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SUMMARY

A magnetohydrodynamic generator of two-disk geometry operating in the Hall mode is considered. The generator working fluid flows radially between the two disks and a constant magnetic field is applied parallel to the axis of the disks. The conditions where the Mach number is small compared to one and where viscous effects can be neglected are analyzed. The power output, power-output density, and efficiency of this generator are determined and compared to the linear Hall generator. The calculations indicate that the linear generator has higher efficiency and power output for a given working fluid and entrance velocity.

INTRODUCTION

A magnetohydrodynamic (MHD) generator operating in the Hall mode (i.e., the Hall current is the load current) is considered. Generators of this type are particularly well suited for operation with large Hall parameters (ref. 1). A Hall generator with the Faraday electric field equal to zero can be shown to give a large amount of nonequilibrium ionization (ref. 2, eq. (23)). In linear generators the Faraday field is made to vanish by segmenting the electrodes and then shorting opposite pairs of segments. The short circuit for the Faraday current may lead to difficulties (ref. 3) so that a geometry having the Faraday field always equal to zero is of interest. Such a configuration is shown in figure 1 where the working fluid flows radially between the two disks with a constant magnetic field applied parallel to the disk axis. The design is functionally similar to that reported in reference 3.

This report analytically compares the performance of a disk and a linear Hall generator in terms of power density and efficiency. Emphasis is placed upon parametric

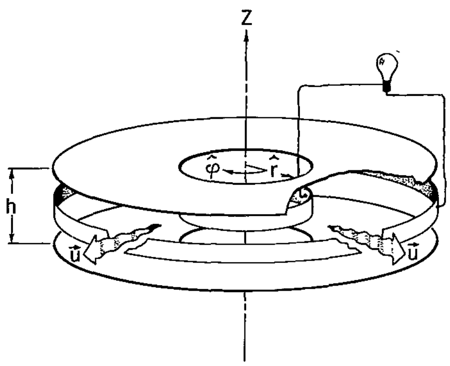

Figure 1. - Sketch of radial-flow generator.

performance evaluation that illustrates the effect of the the geometrical arrangement. Several simplifying assumptions are used; namely, that viscous effects are negligible, magnetic Reynolds number and Mach number are small, and electrical properties are independent of magnetic field and current. Although these assumptions may not always be valid (especially if electron heating is imparted), they are made for both generators so that the analysis is selfconsistent and should permit a reasonable comparison of the configuration effects.

EQUATIONS DESCRIBING GENERATOR

The momentum equation for an inviscid fluid carrying a current and flowing in a magnetic field is (ref. 4)

$$\rho \frac{D\vec{u}}{Dt} + \text{grad } p = \vec{j} \times \vec{B} \tag{1}$$

(a list of symbols is given in appendix A). If the magnetic Reynolds number is small compared to 1 then the induced magnetic fields can be neglected in comparison to the applied magnetic field strength $\vec{B}$ (ref. 4). For this study B will be only in the z direction. (See fig. 1 for coordinate sketch.) The velocity and current density vectors can be written:

$$\vec{u} = v\hat{r} + w\hat{\varphi}$$

$$\vec{j} = j_r\hat{r} + j_\varphi\hat{\varphi}$$

so that equation (1) in component form becomes:

$$\rho\left(v\frac{dv}{dr} - \frac{w^2}{r}\right) + \frac{dp}{dr} = j_\varphi B \tag{2}$$

$$\rho v \frac{1}{r}\frac{d}{dr}(rw) = -j_r B \tag{3}$$

For a flow with the Mach number small compared to one the continuity equation for axially symmetric flow requires:

$$\frac{d}{dr}(rv) = 0 \tag{4}$$

For a plasma with nonzero Hall parameters for ions and electrons and with the current flowing in a plane perpendicular to the magnetic field, Ohm's law becomes (ref. 5, eq. (49)):

$$\vec{j} = \frac{1}{(1 + \beta_e\beta_i)}\left[\sigma_0(\vec{E} + \vec{u} \times \vec{B}) + \beta_e \frac{\vec{B} \times \vec{j}}{B}\right]$$

