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The Electric Field Generated by a Rotating Magnetized Sphere+

by

Edward W. Hones, Jr.*
and
John E. Bergeson**

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*On leave of absence from the Institute for Defense Analyses, Arlington, Virginia.

**Graduate Trainee of the National Aeronautics and Space Administration.

ABSTRACT

The electric field outside a steadily rotating, uniformly magnetized sphere is determined for the general case in which the magnetic and rotational axes, though both passing through the center of the sphere, may be oriented at any angle relative to each other. The sphere is perfectly conducting and is surrounded by a conducting plasma of charged particles which are constrained to move along the magnetic field lines. The electric field generated by the rotating sphere is found to be just that required to cause the surrounding plasma to rotate with the sphere. When the magnetic and rotational axes are parallel or anti-parallel, co-rotation of sphere and plasma is caused by the $\vec{E} \times \vec{B}$ drift. For all other orientations Fermi acceleration plays a role in causing co-rotation. The electric field in a reference frame rotating with the sphere is identically zero for the symmetric (dipole) magnetic field under consideration. Therefore, charged particles in the plasma do not change energy in this frame, although they appear alternately to gain and lose small amounts of energy in a non-rotating frame. It is concluded, however, that the electric field generated by the earth's "wobbling" magnetic axis in the real magnetosphere, distorted by the solar wind, probably

does cause charged particles to experience net energy changes over a number of revolutions around the earth. It thus provides a mechanism for diffusion of plasma and of higher energy particles through the magnetosphere. Calculations of such effects must take into account the high conductivity of the plasma in the magnetosphere if they are to give correct results.

I. INTRODUCTION

The role that the earth's rotation plays in determining the characteristics of the electric and magnetic fields, the low energy plasma, and the high energy trapped radiation in the magnetosphere is essentially unknown. Certain authors, in developing models of the magnetosphere, have included a circulatory pattern of plasma convection driven by the earth's rotation [Axford and Hines, 1961; Johnson, 1960]. Plasma on low-latitude magnetic field lines is assumed to rotate with the earth, whereas plasma on the high-latitude lines which form the magnetospheric tail is assumed to rotate in the opposite direction about an axis in the tail. By mapping electric fields inferred from high-latitude magnetic deviations outward into a model of the magnetosphere, Taylor and Hones [1965] have shown that a plasma circulation pattern of this nature does, indeed, seem to be present.

Davis [1947, 1948] showed that plasma surrounding a rotating, conducting, magnetized sphere will rotate with the sphere if the plasma particles are constrained to move along the magnetic lines of force. Under these conditions charge will flow in the plasma until the electric field

$$\vec{E} = -\frac{1}{c}(\vec{\omega} \times \vec{r}) \times \vec{B} \quad (1)$$

(as seen in a nonrotating reference frame) is established in the plasma. Here $\vec{\omega}$ is the angular velocity of the sphere, $\vec{r}$ is the radial position vector and $\vec{B}$ is the magnetic field vector. The same sphere, rotating *in vacuo*, however, generates a quadrupole electric field in the surrounding space [Swann, 1920; Davis, 1947] and this will *not* cause individual ions and electrons to rotate with the angular speed of the sphere.

In all of the works referred to above, the magnetic and rotational axes of the sphere are assumed parallel (or anti-parallel) and the inductive electric field associated with the wobble of the dipole is ignored. Terletzky [1946] considered the effect of the induction field generated by a rotating, magnetized sphere with nonaligned rotational and magnetic axes and concluded that in the space around the earth particles would be energized to tens of kilovolts by the component of this electric field parallel to the magnetic lines of force--an effect which would be of considerable geophysical importance. However, Terletzky's electric field was simply the field induced by a magnetic dipole of moment $\vec{\mu}$, rotating with angular velocity $\vec{\omega}$:

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} = \frac{1}{cr^3}\left[\vec{r} \times (\vec{\omega} \times \vec{\mu})\right] \tag{2}$$

where $\vec{r}$ is the radial position vector in a nonrotating frame of reference. Terletzky ignored the electric polarization charge generated on the earth and, equally important, the charge generated in the conducting plasma around the earth. One would expect that the plasma's high conductivity parallel to the magnetic field lines essentially cancels the parallel component of the induction field; hence the particle energization process visualized by Terletzky probably does not exist. However, the component of the induction field perpendicular to the magnetic lines is not cancelled out, and may be expected to energize or de-energize particles by its action in the direction of their magnetic gradient and line curvature drifts.

