NASA TECHNICAL MEMORANDUM

NASA TM X-53319
August 16, 1965

N65-35977

(ACCESSION NUMBER) (THRU)

46 (PAGES) 1 (CODE)

(NASA CR OR TMX OR AD NUMBER) 30 (CATEGORY)

FACILITY FORM 602

NASA TM X-53319

CONIC SOLUTIONS TO THE INTERPLANETARY TRANSFER PROBLEM WITH CONSTANT OR MINUMUM INJECTION ENERGY

by LAMAR E. BULLOCK
Aero-Astrodynamics Laboratory

GPO PRICE $ ________

CSFTI PRICE(S) $ ________

Hard copy (HC) 2.00

Microfiche (MF) .50

ff 653 July 65

NASA

George C. Marshall Space Flight Center, Huntsville, Alabama

TECHNICAL MEMORANDUM X-53319

CONIC SOLUTIONS TO THE INTERPLANETARY TRANSFER PROBLEM WITH CONSTANT OR MINUMUM INJECTION ENERGY

By

Lamar E. Bullock

ABSTRACT

George C. Marshall Space Flight Center
Huntsville, Alabama

ABSTRACT

35477

This report describes a method for calculating minimum and constant injection energy, C_3, values for conic interplanetary trajectories. The planets are assumed to move on mutually inclined elliptical orbits about the sun. Lambert's theorem is used to solve the time-constraint on the two-point boundary value problem, and an iterative procedure, with arrival date as the isolation parameter, is used to determine the desired C_3.

Typical results are presented for Earth to Jupiter trajectories in the 1970-71 launch period. The comparisons made with previously used methods indicate the procedures described in the report produce results of comparable accuracy, with a savings in computation time.

Author

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER

TECHNICAL MEMORANDUM X-53319

August 16, 1965

CONIC SOLUTIONS TO THE INTERPLANETARY TRANSFER PROBLEM WITH CONSTANT OR MINIMUM INJECTION ENERGY

By

Lamar E. Bullock

ASTRODYNAMICS AND GUIDANCE THEORY DIVISION
AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

ACKNOWLEDGEMENT

The author would like to express his sincere appreciation to Dozier Johnson of the MSFC Computation Laboratory for preparing the Marshall Interplanetary Conic Trajectory Program utilizing the procedures explained in this report.

TABLE OF CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS (CONT'D)

LIST OF SYMBOLS

Symbol	Definition
a	Semi-major axis
C_3	Energy of launch planet conic
C_3A	Energy of arrival planet conic
DT	Flight time
DAA	Declination of arrival asymptote
DLA	Declination of launch asymptote
e	Eccentricity
ei	Inclination of planet's equatorial plane to its orbital plane
E	Eccentric anomaly
i	Mutual inclination of two heliocentric orbits
M	Mean anomaly
n	Mean motion (mean angular velocity)
pp	Square root of semilatus rectum
r	Orbital radius
$\dot{r}$	Radial velocity
RAA	Right ascension of arrival asymptote
RLA	Right ascension of launch asymptote
T	Date (Julian - 2,400,000)
t	"Normalized" flight time
t_p	Date of perigee passage
v	Velocity
w	True anomaly of ascending node

LIST OF SYMBOLS CONT'D

Symbol	Definition
x, y, z	Components of position
$\dot{x}, \dot{y}, \dot{z}$	Components of velocity
α	Planet's angular distance from ascending node
η	Angle between radius vector and chord joining departure and arrival points
θ	True anomaly
$\dot{\theta}$	Angular velocity
θ_x	True anomaly of arrival planet at transition between Type I and Type II trajectories
μ	Sun's gravitational constant
ξ	Angle between line from empty focus to planet and chord joining departure and arrival points
τ	Orbital period
τ'	True anomaly of planet's "Vernal Equinox"
Φ	Central angle of flight
Ψ	Path angle of heliocentric velocity vector

SUBSCRIPTS

Symbol	Definition
j	Indicates launch planet
k	Indicates arrival planet
$_1$	Refers to launch position on transfer ellipse
$_2$	Refers to arrival position on transfer ellipse
	Non-subscripted variables are elements of the transfer ellipse.

TECHNICAL MEMORANDUM X-53319

CONIC SOLUTIONS TO THE INTERPLANETARY TRANSFER PROBLEM WITH CONSTANT OR MINIMUM INJECTION ENERGY

SUMMARY

The procedures for calculating constant and minimum injection energy, C_3, solutions to the time-constrained two-point boundary value problem of interplanetary flight are presented. The boundary values, position and velocity of the launch planet at launch time and target planet at arrival time, are determined using mean elliptical elements to describe the path of the planets. The time-constraint on the problem is solved using a modified form of Lambert's theorem. The desired injection energy is then determined using an iterative procedure with arrival position as an isolation parameter. The methods used to assure the computation of proper type and class trajectory are described.

Curves are included showing minimum and constant C_3 for trajectories from Earth to Jupiter in the 1970-71 synodic period with flight time, central angle, inclination of transfer plane, right ascension of launch asymptote, declination of launch asymptote, and arrival energy as dependent variables with launch date as independent variable. A discussion of the modification required to solve other interplanetary problems of current interest is also included. A detailed comparison of the computer program utilizing these procedures with other methods in use indicates the accuracy is the same, but this method uses considerably less computer time.

SECTION I. INTRODUCTION

The basic problem presented in this report is the time-constrained two-point boundary value problem of calculating the trajectory which under the influence of only the sun's gravitational force connects the position of the launch planet at launch time to the position of the arrival planet at arrival time. The methods used to solve this problem differ only slightly from those used in earlier work [2,3]; however, the way in which the solutions are presented and the way the solutions are used to advantage in solving additional problems are unique.

The planetary orbits are represented by inclined elliptical orbits, and a form of Lambert's theorem [9] is used to solve the time-constraint. After the conic transfer trajectory is determined, the injection energy is calculated in terms of the impulsive velocity increment required to get from the planetary orbit to the transfer orbit. The object of the procedure to be discussed is to obtain a trajectory requiring a given injection energy. The position of the target planet at arrival is varied as an isolation parameter to obtain the desired injection energy. Normally, it is desirable to solve the problem over a range of launch dates, and it is possible to use the results of each launch date as a basis to extrapolate the first guess to the solution at the next launch date.

A brief discussion of the modifications which must be made to the procedure in order to solve additional problems of current interest is also included. It is felt that the main advantages of the procedures presented are speed, high versatility, and ability to solve the problem in terms of injection energy. Comparisons with some other procedures will be made to emphasize these points.

The description of the problem and the methods used to solve it will, as much as possible, be made without the use of mathematical equations. However, a complete list of the equations used to solve this problem and an indication of how they were obtained will be found in the Appendix.

SECTION II. PLANETARY COORDINATES

It is desirable to designate the launch date; and therefore, it is necessary to have some method of determining the planet's positions and velocities as a function of time. The most accurate method used involves interpolation between values in an ephemeris [3,4]. However, a method which is sufficiently accurate for the conic transfer, and a great deal faster, involves the use of mean conic elements for the planetary orbits [2,4].

It is necessary to have five elements for each planet. The ones chosen are time at perihelion (t_p), semi-major axis (a), eccentricity (e), inclination (i) of launch planet's orbital plane to that of the target planet, and the true anomaly of the ascending node (w) of each planet with respect to the other. From these five basic elements, all additional heliocentric constants which may be required can be determined.

