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SINGLE PARAMETER TESTING

FINAL REPORT

NAS 8-11715, Part III

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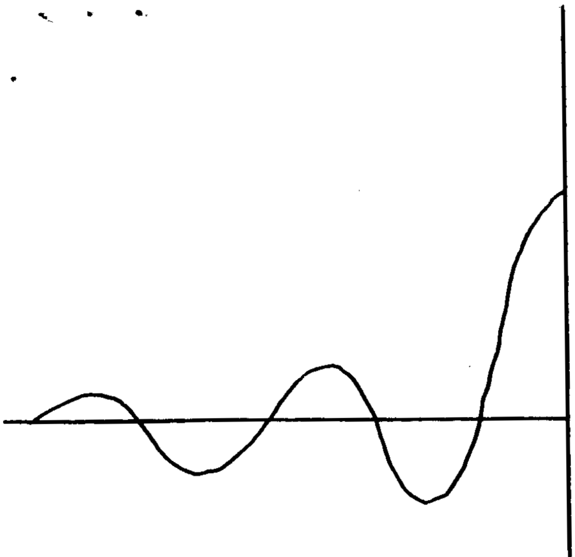


SINGLE PARAMETER TESTING

"There is a better way to conduct testing"

SUMMARY

This report gives the final results obtained on the NAS 8-11715 contract, in the area of single parameter testing. The main objective of the study is to put into operation better ways of testing transfer functions. The expected savings are faster checkout time, better accuracy and less degradation of performance due to the testing.

The results of this study are positive. We can test linear passive and active transfer functions. The savings are faster checkout time, faster isolation of parameters out of tolerance, and less degradation of performance due to testing.

The technical areas investigated have confirmed two techniques which are directly applicable to the measurement of the parameters of a transfer function. These techniques are:

1. Growing exponential probing signals matched to the partial system responses and filtering.

2. Growing impulse probing signals and time sampling.

The second method is directly applicable to confidence sampling for GO-NO-GO testing of transfer functions. Each of these techniques have their advantages. Growing exponentials have the advantage of accuracy, while the second method is simple and requires less equipment for implementation.

This report presents the theory of growing exponentials and the results obtained in measuring first, second and sixth order transfer functions and also the pen position control system of an X-Y plotter. Practical problems in implementation were encountered with the X-Y plotter but with proper design of the test equipment these problems can be eliminated.

The techniques developed in this program will be applied to a non-linear system model (the Saturn IB thrust vector control system) and the results will be published in a supplement to this report which will be issued October 1, 1965.

CONCLUSION

The main objective of the study has been met. We have established a method for the testing of active and passive transfer functions. This method was used on several transfer functions and accuracy and measurement time proved to be as good or better than present methods of checkout. Faster checkout time is a direct result of the methods studied. Less degradation due to performance is a direct result of using smooth probing signals. Accuracy is acceptable over a parameter range of $\pm$ 10% or more.

1.0 INTRODUCTION

The objective of single parameter testing is to test several individual parameters of a system with one testing signal, thereby obtaining faster checkout time, better accuracy, and less degradation of performance due to testing.

The study program to achieve this objective was divided into three specific tasks:

Phase A: The development of methods to test simple first and second order linear passive networks whose transfer functions resemble those of actual systems.

Phase B: The investigation and selection of criteria developed in Phase A. Extend the application of the method to include linear active networks.

Phase C: Investigate testing implementation problems, by studying the pen position control system of an X-Y plotter with the techniques developed in Phases A and B. Extend the testing technique to higher order systems.

To briefly outline the steps necessary to implement the single parameter testing technique which was developed:

1. Develop a nominal system response. This response can be determined by the statistical measurement of a number of good systems. Once the nominal response is determined it can be stored on tape.

2. Develop a system model which can be used in the determination of an estimator. Good methods are available for this system transfer function determination.

3. The estimator is determined by a theoretical method as described in Section 2 for first and second order transfer functions or by experimental techniques for higher order systems.

4. The fourth step is the implementation of the technique with the actual hardware to be tested keeping in mind impedance and signal level matching considerations.

The general technical approach of the study was limited to systems for which continuous transfer functions can be written and restricted to their linear regions. The techniques developed in this program will be applied to a nonlinear system model (the Saturn IB thrust vector control system described in Section 3.5) and these results will be published October 1, 1965, as a supplement to this report.

SECTION 2

THE THEORY OF SINGLE PARAMETER TESTING WITH GROWING EXPONENTIALS

2.1 INTRODUCTION

In this section, the theory will be presented which allows single parameter testing with growing exponentials. The use of growing exponentials has been investigated by Huggins, et al, in such applications as electrocardiography, in identification of static nonlinear operators, and in system identification problems.(1-15)

This method was actively studied in detail with the objective of applying it to dynamic systems. The general instrumentation scheme for measurement is shown in Figure 2-1.

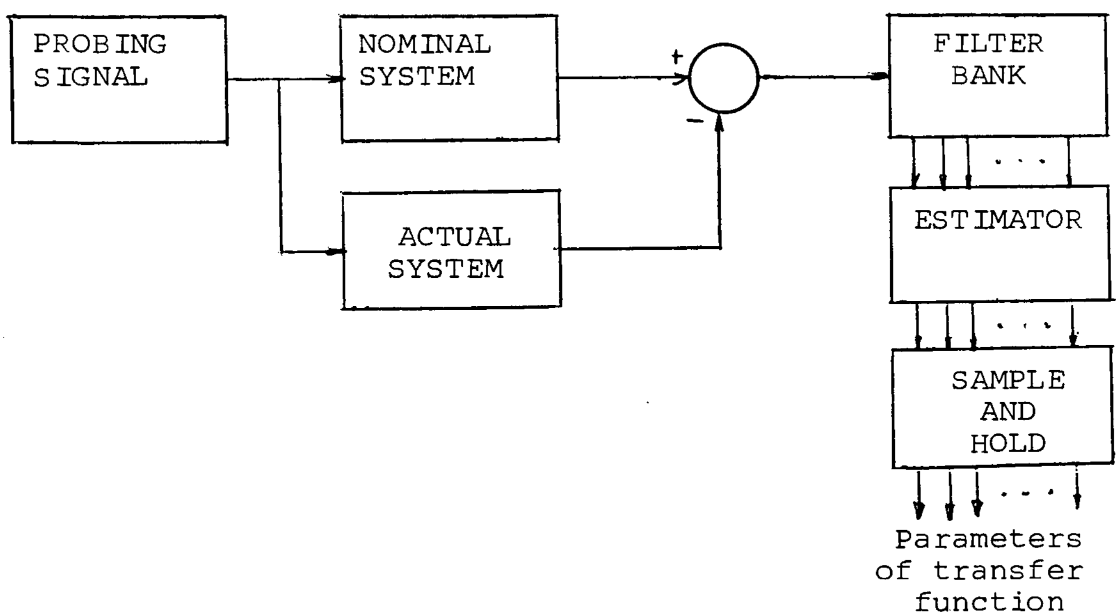


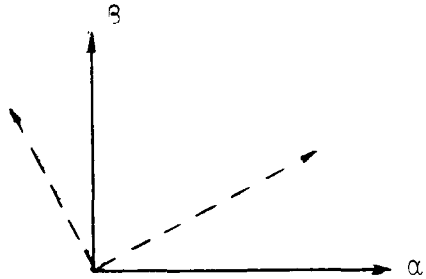
Figure 2-1

Instrumentation Scheme

In a generalized point of view, the signals are a collection of vectors in "n" dimensional space. Each of these vectors is orthogonal over the interval of interest, and each vector represents an independent coordinate.

If the system transfer function can be expanded in terms of orthogonal linear stationary operators, then the measurement of these operators can be accomplished by measuring the projections of the signal vectors on to the linear operator space.

For example, assume that the operator space is as illustrated in Figure 2-2.



Orthogonalized Portions of a Transfer Function

Figure 2-2

where α and β are two orthogonalized portions of some transfer function. Also assume any pair of signals from <u>any</u> orthogonal set as illustrated by the dashed lines in Figure 2-2. Then the projections of α and β, the signal vectors, on to the operator space will, when multiplied by scale factors, measure the magnitude of α and β. For purposes in this report α and β would be proportional to a change in some parameter.

2.2 ORTHOGONALIZED SIGNALS

To obtain orthogonalized signals, it is sufficient to have the time average of the innerproduct of the signals zero. Let $f_i(t)$ be the $i^{\underline{th}}$ signal, then

$$\int_{-\infty}^{+\infty} f_i^*(t)\, f_j(t)\, dt = 0 \tag{2-1}$$

when $i \neq j$, otherwise

$$\int_{-\infty}^{+\infty} f_i^*(t)\, f_i(t)\, dt = 1. \tag{2-2}$$

Now in order to obtain orthogonal signals, exponentials may be considered. Exponentials have several advantages over other sets of orthogonal functions. These include relatively short time bases, and capabilities of being matched to the system to be tested.

Orthogonalization of the exponentials may be accomplished by the Kautz method.(10) This method allows the approximation of the impulse response of any network by sums of orthogonalized signals. These signals, for a transfer function with all real poles and a higher order demoninator than numerator, become:

$$\Phi_n(S) = \sqrt{-S_n - \bar{S}_n} \qquad \frac{(S + \bar{S}_1) \cdot\cdot\cdot\cdot (S + \bar{S}_{n-1})}{(S - S_1) \cdot\cdot\cdot\cdot (S - S_n)} \tag{2-3}$$

where S_n = complex frequency with a <u>negative</u> real part of the $n^{\underline{th}}$ exponential component

$\bar{S}_n$ = conjugate of S_n

For example, let the transfer function be

$$H(S) = \frac{1}{S + K} \tag{2-4}$$

Then the set of orthogonalized exponentials are:

$$\Phi_1(S) = \sqrt{2K} \quad \frac{1}{S + K} \tag{2-5}$$

$$\Phi_2(S) = \sqrt{2K} \quad \frac{(S - K)}{(S + K)^2}$$

$$\Phi_3(S) = \sqrt{2K} \quad \frac{(S - K)^2}{(S + K)^3}$$

$$\vdots$$

In the time domain the set of signals become:

$$f_1(t) = \sqrt{2K} \quad \exp(-K\ t) \tag{2-6}$$

$$f_2(t) = \sqrt{2K} \quad (1 + 2\ t)\ \exp(-K\ t)$$

$$\vdots$$

In the following text we will be concerned with <u>negative</u> time functions and sampling at time zero. When considering negative time, the set of orthogonalized components become

$$\Phi_1(s) = \frac{\sqrt{2K}}{-S + K} \tag{2-7}$$

$$\Phi_2(s) = \frac{\sqrt{2K}\ (-S\ -K)}{(-S + K)^2}$$

$$\vdots$$

And in the time domain

$$f_1(t) = \sqrt{2K} \quad \exp(K\,t) \text{ for } t < 0 \tag{2-8}$$

$$f_2(t) = \sqrt{2K} \quad \exp(K\,t)\,(1 + 2t) \text{ for } t < 0$$

$$\vdots$$

When the transfer function has complex poles, then the orthogonalization takes the form of(10)

$$\frac{\Phi_{2v-1}(S)}{\Phi_{2v}(S)} \sqrt{2\alpha_v} = \frac{\left[(S - \alpha_1)^2 + \beta_1^{\,2}\right] \ldots \left[(S - \alpha_{v-1})^2 + \beta_{v-1}^{\,2}\right] \left(S \pm |S_v|\right)}{\left[(S + \alpha_1)^2 + \beta_1^{\,2}\right] \ldots \left[(S + \alpha_{v-1})^2 + \beta^2_{v-1}\right]\left[(S + \alpha_v)^2 + \beta_v^{\,2}\right]} \tag{2-9}$$

where $v = 1, 2 - - - n/2$

and the poles are at

$S_v = -\alpha_v - j\beta_v$ and $\bar{S}_v = -\alpha_v + j\beta_v$.

The upper (plus) sign in Equation 2-9 pertains to Φ_{2v-1} and the lower (minus) sign pertains to $\Phi_{2v}(S)$. Note, two signals are assigned to each second order pole pair, one for each pole.

2.3 ORTHOGONAL SEPARATION OF THE SIGNALS

To separate these orthogonalized signals, we only need to accomplish the integration

$$\int_{-\infty}^{+\infty} f_i^*(t)\, f_j(t)\, dt = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{2-10}$$

This can be accomplished by performing the contour integration in the frequency domain, i.e., Parseval's Theorem for aperiodic functions.[20]

$$\int_{-\infty}^{+\infty} f_i^*(t)\, f_j(t)\, dt = \int_c \Phi_i^*(-S)\, \Phi_j(S)\, \frac{dS}{2\pi j} \qquad (2\text{-}11)$$

Note that $\Phi_i^*(-S)$ is a real filter which has an impulse response $f_i(t)$ in positive time. The integration in the complex plane is equivalent to sampling the results at time $t = 0$.

2.4 ORTHOGONALIZED TRANSFER FUNCTION

Consider a system $H(S)$ as a function of its parameter variations around some specified nominal design value. Then a Taylor's series expansion can be written as:

$$H(S) = H_o(S) + \frac{\partial H(S)}{\partial \alpha_1}\Delta\alpha_1 + \frac{\partial H(S)}{\partial \alpha_2}\Delta\alpha_2 + \ldots \tfrac{1}{2}\frac{\partial^2 H}{\partial^2 \alpha_1}\Delta\alpha_1^2 + \ldots. \qquad (2\text{-}12)$$

where $H_o(S)$ = the specified nominal system and

$\frac{\partial H(S)}{\partial \alpha_i} = H_i(S)$ = first partial derivative of the system with respect to the $i^{\underline{th}}$ parameter.

Thus, for <u>small</u> deviations in the parameter values, the actual system can be broken into the sum of the partial systems, $H_i(S)$.

Now, let the transfer function be represented by polynomials as:

$$H(S) = \frac{N(S)}{D(S)} = \frac{\sum_{n=0}^{N} c_n S^n}{\sum_{n=0}^{N} d_n S^n} \qquad (2\text{-}13)$$

H(S) can be expanded in terms of the Taylor series expansion for small variations in c_n and d_n as

$$H(S) \approx H_o(S) + \sum_{n=0}^{n} \frac{\partial H(S)}{\partial c_n} \Delta c_n + \sum_{n=0}^{n} \frac{\partial H}{\partial d_n} \Delta d_n \qquad (2\text{-}14)$$

Any given $\frac{\partial H(S)}{\partial c_i}$ is orthogonal to any other partial derivative $\frac{\partial H(S)}{\partial c_j}$. Likewise, any given $\frac{\partial H(S)}{\partial d_i}$ is orthogonal to any other partial derivative $\frac{\partial H(S)}{\partial d_j}$.

This property allows independent measurement of the relative magnitude of all the c_i's or d_i's when only the c_i's or d_i's are measured. If the partial systems of the c's and d's are also independent, then separation between the c's and d's can be accomplished.

2.5 PARAMETER EFFECTS UPON THE MEASURED SIGNALS

Each of the testing signals when passed through the system will be filtered by each partial system. The outputs of these partial systems are combined and then passed through the output filters. The process is illustrated in Figure 2-3.

The mathematical description of passing the inputs through the particular system, filter and estimator will now be given in detail. This analysis will be general and apply to n^{th} order transfer functions.

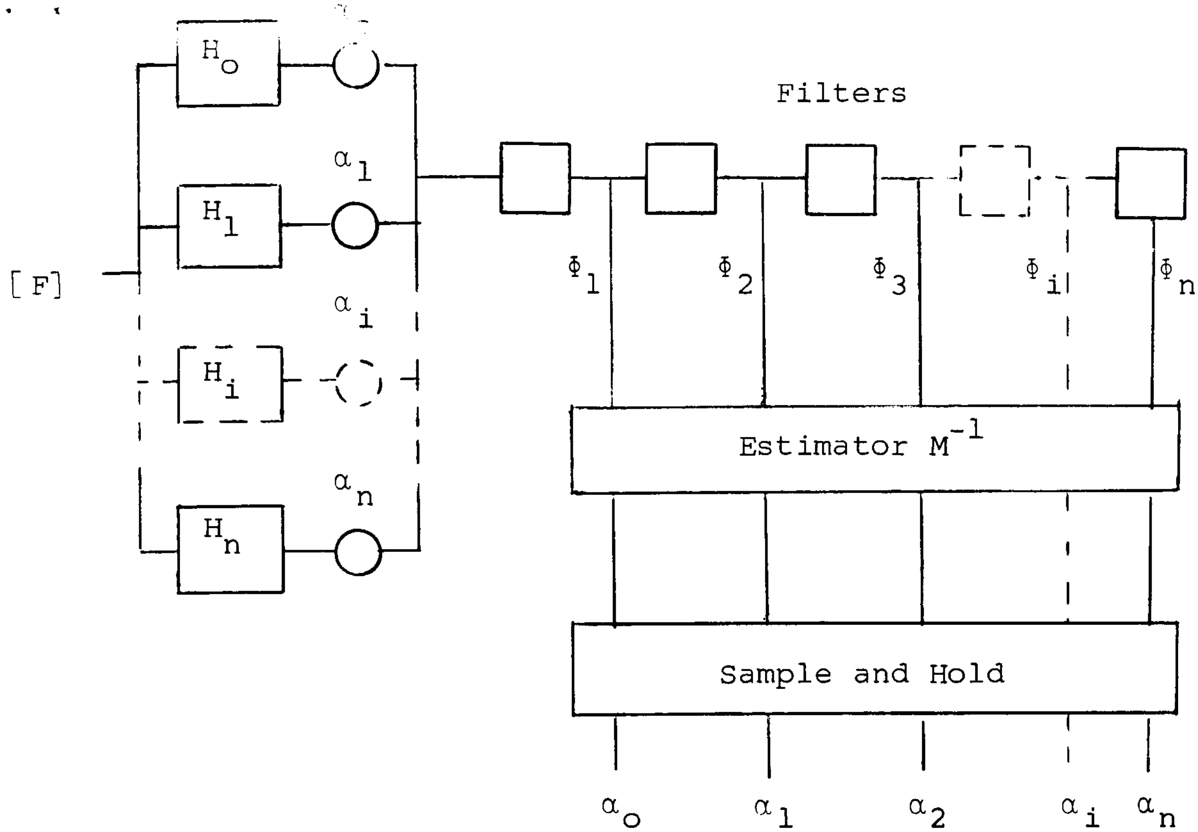


Figure 2-3

Testing Signal Being Processed

The signals appearing at the output of the $H_i(S)$ component system are

$$H_i(S)\ [F] \tag{2-15}$$

where [F] is a row matrix of the input probing signals. The output at the $j^{\underline{th}}$ filter in the filter bank is

$$\Phi_j^*(-S)\ \ H_i(S)\ [F], \tag{2-16}$$

which is a row matrix denoted H_j.

The collection of these row matrices denotes a modulation matrix, H_α. H_α describes the effects of $H_i(S)$ on the outputs of the filters under the influence of the input probing signals. The object will be **to** sample outputs of these filters at a particular time. These samples will represent the variation in the parameters tested.

The output of $j^{\underline{th}}$ filter can be obtained at any particular time by performing the following integration in the complex plane:

$$h_{jk}(\tau) = \int_c \exp(S\tau) \quad \Phi_j^*(-S)\ H_i(S) \quad \Phi_k(S)\ \frac{dS}{2\pi j} \tag{2-17}$$

where τ is a delay variable.

This equation is the transform of the convolution integral, i.e.,

$$h_{ji}(\tau) = \int_{-\infty}^{+\infty} h_j(t) \quad f_i(\tau - t)\ dt \tag{2-18}$$

where $h_j(t)$ is the weighting function of the filter $H_i(S)\ \Phi_j^*(-S)$ and $f_i(t)$ is the $i^{\underline{th}}$ input signal.

Letting $\tau = 0$; the equation gives the value of a sample of the output signal at $\tau = 0$. Thus we obtain,

$$h_{jk} = \int_{-\infty}^{+\infty} h_j(t)\ f_i(t)\ dt = \int_c \Phi_j^*(-S)\ H_i(S)\ \Phi_k(S) \quad \frac{dS}{2\pi j}\ . \tag{2-19}$$

These values of h_{jk} represent the results of the input signal acting on the transfer function $H_i(S)$ and the measuring filter $\Phi_j^*(-S)$ at the time $\tau = 0$. Forming the H_α matrix with these values of h_{jk} as the elements, we have:

$$H_\alpha = \begin{array}{cc} & k \longrightarrow \\ j \downarrow & \begin{bmatrix} h_{11} & h_{12} & --- & h_{1n} \\ h_{21} & h_{22} & --- & h_{2n} \\ \vdots & & & \vdots \\ h_{m1} & --- & & h_{mn} \end{bmatrix} \end{array} \qquad (2\text{-}20)$$

where each row represents the output of a filter and each column represents an input signal.

