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EXPERIMENTS AT 40 MEV

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ABSTRACT

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Recently, qualitatively different phase-shift analyses of the 40 MeV proton-alpha elastic scattering and polarization data have appeared in the literature. The question arises whether further experimental data can decide in favor of one or the other of these analyses. Two classes of experiments, measurement of the triple scattering parameters R and A and measurement of the polarization P in the region of the Coulomb interference scattering cross-section minimum, have been examined to see if these provide additional information to resolve the ambiguity. We conclude that a measurement of the polarization near the Coulomb interference minimum would serve to establish on a purely experimental basis the complete scattering amplitude in the forward direction. This information would remove the ambiguity in the phase-shift analysis

*Consultant to Lewis Research Center; work supported by U.S. Atomic Energy Commission.

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of this data. No further information could be gained from triple scattering experiments unless these experiments were of very high precision.

Author

1. INTRODUCTION

Recently, several different phase shift analyses[1,2] of the 40 MeV proton-alpha scattering[3] and polarization[4] data have appeared in the literature. The present authors have also produced a number of sets of phase shifts that fit this data equally well. It is thus self-evident that there does not exist a unique phase-shift analysis for this data. Furthermore, a brief glance at the scattering amplitudes themselves assures us that this lack of uniqueness does not result from any inherent ambiguity in the expression of the scattering amplitude in terms of phase shifts. The scattering amplitudes themselves differ appreciably.

The question then arises whether this ambiguity can be resolved with the aid of further experimental data. Wolfenstein[5] has shown that in principle measurement of the rotation of polarization can provide additional information. Such experiments are, of course, exceedingly difficult. We therefore wish to examine in advance what might be learned from such an experiment in the case of the scattering of protons by alpha particles at about 40 MeV.

2. PROCEDURE

Consider a 100-percent polarized beam of spin 1/2 particles, with the polarization vector normal to the incident beam and in the plane of scattering as shown in fig. 1. If a scattering experiment were to be performed with such an initial beam, the polarization of the scattered beam could be measured. In such an experiment the component of polarization of the scattered beam in the plane of scattering and normal to the scattering direction is the rotation of polarization parameter R. The sign of R is purely conventional. We use the convention

$$\underset{\sim}{R} = \langle \underset{\sim}{\sigma} \rangle \cdot \underset{\sim}{S} \tag{1}$$

where

$$\underset{\sim}{S} = \frac{(\underset{\sim}{k}_i \times \underset{\sim}{k}_f) \times \underset{\sim}{k}_f}{|(\underset{\sim}{k}_i \times \underset{\sim}{k}_f) \times \underset{\sim}{k}_f|} \tag{2}$$

as shown in fig. 1, which is drawn in the manner of Wolfenstein[5].

A straightforward calculation yields the standard result[5]

$$R = (1 - P^2)^{1/2} \cos (\beta - \theta_{lab}) \tag{3}$$

where θ_{lab} is the laboratory scattering angle, and the parameters P and β are given by the relations

$$\left.\begin{aligned}
P(\theta) &= \frac{2\mathrm{Re}\left[g^*(\theta)h(\theta)\right]}{\left[|g(\theta)|^2 + |h(\theta)|^2\right]} \\
\cos\beta &= \frac{|g(\theta)|^2 - |h(\theta)|^2}{\left[|g(\theta)|^2 + |h(\theta)|^2\right]\left[1 - P(\theta)^2\right]^{1/2}} \\
\sin\beta &= \frac{2\mathrm{Im}\left[g(\theta)h^*(\theta)\right]}{\left[|g(\theta)|^2 + |h(\theta)|^2\right]\left[1 - P(\theta)^2\right]^{1/2}}
\end{aligned}\right\} \tag{4}$$

where θ is the center-of-mass scattering angle. The quantities $g(\theta)$ and $h(\theta)$ are the spin-independent and spin-dependent scattering amplitudes, respectively. In terms of phase shifts, $g(\theta)$ and $h(\theta)$ can be expressed as

$$g(\theta) = f_{coul} + \frac{1}{k}\sum_{l} e^{2i\sigma_l}\left[(l+1)e^{i\delta_l^+}\sin\delta_l^+ + l e^{i\delta_l^-}\sin\delta_l^-\right]P_l(\cos\theta) \tag{5}$$

$$h(\theta) = \frac{1}{2k}\sum_{l} e^{2i\sigma_l}\left(e^{2i\delta_l^+} - e^{2i\delta_l^-}\right)\sin\theta\,\frac{dP_l(\cos\theta)}{d(\cos\theta)} \tag{6}$$

where f_{coul} is the Coulomb scattering amplitude in the absence of the nuclear force, σ_l is the Coulomb phase shift, and δ_l^+ and δ_l^- are the nuclear phase shifts for $J = l + \frac{1}{2}$ and $J = l - \frac{1}{2}$, respectively.

We may substitute the phase shifts generated by Suwa and Yokosawa[1] (SY-A) into the previous expressions, calculate R and compare the results with those of reference 2 (GMT) and with those of the present authors (GMT-B). Three sets of phase shifts are presented in table 1. The results for β and R are plotted for two different cases in fig. 2 and for three different cases in fig. 3, respectively.

