N65-35269

X-640-64-164

TM X-55061

MAGNETOHYDRODYNAMIC SHOCK STRUCTURE IN A MOVING MAGNETIC FIELD

JUNE 1964

GODDARD SPACE FLIGHT CENTER

GREENBELT, MARYLAND

MAGNETOHYDRODYNAMIC SHOCK STRUCTURE

IN A MOVING MAGNETIC FIELD

by

David B. Beard†

University of California

Davis, California

Submitted to The Physics of Fluids

†Present address: University of Kansas, Lawrence, Kansas
Consultant to Goddard Space Flight Center, Greenbelt, Maryland

MAGNETOHYDRODYNAMIC SHOCK STRUCTURE IN A MOVIN G MAGNETIC FIELD

ABSTRACT

The shock structure around a cylinder immersed in a cold collisionless plasma stream containing a weak magnetic field has been investigated in two dimensions. The stream pressure on the boundary of the sheath is proportional to $\cos^2\psi$ where ψ is the angle between the stream velocity and the normal to the sheath. The thickness of the sheath and particle density in the sheath on the upstream side of the cylinder have been found to be nearly constant. A tear drop shaped vacuum is formed downstream from the cylinder. The plasma electrons have essentially all the ion stream kinetic energy in the transition sheath upstream of the cylinder but the ion stream energy is regained downstream from the cylinder. The importance of this shock structure when applied to the earth's magnetosphere and the generation of Type I comet tails is discussed briefly.

I. Introduction

The solar wind, consisting of a cold plasma flowing constantly at high velocity away from the sun, is now known to contain a weak magnetic field[1,2] oriented somewhat perpendicular to the stream velocity vector. Though the magnetic field carried along in and moving with the stream velocity is so weak that the magnetic field pressure is only one percent of the stream pressure,the field prevents the particles from moving independently of one another. The result is that any object larger than a few ion gyroradii will create a magnetohydrodynamic shock when immersed in the solar wind.[3-9]

The distinctive features of shocks generated in the solar wind are caused by the shape of the object creating the shock and the effect of arresting the velocity of the magnetic field lines which initially must move with the wind velocity since the stream pressure is overwhelmingly greater than the magnetic pressure. The plasma and its field are compressed on the upstream surface of the obstacle and much of the stream kinetic energy is diverted to gyromotion perpendicular to the compressed magnetic field. A simple extension of ordinary M theory of the magnetic pinch effect shows that on entering the compressed field region through the charge separation electric field on the surface, the plasma electrons will acquire essentially all of the kinetic energy of the ion stream in the solar wind. This energy is returned to the ion stream after the plasma in the compressed field region moves around to the downstream side of the obstacle.

In the work presented here a simple model is discussed in which a cylinder of intense magnetic field parallel to its axis presents an obstacle to a cold plasma stream containing a weak magnetic field parallel to the cylinder axis. Besides the inherent interest of the model in understanding magnetohydrodynamic shock structure under such unusual conditions, the model illuminates some gross aspects to be expected in astrophysical problems which occur, for example, in the boundary of the earth's magnetic field and in the tail rays of comets.

II. Physical Characteristics of the Transition Sheath Surrounding a Cylinder in a Magnetized Plasma Stream

The simple idealized model considered here is illustrated in Fig. 1. A proton-electron plasma, collisionless and with no thermal motion, is streaming parallel to the polar axis with velocity $\underset{\sim}{v}_o$, and encounters a cylinder of radius R containing an arbitrarily intense magnetic field sufficient to exclude the incident plasma stream. (In general the obstacle need $\boxed{\text{only}}$ $\boxed{\text{furnish}}$ a spatially varying magnetic field pressure, $H\cancel{g}^2/8\pi$, but a cylindrical shape has been introduced at the start to simplify the discussion of the paramount physical features resulting from the analysis.) Finally, the incident plasma stream is assumed to contain a weak magnetic field, parallel to the cylinder axis, which is carried along with the plasma. The stream magnetic field exerts a pressure of only one percent of the stream pressure and the field traveling with the stream velocity $\underset{\sim}{v}_o$ is maintained by the small gyromotion of the plasma in the moving velocity frame of the plasma stream. In this connection it might be noted that the preliminary analysis by Dr. M. Neugebauer of the Mariner R plasma data yielded an effective plasma thermal energy of the order of 10-30 electron volts in a stream energy of the order of a kilovolt. Dr. Neugebauer's analysis was made by examining the energy spread parallel to the stream motion, but the presence of a magnetic field and the gyromotion it causes guarantees that the result is valid for the particle energy component perpendicular to the stream motion as well.

The particle pressure on the outer boundary of the transition sheath is given by

$$p_p = (n_o v_o \cos\psi)(m_p v_o \cos\psi) = n_o m_p v_o^2 \cos^2\psi \tag{1}$$

where n_o is the proton density in the stream m_p is the proton mass and ψ is the angle between the normal to the outer boundary surface of the transition sheath and the stream velocity vector. The second parenthetical factor represents the momentum change of the incident particles at the boundary as they enter the sheath. The external pressure confining the particle and enhanced magnetic field pressure in the sheath acts perpendicularly to the surface. Hence there is no change in particle center of motion momentum component parallel to the boundary which would result in a momentum change factor not linearly dependent on $\cos\psi$.

There is, of course, a very important change in the particle motion on crossing the boundary of the sheath and that is that near the stagnation point, in particular, much of the stream motion is diverted to gyromotion perpendicular to the magnetic field. Although the magnetic field is too weak to contribute effectively to the pressure near the stagnation point, it does cause the charges to rotate about the lines of force. Particle pressure, however, dominates every other consideration and the guiding center of the particles are caused to move towards decreasing pressure gradients "dragging" the magnetic field lines along with them. The particle pressure within the transition sheath is given by

$$p_p = \frac{1}{2} n_s m_p v_p^{\ 2} = \frac{1}{2} n_s m_p (v_o^{\ 2} - v_m^{\ 2}) \tag{2}$$

where n_s is the ion density within the sheath, v_p is the particle velocity component perpendicular to the field, and v_m is the velocity of the guiding center,that is,the velocity of the magnetic lines of force. The factor 1/2 arises because of the two dimensional motion of the particles within the sheath as opposed to the monodirectional motion of the stream pressure. In three dimensions,where the particle motion is also along the lines of force,the factor might be 1/3.

