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RADIO VISIBILITY - ANTENNA LOOK ANGLE FORMULATION FOR AN APOLLO ATMOSPHERIC ENTRY

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SUMMARY

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The formulation of the antenna look angle, δ_{ij}, which is defined as the angle between the axis of propagation of a vehicle-borne antenna and the negative of the line of sight vector from an arbitrary radar station is presented in this paper. The application of this angle to "radio visibility" during entry is described and results from a guided Hawaiian entry are shown in Figures 5, 6 and 7. Other applications of the technique are discussed in Section IIB.

From this look angle and the assumed half cone angle of the vehicle antenna radiation pattern, it is possible to predict whether the vehicle is "radio visible" at a tracking site or not.

author

GLOSSARY

$\vec{A}_i^0$ Unit vector in body axis system used to represent propagation axis of body fixed antennas

δ_{ij} antenna look angle for the ith antenna and the jth radar station (See Fig. 1)

θ_b antenna orientation angle. See Figure 2.

ϕ_i antenna orientation angles i = 1 ... 4. See Figure 2. These angles describe the antenna orientation in the body axis system.

$\vec{R}_j^0$ unit radius vector from jth radar station to vehicle

α_T trim angle of attack of entry vehicle

γ vehicle flight path angle

Λ_Z vehicle azimuth angle

β vehicle bank angle

E_j elevation angle of entry vehicle as seen by a given radar station

ϕ_G geodetic latitude of entry vehicle (subvehicle point)

λ_G earth fixed longitude of entry vehicle (subvehicle point)

λ_E longitudinal position of Greenwich meridian relative to the Vernal Equinox

T_{B2G} transformation matrix used to transform from body to geodetic frame. The notation means transformation <u>from</u> body <u>to</u> geodetic

T_{S2G} transformation matrix used to transform <u>from</u> station <u>to</u> geodetic frame

i subscript of antenna number

j subscript of radar station number

B subscript denoting body frame

G subscript denoting geodetic frame

I subscript denoting inertial frame

S subscript denoting station coordinate system

RADIO VISIBILITY – ANTENNA LOOK ANGLE FORMULATION
FOR AN APOLLO ATMOSPHERIC ENTRY

INTRODUCTION

The purpose of this report is twofold, namely,

a. To present the formulation of the antenna look angle, which is defined as the angle between the axis of propagation of a vehicle-borne antenna and the negative of the line of sight vector from an arbitrary radar station. (See Figure 1).

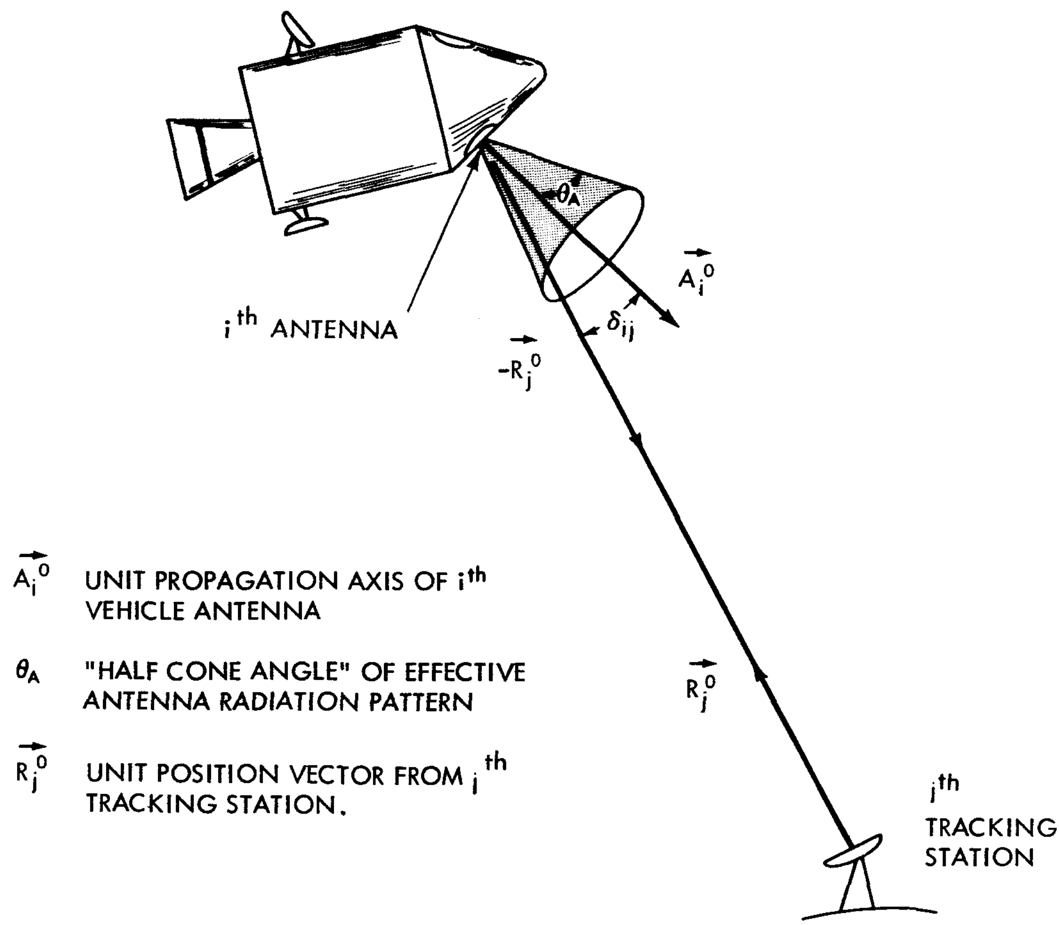

Figure 1–Antenna look angle, δ_{ij} for an arbitrary vehicle

b. To present applications of the antenna look angle concept. Incorporated in this paper are results which were achieved when the antenna look angle formulation was coupled to a guided entry trajectory program. Also included is a discussion of the general applications of this technique.