Backus [1956] studied the rotating, magnetized sphere with arbitrarily aligned rotation and magnetic axes. Though he included the sphere's polarization charge in his calculation, he, too, ignored the important effect of the sphere's immersion in a conducting plasma. He alludes to this deficiency in his discussion, stating that "the medium outside the stars is a good conductor so that the results of this paper cannot be applied to the computation of the electric field outside a rotating magnetic star."

In the present paper we treat the case of a steadily rotating, uniformly magnetized, and perfectly conducting sphere with arbitrarily aligned axes of rotation and magnetization. The sphere is surrounded by, and is in direct contact with, a tenuous plasma in which the conductivity parallel to the magnetic field lines is infinite and that perpendicular to the magnetic lines is zero. The electric field in a non-rotating frame of reference is calculated and then used to determine the $\vec{E} \times \vec{B}$ drift and energy changes of the plasma particles. As in the case with aligned (antiparallel) axes treated by Davis [1948], we find that the charged particles co-rotate with the sphere. Here, however, co-rotation involves a more subtle mechanism than in the case with aligned axes, requiring that a mirroring particle travel faster (as viewed from the rest-frame) during the half-bounce when its velocity along a magnetic line has a component in the direction of the sphere's rotation than during the other half-bounce. It is found that the electric field seen in the rest-frame produces exactly the increases and decreases of kinetic energy necessary to achieve this effect.

II. DERIVATION OF THE ELECTRIC FIELD

A. Formulation of Problem and Method of Solution

A perfectly conducting sphere of radius, a, rotates about an axis through its center with angular velocity, $\vec{\omega}$. The sphere is uniformly magnetized and its internal magnetization vector $\vec{\mu}$ is inclined at an angle γ with respect to $\vec{\omega}$. Let $\hat{k}$ and $\hat{m}$ be unit vectors parallel to $\vec{\omega}$ and $\vec{\mu}$, respectively, as shown in Figure 1. Unit vectors $\hat{i}$ and $\hat{j}$ lie in the plane normal to the rotational axis of the sphere but do not rotate with it. The vector $\vec{r}$ specifies a fixed observation point. The sphere is immersed in a tenuous plasma of infinite conductivity parallel to the external magnetic field lines and of zero conductivity normal to these lines. The anisotropic conductivity of the plasma is equivalent to the condition that $\vec{E}\cdot\vec{B} = 0$, where $\vec{E}$ and $\vec{B}$ are the external electric and magnetic fields. The problem is to determine the $\vec{E}$ field external to the sphere, as seen by an inertially fixed observer.

A solution for the external $\vec{E}$ field proceeds as follows:

1. Determine the sphere's internal electrostatic potential in order to establish a boundary condition on the potential at the surface of the sphere.

2. Use the constraint $\vec{E}\cdot\vec{B} = 0$ (in the nonrotating reference frame) to obtain a differential equation for the unknown external potential Φ.

3. Solve the resulting differential equation for Φ subject to the boundary condition at the sphere's surface.

B. Internal Potential

Backus [1956] has shown that the general solution for the potential inside a rotating, conducting magnet is (to within an arbitrary constant)

$$V = \frac{1}{c}\ (\vec{u}\cdot\vec{A}) \tag{3}$$

where $\vec{u}$ is the linear velocity of an interior point which rotates with the sphere,

$\vec{A}$ is a vector potential whose curl gives the internal (and, in our case, uniform) magnetization, and

c is the speed of light.

