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AEROSPACE PRESSURE VESSEL DESIGN SYNTHESIS

by George Gerard

Prepared under Contract No. NASw-928 *by*
ALLIED RESEARCH ASSOCIATES, INC.
Concord, Mass.

for

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Summary

A governing structures-materials-design synthesis relationship is derived for the primary structural weight of membrane type pressure vessels. The structural efficiency is associated with the configuration and the failure law characterizing the material used. Closed pressure vessels of various shapes utilizing monolithic and filamentary materials are examined in some detail to establish optimum designs.

The structural strength/weight ratio has a profound influence upon the pressure vessel efficiency. Values of this ratio realized currently in monolithic and filamentary designs are evaluated. Likewise, the potential of anisotropic metals, filamentary-monolithic composites and whisker composites is studied. The configuration and material efficiencies are then combined to investigate the comparative efficiencies of pressure vessels of various shapes and materials concepts.

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Symbols

a	texture hardening coefficient, $a = \Sigma_1/\Sigma_3$
A	surface area
b	mechanical anisotropy coefficient, $b = \Sigma_1/\Sigma_2$
c_c	filament cross-over coefficient
C	structural configuration efficiency coefficient
d	diameter
$\bar{e}$	ductility ratio
h	ellipsoidal closure minor diameter
k_e	elastic stress concentration factor
k_p	plastic stress concentration factor
L	overall length of pressure vessel
p	pressure, psi
r	radial coordinate
R	principal radius of curvature
s	arc length
S	uniaxial structural strength, psi
t	thickness
$\bar{t}$	average thickness
V	volume, in ³
w	weight penalty coefficient
W	weight, lbs.
z	axial coordinate
α	thickness coefficient
ϵ	strain
ρ	density, pci
σ	stress, psi
Σ	uniaxial strength, psi
ϕ	angle

Subscripts

a	anisotropic
c	cylinder
e	ellipsoidal
f	filamentary
h	hemispherical
i	isotropic
m	monolithic
tu	tension ultimate
1, 2, 3	principal directions

AEROSPACE PRESSURE VESSEL DESIGN SYNTHESIS

1. Introduction

The utilization of pressure vessels in aerospace applications is manifold. Consequently, it is the objective here to examine systematically those parameters which have a major influence upon the weight of thin wall pressure vessels under specified design conditions.

Since our interest here is in a broad design synthesis viewpoint which is generally applicable in the preliminary design stage, we shall be concerned with the primary structural weight associated with optimum membrane type pressure vessels. It is assumed that the secondary weight comprises the additional material associated with nonoptimum membrane thicknesses, discontinuities, joints, cutouts and fittings.

Accordingly, Section 2 presents a generalized treatment of the governing primary weight equation which relates the structural configuration efficiency, the material efficiency and the prescribed design conditions for several different failure criteria. The configuration efficiencies of various pressure vessel shapes are treated in Section 3 and encompasses both simple shapes as well as cylindrical vessels with closures.

In Section 4, the structural strength/weight ratios attained with current monolithic metallics, filamentary composites and inflatable structures are evaluated. The potential of newer materials concepts such as anisotropic materials, filamentary-monolithic composites, and whisker composites is evaluated in Section 5.

The configuration efficiencies and material efficiencies considered separately in Sections 3-5 are combined in Section 6 to treat the overall efficiency of pressure vessels utilizing various shapes and materials. The results of a comparative efficiency study are presented in a design synthesis chart which summarizes the major results of this investigation.

2. Structures-Materials-Design Synthesis*

The optimum design problem for pressure vessels can be stated in the following manner: for prescribed pressure (p) and volume (V) determine the structural configuration and material that results in a minimum weight design. The following development is based upon the simplifying assumption that the pressure vessel can be treated as a membrane and, therefore, represents the primary structural weight as defined in the preceding section.

This problem has been considered in various aspects by Schuerch¹, Hoffman², Pipkin and Rivlin³, and Brewer and Jeppeson⁴ for filamentary isotensoids and also for monolithic membranes. In the following, a systematic development of the design synthesis equation is presented for three cases: optimum monolithic membranes that fail according to the maximum shear law, filamentary membranes, and monolithic membranes that fail according to the octahedral shear law. The latter results, which were not obtained in the above cited references, can represent an improvement in structural efficiency as compared to the maximum shear case.

Governing Equations

In general form, we have the following relationships for the membrane of revolution shown in Fig. 1. The weight of an elemental ring of radius, r, and width, ds, is

$$dW = 2\pi r t \, ds \quad (1)$$

The equations of equilibrium in terms of principal stresses and radii of curvature are as follows:

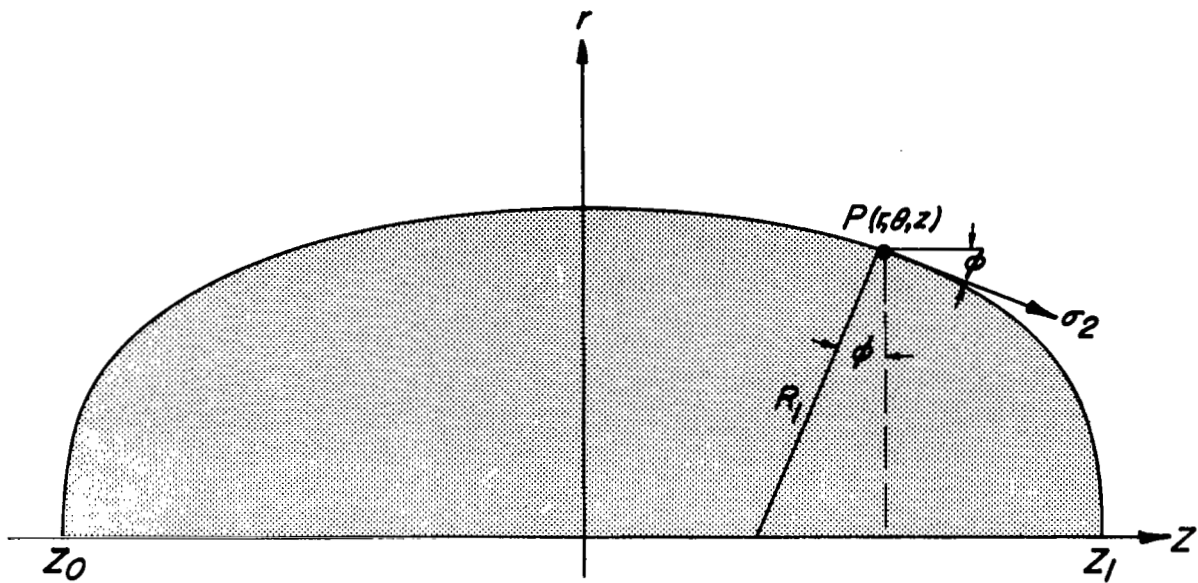
$$(\sigma_1 t / R_1) + (\sigma_2 t / R_2) = p \quad (2)$$

$$\sigma_2 = p R_1 / 2t \quad (3)$$

By substituting Eq. (3) into (2), we obtain

$$\sigma_1 = (p R_1 / 2t)(2 - R_1 / R_2) \quad (4)$$

*The contributions of C. Lakshmikantham to Sections 2 and 3 are gratefully acknowledged.



Note: R_2 is principal radius of curvature of profile $r(z)$

Figure 1 Pressure Vessel Membrane

For σ_1 to be positive (tension), the following condition is imposed upon Eq. (4)

$$2R_2 \geq R_1 \quad (5)$$

This condition is necessary for an isotensoid filamentary membrane and is also desirable for a monolithic membrane to avoid buckling. Furthermore, in the following development for monolithic membranes it is convenient (although not essential) that $\sigma_1 \geq \sigma_2$. For this purpose we can impose the more restrictive condition on Eq. (4)

$$R_2 \geq R_1 \quad \text{for} \quad \sigma_1 \geq \sigma_2 \quad (6)$$

In order to determine the minimum weight design for a prescribed pressure and volume in a general manner, we can integrate Eq. (1)

$$W/\rho = 2\pi \int r t ds \quad (7)$$

Now, if $\bar{t}$ represents the thickness averaged over the surface area, then Eq. (7) can be written as

$$W/\rho V = \bar{t} A/V \quad (8)$$

Note that Eq. (8) represents the volume of structural material relative to the enclosed volume and as such is equivalent to the solidity familiarly used in the minimum weight analysis of compression structures.

For a given shape with $\sigma_1 = \Sigma_1$, where Σ_1 represents the failure strength of the material, Eq. (4) can be put in the following form,

$$\bar{t} = \alpha p / \Sigma_1 \quad (9)$$

Substituting Eq. (9) into (8)

$$W = C(\rho/\Sigma_1)pV \quad (10)$$

$$\text{where: } C = \alpha A/V$$

It can be observed that the structural configuration efficiency factor (C) is a non-dimensional function of the membrane shape. In addition to C, Eq. (10) contains the material efficiency parameter (ρ/Σ_1) and the design index (pV) representing the prescribed design conditions.

In order to determine values for C, a failure law descriptive of the material is required to obtain the minimum weight design. In the following, three different failure laws are examined in conjunction with the assumption that each point on the surface is subjected to the local failure strength and thus optimum thickness is achieved.

Monolithic Membranes - Maximum Shear Law

As perhaps the simplest example, we consider first a monolithic membrane designed according to the maximum shear law as the failure criterion. This criterion can be used for yield or fracture strength according to the behavior of the material under consideration. Denoting the failure strength as Σ_1 as indicated in Fig. 2, and assuming that each element on the membrane surface is subject to Σ_1 simultaneously, then the optimum thickness is obtained directly from Eq. (4) since $\sigma_1 = \Sigma_1$.

$$t = (pR_1/2\Sigma_1)(2-R_1/R_2) \quad (11)$$

By substituting Eq. (11) into (1) and integrating, we obtain

$$W = (\rho/\Sigma_1)p\pi \int rR_1(2-R_1/R_2)ds \quad (12)$$

Eq. (8) can conveniently be written in the form of Eq. (10), where now

$$CV = \pi \int R_1^2 (2-R_1/R_2)dz \quad (13)$$

In obtaining Eq. (13), the relation $r = R_1(dz/ds)$ was utilized.

From Eq. (13) we can immediately obtain the following results: for a long cylinder $R_2 \rightarrow \infty$ and $C = 2$; for a sphere, $R_1 = R_2$ and $C = 3/2$. For other axisymmetric shapes, it is more convenient to utilize the r-z coordinates.

$$\begin{aligned} R_1 &= r[1 + (r')^2]^{1/2} \\ R_2 &= - \frac{[1 + (r')^2]^{3/2}}{r''} \end{aligned} \quad (14)$$

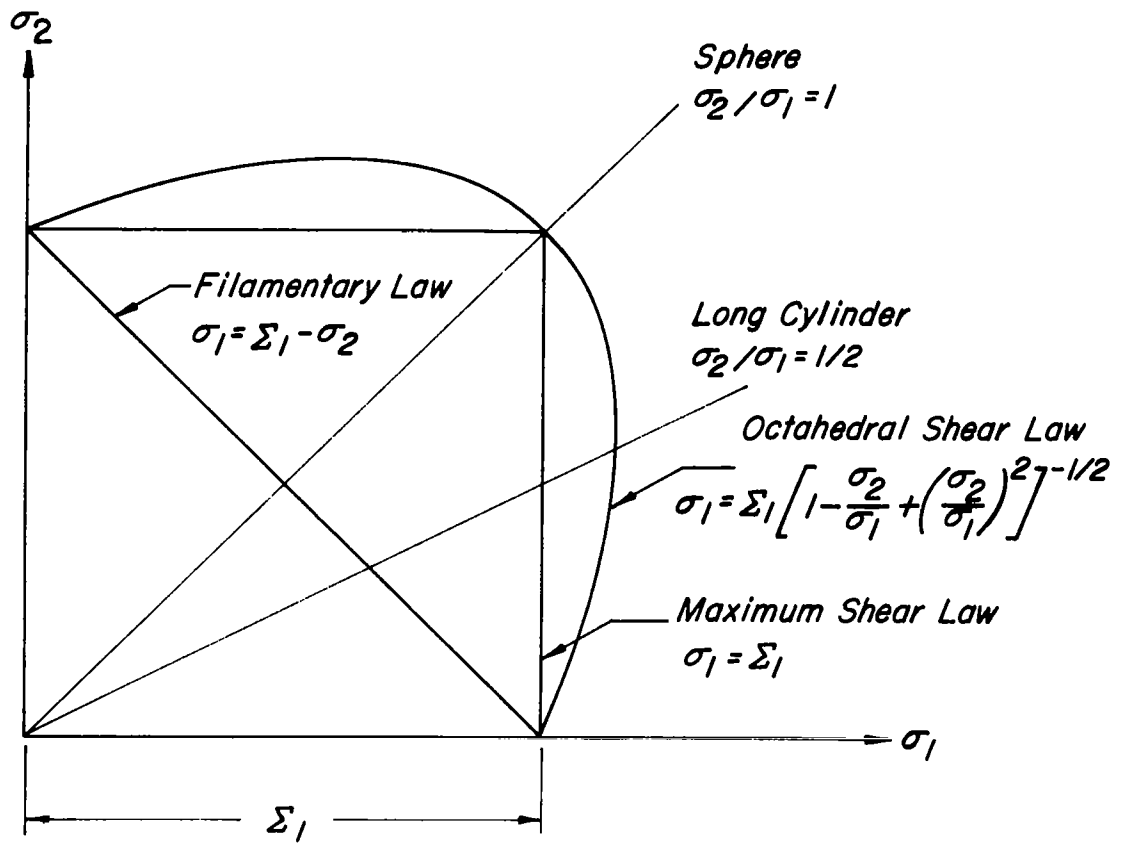


Figure 2 Failure Laws for Monolithic and Filamentary Membranes

Differentials with respect to the z coordinate are indicated by the primes. By substituting Eqs. (14) into (13)

$$CV = \pi \int [2r^2 + 2r^2(r')^2 + r^3 r''] dz \quad (15)$$

In an alternate form

$$CV = 2\pi \int_{z_0}^{z_1} r^2 dz - \pi \int_{z_0}^{z_1} r^2 (r')^2 dz + \pi \int_{z_0}^{z_1} (d/dz)(r^3 r') dz \quad (16)$$

For membranes which are closed and symmetric with respect to the plane $z = 0$, Pipkin and Rivlin³ have shown that the last integral in Eq. (16) vanishes. As a consequence, Eq. (16) reduces to the following form

$$C = 2[1 - \int_{z_0}^{z_1} r^2 (r')^2 dz / (2 \int_{z_0}^{z_1} r^2 dz)] \quad (17)$$

Eq. (17) applies to a closed membrane of revolution symmetrical about the equatorial plane, for which each point on the surface fails according to the maximum shear law.

Filamentary Isotenoid Membranes

Under a combined tensile stress field, where σ_1 and σ_2 are the principal stresses, an isotenoid filamentary network can be oriented along the principal stress directions or a specific optimum angle with the σ_1 direction given by

$$\phi = \pm \tan^{-1} (\sigma_2 / \sigma_1)^{1/2} \quad (18)$$

For these conditions, the principal stresses are related to the failure strength of a filamentary isotenoid membrane by the following relationship

$$\sigma_1 + \sigma_2 = \Sigma_1 \quad (19)$$

This failure law is illustrated in Fig. 2.

By adding Eqs. (3) and (4) and utilizing Eq. (19), the thickness required at any point on the membrane is

$$t = (pR_1/2\Sigma_1)(3-R_1/R_2) \quad (20)$$

Substituting Eq. (20) into Eq. (1), Eq. (10) is obtained where now

$$CV = 3\pi \int_{z_0}^{z_1} r^2 dz + \pi \int_{z_0}^{z_1} (d/dz)(r^3 r') dz \quad (21)$$

As discussed following Eq. (16), the last integral vanishes for closed symmetric membranes and, therefore, Eq. (21) reduces simply to

$$C = 3 \quad (22)$$

Thus, for closed filamentary isotenoid membranes of revolution, symmetrical about the equatorial plane and designed for a prescribed pressure and volume, the structural configuration efficiency factor is independent of shape and has a constant value of 3. This result is in contrast with that obtained for the monolithic membrane.