Now form a column matrix C

$$\lfloor F_1 \; F_2 \cdots\cdot F_n \rfloor \begin{bmatrix} C_1 \\ \vdots \\ C_n \end{bmatrix} \qquad (2\text{-}21)$$

which represents the magnitude of the probing signal components. An additional requirement from practical considerations is that energy be constant,

$$\sum_{n=1}^{N} C_n^2 = 1. \qquad (2\text{-}22)$$

Multiplying the H_α matrix by this column matrix $[C]$ will give a column matrix

$$M_\alpha = \left[H_\alpha\right]\left[C\right] \qquad (2\text{-}23)$$

which is the representative of the signal appearing at the output of each filter due to the partial system $H_i(S)$.

The collection of these columns may be arranged to form a matrix, M, called the modulation matrix.

$$M = \left[M_o, \; M_1, \; \ldots M_\alpha \; \ldots M_n \right] \tag{2-24}$$

By arranging the parameter deviations as a column array

$$[A] = \begin{bmatrix} \alpha_o \\ \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} \tag{2-25}$$

the total system response, G, can be represented as

$$[G] = [M] \cdot [A] = \left[M_o \right] \alpha_o + \left[M_1 \right] \alpha_1 + \ldots . \tag{2-26}$$

The values of α_o through α_n can be determined by solving this matrix equation, i.e.,

$$\left[M^{-1} \right] [G] = [A] \tag{2-27}$$

Since $\left[M^{-1} \right]$ is composed of the input signal magnitude [C], these magnitudes can be adjusted to maximize the estimate of the parameter where they are subjected to noise.

The minimization of white noise can be accomplished from least-square statistical theory. The criterion selected for optimization is the minimization of the covariance of the error in the parameter estimates, as expressed by the minimum variance estimator $(\bar{M} \; M)^{-1}$.

This can be minimized by maximizing the determinate

$$\left| \bar{M} \; M \right| = \left| M \right|^2 \tag{2-28}$$

2.6 SINGLE PARAMETER TESTING THEORY APPLIED TO A FIRST ORDER TRANSFER FUNCTION

The general first order transfer function is

$$H(S) = \frac{c_1 S + c_o}{d_1 S + d_o} \tag{2-29}$$

By differentation of this function with respect to the coefficients, we obtain

$$\frac{\partial H(S)}{\partial c_1} = H_{c_1}(S) = \frac{S}{d_1 S + d_o} = \frac{1}{d_1} - \frac{d_o/d_1}{d_1 S + d_o} \tag{2-30}$$

$$\frac{\partial H(S)}{\partial c_o} = H_{c_o}(S) = \frac{1}{d_1 S + d_o} \tag{2-31}$$

$$\frac{\partial H(S)}{\partial d_1} = H_{d_1}(S) = \frac{-S\ (c_1 S + c_o)}{(d_1 S + d_o)^2} \tag{2-32}$$

$$\frac{\partial H(S)}{\partial d_o} = H_{d_o}(S) = \frac{c_1 S + c_o}{(d_1 S + d_o)^2} \tag{2-33}$$

Two testing exponentials were determined by the Kautz relation. These are

$$\Phi_1(S) = \sqrt{2\ d_o/d_1}\ \frac{1}{-S + d_o/d_1} \tag{2-34}$$

$$\Phi_2(S) = \sqrt{2\ d_o/d_1}\ \frac{-S - d_o/d_1}{(-S + d_o/d_1)^2}$$

where $\Phi(S)$ is the representation of this negative time function in the frequency domain.

In the time domain

$$f_1(t) = \sqrt{2\, d_o/d_1} \quad \exp\left(\frac{d_o}{d_1}\, t\right) \quad \text{for } t < 0 \tag{2-35}$$

$$f_2(t) = \sqrt{2\, d_o/d_1} \quad (1 + 2t)\, \exp\left(\frac{d_o}{d_1}\, t\right) \quad \text{for } t < 0$$

The matrix representation of the signals appearing at the filter outputs of the partial systems are:

$$H_{c_1} = \begin{bmatrix} \frac{1}{2d_1}, & \frac{1}{d_1} \\ 0 & -\frac{1}{2d_1} \end{bmatrix} \tag{2-36}$$

$$H_{c_o} = \begin{bmatrix} \frac{1}{2d_o}, & -\frac{1}{2d_1} \\ 0 & +\frac{1}{2d_o} \end{bmatrix} \tag{2-37}$$

$$H_{d_1} = - \begin{bmatrix} \frac{c_1\, d_o + c_o\, d_1}{4\, d_1^{\,2}\, d_o}, & -\frac{c_1}{2d_1^{\,2}} \\ 0 & \frac{c_1\, d_o + c_1}{4\, d_1^{\,2}\, d_o} \end{bmatrix} \tag{2-38}$$

$$H_{d_o} = - \begin{bmatrix} \frac{d_o\, c_1 + c_o\, d_1}{4\, d_o^{\,2}\, d_1}, & \frac{c_o}{2d_o^{\,2}} \\ 0 & \frac{c_1\, d_o + c_o\, d_1}{4\, d_o^{\,2}\, d_1} \end{bmatrix} \tag{2-39}$$

Forming the matrix M and maximizing its determinant leads to the result that only the second signal $f_2(t)$ is required.

$$\left.\begin{aligned} c_1 &= 0 = \cos\psi \\ c_2 &= 1 = \sin\psi \end{aligned}\right. \qquad \psi = \pi/2 \tag{2-40}$$

The estimators for the parameters c_n and d_n are

$$M_d^{-1} = \frac{-2\, d_o^3\, d_1^3}{c_1^2\, d_o^2 - c_o^2\, d_1^2} \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} \dfrac{c_1\, d_o + c_o\, d_1}{4\, d_o^2\, d_1}, & \dfrac{c_o}{d_o^2} \\ & \\ \dfrac{c_1\, d_o + c_o\, d_1}{d_1^2\, d_o}, & \dfrac{2c_1}{d_1^2} \end{bmatrix} & \begin{array}{c} \Delta d_1 \\ \\ \Delta d_o \end{array} \end{array} \tag{2-41}$$

$$M_c^{-1} = \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} \dfrac{4d_1}{6} & \dfrac{4d_1}{6} \\ & \\ \dfrac{-4d_o}{6} & \dfrac{+4d_o}{3} \end{bmatrix} & \begin{array}{c} \Delta c_1 \\ \\ \Delta c_o \end{array} \end{array} \tag{2-42}$$

Normalizing the estimator is accomplished by dividing each row by the parameter which that row estimates.

$$M_d^{-1} = \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} -\dfrac{2d_o\, d_1}{c_1\, d_1 - c_o\, d_1} & -\dfrac{4c_o\, d_o\, d_1^2}{c_1^2\, d_o^2 - c_o^2\, d_1^2} \\ & \\ -\dfrac{2d_o\, d_1}{c_1\, d_1 - c_o\, d_1} & +\dfrac{4c_1\, d_1\, d_o^2}{c_1^2\, d_o^2 - c_o^2\, d_1} \end{bmatrix} & \begin{array}{c} \Delta d_1/d_1 \\ \\ \Delta d_o/d_o \end{array} \end{array} \tag{2-43}$$

$$
M_c^{-1} = \begin{array}{cc} \begin{array}{cc} \quad 1 & \qquad\qquad 2 \end{array} & \\ \left[\begin{array}{cc} \frac{2}{3}\,\frac{d_1}{c_1} & \frac{2}{3}\,\frac{d_1}{c_1} \\ & \\ -\frac{2}{3}\,\frac{d_o}{c_o} & +\frac{4}{3}\,\frac{d_o}{c_o} \end{array}\right] & \begin{array}{c} \Delta c_1/c_1 \\ \\ \Delta c_o/c_o \end{array} \end{array} \tag{2-44}
$$

Thus, using this result for estimation and $f_2(t)$ as a probing signal, any first order transfer function can be measured to obtain $\frac{\Delta d_1}{d_1}$, $\frac{\Delta d_o}{d_o}$, $\frac{\Delta c_1}{c_1}$, $\frac{\Delta c_o}{c_o}$ with one probing signal.

2.7 SINGLE PARAMETER TESTING THEORY APPLIED TO SECOND ORDER TRANSFER FUNCTIONS

The general second order transfer function is

$$
H(S) = \frac{c_1 S + c_o}{S^2 + d_1 S + d_o} \tag{2-45}
$$

This transfer function can also be written as

$$
H(S) = \frac{c_1 S + c_o}{(S + \alpha)^2 + \beta^2} \tag{2-46}
$$

The partial derivatives of H(S) with respect to the parameters c_1 and c_o are the component partial systems corresponding to c_o and c_1:

$$
\frac{\partial H}{\partial c_1} = \frac{S}{(S + \alpha)^2 + \beta^2} = H_{c_1}(S) \tag{2-47}
$$

$$
\frac{\partial H}{\partial c_o} = \frac{1}{(S + \alpha)^2 + \beta^2} = H_{c_2}(S)
$$

The partial derivatives of $H(S)$, giving the component partial systems corresponding to d_o and d_1, are

$$\frac{\partial H}{\partial c_1} = \frac{-S(c_1 S + c_o)}{\left[(S+\alpha)^2 + \beta^2\right]^2} = H_{d_1}(S) \tag{2-48}$$

$$\frac{\partial H}{\partial d_o} = \frac{-(c_1 S + c_o)}{\left[(S+\alpha)^2 + \beta^2\right]^2} = H_{d_2}(S)$$

The probing signals are then found from the general equation 2-9

$$\Phi_1(S) = \sqrt{2\alpha}\ \frac{-S + \sqrt{\alpha^2+\beta^2}}{(-S+\alpha)^2 + \beta^2} \tag{2-49}$$

$$\Phi_2(S) = \sqrt{2\alpha}\ \frac{-S - \sqrt{\alpha^2+\beta^2}}{(-S+\alpha)^2 + \beta^2}$$

$$\Phi_3(S) = \sqrt{2\alpha}\ \frac{\left[-S + \sqrt{\alpha^2+\beta^2}\right]\left[(-S-\alpha)^2 + \beta^2\right]}{\left[(-S+\alpha)^2 + \beta^2\right]^2}$$

$$\Phi_4(S) = \sqrt{2\alpha}\ \frac{\left[-S - \sqrt{\alpha^2+\beta^2}\right]\left[(-S-\alpha)^2 + \beta^2\right]}{\left[(-S+\alpha)^2 + \beta^2\right]^2}$$

The matrix components for the partial systems are

$$h_{jk} = \frac{1}{2\pi j}\int_c \Phi_j^*(-S)\ H_i(S)\ \Phi_k(S)\ dS \tag{2-50}$$

The matrix elements for the partial system $\frac{\partial H}{\partial c_1}$ are found as follows:

$$(h_{11})_{c_1} = \frac{1}{2\pi j}\int_c \Phi_1(-S)\ \frac{\partial H(S)}{\partial c_1}\ \Phi_1(S)\ dS \tag{2-51}$$

$$= \frac{1}{2\pi j}\int_c \frac{\sqrt{2\alpha}\ (S+\sqrt{\alpha^2+\beta^2})}{(S+\alpha)^2+\beta^2}\ \frac{S}{(S+\alpha)^2+\beta^2}\ \frac{\sqrt{2\alpha}(-S+\sqrt{\alpha^2+\beta^2})}{(-S+\alpha)^2+\beta^2}\ dS$$

This integral is evaluated by integrating around the left half-plane, finding the residues at the left half-plane poles. The residue at the second order pole at $(-\alpha - j\beta)$ is

$$R_1 = \lim_{S \to (-\alpha - j\beta)} \frac{d}{dS}\ \frac{S\ (S^2-\alpha^2-\beta^2)}{(S+\alpha-j\beta)^2\left[(S-\alpha)^2+\beta^2\right]} = -\frac{1}{16\alpha^2} \tag{2-52}$$

The residue R_2 at $(-\alpha + j\beta)$ is the complex conjugate of R_1:

$$R_2 = -\frac{1}{16\alpha^2}$$

The sum of the residues is $-\frac{1}{8\alpha^2}$, and so

$$(h_{11})_{c_1} = (-2\alpha)\ (-\frac{1}{8\alpha^2}) = \frac{1}{4\alpha}\ .$$

The matrix elements for $\frac{\partial H}{\partial c_o}$ are found the same way and the resulting maxtrixes are

$$H_{c_1} = \begin{bmatrix} \frac{1}{4\alpha} & 0 & -\frac{1}{4\alpha} & 0 \\ 0 & \frac{1}{4\alpha} & 0 & -\frac{1}{4\alpha} \\ 0 & 0 & \frac{1}{4\alpha} & 0 \\ 0 & 0 & 0 & \frac{1}{4\alpha} \end{bmatrix} \tag{2-53}$$

$$
H_{c_o} = \begin{bmatrix}
\frac{1}{4(\alpha^2+\beta^2)} & \frac{\alpha+\sqrt{\alpha^2+\beta^2}}{4\alpha(\alpha^2+\beta^2)} & \frac{-1}{4(\alpha^2+\beta^2)} & \frac{\alpha-\sqrt{\alpha^2+\beta^2}}{4\alpha(\alpha^2+\beta^2)} \\
\frac{\alpha-\sqrt{\alpha^2+\beta^2}}{4\alpha(\alpha^2+\beta^2)} & \frac{1}{4(\alpha^2+\beta^2)} & \frac{\alpha+\sqrt{\alpha^2+\beta^2}}{4\alpha(\alpha^2+\beta^2)} & -\frac{1}{4(\alpha^2+\beta^2)} \\
0 & 0 & \frac{1}{4(\alpha^2+\beta^2)} & \frac{\alpha+\sqrt{\alpha^2+\beta^2}}{4\alpha(\alpha^2+\beta^2)} \\
0 & 0 & \frac{\alpha-\sqrt{\alpha^2+\beta^2}}{4\alpha(\alpha^2+\beta^2)} & \frac{1}{4(\alpha^2+\beta^2)}
\end{bmatrix} \tag{2-54}
$$

Of the four probing signals being considered only two of the four signals need be used. For example $\Phi_1(S)$ and $\Phi_2(S)$ can be expressed as a linear combination of two parts

$$\Phi_1(S) = \sqrt{2\alpha}\left[\Phi_x(S) + \Phi_y(S)\right] \tag{2-55}$$

$$\Phi_2(S) = \sqrt{2\alpha}\left[\Phi_x(S) - \Phi_y(S)\right]$$

where

$$\Phi_x(S) = \frac{-S}{(-S+\alpha)^2 + \beta^2} \tag{2-56}$$

$$\Phi_y(S) = \frac{\sqrt{\alpha^2+\beta^2}}{(-S+\alpha)^2 + \beta^2}$$

Testing with either $\Phi_1(S)$ or $\Phi_2(S)$ will have the same information since the results will be linear combinations of the parts. This effect can be seen in the matrix elements for $\frac{\partial H}{\partial c_1}$ and $\frac{\partial H}{\partial c_o}$, notice the h_{ij} elements for $i = j$ are all equal. This is the result of the $\Phi_x(S)$ component. The elements h_{ij} for $i = j \pm 1$ have terms which alternate in sign. This is the result of the alternating sign of $\Phi_y(S)$ in $\Phi_1(S)$ and $\Phi_2(S)$.

We therefore can reduce the consideration of signals to $\Phi_1(S)$ and $\Phi_4(S)$ and be confident that all of the resulting information concerning the two parameters c_1 and c_o can be extracted.

Since there are now only two probing signals, the coefficients of $\Phi_1(S)$ and $\Phi_4(S)$ are chosen to be $\cos\psi$ and $\sin\psi$ as in the first order example. The calculation of the parameter modulation matrix M is as follows:

$$M_c = \begin{bmatrix} M_{c_1} & M_{c_o} \end{bmatrix} \tag{2-57}$$

$$M_{c_1} = \begin{bmatrix} \frac{1}{4\alpha} & 0 \\ 0 & \frac{1}{4\alpha} \end{bmatrix} \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix} = \begin{bmatrix} \frac{\cos\psi}{4\alpha} \\ \frac{\sin\psi}{4\alpha} \end{bmatrix} \tag{2-58}$$

$$M_{c_o} = \begin{bmatrix} \frac{1}{4(\alpha^2+\beta^2)}, & \frac{\alpha - \sqrt{\alpha^2+\beta^2}}{4\alpha(\alpha^2+\beta^2)} \\ 0 & \frac{1}{4(\alpha^2+\beta^2)} \end{bmatrix} \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix} \tag{2-59a}$$

$$M_{c_o} = \begin{bmatrix} \dfrac{\alpha\cos\psi + \sin\psi\left[\alpha - \sqrt{\alpha^2 + \beta^2}\right]}{4\alpha(\alpha^2 + \beta^2)} \\ \\ \dfrac{\sin\psi}{4(\alpha^2 + \beta^2)} \end{bmatrix} \qquad (2\text{-}59b)$$

Then

$$M_c = \begin{bmatrix} \dfrac{\cos\psi}{4\alpha}, & \dfrac{\alpha\cos\psi + \sin\psi\left[\alpha - \sqrt{\alpha^2 + \beta^2}\right]}{4\alpha(\alpha^2 + \beta^2)} \\ \\ \dfrac{\sin\psi}{4\alpha}, & \dfrac{\sin\psi}{4(\alpha^2 + \beta^2)} \end{bmatrix} \qquad (2\text{-}60)$$

The maximum value of the determinant $\left|\overline{M}_c M_c\right|$ occurs when $\psi = 90^\circ$. Substitution of $\psi = 90^\circ$ in the expression for M_c then yields

$$M_c = \begin{bmatrix} 0 & \dfrac{\alpha - \sqrt{\alpha^2 + \beta^2}}{4\alpha\ (\alpha^2 + \beta^2)} \\ \\ \dfrac{1}{4\alpha} & \dfrac{1}{4(\alpha^2 + \beta^2)} \end{bmatrix} \qquad (2\text{-}61$$

The matrix components for the denominator partial systems are found in the same way, but the work involved is much greater because of the complexity of the integrands. As an example, the matrix component $(h_{11})_{d_1}$ is found from

$$(h_{11})_{d_1} = \frac{1}{2\pi j}\int_c \left[\left(\frac{\sqrt{2\alpha}\,(S+\sqrt{\alpha^2+\beta^2})}{\left[(S+\alpha)^2+\beta^2\right]}\right)\left(\frac{-S(c_1 S + c_o)}{\left[(S+\alpha)^2+\beta^2\right]^2}\right)\left(\frac{\sqrt{2\alpha}\,(-S+\sqrt{\alpha^2+\beta^2})}{\left[(-S+\alpha)^2+\beta^2\right]}\right)\right] dS \qquad (2\text{-}62)$$

Evaluation of these types of integrals is extremely tedious and necessitates a long procedure of solution and recheck cycles.

The final matrix result is

$$H_{d1} = \begin{bmatrix} -\frac{1}{16\alpha^2}\left(c_1 + \frac{c_o\alpha}{\alpha^2+\beta^2}\right), & \left[\frac{1}{32\alpha^2\,(\alpha^2+\beta^2)}\left(c_1\alpha^2\,\frac{2\alpha\sqrt{\alpha^2+\beta^2}}{\alpha^2+\beta^2} - c_1\alpha^2 - \frac{c_o\,\beta^2}{\sqrt{\alpha^2+\beta^2}}\right)\right] \\ 0, & -\left(\frac{1}{16\alpha^2}\right)\left((c_1 + \frac{c_o\,\alpha}{\alpha^2+\beta^2})\right) \end{bmatrix} \qquad (2\text{-}63)$$

and

$$H_{do} = \begin{bmatrix} \dfrac{-1}{4\alpha^2(\alpha^2+\beta^2)} \left[c_1\alpha + c_o \dfrac{\alpha^2-\beta^2}{\alpha^2+\beta^2} \right], & \dfrac{-1}{8\alpha^2(\alpha^2+\beta^2)} \left[c_1 \sqrt{\alpha^2+\beta^2} - \dfrac{2\, c_o\alpha^2}{\alpha^2+\beta^2} \right] \\ 0, & \dfrac{-1}{4\alpha^2(\alpha^2+\beta^2)} \left[c_1\alpha + c_o \dfrac{\alpha^2-\beta^2}{\alpha^2+\beta^2} \right] \end{bmatrix} \qquad (2\text{-}64)$$

The coefficients of $\Phi_1(S)$ and $\Phi_4(S)$ are chosen to be cos ψ and sin ψ from constant energy considerations and the parameter modulation matrix is formed as $M_d = \left[M_{d1} \; M_{do} \right]$

where

$$M_{d1} = \left[H_{d1} \right] \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix}$$

and

$$M_{do} = \left[H_{do} \right] \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix}$$

The maximum value of $\left| \overline{M}_d M_d \right|$ occurs when $\psi = 90^\circ$.

