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MISMATCH STRESSES IN PRESSURE VESSELS

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In the manufacture of pressure vessels it is often necessary to join
segments together at circumferential joints by welds or other means. Such
connections may be made in lightweight flight tanks by lapping the seg-
ments and spot welding (fig. 1(a)) or by butt welding the adjacent seg-
ments (fig. 1(b)). The purpose of this note is to present an approximate
analysis for the stresses arising at such a joint because of the mismatch
or nonconcurrence of the middle surfaces of the adjoining segments that
might occur. The method used is applicable to any nonshallow shell of
revolution in which the meridional tangents of the two segments are paral-
lel to each other at the junction.

The methods presented offer the following advantages: a simple
closed-form solution is obtained that compares favorably with the re-
sults computed from linear edge-influence coefficients, solutions are
found that are applicable to both axisymmetric and local mismatches, and
an easily remembered approximate formula is suggested. The use of the
equations presented herein avoids the necessity of using edge-loaded
shell equations for the particular geometry being considered.

In figure 1, N_ξ is the meridional membrane force per unit length
in the shells at their junction and d is the distance between the mid-
dle surfaces (mismatch). The bending moment at the junction due to the
eccentricity of the membrane forces is then

$$M_e = N_\xi d \tag{1}$$

If the shells are thin-walled and have equal thicknesses h, the eccentric moment will be distributed equally between the two shells. The maximum meridional stress occurs at the junction and is given by

$$\sigma_{\xi,\max} = \frac{N_\xi}{h} + \frac{6}{h^2}\frac{M_e}{2} \tag{2}$$

The maximum circumferential stress also occurs at the junction and for non-shallow shells is given by

$$\sigma_{\theta,\max} = \frac{N_\theta}{h} + \frac{6\nu}{h^2}\frac{M_e}{2} \tag{3}$$

where N_θ is the circumferential membrane force per unit length and ν is Poisson's ratio.

If the joint is a junction between two cylinders of radius a with internal pressure p, the foregoing equations can be written as

$$M_e = \frac{pa}{2}\, d \tag{1a}$$

$$\sigma_{\xi,\max} = \frac{pa}{2h}\left(1 + 3\frac{d}{h}\right) \tag{2a}$$

$$\sigma_{\theta,\max} = \frac{pa}{2h}\left(2 + 3\nu\frac{d}{h}\right) \tag{3a}$$

If the distortion-energy theory is used as a yield criterion, the maximum effective stress is

$$\sigma_e = \sqrt{\sigma_\xi^2 + \sigma_\theta^2 - \sigma_\xi\sigma_\theta} \tag{4}$$

Substituting equations (2a) and (3a) into (4) gives

$$\sigma_{e,max} = \frac{\sqrt{3}}{2}\frac{pa}{h}\sqrt{1 + 3\nu\frac{d}{h} + 3(1 - \nu + \nu^2)\left(\frac{d}{h}\right)^2} \qquad (5)$$

for the case of mismatch in a cylinder of constant wall thickness. From equation (4) the effective stress in the membrane region of a pressurized thin cylinder $\sigma_{e,mem}$ can be shown to be $\sqrt{3}\,pa/2h$. Therefore, the quantity under the radical sign in equation (5) can be considered a stress concentration factor due to the mismatch. Dividing equation (5) by $\sqrt{3}\,pa/2h = \sigma_{e,mem}$ gives the stress concentration factor

$$scf = \frac{\sigma_{e,max}}{\sigma_{e,mem}} = \sqrt{1 + 3\nu\frac{d}{h} + 3(1 - \nu + \nu^2)\left(\frac{d}{h}\right)^2} \qquad (6)$$

Equation (6) is plotted in figure 2.

The results computed from the approximate analysis presented in equation (6) agree very favorably with the results computed from the linear edge influence coefficients described in reference 1; for example, the error was less than 2 percent for 100 percent mismatch and $a/h = 75$. The use of equation (6) precludes the necessity of determining edge influence coefficients and using the more rigorous equations indicated in reference 1. Contrary to the usual results obtained from edge influence coefficients the stress concentration factor is independent of the cylinder radius.

In general, the mismatch would not be uniform around the circumference; however, if it is approximately constant for several characteristic lengths $\sqrt{ah}$, the results should be sufficiently accurate for engineering purposes. If the mismatch is more of a local phenomenon, it might be more accurate to exclude the stresses due to the Poisson effect because the radial planes

are no longer constrained to remain radial by axial symmetry, and local
distortion can take place. Poisson strain due to bending is present,
therefore, but not induced stress due to the Poisson effect. Conse-
quently, for local mismatch let ν equal zero. Then

$$scf = \sqrt{1 + 3\left(\frac{d}{h}\right)^2} \tag{7}$$

The result is identical to the one obtained from reference 2 if the over-
lap distance equals zero. Equation (7) is also plotted in figure 2. The
radially symmetric case (eq. (6)) can be considered an upper bound and is
not much greater than the case for local mismatch if the mismatch is a
relatively large percentage of the shell thickness.

