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ABSTRACT

THE ZONE OF SUPERSONIC FLOW CAUSED BY LARGE METEORITES STRIKING THE LUNAR SURFACE

by

M. P. Batra

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The zone at the supersonic flow caused by the large meteorite striking the lunar surface has been computed. We have applied the blast-wave theory, utilizing the self similarity technique, for solving the equations of the Fluid-Mechanical model. The penetration of the meteorite to the depth when its velocity becomes acoustic has been found to be equal to 4 to 6 meteorite diameters, depending upon the impact velocity and the mass of the meteorite. This confirms the prediction made by Baldwin for the penetration of the meteorite in the formation of craters with Central Mountain peak. This limit of penetration is called the 'sonic-crater depth'. The sonic-crater depth also happens to be approximately equal to the depth of Central Mountain peak from the level ground. The sonic-crater depths can be scaled for different velocities, different masses and different substances using Bjork's scaling laws. The sonic depth, after having been scaled for iron and tuff, agrees fairly well with that computed by Bjork for the meteor crater Arizona. We have also confirmed the 2/3 power law of Eichelberger & Gehring. and Walsh & Tillotson.

author

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THE ZONE OF SUPERSONIC FLOW CAUSED BY LARGE METEORITES STRIKING THE LUNAR SURFACE

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Introduction

Gilbert (1893) and Gifford (1929, 1930) pioneered the idea of the formation of lunar craters by the explosions associated with the impact of large meteorites. Baldwin (1949) has produced experimental evidence to support the impact theory. Preliminary estimates of the impact phenomenon were made by Gilvarry and Hill (1956) who showed that the pressures and temperatures did indeed reach explosion magnitude. Thus shock wave must be produced.

For simplicity, the lunar surface and the meteorite were supposed to be composed of the same substance. A hypothetical element "averagium" was defined (Gilvarry and Hill 1956) by determining the average atomic number of elements, weighted by their gross relative abundance by mass over the silicate, sulphide, and metal phases of meteorites. The atomic number of "averagium" was found to be 18.5 from the data of Brown (1949). Then the impact of an averagium meteorite on an averagium surface was considered.

Theory

The meteoritic impact velocities range from 15 km/sec to 75 km/sec and the masses to vary from 10^4 to 10^{14} kgm (Hawkins 1963). Such hypervelocity impacts generate pressures of the order of megabars which are far greater than the shear strengths of the materials, so that matter behaves as a fluid. We can therefore set up a fluid dynamical model for solving the problem of explosive impact. We will assume viscosity is negligible. The justification for this will follow later.

The first step in the analysis is that of defining the important physical processes and the associated constitutive equations which should be included in the theory. The neglect of shear strength makes possible the use of simple equations for pressure, density, energy and state. The neglect of thermal conduction is justified by the following order of magnitude consideration (Brode and Bjork 1960, Walsh and Tillotson 1963):

> The time for the duration of hydrodynamic phase of interaction is of the order of $\frac{L}{V_o}$ where L is the length of the projectile and V_o is the projectile velocity. The thermal diffusion distance x is of the order of $\sqrt{\lambda_c t}$, λ_c being the diffusivity. So for impact of a meteorite of dimension L = 15 meters with $\lambda_c = 1$ cm^2/sec, $V_o = 15$ km/sec, the ratio of diffusion distance to L is of the order of

$$x/L = (\lambda_0/V_oL)^{1/2}$$

$$\simeq 2 \times 10^{-5}$$

On the basis of this fluid-mechanical model, the problem of determining the response of the target material becomes essentially that of solving the general equations of an inviscid, non-conducting fluid which are

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{q} = o \qquad \text{mass conservation} \qquad (1\text{-A})$$

$$\rho\frac{D\vec{q}}{Dt} + \vec{\nabla} p = o \qquad \text{Momentum conservation} \qquad (1\text{-B})$$

$$\frac{De}{Dt} + p\,\frac{D}{Dt}\left(\frac{1}{\rho}\right) = o \qquad \text{entropy conservation} \qquad (1\text{-C})$$

where ρ = density, p = pressure, s = specific entropy, (i.e. entropy per unit mass), $\vec{q}$ = velocity vector which in spherical coordinates may be written as $u\hat{r} + V\hat{\theta} + W\hat{\phi}$, $\frac{D}{Dt} = \frac{\partial}{\partial t} + q \cdot \vec{\nabla}$ where t is the time, $\vec{\nabla}$ is the gradient operator.

In addition we need the equation of state of the medium which will be written at first in general form as

$$e = f(p,\rho) \qquad (1\text{-D})$$

where e is the specific internal energy.

It is interesting to note that the hydrodynamical equations remain the same if characteristic length and time are scaled because the terms in (1-A) have the same dimensions, and the same holds for (1-B), (1-C). Therefore we can say the condition of the shock front and the flow equations are homogeneous in distance and time. However the inclusion of conduction or viscosity would introduce second derivatives and scaling would no longer apply. By experiment the departure from simple scaling has been found negligible (Hermann and Jones 1961, Eichelberger and Gehring 1962), which justifies our neglect of the viscosity and thermal conduction terms in the problem of impact.

Thus we assume the fluid is inviscid and non heat-conducting, and that the motion does not involve any kind of physical and chemical change. Moreover the fluid is supposed to be baratropic having a polytropic form of the equation of state, $p\rho^{-\gamma}$ = constant. Then under isentropic conditions (1-D) becomes $e = \frac{(\gamma-1)^{-1}p}{\rho}$. No energy changes due to radiation is taken into account.

These differential equations contain spatial variables as well as time and the problem of solving them is extremely difficult. To date, the only solutions are those of Bjork's (1958, 1961) which are numerical calculations.

The general equations may be simplified by assuming spherical symmetry. The equations then become:

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2u\rho}{r} = 0 \qquad (2\text{-A})$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \qquad (2\text{-B})$$

$$\frac{\partial}{\partial t}(p\rho^{-\gamma}) + u\frac{\partial}{\partial r}(p\rho^{-\gamma}) = 0 \qquad (2\text{-C})$$

Bjork's calculations (1958, 61) show that the meteoritic impact is so rapid that the time required for the compression pressure to reach its maximum is negligible. The problem may therefore be reduced to that of an explosion at the center of impact. On this basis we further suppose that the shape of the projectile is immaterial.

Equation (2-C) implies that entropy remains constant along the path followed by an element of the fluid provided that it does not traverse a shock front. From equation (2-A) we can note the spherical attenuation, an important point of difference between the cases of spherical and plane wave.

(a) Similarity conditions

Sedov (1959) has shown that a simple solution to the hydrodynamic equations (2-A,B,C) exists under certain conditions. If from the given variables and parameters defining the initial conditions we can form two and only two dimensionless quantities, then time, one of the independent variables can be suppressed in the equations. In a blast wave the initial conditions are defined by the total energy E_o, the pre-blast pressure p_o, and the initial density of the medium ρ_o. In the equations themselves are the independent variables r and t, and the constant γ. From these six quantities Sedov has shown that there exists the dimensionless constant (γ) and the dimensionless variables (λ), (τ). Where

$$\lambda = (\rho_o^{1/5} r) \Big/ (E_o t^2)^{1/5}$$

and

$$\tau = \frac{p_o^{5/6} t}{E_o^{1/3} \rho_o^{1/3}}$$

Now for the impact of a large meteorite on the moon, with $E_o = 10^{30}$ ergs, $\lambda \simeq 10^{-6} r/t^{2/5}$ and $\tau = 10^{-1}$ t. Thus for material at depths less than 10^4 cm and times less than 10^{-2} sec, $\lambda \sim 10^2 \tau$ and therefore $\lambda \gg \tau$. Under these conditions the variable τ is negligible in comparison with λ, and also the constant γ since $\gamma = 6$. We may therefore proceed on the assumption that self-similarity will hold at least as a first approximation. This assumption is good for times $< 10^{-2}$ sec., but begins to break down after that time.

