.m.
ℓ = 0.94 inches	n.m.
μA = 0.582	n.m.
x_o = 0.017 inches	0.015
E = 24 v dc	24
h = 0.56	0.602
α = 0.036 inches	n.m.
a = 2.105	n.m.
g_r = 0.692	n.m.
g_n = 0.639	n.m.
ρ = 10.37 ohms/cir.mil.-ft.	n.m.
Design values calculated using given parameters	
δ = #34 wire	n.m.
k = 5.2 ms.	3.5 ms.
N = 4080 turns	n.m.
P = 3.8 watts	3.97
R_c = 151 ohms	145
R_s = 1 ohm	1
R_t = 152 ohms	146
s = 0.712 inches	n.m.
t_p = 10.1 ms.	8.3
t_s = 15.3 ms.	11.8 ms.

* n.m. = not measured. It was assumed that these values were as specified within reasonable tolerances.

A comparison of the design values with the measured values show some differences. These are to be expected since certain tolerances must exist on the physical device. In the case of P_o it was assumed all the contacts involved in a form C set were the same. The results show about a 6% difference. Since K was not measured on the modified relay it was assumed that it was as assumed. The air gap x_o was not set exactly at the design value giving about a 10% difference. The stability factors were about 10% different. The transit times differed about 30% which could be caused by the deviations noted above. However, the assumptions used in developing the design equation for k were such that the design value should normally be larger than actual values, which is the case here.

The measured power P is a little high caused by the coil resitance R_c being lower than the design value. The pick-up times differed by about 20% which could be caused by the differences in P_o and x_o and other tolerances. Again the design value is larger so that the actual seating time t_s is less than the design value. These noted deviations explain why a stability factor of about 0.5 is necessary. These deviations included with the regulation of the power supply and the increase in ambient temperature all go toward the determination of the stability factor h.

This section of the paper indicates some of the problems that exist in a relay design modification. The important point is that a limited number, namely N-V, of the design parameters can be selected. These N-V parameters must include those that are to be changed and those that are to be maintained the same. The other or V parameters must be allowed to vary in order to satisfy the requirements on the N-V parameters.

CONCLUSIONS

It is seen that the number of items which may be specified is fixed with a given number of parameters and relationships. Many abitrary specifications have been shown to be impossible. This has been accomplished by a logical procedure. Conflicting specifications may not be evident until the items have been checked by the logical procedure.

Before a design may be started it is necessary to formulate the relationships with the parameters which are involved in the device to be designed.

The design theory presented is that of organization. The ideas used are those of "system coherence" and "set independance." A simple system matrix is constructed by using sets of elements from physical laws and arbitrary restrictions. From the system representation and some given specifications an orderly technique is used to determine if these specifications can be satisfied. The distinct difference between the numerical problems of a solution, inevitable in an actual design, from the general solution which indicates how to solve the numerical problems are illustrated by specific examples. It is the relation between the general solution of a system and a particular solution which can be used to increase the efficiency of solving design problems.

REFERENCE

* "Theory of Design", C. F. Cameron, D. D. Lingelbach & C. C. Freeny; Ninth National Conference on Electromagnetic Relays, 1961.

	δ						δ	δ									
	η	η	η	η										2			
	$\mathcal{R}_p$									$\boxed{\mathcal{R}_p}$				10			
	E	E		E	E									3			
	k			k								k					
	K								$\boxed{K}$					9			
	ℓ						ℓ	ℓ						5			
	M			M										4			
	N	N	N					N									
	P				P												
	P_o	P_o							P_o					1			
	R_c					R_c	R_c										
	R_t	R_t	R_t	R_t	R_t	R_t											
	R_S					R_S								11			
	s						s	s			$\boxed{s}$			7			
	t_p		t_p									t_p					
	t_S											t_S					
	x_o	x_o	x_o	x_o					x_o	x_o				8			
	μA	μA	μA							μA	μA			6			
		D S_1				D S_2	D S_3	D S_4	B	C	A						
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 4

△	δ						δ	δ						12			
	η	η	η	η										2			
□	R_p									$\boxed{R_p}$				10			
○	E	E		E	E									3			
	k			k								k					
□	K								$\boxed{K}$					9			
○	ℓ						ℓ	ℓ						5			
○	M			M										4			
△	N	N	N					N						13			
	P				P												
○	P_o	P_o							P_o					1			
△	R_c					R_c	R_c							14			
△	R_t	R_t	R_t	R_t	R_t	R_t								15			
○	R_s					R_s								11			
□	s						s	s			$\boxed{s}$			7			
	t_p		t_p									t_p					
	t_s											t_s					
○	x_o	x_o	x_o	x_o					x_o	x_o				8			
○	μA	μA	μA							μA	μA			6			
		D S_1				D S_2	D S_3	D S_4	B	C	A						
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 5

△	δ						δ	δ						12			
○	η	η	η	η										2			
□	$\mathcal{R}_p$									$\boxed{\mathcal{R}_p}$				10			
○	E	E		E	E									3			
□	k			$\boxed{k}$								k		17			
□	K								$\boxed{K}$					9			
○	ℓ						ℓ	ℓ						5			
○	M			M										4			
△	N	N	N					N						13			
□	P				$\boxed{P}$									18			
○	P_o	P_o							P_o					1			
△	R_c					R_c	R_c							14			
△	R_t	R_t	R_t	R_t	R_t	R_t								15			
○	R_s					R_s								11			
□	s						s	s			$\boxed{s}$			7			
□	t_p		$\boxed{t_p}$									t_p		16			
□	t_s											$\boxed{t_s}$		19			
○	x_o	x_o	x_o	x_o					x_o	x_o				8			
○	μA	μA	μA							μA	μA			6			
		D S_1	E	F	G	D S_2	D S_3	D S_4	B	C	A	H					
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 6

SECTION I

PRELIMINARY CONTACTOR REDESIGN

Preliminary vibration testing of four contactors with ratings from 25 amperes to 200 amperes show that the plunger is moving when the coil is de-energized. The motion of the plunger occurs at different G levels for the different contactors. However, this G level is below the specified 20 g. For the de-energized case the plunger showed motion before contact chatter was indicated. This suggests that the plunger motion influences the contact chatter in the de-energized case. Any change in the design to stop the plunger motion will influence the other characteristics of the contactor, therefore a preliminary redesign should be made to determine some idea of the changes involved.

To stop the plunger motion when the contactor is being vibrated at 20 g with the coil de-energized requires that the initial back tension on the plunger be at least 20 times the mass of the moving parts. Since an opened 200 A contactor was supplied, it will be used to demonstrate the redesign procedure. The redesign for the other contactors would be essentially the same but with different numerical values.

For the opened 200 A contactor provided, the total mass of the moving part is 85 grams. Since this is to withstand 20 $(\sqrt{2})$ g peak, this member must be preloaded mechanically against the back stop with a force of at least 29 x 85 or 2460 grams. The initial back tension existing on the contactor presently is about 1200 grams. A mathematical model of a contactor will be used to determine the overall change in the characteristics when the back tension is increased. The mathematical model being used allows only 8 parameters to be changed or held fixed. Some parameters are easier to change than others because this is a redesign and changing the mechanical quantities may be more difficult than changing electrical quantities. With this in mind the following paramteres are selected to be held fixed: E, ℓ, s, M, x_o, μA, η. The parameter being changed is P_o, the back tension. This set then includes 8 parameters which are all that can be specified, in general. Before checking for dependencey of the parameters, the reasoning behind selecting these, is in order at this point. The supply voltage E was selected as one parameter since the contactor application was to be the same. The parameters ℓ and s were selected since these represent the coil dimensions and if possible it is desirable to use the same enclosure. The mass of the moving parts is to be unchanged if possible at this point so the parameter M was selected. No change was to be made in the magnetic circuit so x_o and μA were selected. The overall operating stability was to be the same or improved so η was selected as one parameter. Table I gives a list of the symbols and their definitions.

To check to see if the parameters are independent as far as the mathematical model is concerned, the design matrix given in Figure 1 is used. This design matrix gives the parameters as row positions and the relationships relating these parameters as column positions numbered f_1 to f_{11} inclusive. Table II gives the mathematical form of each relationship f_1 through f_{11}. To use the design matrix shown in Figure 1, each parameter of the 8 selected is indicated with a circle in column one at the row corresponding to the parameter. This procedure is shown in Figure 2. After or during the marking of the 8 parameters, each relationship (column) is checked to see if all of the parameters in that relationship are circled (selected). If all the parameters in any relationship are circled (selected), then all of the parameters in that relationship are not independent. This means that if the relationship contains m parameters, only m-1 can be independent. In other words this means that any m-1 parameters of an m parameter relationship determines the remaining parameter by the use of that relationship.