The triple scattering experiments are so difficult to perform that we shall restrict our attention to the forward angles, since the cross section falls very rapidly with increasing angle as shown in fig. 4. Very forward angles are essentially pure Coulomb scattering and hence yield no information. From fig. 2, we see that β is nearly a linear function of θ for values of θ from about 10° to 50°. It will be shown that this linearity is theoretically necessary. Hence any measurement in the linear region is equivalent to any other. This implies that the measurements should be made as far forward as possible in order to maximize the number of events that can be observed. In the rotation experiment, however, this is the region where the value of R is least sensitive to the differences in β. For example, a 20-percent experiment at $\theta = 30^{\circ}$ could not distinguish between the two cases plotted in fig. 3.

Ignoring Coulomb effects, we may estimate $g(\theta)$ and $h(\theta)$ to first order in θ to be

$$g(\theta) = \frac{1}{k} \sum_{l} \left[(l+1) e^{i\delta_l^+} \sin \delta_l^+ + l e^{i\delta_l^-} \sin \delta_l^- \right] + O(\theta^2) = g(\theta = 0) + O(\theta^2) \tag{7}$$

and

$$h(\theta) = \frac{1}{2k} \sum_{l} \left(e^{2i\delta_l^+} - e^{2i\delta_l^-} \right) \theta P_l'(\theta = 0) + O(\theta^3) = \left(\frac{dh}{d\theta} \right)_{\theta=0} \theta + O(\theta^3) \tag{8}$$

The quantities $g(\theta = 0)$ and $\left[\frac{dh(\theta)}{d\theta} \right]_{\theta=0}$ calculated according to eqs. (7) and (8) are also given in table 2 where they are listed as $|G_0| e^{i\gamma}$ and $|H_0| e^{i\lambda}$, respectively. When eqs. (7) and (8) are applied to eq. (6) the following form for β is obtained:

$$\beta = \left(\frac{d\beta}{d\theta} \right)_{\theta=0} \theta + O(\theta^3) \tag{9}$$

This accounts for the nearly linear region in fig. 2. In this region, for the two cases plotted in fig. 2, the ratio of the calculated values of β is

$$\frac{\beta_{GMT}}{\beta_{SY\text{-}A}} = \frac{-0.81\theta}{-0.15\theta} = 5.4 \tag{10}$$

We see, however, from eq. (3) that this large difference is somewhat masked by the fact that the rotation of polarization depends on β through the factor $\cos(\beta - \theta_{lab})$.

Other triple scattering experiments are possible. In the case at hand where we are concerned with the scattering of spin 1/2 particles from spin zero targets, these experiments are not useful for obtaining additional information. The so-called depolarization parameter is always unity and is, hence, of no interest. The triple scattering parameters R' and A depend on $P(\theta)$, $\beta(\theta)$, and θ_{lab} according to the relations

$$A = \left[1 - P(\theta)^2\right]^{1/2} \sin(\beta - \theta_{lab}) = - R' \tag{11}$$

These experiments are inherently more difficult to perform than the R experiment since they involve the rotation of the spin vector by 90° in a magnetic field. Thus the A and R' experiments become interesting only if β can be determined in no other way.

The meaning of the quantities R' and A is discussed by Wolfenstein[5] and is illustrated in fig. 1. The values of A calculated for the various sets of phase shifts under discussion are illustrated in the two most different cases in fig. 5.

Both R and R' depend on $\beta(\theta)$ in the combination $\beta - \theta_{lab}$. For fairly small angles $\beta(\theta) \approx \frac{d\beta}{d\theta}(\theta = 0)\theta$ and $\theta_{lab} \approx 0.8\theta$, so that $\beta(\theta) - \theta_{lab} \approx \left[\frac{d\beta(0)}{d\theta} - 0.8\right]\theta$. For the two cases we have chosen as illustrations, the values of $d\beta(0)/d\theta$ appear in eq. 10. For these values

$$\beta_{GMT}(\theta) - \theta_{lab} \approx - 1.61\theta \tag{12}$$

and

$$\beta_{SY\text{-}A}(\theta) - \theta_{lab} \approx - 0.95\theta \tag{13}$$

so that

$$\frac{R_{GMT}}{R_{SY\text{-}A}} \approx 1 - \frac{1}{2}(1.61^2 - 0.95^2)\theta^2 = 1 - 0.84\theta^2 \tag{14}$$

and

$$\frac{R'_{GMT}}{R'_{SY-A}} \approx \frac{1.61}{0.95} = 1.7 \tag{15}$$

Neither difference could be seen for angles less than 30° in a 20-percent experiment.

The information that could have been gained by high-precision triple scattering experiments in the forward cone can in actual fact be attained more readily through a double scattering experiment in the region of the cross-section dip due to interference between the coulomb and the nuclear scattering amplitudes. In a forward cone, which includes the Coulomb interference region, the spin-independent amplitude is nearly constant, while the spin-dependent amplitude is nearly a constant times θ. If the Coulomb terms are included, eqs. (7) and (8) become