The dimensionless variable λ may be simplified to

$$\lambda = \lambda_1 \bar{\xi}$$

where λ_1 is a constant $\left(\frac{E_o}{\rho_o}\right)^{1/5}$, the value where r = t = 1.

The variable part of λ is now defined purely in terms of the physical variables r and t since $\bar{\xi} = rt^{-2/5}$.

Following Courant and Friedrichs (1948) we introduce the dimensionless functions of reduced velocity, $U(\bar{\xi})$, reduced density $D(\bar{\xi})$, reduced pressure $P(\bar{\xi})$ and reduced internal energy $G(\bar{\xi})$. The physical variables may now be written to exclude r as follows:

$$u = t^{\alpha-1} \bar{\xi} U(\bar{\xi}) \quad \text{(3-A)}$$

$$\rho = \rho_o D(\bar{\xi}) \quad \text{(3-B)}$$

$$p/\rho = t^{2(\alpha-1)} \bar{\xi}^2 P(\bar{\xi}) \quad \text{(30C)}$$

$$e = t^{2(\alpha-1)} \bar{\xi}^2 G(\bar{\xi}) \quad \text{(3-D)}$$

where $\alpha = 2/5$

(b) Conditions at the shock front

A shock front is a discontinuity in physical conditions, particularly density. From (3-B) it is clear that any constant value of $\bar{\xi}$ defines the movement of a surface of constant density through the medium. It is now necessary to identify one of these surfaces, i.e. a certain value of $\bar{\xi}$, with the shock wave surface.

If the pressure, density, velocity and internal energy of the undisturbed medium are p_o, ρ_o, u_o, e_o, and the values immediately behind the shock front are p^*, ρ^*, u^* and e^* then these quantities are related by the Rankine-Hugoniot equations:

$$\rho_o c = \rho^*(c-u^*) \qquad \text{(4-A)}$$

$$p^* - p_o = \rho_o u_s u^* \qquad \text{(4-B)}$$

$$e^* - e_o = 1/2(p_o + p^*)\left(\frac{1}{\rho_o} - \frac{1}{\rho^*}\right) \qquad \text{(4-C)}$$

where c is the velocity of sound in the medium and u_s is the velocity of propagation of the shock wave. Thus $\bar{\xi} = \xi_o$ = constant can be defined as the shock front if equations 4-A, B and C are satisfied.

This is so because $u_s = \left.\frac{dr}{dt}\right|_{\bar{\xi}=\xi_o} = \alpha\, \xi_o\, t^{\alpha-1}$.

It is convenient to work in terms of the reduced pressure, density, velocity and internal energy which are P*, D*, U* and G* immediately behind the shock front. Substituting these values the Rankine-Hugoniot equations become:

$$D^*(\alpha - U^*) - \alpha = o \tag{5-A}$$

$$D^*\ U^*(\alpha - U^*)\ -D^*P^* = \frac{p_o}{\rho_o(u_s/\alpha)^2} \tag{5-B}$$

$$G^* = \frac{1}{2}\,\frac{p_o}{\rho_o(c/\alpha)^2} + D^*P^*\ (1 - \frac{1}{D^*}) \tag{5-C}$$

But in the blast wave problem under consideration $p_o << \rho_o({}^{u}s/\alpha)^2$.

Here the form of the equation of state chosen is $e = p\,\varphi(\rho)$ where $\varphi(\rho) = \frac{1}{(\gamma-1)\rho}$. Thus we can write from (3-C) and (3-D) a second equation for G* at the shock front:

$$G^* = \rho_o D^*P^*\ \varphi(\rho_o D) \tag{6-A}$$

from (5-A):

$$D^*(\alpha - U^*) = \alpha$$

$$\frac{U^*}{\alpha} = 1 - \frac{1}{D^*} \tag{6-B}$$

Again from (5-B), and (5-C) since p_o is small, we find

$$D^*P^* = \alpha^2 (1 - \frac{1}{D^*}) \tag{6-C}$$

$$G^* = \frac{1}{2}\,\alpha^2\,(1 - \frac{1}{D^*})^2 \tag{6-D}$$

Substituting (6-B,C,D) in (6-A) we get

$$\frac{1}{2}\,(1 - \frac{1}{D^*}) = \rho_o\,\varphi(\rho_o D^*) \tag{6-E}$$

$$D^*(\xi_o) = (\gamma+1) \Big/ (\gamma-1)$$

Thus it can be seen that a single real value of ξ_o exists at the shock front. The actual value of ξ_o can be found from energy considerations.

For the fast processes like meteoritic impacts we can reasonably assume the total energy to be constant during the crater formation. For $\xi = \xi_o$ representing the shock at time 't' , the total energy of the fluid shell (kinetic + internal) is given by the volume - integral of the total energy per unit volume

$$E(t) = \int_{o}^{R_s(t)} \left\{\frac{1}{2} u^2 + \frac{p}{\rho(\gamma-1)}\right\} 2\pi r^2 \rho dr \qquad (7\text{-A})$$

when $R_s(t)$ is the shock radius at time 't', $\frac{p}{(\gamma-1)\rho}$ = internal energy per gm for the baratropic medium.

We know that for $r = R_s$, $\bar{\xi} = \xi_o$, so by substituting $r = \xi_o t^{2/5} \xi$, we get

$$E(t) = 2\pi\rho_o \xi_o^{\,5} \int_{o}^{1} \left\{\frac{1}{2} U^2 + \frac{P}{\gamma-1}\right\} D \xi^4 d\xi \qquad (7\text{-B})$$

since at the shock front $\xi = 1$.

The value of ξ_o can be evaluated from this equation. Although ξ_o is of critical interest in the problem, since it gives the depth of penetration of the shock front, this integral cannot immediately be evaluated. It is necessary to explore the functional form of U, P and D and evaluate the integral numerically.

Mathematical treatment

(a) Self-similar hydro-dynamical equations:

The solution of the hydrodynamic equations (2-A,B,C) becomes simpler on substituting eq. (3-A,B,C) in them because of the elimination of the variable 't' and we are thus led to the following set of ordinary differential equations instead of the partial ones

$$\xi D'/D = -(\xi U' + 3U) \Big/ (U - \alpha) \tag{8-A}$$

$$\xi U' = \left[- U(U - \alpha)\ (U - 1) + (2\beta + 3U\gamma)\ P\right] \Big/ \left[(U - \alpha)^2 - \gamma P\right] \tag{8-B}$$

$$\xi P'/P = \Big\{(\gamma-1)\ U\ (U-1) - (U-\alpha)\ \left[(3\gamma-1)\ U-2\right] + \left[2\beta/(U-\alpha) + 2\gamma\right]\ P\Big\} \Big/ \Big\{(U-\alpha)^2 - \gamma P\Big\} \tag{8-C}$$

where the primes denote differentiation with respect to single independent variable ξ, and $\beta = \alpha - 1$.

Dividing eq. (8-C) by eq. (8-B) gives

$$\xi P'/\xi U' = \frac{dP}{dU} = P\ \left[N(U) + PQ(U)\right] \Big/ \left[R(U) + PS(U)\right] \tag{8-D}$$

where

$$N(U) = \gamma U(3\alpha - 1 - 2U) + (3 - \alpha)\, U - 2\alpha$$

$$Q(U) = 2\beta/(U - \alpha) + 2\gamma$$

$$R(U) = U(U - \alpha)\,(1 - U)$$

$$S(U) = 2\beta + 3U\gamma$$

We will first solve (8-D) and then (8-A) and (8-B). It is clear that since no closed form of solution is possible numerical methods must be used.