SECTION III. INTERPLANETARY TRAJECTORY

Once the positions of the launch and target planets have been established, the problem is to determine if there exists an elliptical trajectory about the sun which will connect the two points in the required time. There is a one parameter family of ellipses which pass through the two points with the sun at their focus. Considering only direct or counterclockwise flight of less than 360°, at most one of these ellipses will have the required flight time. Lambert's theorem is used to select the required ellipse. The over-all geometry of the planetary transfer is shown in Figure 1.

Lambert's theorem expresses the transfer time, t, as a transcendental function of the radius to the launch and target planets (r_s and r_k), the central angle of flight (Φ), and the semi-major axis of the transfer ellipse (a). A complete description of Lambert's theorem and its use is given by Breakwell [2]; however, the following description should be sufficient to provide a general understanding of the theorem. To facilitate the solution of the problem, two parameters are defined. The first parameter, K, has a range of 0 to 1 and depends only on the planetary geometry (i.e., $K = f_1(r_s, r_k, \Phi)$). The second parameter, E, has a range of -1 to 0 and depends on the semi-major axis of the transfer ellipse as well as the planetary geometry (i.e., $E = f_2(r_s, r_k, \Phi, a)$). The transfer time, t, is then expressed as a function of the two parameters, K and E (i.e., $t=f(K,E)$). The function f is normally expressed in terms of inverse trigonometric functions, but for values of E near 0, and t small, trigonometric formulation is neither accurate nor the most rapid. Thus for E greater than -.2 and t less than t_b a series formulation for f is used. The quantity t_b is explained in the following paragraph.

Plots of t versus E for various values of K are shown in Figure 2. The minimum time for an elliptical transfer is the time, t_a, at E=0. Transit times smaller than t_a require a hyperbolic transfer. As shown in the figure, for values of K greater than 0, there are four values of t for each value of E. The following rules are used to determine which value is correct for each problem. Branch 1 is used for central angles less than 180° and Branch 2 for central angles greater than 180°. The time, t_b, can be calculated by setting E=-1; then the lower portion of the curve is used if t is less than t_b and the upper portion if t is greater than t_b. Since each portion of the curve is monotonic, a two-point interpolation scheme can be used by providing that interpolated values of E are restricted to the range -1 to 0. The first interpolation is made between the points $(-1, t_b)$ and (E_1, t_1) where E_1 is a first guess at the solution and t_1 is the associated value of t. Henceforth, the interpolation is made between the last two points calculated until the calculated value of t is within the desired tolerance of the actual t. Once this condition is reached, the transfer ellipse is completely determined.

SECTION IV. INJECTION ENERGY

The injection energy, C_3, is the energy of the launch planet centered conic. It can be calculated as the square of the hyperbolic excess velocity vector (v_n). This velocity vector is defined, as shown in Figure 3, as the difference between the launch planet velocity vector (v_p) and the velocity on the transfer ellipse at launch time $(\bar{v}_t)$. Two implicit assumptions being made here are that the velocity addition is impulsive and that it is made at the center of the launch planet. The error introduced by these assumptions is sufficiently small for the purpose of this problem.

Graphs of various trajectory parameters versus launch date with C_3 as a parameter, such as those in Figures 12 through 17, have been very useful in mission planning projects and in projects involving general surveys of interplanetary trajectories. Previously, these curves were generated by running a large number of trajectories at discrete intervals of flight time on each of many launch dates. Then the constant C_3 can be established using a curve fit interpolation scheme. This interpolation must be performed on each trajectory parameter on each launch date. It seems, however, that it is more efficient and desirable to calculate these constant C_3 curves directly.

It is interesting and necessary to calculate the minimum injection energy for a given launch date. The first reason is that minimum injection energy could be useful in accomplishing a given mission with maximum payload, or in preliminary planning stages, it could give an idea of the absolute minimum energy characteristics required of the launch vehicle. The second reason is that in order to calculate constant C_3 values over a range of launch dates, it is necessary to know first, for each of the days, that the minimum C_3 for that day is less than or equal to the constant C_3 value being investigated. A plot of C_3 versus arrival planet's position (θ_K), for a given launch date, is shown in Figure 4. As shown, there are two relative minimums. The trajectories for central angles less than 180° are called Type I and those with a central angle greater than 180° are called Type II [3]. Thus, there is a Type I and a Type II minimum.

The method used to calculate the minimum is to take a first guess at the θ_K which will give the minimum C_3. Then the C_3 corresponding to $\theta_K \pm \Delta$, where Δ is a small increment on θ_K, is calculated. Next, a parabola is passed through these three points, and the minimum point on this parabola is used as the next guess at θ_K. A restriction is placed on θ_K so that it will provide an elliptical trajectory of the proper type. The two lowest of the last three points plus the new point are again fit with a parabola to obtain a better guess at θ_K, and the procedure is repeated until the minimum is obtained to a satisfactory accuracy. Once the solution is obtained, it is saved for use in extrapolating the first guess on θ_K at the next launch date. The minimums for successive launch dates are calculated over the required range of launch dates. These minimums determine the curves shown in Figures 5 through 11.

As indicated above, the launch date range of constant C_3 curves is determined from the minimum C_3 curves. For example, it might be desirable to calculate a Type I curve of 100 km^2/sec^2 for an Earth-to-Jupiter transit in the 1971 synodic period. Referring to Figure 5, Type I trajectories with a $C_3 = 100$ km^2/sec^2 are available only between January 11 and February 28. For launch dates outside this range, a higher C_3 is required. To illustrate how a point on a given C_3 curve is calculated, it will again be necessary to refer to Figure 4 where it can be seen that there are two points with a C_3 of 100 km^2/sec^2 for each type trajectory at the indicated launch date. These two solutions are identified as Class I and Class II, Class I always being the trajectory with the shorter flight time. It is apparent that the two classes are coincident for the minimum C_3 on any launch date. The slope of the C_3 curve is used to determine which class is which. If the slope is negative, the solution is Class I; if the slope is positive, the solution is Class II.

The method used to isolate a given C_3 value is to take a first guess at θ_K and to calculate the C_3 for this θ_K and for $\theta_K+\Delta$. Then a linear interpolation scheme is used between the last two points calculated until the desired C_3 is established within the given tolerance. During this iteration the points are restricted such that the proper type and class trajectory will be computed. Again, the solution from one case is used to establish the first guess at the solution to the following case. Figures 12 through 17 show typical plots of launch date versus various dependent variables for several C_3 values.

SECTION V. RESULTS AND CONCLUSIONS

Extensive comparisons between this computer program and the JPL and Lockheed programs were made during the development and checkout of this deck. The JPL program used for comparison was their Heliocentric Conic Program [3] which uses ephemerides to determine the positions and velocities of the planets. It was found that this deck requires about one second to compute each interplanetary trajectory. The Lockheed program used for comparison was their Medium-Accuracy Interplanetary Transfer Program [2] which uses mean conic elements of the planets to determine the planets' positions and velocities. It was found that this program requires about .1 second for each interplanetary trajectory. Table 1 summarizes the deviation of nine parameters for 100 arbitrarily chosen trajectories from Earth to Venus, Mars, and Jupiter. The only restriction placed on these trajectories was that the central angle of flight not be between 170° and 190°. This was done because the rate of change of energy and transit plane inclination with respect to planetary position is very high in the near 180° central angle region. Thus, small discrepancies in planetary position, which do appear due to the different methods of calculating position, cause large discrepancies in energy and inclination. This difficulty is not felt to be a major drawback, first, because even though the values are not the same, the trends are the same in all three decks. Secondly, the problems occur at values of C_3 much higher than those which are practically feasible.