TABLE I

Parameter Symbols and Definitions

Map parameters

δ = diameter of bare wire

η = design stability point

$\mathcal{R}_p$ = reluctance of magnetic circuit presented to the coil with the plunger in the pick-up position

E = open circuit supply voltage

k = armature transit or travel time

K = effective spring constant of spring system

ℓ = coil length

M = effective plunger mass

N = coil turns

P = steady state total power supplied to relay coil circuit by source E

P_o = back tension at pick-up

R_c = coil resistance

R_t = total resistance of relay coil circuit presented to voltage source E

$R_s = R_t - R_c$

s = outside coil diameter

t_p = armature pick-up time

t_s = armature seating time and equals $t_p + k$

μA = permeability of free space times the effective cross sectional area of the working air gap

x_o = plunger working air gap

Restricted parameters

β = ratio of core diameter to outside coil diameter

$\mathcal{R}_i$ = magnetic reluctance of the non-working magnetic circuit

ρ = resistivity of conductor material

α = air equivalent of the non-working part of magnetic circuit when using a series representation

σ = ratio of twice the coil bobbin wall thickness to the outside coil diameter

gr = resistance winding space factor

gn = turns winding space factor

a = ratio of pole face diameter (d') to the core diameter (d)

u = twice the thickness of the coil bobbin

d = core diameter

TABLE II

Mathematical Form of Relationships in Mixed Units

Units involved are:

mass (M) in grams

force (P_o) in grams (gram gage measures force in grams)

length in inches

voltage in volts

current in amperes

resistance in ohms

permeability of free space (μ) is 3.19 lines/amp-turn inch

magnetic flux in lines

magnetomotive force in ampere-turns

power in watts

time in seconds

magnetic reluctance ($\mathcal{R}_p$) in amp-turns/line

Relationships:

$$(f_1) \qquad \eta - (157.5)\,\frac{R_t(x_o + \alpha)\sqrt{2\,P_o}}{E\,N\,\sqrt{\mu A}} = 0$$

$$(f_2) \qquad t_p - (10^{-8})\,\frac{N^2\,\mu A}{(x_o+\alpha)R_t}\,\ln\frac{1}{1-\eta} = 0$$

$$(f_3) \qquad k - (8.66 \times 10^{-3})\left[\frac{18\,M\,x_o^2\,R_t}{E^2\,\eta\,(1-\eta)[1 - \nabla^2(1+\frac{Kx_o}{P_o})]}\right]^{1/3} = 0$$

(f₄) $P - E^2/R_t = 0$

(f₅) $R_t - (R_s + R_c) = 0$

(f₆) $R_c - \frac{(0.865 \times 10^{-6})\, g r s^2\, (1 - \beta - \sigma)(1 + \beta + \sigma)\, \ell}{\delta^4} = 0$

(f₇) $N - \frac{0.637\, g n\, (1 - \beta - \sigma)\, \ell s}{\delta^2}$

(f₈) $K - \frac{2\, P_o}{x_o + \alpha} < 0$

(f₉) $R_p - \frac{x_o + \alpha}{\mu A} = 0$

(f₁₀) $\mu A - \frac{\pi a^2 s^2 \beta^2 \mu}{4} = 0 \qquad (\mu = 3.19)$

(f₁₁) $t_s - (t_p + k) = 0$

Auxiliary relations

$\nabla = \frac{\alpha}{\alpha + x_o}$

$\alpha = \mu A R_i$

$\beta = d/s$

$\sigma = u/s$

$a = d'/d$

Examination of Figure 2 shows that in relationship f_{10}, all the parameters are selected (circled). This means that in general both s and μA can not be selected, i.e., one determines the other. Since only one of these (s or μA) can be selected, then another parameter must be selected. The parameter R_s, which is the resistance in the supply, will be selected instead of s, since the application of the contactor is fixed. The 8 parameters now are: E, ℓ, M, x_o, μA, η, R_s & P_o. Using these 8 parameters, the design matrix in Figure 3 is obtained with circles used to indicate the original 8 selected parameters. Examination of the design matrix shows that relationships f_8, f_9 and f_{10} have all but one parameter selected. A square is used to indicate that a parameter has been determined by the use of one of the relationships. After marking these newly determined parameters the design matrix shown in Figure 4 is obtained. Examination of this Figure 4 shows no relationship having all but one parameters marked. This might suggest that another parameter needs to be selected but if this is tried, a conflict will exist where two relationships will determine the same parameter. Closer examination of the design matrix in Figure 4 will reveal that their are four relationships involving four unselected (un-marked) parameters. These four relationships are f_1, f_5, f_6 and f_7 and the parameters are N, R_t, R_c and δ. Four equations and four unknowns suggest simultaneous solution of the equations to obtain a solution. At this point in the design the mathematical form of the equations must be used. Up to this point, only the variables or paramters involved were used. Relationship f_1 could be solved for N or R_t, f_5 for R_t or R_c, f_6 for R_c or δ and f_7 for N or δ. Solving f_5 for R_c and substituting into f_6 eliminates R_c. Then substituting for R_t in f_1 by using f_6 gives two equations involving N and δ. These equations are:

$$N = \frac{.637\ g_n\ (1-\beta-\sigma)\ s\ell}{\delta^2} \tag{1}$$

$$N = \frac{.865 \times 10^{-6}\ gr\ (1-\beta-\sigma)(1+\beta+\sigma)s^2\ell\ (x_o+\alpha)\sqrt{2P_o}}{\delta^4\ E\ \eta\ \mu A} \tag{2}$$

Solving these two equations for δ^2 gives:

$$\delta^2 = \frac{.865 \times 10^{-6}\ gr\ (1+\beta+\sigma)\ s\ (x_o+\alpha)\ \sqrt{2\ P_o}}{.637\ E\ \eta\ \sqrt{g_n}} \tag{3}$$

This shows that the parameters N, R_t, R_c and δ are determined by those already marked in Figure 4. These parameters are marked as shown by the design matrix in Figure 5 and the other relationships f_2, f_3, f_4 and f_{11} have all but one parameter selected (marked). The result shown in Figure 5 means that in general the 8 parameters E, ℓ, M, x_o, μA, η, R_s and P_o can be used to determine the other 11 parameters. Figure 5 also shows the order in which the relationships are to be solved and the particular parameter solved for. This is indicated by the alphabetical lettering on the bottom row. The order in which the parameters are used is shown by the numerical sequence in the column at the right.

Only one parameter of the original 8 is being changed, the others are held fixed. At the present time no drawing or information is available about the coil size and magnetic circuit so the redesign will be done in terms of the percent change in the parameters. In other words, the new values will be given in terms of the old values as a percent change. The design matrix indicates that f_{10} is the first relationship used. This relationship is used to solve for s. Since μA was to be unchanged so will s be unchanged since a and β are fixed.

Relationship f_9 indicates that the parameter R_p is unchanged since x_o, μA and α are unchanged. From f_8 the value K must be less than $2\ P_o/(x_o + \alpha)$ and since P_o is the only one changed and it is increased then using the same springs as on the original will still satisfy f_8 so K can remain unchanged.

Equation 3 developed in this discussion is used to determine the new value of δ^2. The term gr is the resistance winding factor and may be assumed essentially constant for small changes in wire size. All the other parameters except P_o are constant therefore the new value of δ^2, say δ_1^2, will be:

$$\frac{\delta_1^2}{\delta^2} = \sqrt{\frac{2460}{1200}} = 1.43 \qquad (4)$$

Therefore the new circular mil size of the wire should be 43% larger than the old or $\delta_1^2 = 1.43\ \delta^2$. Equation 3 was the result of solving four equations simultaneously, therefore, the other 3 parameters involved must be solved for next. The parameter N (coil turns) can be solved for by using f_7, which was one of the 4 relationships used for simultaneous solution. Examination of f_7

shows that all the parameters are constant except δ^2 (wire size). Since the new value of wire size is 1.43 times the original then the new N (or N_1) will be given as

$$\frac{N_1}{N} = \frac{1}{1.43} \quad \text{or} \quad N_1 = .70\ N \tag{5}$$

This means the new number of turns will be 70% of the original number of turns.