2.8 ACTIVE NETWORKS

The technique normally applied to analyzing an active network consists of picking a region of operation and an appropriate model that describes the operation over a specific dynamic range. The model then depends on the physical processes and the ultimate answers that are desired from the model. Since an accurate functional model can abstract the internal operation of the physical process only over a dynamic range, limits must be specified for the validity of the parameters in the functional model.

Now the concept of a small signal model can be clearly stated. The dynamic range is a small signal specified in terms of input-terminal signal condition, and the model parameters are the network parameters. This in turn allows the discussion of a linear active network model without describing the complex physical process. Thus, it allows the non-linearities in the physical process to be considered outside of the dynamic range and detailed questions regarding these non-linearities must be referred to another model.

Within this framework of a limited dynamic range for a particular model, all of the concepts of linear system analysis can be applied. That is, linear differential equations and corresponding Laplace and Fourier transforms can be used to evaluate the state of the model. At this point, questions regarding the existence of a transform, stability properties, system bandwidth, available output power and measureability can be considered. In order to limit the scope of the general system problem a specific interpretation must be considered.

The interpretation of an Active System used in this study is in reference to a device that exhibits a unilateral transfer characteristic. A simple amplifier is the most common example exhibiting this unilateral characteristic. Consequently, an amplifier circuit will be evaluated in terms of the proposed method of single parameter testing.

A number of amplifier circuits were considered.[22] One amplifier circuit to be investigated is shown schematically in Figure 2-4 along with the appropriate equivalent circuit or small signal model.

The transfer function relating input to output is derived in Reference 22.

$$\frac{e_o}{e_{in}} = \frac{S\, R_i\, \mu}{(R_g + R_i)\left[(1 + r_p/R_L + r_p/R_0)\, S + \dfrac{R_L + r_p}{C\, R_0 R_L}\right]} \qquad (2\text{-}65)$$

or in standard form

$$\frac{e_o}{e_{in}} = \frac{K\, S}{d_1 S + d_0}$$

r_p = Tube dynamic plate resistance

μ = Tube amplification ratio

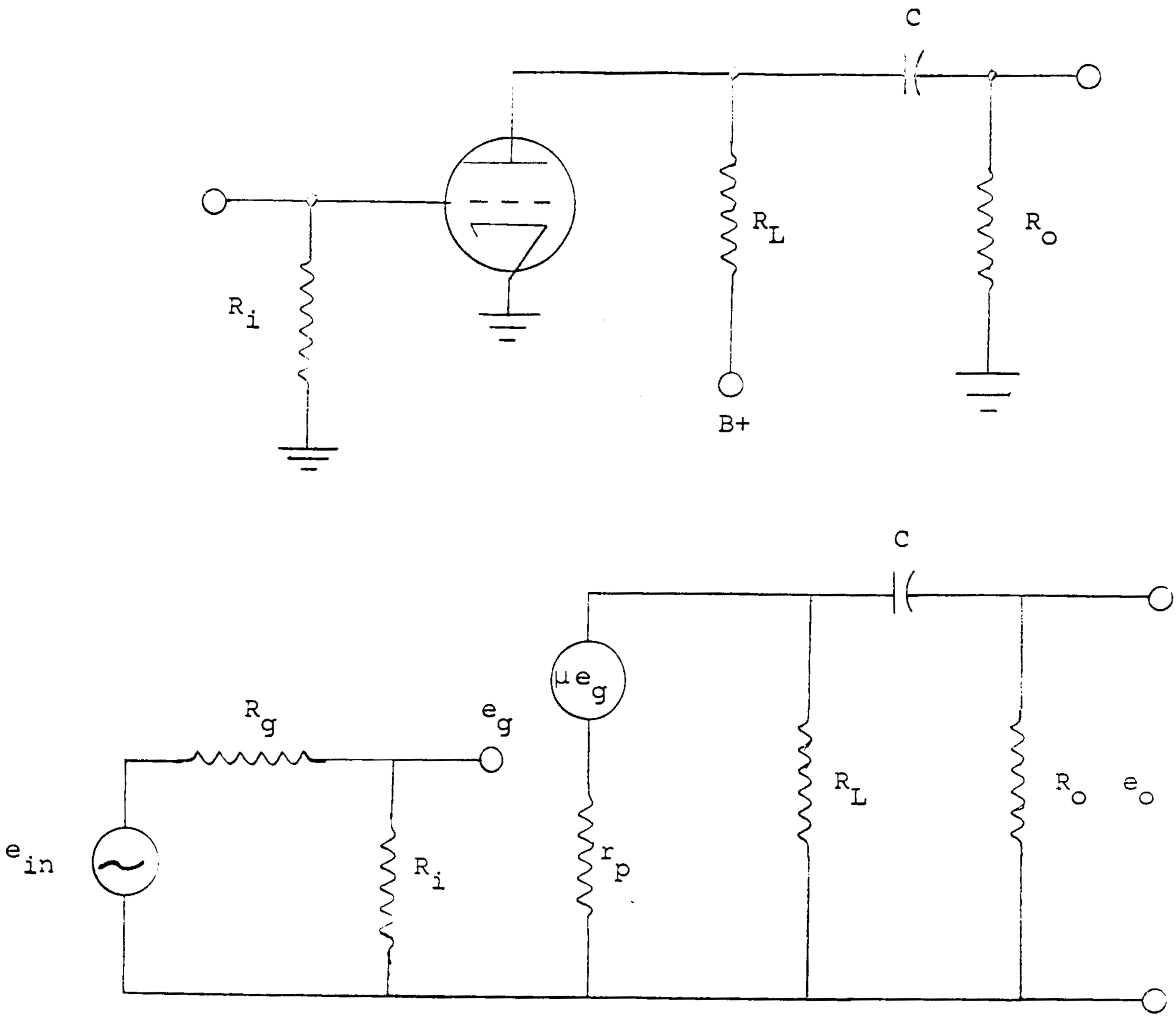


FIGURE 2-4

TRIODE AMPLIFIER AND EQUIVALENT CIRCUIT

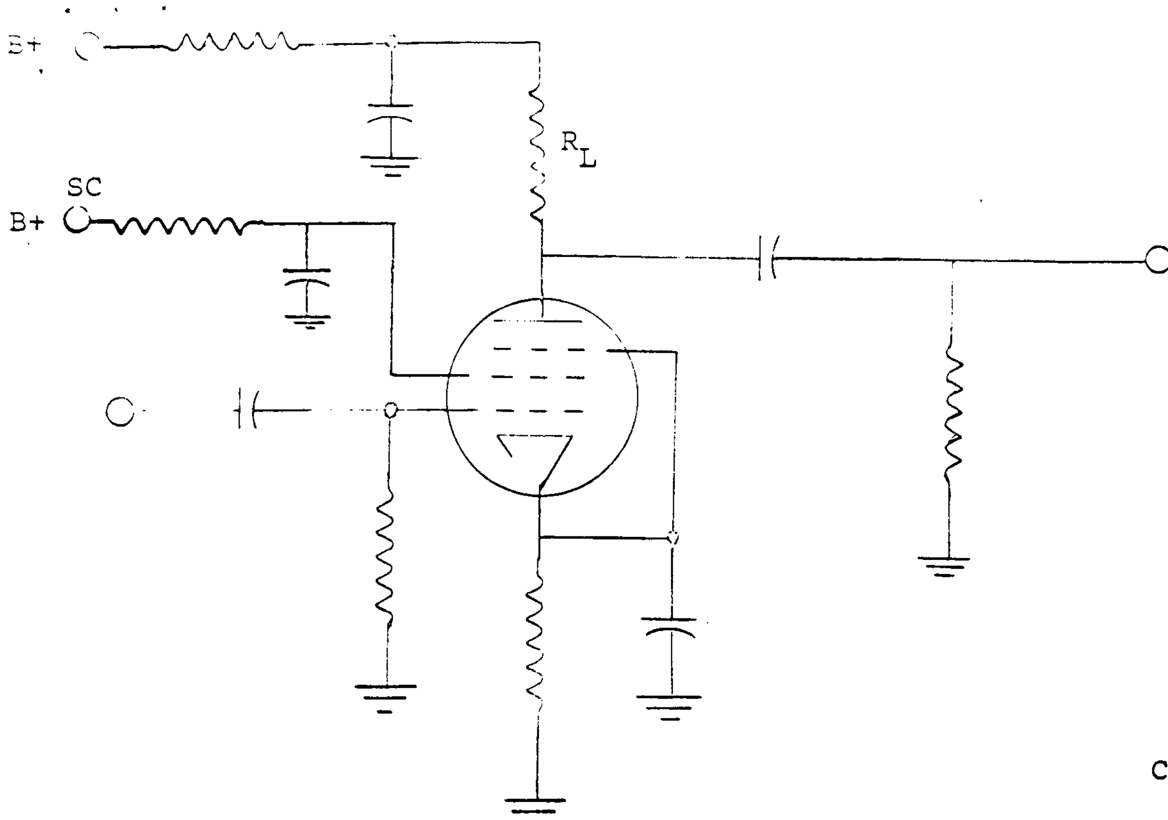
$$K = \frac{R_i \mu}{(R_g + R_i)}$$

$$d_1 = 1 + r_p/R_L + r_p/R_0$$

$$d_0 = (R_L + r_p)/C\ R_0 R_L$$

Clearly this amplifier falls into the general category investigated under the definition of first order transfer functions. The fact that "r_p" is included in both "d_1" and "d_0" complicates the testing required to interrogate the plate resistance. However, "μ" the tube amplification ratio is a linear function in the gain and clearly fits into the parameters already measured. Another note worthy fact in this transfer function is equal order of "S" in both numerator and denominator.

A major assumption in the amplifier transfer function concerns the validity of the model for the tube operation. Consequently, testing signals must be used that satisfy the dynamic range of the model in order to legitimately test the particular transfer function. Let's proceed by obtaining a transfer characteristic for a pentode amplifier as shown in Figure 2-5. The appropriate small signal equivalent circuit in Figure 2-5 is somewhat non-conventional, but for comparison it is desirable to obtain a relation similar to a triode with tube amplification and plate resistance as parameters.



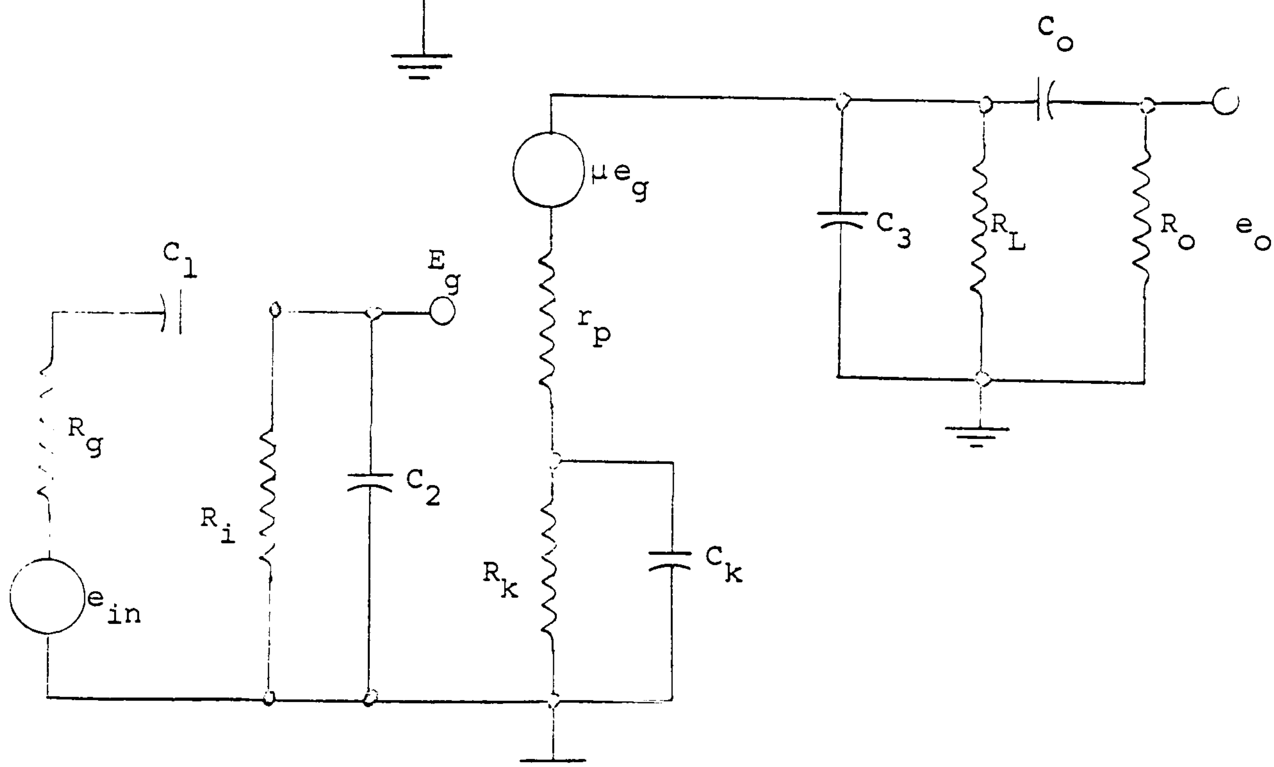


FIGURE 2-5

PENTODE AMPLIFIER AND EQUIVALENT CIRCUIT

The transfer function for the pentode amplifier is derived in Reference 22.

$$\frac{e_o}{e_{in}} = \frac{\mu\, R_i\, C_1\, (1 + R_k\, C_k\, s)s^2}{(d_5\, s^2 + d_4\, s + 1)\,(d_3\, s^3 + d_2\, s^2 + d_1\, s + d_0)} \tag{2-66}$$

where

$$d_0 = \frac{1}{C_0 R_0}\left(1 + \frac{R_k + r_p}{R_L}\right)$$

$$d_1 = 1 + \frac{r_p + R_k}{R_0} + \frac{r_p + R_k}{R_L} + \frac{R_k C_k}{R_0 C_0}\left(1 + \frac{r_p}{R_L}\right)$$

$$d_2 = r_p\, C_3 + R_k\, C_k\left(\frac{C_3}{C_k} + 1 + \frac{r_p}{R_0} + \frac{r_p}{R_L}\right)$$

$$d_3 = R_k\, C_k\, r_p\, C_3$$

$$d_4 = R_g\, C_1 + R_i\, C_1 + R_i\, C_2$$

$$d_5 = R_i\, R_g\, C_1\, C_2$$

It is interesting to see that plate resistance is contained in every coefficient of the cubic. However, tube transconductance ($G_m = \mu/r_p$) is normally given for a pentode and the possibility exists of examining the active tube parameter as a gain coefficient in this transfer function. Also note that the order of the denominator is high, and it might be difficult to measure each of the circuit parameters because of the laborious calculating required. One other factor involved in this model is signal dynamics, which again just remain within the bounds of the model.

Next, let's examine a transistor amplifier in a similar fashion to the tube circuits in order to allow some comparison of active parameter measurement requirements for transistors. A simple audio amplifier circuit with the equivalent circuit is shown in Figure 2-6. The by-pass around the emitter bias resistor is assumed perfect in order to simplify that algebra.

The transfer function for this transistor amplifier is derived in Reference 22.

$$\frac{e_o}{e_{in}} = \frac{K\ S^2}{(d_3\ S + 1)\ (d_2\ S^2 + d_1\ S + d_0)} \tag{2-67}$$

where

$$K = \frac{R_L\ R_0\ C_0\ \alpha_0}{r_e}$$

$$d_0 = \left(\frac{R_{BB} + R_i}{R_{BB}\ R_i}\right) \frac{1}{C_1} + \left(\frac{1 - \alpha_0}{r_e}\right) \frac{1}{C_1}$$

$$d_1 = \frac{C_e}{C_1} + \left(\frac{R_{BB} + R_i}{R_{BB}\ R_i}\right) R_g + \left(\frac{1 - \alpha_0}{r_e}\right) R_g + 1$$

$$d_2 = C_e\ R_g$$

$$d_3 = (R_L + R_0)\ C_0$$

The active device parameters appear in the gain and two coefficients of the quadratic denominator term. Consequently,

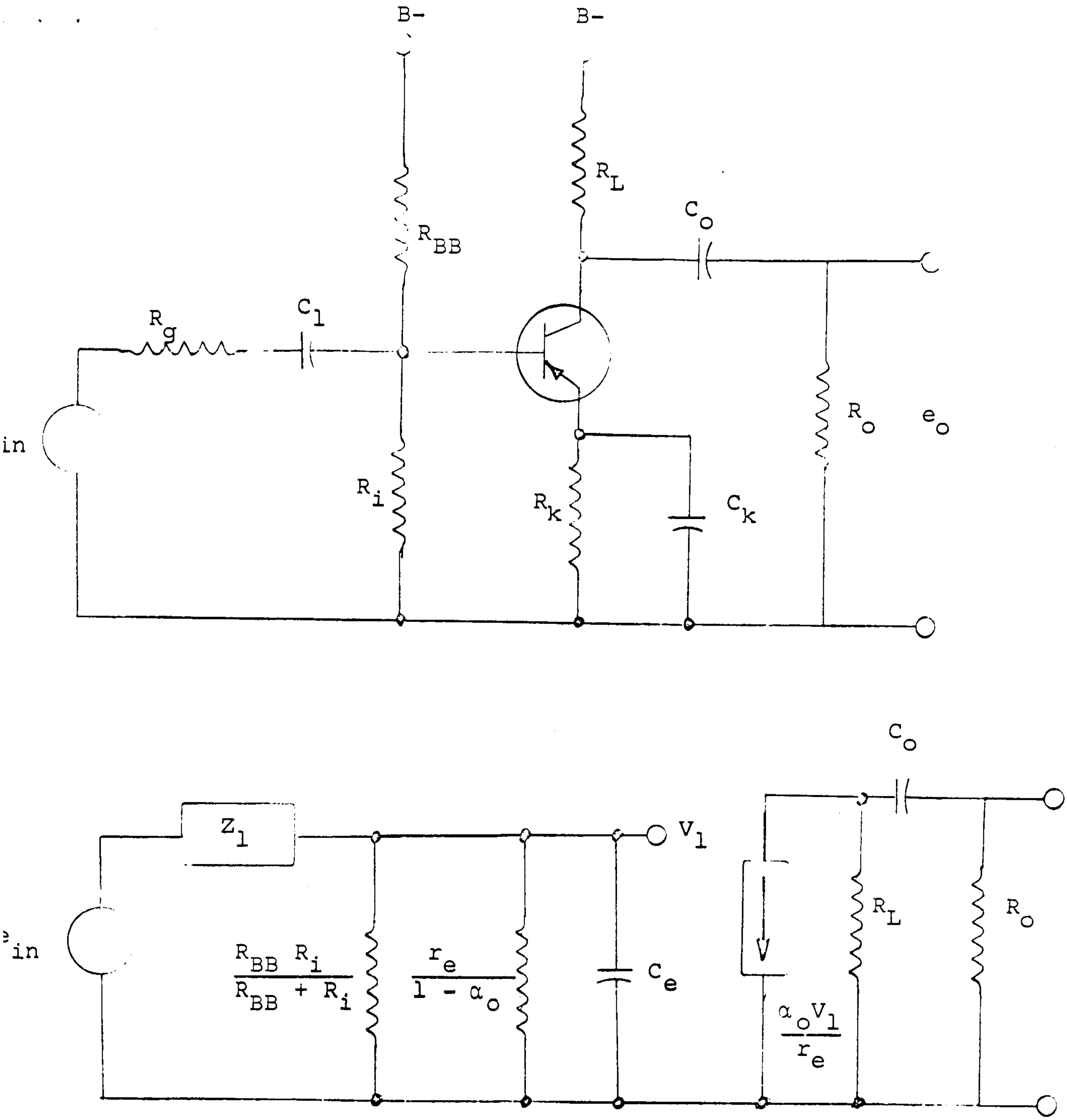


FIGURE 2-6

TRANSISTOR AUDIO AMPLIFIER AND EQUIVALENT CIRCUIT similar measurements to those required for the pentode amplifier can be applied to this transistor amplifier. In spite of the circuit loading due to the transistor, the transfer characteristic comes out rather simple in terms of the active parameters.

The governing assumption for the development of all three transfer functions is certainly the model validity and the ability to keep signals within model restrictions. A common property clearly contained in all three characteristics is the active parameter in the gain coefficient. In the pentode amplifier and the transistor audio amplifier the third order denominator term and the second order term can be compared if the cubic was factored into a quadratic and first order term. Then both would exhibit second order denominator terms with the active element clearly contained in coefficients.