(b) The Initial Conditions:

Mathematically we will solve the problem by starting at a known point (U^*, D^*, P^*). These starting conditions, at the shock-front, are characterized by the Rankine-Hugoniot equations which for very high Mach number are

$$\frac{\rho_1}{\rho_o} = \frac{\gamma+1}{\gamma-1} \qquad (9\text{-A})$$

$$u^* = \frac{2u_s}{\gamma+1} \qquad (9\text{-B})$$

$$p^* = p\, u_s^{\,2}\, \frac{2}{\gamma+1} \qquad (9\text{-C})$$

On the shock front $\xi = \xi_o$ = constant, and so these equations can be expressed in reduced form by using (3-A,B,C).

$$U^* = U(\xi_o) = 2\alpha \Big/ (\gamma+1) \tag{9-D}$$

$$D^* = D(\xi_o) = (\gamma+1) \Big/ (\gamma-1) \tag{9-E}$$

$$P^* = P(\xi_o) = 2\alpha^2(\gamma-1) \Big/ (\gamma+1)^2 \tag{9-F}$$

where $\alpha = 2/5$ according to the total energy conservation condition.

Since the constants α and γ are known we can calculate the initial values $U(\xi_o)$, $D(\xi_o)$ and $P(\xi_o)$ and can proceed with the numerical solution of 8-D. The details of the numerical solution are given in the appendix. Now for a medium for which $\gamma = 6$ the corresponding reduced variables are given as $D(\xi_o) = 1.4$, $U(\xi_o) = 0.114285$, $P(\xi_o) = 0.03265$.

The starting point for the solution of eq. (8-D) has been fixed as (0.114285, 0.03265). The equations have been solved numerically (for details see appendix). The functional form of $U(\xi)$, $P(\xi)$, $D(\xi)$ having been thus calculated, the energy integral in (7-B) is computed numerically using Simpson's Rule. So the value of ξ_o is determined as follows:

We know

$$E = 2\pi\rho_o \xi_o^5 I(\gamma) \quad \text{where} \quad I(\gamma) = \int_o^1 \left[\frac{1}{2} U^2 + \frac{P}{\gamma - 1}\right] D \xi^4 d\xi$$

But $E = f \cdot \frac{1}{2} mv^2$ where f is the fraction of the impact energy converted into the shock energy.

$$\xi_o = \left[\frac{1}{2\pi\rho_o I(\gamma)}\right]^{1/5} \left(f \cdot \frac{1}{2} mv^2\right)^{1/5}$$

$$= 5.678 \times 10^{-1} \left[f \cdot \frac{1}{2} mv^2\right]^{1/5} \qquad (9\text{-}G)$$

The velocity v is supposed to be the vertical velocity of meteorite, and the direction of meteorite velocity is unspecified. Experiments have shown that the crater dimensions depend only upon the normal component of the velocity up to the angles of incidence of about 55° (Summers and Charters 1959). For mathematical convenience we will choose $f = 1$.

(c) Limit of Penetration

We want to find the depth at which the penetration velocity becomes sonic. It is convenient to consider the ratio between the pressures just behind and just in front of the shock in order to establish the above mentioned sonic criterion. The pressure ratio can be derived from the Rankine-Hugoniot shock relations as follows:

$$\frac{u_s^{\,2}}{c^2} = \frac{1}{2}\left\{(\gamma-1) + (\gamma+1)\frac{p^*}{p_o}\right\} \qquad (10A)$$

where 'c' is the sound speed in the undisturbed medium.

$$\frac{u^*}{u_s} = \frac{2\left(p^*/p_o - 1\right)}{(\gamma-1) + (\gamma+1)\, p^*/p_o} \qquad (10\text{-}B)$$

where u^* is the particle velocity just behind the shock front.

From the above two equations we get

$$\frac{u^{*2}}{c^2} = \frac{u^{*2}}{u_s^{\,2}} \times \frac{u_s^{\,2}}{c^2} = \frac{2\left(p^*/p_o - 1\right)^2}{\gamma\left\{(\gamma-1) + (\gamma+1)\, p^*/p_o\right\}} \qquad (10\text{-}C)$$

When $\frac{u^*}{c} = 1$, we get a quadratic in ψ $(= p^*/p_o)$

$$2\psi^2 - \psi\left[\gamma(\gamma+1) + 4\right] + \left[2 - \gamma(\gamma-1)\right] = 0 \qquad (10\text{-}D)$$

$$\psi = \left[\frac{\gamma(\gamma+1)}{4} + 1\right] + \frac{1}{4}\left\{\left[\gamma(\gamma+1) + 4\right]^2 - 8\left[2 - \gamma(\gamma-1)\right]\right\}^{1/2} \qquad (10\text{-E})$$

Only the +ve sign for the root of the quadratic equation has been kept in order that it should be in accord with shock Mach number $M > 1$.

The limit of penetration of the meteorite (we shall call it the sonic crater depth) is determined as follows.

The pressure behind the shock wave is given by (3-C) as

$$p = t^{-6/5} \rho_o D^* P^* \xi_o^{\,2} \qquad (3\text{-C})$$

Now $p = \psi p_o$ where $\psi = 23.6$ (from 10-E) for a medium with $\gamma = 6$, and p_o, the pressure on the undisturbed side of the medium = 200 kilobars, corresponding to the crushing strength of averagium. Now from our definition, ξ applied to the shock front where $\bar{\xi} = \xi_o \xi = \xi_o$; $r = \xi_o \, t^{2/5}$. Eliminating 't' from the above two equations, we get

$$r = \xi_o^{\,8/3} \left[\frac{\rho_o D^* P^*}{p}\right]^{1/3} = \xi_o^{\,8/3} \left[\frac{\rho_o D^* P^*}{\psi p_o}\right] \qquad (10\text{-F})$$

where 'r' is the sonic crater depth.

ξ_o is known from (9-G). So all the quantities of right side of eq. 10-F are known. Therefore 'r' can be computed.

(d) <u>Justification of Self Similarity</u>

We know that the fundamental pre-requisite for the flow field to be self-similar is that the density ratio across the shock, i.e. D*, should remain constant. This will hold good for a small interval of time after which the flow will no longer be self similar. Now let us see whether the self similar condition holds good in the time interval during which the particle velocity decays to that of the sound.

We know $\frac{u^*}{c}$ can be expressed in terms of Mach number as follows:

$$\frac{u^*}{c} = \frac{2(M^2-1)}{(\gamma+1)M}$$

The Mach number M of the shock wave corresponding to the conditions where the velocity of the material has decreased to c is given by the expression

$$M = \frac{\gamma+1}{4}\ \frac{u^*}{c} + \left\{1 + \left[\frac{(\gamma+1)\ u^*}{4c}\right]^2\right\}^{1/2} \qquad \text{(11-A)}$$

Only the positive sign has been retained in order that $M > 1$.

The Mie-Gruneisen equation of state for 'Averagium' has been picked such that $\gamma = 6$. So we find

$$\frac{\rho^*}{\rho_o} = \frac{\gamma+1}{\gamma-1 + 2/M^2}$$

Thus at the start the condition is

$$\frac{\rho^*}{\rho_o} = \frac{\gamma+1}{\gamma-1} = 1.4$$

when the particle velocity decays to sonic velocity, $M = 3.75$, and the density ratio across the shock front $\frac{\rho^*}{\rho_o}$, reduces to 1.37 corresponding closely to the initial value of 1.4. So the similarity condition is approximately satisfied, because the density ratio across the shock remains approximately constant.

RESULTS:

The constants for the polytropic equation of state used in our calculations have been fixed from extrapolation of the experimental results of Walsh et al (1957) and from the lower part of the theoretical curves of Gilvarry and Hill (1956). The latter have used the Thomas-Fermi equation of state for these meteoritic velocities, and the theory is probably reliable for the high pressures existing before the particle velocity decreases to the velocity of sound. The differential equations were solved by Runge-Kutta method and the energy integral evaluated numerically using the Simpson rule. (The details are given in the appendix.)

The (U,P) solution curve, (U,ξ) and (ξ,D) curves are as shown in figs. 1, 2, 3. We find a sharp decrease in D just behind the shock front, but it goes gradually to zero as we move away from the shock front. It should be noted from fig. 3 that at the position $\xi = 0.15$ behind the shock front density is zero. This corresponds to a rarefaction or cavitation and has been previously noted by Davids and Huang (1962).