The antenna look angle was developed as an aid in determining "radio visibility" regions for a spacecraft or other vehicle. These equations may easily be incorporated into any program which can determine the vehicle position, velocity and attitude as a function of time.

I. DERIVATION OF ANTENNA LOOK ANGLE

The antenna look angle, δ_{ij}, is defined to be the angle between the i^{th} antenna propagation axis ($\vec{A}_i^0$) of a vehicle-borne antenna and the negative of the line of sight vector from the j^{th} radar station ($\vec{R}_j^0$). Refer to the glossary for a complete definition of symbols.

$$\delta_{ij} = \cos^{-1} (-\vec{A}_i^0 \cdot \vec{R}_j^0) \tag{1}$$

The subscripts i and j denote the antenna number and radar station number, respectively. The geometry of δ_{ij} is shown in Figure 1.

In order to derive $\vec{A}_i^0$, we go through the following algebra. (See Figure 2). A unit vector $\vec{A}_i^0$ analogous to the propagation axis of an antenna may be defined in a body axis system

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}$$

which is attached to the body and moving with it as

$$\vec{A}_i^0 = -\sin\theta_b \, \vec{i} + \cos\theta_b \sin\phi_i \, \vec{j} + \cos\theta_b \cos\phi_i \, \vec{k} \tag{2}$$

by the following technique:

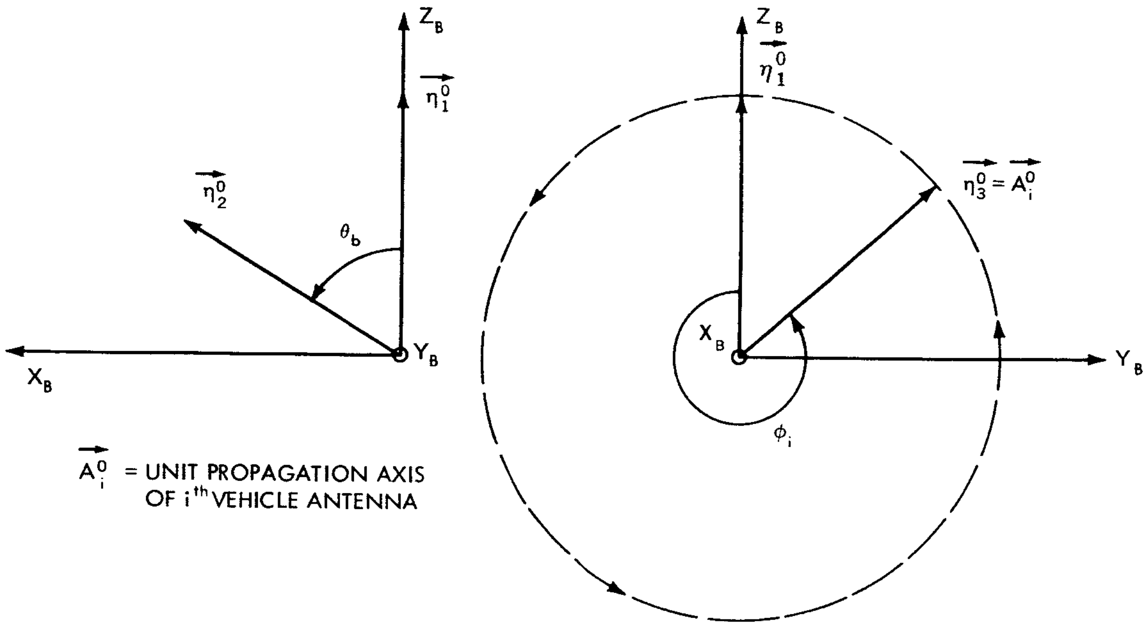

Figure 2–$\vec{A}_i^0$ in body borne coordinate system; formed by rotating $\vec{\eta}_1^0$ through θ_b and ϕ_i about Y_B and X_B respectively

Define a unit vector $\vec{\eta}_1^0$ in the body axis system. This unit vector is along the negative of the position vector when the vehicle is in a zero roll, zero flight path attitude with zero angle of attack.

$$\vec{\eta}_1^0 = \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} \tag{3}$$

Rotate $\vec{\eta}_1^0$ about Y_B through θ_b to form $\vec{\eta}_2^0$ using the relationship that

$$\vec{\eta}_2^0 = \begin{bmatrix} \cos\theta_b & 0 & -\sin\theta_b \\ 0 & 1 & 0 \\ \sin\theta_b & 0 & \cos\theta_b \end{bmatrix} \vec{\eta}_1^0 \tag{4}$$

Rotate $\vec{\eta}_2^{\,0}$ about X_B through ϕ_i to form $\vec{\eta}_3^{\,0}$

$$\vec{\eta}_3^{\,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_i & +\sin\phi_i \\ 0 & -\sin\phi_i & \cos\phi_i \end{bmatrix} \vec{\eta}_2^{\,0} \tag{5}$$

Refer to Figure 2 for an illustration of the above rotations. We now have

$$\vec{\eta}_3^{\,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_i & +\sin\phi_i \\ 0 & -\sin\phi_i & \cos\phi_i \end{bmatrix} \begin{bmatrix} \cos\theta_b & 0 & -\sin\theta_b \\ 0 & 1 & 0 \\ \sin\theta_b & 0 & \cos\theta_b \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ +1 \end{bmatrix} \tag{6}$$

which when expanded may be seen to be

$$\vec{\eta}_3^{\,0} \equiv \vec{A}_i^{\,0} = \begin{bmatrix} -\sin\theta_b \\ \cos\theta_b \sin\phi_i \\ +\cos\theta_b \cos\phi_i \end{bmatrix} \equiv -\sin\theta_b\,\vec{i} + \cos\theta_b \sin\phi_i\,\vec{j} + \cos\theta_b \cos\phi_i\,\vec{k}$$

which is Equation (2). $\vec{A}_i^{\,0}$ is a unit vector in a body axis system and may be arbitrarily defined by choosing θ_b and ϕ_i . This unit vector is analogous to the propagation axis of an antenna.