With

$$\vec{u} = \vec{\omega} \times \vec{r} \qquad \text{and} \qquad \vec{A} = \frac{1}{a^3}\ (\vec{\mu} \times \vec{r})$$

$$V = \frac{1}{ca^3}\ (\vec{\omega} \times \vec{r}) \cdot (\vec{\mu} \times \vec{r})\ . \tag{4}$$

Expansion of the dotted cross products gives

$$(\vec{\omega} \times \vec{r}) \cdot (\vec{\mu} \times \vec{r}) = (\vec{r} \cdot \vec{r})\,(\vec{\omega} \cdot \vec{\mu}) - (\vec{r} \cdot \vec{\mu})\,(\vec{\omega} \cdot \vec{r})$$

$$= r^2 \left|\vec{\mu}\right| \left|\vec{\omega}\right| (\cos\gamma - \cos\psi \cos\Theta) \tag{5}$$

where, from Figure 1, (r, Θ, ϕ) are the spherical polar coordinates of an interior observation point in the direction of the unit vector $\hat{n}$, and ψ is the angle between $\hat{n}$ and $\hat{m}$. By the spherical trignometric addition formula,

$$\cos\psi = \cos\Theta \cos\gamma + \sin\Theta \sin\gamma \cos(\phi - \omega t)\ . \tag{6}$$

Hence,

$$V = \frac{1}{ca^3} \left|\vec{\mu}\right| \left|\vec{\omega}\right| r^2 \left\{ \cos\gamma - \cos\Theta \left[\cos\Theta \cos\gamma + \sin\Theta \sin\gamma \cos(\phi - \omega t)\right] \right\}$$

$$= \frac{\mu\omega r^2}{ca^3} \sin\Theta \left\{ \sin\Theta \cos\gamma - \cos\Theta \cos(\phi - \omega t) \sin\gamma \right\}. \tag{7}$$

The expression above gives the interior potential of the rotating sphere. At the surface $(r = a)$

$$V = \frac{\mu\omega}{ca} \sin\Theta \left\{ \sin\Theta \cos\gamma - \cos\Theta \cos(\phi - \omega t) \sin\gamma \right\} \tag{8}$$

C. Differential Equation for Φ

The constraint $\vec{E}\cdot\vec{B} = 0$ provides a differential equation for the external potential Φ. A general solution of Maxwell's equations for $\vec{E}$ is

$$\vec{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

where $\vec{A}$ is a vector potential for the magnetic field $\vec{B}$. (We shall have occasion, later, to refer to the first term on the right hand side of this equation as the electrostatic field, $\vec{E}^S$, and to the second term as the induction field, $\vec{E}^I$.) With the expression above for $\vec{E}$, and with $\vec{B} = \nabla \times \vec{A}$,

$$\vec{E}\cdot\vec{B} = -\left(\nabla\Phi + \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right)\cdot\left(\nabla\times\vec{A}\right) = 0 . \qquad (9)$$

A rigorous solution for Φ in the presence of plasma would demand that $\vec{A}$ be treated as an unknown, since a time varying external charge density contributes to the spatial dependence of $\vec{A}$. Condition (9) would then be solved for the two unknowns Φ and $\vec{A}$ subject to known (or assumed) boundary conditions on both potentials at the earth's surface. The Lorentz gauge,

$$\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\Phi}{\partial t} = 0$$

could be used as a second relationship between the two unknowns Φ and $\vec{A}$. In the work reported here, however, an approximate solution for Φ in the presence of plasma was obtained by assuming that $\vec{A}$ is a vector potential for a magnetic dipole, i.e.,

$$\vec{A} = \frac{\vec{\mu} \times \vec{r}}{r^3}$$

from which

$$\vec{B} = |\vec{\mu}| \quad \frac{3(\hat{n} \cdot \hat{m})\, \hat{n} - \hat{m}}{r^3} \quad . \tag{10}$$