We thus obtain an expression for the pressure equality on the boundaries of the sheath.

$$n_o m_p v_o^{\ 2} \cos^2\psi + H_o^2/8\pi = \frac{1}{2} n_s m_p (v_o^{\ 2} - v_m^{\ 2}) + (n_s^{\ 2}/n_o^{\ 2}) H_o^2/8\pi = H_c^{\ 2}/8\pi \tag{3}$$

where H_c is the magnetic field intensity in the cylinder at the inner boundary. Since the plasma is collisionless,the magnetic field intensity is directly proportional to the particle density,and thus the magnetic field intensity anywhere in the sheath is $(n_s/n_o)H_o$. Particle thermal pressure would be easy to include for similar reasons. Since n is proportional to H and the square of the thermal velocity, v_{pt}^2, is proportional to H if the motion is adiabatic, the particle thermal pressure is a constant multiple of the magnetic pressure and may be included by multiplying H_o^2 by an appropriate factor.

The change in the external pressure, P, as a function of the distance along the boundary of the sheath, l, results in a pressure gradient which accelerates the plasma within the sheath.

$$n_s m_p \frac{dv_m}{dt} = n_s m_p v_m \frac{dv_m}{dl} = -\frac{dP}{dl} + \frac{d}{dl}\left(\frac{n_s^2}{n_o^2}\frac{H_o^2}{8\pi}\right) + \frac{m_p}{2}(v_o^2 - v_m^2)\frac{dn_s}{dl} \qquad (4)$$

which may also be obtained by differentiating Eq. 3 with respect to l. Throughout most of the upstream side of the cylinder H_o^2 is negligible compared with the particle pressure and Eq. 3 may be approximated by

$$n_s = 2P/m_p(v_o^2 - v_m^2)$$

which satisfies Eq. 4 since the last two terms in Eq. 4 are negligible in these circumstances. Thus, $v_m \sim v_o \sin\psi$ throughout the transition sheath on the upstream side and n_s is nearly constant and equal to $2n_o$. As ψ approaches $\pi/2$, however, the magnetic pressure becomes significant and n_s decreases with l depending on ψ as

$$n_s \sim n_o \left[1 + \frac{1}{4}(8\pi n_o m_p v_o^2/H_o^2)\cos^2\psi - (3/32)(8\pi n_o m_p v_o^2/H_o^2)^2\cos^4\psi\right] \qquad (5)$$

Using the approximations valid on the upstream side of the cylinder the position of the outer boundary to the transition sheath is easily obtained by requiring particle conservation. The number of particles arriving per second per unit cylinder length within a distance, y_2, from the polar axis is

$n_o y_2 v_o$; the number leaving the transition region per second per unit cylinder length is $n_s(r_2-r_1)v_o \sin\psi$ (see Fig. 2).

$$r_2 - r_1 \sim r_2 \sin\theta_2/2 \sin\psi \qquad (6)$$

For a cylindrical obstacle for which r_1 is constant, $\theta_2 = \psi$, and

$$r_2 = 2r_1 \qquad (7)$$

If the obstacle were not cylindrical a differential form of the constant flux condition, Eq. 6, would be required

$$n_o v_o d\,l \cos\psi = n_s(r_2-r_1)v_o d(\sin\psi) \quad \frac{d\psi}{d\,l} = \frac{n_o}{n_s}\;\frac{1}{r_2-r_1} = \frac{1}{r_2}\frac{d\psi}{d\theta} \qquad (8)$$

which yields Eq. 7 in the region where $n_s = 2n_o$ if r_1 and r_2 are now regarded as functions of θ. If n_s is a decreasing function of θ, as it is near $\theta \sim \psi \sim \pi/2$, r_2-r_1 increases faster than r_2 and $\frac{d\psi}{d\theta}$ is less than unity. In this region, the parametric equations 5 (or rather 3) and 8 must be solved numerically.

The magnetic pressure cannot be neglected in the region

$$\theta \sim \psi \gtrsim \cos^{-1}\left[H_o/(8\pi n_o m_p v_o^2)^{1/2}\right] \sim 84^o \qquad (9)$$

where the stream velocity is

$$v_m \sim v_o \sin\psi \sim 0.99\ v_o \qquad (10)$$

and

$$(v_o^2 - v_m^2)^{1/2} \sim 0.10\ v_o$$

Therefore, on the downstream side of the cylinder, the guiding center or stream velocity in the transition sheath is again approximately v_o, and the velocity perpendicular to the field lines and v_o is less than $0.10\ v_o$. The stream expands freely into the vacuum behind the cylinder so as to form a diffuse triangle shaped tail, based on simple kinematics, which closes more than ten cylinder radii downstream and leaves a turbulent wake as illustrated in Fig. 3.

III. The Shock Produced Energetic Electrons in the Transition Sheath

The preceding section has given us a general description of our simple two dimensional model problem as illustrated in Figs. 1-3. A weakly magnetized plasma moving with velocity $\underset{\sim}{v}_0$ enters a transition sheath where the velocity of the particle's guiding centers, that is, the velocity of the magnetic lines of force, changes abruptly at the outer boundary of the transition sheath to a lower velocity, $\underset{\sim}{v}_m$. Ferraro[10] and especially Dungey[11] and Rosenbluth[12] have shown how a charge separation layer will be created at any boundary where the magnetic field is suddenly increased on the side away from the plasma. Beard,[13] Grad,[14] and Sestero[15] have further developed the theory of the electric field on the boundary which serves to energize the electrons at the expense of the positive ions. The electric field arises due to the difference in momentum between the electrons and the positive ions, and thus there is no field in any region where the momenta of the opposite charges are equal.

In the simple two-dimensional problem discussed in this paper, it might be anticipated that when the plasma enters and passes through the transition sheath the charges will have gained or lost potential energy (depending on the sign of their charge) as though they had passed through permeable condenser plates at the boundary. Hence, if there is no reverse electric field, the electrons will have acquired essentially all of the initial stream kinetic energy of the positive ions.

Following Rosenbluth's[12,13] analysis in which the plasma did not enter the region of increased field intensity, we write, for the changed conditions of this problem, the equations of motion of a charge initially moving parallel to the x axis

$$\ddot{x} = \frac{q}{m} E(x) + \frac{q}{mc} H\dot{y} \tag{11}$$

$$\ddot{y} = -\frac{q}{mc} H(\dot{x} - v_m) \tag{12}$$

where the coordinate system is illustrated in Fig. 4, q and m are the charge and mass of the charged particle, E is the electric field perpendicular to the boundary which is tangent to the y,z plane, H is the magnetic field and is parallel to the Z axis, and v_m is the velocity of the guiding centers (i.e. the magnetic lines of force) which are assumed to be moving entirely parallel to the x axis for this part of the analysis. Eq. 12 is easily integrated and yields the y component of the particle velocities.

$$\dot{y} = \frac{-q}{mc} \int_{-\infty}^{t} H \, (\dot{x} - v_m) \, dt' \tag{13}$$

Hence the y component of the particle momenta is the same for both electrons and protons.