In order to compute δ_{ij} , Equation (1), it is necessary to express $\vec{A}_i^{\,0}$ and $\vec{R}_j^{\,0}$, the line of sight vector from the j^{th} station to the entry vehicle, in a common reference frame. A reasonable choice might be the typical geodetic or G system that is shown in Figure 3. The G frame is a right-handed orthogonal coordinate system with origin at the surface of the Earth at the subvehicle point with the Z_G axis positive inward along the radius vector from the vehicle to the Earth. For the sake of generality, no specific transformation will be defined; and only an operational form of the necessary transformation matrices will be used.

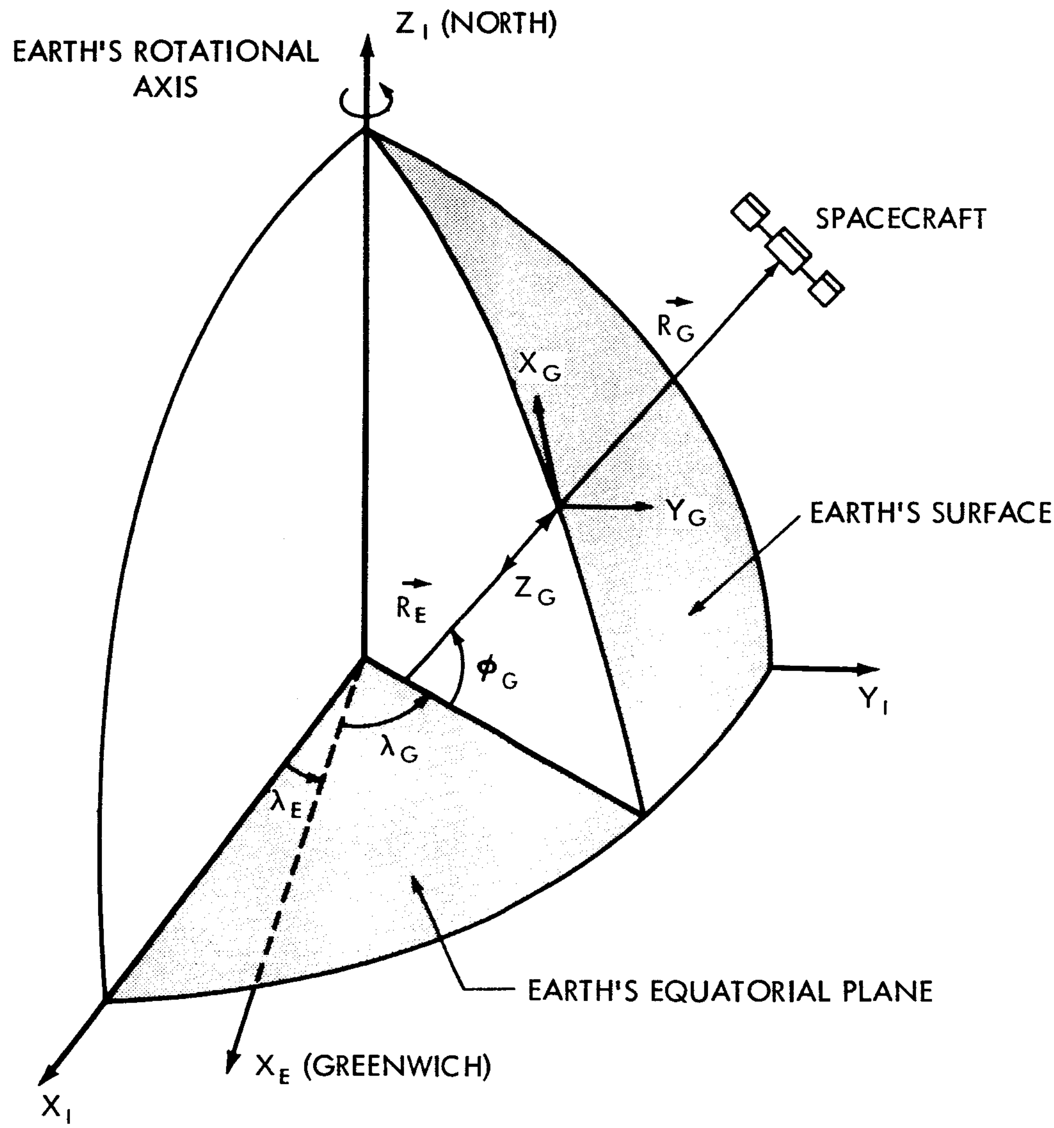

Figure 3–Typical geodetic coordinate system relative to inertial system

It is possible to convert both $\vec{A}_i^0$ and $\vec{R}_j^0$ to a geodetic frame by defining: T_{B2G}[1] as a (3×3) transformation matrix used to transform from the body

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}$$

system to the geodetic

$$\begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix}$$

system and T_{S2G} as a transformation from the station coordinate system to the G frame. T_{B2G} can be expressed as a function of γ, A_z, α_T and β. See the Glossary for a definition of symbols. T_{S2G} can be expressed as a function of ϕ_G, λ_G, and λ_E. $\vec{A}^0_{G_i}$ and $\vec{R}^0_{G_j}$ can now be expressed as follows:

$$\vec{A}^0_{G_i} = T_{B2G}\vec{A}_i^0 \tag{7}$$

and

$$\vec{R}^0_{G_j} = T_{S2G}\vec{R}^0_{S_j} \tag{8}$$

The antenna look angle, Equation (1), is shown in Figure 1 and is expressed as:

$$\delta_{ij} = \cos^{-1}\left(-\vec{A}^0_{G_i} \cdot \vec{R}^0_{G_j}\right)$$

[1]The notation T_{B2G} means transformation from body to geodetic.