In spherical coordinates, (r, θ, ϕ), the components of this field are:

$$B_r = \frac{2\mu}{r^3} [\cos \theta \cos \gamma + \sin \theta \sin \gamma \cos (\phi - \omega t)]$$

$$B_\theta = \frac{\mu}{r^3} [\sin \theta \cos \gamma - \cos \theta \sin \gamma \cos (\phi - \omega t)]$$

$$B_\phi = \frac{\mu}{r^3} [\sin (\phi - \omega t) \sin \gamma] \, . \tag{10a}$$

Since $\vec{r}$ specifies an inertially fixed observation point,

$$\frac{\partial \vec{A}}{\partial t} = \frac{1}{r^3} \left[\frac{\partial \vec{\mu}}{\partial t} \times \vec{r} \right]$$

$$= \frac{1}{r^3} [(\vec{\omega} \times \vec{\mu}) \times \vec{r}]$$

or,

$$\vec{E}^{I} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} = \frac{1}{cr^3}\,[\vec{r} \times (\vec{\omega} \times \vec{\mu})] \tag{11}$$

which is the electric field used by Terletzky [1946] in his calculation of the energization of particles in the wobbling dipole field of the earth.

The components of this field in spherical coordinates are:

$$\begin{aligned} E_r^I &= 0 \\ E_\theta^I &= \frac{\mu\omega}{cr^2}\sin\gamma\cos(\phi - \omega t) \\ E_\phi^I &= \frac{\mu\omega}{cr^2}\sin\gamma\cos\theta\sin(\phi - \omega t)\,. \end{aligned} \tag{12}$$

Use of (10) and (11) in (9) and the expansion (6) leads to the partial differential equation

$$\begin{aligned} &2\,[\cos\theta\cos\gamma + \sin\theta\sin\gamma\cos(\phi - \omega t)\,]\,\frac{\partial\Phi}{\partial r} \\ &\quad + \frac{1}{r}\,[\sin\theta\cos\gamma - \cos\theta\sin\gamma\cos(\phi - \omega t)]\,\frac{\partial\Phi}{\partial\theta} \\ &\quad + \frac{1}{r\sin\theta}\,[\sin\gamma\sin(\phi - \omega t)]\,\frac{\partial\Phi}{\partial\phi} \\ &\quad = \frac{\mu\omega}{cr^2}\,\sin\gamma\,[\cos\theta\sin\gamma - \sin\theta\cos\gamma\cos(\phi - \omega t)] \end{aligned} \tag{13}$$

for the external potential Φ in spherical coordinates.

D. External Potential and Field

A solution of (13) for Φ which satisfies the boundary condition (8) is

$$\Phi = \frac{\mu\omega}{cr} \sin\Theta \, [\sin\Theta \cos\gamma - \cos\Theta \cos(\phi - \omega t) \sin\gamma]. \tag{14}$$

The negative gradient of this potential is the electrostatic field $\vec{E}^s$ necessary to cancel the component of the induction field $\vec{E}^I$ along the magnetic lines of force. Explicitly, the components of $\vec{E}^s$ in spherical coordinates are

$$E_r^s = \frac{\mu\omega}{cr^2} \sin\Theta \, [\sin\Theta \cos\gamma - \cos\Theta \sin\gamma \cos(\phi - \omega t)]$$

$$E_\Theta^s = -\frac{\mu\omega}{cr^2} [\sin 2\Theta \cos\gamma - \cos 2\Theta \sin\gamma \cos(\phi - \omega t)]$$

$$E_\phi^s = -\frac{\mu\omega}{cr^2} \cos\Theta \sin\gamma \sin(\phi - \omega t) \tag{15}$$

Finally, the complete external field, $\vec{E}$, in the presence of conducting plasma is

$$\vec{E} = \vec{E}^I + \vec{E}^s = -\left(\frac{1}{c}\frac{\partial \vec{A}}{\partial t} + \nabla \Phi\right).$$

From (12) and (15) the components of the complete $\vec{E}$ field in spherical coordinates are

$$E_r = \frac{\mu\omega}{cr^2} \sin\Theta \,[\sin\Theta \cos\gamma - \cos\Theta \cos(\phi - \omega t) \sin\gamma]$$

$$E_\Theta = -\frac{2\mu\omega}{cr^2} \sin\Theta \cos\psi$$

$$E_\phi = 0 \tag{16}$$

where the angle ψ between $\vec{\mu}$ and $\vec{r}$ is given by (6).