$$\begin{aligned} g(\theta) &= g_{coul}(\theta) + g_{nuc}(\theta) \approx g_{coul}(\theta) + g_{nuc}(\theta) \\ &= \frac{\eta}{2k\ \sin^2(\theta)}\ e^{2i\left[\sigma_0 - \eta\, ln(\sin\ \theta/2) + (\pi/2)\right]} \\ &\qquad + \frac{1}{k}\sum_l e^{2i\sigma_l}\left[(l+1)e^{i\delta_l^+}\ \sin\ \delta_l^+ + l e^{i\delta_l^-}\ \sin\ \delta_l^-\right] \\ &\equiv g_{coul}(\theta) + |G_0|\, e^{i\gamma} \equiv |g(\theta)|\, e^{i\Gamma(\theta)} \end{aligned} \tag{16}$$

and

$$\begin{aligned} h(\theta) &\approx \left(\frac{dh}{d\theta}\right)_{\theta=0}\theta = \frac{1}{2k}\sum_l e^{2i\sigma_l}\left(e^{2i\delta_l^+} - e^{2i\delta_l^-}\right)P_l'(\theta = 0)\theta \\ &\equiv |H_0|\, e^{i\lambda}\theta \end{aligned} \tag{17}$$

To lowest order in

$$\sigma(\theta) \approx \sigma_{coul} + 2|G_0| \mathrm{Re}\left[e^{-i\gamma} g_{coul}(\theta)\right] + |G_0|^2 \tag{18}$$

Examination of the experimentally determined angular distribution in the vicinity of the Coulomb interference minimum can now serve to identify $|G_0|$ and γ. When using the data at 6° and at 9°, we find

$$|G_0| = 4.9 \tag{19}$$

and

$$\gamma = 59^\circ \tag{20}$$

The calculated values for $|G_0|$ and γ for the scattering amplitudes in the present discussion appear in table 2.

If we were able to determine $\left[\frac{\partial h(\theta)}{\partial \theta}\right]_{\theta=0}$ in a similar fashion, we would have all the information necessary to give us the complete scattering amplitude in the forward cone. Polarization data in the Coulomb interference region can give us this information. For small angles the polarization is

$$P(\theta) = \frac{2\mathrm{Re}\left[g^*(\theta)h(\theta)\right]}{|g(\theta)|^2 + |h(\theta)|^2} \approx \frac{2|H_0| \cos\left[\Gamma(\theta) - \lambda\right]}{\sqrt{\sigma(\theta)}}\,\theta \tag{21}$$

where $|g(\theta)|$ and $\Gamma(\theta)$ are as defined in eqs. (16), (19), and (20). An experimental knowledge of the polarization at two angles in the Coulomb interference region will then serve to determine $|H_0|$ and λ.

The polarization for the cases under discussion is shown in fig. 6. The GMT and GMT-B cases differ significantly from the SY-A case in the forward cone.

Let us suppose that we had polarization data at 8° and 16°. The values of $\Gamma(\theta)$ are

$$\Gamma(\theta = 8^\circ) = 125^\circ \tag{22}$$

and

$$\Gamma(\theta = 16^{\circ}) = 72^{\circ} \tag{23}$$

The values of $\sqrt{\sigma(\theta)} = |g(\theta)|$, taken from ref. 3, are

$$g(\theta = 8^{\circ}) = 3.8 \text{ f} \tag{24}$$

$$g(\theta = 16^{\circ}) = 4.2 \text{ f} \tag{25}$$

They give

$$P(\theta = 8^{\circ}) = 0.073|H_0| \sin\left(125^{\circ} - \lambda + \frac{\pi}{2}\right) \tag{26}$$

and

$$P(\theta = 16^{\circ}) = 0.132|H_0| \sin\left(72^{\circ} - \lambda + \frac{\pi}{2}\right) \tag{27}$$

The values of $|H_0|$ and λ for the three illustrative cases under discussion appear in table 2. From that table one may see that the principal qualitative difference between the SY-A and the GMT results is that the SY-A value for $|H_0|$ is small. Thus, no matter what the phase of the spin-dependent forward scattering amplitude, a small polarization may be expected everywhere in the forward cone.

On the other hand, the GMT value for $|H_0|$ is large. The existing polarization data in the forward cone indicates that the polarization is small beyond the Coulomb interference region. If $|H_0|$ is large, this can come about only if the spin-dependent amplitude is $\sim \pi/2$ out of phase with the spin-independent amplitude. This is, in fact, true for all cases under consideration. From table 2 we see that $0 < \left(\gamma - \lambda + \frac{\pi}{2}\right) < 0.2$ for all cases under discussion. If $|H_0|$ is sufficiently large, however, substantial polarization in the Coulomb interference region may be expected. At the Coulomb interference minimum ($\theta = 8^{\circ}$) the phase of the spin-independent scattering amplitude is 125°. If the phase λ of the spin-dependent amplitude were such that $\left(\Gamma - \lambda + \frac{\pi}{2}\right)$ turned out to be near $\pi/2$ in the Coulomb interference region, then a large polarization could result. From eq. (26) we see that for $|H_0| \sim 2.5$ f a maximum polarization at $\theta = 8^{\circ}$ of 18 percent could result for $\Gamma = \lambda$. For $|H_0| \sim 0.4$ f there can be a maximum polarization of no more than 3 percent.

3. CONCLUSION

In view of the previous arguments, we may conclude that a measurement of the polarization in the vicinity of the Coulomb interference minimum in proton-alpha scattering at 40 MeV will serve to establish on a purely experimental basis the complete scattering amplitude in the forward direction. Furthermore, this additional information would serve to remove the ambiguity in the complete phase shift analysis of this data. No further information can be gained through the more difficult triple scattering experiments unless these were of high precision.