The particle densities are inversely proportional to the x component of the average velocity of the particles. To avoid the impossibly large electric fields resulting from excessive charge separation the x component of the average velocities of the particles must be kept equal everywhere. This requires that in being slowed from an initial velocity v_o to the final guiding center velocity of V_m, the x-component of the deceleration of the electrons must be equal to the deceleration of the ions. Thus Eq. 11 and 13 yield the equality

$$\frac{e}{m_e} E(x) - \left(\frac{e}{m_e c}\right)^2 H \int_{-\infty}^{t} H \, (\dot{x} - v_m) \, dt' = - \frac{e}{m_i} E(x) - \left(\frac{e}{m_i c}\right)^2 \int_{-\infty}^{t} H \, (\dot{x} - v_m) \, dt'$$

where the subscripts e and i refer to electron and positive ion and $q_e = -e, q_i = e$. Neglecting the right hand side of this equation as being of order m_e/m_i smaller than the left hand side we obtain the familiar M theory result

$$E(x) = \frac{e}{m_e c^2} H \int_{-\infty}^{t} H \, (\dot{x} - v_m) \, dt' \tag{14}$$

If Eq. 13 and 14 are inserted into Eq. 11, then the integral of Eq. 11 after multiplication by $\dot{x}$ is to zero order in m_e/m_i

$$\dot{x}_i^2 = v_o^2 - \frac{2e}{m_i}\int_{-\infty}^{x} E(x')dx'$$

Let X_M represent the position at which the incident proton first reaches the final guiding center velocity, V_m, (i.e. the position beyond which the compressed field is constant.) then,

$$\frac{1}{2} m_i (v_o^2 - V_m^2) = e\int_{-\infty}^{X_M} E(x')dx' \qquad (15)$$

Eq. 15 has the very obvious physical interpretation that the total potential drop for $-\infty < x < x_M$ is equal to the change in the ion stream kinetic energy divided by the ion charge. For $x > x_M$ charge neutrality does not require $\dot{x}_e = \dot{x}_i$; only their averages, V_m, must be equal.

Since the momenta of the electrons and protons are equal, no further charge separation takes place; the gyroradii of the particles are equal. The momentum of an ion in the stream $m_i v_o$ has been reduced to $(m_e/m_i)^{1/2} m_i v_o$ in exerting the pressure on the boundary needed to confine the enhanced magnetic field interior to the boundary. The kinetic energy of the ions has been reduced, however, from $\frac{1}{2} m_i v_o^2$ to $\frac{1}{2} m_e v_o^2 + \frac{1}{2} m_i v_M^2$. The considerations of the previous section are still valid since the total particle pressure in the transition layer is not affected by the electrical potential at the boundary since the electron energy in the transition layer is $\frac{1}{2} m_i v_o^2 - \frac{1}{2} m_i v_M^2$.

A free hand drawing of the particle motion as it crosses the boundary is shown in Fig. 5 for conditions near the stagnation point. The conclusions are essentially unchanged at other points on the boundary as illustrated in

Fig. 6 if one considers only the component of the guiding center motion perpendicular to the boundary. The component of the stream velocity parallel to the boundary is the same on both sides of the boundary and therefore the problem reduces to the stagnation point case in a coordinate frame which moves with the velocity $v_o \sin \psi$ parallel to the boundary and $v_o \cos\psi$ supplants v_o in the stagnation point considerations. In accelerating to nearly the free stream velocity again in its motion away from the stagnation point, the motion of the plasma in the transition sheath causes the initial ion kinetic energy to be recovered at the expense of the electron energy.

Conclusions

A cold plasma stream containing a weak magnetic field (field pressure much less than particle stream pressure) will be compressed on the upstream surface of a cylindrical obstacle whose axis is parallel to the magnetic field. The particle density and magnetic field intensity within the transition sheath on the upstream side are double their free stream value. The thickness of the sheath is equal to the cylinder radius over almost all of the upstream surface and thickens slightly near the joining of the upstream and downstream surfaces due to the additional magnetic pressure, however small, in the transition sheath. The enhanced magnetic pressure in the transition sheath, however slight, prevents the ions from returning completely to their initial free stream motion on the downstream side of the cylinder. As a result, a plasma stream, even though the particles initially have a unique velocity, will expand into the vacuum behind the cylinder. In the case of a plasma with some thermal velocity $(kT/2\pi m)^{1/2}$ the vacuum downstream would be a diffuse isoceles triangle whose height would be $v_o \ (2\pi m/kT)^{1/2} R$ where v_o is the initial stream velocity and R is the cylinder radius (1/2 the triangle base.)

The enhanced magnetic field in the transition sheath creates a charge separation layer at the boundary of the undisturbed plasma stream. The total change in electrical potential across this layer is equal to half the stream pressure on the transition sheath. The plasma stream moves through the charge separation layer, and, on entering the transition sheath, the electrons acquire all the kinetic energy the ions would have had perpendicular to the magnetic field in the absence of the charge separation layer. At the stagnation point the electron kinetic energy in the transition sheath equals the initial ion kinetic energy in the free stream, but elsewhere in the transition sheath the electron kinetic energy equals the difference between the ion free stream kinetic energy and the ion stream energy in the transition layer.

Astrophysical Applications

The application of this work to Type I comet tails is straight forward and will be reported elsewhere. The slight ionization of massive cometary molecules through charge transfer with protons in the solar wind causes the interplanetary magnetic field lines to slow down drastically in overcoming the inertia of the massive cometary ions initially at rest. The energetic electrons in the transition sheath which forms on the surface of the "slowed" field lines then rapidly and efficiently ionizes hundreds of cometary molecules per cc. The energetic electrons will further rapidly diffuse out along the field lines (as suggested by Axford in private conversion) and pull the ions out along the field lines so as to form the striking ray structure known as Type I comet tail rays.

