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Abstract

This document is the Second Quarterly Report on NASA Contract No. NAS 8-11050, Special AROD System Studies. The study is being performed at the Communications Systems Department of the IBM Corporation located in Bethesda, Maryland. The material presented in Sections 1 (Computer Requirements) and 4 (Oceanborne Transponders) represents the essentially completed study efforts on Items 1 and 2 of the Statement of Work and Work Specification under the contract. Sections 2 (AROD System Interface Investigations) and 3 (Investigations of GDOP) are interim reports on study efforts that are in progress.

The principal result of the Computer Requirements effort is an estimate of requirements for basic and auxiliary computations. The estimates, based on the characteristics of the Saturn V Guidance Computer and Data Adapter, for all AROD-related computation time and storage requirements are 0.15 seconds and 1500 data word locations. This estimate includes allowances for additional computations related to AROD command and control functions. A preliminary investigation of these functions indicate that their impact upon computer time and storage is strongly dependent upon the station selection process that is adopted for AROD operation.

Section 2 reports progress on the investigation of AROD/Vehicle interfaces. This effort was directed towards optimizing the AROD sensor extraction elements to the Saturn V guidance system. The principal conclusion concerns the desirability of providing a continuous doppler readout capability. Operational-digital techniques are discussed as a means of implementing this capability, which if successful, will obviate the need for additional hardware for buffer storage.

Investigations of Geometrical Dilution of Precision associated with a translunar orbit injection, effects of transponder geometry on system errors,

and effects of transponder survey errors made at the request of the Contracting Officer's Representative are reported on in Section 3.

The material on Oceanborne Transponders, Section 4, deals with the limitations of existing buoy designs in the environments and conditions that exist in the ocean region of interest. Their inability to meet the position keeping and stability requirements that are desirable for AROD is shown.

A triply moored lightweight buoy is described which, on the basis of a preliminary analysis, appears capable of maintaining its position with 30 feet of a mean, unperturbed position under environments that will not be exceeded in severity more than five percent of the time. Recommendations concerning additional study of this design approach are presented in order to determine design feasibility.

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Section 1

COMPUTER REQUIREMENTS STUDY

This section contains the final report on the computer requirements for all computations leading to the determination of vehicle position and velocity. The method selected for performing the basic computations is presented in Section 1.1. The auxiliary computations essential to processing and correcting the AROD measurements to a form suitable for use in the basic computations are also defined (section 1.2). Additional auxiliary computations, primarily for command and control functions, are discussed in Section 1.3. A summary of the computer requirements for the basic and auxiliary computations, including estimates of the command and control computations, is given in Section 1.4. These requirements are based on the most recent specification status for the Saturn V Guidance Computer and its associated Data Adapter as reported in IBM Technical Document No. 63-928-130, titled Apollo Study Report - Volume II prepared for MSFC on contract NAS 8-5296/5276.

1.1 Basic AROD Computations

The AROD position and velocity navigation equations were presented in the SASS First Quarterly Report. Three different methods of solution for the position equations were proposed. These methods are; Direct, Differential Correction, and Analytic Geometry. Section 1.1.1 presents a determination of the Saturn V Guidance Computer requirements estimates for these three methods of position computation and the subsequent velocity computation. Section 1.1.2 follows with recommendations based on these estimates.

1.1.1 Determination of Computation Requirements

The storage and computation time estimates required for the velocity computation and the three methods of computing position are given in Table 1-1. To obtain these estimates, it is first necessary to write Saturn V Guidance Computer type preliminary programs for these computations. Although these programs are quite detailed, they do not consider scaling which depends to a great extent on the magnitude of the actual data. However, past experience with similar computers has indicated that an additional 15 percent in storage and 20 percent in instruction cycles adequately compensates for this initial simplification.

The total storage requirement for each program is obtained by counting the total number of instructions required including sine-cosine and square root sub-routines. Add the 15 percent for scaling to obtain the total number of instruction cycles in each program. The storage requirement for each program is one-half the total number of instructions (i.e., two instructions fit into each storage location) plus an additional data storage block in the memory. An adequate allocation for data storage is 80 locations. This number does not include storage for transponder location information. Assuming a maximum of 50 transponders this gives, for three storage locations each, an additional 150 storage locations. This brings the total non-instruction storage to 230 locations.

The solution time for each program is obtained by first counting the total number of instruction cycles required by each program. Two solution times are given for each position computation. For the Direct Method and the Analytic Geometry Method of position computation, two intermediate answers result from the necessity of selecting the correct root of a quadratic equation. The smaller solution time indicates that the first root selected was the correct one while the large solution time indicates that it was necessary to compute the second root.

In the case of the Differential Correction method, the smaller number indicates that only one iteration was required to obtain an answer of sufficient accuracy while the larger number assumes two iterations. Each subsequent iteration will require an additional 506 instruction cycles.

Table 1-1

STORAGE AND COMPUTATION TIME ESTIMATES FOR THE AROD EQUATIONS

	Direct Method	Differential Correction Method	Analytic Geometry Method	Velocity
<u>Storage Requirements</u>				
Program Instructions	404	482	539	186
15% of Total for Scaling	<u>61</u>	<u>73</u>	<u>81</u>	<u>28</u>
Total 13 Bit Words	465	555	620	214
Total 26 Bit Words	233	278	310	107
Data Storage Requirements	230	230	230	
Total 26 Bit Word Storage	<u>463</u>	<u>508</u>	<u>540</u>	<u>107</u>
<u>Solution Time</u>				
Instruction Cycles				
1st Correct	485	677	673	303
2nd Correct	521	1183	683	
Sine-Cosine/Square Root Routines	285	210	510	
Total Executions				
1st Correct	<u>770</u>	<u>887</u>	<u>1183</u>	<u>303</u>
2nd Correct	806	1396	1193	
20% of Total for Scaling	162	279	239	61
Total Instruction Cycles Executed at 82 Microseconds per In- struction Cycle				
1st Correct	<u>932</u>	<u>1166</u>	<u>1422</u>	<u>364</u>
2nd Correct	968	1672	1432	
Total Time Required				
1st Correct	.078 sec.	.098 sec.	.119 sec.	.031 sec.
2nd Correct	.081 sec.	.140 sec.	.120 sec.	

To these figures, it is necessary to add the number of instruction cycles required by the sine-cosine and square root sub-routines for each time they are used. The sine-cosine sub-routine estimate assumes that a 26 bit accuracy is required. Since the square root sub-routine uses an approximation method, the number of instruction cycles required will depend upon how close an initial estimate of the answer is to the actual value. A reasonably close estimate is assumed since the result does not change radically from one range computation to the next.

Adding in the 20 percent scaling factor, the total number of instruction execution cycles required is obtained. The solution time for each program is now obtained by multiplying the number of instruction cycles by the time required for each cycle.

1.1.2 Recommendations

An inspection of Table 1-1 indicates that the Direct Method is clearly the best approach for computing vehicle position. Although the Differential Correction Method has approximately the same solution time for one iteration, it is not likely that one iteration will be sufficient. It is therefore recommended that the Direct Method of position computation be used in conjunction with the velocity computation. This gives a total storage requirement of 420 locations and solution time of 0.112 seconds.

1.2 Auxiliary Computations

The measured quantities made available to the Saturn V Guidance Computer by the AROD Sensor subsystem are somewhat different in form from the range and range rate data required by the basic computation method used to determine the position and velocity of the vehicle. This section discusses the transformation and correction computations required to generate range (R) and range rate ($\dot{R}$) from the unprocessed or "raw" AROD Sensor measurements. Sections 1.2.1 and 1.2.2 analyze the computations that are not dependent on the nature of the propagating medium. Section 1.2.3 considers the propagation corrections needed to complete the

transformation to the accurate estimates of R and $\dot{R}$ required by the AROD navigational equations. The final section determines the computer requirements for these auxiliary computations.

1.2.1 Transformation of Unprocessed Data to Virtual Range

It is the purpose of this section to determine the necessary computations for the conversion from the unprocessed data to the virtual range data obtained by assuming free space propagation. The basic data available at the AROD/computer interface for the computation of the transponder ranges will be the two-way transmission delays (τ_D) plus their common measurement time, t_m . Once τ_D has been converted to range

$$\Delta s = c\tau_D \quad (1-1)$$

where c = free space propagation velocity, it is necessary to correct for errors caused by the motion of the vehicle during the range measurement interval as discussed in Sections 1.2.1.1 and 1.2.1.2.

1.2.1.1 Interpolation

Since the vehicle can move a considerable distance during the range measurement interval, the instantaneous range at the measurement time may vary considerably from the "average" range actually measured. This problem can be more clearly seen by considering Figure 1-1. In the AROD system, range is determined by measuring the phase difference between the transmitted and received side-tone signals. That part of the RF carrier from which the phase difference is being measured at point ② and time t_m can be thought of as having been originally transmitted at point ① and time t'_1 .

The propagation time $\tau_D = t_m - t'_1$ will be this measured phase difference. Since only the two-way transmission delay is available to the computer, it has knowledge only of the two-way range,

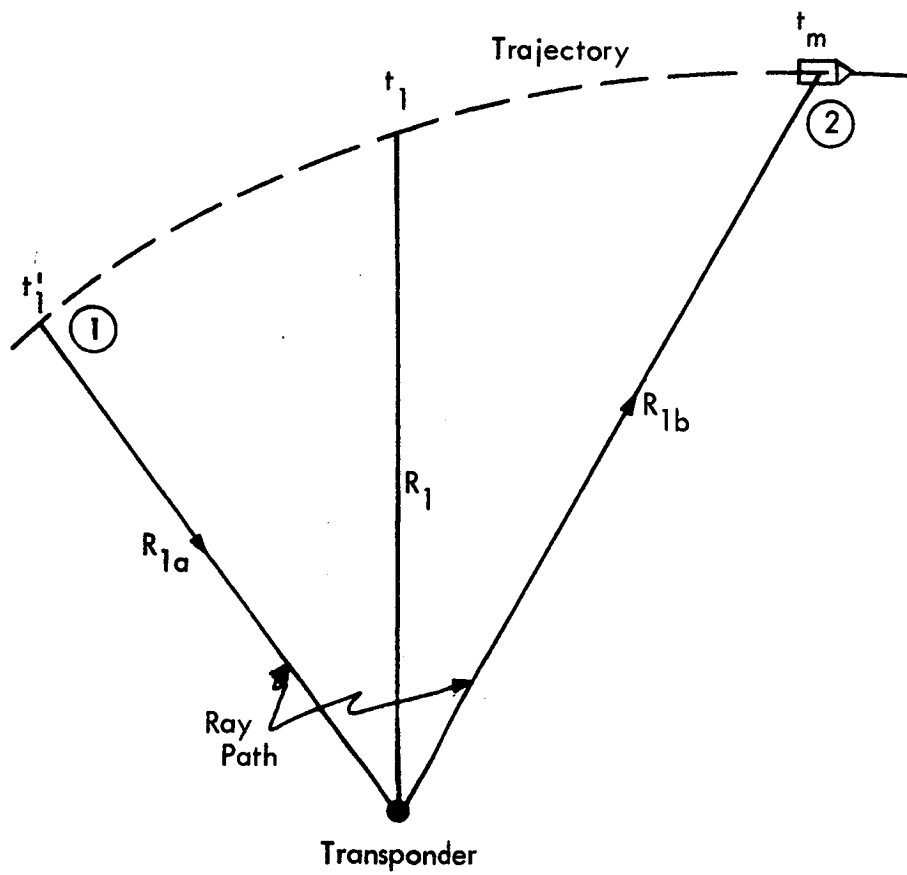


Figure 1-1. Interpolation

$$\Delta S = c(t_m - t'_1) \quad (1-2)$$

To indicate the extent of this problem, one can assume a typical range R_{1a} and vehicle trajectory and compute the difference between R_{1a} and R_{1b} . Using a Taylor Series expansion **,

$$R_{1b} - R_{1a} = \tau_D \dot{R}_{1a} + 1/2 \tau_D^2 \ddot{R}_{1a} + \dots \quad (1-3)$$

Where $\dot{R}$ and $\ddot{R}$ are the first and second time derivatives respectively of the slant range R .

As a typical example, consider the vehicle to be in a 90 nautical mile circular orbit with a 5° elevation angle with respect to the transponder. Using values for $\dot{R}$ and $\ddot{R}$ as given in Table 1 of Appendix I, the maximum range difference is found to be approximately $R_{1b} - R_{1a} \simeq 50$ meters. Since errors of this magnitude are large in comparison to the AROD system errors under current consideration,* it will be necessary to determine a more accurate result.

This problem of selecting some R_1 and its associated time t_1 (see Figure 1-1) can be approached by first choosing the range and then determining the associated time.

Assuming no a priori knowledge of the relative magnitudes of R , $\dot{R}$, $\ddot{R}$, etc. one intuitively expects the average range

$$R_1 = \frac{R_{1a} + R_{1b}}{2} = \frac{C}{2} (t_m - t'_1) \quad (1-4)$$

to be the best guess. The following analysis uses worst case values of $\dot{R}$, $\ddot{R}$, etc., to prove that this assumption causes a negligible error. The equation for the range at time t_1 given the range at time t'_1 will be

$$R(t_1) = R(t'_1) + (t_1 - t'_1) \dot{R}(t'_1) + \frac{1}{2} (t_1 - t'_1)^2 \ddot{R}(t'_1) + \dots \quad (1-5)$$

*The AROD Design Feasibility Report discusses system errors of 3 m/sec and 0.03 m/sec².

**See Reference Data for Radio Engineers ITT Corp.

It can easily be shown that the magnitude of the third term of Equation (1-5) and therefore all higher order terms will be negligible when compared to the first two terms. To do this, one computes minimum values for the first two terms and a maximum value for the third. The maximum value of $\ddot{R}$ is computed in Appendix I to be $\ddot{R}_m = 360 \text{ m/sec}^2$. The minimum value of $\dot{R}$ is zero while the minimum value of R will be about 50 km.**

These maximum range derivatives occur for the minimum altitude of 90 nautical miles as discussed in Appendix I. The time delay $\tau = (t_1 - t'_1)$ associated with these range derivatives is at worst 10 msec. It is not necessary to use the maximum time delay possible with any AROD mission (i.e., 50 msec associated with ranges of 4000 nautical miles) since the range derivatives decrease with increasing range at a greater rate than $(t_1 - t'_1)$ increases.

Using these figures, the third term of Equation (1-5) will have a maximum magnitude of approximately 1.8×10^{-2} meters. Therefore the ratio of the first term to the third term (the second term is assumed zero) will be a number less than 10^{-6} .* Since the current AROD objectives assume errors of the order of 10^{-6} to be negligible, Equation (1-5) can then be re-written as,

$$R(t_1) = R(t'_1) + (t_1 - t'_1) \dot{R}(t'_1) \quad (1-6)$$

Since $R(t'_1) = R_{1a}$ and $R(t_1) = R_1$, Equation (1-6) can be written as

$$R_1 = R_{1a} + (t_1 - t'_1) \dot{R}_{1a} \quad (1-7)$$

*It should be emphasized that this error is truly an upper bound since all the worst cases were assumed to occur simultaneously when this in fact is not possible.

**This assumes the AROD system will have sufficiently good geometry when the vehicle reaches an altitude of 50 km to allow accurate determination of position and velocity.

A similar equation can also be written for the range at time t_m in terms of the range at time t_1' .

$$R_{1b} = R_{1a} + (t_m - t_1') \dot{R}_{1a} \quad (1-8)$$

Substituting Equation (1-4) in Equation (1-7) yields

$$\frac{R_{1a} + R_{1b}}{2} = R_{1a} + (t_1 - t_1') \dot{R}_{1a} \quad (1-9)$$

Finally, substituting Equation (1-8) into Equation (1-9) and solving for t_1 yields,

$$t_1 = \frac{t_1' + t_m}{2} \quad (1-10)$$

Thus the time t_1 associated with the average range R_1 will be the midpoint of the range measurement interval. This is not surprising since by neglecting the effects of $\ddot{R}$ one is assuming $\dot{R}$ to be a constant over the measurement interval. Therefore the midpoint of the path traveled by the vehicle during the range measurement interval will have associated with it, the midpoint of the time interval.

1.2.1.2 Synchronization

In general, the times (t_1 , t_2 , & t_3) associated with the ranges (R_1 , R_2 , & R_3) for the three transponders will not be coincident. To illustrate this problem Figure 1-2 superimposes three sketches of the type shown in Figure 1-1. T_1 , T_2 , & T_3 represent three transponders on the earth's surface. The dashed line is any arbitrary trajectory. Time t_m is the common range measurement time. Since the mathematical equations for computing position and velocity as discussed in Section 1.1.1 assume that the range readings are taken simultaneously, it will be necessary to synchronize these range readings to some common time, t_d . This can be done using a series

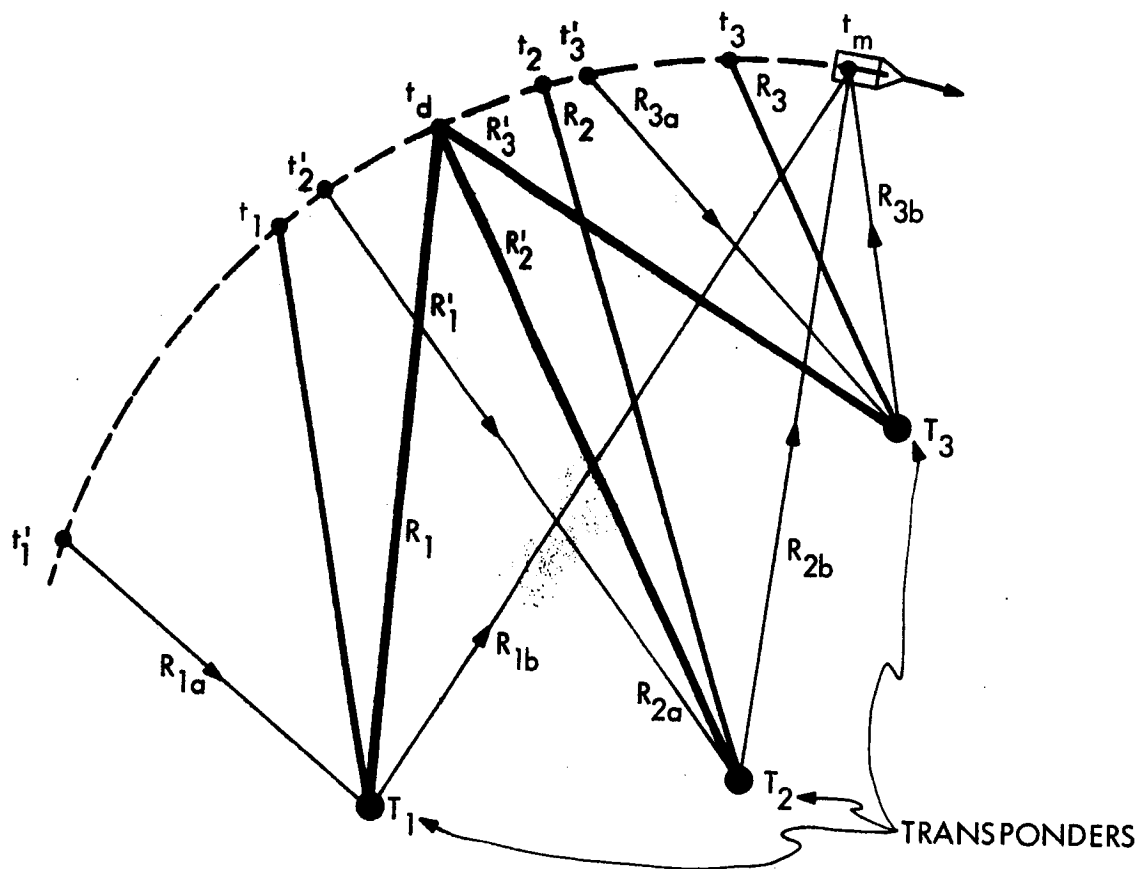


Figure 1-2. Synchronization

expansion of the same type as that discussed in the Section 1.2.1.1. As was proven in the previous section, if t_d is chosen in the region between t_1' and t_m , the time differences $(t_d - t_1)$, $(t_d - t_2)$ and $(t_d - t_3)$ will be small enough that only the first two terms of a series expansion need be retained. Therefore, the synchronized ranges R_1' , R_2' , and R_3' , will be given by the equations.

$$R_1' = R_1 + (t_d - t_1) \dot{R}_1 \quad (1-11)$$

$$R_2' = R_2 + (t_d - t_2) \dot{R}_2 \quad (1-12)$$

$$R_3' = R_3 + (t_d - t_3) \dot{R}_3 \quad (1-13)$$

1.2.2 Transformation of Unprocessed Data to Virtual Range Rate

1.2.2.1 Doppler Equation

The unprocessed data made available to the computer by the AROD Sensor for the computation of $\dot{R}$ is in the form of cycle counts. The equation for $\dot{R}$ as a function of the doppler effect associated with each transponding station is developed in Appendix II. The equation is

$$\dot{R} = \frac{A + Cx}{B + x}$$

where x is the measured quantity which is related to the doppler frequency shift, and A , B , and C are precomputed constants.

1.2.2.2 Interpolation

Since the range rate measurements must be made over a significant observation interval,* it will be necessary to determine the correct

*The doppler counting interval currently contemplated for the AROD equipment is of the order of 0.25 sec.

time to associate with the measured range rate* in a similar manner to that used for the range measurements. Assuming no a priori knowledge of the magnitudes of $\dot{R}$, $\ddot{R}$, $\dddot{R}$, etc. during the measurement interval, one intuitively expects that associating this average range rate with the midpoint of the measurement interval would provide the most accurate estimate. If the error in making this assumption is significant, it will be necessary to measure or compute at least an approximate estimate of $\ddot{R}$.

To determine the magnitude of the error caused by assuming the average $\dot{R}$ measurement to be associated with the midpoint of the measurement interval, it is convenient to expand $\dot{R}$ in a Taylor Series. The instantaneous value of $\dot{R}$ at the middle of the measurement interval, $\dot{R}_2$, as a function of the instantaneous $\dot{R}$ at the beginning of the measurement interval, $\dot{R}_1$, will be

$$\dot{R}_2 = \dot{R}_1 + \Delta t \ddot{R}_1 + \frac{1}{2} (\Delta t)^2 \dddot{R}_1 + \dots \quad (1-14)$$

If the third term of Equation (1-14) can be shown to be negligible, then $\dot{R}$ varies linearly with $\ddot{R}$ and the AROD system need not concern itself with measuring $\ddot{R}$.

As discussed in Appendix I, a significant contribution to $\dot{R}$ may be introduced by the thrust during powered flight. The key factor is the thrust to weight ratio since,

$$\dot{a} \approx \frac{T}{W} \quad ** \quad (1-15)$$

where T/W = thrust to weight ratio.

The rate of change of acceleration, $\dot{a}$, will be a maximum just before the cut-off of each powered stage (when the thrust to weight ratio is the smallest). In the worst case, with the vehicle moving along the slant range, $\dot{a} = \ddot{R}$.

*The measured range rate is an average over the counting interval.

**This equation assumes constant thrust and specific impulse.

For example, if the thrust to weight ratio is 2, the maximum $\ddot{\ddot{R}}$ will be 0.1 m/sec^3 . Assuming a measurement time of 0.25 seconds, an error of approximately 0.003 m/sec results from neglecting the third and higher order terms in Equation (1-14). This error is negligible as compared to the range rate error of 0.03 m/sec that is currently under consideration. Greater thrust to weight ratios or longer measurement times might cause errors (worst-case) that are significant.

An examination of available classified literature indicates that the contribution to $\ddot{\ddot{R}}$ caused by the thrust from the S-IV B is not large enough to warrant consideration of third order or higher terms. Therefore, this report will assume the measured range rate to be associated with the midpoint of the measurement interval.

1.2.3 Propagation Corrections

To obtain measurement accuracy of the same magnitude as that given in the AROD Design Feasibility Report (i.e., 3 m and 0.03 m/sec) it will be necessary to correct for any deviation from free space propagation due to the troposphere or the ionosphere. These corrections depend upon a number of parameters. Although the most important of these are vehicle altitude and vehicle-transponder range, other parameters such as geographical location, time of day, season, and sun-spot phase are also important.

A careful search of the literature was undertaken to determine what work has been done on the specific problem of computing R and $\dot{R}$ corrections and estimating the residual error after correction. The most applicable work available at this time was performed in 1958 by V. A. Counter and is discussed at length in Appendix B of the AROD Design Feasibility Report (DFR). The major limitation in his work was the lack of accurate ionospheric data at the time his study was performed. Since that time, there have been significant advances in the knowledge of the atmosphere. However, this new data has not been used to compute more accurate values of the range and range rate corrections or the errors remaining after correction.

The atmosphere has both a bending and a retarding influence on the propagating wave. Although $\dot{R}$ is only affected by the bending, R is affected by both. Fortunately, the bending effect on R can be shown to be negligible (see Appendix B of AROD DFR). For brevity, the range and range rate corrections will be denoted by the symbols $\chi(\Delta R)$ and $\chi(\Delta \dot{R})$ respectively.

One likely method of implementing the R and $\dot{R}$ corrections is to store the corrections in tabulated form with retrieval by table lookup. Such a procedure will minimize computer time but will maximize storage requirements. As an alternative, it may be possible to derive analytical expressions for $\chi(\Delta R)$ & $\chi(\Delta \dot{R})$ as functions of range, height, or other parameters. A trade-off must then be performed between the computer time necessary to solve the analytic expressions and the storage requirements for the table

lookup method of error correction. The optimum method may well involve a combination of the table lookup and analytical techniques. Such a trade-off analysis will require a more detailed knowledge of $\chi(\Delta R)$ and $\chi(\Delta \dot{R})$ than is now available.

To determine the storage requirements for the table lookup method, the primary concern is the range of the corrections $\chi(\Delta R)$ and $\chi(\Delta \dot{R})$ as a function of such parameters as range, altitude, time, and geographical location. Fortunately, not all of these parameters need be considered in making the computer estimates. Any parameter with negligible variation during the mission can be accounted for prior to launch. Such factors as season and phase of sun-spot cycle fall into this category; other factors such as longitude and latitude may or may not; while range and altitude definitely cannot be accounted for by pre-launch computation. Taking these considerations into account, reasonable estimates of computer storage can be made.

The total storage requirements may be estimated by determining the size of the table of corrections as a function of R and h for given values of longitude, latitude, etc. Since Counter's data gives a good indication of how $\chi(\Delta R)$ and $\chi(\Delta \dot{R})$ vary with R and h for given values of the other parameters, his data, as given in Appendix B of the AROD DFR, will be used for this purpose.

The design of the R/h correction table will depend on two factors. The first of these is the range of numerical values of $\chi(\Delta R)$ and $\chi(\Delta \dot{R})$. These may vary depending upon the trajectory and transponder locations. The second factor is the ~~minimum~~ resolution requirements on $\chi(\Delta R)$ and $\chi(\Delta \dot{R})$. These requirements are set by overall system considerations such as the position and velocity error ellipsoids.