ACKNOWLEDGEMENT

We wish to thank Mrs. D. V. Renkel for her efficient work in carrying out the numerical computations.

TABLE 1. - PHASE SHIFTS THAT FIT PROTON-ALPHA ELASTIC SCATTERING AND POLARIZATION DATA AT APPROXIMATELY 40 MeV

Angular momentum quantum number, l	Real		Imaginary		Predicted reaction cross section, fermis^2	Phase shift from ref.
	δ_l^+	δ_l^-	δ_l^+	δ_l^-		
0	1.289	------	0.0877	-----	11.6	1 (SY-A)
1	1.318	0.8835	0.160	0.023		
2	0.397	0.228	0.0554	0.140		
3	0.209	0.186	0.0599	0.065		
0	0.987	------			0	2 (GMT)
1	1.144	0.572				
2	0.432	0.156				
3	0.233	0.115				
4	0.075	0.001				
0	1.161	------	0.091	-----	10.3	Present work (GMT-B)
1	1.299	0.5897	0.029	0.026		
2	0.416	0.1142	0.143	0.068		
3	0.224	0.1237	0.082	0.030		
4	0.062	0.005				
5	0.026	0.016				

TABLE 2. - VALUES OF SCATTERING AMPLITUDES AS DEFINED IN EQS. (16) AND (17). FIRST THREE COLUMNS CONTAIN SCATTERING AMPLITUDES AS CALCULATED USING APPROXIMATIONS TO LOWEST ORDER IN θ. VALUES OF σ AND P USED TO MAKE THESE APPROXIMATIONS ARE COMPUTED EXACTLY FROM PHASE SHIFTS OF TABLE 1. FOURTH COLUMN LISTS SCATTERING AMPLITUDES AS COMPUTED FROM CROSS-SECTION DATA OF REF. 3

	Calculated phase shift			Data at 6° and 9°
	SY-A	GMT	GMT-B	
$\lvert G_0 \rvert$	5.46	5.54	5.48	4.9
γ	56°	40.2°	53.2	59°
$\lvert H_0 \rvert$	0.42	2.24	2.64	------
λ	144.3°	129.3°	132.6°	------
$\gamma - \lambda + \frac{\pi}{2}$	1.7°	0.9°	10.6°	------
$g(8^\circ)$	3.84	2.57	3.62	3.8
$\Gamma(8^\circ)$	117.2°	104.3°	115.6°	125°
$\Gamma(8) - \lambda + \frac{\pi}{2}$	62.9°	65.0°	73.0°	------
$P(8^\circ)$	0.03	0.21	0.18	------
$g(16^\circ)$	4.26	4.00	4.15	4.2
$\Gamma(16^\circ)$	69.8°	51.8°	67.6°	72°
$\Gamma(16) - \lambda + \frac{\pi}{2}$	15.7°	12.5°	25°	------
$P(16^\circ)$	0.01	0.05	0.11	------

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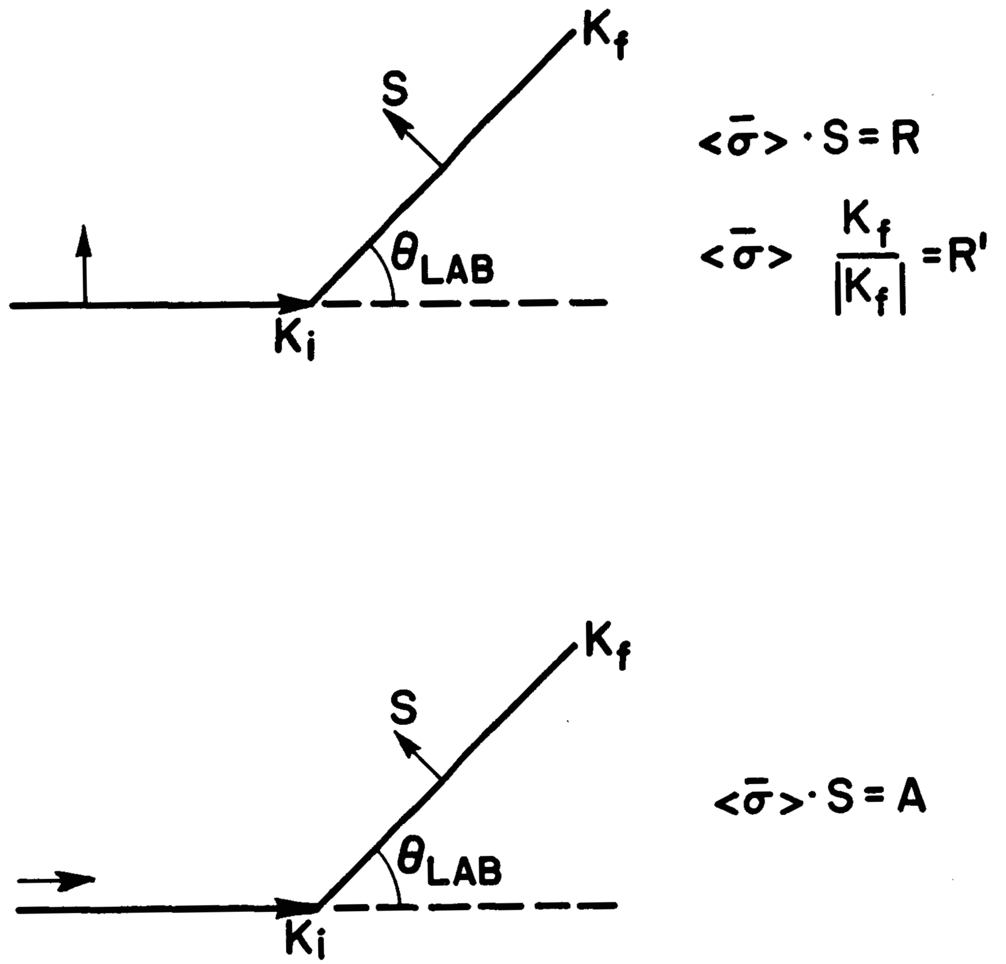