The simple model examined in this paper also yields a good qualitative understanding of the phenomena observed at the edge of the earth's magnetic field and enables predictions about the energy of the charges in the magnetosheath to be made. The sphericity of the earth's magnetosphere introduces a new physical dimension. In the plane containing the stagnation point and the magnetic field lines the particle pressure both inside and outside the outer boundary of the transition sheath (A. J. Dessler has suggested the name magnetosheath) is still proportional to $\cos^2\psi$. The particles are decoupled, however, from the magnetic field in the direction of the field, and conservation of magnetic flux requires that the magnetic field intensity be approximately independent of distance from the stagnation point or $\cos\psi$ in the case of a cylinder and to <u>increase</u> away from the stagnation point in the case of a sphere. Therefore, in this plane the shock boundary around a circular surface would expand faster than in the plane at right angles to the interplanetary magnetic field. The particle density in this plane away from the stagnation point is less than in the plane perpendicular to the magnetic field. The particle pressure in this plane is <u>not</u> communicated along the field lines and this introduces a complication in the analysis depending on the isotropy of the particle velocity discussed below. Free hand drawings of the field lines are illustrated in Figs. 7 and 8.

Another aspect of the magnetosphere problem is that turbulence on the surface or the sharp kinks produced in magnetic field lines where the interplanetary field lines are joined to the lines in the magnetosheath will invalidate the simple two dimensional analysis presented in this work. The particle motion in the magnetosheath will be more nearly isotropic in three dimensions instead of being isotropic in two dimensions. This causes the particle density and field intensity to be three times the interplanetary values and the thickness of the sheath near the stagnation point to be more nearly one-half of the radius of curvature of the magnetosphere.

Independent of these considerations the sudden increase of the magnetic field intensity from the interplanetary field value to the magnetosheath value will cause the magnetosheath electrons to have the same momentum perpendicular to the field lines as the protons do. Thus the particle energy and pressure perpendicular to the field lines will be monopolized by the electrons. The particle energy and pressure parallel to the field lines will be monopolized by the protons, however, since charge separation effects will cause the electron and proton velocities to equalize.

Acknowledgement

I would like to thank Dr. John Banister of Sandia Corporation for a helpful discussion of this problem. I would also like to thank both the Sandia Corporation and the Goddard Space Flight Center for their hospitality and support of this work.

REFERENCES

1. P. J. Coleman Jr., L. Davis Jr., E. J. Smith, and C. P. Sonett Science, 138 1099, (1962)

2. N. F. Ness, C. S. Scearce, and J. B. Seek, J. Geophys. Res., in press, (1964)

3. F. de Hoffman and E. Teller, Phys. Rev. 80, 692, (1950)

4. P. L. Auer, H. Hurwitz Jr., and R. W. Kilb, Phys. of Fluids, 5, 298 (1962)

5. P. J. Kellogg, J. Geophys. Res., 67, 3805, (1962)

6. J. R. Spreiter, and W. P. Jones, J. Geophys. Res., 68, 3555, (1963)

7. W. I. Oxford, J. Geophys. Res., 67, 3791, (1962)

8. D. B. Beard, J. Geophys. Res., 69, 1159, (1964)

9. L. Lees, Proc. AIAA New York meeting, Jan. 1964 and Cal. Inst. of Tech. preprint

10. V.C.A. Ferraro, J. Geophys. Res., 57, 15, (1952)

11. J. W. Dungey, Cosmic Electrodynamics, Cambr. Univ. Press, Cambridge, England, 132-152, (1958)

12. M. N. Rosenbluth, Dynamics of a Pinched Gas contribution to Magnetohydronamics edited by R. Landshoff, Stanford Univ. Press, Palo Alto, 57-66, (1957)

13. D. B. Beard, J. Geophys Res., 65, 3559, (1960)

14. H. Grad, Phys. Fluids, 4, 1366, (1961)

15. A. Sestero, Phys. Fluids, 7, 45, (1964)

FIGURE CAPTIONS

Fig. 1 Illustration of solar wind containing a weak magnetic field perpendicular to the plane of the figure incident on a cylinder of radius R. The arrows represent the velocity of the field lines (particle guiding centers) in the free stream $\underset{\sim}{v}_o$, and in the transition sheath, $\underset{\sim}{v}_m$. The circles and loops represent partial trajectories of the particles while in the sheath.

Fig. 2 Diagram of use of constant flux condition in determining the sheath thickness, $r_2 - r_1$, by which particles leave a volume of unit height after entering the volume within the length y_2.

Fig. 3 The shock structure behind a cylinder of radius R. The length of the vacuum downstream from the cylinder $L \sim R\ V_o / (v_o^2 - v_m^2)^{1/2}$.

Fig. 4 The coordinate system used in analyzing particle motion in a magnetic field parallel to the z axis whose field lines travel with a velocity $\underset{\sim}{v}_o$ to the left of the shock boundary and with velocity $\underset{\sim}{v}_m$ to the right of the shock boundary.

Fig. 5 Electron and proton trajectories initially traveling with a moving magnetic field which is slowed down and compressed, downstream from the shock boundary.

Fig. 6 The geometry of the guiding center or field line motion on both sides of the shock boundary for oblique incidence.

Fig. 7 Magnetic field lines indicated by dashed lines in free interplanetary space and in the transition sheath in the plane containing the field direction.

Fig. 8 Magnetic field lines, indicated by dashed lines, viewed in projection from the upstream side as they diverge and slide around a spherical obstruction bowing out most pronouncedly in the equatorial plane.