Table 2 shows a comparison between these three conic programs and the JPL Space Trajectories Program which is a very accurate n-body integration program. The trajectory parameters chosen for comparison give an indication of the accuracy of the conic programs at both the injection and terminal ends of the trajectory. The approximate computation times given in the chart indicate the tremendous savings in time accomplished by using conic calculations. The time per trial given for the integrating deck is for a single trajectory calculated with a given set of initial conditions. Using initial conditions from one of the conic decks, the integrating deck will isolate the desired terminal condition in about 20 trials. If less accurate initial conditions are used, more trials will be required.

The graphs of Figures 5 through 17 are examples of the type of material that can be produced directly from the output of this program. Each of the graphs with constant C_3 curves was drawn using 450 points with a total computation time on the IBM 7094 of about 31 seconds. The minimum C_3 curves used an additional 84 points and required an additional 16 seconds of computation time. It is estimated that it would require at least 1000 trajectories to produce these same curves by the old method of interpolating between points in a matrix of values. Additional computations would also be required to perform the interpolation for constant values of C_3. It is apparent then that not only is the program more efficient at solving the basic interplanetary transfer problem, but, additionally, it is more efficient at solving the over-all problem of determining constant C_3 curves because of the new approach to the problem.

SECTION VI. OTHER USES

With minimum modification, the principles used in this procedure can be used to solve other interplanetary trajectory problems of interest. For interplanetary orbiting or landing missions, total velocity increment, that is, injection velocity plus braking velocity at the target planet, is of more interest than C_3. Thus, by calculating this total velocity and using it in place of C_3, the same procedure can be used to generate minimum and constant total velocity data. In much the same way, payload mass at the target planet can be approximated by impulsive calculations if the characteristics of the vehicle system are known.

Round-trip fly-by missions and "grand tour" missions, where no major propulsion is used after injection, can also be handled quite easily. For example, suppose it is desirable to fly a round-trip fly-by mission to Mars. The first step is to calculate minimum energy requirements for Earth-to-Mars and for Mars-to-Earth trajectories. Then the constant C_3 range for launch at Earth can be chosen and a trajectory from Earth-to-Mars with the desired C_3 can be established. Now the energy of the Mars centered conic at arrival can be determined, and this can be used to calculate a Mars-to-Earth leg with the same energy, provided this energy is not less than the minimum energy for this particular Mars passage date. The hyperbolic excess velocity vector of approach to Mars is then rotated into the hyperbolic excess velocity vector of departure by calculating the proper radius vector from the center of Mars to the pericenter of the Mars centered conic. This same procedure could be used for "grand tour" type trajectories except that more than two legs would be involved, and the conics would have to be matched at each of the planet passages.

It is felt that the speed and flexibility of this program will make it very useful in approximating trajectories for any interplanetary mission, or, in a broader sense, in calculating free flight transfer trajectories between any two orbits around any central body.

REFERENCES

1. Aero-Astrodynamics Laboratory, "Flight Path, Performance and Tracking," MTP-M-63-2.3, dated October 30, 1963, pages 3 - 33.

2. Breakwell, J. V., Gillespie, R. W., Ross, S., "Researches in Interplanetary Transfer," ARS Journal, p. 201-208, February 1961.

3. Clarke, V. C., Bollman, W. E., Roth, R. Y., Scholey, W. J., "Design Parameters for Ballistic Interplanetary Trajectories. Part 1, One-Way Transfers to Mars and Venus," Jet Propulsion Laboratory, Technical Report No. 32-77, 16 January 1963.

4. H. M. Nautical Almanac Office, "Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac," London, 1961.

5. H. M. Nautical Almanac Office, "Planetary Co-ordinates for the Years 1960-1980, London, 1958.

6. Martin Company for MSFC, "Orbital Flight Handbook - Vol. 1," 1963.

7. Milne, W. E., "Numerical Calculus," Princeton University Press, Princeton, New Jersey, 1949.

8. NASA, SP-35, Part 1, Space Flight Handbooks - Vol. 3, Planetary Flight Handbook, 1963.

9. Plummer, H. C., "An Introductory Treatise on Dynamical Astronomy," Dover Publications, Inc., N. Y., 1960.

TABLE 1

DEVIATIONS BETWEEN MARSHALL, LOCKHEED, AND JPL CONIC PROGRAMS

	EARTH - VENUS			EARTH - MARS			EARTH - JUPITER		
	Marshall JPL	Marshall Lockheed	JPL Lockheed	Marshall JPL	Marshall Lockheed	JPL Lockheed	Marshall JPL	Marshall Lockheed	JPL Lockheed
C_3	.1424	2.4037	2.4458	.1412	.2561	.3100	.2219	.5445	.4192
Φ	.07746	1.4592	1.4046	.05472	.1992	.2440	.2012	.5134	.3175
RLA	.5128	2.2296	1.8338	.4137	.1883	.5288	.3493	.2923	.08790
DLA	.3091	1.7957	1.9947	1.0325	.2536	1.1735	.6331	.2703	.4455
θ_1	.06	.9976	.9556	.1216	.3121	.4315	.2107	.5294	.3310
a	.03516	.1980	.1909	.02418	.04030	.03998	.2123	.3922	.5186
e	.0003589	.001990	.001925	.0003276	.0003276	.0005377	.0000860	.0003658	.0003277
i	.01877	.4353	.4174	.01181	.04862	.05961	.1429	.3621	.2216
C_3A	.3408	3.6454	3.7453	.1446	.1558	.2139	.1579	.2245	.1707

$$\text{Deviation} = \sqrt{\frac{\sum_{i=1}^{N} (x_i - x_i')^2}{N}}$$

where x_i is the value of the parameter for the first deck, x_i' is the value for the second deck, and N is the number of trajectories used. (Earth-Venus, N=25; Earth-Mars, N=33; Earth-Jupiter, N=42.)

TABLE 2

COMPARISON BETWEEN N-BODY AND CONIC CALCULATIONS

TRAJECTORY IDENTIFICATION	Parameter	JPL N-Body	JPL Conic	Marshall Conic	Lockheed Conic
Earth to Venus Launch on Aug. 3, 1970 With Flight Time of 130 Days	C_3 RLA DLA C_3A	9.60 242. - .500 30.6	9.52 242. - .732 31.2	9.39 241. - 1.40 30.6	9.95 245. - 3.80 32.3
Earth to Venus Launch on Sep. 2, 1970 With Flight Time of 180 Days	C_3 RLA DLA C_3A	12.6 274. - 36.6 42.6	12.2 273. - 37.2 42.1	12.3 272. - 37.5 41.8	12.1 274. - 38.0 40.2
Earth to Mars Launch on June 8, 1971 With Flight Time of 230 Days	C_3 RLA DLA C_3A	9.31 331. - 1.15 10.3	9.26 330. - .776 10.3	9.18 331. - 1.96 10.2	9.04 331. - 2.10 10.1
Approximate Running Time per Case on 7094 Computer		10 min. (30 sec. per trial)	1 sec.	.05 sec.	.1 sec