Monolithic Membranes - Octahedral Shear Law

Returning to the monolithic membrane now, it is assumed that it is designed according to the octahedral shear law as the failure criterion rather than the maximum shear law. We then have the interesting situation that the failure strength in general depends upon the location on the membrane surface since, as indicated in Fig. 2, strength is a function of σ_2/σ_1 .

According to the octahedral shear law

$$\sigma_1 = \Sigma_1 [1 - (\sigma_2/\sigma_1) + (\sigma_2/\sigma_1)^2]^{-1/2} \quad (23)$$

From Eqs. (3) and (4)

$$\sigma_2/\sigma_1 = (2 - R_1/R_2)^{-1} \quad (24)$$

Consequently, σ_1 in Eq. (24) is a function of the shape of the membrane and, in general, $\sigma_1 = \sigma_1(z)$. The optimum thickness required at any point is

$$t = (pR_1/2\sigma_1)(2 - R_1/R_2) \quad (25)$$

By substituting Eq. (23) into Eq. (1), Eq. (10) is obtained where now

$$CV = \pi \int (\Sigma_1/\sigma_1) R_1^2 (2 - R_1/R_2) dz \quad (26)$$

Using Eqs. (23) and (24) in conjunction with Eq. (26), we can obtain the following results: for a sphere $C = 3/2$, the same result obtained using the maximum shear law, whereas for a long closed cylinder $C = 1.732$, a significant reduction as compared to the result obtained from using the maximum shear law.

For other shapes, we utilize the r - z coordinates in conjunction with the following approximation for Eq. (23)

$$\Sigma_1/\sigma_1 = 1 - 0.6 (\sigma_2/\sigma_1)(1 - \sigma_2/\sigma_1) \quad (27)$$

Substituting Eq. (24) into (27)

$$\Sigma_1/\sigma_1 = 1 - 0.6 (1 - R_1/R_2)(2 - R_1/R_2)^2 \quad (28)$$

Utilizing Eqs. (14), (26) and (27), we obtain

$$\begin{aligned} CV = \pi \int [2r^2 + 2r^2(r')^2 + r^3 r''] dz \\ - 0.6\pi \int r^2 [1 + (r')^2] (1 + rr'')(2 + rr'')^{-1} dz \end{aligned} \quad (29)$$

Following the argument used with Eq. (16), Eq. (29) reduces to

$$CV = \pi \int [2r^2 - r^2(r')^2] dz - 0.6\pi \int r^2 [1 + (r')^2] (1 + rr'')(2 + rr'')^{-1} dz \quad (30)$$

In comparing Eqs. (16) and (30), it can be observed that octahedral shear values of C will always be lower than such values for the maximum shear case by virtue of the negative value of the last integral in Eq. (30).

3. Structural Configuration Efficiencies

In Section 2, the following design synthesis relation, Eq. (10), was shown to apply to monolithic and filamentary membranes of revolution.

$$W = C(\rho/\Sigma_1) pV \quad (31)$$

In a strict sense, the structural configuration efficiency coefficient, C is a function of the failure law as well as the shape. However, since effects of biaxiality upon the failure law as represented by σ_2/σ_1 can be directly related to the configuration, it is convenient to incorporate them directly in C . Thus, the coefficient C represents all shape effects and the strength Σ_1 in Eq. (31) represents the uniaxial tensile strength in all cases.

Values of C for optimum thickness spheres and cylinders were given in Section 2 as illustrative examples. Here, we consider in some detail the configuration efficiency of other pressure vessel shapes of interest. In addition, closures of various shapes for cylinders of different lengths are considered in terms of their comparative efficiencies.

Basic Configurations

By utilizing Eq. (17) and Eq. (26) or their equivalents in r - z coordinates, configuration efficiency coefficients were computed for long cylinders, spheres and ellipsoids. It is noted that Johnston⁵ has previously treated the ellipsoid for the maximum shear case. The formulas for C are presented in Table 1 and numerical results for optimum thickness membranes are presented in Fig. 3 in terms of the parameter L/d . For all closed filamentary membranes of optimum thickness $C = 3$. In connection with Fig. 3 it is to be noted that because of the equal volume requirement associated with Eq. (31), comparative values of C at the same L/d do not necessarily reflect relative efficiencies.

Also given in Table 1 are C values based upon the maximum rather than the optimum thickness. Such results are of interest when optimum tapering may not be practical. Both results are obviously identical for long cylinders and spheres but not for other shapes. For the latter, the maximum thickness is determined from Eqs. (11) and (25). In conjunction with Eq. (9), where now $\bar{t} = t_{\max}$, the value for C is determined as indicated in Eq. (10).

Table 1
Configuration Efficiency Coefficients for Monolithic Membranes

Configuration	Maximum Shear Law	Octahedral Shear Law														
Long Cylinder: t_o and t_{max}	2	1.732														
Sphere: t_o and t_{max}	1.5	1.5														
Ellipsoids:																
$0 \leq (d/L) \leq 1; t_o$	$2 - (1/2) (d/L)^2$	<table> <tr> <td>d/L</td> <td>0</td> <td>0.2</td> <td>0.4</td> <td>0.6</td> <td>0.8</td> <td>1.0</td> </tr> <tr> <td>C</td> <td>1.732</td> <td>1.711</td> <td>1.665</td> <td>1.600</td> <td>1.534</td> <td>1.500</td> </tr> </table>	d/L	0	0.2	0.4	0.6	0.8	1.0	C	1.732	1.711	1.665	1.600	1.534	1.500
d/L	0	0.2	0.4	0.6	0.8	1.0										
C	1.732	1.711	1.665	1.600	1.534	1.500										
$0.707 \leq (L/d) \leq 1; t_{max}$	$1 + (1/2) (d/L)^2$	<table> <tr> <td>L/d</td> <td>1.0</td> <td>0.9</td> <td>0.8</td> <td>0.707</td> </tr> <tr> <td>C</td> <td>1.500</td> <td>1.518</td> <td>1.611</td> <td>1.850</td> </tr> </table>	L/d	1.0	0.9	0.8	0.707	C	1.500	1.518	1.611	1.850				
L/d	1.0	0.9	0.8	0.707												
C	1.500	1.518	1.611	1.850												
$0 \leq (d/L) \leq 1; t_{max}$	$(3/4) [2 - (d/L)^2] [(d/L) + (1/\lambda) \sin^{-1} \lambda]$	$(3/4) [3 - 3(d/L)^2 + (d/L)^4]^{1/2} [(d/L) + (1/\lambda) \sin^{-1} \lambda]$														
$0.707 \leq (L/d) \leq 1; t_{max}$	$(3d/4L) [(d/L) + (1/2\lambda) \log \frac{(d/L) + \lambda}{(d/L) - \lambda}]$	Same as Maximum Shear Case														
where: $\lambda^2 = 1 - (d/L)^2$																

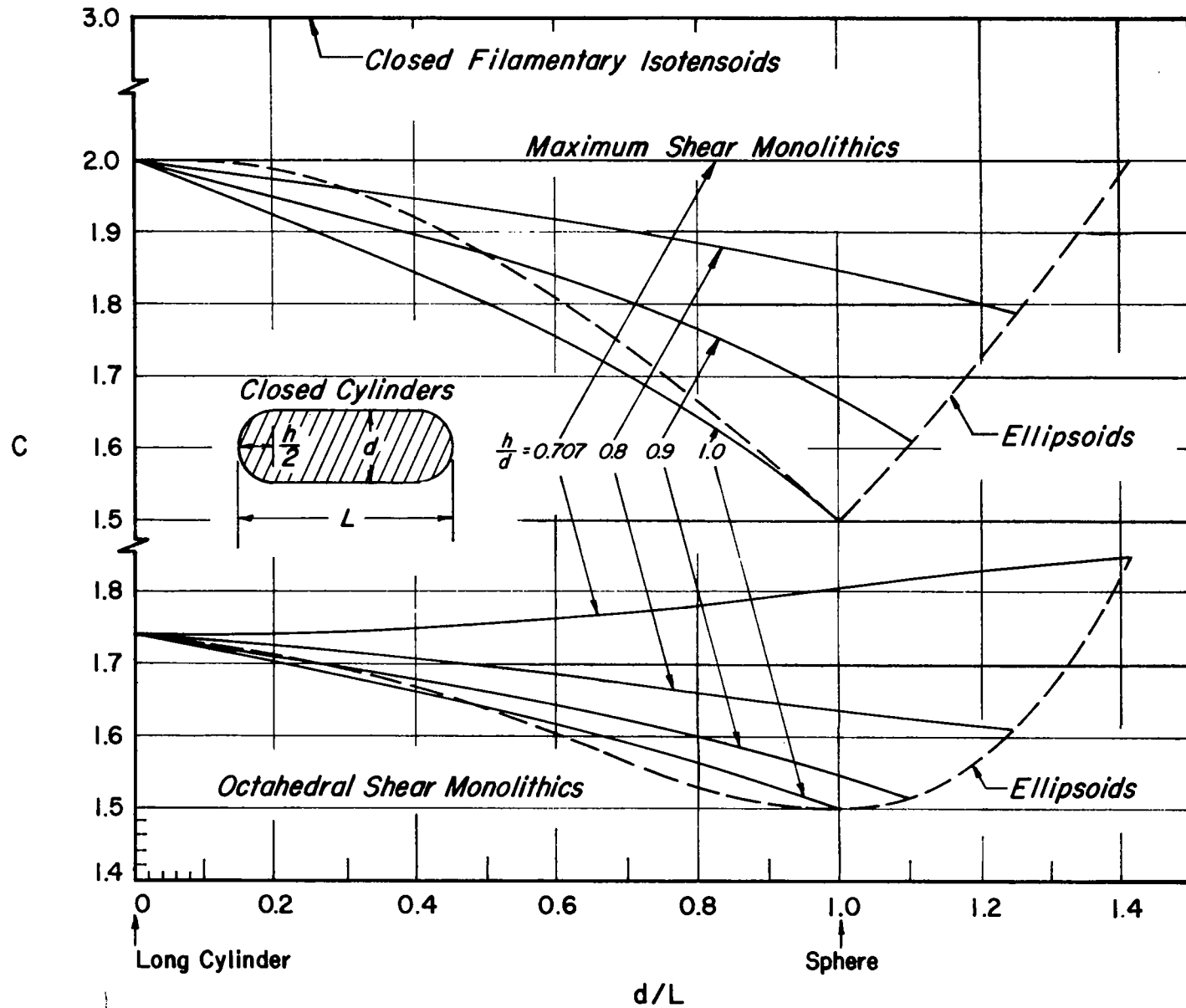


Figure 3 Configuration Efficiencies for Optimum Thickness Membranes

The weight penalty for using the maximum thickness relative to the optimum thickness is given by the coefficient

$$w_t = C(\text{for } t_{\max})/C(\text{for } t_o) \quad (32)$$

Results for the ellipsoid are given in Fig. 4.

It is interesting to observe from Eq. (31), that for a given shape, no weight penalty is incurred if the volume is divided among several pressure vessels. This fact may be useful in certain design situations where space limitations may be of importance. In this connection, it is possible to use a series of spheres in place of a cylinder and obtain the inherently greater efficiency associated with the sphere. This is the limiting case for a segmented sphere design.

Cylinders with Closures

The minimum weight design of monolithic membrane end closures for cylindrical pressure vessels has been considered in some generality by Hoffman⁶ and Bert⁷, among others. The design problems that they considered can be stated in several different ways:

- a) Find the minimum weight closure for prescribed pressure and diameter.
- b) Find the minimum weight closure for prescribed pressure and volume.
- c) Find the minimum weight design of closures and cylinders for prescribed pressure and volume.

Hoffman and Bert have considered (a) and (b) and an extension of (c) which includes consideration of minimum skirt length. Because of our interest in the complete pressure vessel in terms of the configuration efficiency coefficient, design problem (c) is the most meaningful here and accordingly is used in the following. For our purposes we shall restrict our attention to hemispherical and ellipsoidal closures of optimum design. Other closure shapes may be slightly more efficient than the ellipsoid but are generally more complex to treat analytically.

The configuration efficiency coefficients for cylinders with hemispherical and ellipsoidal closures of various overall L/d ratios can be determined by summing the respective CV values for the cylinder and closure and dividing by the total volume.

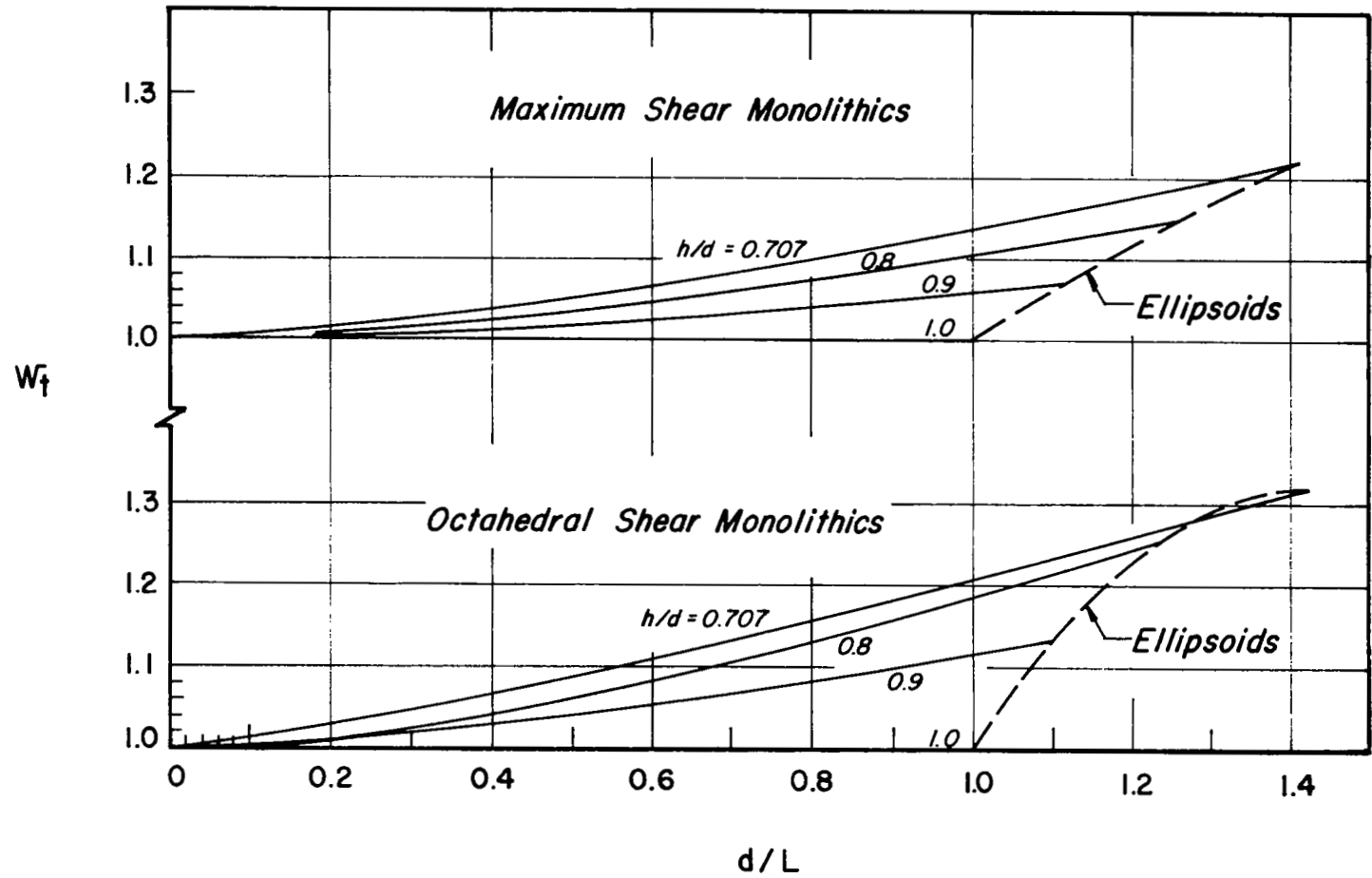


Figure 4 Weight Penalty for Maximum Thickness Ellipsoidal Closures

$$C = \frac{\frac{\pi d^3}{6} \frac{h}{d} C_e + \frac{\pi d^3}{4} \left(\frac{L}{d} - \frac{h}{d} \right) C_c}{\frac{\pi d^3}{4} \left(\frac{L}{d} - \frac{h}{3d} \right)} \quad (33)$$

In Eq. (33), C_e is the ellipsoidal closure value and C_c is the cylinder value as given in Table 1. Numerical values of C for closed cylinders are given in Table 2 and are illustrated in Fig. 3. Also shown in Fig. 4 are the weight penalties associated with using a constant rather than optimum thickness closure.