The relative importance of coefficients in the denominator of a second order transfer function are equivalent to the importance of parameters in an active device. Consequently, a thorough evaluation in any experimental sense of variations in denominator coefficients of a second order system, is required.

Normally, the transistion from one model of an active network to another model is based on properties of the active parameters. For example, a tube equivalent circuit is modified when either dynamic plate resistance or tube gain vary appreciably over the dynamic range of interest. In the mathematical model some of the parameter variations can be included with a non-linear or non-constant function representing the parameter. In a strict mathematical sense, this non-linear effect is difficult is include but experimental observations can be made on the analog computer simulation. If the basic relations obtained for a constant parameter second order system are used in conjunction with a saturation effect type non-linearity, some small range for the model can be established.

2.8.1 Initial Condition in a System

Initial conditions and constant source values, such as a D-C bias, both enter a transfer function in a very similar way as many text books prove. The question here is one of measureability of this source value and where it appears in the transfer characteristic. To observe these properties, consider the definition of a Laplace transform.

$$H(S) = \int_{0}^{\infty} f(t) \exp(-St)\, dt \qquad (2\text{-}67)$$

where S = Laplace transform operator

$f(t)$ = Time function of the source

Normally an initial value theorem is stated so that the initial value depends upon a limiting process where "S" approaches infinity. Then this transfer function can only take on non-vanishing values if the exponential term does not vanish. This can occur only for very small values of it, and the result in the limit is the initial condition.

Consider a typical source in the general form

$$f(t) = M t^n$$

where

n = 0, 1, 2, ... a

M = Scale factor

clearly

$$H(S) = \frac{M\, n!}{S^{n+1}}$$

If f(t) is merely a bias contained in a voltage equation, n = 0 and

$$\lim_{S \to \infty} H(S) = \lim_{S \to \infty} \frac{M}{S} = 0$$

Since S contains a real and imaginary part ($S = \alpha + j w$) this clearly shows that the real part of S is required to go to infinity as well as the imaginary part. This is required because the integral of an exponential along only a vertical line would become an undefined quantity (except in a special impulse sense).

This would not be clear if the theorem is written as follows:

$$\lim_{S \to \infty} S^{n+1} F(S) = \left[\frac{d^n f(t)}{dt^n}\right] = f^{(n)}(0)$$

From this discussion it is apparent that initial conditions appear normally as poles at the origin.

The next concern is the measureability of these poles. Since the growing exponential technique is based on a transient responce. The non-transient, D-C or steady values can not be measured by applying this technique. However, the opposite question concerning the use of growing exponentials in testing a system with a pole at zero does arise. The normal approach in testing a system with a D-C value is too block the D-C out of the measuring circuit. This can be accomplished in an electrical network with a capacitor in series with the output to the measuring equipment. A typical transfer characteristic for this blocking action is

$$H_B(S) = \frac{C\ R_0\ S}{C\ R_0\ S+1} = \frac{S}{S + \frac{1}{C\ R_0}}$$

Obviously the pole at the origin is removed and replaced with a pole at

$$P = \frac{1}{C\ R_0}$$

This complicates the mathematics in obtaining an estimator for the growing exponential technique in that it adds a pole to be considered in evaluation of residues. However, the basic mathematics is in no way limited since the presence of additional poles is not presently considered a limiting factor.

One requirement inferred in a system in this discussion is stability. In order to test with growing exponential signals, a basic requirement is that bounded or controlled input signals produce controlled or bounded output signals. With this in mind, problems associated with stability and initial conditions must be assumed answered by another means.

3.0 THE RESULTS OF SINGLE PARAMETER TESTING

The results presented in Section 3.1 through 3.4 are in summary form. More complete details can be found in the previous phase reports (References 21, 22, and 23). The results of testing the sixth order transfer function are being reported for the first time and are therefore given in more detail (Section 3.5).

3.1 THE FIRST ORDER TRANSFER FUNCTION

The first transfer function tested with the Single Parameter Testing setup as shown in Figure 2-1 was

$$H(S) = \frac{1}{d_1 S + d_o} .$$

Results of the parameter estimator outputs as a function of time are given in Figure 3-1 for incremental parameter changes of 3%, i.e., 3%, 6%, 9%, 12%, 15%, 18%, etc. Measurements of the parameter errors were taken when the probing signal stopped. Notice that when only one parameter is varied, the estimate of the other parameter is approximately zero. Notice also that an incremental change in a parameter results in a similar incremental change in the estimate. The estimates were accurate up to approximately a $\pm$ 40% change in the parameter. After 40%, there was an interaction between the estimators for d_1 and d_o. This is not surprising because the approximation of the transfer function is based on a Taylor series expansion, which is only accurate within a limited region, when higher order terms are neglected.

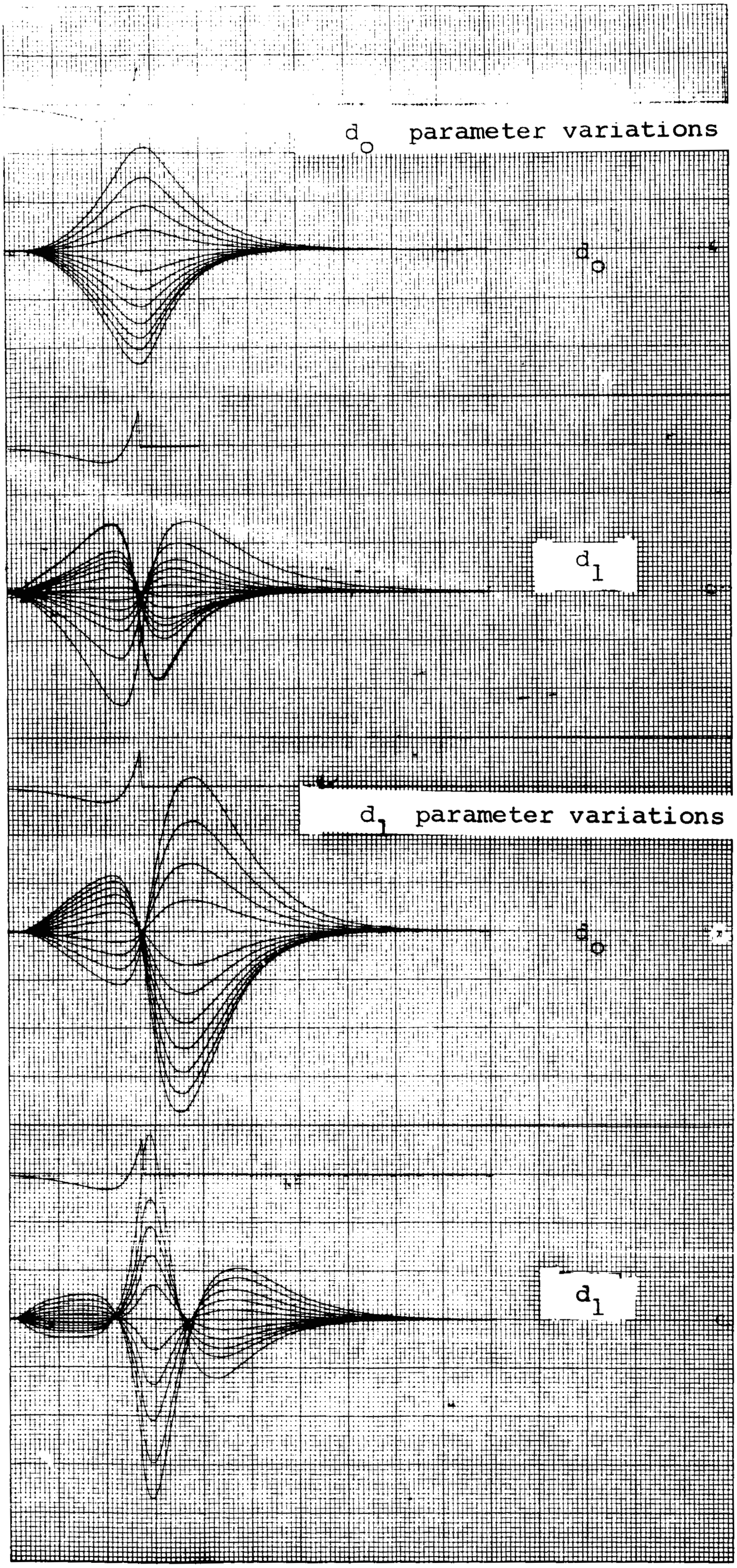


Figure 3-1 d_o Parameter variations with no variations in d_1 and d_1 Parameter with no variations in d_o

If both parameters are varied at the same time, the range of parameter variation over which an accurate estimate can be obtained, is decreased.

3.1.1 Noise Experiment

A noise experiment was conducted to determine the sensitivity of the parameter measurement accuracy to noise. The experiment was conducted by inserting independent band limited noise along with the probing signal into both the nominal and actual system under test. The signal to noise ratio was measured by the ratio

$$\frac{\text{peak voltage of signal}}{\text{rms noise voltage}} .$$

By observing the results of measuring a parameter variation of 10% in d_1, it was concluded that the testing signal should be greater than 26 db above the rms noise level. With a 26 db voltage ratio the range of indication of d_1 was 10% $\pm$ 1.5%.

3.1.2 Time Varying Parameters

To study the effects of time varying parameters upon the measurement accuracy, d_1 was allowed to vary sinusoidally, as

$$d_1 + \Delta d_1 \sin \omega_o t.$$

The radian frequency ω_o was varied and the d_1 estimator output observed. The results show that in tests of time varying parameters, good measurement accuracy can be obtained at radian freouencies below one half of the value of the transfer function pole.

3.1.3 A Second Example of a First Order Transfer Function

Another first order transfer function of the form

$$\frac{c_1 s + c_o}{d_1 s + d_o}$$

was tested using the theory developed in Section 2.6. The experiment was conducted on this transfer function to establish the measurability of coefficients in the numerator and denominator. The previous example (Section 3.1) only measured the effect of coefficients in the denominator.

The results showed that the numerator estimator outputs were orthogonal to each other for about a $\pm$ 20% change in either numerator parameter. The denominator estimator outputs also were orthogonal to each other. However, there was an interaction between numerator term changes and the denominator term output estimates, and vice-versa. That is, for a 10% change in c_1, the four estimator outputs would be: c_1-10%, c_o-0% and d_1 and d_o would read a value other than the desired 0%. This interaction problem between the numerator and denominator terms could have been solved by adding a third signal to the two signals which make up the probing signal (see Section 2.6). This third signal is an impulse at time t = 0. The output of the subtraction circuit is then used as another filter bank output. This impulse is required since the order of the numerator is the same as the order of the denominator. This interaction between the numerator and denominator terms would also not be a problem if the time sampling technique to be described in Section 3.5.9 were used.

3.2 SECOND ORDER SYSTEM

A second order transfer function of the form

$$\frac{c_1 S + c_o}{S^2 + d_1 S + d_o}$$

was tested using the theory which was developed in Section 2.7. The results of the experimentation showed that parameter variations in c_o and c_1 could be measured independently. The following two figures 3-2 and 3-3 illustrate the data obtained. The parameter variations are for $\pm$ 10%, $\pm$ 20%, $\pm$ 30%. In Figure 3-2, c_o was varied, while c_1 was held at its nominal value. In Figure 3-3, c_o was held constant while c_1 was varied. The trace in the middle of each figure is the optimum probing signal.

The system was also tested with variations of d_1 and d_o up to 30% under otherwise nominal conditions (See Figures 3-4 and 3-5). Variations in d_o were readily measured over a $\pm$ 10% range. The measurement of the parameter d_1, however, was influenced by variation in the parameter d_o as well as the parameter d_1. However, by plotting d_1 versus d_o on a set of contours of constant d_1 and d_o, variations of both parameters can be measured. Alternatively, when limits have been set on d_o and d_1, contours such as those in Figure 3-6 can be used to decide whether or not the system is acceptable.

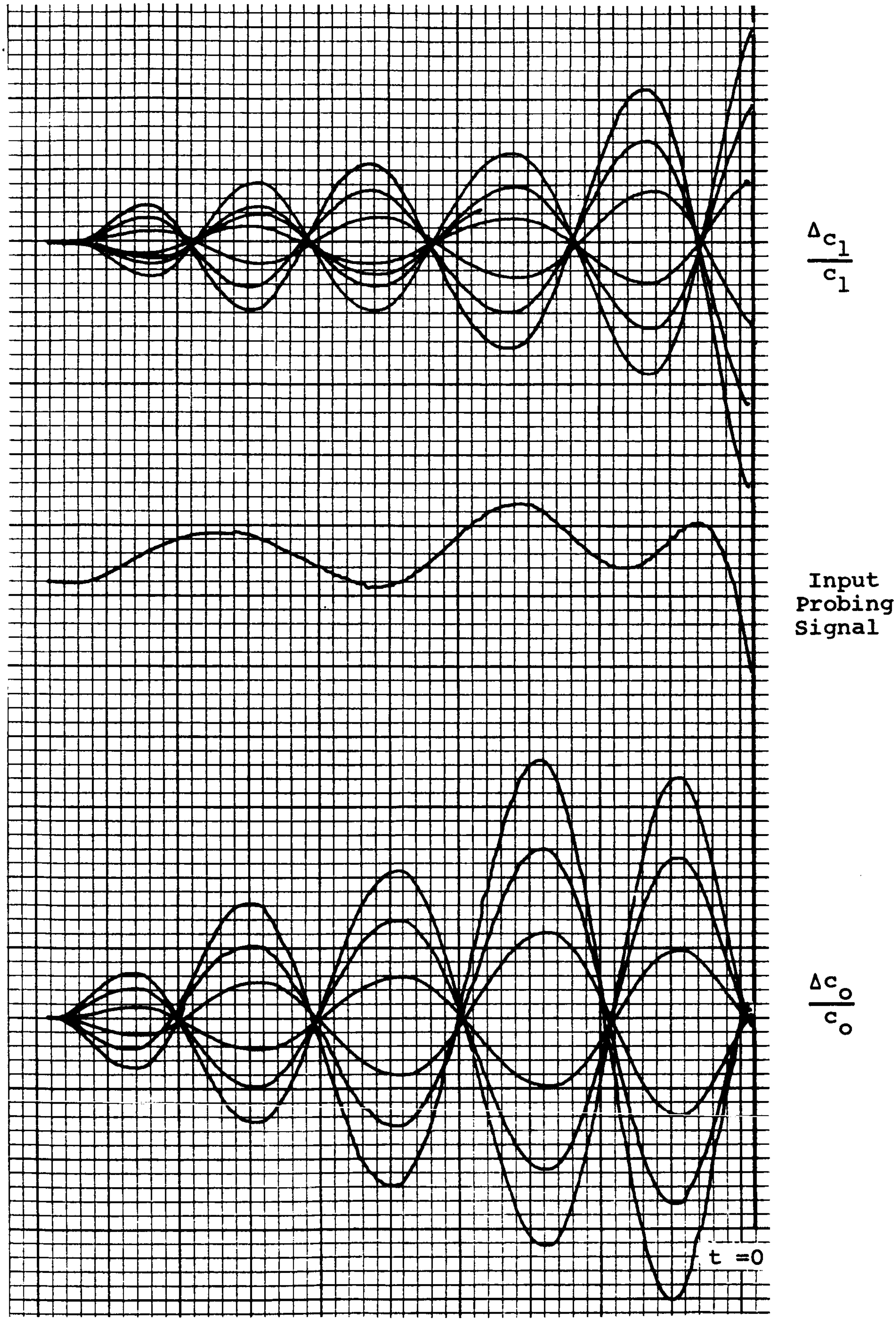


Figure 3-2 Parameter variations of $\frac{\Delta c_o}{c_o}$ with $\frac{\Delta c_1}{c_1} = 0$

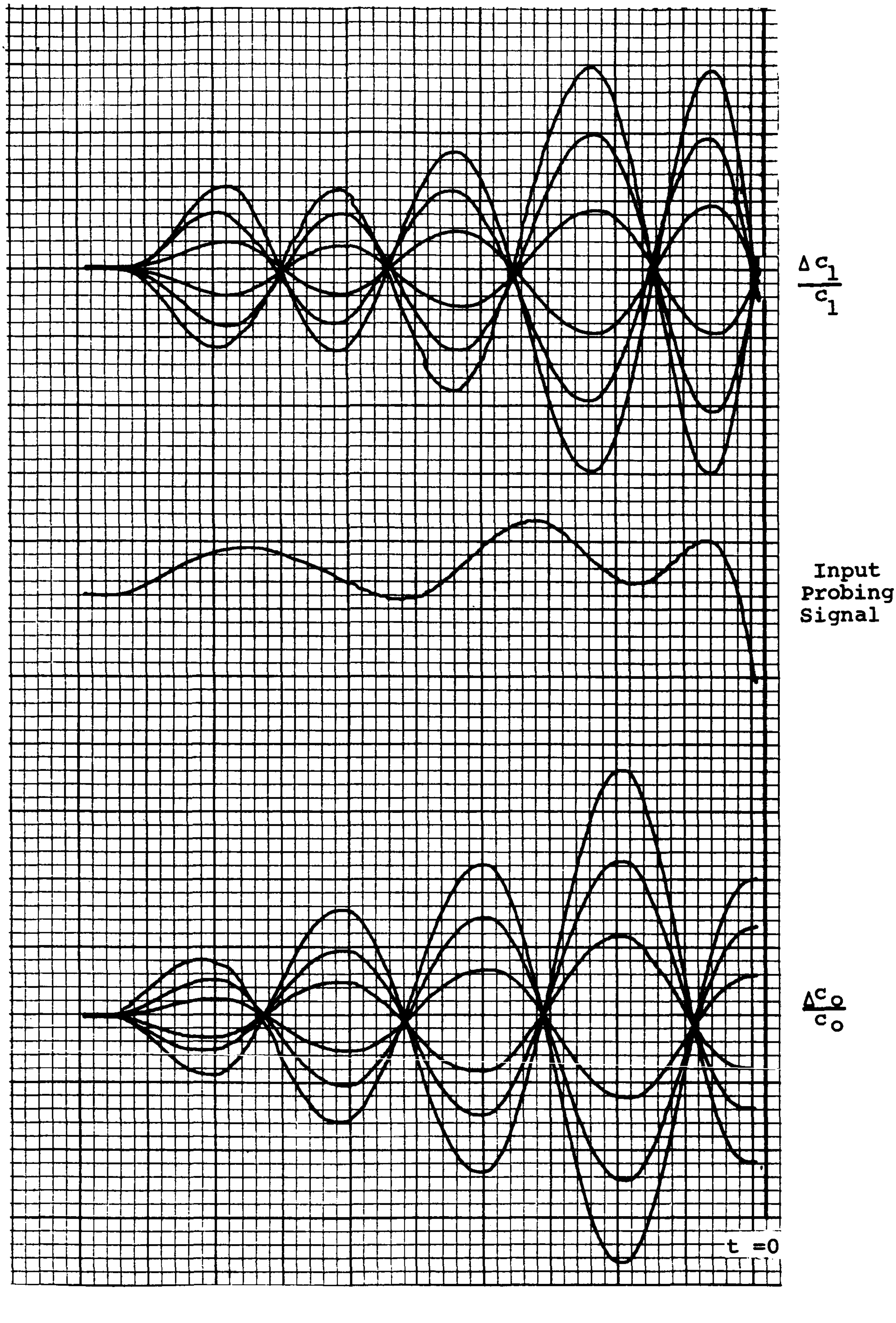


Figure 3-3 Parameter Variations of $\frac{\Delta c_1}{c_1}$ with $\frac{\Delta c_o}{c_o} = 0$