An estimate of the sonic crater size has been made by applying the sonic criterion of the pressure ratio across the shock front which is equal to 23.6 for $\gamma = 6$. The value of p_o i.e. the pressure of the undisturbed medium has been assumed to be equal to the shear modulus of 'averagium' which has been taken as 200 kilobars. The energy has been assumed to remain constant throughout the process. The impact kinetic energy of the projectile manifests itself as heat, light, breaking and throwing of the materials, elastic and plastic flow, and the shock wave. For the time being we have supposed the whole input energy to be converted into shock energy, corresponding to $f = 1$. Further work must be done to obtain a more realistic value because certainly f is less than 1.0.

The particle velocity represents the rate of penetration of the meteorite, the sonic radius, therefore, is the depth at which the meteorite has been decelerated to the speed of sound.

At this condition the pressure just behind the shock front is approximately 20 times greater than the shear strength of the material of the moon surface. Let us examine how the fluid model should behave when the particle velocity has become equal to the sound velocity.

The pressure exerted on the shocked side of the front is 23 times the pressure on the undisturbed side. If we suppose as a very rough approximation that the self-similarity remains valid after the shock wave decays into the sound wave then according to (3-C), the time required for the pressure to decay to the shear modulus would be almost 12 times the sonic time interval. At this time the crater radius would be 2.7 times the sonic crater depth. This would be true only if the flow field remains similar and the medium remains fluid throughout. It is nevertheless a useful approximation and is probably more close to the upper limit than the lower limit because 100% efficiency is assumed in the self similar calculations.

We can see that at the sonic depth, the difference in the internal energies of the shocked and undisturbed media is 16×10^{10} ergs/gm which is almost 15 times the latent heat of fusion of iron. So the shocked medium is definitely in a fluid state at this time. Spherical divergence is a cause of shock decay and this becomes quite dominant at large radii. At the sonic depth the energy difference across the shock is about 1.6×10^{11} ergs/gm which is sufficient to melt the lunar material. However, after penetrating further by a factor of 2.7, the energy is no longer able to fuse and melt the material. Somewhere between these limits, the liquid hypervelocity equations fail. It is convenient to take this change of mechanism at the sonic crater depth.

So from now onward the hydrodynamic model will no longer be used. The solid-solid, shock-solid, and the liquid-solid impact should be sufficiently valid approximations. Cracking, crushing and disturbance of the lunar material under brittle conditions (Bowden and Brunton 1961) will be considered in a later report.

The penetration depth of the meteorite, defined as the sonic depth, varies from 4 to 6 meteorite diameters. This confirms Baldwin's (1962) prediction regarding the formation of the central mountain peaks.

The sonic crater depths have been calculated for various masses of meteors ranging from 10^4 to 10^{14} kgm (Hawkins 1963) and for the meteorite velocity between 15 to 75 km/sec. The plots for the impact velocity vs. sonic crater depth and the meteoritic mass vs. the sonic crater depth are shown in Figs. (4, 5, 6). The slope of this straight line is .68 which agrees with 2/3rd power law of Eichelberger and Gehring (1962) obtained by curve fitting, and with Walsh and Tillotson (1963) relation $\mathcal{P}/d = k(V_o/C_o)^\delta$ when $\mathcal{P}$ is crater penetration depth, d the projectile dimension, V_o the projectile velocity, K and C_o being constant and the value of δ is as follows:

$\delta = .61 \pm .02$ for the velocity range 10^7 to 2.5×10^7 cm/sec

$\delta = .62 \pm .03$ for the velocity range 10^6 to 4×10^6 cm/sec

These values are not in bad agreement with our calculation of $\delta = .68$.

The fact that the self-similarity approximation for determining the sonic crater depth holds fairly good, is corroborated by the velocity-depth profiles for the meteor crater, Arizona, as calculated by Bjork (1961). For a meteorite mass 12,000 tons and an impact velocity 30 km/sec, the sonic crater depth for iron-tuff impact, using Olshaker and Bjork (1962) scaling laws, is found to be 43.7 meters by our calculations, corresponding to Bjork's 57 meters. This latter value is a sum of sonic depth and projectile height, which was taken as 12 meters.

So for the early phases our results seem to be in good agreement with other methods. The lunar central mountain peaks are supposed to be formed during the earlier stages when the particle velocity is sufficient to melt and vaporize the material by a hypervelocity explosion.

In our calculations we considered the Grüneisen's constant to be the same for all impact velocities and this assumption might be questioned. Moreover we have not taken into account the increase in the crater depth due to later fusion or vaporization of the medium caused by the shock wave after the projectile has slowed down to a velocity less than the speed of sound.

Concluding Remarks:

We have dealt with a symmetric problem of a point release of energy in a given medium. There is need for introduction of two spatial coordinates in our hydrodynamical equations.

The similarity assumption restricts the equation of state to a special form, and in relatively weaker stages of shock propagation this assumption is poor. Therefore in the later stages, the shock propagation is characteristically non-similar. So quasi-similarity technique (Oshima 1960) for solving the problem should be applied (Rae and Kirchner 1963.)

It is necessary that the velocity corresponding to energy conservation (in explosion case) and the momentum conservation should be matched and used in quasi similarity calculations.

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I am grateful to Dr. G. S. Hawkins, Professor of Astronomy, Boston University, under whose direction the work was done. Without his help, this report could not have been prepared.

Appendix (A):

The polytropic form of the equation of state which has been chosen is amenable to the similarity solution of the fluid-dynamical equations. The similarity solution holds good for the earlier stages of the impact. Non-similar solution would give the proper description at a later time. As we are interested in the time interval during which the particle velocity becomes sonic, we have fixed the value of $\gamma = 6$, $\rho_o = 4$, $p_o = 200$ kilobars. So for such a medium the initial values have been fixed as $P(\xi_o) = 0.03265$, $U(\xi_o) = 0.114285$, $D(\xi_o) = 1.4$. These values can give us the initial point to start the solution of the differential equations.

The differential Eq. (8-D)

$$\frac{dP}{dU} = P\left[N(U) + PQ(U)\right] \Big/ \left[R(U) + PS(U)\right] \qquad \text{(8-D)}$$

has singularities at $(P = o, U = o)$, $(P = o, U = \alpha)$, $(P = o, U = 1)$ and at the shock surface where $P = 0.03265$, $U = 0.114285$. The equation becomes singular where $\frac{dP}{dU} = \frac{o}{o}$. For details see Davids and Calvit, 1962.

In order that the above equation be amenable to Runge-Kutta method, (Romanelli 1959) the initial points to start numerical calculations should not be a singular point. But the initial conditions fixed are those for the shock point; so the Runge-Kutta method cannot be applied as it is. One way of solving the equation is to start from $P = o$, $U = \alpha$ and determine the limiting value of slopes (Davids and Calvit, 1962). The values of the slope are ∞, o, $\frac{\alpha-1}{3}$. We put for $\alpha = 4$, the slope = -0.2. So the starting point for the Runge-Kutta numerical integration can be taken as U = .399, P = 0.0002 and we can go back in steps till we hit the shock-point. Then we can fix the initial conditions for the other two equations.

But we can evaluate the slope $\frac{dP}{dU}$ at the shock points by taking limits and then exclude the unstable zone and fix up the starting point by linear extrapolation using the suitable limiting value of the slope. The limit can be evaluated as follows:

$$\lim_{\substack{U \to U_s \\ P \to P_s}} \frac{dP}{dU} = \frac{o}{o}$$ when (U_s, P_s) represents the shock point of

P-U plane. Now applying L'Hopital's Rule at the shock front

$$\lim_{\rightarrow} \frac{dP}{dU} = \lim_{\rightarrow} \frac{dP/dU\ [N + 2PQ] + P\ [dN/dU + PdQ/dU]}{dR/dU + PdS/dU + SdP/dU} \tag{12-A}$$

From this we get

$$A\,X^2 + BX + C = o \tag{12-B}$$

where: $X = \lim_{\rightarrow} \frac{dP}{dU}$

$$A = S$$

$$B = \lim_{\rightarrow} \left[\frac{dR}{dU} + P\,\frac{dS}{dU} - N - 2PQ\right]$$

$$C = \lim_{\rightarrow} - \left[dN/dU + PdQ/dU\right] P$$

The solution of this quadratic equation gives two real roots for $\gamma = 6$, and the roots are $X = 0.284165$ and $X = -0.206069$, indicating a discontinuity at the shock front which satisfies the physical condition of a discontinuity in velocity, pressure, hence density, etc.