A direct comparison of the relative efficiencies of the hemispherical and ellipsoidal closures cannot be obtained from Fig. 3 since the L/d ratios are slightly different for the same volume. For the latter condition

$$\frac{\pi d^3}{4} \left[\left(\frac{L}{d} \right)_e - \frac{h}{3d} \right] = \frac{\pi d^3}{4} \left[\left(\frac{L}{d} \right)_h - 1/3 \right] \quad (34)$$

In Eq. (34), the subscripts e and h represent ellipsoidal and hemispherical closures, respectively. For the same diameter, d ,

$$\left(\frac{L}{d} \right)_h = \left(\frac{L}{d} \right)_e + (1/3)(1 - h/d) \quad (35)$$

The weight penalty w_e , associated with an ellipsoidal as compared to an hemispherical optimum closure is obtained by using the C values given in Table 2 for these cases in conjunction with Eq. (35). Numerical results for both the maximum shear and octahedral shear cases are shown in Fig. 5. It can be observed that the hemispherical closure results in the most efficient pressure vessel for a prescribed volume.

Summary of Results

Of the monolithic structures considered, the sphere is the most efficient by virtue of the least surface area per unit volume and favorable thickness distribution. Other monolithic shapes such as closed cylinders and ellipsoids are somewhat less efficient depending upon their L/d ratio and the failure law characterizing their behavior.

In terms of the configuration efficiency coefficient, filamentary shapes are considerably less efficient than corresponding monolithic shapes by a factor as high as 2 for the sphere. This significant difference is attributable to the fact that for a biaxial stress field, two separate sets of filaments are required, whereas in a monolithic membrane the minor principal stress is carried without any additional thickness requirement.

Table 2
Configuration Efficiency Coefficients for Monolithic Cylinders
With Ellipsoidal Closures

Case	h/d	d/L =					
		0	0.2	0.4	0.6	0.8	1.0
Max. Shear	0.707	2.000	2.000	2.000	2.000	2.000	2.000
t_o	0.8	2.000	1.975	1.945	1.915	1.880	1.840
	0.9	2.000	1.950	1.910	1.830	1.760	1.670
	1.0	2.000	1.930	1.845	1.750	1.635	1.500
Oct. Shear	0.707	1.732	1.741	1.760	1.769	1.788	1.806
t_o	0.8	1.732	1.719	1.705	1.688	1.670	1.648
	0.9	1.732	1.705	1.674	1.638	1.597	1.549
	1.0	1.732	1.702	1.661	1.618	1.562	1.500
Max. Shear	0.707	2.000	2.043	2.090	2.143	2.203	2.268
t_{max}	0.8	2.000	2.004	2.009	2.015	2.020	2.028
	0.9	2.000	1.965	1.926	1.881	1.829	1.768
	1.0	2.000	1.930	1.845	1.750	1.635	1.500
Oct. Shear	0.707	1.732	1.802	1.879	1.964	2.059	2.166
t_{max}	0.8	1.732	1.767	1.805	1.849	1.898	1.955
	0.9	1.732	1.732	1.731	1.731	1.731	1.730
	1.0	1.732	1.702	1.661	1.618	1.562	1.500

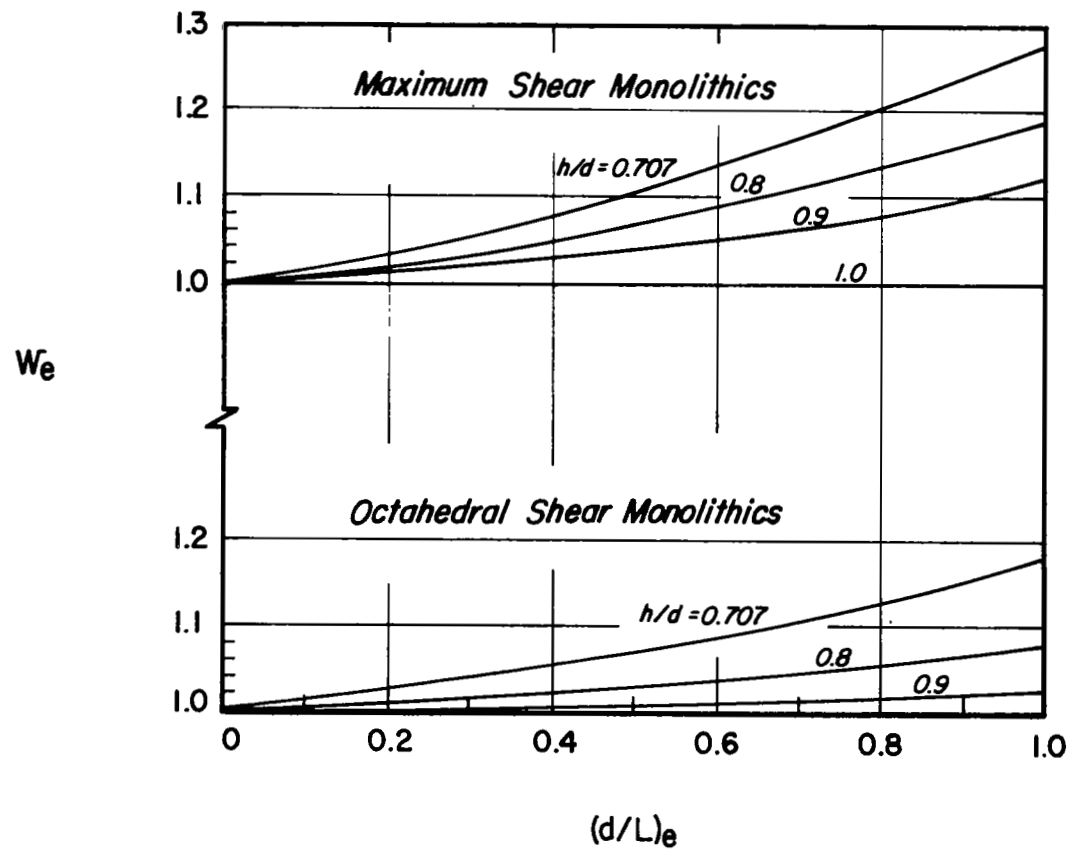


Figure 5 Weight Penalty for Cylinder with Optimum Ellipsoidal Closures as Compared to an Equal Volume and Diameter Cylinder with Hemispherical Closures

4. Efficiencies of Materials

In the preceding section, the efficiencies of various pressure vessel configurations were investigated and it was shown that the overall weight can be affected by a factor as large as 2. As indicated by Eq. (31), the only other factor affecting the weight for prescribed design conditions (pV) is the material efficiency parameter (ρ/Σ_1). This factor obviously has a most profound effect upon the overall efficiency. Consequently, we shall examine in some detail various aspects of weight/strength levels that can be achieved with materials characteristically used in pressure vessel applications.

At the outset, it is important to recognize that there can be significant differences between the tensile strength of materials and the structural strength levels achieved in pressure vessels, particularly when high strength materials are used. Accordingly, we shall be concerned in this section with structural strength levels. However, for reference purposes, Table 3 lists representative values of room temperature material strength/weight ratios for various classes of materials as an indication of their potential.

Table 3
Representative Strength/Weight Levels of Materials
At Room Temperature

Material	Σ_1/ρ (psi/pci)	Type
metals	1.2×10^6	monolithic
monofilaments	5-8	filamentary
films	0.5	monolithic
fabrics	2.5	filamentary

The appropriate strength/weight ratio to be used in the design synthesis relation, Eq. (31) is the uniaxial value since any effects of biaxiality have been incorporated into the configuration efficiency coefficient. Furthermore, this ratio should be

the structural strength/weight (S/ρ) rather than that associated with the material strength/weight (Σ_1/ρ) such as given in Table 3. In general, S/ρ is less than Σ_1/ρ and we shall evaluate in the following, structural strength levels achievable in monolithic and filamentary structures.

High Strength Sheet Metals

One of the major factors limiting the use of high strength sheet metals in pressure vessel applications is their loss of ductility as the strength level increases. Ductility is required to reduce by plastic behavior the stress concentrations resulting from geometric discontinuities and fabrication processes and thus permit the structural strength to approach the strength of the material used. The problem is reasonably well recognized and terms such as fracture mechanics, notch toughness, fracture initiation and fracture propagation are associated with various aspects of this problem. We shall be concerned here with the fracture initiation phase since this appears to be the governing factor in achieving satisfactory structural strength levels.

The simplest representation of a tensile structure is a flat strip similar to the smooth tensile specimen used to obtain the strength of a material, but containing a suitable stress concentration. By testing to failure specimens containing a range of elastic stress concentration factors, the plastic stress concentration factor can be determined. As shown in Refs. 8 and 9, these data can be plotted in a form which yields the ductility ratio, a quantity which can be looked upon as a basic mechanical property that provides a meaningful measure of ductility in a structural sense. The ductility ratio has a value of unity for a completely brittle material and a value of zero for a completely ductile material. In general, the ductility ratio

$$\bar{e} = \epsilon_b / \epsilon_f \quad (36)$$

In Eq. (36), $\epsilon_b = \Sigma_{tu}/E$ and is the "brittle material" strain while ϵ_f is the local strain or zero gage length strain at fracture.

Ductility ratio data obtained from such tests on various steels, titanium alloys and beryllium are shown in Fig. 6 in terms of the material strength/weight ratio. The data tend to follow the line shown in the figure within ten percent limits and thus reflect the following convenient strength/weight-ductility ratio "law" that hardly could have been anticipated.

$$\Sigma_{tu}/\rho = 1.6 \times 10^6 \bar{e}^{1/6} \quad (37)$$

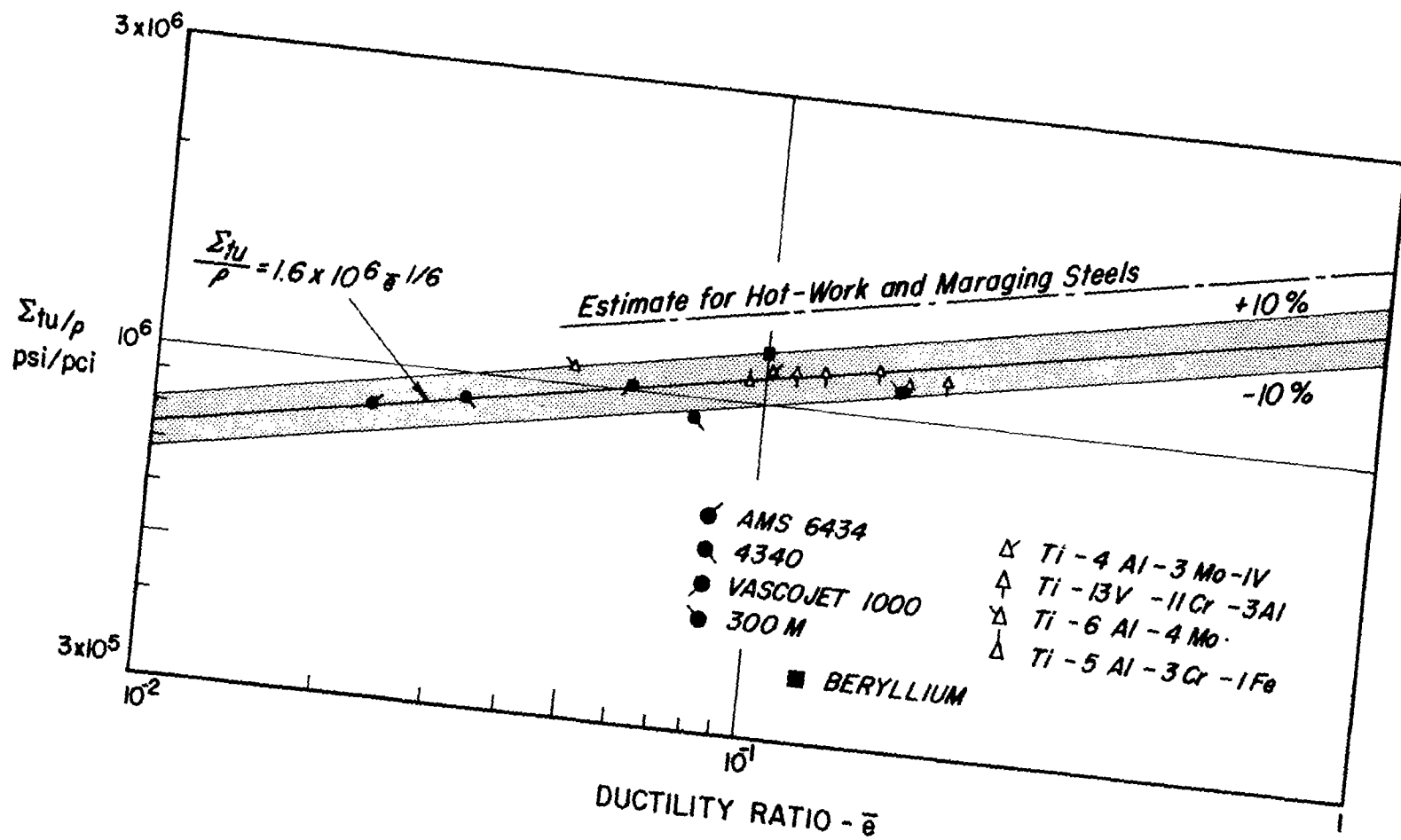


Figure 6 Strength/Weight Ratio of Various Materials at Room Temperature as a Function of Ductility Ratio

Also shown in Fig. 6 is an estimate of the improvement in ductility ratio that may be associated with the more recent hot-work and maraging ultrahigh strength steels.

By use of such data, it is possible to estimate the influence of ductility and stress concentrations upon structural strength. For such purposes, we utilize the following development of Ref. 8. The structural strength

$$S = \Sigma_{tu}/k_p \quad (38)$$

The plastic stress concentration factor (k_p) and elastic stress concentration factor (k_e) are related by the ductility ratio as follows:

$$k_p = 1 + (k_e - 1) \bar{\epsilon} \quad (39)$$

By utilizing Eqs. (37) and (39), Eq. (38) becomes

$$S/\rho = \frac{\Sigma_{tu}/\rho}{1 + (k_e - 1) (\Sigma_{tu}/1.6 \times 10^6 \rho)^6} \quad (40)$$

The results presented in Fig. 7 are obtained from Eq. (40) where structural strength/weight is plotted as a function of material strength/weight for various reference values of the elastic stress concentration factor, k_e . It is most interesting to observe that for each value of k_e , the structural strength reaches a maximum and then declines with further increases in the material strength/weight ratio. This result is associated with the reduced ductility as the strength level of the metal is increased. The results shown in Fig. 7 indicate that there is an optimum Σ_{tu}/ρ for each elastic stress concentration at which S/ρ has a maximum value. Departures to either side of this strength level result in a decrease in structural strength.

In order to confirm these predictions, burst pressure test data on welded steel cylinders heat treated to various strength levels from Ref. 10 are shown in Fig. 8. Also shown is the predicted trend based on the use of Fig. 6 and Eq. (40). It can be observed that a maximum structural strength (S) and optimum Σ_{tu} are indeed obtained.