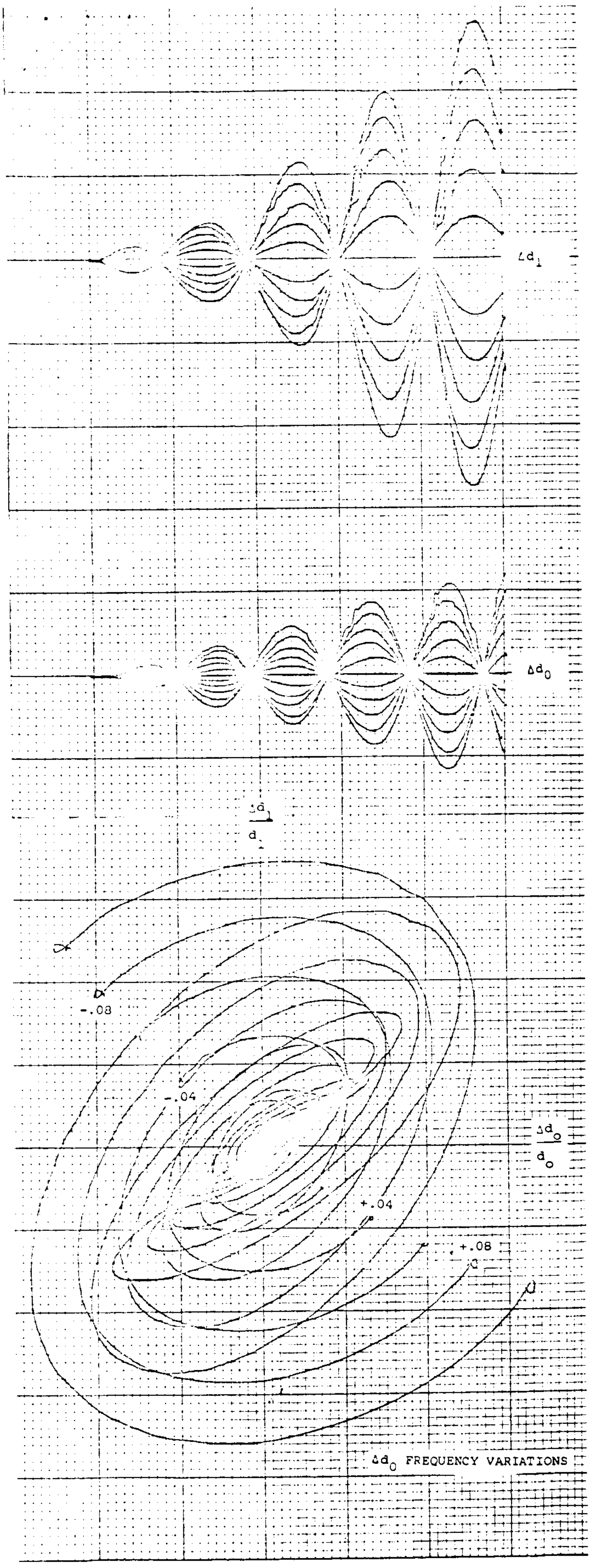


FIGURE 3-4 Δd_0 FREQUENCY VARIATIONS WITH NO VARIATION IN d_1 (SMALL VARIATIONS)

FIGURE 3-5 Δd_1 DAMPING VARIATIONS WITH NO VARIATIONS IN d_0 (SMALL VARIATION)

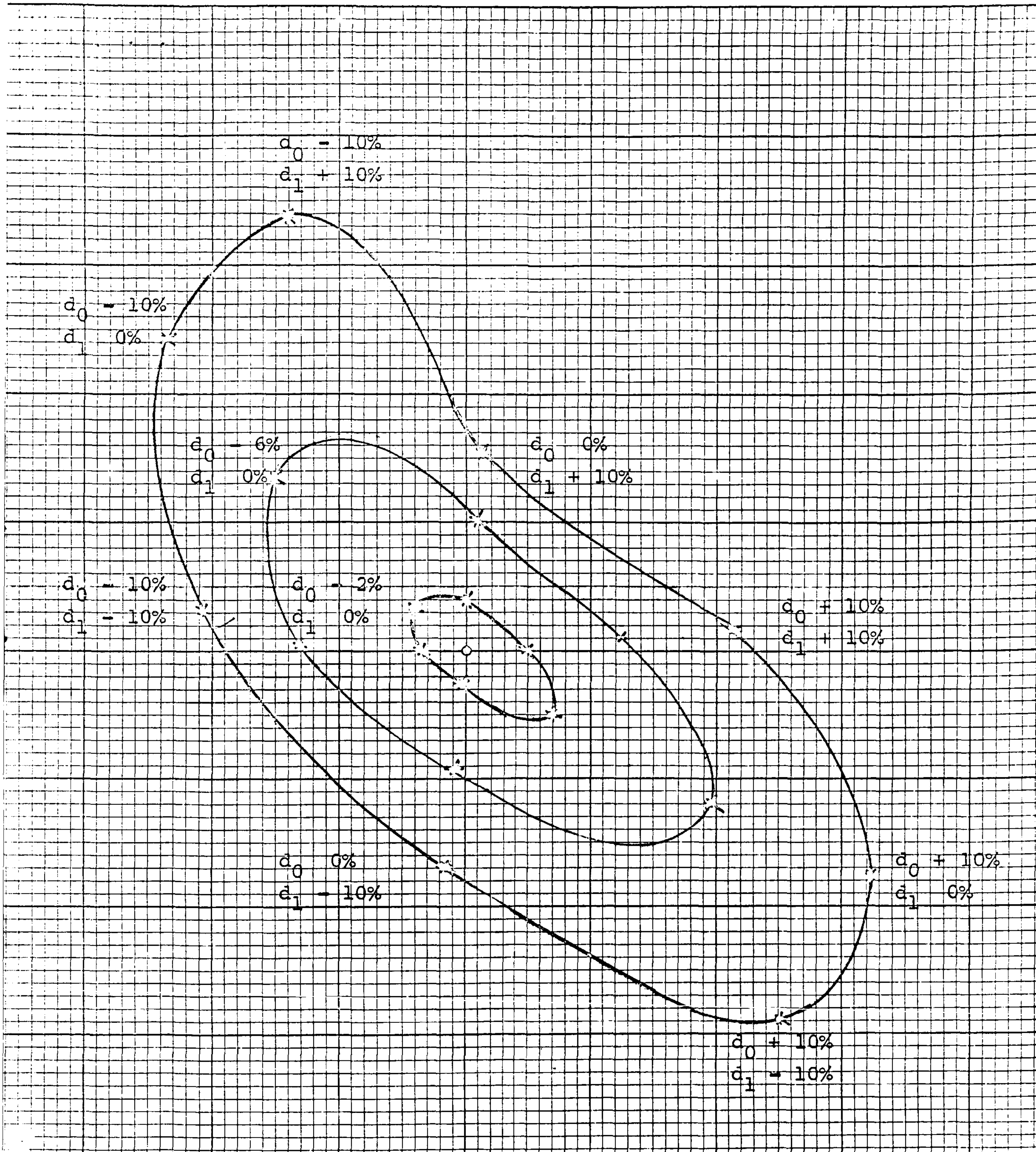


FIGURE 3-6. CONTOURS RELATING LIMITS OF PARAMETERS TO GO-NO-GO TESTING

As was the case of the first order system described in Section 3.1.3 there was an interaction between numerator term changes and the denominator term output estimates, and vice versa. This interaction problem could have been solved by using a probing signal consisting of four signals instead of the two signals which were used. These four signals are given in Equation 2-49. The filter bank would also be expanded to four filters. This interaction between the numerator and denominator terms would also not be a problem if the time sampling technique to be described in Section 3.5.9 were used.

3.3 SECOND ORDER SYSTEM WITH A NONLINEARITY

The gain of the coefficient d_1 of the second order system was varied nonlinearly as in Figure 3-7. Plots comparing the measurements of parameter variation are shown in Figure 3-8. The effect of the damping nonlinearity was to offset the measurements by an approximate value without appreciably affecting the slopes of the plots of estimated versus actual variations of the parameters.

The isolation of the measurement of d_o variations from d_1 variations was somewhat deteriorated by inclusion of the nonlinearity where there is some variation of the d_o measurement from an initial offset value. As mentioned in Section 3.2, the measurement of d_1 could not be isolated from d_o variations, even with a linear system.

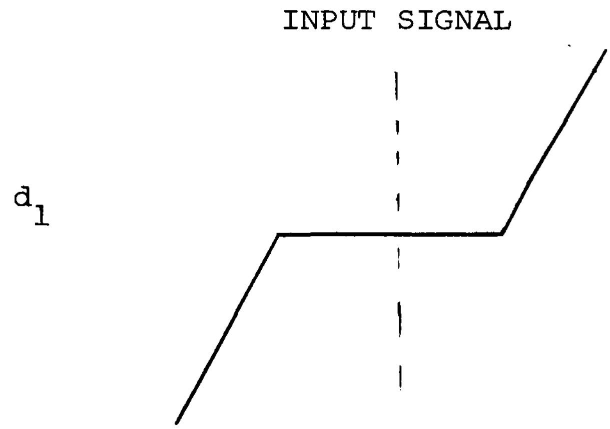


Figure 3-7

Non-Linear Deadband On Damping Coefficient

3.4 TESTING AN X-Y PLOTTER SERVO SYSTEM

The technical approach of single parameter testing with growing exponentials signals was applied to the servo-loop controlling pen position on an X-Y plotter. The primary purpose of the test was to establish the test procedure for a physical system and gain insight into practical problems. The testing was necessary for a theoretical examination of more complex systems because evaluation of various poles ignored in the model development could show their practical effect.

In general the results of the tests on the X-Y plotter are weaker than initially expected since no specific quantitative results can be stated. However, qualitative results pertinent to the general testing philosophy and the observations made during the tests showed an improved testing procedure for more complex systems.

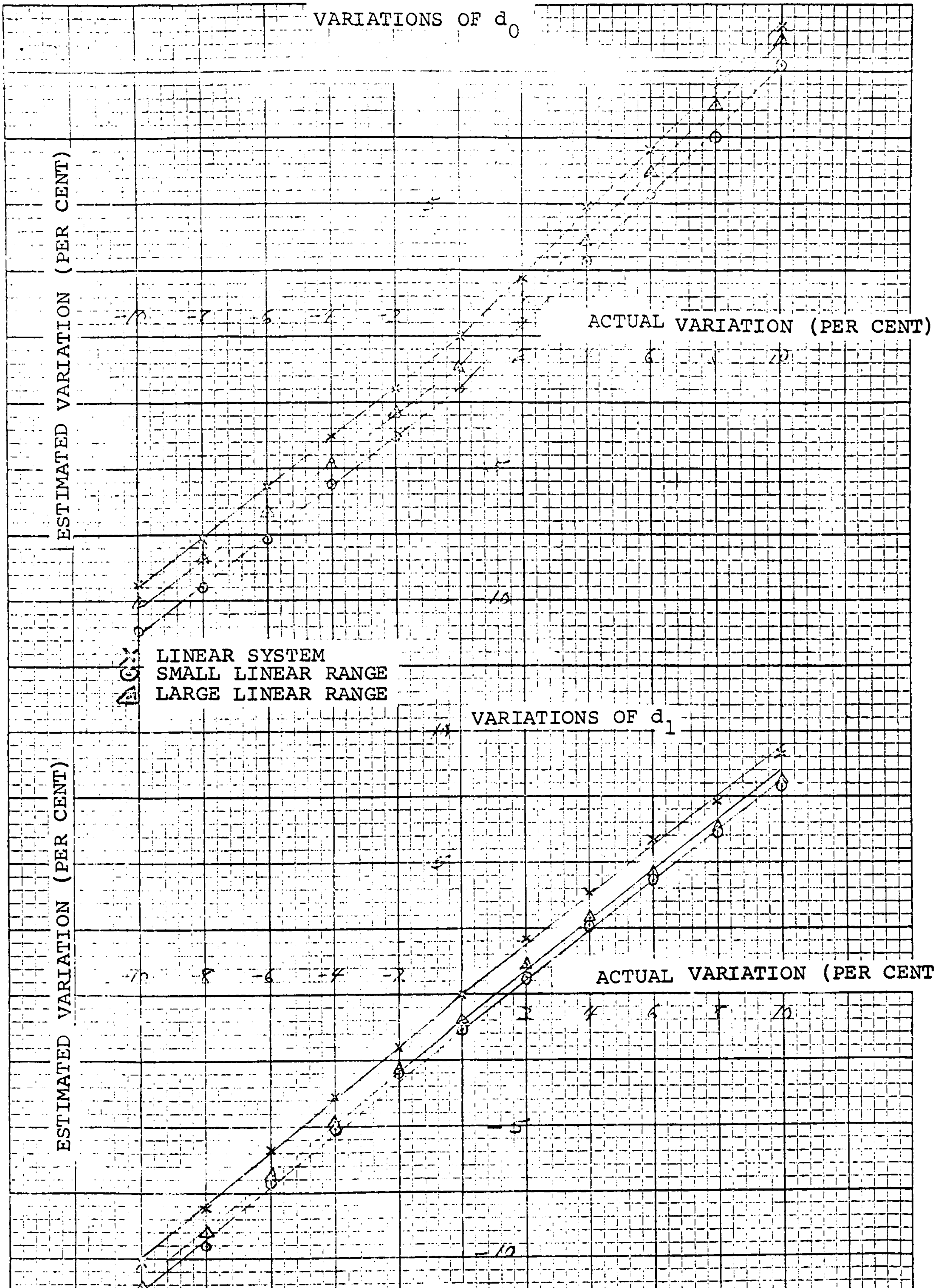


FIGURE 3-8 LINEARITY COMPARISON OF ESTIMATED PARAMETER VARIATIONS

The main conclusions obtained from the experimental work on the X-Y plotter arm position control loop are:

a. The practical sensitivity of the growing exponential technique to variations in parameter values depends strongly on how close the input probing signal is matched to the partial derivative of the system function. This signal match requires accurate coefficient values in the transfer function of the nominal system. Some systems may require an extensive evaluation of the system operation to obtain these transfer function coefficients.

b. The instrumentation for implementing the method must be compatible with input/output impedance relations of the system under evaluation. The use of isolation devices between the testing equipment and the system to be tested would alleviate the impedance matching problem.

c. Signal levels between the testing equipment and the transfer function under evaluation must be compatible. This was a problem with the X-Y plotter which operates with millivolt signals and the analog computer which operates from 10 millivolts to 100 volts. It would have been desirable to test the X-Y plotter with a transistorized testing system designed to operate in the low millivolt region.

The data shown in Figure 3-9 includes the input probing signal matched to H(S) and the difference in the response of each of the two X-Y plotter servo loops. (For a nominal system, a second X-Y plotter servo loop was used.)

Figure 3-10 illustrates the response from each of the servo loops separately. Figure 3-11 shows a gain change in each servo loop observed at the estimator output terminal. Figures 3-12 and 3-13 indicate the respective changes in each recorder (+ 100 percent, 0, -50 percent). Finally, Figure 3-14 shows two separate levels of limiting the servo travel.

The second order transfer function for the servo loop investigated is:

$$H(S) = \frac{K}{S^2 + 15.08S + 184.4}$$

The total transfer function obtained by a detailed analysis and measured data is:

$$H_T(S) = \frac{K(S + 275)}{(S^2 + 15.08S + 184.4)\ (S + 81.5)\ (S + 279)}$$

In review, it can be stated that the following reasons accounted for the difficulties in obtaining quantitative results:

a. The approximation of the transfer function in forming the matching input signal.

b. Nonlinear effects introduced by the chopper and motor modulation demodulation characteristic.

c. Signal levels from the servo loop required large gains to be compatible with the computer signal levels.

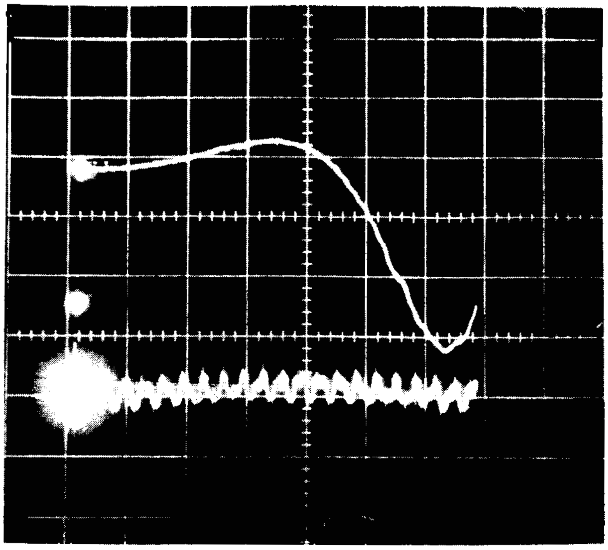


Figure 3-9. Input Probing Signal
System A Minus
System B Response

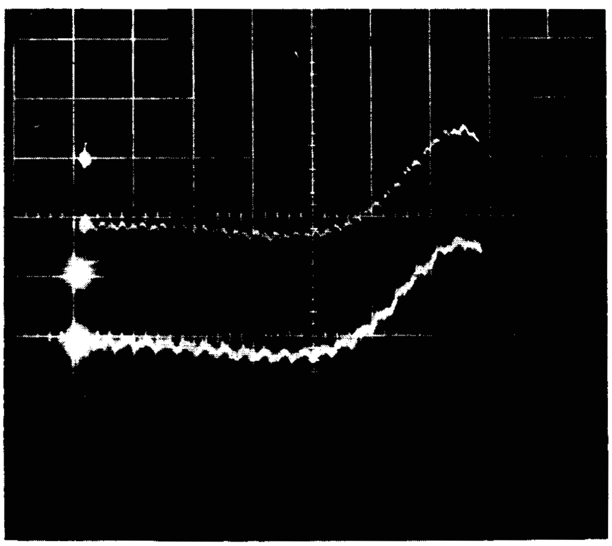


Figure 3-10. Servo-Loop Response
a. System A Response
b. System B Response

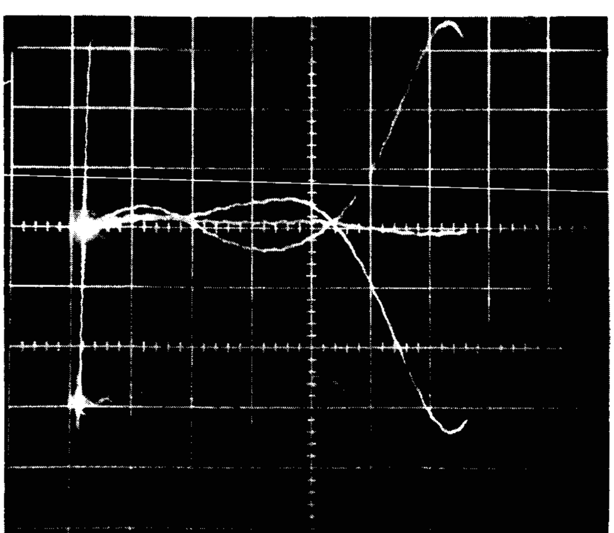


Figure 3-11. Gain Change at Estimator
Output

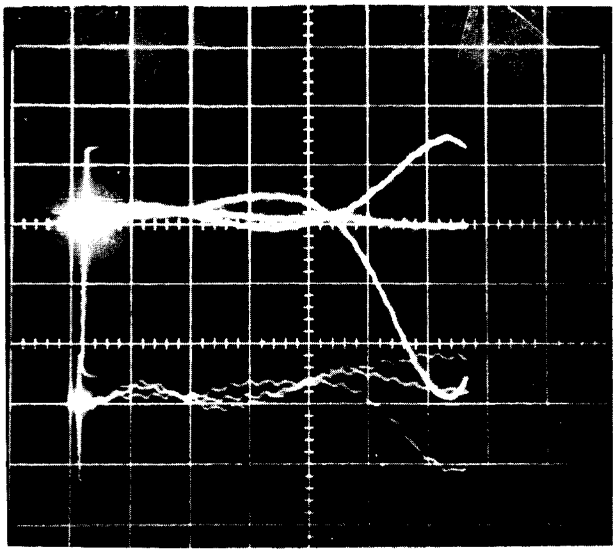


Figure 3-12. Gain Change at Estimator Output - System A

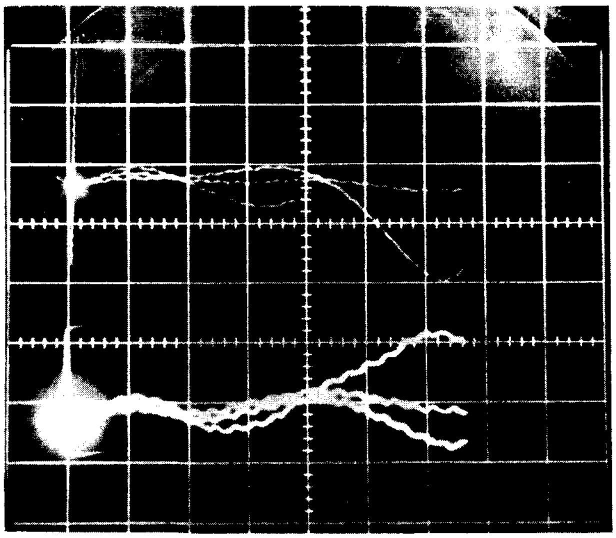


Figure 3-13. Gain Change at Estimato Output - System B

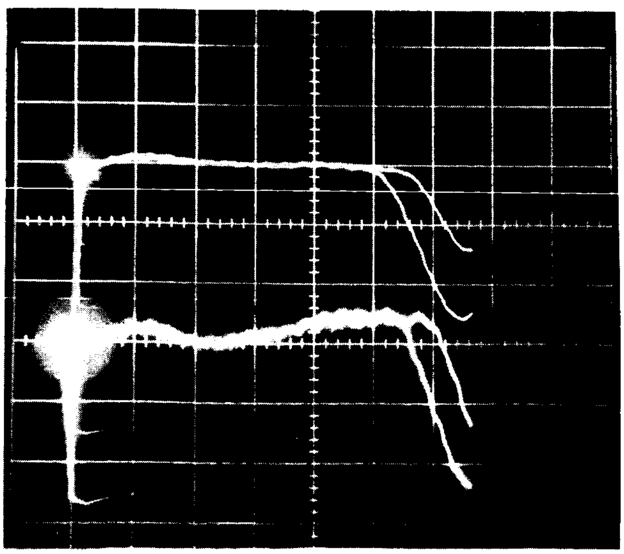


Figure 3-14. Position Limiting

3.5 SIXTH ORDER SYSTEM

3.5.1 Introduction

The transfer function of an actual Saturn-IB system was selected for testing using the growing exponential technique. The thrust vector control system uses a Moog's Model 16-120A dynamic pressure feedback servovalue and a Moog's Model 17-150 actuator. The linearized closed loop transfer function of the system derived from empirical data is

$$G_7 = \left[(1 + S/\omega_1)(1 + 2\delta_2 S/\omega_2 + \omega_2{}^2)(1 + S/\omega_3)(1 + 2\delta_4 S/\omega_4{}^2)\right]^{-1}$$

where $\omega_1 = 21.02$ rad/sec

$\omega_2 = 49.52$ rad/sec

$\omega_3 = 302.5$ rad/sec

$\omega_4 = 262.7$ rad/sec

$\delta_2 = .202$

$\delta_4 = .528$

The study of this control system and derivation of the transfer function is found in Reference 24. Also in this reference is a specification of the permissable amplitude ratio and phase lag of the transfer function. The range of changes in each parameter to be studied was determined from these specifications to be $\omega_1 \pm 10\%$

$\omega_2 \pm 5\%$

$\omega_3 \pm 10\%$

$\delta_2 \pm 5\%$

The investigation of dominant poles of the transfer function can be accomplished by evaluating the impulse response of the system transfer function, G_7.

$$f(t) = 39.6 \ \exp(-21.02t) - .694 \ \exp(-302.5t)$$
$$+ 37.05 \ \exp(-10t) \sin(44.2t - 98.2^{\circ})$$
$$+ 1.675 \ \exp(-138.8t) \sin(180t + 53.5^{\circ})$$

From this time response, it can be observed that the exponential 39.6 exp (-21.02t) is the largest term. The complex poles at $\omega = 49.52$ also contribute to a large part of the time response. The parameters which were therefore selected for testing were ω_1, ω_2, δ_2, the open loop gain K and ω_3. The additional pole ω_3 was selected because the servo-valve which it represents is subject to large changes from system to system.

