NASA TECHNICAL NOTE

NASA TN D-3019

c.1

NASA TN D-3019

STUDY OF DIRECT-RETURN RENDEZVOUS TRAJECTORIES USING CONSTANT-THRUST ENGINES AND RENDEZVOUS OR MANEUVERING THRUSTERS

by James L. Williams and Herman S. Fletcher

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1965

NASA TN D-3019

STUDY OF DIRECT-RETURN RENDEZVOUS TRAJECTORIES USING CONSTANT-THRUST ENGINES AND RENDEZVOUS OR MANEUVERING THRUSTERS

By James L. Williams and Herman S. Fletcher

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – Price $1.00

STUDY OF DIRECT-RETURN RENDEZVOUS TRAJECTORIES USING CONSTANT-THRUST ENGINES AND RENDEZVOUS OR MANEUVERING THRUSTERS

By James L. Williams and Herman S. Fletcher
Langley Research Center

SUMMARY

An analytical study has been made to determine the effect of using two separate engines in the lunar excursion module or lander for direct-return rendezvous with a vehicle in an 80-nautical-mile circular orbit around the moon. A high-thrust engine was used for ascent to establish a nonimpact transfer orbit. After a period of coast, a low-thrust engine was used to complete rendezvous. Only coast trajectories with pericynthion altitudes greater than 25 000 feet (7620 meters) were considered, so that in the event of failure of the low-thrust engine the lander would follow a nonimpact orbit.

It was found that characteristic velocity (a measure of the fuel consumption), angular distance traveled to rendezvous, and the magnitude of the separation angle between orbiter and lander at lift-off were primarily functions of the initial thrust-weight ratio. The magnitude of the low thrust used for rendezvous had little effect on these parameters in terms of the complete direct-return rendezvous trajectory. From the viewpoint of man-machine consideration, the increased thrust durations associated with low-thrust engines permit more precise control during the final thrusting phase of the direct-return rendezvous.

INTRODUCTION

Several approaches to performing the manned lunar mission have been under consideration for some time. Of these approaches the lunar orbit rendezvous has been selected by this nation as the primary method of accomplishing the lunar exploration mission. The lunar-orbit-rendezvous operation involves a rendezvous with a vehicle in orbit around the moon. Many studies have been made to determine the implication of the required rendezvous on various phases of the lunar mission. A summary of some of these studies is presented in reference 1.

Some consideration has been given to the use of single-stage constant-thrust engines for performing the ascent operation from the surface of the moon to establish a circular orbit. (See ref. 2.) However, constant-thrust engines are not particularly efficient for use from launch to orbit (refs. 3 and 4). This inefficiency suggests the use of a high-thrust engine to establish a non-impact trajectory (ballistic transfer orbit) and a low-thrust engine for rendezvous. The purpose of this paper is to examine the lunar take-off to rendezvous phase of the lunar mission and to determine the characteristics and efficiency of two engines with different thrust levels.

SYMBOLS

The units used for the physical quantities defined in this section are given in both U.S. Customary Units and in the International System of Units (SI). (See ref. 5.)

F thrust, pounds (newtons)

F_o initial thrust of lander at lunar surface, pounds (newtons)

g_e gravitational acceleration at surface of earth, 32.2 feet/second^2 (9.8 meters/second^2)

g_m gravitational acceleration at surface of moon, 5.32 feet/second^2 (1.62 meters/second^2)

h altitude, feet (meters)

h_p altitude at pericynthion, feet (meters)

I_{sp} specific impulse, 305 seconds

m mass of lander at any point in trajectory, slugs (kilograms)

m_f mass of fuel used, slugs (kilograms)

m_o initial mass of lander at lunar surface, slugs (kilograms)

r radial distance from center of moon, feet (meters)

$\dot{r}$ radial velocity component, feet/second (meters/second)

$\ddot{r}$ radial acceleration, feet/second^2 (meters/second^2)

r_m radius of moon, 5 702 000 feet (1.738×10^3 kilometers)

t time, seconds

t_f time during which rocket is firing, seconds

ΔV	characteristic velocity, $I_{sp}g_e \log_e \frac{m_o}{m_o - m_f}$, feet/second (meters/second)
$(\Delta V)_{asc}$	characteristic velocity for ascent, feet/second (meters/second)
$(\Delta V)_{ren}$	characteristic velocity for rendezvous, feet/second (meters/second)
W	weight of lander, pounds (newtons)
W_o	initial weight of lander at lunar surface, $m_o g_e$, pounds (newtons)
β	thrust vector angle (measured from local horizontal), degrees or radians
γ	vehicle flight-path angle, degrees
θ	angular travel over lunar surface, degrees or radians
θ_c	angular travel for coast, degrees or radians
θ_R	total range angle to rendezvous, $\theta_1 + \theta_c + \theta_3$, degrees or radians
$\dot{\theta}$	angular rate, radians/second or degrees/second
$\ddot{\theta}$	angular acceleration, radians/second^2 or degrees/second^2
$\Delta\theta$	separation angle between orbiter (control module and service module) and lander (lunar excursion module) at lift-off, degrees

Subscripts:

1	conditions at thrust cut-off prior to coast
2	conditions at thrust initiation
3	conditions after final thrusting period

ANALYSIS

In this investigation an examination is made of the rendezvous trajectories obtained by utilizing high-thrust engines for lift-off and low-thrust engines for rendezvous or orbit establishment. All computations were made with the restricted two-body equations of motion. In addition to the lunar gravity, these equations included the vehicle thrust forces. The moon was assumed to be a homogeneous sphere with a radius of 5 702 000 feet $(1.738 \times 10^3$ kilometers) and a surface gravity acceleration of 5.32 feet per second per second

(1.62 meters per second per second). The following equations of motion were used:

$$\ddot{r} - r\dot{\theta}^2 = \frac{F}{m}\sin\beta - g_m\left(\frac{r_m}{r}\right)^2 \tag{1}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -\frac{F}{m}\cos\beta \tag{2}$$

where

$$m = m_o + \int \dot{m}\, dt_f \tag{3}$$

and

$$\dot{m} = \frac{F}{I_{sp}g_e} \tag{4}$$

These equations of motion were solved on an electronic digital computer. The directions of the angles and vectors involved are shown in figure 1.

The procedure used in this study was to initiate numerical integration of equations (1), (2), and (3) from circular orbit conditions with increasing mass. In the coast phase (no thrust applied) the standard orbit equations for motions in a central force field were used to determine orbit characteristics at various altitudes. An iteration process was used to obtain desired end conditions of near-zero velocity in the vicinity of the lunar surface. A complete trajectory was obtained by patching the coast phase (ballistic transfer orbit) with the appropriate thrusting phase. These trajectories are used as rendezvous trajectories from the lunar surface to a vehicle in a circular orbit.

Several trajectories from the lunar surface can be used to effect rendezvous with a vehicle in orbit. In the present study the direct-return rendezvous maneuver was of primary interest. The direct rendezvous trajectory simulated herein involves three phases: ascent, coast, and rendezvous. The ascent phase was a gravity-turn thrusting maneuver from lift-off to thrust cut-off. The position and velocity conditions at thrust cut-off were such that the pericynthion altitude of the resulting coast phase (ballistic transfer orbit) was greater than 25 000 feet (7620 meters).

After an interval of flight under zero thrust conditions a low thrust was initiated parallel to the velocity vector from a position along the coast phase. The gravity-turn trajectory associated with this powered portion of the flight terminated when rendezvous was attained with the orbiting vehicle in an 80-nautical-mile circular orbit. An illustration of the direct ascent to rendezvous trajectory is shown in figure 2. In the present study rendezvous trajectories were obtained for selected values of $\frac{F_o}{W_o}$ and $\frac{F_2}{F_o}$. Values of $\frac{F_o}{W_o}$ for the ascent phase (first thrusting mode) and of $\frac{F_2}{F_o}$ for the rendezvous mode (second thrusting mode) are as follows:

First gravity-turn maneuver $\left(\frac{F_o}{W_o}\right)$: 0.352, 0.431, 0.465, 0.509

Second gravity-turn maneuver $\left(\frac{F_2}{F_o}\right)$: 0.01 to 0.04, 0.2 to 0.3, 1.0

The smaller values of $\frac{F_2}{F_o}$ simulated the expected thrust characteristics of the maneuvering thrusters.