Fig. 1. Triple scattering experiments. Arrow on incident beam indicates direction of polarization on second scatterer. Arrow on outgoing beam indicates normal S to third scattering plane. Equations for measured component of polarization are given for case in which incident beam is completely polarized.

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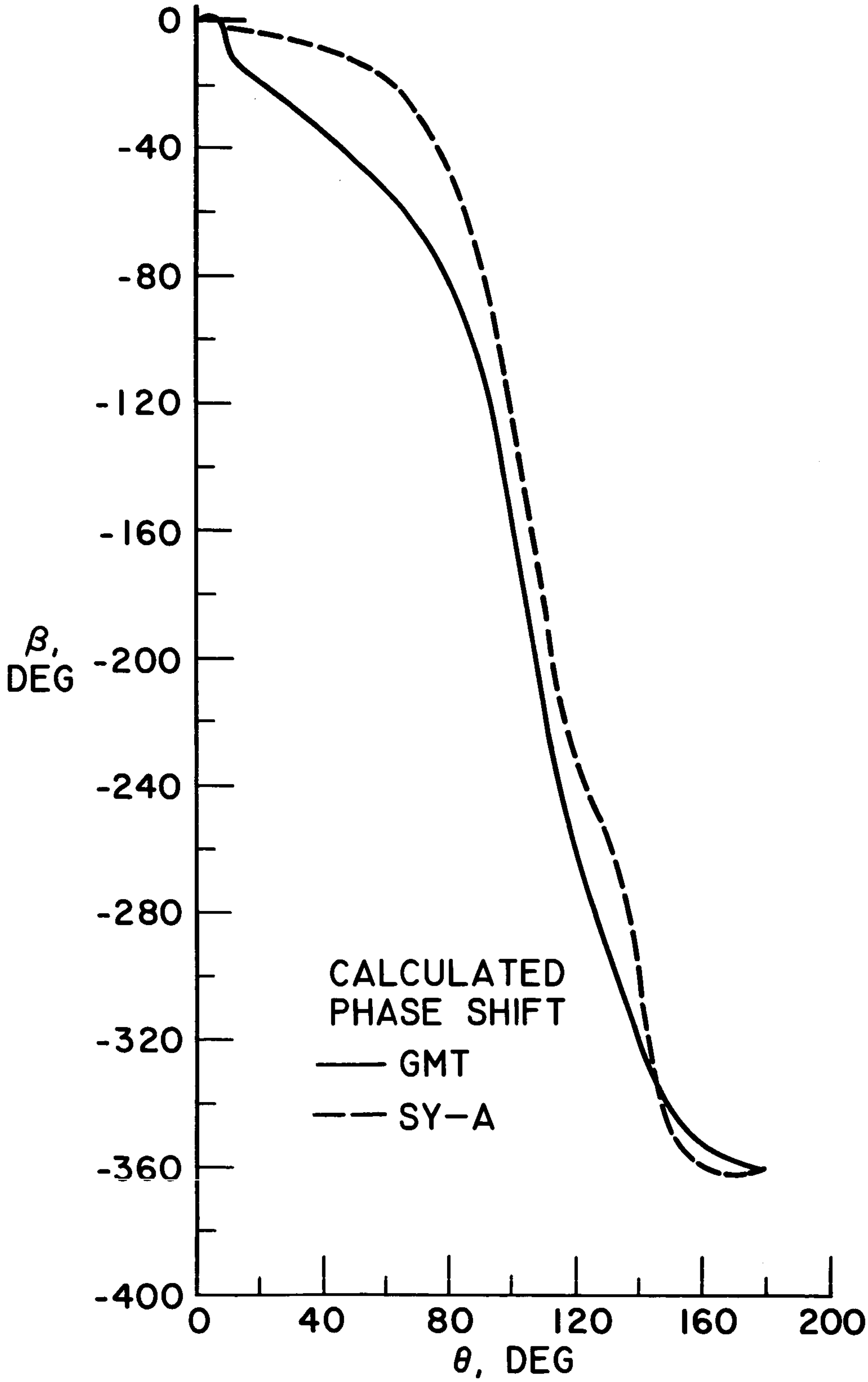

Fig. 2. Rotation angle β as function of center-of-mass scattering angle for two sets of phase shifts.