This equation can be solved for $\vec{j}$ (ref. 5, eq. 50):

$$\vec{j} = \sigma\left[(1 + \beta_e\beta_i)(\vec{E} + \vec{u} \times \vec{B}) + \beta_e \frac{\vec{B} \times (\vec{E} + \vec{u} \times \vec{B})}{B}\right]$$

where

$$\sigma = \frac{\sigma_0}{(1 + \beta_e\beta_i)^2 + \beta_e^2}$$

For the coordinate system used herein, the equation for $\vec{j}$ can be written in component form as shown:

$$j_r = \sigma\left[(1 + \beta_e\beta_i)(E_r + wB) + \beta_e vB\right] \tag{5}$$

$$j_\varphi = \sigma\left[(1 + \beta_e\beta_i)(-vB) + \beta_e(E_r + wB)\right] \tag{6}$$

The requirement that $\operatorname{div} \vec{j} = 0$ is

$$\frac{d}{dr}(rj_r) = 0$$

which yields

$$rj_r = r_1 j_1 \tag{7}$$

This value for rj_r may be substituted into equation (5) which, when solved for E_r yields

$$E_r = \frac{\frac{r_1 j_1}{r} - \sigma\beta_e vB}{\sigma(1 + \beta_e\beta_i)} - wB \tag{8}$$

Using this value for E_r the azimuthal current j_φ may be obtained from equation (6)

$$j_\varphi = \frac{\beta_e j_r - \sigma_0 vB}{1 + \beta_e\beta_i} \tag{9}$$

Equations (4) and (3) can be solved for v and w

$$v = v_1 \frac{r_1}{r} \tag{10}$$

$$w = \frac{j_1 B}{\rho v_1}\left(\frac{r_1^2 - r^2}{2r}\right) \tag{11}$$

Equations (7), (8), (9), (10), and (11) express all components of current, electronic field, and velocity as functions of radius in terms of initial conditions at $r = r_1$.

POWER-OUTPUT DENSITY

The equations in the preceding section describe a generator (positive power output), only for a certain range of initial current density, j_1. The smallest j_1 of interest is the open-circuit value, namely zero. The largest value is the short-circuit current, that is, the current which flows when the load voltage is zero. The load voltage can be expressed as

$$V = \int_{r_1}^{r_2} (-E_r)dr = \frac{r_1 \ln \frac{r_2}{r_1} \beta_e v_1 B}{1 + \beta_e\beta_i}\left\{1 - \frac{j_1\left[1 + (1 + \beta_e\beta_i)\delta f\right]}{\sigma v_1 B\beta_e}\right\} \tag{12}$$

where

$$f = \frac{\left(\frac{r_2}{r_1}\right)^2 - 1}{\ln\left(\frac{r}{r_1}\right)^2} - 1$$

$$\delta = \frac{\sigma_0 B^2 r_1}{2\rho v_1 \left[(1 + \beta_e \beta_i)^2 + \beta_e^2\right]}$$

It is evident from equation (12) that the short-circuit current is

$$(j_1)_{sc} = \frac{\sigma v_1 B \beta_e}{1 + \delta f(1 + \beta_e \beta_i)}$$

It is convenient to express the load voltage as a fraction of the open-circuit voltage. Let this fraction be called the load parameter and be denoted by K, so that from equation (12)

$$K \equiv \frac{V}{V_{oc}} = 1 - \frac{j_1 \left[1 + \delta f(1 + \beta_e \beta_i)\right]}{\sigma v_1 B \beta_e} \tag{13}$$

Thus the range on j_1 can be specified in terms of the load parameter as

$$j_1 = \frac{\sigma v_1 B \beta_e (1 - K)}{1 + \delta f(1 + \beta_e \beta_i)} \qquad 0 \leq K \leq 1 \tag{14}$$

The power output can be represented as the volume integral of $\left[\vec{j} \cdot (-\vec{E})\right]$

$$P = \int_{vol} \left[\vec{j} \cdot (-\vec{E})\right] d\tau = \int_{r_1}^{r_2} (2\pi h j_r)(-E_r) r \, dr$$

With equations (7), (12), and (14), this power becomes

$$P = 2\pi h r_1 j_1 V = \frac{\pi h r_1^2 \ln\left(\frac{r_2}{r_1}\right)^2 \sigma v_1^2 B^2 \beta_e^2 K(1 - K)}{(1 + \beta_e \beta_i)\left[1 + \delta f(1 + \beta_e \beta_i)\right]}$$

Since the volume of the generator is $\pi h\left(r_2^2 - r_1^2\right)$ the power density becomes

$$\Pi = \frac{P}{\pi h\left(r_2^2 - r_1^2\right)} = \frac{\sigma v_1^2 B^2 \beta_e^2 K(1 - K)}{(1 + \beta_e \beta_i)\left[1 + \delta f(1 + \beta_e \beta_i)\right](1 + f)} \qquad (15)$$

The maximum value for Π occurs for $K = 1/2$ for the case of constant conductivity.