Two pericynthion altitudes were examined: approximately 31 000 feet and 55 000 feet (9448.8 and 16 764 meters). A specific impulse of 305 seconds was assumed for the computations. Also, it was assumed that there were no engine restarts except for $\frac{F_2}{F_o} = 1.0$. The rendezvous trajectories were examined in terms of characteristic velocity, angular separation between the orbiter and lander at lift-off, angular distances traveled to rendezvous, and thrusting time requirements for powered flight.

RESULTS AND DISCUSSION

Direct-return rendezvous trajectories are examined to determine the feasibility of using high-thrust engines for the powered ascent mode and relatively low-thrust engines for the rendezvous mode. The ascent mode is followed by a coast phase (ballistic transfer orbit).

Rendezvous Trajectories

Selected trajectories are presented in figure 3 in the form of altitude plotted against angular range over the lunar surface. Results are presented for two values of initial thrust-weight ratio $\frac{F_o}{W_o}$ and two values of final-to-initial thrust ratio $\frac{F_2}{F_o}$. Trajectories corresponding to values of $\frac{F_o}{W_o}$ of 0.352 and 0.509 for the ascent phase and values of $\frac{F_2}{F_o}$ of 0.03 and 0.02 for the rendezvous phase are presented in figure 3(a). These $\frac{F_2}{F_o}$ values simulated rendezvous using maneuvering thrusters. The pericynthion altitude of the coast phase associated with these lunar take-off trajectories was about 31 000 feet (9448.8 meters). In figure 3(b) trajectories are presented for $\frac{F_o}{W_o}$ values of 0.352 and 0.465 for the ascent mode and $\frac{F_2}{F_o}$ values of 0.3 and 0.2 for the rendezvous mode. These $\frac{F_2}{F_o}$ values simulated engines different from and larger than the maneuvering thrusters. The pericynthion altitude of the coast phase associated with these lunar take-offs was about 55 000 feet (16 764 meters). It can be seen from figure 3 that for a gravity-turn ascent an important factor associated with the magnitude of the angular range to rendezvous is the combination of $\frac{F_o}{W_o}$ and $\frac{F_2}{F_o}$ used in the direct ascent maneuver. A more detailed plot of this effect is given in figure 4, wherein $\frac{F_2}{F_o}$ is shown as a function of θ_R for values of $\frac{F_o}{W_o}$ of 0.352, 0.431, and 0.509. The results shown in this figure correspond to a 31 000-foot (9448.8 meters) pericynthion altitude ballistic trajectory. A study of the figure shows that $\frac{F_o}{W_o}$ was a prime factor in determining the magnitude of the angular range for direct ascent to rendezvous. The change in θ_R is due directly to a change in the length of the ballistic trajectory which increased with an increase in $\frac{F_o}{W_o}$. Somewhat smaller changes in θ_R can also be effected through variation of $\frac{F_2}{F_o}$. This effect can also be seen in figure 4 for constant $\frac{F_o}{W_o}$ values, wherein the decrease in $\frac{F_2}{F_o}$ from about 0.035 to 0.01 increased the total range angle by about 5^o. The range of $\frac{F_2}{F_o}$ presented in figure 4 was assumed to be representative of maneuvering thrusters.

The total range angle can be expressed as the sum of the angular distance traveled in the coast and thrusting phases. The effects of range angles associated with the thrusting phases (gravity turn) on initial thrust-weight ratio $\frac{F_o}{W_o}$ and on thrust-weight ratio at the end of coast $\frac{F_2}{W_2}$ are shown in figures 5(a) and 5(b), respectively. The magnitudes of the angles presented in figure 5 are for the range from lunar lift-off to thrust termination for ascent (fig. 5(a)) and for thrust initiation after coast to rendezvous (fig. 5(b)). As can be seen in figure 5(a) the angular range for ascent θ_1 was relatively insensitive to a change in pericynthion altitude from 31 000 to 55 000 feet

(9448.8 to 16 764 meters). Also, a change in $\frac{F_o}{W_o}$ directly changed the angular travel of the ascent. An indication of range angle magnitude for the second thrusting mode can be obtained from figure 5(b) where it can be seen that a change in $\frac{F_2}{W_2}$ from 0.03 to 0.01 extended the second-thrusting-mode range angle from 5^o to 15^o. As would be expected, the results of figure 5(b) exhibited trends similar to the results for the ascent mode in figure 5(a). These results, however, are not unexpected since small-size engines require longer thrusting periods to attain specified end conditions.

The angular velocity of a vehicle in an 80-nautical-mile circular orbit is about 3^o per minute. For direct-return rendezvous, the combinations of $\frac{F_o}{W_o}$ and $\frac{F_2}{F_o}$ specify that lift-off be initiated with specific separation angles between orbiter and lander. The effect of $\frac{F_2}{F_o}$ on separation angle $\Delta\theta$ is shown in figure 6 for $\frac{F_o}{W_o}$ values of 0.352, 0.431, and 0.509. The pericynthion altitude of the ballistic trajectory was about 31 000 feet (9448.8 meters). Negative values of $\Delta\theta$ indicate orbiter location behind the lander at lift-off. Since the thrust-weight ratio is proportional to acceleration, it would be expected that relatively large values of $\frac{F_o}{W_o}$ would result in smaller values of $\Delta\theta$ than those obtained for smaller values of $\frac{F_o}{W_o}$; this can be seen in figure 6 where an increase in $\frac{F_o}{W_o}$ from 0.352 to 0.509 decreased the separation angle $\Delta\theta$ from about 8^o to less than 1^o. A study of these results (fig. 6) for constant $\frac{F_o}{W_o}$ show that varying the magnitude of $\frac{F_2}{F_o}$ had only a small effect on $\Delta\theta$.

Characteristic Velocity

An indication of the efficiency of performing the rendezvous maneuver with certain combinations of $\frac{F_o}{W_o}$ and $\frac{F_2}{F_o}$ values can be obtained from a study of the characteristic velocity (a measure of fuel comsumption). The characteristic velocities calculated for the ascent and rendezvous gravity-turn maneuvers are plotted against $\frac{F_o}{W_o}$ and $\frac{F_2}{F_o}$ in figure 7. The significant points to be noted from figure 7(a) are that the gravity-turn ascent was more efficient (less ΔV) when larger values of $\frac{F_o}{W_o}$ were used than when lower values of $\frac{F_o}{W_o}$ were used and that an increase in ΔV of about 10 feet per second (3 meters per second) was associated with the increase in pericynthion altitude from 31 000 to 55 000 feet (9448.8 to 16 764 meters). The characteristic velocities for the second thrusting (rendezvous) mode are presented for $\frac{F_2}{F_o}$ values between 0.01 and 1.0 in figure 7(b).

The results in figure 7(b) $\left(h_p = 31\ 000\text{ feet or }9448.8\text{ meters}\right)$ show that the magnitude of ΔV for rendezvous was generally independent of $\frac{F_2}{F_o}$. The magnitude of the characteristic velocity required for the rendezvous maneuver is about 2 percent of that required for ascent from the lunar surface.

The total characteristic velocity required for a direct-return rendezvous with a nonimpact coast trajectory $\left(h_p = 31\ 000\text{ feet or }9448.8\text{ meters}\right)$ can be estimated from figure 7. As an example, assume a value of $\frac{F_o}{W_o}$ of 0.400 for the ascent gravity turn and a value of $\frac{F_2}{F_o}$ of 0.300 for the second gravity turn. From figure 7(a) a ΔV value of 6070 feet per second (1850.1 meters per second) is obtained for ascent and from figure 7(b) a ΔV value of 101.6 feet per second (30.97 meters per second) is obtained for the second thrust mode. A total characteristic velocity of 6171.6 feet per second (1881.1 meters per second) then is obtained for the direct-return maneuver.

Thrusting Time Requirement

The direct ascent-to-rendezvous maneuver considered in this study consisted of two separate gravity-turn thrusting modes connected by a nonimpact ballistic trajectory. Each gravity turn was characterized by a specific thrust duration. The magnitude of these thrust durations is of interest since certain types of engines have thrusting time restrictions because of cooling problems. An indication of the magnitude of the thrusting period associated with 31 000- and 55 000-foot-pericynthion-altitude (9448.8 and 16 764 meters) coast phases can be seen in figures 8, 9, and 10. The thrust duration for the initial gravity-turn maneuver is presented in figure 8 and that associated with the second thrusting maneuver in figure 10.

