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ENTHALPY CALCULATED FROM PRESSURE AND FLOW-RATE MEASUREMENTS IN HIGH-TEMPERATURE SUBSONIC STREAMS

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SUMMARY

A method of calculating the enthalpy of a high-temperature subsonic stream from measurements of the total pressure, the static pressure at the nozzle exit, the nozzle area, and the mass-flow-rate is presented. The enthalpy calculated by this method is compared with the enthalpy determined from the free-stream temperature as measured with a spectrometer and the enthalpy determined by an energy balance.

The average difference between the values of free-stream enthalpy determined by the method presented and those determined by the spectrometric method is less than 10 percent at a flow rate of 0.011 slug/s (0.16 kg/s). Lower mass-flow rates produce nonuniform pressure distributions across the stream and tend to increase the deviation between the enthalpy values.

INTRODUCTION

Since large subsonic arc-jet facilities are currently being used to investigate material properties, it is desirable to characterize the test stream. The stagnation temperature of such streams is well above the value that can be measured with conventional thermocouples and therefore other approaches are necessary. The spectrometric techniques described in reference 1 require an elaborate setup and an exercise of judgment in reduction of data. The energy-balance technique requires accurate measurement of power input, water-flow rates, water temperatures, and test medium mass-flow rate; also the reduction of data is time consuming and laborious.

A thermodynamic approach calculating the enthalpy for subsonic flow from mass-flow rate and pressure measurements is developed, with several simplifying assumptions. This approach, which is roughly parallel to the sonic-flow calculation presented in reference 2, may generally be used in conjunction with any subsonic facility in which the total arc-chamber pressure, static pressure, mass-flow rate, and nozzle area are either known or measurable.

The thermodynamically derived enthalpy is compared with enthalpy determined by the spectrometric technique and enthalpy determined by an energy balance. The limitations of the simplifying assumptions are enumerated.

SYMBOLS

The units used for the physical quantities in this paper are given both in the U.S. Customary Units and in the International System of Units (SI). Factors relating the two systems are given in reference 3, and those used in the present investigation are presented in the appendix.

A nozzle area, ft^2 or m^2

c_p heat capacity at constant pressure, Btu/lb-°R or J/kg-°K

c_v heat capacity at constant volume, Btu/lb-°R or J/kg-°K

F_c compressibility factor

h enthalpy, Btu/lb or J/kg

M Mach number

$\dot{m}$ mass-flow rate, slug/s or kg/s

p pressure, lb/ft^2 or N/m^2

R gas constant, ft^2/s^2-°R or m^2/s^2-°K

T temperature, °R or °K

u velocity, ft/s or m/s

Z ratio of molecular weights

γ ratio of specific heats, c_p/c_v

ρ density, $slug/ft^3$ or kg/m^3

Subscripts:

∞ free stream

t total

METHOD OF ANALYSIS

An average free-stream enthalpy may be calculated in a subsonic stream from the pressure differential and the mass-flow rate through the nozzle. The compressibility factor is

$$F_c = \frac{p_t - p_\infty}{\rho_\infty u_\infty^2/2} \tag{1}$$

Thus

$$\frac{p_t - p_\infty}{F_c} = \frac{\rho_\infty u_\infty^2}{2} \tag{2}$$

where

$$F_c = \frac{2}{\gamma M^2}\left[\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

The compressibility factor F_c is shown as a function of Mach number M and ratio of specific heats γ in figure 1. In addition the ratio of the static pressure to the total chamber pressure p_∞/p_t is shown as a function of M and γ.

If a uniform velocity and density at the nozzle exit is assumed, the velocity may be calculated from the continuity equation

$$u_\infty = \frac{\dot{m}}{\rho_\infty A} \tag{3}$$

Substituting equation (3) into equation (2) yields

$$\frac{p_t - p_\infty}{F_c} = \frac{1}{2\rho_\infty}\left(\frac{\dot{m}}{A}\right)^2 \tag{4}$$