The next parameter involved in the simultaneous solution is R_c. Relationship f_6 is used to determine R_c. Relationship f_6 shows that only δ^4 has changed in this case. Therefore the coil resistance is given as:

$$\frac{R_c^{\,1}}{R_c} = \frac{1}{(1.43)^2} = .487 \quad \text{or} \quad R_{c1} = .487\ R_c \tag{6}$$

This gives the new total resistance as 48.7% of the original total resistance. The value of R_s is the same but the total resistance R_t will decrease because $R_t = R_s + R_c$ and R_c is 48.7% of its original value. The total resistance R_t is the last parameter of those solved simultaneously. Figure 5 indicated that t_p is the next parameter solved for and it is obtained by relationship f_2. This relationship involves two parameters which have been changed. These are N and R_t. Using the relationship f_2 gives the new value of pick-up time (t_{p1}) in terms of the original as:

$$t_{p1} = \frac{10^{-8}[(.70N)^2]\ \mu A}{(x_o + \alpha)[.487R_t]}\ \ln \frac{1}{1-\eta} \tag{7}$$

$$t_{p1} = \frac{(.70)^2}{.487}\ t_p = t_p. \tag{8}$$

The result shown by equation 8 means that the new pick-up time (t_{p1}) is the same as the original pick-up time.

The transit time k is the next parameter calculated and this is accomplished by using relationship f_3. Inspection of relationship f_3 shows only two of the parameters are changed. These two are R_t and P_o. The manner in which P_o changes k is not a direct variation because the operations involved are sum and differences. Past experiments have indicated that the influence of P_o on k is normally not great. Also since the numerical value of ∇ has not been determined, it is desirable to not consider the effect of P_o in this particular relationship. Therefore the new value of transit time (k_1) will be given as

$$k_1 = \sqrt[3]{.487} \qquad k_1 = .785\ k. \tag{9}$$

This gives the new value of transit time (k_1) as 78.5% of the original value.

The coil power is computed next using relationship f_4. Since R_t is the only parameter changed, then the new value of coil power (P_1) is:

$$P_1 = \frac{1}{.487} P = 2.05\ P \quad \text{if } R_s = 0 \tag{10}$$

The new coil power is 105% larger than the original. With this increase in coil power, a re-evaluation of the thermo-dissipation may be necessary. Design equations for the heat dissipation have not as yet been developed.

The last parameter calculated is the total seating time t_s. The relationship used is f_{11} and since its form is the sum of t_p and k, only the relative change can be given. Since the new value of the pick-up time is the same as the original and since the new transit time is less than the original, then the total seating time (t_{s1}) will be less than the original. Table III gives a summary of the results of a new design calculation in terms of the original values. Percentage changes have been given when possible and relative changes for the other cases.

TABLE III

Comparison of the new values of the contactor parameters in terms of the original values when E, ℓ, M, x_o, μA, η, R_s are fixed and P_o increased.

Design parameters	
δ^2 = area of bare wire	43% larger
η = design stability point	specified same
$\mathcal{R}_p$ = reluctance of the magnetic circuit presented to the coil with the plunger in the pick-up position	no change
E = open circuit supply voltage (dc)	specified same

k	= plunger transit time	21.5% smaller
K	= effective spring constant of spring system	no change needed but maybe increased 100% if necessary to obtain desired P_o
ℓ	= coil length	specified same
M	= effective plunger mass	specified same
N	= coil turns	30% smaller
P	= total steady state power to contactor from voltage supply	105% larger
P_o	= effective back tension at pick-up	specified 105% larger
R_c	= coil resistance	51.3% smaller
R_t	= total resistance of contactor circuit presented to voltage supply	51.3% smaller if $R_s = 0$
R_s	= $R_t - R_c$	specified same
s	= outside coil diameter	no change
t_p	= plunger pick-up time	no change
t_s	= plunger seating time	less
x_o	= plunger working air gap	specified same
μA	= permeability of free space times effective cross sectional area of working air gap	specified same

Restricted parameters

β	= ratio of core diameter to outside coil diameter	assumed constant
ρ	= resistivity of conductor material	assumed constant
α	= air equivalent of non-working part of magnetic circuit when using a series representation	assumed constant
σ	= ratio of twice the coil bobbin wall thickness to the outside coil diameter	assumed constant
gr	= resistance winding space factor	assumed constant
gn	= turns winding space factor	assumed constant
a	= ratio of pole face diameter (d') to the core diameter (d).	assumed constant

A study of Table III may suggest that the parameters that were selected be held fixed, are not the most desirable. Particularly since the coil power required has more than doubled, it might be desirable to consider holding the coil power constant and letting the coil height vary instead. With this change the second set of selected parameters would be η, E, M, P, P_o, R_s, x_o and μA. Figure 6 shows the parameters marked on the design matrix. The results show that f_6 and f_7 must be solved simultaneously for ℓ and δ. Solution of these two equations gives

$$\delta^2 = \frac{0.865 \times 10^{-6}\ grs(1 + \beta + \sigma)N}{.637\ gn\ R_c} \tag{11}$$

The solution order is given in Figure 6 showing that f_{10} is used first to solve for s. Since μA is constant then the new value of s(given as s_2) is the same as the original. Therefore, $s_2 = s$. Relationship f_8 indicates that K can be unchanged but maybe increased 100% if necessary to get the new value of P_o. The magnetic reluctance R_p as given by f_9 is unchanged since x_o and μA are constant. The value of R_t is unchanged because E and P are constant. The value of R_c is the same since R_t and R_s are constant. Relationship f_1 indicates that N will change since P_o is changed. The new value N_2 of the number turns is given as

$$N_2 = \sqrt{\frac{2640}{1200}}\ N = 1.43N. \tag{12}$$

This means the number of turns required is 43% greater than the original number.

The new value of t_p is given by f_2 and varies as the square of N. Therefore, the new value t_{p2} of the pick-up time is

$$t_{p2} = (1.43)^2\ t_p = 2.05\ t_p. \tag{13}$$

This means that the pick-up time is essentially doubled. Relationship f_3 indicates that k is unchanged since R_t and the other specified parameters are unchanged.

The value of the total seating time t_s is increased as given by f_{11}. The numerical value of the increase is not known since this equation is the sum of t_p and k. Since t_p was doubled and k was unchanged the increase in t_s would likely be about 50%.

The use of equation (11) will give the new value of δ^2. This gives the new value $\delta_2{}^2$ as

$$\delta_2{}^2 = 1.43\delta^2. \tag{14}$$

This means a 43% increase in the circular mil size of the wire.

Either f_6 or f_7 may now be used to determine the value of ℓ. The new value ℓ_2 of the coil length is given as

$$\ell_2 = (1.43)^2\ell = 2.05\ell. \tag{15}$$

This means the coil length would have to be doubled.

A summary of these results is given in Table IV.

TABLE IV

Comparison of the new values of the contactor parameters in terms of the original values when E, P, M, x_o, μA, η, R_s are fixed and P_o increased.

Design parameters

δ^2	43% increase
η	specified same
R_p	no change
E	specified same
K	no change needed but maybe increase 105% to obtain desired P_o
ℓ	105% increase
M	specified same
N	43% increase
P_o	specified 105% increase
R_c	no change
R_t	no change

P	specified same
R_s	no change
s	specified same
t_p	105% increase
k	no change
t_s	increased
x_o	specified same
μA	specified same

All restricted parameters assumed constant.

This section has shown the problems involved when it is desired to change the value of the plunger back tension. From the preliminary vibration tests it was found that the plunger was in motion when the coil was deenergized and when the contactor was being vibrated along its axis of operation. This action indicated that increased back tension would be necessary. To obtain an idea of the problems involved, a set of 7 parameters was selected as being desirable not to change. These 7 along with P_o, the parameter being changed, give the number that can be specified with the mathematical model being used.

In the first computation the coil length was selected as fixed but the coil power was not. The results of the computation shows that the power must be essentially doubled to essentially double the back tension. Increasing the coil power without changing the heat dissipating surface can result in over heating. Since the heat dissipating relationship has not been developed, no check could be made to determine the temperature change that might result.

As an alternative it was decided that a second computation involving the coil power as fixed and the coil length as variable would be desirable. The results indicated here that the coil length must be essentially doubled. There were other changes which might make this set of fixed parameters undesirable. The main change was a 100% increase in the pick-up time.

Whether this is critical or not depends on the application.

These two sample computations should point out the basic restrictions involved in a redesign and that is:only a limited number of parameters can be fixed or changed. A realization of this restriction usually means that a critical evaluation must be made of the parameters that must be fixed. An increase in the required back tension usually means an increase in coil power or an increase in coil volume or a combination of both.