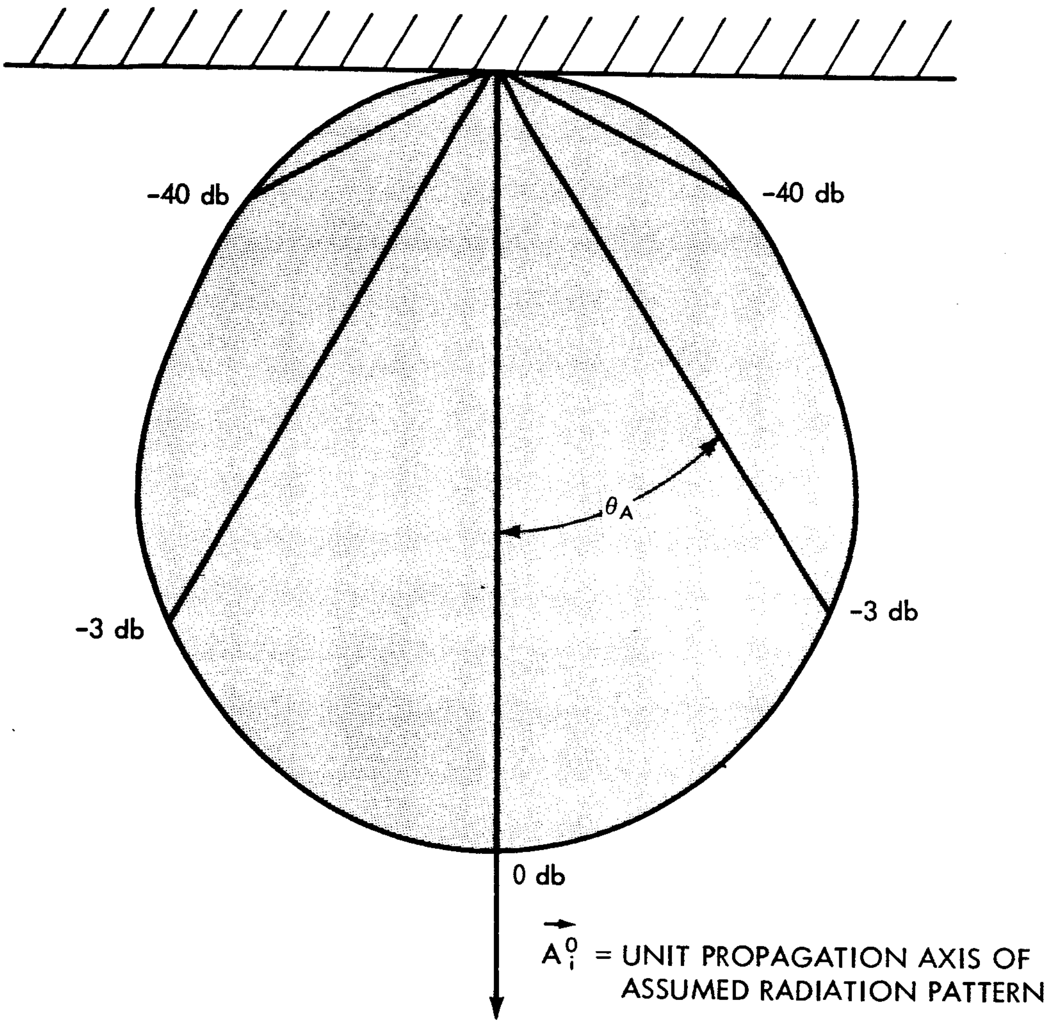

Figure 4—Relationship between antenna gain and "half cone angle" θ_A, for an arbitrary antenna

The criterion for a entry vehicle to be "radio" visible with respect to a given radar station is

$$\delta_{ij} \leq \theta_A \tag{9}$$

where θ_A is the half cone angle of the effective antenna radiation pattern. The half cone angle of the effective radiation pattern is chosen according to the known antenna characteristics. Figure 4 shows the relationship of antenna gain to θ_A for an arbitrary antenna. For this antenna one would choose $\theta_A = 30°$ in order to represent the point where the gain is 3 db down. Similarly, $\theta_A = 60°$ would represent the 40 db down point for this arbitrary pattern. On the basis of a priori knowledge, one can choose an appropriate θ_A to correspond to the necessary antenna gain that is required for the signal to be above the threshold level at the ground station. Since "radio" visibility is dependent on the radius vector from the ground station being contained in the "effective" radiation cone, optical visibility in no way implies radio visibility.

II. APPLICATIONS OF ANTENNA LOOK ANGLE FORMULATION

A. Entry

The formulation in Section I for δ_{ij}, the antenna look angle, has been incorporated in the General Electric Missile and Satellite Systems Program (Ref. 1, hereafter abbreviated as G. E. Mass). The version of the G. E. Mass program to which this calculation was added has an entry guidance (Ref. 2) and blackout prediction (Ref. 3) capability. The addition of the antenna look angle computation greatly increases the usefulness of this program for tracking communications planning.

The present program has the capability of inputing up to four antennas arbitrarily oriented about the longitudinal axis of the vehicle by the angle ϕ_i. For convenience in adding the look angle formulation to the G. E. Mass program, $\vec{A}_i^0$, was defined to be along the local vertical when the vehicle is in a zero roll, zero flight path attitude with zero angle of attack. This was done so that the antenna roll orientation angle ϕ_i corresponds to the vehicle roll angle, β.

To illustrate the utility of the revised Mass program, a 2,200 n. mi. guided (Hawaiian) entry trajectory was run. Figure 5 shows a plot of altitude vs range to go for this trajectory. Also included on this plot are station visibility and predicted blackout bounds. Figure 6 shows the ground track of this trajectory and the station locations.