The density for air, nitrogen, and nitrogen-oxygen mixtures may be calculated from the relationships obtained from references 4 and 5:

$$\rho_\infty = \frac{p_\infty}{Z_\infty R T_\infty} \tag{5}$$

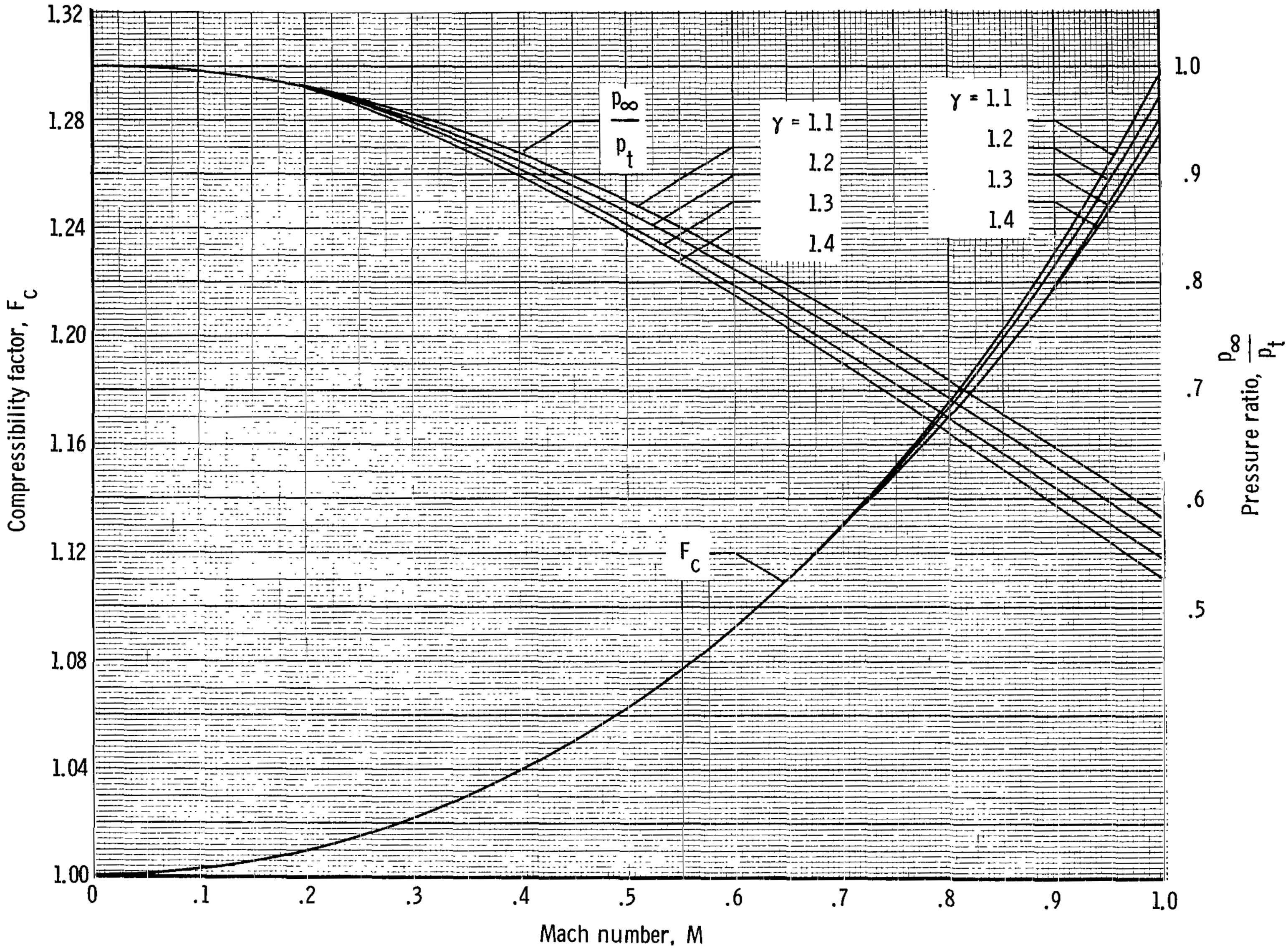

Figure 1.- Compressibility factor F_c and pressure ratio as a function of Mach number and γ.