From the viewpoint of human response, the results in figure 10 suggest that low-thrust engines with their extended thrust duration would permit more precise control during the second thrusting mode than high-thrust engines. The coast period associated with the two powered trajectories is presented in figure 9. In this figure, the initial thrusting period t_1 is plotted against the coast period t_2 for several values of the final thrusting period. The larger values of t_3 (319 seconds, 159 seconds, and 106 seconds) were obtained for simulated rendezvous using the maneuvering thrusters, whereas the value of t_3 of 15 seconds (for the coast trajectory with a 55 000-foot (16 764 meters) pericynthion altitude) was obtained for simulated rendezvous using engines considerably larger than the maneuvering thrusters.

CONCLUDING REMARKS

An analytical study has been made to determine the effect of using two separate engines in a lander for direct-return rendezvous with a vehicle in an 80-nautical-mile circular orbit around the moon. All rendezvous trajectories utilized two thrusting periods. An initial thrusting period during which high thrust was utilized for ascent to establish a nonimpact coast trajectory and a second thrusting period during which low thrust or maneuvering thrusters were used to complete rendezvous. Only coast trajectories with pericynthion altitudes greater than 25 000 feet (7620 meters) were considered so that in the event of failure of the low-thrust engine the lander would follow a nonimpact orbit. The procedure used in this study was to examine the direct-return rendezvous trajectories in terms of characteristic velocity, relative angular separation of the vehicles at lift-off, angular distance traveled to rendezvous, and the thrusting time requirements for powered flight.

It was found that characteristic velocity (a measure of the fuel consumption), angular distance traveled to rendezvous, and the magnitude of the separation angle between orbiter and lander at lift-off were primarily functions of the initial thrust-weight ratio. The use of relatively small maneuvering thrusters for rendezvous had, in general, little effect on these parameters in terms of the complete direct ascent trajectory. From the viewpoint of man-machine consideration, the increased thrust durations associated with low-thrust engines permit more precise control during the final thrusting phase of the direct-return rendezvous operation.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., July 29, 1965.

REFERENCES

1. Houbolt, John C.; Bird, John D.; and Queijo, Manuel J.: Guidance and Navigation Aspects of Space Rendezvous. Proceedings of the NASA-University Conference on the Science and Technology of Space Explorations, Vol. 1, NASA SP-11, 1962, pp. 353-366. (Also available as NASA SP-17.)

2. Burns, Rowland E.; and Singleton, Larry G.: Ascent From the Lunar Surface. NASA TN D-1644, 1963.

3. Queijo, M. J.; and Miller, G. Kimball, Jr.: Analysis of Two Thrusting Techniques for Soft Lunar Landings Starting From a 50-Mile Altitude Circular Orbit. NASA TN D-1230, 1962.

4. Williams, James L.; and Barker, L. Keith: Analytical Study of Lunar Landing Trajectories With Reference to the Lunar-Orbit-Rendezvous Mode and Possible Abort Situations. NASA TN D-2016, 1963.

5. Mechtly, E. A.: The International System of Units - Physical Constants and Conversion Factors. NASA SP-7012, 1964.

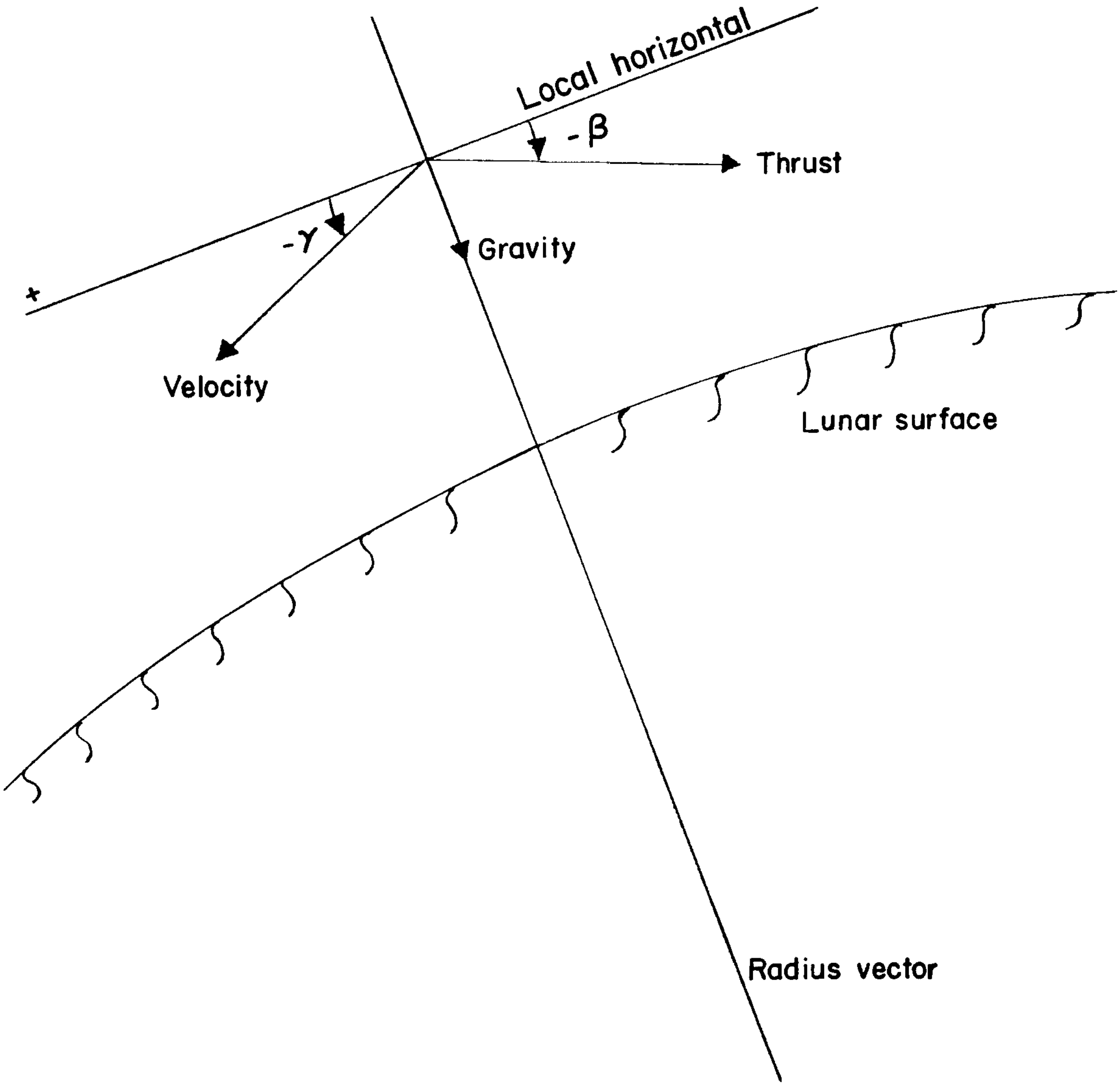


Figure 1.- Illustration of angles and vectors.