Figure 7 depicts the time history of the antenna look angle for four antennas. Also included on this plot are the time histories of the elevation and bank angles for the guided entry trajectory which is described above. These plots are in order of station visibility. The elevation angle plots are labeled to identify the tracking station. Figure 8 shows how Radio Visibility regions can be determined from the antenna look angle time history.

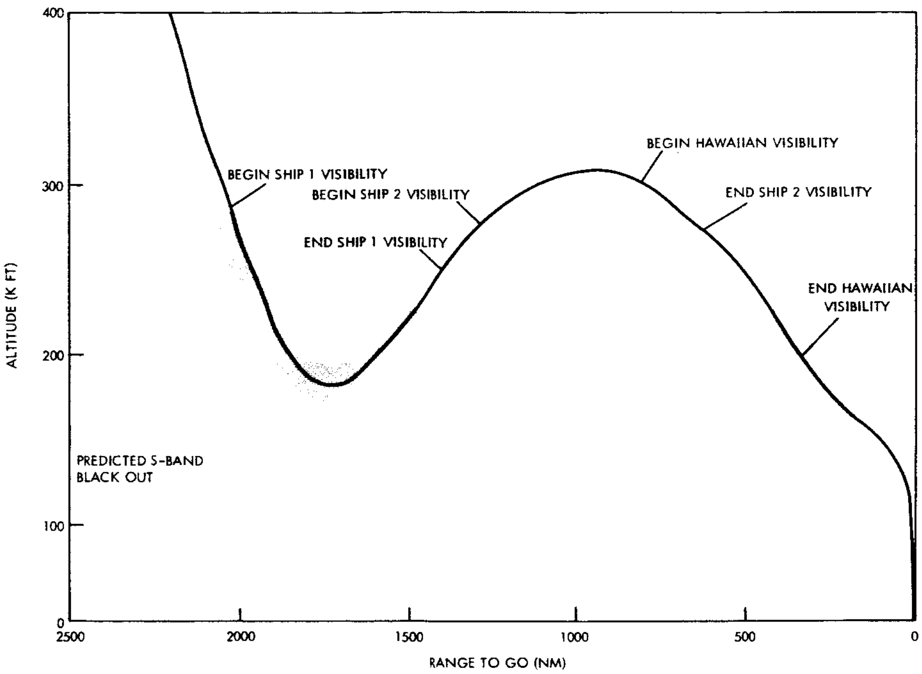

Figure 5—2200 NM Hawaii entry trajectory showing predicted S-band blackout regions and ground station optical visibility assuming 5° horizon

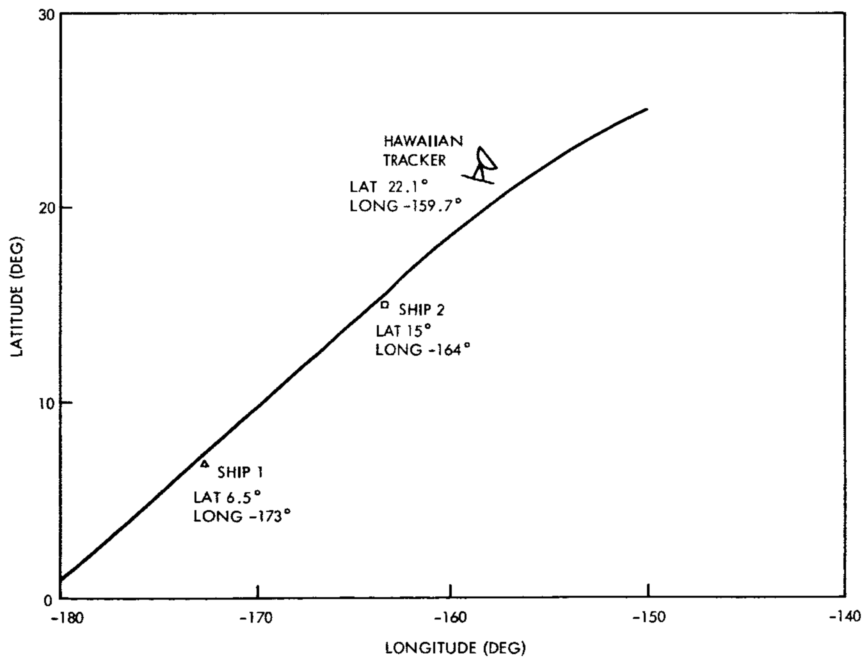

Figure 6—Ground track of Hawaiian entry showing ships and and Hawaiian tracker

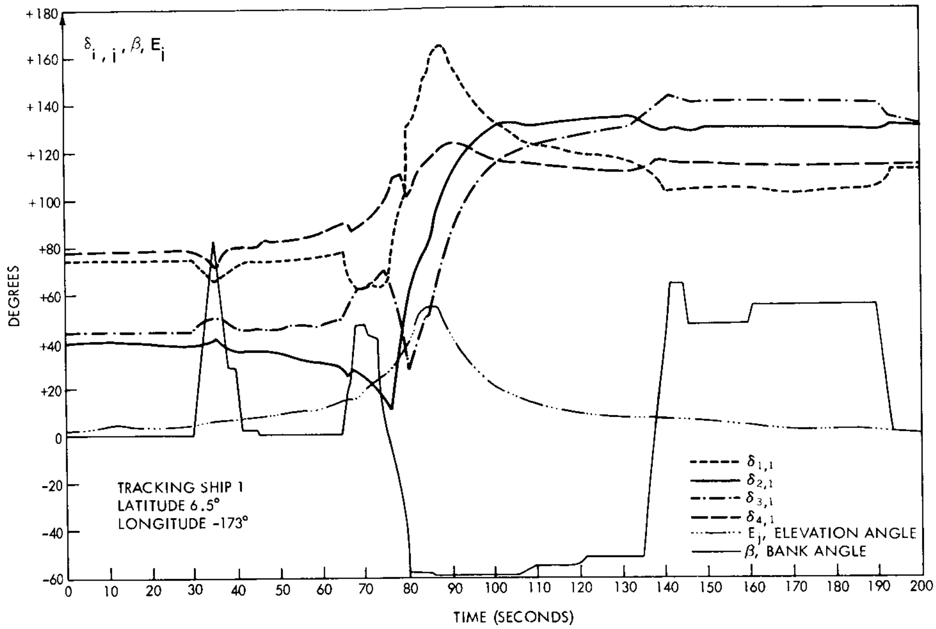

Figure 7a—Time history of antenna look angles, δ_{ij}, for four vehicle borne antennas during pass over ship 1 in Hawaiian entry