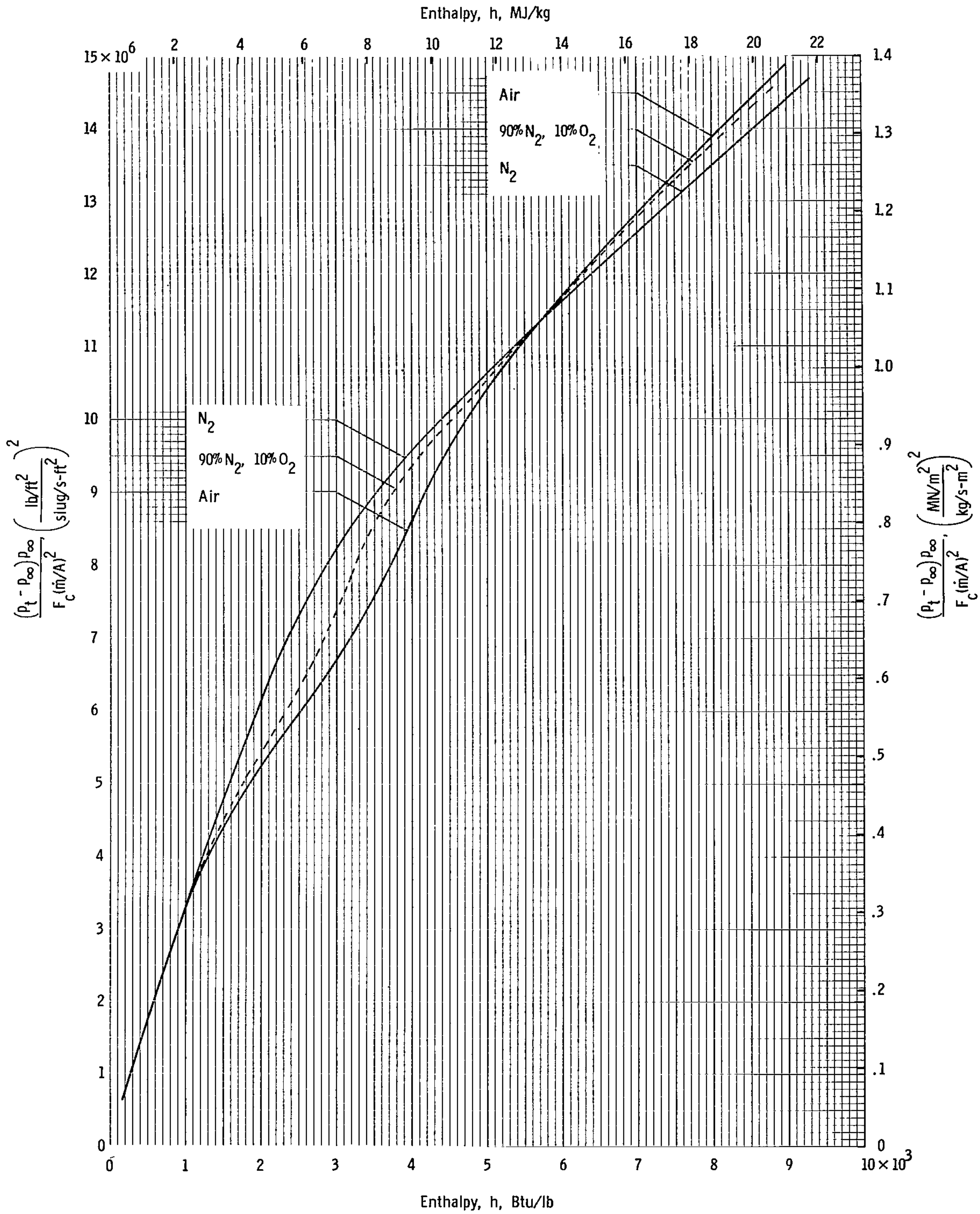

Figure 2.- Pressure-flow parameter as a function of enthalpy at a free-stream pressure of 1.0 atmosphere.