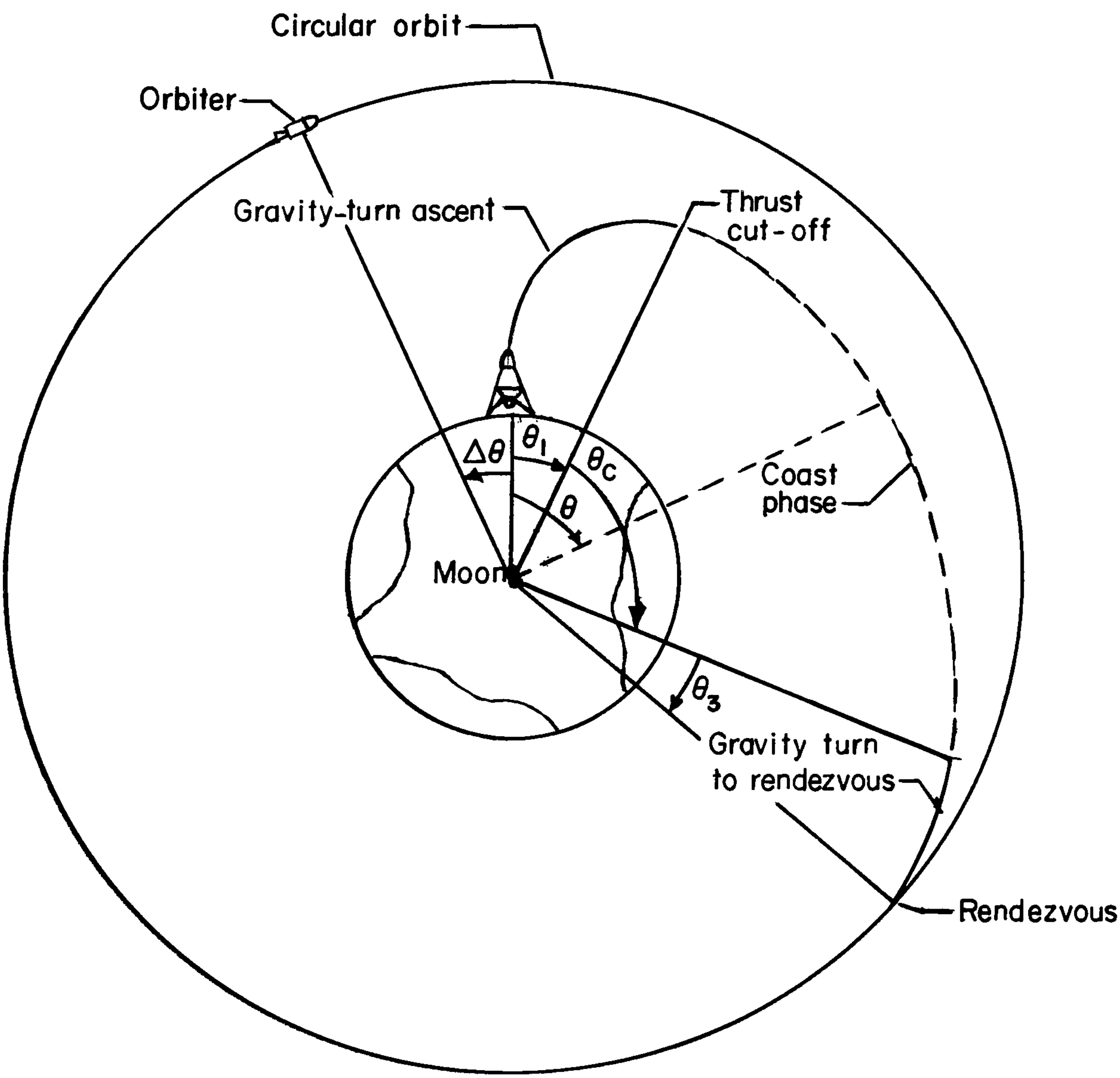
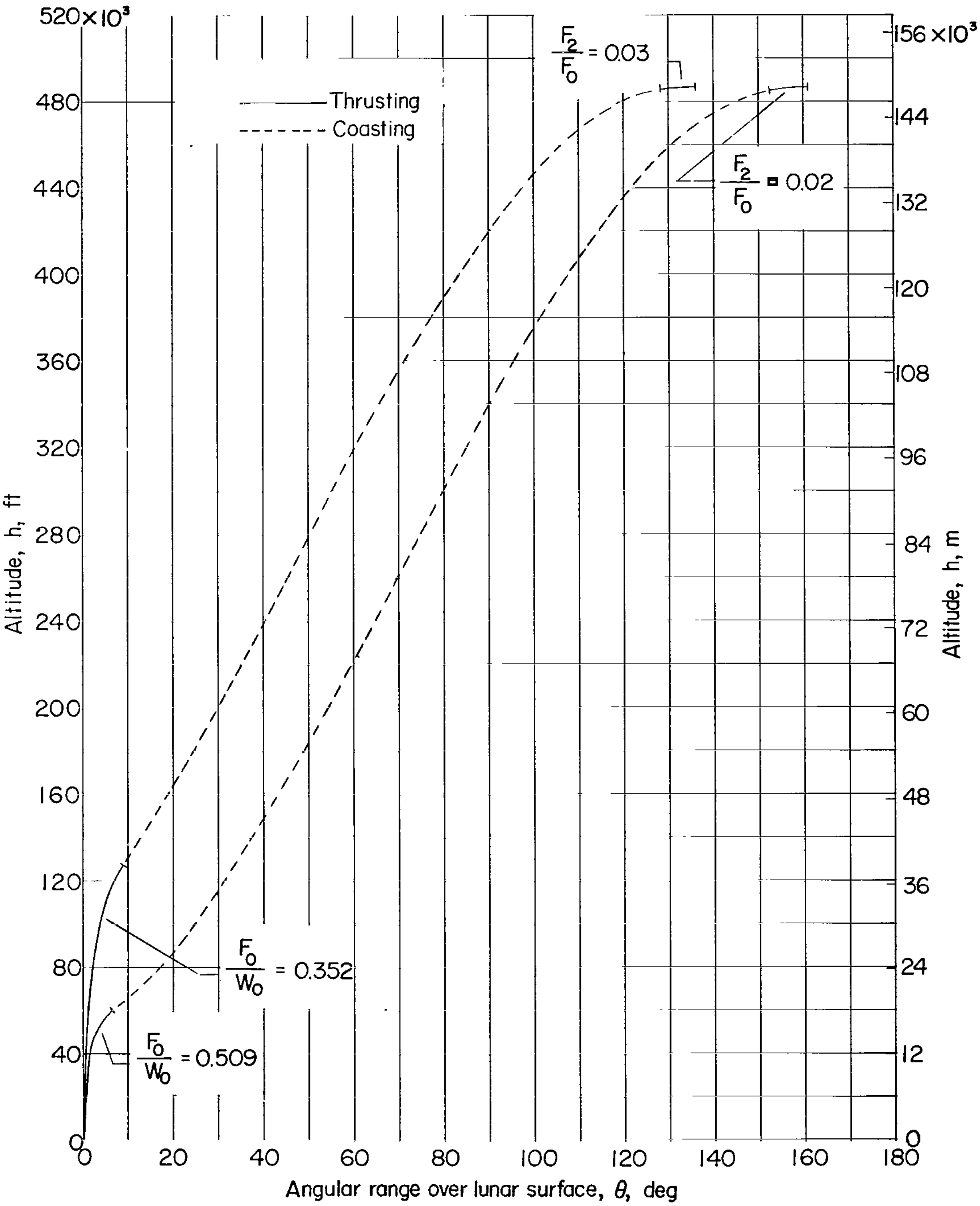
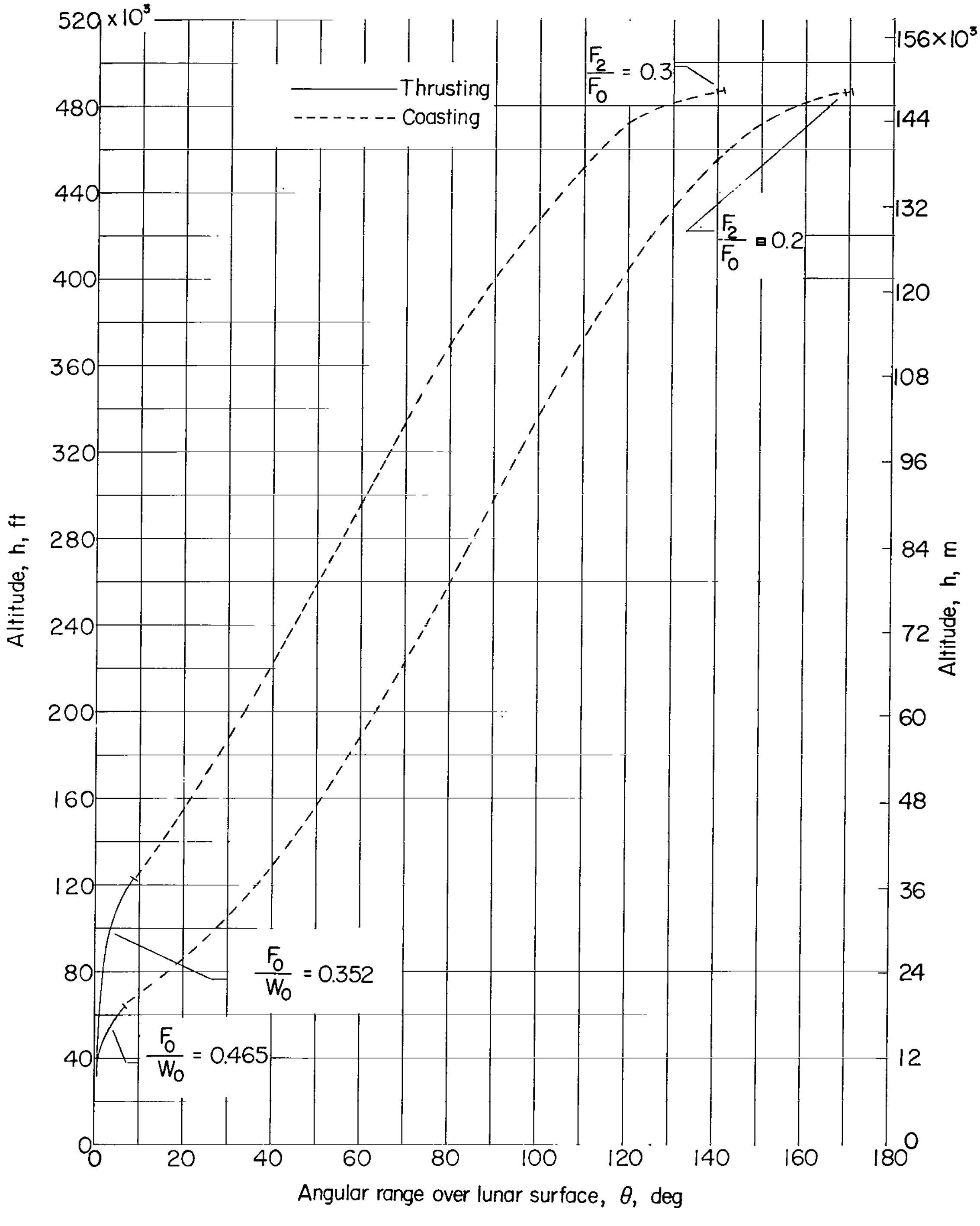