	δ						δ	δ									
	η	η	η	η													
	$\mathcal{R}_p$									$\mathcal{R}_p$							
	E	E		E	E												
	k			k								k					
	K								K								
	ℓ						ℓ	ℓ									
	M			M													
	N	N	N					N									
	P				P												
	P_o	P_o							P_o								
	R_c					R_c	R_c										
	R_t	R_t	R_t	R_t	R_t	R_t											
	R_s					R_s											
	s						s	s			s						
	t_p		t_p									t_p					
	t_s											t_s					
	x_o	x_o	x_o	x_o					x_o	x_o							
	μA	μA	μA							μA	μA						
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Parameters

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{Relationships}}$

Figure 1. Contactor Design Matrix

	δ						δ	δ									
○	η	η	η	η										7			
	$\mathcal{R}_p$									$\mathcal{R}_p$							
○	E	E		E	E									1			
	k			k								k					
	K								K								
○	ℓ						ℓ	ℓ						2			
○	M			M										4			
	N	N	N					N									
	P				P												
○	P_o	P_o							P_o					8			
	R_c					R_c	R_c										
	R_t	R_t	R_t	R_t	R_t	R_t											
	R_S					R_S											
○	s						s	s			s			3			
	t_p		t_p									t_p					
	t_S											t_S					
○	x_o	x_o	x_o	x_o					x_o	x_o				5			
○	μA	μA	μA							μA	μA			6			
	0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 2

	δ						δ	δ									
	η	η	η	η										6			
	$\mathcal{R}_p$									$\mathcal{R}_p$							
	E	E		E	E									1			
	k			k								k					
	K								K								
	ℓ						ℓ	ℓ						2			
	M			M										3			
	N	N	N					N									
	P				P												
	P_o	P_o							P_o					8			
	R_c					R_c	R_c										
	R_t	R_t	R_t	R_t	R_t	R_t											
	R_s					R_s								7			
	s						s	s			s						
	t_p		t_p									t_p					
	t_s											t_s					
	x_o	x_o	x_o	x_o					x_o	x_o				4			
	μA	μA	μA							μA	μA			5			
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 3

	δ						δ	δ									
	η	η	η	η													
	$\mathcal{R}_p$									$\boxed{\mathcal{R}_p}$							
	E	E		E	E												
	k			k								k					
	K								$\boxed{K}$								
	ℓ						ℓ	ℓ									
	M			M													
	N	N	N					N									
	P				P												
	P_o	P_o							P_o								
	R_c					R_c	R_c										
	R_t	R_t	R_t	R_t	R_t	R_t											
	R_s					R_s											
	s						s	s			$\boxed{s}$						
	t_p		t_p									t_p					
	t_s											t_s					
	x_o	x_o	x_o	x_o					x_o	x_o							
	μA	μA	μA							μA	μA						
									C	B	A						
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 4

△	δ						δ	δ						12
○	η	η	η	η										6
□	R_p									$\boxed{R_p}$				10
○	E	E		E	E									1
□	k			$\boxed{k}$								k		17
□	K								$\boxed{K}$					11
○	ℓ						ℓ	ℓ						2
○	M			M										3
△	N	N	N					N						13
□	P				$\boxed{P}$									18
○	P_o	P_o							P_o					8
△	R_c					R_c	R_c							14
△	R_t	R_t	R_t	R_t	R_t	R_t								15
○	R_s					R_s								7
□	s						s	s			$\boxed{s}$			9
□	t_p		$\boxed{t_p}$									t_p		16
□	t_s											$\boxed{t_s}$		19
○	x_o	x_o	x_o	x_o					x_o	x_o				4
○	μA	μA	μA							μA	μA			5
		D_{S_1}	E	F	G	D_{S_1}	D_{S_1}	D_{S_1}	C	B	A	H		
	0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}		

Figure 5

□	δ						δ	δ						19			
○	η	η	η	η										1			
□	$\mathcal{R}_p$									$\boxed{\mathcal{R}_p}$				11			
○	E	E		E	E									2			
□	k			$\boxed{k}$								k		16			
□	K								$\boxed{K}$					10			
□	ℓ						ℓ	ℓ						18			
○	M			M										3			
□	N	$\boxed{N}$	N					N						14			
○	P				P									4			
○	P_o	P_o							P_o					5			
□	R_c					$\boxed{R_c}$	R_c							13			
□	R_t	R_t	R_t	R_t	$\boxed{R_t}$	R_t								12			
○	R_s					R_s								6			
□	s						s	s			$\boxed{s}$			9			
□	t_p		$\boxed{t_p}$									t_p		15			
□	t_s											$\boxed{t_s}$		17			
○	x_o	x_o	x_o	x_o					x_o	x_o				7			
○	μA	μA	μA							μA	μA			8			
		F	G	H	D	E	J_{S_1}	J_{S_1}	B	C	A	I					
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 6

CONTINUATION OF PRELIMINARY CONTACTOR REDESIGN
Report No. 3 Section No. I

Section I of the Interim Report for 1 March to 30 April 1962 was a discussion of the changes involved in increasing the back tension on the contactor plunger. The results presented in that discussion were for two sets of fixed parameters. In one set of parameters, the coil dimensions were held fixed along with six other parameters specified and the results indicated that the coil power would have to vary directly with the back tension. In the other set of parameters, the coil power was fixed along with the same seven other parameters but the coil length was allowed to vary. The results in this case indicated that the coil length varied directly with the back tension. These results are a function of the particular set of parameters specified.

It now appears that some combination of increased coil power and coil length might be the most feasible, therefore additional calculations are given in this section to show the result of increasing the back tension by a combination of coil power and coil length. To present the results in a more enlightening manner the values of the various parameters are plotted against the coil power. The results are given in per unit value which is the ratio of the new value to the original value of the parameter. The original value of the parameter being the value existing on the contactor furnished by the contractor. To date no numerical information has been received from the manufacturer of the contactors furnished but measurements have been made when numerical values were needed. Fortunately, the form of most of the relationships do not require absolute values of the parameters. The relative or per unit value can be used in most of the equations.

The set of parameters specified is; η, E, M, P, P_o, R_S, x_o and μA.

Tables I and II given in the previous report are repeated here for convenience of discussion. The value of back tension P_0 is to be doubled and the coil power is increased in increments of 25% to a total of twice the original value. The other specified parameters are held fixed at their original values. Using these parameters on the design matrix gives the result shown in Figure 1. The order for solving the relationships is given by the alphabetical list at the bottom of Figure 1. The solution becomes:

f_{10} for s where $s^2 = \dfrac{4\mu A}{\mu \pi a^2 \beta^2}$

f_8 for K where $K < \dfrac{2P_0}{x_0 + \alpha}$

f_9 for R_p where $R_p = \dfrac{x_0 + \alpha}{\mu A}$

f_4 for R_t where $R_t = E^2/P$

f_5 for R_c where $R_c = R_t - R_s$

f_1 for N where $N = \dfrac{R_t\ (x_0 + \alpha)\sqrt{2P_0}\ (157.5)}{E\ \eta\ \sqrt{\mu A}}$

f_2 for t_p where $t_p = \dfrac{10^{-8} N^2 \mu A}{(x_0 + \alpha) R_t}\ \ln \dfrac{1}{1-\eta}$

f_3 for k where $k = 8.66 \times 10^{-3} \left[\dfrac{18\ M x_0{}^2 R_t}{E^2 \eta (1-\eta)[1-\nabla^2(1 + K x_0/P_0)]}\right]^{1/3}$

f_{11} for t_s where $t_s = t_p + k$

f_6 and f_7 for δ where $\delta^2 = \dfrac{0.865 \times 10^{-6} g_r s(1 + \beta + \sigma) N}{0.637\ g_n\ R_c}$

f_6 and f_7 for ℓ where $\ell = \dfrac{N\delta^2}{0.637\ g_n(1-\beta-\sigma)s}$

These relationships give the variation in each parameter in terms of the specified parameters. In this case the two parameters P_0 and P are being increased and η, E, M, R_s, x_0 and μA are constant. With these conditions the changes that exist for the other parameters are:

s is unchanged,

K varies directly with P_0,

R_p is unchanged,

R_t varies inversely with P,

R_c varies inversely with P when $R_s = 0$,

N varies directly with the product $R_t\sqrt{2P_0}$,

t_p varies directly with N^2 and inversely with R_t,

k varies as the cube root of R_t,

t_s varies as the sum of t_p and k,

δ^2 varies directly with N and inversely with R_c,

ℓ varies directly with the product $N\delta^2$

The value of P_0 was increased to 2 per unit and the value of P was incremented 0.25 per unit from 1 to 2 per unit. The results of these changes on the other parameters are given in Table III. The data in Table III are plotted in Figure 2 with the coil power P as the variable. These curves show the value of each parameter as the coil power is changed in order to double the back tension P_0. One interesting result is that the product of coil power P and coil length ℓ is a constant having a value of two. Once this result is noted from the curve it can also be shown, by using the relationships for ℓ, δ^2, N, R_c and R_t with $R_s = 0$, that the product ℓP is

$$\ell P = 2P_0(x_0 + \alpha)^2 \left[\frac{0.865 \times 10^{-6} g_r(1 + \beta + \sigma)}{(0.637g_n)^2(1-\beta-\sigma)}\right] (157.5)^2 .$$

This product ℓP varies directly with P_0 since x_0 is a fixed parameter and all the other variables are restricted parameters. A result of this kind suggests that a balanced rotary type armature would reduce this factor of ℓP since a smaller value of P_0 could be employed to hold the armature open when the coil is deenergized. With the present design the value of P_0 must be such to hold the total plunger and movable contact mass against the acceleration specified. With a balanced rotary arrangement the value of P_0 would be determined primarily by the opening contact force required.