III. EXTERNAL CHARGE AND CURRENT DENSITY

The external volume charge density corresponding to the $\vec{E}$ field in (16) is given by

$$\rho = \frac{1}{4\pi} \vec{\nabla} \cdot \vec{E}$$

or

$$\rho = - \frac{\mu\omega}{2\pi c r^3} \left\{ [3 \cos^2 \theta - 1] \cos \gamma + [3 \sin \theta \cos \theta \, (\phi - \omega t)] \sin \gamma \right\} . \qquad (17)$$

The charge density varies both spatially and temporally at an inertially fixed observation point. The absolute value of the maximum charge density is, for the parameters r, μ, ω, and γ applicable to the earth, approximately

$$\frac{1.2 \times 10^{-16}}{n^3} \text{ statcoulombs/cm}^3$$

where n is the radial distance in earth radii. This requires a difference of only $\sim 10^{-9}/\text{cm}^3$ between the number densities of protons and electrons in the plasma at $\sim 5\ R_e$, or only $\sim 10^{-11}$ of the ambient particle density at this distance.

The continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

may be used to determine the current system corresponding to the time varying charge density in (17). Taking the partial time derivative of (17), one finds that

$$\vec{\nabla} \cdot \vec{J} = \frac{G}{r^3} \sin \gamma \sin \theta \cos \theta \sin (\phi - \omega t) \qquad (18)$$

where $$G = \frac{3\mu\omega^2}{2\pi c} .$$

Hence the maximum current density will be on the order of $\dfrac{G \sin \gamma}{r^2}$. For values of μ, ω, r, and γ applicable to the earth the maximum current at 1 R_e becomes

$$\left| \vec{J}_{max} \right| \sim 1 \times 10^{-11} \ \mu amp/m^2,$$

a current so feeble that its perturbing effect on the dipole $\vec{B}$ field is completely negligible, as was assumed in solving for $\vec{E}$.

IV. PARTICLE MOTION AND ENERGY CHANGES

The electric field (16) would be seen by a stationary observer at point (r, θ, ϕ). Its effect on charged particles in the magnetic field (i.e., on the low energy plasma particles themselves and also on more energetic particles which may be present) is most easily understood by noting that the field of (16) vanishes when transformed to a reference frame rotating with the sphere. The field, $\vec{E}^1$, in the rotating frame is (for $\omega r \ll c$):

$$\vec{E}^1 = \vec{E} + \frac{1}{c}\left[(\vec{\omega} \times \vec{r}) \times \vec{B}\right] . \qquad (19)$$

Using $\vec{\omega} \times \vec{r} = (r\,\omega \sin\theta)\,\hat{e}_\phi$ and making use of (10a) for $\vec{B}$, one finds that $\frac{1}{c}\left[(\vec{\omega} \times \vec{r}) \times \vec{B}\right]$

$$= \left\{ \frac{\mu\omega}{r^2} \sin\theta \left[\sin\gamma \cos\theta \cos(\phi - \omega t) - \cos\gamma \sin\theta\right] \right\} \hat{e}_r$$

$$+ \left\{ \frac{2\mu\omega}{r^2} \sin\theta \left[\sin\gamma \sin\theta \cos(\phi - \omega t) + \cos\gamma \cos\theta\right] \right\} \hat{e}_\theta \qquad (20)$$

where $\hat{e}_r$, $\hat{e}_\theta$, and $\hat{e}_\phi$ are a set of unit vectors in the spherical coordinate system. But the components of $\vec{E}$ given by equation (16) are just the negative of the components above when the expansion (6) is used for cos ψ. Therefore, in the rotating frame $\vec{E}^1 = 0$, and charged particles undergo no $\vec{E} \times \vec{B}$ drift or energization but appear to move just as they normally would in a static dipole magnetic field. In the special case when $\vec{\mu}$ and $\vec{\omega}$ are antiparallel (the situation treated by Davis [1948]) $\sin\gamma = 0$, $\cos\gamma = -1$, and the electric field (16) in the rest frame becomes:

$$\vec{E} = -\left(\frac{\mu\omega}{r^2}\sin^2\theta\right)\hat{e}_r + \left(\frac{2\mu\omega}{r^2}\sin\theta\cos\theta\right)\hat{e}_\theta \tag{21}$$

which is Davis' result.