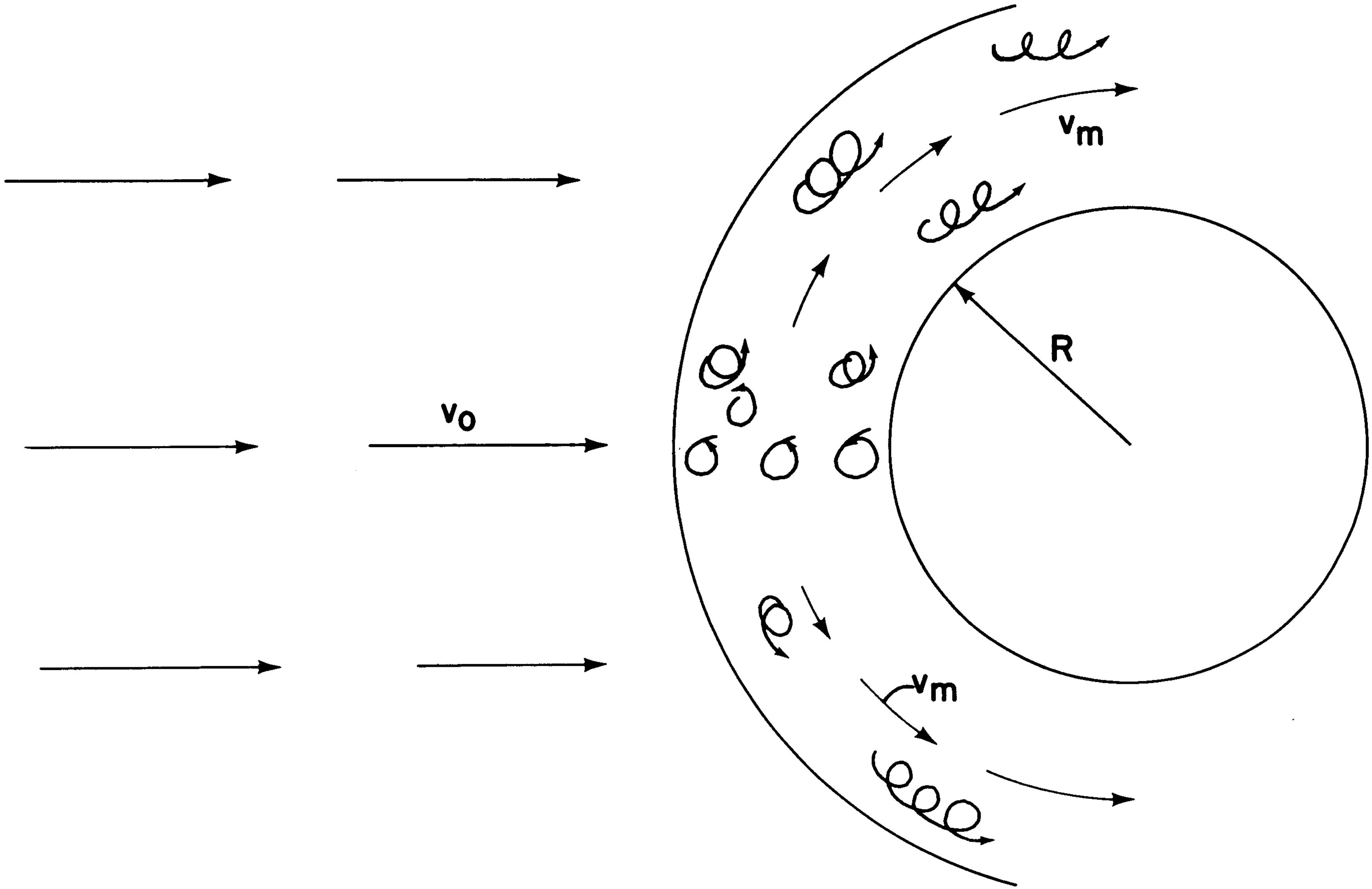

Figure 1 – Illustration of solar wind containing a weak magnetic field perpendicular to the plane of the figure incident on a cylinder of radius R. The arrows represent the velocity of the field lines (particle guiding centers) in the free stream v_o, and in the transition sheath, v_m. The circles and loops represent partial trajectories of the particles while in the sheath.

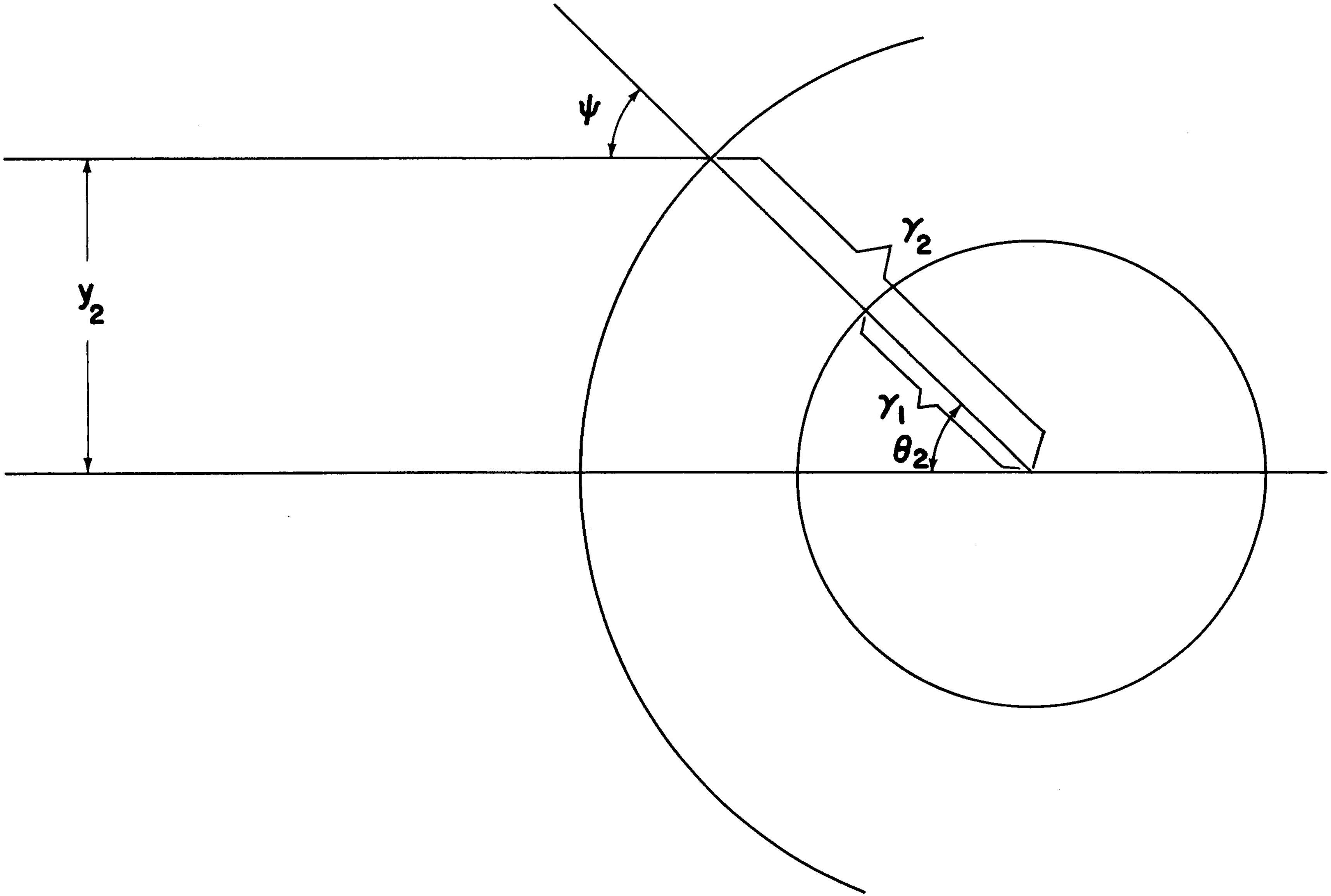

Figure 2 - Diagram of use of constant flux condition in determining the sheath thickness, $r_2 - r_1$, by which particles leave a volume of unit height after entering the volume within the length y_2.