Substituting equation (5) into equation (4) and rearranging to have only enthalpy-dependent terms at constant pressure on the right-hand side gives

$$\frac{\left(p_t - p_\infty\right)p_\infty}{F_c\left(\frac{\dot{m}}{A}\right)^2} = \frac{Z_\infty R T_\infty}{2} \tag{6}$$

The compressibility factor F_c on the left-hand side of equation (6) is a weak function of the ratio of specific heats γ (hence enthalpy) and a strong function of Mach number. (See fig. 1.) Equation (6) is plotted in figure 2 as a function of enthalpy at a free-stream pressure of 1.0 atmosphere for air, nitrogen, and a mixture of 90 percent nitrogen and 10 percent oxygen. As long as the assumption of a uniform total-pressure distribution across the stream is valid, the arc-chamber pressure may be assumed to be equal to the total pressure. Thus the enthalpy may be determined from figure 2 and the measured values of the arc-chamber total pressure p_t, the test-section static pressure p_∞, the test-medium mass-flow rate $\dot{m}$, and the nozzle exit area A.

Since F_c is dependent on both M and γ, an iteration is required. A first approximation for the Mach number may be obtained from the measured value of p_∞/p_t and an estimated value of γ from figure 1. The iteration may be arbitrarily terminated when two successive values of the Mach number differ by no more than 0.01.

APPARATUS AND TESTS

2500-Kilowatt Arc Jet

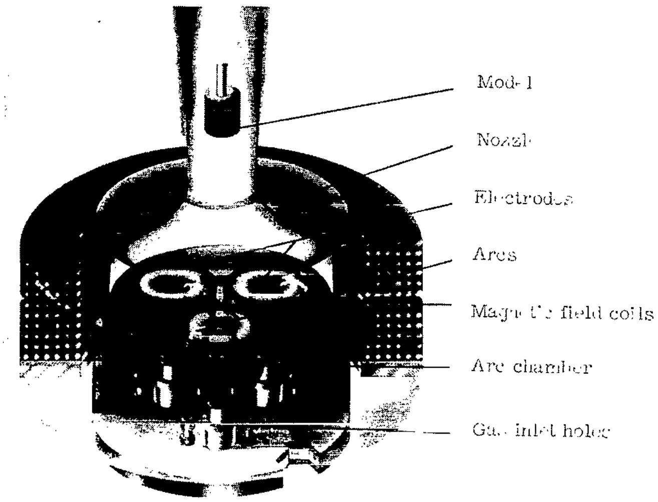

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Figure 3.- Phantom view of 2500-kilowatt arc-jet facility.

The arc chamber of the 2500-kilowatt arc-jet facility (see fig. 3) contains three equally spaced sets of electrodes, each set comprising two concentric water-cooled copper rings. A 3-inch-diameter (7.6-cm) center electrode is connected to each phase of the three-phase, alternating-current, power supply (7500 kilovolt-ampere and 2400 volts phase to ground) which provides the energy to heat the test gas. These electrodes pass through a water-cooled copper base plate and are electrically insulated from the base plate. The three 6-inch-diameter (15.2-cm) outside electrodes are mounted directly to the base plate through which they have a common connection to the grounded neutral of the power supply. The arc chamber is a double-walled, water-cooled copper cylinder, 20 inches (51 cm) in diameter and 15 inches (38.1 cm) in length. A pair of direct-current magnetic coils is mounted externally around the arc chamber so that a magnetic field is applied parallel to the axis of the arc unit. The arc rotation (approximately 360 rps) changes direction as the arc current reverses because of the interaction of the arc with the constant-direction magnetic field.

The test gas is injected into the arc chamber through holes at the base of each electrode. A baffle, with a diameter equal to that of the outer electrodes, in the form of a helix tightly wound from water-cooled copper tubing (not shown in fig. 3) is used for each electrode pair to direct the test gas through the arc region. The test gas is heated and discharged to the atmosphere through a water-cooled, copper subsonic nozzle. Nozzles of various diameters may be attached to the arc-jet top; for this investigation, only the nozzles with 4-inch (10.2-cm) and 2-inch (5.1-cm) diameters were used.

The outermost radius of the entrance nozzle was fitted with three pressure taps 0.05 inch (0.13 cm) in diameter which lead to a common fitting. The arc-chamber pressure was measured by a manometer connected to this common fitting. After the test conditions stabilized, a photograph of the manometer board was taken, and subsequently analyzed. The test gas was supplied from a high-pressure storage tank, regulated by a throttling valve and measured by an orifice meter downstream of the valve.