3.5.2 Partial Systems

A change of the transfer function as a result of a particular parameter change can be described by the Taylor series expansion:

$$H(S) = H_o(S) + \frac{\partial H(S)}{\partial \alpha_1} \Delta\alpha_1 \cdots \tfrac{1}{2} \frac{\partial^2 H(S)}{\partial^2 \alpha_i} \Delta\alpha_i^{\,2} + \cdots$$

where $H_o(S)$ = the specified nominal system.

$\frac{\partial H(S)}{\partial \alpha_i}$ = first partial derivative of the system with respect to the ith parameter.

α_i = the particular parameter under investigation.

The partial derivatives which will be considered are those associated with K, $\frac{1}{\omega_1}$, $\frac{1}{\omega_2}$, δ_2 and $\frac{1}{\omega_3}$. These are:

$$\frac{\partial G_7}{\partial(1/\omega_1)} = (G_7)\ \frac{-S}{(1 + S/\omega_1)}$$

$$\frac{\partial G_7}{\partial(1/\omega_2)} = (G_7)\ \frac{-2S^2/\omega_2 - 2\delta_2\ S}{(1 + 2\delta_2 S/\omega_2 + S^2/\omega_2{}^2)}$$

$$\frac{\partial G_7}{\partial(\delta_2)} = (G_7)\ \frac{-2S/\omega_2}{(1 + 2\delta_2 S/\omega_2 + S^2/\omega_2{}^2)}$$

$$\frac{\partial G_7}{\partial\ (K)} = \frac{(G_7)}{HK^2}\ \frac{S}{(1 + S/\omega_1)}\ \frac{(1 + S/314)}{(1 + S/\omega_3)}\ \frac{1 + \frac{2(.25)S}{53.38} + \frac{S^2}{53.38^2}}{1 + 2\delta_2 S/\omega_2 + S^2/\omega_2{}^2}$$

$$\frac{1 + \frac{2(.52)S}{270} + \frac{S^2}{270^2}}{1 + 2\delta_4 S/\omega_4 + S^2/\omega_4{}^2}$$

$$\frac{\partial G_7}{\partial(1/\omega_3)} = (G_7)\ \frac{-S}{(1 + S/\omega_3)}$$

The partial derivative systems were simulated and the impulse responses taken. These impulse responses are illustrated in

Figure 3-15. It can be seen from these impulse responses that $\frac{\partial G_7}{\partial K}$ is approximately the negative of $\frac{\partial G_7}{\partial (1/\omega_1)}$. This means that it will be difficult to separate the effect of an error due to $(\frac{1}{\omega_1})$ from an error due to a change in K.

3.5.3 The Orthogonalized Signals

The orthogonalized signals which were used to study the sixth order transfer functions were generated with the filter bank shown in Figure 3-16. The signal generation procedure was to apply an impulse to the filter bank and record the twelve filter outputs. These twelve time responses are shown in Figures 3-17 and 3-18. To obtain the growing exponentials which were used in the system testing, the twelve impulse responses were recorded on tape and then the tape was reversed end for end. Thus if Figures 3-17 and 3-18 are turned upside down, they show the twelve growing exponential signals.

These signals were then tested to establish orthogonality. This was accomplished by inserting the signals into the generator filter bank and observing the outputs at the time that the growing exponential ends. Figures 19 through 20 give sample results of applying the test signals to the filter bank. Notice that all signals were orthogonal to each other.

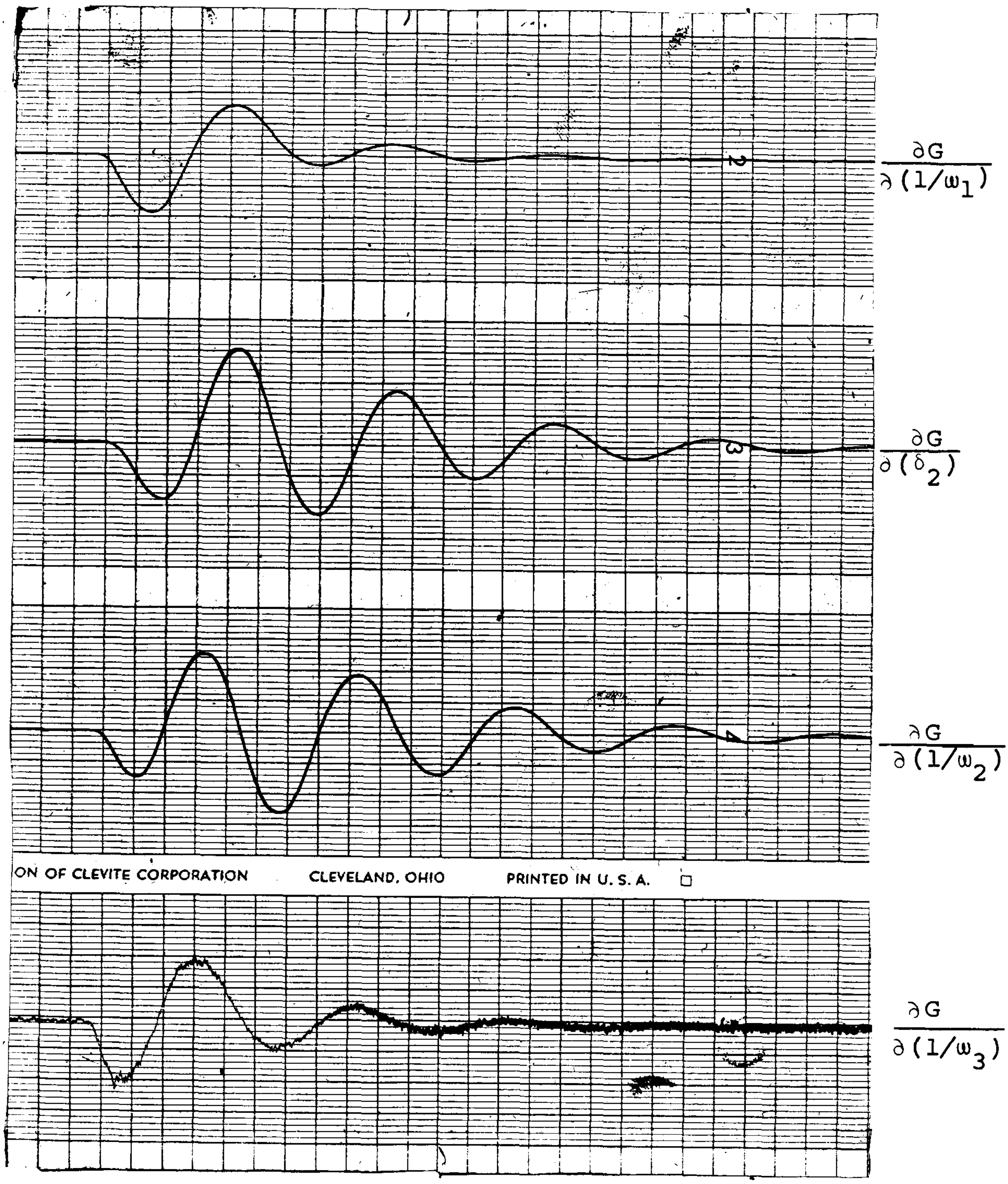


Figure 3-15

Sixth Order System Partial System Impulse Responses

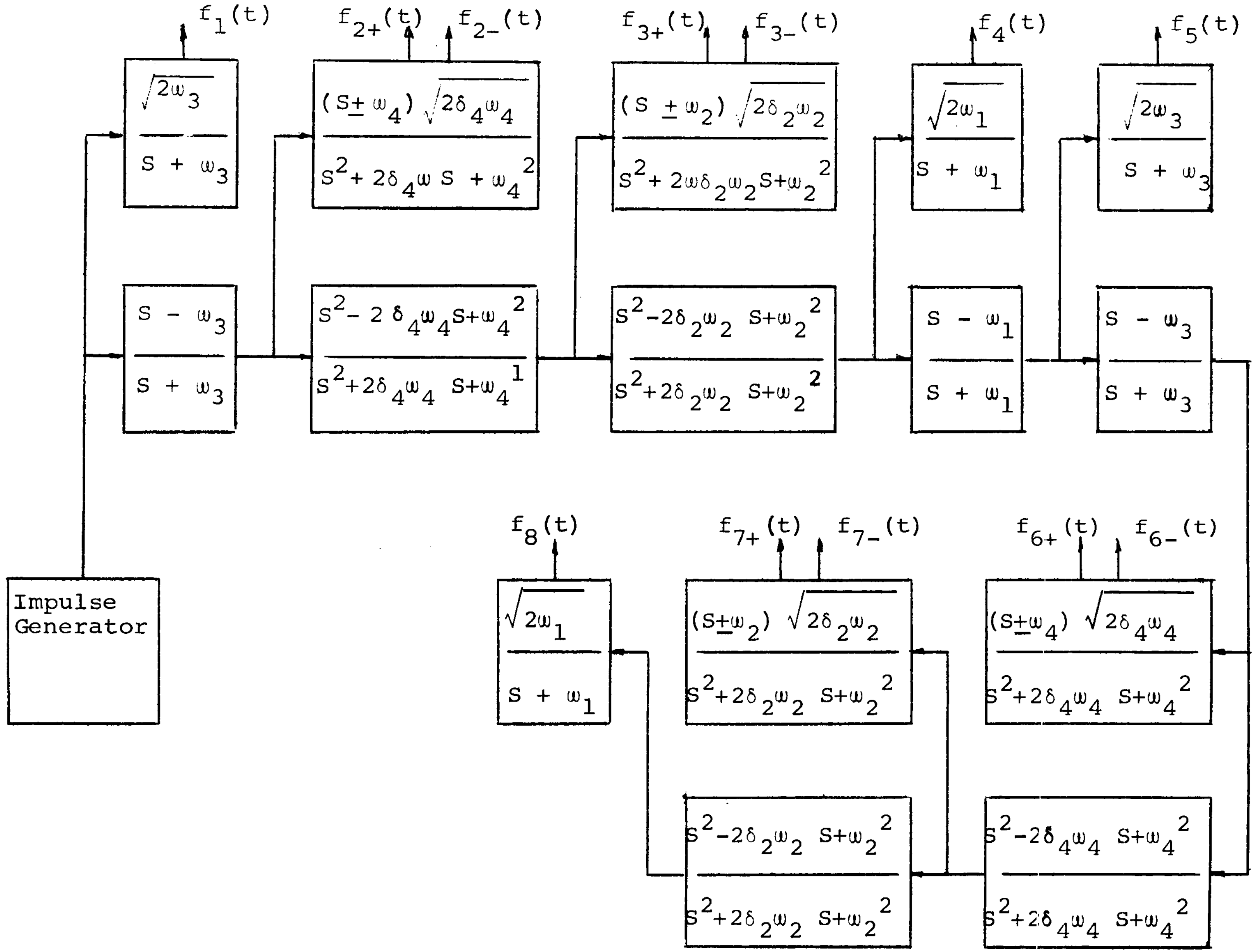


Figure 3-16. Filter Bank Block Diagram (Signal Generator)