Figure 2 also shows that the other parameters either remain constant or decrease as the coil power is increased. The parameters that were specified as being fixed were not shown in Figure 2.

Another alternative in modifying a design is to consider making the coil more efficient. This can be accomplished by changing the ratio of the iron core diameter to the coil outside diameter. This ratio is one of the restricted parameters and is designated by the symbol β. When the coil bobbin insulation is negligible compared to the coil outside diameter the value of β to make the pull per watt a maximum is 0.6. Measurements made on the coil of the 200 amp contactor shows that β has a value of 0.435. Therefore, a change in β should give some improvement in the coil efficiency. Since β will now be one of the selected parameters, the design matrix given in Figure 1 will have to be modified to include the parameter β. This is accomplished by examining the relationships to see if they contain β. Those relationships involving β will now have β entered in the column for that relationship. Only three relationships contain β and they are f_6, f_7 and f_{10}. Figure 3 shows the modified design matrix containing the additional parameter β. Now the design matrix contains twenty parameters minus eleven relationships or nine parameters may be specified. Adding β to the original eight parameters will give the nine parameters needed. Figure 3 shows the nine parameters mapped on the design matrix and gives the order of solution. The order of solution is as follows:

f_4 for R_t where $R_t = E^2/P$

f_5 for R_c where $R_c = R_t - R_s$

f_{10} for μA where $\mu A = \dfrac{\pi a^2 s^2 \beta^2 \mu}{4}$

f_8 for K where $K < \dfrac{2P_0}{x_0 + \alpha}$

f_9 for $\mathcal{R}_p$ where $\mathcal{R}_p = \dfrac{x_0 + \alpha}{\mu A}$

f_1 for $N = \dfrac{157.5\ R_t\ (x_0 + \alpha)\sqrt{2P_0}}{E\eta\sqrt{\mu A}}$

f_2 for t_p where $t_p = \dfrac{10^{-8} N^2 \mu A}{(x_0 + \alpha)\ R_t}\ \ell n\ \dfrac{1}{1-\eta}$

f_3 for k where $k = 8.66 \times 10^{-3} \left[\dfrac{18\ M x_0{}^2 R_t}{E^2 \eta(1-\eta)[1-\nabla^2(1 + K x_0/P_0)]}\right]^{1/3}$

f_{11} for t_s where $t_s = t_p + k$

f_6 and f_7 for δ^2 where $\delta^2 = \dfrac{0.865 \times 10^{-6} g_r s(1 + \beta + \sigma)N}{0.637\ g_n\ R_c}$

f_6 and f_7 for ℓ where $\ell = \dfrac{N\delta^2}{0.637\ g_n(1-\beta-\sigma)s}$.

In these equations the parameter β is involved as a sum with the variable σ. Therefore, numerical values of β and σ must be used. The dimensions of the coil result in $\beta = 0.435$ and $\sigma = 0.13$. The values used for these calculations are: s = 1 - 7/16 inch, d = 5/8 inch and u = 3/16 inch. The value of s is to remain constant so to change β requires that d be changed. The new value of β is to be 0.6 or a change of 1.38 per unit. This means the factor $(1-\beta-\sigma)$ changes from 0.435 to 0.27 or a 0.62 per unit value. Also the factor $(1 + \beta + \sigma)$ changes from 1.56 to 1.73 or a 1.11 per unit value. Using these values and a value of 2 per unit for P_0 gives the results shown in Table IV. The data shown in Table IV are plotted in Figure 4.

Comparison of Figures 2 and 4 shows that a value of β of 0.6 instead of the existing value 0.435 results in a decrease in the coil length at a given power. Conversely, for a given length, less increase in coil power is required to double the back tension. These results show the effect of increased coil efficiency. In addition to decreasing the coil length, other decreases were noted such as the pick-up time t_p, coil turns N and wire size δ^2. The coil volume was decreased because in this case the core diameter was increased while the outside coil diameter was constant.

The improvement resulting from increasing β from 0.453 to 0.6 may not appear to be significant with respect to some of the variables. However, changing β in the opposite direction does result in a drastic change in some of the parameters, especially the coil length ℓ. Changing β from 0.435 to 0.2 gives the results shown in Table V when the back tension P_0 is doubled. Inspection of the values in Table V shows that, for no increase in coil power P_c to double the back tension P_0 when $\beta = 0.2$ requires the coil length ℓ to be 9.45 times the original value. In addition the coil volume increases because the wire size and turns are both much larger.

The results presented in Tables IV and V show that the coil length ℓ is materially influenced by the value of β. Because of the manner in which β determines ℓ, one is lead to suspect that there is a value of β which will make ℓ a minimum for the parameters fixed in this discussion. The relationship between ℓ and β can be obtained from relationships f_1, f_4, f_5, f_6, and f_7. For given values of the parameters E, P, P_0, η and x_0 the five relationships show that ℓ is related to β by

$$\ell = C\left[\frac{1+\beta+\sigma}{\beta^2(1-\beta-\sigma)}\right]. \qquad (2)$$

Where: C is a constant involving the parameters E, P, P_0, η, x_0 and the restricted parameters. Equation 2 is a function of σ and with σ = 0.13 equation 2 becomes

$$\ell = C\left[\frac{1.13 + \beta}{\beta^2(.87-\beta)}\right]. \tag{3}$$

To obtain the value of β to make ℓ a minimum, set $d\ell/d\beta = 0$. This gives

$$\beta^2 + 1.26\beta - .985 = 0$$

or

$$\beta = 0.545. \tag{4}$$

This value of β is smaller than the 0.6 used since in this case σ was not negligible with respect to unity. To obtain an idea of the manner in which ℓ varies with β, equation 3 is plotted and shown in Figure 5. It will be noticed that the curve is fairly flat in the region of β = 0.55 and that a value of β = 0.6 gives a value of ℓ only slightly larger than when β = 0.55. However, β of 0.6 results in less copper volume for a given value of outside coil diameter s.

Since equation 2 was also a function of σ, some improvement may be obtained by decreasing σ. There is a lower limit on σ since it is the ratio of twice the bobbin thickness u to the outside coil diameter s. For the original coil the value of u was 3/16 inch which could be reduced to 3/32 inch by careful design. Assume that it can be reduced to 3/32 inch giving a change of 0.5 per unit. Let β = 0.6 again for comparison and compute the value of the unspecified parameters. Inspection of the relations shows that σ appears in only two of them. These are f_6 and f_7 and these determine δ^2 and ℓ. Figure 6 shows a plot of the unspecified parameters when β = 0.6 and σ = 0.065. This shows that doubling the back tension P_0, with β = 0.6 and σ = 0.065, requires only a 47% increase in the coil length ℓ or coil power P instead of the 100% increase when $\sigma = 0.13$ and $\beta = 0.435$.

Figure 6 shows that decreasing σ can result in a significant decrease in the value of the coil length ℓ required when other things are equal. In the case of a plunger arrangement small values of σ are hard to obtain because of the needed mechanical clearance.

This discussion has brought out several things which can be used to improve the efficiency of the electrical to mechanical energy conversion. One important factor is the ratio β of the iron core diameter d to the outside coil diameter s. For the insulation thickness used the value of β to minimize the value of coil length ℓ is 0.545. However, Figure 5 shows that any value of β between 0.5 and 0.6 will almost give the minimum value of ℓ.

Figures 2, 4 and 6 show the influence of various values of coil power P upon the unspecified parameters when the parameters P_0, T, E, M, R_S, x_0 and μA are fixed. Also the influence of β and σ, two of the restricted parameters, upon the unspecified parameters is presented by comparing the results shown in Figures 2, 4 and 6. It was shown that a non zero value of β exists which will make the coil length ℓ a minimum. The parameter σ has no non zero value which will make the coil length a minimum but the smaller the value of σ the less the coil length.