Figure 2.- Typical ascent to rendezvous trajectory.



(a) h_p = 31 000 feet (9448.8 meters).

Figure 3.- Variation of altitude with range angle for selected rendezvous trajectories.



(b) h_p = 55 000 feet (16 764 meters).

Figure 3.- Concluded.

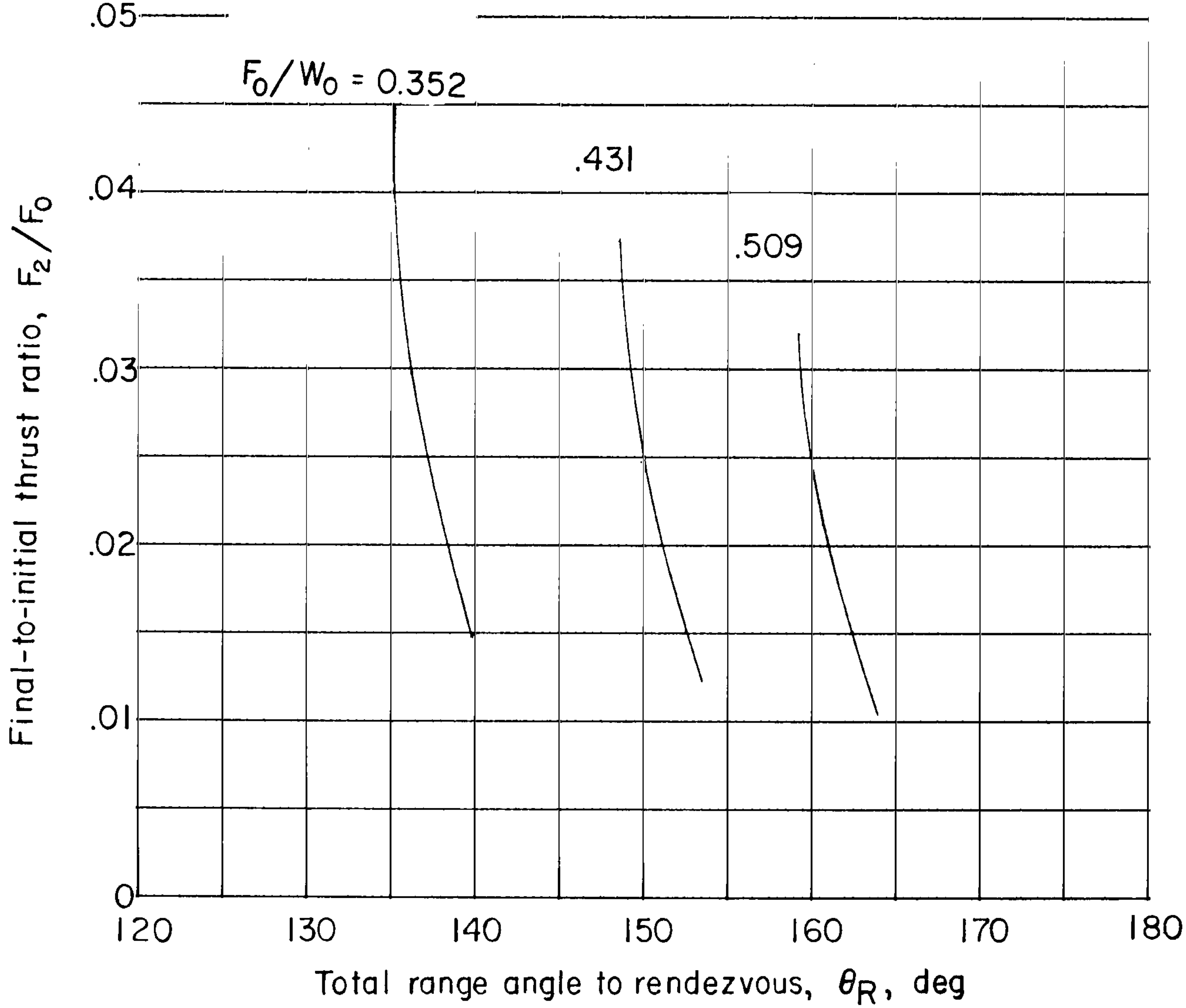
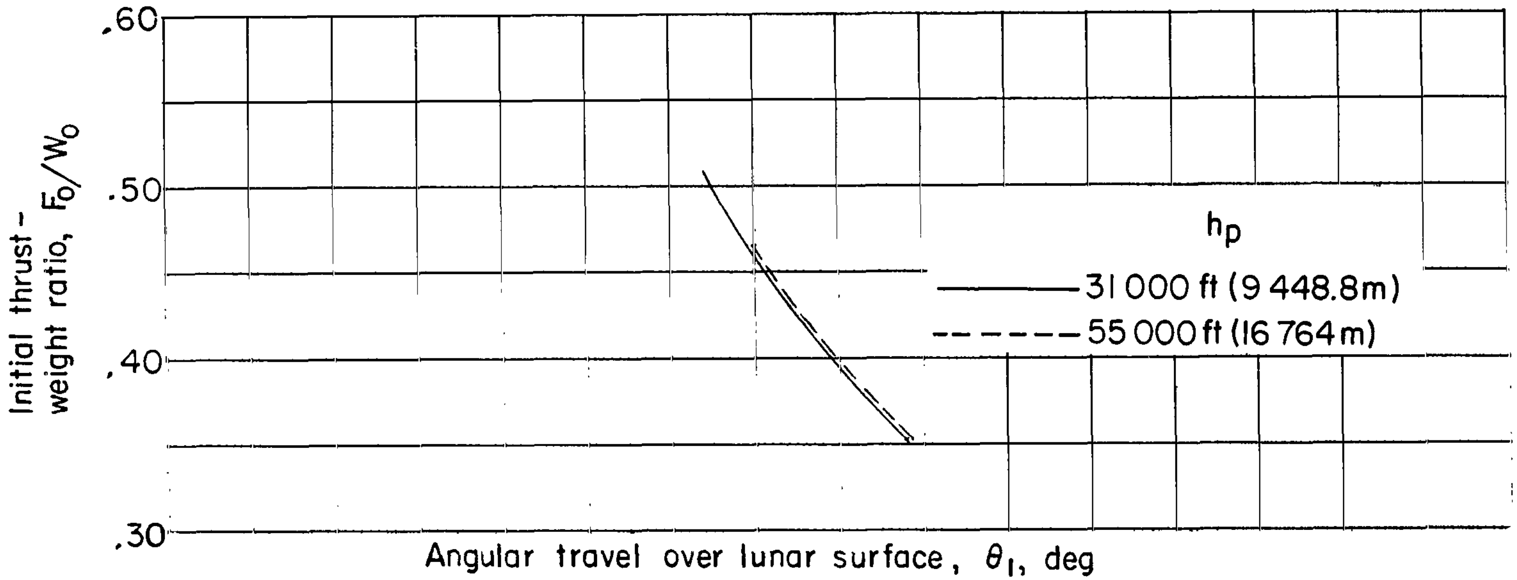
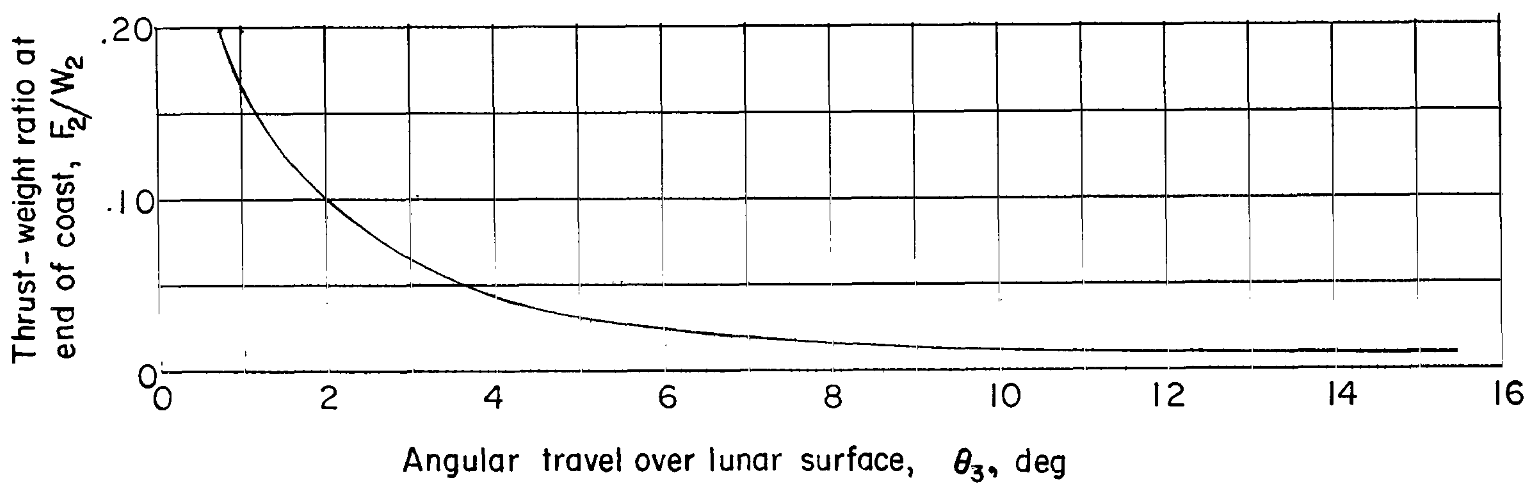


