[redacted]

NASA CR 66006

[redacted]

[redacted]

GPO PRICE $ ______

CFSTI PRICE(S) $ ______

Hard copy (HC) ______

Microfiche (MF) ______

ff 653 July 65

FACILITY FORM 602

N65-35024 (ACCESSION NUMBER)	(THRU)
31 (PAGES)	1 (CODE)
CR-66006 (NASA CR OR TMX OR AD NUMBER)	30 (CATEGORY)

ORBIT SELECTION STUDY FOR PAGEOS SATELLITE SUMMARY REPORT

Contract No. NAs 1-4614

By S. Worley, J. Miller, and
G. Linsenmayer

Westinghouse Electric Corporation
Aerospace Division
Baltimore, Maryland

[redacted]

[redacted]

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

[redacted]

ORBIT SELECTION STUDY FOR PAGEOS SATELLITE SUMMARY REPORT

Contract No. NAs 1-4614

By S. Worley, J. Miller, and G. Linsenmayer

Westinghouse Electric Corporation
Aerospace Division
Baltimore, Maryland

SUMMARY

An investigation was made to select a suitable orbit for the PAGEOS Geodetic Satellite program. Selection was based on obtaining a maximum number of satisfactory observation opportunities, subject to certain initial orbit constraints. This summary report contains a brief discussion of the approach, a discussion of the final orbit selected, and reasons for its selection.

INTRODUCTION

The PAGEOS program is a program for measuring the shape of the earth by photographing an Echo satellite against the starfield background. Photographs are to be taken from camera stations in a network of approximately 35 or 36 stations which are distributed more or less evenly over the earth's surface. (Table I lists the station locations considered.) The direction of a camera/satellite sightline at the time of exposure is determined by interpolating the images of the satellite into the celestial coordinate system of the starfield background, with appropriate coordinate system rotation as determined by the sidereal time of the exposure. Simultaneous observation of a satellite from two adjacent camera stations determines a plane containing the baseline connecting them, and a second set of simultaneous observations determines a second plane whose intersection with the first gives the direction of the baseline. Other baselines are similarly found and are combined in an appropriate adjustment to give the shape (except for scale factor) of the observation station network. Observations may be made simultaneously from two or three stations; three-station observations are to be preferred since information is gained relating to all three baselines in the observation station triangle, whereas two-station observations yield data only relating to one baseline.

TABLE I

STATION LOCATIONS

No.	Station Name	Latitude (degrees)	Longitude (degrees)
1	Greenland, Thule AFB	76.5 N	68.7 W
2	U.S.A, Aberdeen, Md.	39.5 N	76.1 W
3	U.S.A, Larson AFB, Wash.	47.2 N	119.3 W
4	U.S.A, Aleutian Is., Shemya I.	52.7 N	174.1 E
5	U.S.S.R, Tura, Siberia	64.8 N	101.0 E

TABLE I (Continued)

No.	Station Name	Latitude (degrees)	Longitude (degrees)
6	Finland, Kuopio	62.7 N	28.0 E
7	Azores Is., Pico I.	39.0 N	28.5 W
8	Dutch Guiana, Paramaribo	05.5 N	55.2 W
9	Ecuador, Quito	00.1 S	78.5 W
10	Clipperton I.	10.3 N	109.2 W
11	U.S.A., Hilo, Hawaii	19.8 N	155.0 W
12	Wake Island	19.7 N	166.2 E
13	Japan, Kagoshima	31.7 N	130.6 E
14	India, Gauhati	26.2 N	91.7 E
15	Iran, Sabzawar	36.5 N	57.5 E
16	Libya, Syrte	31.7 N	16.4 E
17	Liberia, Roberts Field	06.8 N	10.2 W
18	Trindade Island	20.5 S	29.4 W
19	Argentina, Villa Dolores	32.0 S	65.1 W
20	Sala y Gomez Island	26.6 S	105.2 W
21	Pukapuka Island	14.7 S	138.8 W
22	Wallis Is., Uvea I.	13.2 S	176.3 W
23	New Guinea, Kikori	07.3 S	144.2 E
24	Sumatra, Palembang	03.0 S	105.0 E
25	Maldive Is., Male′	04.2 N	73.3 E
26	Sudan, Juba	04.8 N	31.6 E
27	Southwest Africa, Bogenfels	27.8 S	15.8 E
28	So. Sandwich Is., Saunders I.	58.4 S	26.7 W
29	Antarctica, Peter I.	69.2 S	90.0 W
30	So. Pacific Ocean, Shoal	41.5 S	148.6 W
31	New Zealand, Queenstown	45.0 S	168.2 E
32	Australia, Denmark	35.0 S	117.3 E
33	St. Paul Island	38.7 S	77.0 E
34	Madagascar, Fort Dauphin	25.0 S	47.1 E
35	Antarctica, U.S.S.R Station	68.0 S	46.4 E
36	Antarctica, France Station	67.0 S	139.0 E

To be practical and suitable for such a program, the orbit of the satellite must satisfy a number of conditions. These are:

a. The orbit must initially be totally sunlit for a period of 14 days to ensure proper inflation of the echo satellite.

b. The launch window must be at least 1 hour.

c. The initial apogee altitude must be in the range of 4000 to 4500 kilometers.

d. The initial eccentricity must be in the range 0 to 0.04.

e. The inclination must be in the range 80 to 90 degrees.

f. The argument of the perigee may be in the range 0 to 360 degrees.

g. The launch interval may be the entire year.

h. The lifetime of the orbit must be at least 5 years.

i. The orbit must provide a large number of suitable two-station and three-station observation opportunities.

The purpose of the orbit selection study was to choose, within the constraints of the first seven conditions, an orbit suitably satisfying the last two.

This choice is complicated by the fact that the effects of solar pressure on a high area/mass ratio satellite of the echo type lead to complex variations in eccentricity, inclination, and period. These variations are difficult to relate to the frequency of acceptable viewing opportunities without extensive computation. A brief summary of the approach used in this selection and a discussion of the selected orbit and the reasons for its selection are presented in later sections. The criteria employed to judge the acceptability of a viewing opportunity were:

a. The viewing station must be in darkness (i.e., the sun is at least 18 degrees over the horizon) while the satellite is sunlit.

b. The elevation angle from the station to the satellite must be at least 30 degrees. (Elevation angles of 25 degrees are considered as marginal.)

c. The sun-satellite-camera included angle must not exceed 135 degrees.

d. The viewing conditions must be acceptable at two or three stations simultaneously for a period of at least 2 minutes.

e. The satellite must not exceed an altitude which would seriously degrade the resolution of the photograph (i.e., 5000 km).

f. To complete the observations for each baseline, at least two observations must be made to define planes intersecting the baseline at an angle of 60 degrees or greater.

APPROACH

Several computer programs were used to aid in the orbit selection process. The use of these programs is illustrated by the block diagram of figure 1 and is explained in complete detail in the final report. Running time proved to be a severe problem in the single-station observation program, so that the following approach to the selection of an orbit was chosen to permit efficient use of computer time.

The first step in the selection process was to examine a number of possible orbits (i.e., orbits satisfying the initial launch restrictions) using the Lifetime-18 program. Of these orbits, those having poor lifetimes, large eccentricity variations, or other objectionable characteristics were rejected, and six of the most promising orbits were chosen for further study.

To obtain a further comparison of these six orbits, an abbreviated problem was defined. In this problem, a network of seven representative stations (stations 8, 17, 18, 27, 28, 29, and 35) were chosen from the complete network. The numbers of viewing opportunities were found for this abbreviated network by using the single-station observation program and the simultaneous observation program; however, these calculations were performed only for 1 month out of 3, over a period of 3 years. It was believed that such a sampling would provide a suitable comparison of the orbits. A tentative comparison and selection were made by tabulating the results and selecting the orbit which gave the best overall performance, as explained in detail in the section entitled "Results and Discussion."

A complete 5-year run was then made using the full 36-station network to ensure that the selected orbit was satisfactory for the complete problem.