Again for other equations

$$\frac{\partial U}{\partial(\ln\xi)} = \left[(U - \alpha)^2 - \gamma P\right] \Big/ \left[R(U) + S(U)\right] \qquad (8\text{-B})$$

Applying L'Hopital's Rule

$$\lim_{\rightarrow} \frac{\partial U}{\partial(\ln\xi)} = \lim_{\rightarrow} \frac{2(U-\alpha) - \gamma\, dP/dU}{\frac{dR}{dU} + \frac{dS}{dU}} \qquad (12\text{-C})$$

$= -3.1569$ at the shock point (U_s, P_s)

ξ can be taken as 1 at the shock, hence the suitable starting point for numerical solution of the above differential equation, can be computed using this limiting value of the slope $\frac{\partial U}{\partial(\ln \xi)}$.

The solution of the third differential equation now becomes a straightforward problem of numerical analysis.

The computations were performed on IBM 1620 machine at the Boston University Computing Center. The details of the programs appear in Appendix B.

```
CC          APPENDIX (B)
C      PROGRAM 1
C      LOCATION OF ACOUSTIC MASS FRONT IN SHOCK WAVE ANALYSIS
       DIMENSION      Y(4,5),YP(4,5),Q(4,5),YK(4,5),SA(5),SB(5),SC(5)
       DIMENSION Z(4),X(4),YA(4)
       SA(2)=.5
       SA(3)=1.-1./SQRTF(2.)
       SA(4)=1.+1./SQRTF(2.)
       SA(5)=1./6.
       SB(2)=2.
       SB(3)=1.
       SB(4)=1.
       SB(5)=2.
       SC(2)=1./2.
       SC(3)=SA(3)
       SC(4)=SA(4)
       SC(5)=1./2.
       PB=1.
       DELTA=1./100.
       M=100
       L=4
       H=0.001
       G=6.
       AL= .4
       W=(2.*AL)/(G+1.)
       X(1)=1.
       X(2)=W
       X(3)=2.*AL*AL*(G-1.)/(G+1.)**2
       X(4)=(G+1.)/(G-1.)
       YI=4.*(3.1416)* (.5*X(2)*X(2)+X(3)/(G-1.))*X(4)*(X(1))**4
       PRINT 302,YI
  302 FORMAT( 4H YI=E18.4)
       Y(1,1)= 0.1190028
       Y(2,1)=0.031681698
       Y(3,1)=-0.0157745
       Y(4,1)=-.00001403
       YP(1,1)=1.
       Q(1,1)=0.
       Q(2,1)=0.
       Q(3,1)=0.
       Q(4,1)=0.
   65 DO 70 I=1,L
       YA(I)=X(I)
   70 CONTINUE
       Z(1)=1.-PB*DELTA
   12 J=2
   13 I=1
       KM=1
    2 U=Y(1,J-1)
       P=Y(2,J-1)
       AA=3.*AL-1.-2.*U
       AB=(AL-1.)*P/(U-AL)
       AF=3.-AL
       UN=G*U*AA+AF*U-2.*AL
       QU=2.*AB+2.*G*P
       FF=P*(UN+QU)
       RU=U*(U-AL)*(1.-U)
       SU= (2. *(AL-1.)+3.*G*U)*P
       GG=RU+SU
       YP(2,J)=FF/GG
```

```
      YP(3,J)=((U-AL)**2-G*P)/(RU+SU)
      YP(4,J)=(1.+3.*U*YP(3,J))/(AL-U)
      YP(1,J)=1.
      GO TO (4,91),KM
    4 YK(I,J)=YP(I,J)
    5 IF(I-L)15,16,16
   15 I=I+1
      GO TO 4
   16 I=1
   17 Y(I,J)=Y(I,J-1)+H*SA(J)*(YK(I,J)-SB(J)*Q(I,J-1))
      Q(I,J)=Q(I,J-1)+3.*SA(J)*( YK(I,J)-SB(J)*Q(I,J-1))-SC(J)*YK(I,J)
      IF(I-L)18,19,19
   18 I=I+1
      GO TO 17
   19 IF(J-5)20,21,21
   20 J=J+1
      GO TO 13
   21 DO 90 I=1,L
      Y(I,J-1)=Y(I,J)
   90 CONTINUE
      X(1)=EXPF(Y(3,J))
      X(4)=EXPF(Y(4,J))
      X(2)=Y(1,J)
      X(3)=Y(2,J)
      KM=2
      GO TO 2
   91 IF(Y(1,1)-.399) 26,26,630
   26 PRINT 300, (I, YP( I,J),Y(I,J),Q(I,J),I=1,L)
      PRINT 303,X(1),X(2),X(3),X(4)
  303 FORMAT(6H X(1)=E18.4,6H X(2)=E18.4,6H X(3)=E18.4,6H X(4)=E18.4)
  300 FORMAT(3H I=I1,3E18.4/3H I=I1,3E18.4)
      DO 130 I=1,L
      Y(I,1)=Y(I,5)
      Q(I,1)=Q(I,5)
  130 CONTINUE
      IF(Z(1)-X(1))12,550,550
  550 DO 560  I=2,L
      Z(I)= X(I)+(X(I) -YA(I))* (Z(1)- X(1))/(X(1)-YA(1))
  560 CONTINUE
      YB=4.*(3.1416)* (.5*Z(2)*Z(2)+Z(3)/(G-1.))*Z(4)*(Z(1))**4
      LN=PB
      PRINT 209,YB,LN,Z(1)
  209 FORMAT(4H YB=E18.4,4H LN=I3   ,6H Z(1)=E18.4)
      PP=PB/2.
      KB=PB
      KC=KB/2
      PC=KC
      IF(KB-M)590,630,630
  590 IF(PP-PC)620,620,600
  600 YZ=4.*YB
      GO TO 625
  620 YZ=2.*YB
  625 YS=YI+YZ
      YI=YS
      PRINT 200,YS
  200 FORMAT(4H YS=E18.4)
      IF(Z(1)-.001)630,400,400
  400 IF(Z(4)-0.001)630,450,450
  450 PB=PB+1.
      KB=KB+1
```

```
      IF(Z(1)-X(1)-DELTA)65,80,80
   80 Z(1)=1.-PB*DELTA
      GO TO 550
  630 SUM=(1./3.)*DELTA*(YS+YB)
      PRINT 201,SUM
  201 FORMAT(5H SUM=E18.4)
   27 STOP
      END
```

```
C     PROGRAM 2
C     COMPU TATION OF SONIC CRATER DEPTH
  210 FORMAT(28H ACOUSITIC VELOCITY(CM/SEC)=E16.2)
  209 FORMAT(45H DECAY TIME   VELOCITY(CM)   SONIC RADIUS(CM))
  207 FORMAT(F12.7,E16.2,E18.4)
  206 FORMAT(E18.4,2F3.1,2E16.4)
  208 FORMAT(21H MASS OF METEOR(GMS)=E15.4)
  205 FORMAT(34H COMPUTATION OF SONIC CRATER DEPTH)
      PRINT 205
      READ 206,SUM,G,RHO,A,V
      AL=0.4
      U=2.*AL/(G+1.)
      D=(G+1.)/(G-1.)
      PR=2.*.16*(G-1.)/(G+1.)**2
      PS=2.*(10.)**11
      DO 301  N=1,11
      Q=(A/10.)*(10.)**N
      PRINT 208,Q
      PRINT 209
      DO 301  IK=1 ,5
      BIK=IK
      X=BIK*V
      ZZ=Q*X*X/(RHO*SUM)
      ZI=(ZZ)**(1./5.)
      T=(D*PR*RHO*ZI*ZI)/(23.5*PS)
      TT=(T)**(5./6.)
      UU=U*ZI*((TT)**(-3./5.))
      RC=(ZI) *((T)**(1./3.))
      PRINT 207,TT,X,RC
  301 CONTINUE
      PRINT 210,UU
      STOP
      END
     3228.2380E-056.04.0         1.00E 07           1.50E 06
```