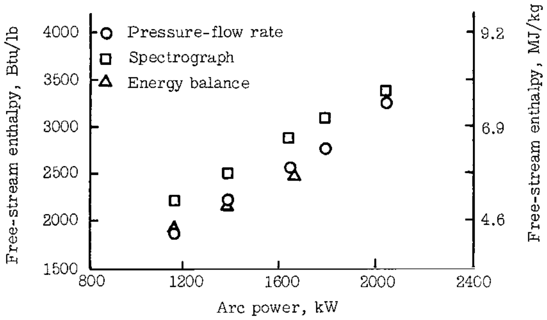
Spectrometer

The spectrometric instrumentation consists of a glass-prism spectrograph modified to provide photoelectric data readout. Data are displayed on an oscilloscope and recorded on film by means of an oscilloscope camera. A more detailed description of the instrumentation and the modifications may be found in reference 1. The free-stream temperature 1 inch (2.5 cm) downstream of the nozzle exit is measured by the atomic-line-intensity-ratio method in which the electronic excitation spectrum of copper that appears in the gas from the small electrode contamination (approximately 0.05 percent by weight) is used. The temperature of the stream, if thermodynamic equilibrium is assumed, is a function of the intensity ratio. The external optical system of reference 1 has been modified to observe, alternately, the arc-jet stream and a tungsten-ribbon-filament calibration lamp. Frequency of signal alteration between these two sources is about 20 cps, which is fast enough to provide simultaneously a calibration signal and a signal from the test source. This calibration signal allows adjustment of the oscilloscope gain controls during a test run for the most advantageous signal display. The intensity-ratio signals from the two sources are observed as two lines on the oscilloscope.

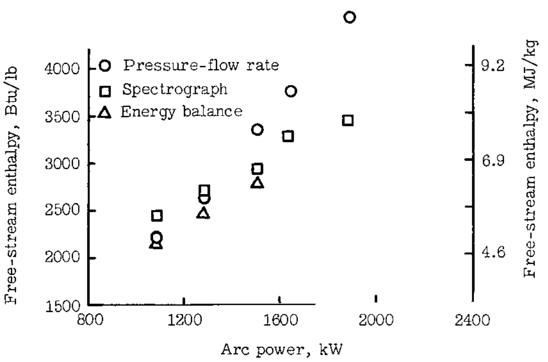
RESULTS AND DISCUSSION

Previous tests in the arc-jet facility used in this investigation indicated a linear variation of calorimeter heat-transfer rate with power input to the arc. Since heat-transfer rate is proportional to the enthalpy of the test medium, it is expected that the enthalpy would be proportional to arc power. When nitrogen was used as the test medium, the enthalpy indicated by the spectrometric method increased approximately in proportion to the arc power, as expected. However, when air was used as the test medium, the spectrometric method indicated a relatively constant enthalpy above a certain power level. Because of this unresolved behavior, nitrogen was used as the test medium for the investigation reported herein. Test results are summarized in table I.

The free-stream enthalpy as a function of arc power for the 2500-kilowatt arc jet is shown for the two test mass-flow rates and for the two nozzle diameters in figures 4 and 5. In both of these figures, the circular symbols are the calculated values obtained from the measured flow quantities and from figures 1 and 2. The triangular symbols are the energy-balance data that were taken on different runs, and the square symbols represent the values obtained by the spectrometric technique. Since the Mach number in the 2-inch-diameter (5.1-cm) stream was always 0.4 or greater, it was necessary to consider compressibility ($F_c > 1.04$) for all the flow calculations for that nozzle.

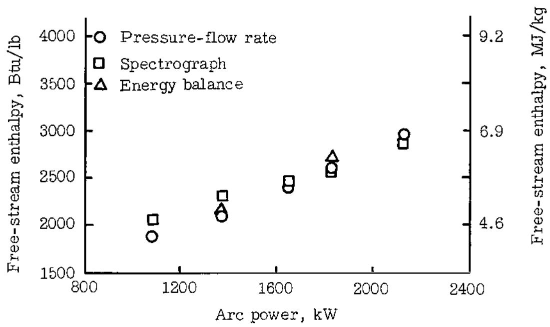


(a) Mass-flow rate, 0.011 slug/s (0.16 kg/s).