The results presented in Fig. 7 can be synthesized to provide some approximate guidelines for the use of high strength metals in pressure vessel applications. By using the elastic stress concentrations factor as a reference value which characterizes the efficiency of the structural design and its fabrication, the results shown

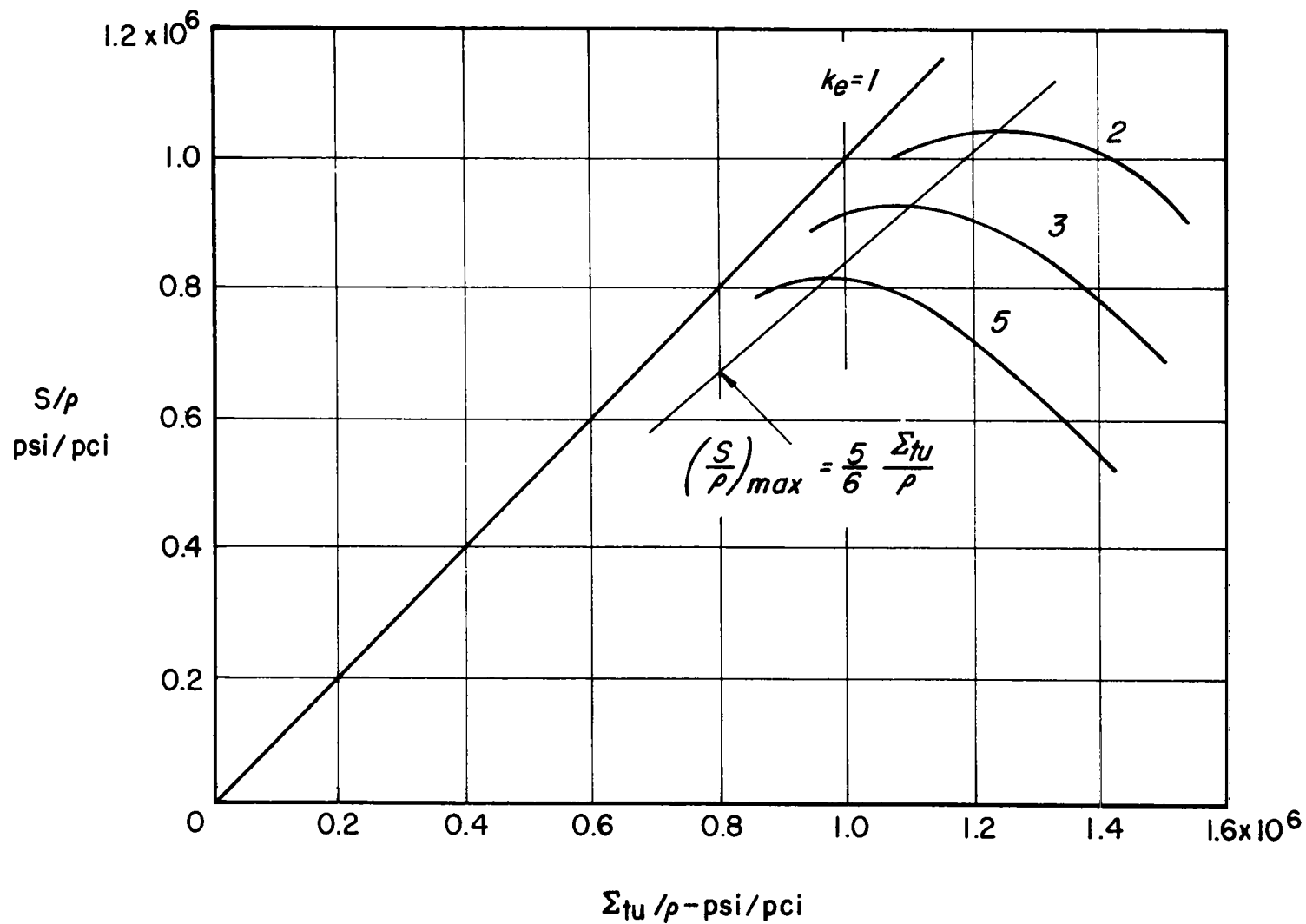


Figure 7 Structural Strength/Weight as a Function of Material Strength/Weight for Various Elastic Stress Concentration Factors

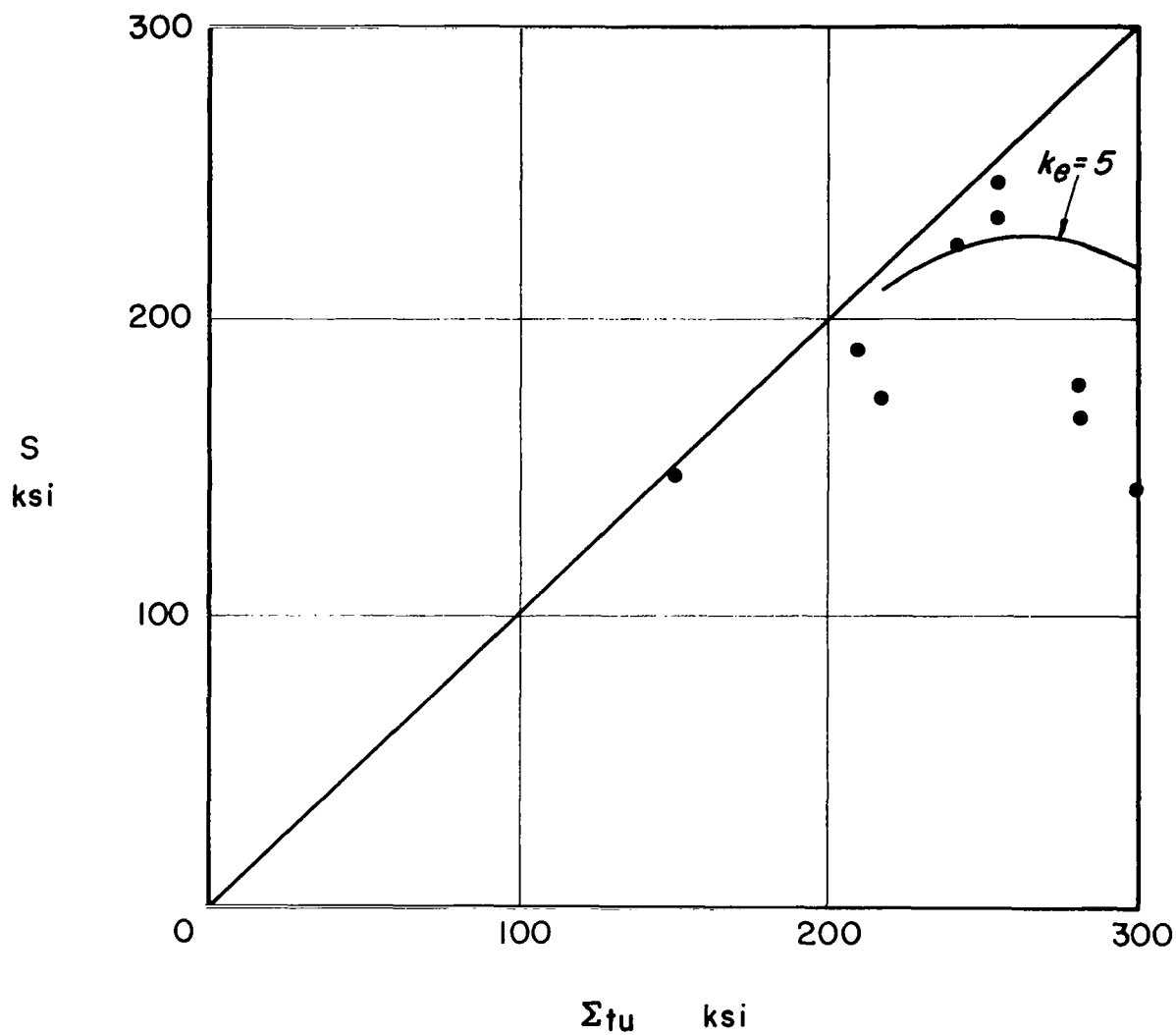


Figure 8 Structural and Material Strengths of Welded Cylinders Fabricated from High Strength Steels. Test Data from Ref. 10.

in Fig. 9 are obtained. On the left scale, the optimum material strength/weight ratios and the associated maximum attainable structural strength/weight levels are shown. On the right scale, the minimum required ductility for a given elastic stress concentration factor is shown. It is to be noted that as shown in Ref. 9, the ductility is associated with the zero gage length fracture strain.

The results shown in Fig. 9 are presented in terms of the elastic stress concentration factor, k_e , because it is believed that this factor can provide a meaningful characterization of the efficiency of the structural design and fabrication. For example, the maximum k_e resulting from geometric discontinuities in the structure can be established analytically or by experimental techniques such as photoelasticity, strain gages or coatings. The stress concentrations arising from fabrication such as tolerance mismatches or the minimum detectable flaw size can also be represented in terms of an effective elastic stress concentration factor. Thus, k_e can be used as a basic design parameter to characterize the efficiency or quality of the structural design and fabrication.

It is for this reason that the horizontal scale of Fig. 9 is somewhat arbitrarily divided into three "quality" regions as follows:

Region	k_e Range	Requirements
Quality A	1-3	meticulous design and fabrication
Quality B	3-8	careful design and fabrication
Quality C	> 8	routine design and fabrication

These regions are to be looked upon as conceptual rather than quantitative at this stage of development and were selected primarily for the purpose of providing some guidelines as to the minimum ductility that is required in each of these regions.

In the Quality C region which is associated with stress concentration factors greater than 8, structural strength/weight levels of approximately 0.7×10^6 psi/pci can be realized using 0.85×10^6 psi/pci strength/weight metals of adequate ductility. A rough estimate of the minimum required zero gage length ductility is approximately 30 percent as indicated in Fig. 9. For this region, it is anticipated that rather routine aerospace design and fabrication techniques can be employed because of the relatively large ductility requirements.

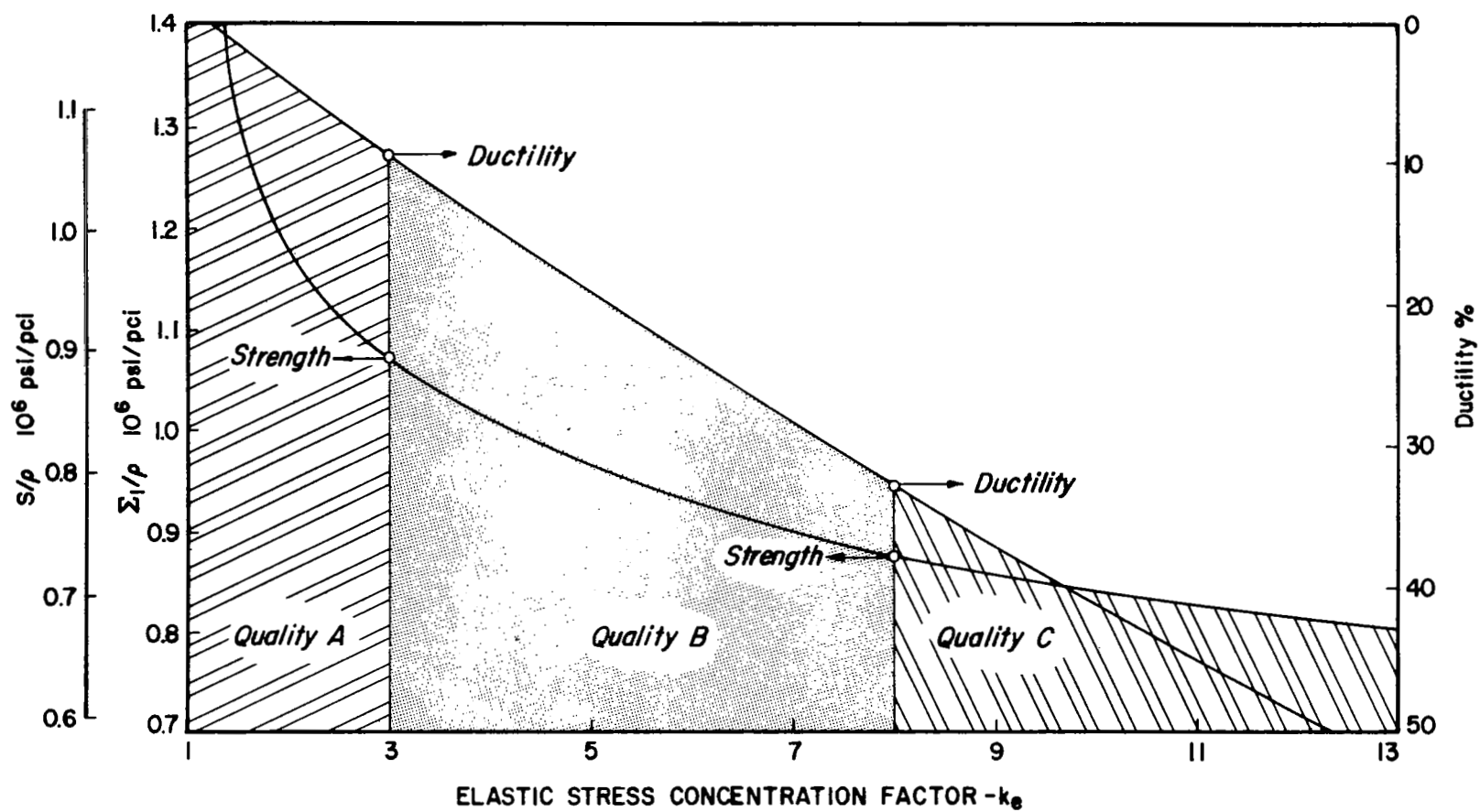


Figure 9 Optimum Strength Levels and Required Ductility for Various Elastic Stress Concentrations

The Quality B region requires rather careful design and fabrication techniques to achieve elastic stress concentration factors in the 3 to 8 range. For $k_e = 3$ and 10 percent zero gage length ductility, 0.9×10^6 psi/pci structural strength/weight levels appear to be attainable with 1.1×10^6 psi/pci ultimate tensile strength/weight metals.

Meticulous design and fabrication techniques are required to operate in the Quality A region because of the relatively low stress concentration factors associated with this region. The required ductility values become quite low and the structural strength level reflects a dangerous sensitivity to small changes in stress concentration. These tentative conclusions are based upon the main trend line shown in Fig. 6. It is believed that a significant improvement in this picture can be realized with the newer hot-work and maraging steels for which an estimate of improvement in ductility is shown in Fig. 6.

With regard to the further development of metallic materials, it is quite apparent that improvements in the zero gage length ductility particularly in the Quality A and B regions are most desirable. More important, perhaps, is the concept that optimum heat treatment procedures should not be based upon achieving the highest tensile strength of the material, but upon achieving the highest structural strength for an elastic stress concentration representative of the quality region of interest. This concept, which is illustrated in Fig. 10, accounts for ductility and its effect upon stress concentrations and could lead to an effective increase in the structural strength level of existing high strength sheet metals.

In summary, it is quite obvious from Figs. 7, 8 and 9 that the structural designer must strive to reduce stress concentrations in order to achieve maximum structural strength levels compatible with the material selected. If relatively low stress concentration factors cannot be achieved there is obviously no point in using ultrahigh strength materials. In fact, their use could lead to lower structural strength than by use of a lower strength, more ductile material. Data such as presented in Figs. 7, 8 and 9 may be used to provide estimates of the appropriate values of S/ρ to be used in conjunction with Eq. (31).