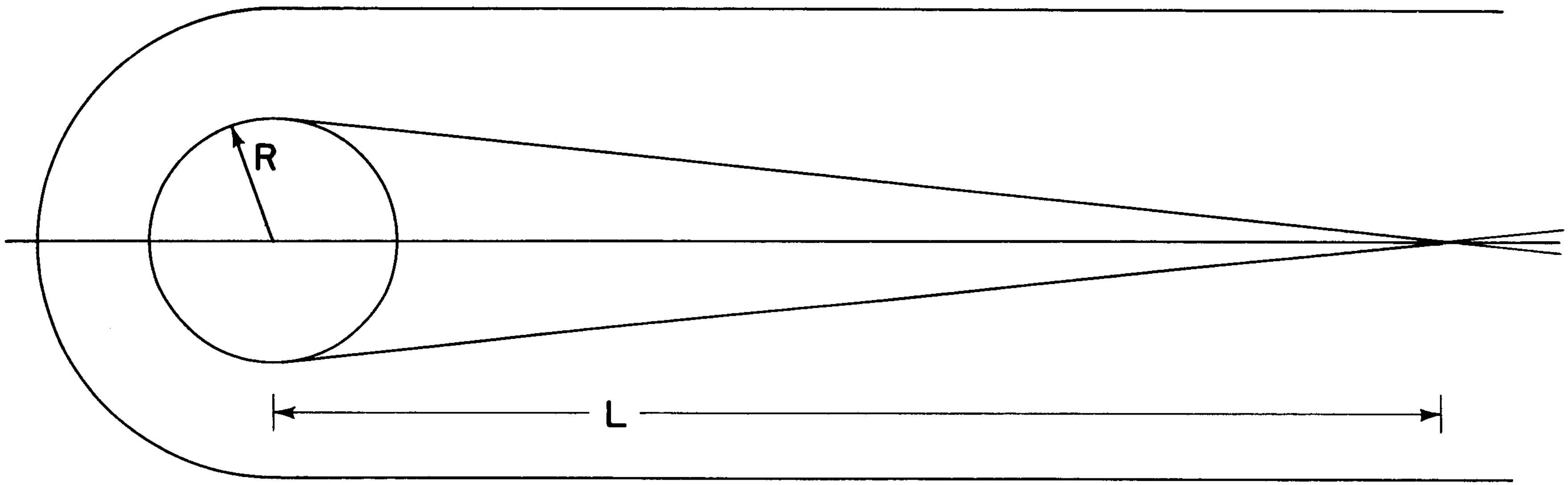

Figure 3 - The shock structure behind a cylinder of radius R. The length of the vacuum downstream from the cylinder $L \sim R V_o/(v_o^2 - v_m^2)^{1/2}$.

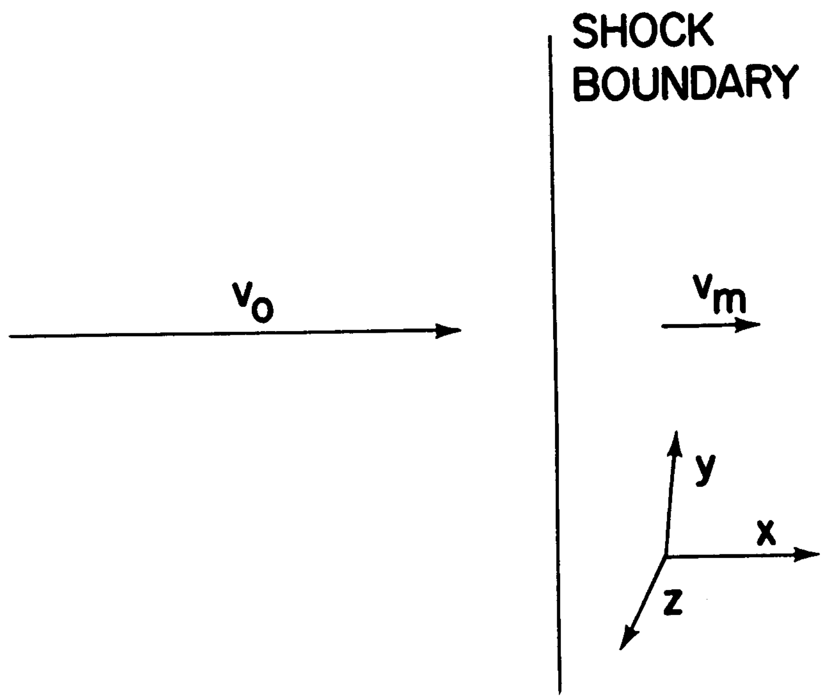

Figure 4 - The coordinate system used in analyzing particle motion in a magnetic field parallel to the z axis whose field lines travel with a velocity v_o to the left of the shock boundary and with velocity v_m to the right of the shock boundary.