It is interesting to examine the motion of particles in the combined electric and magnetic fields of the non-rotating reference frame to see how co-rotation of the plasma with the sphere is achieved. The electric drift velocity, $\vec{V}_e$, is:

$$\begin{aligned}
\vec{V}_E &= \frac{c\,(\vec{E} \times \vec{B})}{B^2} = -\frac{c\,(\vec{E}^1 \times \vec{B})}{B^2} \\
&= -\frac{c}{B^2}\,\frac{1}{c}\left[\,(\vec{\omega} \times \vec{r}) \times \vec{B}\,\right] \times \vec{B} \\
&= -\hat{B}_o \times \left[\,\hat{B}_o \times (\vec{\omega} \times \vec{r}\,)\,\right] \\
&= (\vec{\omega} \times \vec{r}\,)_\perp
\end{aligned} \tag{22}$$

where $\hat{B}_o$ is a unit vector in the direction of the magnetic field. Expression (22) states that the electric drift velocity is equal to the component of $(\vec{\omega} \times \vec{r})$ which is perpendicular to $\vec{B}$. Though this implies co-rotation of sphere and plasma in the <u>special case</u> when magnetic and rotational axes are aligned (parallel or antiparallel), it does not, by itself, imply co-rotation in the general case considered here.

The explicit expression for $\vec{V}_E$, with equations (16) for $\vec{E}$ and (10a) for $\vec{B}$, is:

$$
\begin{aligned}
\vec{V}_E &= \frac{c\,(\vec{E} \times \vec{B})}{B^2} \\
&= \frac{\omega r \sin \Theta}{(3\cos^2 \psi + 1)} \Big\{ [\, -2\cos \psi \sin \gamma \sin (\phi - \omega t) \,]\, \hat{e}_r \\
&\quad + [\, \sin \gamma \sin (\phi - \omega t) \,]\, [\, \sin \gamma \cos \Theta \cos (\phi - \omega t) \\
&\qquad\qquad - \cos \gamma \sin \Theta \,]\, \hat{e}_\Theta \\
&\quad + [\, 3 \cos^2 \psi + \cos^2 \gamma + \sin^2 \gamma \cos^2 (\phi - \omega t) \,]\, \hat{e}_\phi \Big\}
\end{aligned}
\tag{23}
$$

This reduces to $(\omega r \sin \Theta)\, \hat{e}_\phi$--indicating co-rotation--for $\gamma = 0$ or π (i.e., when rotational and magnetic axes are parallel or antiparallel) or for $(\phi - \omega t) = 0$ or π (i.e., at points lying in the plane which contains the rotational and magnetic axes). For all other orientations $\vec{V}_E \neq (\omega r \sin \Theta)\, e_\phi$; that is, in general the $\vec{E} \times \vec{B}$ drift alone does not cause co-rotation of plasma and sphere. Nevertheless, the plasma particles do, on the average, rotate with the sphere; they do so because their kinetic energy as seen in the rest frame is slightly greater when the ϕ-component of their motion along the lines of force is in the direction of the sphere's rotation than when it is in the opposite direction. But this is simply a special instance of Fermi acceleration wherein a particle is reflected from regions of magnetic field moving, alternately, in the same and opposite directions as the particle itself.