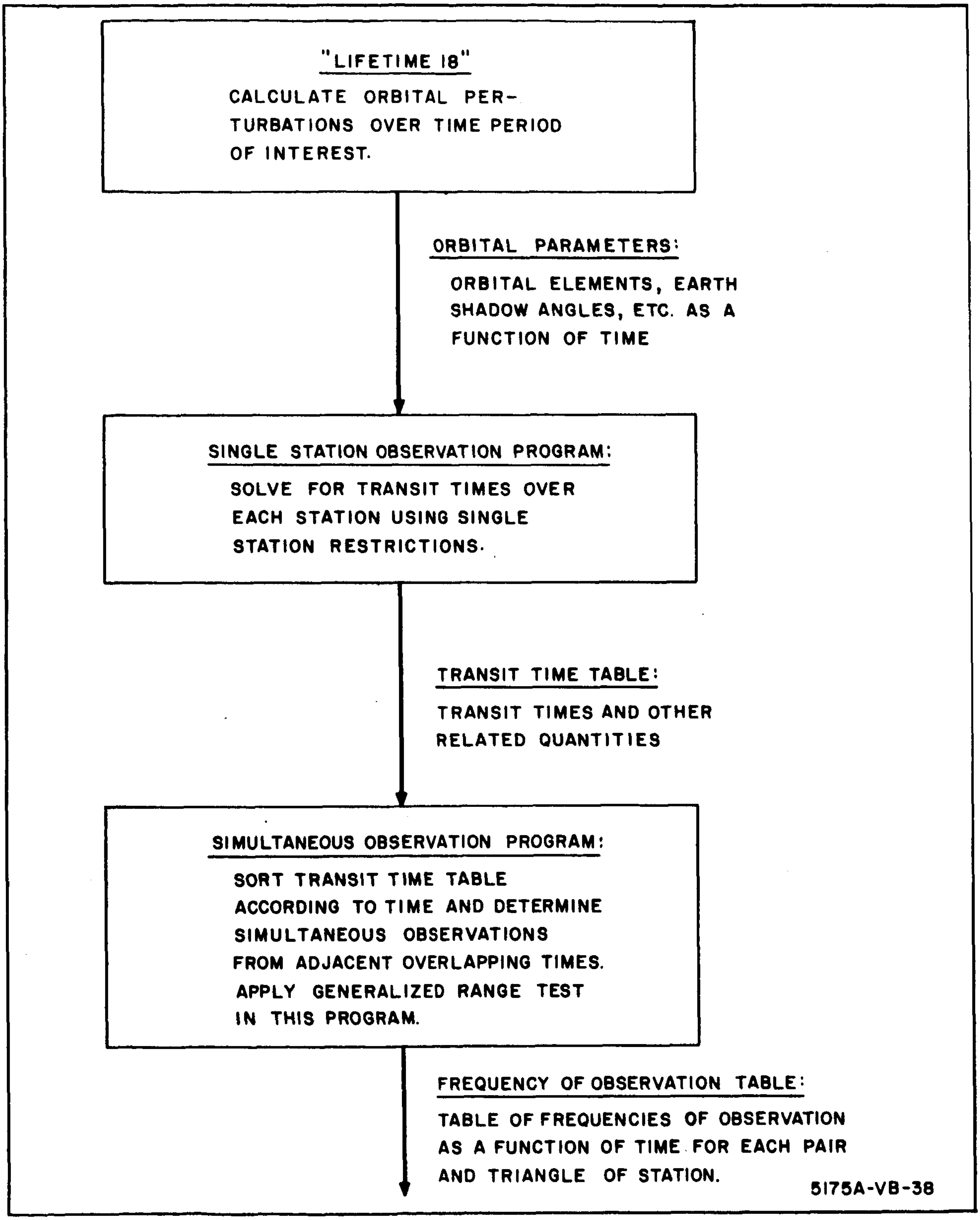


Figure 1. Block Diagram of Digital Computer Program Interfaces for Use in Passive Geodetic Satellite Orbit Selection

In addition, this run was expected to provide a check of the assumption that the abbreviated problem was a reasonable approximation to the full problem. Finally, using Lifetime-18, variations within the launch window of the selected orbit were studied to determine their effect on lifetime, eccentricity changes, etc.

CONCLUSIONS

Six promising orbits were investigated in terms of the relative number of viewing opportunities provided to a representative 7-station network, using a time sample of 1 month in 3. The best of these orbits was then selected as a final orbit; this orbit has the following characteristics at launch:

Altitude - 4250 km (circular)

Inclination - 87 degrees

Right Ascension of the Ascending Node - 345 degrees

Launch Date - 1 June 1966

(Launch time approximately 6:30 AM Pacific Standard Time if launched from Pacific Missile Range)

It was noted during this selection that all orbits appeared to give satisfactory viewing opportunities, with only small variations from orbit to orbit.

A complete 5-year run using the selected orbit showed that it meets all observation requirements, and that the results of the short run agree reasonably well with those of the complete run. An investigation of variations in the launch characteristics, in terms of the resulting maximum eccentricity attained by the orbit during a 5-year period, revealed only slight changes for initial eccentricities of 0.02, for altitude variations of ±100 km, and for inclination variations of ±1 degree.

An additional facet of the study was an investigation of the maximum eccentricity attained during a 5-year period as a function of launch right ascension and launch date. Choosing the launch right ascension as a function of launch date such that this maximum eccentricity is made as small as possible, the resulting value of maximum eccentricity varies with launch date, and is smallest for June and December launches.

RESULTS AND DISCUSSION

Selection of Orbit

As a result of data obtained from preliminary analysis with the Lifetime-18 program, six of the most promising orbits were chosen for further study. Details of this choice are explained in the final report; the altitude (circular orbits), inclination, longitude of the ascending node, and launch date of the chosen orbits are listed in table II below.

TABLE II

ORBIT DATA

Orbit	Altitude (km)	Inclination (degrees)	Ω (degrees)	Launch date
1	4250	87	345	1 June 1966
2	4250	89	10	1 June 1966
3	4500	90	10	1 June 1966
4	4000	85	10	1 June 1966
5	4500	80	0	1 June 1966
6	4250	87	80	1 October 1966

These orbits were analyzed further using an abbreviated problem in which only seven stations were considered and in which the viewing opportunities were found for periods of 1 month every 3 months, over a time interval of 3 years. The seven stations of the abbreviated problem are listed in table III below. (The "old" numbers refer to the numbers of table I; the "new" numbers are those adopted for analysis of the abbreviated problem.) These particular stations were chosen because they formed a network which appeared to include a particularly representative selection of baselines and triangles.

The locations of these stations (and also of the stations of the complete network) are shown on figure 2.

TABLE III

SEVEN STATIONS OF THE ABBREVIATED PROBLEM

"Old" number	Station	"New" number
8	Dutch Guiana, Paramaribo	1
17	Liberia, Roberts Field	2
18	Trindade Island	3
27	Southwest Africa, Bogenfels	4
28	So. Sandwich Is., Saunders I.	5
29	Antarctica, Peter I.	6
35	Antarctica, U.S.S.R. Station	7

To evaluate the data obtained through consideration of the abbreviated problem, tables were prepared using results from the output of the simultaneous observation program.

Tables IV and V illustrate the results obtained for orbit 1 (which was the orbit finally selected). In these tables tabulated entries are:

a. Number of good observations (G) in each time interval. (Time intervals were 1 month in duration at intervals of 3 months.)

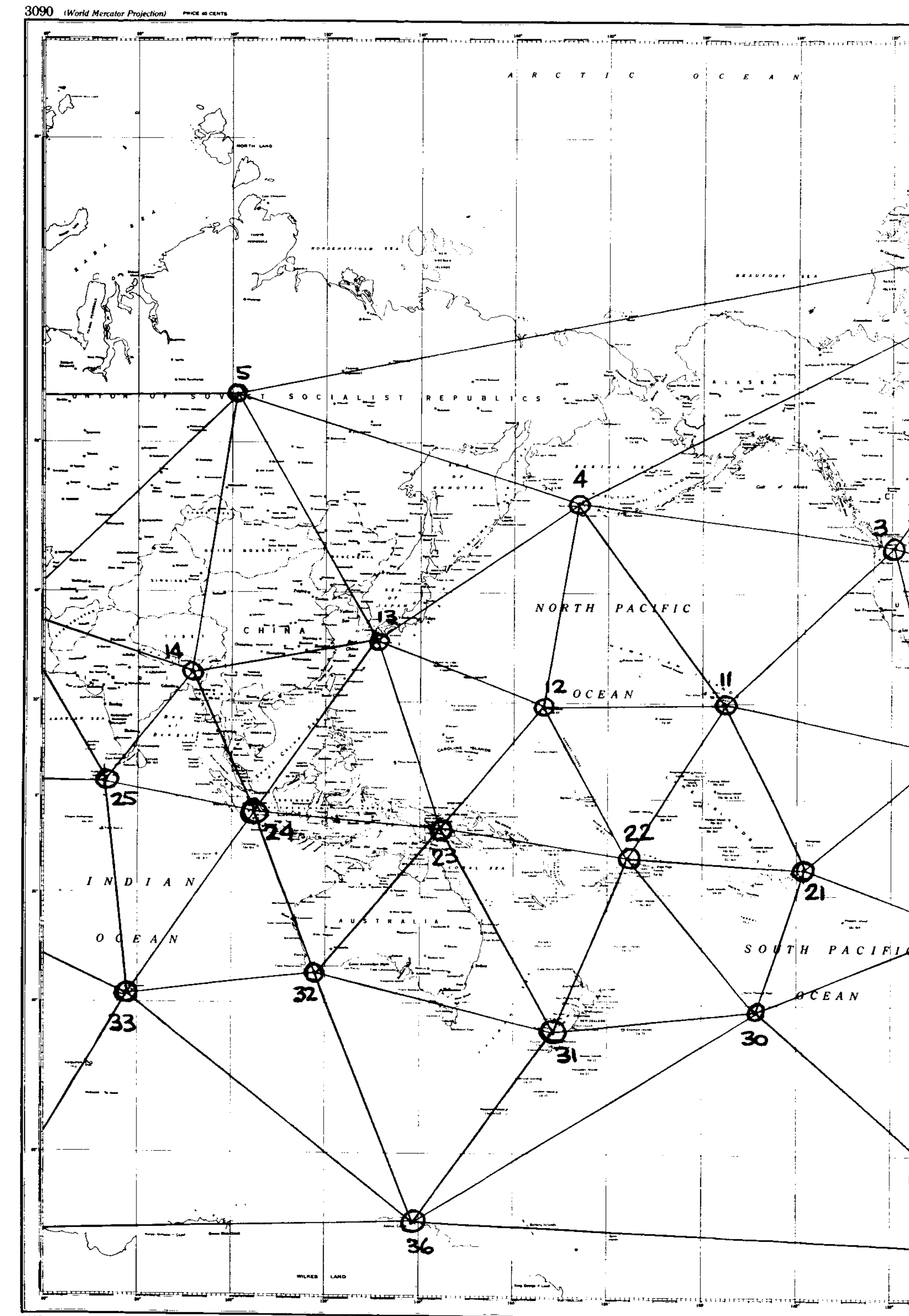
b. Number of marginal observations (M) in each time interval.

c. Number of observations in which the angle of the observation plane is greater than 30 degrees from vertical (+).

d. Number of observations in which the angle of the observation plane is less than -30 degrees from vertical (-).

e. Totals for the 3-year period.