The effect of this constant conductivity restriction can be illustrated by comparison with results in reference 5. In figure 5 of that reference, the power density for a generator with a plasma whose properties are calculated with the theory of electron heating is shown to be a maximum for load parameter of about 0.42. This difference must be noted when the generator is operating under conditions such that electron heating is likely to be important.

EFFICIENCY

The efficiency of this generator may be defined as the ratio of the output power to the flow work per unit time done by the fluid. Since the only force considered herein is the electromagnetic-body force, the flow work per unit time is $\left[-\vec{u} \cdot (\vec{j} \times \vec{B})\right]$ so that the efficiency η becomes

$$\eta = \frac{\int_{vol} \vec{j} \cdot (-\vec{E}) d\tau}{\int_{vol} \left[-\vec{u} \cdot (\vec{j} \times \vec{B})\right] d\tau} = \frac{\eta_0 K(1 - K)}{(\epsilon - \lambda)K^2 + \lambda K + 1} \qquad (16)$$

where

$$\eta_0 = \frac{\beta_e^2 \left[1 + \delta f(1 + \beta_e \beta_i)\right]}{(1 + \beta_e \beta_i)^2 \left[1 + \delta f(1 + \beta_e \beta_i)\right]^2 + \left[\beta_e \delta f(1 + \beta_e \beta_i)\right]^2}$$

$$\lambda = \frac{1 + 3(1 + \beta_e\beta_i)\delta f}{1 + (1 + \beta_e\beta_i)\delta f}\,\eta_0$$

$$\epsilon = \frac{\lambda + \eta_0}{2} = \frac{1 + 2(1 + \beta_e\beta_i)\delta f}{1 + (1 + \beta_e\beta_i)\delta f}\,\eta_0 \tag{17}$$

Differentiation of equation (16) shows that the efficiency is a maximum when

$$K = \frac{1}{\sqrt{1 + \epsilon} + 1}$$

and has a value given by

$$\eta_{max} = \frac{\eta_0}{\lambda + 2(\sqrt{1 + \epsilon} + 1)} \tag{18}$$

SPECIFICATION OF MAGNETIC FIELD

Although the parameters appearing in the expressions for the generator are arbitrary, there are some optimum values that can be specified. Because of ion slip there is a magnetic field which maximizes the efficiency and power density (ref. 6, figs. 5, 6, and 7). To examine this, introduce the following dimensionless variables:

$$\delta_c = \frac{\sigma_0 r_1}{2\mu_e\mu_i\rho v_1} \tag{19a}$$

$$\mu \equiv \frac{\mu_e}{\mu_i} \tag{19b}$$

$$z \equiv \beta_e\beta_i = (\mu_e B)(\mu_i B) \tag{19c}$$

where δ_c is a modified interaction parameter; μ is the ratio of electron to ion mobility; and z is an ion slip term. The parameters δ_c and μ do not depend upon B since the plasma properties μ_e, μ_i, and σ_0 are assumed constant. Then by substitution in equation (15) the power density may be written as

$$\Pi = \frac{2K(1 - K)\rho v_1^3 \mu \delta_c z^2}{r_1(1 + z)(1 + f)\left[(1 + z)^2 + (\mu + \delta_c f)z + \delta_c f z^2\right]} \tag{20}$$