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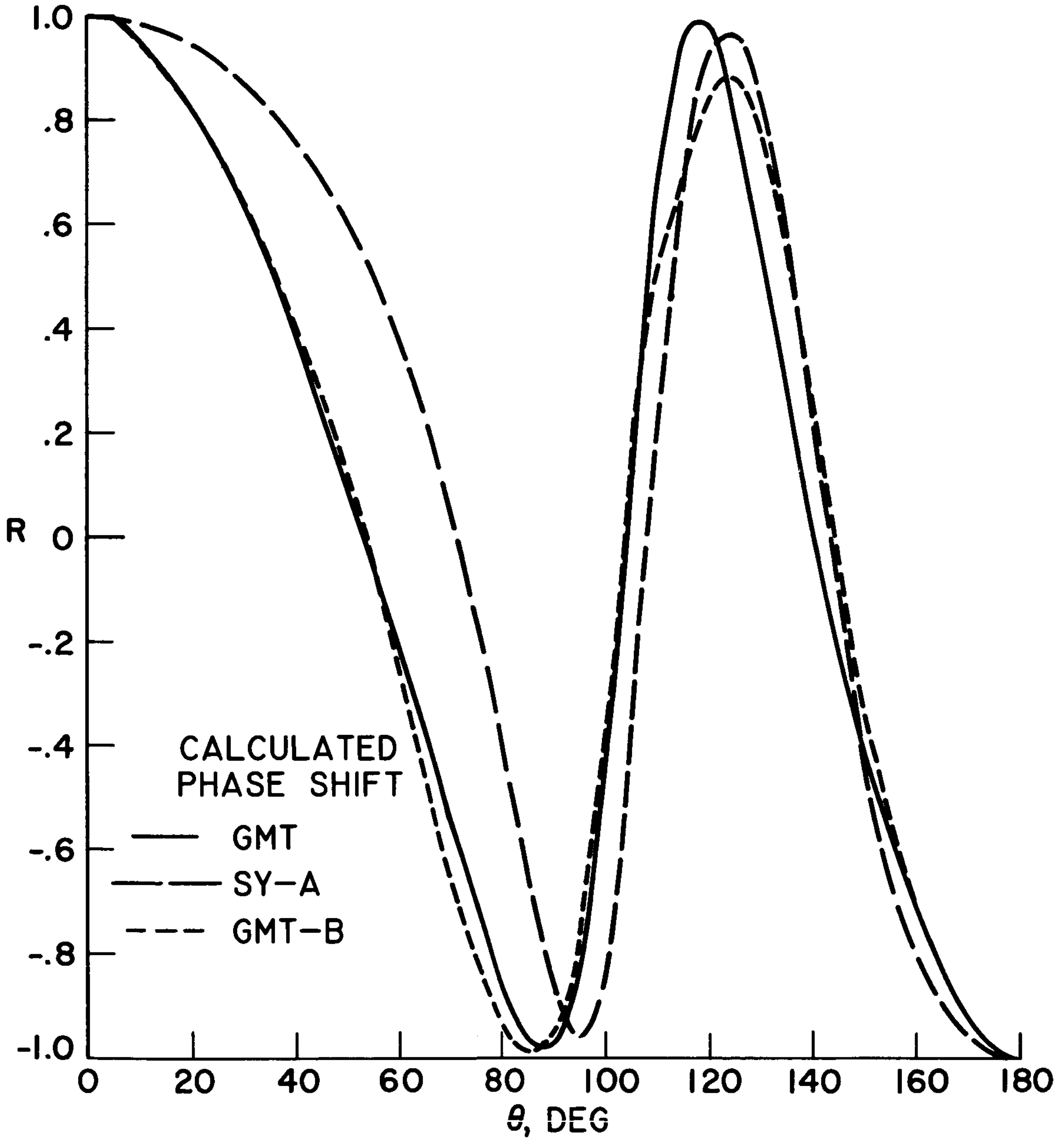

Fig. 3. Rotation parameter as function of center-of-mass scattering angle for three sets of phase shifts.