In table V, the good (G) and marginal (M) entries apply to the whole triangle, while the (+) and (-) entries apply to the baselines whose endpoints are the 1st and 2nd, 1st and 3rd, and 2nd and 3rd stations listed, respectively. For example, the (+), (-) entries associated with triangle 2-3-4 refer to baselines 2-3, 2-4, and 3-4 respectively. Negative entries are associated with


3090 (World Mercator Projection)
ARCTIC OCEAN
UNION OF SOVIET SOCIALIST REPUBLICS
CHINA
NORTH PACIFIC OCEAN
SOUTH PACIFIC OCEAN
INDIAN OCEAN
AUSTRALIA
WILKES LAND
5
4
3
13
14
12
11
25
24
23
22
21
33
32
31
30
36
3090

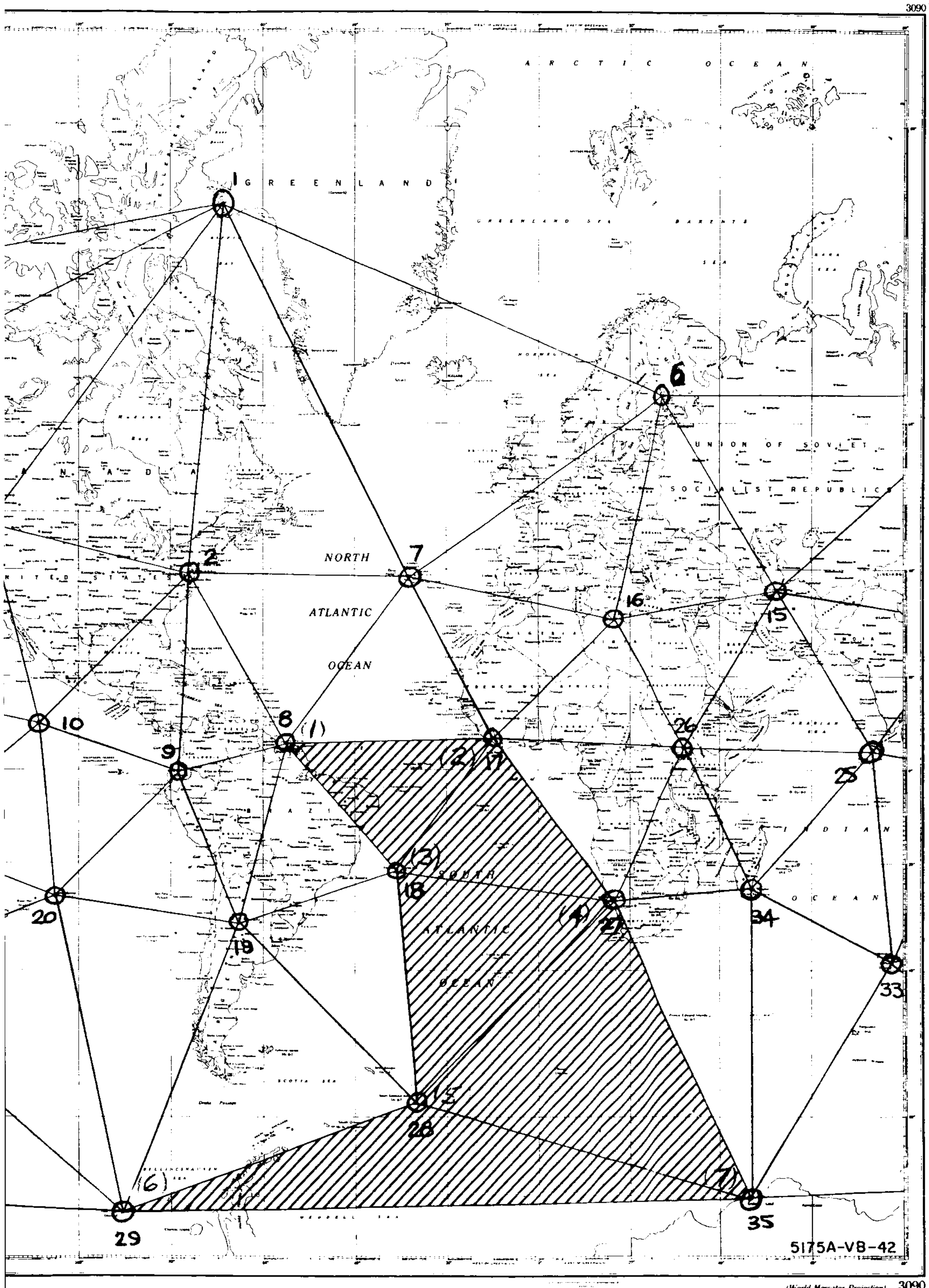


Figure 2. 36- and 7-Station Network

TABLE IV

BASELINE RESULTS

Baselines		End of Time Interval in Days from 1 January 1965												
		607	698	789	880	970	1062	1153	1244	1335	1426	1517	End	Total
1-2	G				3	1								4
	M		4		1						1			6
	+													
	-				2									2
1-3	G	3		1	3	6	10		1			11		35
	M				1	1		11				3		16
	+	2		1	1	2						2		8
	-						7		1			2		10
2-3	G	2		8	1	5	7				11	11	4	49
	M	1		4	1	7		2			3		1	19
	+	2		7		2					2			13
	-				2		5					5		12
2-4	G	3			16	2	15		4			9		49
	M	2			2	4	3	1	2		1	2		17
	+	1			2		4							7
	-				8		2		7					17
3-4	G				16			2				2		20
	M		1											1
	+				4									4
	-				16									16
3-5	G	21	22			6	14						22	85
	M	6	3			2	4				1		5	21
	+	11				1								12
	-	5	7				5						4	21
4-5	G	5	1		12								16	36
	M	2	1										2	5
	+	3											8	11
	-				9									9
4-7	G	2	21		16								2	41
	M	4	1		5		10							20
	+		2											2
	-													
5-6	G	43	12			38			2	35			9	139
	M		1			1				2				4
	+	60	4			27			4	17			2	114
	-	3	9			10							9	31
5-7	G	36	9			22				25			5	97
	M					2								2
	+													
	-	59	11			44				23			4	141
6-7	G	1				3								4
	M													
	+													
	-													
Total	G													559
	M													111
	+													
	-													

ORBIT 1: 4250 km, 87-degree inclination
Ω = 345 degrees, 1 June 1966 launch

TABLE V

TRIANGLE RESULTS

Triangles and Baselines		End of Time Interval in Days from 1 January 1965: 607	698	789	880	970	1062	1153	1244	1335	1426	1517	End	Total
1 2 3	G				1		7							8
	M					1	1					1		3
	+				1		6							7
	-						3							3
	+				1		7							8
	-													
	+													
	-				2		9							11
2 3 4	G													
	M				2							4		6
	+													
	-													
	+													
	-													
	+													
	-													
3 4 5	G		5										1	6
	M		3										3	6
	+													
	-		5										1	6
	+		4										2	6
	-													
	+													
	-		3											3
4 5 7	G	2	8										1	11
	M	4	9			4							6	23
	+	2	4										1	7
	-													
	+													
	-		6											6
	+	2	8										1	11
	-	2	7										1	10
5 6 7	G	39				34				3				78
	M	1												1
	+	64				63				3				130
	-	1												1
	+	3				8								11
	-	58				35				4				7
	+					4				1				5
	-													
Total	G													103
	M													39
	+													
	-													
	+													
	-													
	+													
	-													

ORBIT 1: 4250 km, 87-degree inclination
Ω = 345 degrees, 1 June 1966 launch

observation planes which slant upward and to the right when the higher numbered station is viewed from the lower numbered station, positive entries with planes slanting to the left. These entries were tabulated for all baselines (table IV), and for each triangle and all baselines of each triangle (table V). Observations shown on table IV are in addition to those of table V, so that the total number of observations associated with any given baseline may be obtained by summing the information in the two tables. A lower bound on the number of observations intersecting at angles of 60 degrees or greater may be obtained by summing the (+) and (-) entries. (This is only a lower bound; for example, a 60-degree intersection could be obtained with observation plane angles of +40 and -20 degrees, but this latter observation would not be tabulated in the (-) category since it is greater than -30 degrees.)

A summary table was then prepared (table VI), using data for all six orbits considered. The entries in this table are:

a. Total number of good/marginal observations for all baselines (excluding observations for triangles).

b. Number of observations for the most observed and the least observed baselines (including triangle observations).

c. Total number of good/marginal observations for all triangles.

d. Number of baselines for which observations were not obtained in both the (+) and the (-) categories.

e. Number of triangles and baselines unobserved.

Examination of table VI reveals that, of the first 5 orbits, orbits 2 and 3 are relatively undesirable, due to a large number of "no (+) and (-)" occurrences and because the number of observations associated with the least observed baselines are rather low. Of the remaining 1 June orbits (numbers 1, 4, 5), orbit No. 1 was chosen for the following reasons:

a. Orbit 4 scores somewhat low in terms of total baseline observations and number of observations of the least observed baseline.

b. Preliminary graphical analysis (which is reported in detail in the final report) indicates the possibility of difficulties in obtaining satisfactory

TABLE VI

PRELIMINARY ORBIT RESULTS

Orbit	Baselines			Triangles[1]	No (+) and (-)	Misses	
	Total[1]	Max[2]	Min[2,3]			Triangles	Baselines
1	559/111	217	12/26	103/39	1	1	0
2	590/102	222	1/5/6	80/38	5	1	0
3	460/127	124	3/10	55/39	6	1	0
4	535/101	207	5/27	127/38	0	0	0
5	601/84	162	10/40	107/48	1	0	0
6	662/108	116	13/14	44/24	0	0	0

NOTES:
1. Good/Marginal
2. Triangles plus baselines, good observations only
3. Listed in order of number of observations, with minimum first

observation of all near-polar baselines with an orbit inclination of 80 degrees. Although this was not indicated during study of the abbreviated problem, it is possible that difficulties might arise with consideration of the complete network. For this reason, the more highly inclined orbit 1 appeared to be a more conservative choice.

In addition to the five orbits discussed above, a sixth run was made using the same altitude and inclination as used for orbit 1, but with an October 1 launch date. Comparing these results with those for the June 1 launch, the October 1 launch is also seen to give good performance, having more total baseline observations and fewer triangle observations.

After selection of orbit 1 based on the considerations previously discussed, a complete 5-year run was made for this orbit. In this run, the 36-station network was used and time sampling was not employed. The results of this run, presented in the form of tables similar to tables IV and V, may be found in the final report. In summarizing these results, several points were noted:

a. All required observation conditions were fulfilled. Simultaneous three-station observations were obtained at all triangles except the polar triangles. However, the baselines of the polar triangles were individually observed as required.

b. A definite pattern of observation opportunities at high northern and southern latitudes was observed. Observations were poor at high northern latitudes during the summer months (June, July, August) and good during winter months, with the conditions at southern latitudes exactly reversed. This was expected, due to the variation in the lengths of day and night as a function of season.

c. Observations were more common during the first year than during later years. This is due to the fact that the orbit is initially circular, allowing observations at all points of the orbit; however, in later years, the orbit becomes eccentric until portions of the orbit exceed the 5000 km altitude limitation, reducing the number of observation opportunities possible.

d. The results of the 5-year run agreed reasonably well with those of the corresponding sampled run. This point is discussed more fully at the end of this section.