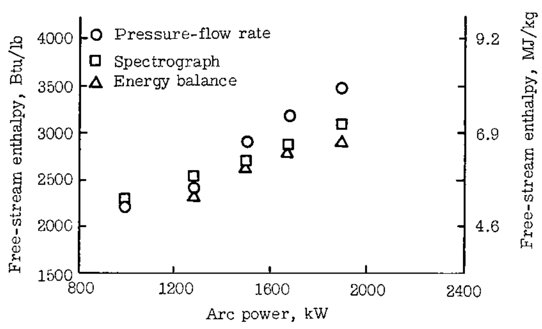


(b) Mass-flow rate, 0.0078 slug/s (0.11 kg/s).

Figure 4.- Enthalpy determined by three methods on a 4-inch-diameter (10.2-cm) nozzle for two flow rates.



(a) Mass-flow rate, 0.011 slug/s (0.16 kg/s).



(b) Mass-flow rate, 0.0078 slug/s (0.11 kg/s).

Figure 5.- Enthalpy determined by three methods on a 2-inch-diameter (5.1-cm) nozzle for two flow rates.

TABLE I.- TEST RESULTS

(a) Nozzle diameter, 4 inches (10.2 cm)

Arc power, kW	Chamber pressure		Pressure-flow method				Spectrometric free-stream enthalpy		Energy-balance enthalpy	
			Free-stream enthalpy		Stagnation enthalpy					
	lb/ft^2	kN/m^2	Btu/lb	MJ/kg	Btu/lb	MJ/kg	Btu/lb	MJ/kg	Btu/lb	MJ/kg
Mass-flow rate, 0.011 slug/s (0.16 kg/s)										
1150	2158	103	1850	4.30	1850	4.30	2200	5.11	1900	4.42
1380	2165	104	2200	5.11	2200	5.11	2500	5.81	2175	5.06
1630	2171	104	2550	5.93	2550	5.93	2850	6.62	2450	5.69
1780	2174	104	2750	6.39	2750	6.39	3060	7.11		
2030	2180	104	3250	7.55	3250	7.55	3360	7.81		
Mass-flow rate, 0.0078 slug/s (0.11 kg/s)										
1080	2140	102	2200	5.11	2200	5.11	2450	5.69	2150	5.00
1280	2144	103	2600	6.04	2600	6.04	2700	6.28	2450	5.69
1500	2148	103	3350	7.79	3350	7.79	2930	6.81	2750	6.39
1630	2150	103	3750	8.72	3750	8.72	3280	7.62		
1880	2154	103	4520	10.51	4520	10.50	3420	7.95		

(b) Nozzle diameter, 2 inches (5.1 cm)

Arc power, kW	Chamber pressure		Pressure-flow method				Spectrometric free-stream enthalpy		Energy-balance enthalpy	
			Free-stream enthalpy		Stagnation enthalpy					
	lb/ft^2	kN/m^2	Btu/lb	MJ/kg	Btu/lb	MJ/kg	Btu/lb	MJ/kg	Btu/lb	MJ/kg
Mass-flow rate, 0.011 slug/s (0.16 kg/s)										
1080	2907	139	1900	4.42	2025	4.71	2100	4.88		
1380	2962	142	2100	4.88	2260	5.25	2360	5.49	2175	5.06
1650	3113	149	2450	5.69	2650	6.16	2490	5.79	2500	5.81
1830	3178	152	2650	6.16	2850	6.62	2600	6.04	2750	6.39
2130	3286	157	3000	6.97	3280	7.62	2900	6.74		
Mass-flow rate, 0.0078 slug/s (0.11 kg/s)										
1000	2535	121	2200	5.11	2320	5.39	2280	5.30		
1280	2579	123	2400	5.58	2550	5.93	2520	5.86	2350	5.46
1500	2651	127	2900	6.74	3020	7.02	2680	6.23	2650	6.16
1670	2679	128	3150	7.32	3260	7.58	2860	6.65	2800	6.51
1900	2710	130	3450	8.02	3625	8.43	3080	7.16	2900	6.74