If the junction between the two shells is a lap joint, additional
discontinuity stresses arise because the increased thickness acts as a
girdle, restricting expansion due to internal pressure. These additional
stresses are not accounted for in this analysis. This effect may not be
serious, however, because the restriction on the radial expansion tends
to reduce the hoop stress, which is generally larger than the meridional
stress. Thus, the stress concentration factor is probably reduced. Also,
no effect of stress concentration due to sharp corners at the mismatch is
included. If the joint is butt welded, there will probably be a fillet
between the two sections that minimizes this effect.

One approximation of the effect of mismatch, which is easy to re-
member, is that the effective stress concentration factor is 1 plus the
mismatch given as a fraction of the thickness. For example, if there is
50-percent mismatch, this approximation would give a stress concentration

factor of 1.5. For axisymmetric or relatively wide mismatch, equation (6) gives 1.43 for the stress concentration factor. For relatively local mismatch, however, equation (7) gives 1.32. In figure 2, it is apparent that this simple approximation is reasonably accurate for large percentages of mismatch but is less accurate for small mismatch.

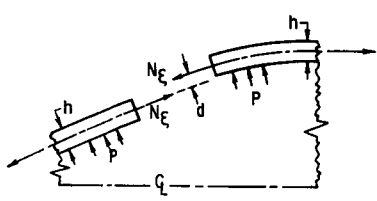<div align="center">REFERENCES</div>

1. Johns, Robert H., and Orange, Thomas W.: Theoretical Elastic Stress Distributions Arising from Discontinuities and Edge Loads in Several Shell-Type Structures, Appendix L. NASA TR R-103, 1961.

2. Sechler, E. E.: Stress Rise Due to Offset Welds in Tension. Rep. EM 9-18, Space Tech. Labs., Inc., Aug. 1959.
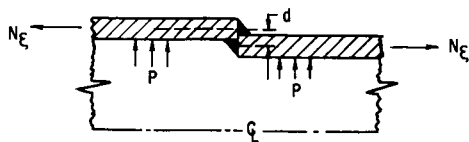
(a) Butt joint with mismatch.
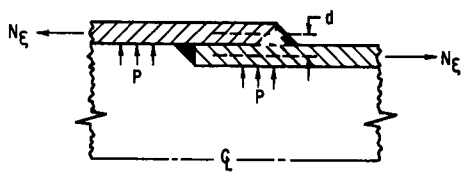
(b) Lap joint.

(c) Joint with mismatch in general shell of revolution.

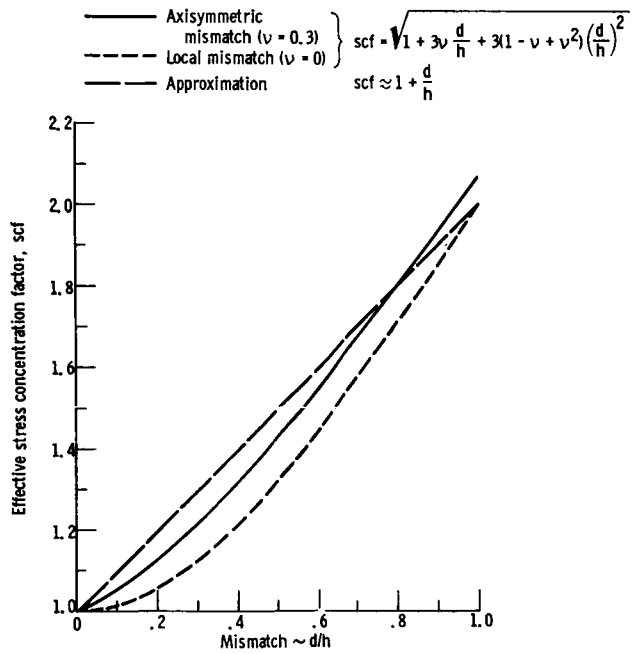Figure 1. – Mismatch joints and loading.

Axisymmetric
    mismatch ($\nu = 0.3$)
Local mismatch ($\nu = 0$)

$\text{scf} = \sqrt{1 + 3\nu \frac{d}{h} + 3(1 - \nu + \nu^2)\left(\frac{d}{h}\right)^2}$

Approximation $\text{scf} \approx 1 + \frac{d}{h}$

Figure 2. – Maximum effective stress due to mismatch in cylinder.