Time of Day of Launch

To determine the exact time of launch required to achieve a given right ascension, it is necessary to consider the exact launch trajectory from launch to orbit injection. For the 1 June launch, an approximate launch time was calculated ignoring the launch trajectory; the procedure of this calculation may be found in the appendix. The approximate launch time is 6:30 a.m., Pacific Standard Time, for launch 1 June 1966.

Satellite Orbit Resonance

If the period of a satellite orbit is an integral fraction of a sidereal day, the satellite would appear at the same position at the same time each day, resulting in a poor pattern of coverage. The altitude of the selected orbit is quite close to the altitude of such a resonant orbit, that altitude being 4160 km for a 3-hour orbit. However, even a small variation in altitude provides a satisfactory degree of nonresonance. For an altitude of 4250 km, the earth position under the satellite shifts about 3 degrees per day; thus, the total earth surface is covered in 15 days. (The orbit period is about 3 hours, corresponding to 8 revolutions per day or 45 degrees of earth revolution per orbit; 45 degrees/3 degrees per day = 15 days.)

During the 5-year period, large variations in orbit elements are encountered, as much as 60 km in altitude and 3 degrees in inclination. Because of these variations, even if the orbit initially has a 3-hour period, this condition would not long persist and a satisfactory degree of nonresonance would be attained. It was concluded from these considerations that resonant conditions as described above will not exist for any significant length of time and will not be an important problem.

Stability of Selected Orbit

With the nominal launch conditions chosen to be a 4250-kilometer circular orbit at 87-degree inclination, 345-degree right ascension and a date of 1 June 1966, a study was made to determine the orbital stability of the satellite as a function of deviations from the nominal conditions. The Lifetime-18 orbital prediction program was used to determine the orbital stabilities in the study. The time variations of the orbital elements of the nominal orbit have also been plotted in detail and are shown later in this section.

In figures 3 through 6 are shown the variations of maximum eccentricity and the initial period of continuous sunlight with respect to the launch right ascension (time of day). This information is given for 12 launch dates throughout the year (first day of each month) assuming a nominal conditions on the launch altitude, eccentricity, and inclination. As can be seen, the maximum eccentricity curves retain the same general shape for changes in the launch date. The initial periods of continuous sunlight simply slide along the launch right ascension axis.

In figure 7 is shown a cross plot of the information given in figures 3 through 6 for several selected launch right ascensions. As can be seen, in all cases the maximum eccentricity varies nearly sinusoidally with respect to the launch date. This allows a relatively accurate interpolation of this information for launch dates between the actually computed dates. The sinusoid for the 320-degree launch right ascension is seen to be approximately 180 degrees out of phase with respect to the other launch right ascensions. At approximately a launch right ascension of 340 degrees, the curve appears to flatten out and shift its phase relationship. The flattening of the 120-degree curve indicates the approaching of another transition point where the phase relationship will be shifted again.