Although R may be a more convenient parameter to use, the correction tables will be discussed here as a function of elevation angle E to conform with the data given in the AROD DFR.

The range correction, $\chi(\Delta R)$, given in figure B-9 of the DFR is redrawn on a linear scale in Figure 1-3 to facilitate the storage estimates.

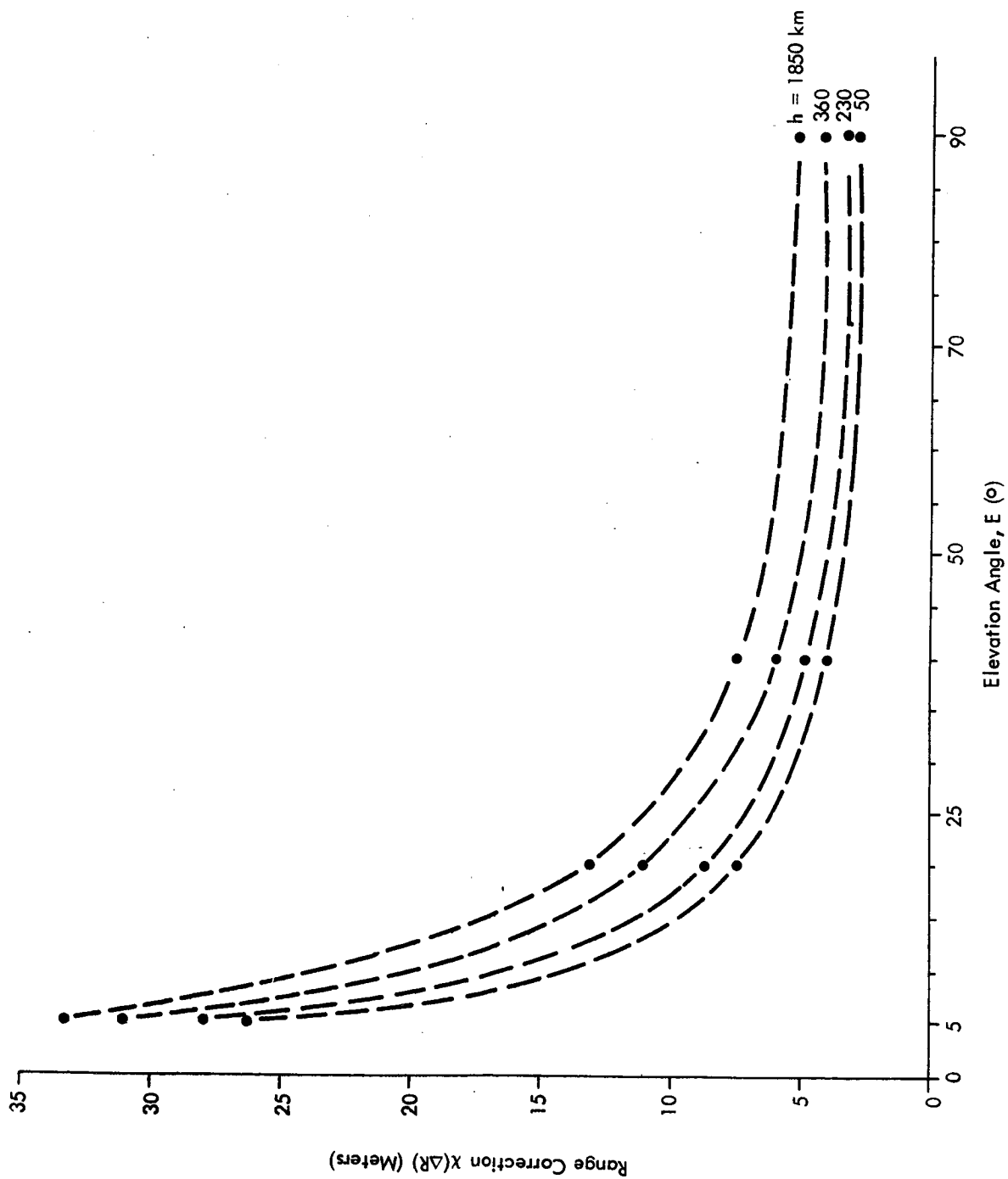


Figure 1-3. Range Correction $X(\Delta R)$ as a Function of Elevation Angle and Altitude

It is the purpose of this graph to depict $\chi(\Delta R)$ as a function E and h for the types of missions under consideration.

In particular, one is concerned with elevation angles greater than 5° and range less than 3700 nautical miles. The minimum elevation angle is determined by propagation considerations such as multipath while the maximum range derives from assuming a maximum altitude of 2000 nautical miles (i. e., at 5° elevation angle, the slant range will be 3700 naut. miles).

The process of determining the number of values of $\chi(\Delta R)$ is straightforward once a resolution in $\chi(\Delta R)$ and a range of values of h have been specified. The following two examples will be given to indicate the order of magnitude of the computer time and storage requirements. It will be shown from these examples that the computer requirements will not represent a serious problem. Consider a range of altitudes from 50 km to 185 km, as might be the case in the launch phase, and a desired range resolution of one meter. From Figure 1-3, it may be observed that the largest variation in $\chi(\Delta R)$ with h is about 4 meters. One must therefore store four curves of $\chi(\Delta R)$ vs E . Also by inspection from Figure 1-3, the maximum variation of $\chi(\Delta R)$ with E for a given h will be about 6 m per degree. Therefore, the total table of $\chi(\Delta R)$ will consist of four sub-tables with $85 \times 6 = 510$ entries for a total of 2040 entries. This first case assumes a simple table lookup using no knowledge of the nature of the data. However, it is obvious from Figure 1-3 that the rapid variation of $\chi(\Delta R)$ with E and h occurs only for small values of E . The total number of entries in the table can be greatly reduced with only a minor increase in program complexity by subdividing the table into two or more regions. For example, consider that the table is divided into two sections - one from $E=5^\circ$ to $E=20^\circ$ and the other from $E=20^\circ$ to $E=90^\circ$. As previously discussed, the first section will contain 360 entries while the second section will contain about 70 entries for a total of approximately 430 entries. Increasing the number of subdivisions will increase the computation time and decrease the storage required. This process of subdividing the table can be extended until a proper trade-off between computation time and storage is reached.

The actual number of computer storage locations required will depend not only upon the number of entries but also upon the required resolution. For accuracies in $\chi(\Delta R)$ of 1 meter over a range of about 30 meters (as in the case for this example) 5 bits will be adequate to describe each correction. Since the Saturn Guidance Computer uses 26 bit storage locations, a packing factor of five can be used for this example problem. Therefore, an approximate total of 2000 storage locations will be required in the worst case where a simple table lookup and no packing is used (i. e., Case 1) and 90 storage locations are required where a double table lookup with packing is used, (i.e., Case 2). The total storage requirements would, of course, depend on the number of R/h tables required for a complete mission. For instance, in the launch phase example just given, it might possibly be necessary to have four such tables - - one each for the launch, pre-Bermuda, Bermuda, and S IV B cut-off areas.

Another approach to the range correction problem would be to develop analytical expressions for $\chi(\Delta R)$ as a function of R and h. In this case the storage requirements would be negligible but the computation time would be increased. Although the smoothed approximate curves in Figure 1-3 are adequate for storage estimation purposes, they are not adequate for curve fitting to determine analytical expressions. However, the general shape of the curves does indicate that reasonable simple analytical expressions may be obtainable.

Although it is difficult to sketch curves for $\chi(\Delta \dot{R})$ similar to those given in Figure 1-3 for $\chi(\Delta R)$, a reasonable estimate of the storage requirements for the range rate correction can still be given. Based on Counter's data for $\chi(\Delta \dot{R})$ as given in Appendix B of the AROD DFR, the storage requirements will be about three times that for the $\chi(\Delta R)$ data. This assumes a range rate correction resolution of 0.01 m/sec with a maximum $\chi(\Delta \dot{R})$ of 0.5 m/sec. Counter's work, which is the basis for these analyses, includes effects of all seasons at a particular location. Since the storage requirements will depend upon the size of the geographical area for which a given table will hold, additional data should be obtained as it becomes available to include these atmospheric effects.

1.2.4 Auxiliary Computation Estimates

This section will discuss the Saturn V Guidance Computer time and storage requirements for the auxiliary computations discussed in Section 1.2. The necessary computations for determining virtual range and range rate (i.e., corrections for non-propagation effects) as discussed in Sections 1.2.1 and 1.2.2 are complete. The computer time and storage estimates given in this section for these corrections will therefore be complete. In addition, some partial results for the propagation corrections will also be given.

The complete set of equations for the non-propagation computations are summarized below.

$$t_d = t_c + \Delta t_c \quad (1-16)$$

$$R_i = \frac{Cx_i + B_i}{x_i + A} \quad i = 1, 2, 3 \quad (1-17)$$

$$t_i = t_m - \frac{\Delta t_i}{2} \quad (1-18)$$

$$R_i = \frac{c}{2} \Delta t_i + (t_i - t_d) \dot{R}_i \quad (1-19)$$

Where the input parameters are,

- t_c = Doppler count start time
- t_m = Range measurement time
- Δt_i = Range time increment
- Δt_c = one half the doppler counting time
- x_i = doppler cycle count
- $A, B_i, \& C$ = Constants

and the output results are

- t_i = Time associated with i^{th} range measurement
 R_i = range measurement for i^{th} transponder
 t_d = Time associated with all range rate measurements
 $\dot{R}_i$ = range rate measurement for i^{th} transponder

A detailed computer storage and time estimate for these equations is given in Table 1-2. A detailed description of the various terms used in this table is given with Table 1-1 in Section 1.1.1. As can be seen from Table 1-2, the storage requirement is 64 locations while the solution time is 0.012 seconds.

TABLE 1-2
AUXILIARY COMPUTATION OF VIRTUAL R AND $\dot{R}$

Storage Requirements

Program Instructions	84
15% of Total for Scaling	<u>13</u>
Total 13 bit words	97
Total 26 bit words	49
Data Storage Requirements	15
Total 26 Bit Word Storage	<u>64</u>

Solution Time

Instruction Execution	120
20% of Total for Scaling	24
Total Instruction Cycles	
Executed at 82 Microseconds per Instruction Cycle	144
Total Time Required	.012 seconds

The corrections for propagation effects, as discussed in Section 1.2.3, used two example cases to indicate the order of magnitude of the time and storage requirements for the table lookup method. Table 1-3 summarizes the results for the storage estimates given in Section 1.2.3 in addition to the computer time required for the table lookup. The figures given are for the combined R and R corrections for a typical launch mission (see Section 1.2.3 for specific details).

Case 1 represents a worst case where no assumptions are made as to the nature of the data in the correction tables. Case 2 utilizes the knowledge of the nature of the data in this correction table to reduce the storage requirements by increasing the complexity of the table lookup routine. As can be seen, Case 2 gives a dramatic decrease in storage requirements at only a modest increase in computer running time.

It should be noted here that the table lookup times given in Table 1-3 are not significantly affected by the size of the tables. Therefore, even though the storage figures represent only lower bounds on the storage estimates, the table lookup times are firm estimates.

Table 1-3

**ESTIMATE OF PROPAGATION CORRECTIONS
FOR TABLE LOOKUP METHOD**

<u>Storage Requirements</u>	<u>Case #1*</u>	<u>Case #2*</u>
Program Instructions	8000	360
15% of Total for Scaling	<u>1200</u>	<u>54</u>
Total 13 bit words	9200	414
Total 26 bit words	4600	207
Program Storage Requirements	<u>10</u>	<u>50</u>
Total 26 Bit Word Storage	4610	257
<u>Solution Time</u>		
Instructions Executed	26	65
20% Total for Scaling	6	14
Total Instruction Cycles	32	79
Total Time Required	0.002 sec	0.007 sec

* Case #1 and Case #2 are for the same example, but Case #2 assumes a more sophisticated table lookup procedure and data packing.

1.3 Command and Control Computations

In addition to the processing required to transform the unprocessed or "raw" AROD Sensor measurements into the vehicle position and velocity, the Saturn V Guidance Computer will be required to perform a number of functions to enable rapid and accurate selection of optimum sensor measurement data for use in the basic and auxiliary computations. This section discusses the functions which will be required in this process, and includes estimates of the computer requirements for their performance. Section 1.3.1 discusses the station selection process, and considers three alternate methods for selecting the optimum triad of ground transponder stations from which the raw measurement data should be obtained. Sections 1.3.2 and 1.3.3 consider the computer requirements for commanding and facilitating actions required to implement the decisions reached in the station selection process. Section 1.3.4 discusses computer functions associated with the detection of error conditions and the corrective actions which may be taken when malfunctions are indicated.

1.3.1 Station Selection Process

Throughout the entire mission, the computer will determine which transponder ground stations will be tracked at any instant. Three stations will be tracked, and their range and range rate measurements used to determine the vehicle's position and velocity. A fourth transponder station will be tracked so that the vehicle tracking filters may lock onto the received signal before range and range rate information from that station is selected for use in the calculations.

The problem of determining which transponders should be selected at any given point in the trajectory is quite complex. Only those transponders which have a line-of-sight elevation to the vehicle of 5° or more should be considered. However, it is quite possible that four or more stations will meet this criterion at one time. It will be necessary to choose from among the currently available stations a combination of three which will result in a minimum, or at least acceptable, AROD system error.

There are a number of factors which have a bearing upon which stations provide the optimum combination at any given time. Among these are:

- Signal-to-noise ratio
- Estimated station coordinate errors
- Geometrical relationships among the stations and between the stations and the vehicle

The signal-to-noise ratio of the signal received from any transponder station tentatively selected for use should be checked to ascertain that it exceeds the design threshold of the system. Ratios in excess of this may enhance system performance by increasing measurement accuracy and decreasing tracking filter acquisition time. The increase in accuracy, however, will be small compared to the geometrically induced effects upon system accuracy. Any station with a line-of-sight elevation to the vehicle of greater than 5° should provide a signal-to-noise ratio in excess of the design threshold—unless there is a malfunction. If a selected station is found to have an inadequate signal-to-noise ratio, the vehicle tracking filter may be unable to remain locked onto the signal. Another station should be selected in its place, if possible.

The remaining factors are the same as those which were considered in the GDOP programs. The optimum configuration at any instant would be chosen so as to minimize the total AROD system position and velocity errors considering the expected measurement errors to each transponder, the station coordinate errors, and the geometrical relationships among the transponders and between the transponders and the vehicle.

Three approaches were considered for the station selection process. One involves selection of transponder stations in accordance with a pre-stored program based upon the results of GDOP computations performed on the ground. Another involves selection of stations based on GDOP analyses performed by the guidance computer. A third approach would enable selection of transponder stations by the vehicle guidance computer based upon a more simplified method of selection.

1.3.1.1 Preprogrammed Approach

The preprogrammed approach to the station selection process would take advantage of the ability to perform GDOP computations in advance and on the ground. These computations would assume a nominal flight trajectory. By comparing the AROD system errors associated with potential transponder combinations, the optimum triad could be determined for each point in the trajectory. These combinations could then be stored in the Saturn V Guidance Computer memory, together with the time after liftoff for which each is applicable. Thus, the station selection process would be performed by a table lookup procedure employing an optimum schedule, taking into full account all the diverse factors included in the GDOP programs.

This method thus provides a basis for selecting an optimum combination of stations, with a minimum of computer logic requirements. Computer storage requirements will be large, compared to the other methods. These storage requirements may be estimated by assuming that (1) during an orbit every time the vehicle comes within 5° elevation of a new transponder, it is necessary to select a new optimum triad of transponder stations; and that (2) the use of one memory location is required to store this optimum triad. Then the number of storage locations, S , is given by

$$S = \frac{T_u p_s N_T}{T_o}, \text{ where}$$

T_u = Time between updating of memory from ground

p_s = Probability of sighting any transponder station during orbit

N_T = Number of transponder stations

T_o = Time for one orbit

T_u is an operational parameter whose optimum value may be determined by trading off computer storage requirements against ease of updating the computer memory. The probability of sighting a transponder station is a function of the height of the trajectory, the ground track, and the locations of the transponder stations.

If it is assumed that $T_u = 24$ hours, $p_s = 1$, $N_T = 50$, and $T_o = 90$ minutes, then the number of storage locations required would be 900. These figures, however, involve the unrealistic assumption that the probability of sighting a transponder station during each 90-mile orbit is 1.0. This probability could be attained only if transponder stations were lying in a belt around the equator, and the orbit lay in the equatorial plane. For trajectories launched from Cape Kennedy, a more reasonable estimate of the upper bound on the number of storage locations would be obtained by using $N_T = 50$, $T_u = 120$ minutes, and $p_s = 0.8$. The storage requirement for a 24 hour period under these conditions is 480 locations.

Storage requirements may be more accurately determined after the locations of the stations in the transponder network are determined. Nominal flight trajectories for various altitudes may be programmed into the GDOP computations, and the actual number of optimum combinations may be counted for any period of time desired.

Assuming there are no more than 64 transponder stations, each transponder could be adequately identified by a 6-bit code. Thus, 18 bits per memory location would be required to identify each triad of stations. Each combination would also require a time tag indicating at what point it should be selected as optimum.

1.3.1.2 GDOP Computations Approach

The second approach to the station selection process involves performing GDOP computations in the vehicle guidance computer. The applicable GDOP programs are described in Appendix A of the Design Feasibility Report for AROD.

The first step in the process of station selection for this approach is to determine which transponder stations instantaneously are at an elevation angle to the vehicle of 5° or more. This may be accomplished by computing the instantaneous range to each transponder station, and then inserting each such range into

$$E_L = \sin^{-1} \left[\frac{r^2 - R_i^2 - r_i^2}{2R_i r_i} \right] \geq 5^\circ$$

Let N be the number of transponders currently determined to have an elevation angle of 5° or more to the spacecraft. For each possible triad which may be constructed from this set of N transponders, the AROD system error will be calculated based on the expected measurement error to each transponder and the rms station coordinate error of the transponder stations. The number of triads which must be so tested is given by $N!/[3!(N-3)!]$.

The position system error may be computed from

$$\Lambda_p = M^{-1} (\Lambda_r + M_1 \Lambda_{S_1} M_1^T + M_2 \Lambda_{S_2} M_2^T + M_3 \Lambda_{S_3} M_3^T) (M^{-1})^T,$$

where each term is as defined in equation A-17 of the Design Feasibility Report, Vol. II. The triad configuration with the smallest indicated error would be chosen for use in the position and velocity computations.

This mode of operation would enable station selection to be made based upon an accurate GDOP analysis, taking into account any deviations from the nominal trajectory. In addition, full efficiency in station selection is maintained even when one or more transponders are inoperative. The optimum triad which may be formed from the remaining stations may be identified and used. Storage requirements for this approach are small. Computational requirements, however, have not yet been estimated. It is possible that they will be prohibitively large.

1.3.1.3 Simplified Empirical Approach

The third approach to the station selection process involves the introduction of simplifying assumptions into the GDOP programs, or the development of approximations to the GDOP computations. To develop a simplified method for selecting optimum combinations of transponder stations, it is first necessary to understand or identify significant factors influencing the AROD system errors. Analysis of the GDOP computations which have been performed to date has indicated a number of these factors. It has been determined that a minimum system error tends to result when:

1. The triangle formed by the transponder stations is equilateral
2. The base lines of this triangle are large compared to the altitude of the vehicle
3. The ground track of the vehicle is near the center of the triangle formed by the transponders
4. The station coordinate errors of the transponder stations are small

In actual practice, tradeoffs are required between these factors to select the optimum configuration from the possible combinations. It may be possible to develop empirically an effective algorithm with modest computational requirements for selecting the optimum configuration. Such an algorithm would be based upon consideration of the instantaneous geometry between the vehicle and the transponders and the station coordinate errors. One such promising approach is described below as an example of the type of algorithm which may be used.

Examination of GDOP computational results indicate that it will be possible to select one near optimum configuration for both position and velocity from the potential triads by considering either the AROD system position or velocity error. In most cases, selection of the same triad will result regardless of which factor is considered. In a few cases, for a short period of time separate consideration of position and velocity errors would result in selection of different triads. In such cases, however, the performance of the triad selected on the basis of position only would be slightly

inferior in velocity performance to the triad selected on the basis of velocity only. The following algorithm was developed primarily as a result of considering positional errors.

It has been observed that for station coordinate errors up to 300 meters, and for triads with base lines of 100 km or more, the minimum positional system error varies approximately linearly with the sum of the expected rms station coordinate errors and range errors to each transponder station (see Figure 1-4). This indicates that if a figure of merit inversely proportional to the expected position error is assigned to each potential triad, then the figure of merit would be inversely proportional to the sum of the station coordinate and range errors.

The rms stations coordinate errors will be known in advance for each transponder, and may be stored in memory. It is possible that the station coordinate errors for all transponder stations will fall within a few categories. For example, it may be possible to represent the coordinate errors of all transponders located on large land masses with one figure, on small islands with a second figure, and oceanborne transponders with a third figure.

The range measurement error will be a function of the range and elevation angle. The functional relationship is not precisely known at this time. From the error budget presented in the Design Feasibility Report, it appears that the range measurement error to a transponder could be estimated to about 10 percent accuracy by the function $1 + 1.8 (\text{Range}/1000)$. The combined range measurement error to the three transponder stations could thus be represented by $3 + 1.8 \cdot 10^{-3} (R_1 + R_2 + R_3)$ meters, where R_1 , R_2 , and R_3 are the ranges to the three transponder stations. The figure of merit for a particular triad will therefore be inversely proportional to the quantity $[\sigma_1 + \sigma_2 + \sigma_3 + 3 + 1.8 \cdot 10^{-3} (R_1 + R_2 + R_3)]$, where σ_1 , σ_2 , and σ_3 are the station coordinate errors associated with the transponders.

The amount by which the actual positional system error will exceed the minimum will be determined by the geometrical relationships between

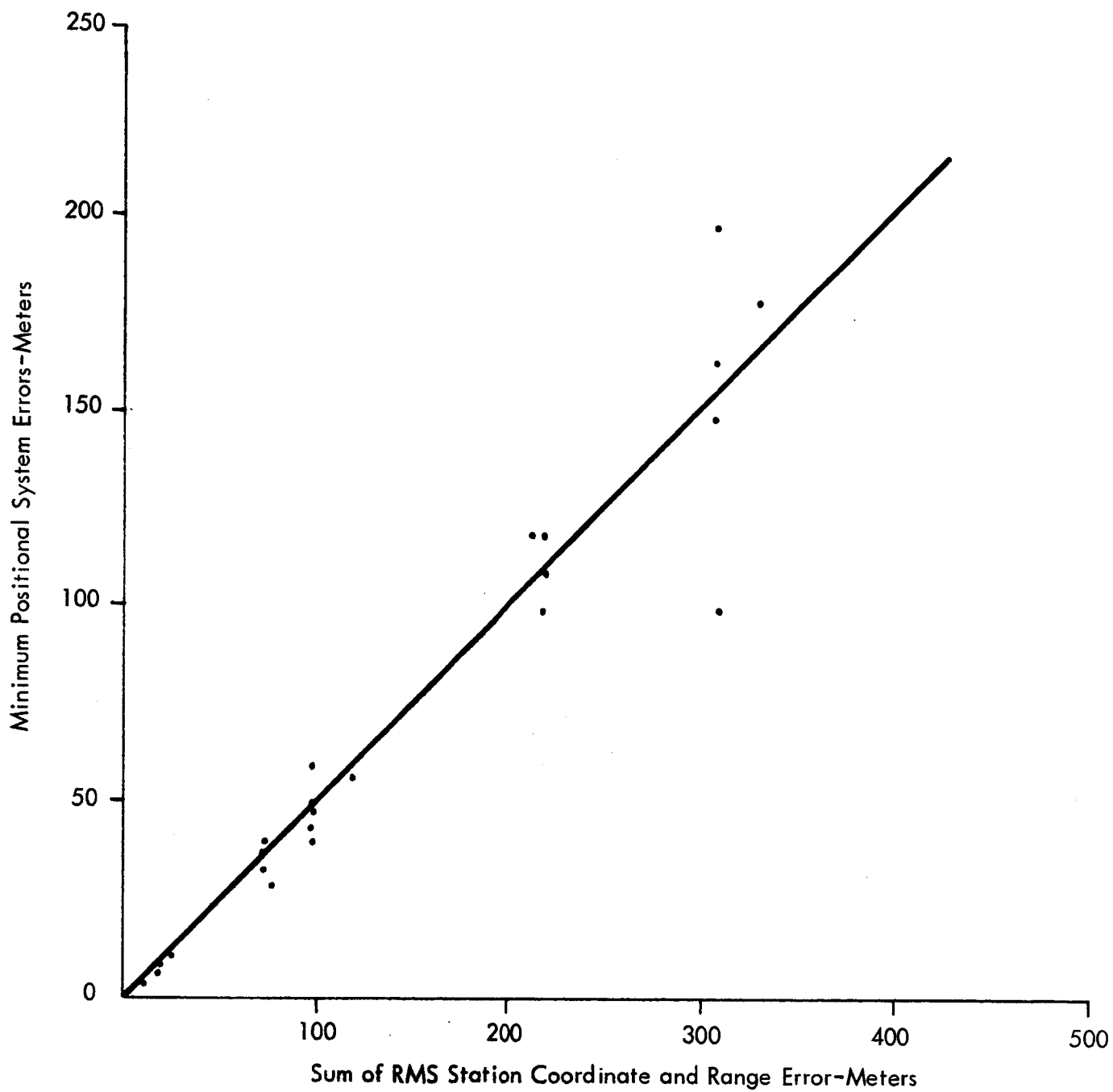


Figure 1-4. Minimum Position Error vs Sum of Station Coordinate and Range Errors

the vehicle and the transponder stations. The minimum error will occur whenever three lines drawn between the vehicle and the transponder would intersect orthogonally at the vehicle. When these lines do not intersect orthogonally to each other, there will be an increase in the system error, and there should be a corresponding decrease in the figure of merit ascribed to such a triad. This suggests that some function be found which relates the figure of merit to the nearness to orthogonality of the three angles. One function which appears to express the desired relationship is the product of $\sin \alpha \sin \beta \sin \gamma$, where α , β , and γ are the three angles in question. When each of these is 90° , the maximum value of one is achieved. As any one of these angles approaches 0° , a minimum value of 0 is approached, indicating correctly that the position of the vehicle becomes indeterminate and the error approaches infinity.