The free-stream enthalpy calculated from the measured flow quantities in the 4-inch-diameter (10.2-cm) stream at a mass-flow rate of nitrogen of 0.011 slug/s (0.16 kg/s) is shown as a function of arc power in figure 4(a). Since the Mach number is low ($\approx$0.2), the free-stream static enthalpy is approximately equal to the total enthalpy. If a constant total-pressure distribution is assumed, the calculated enthalpy must be taken as an average value. The spectrometric enthalpy measurements are weighted strongly toward the highest enthalpy along the line of observation; this point should be at the axis of the stream. Thus, it is reasonable to expect the spectrometrically determined enthalpy to be somewhat higher than that obtained from pressure measurements, as is confirmed by the data of figure 4. It is also reasonable to expect the enthalpy obtained from the pressure measurements to agree with that determined by the energy balance. This agreement is indicated in figure 4(a). However, as the mass-flow rate is decreased through the large-diameter nozzle, the data of the spectrometrically indicated enthalpy has a different slope from the pressure data, as seen in figure 4(b). At the higher arc powers, the spectrometer indicates a lower enthalpy than the pressure data.

The discrepancy between the results obtained by the two methods of enthalpy determination are attributed mainly to two causes. First, a water-cooled, total-pressure probe indicated that the total-pressure distribution across the 4-inch (10.2-cm) nozzle exit was uniform for the 0.011-slug/s (0.16-kg/s) flow rate but contained a gradient of as much as 20 percent for the 0.0078-slug/s (0.11-kg/s) flow rate. A further decrease in flow rate to 0.0046 slug/s (0.067 kg/s) resulted in a pressure gradient so severe that the pressure increment $(p_t - p_\infty)$ at a distance 1/4 inch (0.64 cm) from the nozzle wall was 2.8 times greater than the pressure at the center line. Since the method of calculation contains the inherent assumption of no total-pressure gradient, the calculated enthalpy is not realistic for flow rates below 0.0078 slug/sec (0.11 kg/s) for the large-diameter nozzle.

The second factor contributing to the discrepancy between results obtained by the two methods of enthalpy determination lies in two assumptions made in the spectrometric measurements. The first assumption - that the test stream contains no temperature gradients along the line of observation - is not true in the facility used in this investigation and the spectrometric results yield a temperature strongly weighted toward the highest temperature along the line of sight. The second, and possibly more important, assumption is that the arc stream is optically thin (no self-absorption). Preliminary measurements indicate that the arc stream is not optically thin and show that the optical thickness varies with both arc power and mass-flow rate. Furthermore, the variations are not the same for both of the spectral lines under observation. Thus the measured spectrometric values of enthalpy given in figures 4 to 6 are lower than those that would be obtained in an optically thin stream. In addition, the correction factor will be greater for the high powers and low mass-flow rates than for the low powers and high mass-flow rates (that is, when the contamination is the greatest).

This type of variation of optical thickness with power and mass-flow rate cannot be explained solely by attributing it to the increase in gas temperature, but is caused by a combination of both the gas-temperature increase and the level of contamination from the electrodes. Previous measurements of electrode weight losses (hence, relative contamination levels) as a function of power over extended time periods showed that the average weight loss increased by a factor of about 6 when the arc power was doubled and that the contamination level also increased when the flow rate was decreased. Simultaneous spectrometric measurements indicated variations in contamination levels of the same order of magnitude.

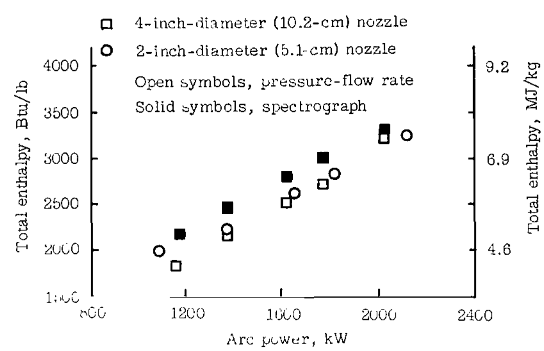
The enthalpies for the 2-inch-diameter (5.1-cm) nozzle were evaluated in the same manner as those for the 4-inch-diameter (10.2-cm) nozzle as shown in figure 5. The free-stream enthalpies shown in these figures cannot be compared directly with those of the 4-inch (10.2-cm) nozzle because of the different velocities (hence Mach number) for a given mass-flow rate. However, a direct comparison of the total enthalpies may be made. It can be seen in figure 6(a) that the total enthalpy for each nozzle was approximately the same at the 0.011-slug/s (0.16-kg/s) flow rate. Figure 6(b) shows the same comparison for the 0.0078-slug/s (0.11 kg/s) flow rate. In this comparison, the spectrometric values were used for the data for the 4-inch (10.2-cm) nozzle because of the poor total-pressure profile for this low mass-flow rate. The average difference between the enthalpy determined by the pressure method and the enthalpy determined by the spectrometric method is less than 10 percent at a mass-flow rate of 0.011 slug/s (0.16 kg/s). Lower mass-flow rates produce nonuniform pressure distributions and tend to increase the deviation.