Another thing pointed out was that the product of the coil length ℓ and the coil power P is a constant for the specified parameters used in this presentation. In fact, additional examination of equation 1 shows that the combination $\ell P/P_0$ is a constant when x_0 is fixed and a given set of restricted parameters is used.

TABLE I

Parameter Symbols and Definitions

Map parameters

δ = diameter of bare wire

η = design stability point

$\Re_p$ = reluctance of magnetic circuit presented to the coil with the plunger in the pick-up position

E = open circuit supply voltage

k = armature transit or travel time

K = effective spring constant of spring system

ℓ = coil length

M = effective plunger mass

N = coil turns

P = steady state total power supplied to relay coil circuit by source E

P_0 = back tension at pick-up

R_c = coil resistance

R_t = total resistance of relay coil circuit presented to voltage source E

$R_s = R_t - R_c$

s = outside coil diameter

t_p = armature pick-up time

t_s = armature seating time and equals $t_p + k$

μA = permeability of free space times the effective cross sectional area of the working air gap

x_0 = plunger working air gap

Restricted parameters

β = ratio of core diameter to outside coil diameter

$\Re_i$ = magnetic reluctance of the non-working magnetic circuit

ρ = resistivity of conductor material

α = air equivalent of the non-working part of magnetic circuit when using a series representation

σ = ratio of twice the coil bobbin wall thickness to the outside coil diameter

g_r = resistance winding space factor

g_n = turns winding space factor

a = ratio of pole face diameter (d') to the core diameter (d)

u = twice the thickness of the coil bobbin

d = core diameter

TABLE II

Mathematical Form of Relationships in Mixed Units

Units involved are:

mass (M) in grams

force (P_0) in grams (gram gage measures force in grams)

length in inches

voltage in volts

current in amperes

resistance in ohms

permeability of free space (μ) is 3.19 lines/amp-turn inch

magnetic flux in lines

magnetomotive force in ampere-turns

power in watts

time in seconds

magnetic reluctance ($\mathcal{R}_p$) in amp-turns/line

Relationships:

$$(f_1) \qquad \eta - (157.5)\,\frac{R_t(x_0 + \alpha)\sqrt{2\,P_0}}{E\,N\sqrt{\mu A}} = 0$$

$$(f_2) \qquad t_p - (10^{-8})\,\frac{N^2\,\mu A}{(x_0+\alpha)R_t}\,\ell n\,\frac{1}{1-\eta} = 0$$

$$(f_3) \qquad k - (8.66 \times 10^{-3}) \left[\frac{18\,M\,x_0^2\,R_t}{E^2\,\eta\,(1-\eta)[1 - \nabla^2(1+\frac{Kx_0}{P_0})]}\right]^{1/3} = 0$$

(f_4) $P - E^2/R_t = 0$

(f_5) $R_t - (R_s + R_c) = 0$

(f_6) $R_c - \dfrac{(0.865 \times 10^{-6})\ g r s^2 (1 - \beta - \sigma)(1 + \beta + \sigma)\ \ell}{\delta^4} = 0$

(f_7) $N - \dfrac{0.637\ g n\ (1 - \beta - \sigma)\ \ell s}{\delta^2} = 0$

(f_8) $K - \dfrac{2\ P_0}{x_0 + \alpha} < 0$

(f_9) $\Re_p - \dfrac{x_0 + \alpha}{\mu A} = 0$

(f_{10}) $\mu A - \dfrac{\pi a^2 s^2 \beta^2 \mu}{4} = 0$ $(\mu = 3.19)$

(f_{11}) $t_s - (t_p + k) = 0$

Auxiliary relations

$$\nabla = \frac{\alpha}{\alpha + x_0}$$

$$\alpha = \mu A \Re_i$$

$$\beta = d/s$$

$$\sigma = u/s$$

$$a = d'/d$$

Table III $\beta = 0.435$ other values in per unit

P	1.00	1.25	1.50	1.75	2.00
P_0	2.00	2.00	2.00	2.00	2.00
#η	1.00	1.00	1.00	1.00	1.00
#E	1.00	1.00	1.00	1.00	1.00
#M	1.00	1.00	1.00	1.00	1.00
#R_s	1.00	1.00	1.00	1.00	1.00
#x_0	1.00	1.00	1.00	1.00	1.00
#μA	1.00	1.00	1.00	1.00	1.00
s	1.00	1.00	1.00	1.00	1.00
K	<2	<2	<2	<2	<2
$\mathcal{R}_p$	1.00	1.00	1.00	1.00	1.00
R_t	1.00	0.80	0.67	0.57	0.50
$\mathcal{R}_c$	1.00	0.80	0.67	0.57	0.50
N	1.41	1.13	0.95	0.81	0.71
t_p	2.00	1.60	1.34	1.14	1.00
k	1.00	0.93	0.88	0.83	0.80
*t_s	1.50	1.27	1.10	0.99	0.90
δ^2	1.41	1.41	1.41	1.41	1.41
ℓ	2.00	1.60	1.34	1.14	1.00

#Fixed parameters
*Based on $t_p = k$

Table IV Values in per unit $\beta = 0.6$

P	1.00	1.25	1.50	1.75	2.00
P_0	2.00	2.00	2.00	2.00	2.00
θ	1.38	1.38	1.38	1.38	1.38
# η	1.00	1.00	1.00	1.00	1.00
# E	1.00	1.00	1.00	1.00	1.00
# M	1.00	1.00	1.00	1.00	1.00
# R_s	1.00	1.00	1.00	1.00	1.00
# x_0	1.00	1.00	1.00	1.00	1.00
# s	1.00	1.00	1.00	1.00	1.00
R_t	1.00	0.80	0.67	0.57	0.50
R_c	1.00	0.80	0.67	0.57	0.50
μA	1.90	1.90	1.90	1.90	1.90
K	2.00	2.00	2.00	2.00	2.00
$\mathcal{R}_p$	0.56	0.56	0.56	0.56	0.56
N	1.03	0.82	0.69	0.59	0.51
t_p	1.95	1.60	1.33	1.14	1.00
k	1.00	0.93	0.88	0.83	0.80
* t_s	1.48	1.27	1.10	0.98	0.90
δ^2	1.14	1.14	1.14	1.14	1.14
ℓ	1.89	1.50	1.26	1.07	0.94

#Fixed parameters
*Based on $t_p = k$

Table V Per unit values. $\rho = 0.2$

P	1.00	1.25	1.50	1.75	2.00
P_0	2.00	2.00	2.00	2.00	2.00
#η	1.00	1.00	1.00	1.00	1.00
#E	1.00	1.00	1.00	1.00	1.00
#M	1.00	1.00	1.00	1.00	1.00
#R_s	1.00	1.00	1.00	1.00	1.00
#x_0	1.00	1.00	1.00	1.00	1.00
#s	1.00	1.00	1.00	1.00	1.00
R_t	1.00	0.80	0.67	0.57	0.50
R_c	1.00	0.80	0.67	0.57	0.50
μA	0.21	0.21	0.21	0.21	0.21
K	2.00	2.00	2.00	2.00	2.00
$\mathcal{R}_p$	4.73	4.73	4.73	4.73	4.73
N	3.07	2.46	2.05	1.75	1.54
t_p	2.00	1.60	1.33	1.14	1.00
k	1.00	0.93	0.88	0.83	0.80
δ^2	2.62	2.62	2.62	2.62	2.62
ℓ	9.45	7.55	6.30	5.38	4.73

#Fixed parameters

△	δ						δ	δ						19
○	η	η	η	η										1
□	R_p									$\boxed{R_p}$				11
○	E	E		E	E									2
□	k			$\boxed{k}$								k		16
□	K								$\boxed{K}$					10
△	ℓ						ℓ	ℓ						18
○	M			M										3
□	N	$\boxed{N}$	N					N						14
○	P				P									4
○	P_o	P_o							P_o					5
□	R_c					$\boxed{R_c}$	R_c							13
□	R_t	R_t	R_t	R_t	$\boxed{R_t}$	R_t								12
○	R_s					R_s								6
□	s						s	s			$\boxed{s}$			9
□	t_p		$\boxed{t_p}$									t_p		15
□	t_s											$\boxed{t_s}$		17
○	x_o	x_o	x_o	x_o					x_o	x_o				7
○	μA	μA	μA							μA	μA			8
		F	G	H	D	E	J s_1	J s_1	B	C	A	I		
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}		

Figure 1. Map obtained with η, E, M, P, P_o, R_s, x_o and μA Specified.