The overall figure of merit, F , instantaneously assigned to any potential triad may thus be computed from

$$F = \frac{\sin \alpha \sin \beta \sin \gamma}{\sigma_1 + \sigma_2 + \sigma_3 + 3 + 1.8 \cdot 10^{-3} (R_1 + R_2 + R_3)}$$

The value for $\sin \alpha$, $\sin \beta$, and $\sin \gamma$ may be computed as follows. If α is the angle between the lines from the vehicle to transponder stations 1 and 2, then $\cos \alpha = [R_1^2 + R_2^2 - b_{12}^2] / 2R_1 R_2$, where R_1 and R_2 are the ranges to transponders 1 and 2, respectively, and where b_{12} is the distance between the two transponder stations. This distance could be computed from knowledge of the station coordinates, or could be stored in memory.

$$\text{Now, } \sin \alpha = \sqrt{1 - \cos^2 \alpha}, \text{ or}$$

$$\sin \alpha = \left[1 - \left(\frac{R_1^2 + R_2^2 - b_{12}^2}{2R_1 R_2} \right)^2 \right]^{1/2}$$

Similarly,

$$\sin \beta = \left[1 - \left(\frac{R_2^2 + R_3^2 - b_{23}^2}{2 R_2 R_3} \right)^2 \right]^{1/2}$$

$$\sin \gamma = \left[1 - \left(\frac{R_1^2 + R_3^2 - b_{13}^2}{2 R_1 R_3} \right)^2 \right]^{1/2}$$

The triad which has the highest figure of merit would be selected as optimum.

There are many other methods which could be proposed for the station selection process. Some of them might prove equal or superior in effectiveness to the method previously described. Each could be tested by comparing the selections indicated with the optimum combinations as determined by GDOP computations for the same vehicle and transponder locations while using the same values for station-coordinate and range measurement errors.

1.3.1.4 Summary

The preprogrammed approach appears particularly attractive during the launch phase, when system accuracy will be extremely important and frequent changes in station configurations will be required due to the low altitude of the vehicle. During this period, the nominal trajectory is well defined and any major deviation will probably result in an abort situation. Storage requirements to control the station selection process during the launch phase should be modest.

The preprogrammed approach does not appear so attractive for extended orbital missions, however. In addition to large computer storage requirements and possible difficulties in updating the memory, there are two other difficulties which may be foreseen in this type of operation. If a malfunction is discovered in one of the preselected transponders, it will be necessary to switch, if possible, to another triad. If this is to be done under control of the stored program, alternative configurations would have to be stored for each segment of the trajectory. Since the alternative

configuration would be determined by the inoperative transponder, it might be necessary to store up to three alternatives to allow for malfunctions in any one of the preselected transponder stations. Secondly, it is possible that the mission profile may differ markedly from the planned trajectory without necessitating an abort. In such cases, the stored program might switch to transponder configurations which are either inaccurate or completely out of range.

For post-launch phases of extended orbital missions, the simplified empirical approach appears to be most attractive. It is recommended that further study be given to the development of an optimum station selection algorithm. The analysis so far performed has indicated that it will be possible to develop an effective program with small computational requirements.

1.3.2 Initiation of Commands to Transponder Stations

Before any transponder station may be tracked, it will be necessary for the tracking filter of that station to acquire the signal transmitted from the vehicle, and for the transponder transmitter to be turned on. If both these actions are commanded at the same time it will be necessary to wait for the transponder tracking filter to lock onto the vehicle signal. This process may take up to 10 seconds. To avoid this delay, it is recommended that the vehicle's VHF command link continuously transmit a message directing all transponders within its range to search for and lock onto the received signal. These transponders would then be ready to commence transmission upon receipt of a command message. Since the VHF coverage envelope is considerably greater than the S-band envelope, the transponder will initiate its search prior to receiving an S-band signal of adequate strength. By taking advantage of the fact that for initial acquisition the doppler frequency will always be higher than the transmitted frequency, the frequency range to be searched may be reduced by a factor of two.

As the vehicle passes out of VHF range of a given transponder, the standby message command signal would be no longer received, and the transponder would revert to its normal condition.

When it is desired to command a transponder to commence S-band transmission, a coded signal must be addressed to the desired transponder indicating which of four translation frequencies should be selected. This frequency will be chosen to match the frequency band corresponding to the vehicle's available receiving channel. When the transponder is no longer required, a second coded signal must be initiated to command the transponder to cease transmission.

One memory location per transponder station would be sufficient for the storage of the command signals. Logic requirements for the initiation of these command signals are negligible, and may be readily performed by the vehicle guidance computer.

1.3.3 Pre-Acquisition Computation of Doppler Frequency

Before a new transponder station may be utilized to determine range and range rate, the vehicle tracking filters must acquire the transponder transmitted signal. To reduce the pull-in time for these filters, it may be desirable to pre-set their VFO's to the anticipated frequency. This frequency may be determined by means of computations performed in the computer.

The anticipated received frequency, f_r , is given by

$$f_r = \frac{2\dot{r}}{c-\dot{r}} f_{t_s} + \sqrt{\frac{c^2 - \dot{r}^2}{c-\dot{r}}} f_T, \quad (1)$$

where,

c = velocity of propagation

f_{t_s} = transmitted carrier frequency

f_T = transponder translation frequency

$\dot{r}$ = range rate

The range rate may be determined from the vehicle position (x, y, z) and velocity $(\dot{x}, \dot{y}, \dot{z})$, and the transponder position (x_t, y_t, z_t) and velocity $(\dot{x}_t, \dot{y}_t, \dot{z}_t)$. The equation for this relationship, as presented in the First Quarterly SASS Report, is:

$$\dot{r}_t = \frac{1}{r_t} \left[(x - x_t) (\dot{x} - \dot{x}_t) + (y - y_t) (\dot{y} - \dot{y}_t) + (z - z_t) (\dot{z} - \dot{z}_t) \right] \quad (2)$$

where

$$r_t = \left[(x - x_t)^2 + (y - y_t)^2 + (z - z_t)^2 \right] \quad (3)$$

Since the coordinate system is space-fixed, the position and velocity components of the transponder must be determined prior to computing r_t . These may be derived from:

$$x_t = R \cos \psi \cos \lambda(t)$$

$$y_t = R \cos \psi \sin \lambda(t)$$

$$z_t = R \sin \psi = \text{constant}$$

$$\dot{x}_t = -\Omega_E R \cos \psi \sin \lambda(t)$$

$$\dot{y}_t = \Omega_E R \cos \psi \cos \lambda(t)$$

$$\dot{z}_t = 0$$

$$\lambda(t) = \lambda_0 + \Omega_E (t - t_0)$$

R and ψ are constants indicating, respectively, the distance from the center of the earth to the transponder and the latitude. λ_0 is the initial ($t = t_0$) value of the transponder's longitude, and Ω_E is the known value of the earth's rotation.

The reduction in pull-in time will be determined by how closely the computed value of frequency (at IF) approaches the actual received frequency (at IF). This, in turn, will be determined by the accuracy of the computed value of $\dot{r}_t$.

The range rate measurement error, $\Delta \dot{r}_t$, can be expressed in terms of the vehicle position errors ($\Delta x, \Delta y, \Delta z$), the vehicle velocity errors ($\Delta \dot{x}, \Delta \dot{y}, \Delta \dot{z}$), the transponder position errors ($\Delta x_t, \Delta y_t, \Delta z_t$), and transponder velocity errors ($\Delta \dot{x}_t, \Delta \dot{y}_t, \Delta \dot{z}_t$). From the AROD velocity navigation equation,

$$\dot{r}_t \dot{r}_t = (x - x_t) (\dot{x} - \dot{x}_t) + (y - y_t) (\dot{y} - \dot{y}_t) + (z - z_t) (\dot{z} - \dot{z}_t) \quad (4)$$

Taking the differential of this equation yields:

$$\begin{aligned} \dot{r}_t \Delta \dot{r}_t + \dot{r}_t \Delta r_t = & (x - x_t) (\Delta \dot{x} - \Delta \dot{x}_t) + (\Delta x - \Delta x_t) (\dot{x} - \dot{x}_t) \\ & + (y - y_t) (\Delta \dot{y} - \Delta \dot{y}_t) + (\Delta y - \Delta y_t) (\dot{y} - \dot{y}_t) \\ & + (z - z_t) (\Delta \dot{z} - \Delta \dot{z}_t) + (\Delta z - \Delta z_t) (\dot{z} - \dot{z}_t) \end{aligned} \quad (5)$$

Solving Equation (5) for $\Delta \dot{r}_t$ and rearranging terms,

$$\begin{aligned} \Delta \dot{r}_t = & \frac{(x - x_t) (\Delta \dot{x} - \Delta \dot{x}_t)}{\dot{r}_t} + \frac{(y - y_t) (\Delta \dot{y} - \Delta \dot{y}_t)}{\dot{r}_t} + \frac{(z - z_t) (\Delta \dot{z} - \Delta \dot{z}_t)}{\dot{r}_t} \\ & + \frac{(\Delta x - \Delta x_t) (\dot{x} - \dot{x}_t)}{\dot{r}_t} + \frac{(\Delta y - \Delta y_t) (\dot{y} - \dot{y}_t)}{\dot{r}_t} + \frac{(\Delta z - \Delta z_t) (\dot{z} - \dot{z}_t)}{\dot{r}_t} \\ & - \frac{\dot{r}_t \Delta r_t}{\dot{r}_t} \end{aligned}$$

For a ground based transponder station, $\Delta \dot{x}_t = \Delta \dot{y}_t = \Delta \dot{z}_t = 0$. Eliminating these terms,

$$\begin{aligned} \Delta \dot{r}_t = & \frac{(x - x_t) \Delta \dot{x}}{\dot{r}_t} + \frac{(y - y_t) \Delta \dot{y}}{\dot{r}_t} + \frac{(z - z_t) \Delta \dot{z}}{\dot{r}_t} \\ & + \frac{(\Delta x - \Delta x_t) (\dot{x} - \dot{x}_t)}{\dot{r}_t} + \frac{(\Delta y - \Delta y_t) (\dot{y} - \dot{y}_t)}{\dot{r}_t} + \frac{(\Delta z - \Delta z_t) (\dot{z} - \dot{z}_t)}{\dot{r}_t} - \frac{\dot{r}_t \Delta r_t}{\dot{r}_t} \end{aligned} \quad (6)$$

This equation may be used to derive reasonable estimates of the maximum error in range rate by determining the maximum contribution to the error of groups of terms in the equation. Substitution of values for parameters tending to maximize the range rate error will then yield maximum values for $\Delta \dot{r}_t$. These parameters were chosen for a 90-mile orbit, since this orbit results in the greatest error. From this, the maximum doppler frequency error was derived by using Equation (1).

A more significant measure of the range rate error would be the mean square $\sigma_{\dot{r}_t \dot{r}_t} = E \{ (\Delta \dot{r}_t - \Delta \dot{r}_t)^2 \}$. Assuming that the vehicle and transponder errors are statistically independent with zero mean, the mean square range rate error will contain no cross terms such as $E (\Delta x \Delta y)^2$, $E (\Delta x \Delta \dot{x})^2$, etc., since they will be zero. The expression for $\sigma_{\dot{r}_t \dot{r}_t}$ will be

$$\begin{aligned} \sigma_{\dot{r}_t \dot{r}_t} = & a_1 (\sigma_{xx} - \sigma_{x_t x_t}) + b_1 (\sigma_{\dot{x}\dot{x}} - \sigma_{\dot{x}_t \dot{x}_t}) + a_2 (\sigma_{yy} - \sigma_{y_t y_t}) \\ & + b_2 (\sigma_{\dot{y}\dot{y}} - \sigma_{\dot{y}_t \dot{y}_t}) + a_3 (\sigma_{zz} - \sigma_{z_t z_t}) + b_3 (\sigma_{\dot{z}\dot{z}} - \sigma_{\dot{z}_t \dot{z}_t}) \\ & - \frac{\dot{r}_t}{r_t} \sigma_{r_t r_t} \end{aligned}$$

The quantities σ_{xx} , σ_{yy} , σ_{zz} , $\sigma_{\dot{x}\dot{x}}$, $\sigma_{\dot{y}\dot{y}}$, $\sigma_{\dot{z}\dot{z}}$ are the diagonal terms of the position and velocity covariance matrices as computed by the GDOP computer program discussed in Appendix A of the AROD Design Feasibility Report. They may be interpreted physically as the square of the semi-axws of the position and velocity error ellipsoids coincident with an xyz coordinate system at the center of the ellipsoids. The quantities

$\sigma_{x_t x_t}$, $\sigma_{y_t y_t}$, $\sigma_{z_t z_t}$, $\sigma_{\dot{x}_t \dot{x}_t}$, etc. are the mean squares of the transponder location and velocity errors. The velocity errors are zero for a ground based transponder. $\sigma_{r_t r_t}$ is the mean square AROD range measurement

error. Due to the complexity of the above equation, it is difficult to gain sufficient insight to the problem without direct computation. Such calculation would require a computer run similar in principle to the GDOP program.

The first three terms of Equation 6 represent the contribution to the range rate error due to errors in the knowledge of the vehicle velocity. The quantities Δx , Δy , Δz , may be considered to be the direction components of the vehicle velocity error at any given instant of time. This error vector is the vector between the measured vehicle velocity and the true vehicle velocity. Thus, these quantities may be written as $\cos \alpha \Delta V$, $\cos \beta \Delta V$, and $\cos \gamma \Delta V$, where $\cos \alpha$, $\cos \beta$, and $\cos \gamma$ are the direction cosines of the velocity error vector of magnitude ΔV . Similarly, $(x - x_t)/r_t$, $(y - y_t)/r_t$, and $(z - z_t)/r_t$ may be written as $\cos \alpha'$, $\cos \beta'$, and $\cos \gamma'$, respectively, where these are the direction cosines of the vector from the vehicle to the transponder. The first three terms may now be written as:

$$(\cos \alpha \cos \alpha' + \cos \beta \cos \beta' + \cos \gamma \cos \gamma') \Delta V$$

If θ is the angle between the two vectors, it can be shown that

$\cos \theta = \cos \alpha \cos \alpha' + \cos \beta \cos \beta' + \cos \gamma \cos \gamma'$. The first three terms may therefore be written as $\cos \theta V$.

The fourth through sixth terms of Equation 6 may be considered as follows. The quantities $(\Delta x - \Delta x_t)$, $(\Delta y - \Delta y_t)$, and $(\Delta z - \Delta z_t)$ may be considered as direction components of the vector which is the difference between the position error vector of the vehicle and the position error vector of the transponder. These terms may be written as $\cos \xi \Delta P$, $(\cos \rho) \Delta P$, and $(\cos \mu) \Delta P$, where ΔP is the magnitude of the net position error vector and $\cos \xi$, $\cos \rho$, $\cos \mu$ are the direction cosines of that vector. Similarly, the quantities $(\dot{x} - \dot{x}_t)$, $(\dot{y} - \dot{y}_t)$ and $(\dot{z} - \dot{z}_t)$ may be written as $(\cos \xi') V$, $(\cos \rho') V$, $(\cos \mu') V$, where V is the combined velocity vector of the vehicle and transponder. Since the velocity of the vehicle is very much greater than the velocity of the transponder, V will be approximately equal to the vehicle velocity. Thus, three terms may be written as:

$$\frac{(\cos \xi \cos \xi' + \cos \rho \cos \rho' + \cos \mu \cos \mu') (\Delta P) V}{r_t}$$

If η is the angle between the net position error vector and the velocity vector, then these three terms may be written as: $\frac{\cos \eta (\Delta P) V}{r_t}$

Thus, Equation 6 may be written as

$$\Delta \dot{r}_t = \cos \theta \Delta V + \frac{\cos \eta (\Delta P) V}{r_t} - \frac{\dot{r}_t \Delta r_t}{r_t} \quad (7)$$

Since it is desired to estimate the maximum probable values for $\Delta \dot{r}_t$, values tending to maximize the function will be assumed for the orbital parameters. For a 90-mile circular orbit:

$$\dot{r}_t \text{ max} = 7.54 \cdot 10^3 \text{ meters/sec}$$

$$r_t \text{ min} = 1.67 \cdot 10^5 \text{ meters}$$

$$V \text{ max} = 7.8 \cdot 10^3 \text{ meters/sec}$$

$$\cos \theta \text{ max} = 1$$

$$\cos \eta \text{ max} = 1$$

$$\text{Equation 7 can now be written as } \Delta \dot{r}_t \leq \Delta V + 4.66 \cdot 10^{-2} \Delta P - 4.51 \cdot 10^{-2} \Delta r_t \quad (8)$$

The quantity ΔV is the magnitude of the vehicle velocity error. The probability that such an error is less than the maximum axis of the velocity error ellipsoid, as computed in the GDOP computations, is greater than 0.62. The magnitude of the maximum axis will therefore be substituted for ΔV in Equation 8.

For oceanborne transponder systems, the GDOP analyses showed that this maximum axis is approximately equal to $0.02 \sigma_\ell$, where σ_ℓ is the station coordinate error in meters, for $\sigma_\ell \geq 30$ meters. Similarly, the quantity ΔP is the vector difference of the vehicle and station position errors. The maximum axis of the vehicle position error ellipsoid was shown in the First Quarterly Report to be approximately $2 \sigma_\ell$. A conservative approximation for ΔP will then be $3 \sigma_\ell$.

The maximum value for Δr_t will be ΔP , and occurs when the position error lies in the direction of the axis between the vehicle and the transponder.

Substituting the above values into Equation 8, gives $\Delta \dot{r}_t < 0.3 \sigma_\ell$.

If, for example σ_ℓ equal to 100 meters is assumed, then $\Delta \dot{r}_t < 33$ meters / sec. This would result in a doppler frequency error of approximately 400 cycles. From the AROD Design Feasibility Report, Volume 1, page 129, assuming the use of a Type I, 2nd order loop, the formula for estimating acquisition time is

$$T_{P1} = \frac{\tilde{A}^2}{2\xi\omega_n^3}$$

In this case, $A = 2 \Pi 400$

For a 90-mile circular orbit, the following values were assumed:

$$\xi = 0.5 \quad \sqrt{10} \approx 1.57$$

$$W_n = 200 \quad \sqrt{10} \approx 633 \text{ rad/sec}$$

For these parameters, a pull-in time of 16 ms would result. This would be a decrease of almost three orders of magnitude below the worst case 10 second pull-in time if no presetting of filters is used.

For a ground based transponder system, the range rate error and doppler frequency error will be much smaller. For such system, the following values may be assumed:

$$\Delta V = 0.3 \text{ meters/second}$$

$$\Delta P = \text{vehicle position error and transponder coordinate error}$$

$$\Delta P = 10 \text{ meters}$$

$$\Delta \dot{r}_t = 10 \text{ meters}$$

Substituting these values into Equation 8 yields a value for $\Delta \dot{r}_t$ of 0.97 meters/sec. Substituting this value into Equation 1 would indicate a doppler frequency error of approximately 10 cycles/sec. This error would be negligible, since it will be impractical to set the Φ LL tracking filter VFO's to this precision.

It is concluded, therefore, that presetting of the tracking filters to the anticipated frequency will reduce maximum pull-in time by at least

an order of magnitude, to a fraction of a second. This will enable essentially instantaneous changeover from one transponder station to another when desired.

1.3.4 Error Detection/Control Functions

It is possible that highly inaccurate position or velocity parameters might be indicated by the AROD computations. Such inaccuracies could result from malfunctions in the ground-based transponders, from malfunction in equipment in the vehicle, or from transient conditions such as temporary loss of lock by any of the system tracking filters. The AROD command program in the Saturn V Guidance Computer may be used to detect such errors, and to take appropriate corrective action.

1.3.4.1 Error Indications

There are two sources of error indications: equipment alarms and reasonableness checks. The equipment alarm may take the form of a deliberately generated, invalid signal in one of the AROD accumulator registers, such as all zeros or all ones for the values of range and range-rate for the channel with the trouble indication. Such a signal would be generated when a malfunction or operational deficiency is detected. These could occur due to inadequate signal-to-noise ratio from the transponder, loss of tracking on either the carrier or one or more of the transponder tones, or detection of vehicle equipment malfunctions. If a transponder station loses tracking filter lockon to the vehicle's signal, or if any other equipment malfunction occurs in the transponder station, this may be indicated by ceasing transmission. This will result in an equipment alarm in the airborne AROD equipment, which will be unable to continue tracking the transponder signals.

The computational capability of the guidance computer will enable detection of many other error conditions by means of reasonableness checks. These checks may take any of the following forms:

1. Comparison of the range and range-rate to each transponder station with the previous measurement. The new values must be within a given range of the previous values, or it may be assumed that an error of some type has occurred. The range within which the new values must be may readily be determined from the maximum velocity and acceleration of the vehicle. This comparison will indicate any errors due to a sudden malfunction.

2. Comparison of the most recently calculated values for vehicle position and velocity with previously computed values. This comparison will be particularly significant when there is a change in the triad of ground stations used in the position and velocity calculations.

3. Comparison of the measured and calculated range and range-rate to the fourth transponder, using as input the calculated present position and velocity of the vehicle. This check may be made only when four or more transponders lie within line-of-sight of the vehicle. If the measured and calculated values do not lie within a preselected, reasonable range (determined by the expected maximum AROD system errors), it may be assumed that there is an error either in the readings or calculations upon which the position and velocity of the vehicle are based, or in the calculations or measurements upon which the range and range-rate to the fourth transponder are based.

4. Comparison of the altitude indicated by the Saturn V pulsed radar altimeter with an altitude computed from the present calculated position.

The calculations required to perform the above checks involve straightforward manipulation of the basic AROD navigational equations. The computation required for their performance will be negligible.

1.3.4.2 Error Control Functions

When an error indication is noted, there are three actions which may be required of the AROD command program.

First, the position and velocity calculations associated with error indications should be tagged as questionable. This will enable consideration of possible inaccuracies before any action is taken, based on the questionable data.

Second, an effort should be made to bypass the trouble indicated, while maintaining full AROD capability. This may be attempted by switching vehicle channel/transponder while noting and comparing the consecutive vehicle position and velocity calculations and the range and range-rate to each transponder. As a result of such switching, it may be found that the malfunction is located in:

- a) One complete transponder
- b) One translation frequency of a transponder
- c) One vehicle channel

Appropriate action may be taken to preclude use of the malfunctioning equipment for future calculations. If three functioning channels and three functioning transponders are located by the switching process, they may be used for future calculation.

As a result of a transponder malfunction when only three transponders are within line-of-sight, or if two or more AROD channels are malfunctioning, full AROD capability may not be maintained. In this event, it may be possible to utilize a truncated AROD system. Range and range-rate measurements taken at different times from the same transponder may be substituted for simultaneous measurements to three transponder stations. The interpolation and synchronization errors associated with such a mode of operation will be significant, and a corresponding increase in AROD system error may be expected.

1.4 Summary of Saturn V Guidance Computer Time and Storage Requirements

The major objective of the work reported on in Section 1 is the determination of the computer requirements for the basic and auxiliary computations for AROD. Summarized here is the result of that work. The solution times and storage requirements, as given in Sections 1.1 and 1.2, are restated in the following table.

<u>Solution Time</u>		<u>Storage</u>
Basic	0.11 sec	570
Auxiliary	<u>0.02 sec</u>	<u>320</u>
Total	0.13 sec	890

The auxiliary time and storage estimates assume the modified table lookup method (see Section 1.2.3) which minimizes solution time with an increase in storage requirements. The totals given above do not include the command and control functions discussed in Section 1.3. When estimates for these functions are included, the total time and storage requirements for all AROD related computations are $0.15 \pm .05$ seconds* and 1500 ± 750 storage locations.* The lower bound on these computer estimates (i.e., 0.10 sec and 750 locations) is less than the totals given above because no attempt was made to optimize the Saturn V Guidance Computer programs that were written for estimation purposes. It should be stated that the main contribution to the tolerances in the estimates (i.e., $\pm .05$ sec and ± 750 loc) is the wide variation in the requirements for the various missions to which AROD might be applied.

* Based on IBM's past experience, the limits on the computer time and storage requirements are 90% confidence levels

Section 2

AROD SYSTEM INTERFACE INVESTIGATIONS

This section considers both functional and physical problems associated with AROD interfaces with the vehicle guidance system. The overall concept of AROD is that of an essentially self-contained Space Radio Navigation system. This may be broken down into two major functions: (1) sensing of range and range rate, relative to a complex of ground stations of known locations, and (2) computation to determine vehicle position and velocity from these measurements and known locations. The sensing function is accomplished by a cooperative radar sensor for measurements of doppler frequency shifts (f_d) and time delays (τ_D) of RF energy transmitted between the vehicle and transponding ground stations. It is appropriate, therefore, to term the AROD sensing function as well as the physical equipment as the AROD Sensor Subsystem.

The second function, the AROD Computation Subsystem, represents the time-shared usage of the vehicle's guidance system computer. For purposes of this study this was taken to be the digital computing system provided by the Saturn V Guidance Computer and its associated Data Adapter, the latest specified status for which is reported in IBM Tech. Doc. No. 63-928-130, "Apollo Study Report—Vol. II." In the absence of an on-board guidance computer (e.g., during test flights and installation in other vehicles where the self-contained feature is not required), the AROD equipment (i.e., the Sensor Subsystem), can also be employed. In such applications unprocessed tracking information (see Section 1.2) is telemetered to a ground-based computation system for determination of navigational parameters (vehicle position and velocity). For the present study we have considered the interface of the operational AROD system in Saturn V as the principal requirement.

Section 2.1 will consider the AROD mission and the resultant impact on system interfaces. The characteristics of the Saturn V Data Adapter, which are pertinent to the AROD system data processing requirements for computation, will be discussed in Section 2.2. In Section 2.3 the nature of the data extraction operations performed as the final step in the AROD Sensor Subsystem measurement chain will be discussed from a point of view of influences on the interface. Certain problems of signal characterization and timing/synchronization made pertinent by the Saturn V vehicle environment and the present specification status of the Saturn V Guidance Computer and Data Adapter will be discussed in Section 2.4. Section 2.5 will present recommendations for interfacing configurations reflecting these considerations.

2.1 Projected Mission of AROD Equipment

The definition of various interfaces between the AROD Sensor equipment and other equipment elements of the Saturn V launch vehicle must be preceded by either statement (or assumption) of the basic operational use of the AROD system, as well as specific flight purposes within the context of that usage. In addition, the subtle differences sometimes manifested between functional interfaces and physical interfaces must be recognized and understood.

Projecting the role of the AROD system as a vehicle position and velocity (i.e., navigation) sensor to back up the inertial platform (IMU) within the Saturn V Vehicle Guidance System permits some definition of requirements. Central to this is the use of the Saturn V Guidance Computer (SVGC) and Data Adapter (SVDA) for performing the AROD Computation Subsystem requirements as discussed in Section 1. These computer requirements include performance of the basic AROD computations, auxiliary computations, and computation for various command and control functions associated with the AROD Sensor Subsystem operation. A further aspect of the use of the computer is that a separately defined interface between AROD Sensor outputs and the Saturn V vehicle telemetry system

is not required. This is due to the basic Data Adapter design philosophy as discussed further in Section 2.2. It should be pointed out, however, that housekeeping analog telemetry from the AROD equipment will be interfaced into the T/M system separately. Since the AROD command and control links (up and down) are to be implemented within the AROD Sensor equipment (separate VHF transmitter for command downlink and utilization of sensor tracking uplink for limited communications associated with command and control of ground transponder network), and the bulk of the decision logic for command and control are computer generated, a specifically organized physical interface can be said not to exist as such. Note, however, that a functional interface does indeed exist in terms of the programmed time-sharing of the computer and usage of certain physical elements between the computer (actually Data Adapter) and the AROD equipment.