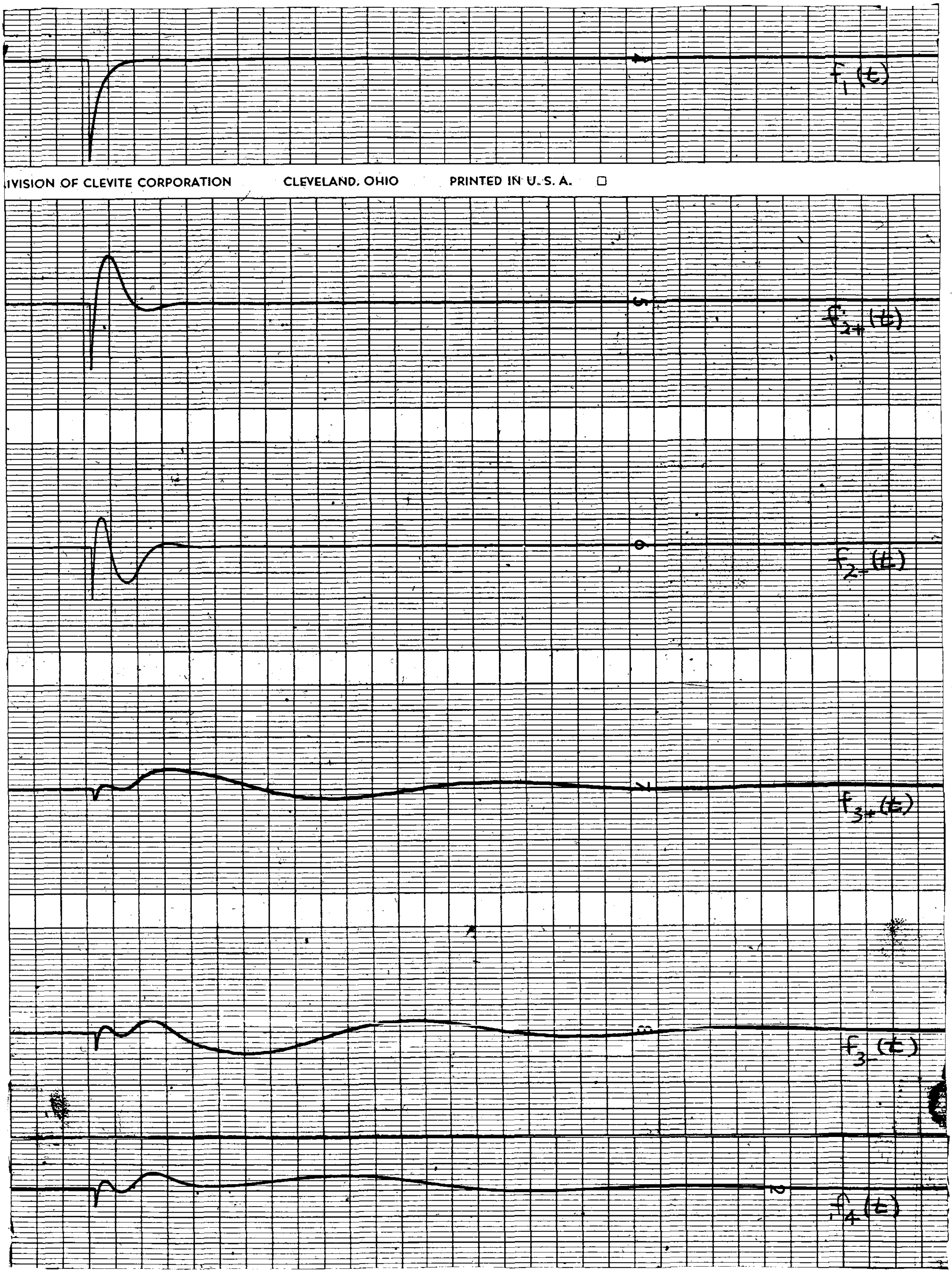


Figure 3-17 Filter Bank Impulse Responses

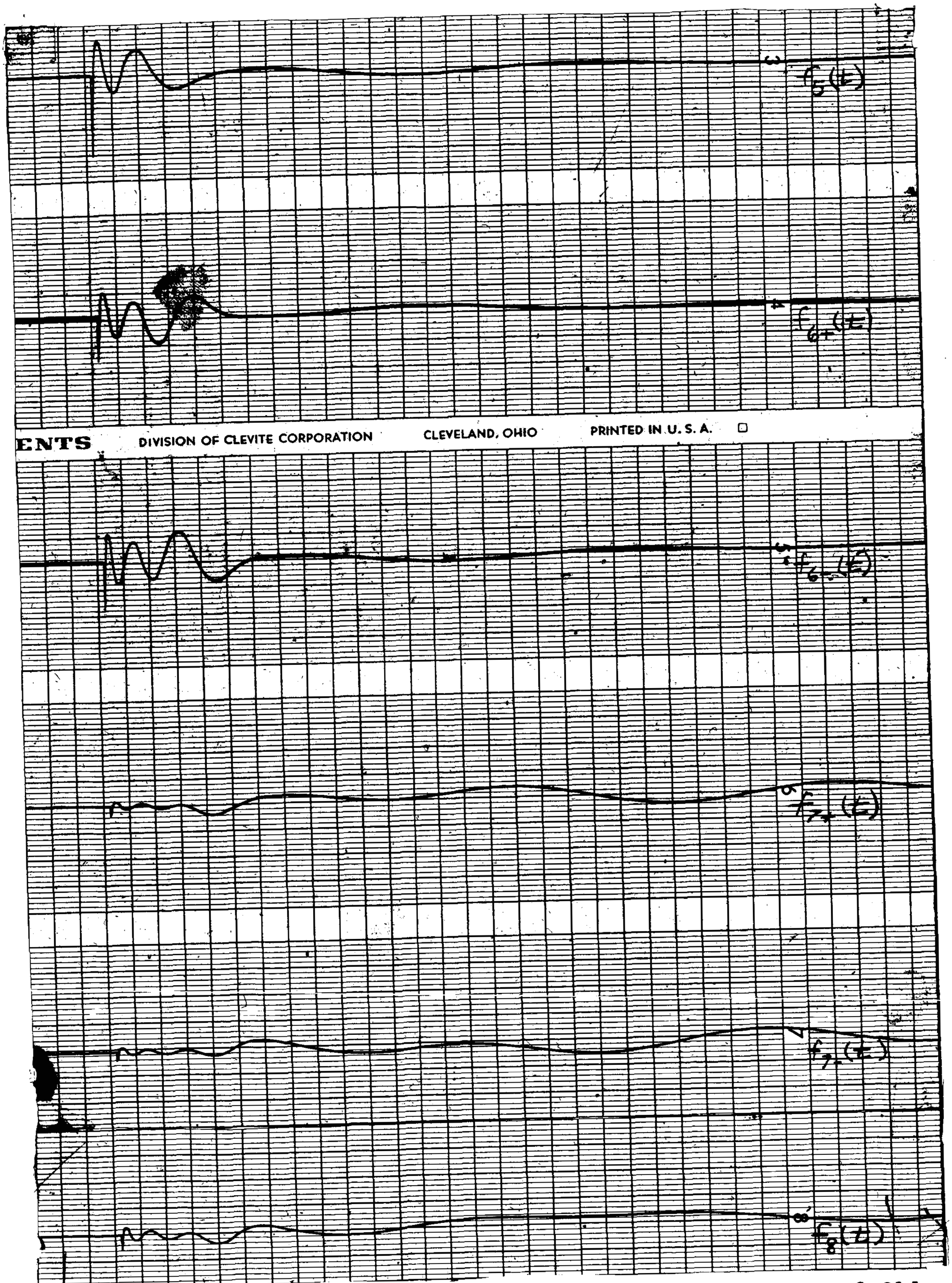


Figure 18 Filter Bank Impulse Responses

Figure 3-19

Results of Testing the Cross-Correlation Characteristics of the Responses

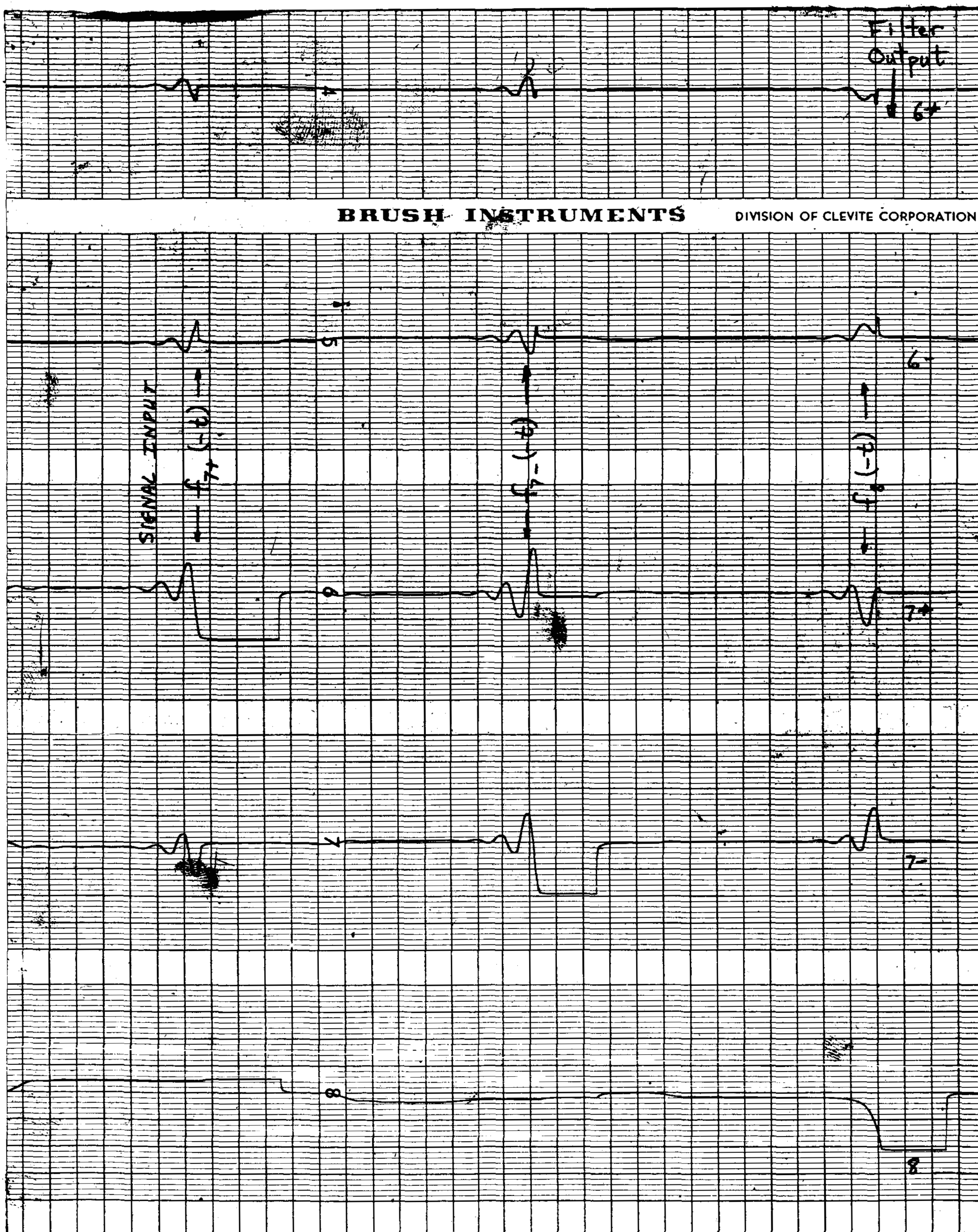


Figure 3-20

Results of Testing the Cross-Correlation Characteristics of the Responses

3.5.4 Determination of the H Matrices

The H matrices were determined using the test setup shown in Figure 21.

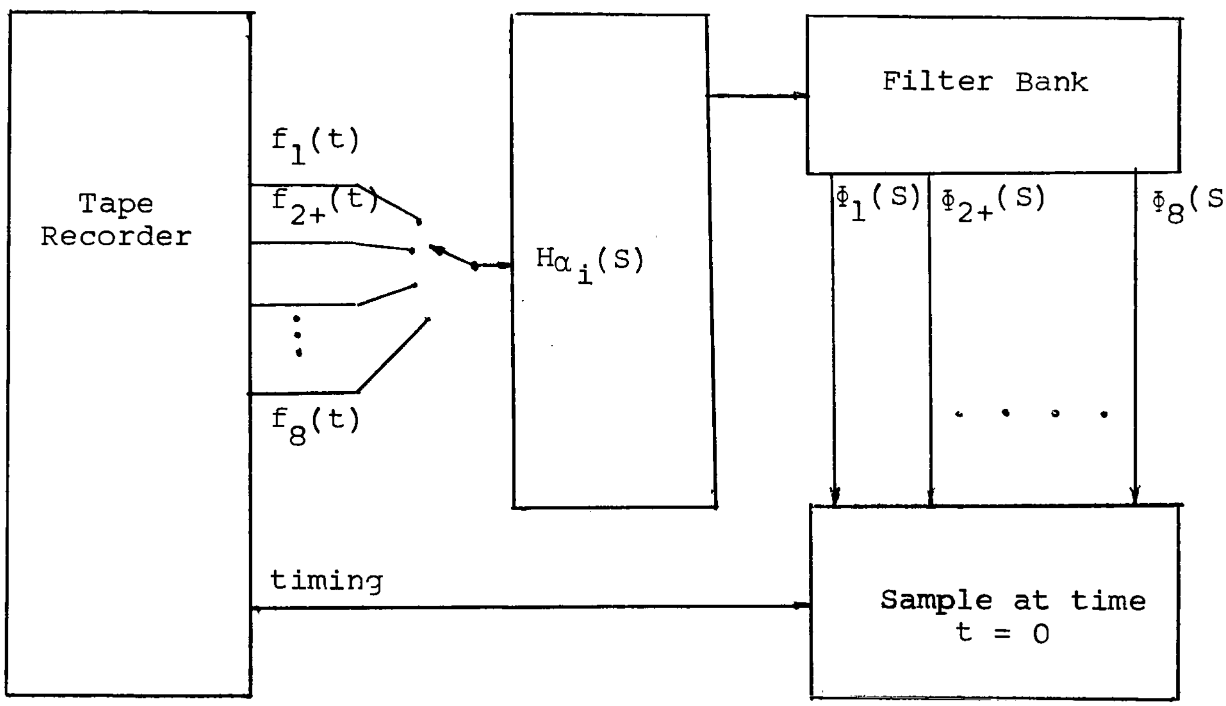


Figure 21

Determination of the H Matrices Test Setup

Thus for each of the **five** partial systems given in Section 3.5.2, a 12 x 12 matrix is formed representing the sampled 12 filter output response values to the 12 input signals. Each element of the table gives the result of

$$h_{jk} = \int_c \Phi_j^*(-S)\ H_i(S)\ \Phi_k(S)\ \frac{dS}{2\pi j}$$

where the value of h_{jk} represents the results of the input signal acting on the transfer function $H_i(S)$ and the measuring filter

$\Phi_j^*(-S)$ at time $t = 0$. There are five of these matrices corresponding to the five partial systems of interest. By combining these five H_α matrices a modulation matrix can be obtained. Because of the size of the matrices no optimization was conducted to arrive at an optimum probing signal, rather, engineering judgement was used to arrive at a modulation matrix. While an optimization process could be conducted, it would give results for only the first term in the Taylor series expansion. Higher order interactions should determine a portion of the size of the covariance terms. Any optimization to minimize the covariance terms based on the first order terms only would be misleading. An example of this was observed in the testing of the second order system studied in Phase B and in the testing of the X-Y plotter. In both cases estimators based on experimental results which included the second order results were better than that predicted from the optimization.

One observation about the matrices that was made, was that the values in the table corresponding to $(\frac{1}{\omega_1})$ are the negative of the values in the K table. Because of the close correlation between a change in $(\frac{1}{\omega_1})$ and K the two parameters will be hard to separate.

3.5.5 The Results Using Modulation Matrix One

The first modulation matrix was selected to measure the parameters K, $\frac{1}{\omega_1}$, $\frac{1}{\omega_2}$, and δ_2. The parameter $\frac{1}{\omega_3}$ was not considered in this modulation matrix because of its low contribution to the system response. The test signal, $f_T(t)$ used to obtain this modulation matrix was

$$f_T(t) = f_{7+}(t) + .5\ f_{7-}(t) + f_{3-}(t).$$

When this modulation matrix was tested it was found that the subtraction of large signals was required to separate the parameter changes. This resulted in large errors. To reduce these errors, the modulation matrix was remeasured to obtain an accuracy of $\pm$ .1%. This resulted in only slightly improved results. It was decided that the numbers in the modulation matrix would have to be known to within .01% if accurate measurements were to be made. This level of repeatability could not be attained with the testing model and could not be expected in a practical situation.

3.5.6 The Results Using Modulation Matrix Two

To bypass the problem of subtracting large signals, an approach was tried which used a filter bank output to directly represent a parameter change. By doing this no estimator is needed, and therefore no subtraction of large signals is necessary. To do this required two sequential testing signals. A test signal $f_{T1}(t) = f_8(t)$ to perform the measurement of K (or $-\frac{1}{\omega_1}$), $\frac{1}{\omega_2}$ and δ_2. A second test signal, $f_{T2}(t) = f_{7+}(t)$ was then used to resolve the difference between K and $(-\frac{1}{\omega_1})$.

The experimentation with this technique showed an additional consideration which had not been taken into account. This was that the size of the second derivative is not included in the estimates of the parameters. In some cases the second derivative is large and does represent a significant contribution to the output of filters. These results indicated that either experimental H matrices should be taken on the second derivatives, or that the H matrices should be determined with the actual system with fixed changes of parameters. The results would still be further complicated by the inclusion of the cross derivatives, which would provide interactions in the outputs of the corresponding filters.

3.5.7 The Results Using Modulation Matrix Three

In order to include higher order effects in the parameter prediction process, a new approach was taken. The signal $f_8(t)$ was used as the input testing signal and an experimental design was conducted to express the output of each filter at the sampled time, as a function of changes in the parameters of interest. That is, an equation of the form

$$\Phi_j = \beta_o + \sum_{m=1}^{4} (\beta_m \alpha_m + \gamma_m \alpha_m^2) + E_j$$

where the α_m are the parameters of interest. The β and γ coefficients are determined from the experimental design data and E_j is an error term which expresses the result of the neglected higher order terms such as α^3 and cross product terms like $\alpha_1\alpha_2$. The experimental design plan which was performed is shown in Table 3-1. For each of the 25 runs the sampled final value of the twelve filter outputs was recorded. The equations used to process this data to determine the β and γ coefficients are given in Table 3-2. A further discussion of the experimental design plan theory can be found in Reference 25. By a careful selection among the possible twelve filter output equations it was found that because some of the β and γ terms were relatively low that the four parameters could be determined by using only four of the filter outputs. In the general case it would be necessary to use eight of the filter outputs. The matrix which was determined is given in Table 3-3.

Example results obtained with this estimator are given in Table 3-4. Each run in the table gives the actual parameter changes and the results of the parameter prediction after performing the single parameter testing. All numbers in the table are expressed in percent. The changes in all four parameters could be estimated with an average error of about 3% when the magnitude of a parameter error (or errors) was less than 10%. When parameter error was greater than 10%, the prediction accuracy decreased.

The prediction accuracy for three of the parameters was much better than the accuracy of the ω_3 prediction. The parameters ω_1, ω_2 and δ_2 can be predicted with an average accuracy of 1% when any of the parameters ω_1, ω_2, ω_3, ω_4, δ_2, δ_4 vary within a $\pm$ 10% range. Trying to also predict ω_3 though, decreases the average accuracy from 1% to 3%. The reason for the difficulty in predicting ω_3 is that the partial derivative of G_7 with respect to ω_3 is small relative to the derivatives with respect to ω_1 or ω_2 (see Section 3.5.2). Therefore, the system error output is much larger for a given percentage error in ω_1 than ω_3. In the estimator, to determine ω_3 requires a cancellation of large signals, and a small effect (for example a α^3 order effect) in one of the large signals, when subtracted from another large signal, will produce a small difference, but this difference can be of the same size as the ω_3 error signal which it is desired to measure. This causes the accuracy difficulty in measuring ω_3.

RUN	PARAMETER CHANGE				FILTER* OUTPUT
n	X_1	X_2	X_3	X_4	
1	-1	-1	-1	-1	a_1
2	-1	-1	-1	1	a_2
3	-1	-1	1	-1	a_3
4	-1	-1	1	1	a_4
5	-1	1	-1	-1	a_5
6	-1	1	-1	1	a_6
7	-1	1	1	-1	a_7
8	-1	1	1	1	a_8
9	1	-1	-1	-1	a_9
10	1	-1	-1	1	a_{10}
11	1	-1	1	-1	a_{11}
12	1	-1	1	1	a_{12}
13	1	1	-1	-1	a_{13}
14	1	1	-1	1	a_{14}
15	1	1	1	-1	a_{15}
16	1	1	1	1	a_{16}
17	0	0	0	0	a_{17}
18	2	0	0	0	a_{18}
19	-2	0	0	0	a_{19}
20	0	2	0	0	a_{20}
21	0	-2	0	0	a_{21}
22	0	0	2	0	a_{22}
23	0	0	-2	0	a_{23}
24	0	0	0	2	a_{24}
25	0	0	0	-2	a_{25}

* There are 12 filter outputs to be recorded for each run

$X_1 = \frac{\Delta\omega_1}{.1}$, $X_2 = \frac{\Delta\omega_2}{.05}$

$X_3 = \frac{\Delta\delta_2}{.05}$, $X_4 = \frac{\Delta\omega_3}{.1}$

Table 3-1

Experimental Design Plan

$$\sum_{n=1}^{N=25} X_{1n}\, a_n = \beta_1 \sum_{n=1}^{N=25} X_{1n}$$

$$\sum X_{2n}\, a_n = \beta_2 \sum X_{2n}$$

$$\sum X_{3n}\, a_n = \beta_3 \sum X_{3n}$$

$$\sum X_{4n}\, a_n = \beta_4 \sum X_{4n}$$

$$\sum a_n = \beta_o N + \gamma_1 \sum X_{1n}^2 + \gamma_2 \sum X_{2n}^2 + \gamma_3 \sum X_{3n}^2 + \gamma_4 \sum X_{4n}^2$$

$$\sum X_{1n}^2 a_n = \beta_o \sum X_{1n}^2 + \gamma_1 \sum X_{1n}^4 + \gamma_2 \sum X_{1n}^2 X_{2n}^2 + \gamma_3 \sum X_{1n}^2 X_{3n}^2 + \gamma_4 \sum X_{1n}^2 X_{4n}^2$$

$$\sum X_{2n}^2 a_n = \beta_o \sum X_{2n}^2 + \gamma_1 \sum X_{2n}^2 X_{1n}^2 + \gamma_2 \sum X_{2n}^4 + \gamma_3 \sum X_{2n}^2 X_{3n}^2 + \gamma_4 \sum X_{2n}^2 X_{4n}^2$$

$$\sum X_{3n}^2 a_n = \beta_o \sum X_{3n}^2 + \gamma_1 \sum X_{3n}^2 X_{1n}^2 + \gamma_2 \sum X_{3n}^2 X_{2n}^2 + \gamma_3 \sum X_{3n}^4 + \gamma_4 \sum X_{3n}^2 X_{4n}^2$$

$$\sum X_{4n}^2 a_n = \beta_o \sum X_{4n}^2 + \gamma_1 \sum X_{4n}^2 X_{1n}^2 + \gamma_2 \sum X_{4n}^2 X_{2n}^2 + \gamma_3 \sum X_{4n}^2 X_{3n}^2 + \gamma_4 \sum X_{4n}^4$$

Table 3-2

Experimental Design Equations

$$
\begin{bmatrix} \Phi_{6+} \\ \Phi_{2-} \\ \Phi_{7+} \\ \Phi_{8} \end{bmatrix} = 10^{3} \begin{bmatrix} 0 & 0 & .7265 & 0 & 0 & 0 \\ -.206 & 0 & .04 & .0049 & 0 & 0 \\ -.968 & .0091 & .948 & .0229 & .322 & .0774 \\ -.73 & 0 & -.288 & .0018 & .0935 & -.0944 \end{bmatrix} \begin{bmatrix} X_1 \\ X_1^2 \\ X_2 \\ X_2^2 \\ X_3 \\ X_4 \end{bmatrix}
$$

Table 3-3

Modulation Matrix Three

Run	$\Delta\omega_1$ Actual	$\Delta\omega_1$ Predicted	$\Delta\omega_2$ Actual	$\Delta\omega_2$ Predicted	$\Delta\delta_2$ Actual	$\Delta\delta_2$ Predicted	$\Delta\omega_3$ Actual	$\Delta\omega_3$ Predicted	$\Delta\omega_5$ Actual	$\Delta\delta_5$ Actual
0	0%	0%	0%	0%	0%	0%	0%	+1%	0	0
1	10	9	0	0	0	0	0	+2		
2	-20	-24	0	1	0	-11	0	-17		
3	40	30	0	1	0	-11	0	-3		
4	0	0	0	0	0	-1	-10	-6		
5	0	0	0	+1	0	0	20	+13		
6	0	-2	0	-2	0	-2	-40	-35		
7	0	1	5	5	0	1	0	0		
8	0	2	-10	-11	0	2	0	-7		
9	0	-33	20	16	0	*	0	*		
10	0	0	0	0	-5	-5	0	0		
11	0	-1	0	0	10	8	0	6		
12	0	-2	0	1	-20	-23	0	-2		
13	10	10	0	0	0	0	10	9		
14	10	9	0	-1	0	-1	-10	-6		
15	-10	-11	0	0	0	-4	-10	-10		
16	-10	-11	0	1	0	-3	10	3		
17	-10	-11	5	5	0	-3	0	-3		
18	-10	-12	-5	-5	0	-4	0	+3		
19	10	10	5	5	0	1	0	3		
20	10	6	-5	-6	0	-5	0	13		
21	0	-1	5	5	5	3	0	9		
22	0	2	5	6	-5	-2	0	-6		
23	0	-4	-5	-6	-5	-11	0	15		
24	0	-1	-5	-5	+5	2	0	6		
25	0	-7	10	8	10	-2	0	42		
26	0	-1	10	11	-10	-10	0	7		
27	20	17	0	1	0	-2	20	17		
28	-20	-24	0	2	0	-12	20	-5		
29	0	0	0	1	0	-1	0	23	10	
30	0	0	0	0	0	0	0	12	0	-10

* Very Large

Table 3-4

Actual Versus Predicted Parameter Changes

3.5.8 Single Parameter Testing With Gaussian White Noise

In order to better understand some of the results which were obtained with modulation matrix one (Section 3.5.5), the Single Parameter testing scheme was modified as shown in Figure 3-22 to use Gaussian white noise as the input signal.