The value of z for which Π as well as the total power output is a maximum is determined from $\partial\Pi/\partial z = 0$

$$\delta_c fz(1 + z)(1 - z) + \mu z = (z + 1)^2(z - 2) \tag{21}$$

The power output of the generator (but not the power density) also has a maximum with respect to variation in the radius ratio. This maximum occurs when

$$\delta_c\left[(y - 1) - 2f\right]z(z + 1) - \mu z = (1 + z)^2 \tag{22}$$

where

$$y = \left(\frac{r}{r_1}\right)^2$$

Equations (21) and (22) determine conditions for which the power output is maximized. Thus, for specified δ_c and μ (which are parameters determined by the fluid properties, generator inlet size, and generator operating conditions) the optimum magnetic field and outside radius may be obtained from

$$\delta_c = \frac{(z^2 - 1)}{z\left[y - 1 - (z + 1)f\right]} \tag{23}$$

$$\mu = \left[\frac{(z - 1)(y - 1 - 2f)}{y - 1 - (z + 1)f} - 1\right]\frac{(z + 1)^2}{z} \tag{24}$$

These functions are shown in figure 2. The resulting maximum power output (divided by the input kinetic-energy rate) is given by

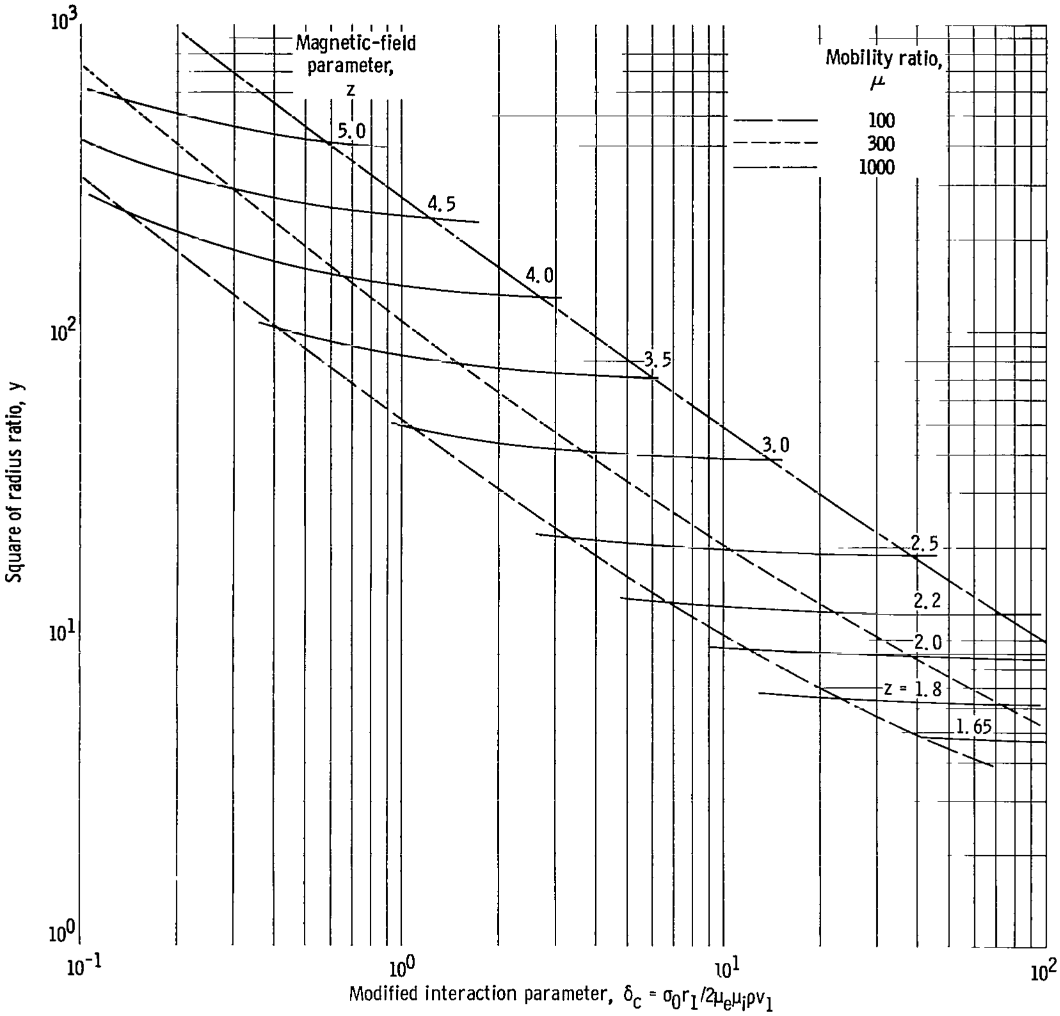

Figure 2. - Radius ratio as function of modified interaction parameter for specified mobility ratio and magnetic field parameter.

$$\frac{P}{2\pi h r_1 \rho v_1^3} = \frac{\delta \mu z \ln y}{4(1 + z)\left[1 + (1 + z)\delta f\right]} \tag{25}$$

and is shown in figure 3. The maximum efficiency, calculated from equation (18), is shown in figure 4. The ranges shown for the dimensionless variables are those that might occur in actual generators.