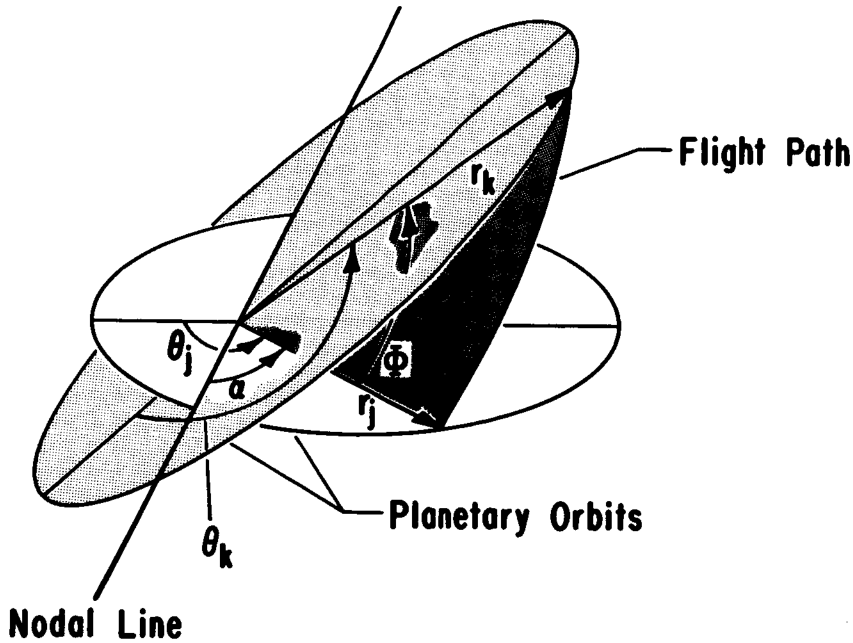


FIG. 1. PLANETARY TRANSFER GEOMETRY

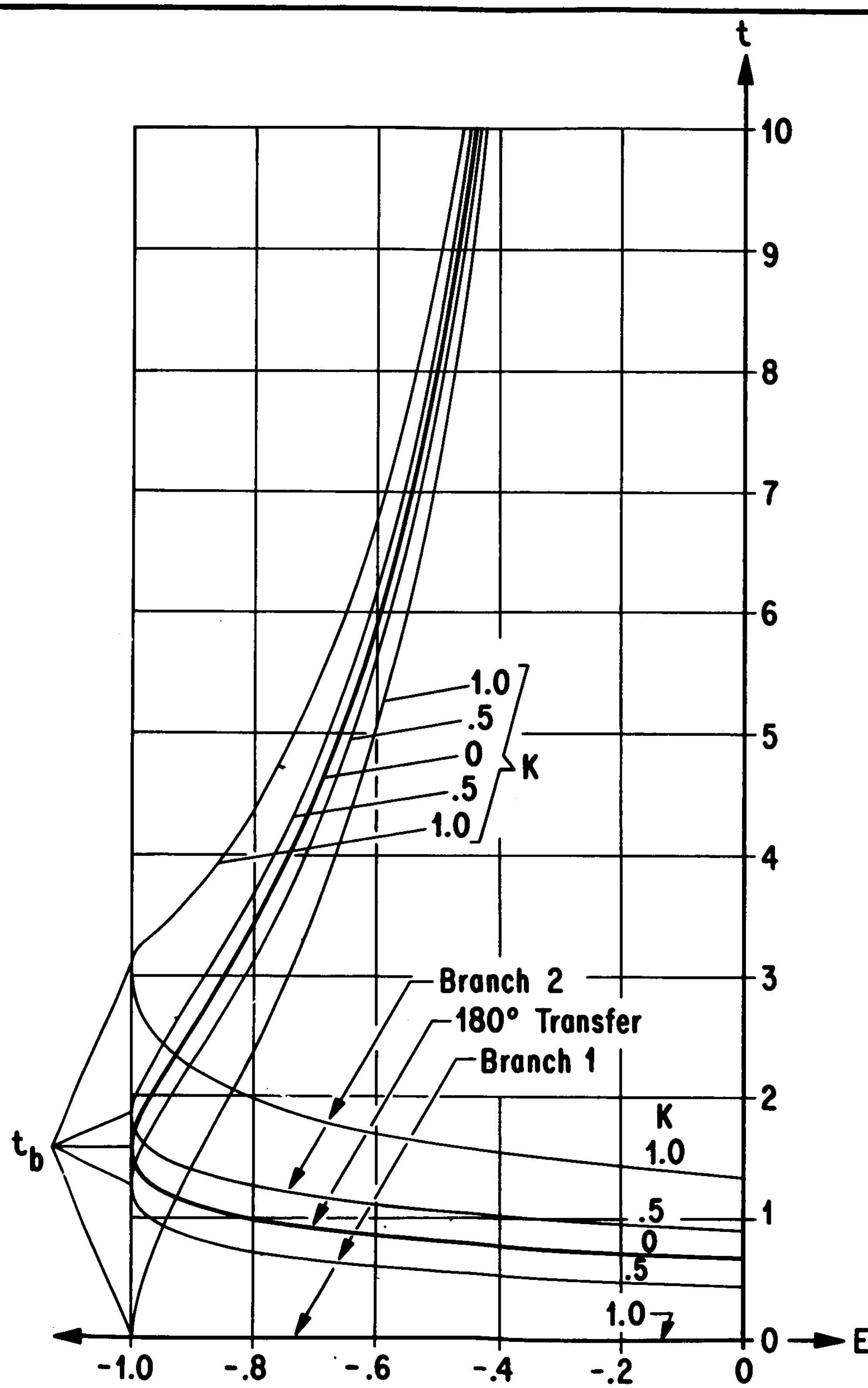


FIG. 2. PARAMETER CURVES FOR THE SOLUTION OF LAMBERT'S THEOREM

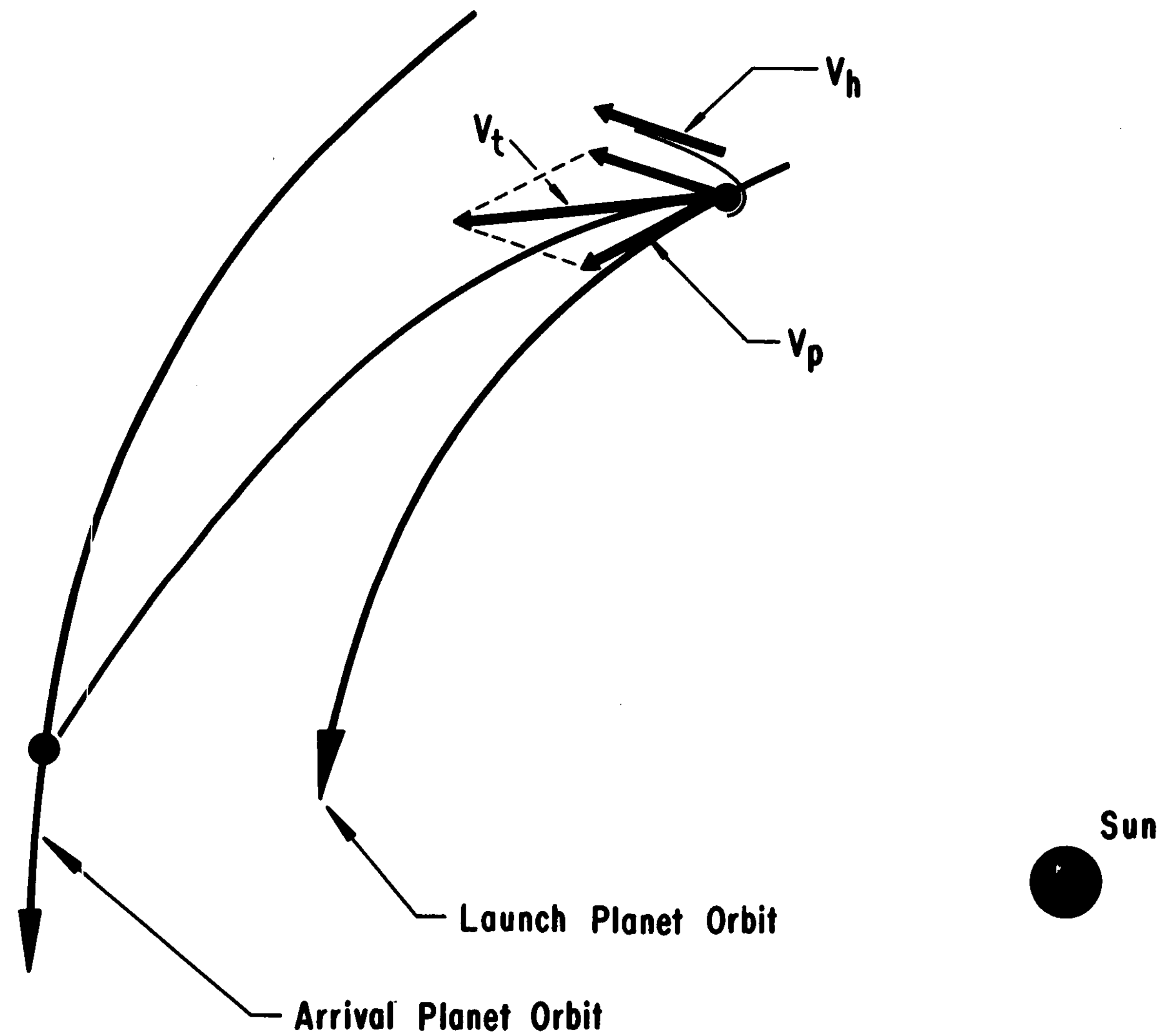