Filamentary Composites

It is a well known fact that filamentary composites, such as the glass-epoxy composites currently used in pressure vessel applications, realize only a fraction of their monofilament strength potential. In a manner somewhat akin to stress con-

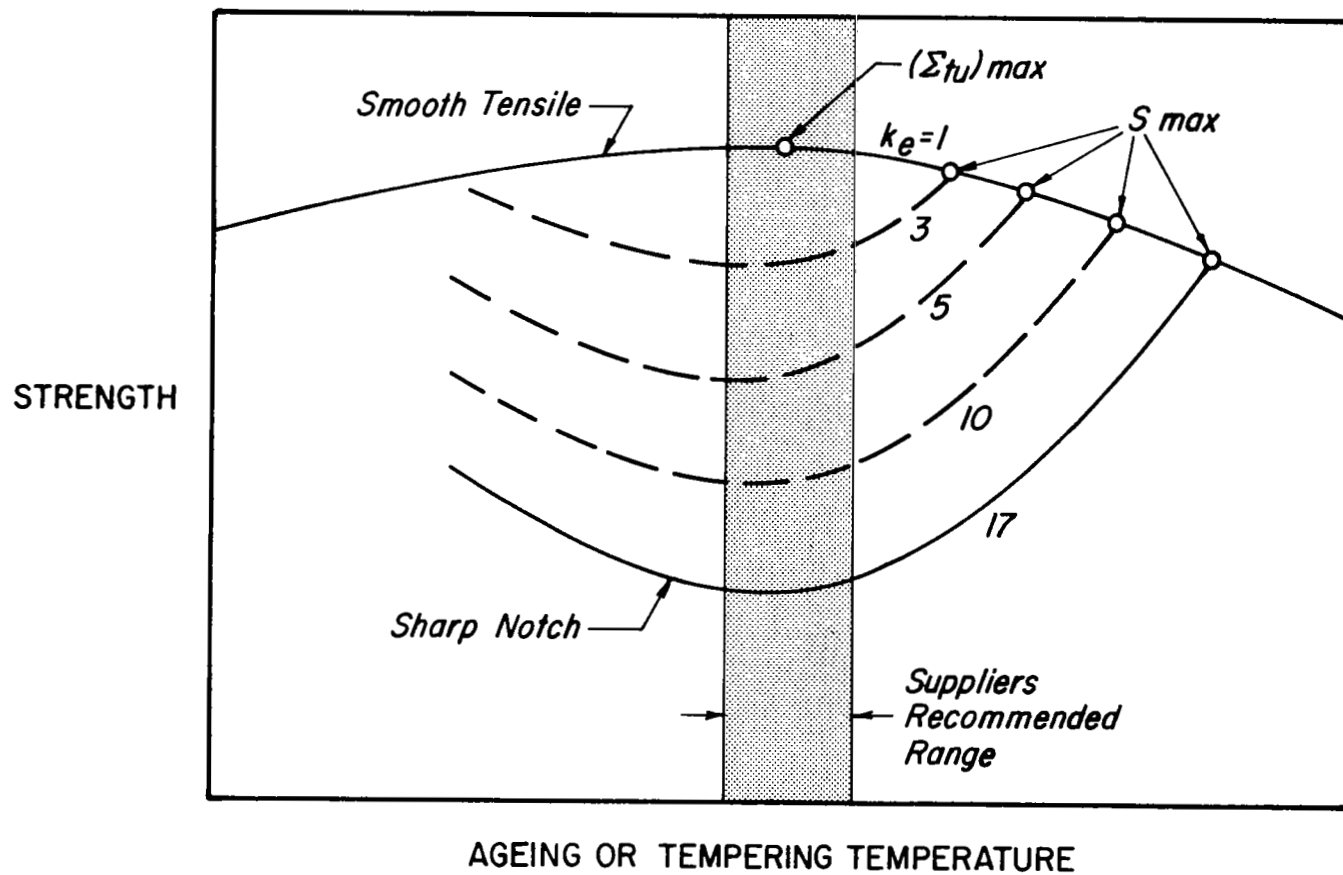


Figure 10 Schematic Illustration that Heat Treatment Should be Selected to Provide S_{max} Rather Than $(\Sigma t_u)_{max}$.

centrations in metallic materials, the formation of filaments into strands and rovings and the cross-over of the rovings in the composite act to reduce the useable structural strength. The composite becomes a structural material for pressure vessel applications by virtue of the fact that the filaments provide the load carrying function while the matrix basically provides the contouring and sealing functions. Thus the degradation of the monofilament strength is to some extent associated with the structural functions required of the composite.

Although it is not now possible to analyze the structural strength of filamentary composites in a manner similar to that used for monolithic metallics, it is possible to obtain an insight into the factors which tend to affect the structural strength of composites. For this purpose, we shall utilize the data presented by Morris¹¹ in his rather comprehensive survey of cylindrical glass-epoxy composite pressure vessels.

In this evaluation, it is important to recognize that there are several different strength levels that are significant; monofilament strength, roving strength, uniaxial composite strength and biaxial composite strength. The uniaxial composite strength is the proper structural strength value to be used in conjunction with the design synthesis relation, Eq. (31).

From data presented by Morris for E glass and S-994 glass-epoxy composites the information presented in Table 4 has been assembled. An evaluation of these data indicate the following:

- a) The average roving strength is 0.7 of the average monofilament strength. This reduction is probably associated with local contact stresses among filaments.
- b) The uniaxial composite strength/weight ratio (small scale) is approximately 0.73 of the average roving strength/weight ratio. For a 67 vol. % glass - 33 vol. % epoxy composite this value should be 0.8 for a non-load carrying matrix, thereby indicating some loss probably associated with cross-over of the glass filaments.
- c) The biaxial composite strength is approximately 2/3 of the uniaxial composite strength. This factor corresponds directly with that predicted by Eq. (19) for a cylinder ($\sigma_2/\sigma_1 = 1/2$).

Table 4
Test Data on Glass-Epoxy Composites
At Room Temperature (Ref. 11)

Property	E-glass		S-994 glass	
	S/ ρ *		S/ ρ *	
density	0.092 pci		0.088 pci	
monofilament average Σ_{tu}	500 ksi	5.45×10^6 in.	650 ksi	7.38×10^6 in.
roving average Σ_{tu}	350 ksi	3.81×10^6	450 ksi	5.11×10^6
composite density	0.076 pci		0.073 pci	
uniaxial composite strength (small scale)	220 ksi	2.90×10^6	260 ksi	3.56×10^6
biaxial composite strength (cylinders-small scale)	120 -	1.58 -	170 -	2.33 -
	150 ksi	1.97×10^6	180 ksi	2.47×10^6
biaxial composite strength (cylinders-full scale)	125 ksi	1.64×10^6	155 ksi	2.47×10^6
uniaxial composite strength (calculated)	187 ksi	2.46×10^6	232 ksi	3.18×10^6

*S/ ρ = strength/weight ratio.

- d) The full scale cylinder biaxial composite strength data shown in Table 4 are somewhat lower than the small scale data. The corresponding uniaxial composite strength was calculated by multiplying the biaxial data by 3/2. On this basis, the full scale uniaxial composite strength/weight is approximately 0.63 of the average roving strength/weight.

To summarize these data on full scale glass-epoxy composite pressure vessels, the following uniaxial structural strength/weight ratios are representative of current practice:

$$S/\rho = 0.44 \Sigma_1/\rho \text{ (monofilaments)} \quad (41)$$

$$S/\rho = 0.63 \Sigma_1/\rho \text{ (rovings)} \quad (42)$$

The appropriate value to be used depends upon what one considers to be the raw material. Eq. (41) is representative of the potential of the material whereas Eq. (42) is realistic in terms of the material currently used in the fabrication process.

There are other filamentary materials such as high strength metallic wires that can be utilized for pressure vessel applications particularly in the form of filamentary-monolithic composites. Although we shall consider the efficiencies of such composite in the next section, it is advantageous to consider the structural strength/weight of the filamentary materials here.

For this purpose, the representative material tensile strengths (Σ_1) given in Table 5 were assembled from available literature (Refs. 1, 11, 12). Also given in Table 5 are the material strength/weight ratios (Σ_1/ρ) and the theoretical uniaxial composite strength/weight ratios (Σ_1/ρ)_c based upon a 67 vol.% filament - 33 vol.% epoxy composite. The last column lists the S/ρ values based upon 90% of the theoretical uniaxial composite strength/weight ratio. This number was cited by Schuerch¹ as that typically obtained in filament wound structures utilizing hoop windings only.

Table 5

Filamentary Materials and Composites at Room Temperature

Type	Σ_1 (ksi)	ρ (pci)	Σ_1/ρ (psi/pci)	$(\Sigma_1/\rho)_c$	S/ρ
beryllium wire (5 mils)	200	0.066	3.1×10^6	2.3×10^6	2.1×10^6
S-994 glass roving	450	0.088	5.1	4.1	3.7
boron filament	500	0.090	5.6	4.5	4.0
titanium wire (5 mils)	280	0.174	1.6	1.4	1.3
steel wire	575	0.278	2.1	1.9	1.7

It can be observed from Tables 4 and 5 that filamentary composites as a class have a considerably higher S/ρ potential than monolithic metallics for certain pressure vessel applications. This observation is based upon room temperature and short time load applications for which the data given herein apply. It is to be noted, however, that an evaluation of the relative efficiencies of monolithic and filamentary materials for pressure vessels cannot be obtained from a direct comparison of their respective S/ρ values since the configuration efficiency coefficients must also be considered.

Materials for Inflatable Structures

For inflatable structures applications where packaging requirements are important, monolithic plastic films and filamentary fabrics have been employed. Because of the fact that pressurization is used to expand the packaged structure and then maintain the expanded shape, inflatable structures are essentially pressure vessels. In this case, the various structural functions are performed as follows:

Function	Film	Fabric
load carrying	film	cloth
contouring	pressurization	pressurization
sealing	film	sealant

Because of these functional requirements, the overall efficiency of inflatable structures when considered as pressure vessels should include the weight of the pressurization equipment and that associated with the sealant required for fabrics. In addition, there is an inherent penalty on the configuration efficiency coefficient since maximum rather than optimum thickness structures may be required when using films and fabric. From a materials standpoint, joining of film and fabric segments to achieve the desired shape causes a significant degradation of the structural strength/weight as compared to the material strength/weight when the weight penalty associated with seams is considered.

Brewer and Jeppeson⁴ have considered these factors in considerable detail. Because of the form in which they present their data, it is not possible to ascertain structural strength/weight levels for films and fabrics in the sense used herein. Consequently, further consideration of the efficiency of inflatable structures is reserved for discussion in a subsequent section.

Comparative Efficiencies of Materials

Fig. 11 has been prepared to summarize the evaluation presented in this section. On the horizontal scale, the uniaxial material tensile strength/weight ratios are indicated. For glass filaments, this ratio is based upon the strength of the rovings. The vertical scale represents the uniaxial structural strength/weight ratios which can be achieved by application of the best current technology. Although filamentary composites appear to be superior to monolithic construction, it must be noted that the configuration efficiency coefficient must also be considered when evaluating overall pressure vessel efficiencies. Consequently, Fig. 11 does not permit a direct comparison of the relative efficiencies of materials as used in pressure vessels.

It is also to be noted that Fig. 11 is based upon short time load applications at room temperature. Consideration of cryogenic and elevated temperatures and other environmental factors can change the relative efficiencies of monolithic and filamentary materials substantially.

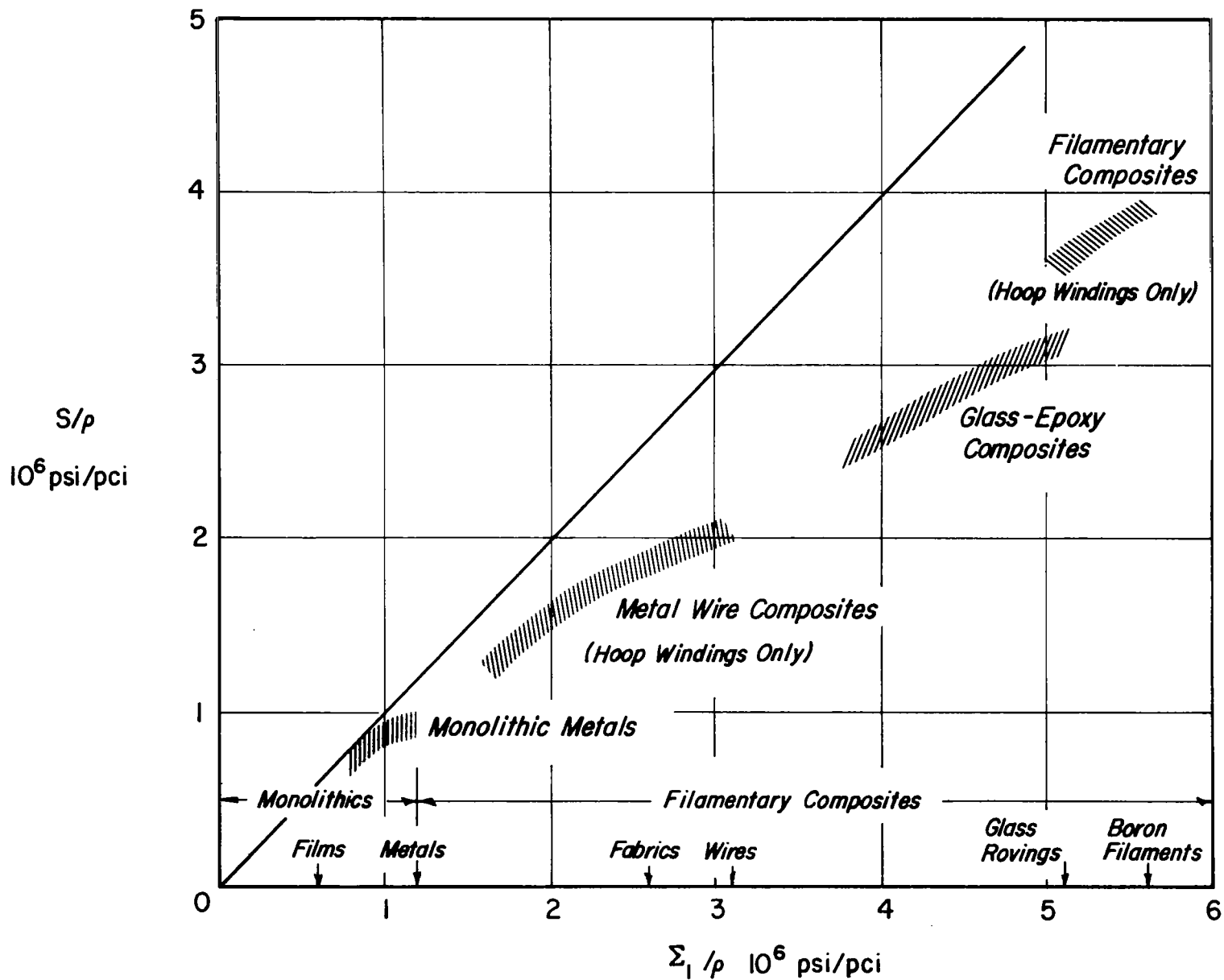


Figure 11 Comparative Structural Efficiencies of Various Materials in Pressure Vessel Applications at Room Temperature

5. Potential of Newer Materials Concepts

Materials characteristically employed in aerospace pressure vessel applications were considered in the previous section. Here, we shall be concerned with the potential efficiencies of certain newer material concepts for such applications: anisotropic metals for monolithic construction, combined monolithic and filamentary designs, and whisker composites.

Obviously, there may be many problems in the application of these concepts to the production of pressure vessels and many of the factors which result in a reduction of the material strength to the structural strength levels discussed in the preceding section will operate here also. Although the full potential represented by the material strength/weight ratio may not be realizable, these newer concepts could result in significant increases in structural strength/weight levels in the future.

Anisotropic Metals

Although theories of yielding and plastic flow of anisotropic metals have been available for some time, Backofen et al¹³ appear to have been the first to observe that significant strengthening effects are predicted by such theories for combined loadings typical of pressure vessels. Anisotropy of mechanical properties is inherent in metallic materials as a result of their basic crystalline form and also as a result of differences in deformation along various rolling axes in processing the material into sheet form. In fact, metal producers expend considerable effort to achieve as nearly isotropic a product as practical. Conversely, it should be possible to produce sheets with controlled anisotropy for pressure vessel applications.

Anisotropy due to a preferred orientation or texture of the crystal structure was suggested by Backofen¹³ as a method of increasing the yield strength in the thickness direction of sheet. Particularly for hexagonal close-packed metals such as titanium and beryllium, the slip systems can be so oriented as to result in a significant increase in yield strength in the thickness direction. Other important forms of anisotropy can be obtained by unidirectional plastic working of the sheet as a result of rolling or stretching.

Hill¹⁴ has presented a generalization of the octahedral shear law for anisotropic behavior. In terms of the principal stresses, his relation reduces to the following for plane stress:

$$\frac{\Sigma_1^2}{\sigma_1^2} = 1 - \left(1 + \frac{\Sigma_1^2}{\Sigma_2^2} - \frac{\Sigma_1^2}{\Sigma_3^2}\right) \frac{\sigma_2}{\sigma_1} + \frac{\Sigma_1^2}{\Sigma_2^2} \left(\frac{\sigma_2}{\sigma_1}\right)^2 \quad (43)$$

In Eq. (43), Σ_1 , Σ_2 and Σ_3 represent the uniaxial strengths in the principal stress and thickness directions, respectively.