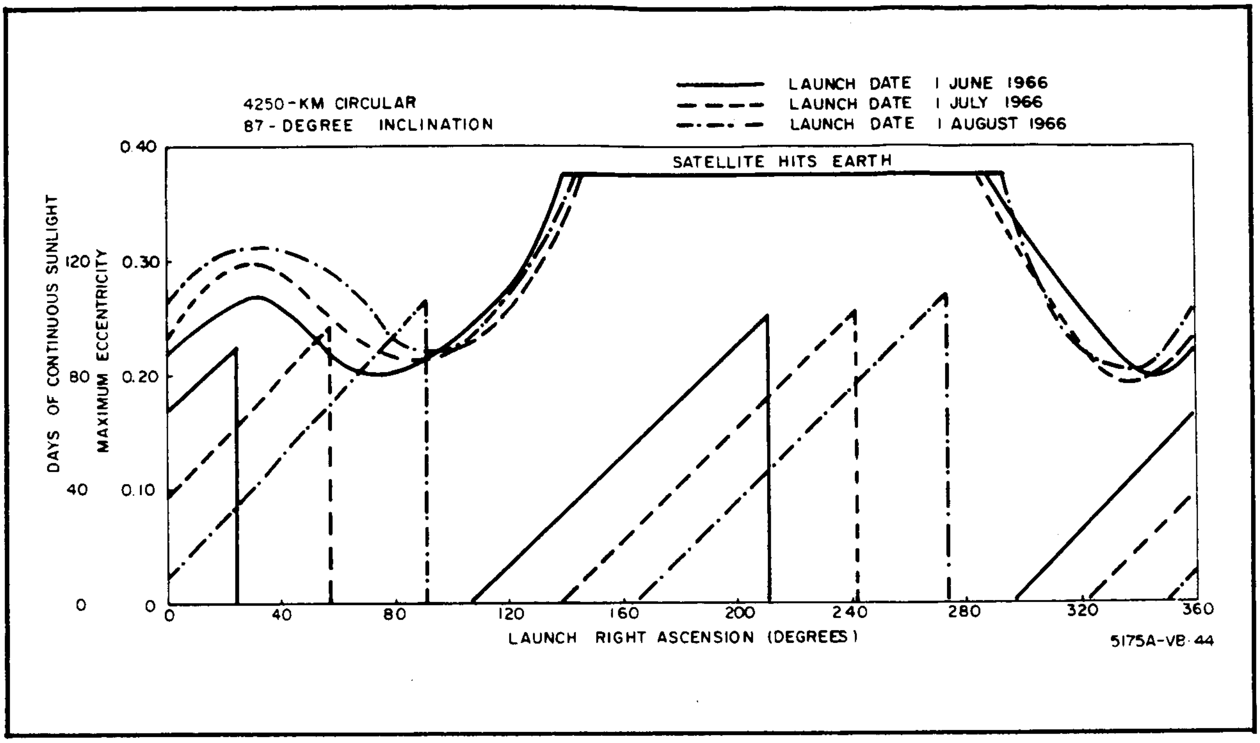


Figure 3. Maximum Eccentricity vs Launch Right Ascension - June, July, August

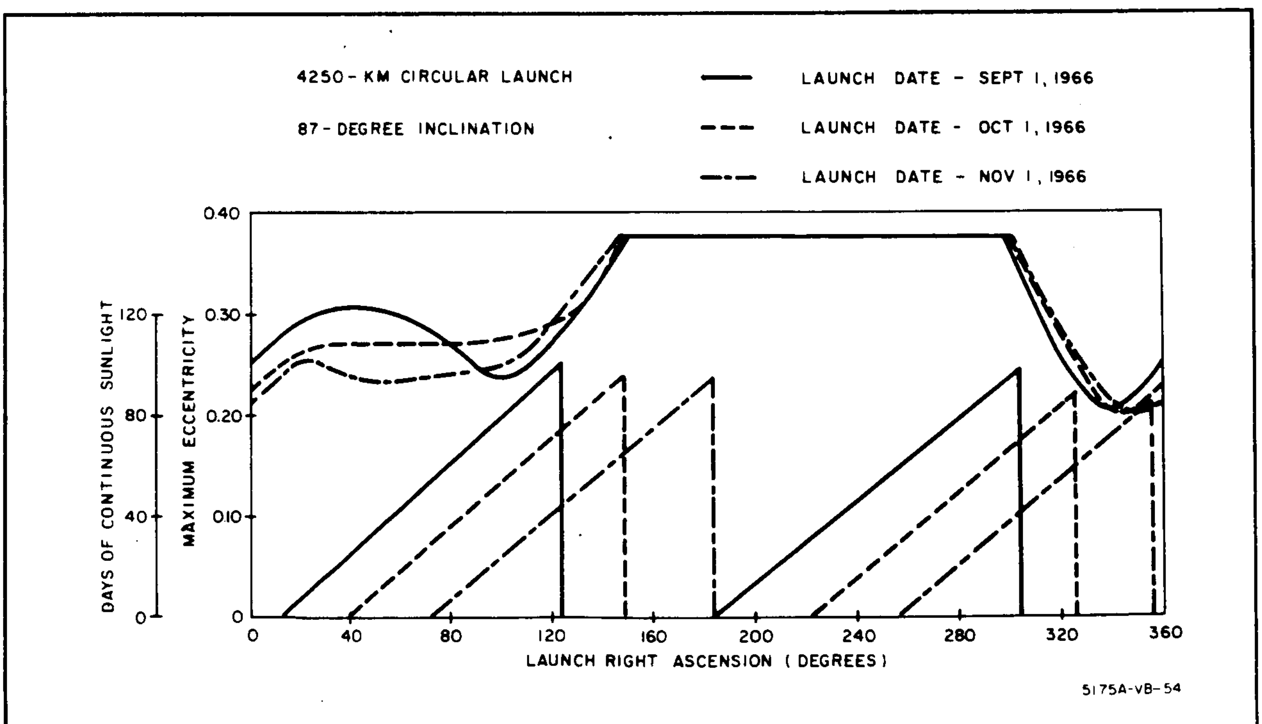


Figure 4. Maximum Eccentricity vs Launch Right Ascension

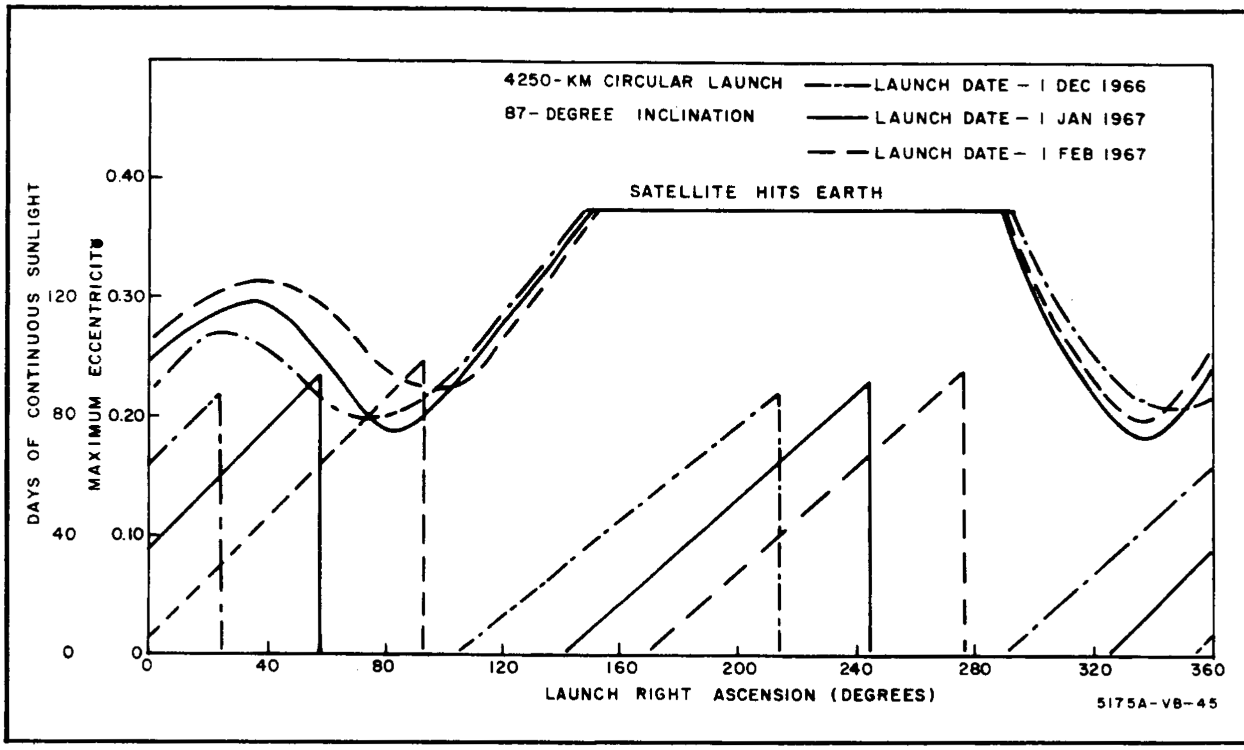


Figure 5. Maximum Eccentricity vs Launch Right Ascension - December, January, February

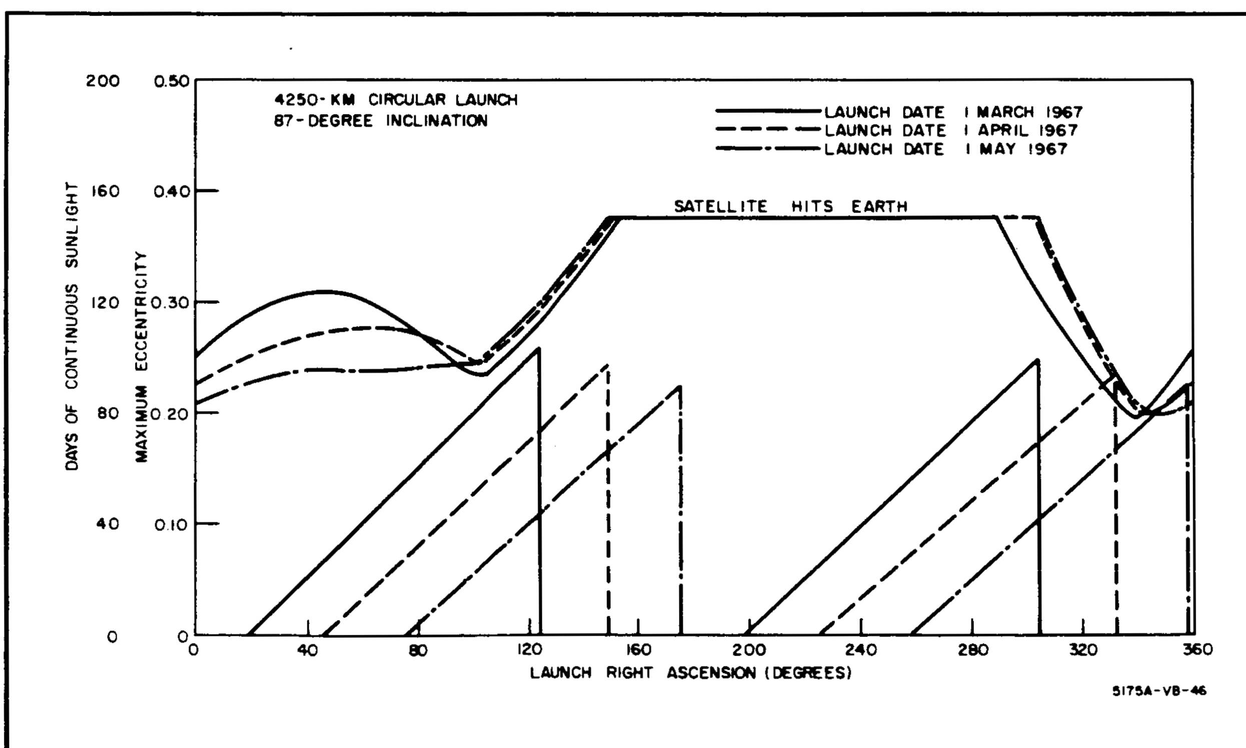


Figure 6. Maximum Eccentricity vs Launch Right Ascension - March, April, May

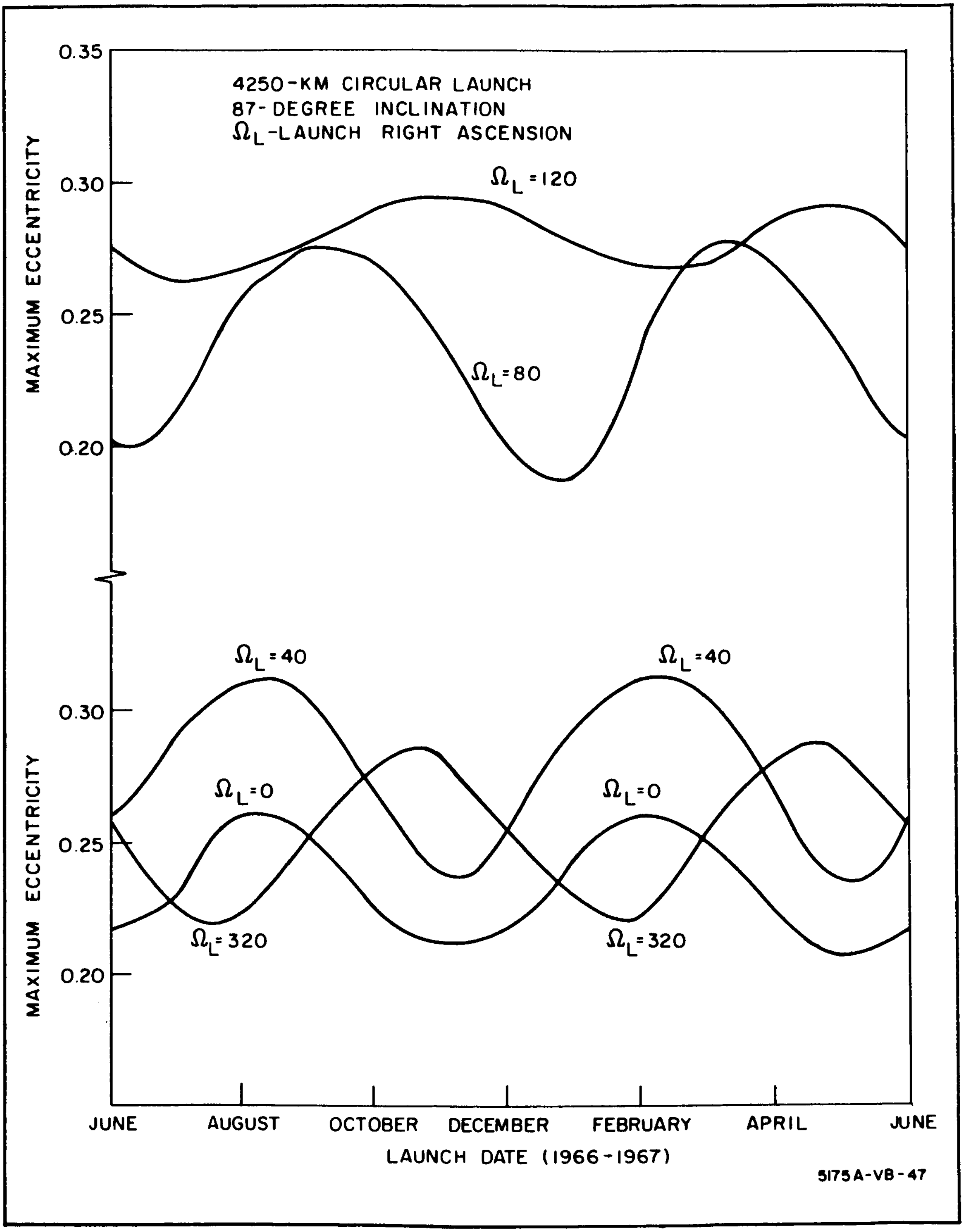


Figure 7. Correlation of Maximum Eccentricity to Launch Date

Figure 8 shows the nominal launch right ascension which should be chosen as a function of the time of year, and the maximum eccentricity associated with that orbit. These right ascensions were chosen from the two considerations of minimum eccentricity and fulfillment of the 14-day continuous sunlight requirement.