FIG. 3. DETERMINATION OF HYPERBOLIC EXCESS VELOCITY

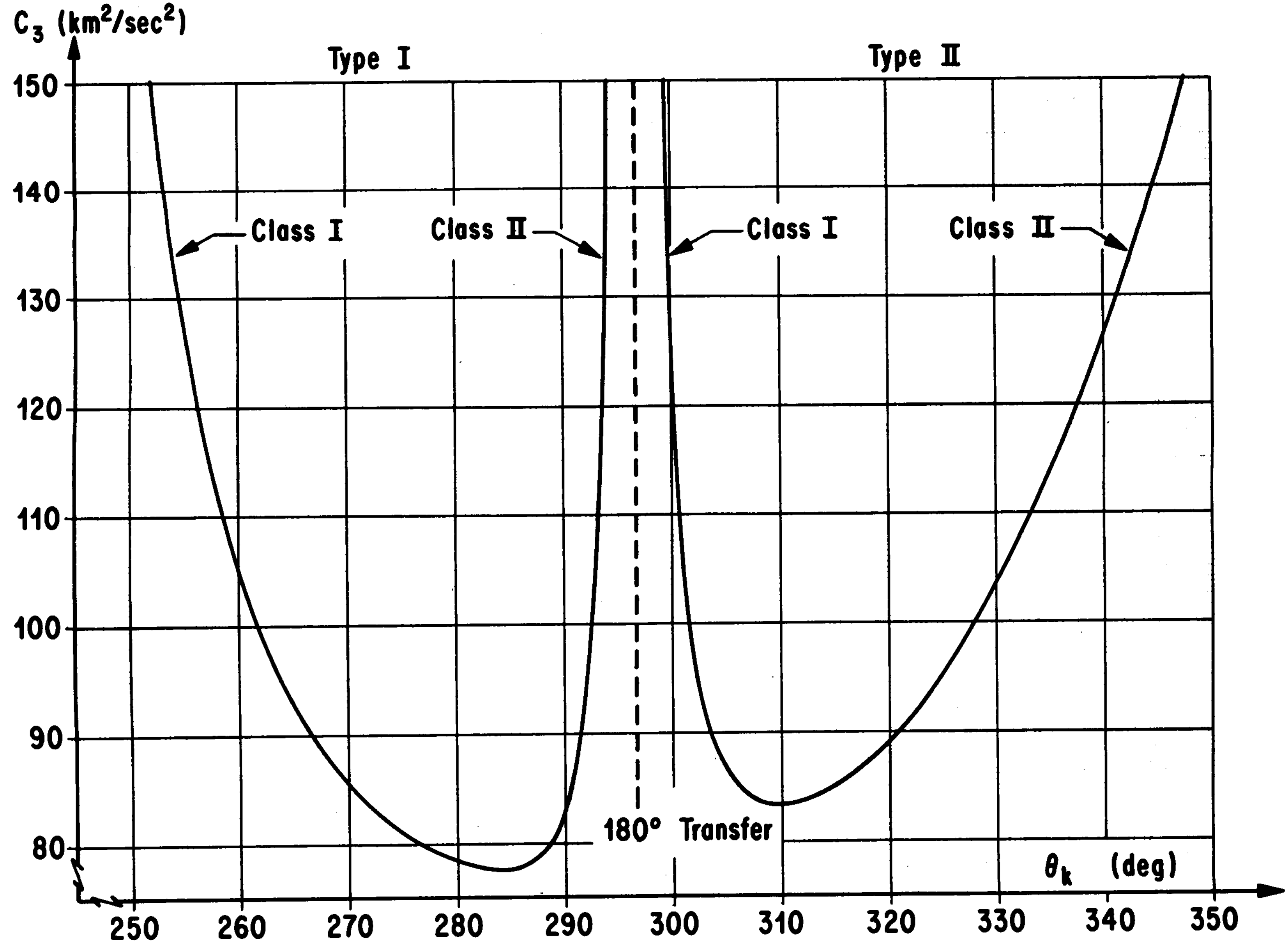


FIG. 4. INJECTION ENERGY VERSUS ARRIVAL PLANET'S POSITION EARTH TO JUPITER TRAJECTORIES FOR LAUNCH ON JAN. 31, 1971

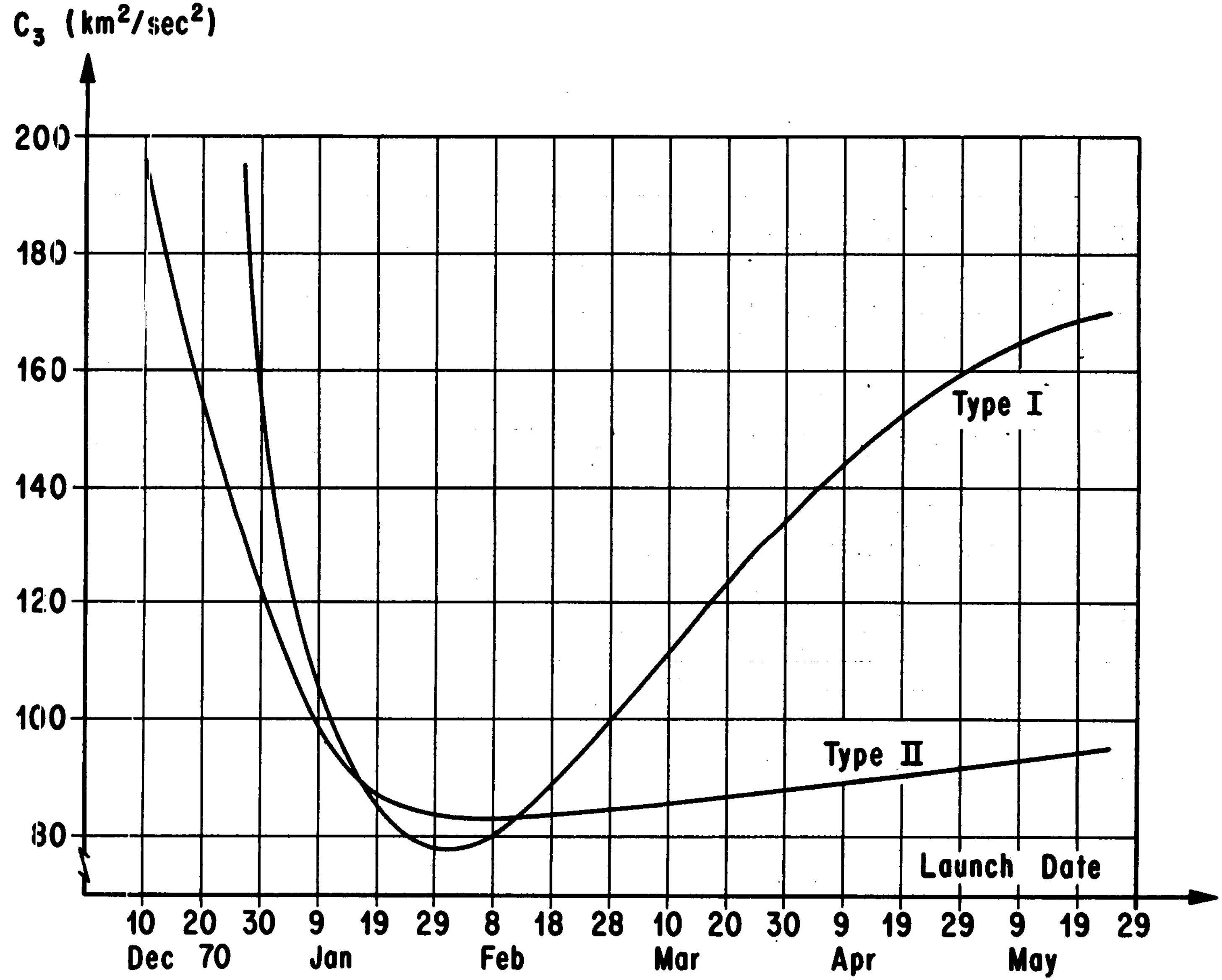