Figure 4.- Effect on total range angle to rendezvous of final-to-initial thrust ratio for several values of initial thrust-weight ratio. h_p = 31 000 feet (9448.8 meters).



(a) Ascent.



(b) Rendezvous.

Figure 5.- Effect on angular travel over lunar surface of initial thrust-weight ratio and of thrust-weight ratio at end of coast.

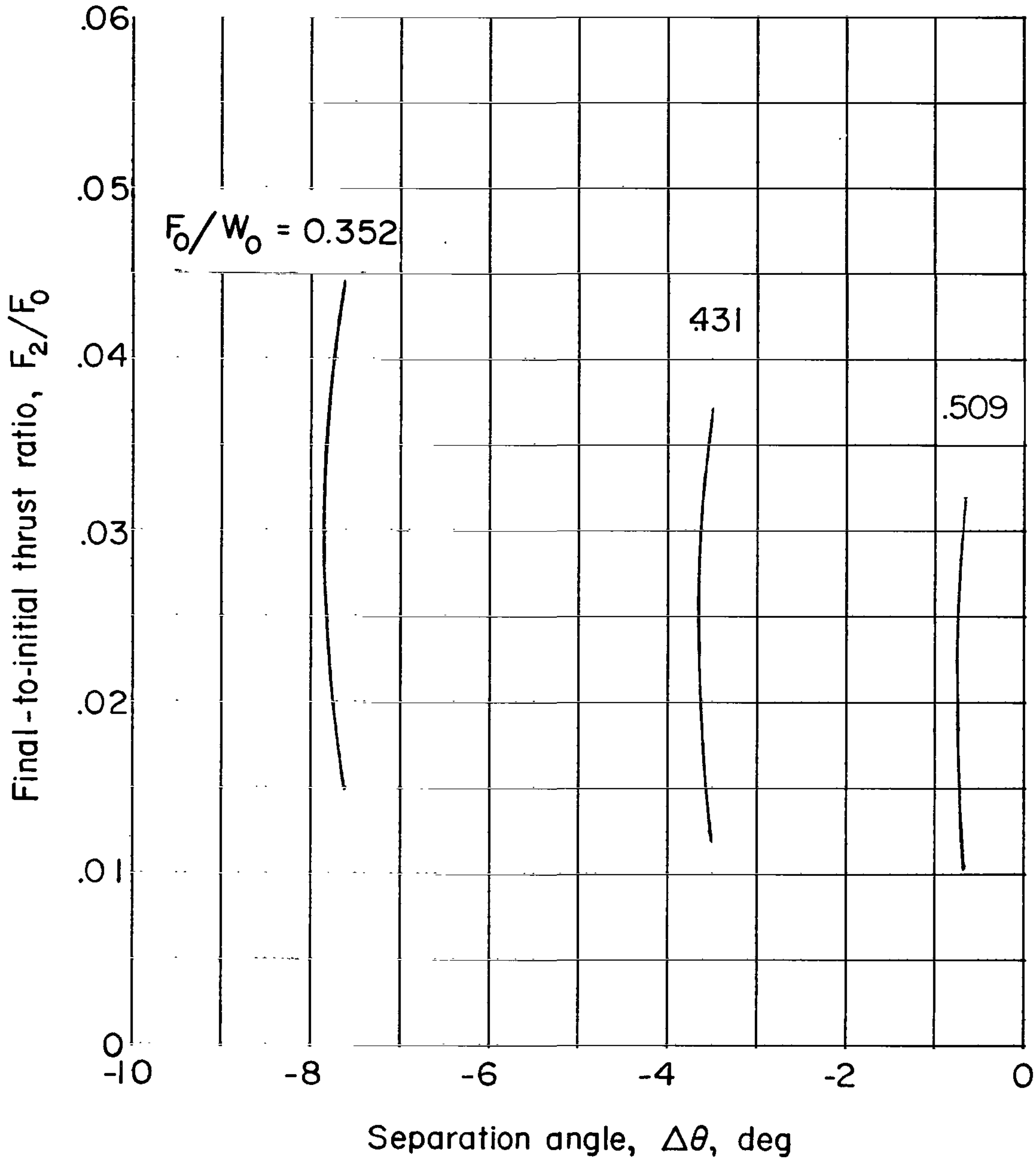