To illustrate this effect, we evaluated the time rate of change in kinetic energy W of a particle moving in the combined electric and magnetic fields as seen in the rest frame. Northrup [1963] gives this rate as:

$$\frac{dW}{dt} = e\,\vec{E} \cdot \vec{U} + M\,\frac{\partial B}{\partial t} \tag{24}$$

where $\vec{U}$ is the velocity of a particle's guiding center and M is the particle's magnetic moment. In the present case $\vec{E}_\parallel = 0$. Therefore, we are concerned only with $\vec{U}_\perp$, the component of $\vec{U}$ perpendicular to the magnetic field, and specifically, only with that part of $\vec{U}_\perp$ other than the $\vec{E} \times \vec{B}$ drift, since the $\vec{E} \times \vec{B}$ drift can contribute nothing to the energy change. We take $\vec{U}_\perp$ to consist of the magnetic gradient and line curvature drifts, which in a curl free magnetic field may be combined to give

$$\vec{U}_\perp = \frac{c}{eB^3} W \left(2 - \frac{B}{B_m}\right) (\vec{B} \times \nabla B) . \qquad (25)$$

With this expression, and using the equations already given for $\vec{B}$, $\vec{E}$, and $\cos \psi$, we obtain:

$$\frac{1}{W} \frac{dW}{dt} = \frac{3\omega \sin \gamma \sin \Theta \sin (\phi - \omega t) \cos \psi}{(1 + 3 \cos^2 \psi)}$$

$$\left\{ 2 \left(2 - \frac{B}{B_m}\right) \left(\frac{1 + \cos^2 \psi}{1 + 3 \cos^2 \psi}\right) + \frac{B}{B_m} \right\} . \qquad (26)$$

This equation reveals several interesting features of particle motion:

(a) The fractional change of energy per unit time has a limiting value, $3\omega \sin \gamma$, which it may approach for certain combinations of the other parameters. For the earth, $3\omega \sin \gamma \approx 5 \times 10^{-5}$/sec.

(b) The rate of energy change is independent of radial distance (except as this enters implicitly in other terms, such as B/B_m) and is independent of the magnetic moment of the sphere.

(c) The rate of energy change is anti-symmetrical about the magnetic equator (because of the cos ψ term) and about the plane containing $\vec{\mu}$ and $\vec{\omega}$ (because of the term sin $(\phi - \omega t)$).

(d) Only a minor part of the energy change (that associated with the last (B/Bm) term in the curly bracket of equation (24)) is due to $M \frac{\partial B}{\partial T}$; the remainder results from $e \vec{E} \cdot \vec{U}$.

The anti-symmetry about the magnetic equator causes a particle to lose (gain) as much energy in going from one mirror to the equator as it gains (loses) in going from the equator to its next mirror. Thus the particle kinetic energy at each mirror is the same but the average kinetic energy is higher during the "forward" bounces than during the "backward" bounces.

V. CONCLUSIONS

A perfectly conducting, uniformly magnetized sphere rotating in a tenuous, conducting plasma causes the plasma to rotate with it, regardless of the relative orientation of the sphere's magnetic and rotational axes. If the axes are parallel or anti-parallel, co-rotation is achieved wholly through the $\vec{E} \times \vec{B}$ drift resulting from the $\vec{E}$ field generated in the rest frame by the rotating sphere. If the axes are not parallel or anti-parallel, co-rotation is achieved through the combined effects of $\vec{E} \times \vec{B}$ drift and Fermi acceleration. In either case the electric field in a reference frame rotating with the sphere is identically zero.