FIG. 5. LAUNCH ENERGY VERSUS LAUNCH DATE
MINIMUM ENERGY EARTH TO JUPITER TRANSFER

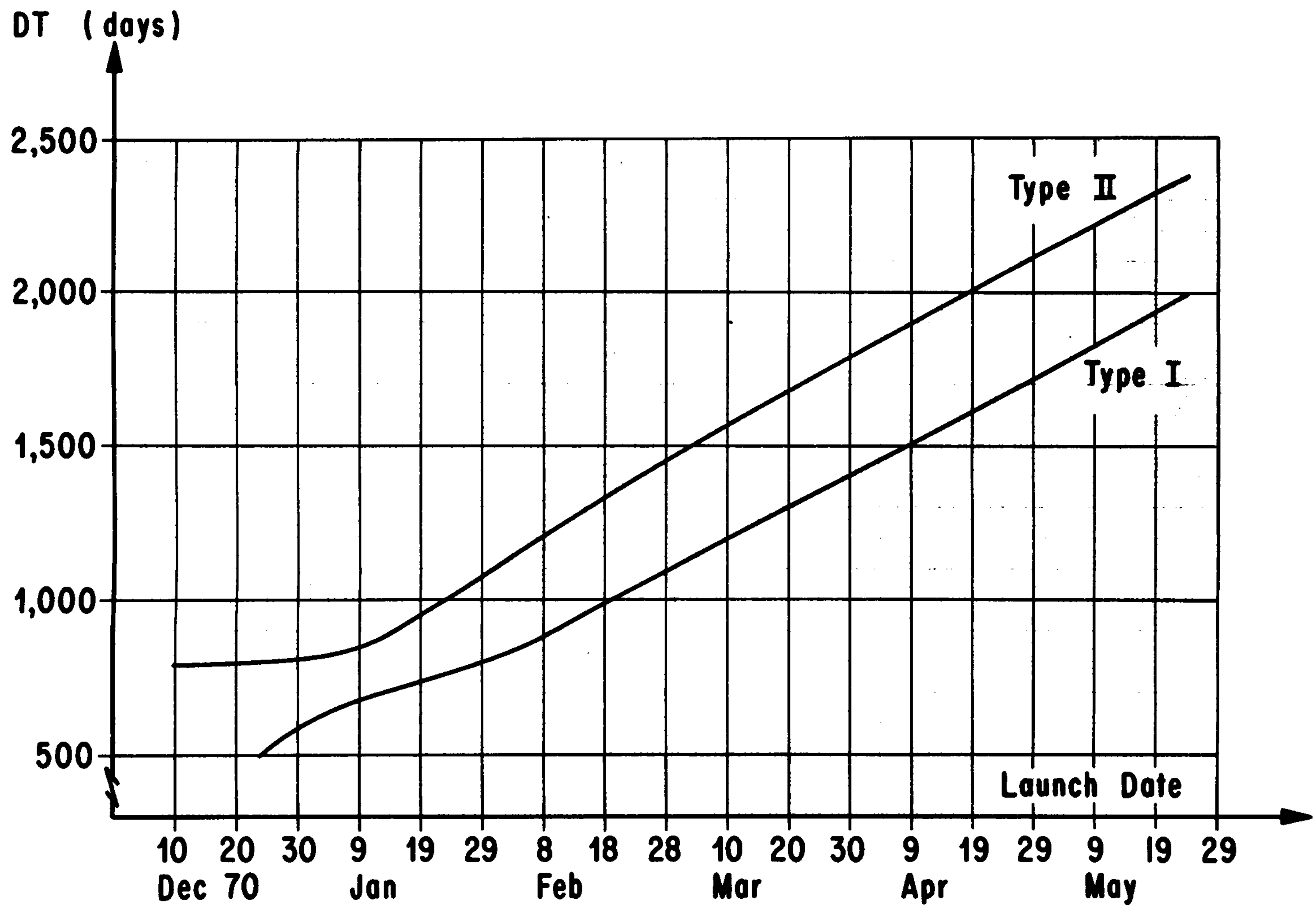


FIG. 6. FLIGHT TIME VERSUS LAUNCH DATE
MINIMUM ENERGY EARTH TO JUPITER TRANSFER

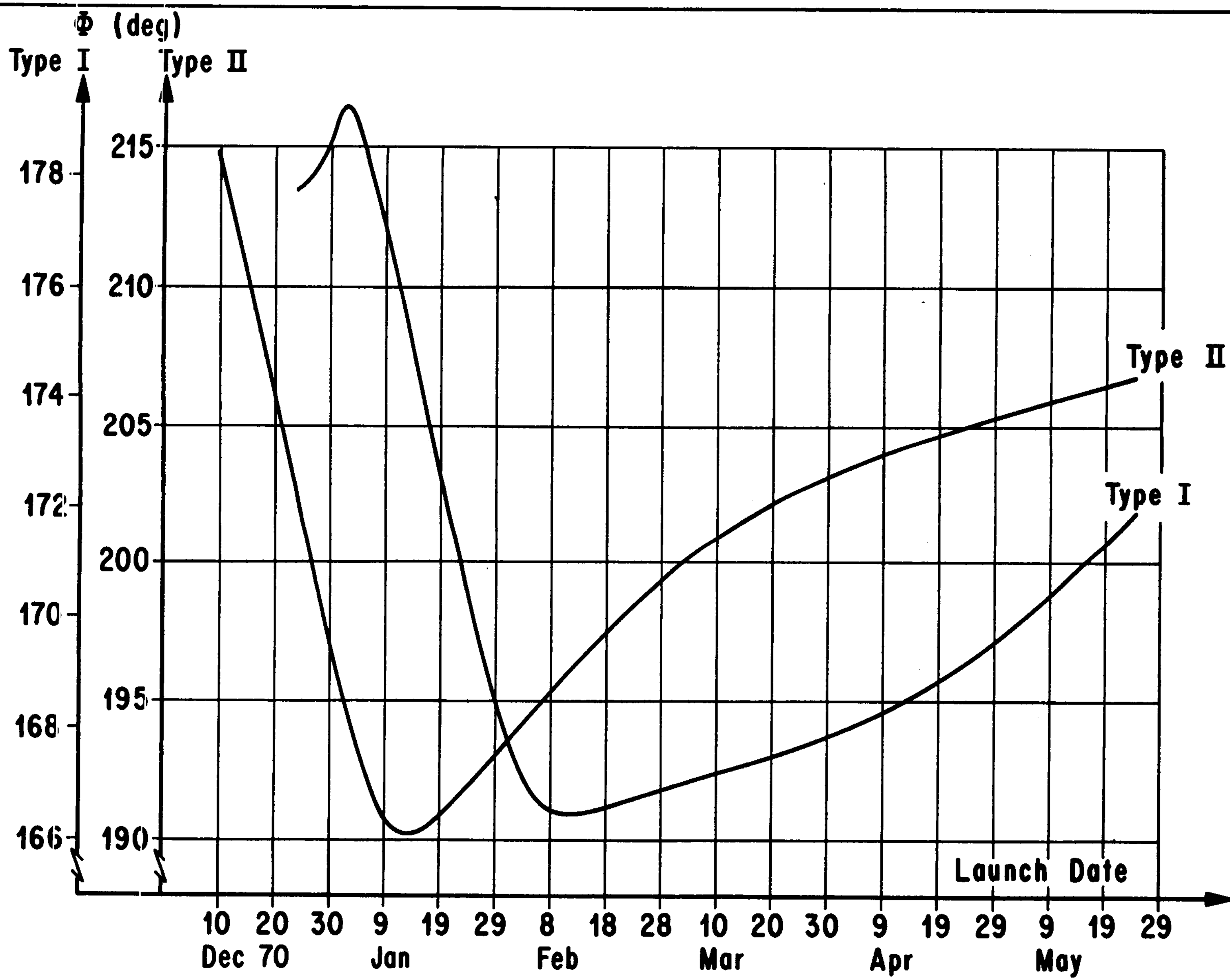