The foregoing considers what might be called the operational mode with an AROD system integrated into the vehicle guidance system as an additional sensor subsystem. In so doing, it can be seen that the system interfaces to be considered on a physical basis, i.e., from the implementation point of view, condense to an AROD/Computer Interface—or more specifically, an AROD/SVDA Interface. The main feature of this interface as far as implementation is concerned, is that simultaneous interfacing of AROD Sensor output to both computer and telemetry is not required (except as noted for housekeeping type of analog telemetry).

Another mode with which the same AROD equipment configuration should be compatible is the flight test mode. In this mode a computer may or may not be available, but in any case the AROD equipment is not in an operational status and should not depend upon a vehicle computer. For flight tests the telemetry interface becomes dominant. However, provisions must be made for performing the transponder network command and control functions by either additional self-contained on-board equipment elements (a pre-programmed Vehicle Command Logic Unit), or uplinking via the Saturn Command Link.

The key feature brought out by the preceding discussion is the essential non-simultaneity of system interface requirements between the flight test of a spaceflight-qualified prototype AROD Sensor equipment and the operational use in the vehicle guidance and navigation system. Since equipment design is addressed to the functional and physical requirements of the intended operational use and not the flight test mode, any additional functional requirements must be implemented externally. This suggests that the interface between the SVDA and AROD Sensor equipment be optimized for primarily operational use, in the context of minimum size and weight and maximum-reliability, with the achievement of flight test objectives a secondary consideration. Special hardware required for flight test operations should be external to the operational equipment. In this sense, the interface design is more than mere agreement on the "number of wires and voltage levels" associated with the physical interconnection, but rather a detailed optimization of the functional equipment elements on adjacent sides of the interface.

To develop the above point further, consideration is directed to the vehicle receiving end of the AROD system tracking link as depicted in Figure 2-1. The elements which are associated with the interfacing of sensor output data are identified as the Data Extraction Processing function on the AROD Sensor side and the Data Adapter on the Computation Subsystem side. Both functional elements will be subsequently discussed in detail, particularly the Data Extraction Processing function, since the Data Adapter is a defined piece of hardware and must be accommodated in all respects.

The function of the Correlation Receiver (or Vehicle Tracking Receiver) is to process the frequency-multiplexed 2GC transponder return signals to deliver the ranging sidetones and some suitable representation of the doppler shift in frequency experienced by the transmitted carrier for each of four channels. These channels are defined by the frequency translations involved at each of the four transponding ground stations. This function is essentially achieved by successive operations by frequency demultiplexing filters,

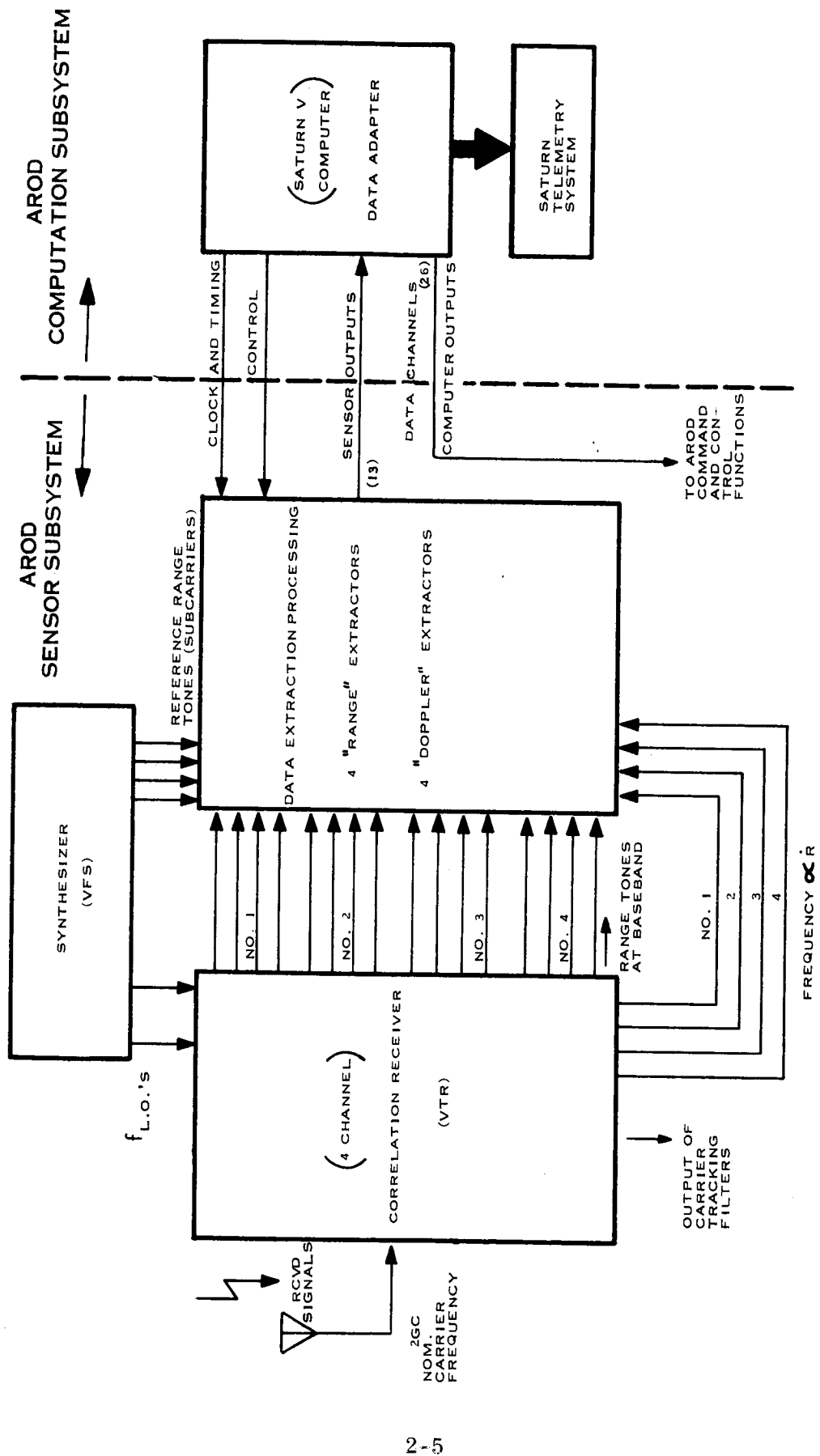


Figure 2-1. AROD System at Vehicle Receiving End of Tracking Link

coherent and synchronous demodulators, and phase-lock loop tracking filters and demodulators. The demodulated range tones, now at baseband, are essentially subcarriers, phase-modulated by the range information sought after; thus, they require further synchronous demodulation with reference tones. This is the primary function of the range extractors, one for each channel. A similar statement can be made with respect to the doppler shift in frequency (a frequency modulation) experienced by the RF carrier * due to the range rate parameter. This information is extracted in the "doppler" extractors—again one for each of the four carriers defined by the four transponding ground stations. As noted in Section 1, the "range" and "doppler" quantities extracted (i.e., demodulated from their respective sensing subcarriers) bear relationships to actual range and range rate (the "tracking" quantities being measured) which is not simply proportional.

The eight extractors, or extraction units (four for "range" and four for "doppler"), are required to provide output in digital form. This follows not only because of the necessity to supply data to a digital computation subsystem, but largely from considerations of precision and accuracy required for the AROD Sensor output range and range-rate quantities. Since these parameters are measured by the AROD Sensor Subsystem in terms of phase shifts due to time delays (phase delay) and rate of change of phase shift (doppler shift in frequency), they exist essentially as time analogs rather than voltage analogs which are inherently amenable to A-D conversion to greater levels of resolution and accuracy. This type of A-D conversion is generally termed "time encoding".

Since the desired output data must be in some form of digital representation, what remains to be settled is how to operate in conjunction with the Data Adapter in the most compatible manner. Important areas in assessing this compatibility, which is really the essence of the interface problem, pertain to buffering or storage requirements and total number

* Also experienced by the range tone modulation components.

of wiring interconnections. An overall interface configuration, representing an optimum design requiring no additional storage implementation and a minimum number of interconnection circuits, can best be achieved through judicious consideration of the characteristics of the "communicating" functions on both sides. This is the underlying consideration in what follows.

2.2 Data Adapter Characteristics

Only those aspects of the Saturn V Data Adapter and its relation to the Guidance Computer, which influences the interface configuration under consideration, will be discussed. Greater detail on internal organization and tabulation of characteristics may be found in the previously referenced document, Apollo Study Report—Vol. II.

The DA is the focal point of the Saturn V Guidance System and may be thought of as a "signal processor" for all sensor subsystems and their relation to the computer (i.e., the central processor). The same relationship exists for the control subsystems. The DA has a single, highly integrated interface with the Guidance Computer which serves to isolate the computer from changes in input-output configuration occasioned by differing mission requirements or different sensor and control configurations. Consequently, the DA design and implementation may be characterized as flexible. This permits substantial guidance system changes, such as addition of an AROD system, to be easily accommodated as minor hardware alteration and additions to the DA (substantial growth capability is an important design objective) and reprogramming of the computing system central processor.

An important aspect of the DA design and implementation is its role as the sole channel for guidance system output of all types to the Saturn PCM Telemetry system. This role is so extensive as to include telemetry of all data input to the guidance computer from the DA (and thus from the various system sensors) as well as all control output of the computer. This function represents a substantial part of the DA organization, but serves to remove the need for separate telemetry interfaces associated with sensors throughout the guidance system. Because of the asynchronous nature of

the DA/Telemetry interface, a large number of buffers are employed in both the DA inputting and outputting facilities.

The input from the DA to the SVGC is accomplished over a single channel as a serial word in two 13-bit "bytes." Actually, an additional parity bit is inserted making the byte 14 bits long. The serial 26 bits of data comprising an input word are generated in response to a PIO (Process-Input-Output) Instruction in the computer program. The address part of this instruction designates the source for one data word up to 26 bits in length. The decoding of this address (9 bits) is accomplished in the DA and selects the desired source. In the case of parallel input to the DA from external equipment, a time multiplexed gating structure associated with the Digital Interface Multiplexer (DIM) function is controlled by the decoded PIO address to deposit one-half of the input word in a 13-bit latch (flip-flop) register, also associated with the DIM. This register is then interrogated by a group of gates actuated by the sequenced output of a 13 position (14) timing distributor, all of which constitutes the Serial-izer function. The net result of the DIM—Serializer action is to generate a serial word precisely synchronized to the computer clock in frequency and phase (discussed in Section 2.4) which is acceptable to the internal organization of the computer. It should be noted that the SVGC is a serial machine with all registers (except those directly associated with memory and instruction control) implemented by glass acoustic delay lines. Buffer storage, in addition to that provided by the DIM latch register, is provided by a group of additional delay line registers located in the DA. The timing of the inputting of this 26-bit word, whether from the Serializer or Delay Line Storage, is such that approximately 82 usec is required. (Details of timing will be discussed in Section 2.4).

Outputting from the computer is essentially the reverse order of the above, except that a staticizer, rather than a Serializer, is provided in the DA. The staticizer is termed the Buffer Register and makes a serial-to-parallel conversion of two successive 13-bit bytes from the computer to a 26-bit word in temporary storage in the register latches. It is from

the Buffer Register that the greatest part of telemetry output interfacing takes place, as well as do the outputs which are required for AROD command and control. The telemetry of input to the computer is taken directly from the DA Output Bus (1 channel), which feeds the serial words to the accumulator register in the computer, by means of the Data Output Monitor (DOM), and is supplied to the Buffer Register for subsequent multiplexing in the TM frame.

Another provision of the DA is an internal Real Time Clock accumulator register which is mechanized into the delay line Buffer Storage previously mentioned. This real time accumulation runs to two seconds before recycling and all guidance system "times" are available by programmed computer requests. To avoid synchronization problems between two independent clocks it is advisable to use only the DA Real Time Accumulator Register for time labeling sensor inputs such as AROD. It should be pointed out at this time that the referenced document indicates that the computer clock oscillator long and short term stability performance will be such that compatibility is also assured for the Apollo navigation application. This is of the order of 1 part in 10^6 or better.

The DA as mechanized includes substantial provisions for "interrupt" inputting to the SVGC. This is contrasted to the "request" inputting capability represented by the action of the PIO Instruction previously mentioned. Interrupt operation should be reserved for high priority input data elements signifying limit or emergency conditions. In this sense, AROD, acting as a "continuous" sensor of vehicle range and range rate, does not qualify as requiring interrupt inputting. As far as the request input action is concerned, a separate PIO Instruction with each input quantity desired will require a separate address code. The implied addressing capability of the nine bits would be 512 separate addresses. However, since input and output must be coded into this address code, along with other DA/GC control actions, the addressing capability is reduced to the order of 32. The AROD Sensor output, if separately addressed, will require between eight to ten 26-bit words. In the future, should insufficient unused address

codes remain, it will be possible to treat the AROD input to the computing system as a frame designated by only one address. This address will initiate a counter in the AROD equipment to step through each data element, one word at a time, in a fixed format. This format would correspond to pre-arranged address sets within the computer memory.

A final note might be made with respect to existing "interface buffering" circuits (i.e., interconnection circuits with storage not necessarily implied) on the DA side of the interface. Three such circuits are presently defined; one is an adaptation of the type of transformer-coupled interface coupling circuits specified for the Apollo computer system. The other two are conductively coupled, but exhibit different characteristics with respect to relationships between signal grounds on both the source and receiver sides of the interface. Section 2.4 will consider these questions with respect to signal characterization.

2.3 AROD Data Extraction Techniques

As pointed out in paragraph 2.1 the achievement of an optimum interface configuration is dependent on the interrelationship of the communicating elements on either side of the interface. Since the DA configuration is fixed for all purposes from the AROD system point of view and can only be altered within the bounds of design philosophy discussed in paragraph 2.2, the burden of achieving compatibility will rest on the AROD equipment implementation. The techniques employed for data extraction processing within the AROD Sensor Subsystem, as well as the inherent nature of the AROD measurement process, impact the interface design in a fundamental manner.

By virtue of the CW (modulated for ranging) transmission spectra on both up and down tracking links it follows that continuous readings of the phase shift time delays (τ_D) and doppler shifts in frequency (f_d) can be made. This is true regardless of whether the data representation is analog or digital. Therefore, the phase delays on the composite range tone structure, as well as the rate of change of this phase delay (the doppler

shift on the range tones), are continuous since they are measures of the motion and position sought after (which are also continuous in nature). This is in contrast to a pulse system which is inherently sampled, not continuous. Continuously varying quantities may be quantized and changed in increments, but this does not necessarily imply that their measurement cannot be absolute.

To clarify these concepts an analogy between the CW Sidetone Ranging principle basic to the AROD system and shaft-position analog-digital converters will be drawn. The conventional binary progression-type of coding, commonly encountered in shaft-position encoding devices, is an example of multiple scale (or "multiple-speed") measurement techniques (such as encountered in multi-speed synchro systems for precision transmission of angular data): it represents a limit case in recognizing or assigning a value to a particular shaft or encoder tracks (i.e., a binary choice). It can be visualized that a like binary progression of ranging sidetones on some suitable RF carrier can be described as a "shaft-position-encoder-rolled-out-in-sky" along which the range measurement is unambiguously made.*

Neglecting for a moment the complexity of implementing such a scheme, it can be seen that the value associated with a fiducial line at a point in space (or time), alongside the binary ranging tone waves in space, is developed by making a binary choice at each track (or tone wave). For any frequency tone, only a decision of phase $0-\pi$ for "1" or $\pi-2\pi$ for a "0", or, vice versa, as per prior established convention. Such a measurement, along with the associated encoding process (A-D conversion), can be characterized as absolute, or, whole number, and as such is immune to propagation of errors made on one reading to any other reading.

In contrast, a system configuration which counted a finely divided reference (i.e., so-called fine-ranging-tone), or the least significant bit track of a shaft position encoder, and stored this count until such time as another

*Note that the sidetones represent subcarriers in that they are phase-modulated by the geometry—path length—under measurement.

increment, in either direction, is registered, would be classed as an incremental measurement technique (or encoding, since the two processes are indistinguishable in this case). Such methods are simple to implement but suffer the serious faults of being prone to propagation of errors for indefinite periods, or, until such time as an index of known value (calibration) is crossed.

Now, if an essentially continuous distance measuring system such as AROD is so configured as to sample the result of the measurements being carried out continuously and encode these "time analog" samples by some time encoding method such as EPUT (Events-Per-Unit-Time) or TIM (Time Interval Measurement), the encoding process becomes absolute in nature. But, and this is important from a data utilization point of view, the outputs are now inherently discontinuous. For the usual case of asynchronous operation of the interfacing elements, some artifice such as buffer storage, which is expensive in hardware for AROD because of the indicated need for at least eight registers, each about 24 bits long, is needed. This approach is unnatural at best and should be avoided if the equipment penalties introduced by a continuous read out device compare favorably against the buffer storage. It should be noted that the AROD system configuration being implemented does not use a binary progression of ranging tones, but rather something of the order of a $2^5:1$ progression. Even though this requires a phase measurement resolution of at least 1 part in 32 on each ranging sidetone in order to resolve ambiguities (rather than a simple binary choice), it can be seen that the discontinuous nature of time encoding still permits the measurement to be absolute in the sense of no possibility of propagating errors.

It is possible to avoid the artificialities in this approach by applying some of the ideas presented in Chapter 2 of "Electronic Time Measurements" by Chance, et. al. (Vol. 20, MIT Rad. Lab. Series). Here, the basic concept involves the use of phase-tracking demodulators to extract the geometry-imposed modulation on the received subcarriers using the transmitted subcarrier as a reference. In the AROD context these subcarriers are the

ranging tones. It can be seen that this involves the basic ideas of phase-lock feedback control systems. The concept is made applicable to AROD, with its stringent accuracy requirements calling for direct transducing to digital form rather than through intermediate voltage analog representations, by applying so-called operational-digital techniques and novel methods of generating quantized, digitally controlled phase shifts (phase modulators). Operational-digital methods are essentially analog methods of processing signals, except that representation is digital, in which the classic time-sharing ideas inherent in digital (i.e., programmed digital) techniques are absent. This means that sampling is no longer required per se, which is the very factor tending to distort the basic nature of the AROD system.

The manner in which these techniques are applied to the ranging sidetone extraction problem is shown in Figure 2-2, which gives the essential features of the arrangement for a multiple-scale phase demodulation process. The feedback control nature is denoted by the R, E, and C symbols. The multiplier function phase-compares R and an analog representation of C, which is obtained through the action of the digital-to-phase converter which acts as a precision phase modulator, to generate a "DC" error signal E. The control unit quantizes this error, with sign, and generates increment pulses to the UP-DOWN (bi-directional) counter stages associated with a particular scale. Note that this constitutes an incremental measurement (and encoding) technique until such time as 32 increments are counted up in any one direction, on any one scale (i.e., a ranging sidetone) at which time an "index" is in effect crossed; any errors accumulated during this incrementing interval are corrected. At this point it is also necessary to interconnect in some suitable manner adjacent "scales" with a signal to resolve ambiguity which is present when two tones cross zero at the same time. A similar problem exists in shaft-position encoders and is solved by equivalent approaches such as "U" or "V" scan, or, by the somewhat different method of using a unit-distance code. The successively ordered stages of the UP-DOWN counter as depicted in Figure 2-2 together form the "range" or, more precisely, the unambiguous phase delay (due to

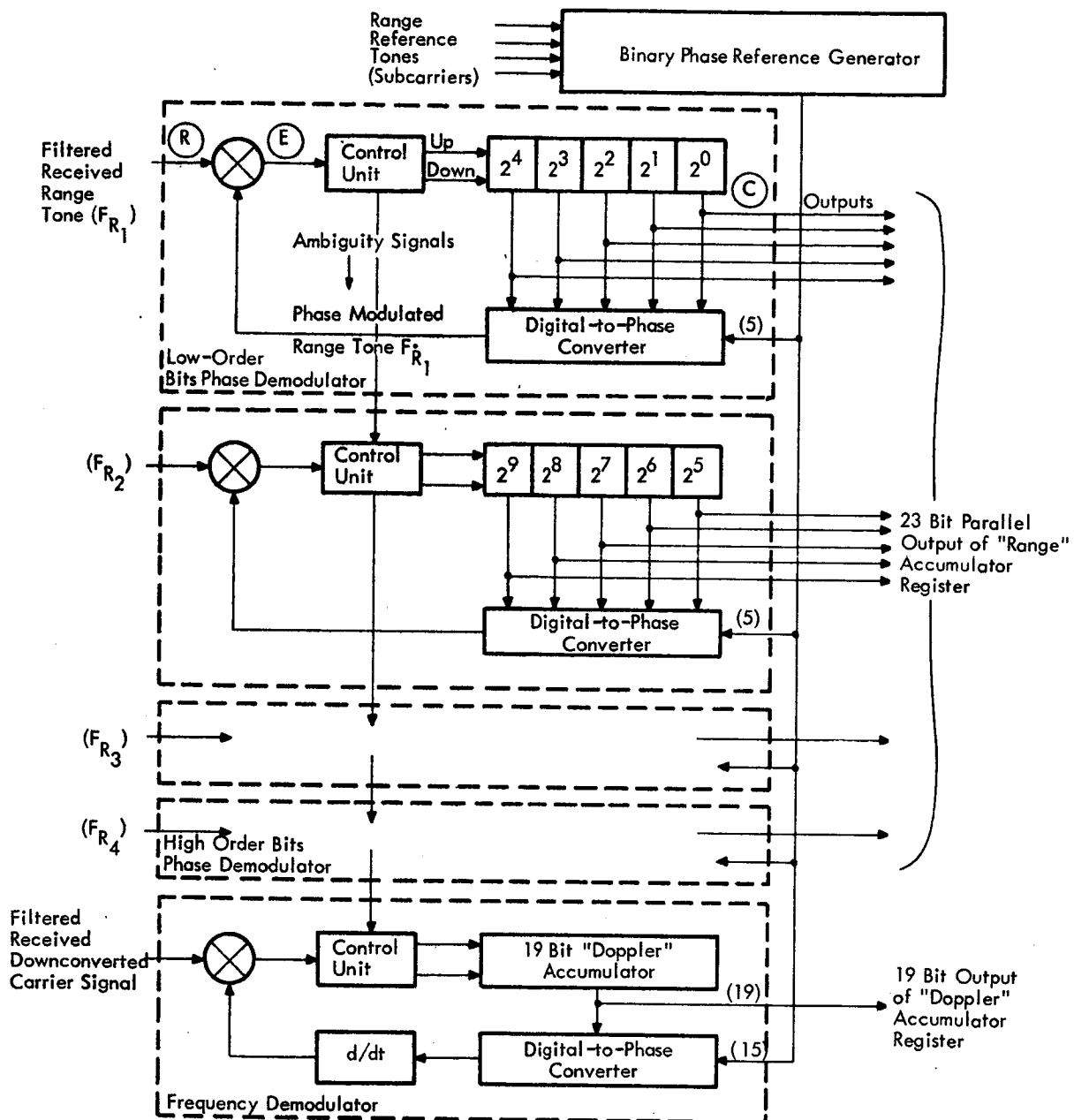


Figure 2-2. Multiple Scale Phase/Single Scale Frequency Demodulation "Servo" Type Data Extraction Processing

2-way transmission time delay τ_D). These constitute the accumulator register of 23-bit capacity required by the AROD requirements on maximum unambiguous range and range quantization.

Except for the time required to propagate an increment in the "range" accumulator register UP-DOWN counters, it can be seen that an outstanding feature of this approach to synchronous demodulation of the range tone sub-carriers is the continuous readout feature. This has been likened to the property of an automobile odometer in that a reading is essentially always available. When range is changing (i.e., range rate, or velocity for the odometer example, is not zero), then an ambiguous reading will exist very briefly. For the "range" accumulator register this eventuality can be avoided by a short anticipating inhibit period applied just prior to taking a digital reading. At first glance it might seem that, in effect, this method also has the serious defect previously cited for the absolute time encoding technique—the need for some form of buffer storage to ensure a valid reading at the time of reading (but slightly stale however). Further thought will reveal that the decisive factor is not that a true reading is not always obtainable, but rather the time period for which such is the case. Since an increment is accumulated directly as the range is changed in the operational-digital technique of range extraction, inhibition of an up or down increment will only provide a stale reading to the extent of one, or at most, several quanta for large range rates.

Considering the advantages in employing this technique for "range" extraction processing, an extension to the "doppler" extraction processing would permit the establishment of a completely non-buffered interface with the DA, with only slight difficulty in inhibiting the various multiple scale loops a short time before accumulator register readout. Figure 2-2 contains a suggested configuration for achieving a single-scale frequency demodulation, showing its relationship to that presently projected for AROD "range" extraction technique. The essential difference is the addition of a differentiation to the analog (quantized) output of the digital-to-phase converter so that frequency will be the tracked quantity. Note that the combination of the

digital-to-phase converter operating on suitable reference signals derived from the Binary Phase Reference Generator function and the differentiator constitute a frequency modulator. It is recommended that further investigation be carried out to establish the feasibility of such an implementation of "doppler" extraction for the potential advantages to be derived in generating a structurally and functionally simple interface with the Saturn V Data Adapter.

2.4 Interface Signal Timing Problems

In the implementation of an interface between separated equipment entities involving the transfer of information in digital form, the most difficult problem encountered is the corruption of data reliability by interference commonly termed "noise." This interference is not random in origin, but in systems of sufficiently large complexity, such as the Saturn V Instrument Unit, the interference takes on an apparently random character when observed at any one point. The complex combination of contributions from many sources, by means of at least three basic forms of coupling, makes any meaningful analysis and prediction very difficult, if possible at all. The usual approach is to establish a consistent set of philosophies and practices and exercise firm specification control at all levels of equipment and system interfacing.

The three basic forms of coupling alluded to may be described as due to induction effects (magnetic and/or electric field coupling), radiation (i.e., due to radiated EM waves), and "ground loops." Of the three basic forms the action of "ground loop" coupling is the most elusive, since in addition to whatever coupling of interfering signals it directly provides, it is also a path for the other two. The effects of inductive and radiative coupling can be controlled by suitable selection of power and impedance levels of the various interface circuits. In so doing, it is well to avoid wide disparity in these characteristics for the various circuits interconnecting the various equipments. As a general rule, low-impedance circuits tend to be less prone to "crosstalk" effects. Radiative coupling

is not subject to any such simple rule because of the overwhelming effect of frequency in such coupling. At this point it might be well to point out that what is often blamed on "radiation" as "pick-up" usually turns out on more detailed examination to be due to induction effects.

The problem of "ground loops," i.e., circuit paths not shown explicitly in the "abstract world" of schematic or wiring diagrams but nonetheless existing in the "real world," is essentially the introduction of interfering signal voltage sources in a complete circuit between the transmitting element (source) and receiving element (sink). The most common technique of minimizing the possibility of these spurious voltage sources from being included in a circuit loop is by establishment of a hierarchy of grounds such as: power ground, chassis ground, and "DC" and/or signal ground.