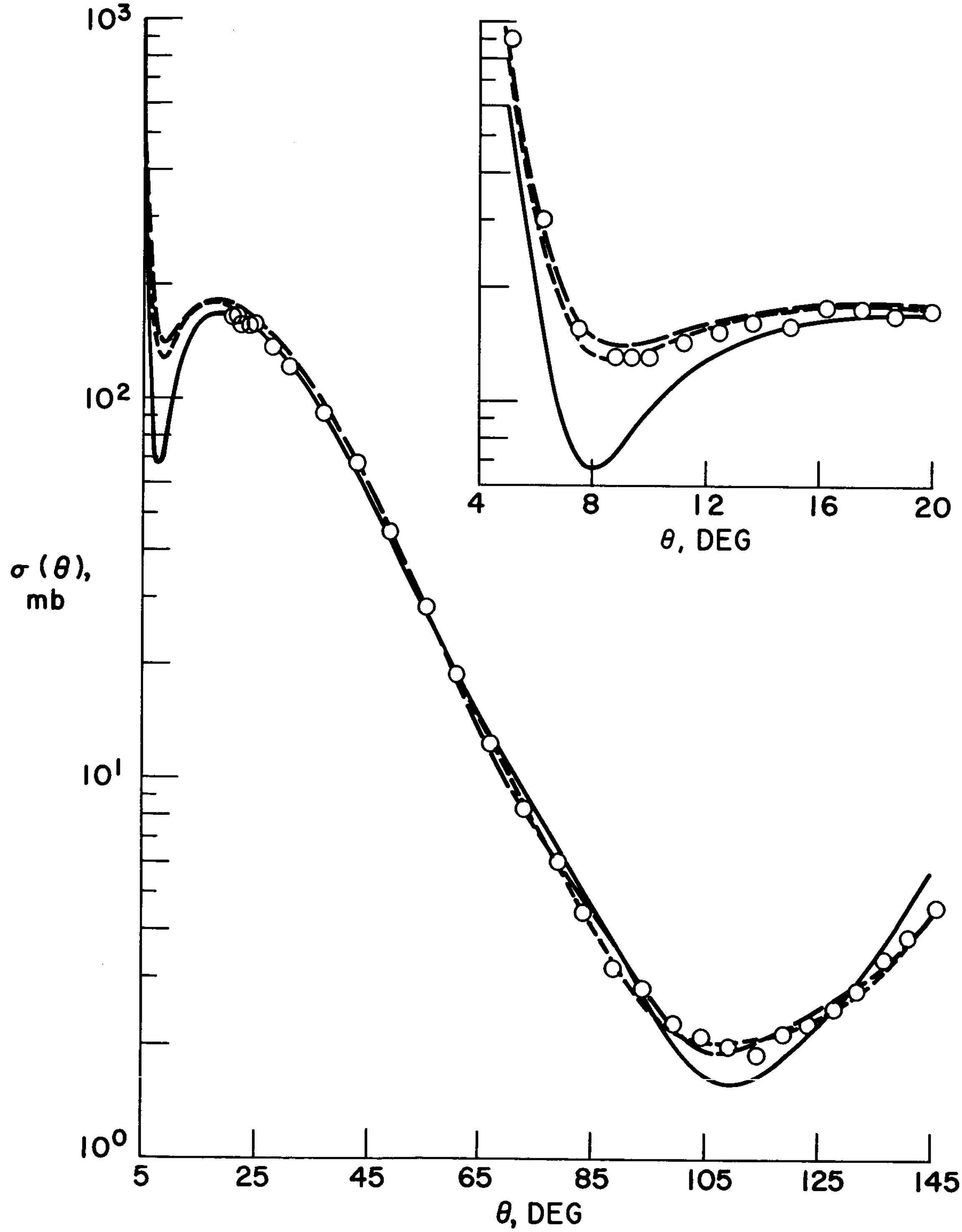

Fig. 4. Elastic cross section as function of center-of-mass scattering angle for three sets of phase shifts compared to data of ref. 3. Circles representing data points are larger than error bars assigned in ref. 3.

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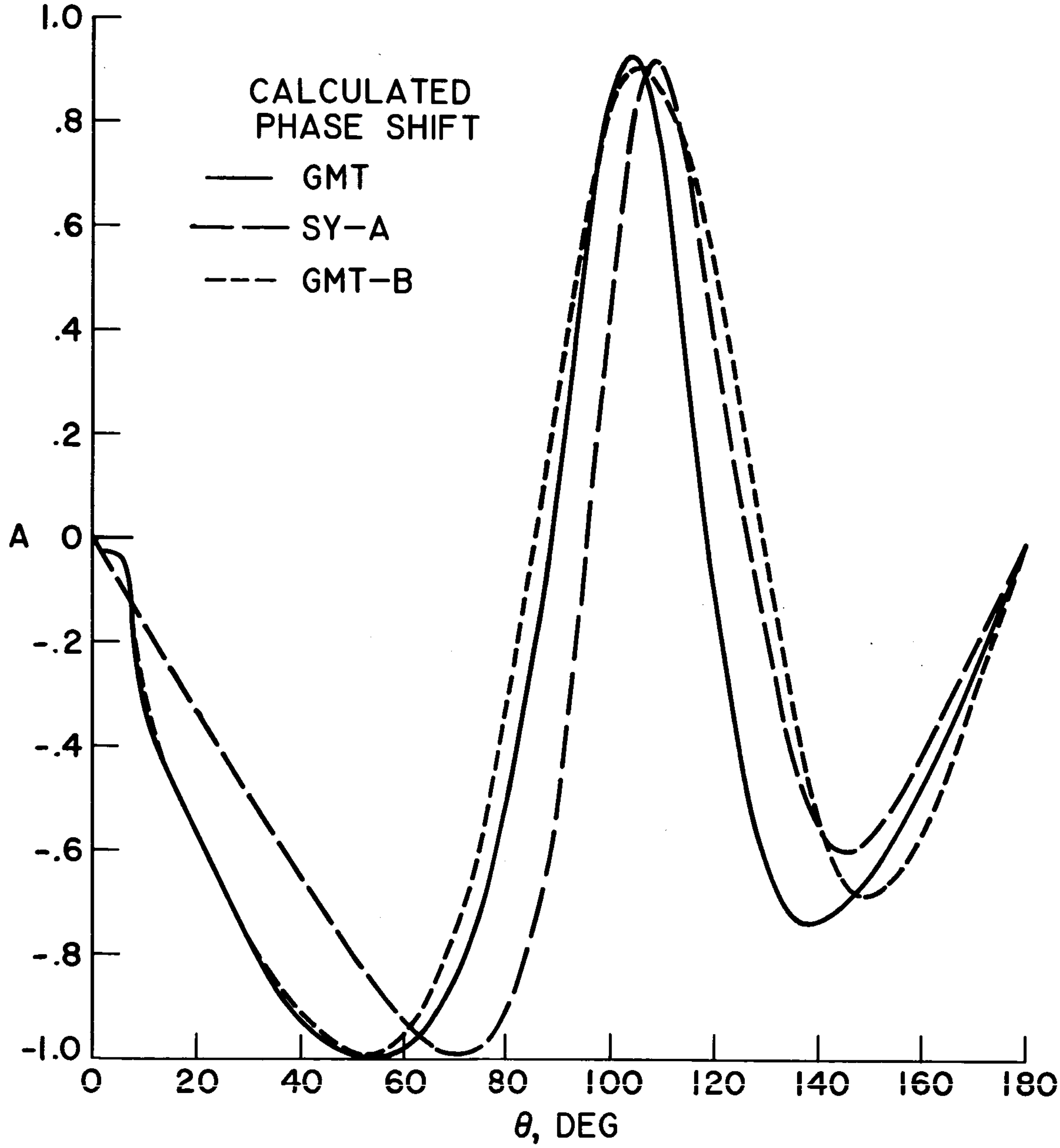