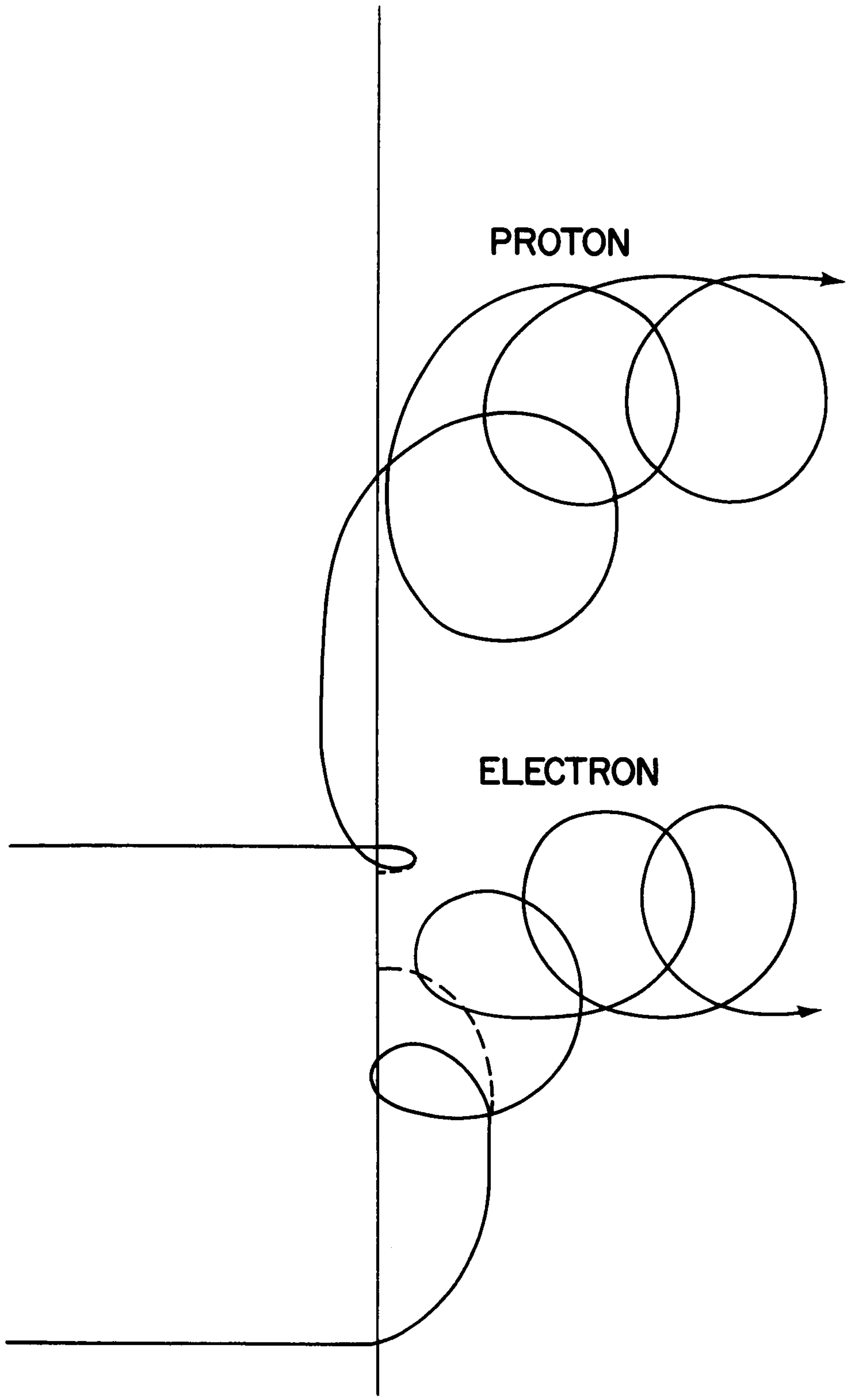

Figure 5 - Electron and proton trajectories initially traveling with a moving magnetic field which is slowed down and compressed, downstream from the shock boundary.

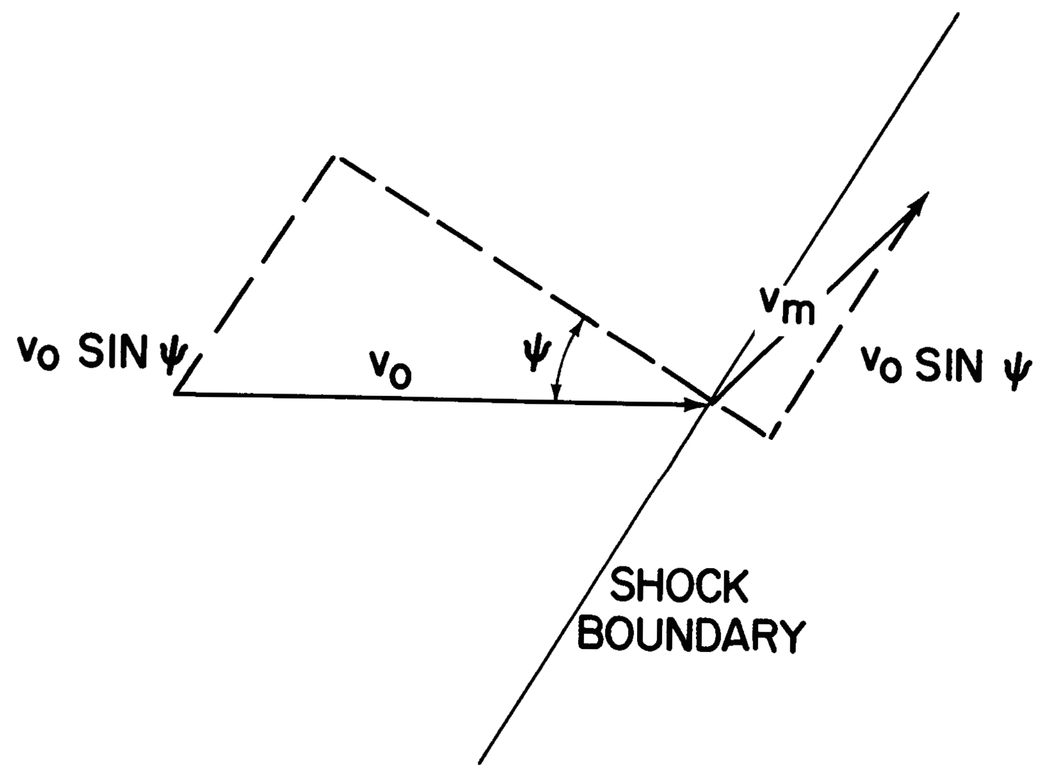

Figure 6 - The geometry of the guiding center or field line motion on both sides of the shock boundary for oblique incidence.