Texture Hardening

To represent texture hardening, we can let $\Sigma_1/\Sigma_2 = 1$ and $\Sigma_1/\Sigma_3 = a$. Thus Eq. (43) becomes

$$\Sigma_1/\sigma_1 = [1 - (2 - a^2)(\sigma_2/\sigma_1) + (\sigma_2/\sigma_1)^2]^{1/2} \quad (44)$$

Tensile strength surfaces for various values of a are illustrated in Fig. 12. Note that strengthening occurs for $a < 1$. For $a > 1$ weakening occurs and, in fact, for $a = 2$, the filamentary strength law, Eq. (19) is obtained. By use of Eq. (26) in conjunction with Eq. (44), we can obtain the following results for the configuration efficiency coefficient of anisotropic monolithic shapes:

$$\text{Sphere:} \quad C = 1.5a \quad (45)$$

$$\text{Long Cylinder:} \quad C = (1 + 2a^2)^{1/2} \quad (46)$$

These results are illustrated in Figs. 13 and 14 and it can be observed that significant improvements in efficiency can be realized by raising Σ_3 relative to Σ_1 . Backofen¹³ has discussed the degree of texture hardening associated with various crystallographic structures and his estimates are indicated in these figures for HCP, BCC and FCC metals. A weight saving potential of roughly 50 percent seems possible for a sphere of properly textured HCP metal. For the long cylinder the weight saving potential is considerably less although still attractive. A hemispherically closed cylinder would lie between these two limiting cases.

It is important to note that Sliney et al¹⁵ have conducted tests on two cylinders fabricated of Ti-5Al-2.5Sn titanium alloy sheet which has a HCP structure. From auxiliary tensile tests, it was established that the anisotropy coefficient (a) had an average value of 2/3. By use of Fig. 14, a weight saving potential of 20% is obtained for $a = 2/3$ as compared to an isotropic material ($a = 1$). Although in the two cylinder tests failure occurred at longitudinal welds, the burst strengths (S) were significantly higher than the uniaxial tensile strength (Σ_1) as indicated in Table 6.

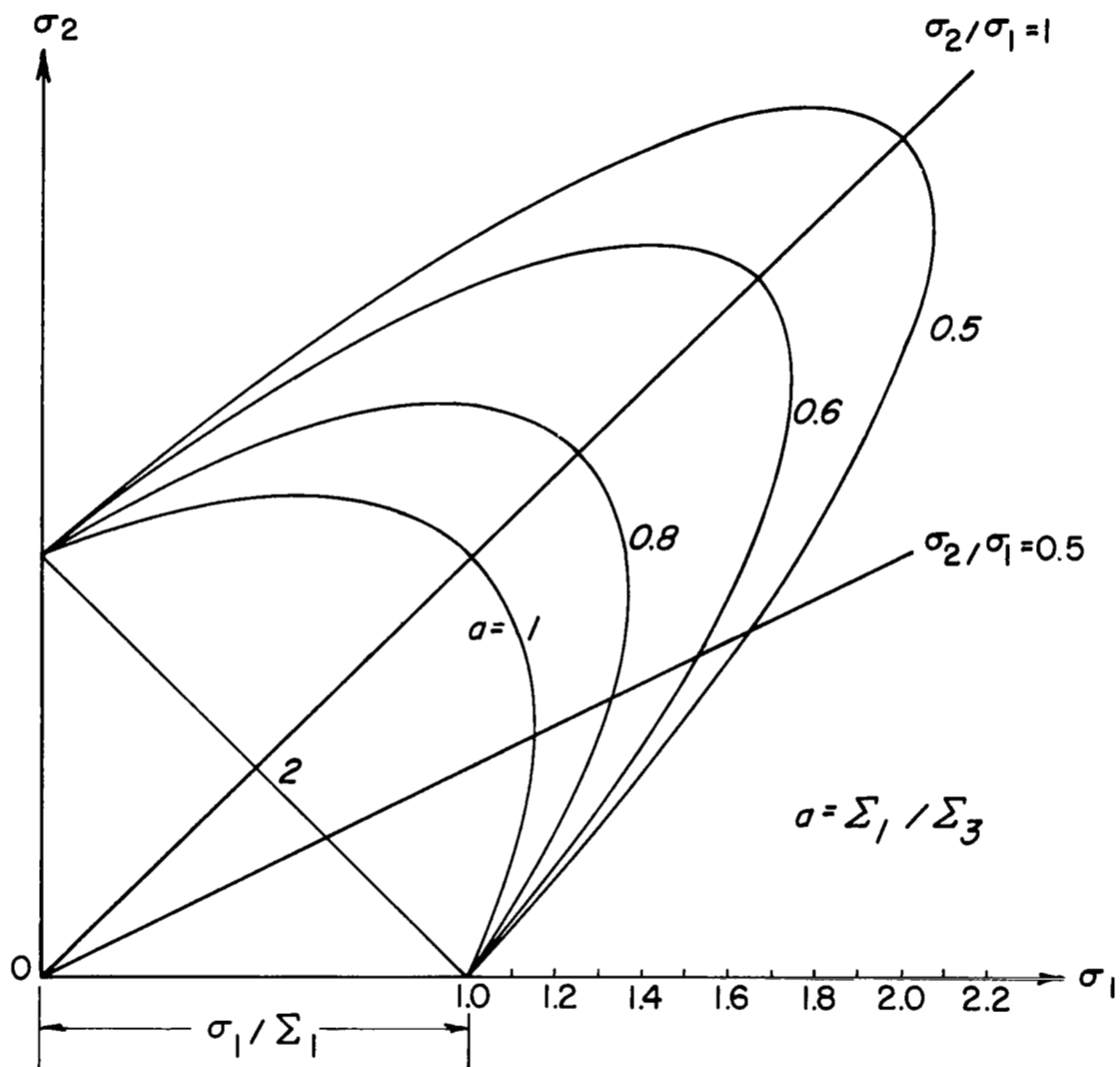


Figure 12 Strength Surfaces Representative of Texture Hardening

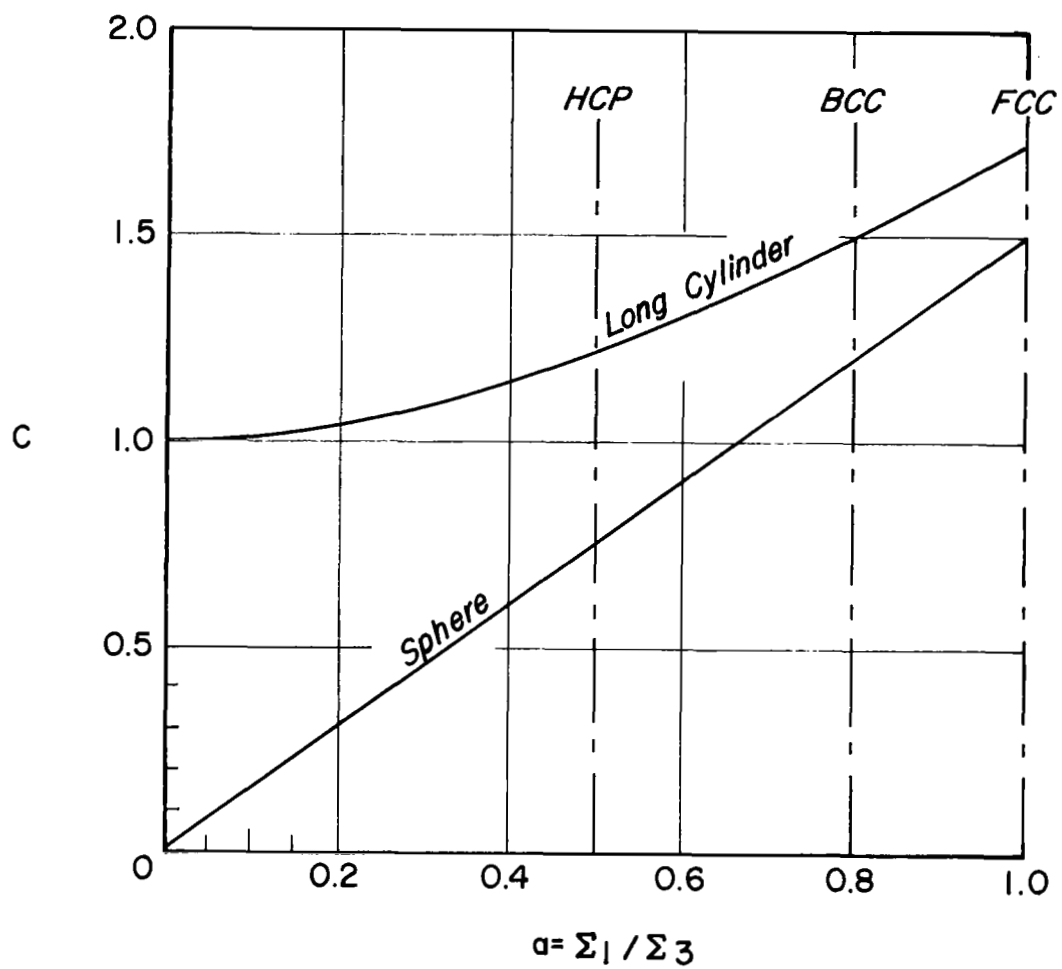


Figure 13 Configuration Efficiency Coefficients for Texture Hardened Metals

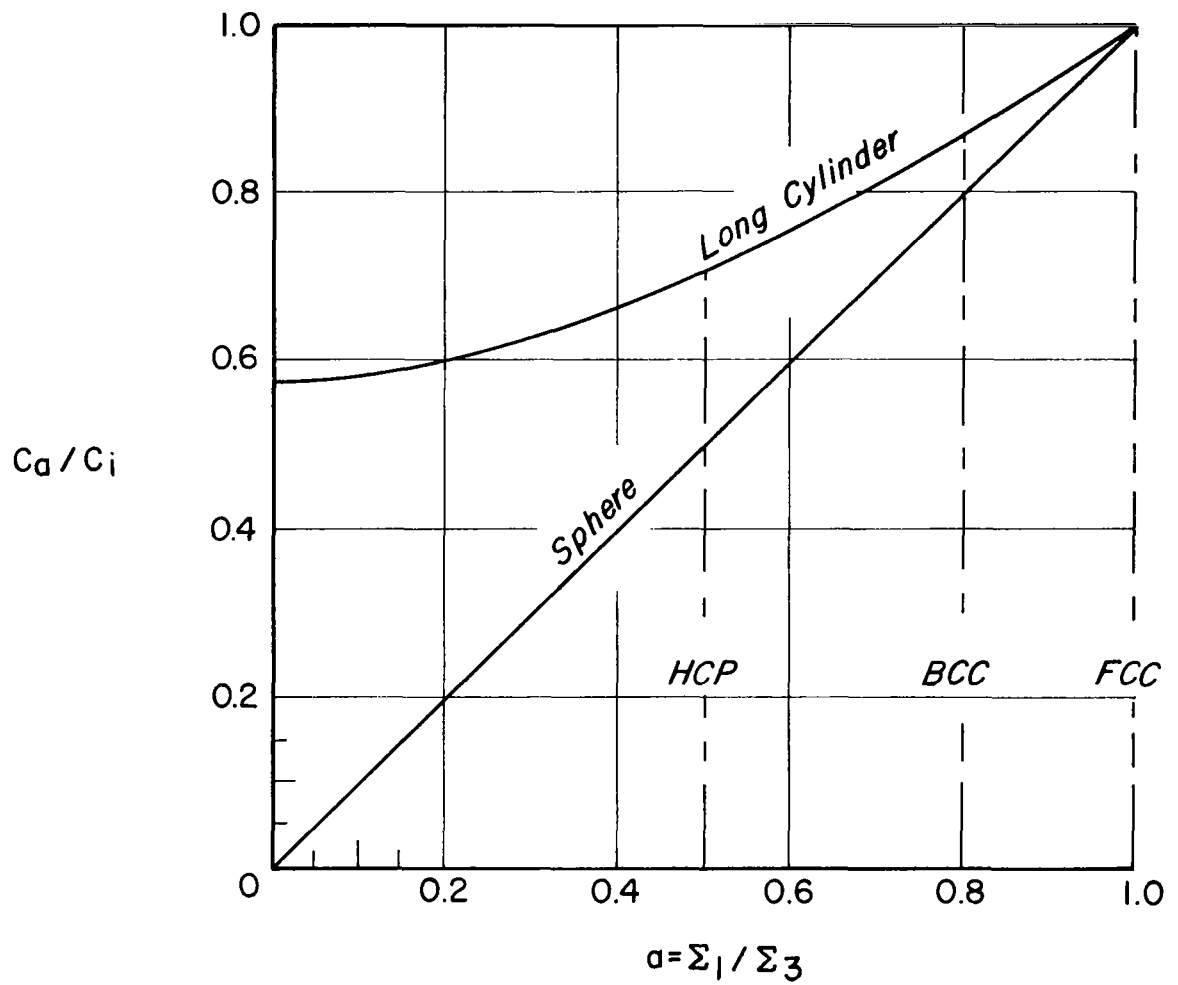


Figure 14 Weight Saving Potential of Texture Hardened Metals
(For Same Σ_1 as Isotropic Metal)

Table 6
Test Data on Anisotropic Titanium Alloy Cylinders
At Room Temperature

Material	Σ_1 (ksi)*	S(ksi)**	S/Σ_1	a	Reference
Ti-5Al-2.5Sn	132.5	156	1.18	0.67	15
	132.5	153	1.16	0.67	15
Ti-6Al-4V	147	195	1.33	--	16
	147	195	1.33	--	16
	147	198	1.35	--	16

*Uniaxial Tensile Strength

**Hoop Stress at Burst

Additional test data by Martin et al.¹⁶ on Ti-6Al-4V cylindrical pressure vessels are presented in Table 6. Although the value of the anisotropy coefficient (a) is uncertain, the burst strengths (S) are significantly higher than the uniaxial tensile strength. These results are particularly encouraging in view of the fact that they are based on failure rather than yield strength.

Mechanical Anisotropy

Another technically interesting form of anisotropy is that obtained by mechanical unidirectional plastic working of the sheet by rolling or stretching. Here the tensile strengths in the plane of the sheet are intentionally different so that $\Sigma_1 > \Sigma_2$. If it is assumed that $\Sigma_1/\Sigma_3 = 1$ and $\Sigma_1/\Sigma_2 = b$ where $b \geq 1$, then Eq. (43) becomes for this case

$$\Sigma_1/\sigma_1 = [1 - b^2 (\sigma_2/\sigma_1) (1 - \sigma_2/\sigma_1)]^{1/2} \quad (47)$$

Tensile strength surfaces for various values of b are shown in Fig. 15.

It is important to note that mechanical anisotropy can improve efficiency by two different mechanisms. The first is the biaxiality effect displayed in Fig. 15. The second is the increase in Σ_1 over that for an isotropic material which presumably can be attained as a result of the decrease in Σ_2 . This mechanism is not shown by the presentation of Fig. 15.