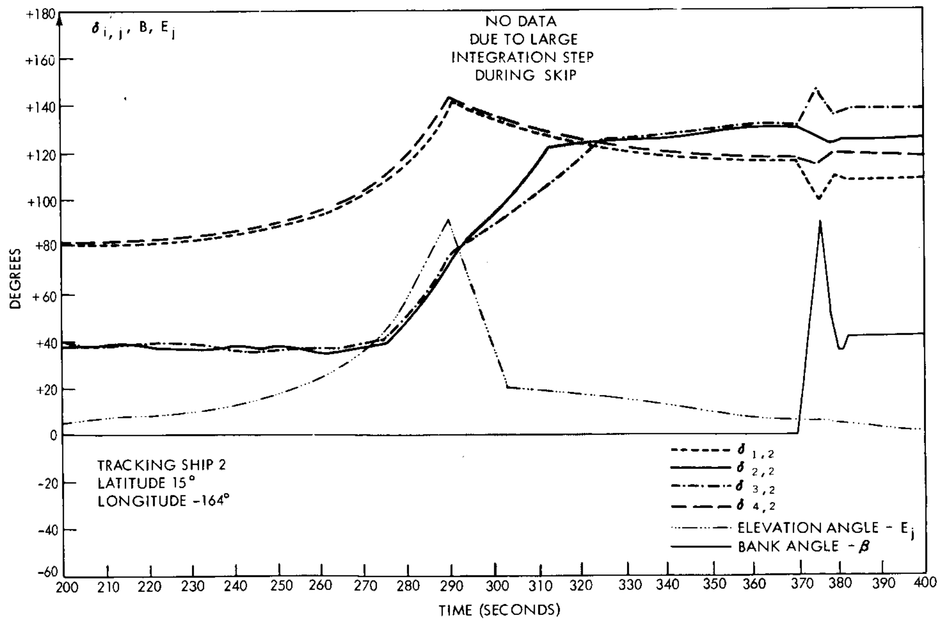

Figure 7b—Time history of antenna look angles, δ_{ij} , for four vehicle borne antennas during pass over ship 2 during Hawaiian entry

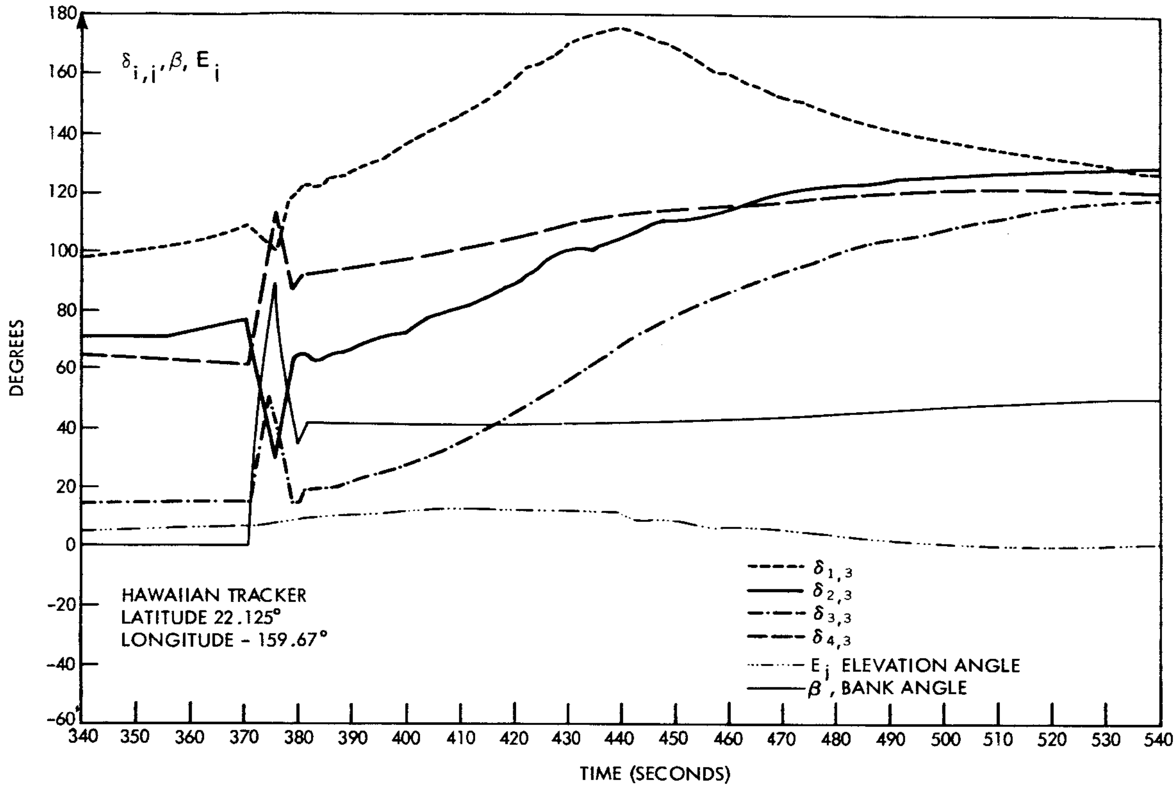

Figure 7c—Time history of antenna look angles, δ_{ij}, for four vehicle borne antennas during pass over Hawaiian tracker