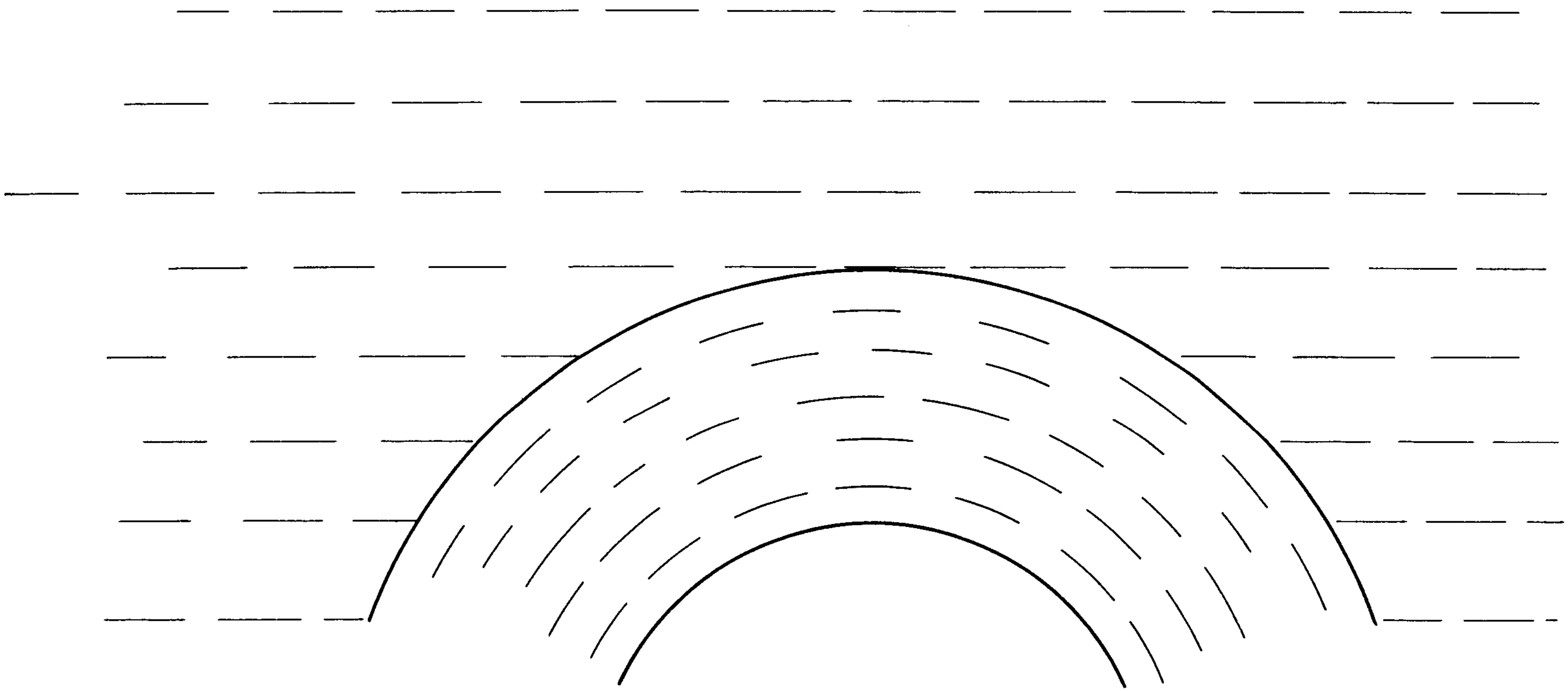

Figure 7 - Magnetic field lines indicated by dashed lines in free interplanetary space and in the transition sheath in the plane containing the field direction.

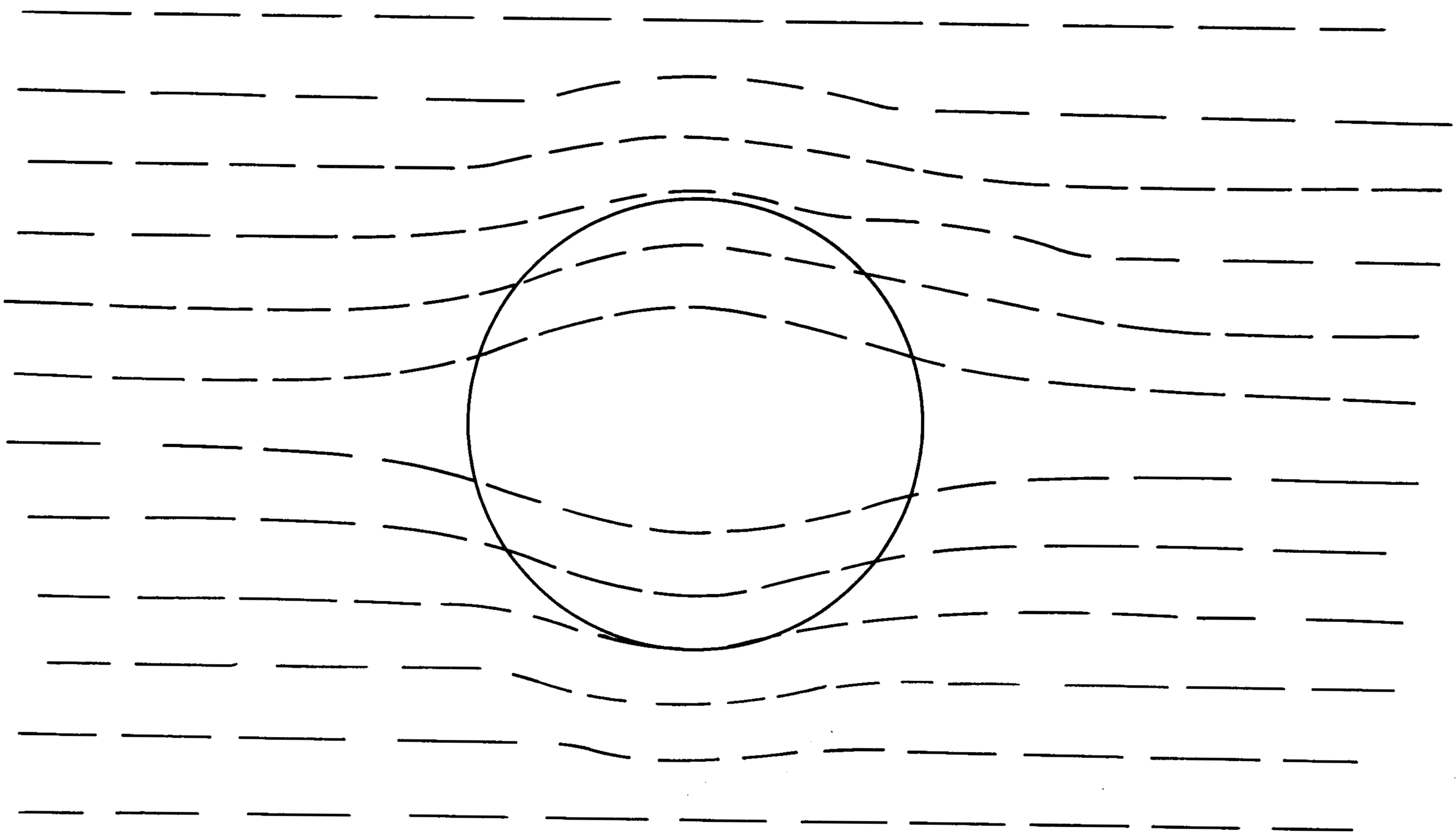

Figure 8 - Magnetic field lines, indicated by dashed lines, viewed in projection from the upstream side as they diverge and slide around a spherical obstruction bowing out most pronouncedly in the equatorial plane.