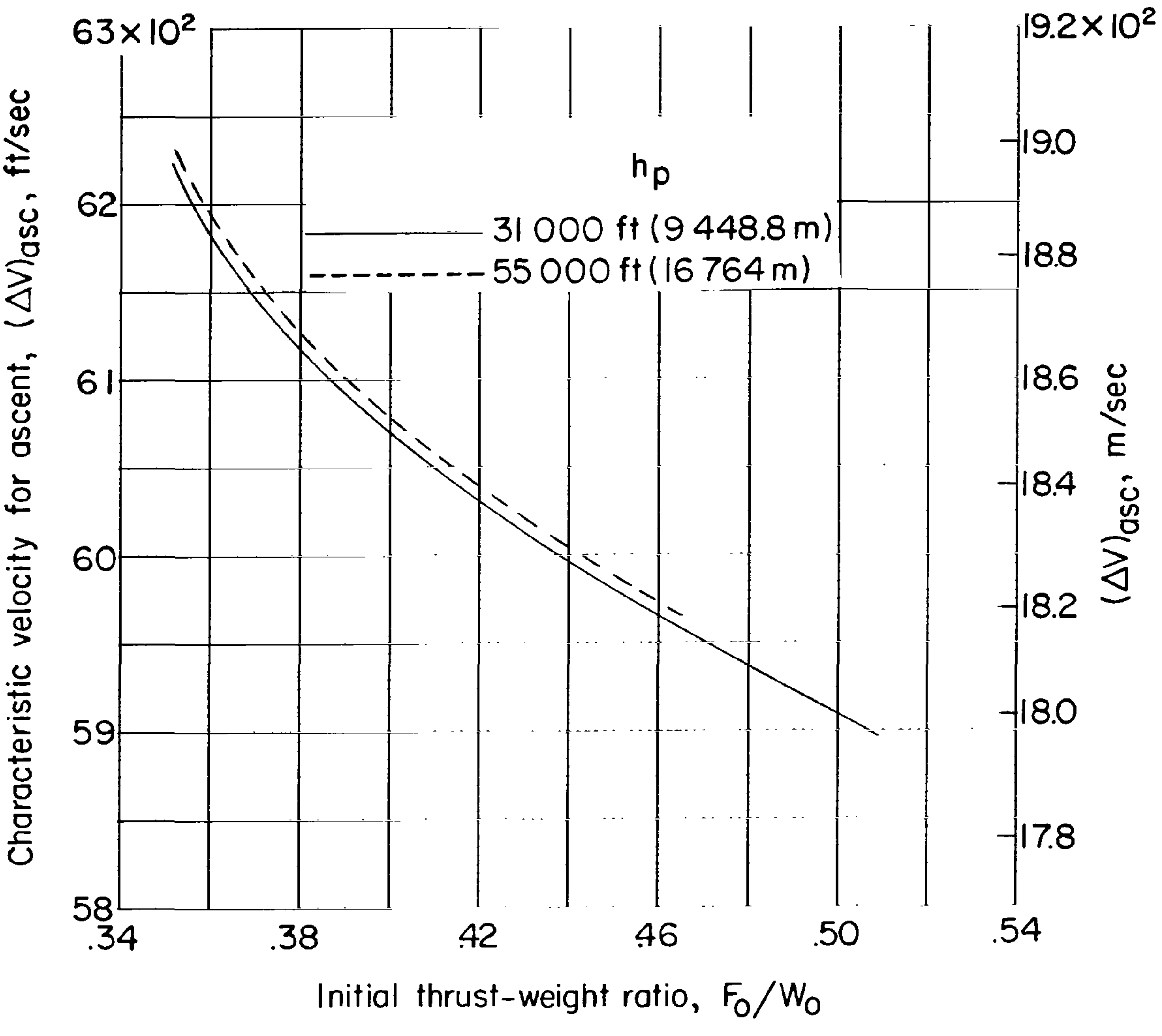
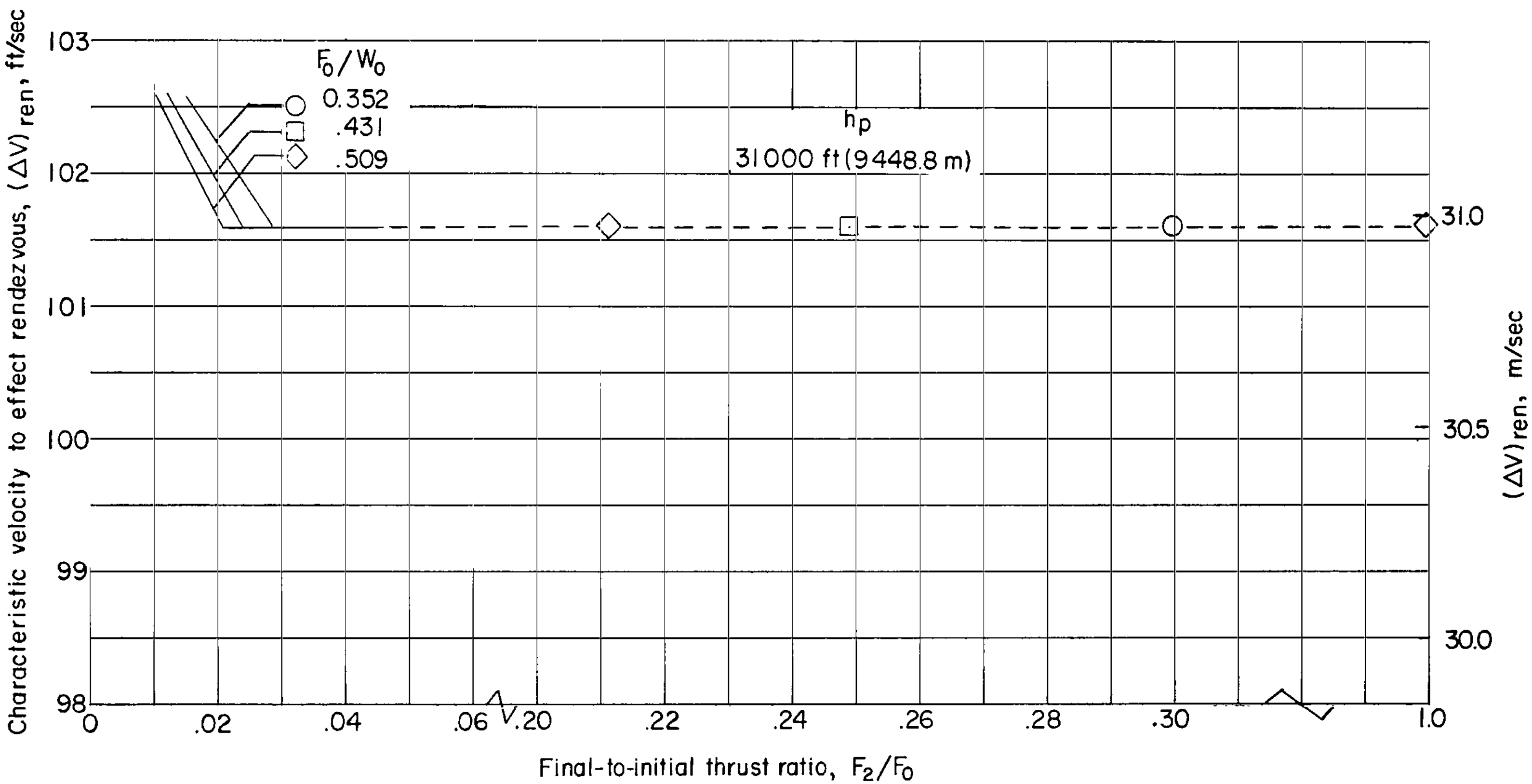


Figure 6.- Variation of final-to-initial thrust ratio with separation angle between orbiter and lander at lander lift-off. h_p = 31 000 feet (9448.8 meters).



(a) Ascent.

Figure 7.- Variation of characteristic velocity with initial thrust-weight ratio and final-to-initial thrust ratio.



(b) Rendezvous.

Figure 7.- Concluded.