COMPARISON WITH LINEAR HALL GENERATOR

The power-output density for a linear Hall generator is given as (ref. 7, eq. (14.43))

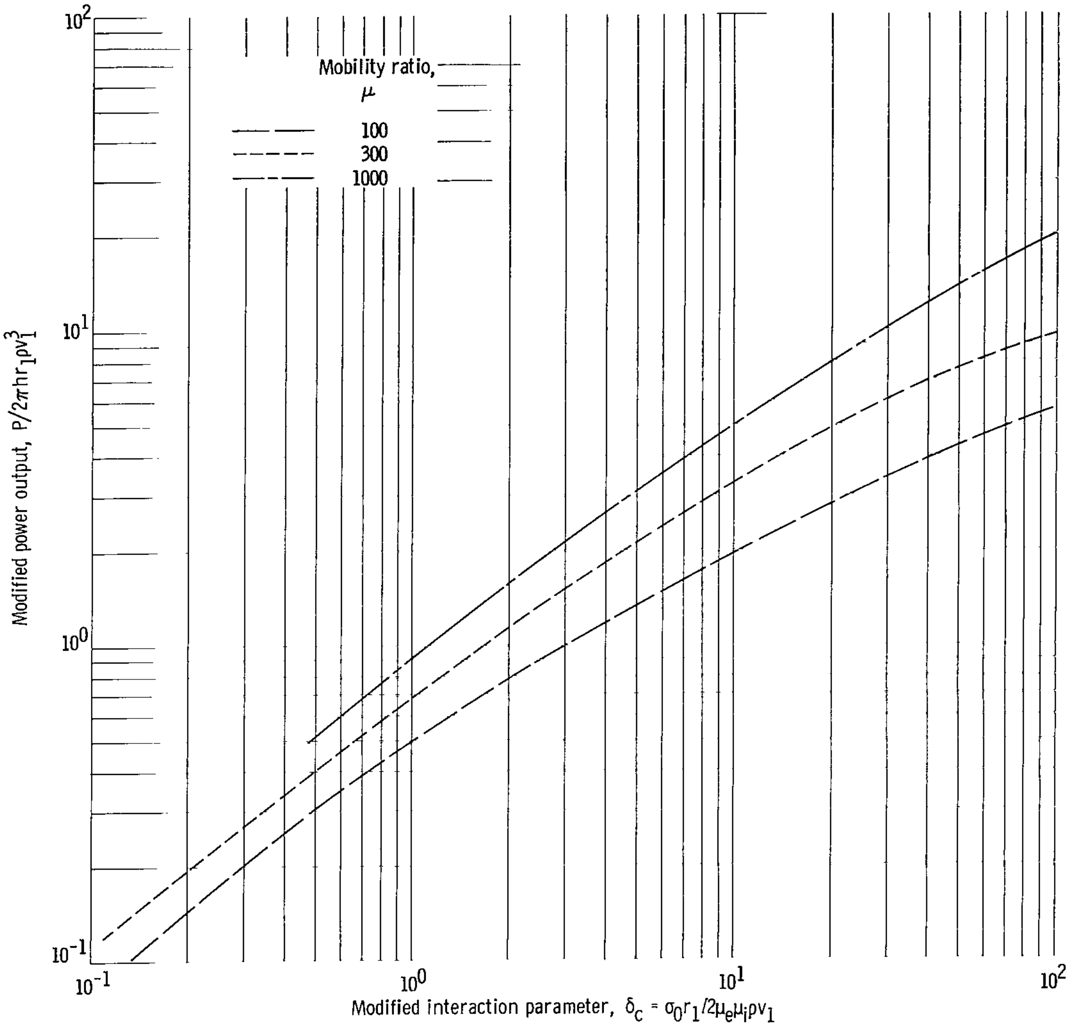

Figure 3. - Power output as function of modified interaction parameter for specified values of mobility ratio.

$$\Pi_\ell = \frac{\sigma K_\ell(1 - K_\ell)V_1^2\beta_e^2B^2}{1 + \beta_e\beta_i} \tag{26}$$

It can be shown from equations (20) and (26) that, since $\delta_c f \geq 0$ and $\beta_e\beta_i \geq 0$, the power density ratio $\Pi/\Pi\ell \leq 1$. This ratio is plotted in figure 5 using the results of equations (23) and (24). It may be seen that the ratio decreases with increasing values of the slip parameter.