The philosophy usually established is to avoid any direct interconnection of these grounds within an equipment and to control carefully the interconnection outside the equipment. The usual difficulty that ensues affects the DC/Signal ground interconnection question. For digital type signals on the interconnection this resolves to a question of levels or pulses. Level type signals must be DC coupled (unless some rather unpalatable artifices of what amounts to building a radio link are employed) on a signal ground basis, and so provide an opportunity for "noise" injection via the "ground loop" route. This is usually addressed by raising signal power levels and/or usually the signal voltage level. Note that if the voltage level of a signal is raised, with the power unchanged, the circuit impedance must also have been raised. This may serve to increase susceptibility to various forms of inductive origin interference. If this is countered by a decrease in impedance level, then the signal power level increases, and the interference causing potential with respect to other circuits is increased.

For the Data Adapter interface problem, three types of interconnecting circuit capability have been provided at present. The Discrete Input (sometimes referred to as a Buffer Interface-IB) circuit is predicated on handling level signals in which the nominal levels are established as 0 volts and 28 volts with very substantial margins either way, such that up to 8 volts of "noise" will not result in false action. The use of this circuit in Saturn V

is based upon no direct unit-to-unit interconnection of DC/Signal grounds, such interconnection being accomplished at only one point in the entire Instrument Unit. Although this technique certainly avoids establishing a ground loop among a group of interfacing equipment, it makes for a very "noisy" signal structure on the interconnections. However, as long as it can be established in practice that this will not exceed some threshold, say the 8 volt margins previously referred to, then all should be well.

Another way of approaching this particular problem is to use a signal structure which does not require continuity between the DC/Signal grounds of the interconnecting equipment. This is simply accomplished by the use of transformer coupling. To keep the size of the coupling transformers down, the circuit impedance levels must be kept low and the signal duration short, say, less than 10 microseconds. This, in essence, establishes a pulse type interface. However, the effect of a wide pulse signal can be made to appear as a level signal by suitable strobe gating in the "receiving" equipment. A circuit embodying these ideas has recently been made available for the Saturn V Data Adapter in connection with the Apollo backup program, since transformer coupled interfaces are specified for all digital quantities in Apollo. For a late-joining equipment, such as AROD might be in the Saturn V Instrument Unit, a transformer coupled interface could help avoid the problems of being a victim, or, being the cause of some last minute interference problems, especially with respect to RFI.

To appreciate the potential timing/synchronization problem area, a few points concerning the GC/DA interface are worth considering. First, the DA timing is exactly as in the central processor. That is to say that three phases of approximately 27.3 microseconds per phase constitute an instruction cycle. The first period, ϕ_A , is for instruction decoding, whereas the next two, ϕ_B and ϕ_C , are associated with serial transfer of the two 13-bit bytes making up one data word. Each bit time is 1.95 microseconds (essentially a 512KC machine), with further subdivision into four periods of 0.49 microseconds each. These periods are the X, Y, Z, W clocks associated with the four-channel time division multiplexing of the glass delay line

registers in both GC and DA. An input to the GC from the DA is delivered at one of the above clock periods, at a rate of 512KC (bit rate), in serial form for input to the accumulator register. This timing must be accurate in order to avoid excessive pulse-splitting in the retiming gates. Note that the basic clock frequency is 2.048 mc ($4 \times 512\text{KC}$), and is crystal-controlled.

One approach to entering quantities into the computer system would involve the generation of suitable serial pulse trains to be compatible with the above timing and synchronization requirements. However, this would move the precise timing requirements across the DA/AROD interface and could thus conceivably jeopardize the reliability of the DA/GC interface. For this reason, in spite of the equipment simplification (and reduced inter-connecting wiring) inherent in a serial approach, such an implementation should be avoided in this case. Instead, a simplified parallel signal interface characterized by minimal timing requirements should be effected. This is considered in Section 2.5.

2.5 Recommended AROD/Computer Interface

The above considerations lead to recommendations for implementing an interface between the existing Data Adapter capability and a projected approach to the AROD equipment. A configuration for the inputting AROD Sensor output data to the DA is shown in Figure 2-3. Similarly, a configuration for outputting computed command and control information is shown in Figure 2-4. Both schemes are based on parallel transfer of data words with timing and control derived solely from the DA.

From the AROD equipment point of view, a minimum hardware configuration would exploit the characteristics of the operational-digital approach to data extraction. The Data Output Multiplexer function represents a requirement for an 8×26 gating matrix and 13 output driving circuits compatible with the existing Discrete Input circuit's required signal structure. This would not materially change if a transformer coupled interconnection scheme were pursued. The Multiplexing Control would only involve a small number of gates and a few control latches if

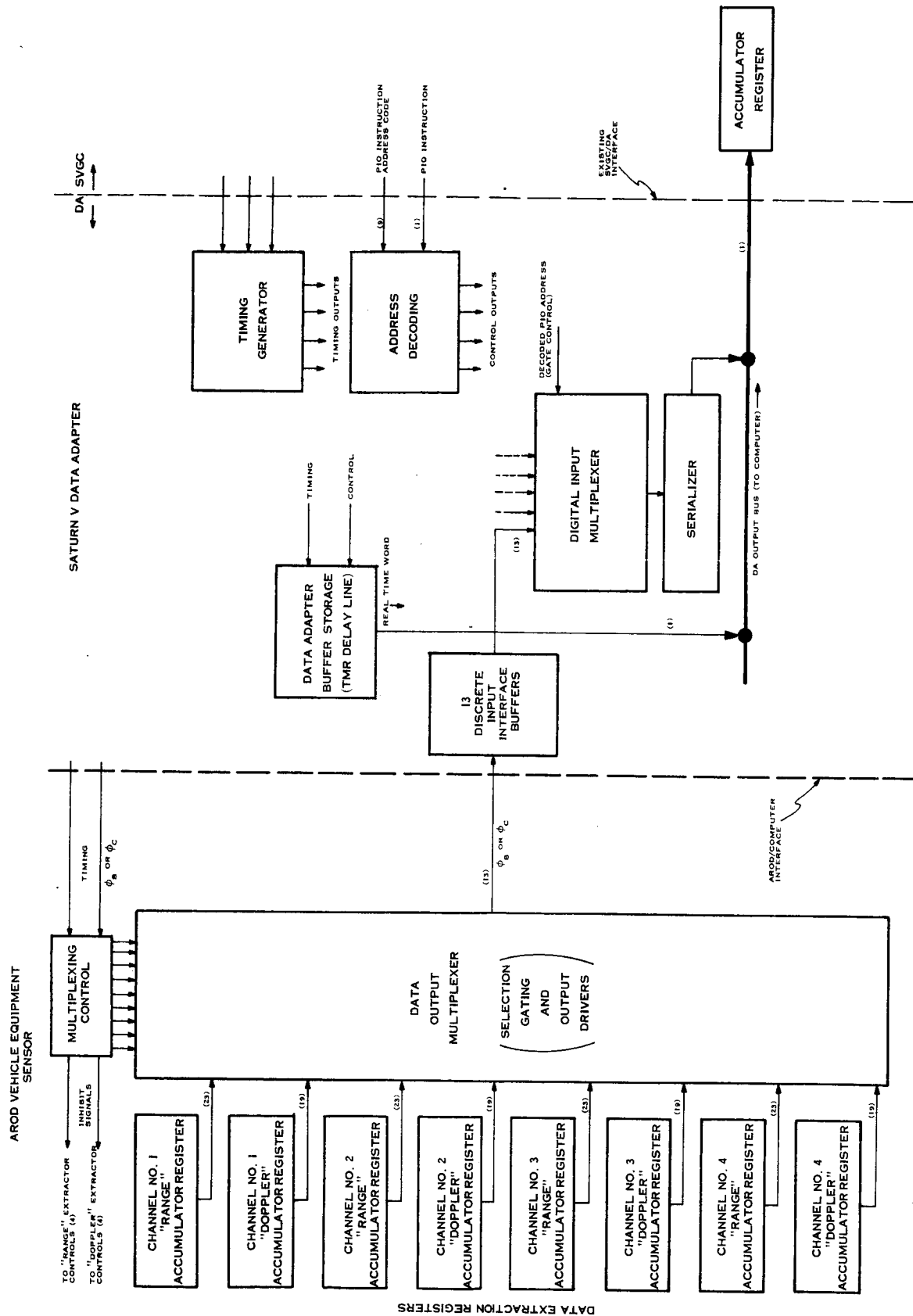


Figure 2-3. Recommended Configuration for Inputting AROD Sensor Output Data

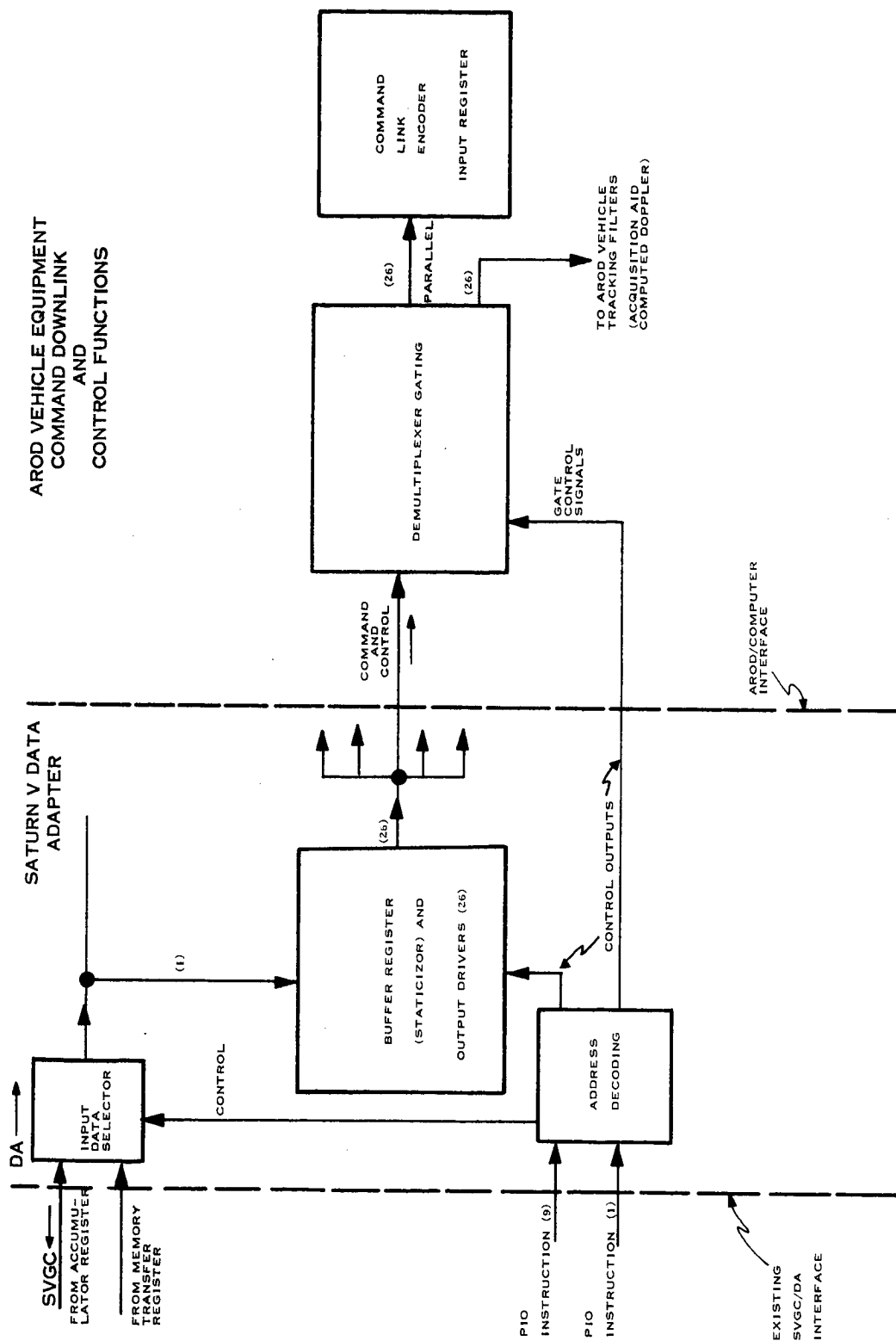


Figure 2-4. Recommended Configuration for Outputting AROD Command and Control Data From Computer

sufficient PIO addresses are available. If not, an eight (or more) step counter with additional lines across the interface to signal an address code would be required to control the output multiplexing. Note that the breakdown of a nominal 26-bit data word into two bytes is accomplished in the AROD equipment by suitable use of ϕ_B and ϕ_C timing signals. This serves to reduce the number of output lines (13 instead of 26) and also the gating to accomplish this action in the DA's Digital Input Multiplexer. The time information required with all AROD measurements is supplied along with the eight data quantities by appropriate gating of the Data Adapter Delay Line Buffer Storage (implementing the Real Time Clock Register) into the DA/GC interface. This gating will be derived from the PIO addresses which enter AROD data. It can be noted that most of the accommodations required of the DA involve control and timing signal gating and not additional multi-bit data channel hardware (13 or 26 bits per function) other than the 13 DI circuits.

The outputting configuration is based on similar ideas with the selection gating again performed in AROD, this time for demultiplexing the quantities pertaining to AROD from the succession of 26-bit data words presented by the Buffer Register. The Demultiplexer Gating function would consist of 26 gates driven by 26 DI type circuits. The control for this gating operation is again determined by the Address Decoding in the DA.

If sufficient PIO Instruction addresses are no longer available, then output quantities would be handled along with the input data in one grouping in which the addressing is carried out by the counting action of Multiplexer Control. The timing requirements of the output interface are, of course, minimal in that clearing of the receiving register in AROD and gating will be simultaneously derived from the inherent timing of the gate control signals.

The most important aspect of the AROD interface implementation is the dependence on the availability of the latest value of correct data in each of the eight accumulator registers at any time a PIO Instruction

request is developed by the computer. That additional buffer storage can always achieve this is obvious. However, even here the question of when and how to update the buffer storage without time conflict between reading and writing presents itself. For the recommended approach based on full utilization of operational-digital data extraction techniques, this problem is handled by inhibit signals derived from the computer timing and control, which prevents updating of the accumulator registers at the time of interrogation by the DA. The required period of the inhibit operation for the "doppler" extractor inflicts a more severe limitation than with the "range" extractor. For this reason it is recommended that an operational digital technique for "doppler" extraction, as adapted from the "range" extraction method presently under development, be further investigated. The potential advantages with respect to the extraction demodulation problem and its favorable impact on the interface with the computation subsystem merit serious consideration.

Section 3

INVESTIGATIONS OF GEOMETRICAL DILUTION OF PRECISION

This section discusses several miscellaneous GDOP analyses performed during the second quarterly period. Detailed descriptions of the computer runs and brief discussions of the resulting data are included. The summary computer printout data associated with these runs will be published in the supplement to this report.

3.1 Translunar Investigation

As indicated in the data made available to IBM by NASA, the translunar mission involves the transition from a circular orbit with an altitude of approximately 185 km to a translunar orbit which begins at an altitude of about 288 km. It was decided that this transition could be adequately approximated by two circular orbits—one at 185 km and the other at 288 km. There are two justifications for the acceptability of this approximation. First, the change of altitude during the transition to a translunar orbit is relatively small (i.e., less than a factor of 2) and therefore would not cause totally unpredictable effects on the AROD system errors. Second, available classified information* indicates that the contribution to $\dot{R}$ caused by the vehicle acceleration during the transition is negligible in comparison to the geometrically induced contribution.** It is therefore concluded that the GDOP data computed for the two circular orbits will set bounds on the system errors at any time during the translunar mission.

*"Saturn V Launch Vehicle Nominal LOR Trajectory," R-ASTR-S 15 Oct 63.

**See Appendix I for a discussion of the geometrically induced $\dot{R}$.

Five separate computer runs were made for each of the two altitudes. Figure 3-1 shows the transponder geometry used, and Table 3-1 describes the parameters used for these runs. For each altitude, two oceanborne (runs 55 and 56 at 185 km and runs 60 and 61 at 288 km) and two land-based (runs 57 and 58 at 185 km and runs 62 and 63 at 288 km) examples were used to demonstrate the effects of wide and medium baselines on the system errors. A third land-based example (run 59 at 185 km and run 64 at 288 km) was used to indicate the improvement in system errors for a translunar mission over a more accurately surveyed region. Figure 3-2 gives the range of maximum axis data for these 10 runs to facilitate their analysis.

3.2 Additional Investigations of Transponder Geometry

Several additional GDOP computer runs were made to investigate the effects of transponder geometry on the AROD system errors for the launch mission. These runs are described in Table 3-2 with the geographical locations shown in Figure 3-3.

Three of these computer runs (48, 49, and 50), in conjunction with run 37 (as reported on in the SASS First Quarterly Report), were made to investigate the effects of baseline on the AROD system errors during the down-range segment of the launch trajectory.

Figure 3-4 gives the combined position and velocity maximum axis data on a logarithmic scale for these runs. Configuration J is for three stations on Bermuda. Since these are land-based transponders with a small baseline (i.e., a maximum of 40 km), relative station location errors of 0.03 m were assumed.* Configurations M, N, and O are concentric oceanborne triads with baselines of 100, 400, and 800 km, respectively.

Three important effects on the shape of the maximum axis data as a function of time can be deduced from Figure 3-4. First, coverage time decreases with increasing baseline by almost a factor of 2 between the

*An additional bias error of approximately 10 meters was assumed associated with the triad.

smallest baseline (40 km) and largest baseline (800 km). Second, the maximum axis data becomes more uniform over the coverage interval as the baseline increases. Third, it should be noticed that, although the minimum error may be smaller* for a shorter baseline over part of the coverage interval, it will be higher* over most of it.

No recommendation can be made as to the best baseline length, since this will depend upon the relative importance of such factors as the coverage time, required minimum or maximum allowable system errors, and average system errors.

An analysis was also made to determine the number of oceanborne transponders necessary for the typical launch trajectory currently being analyzed (see First Quarterly Report). Four runs (51, 52, 53, and 54) in conjunction with runs 30, 31, 32, 33, and 35, as reported in the SASS First Quarterly Report, were used in this investigation. This series of computer runs used three oceanborne transponders. Configurations E, F, and G (see SASS First Quarterly Report for details) appear to give adequate coverage out to the vicinity of Bermuda without the use of oceanborne transponders. Three orientations for the three oceanborne transponders down-range from Bermuda were analyzed. Two of these are shown in Figure 3-5. Configurations Q and S are for the case where the oceanborne triad is near Bermuda, configurations H and I are for the case where they are further down range (see Figure 3-3). As can be seen from Figures 3-5 and 3-6, the critical time is the switchover from configuration G (Cape Hatteras, San Salvador, and Bermuda) to Bermuda and two ships. There seems to be no orientation of the three oceanborne transponders which will give acceptable errors near this transition time that will continue to give good coverage out to SIV-B cutoff..

It is therefore concluded that for this typical launch trajectory, at least four oceanborne transponders will be required to avoid gross geometrical distortions of the AROD system errors.

*Depending, of course, upon the magnitude of the station coordinate errors.

3.3 Additional Investigations of the Effects of Transponder Location Errors

At NASA's request, four additional computer runs were made. These runs (65, 66, 67, 68) have identical geometries to runs 22, 27, 28, and 29, as reported in the SASS First Quarterly Report. The station location errors were increased for these runs, as shown in Table 3-3. It is the purpose of these runs to indicate AROD system errors for transponder location accuracies other than those previously investigated.

Table 3-1

COMPUTER RUN DESCRIPTIONS

Run No.	Description	σ_r (m)	σ_r' (m/s)	Long. & Lat. Errors, rms (m)	Altitude errors rms (m)	Min. Elev. Angle (deg.)
55	Oceanborne Wide baseline 185 km orbit	3	0.05	30	10	5
56	Oceanborne Medium baseline 185 km orbit			30	10	
57	Landbased Wide baseline 185 km orbit			10	10	
58	Landbased Medium baseline 185 km orbit			10	10	
59	Landbased, better geodetics Wide baseline 185 km orbit			3	3	
60	Oceanborne Wide baseline 288 km orbit			30	10	
61	Oceanborne Medium baseline 288 km orbit			30	10	
62	Landbased Wide baseline 288 km orbit			10	10	
63	Landbased Medium baseline 288 km orbit			10	10	
64	Landbased, better geodetics Wide baseline 288 km orbit	3	0.05	3	3	5

Table 3-2

COMPUTER RUN DESCRIPTIONS

Run No.	Config.	Range Meas. Error, σ_r (meters)	Range Rate Meas. Error, $\sigma_{\dot{r}}$ (m/sec)	Oceanborne Long. & Lat. Errors, σ_ℓ (meters)	Oceanborne Altitude Errors, σ_h (meters)	All Land-based Errors, (meters)
48	M	3	0.05	30	10	—
49	N	↑	↑	30	10	—
50	O	↑	↑	30	10	—
51	P	↑	↑	30	10	10
52	Q	↑	↑	30	10	10
53	R	↓	↓	30	10	—
54	S	3	0.05	30	10	—

Table 3-3

COMPUTER RUN DESCRIPTIONS

Run No.	Config.*	Range Meas. Error, σ_r (meters)	Range Rate Meas. Error $\sigma_{\dot{r}}$ (m/sec)	Ocean borne Long. & Lat Errors, σ_ℓ (meters)	Oceanborne Altitude Error, σ_h (meters)	Landbased Long. & Lat. Errors (meters)	Landbased Altitude Errors (meters)
65	D	3	0.05	1000	10	—	—
66	A	↑	↑	300	10	3	3
67	B	↓	↓	300	10	100	10
68	C	3	0.05	1000	10	100	10

* The configuration letters coincide with those given on page 1-42 of the SASS First Quarterly Report.