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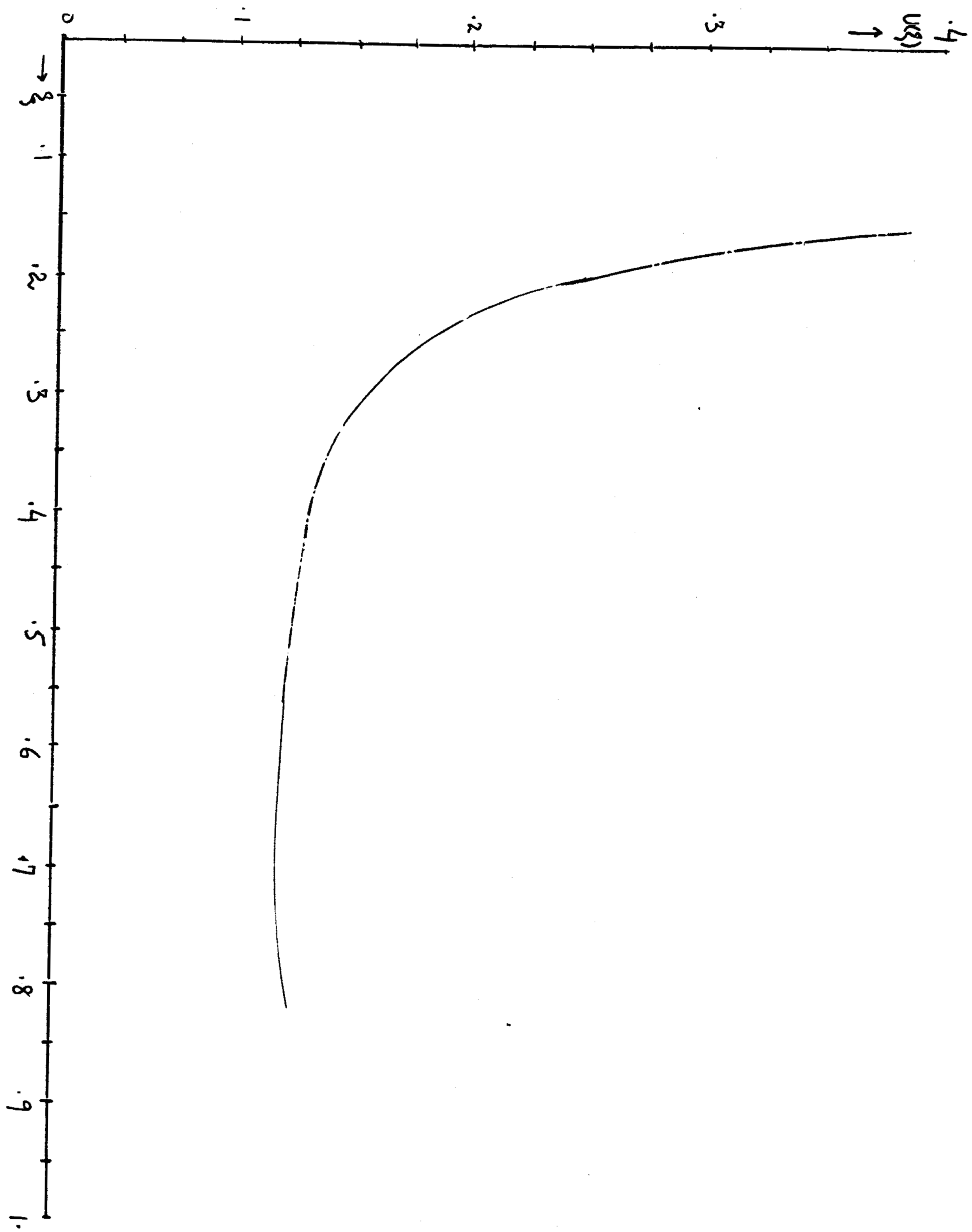