The efficiency of the linear generator is (ref. 7, eq. (14.50))

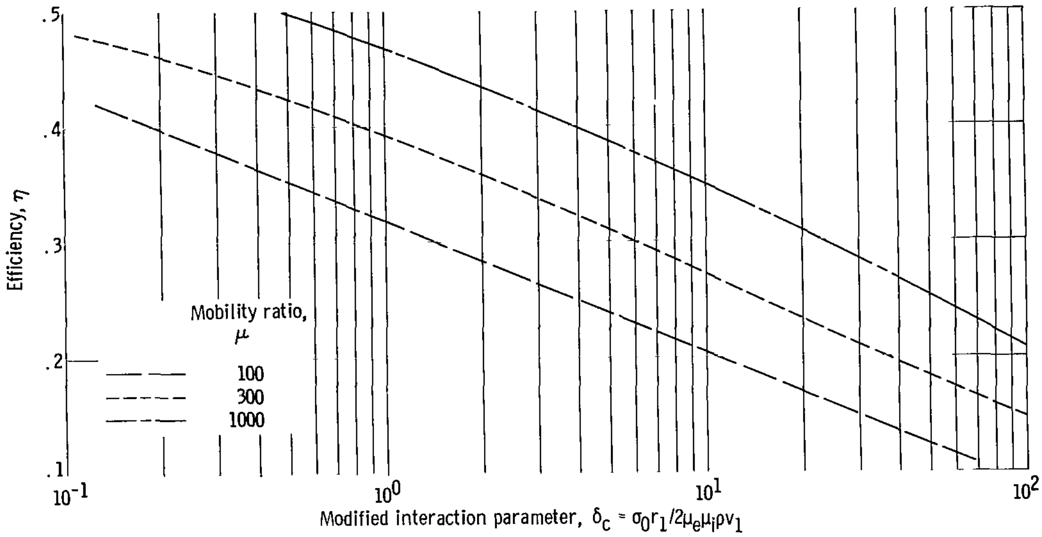

Figure 4. - Efficiency as function of modified interaction parameter for various mobility ratios.

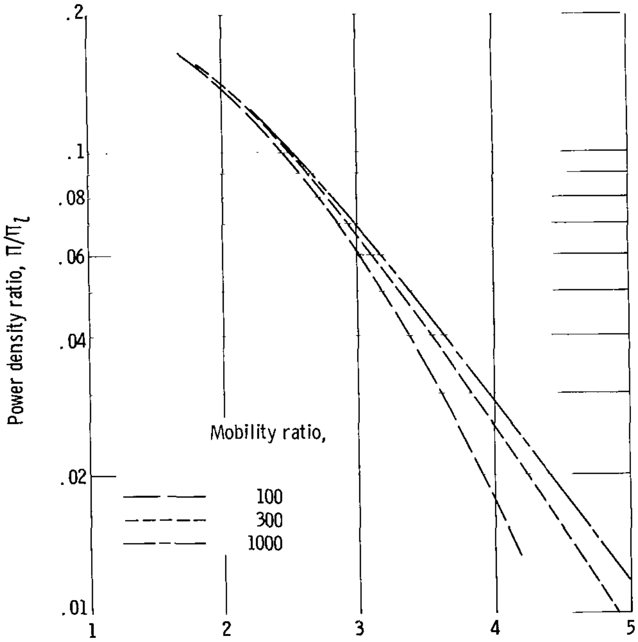

Figure 5. - Power density ratio as function of slip parameter for various mobility ratios at optimum magnetic field and radius ratio.

$$\eta_\ell = \frac{K_\ell(1 - K_\ell)}{\dfrac{(1 + \beta_e\beta_i)^2}{\beta_e^2} + K_\ell}$$