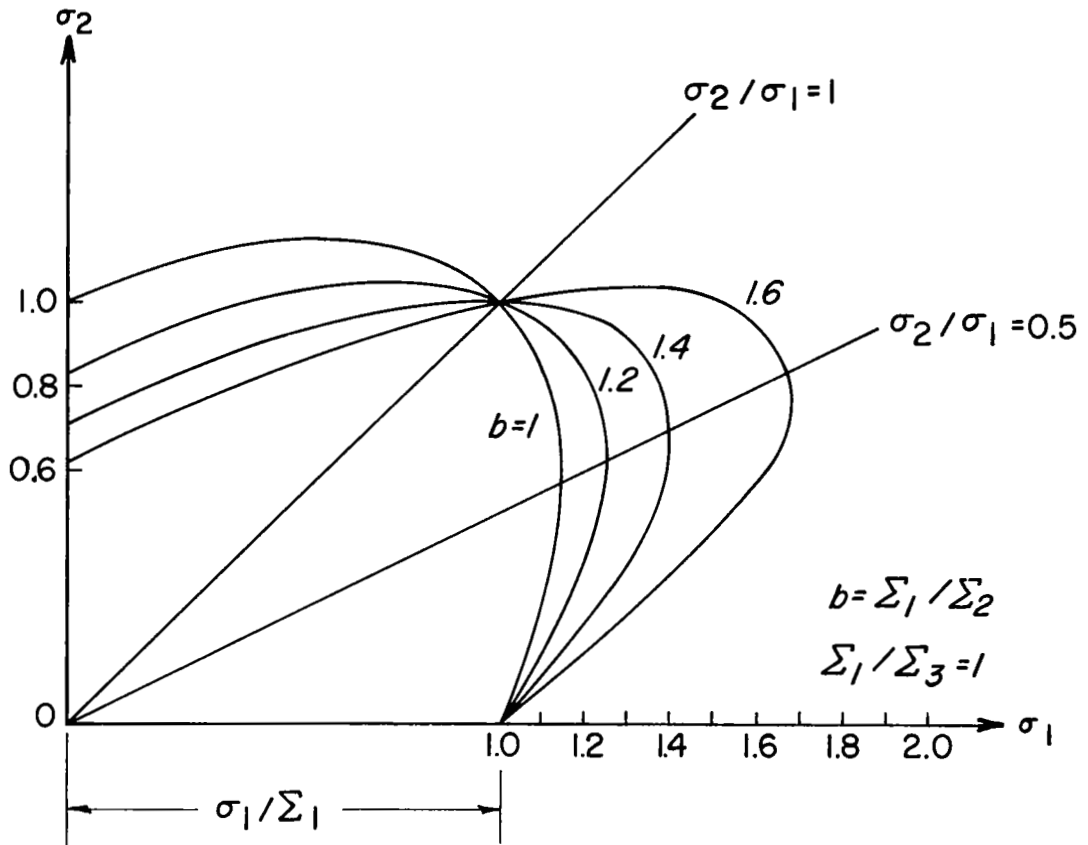


Figure 15 Strength Surfaces Representative of Mechanical Anisotropy

By use of Eq. (26) in conjunction with Eq. (47), the following configuration efficiency coefficients are obtained for monolithic shapes of mechanically anisotropic materials:

$$\text{Sphere:} \quad C = 1.5 \quad (48)$$

$$\text{Long Cylinder:} \quad C = (4 - b^2)^{1/2} \quad (49)$$

It can be observed that the biaxiality effects of mechanical anisotropy do not result in any improvement in efficiency for the sphere. For a long cylinder, on the other hand, significant improvements in efficiency can be obtained. In fact, as b approaches 2, dramatic improvements are predicted.

In comparing the results obtained for the texture hardening and mechanical anisotropy cases for the sphere, significant improvements in efficiency are predicted for texture hardening only. On the other hand, comparable results obtained for the long cylinder indicate that large improvements are predicted for mechanical anisotropy. Thus, the type of anisotropy that may be optimum for a given shape depends specifically upon the configuration.

To account for the possible increase in Σ_1 of the mechanically anisotropic material as compared to that of the isotropic material $(\Sigma_1)_i$, the following assumption is made which appears reasonable for a small degree of anisotropy:

$$\Sigma_1 + \Sigma_2 = 2(\Sigma_1)_i \quad (50)$$

Since $\Sigma_1/\Sigma_2 = b$,

$$b = [2(\Sigma_1)_i/\Sigma_1 - 1]^{-1} \quad (51)$$

By incorporating the increase in tensile strength Σ_1 as indicated by Eq. (50), the corresponding configuration efficiency coefficients become:

$$\text{Sphere:} \quad C = 1.5 (\Sigma_1)_i/\Sigma_1 \quad (52)$$

$$\text{Long Cylinder:} \quad C = \left\{ 4 - [2(\Sigma_1)_i/\Sigma_1 - 1]^{-2} \right\}^{1/2} (\Sigma_1)_i/\Sigma_1 \quad (53)$$

Numerical results based upon Eqs. (52) and (53) are shown in Figs. 16 and 17. It would appear that really significant improvements in efficiency are predicted for long cylinders for moderate degrees of mechanical anisotropy. For spheres, it is apparent that texture hardening has the greater potential for weight saving. Thus, for a hemispherically closed cylinder, mechanically anisotropic materials are indicated for the cylindrical portion and texture hardened materials for the hemispherical closures.

Filamentary-Monolithic Composites

Although the configuration efficiency coefficient is more favorable for monolithic as compared to filamentary shapes, the latter have a greater overall efficiency because of the use of higher strength/weight materials. It is of interest, therefore, to consider filamentary-monolithic composites which would use to advantage the greater configuration efficiency inherent in monolithics with the greater material efficiency of the filamentaries.

Of the practical pressure vessel shapes of interest, the cylindrical portion of a closed cylinder appears to have the most interesting potential as a filamentary-monolithic composite. The monolithic portion of the cylinder forms the inside shell to which the closures are attached. This shell provides the contouring and sealing functions, and it is designed to carry the end loads and one-half the circumferential loads. The filaments are wound on the cylindrical position in the hoop direction only and carry the other half of the circumferential loads. As such, the filaments act in a uniaxial stress field and should not be degraded by the filament cross-over associated with biaxial stress fields. The filaments provide only a unidirectional load-carrying function in the composite. It is assumed that the elastic modulus mismatch between the monolithic and filamentary materials can be accommodated by yielding of the monolithic material.

The weight of the composite cylinder minus the end closure weight (approximately equal to a long cylinder) is given by

$$W = 2\pi RL (\rho_m t_m + \rho_f t_f) \quad (54)$$

Here, the subscripts m and f refer to monolithic and filamentary, respectively. In both cases, $t = pR/2S$, and therefore, Eq. (54) can be written as

$$W = C(\rho/S)_m pV \quad (55)$$

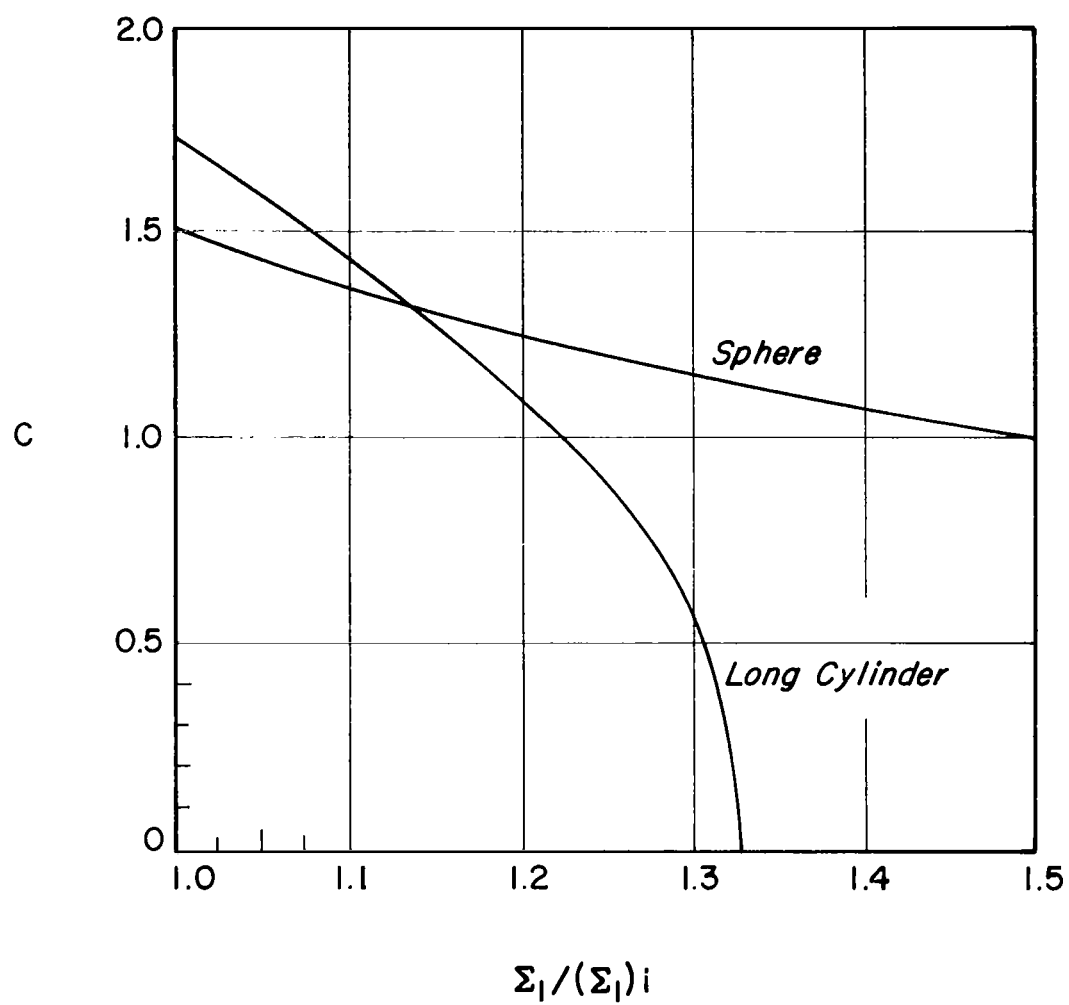


Figure 16 Configuration Efficiency Coefficients for Mechanically Anisotropic Metal

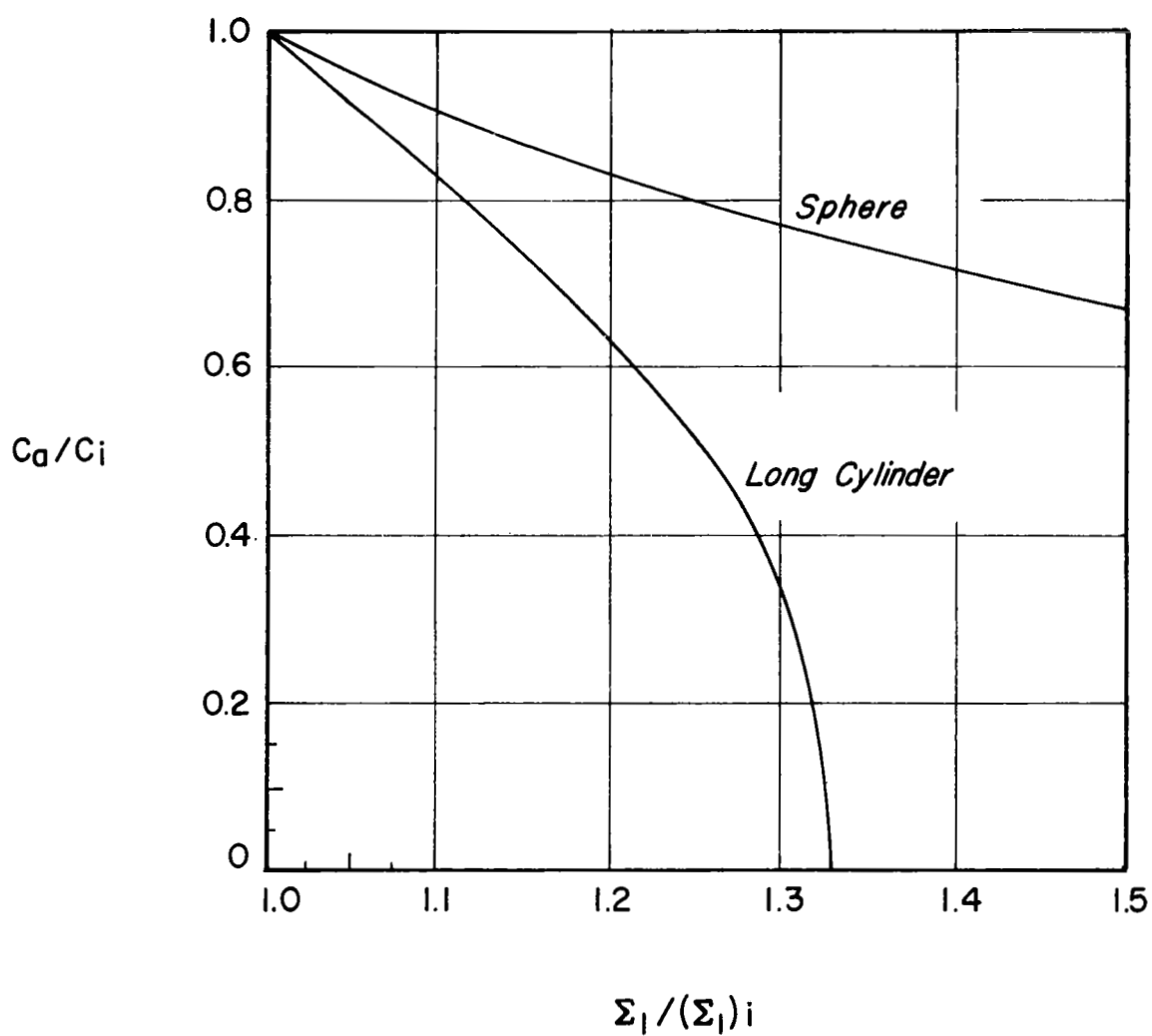


Figure 17 Weight Saving Potential of Mechanically Anisotropic Metals
(For Same $(\Sigma_1)_i$ as Isotropic Metal)

$$\text{where: } C = a + (\rho/S)_f / (\rho/S)_m \quad (56)$$

In Eq. (56), the coefficient a is the anisotropy coefficient for texture hardened monolithic metals. For an isotropic material ($a = 1$) note that $C = 2$ rather than $C = 1.732$ in the limiting case when $(\rho/S)_f = (\rho/S)_m$ because the biaxiality effect is $\sigma_2/\sigma_1 = 1$ for the composite as compared to $\sigma_2/\sigma_1 = 1/2$ in the monolithic design.

Numerical results based upon Eq. (56) are shown in Fig. 18 together with appropriate strength/weight ratios of filamentary composites given in Table 5. The potential increases in efficiency for both isotropic and anisotropic materials are indeed attractive particularly for the glass filaments.

In order to indicate the overall weight saving potential of the composite, it is of interest to compare the weight of the composite with a filament wound cylinder. For the filamentary-monolithic cylinder, Eqs. (55) and (56) are used while the weight of a filament wound cylindrical pressure vessel is taken as

$$W_f = 3c_c (\rho/S)_f pV \quad (57)$$

Since $(\rho/S)_f$ represents the uniaxial tensile composite strength/weight ratio in both Eqs. (56) and (57), the factor c_c is introduced in Eq. (57) to account for the degradation in strength associated with filament cross-over required for a biaxial stress field. The weight ratio is thus

$$W_{fm}/W_f = \left(a + \frac{(\rho/S)_f}{(\rho/S)_m} \right) \left[3c_c \frac{(\rho/S)_f}{(\rho/S)_m} \right]^{-1} \quad (58)$$

Numerical results based upon Eq. (58) are shown in Fig. 19 for isotropic and anisotropic metals. With glass rovings for which $c_c = 1.15$ (S/ρ for S-994 in Tables 4 and 5) is representative of current practice, it can be observed that the filamentary-monolithic composite is more efficient than the filamentary composite only when anisotropic metals ($a = 0.5$) are utilized. For isotropic metals the results are sufficiently close that other considerations may govern the choice of construction.