FIG. 7. CENTRAL ANGLE VERSUS LAUNCH DATE MINIMUM ENERGY EARTH TO JUPITER TRANSFER

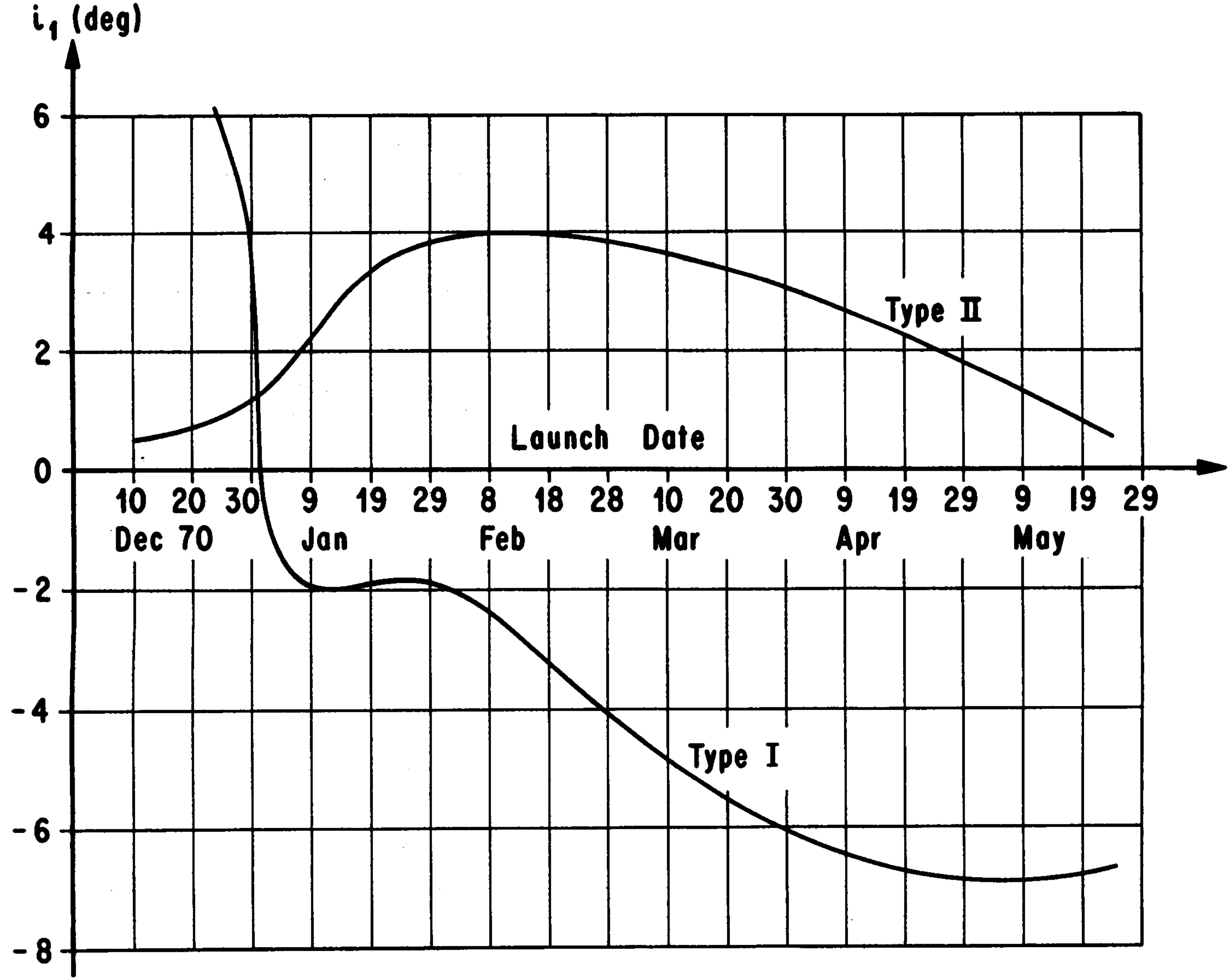


FIG. 8. INCLINATION OF TRANSFER ORBIT VERSUS LAUNCH DATE MINIMUM ENERGY EARTH TO JUPITER TRANSFER

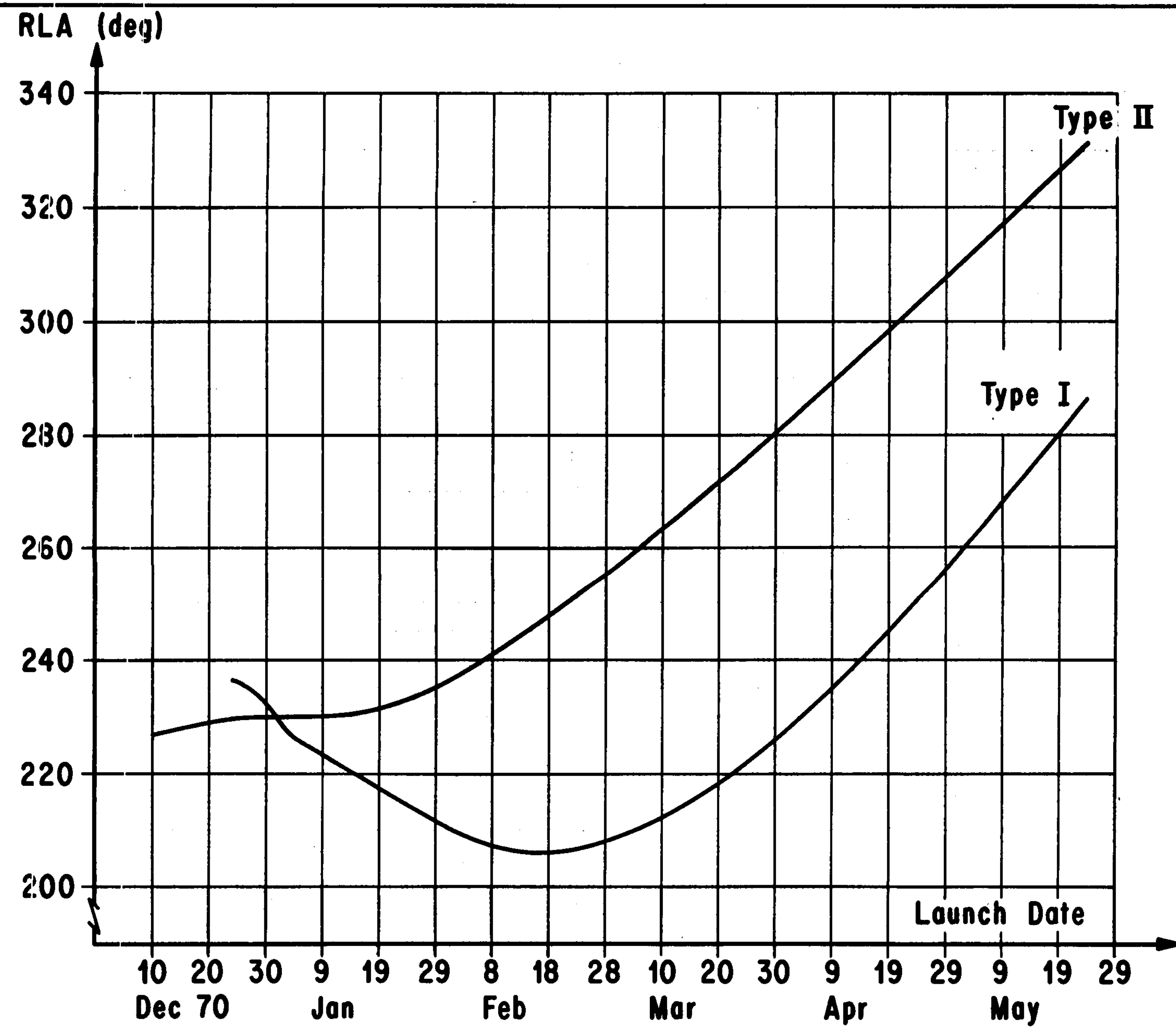