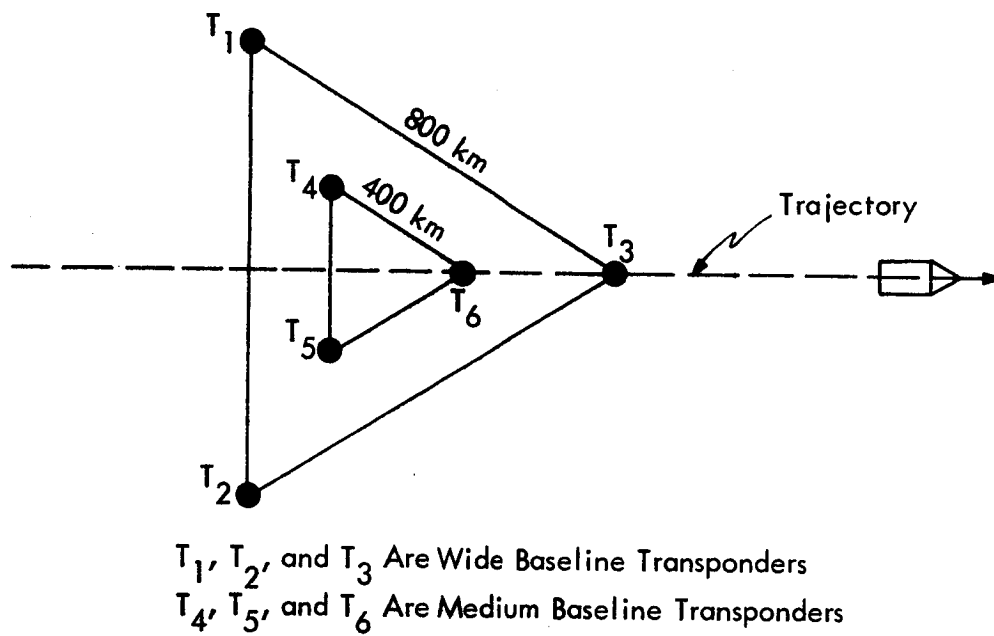


Figure 3-1. Vertical View of Transponder Orientation

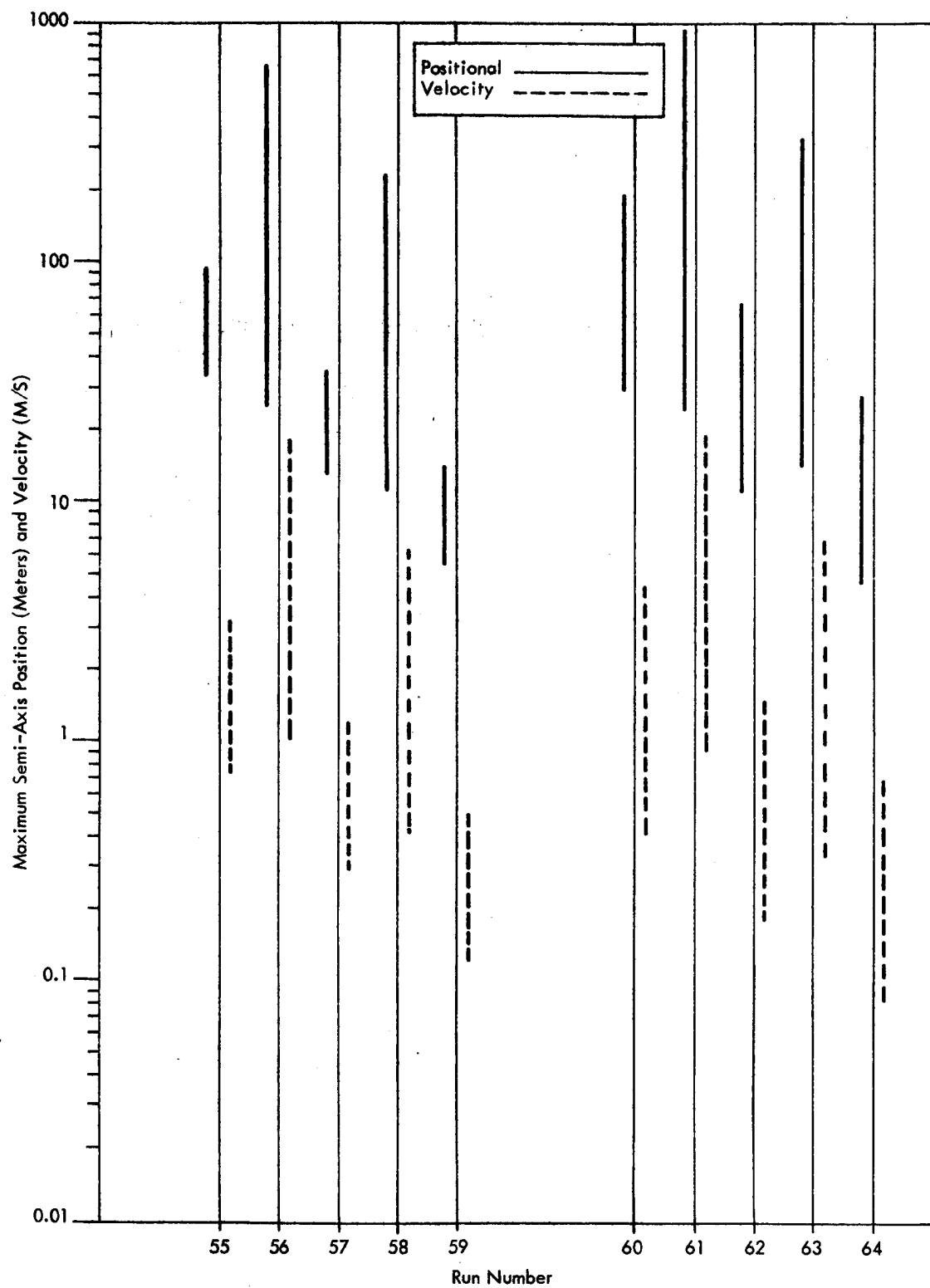


Figure 3-2. Range of Maximum Axis Data

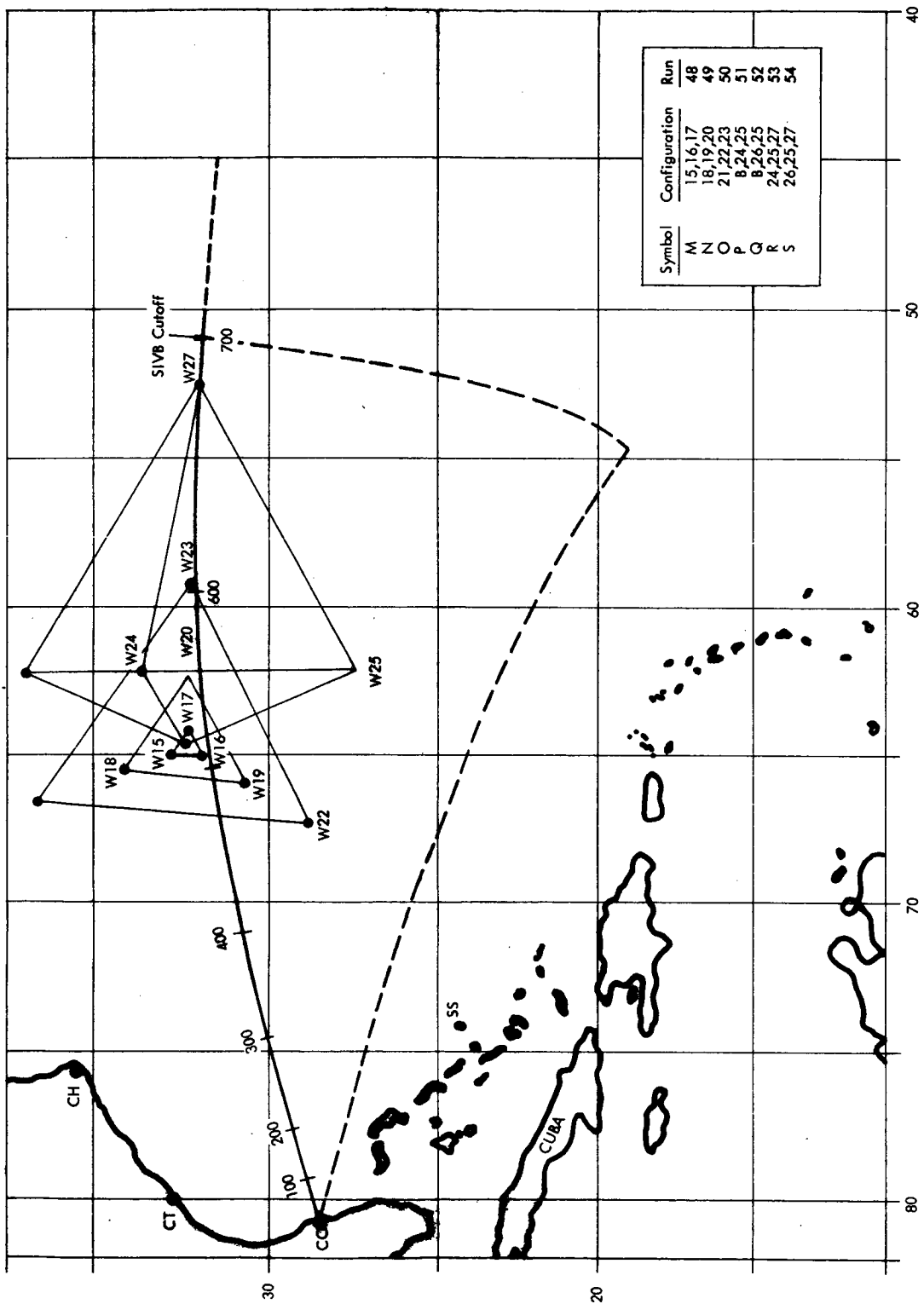


Figure 3-3. Station Locations for GDOP Analyses

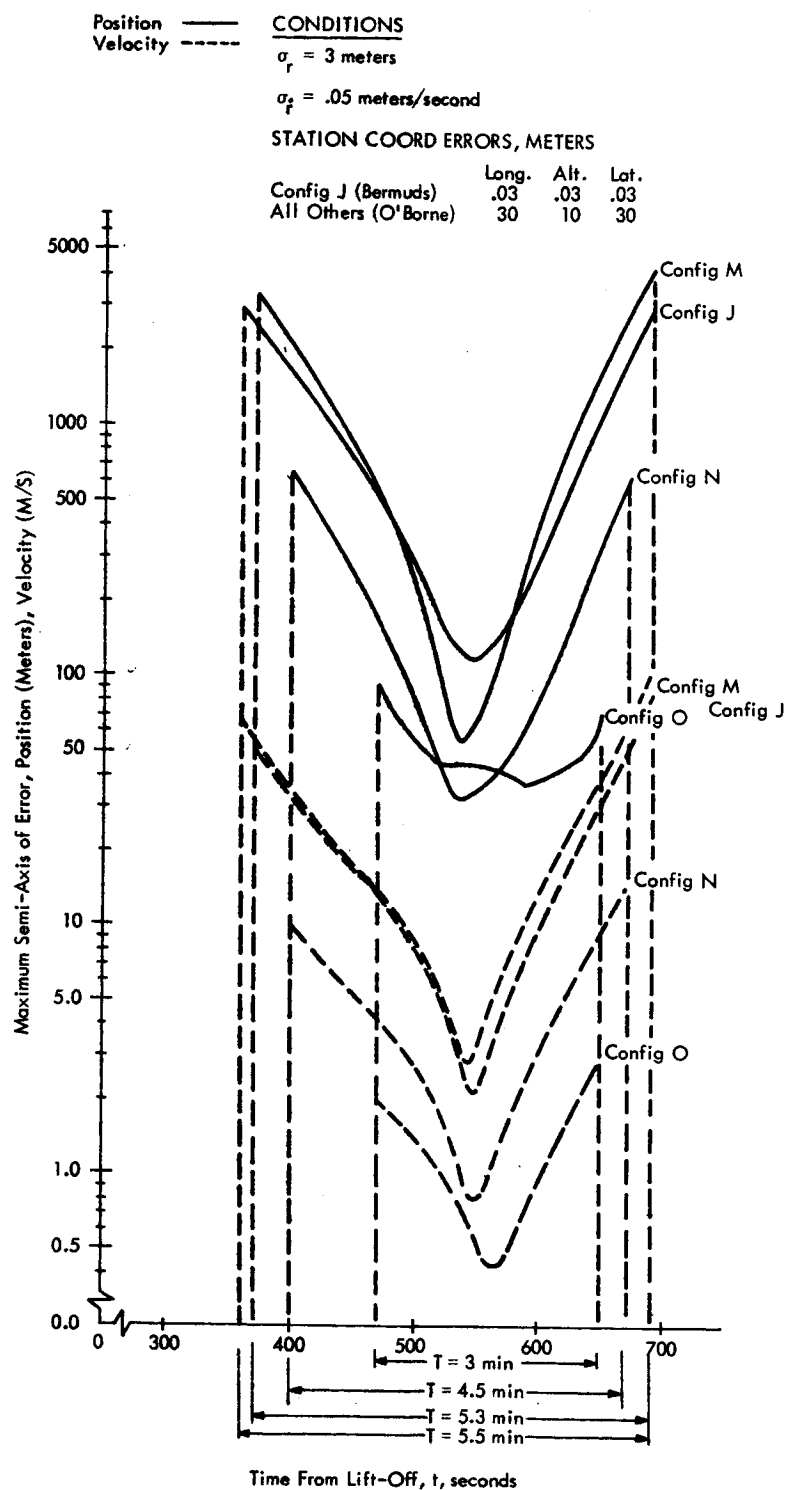


Figure 3-4. Maximum Semi-Axis Data for Concentric Triads

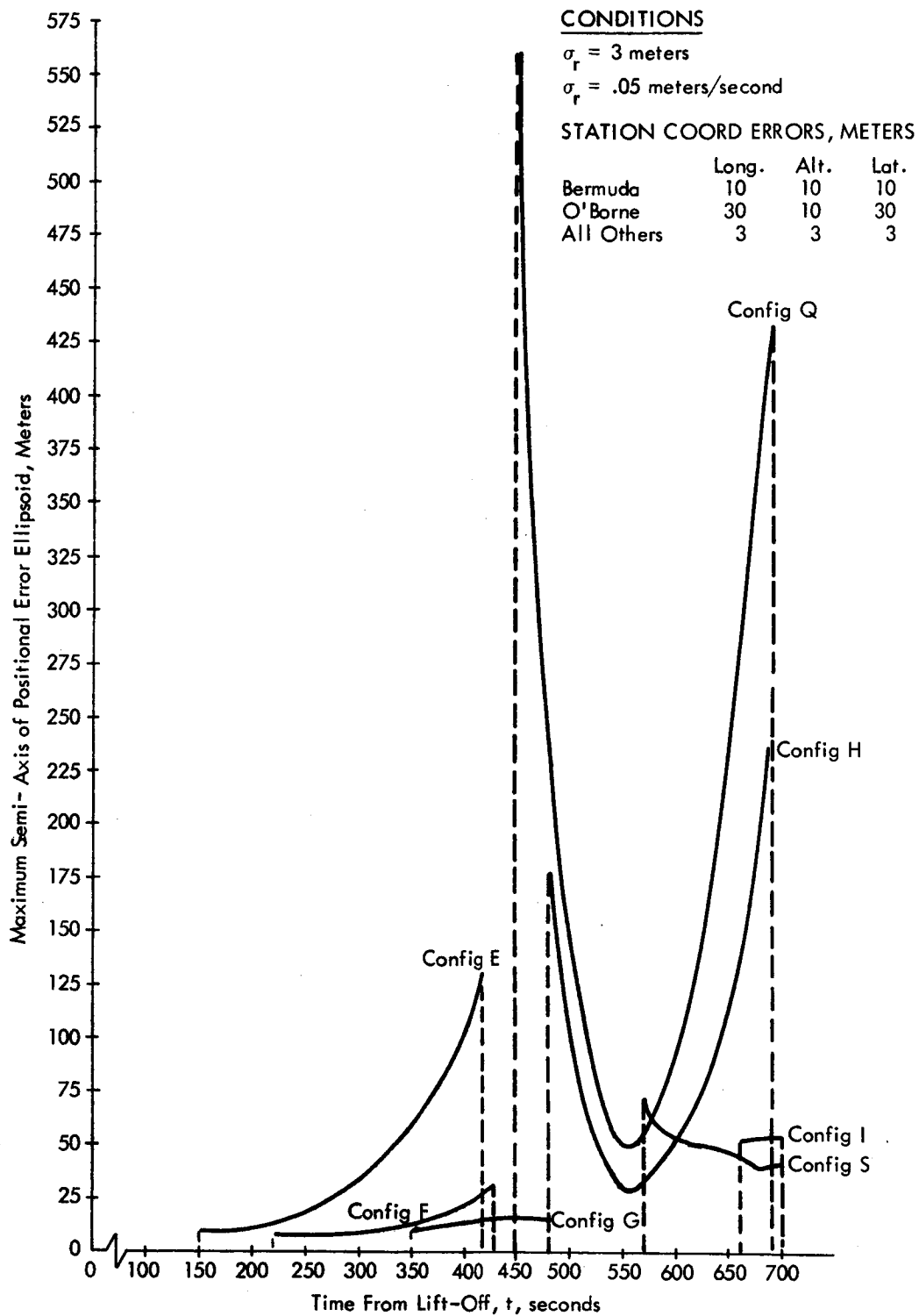


Figure 3-5. Maximum Semi-Axis Data (Positional)
for Coverage Investigation

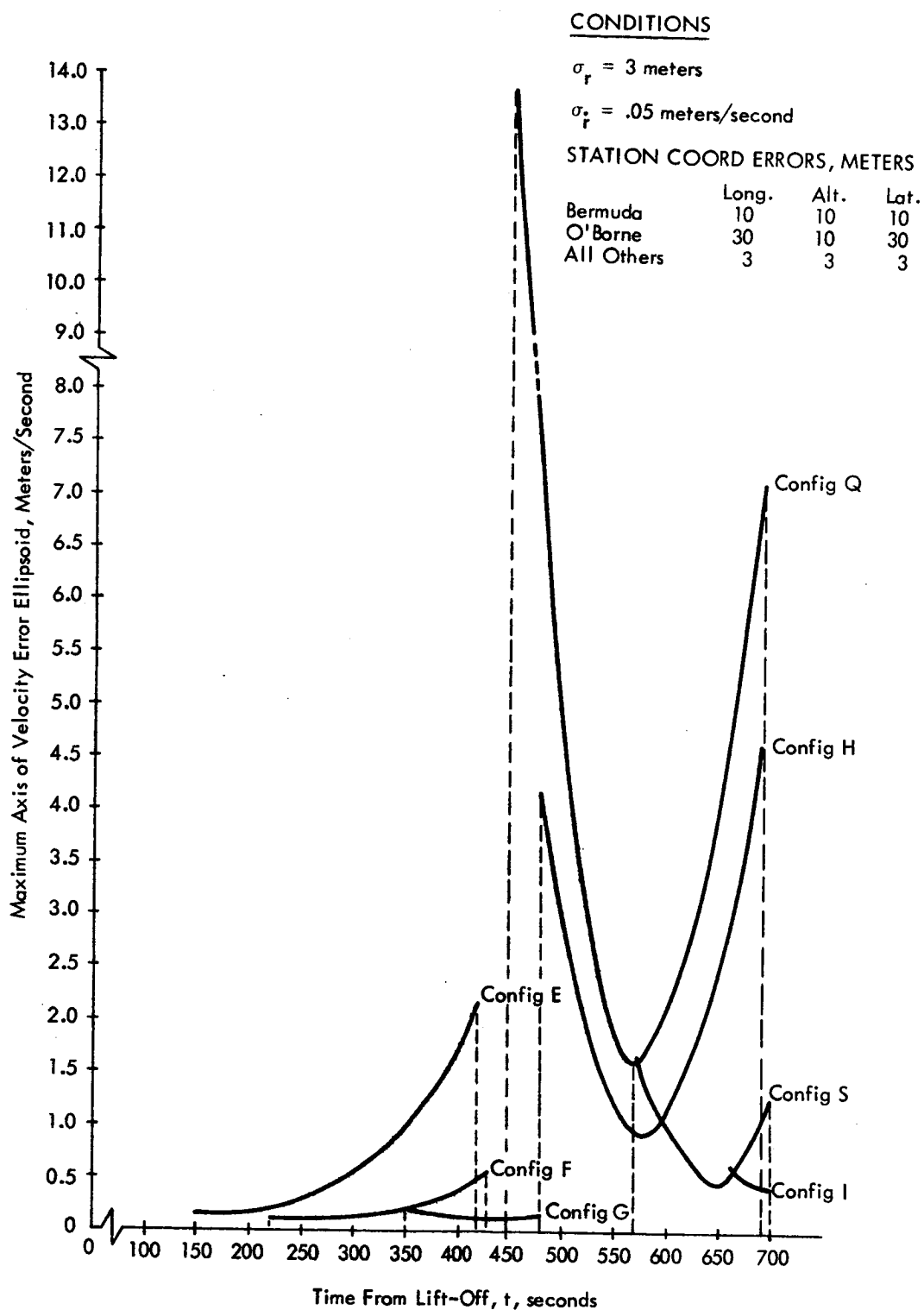


*Figure 3-6. Maximum Semi-Axis Data (Velocity)
for Coverage Investigation*

Section 4

OCEANBORNE TRANSPONDERS

4.1 Introduction

This investigation has been concerned with establishing the technical feasibility of a network of oceanborne AROD transponders in a sector of the North Atlantic bounded by bearings of 75° and 105° from Cape Kennedy to a radius of 3500 km. Conceptual considerations were dominant in the study. A requirement for unmanned transponder operation, with servicing cycles of at least six months, was assumed. This essentially precluded the use of buoys which employ propulsion as a means of station-keeping. The transponders that are discussed, therefore, conform with the basic AROD feature of economical ground installations.

As of this date, there have been no successful oceanborne platform installations of any type, in depths comparable to those encountered in the region of interest, which, even remotely approach the position accuracy demanded of the AROD system. Many design approaches were analyzed and evaluated; the most promising approach (though a drastic departure from other massive, brute force approaches of past and present) appears capable of at least an order of magnitude improvement in station-keeping performance. It is recommended as worthy of further study to determine design feasibility. On the basis of preliminary analysis, this design approach appears to have the desired position-keeping ability in environments (that is, combined winds, currents, and waves) that will be exceeded in severity only five percent of the time. If this design is feasible, the need for an active position determination subsystem will be obviated along with the attendant transponder-to-spacecraft telemetry link. The penalties

aboard the spacecraft, which would be associated with this link, are the additional telemetry hardware plus computer time to update station coordinate storage in the spacecraft.

The position accuracy requirements assumed for the moored platform were keyed to the best geodetic surveying techniques expected to be in existence in the 1965 to 1970 time frame. These techniques were investigated during Phase A of the AROD study, Contract NAS 8-5098, and were reported in Appendix C of the Design Feasibility Report (ref. 1) on that contract. A geodetic accuracy of 30 meters was taken as representative based on systems which photograph an illuminated satellite (or rocket) against a background of reference stars, or a satellite borne line-of-sight electromagnetic navigation system such as Secor. The 30-meter position error propagates a 0.6 meter per second velocity error in the computation of the spacecraft velocity under typical geometries. This velocity error was determined in a test employing the AROD Error Evaluation Computer Program. To avoid significant degradation of the overall system, performance objectives assumed for buoy position and velocity uncertainties were chosen to be less than the expected geodetic errors. Although these uncertainties are large in comparison to the errors encountered when AROD is operating with ground-based transponders, it was deemed sufficiently accurate to make AROD useful as a backup to the inertial system during early portions of the Saturn V flight. A further performance objective for the buoy is to maintain the antenna axis vertical within 2° so as to avoid AROD accuracy degradation due to multi-path effects.

4.2 Design Considerations

4.2.1 Mooring

Moorings that are required to withstand large horizontal drag forces usually are designed with a large "scope," that is, a large ratio of line length to water depth. In very deep water this is often impractical because

of the line's great weight. Also, it does not allow accurate positioning because of the large excursions made possible with changing waves, winds, and currents. The ultimate tensile strength required of a steel line to support its own weight in water can be shown to increase linearly with length. For example, to hold its own weight, a vertical steel line in 20,000 feet of water must have an ultimate tensile strength of 200,000 lbs/in² (see ref. 2). Even greater strengths are useful for applications such as shown in Figure 4-1, where a large vertical tension is used to limit the excursion of a buoy disturbed by horizontal drag forces. The angles that the line makes with respect to vertical references are defined as γ_1 at the top and γ_2 at the bottom.

Following the procedures suggested by Isaacs of the Scripps Institute of Oceanography (ref. 2): the curve formed by the line may be approximated by a circle making angles γ_1 and γ_2 at the top and bottom respectively. The horizontal excursion and vertical dip of the surface end of a line of length ℓ can be written as

$$\text{Excursion} = \frac{\ell}{\gamma_2 - \gamma_1} (\cos \gamma_1 - \cos \gamma_2) \quad (\text{ft.}) \quad (4-1)$$

$$\text{Dip} = \frac{\ell}{\gamma_2 - \gamma_1} (\sin \gamma_2 - \sin \gamma_1) - \ell \quad (\text{ft.}) \quad (4-2)$$

In turn, the angles γ_1 and γ_2 can be expressed in terms of forces acting at the cable ends. Defining:

B = net buoyancy or vertical force at the top

F_{H_1} = total horizontal drag on the buoy

F_{H_2} = total drag on the line

W' = weight of the line (prime denotes in water)

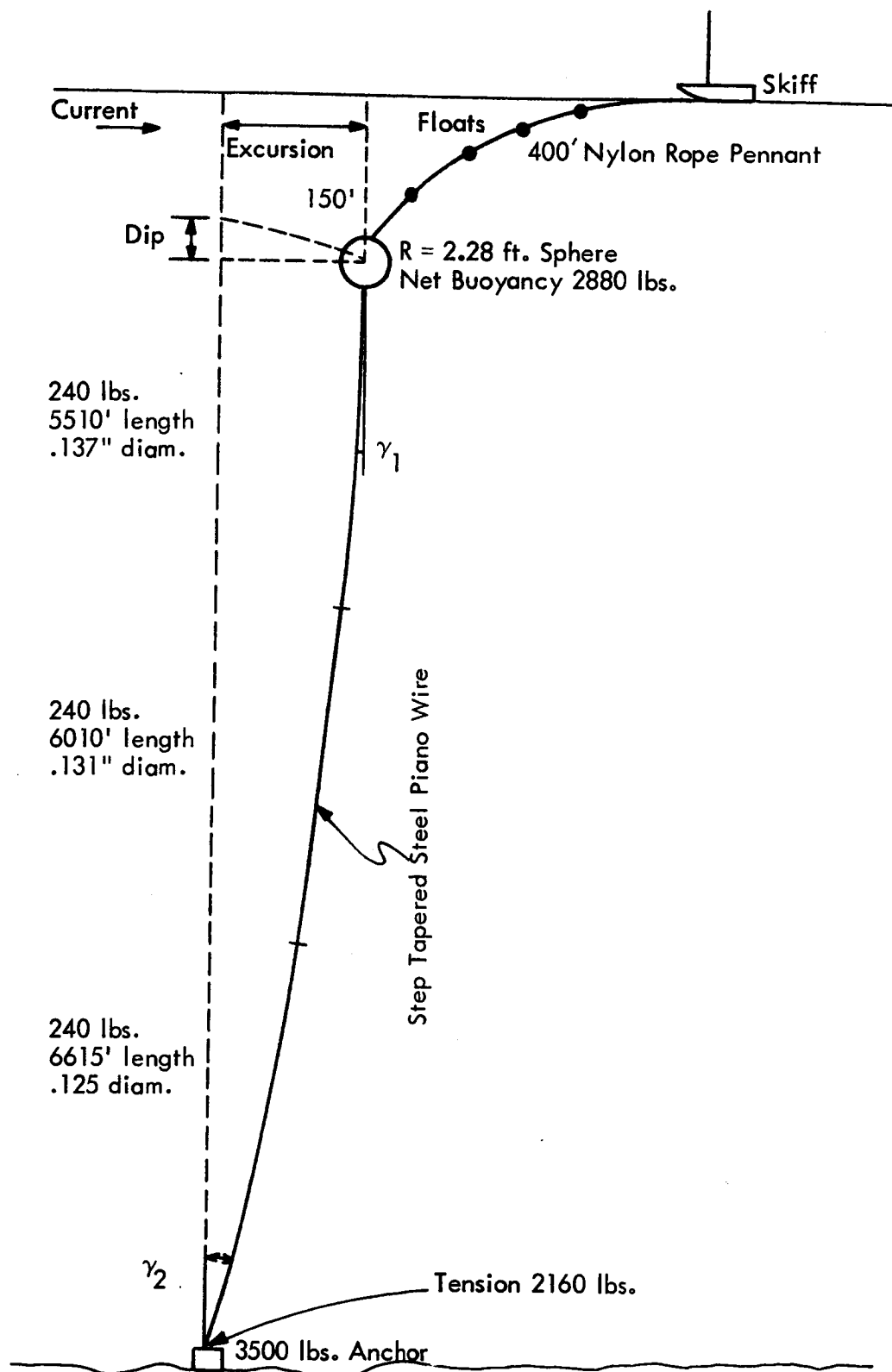


Figure 4-1. Typical Taut Line Buoy Configuration

then the following relationships can be established:

$$\tan \gamma_1 = \frac{F_{H_1}}{B} \quad (4-3)$$

$$\tan \gamma_2 = \frac{F_{H_1} + F_{H_2}}{B - W'} \quad (4-4)$$

It can be seen that the line's weight has a detrimental effect on position accuracy. Two remedies are available:

1. Use of neutrally buoyant lines
2. Use of tapered wire lines

4.2.1.1 Neutrally Buoyant Lines

Line made of polypropylene have a slight positive buoyancy in sea water which prevents snagging or fouling on the bottom when slack. Nylon rope, on the other hand, has a small negative buoyancy. Therefore, a nylon line sinks and is prevented from being fouled by passing ships, or by the buoy itself. An improved line employing polypropylene and nylon has been designed by A. Vine of Woods Hole Oceanographic Institute and has been successfully used in several applications. It consists of using polypropylene for the bottom half, or two thirds, and nylon for the top.

The major disadvantage of neutrally buoyant lines for position holding in deep water is their characteristically low moduli of elasticity and high drag to tensile strength ratio. This property introduces a large amount of stretching per unit stress and incurs large position excursions in these applications. Polypropylene stretches up to 45 percent before breaking, and nylon may stretch 20 percent or more. This property is highly useful in absorbing stresses that a mooring line may be subjected to in an ocean environment, but it is undesirable where fixed station-keeping is of prime importance.

4.2.1.2 Tapered Wire Lines

A tapered wire line, whose cross-section decreases with depth, may be designed so that the rate of decreasing strength matches the decreasing weight of the remaining lower portions. Thus a greater useful tension, than with a constant cross-section line is available with a tapered line. The tapered line affords a significant increase in useful tension or load for any given weight of line.

A practical approximation of this concept can be achieved by using sections of wire of different diameters. If the lowest section of wire has a net weight (W'_1) equal to 10 percent of the total tension for that diameter wire, it has a useful tension of 90 percent of the total tension. The total tension, i.e., the actual working tension, will be taken to be 0.6 of the ultimate tensile strength as has been assumed in successful deep sea moored buoy designs (ref. 2). The next section of wire, above, will have a total tension W'_1 greater than the lower section, so that a length of wire of this diameter, also of weight W'_1 will not exceed 0.6 of the ultimate tensile strength at any point. A third section, with a total tension of $2W'_1$ greater than the first, will be able to support a length of itself having a weight of W'_1 , and the weight of two lower sections, in addition to the original useful tension. To generalize: each section has a weight W'_1 , and each higher section has a total tension W'_1 greater than the one immediately below it.

Steel piano wire is recommended since it offers the highest tension-to-drag ratio known. Its ultimate tensile strength is taken as 320,000 lbs/in. If the desired total tension is 2400 lbs, the breaking tension can be calculated by dividing by 0.6. This yields 4000 lbs. and calls for a wire diameter of 0.125 in.