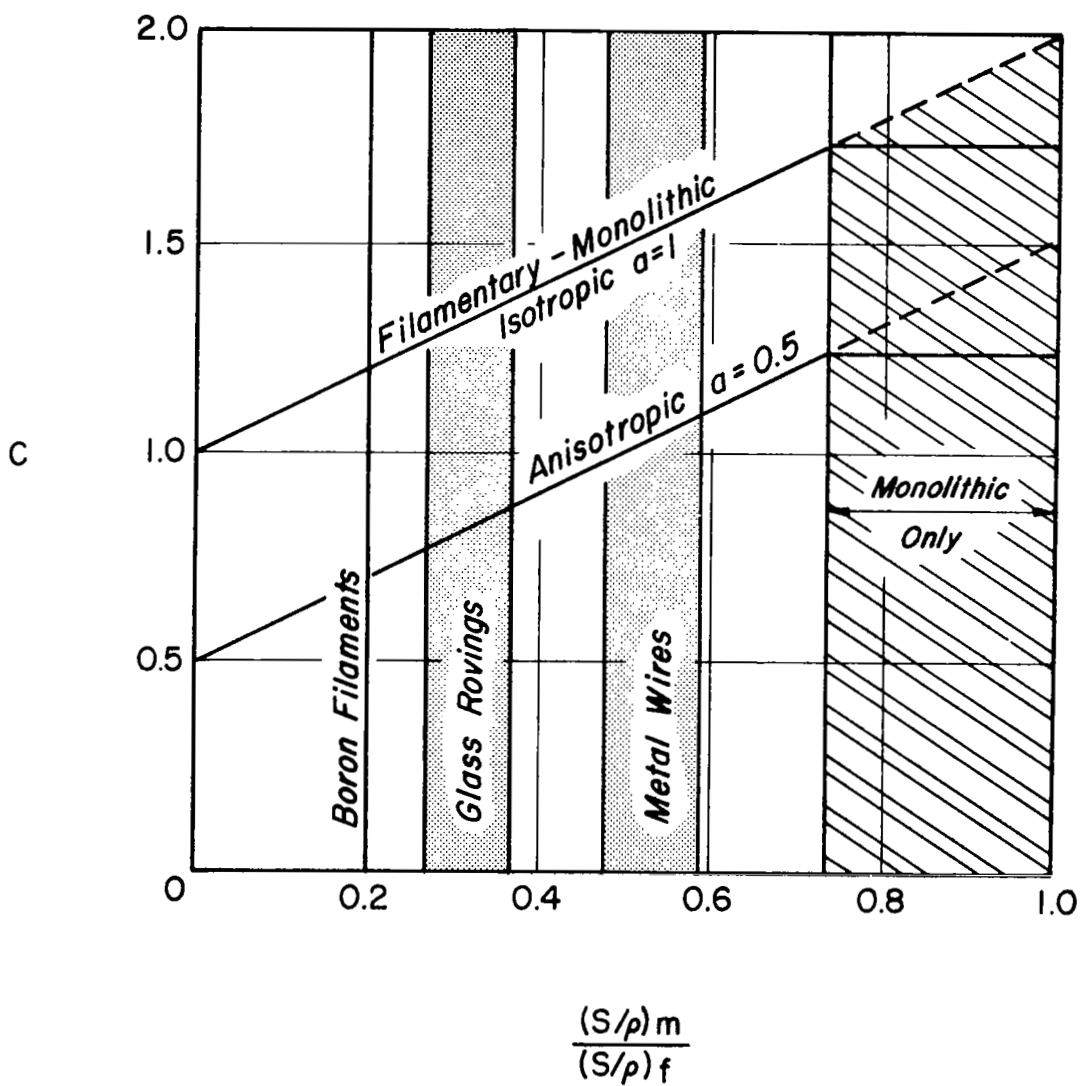


Figure 18 Configuration Efficiency Coefficients for Filamentary Hoop Windings on a Long Monolithic Cylinder $(S/\rho)_m = 10^6$ psi/pci

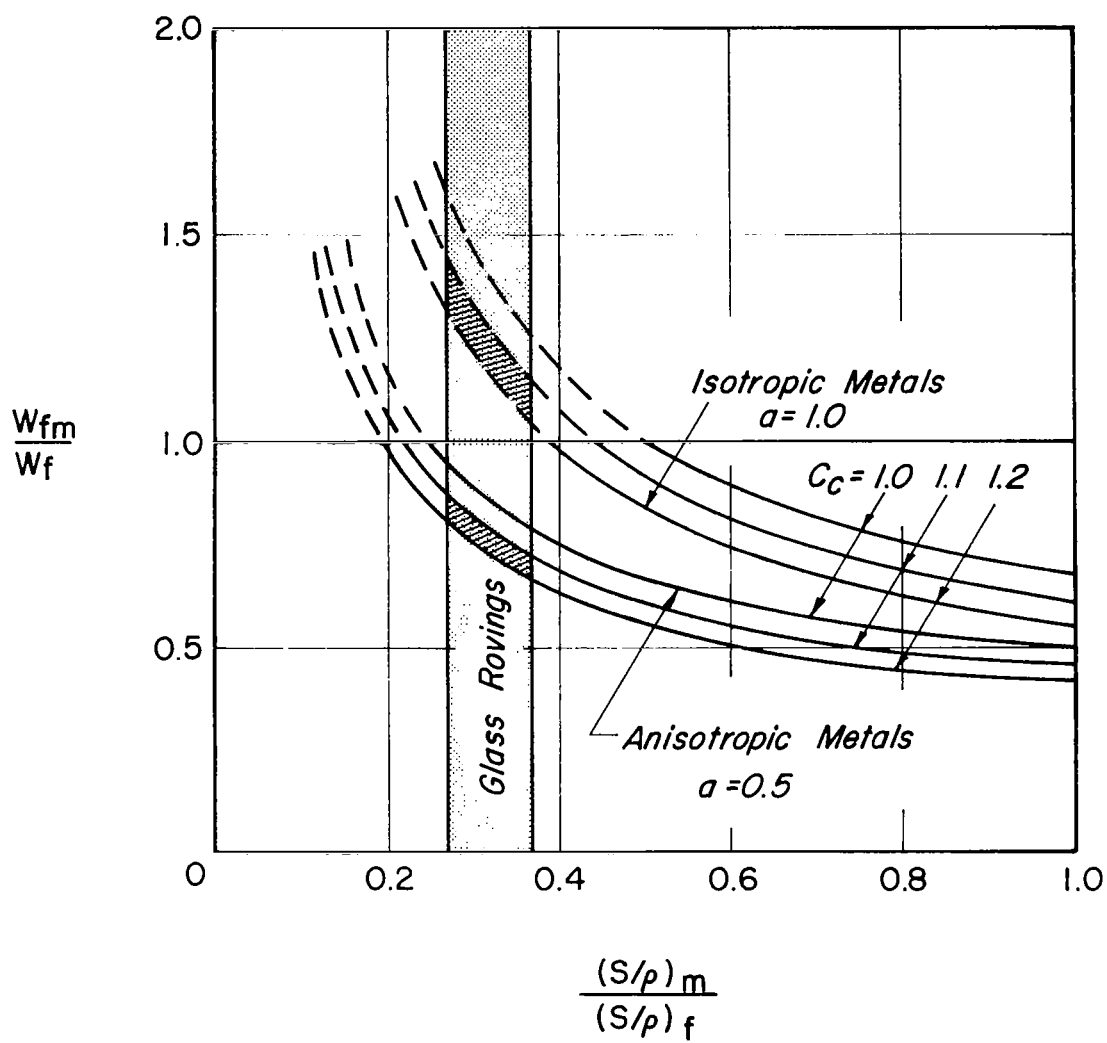


Figure 19 Overall Efficiencies of Filamentary-Monolithic Composites Compared to Filamentary Composites. Long Cylinder
 $(S/\rho)_m = 10^6$ psi/pci

Whisker Composites

The filamentary composites considered up to this point all utilize continuous reinforcements in the form of rovings, monofilaments or fine wires. Hoffman² has discussed, at some length, the interesting potential for pressure vessel applications of composites with discontinuous reinforcements in the form of whiskers. Although the current research work in this area is exploratory in many respects, the achievements are sufficiently encouraging to warrant serious consideration of whisker composites as a potential pressure vessel material. Table 7 lists some representative strength levels of the best whiskers tested.

Table 7
Representative Strength Data on the Best Whiskers
At Room Temperature

Material	Σ_1 (ksi)	ρ (pci)	Σ_1/ρ (psi/pci)	S/ρ (psi/pci)
graphite	3,000	0.07	43×10^6	13.4×10^6
aluminum oxide	1,800	0.13	14	5.3
iron	1,900	0.284	6.7	2.9
silicon	550	0.087	6.3	2.1

Although Table 7 lists the best whisker properties currently achieved, the average properties of a batch of whiskers will be far below these values. For our purposes here, it will be assumed that within a batch of whiskers there is a normal distribution of tensile strengths between the highest values given in Table 7 and the lowest strength which is taken as zero. Hence, the batch tensile strength would be 1/2 of the Table 7 values.

For purposes of comparison with filamentary composites, it is assumed that an epoxy matrix may be suitable for whisker composites. Since the packing density of the whiskers in the composite will probably not be as high as that achieved for filamentary composites, it is assumed that 50 vol. % whiskers and 50 vol. % epoxy may be representative. Using this composition and the batch tensile strength, the

uniaxial composite strength/weight ratios (S/ρ) given in Table 7 were computed. Degradation effects due to whisker cross-over and improper whisker alignment are not accounted for.

A comparison of the S/ρ values with the uniaxial composite strength of a full scale S-994 glass composite (Table 4) of $S/\rho = 3.18 \times 10^6$ (psi/pci) indicates that iron and silicon whisker composites are not competitive on this basis. On the other hand, graphite and aluminum oxide whisker composites are attractive.

In a biaxial stress field, the whisker must be oriented according to Eq. (18) so as to carry both principal stress components in an optimum manner. In this respect whisker (and filamentary) composites are less efficient than monolithic materials. For all whisker composites of optimum design, the configuration efficiency coefficient, $C = 3$.

In the preceding evaluation, the whiskers were assumed to have the optimum orientation associated with filamentary membranes in a biaxial stress field. This may be unrealistic in a practical sense and, therefore, it may be important to consider randomly oriented whisker composites. In this case the composite is to be considered as a monolithic rather than filamentary membrane. By some control of the randomness of whisker orientation in the composite it should be possible to obtain anisotropic monolithic membranes. The analyses of monolithic membrane structures presented previously herein would apply to the randomly oriented whisker composite. In particular, Eq. (44) and its representation in Fig. 12 indicates that this composite must have significant compressive strength in the thickness direction to be an efficient monolithic material for pressure vessel applications.

6. Overall Pressure Vessel Efficiencies

In previous sections, the configuration efficiencies of various pressure vessel shapes were investigated and the efficiencies of various materials were studied in some detail. Now, we combine the two efficiency factors to determine the overall primary structural weight efficiency of membrane type pressure vessels. For this purpose we return to the design synthesis relationship in the following form:

$$W/pV = C_p/S \quad (59)$$

In Eq. (59), the design conditions are represented by the pressure (p) and volume (V). The latter can usually be specified in a completely straightforward manner. The design pressure, on the other hand, is usually taken as the maximum operating pressure multiplied by a suitable safety factor.

For a structural reliability standpoint, the maximum operating pressure is statistical in nature as is the structural strength (S). Consequently, for a prescribed value of structural reliability, which can also be taken as a specified design condition, the specific statistical variations of the structural strength should be charged to the material when comparing the efficiencies of a variety of materials. In this manner we would be comparing pressure vessels designed for the same structural reliability. Unfortunately, there are insufficient data available to permit incorporation of strength distributions in the present investigation.

Based upon the C and S/p values obtained herein, Fig. 20 has been prepared to evaluate the overall efficiencies of monolithic and filamentary materials. The cross-hatched regions in Fig. 20 represent materials that have been utilized in full scale aerospace production components. It is to be noted, however, that the aerospace environment encompasses temperatures other than room temperature upon which Fig. 20 is based.

It can be observed that inflatable structures as a class (based on data of Ref. 4) are inherently much less efficient than metallic and glass-epoxy composites. Based on room temperature properties, isotropic metallics are not as efficient as glass-epoxy composites. Under the best circumstances for each, a weight saving potential of approximately 1/3 can be attained with the glass-epoxy composite.

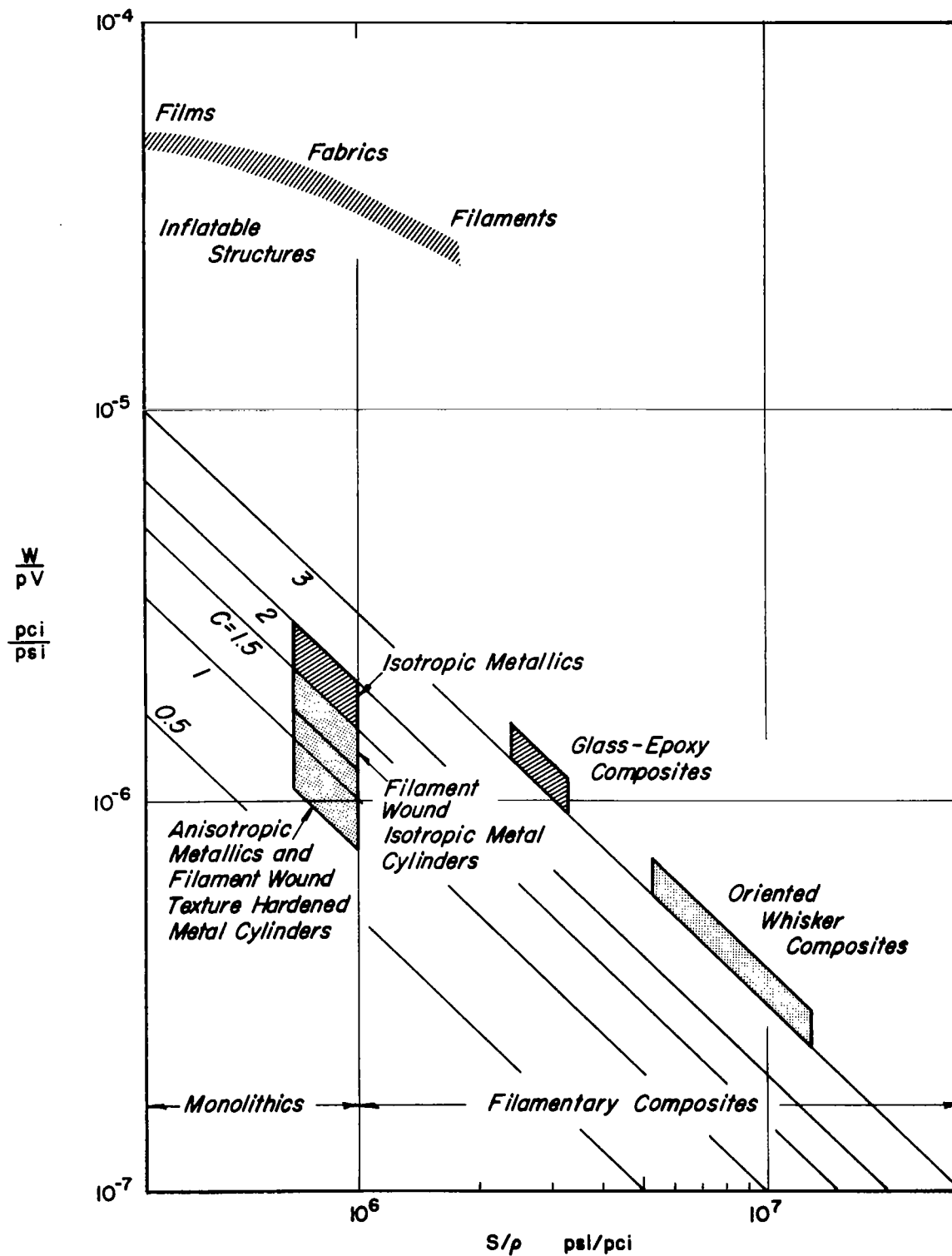


Figure 20 Overall Membrane Efficiencies of Pressure Vessels at Room Temperature

For other materials concepts which have not, as yet, reached the aerospace production stage, filament wound isotropic metal cylinders represent an inherent improvement over monolithic isotropic metallics. However, at room temperature, the glass-epoxy composites still appear to be at an advantage. On the other hand, the development of anisotropic metals can represent a significant weight saving potential as compared to currently used materials. This potential depends strongly upon the degree of anisotropy that can be achieved with high strength metals and the configuration of the pressure vessel. This is also true for filament wound texture hardened metal cylinders.

An important improvement in overall efficiency appears possible with oriented whisker composites. However, on the basis of the analysis used herein the potential of such composites appears to be far less dramatic than predicted by Hoffman². In fact, only the low density whiskers such as graphite and aluminum oxide appear to be attractive when used in the form of oriented whisker composites.

Should it not be possible on a production basis to orient properly the whiskers, then randomly oriented whisker composites may be a practical solution. In this case, the composite would tend to act as an anisotropic monolithic material with a consequent large reduction in structural strength as compared to the oriented composite. In pressure vessel applications this reduction in strength would be compensated to some degree by the inherently more efficient configuration coefficient associated with monolithic materials.

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