Fig. 1

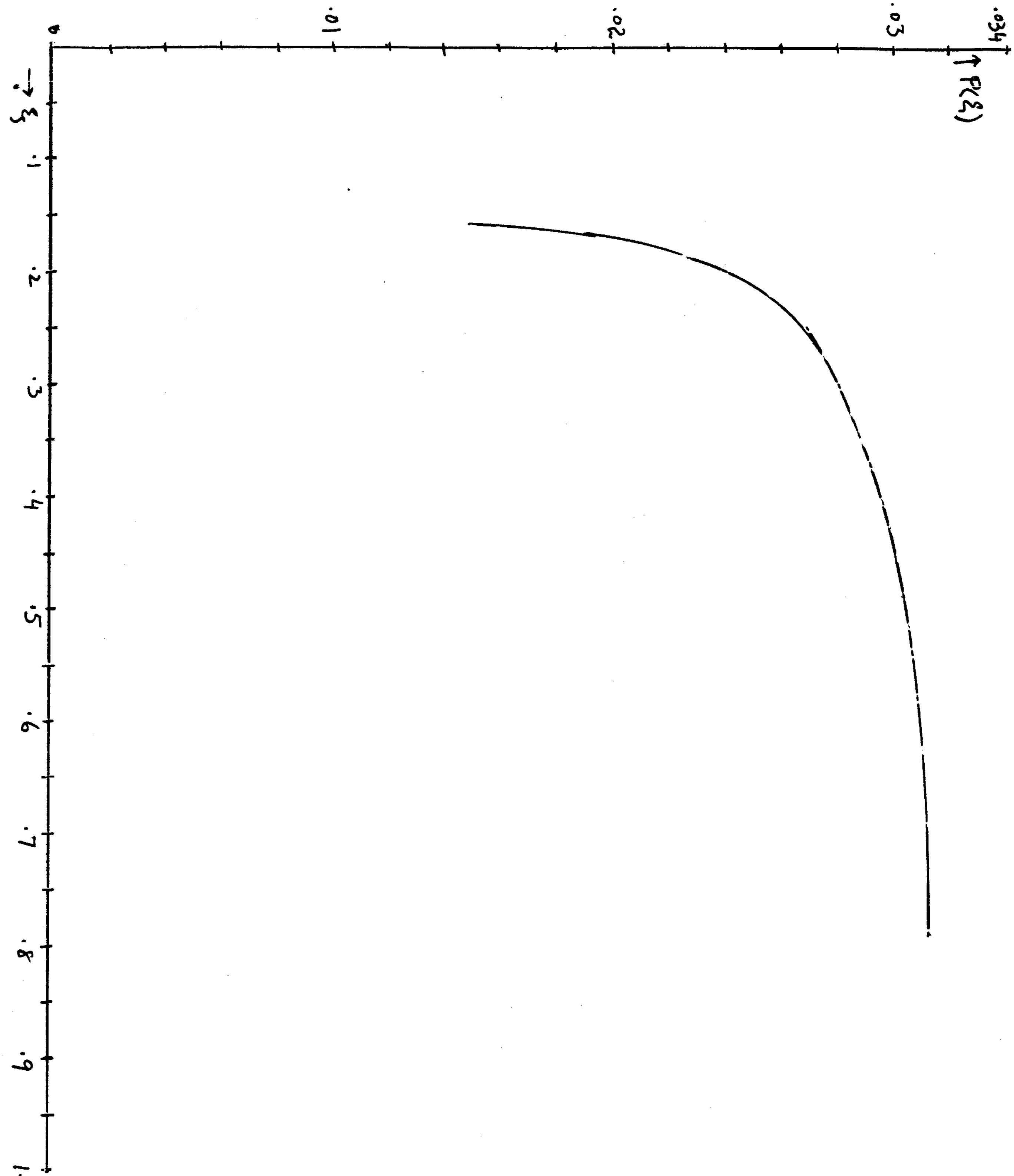

Fig. 2

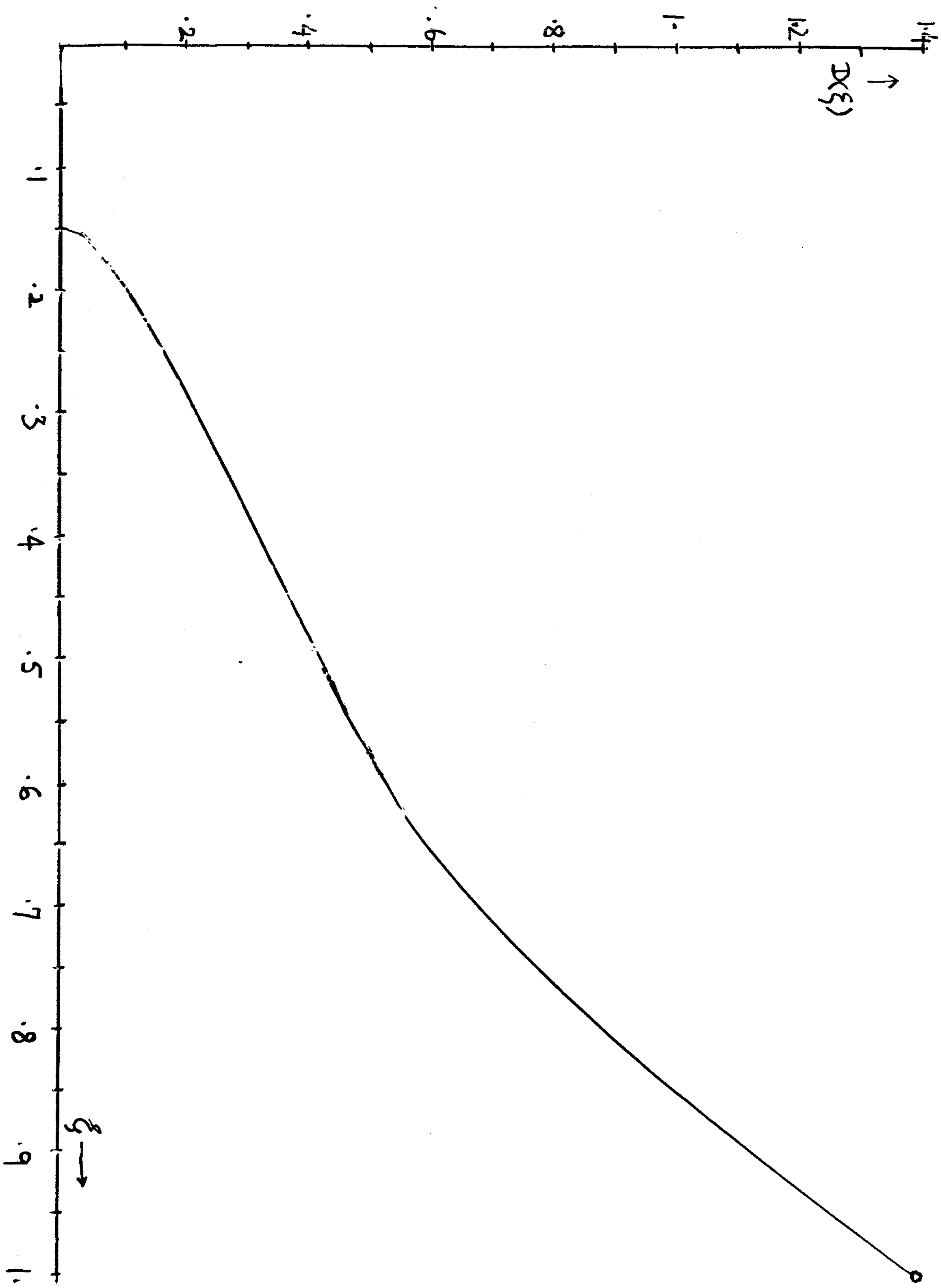

Fig. 3

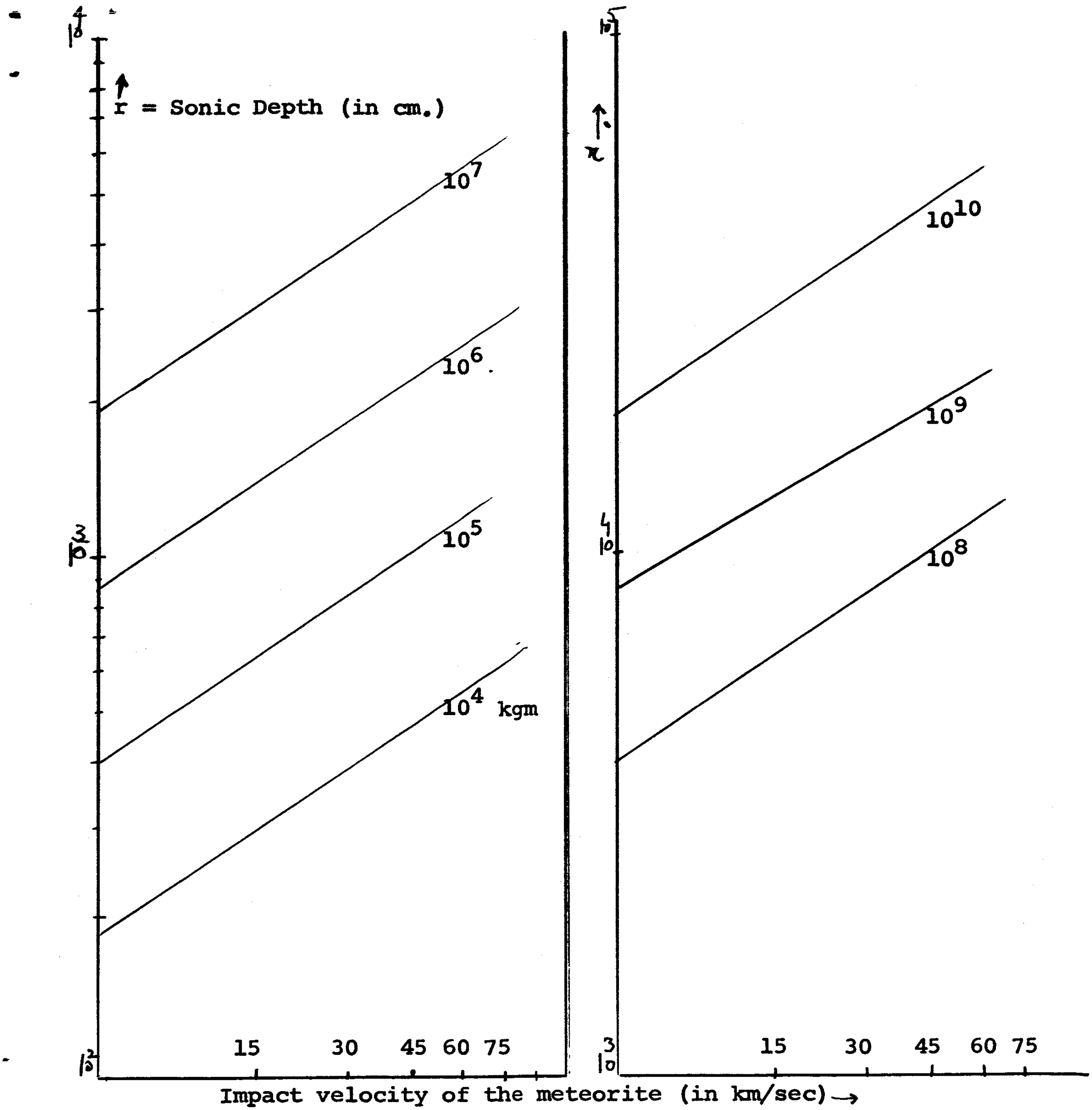

Fig. 4 Log-Log Plot for the Impact Velocity vs. Sonic Depth for different Masses of the Meteorite