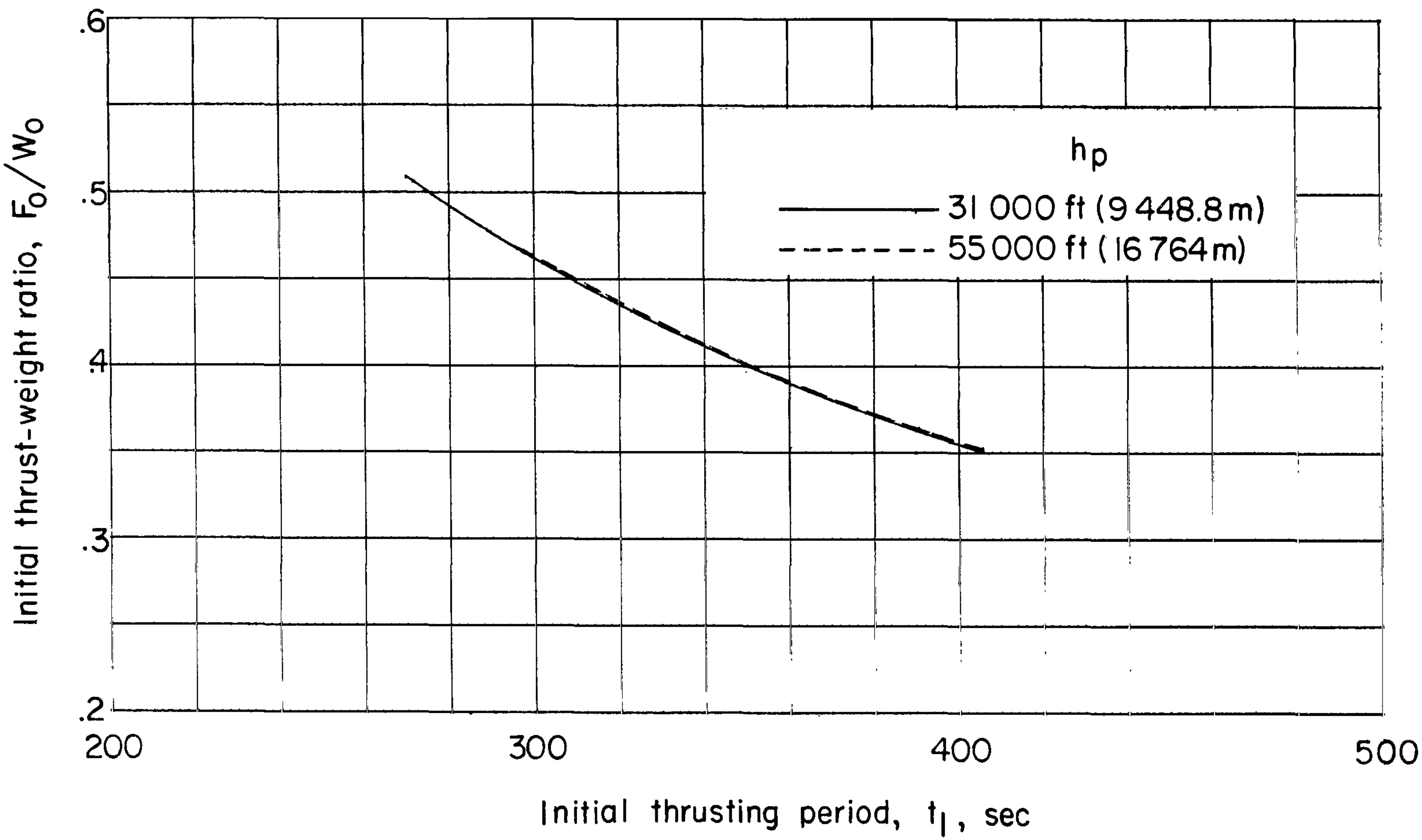


Figure 8.- Variation of initial thrust-weight ratio with initial thrusting period for ascent mode.

NASA-Langley, 1965 L-4251

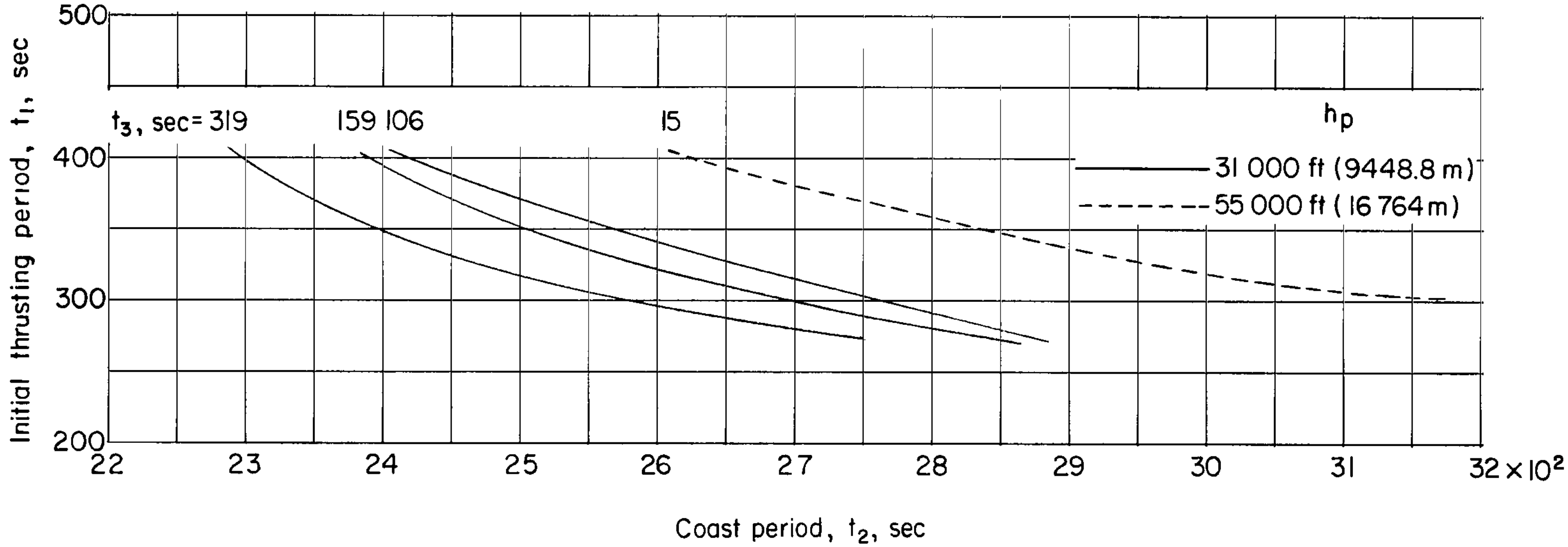


Figure 9.- Variation of initial thrusting period with coast time for several final thrusting or rendezvous periods.

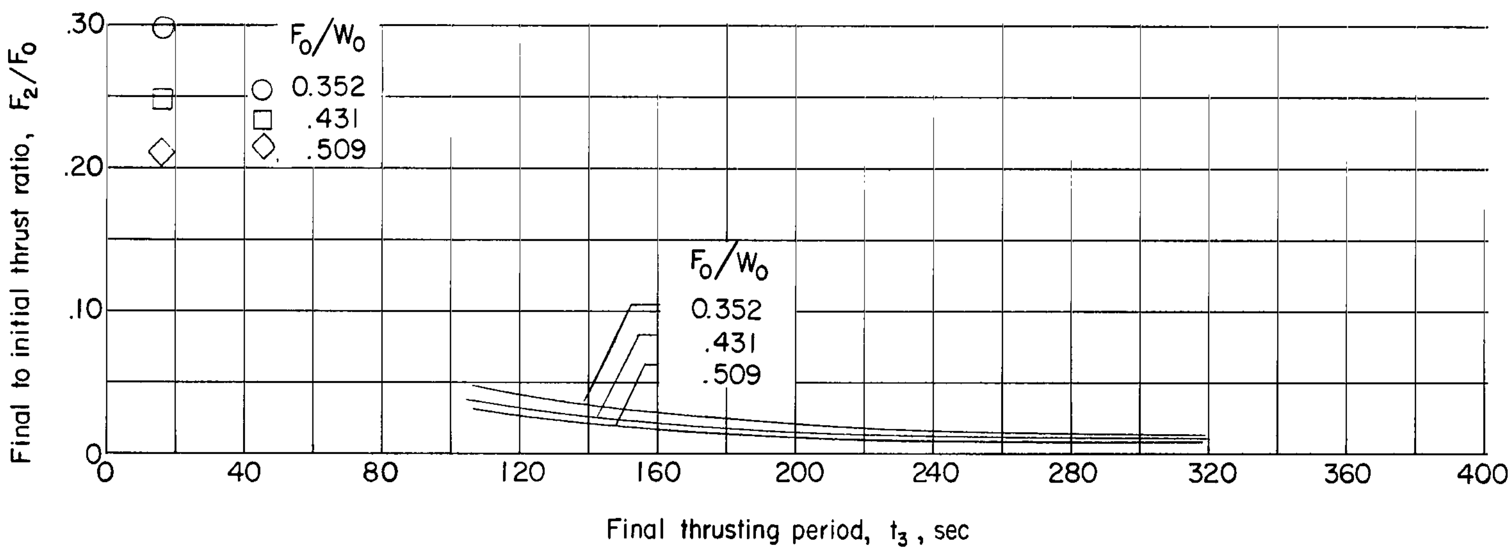


Figure 10.- Variation of final thrusting period with final-to-initial thrust ratio for rendezvous mode. h_p = 31 000 feet (9448.8 meters).

3/18/85

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.