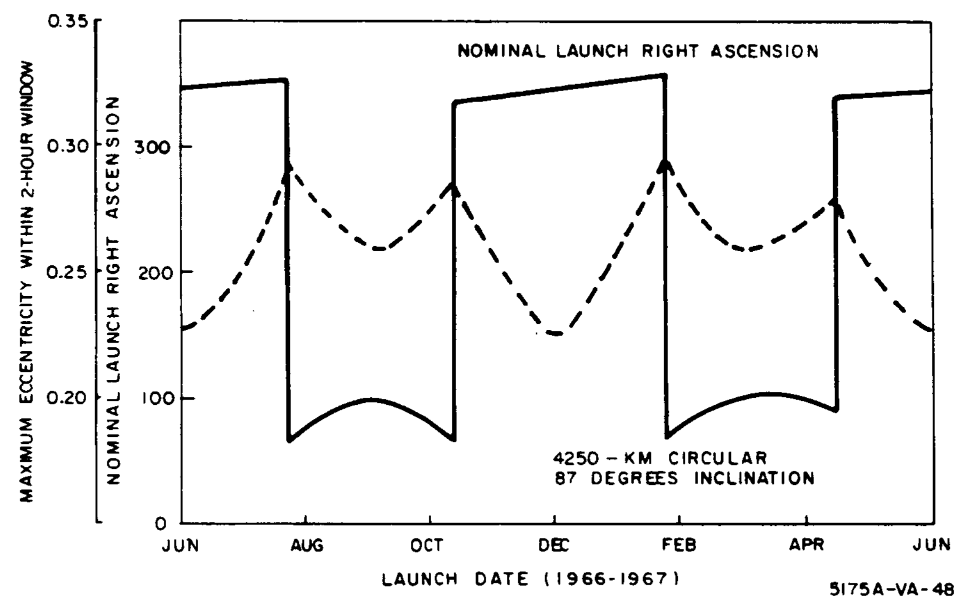


Figure 8. Launch Right Ascension vs Launch Date

Generally, there are two regions of launch right ascension around 340 and 90 degrees where the maximum eccentricity reaches a minimum. In all cases, one or the other of the two regions of continuous sunlight occurs near one of the minimum eccentricity points. Figure 8 was generated, then, by determining the right ascension, which produces the least eccentricity within a 2-hour launch window (30-degree right ascension) and fulfills the continuous sunlight requirement. The points of discontinuity on the curve are points of transition from one sunlight region to the other.

As can be seen, there will be a penalty in terms of eccentricity for launching during certain times of the year. The eccentricity within the 2-hour launch window is seen to vary between about 0.225 and 0.295. The perigee altitude can be calculated by the expression:

$$p = A(1-C) - R_e$$

where A is the semimajor axis of the orbit and R_e is the radius of the earth. For the 4250-kilometer orbit, the perigee altitude will be equal to 2225 kilometers and 1060 kilometers for eccentricities of 0.20 and 0.30, respectively. Considering then, that the Echo I satellite reaches a perigee altitude of approximately 1000 kilometers, even the maximum eccentricity of 0.295 is not considered to seriously affect the lifetime of the satellite. The previously shown results of the station time sampled run for a 1 October launch date indicates that this added eccentricity will not seriously degrade the number of available observations.

In figure 9 are shown the effects of other launch errors on the maximum eccentricity of the orbit. Four different arguments of perigee were examined for a launch eccentricity of 0.02 (over twice the 3-sigma limit). As can be seen, the initial eccentricity is either added or subtracted from the nominal eccentricity depending upon the initial orientation of the perigee. In the case of the altitude error, an error on the low side by 100 kilometers is seen to increase the maximum eccentricity by approximately 0.03, while a 100-kilometer error on the high side does not affect the eccentricity appreciably. For the inclination error, the maximum eccentricity is seen to increase by nearly 0.02 per degree of inclination between 86- and 88-degree inclination. In all cases, the errors studied appear to be well outside the 3-sigma limit on the launch conditions and even for these extreme errors the eccentricity does not increase enough to degrade either the lifetime of the satellite or the number of available observations.

In figures 10 through 13 are shown the actual variations of the orbital elements as a function of time for an orbit with the nominal launch conditions. As can be seen, the altitude varies a total of 60-kilometers and the inclination a total of 3 degrees over the 5-year period of interest.