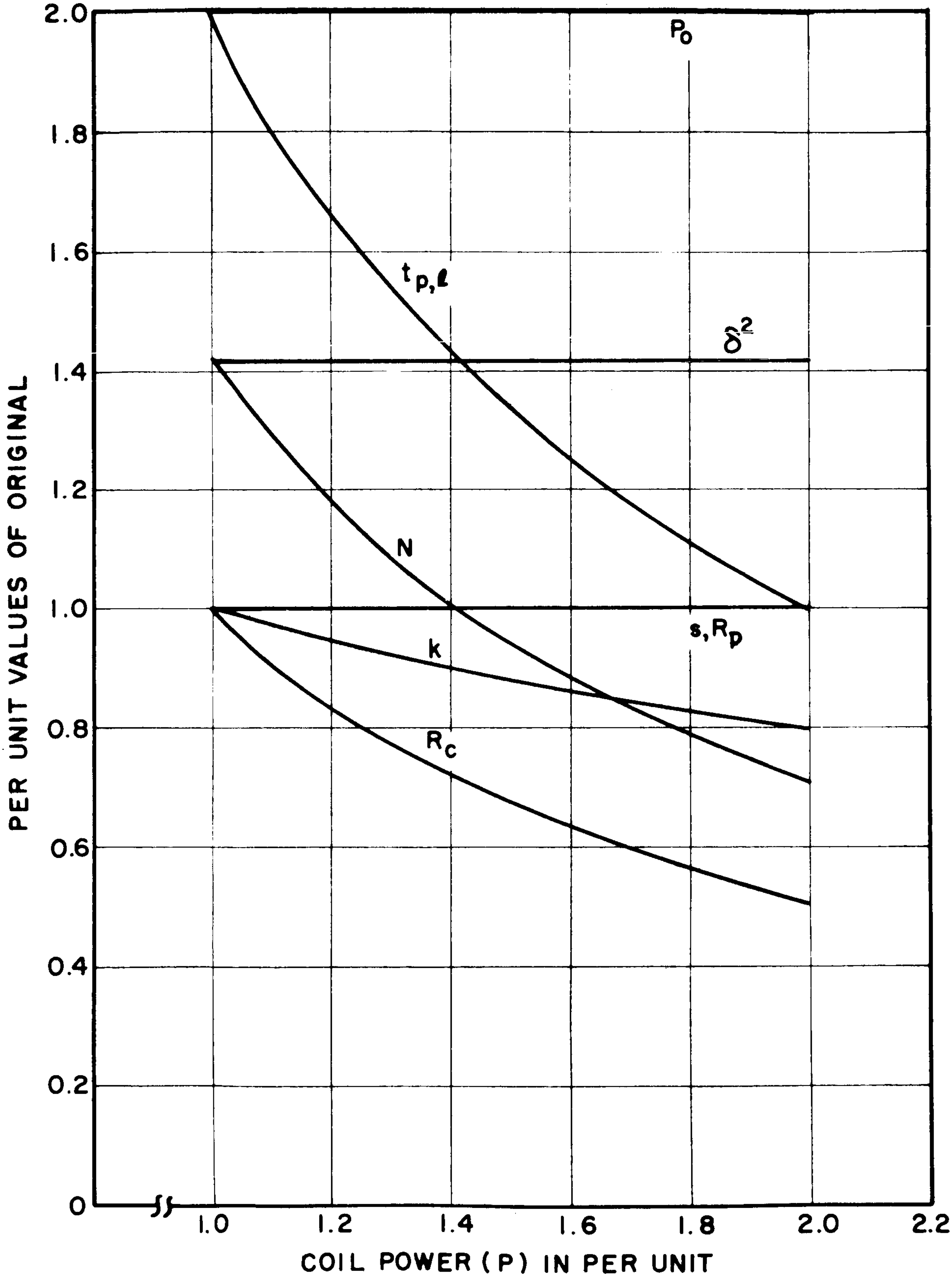

Figure 2 Changes in the Parameters Plotted Versus Coil Power when Back Tension (P_o) is Doubled and η, E, M, R_s, x_o and μ A are Fixed. β = 0.435 and σ = 0.13

□	δ						δ	δ						20			
○	η	η	η	η										4			
□	$\mathcal{R}_p$									$\boxed{\mathcal{R}_p}$				14			
○	E	E		E	E									5			
□	k			$\boxed{k}$								k		17			
□	K								$\boxed{K}$					13			
□	ℓ						ℓ	ℓ						19			
○	M			M										7			
□	N	$\boxed{N}$	N					N						15			
○	P				P									3			
○	P_o	P_o							P_o					2			
□	R_c					$\boxed{R_c}$	R_c							9			
□	R_t	R_t	R_t	R_t	$\boxed{R_t}$	R_t								6			
○	R_s					R_s								8			
○	s						s	s			s			11			
□	t_p		$\boxed{t_p}$									t_p		16			
□	t_s											$\boxed{t_s}$		18			
○	x_o	x_o	x_o	x_o					x_o	x_o				10			
□	μA	μA	μA							μA	$\boxed{\mu A}$			12			
○	β						β	β			β			1			
		F	G	H	A	B	s_1 J	s_1 J	D	E	C	I					
	o	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}					

Figure 3. Design Matrix Modified to Include β

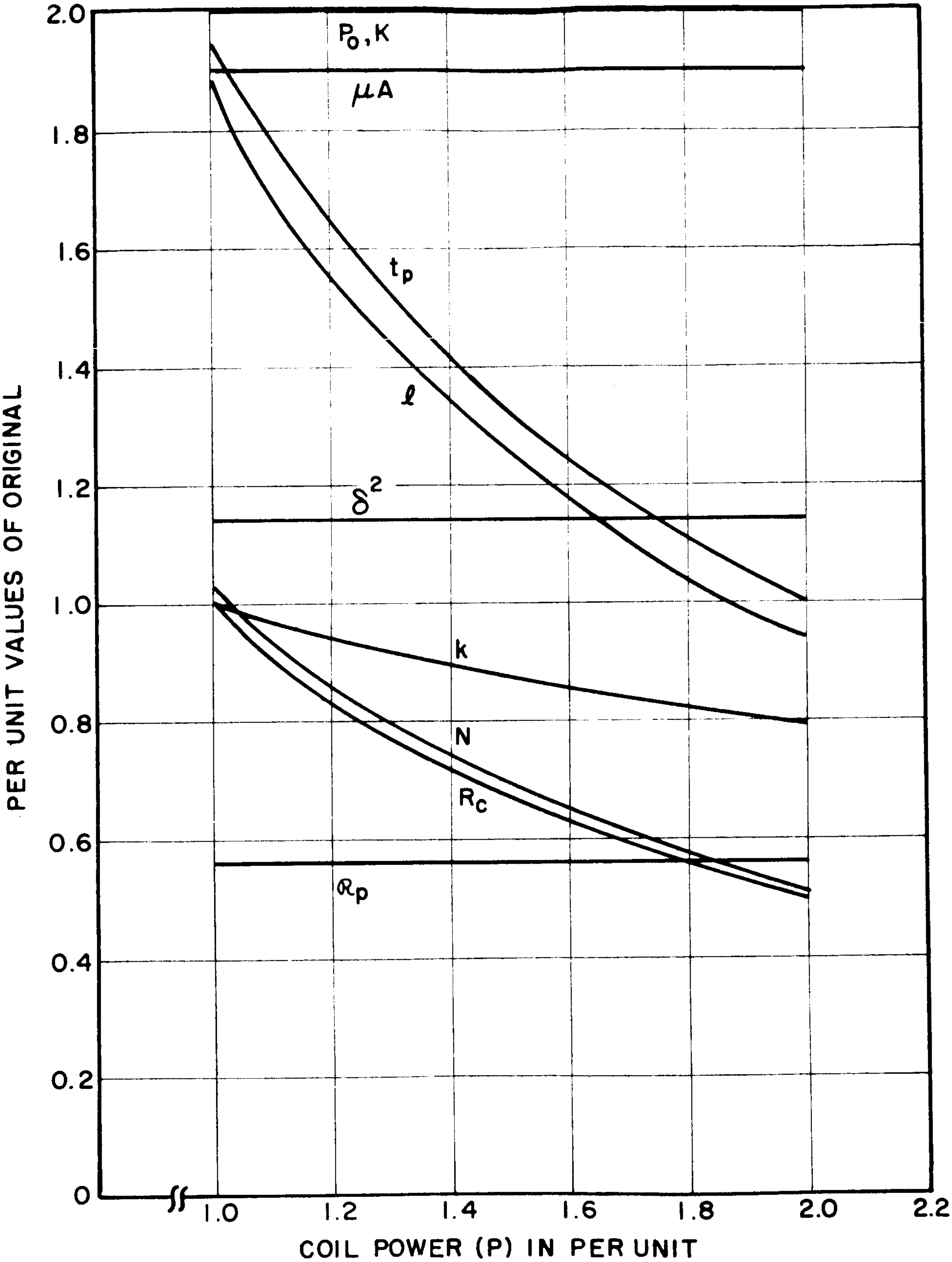

Figure 4 Changes in the Parameters Plotted Versus Coil Power P when Back Tension P_o is Doubled and η, E, M, κ_s, x_o and s are Fixed. β = 0.6 and σ = 0.13