The length of 0.125 in. diameter piano wire weighing 240 lbs (i.e., 10 percent of 2400 lbs) in sea water is 6615 feet. The net useful tension at the bottom is 2160 lbs (i.e., 2400-240 lbs). According to the design

method described, the second 240-lb section would have a diameter of 0.131 in., and would have a length of 6010 ft. A third section, weighing 240 lbs when submerged and 240 lbs stronger than the second (480 lbs stronger than the first), would have a diameter of 0.137 in. and a length of 5510 ft. These three sections together provide a mooring line length of 18,135 ft with a useful tension equal to 90 percent of the maximum tension for the lowest section. If this example is continued with three additional larger diameter sections of wire, each 240 lbs in weight, the useful tension at the bottom will be unchanged and the total length will be 32,245 ft. A constant diameter wire of the same tensile strength, weight and length would have a useful tension at the bottom of only 1470 lbs or 68 percent as great. A wire with the same useful tension of 2160 lbs would weigh 690 lbs more.

4.2.2 Single Taut Line Design Example

An example of a buoy moored by a single line is presented here to illustrate the large position uncertainties that occur with this design approach. The configuration given in Figure 4-1 is used for this example in an environment of a 29-knot wind and 0.5 knot current, prevailing in the same direction. This environment is taken as representative of the worst case in the region of interest for all but five percent of the time. Assumptions bearing on the drag forces that would be encountered with the example buoy are as follows:

Wind Drag: For the tower, a frontal area of 5 ft² is assumed; and for the skiff, an average frontal area above the water surface of 12 ft² is taken as representative of skiffs such as Nomad. Drag force is related to wind velocity and frontal area by:

$$F \text{ (lbs)} = k A \text{ (ft}^2\text{)} V w^2 \text{ (mph)} \quad (4-5)$$

The wind drag coefficient, k , is taken as 0.0025 for the simple shape of the antenna mast, and 0.004 for the complex shape of the skiff. The wind drag forces that result are 14.0 lbs and 52.9 lbs for the mast and skiff respectively.

Water Drag: The two components of water drag are skin drag and frontal drag. For this example, with the low water velocity, skin drag is negligible for surface areas up to several hundred square feet. Frontal drag forces for the submerged elements of the example buoy are assumed as follows:

Skiff (4 ft^2)-----	3.15 lbs
Pennant ($1/2$ inc. diameter x 400 ft)-	13.1 lbs
Floats (4 ft^2)-----	3.15 lbs
Buoy (16.4 ft^2)-----	13.0 lbs

The value of F_{H_1} (see paragraph 4.2.1) is the sum of wind drag and water drag of the above elements, or 99.3 lbs.

The value of F_{H_2} (135 lbs) is based on the assumption of an average current of 0.5 knots, and a tapered piano wire line with a $1/8$ -in. diameter at the bottom. Applying equations (4-3) and (4-4) and then (4-1) and (4-2), the values of excursion (1372 ft) and the depression (54.4 ft) can be determined. This example has assumed no resultant wave forces present. In the event of breaking waves, combers would have been produced. These travel at nearly the phase velocity of the waves, and much larger forces than those calculated would be produced.

An excursion of 1372 ft is unacceptable for the AROD application. A calculation was made to determine the size wire and buoy needed to keep the excursion within the design objective of the study.

Increased vertical tension will reduce the excursion. It has been shown in ref. 2 that excursion is inversely proportional to wire diameter if tensile strength and percentage of tensile strength used remains unchanged.

A steel rod 5.72 in. in diameter, having the same 320,000 psi tensile strength as piano wire, would be capable of maintaining the 4.94 million lbs tension required (60 percent safety factor). The feasibility of producing or handling such a rod is doubtful. Three-inch diameter cables with a tensile strength of 125,000 psi have been reported (ref. 5). Ten of these cables used vertically, and costing 0.7 million dollars, would be required to maintain a position uncertainty of 30 feet for the buoyant sphere, and a somewhat larger uncertainty for the skiff.

4.2.3 Environment

4.2.3.1 Currents

The ocean surface currents in the region of interest (a 2000 n. m. 30 degree pie sector due east of Cape Kennedy, extending from just south of Bermuda to just north of the West Indies) are mostly westerly at less than 0.5 knots. In the northern half of the outer 1000 miles, surface currents are very weak and of variable direction, with speeds averaging less than 0.3 knots (ref. 4). This region is known as the Saragasso Sea. The accuracy of these current measurements is probably nominal, since stationary platforms have not been available to gather data. The currents are at their minimum in the Spring and are strongest in the Fall of the year.

Information on deep ocean currents in the specific region of interest was not available. However, in general locations in the tropics (ref. 2), where surface currents of two knots exist, the high currents usually run to a depth of 200 to 400 ft throughout the year. In the middle-latitudes, high current regions are very shallow in summer, but 300 to 600 ft deep in winter.

The bottom limit of high surface currents is found to coincide with the depth of the mixing layer, below which the current and the temperature diminish rapidly within 100 or 200 ft. Immediately below the transition a current of 0.5 knots is considered high, and from this point to the bottom velocities decrease.

There is evidence of the existence of deep closed circulation cells of water (sometimes called boluses). Turbulent, highly variable currents exist in small regions at the interfaces of these cells (ref. 5). It is highly desirable to measure the vertical current profile at each potential mooring site. Sea mounts and other irregularities are known to create deep turbulence and higher velocity currents (ref. 6).

A complete understanding of deep circulation is only now being obtained by oceanographers; however, the simplest gross pattern that would fit the available data is a broad, sluggish, spreading, and unchanging southern current in the western North Atlantic. Deep ocean currents are considered far more constant in nature than surface currents, since seasonal and storm disturbances are not transmitted beneath the mixing layer which is within a few hundred feet of the surface.

4.2.3.2 Waves

The trochoidal theory (see First Quarterly Report under this contract pp. 2-6 to 2-8) is generally used as a working model by investigators of ocean waves. Under stormy conditions it may be assumed that a large number of trochoidal wave systems are superimposed to create the observed confused sea state.

The usual ratio of wave height (H), measured from trough to crest, to wave length (λ), is about 1:20. For short waves (300 ft) the ratio may be 1:15, or for very long waves (1000 ft), 1:25. The wave period (T), in seconds, is related to wave length, in feet, by the following formula:

$$T = \sqrt{\frac{2 \pi \lambda}{g}} \approx 0.442 \sqrt{\lambda} \quad (4-6)$$

where g is the acceleration of gravity.

The water particles at the surface rotate in a circular path with a radius (R_o), equal to $\frac{H}{2}$. Therefore, the water particle velocity (v) is

$$v = R_o \omega = \frac{2\pi}{T} R_o$$

$$v = \pi \frac{H}{T} \quad \frac{\text{ft}}{\text{sec}} \quad (4-7)$$

If the sea's surface is broken, combers may be formed and the trochoidal model is no longer applicable. Water particles in a comber travel about one-third of a wave length with a velocity given by ref. 2.

$$V_c = 6 \sqrt{H} \quad \text{ft/sec} \quad (4-8)$$

This particle velocity exists at the surface only. Below the surface, the combers do not significantly affect the particle velocity associated with trochoidal waves.

During winter (the worst season), in the region of interest, waves with both a height of 9 ft or greater, and a period of 9 sec or less, can be expected less than five percent of the time (ref. 4). This combination of height and period gives the following set of velocities:

$v = 3.14$ ft/sec circular water particle velocity

$V_c = 18.0$ ft/sec comber velocity

From these relationships simple computations may be made for induced drag forces. For a given wave height the waves are more forceful for the shorter periods due to the higher water velocities. Induced drag on submerged objects is proportional to the square of the velocity. Table 4-1 summarizes the average frequencies with which different wave height conditions prevail at two locations in the mooring area (ref. 4).

4.2.3.3 Winds

The cumulative percentage of time during which winds are less than a given velocity is shown in Figure 4-2 for three regions of interest. In the northernmost region winds exceed 28 knots 6% of the time; in the

Table 4-1

DISTRIBUTION OF WAVES OF VARIOUS HEIGHTS

57.5 longitude 27.5 latitude

	Winter	Spring	Summer	Autumn	Ave
0 - 1 ft	4	10	9	5	7
1 - 4 ft	34	53	51	38	44
4 - 9 ft	41	30	29	43	35.8
9 - 14 ft	9	0	5	6	5
14 - 20 ft	3	0	0	1	1
Greater than 20 ft	1	0	0	0	.2
Undetermined height	8	7	6	7	7

72.5 longitude 27.5 latitude

0 - 1 ft	6	12	12	6	9
1 - 4 ft	43	56	54	43	49
4 - 9 ft	40	26	26	40	33
9 - 14 ft	6	2	3	7	4.5
14 - 20 ft	1	0	1	0	.5
Greater than 20 ft	0	0	0	0	0
Undetermined height	4	4	4	4	4

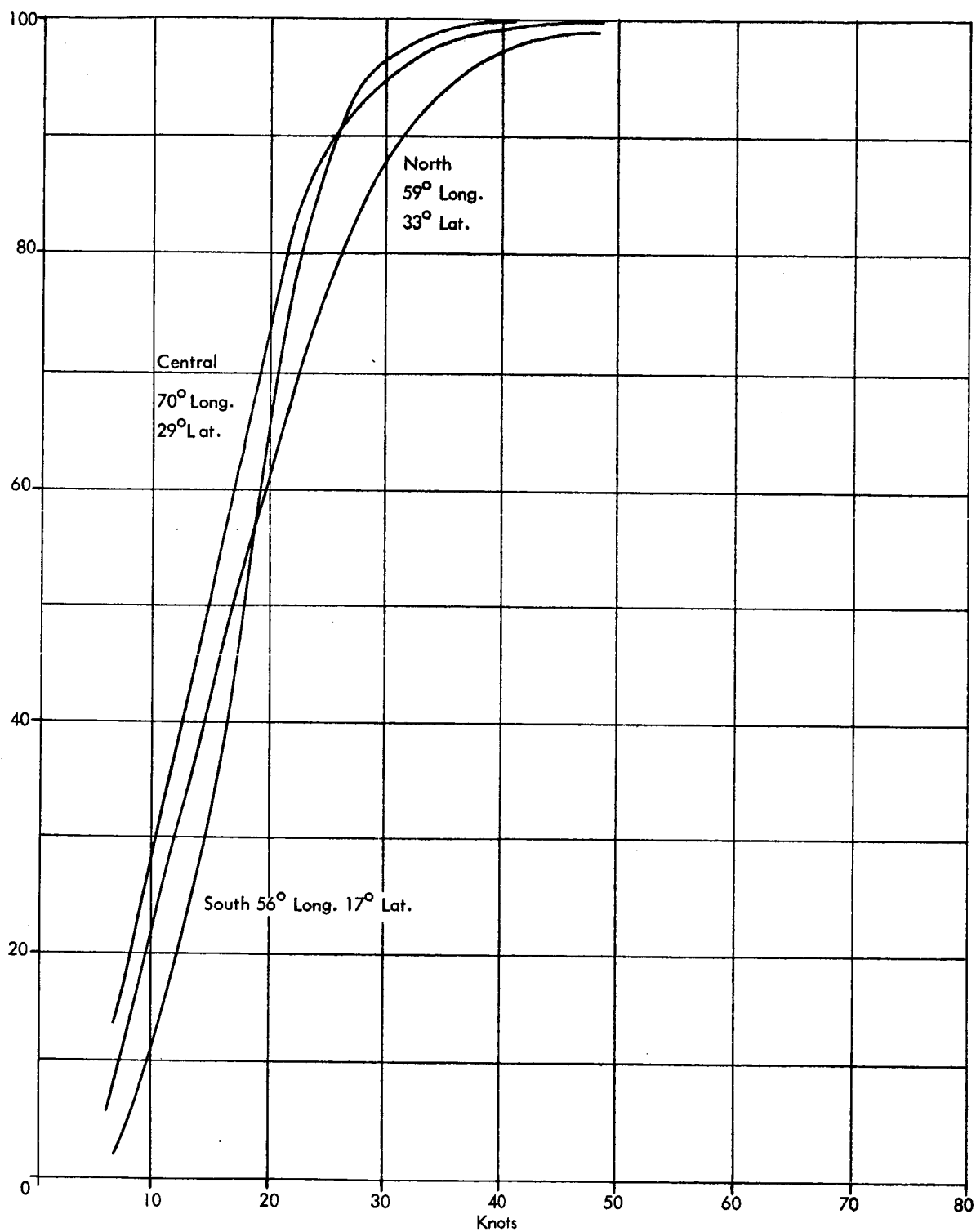


Figure 4-2. Cumulative Distribution of Wind Velocities

central region 2.3% and in the southernmost region, 1% (ref. 4). The wind velocity chosen as the operational design requirement is 29 knots, which is 33.4 statute miles per hour.

If the entire area of interest were covered by a network of 13 buoys, the record of tropical storms (35 knots and above) over the past 70 years would indicate one buoy on the average would be affected each year. Since little traffic passes through much of this region, and since reporting of storms off the shipping lanes has become practical and of only recent interest, it may be presumed that a larger number of storms existed than were recorded. Two or three storm losses per year would seem a reasonable estimate if buoys cannot be designed to withstand the severe strain of tropical storms.

4.2.3.4 Other Design Considerations

All oceanborne platforms, particularly those that protrude above the water surface, are subject to a large number of hazardous and decaying elements. These factors must be addressed in a realistic manner to successfully achieve unattended operation over periods of six months or greater. Among the more important problem areas confronting the buoy designer are:

- 1) The corrosive action of sea water upon structural materials is extremely severe. Even stainless steel is not immune. The use of dissimilar metals in close proximity to each other is all but impossible, due to electrolytic action.

- 2) Many materials are subject to deterioration from bacteria, fungi, and minute boring animals. Skin drag of exposed surfaces has been reported (ref. 7) to increase at rates which are typically 0.5% per day for periods of months, whereby the ultimate skin drag is 300 percent of the initial value by virtue of barnacle and other animal growth. Usually, however, skin drag associated with fixed buoys in low velocity currents is small compared to frontal drag forces.

3) Voracious species of marine life, sharks in particular, have been reported (ref. 4) to bite and break cables and other objects that may attract their attention.

4) A major portion of the oceanographic region of interest is in the Saragasso Sea (longitudes 30° to 70° West and latitudes 20° to 35° North) where extremely massive rafts of floating seaweed have been encountered (see ref. 2). Although the Saragasso Sea is by nature calm and non-turbulent, these rafts could conceivably sweep bodies from their moorings.

5) Highly irregular ocean bottom contours and the high currents, localized at the irregularities, may cause drag of anchors and severing of mooring lines.

6) Probably the most unpredictable hazard is that of man who, out of curiosity, maliciousness, or poor judgment, has been found to tamper with deep ocean buoys. It is a moot question as to whether to mark the buoy with flashing lights or radar reflectors. The increased detectability of the buoy may serve to attract would be rescuers or tamperers to a degree that more than off-sets the advantage of increased warning that is afforded against accidental collision. Marking for purposes of locating the buoy for servicing operations may not be necessary, since the transponder may be commanded to operate for this purpose.

4.2.4 Stability

The usual approach toward stabilizing a floating platform for operation under turbulent sea conditions is to apply a combination of the following techniques:

1) Design a structure with a very large moment of inertia about all axes in the horizontal plane, so that the platform will respond sluggishly to rapidly changing tipping forces.

2) Employ a large, deeply submerged mass, with a minimum structural cross-section at the water-to-air interface region, to diminish the variation in buoyancy force which occurs as waves roll by and which, in turn, causes heaving motions.

Both these approaches aggravate surge, that is, side-way movements, because of the large drag surfaces presented to currents and waves. Accurate mooring, which has already been shown to be a problem for buoys with minimum dimensions (see section 4.2.2) becomes very difficult and expensive, if not impossible, for floating platforms so stabilized.

Surge can be minimized by either of the following platform designs, riding on top of the waves. Floating discus buoys, with a diameter-to-thickness ratio of 15:1 having a beveled edge, produce very low drag except when waves break over the edge. They are under study by General Dynamics for use as telemetry antenna platforms, in sizes up to 40 feet in diameter (ref. 8). Further consideration was not given for AROD use, due to the 15° and greater antenna pitching reported on scale model wave tests. Boat or skiff type buoys, such as NOMAD, produce even less drag resistance—when aligned to current and wave forces—than a discus buoy, but they exhibit very poor roll stabilization for antenna platforms (ref. 9). Both these buoy types require a slack mooring, so that freedom is provided to ride over the waves. As a result, large excursions in position may be expected.

Abstracting from "High Stability Platforms for Range Instrumentation," by Samuel T. Carpenter (ref. 5), a floating platform 160 ft x 160 ft costing \$2.9 million and using an economic combination of minimum pitch and heave factors mentioned above, was shown to require nine 3-inch steel cables costing a total \$0.66 million to withstand the force of 30-ft, 10-sec storm waves in 12,000 ft of water. Less severe 11-ft, 8-sec waves, judged to be a representative limit of 98 percent of the storms, could be withstood with three 3-inch cables—although excursions of up to 25,000 ft were predicted. As a result, powered propulsion was favored over mooring for maintaining station when not prohibited by storm wave forces.

A sonar referencing system, as used in Project MOHOLE, is required to accomplish station-keeping or position determination to within a few hundred feet.

The stability achieved (1° 11' pitch, 0.56 ft heave, 2.1 ft surge and 1.5 ft/sec maximum velocity) in a 1 kt current, 11 ft waves, and 30 mph winds was barely sufficient for AROD velocity measurement requirements; it was quite inadequate for position accuracy requirements.

Additional stability can be gained by providing excess buoyancy to the limit of the mooring cables and anchor available, while holding the structure somewhat submerged. Two such taut line designs were examined during our study, employing deeply submerged buoyant volumes to avoid surge effects of surface waves. The antenna, projecting above the surface, was rigidly attached to the buoy by a very high tower. This approach was deemed inadequate in performance and prohibitively expensive.

4.3 Recommended Design Approach

4.3.1 Description of a Triply Moored Lightweight Buoy

The poor cost-to-effectiveness ratio of buoys moored with a single line and, indeed, the doubtful feasibility of approaching the position-keeping and stability requirements of AROD with these approaches led the study effort toward the consideration of wide baseline, triple mooring lines in order to capitalize on the inherent accuracy of trilateration systems. The lines are stretched under high tension at relative angles of 90° (see Figure 4-3) so that each line is constraining from horizontal and upward movement perpendicular to its axis by the other two, and from downward perpendicular movement by the increased share of buoyant force such movement would incur.

Very high tensile strength wire, such as piano wire (320,000 psi), is desirable to maximize the tension-to-drag ratio. It is a design objective to make the tension a large fraction of the ultimate (breaking) strength, so that wave action and current change forces on the buoy and wires

create only minor fluctuations in the wire tension. As noted previously, a total tension of 60 percent of the ultimate tension has been successfully employed in previous buoy designs (ref. 2). This is necessary also to keep the cost of the system within bounds, since not only the cost of the wire, but also the installation cost of the mooring increase greatly for larger size wire.

To design a rigid mooring with small wire, drag forces must be kept very small. Thus, the buoy and its antenna mast must achieve their stability from line tensions rather than from a massive iceberg-like structure which is inherently stable. Although there is considerable appeal to the idea of submerging the buoyant chamber deeply enough to avoid wave action, and thus to gain stability by minimizing tension fluctuations, the problem of communicating this stability to a surface antenna with a lightweight structure remains. Torques on the antenna mast become excessive when the buoyant chamber is submerged sufficiently to gain appreciably from the exponential decrease in the circular water particle motion with depth. The increased drag of the longer mast would require that a less rigid system of lines be designed to allow for the larger force variations within the safety limits of the lines. This degrades the position-holding performance.

As with the single taut line buoy designs approximation to a tapered wire makes the design of a triply moored buoy more attractive. For orthogonal intersection at the buoy, the three lines must make angles of $54^{\circ}44'$ to the vertical. If held without sag in 18,600 feet of water, the required line length would be 32,245 feet. Six sections 240 lbs each are capable of achieving this length. With bottom and top section diameters of 0.125 in. and 0.153 in. respectively, the six sections weigh 1440 lbs. Together, with the useful tension of 2160 lbs, this weight equals the maximum total tension for the top wire ($3600 \text{ lbs} = 0.6 \times 6000 \text{ lbs}$, the ultimate strength for the 0.153 in. diameter piano wire). When balanced (i.e., equal tension on the three orthogonal lines) the horizontal component of force at the top of each line will be 2940 lbs.

The angles made by each mooring line with respect to the vertical at the top and bottom are the angles γ_1 and γ_2 given in equations (4-3) and (4-4). Assuming no drag force $\gamma_1 = 54^\circ 44'$ and γ_2 is equal to $77^\circ 45'$ in this example.

The tapered mooring line will hang in approximately the shape of a segment of a catenary; actually, a series of segments of catenaries. Upper sections will hang more steeply than a single catenary because their weight is greater than the mean weight, and the lower (lighter) sections will hang less steeply. The actual shape tends toward, and in this report is approximated by, the arc of a circle. Because of the mooring line sag, the buoy will only be 13,000 ft above the ocean floor with the six sections as described above. The majority of the network locations that would be chosen for the oceanborne transponders will be at depths of 13,000 feet or less, particularly since a baseline as large as 800 nautical miles is not unreasonable (see Section 3). Additional sections may be used for operation in deeper water; or, alternatively, the same line lengths may be steepened, sacrificing the intuitively desirable orthogonal intersections. Parametric trade-offs would be in order during the design study phase of a buoy development program to optimize the angles of the lines as a function of depth.

To determine the positional stability in the presence of wind, waves and currents, a tower design as shown in Figure 4-3 has been chosen as a reasonable example for computing drag cross-sections. A steel sphere 2.97 ft in radius will supply the required 6235 lbs net buoyancy and present a drag cross-section of 27.7 ft^2 . If the mooring lines are adjusted to keep the sphere totally submerged when under the trough of the largest waves (which the example system is capable of surviving—20 ft height and 10 sec period), then a 30-ft tower will project 10 ft above maximum wave height and 20 ft above mean sea level. The tower is assumed to present three inches of drag thickness for 20 ft of its length, and two in. for the upper 10 ft. Provision has been made for guying the tower with 300 ft of 1/16 in. in diameter wire to maintain vertical position in spite of turning

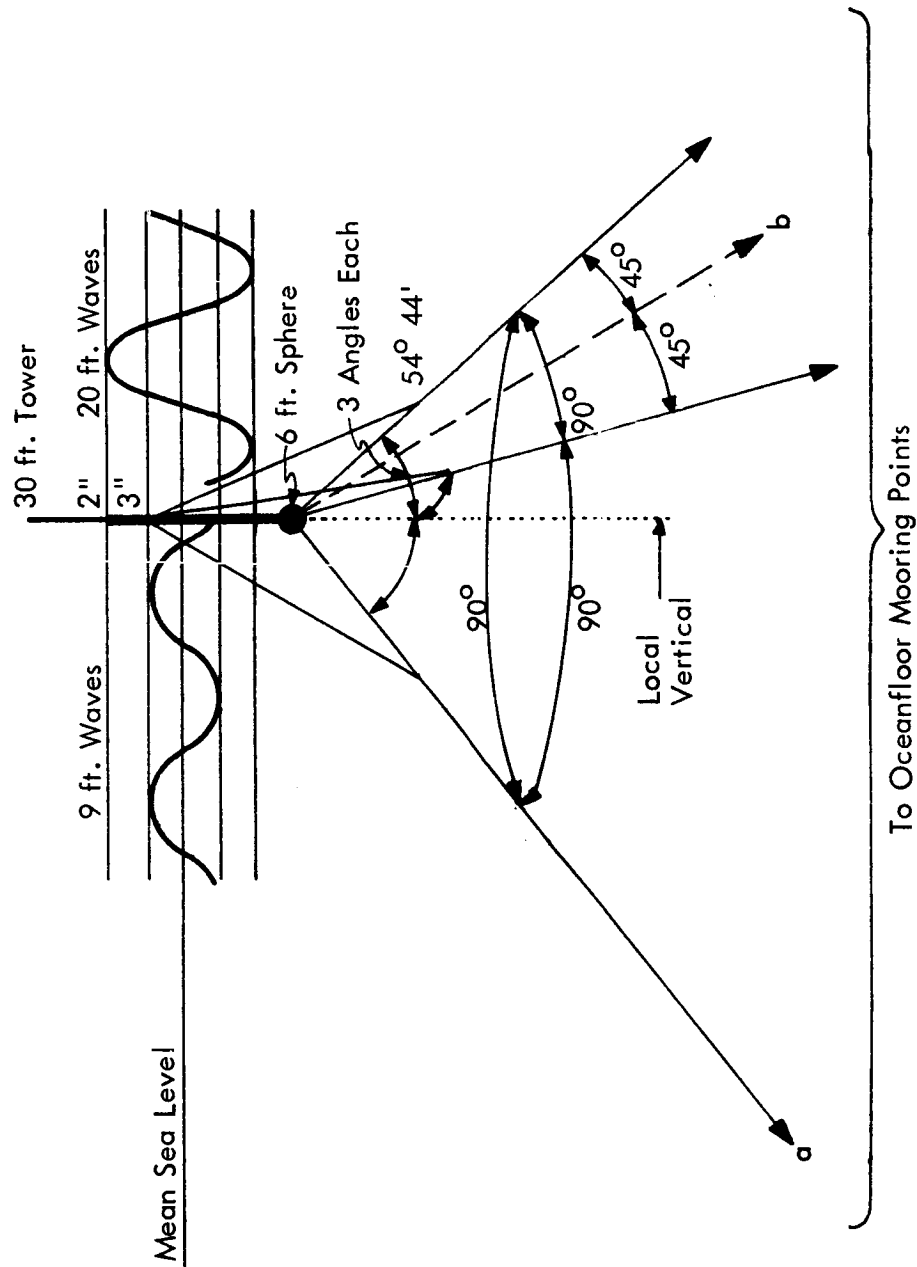


Figure 4-3. Triply Moored-Lightweight Buoy

torques from the wave and wind forces. The tower area exposed to the wind above mean sea level may thus be computed as 4.25 sq. ft. Winds of 29 knots, which are exceeded only five percent of the time, create a force on the tower of 11.85 lbs.

The largest forces acting on the buoy will be those of the circular water flow as associated with waves. All but five percent of the time, the waves with a period of 9 sec may be expected to be less than 9 ft in height, which, from trochoidal wave theory, results in a circular water velocity of less than 3.14 ft/sec. The drag on submerged surfaces near the surface of the ocean will result in 370-lb tension variations in the mooring lines, which, if permitted to exist, would cause movements of over 100 ft. It must be recognized, however, that the water causing the force is itself traveling in a circle whose radius is only 4.5 ft; thus, even an unrestrained buoy would not be moved farther than this by wave action. The computation for viscous damping and harmonic analysis for the tension variation in the mooring lines has not been performed due to the limited scope of the study. It is, however, of considerable interest, since platform stability is a prime requirement and should be accomplished during design and modeling phases of development.