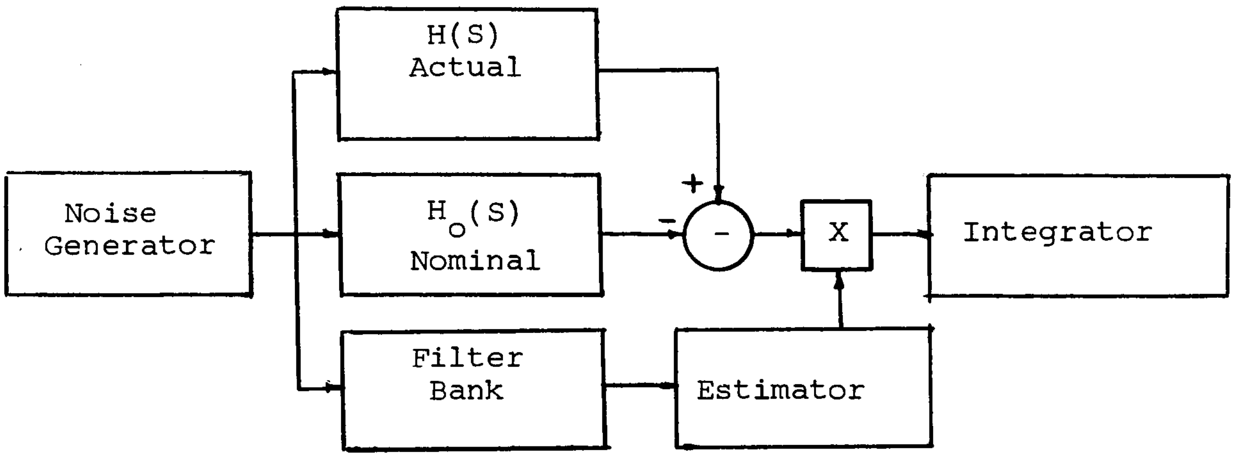


Figure 3-22

Noise Source Testing

Significant parameter testing could not be performed with this test set up in the short time that it was studied. Once again the fact that the estimator coefficients were determined based on just the first order Taylor series expansion terms was the problem.

3.5.9 Single Parameter Testing Using Time Sampling

One other Single Parameter Testing method was used which employed time sampling.[26] A block diagram of the test set up is shown in Figure 3-23.

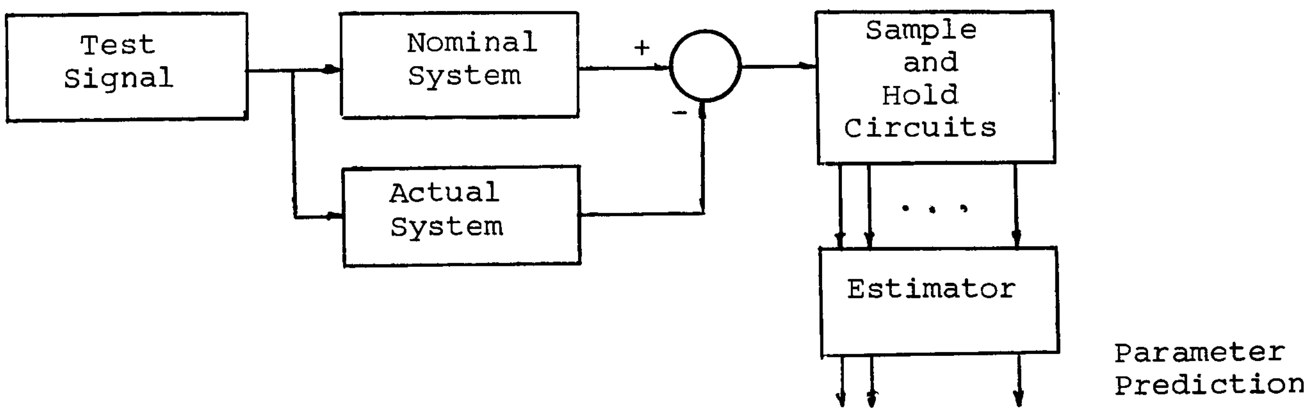


Figure 3-23

The Time Sampling Test Setup

The test signal for this time sampling technique was formed by recording the impulse response of the nominal system on tape and then reversing the tape end-for-end. The sampling times were selected by plotting the difference circuit output as a function of a given parameter change on an X-Y plotter. An example of this plot for changes in a given parameter is shown in Figure 3-24. Note that at time t_1, the value of the error function is zero regardless of the size of the parameter change. This time was then selected as one of the sampling times. This same type of function was plotted out for three of the four parameters of interest which determined three sampling times. The plot for the fourth function (ω_2^2) is shown in Figure 3-25. Note that there is no one time where this function is zero regardless of the parameter change. Two sampling times were selected from the plot as follows. Express the error as

$$E(t) = a(t) \left(\frac{\Delta\omega_2^2}{.1}\right) + b(t) \left(\frac{\Delta\omega_2^2}{.1}\right)^2$$

It is possible to select a time t_2 from the X-Y plot such that $a(t_2) = 0$ At this time

$$-1.25 = a(t_2)\ (1) + b(t_2)\ (1)^2$$

$$-5 = a(t_2)\ (2) + b(t_2)\ (2)^2$$

and solving these two equations

$$a(t_2) = 0$$

$$b(t_2) = -1.25$$

It is also possible to select a time t_3 from the X-Y plot such that $b(t_3) = 0$. At this time

$$-4 = a(t_3)\ (1) + b(t_3)\ (1)^2$$

$$-8 = a(t_3)\ (2) + b(t_3)\ (2)^2$$

and solving these two equations

$$a(t_3) = 4$$

$$b(t_3) = 0$$

This then is how the five sampling times were selected. The modulation matrix which was obtained is given in Table 3-5 and inverting this matrix gives the estimator to be used in the test setup.

Example results obtained with this estimator are given in Table 3-6. Each run in the table gives the actual parameter changes and the results of the parameter prediction after performing the single parameter testing. All numbers in the table are expressed in percent. The changes in all four parameters could be estimated with an average error of about 4% when the magnitude of a parameter error (or errors) was less than 10%. When a parameter error was greater than 10%, the prediction accuracy decreased.

The prediction accuracy for three of the parameters was much better than the accuracy of the ω_3 prediction. The parameters ω_1, ω_2^2, and δ_2 could be predicted with an average accuracy of 2% when any of the parameters ω_1, ω_2^2, ω_3, ω_4, δ_2, δ_4 vary within a $\pm$ 10% range. Trying to also predict ω_3 though, decreased the average accuracy from 2% to 4%. The reason for the difficulty in predicting ω_3 is again the fact that the partial derivative of G_7 with respect to ω_3 is relatively small.

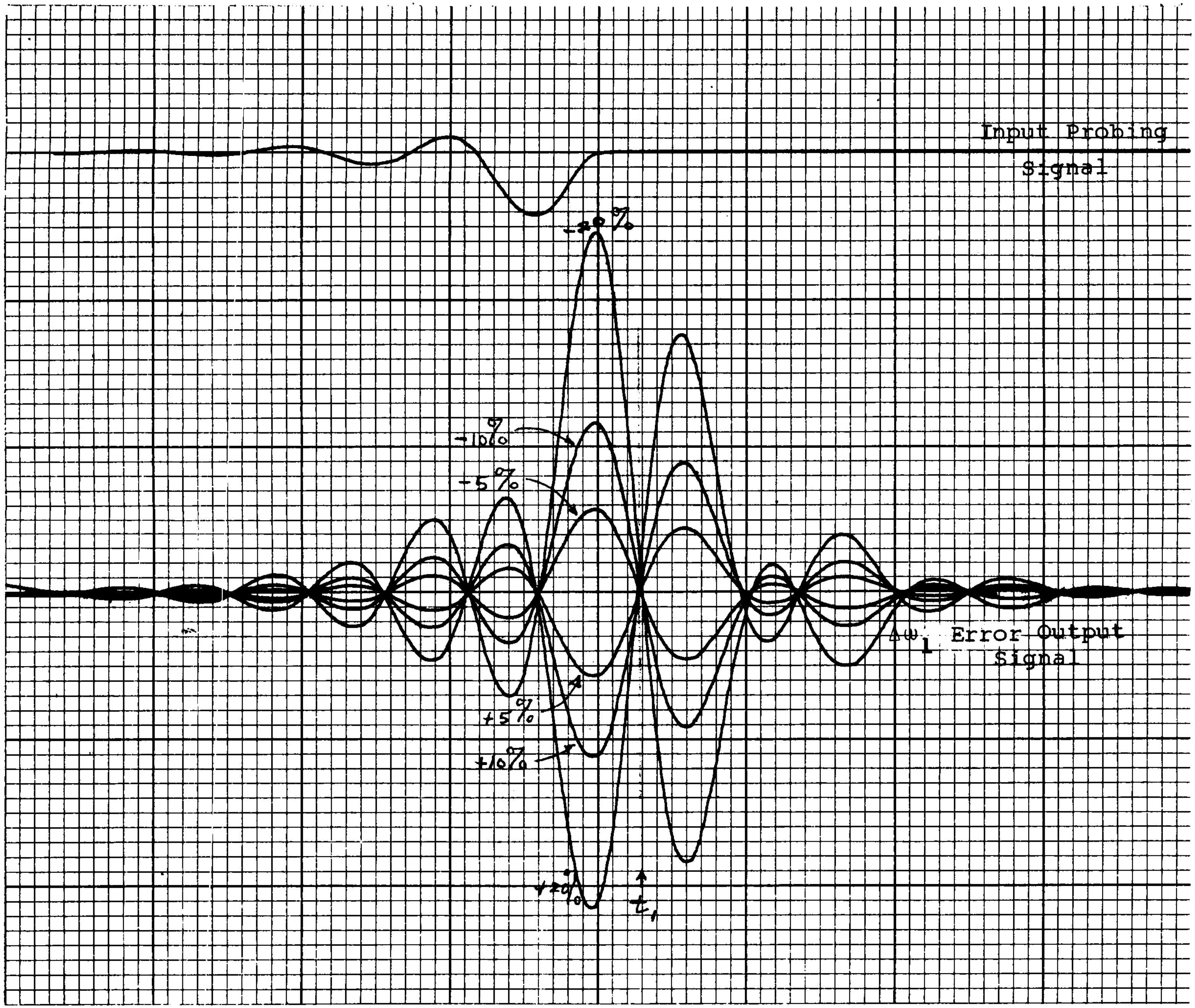


Figure 3-24

Results from the Time Sampling Test Setup

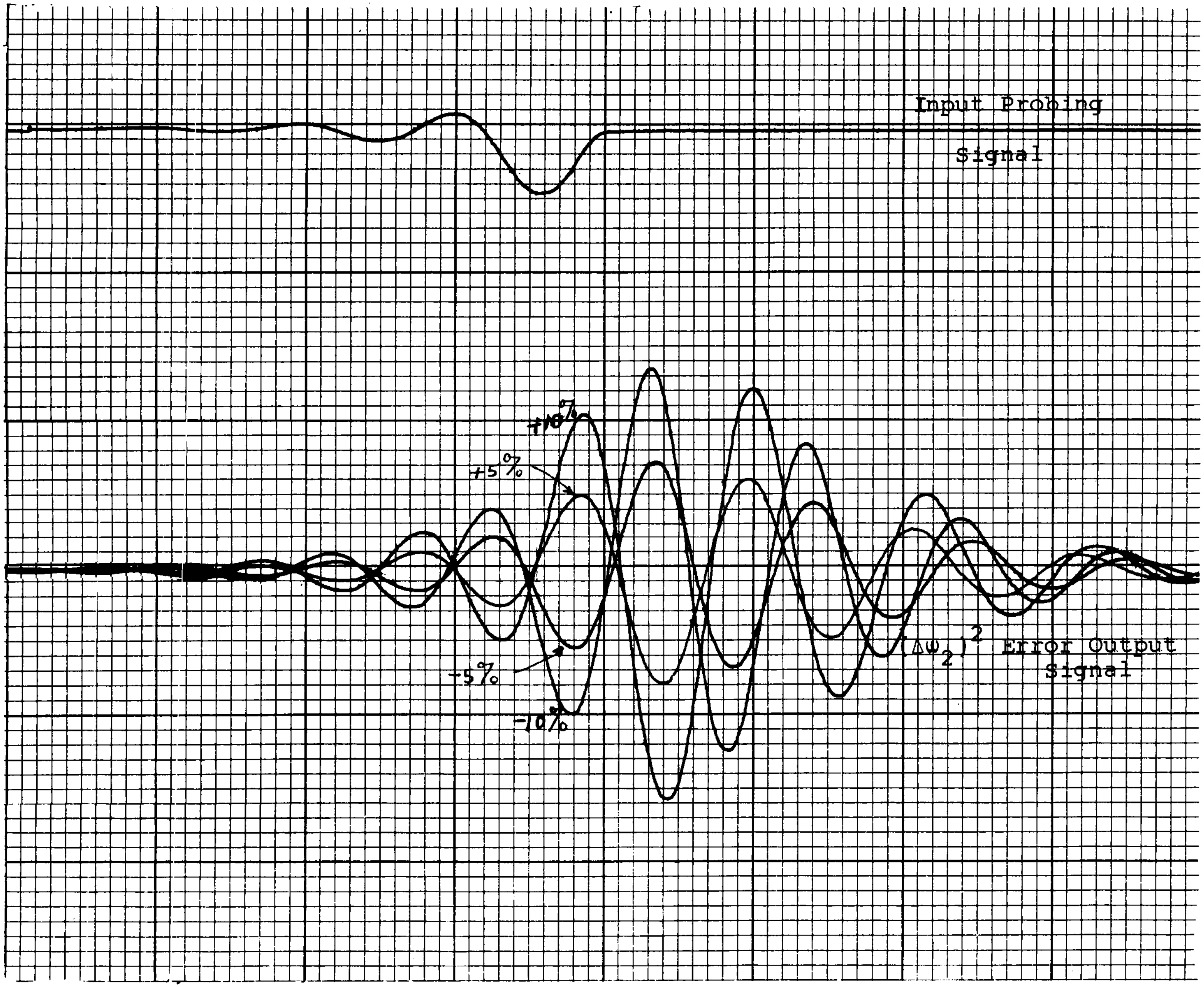


Figure 3-25

Results from the Time Sampling Test Setup

$$\begin{bmatrix} E(t_2) \\ E(t_1) \\ E(t_4) \\ E(t_3) \\ E(t_5) \end{bmatrix} = \begin{bmatrix} 0 & -2.6 & +.20 & -1.25 & +3.2 \\ 5.75 & 0 & +.85 & .25 & 0 \\ .35 & +2.05 & 0 & +.65 & -3.4 \\ -4.0 & +0.3 & -.83 & 0 & -1.55 \\ -1.5 & -0.5 & -.475 & -0.5 & 0 \end{bmatrix} \begin{bmatrix} (\omega_2{}^2/.1) \\ (\delta_2/.1) \\ (\omega_3/.2) \\ (\omega_2{}^2/.1)^2 \\ (\omega_1/.1) \end{bmatrix}$$

Table 3-5

Modulation Matrix for Time Sampling

Run	$\Delta\omega_1$ Actual	$\Delta\omega_1$ Predicted	$\Delta\omega_2^2$ Actual	$\Delta\omega_2^2$ Predicted	$\Delta\delta_2$ Actual	$\Delta\delta_2$ Predicted	$\Delta\omega_2$ Actual	$\Delta\omega_2$ Predicted	$\Delta\omega_5$ Actual	$\Delta\delta_5$ Actual
1	0%	0%	0%	0%	0%	0%	0%	0%	0	0
2	+10	+12	0	0	0	+3	0	-2		
3	-20	-30	0	+1	0	-13	0	+31		
3.5	0	+2	0	0	0	0	+10	+14		
4	0	-3	0	0	0	-1	-10	-12		
5	0	+3	0	0	0	+1	+20	+22		
6	0	+15	+10	+11	0	-2	0	-3		
7	0	-7	-20	-16	0	+37	0	-28		
7.5	0	+3	0	0	+10	+10	0	+1		
8	0	-4	0	0	-10	-12	0	+3		
9	0	+6	0	0	+20	+18	0	+2		
10	+10	+15	0	0	0	+3	+10	+19		
11	+10	+10	0	0	0	+1	-10	-4		
12	-10	-16	0	0	0	-4	-10	-9		
13	-10	-13	0	0	0	-5	+10	+13		
14	-10	-2	+10	+10	0	-6	0	+1		
15	-10	-22	-10	-8	0	+8	0	-7		
16	+10	+4	-10	-11	0	+15	0	-3		
19	0	+14	+10	+13	-10	-10	0	-15		
20	0	-14	-10	-12	-10	-2	0	+4		
21	0	-5	-10	-8	+10	+21	0	-16		
22	0	+33	+20	+16	+20	+23	0	+16		
23	0	-18	-20	-24	-20	+19	0	+5		
24	+20	+27	0	0	0	+1	+20	+56		
25	-20	-38	0	0	0	-12	-20	-6	0	
26	0	+5	0	-1	0	+2	0	+38	+20	
27	0	-7	0	+1	0	-1	0	-50	-20	0
28	0	-3	0	0	0	0	0	-15	0	+20
29	0	+3	0	0	0	+1	0	+17	0	-20

Table 3-6

Actual Versus Predicted Parameter Changes

3.5.9.1 Confidence Sampling

This time sampling technique could also be used for Go-NO-GO confidence sampling. For example, if the time at which the error due to a change in ω_1 is a maximum is selected as the sampling time, than an error voltage output at this time less than a certain threshold limit (determined for this example by that level produced by a 5% error in ω_1) would assure that if only one parameter has varied, the change is

$$|\Delta\omega_1| < 5\%$$

or $$|\Delta\omega_2{}^2| < 6\%$$

or $$|\Delta\omega_3| < 46\%$$

or $$|\Delta\delta_2| < 10\% .$$

One problem with this type of testing is the possibility that two parameters will change in opposite directions such that the resultant error signal is zero at the sampling time.

Another way to perform the confidence testing would be to detect the maximum error signal over the complete time period. If the same threshold limit is selected as in the previous case this would assure that if only one parameter has varied, the change is

$$|\Delta\omega_1| < 5\%$$

or $$|\Delta\omega_2{}^2| < 3\%$$

or $$|\Delta\omega_3| < 30\%$$

or $$|\Delta\delta_2| < 7\% .$$

Again it is possible for two error signals to cancel each other out but the probability of cancellation over the complete time period is much less than at just one particular sampling time.

4.0 SUMMARY OF RESULTS

The theory of growing exponential signals has been shown to be applicable to testing first and second order systems. The theory can also be used for higher order systems using experimental methods to establish the estimator for the parameters to be measured.

The range in which accurate parameter measurements can be made depends upon the complexity of the system, but in all cases the range was at least $\pm$ 10% of the nominal value. This range should be sufficient to cover the desired range of parameter changes and establish undesirable conditions if they are present. The average accuracy of the measurements was within $\pm$ 3% in all cases. The size of the error depended upon the sensitivity of the parameters, and the complexity of the transfer function. This accuracy should be sufficient to establish high confidence in the measurements.

Other results which indicate the applicability of the method are:

a. The minimum signal to noise ratio requirement of 26 db does not restrict the approach.

b. In a brief look taken at a transfer function with a non-linear coefficient it was found that it did not appreciably affect the parameter prediction. Further study of a non-linear model of the thrust vector control system will soon be undertaken and the results published in a supplement report.

c. Single parameter testing can be used to study linear active networks by interpreting them as linear system transfer functions with limited input signal amplitudes.

Implementation problems which were uncovered in the study indicate the importance of the following two items:

a. The testing equipment should be well isolated from the system by buffer stages. These buffer stages should not appreciably affect the transfer function.

b. The testing ecuipment should be compatible with signal levels of the equipment being measured.

These two problems are not limitations to the method, but they must be carefully considered.

The time sampling single parameter testing technique gives results which are comparable to the growing exponential technique. The estimator is easier to determine with the time sampling technique and the equipment involved in the implementation is simpler. The time sampling technique can also be used for GO-NO-GO confidence sampling. All of the comments concerning the applicability of the growing exponential technique apply also to the time sampling approach.

In conclusion, the steps to be taken to implement single parameter testing for a given system are:

1. Develop a nominal system response. This response can be determined by the statistical measurement of a number of

good systems. Once the nominal response is determined it can be stored on tape.

2. Develop a system model which can be used in the determination of an estimator. Good methods are available for this system transfer function determination.

3. The estimator is determined by a theoretical method as described in Section 2 for first and second order transfer functions or by experimental techniques for higher order systems.

4. The fourth step is the implementation of the technique with the actual hardware to be tested, keeping in mind impedance and signal level matching considerations.

5.0 CONCLUSIONS AND RECOMMENDATIONS

The main objective of the study has been met. We have established a method for the testing of active and passive transfer functions. This method was used on several transfer functions and accuracy and measurement time proved to as good or better than present methods of checkout. Faster checkout time is a direct result of the methods studied. Less degradation due to performance is a direct result of using smooth probling signals. Accuracy is acceptable over a parameter range of ± 10% or more.

It is therefore recommended that an extension of this program be carried out. The emphasis during the extension would be placed on the applicability of the testing method. The objectives of the program would be:

1. Apply the methods of single parameter testing to launch vehicle systems, the purpose being to establish the requirements for implementation.

2. Extend the techniques to particular non-linear systems. The intent being to enlarge the class of systems being applicable to single parameter testing.

3. Assess the usefulness of single parameter testing by compiling a list of equipment, with equipment parameters, which could be tested by single parameter testing methods.

4. Recommend an approach to the actual implementation of the testing methods for the equipment on which the technique can be used.

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