This is equal to the disk-generator efficiency when $f = 0$, and has a maximum when the load parameter K_ℓ is

$$K_\ell = \frac{1}{\sqrt{1 + \dfrac{\beta_e^2}{(1 + \beta_e\beta_i)^2}} + 1}$$

The efficiency at this value of K_ℓ becomes

$$(\eta_\ell)_{max} = \frac{\dfrac{\beta_e^2}{(1 + \beta_e\beta_i)^2}}{\dfrac{\beta_e^2}{(1 + \beta_e\beta_i)^2} + 2\left(\sqrt{1 + \dfrac{\beta_e^2}{(1 + \beta_e\beta_i)^2}} + 1\right)}$$

The ratio of efficiencies can be expressed as

$$\frac{\eta}{\eta_\ell} = \frac{\eta_0}{\dfrac{\beta_e^2}{(1+\beta_e\beta_i)^2}} \left[\frac{\dfrac{\beta_e^2}{(1+\beta_e\beta_i)^2} + 2\left(\sqrt{1 + \dfrac{\beta_e^2}{(1+\beta_e\beta_i)^2}} + 1\right)}{\lambda + 2\left(\sqrt{1+\epsilon} + 1\right)}\right]$$

and, since from equation (17)

$$\eta_0 \leq \epsilon \leq \lambda \quad \text{and} \quad \epsilon \leq \frac{\beta_e^2}{(1+\beta_e\beta_i)^2}$$

then

$$\frac{\eta}{\eta_\ell} \leq 1$$

This ratio is plotted in figure 6, using the values from equations (23) and (24). The efficiency ratio is seen to be an increasing function of the slip parameter.

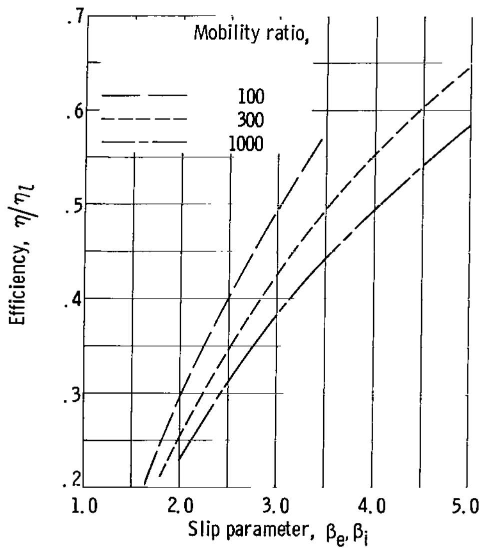

Figure 6. - Efficiency ratio as function of slip parameter for various mobility ratios at optimum magnetic field and radius ratio.

CONCLUDING REMARKS

The results of the calculations indicate that the disk generator is quite inferior in performance to the linear generator. For small values of the slip parameter the efficiency is much less and for large values of the slip parameter the power density is much less. However, it must be remembered that this comparison was made on the basis of maximum power output of the disk generator. Nevertheless, it appears that the disk-generator performance is so inferior that the greater benefits of the electron heating (as indicated in reference 3, (figs. 6 and 10) are inadequate to make its performance competitive with that of the linear generator.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 14, 1965.

APPENDIX - SYMBOLS

$\vec{B}$	magnetic field strength
$\vec{E}$	electric field strength
f	function of radius ratio
h	spacing between disks
$\vec{j}$	current density
K	load parameter
P	power output
p	pressure
r	radial coordinate
$\hat{r}, \hat{\varphi}, \hat{z}$	unit vectors in coordinate directions
$\vec{u}$	velocity
V	load voltage
v	radial velocity
w	azimuthal velocity
y	square of radius ratio $(r/r_1)^2$
z	magnetic-field parameter, eq. (19c)
β_e, β_i	electron and ion Hall parameters
δ, δ_c	interaction parameters, eq. (19a)
$\epsilon, \lambda, \eta_0$	efficiency parameters, eq. (17)
η	efficiency
μ	mobility ratios, eq. (19b)
μ_e, μ_i	electron and ion mobilities
Π	power densities
ρ	density
σ, σ_0	electrical conductivities
$d\tau$	volume element

Subscripts:

ℓ	linear generator
oc	open circuit
sc	short circuit
r	radial component
1	inlet condition
φ	azimuthal component

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