Check points in air gave results that agreed with the tests in nitrogen; that is, for a particular mass-flow and arc-power setting the enthalpy was the same regardless of the test gas. For example, at a test-gas enthalpy of 2980 Btu/lb (6.92 MJ/kg), from figure 2, the following relation should exist:

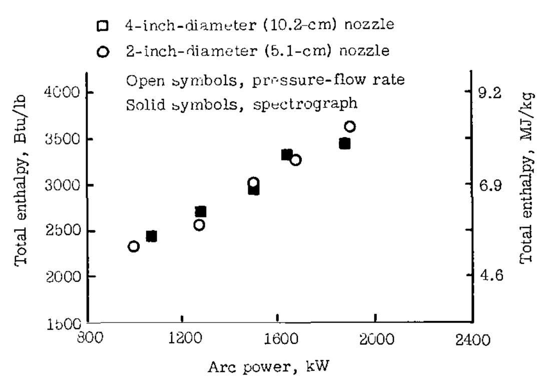
$$\frac{\left(p_t - p_\infty\right)_{nitrogen}}{\left(p_t - p_\infty\right)_{air}} = 1.23$$

The appropriate pressures used to obtain this ratio were measured and the ratio was within 2 percent of 1.23. The energy balance also showed that the enthalpy was independent of the chemical mixture of the test gas.

The method of enthalpy calculation described herein has subsequently been used to compute the enthalpy of a sonic test stream. By making the proper



(a) Mass-flow rate, 0.011 slug/s (0.16 kg/s).



(b) Mass-flow rate, 0.0078 slug/s (0.11 kg/s).

Figure 6.- Comparison of pressure-rise total enthalpy on a 2-inch-diameter (5.1-cm) nozzle with spectrometric total enthalpy on a 4-inch-diameter (10.2-cm) nozzle for two flow rates.

corrections for compressibility and Mach number effects as previously described, the results were within 5 percent of the enthalpy obtained in reference 2. This accuracy is considered to be a basic check on the validity of the assumptions and the method in general.

CONCLUDING REMARKS

A method is presented to calculate the enthalpy of high-temperature subsonic streams from the total pressure, the static pressure at the nozzle exit, the nozzle area, and the mass-flow-rate measurements. The enthalpy calculated by this method is compared with spectrometrically determined and energy-balance enthalpies. The average difference between the enthalpy determined by the method presented and the enthalpy determined by the spectrometric method is less than 10 percent at a mass-flow rate of 0.011 slug/s (0.16 kg/s). Lower mass-flow rates produce nonuniform pressure distributions and tend to increase the deviation.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., May 12, 1965.

APPENDIX

CONVERSION OF U.S. CUSTOMARY UNITS TO SI UNITS

Factors required for converting the U.S. Customary Units used herein to the International System of Units (SI) are given in the following table:

Physical quantity	U.S. Customary Unit	Conversion factor (*)	SI Unit
Area	ft^2	9.29×10^{-2}	$meters^2$ (m^2)
Density	$slug/ft^3$	5.15×10^2	$kilogram/meter^3$ (kg/m^3)
Enthalpy	Btu/lb	2.32×10^3	joules/kilogram (J/kg)
Length	in.	0.0254	meters (m)
Mass-flow rate	slug/s	14.59	kilogram/second (kg/s)
Pressure	lb/ft^2	47.88	$newtons/meter^2$ (N/m^2)

*Multiply value given in U.S. Customary Unit by conversion factor to obtain equivalent value in SI Unit.

Prefixes to indicate multiples of units are as follows:

Prefix	Multiple
centi (c)	10^{-2}
kilo (k)	10^3
mega (M)	10^6

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