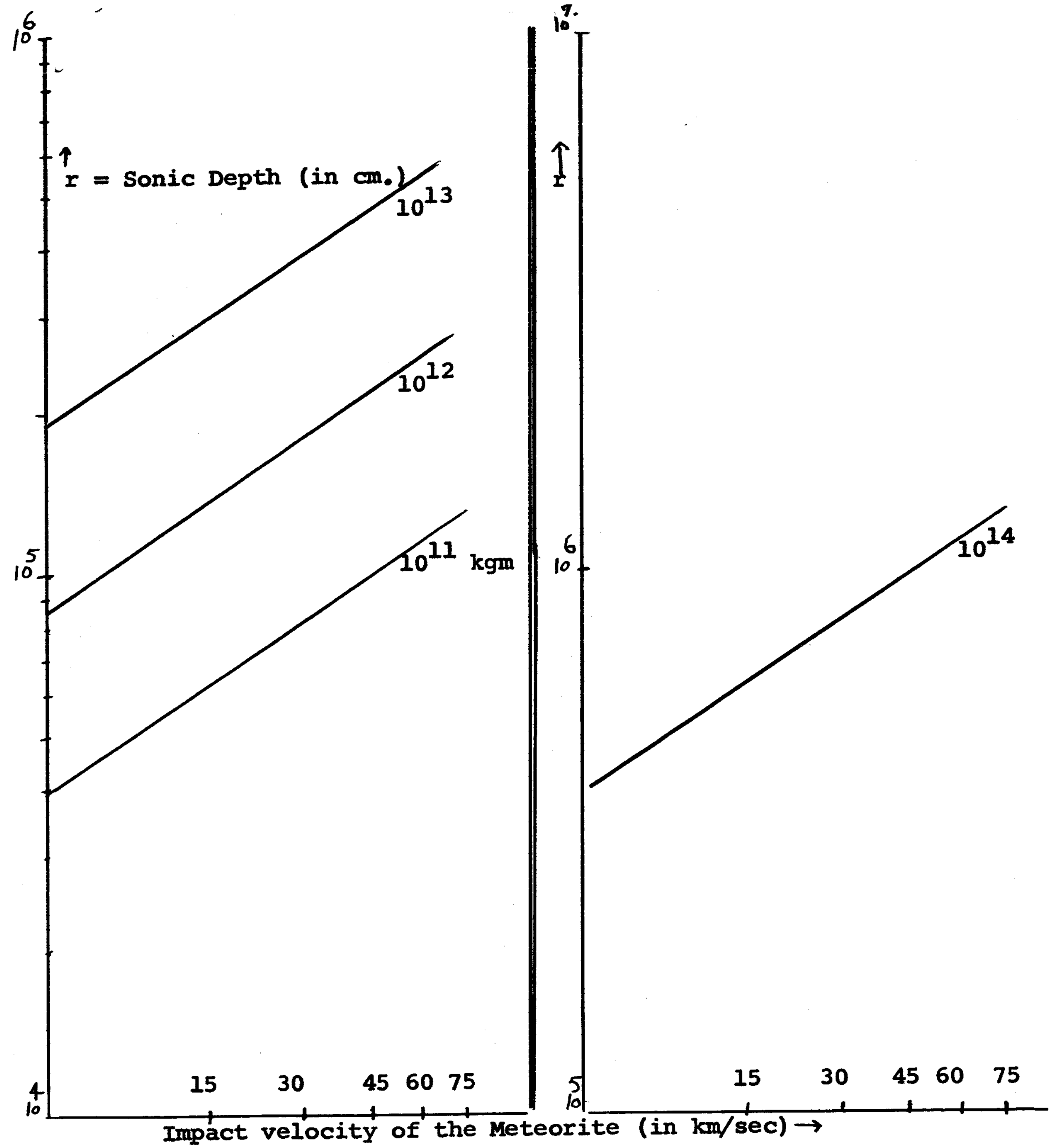

Fig. 5 Log-Log Plot for the Impact Velocity vs. Sonic Depth for Different Masses of the Meteorite

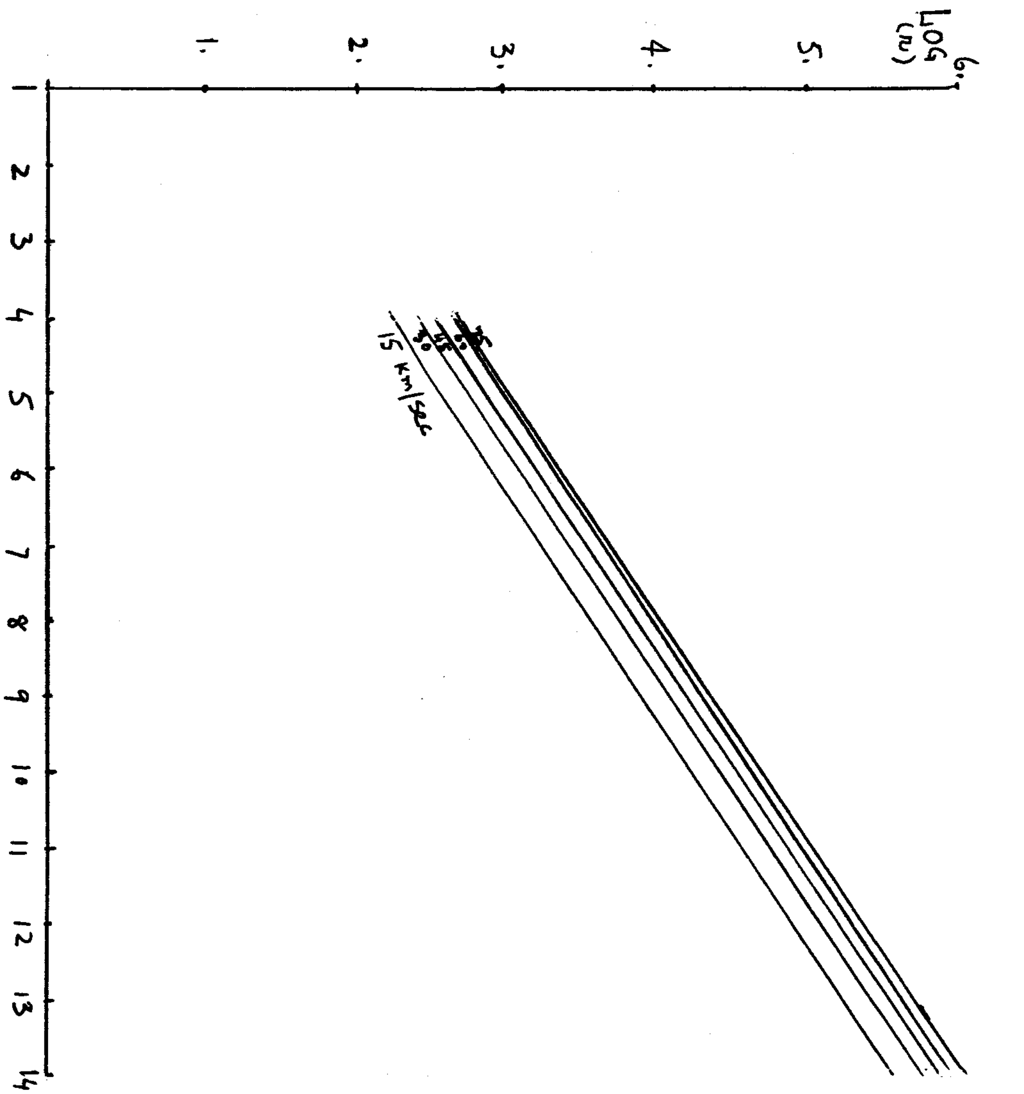

Fig. 6 Log-Log Plot for the mass of the Meteorite vs. the sonic crater depth