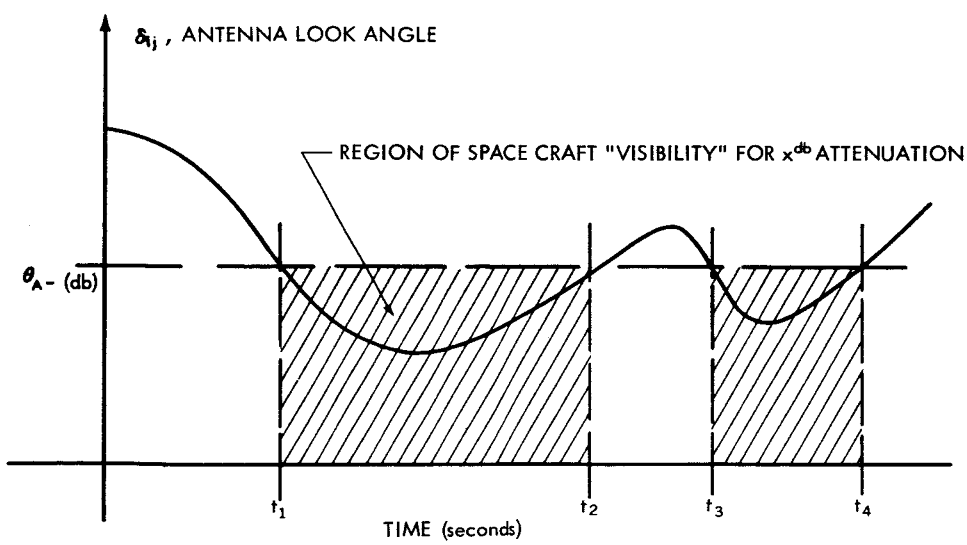

Figure 8—"Radio visibility" regions for a particular ground station

1. Input data for Hawaiian Entry Trajectory

a. Entry conditions

Entry velocity	36093 (ft/sec)
Entry flight path angle	-6.786°
Entry azimuth	50.45°
Entry altitude	399715 ft
Entry latitude	1.652°
Entry longitude	-179.1°
Entry trim angle of attack	20.83°

b. Radar Station Locations

Latitude Ship 1	6.5°
Longitude Ship 1	-173°
Latitude Ship 2	15°
Longitude Ship 2	-164°
Latitude Hawaiian Tracker	22.125°
Longitude Hawaiian Tracker	-159.67°

c. Target Position

Latitude	25°
Longitude	-150°

d. Antenna Orientation Information

θ_B	33°
ϕ_1	45°
ϕ_2	135°
ϕ_3	-135°
ϕ_4	-45°
θ_A	60°

2. Output

The following output is taken directly from the nominal 2,200 n. mi. trajectory whose input is given above:

```
T=  0.37700000E 03   RFT=  0.18843388E 07   RNM=  0.31012291E 03   AZ=  0.48333061E 02   EL=  0.55277950E 01   RAD  2
                     ANG1=  0.10250472E 03  ANG2=  0.12422049E 03  ANG3=  0.14331445E 03  ANG4=  0.11713362E 03
                     IN VIS                 IN VIS                 IN VIS                 IN VIS
T=  0.37700000E 03   RFT=  0.14201894E 07   RNM=  0.23373358E 03   AZ=  0.18479679E 03   EL=  0.88345736E 01   RAD  3
                     ANG1=  0.11011950E 03  ANG2=  0.41731057E 02  ANG3=  0.39448206E 02  ANG4=  0.10854550E 03
                     IN VIS                    VIS                    VIS                 IN VIS
```

The symbols are:

T = Time of flight (sec)

RFT = Range in ft. from station j

RNM = Range in n. mi. from station j

AZ = Azimuth of vehicle as seen from station j (Deg)

EL = Elevation of vehicle as seen from station j (Deg)

ANG1 = $\delta_{1,j}$

ANG2 = $\delta_{2,j}$

ANG3 = $\delta_{3,j}$

ANG4 = $\delta_{4,j}$

INVIS = Key which indicates $\delta_{ij} > \theta_A$ (i.e. stations cannot "see" antenna i)

VIS = Key which indicates $\delta_{ij} \leq \theta_A$

Although concurrent output from two stations is shown above, this information is printed out for a given station only when the elevation angle E_j is greater than a nominal input value.

B. General

The utility of the antenna look angle concept need not be restricted to any particular mission phase. It may be incorporated into any program which can specify a vehicle's position, velocity and attitude as a function of time. This program may be a complete six degrees of freedom program which simulates a vehicle's motion from launch to return landing or impact, or it may simply be a data reduction program which processes flight test data.

A vehicle with an arbitrary roll axis is shown in Figure 9. The orientation of $\vec{A}_i^0$ may be defined by choosing θ_b and ϕ_i as in Section I. Once this has been done, a common coordinate system may be chosen and $\vec{A}_i^0$ and $\vec{R}_j^0$ may both be expressed in this system. The angle between these two vectors is the antenna look angle δ_{ij}.