Tension variations on the mooring lines will be transmitted to the bottom in approximately two seconds and start to change the shape of the sagging line, permitting movement in the direction of the line at the buoy. The amount of movement actually permitted may be negligible. If, upon transient analysis, it is shown that this motion is excessive, perpendicular discs may be attached to the lines at a depth of 1000 ft or more to avoid wave action. These would greatly dampen longitudinal movement of the line and hence reduce any rapid movement of the buoy due to periodic wave motion. In effect, for dynamic purposes, the equivalent line length will approach the length between the buoy and the discs.

Trochoidal wave theory indicates that water movement is in opposite directions in wave crests and troughs. Thus, 9-foot waves acting on the

antenna tower will act on a larger area of the tower in one direction than in the other. This differential area for 9-ft waves consists of 9 ft by 3 in. of tower drag cross-section, and 30 ft of guy wire (2.4 ft^2 frontal area). The wave action would cause a force difference of 26 lbs on the guy wire. If, in addition, a 0.5 knot current is applied to the 33 ft^2 buoy and tower cross-section, a 25.9 lb force will result. The total force, should wind, waves and current all act in the same direction at the same time, is 63.8 lbs. The force on the 32,245 ft of mooring line created by a 0.5 knot current at the surface, assuming a linear decrease-to-0 velocity at the ocean bottom, is 59.2 lbs. The application of these forces to the buoy will result in a tension change on the mooring line which will change the angle at the bottom and top by only a fraction of a degree. As a result, the buoy would change its distance from the mooring line anchor by a distance of approximately 30 ft.

4.3.2 Analysis of the Triply Moored Lightweight Buoy

To analyze the effect of disturbing forces on a triply moored buoy, consider a vertical plane through the buoy and one of the mooring lines (a) as in Figure 4-3. In place of the two mooring lines not in this plane, a single line (b) will be supposed to exert equivalent forces on the buoy (see Figure 4-4). Summing the forces vertically and horizontally for a disturbing drag force F_H to the right, assuming γ_{1a} and γ_{1b} remain fixed, the effect of F_H on the line tensions T_a and T_b can be determined by:

$$B = T_a \cos 54^\circ 44' + T_b \cos 35^\circ 16'$$

$$F_H = T_a \sin 54^\circ 44' - T_b \sin 35^\circ 16'$$

$$T_a = 0.577 B + 0.816 F_H$$

$$T_b = 0.816 B - 0.517 F_H$$

The vertical forces on line (a) are one-third the buoyancy plus 47% of the F_H force, while on line (b) they are two-thirds the buoyancy minus 47% of the F_H force. The horizontal forces on line (a) are 47% of the buoyancy and two-thirds F_H to the right, while on line (b) they are 47% of the buoyancy minus one-third F_H to the left.

The assumption of no angle change presumes no sagging or stretching of the lines. In such a case, the buoy would remain motionless while the tension in line (a) increased by 82% of F_H and the tension in line (b) decreased by 58% of F_H . In the actual case the increased tension reduces the sag by a fraction of a degree on line (a) and increases the sag in line (b), permitting a small displacement of the buoy.

For line (a), $\gamma_{2a} = 77^\circ 44.9'$ without the disturbing force. With $F_H = 63.8$ lb, γ_{2a} becomes $77^\circ 22.1'$ for a decrease in line sag by 22.8' or 0.380° .

For line (b), $\gamma_{2b} = 66^\circ 31.4'$ without the disturbing force, while with $F_H = 63.8$ lb, γ_{2b} is $66^\circ 52.0'$ for an increase in line sag by 20.6' or 0.343° . These angle changes for the bottom ($\Delta \gamma_2$) were computed for a proportioning of F_H which left γ_1 unchanged in both lines. Actually, a

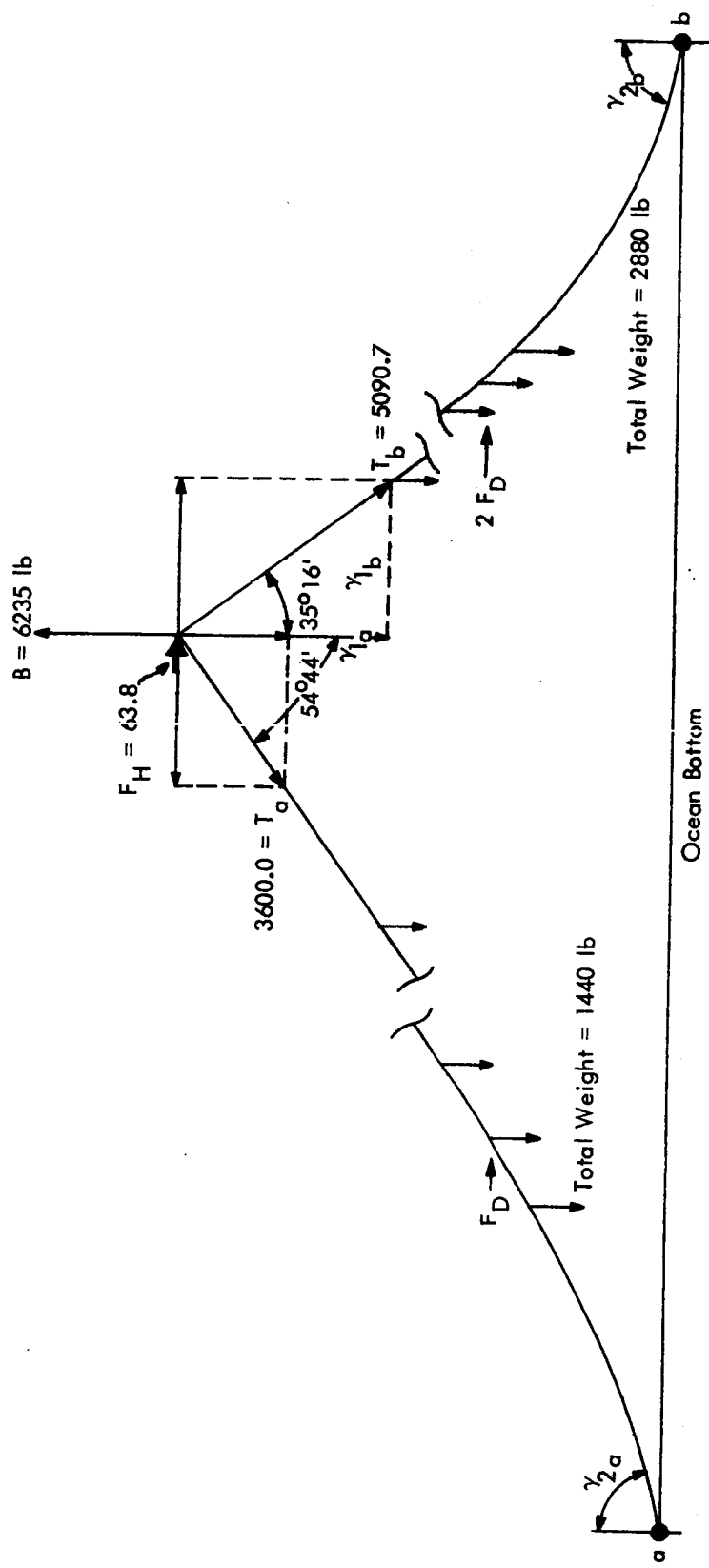


Figure 4-4. Mooring Line Tension Analysis (Vertical Plane)

change in tension which decreases γ_2 will increase γ_1 in that line. Conversely, when γ_2 increases, γ_1 decreases. The relative proportion of angle change in γ_2 and γ_1 is unnecessary in computing displacement as long as the change of their difference is known.

The chord length (C) for an arc length (ℓ) of a circle corresponding to the mooring line length is $C = 2 \left(\frac{\ell}{\Theta}\right) \sin \frac{\Theta}{2}$ where

$$\Theta_a = \gamma_{2a} - \gamma_{1a} = 23^\circ 8' = 0.40166 \text{ radians}$$

$$\Theta_b = \gamma_{2b} - \gamma_{1b} = 31^\circ 15.5' = 0.54556 \text{ radians}$$

where

$$\Delta\Theta_a = -22.8' = -0.006636 \text{ radians}$$

$$\Delta\Theta_b = 20.6' = 0.005994 \text{ radians}$$

The change in chord length corresponding to an increase in tension in line (a), as caused by the 63.8 lb force ΔC_a , is + 7.125 ft.

The decrease in chord length, corresponding to a decrease in tension in line (b) ΔC_b , is - 6.16 ft.

The chord of line (a) makes an angle of $66^\circ 03''$ to the vertical, and the chord of line (b) makes an angle of $51^\circ 04'$ to the vertical. The resulting horizontal displacement is 11.3 ft to the right, and the vertical displacement is 0.98 ft downward.

In addition to the buoy motion from line shape changes, each line will stretch due to a finite modulus of elasticity. The additional tension of + 52 lbs on line (a) and - 36.8 lbs on line (b) due to drag (F_H) stretch these lines + 3.13 ft and - 2.22 ft, respectively. The horizontal displacement, including the effect of stretch, becomes 15.9 ft and total downward displacement is 1.1 ft from the initial, no drag location. The effects of thermal expansion are insignificant since a 1° centigrade change in the temperature of the mooring line over its full length will result in a length change of only 0.6 ft. Water temperature in the ocean depths is very stable, changing by less than 0.1 degree.

The distributed drag forces experienced by each line (F_D), due to deep ocean currents, will change the shape of the lines from that computed above

for the tensions already considered. The total distributed force was computed to be 59.2 lbs for each line, based on a 0.5 knot current at the top decreasing linearly to zero at the bottom. Actual current distributions at transponder locations may be different; however, the one assumed for the example is, in view of available data, quite conservative. Available current profile data indicates that in most cases current velocity decreases markedly below the mixing layer. Further, and perhaps most significantly, ocean currents below the mixing layer, which rarely exceeds 600 ft in depth in the region of interest, are relatively unchanging. Only the varying components contribute to position uncertainty. To stay within safe bounds, and to avoid the use of meaningless figures for ΔF_{D1} which are based on insufficient knowledge of current profiles, it was assumed that the position error introduced by the variable components of line drag forces is at worst far less than that from the larger combined buoy and tower drag. If the displacement due to total line drag is considered equal to buoy and tower displacement and in the same direction, the total position displacement of the triply moored buoy is less than 32 ft in the horizontal plane and 2.2 ft along the vertical axis.

4.3.3 Installation Considerations

Considerable care and planning is required for installing a deep ocean buoy with a piano wire mooring. The tension must be controlled carefully and the bottom anticipated, to slowly decelerate the anchor to avoid jerking, kinking or snagging on the bottom. Standard hydrographic winches are adequate to provide the controlled braking of the unspooling wire during decent. However, the spools should be modified to larger diameters to avoid bending the wire too sharply. Several hundred feet of line adjacent to the anchor and 10 ft adjacent to the buoy should be of a stronger and more flexible wire rope or chain to withstand the greater abuse given to these areas during installation.

The triply moored buoy has proven difficult to install in the past. These difficulties are greatly increased by the desire to use, to the fullest extent,

available line strengths to achieve maximum rigidity. Excess stresses may strain a wire beyond its yield point, permanently changing its length and weakening or even breaking it outright. Accuracy in anchoring positions is the easiest way of achieving the three equal and desired line tensions.

By attaching a sonar transponder to each anchor, accurate knowledge of relative anchor positions can be achieved. It is felt that the expense of self-powered sonar transponders can easily be justified for this purpose. (Bendix, Pacific Div., Model AT-051 at \$4000 is adequate.) The installing ship can then reposition the anchors as necessary, relative to each other and according to depth, before starting the common tie-together of the three lines at the buoy. These precautions will assure that the line tensions are equal and close to the design values when installation is complete.

Bringing the three lines together could most safely be done by separate light craft whose inertia and propulsion systems would be capable of supplying needed tension, while playing out additional line without the likelihood of snapping the lines or dragging an anchor through an error in control. The buoy, tower, and guy wires can all be connected to the three lines at the surface if a fourth line is attached to one of the anchors. This anchor should have been placed somewhat inward of the desired location, so that after connection of the buoy at the surface, this anchor may be moved outward to the planned position. This will submerge the buoy to the proper design depth.

The fourth line may be removed after installation by placing a fast corroding link at the bottom. Its presence could, however, be put to good use by attaching a secondary buoy to it, to measure its distance from the triply moored buoy. Large excursions will occur as a result of small drag force changes. A very sensitive measure is thus available for changes in ocean drag forces and hence changes of drag on the main buoy. The added drag data permits the computation of the triply moored buoy's movement from rest position.

A fourth independent ocean bottom transponder, anchored in the vicinity, could act as a reference for checking the relative positions of the anchors during installation and for future mooring inspections.

A study of sonar equipment was undertaken to ascertain the feasibility for operational determination of the position of a buoy, moored with a single line. The placement of three transponders on the ocean bottom, at approximately equal distances from the anchor point, promises to be a satisfactory way of measuring the present position of a buoy relative to the transponders to 20 or 30 feet with 95 percent confidence. The positions of the transponders must themselves be determined initially by making a group of range soundings from different known surface positions.

Sonar is particularly unsuited to buoy velocity determination. A cycle-counting measurement of the doppler frequency shift is used to measure the range rate in the direction of each transponder. Range rates in three orthogonal directions can be computed from these. The transponder response pulse must be lengthened to one second to provide sufficient time for an accurate count. Measurements of velocity are one-second averages, obtainable at intervals of a few seconds. Such measurements would be of little use for buoy motions. The high duty cycle required for a transponder producing one-second pulses greatly increases its power requirements or, conversely, reduces its operational life.

The major disadvantage in using sonar operationally is the need for large amounts of transmission power at the buoy. A data processing system is also needed at the buoy to interpret the range delay data. Such requirements are readily available on a ship, but to design a self-sufficient buoy to operate reliably for six months to a year with no maintenance is much more difficult and expensive. In fact, it conflicts with the AROD objective of installing inexpensive, simple, and reliable transponders at untended sea locations.

4.4 Conclusions and Recommendations

Upon completion of the preliminary investigation of oceanborne platforms for mounting AROD transponders, the result appears very encouraging for the development of a simple, inexpensive buoy. This concept is strongly recommended for further study to determine design feasibility. The alternative to the recommended design approach is probably a major manned subsystem, such as a ship, with a stabilized antenna platform—if the position-keeping and stability objectives assumed for this study are to be realized. Otherwise, the large position uncertainties (order of 1000 feet) and poor stability attendant with singly moored buoys seem inevitable.

The principal conclusions of this investigation are:

- 1) It appears that a low drag buoy (transponder platform) can be maintained within the established position error tolerance. No active referencing subsystem (such as sonar) is needed to measure operational position and/or velocity errors.
- 2) The buoy should be held in position by three mutually perpendicular mooring lines, each of which is step-tapered and made of high strength piano wire.
- 3) A sonar transponder on each mooring line anchor will simplify accurate installation procedures.
- 4) More exact error and transient analysis is dependent upon having further knowledge of the ocean environment at selected mooring sites.

A design feasibility study should proceed, with efforts directed toward:

- 1) Investigation and analysis of the transient behavior of the mooring lines to consider the varying tensions and the fact that a dampening fluid is present. Finite difference methods for the solution of a system of nonlinear partial differential equations is appropriate. A similar analysis was made at the David Taylor Model Basin (see ref. 10).
- 2) Analysis of the dynamic stability of the buoy and antenna, when subjected to waves, to determine the velocity error introduced into the AROD system.

- 3) Perform a composite, optimized design to determine the most favorable mooring line angles, wire diameter, float depth, tower height, and guying technique for varying ocean depths and currents.
- 4) Establish a requirement with the appropriate government agency to determine with precision the ocean environment at candidate transponder sites.
- 5) Determine the most efficient method of installing the mooring lines and buoy.
- 6) Determine the cost for constructing, placing, and maintaining oceanborne AROD transponders.

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Appendix I

COMPUTATION OF RANGE DERIVATIVES

Various analyses in this report require numerical values for the range (R) and range derivatives ($\dot{R}$, $\ddot{R}$, and $\dddot{R}$). These derivatives vary widely with elevation angle, slant range, and the vehicle velocity component along the slant range. Primary interest is on the maximum values of the range derivatives $\dot{R}_m$, $\ddot{R}_m$, and $\dddot{R}_m$ that will be encountered during a given mission. Three types of missions are being considered in this report: circular, launch, and translunar.

The equations for the range and range derivatives under the circular orbit assumption are easily derived from the geometry (shown in Figure 1). The equation for range will be

$$R = \left[(R_e + h)^2 + R_e^2 - 2 R_e (R_e + h) \cos \theta \right]^{1/2} \quad (1)$$

where R_e = earth radius

h = altitude

θ = central angle

The range derivatives obtained by differentiation are given by Equations (2), (3), and (4)

$$\dot{R} = \frac{R_e (R_e + h) \dot{\theta} \sin \theta}{R} \quad (2)$$

$$\ddot{R} = \frac{R_e (R_e + h) \ddot{\theta} \cos \theta - \dot{R}^2}{R} \quad (3)$$

$$\dddot{R} = -\dot{R} \left[\dot{\theta}^2 + \frac{\ddot{\theta} \dot{R}}{R} \right] \quad (4)$$

The rate of change of the central angle, $\dot{\theta}$, is related to the magnitude of the vehicle velocity by the expression

$$\dot{\theta} = \frac{V}{(R_e + h)} \quad (5)$$

For a circular orbit, the vehicle velocity will be

$$V = \left[\frac{3.986 \times 10^5}{(R_e + h)} \right]^{1/2} \text{ km/sec} \quad (6)$$

Since the limits on visibility for the AROD system are expressed in terms of the elevation angle, E , an expression for θ as a function of E is needed.

$$\theta = 90^\circ - E^\circ - \text{Arcsin} \left\{ \frac{R_e}{R_e + h} \cos E^\circ \right\} \quad (7)$$

For missions using circular orbits, the range derivatives will be greatest for small altitudes, h . For this reason, an altitude of 90 nautical miles will be chosen as typical of the lowest practical circular orbit. Since $\dot{R}$ and $\ddot{R}$ will be a maximum at the smallest allowable elevation angle, and $\ddot{R}$ will be a maximum at the zenith, elevation angles of 5° and 90° will be chosen for the computation. The results of these computations are given in the following table.

Elevation Angle ($^\circ$)	r (km)	$\dot{r}$ (km/sec)	$\ddot{r}$ (km/sec ²)	$\dddot{r}$ (km/sec ³)
5	1.02×10^3	7.54	1.74×10^{-3}	-4.94×10^{-5}
90	1.67×10^2	0	3.56×10^{-1}	0

Therefore, for a circular orbit the maximum values of $\dot{R}_m$, $\ddot{R}_m$, and $\ddot{R}_m$ will be 7.5×10^3 m/sec, 360 m/sec², and 0.05 m/sec³ respectively.

Non-circular trajectories, such as those for launch or translunar missions, affect the geometry shown in Figure I-1 by changing the direction of the velocity vector V and introducing a vehicle acceleration $\dot{V}$. The values of $\dot{R}_m$, $\ddot{R}_m$ and $\ddot{R}_m$, previously given, can be corrected to account for these effects.

The range rate, $\dot{R}$, has a maximum value of V when the vehicle is moving directly toward the transponder. A worst case value of $V = 8 \text{ km/sec}$ can be chosen for $\dot{R}_m$. The contribution to $\ddot{R}$ from the vehicle acceleration $\dot{V}$ will also occur when the vehicle is moving along the slant range. However, the geometrically induced contribution to $\ddot{R}$ given in the above table is a maximum when the vehicle velocity is perpendicular to the slant range (i.e., directly overhead for the circular orbit case). Since the maximum value of V will be less than the geometrically induced acceleration (i.e., only one-third as large for a 10g acceleration) the maximum $\dot{R}$ will be $\dot{R}_m = 360 \text{ m/sec}^2$.

The effect of power flight on the maximum value of $\ddot{R}$ is determined by the rate of change of acceleration of the vehicle, $\dot{a}$. Assuming constant thrust and specific impulse, and ignoring such factors as atmospheric drag, the equation for $\dot{a}$ at any given instant in time will be

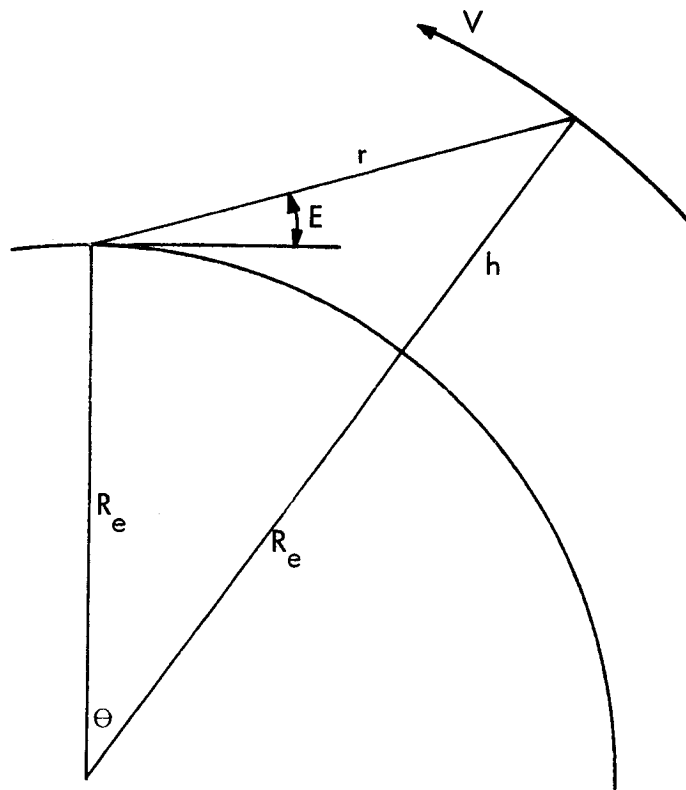
$$\dot{a} = \frac{g}{I} \left(\frac{T}{w} \right)^2 \quad (8)$$

where $g = 32.2 \text{ ft/sec}^2$

I = specific impulse

T/w = Thrust to weight ratio

To maximize the estimate of $\dot{a}$, a minimum practical value for I of 300 sec will be chosen. For a modest maximum thrust/weight ratio of 2, Equation (8) gives $\dot{a} = 0.13 \text{ m/sec}^3$. Therefore, the maximum value of R will depend on power flight considerations rather than geometrical effects.



r = Transponder-to-Vehicle Distance
 Θ = Central Angle
 V = Vehicle Velocity
 E = Elevation Angle
 h = Altitude
 R_e = Mean Earth Radius

Figure I-1. Geometry for Range Derivative Computations

Appendix II

DOPPLER SHIFT EQUATION DERIVED FOR AROD

The exact form of the relationship between the doppler shift in frequency observed at the vehicle receiving end of AROD and the range rate parameter will be derived. This is done to aid in establishing auxiliary computation requirements. The expression, or its derivation, will not explicitly involve the doppler frequency (f_d) as such. What is desired is an expression relating the range rate ($\dot{r}$) to an actually measured quantity and the constants of the measurement situation. The derivation will be carried out with consideration of the effect of the Theory of Special Relativity. This theory is inherently linked with the concept of the velocity of light being a fundamental constant of nature, regardless of the reference frame within which observations are made. While terms of the form $(\dot{r}/c)^2$ are certainly negligible for AROD, terms of the form $(\dot{r}/c)$ are not. Using Special Relativity gives the exact form for two-way shift in the received frequency, which by the use of Classical Relativity principle can only be obtained by approximation. The effects of ionospheric and tropospheric propagation mediums are not considered at this time for purposes of this derivation, for either the Classical or Special Relativity principles.

Consider that the transmitted frequency at the vehicle sensor is f_{t_s} . Due to motion of the vehicle transmitter with respect to the reference frame associated with the AROD ground transponder receiver, the time scale is "slowed" by the Lorentz transformation to account for the effect of Special Relativity:

$$f_{t'_s} \text{ (at transponder)} = f_{t_s} \sqrt{1 - \left(\frac{\dot{r}}{c}\right)^2}.$$

The frequency received is also modified by a term to account for the doppler, i.e., a shortening of the received wavelength due to the source moving towards the observer, as follows:

$$f_{r_g} \text{ (at transponder)} = f_{t_s} \left[\sqrt{1 - \frac{\dot{r}^2}{c^2}} \right] \cdot \left[\frac{c}{c - \dot{r}} \right],$$

where the term $c/c - \dot{r}$ represents this effect. This can be reduced to the form

$$f_{r_g} = f_{t_s} \sqrt{\frac{c^2 - \dot{r}^2}{(c - \dot{r})^2}},$$

by suitable algebraic manipulation.

At the ground station transponder the received signal is processed in such a manner as to develop a new signal frequency effectively offset by a translation frequency, f_T . This signal is then transmitted by the transponder and subsequently received by the vehicle receiver. The same considerations applied to the foregoing downlink are now applied to the uplink. The result for the frequency of the received signal at the vehicle is:

$$f_{r_s} = \left[f_{t_s} \sqrt{\frac{c^2 - \dot{r}^2}{(c - \dot{r})^2}} + f_T \right] \cdot \left[\sqrt{\frac{c^2 - \dot{r}^2}{(c - \dot{r})^2}} \right]$$

Algebraic manipulation and the approximation ($\dot{r}^2 \ll c^2$) applied to the term associated with f_T result in the following expression for the received signal frequency:

$$f_{r_s} \cong f_{t_s} \left(\frac{c + \dot{r}}{c - \dot{r}} \right) - f_T \left(\frac{c}{c - \dot{r}} \right)$$

Note the form of the term associated with f_{t_s} is exact.

The receiver signal processing consists of successive downconversions by first heterodyning against f_{t_s} , and then against an L.O. signal, f_x . This results in a final output signal, the frequency of which is subsequently measured. This frequency is termed f_{out} , and is obtained by manipulation of the expression for f_{r_s} with the respective downconverting frequencies

to arrive at the following:

$$f_{out} = f_{t_s} \left[\frac{2\dot{r}}{c - \dot{r}} \right] - f_T \sqrt{\frac{c^2 - \dot{r}^2}{c - \dot{r}}} - f_x$$

in which f_{t_s} , f_T and f_x are the known quantities.

Solving for $\dot{r}$ as an explicit function of f_{out} , the measured quantity, results in

$$\dot{r} \cong \frac{c(f_{out} - f_x)}{2f_{t_s} + f_{out} - f_x + f_T} \cdot \epsilon = \frac{A' + C'f_{out}}{B' + f_{out}} \cdot \epsilon$$

where ϵ is of the form

$$\epsilon \cong \frac{1}{1 - \frac{1}{4} \frac{\dot{r}}{c} \frac{f_T}{f_{t_s}}}.$$

Considering the values of AROD system parameters as presently defined, ϵ approaches zero rapidly. Consequently, the equation for an auxiliary computation procedure to transform extracted doppler shift information into virtual (see Section 1.2.2) range rate can be expressed as:

$R = r = \frac{A + Cx}{B + x}$ where x is the actually measured quantity, i.e., cycle count or some other suitable form, and A , B , and C are constants reflecting system parameters f_{t_s} , f_x and f_T as defined by the substitution process carried out above. This equation shows that simple scaling is inadequate as a transforming process for doppler shift to range rate.