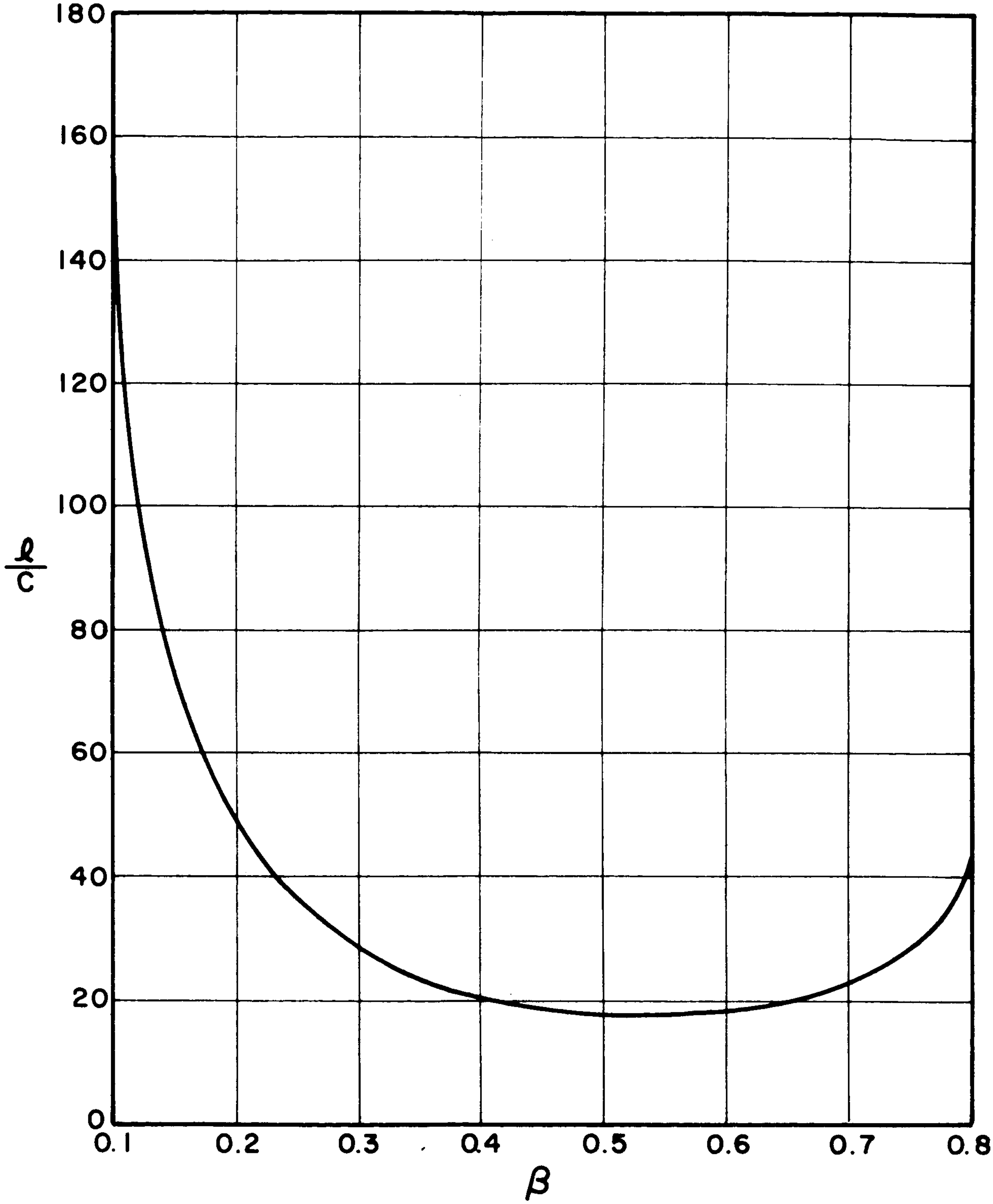

Figure 5 Plot of Equation 3

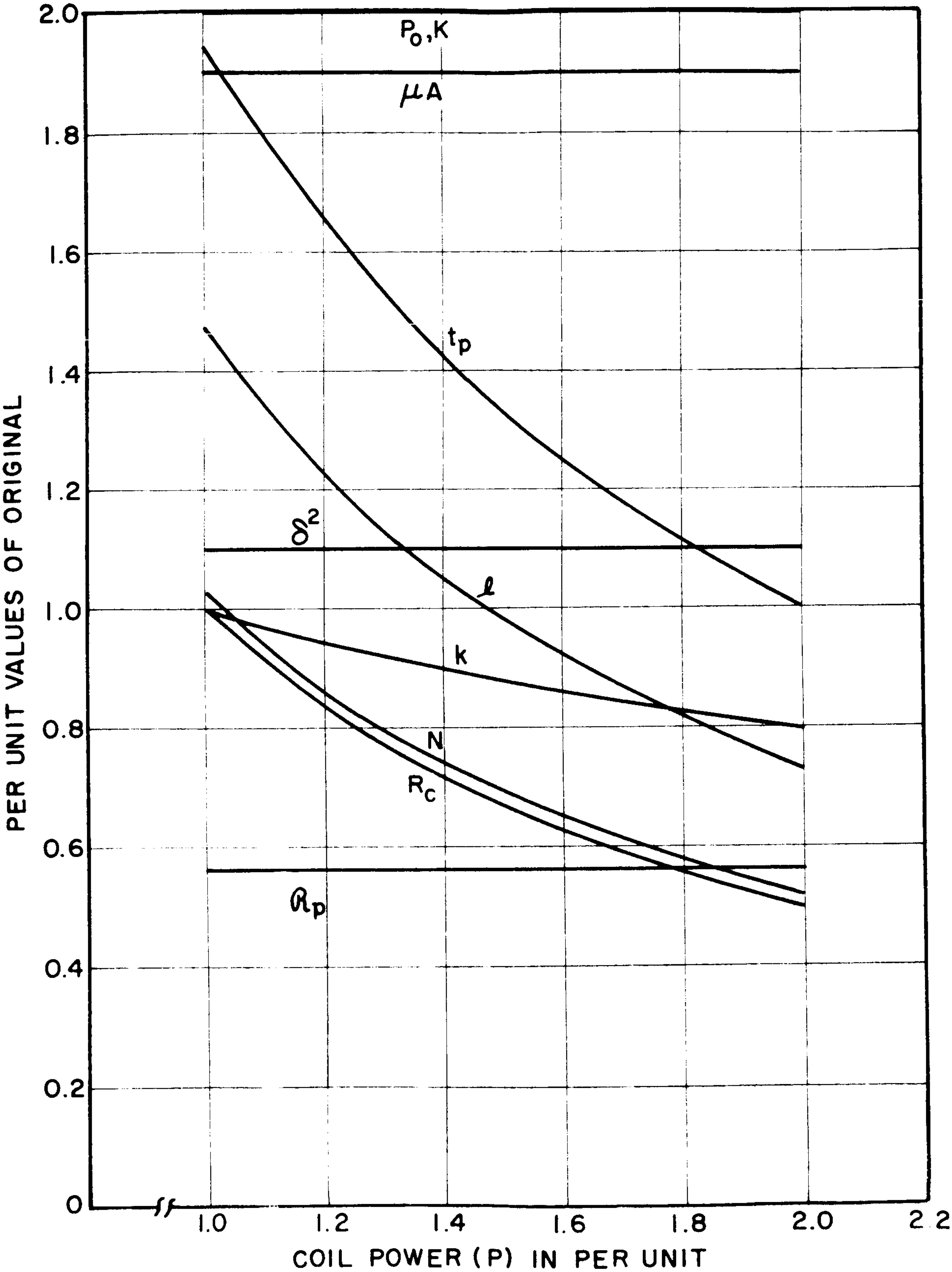

Figure 6 Changes in the Parameters Plotted Versus Coil Power P when Back Tension P_o is Doubled and η, E, M, R_s, x_o and s are Fixed. β = 0.6 and σ = 0.065