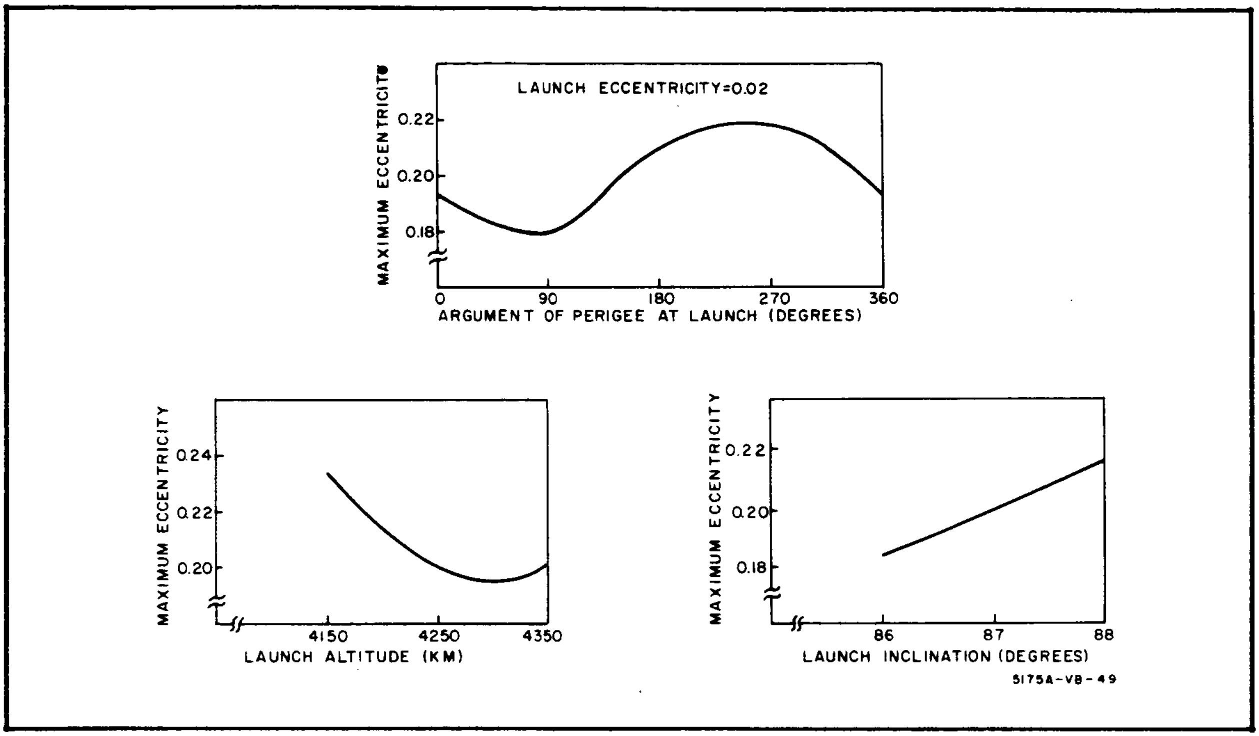


Figure 9. Launch Error Effects on Maximum Eccentricity

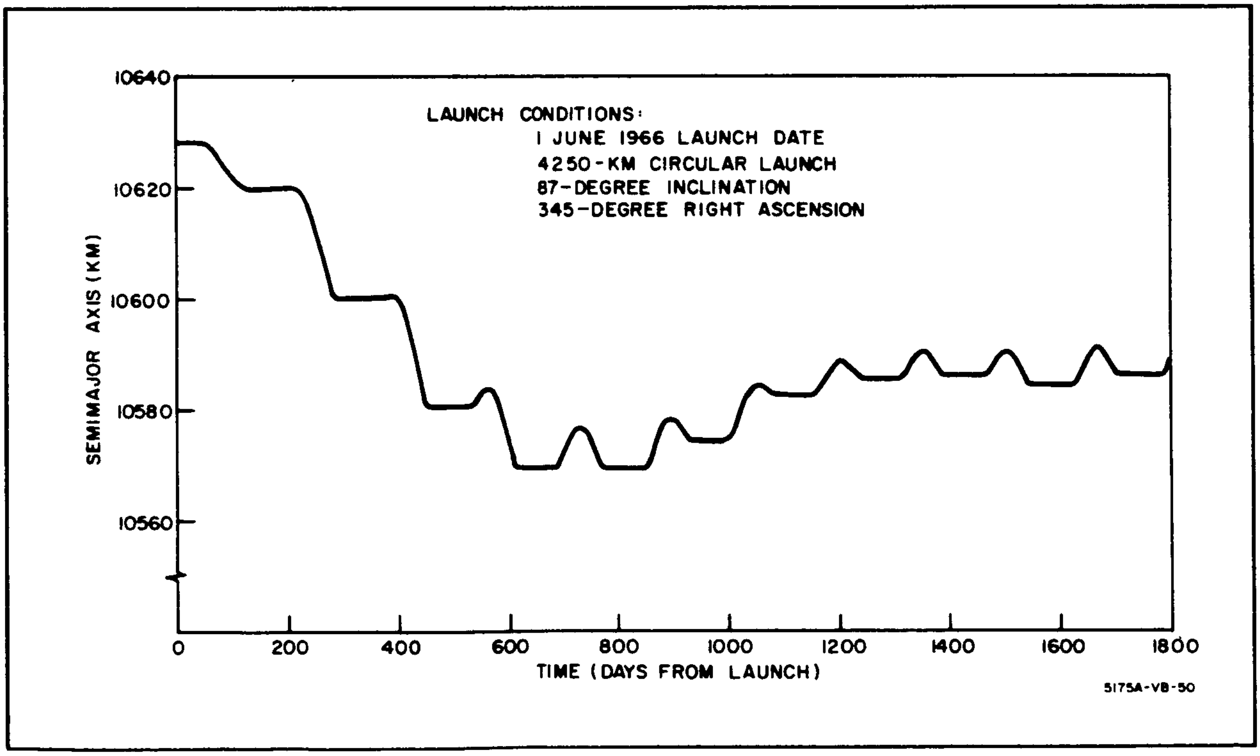


Figure 10. Semimajor Axis of Nominal Orbit as a Function of Time

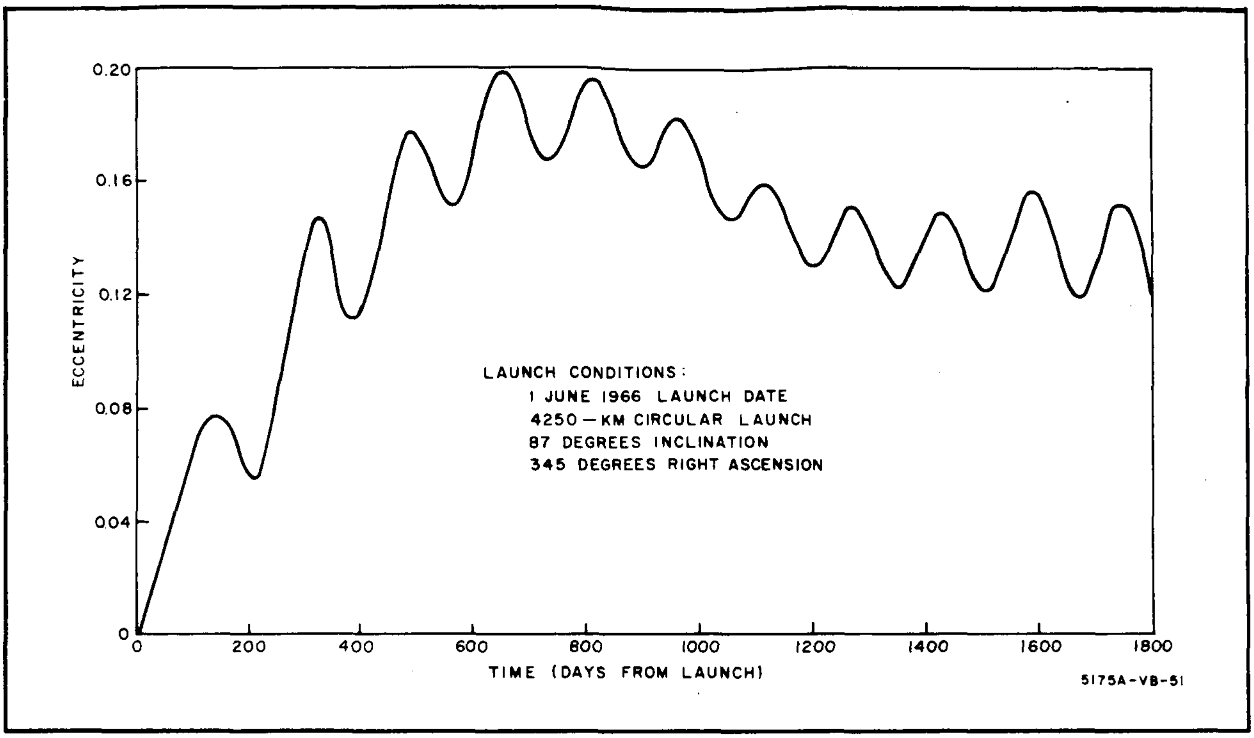


Figure 11. Eccentricity of Nominal Orbit as a Function of Time

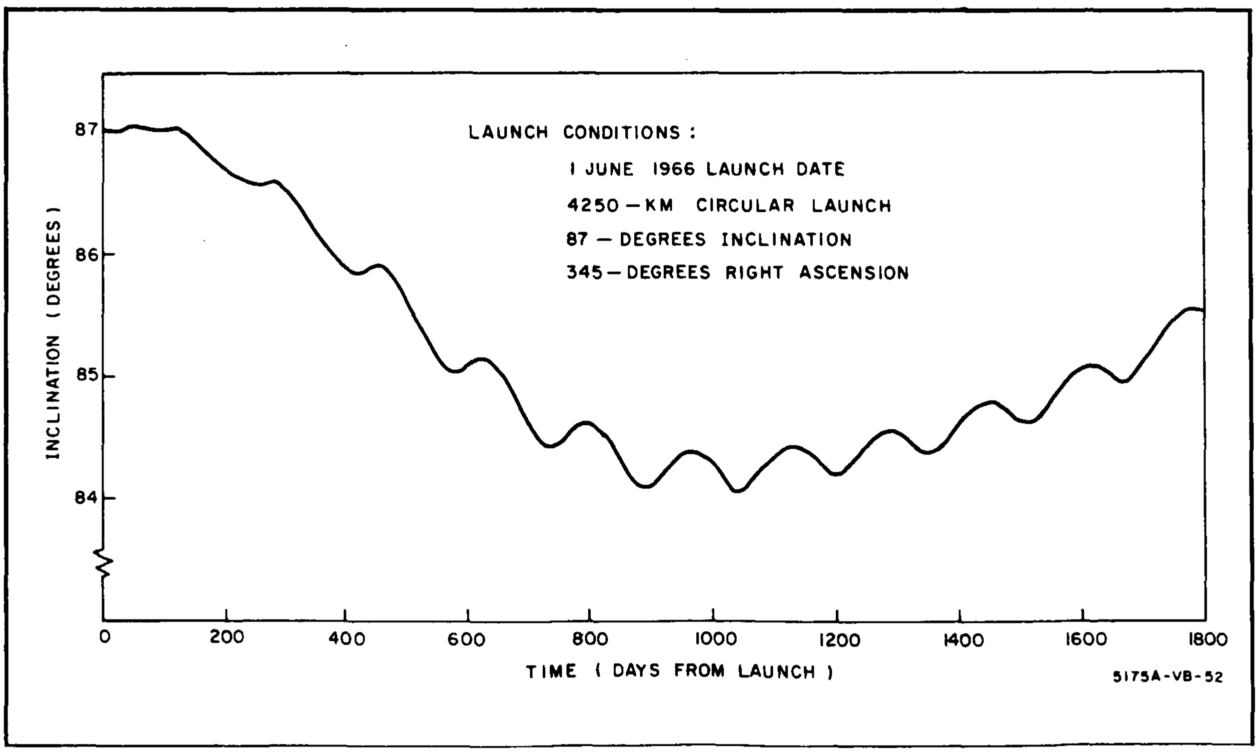


Figure 12. Inclination of Nominal Orbit as a Function of Time

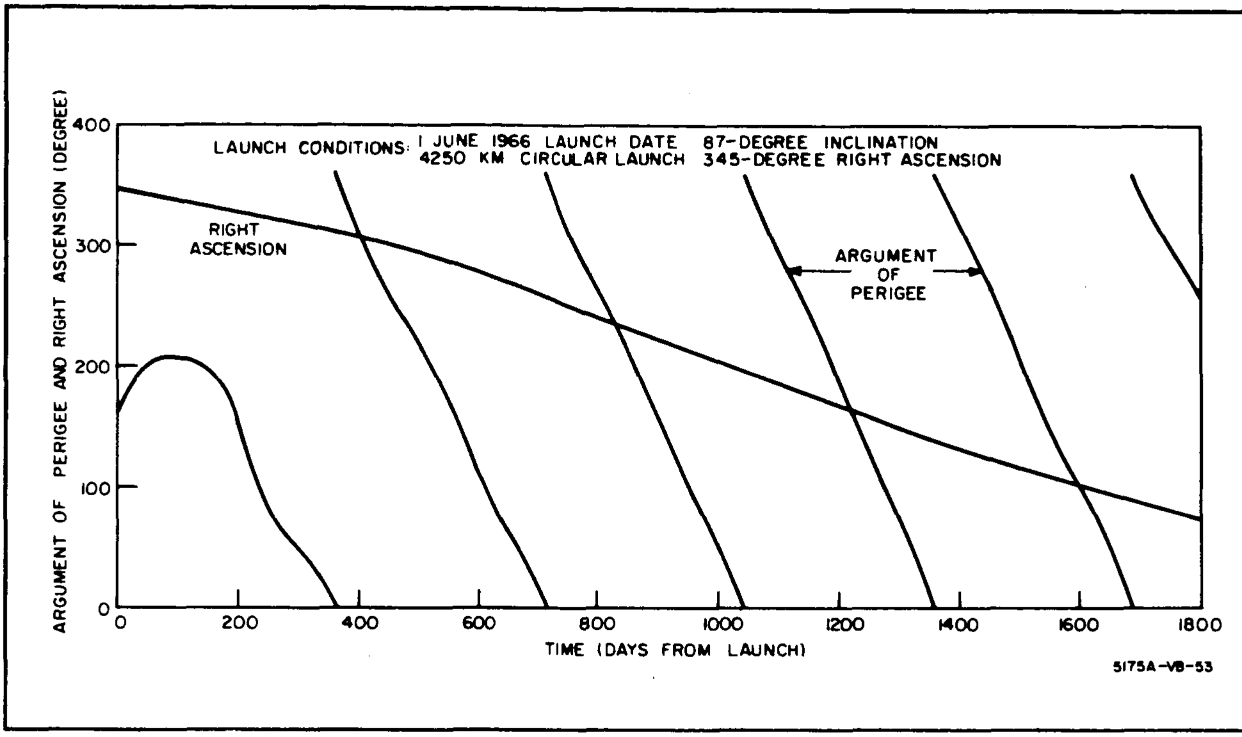


Figure 13. Argument of Perigee and Right Ascension of Nominal Orbit as a Function of Time

Validity of Sampling

Several tables were prepared to check the validity of the sampling techniques employed in the abbreviated runs. Tables VII and VIII present the results of investigating the time sampling procedure. Considering only the four interior baselines and the five triangles of the 7-station problem, results obtained from the sampled run using orbit No. 1 are compared with those obtained from the first 3 years of the full 5-year run (through interval 1510). In these tables, the results of the sampled runs were multiplied by 3, since the sampling interval was 1 month in 3 and were listed in the rows marked S. The corresponding results from the complete run were listed below in the rows marked T, the results being summed over the appropriate 3-month intervals. A fair degree of correspondence may be noted on comparing the total numbers of good observations obtained, although the