Fig. 5. Triple scattering parameter A as function of center-of-mass scattering angle for three sets of phase shifts.

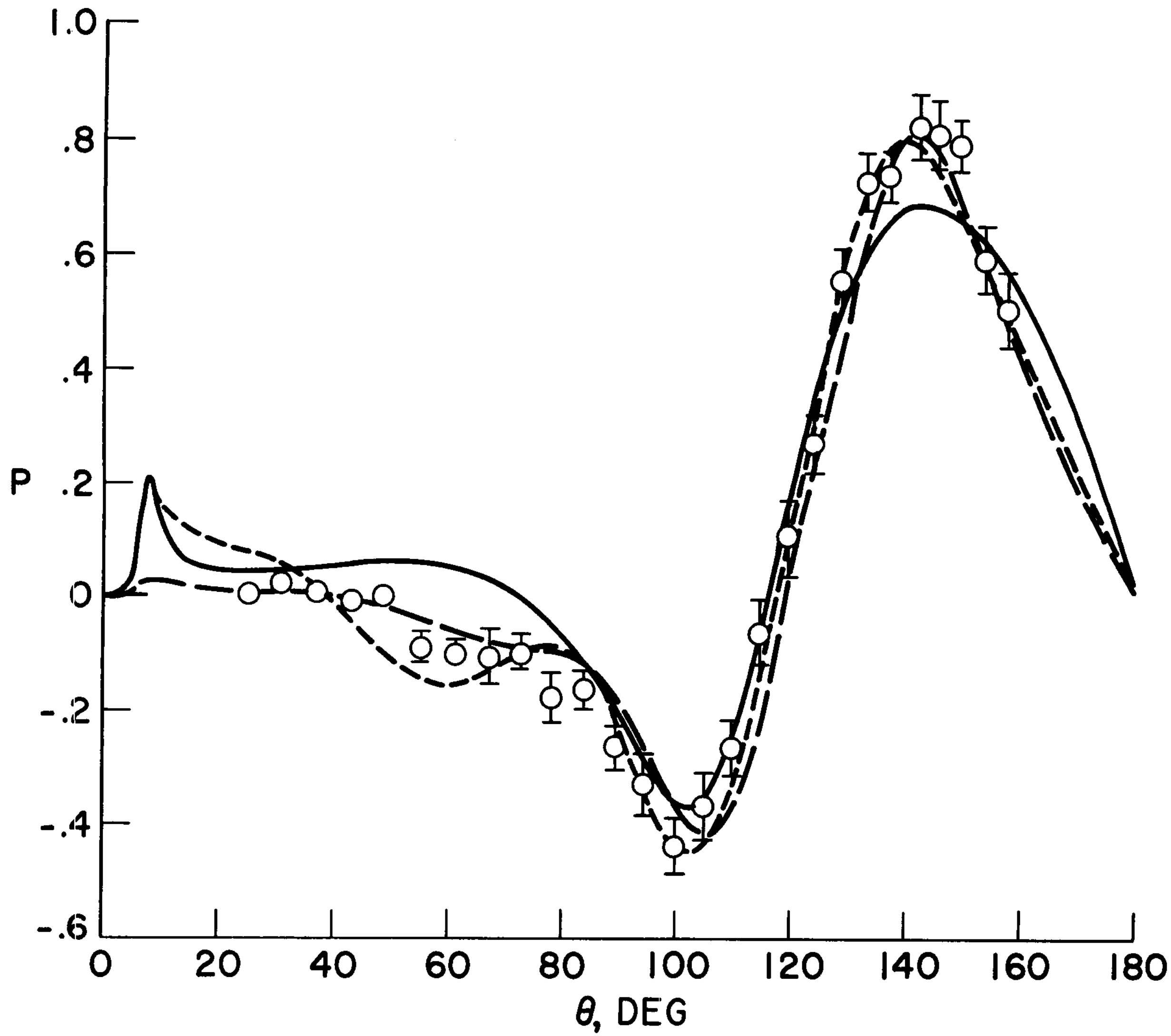

Fig. 6. Polarization P as function of center-of-mass scattering angle for three sets of phase shifts compared to data of ref. 4.