We conclude from our results that plasma surrounding a rotating magnetized body will rotate with the body as shown here, regardless of whether the magnetic field of the body is symmetrical (as in the present case of the uniformly magnetized sphere) or not, so long as the requirements on conductivity are the same as those used here, and so long as the rotating body, itself, is the sole significant magnetic source, for then $\frac{\partial \vec{B}}{\partial t}$ vanishes in the rotating frame. In a distorted magnetic field such as that in the earth's magnetosphere, however, where there are (non-corotating) magnetic sources in addition to the earth itself, there can be no preferred reference frame in which the electric field is identically zero. For even in an irregularly-moving frame which "moves with the magnetic field lines", there is a $\frac{\partial \vec{B}}{\partial t}$ and one expects that particles observed in <u>any</u> reference frame will accumulate or lose energy over many bounces. If the earth's magnetic and rotational axes were aligned and fixed in space in such a manner (i.e., perpendicular to the ecliptic plane) as to provide a magnetosphere with a non-time-varying structure, one could further anticipate that particle energy gains and losses would be anti-symmetrical about the noon-midnight plane [Hones, 1963]. However, in the actual case of a time-varying distortion of the field, it is not obvious that there will be any such anti-symmetry; therefore the electric field generated by the earth's rotation may cause a cumulative change in particle energy and position over many revolutions around the earth, providing a mechanism for diffusion of plasma and of higher energy particles through the magnetosphere.

It is probably very important, when studying the effects of the time-varying configuration of the magnetic field in the magnetosphere to take account of the plasma's tendency to cancel the parallel component of the electric field, as we have done in this paper, since so little charge separation is required to accomplish this (see Section III). The nature of the electric field is completely altered by this effect, and it is likely that conclusions reached regarding particle motions from a model in which conductivity of the plasma is neglected (such as Terletzky's) will be quite misleading.

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Figure Caption

Figure 1. Coordinates used in derivation of electric field around rotating, conducting magnetized sphere.

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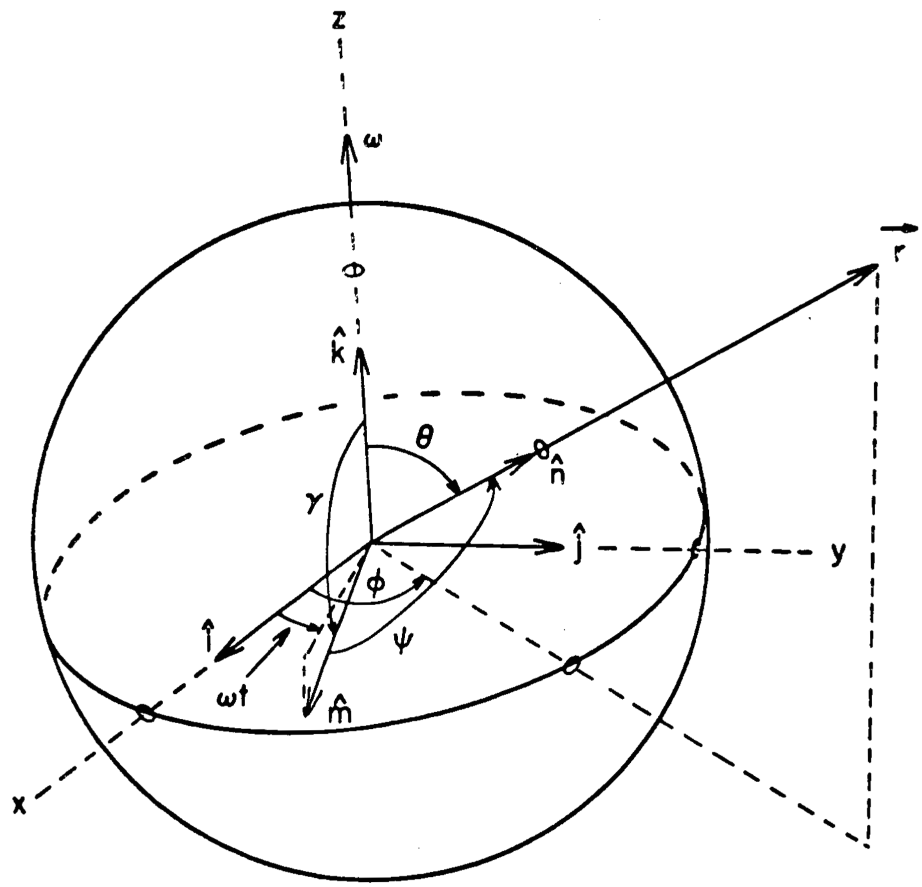

Figure 1