TABLE VII

BASELINE RESULTS

Baselines		End of Time Interval in Days From 1 January 1965: 610	700	790	880	970	1060	1150	1240	1330	1420	1510	Total	1/3 Total	1st Year	2nd Year	3rd Year
17-18	G	6		24	3	15	21				33	33	135		33	36	66
	M	3		12	3	21		6			9		54		18	30	9
(2-3)	+	6		28		6					8		39		34	6	8
S	-				6		15					15	36		6	15	15
17-18	G	6	12	25	4	6	20	10	7	19		2	111		47	43	21
	M	1		7	1		2	5	3	1	1	7	28		9	10	9
(2-3)	+	2		10	1	1	1			3		4	22		13	2	7
T	-		9	2	5	1	10	4	4	12			47		16	19	12
18-27	G				48			6				6	60		48	6	6
	M		3										3		3		
(3-4)	+				12								12		12		
S	-				48								48		48		
18-27	G		2	4	15			6					27		21	6	
	M	2									2		4		2		2
(3-4)	+				3								3		3		
T	-		1	3	14			4					22		18	4	
27-28	G	15	3		36								54		54		
	M	6	3										9		9		
(4-5)	+	9											9		9		
S	-				27								27		27		
27-28	G	6		2	15	6	1		12				42		23	19	
	M	2		1	5				10	6	13		37		8	10	19
(4-5)	+	4			1				1				6		5	1	
T	-			2	9								11		11		
28-35	G	108	27			66				75			276		135	66	75
	M					4							4			4	
(5-7)	+																
S	-	177	33			132				69			411		210	132	69
28-35	G	36	7		48	42	7		50	44			234		91	99	44
	M				1	1							2		1	1	
(5-7)	+	1				3			1	6			11		1	4	6
T	-	66	8		80	45	7		70	41			317		154	122	41
					Total Good Observations				Sampled Run				525	175	270	108	147
									Total Run				414	138	182	167	65

TABLE VIII

TRIANGLE RESULTS

Triangles and Baselines		End of Time Interval in Days From 1 January 1965															
		610	700	790	880	970	1060	1150	1240	1330	1420	1510	Total	1/3 Total	1st Year	2nd Year	3rd Year
8-17-18	G				3		21						24		3	21	
	M					3	3					3	9			6	3
(1-2-3)	+				3		18						21		3	18	
	-						9						9			9	
	+				3		21						24		3	21	
	-																
	+																
S	-				6		27						33		6	27	
8-17-18	G	6	3	1	1			5					16		11	5	
	M	2			1	2				4			9		3	2	4
(1-2-3)	+	3	2		1								6		6		
	-	2	2	1				3					8		5	3	
	+	8	3	1	1			3					16		13	3	
	-																
	+																
T	-	8	3	1	1			7					20		13	7	
18-27-28	G		15										15		15		
(3-4-5)	M		9										9		9		
	+																
	-		15										15		15		
	+		12										12		12		
	-																
	+																
S	-		9										9		9		
18-27-28	G	6	7				1						14		13	1	
(3-4-5)	M	2	2			2	5						11		4	7	
	+	2											2		2		
	-	5	6				1						12		11	1	
	+	8	3				2						13		11	2	
	-																
	+																
T	-	4	8				1						13		12	1	
28-29-35	G	117				102				9			228		117	102	9
(5-6-7)	M	3											3		3		
	+	192				189				9			390		192	189	9
	-	3											3		3		
	+	9				24							33		9	24	
	-	174				105				12			291		174	105	12
	+					12				3			15			12	3
S	-																
28-29-35	G	82			27	65			9	10			193		109	74	10
(5-6-7)	M	2				5							7		2	5	
	+	29				12							41		29	12	
	-	101			37	74			9	11			232		138	83	11
	+	142			38	113			18	14			325		180	131	14
	-	1											1		1		
	+	9				9							18		9	9	
T	-	3								4			7		3		4
Total Good Observations					Sampled Run								267	89	133	123	9
					Total Run								223	74	135	80	10

correlations are not always high in the individual intervals. Several reasons for this were suggested by examination of the results and the computer output data. First, in the single station observation program, a fictitious satellite position is introduced, since Lifetime-18 does not integrate satellite position. These fictitious positions tended to drift apart in the two runs (the sampled run and the complete run), and as a result of this drift the single station and multiple station observation tables were no longer in agreement on a month-to-month basis. This difference should more or less average out over long time periods, and is of questionable significance since the fictitious satellite positions have no real physical meaning at all. Secondly, the sampled intervals always occurred in June, September, December, and March, and thus the large numbers of observations obtained near the poles during the solstices, when multipled by the factor three, tended to bias the sampled results high as compared with the results of the complete run. Since this factor would affect all sampled runs approximately equally, its effect on the comparison of the various orbits is diminished.

Table IX lists results applicable to the investigation of the validity of the 7-station sample. The original 7-station group was labeled group A, while two more 7-station groups were chosen for comparison. These were stations 1, 3, 4, 10, 11, 20, and 21 (group B) and stations 9, 10, 19, 20, 29, 30, 36 (group C). Using the results of the complete 5-year run, the total number of good baseline observations through interval 1510 were calculated for these three groups and for the complete 36-station network. Corresponding averages were found by dividing by the number of baselines. Again, fair correspondence may be noted, although not as good as might be wished.

From the above comparisons it may be concluded that although the sampling may have exaggerated the variations between orbits, the comparison of orbits on the basis of the abbreviated problem is probably valid. However, the possibility is not eliminated that a comparison on the basis of complete runs might have led to a somewhat different ranking of the top few orbits.

TABLE IX

7-STATION SAMPLE RESULTS

	Total Good Observations **	Average Good Observations ** Per Baseline
7 Station Samples *		
Group A	1078	99
Group B	932	85
Group C	789	72
All Stations *	8934	88

* Data obtained from 5-year run
** Through time interval 1510

APPENDIX I

RELATION BETWEEN LOCAL TIME AND LAUNCH RIGHT ASCENSION

The solar time at a point on the earth's surface is the angle from the plane containing the sun and the earth's axis, eastward to the meridian plane of the point; it is AM if the angle is measured from the shadow side of the sun plane and PM if measured from the bright side. This relationship is illustrated in figure I-1. The local standard time depends on the longitude difference between the point and the standard time meridian for the applicable time zone; for pacific standard time this meridian is 120 degrees west and the local solar time is 4 minutes slower than pacific standard time for each additional degree of westward longitude.

To obtain a launch right ascension of Ω, the right ascension of the launch point is approximately $\Omega + \Delta\Omega$, where $\Delta\Omega \approx (\frac{\pi}{2} - i) \tan \phi$ for highly inclined orbits ($i \geq 85°$). The local solar time is then $(\Omega + \Delta\Omega - \theta_s - 180°)$ for the geometry illustrated. For launch from Vandenburg AFB ($\phi \approx 34°$ N, $\lambda \approx 120.6°$W) at an

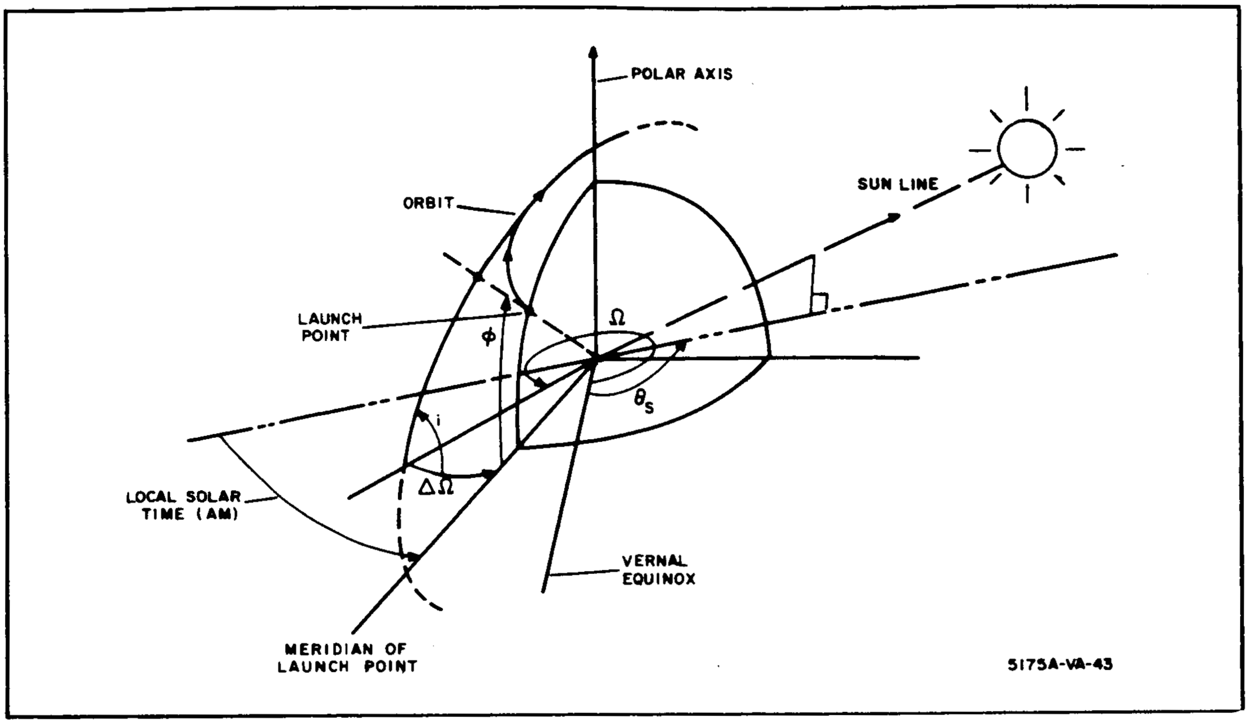


Figure I-1. Local Time/Launch Right Ascension Geometry

inclination of 87°, $\Delta\Omega \approx 2.0°$ and local solar time lags pacific standard time by 2.4 minutes. For a June 1 launch with $\Omega = 345°$, the pacific standard time at launch is thus approximately

$$\text{PST} \approx 2.4 \text{ minutes} + \frac{1}{15} \left(345 + 2 - \frac{71}{91} \times 90 - 180\right) \text{ hours}$$

$$\text{PST} \approx 2.4 \text{ minutes} + 6.46 \text{ hours}$$

PST 6:30.0 AM

Similarly, the approximate launch times of other orbits can be found.