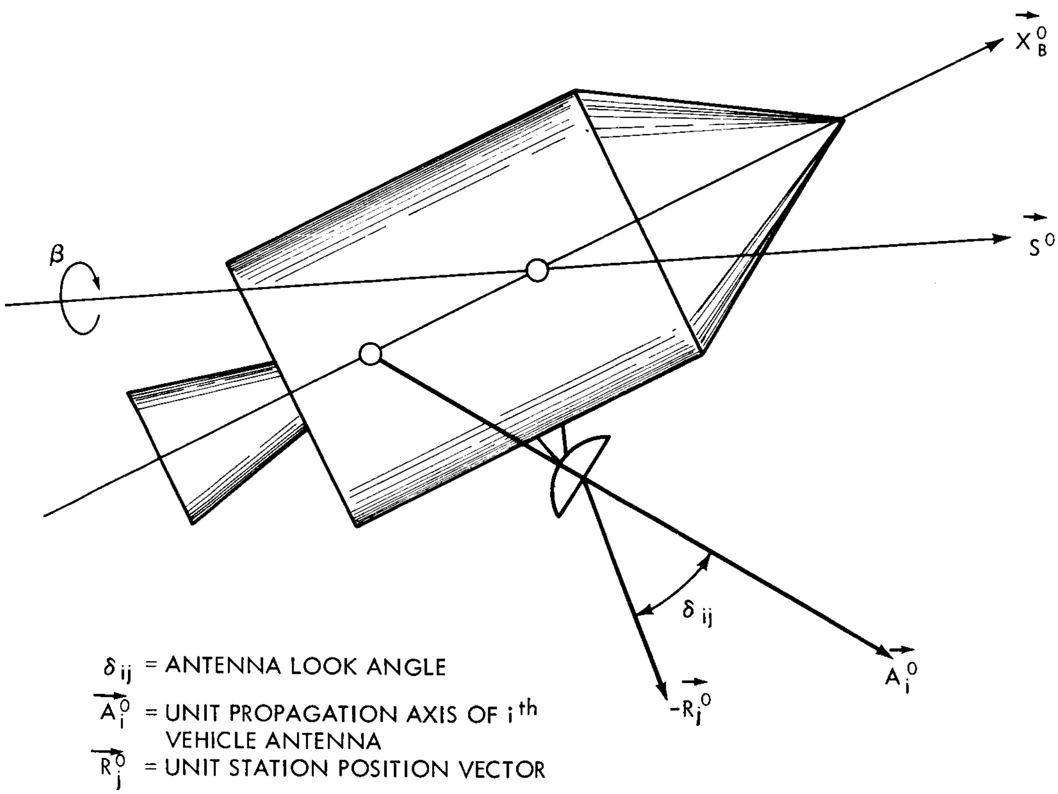

Figure 9–Vehicle with arbritrary roll axis $\vec{S}^0$

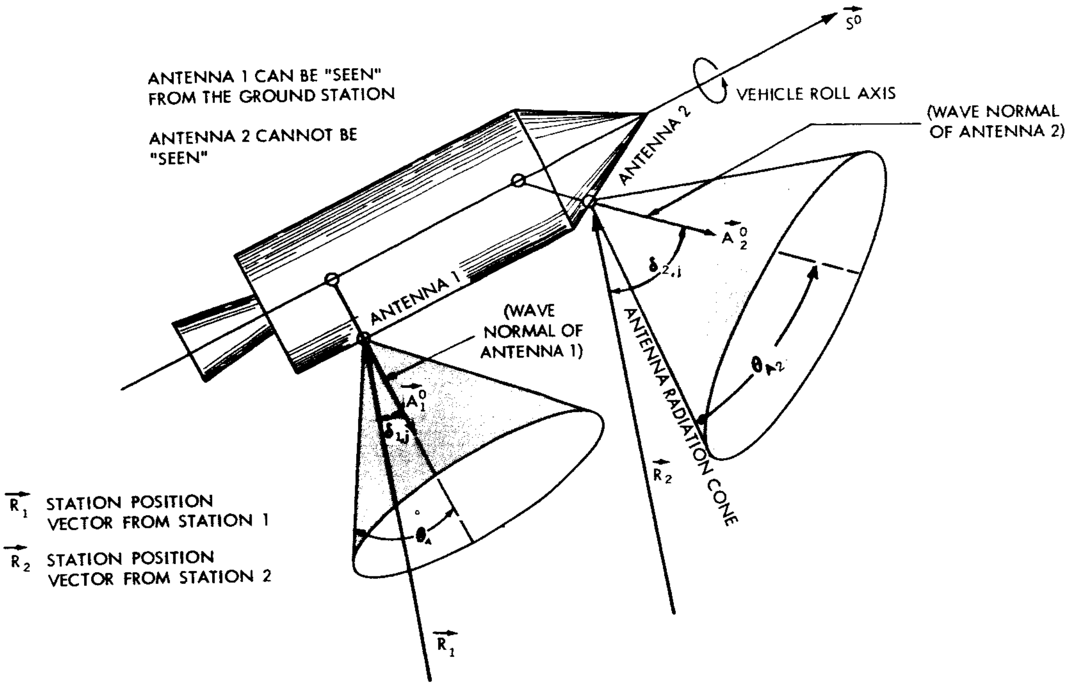

Figure 10–Space vehicle "radio visibility"

Another situation which this technique will handle with ease is depicted in Figure 10. The problem is this: there are two antennas on a spacecraft. Antenna 1 is located on the booster, and antenna 2 is located on the nose of the vehicle. When is each antenna visible with respect to a given ground station? The solution is as follows:

1. Locate these antennas the desired distance apart and orient their propagation axis by specifying θ_{b_1} and ϕ_1, θ_{b_2} and ϕ_2.

2. Define the half cone angle of the effective antenna radiation pattern for each antenna (θ_{A_1} and θ_{A_2}).

3. The antenna look angle for each antenna is:

$$\delta_{1,j} = \cos^{-1} (-\vec{A}_1^{\,0} \cdot \vec{R}_1^{\,0})$$

and

$$\delta_{2,j} = \cos^{-1} (-\vec{A}_2^0 \cdot \vec{R}_2^0)$$

assuming that each vector is in the same coordinate system.

The antenna orientation angles need not be constant, but may be varied or directed by suitable steering equations.

III. CONCLUSIONS

The antenna look angle concept provides a useful tool for the analysis of "radio visibility" regions with respect to a spacecraft and arbitrary tracking stations. The techniques may be applied to any phase of a mission including launch, coast, entry, etc. In order to utilize the antenna look angle formulation, all that is required is that the position, velocity, and attitude of the vehicle be expressed as a function of time.

ACKNOWLEDGMENT

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