FIG. 9. RIGHT ASCENSION OF LAUNCH ASYMPTOTE VERSUS LAUNCH DATE MINIMUM ENERGY EARTH TO JUPITER TRANSFER

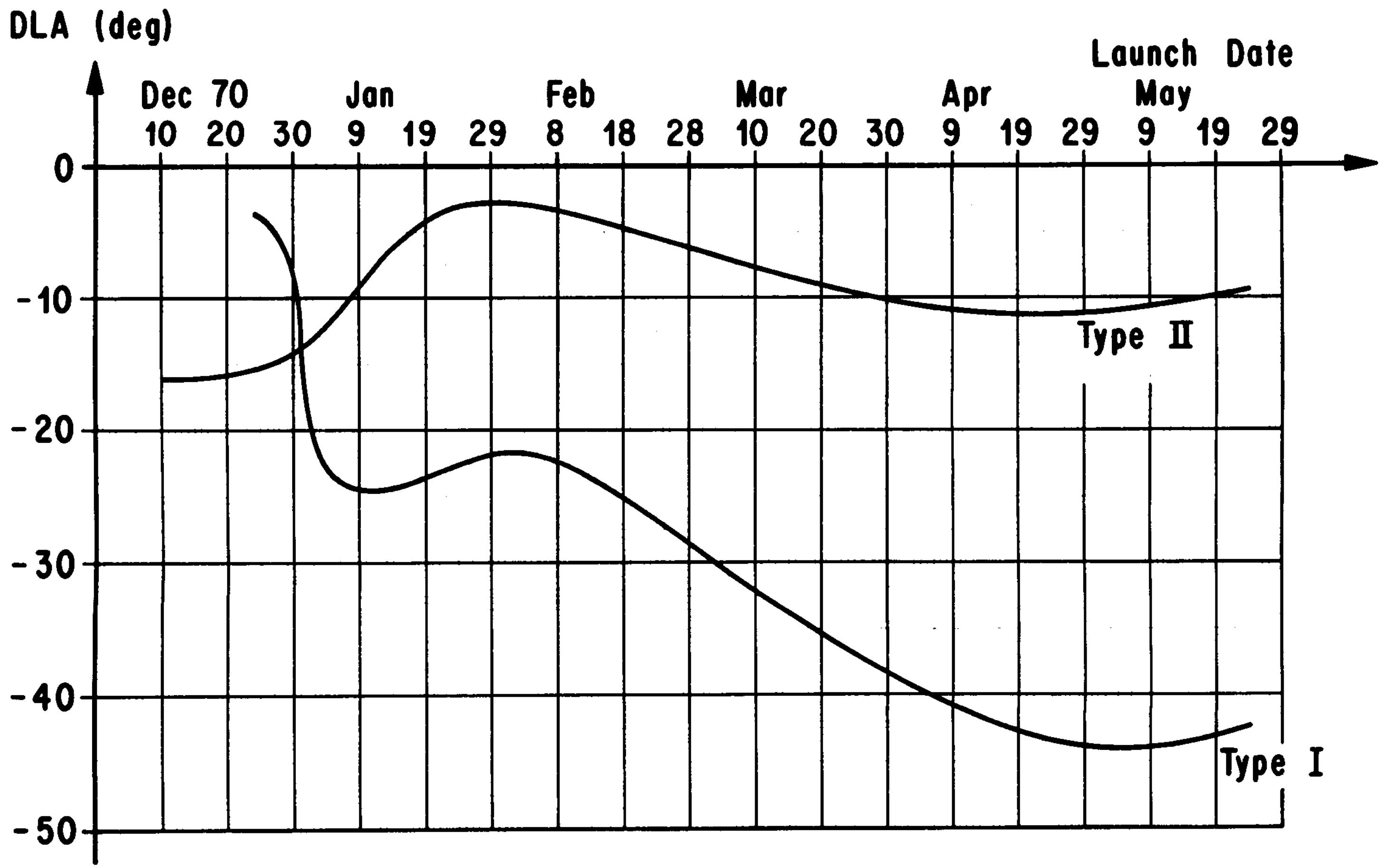


FIG. 10. DECLINATION OF LAUNCH ASYMPTOTE VERSUS LAUNCH DATE MINIMUM ENERGY EARTH TO JUPITER TRANSFER

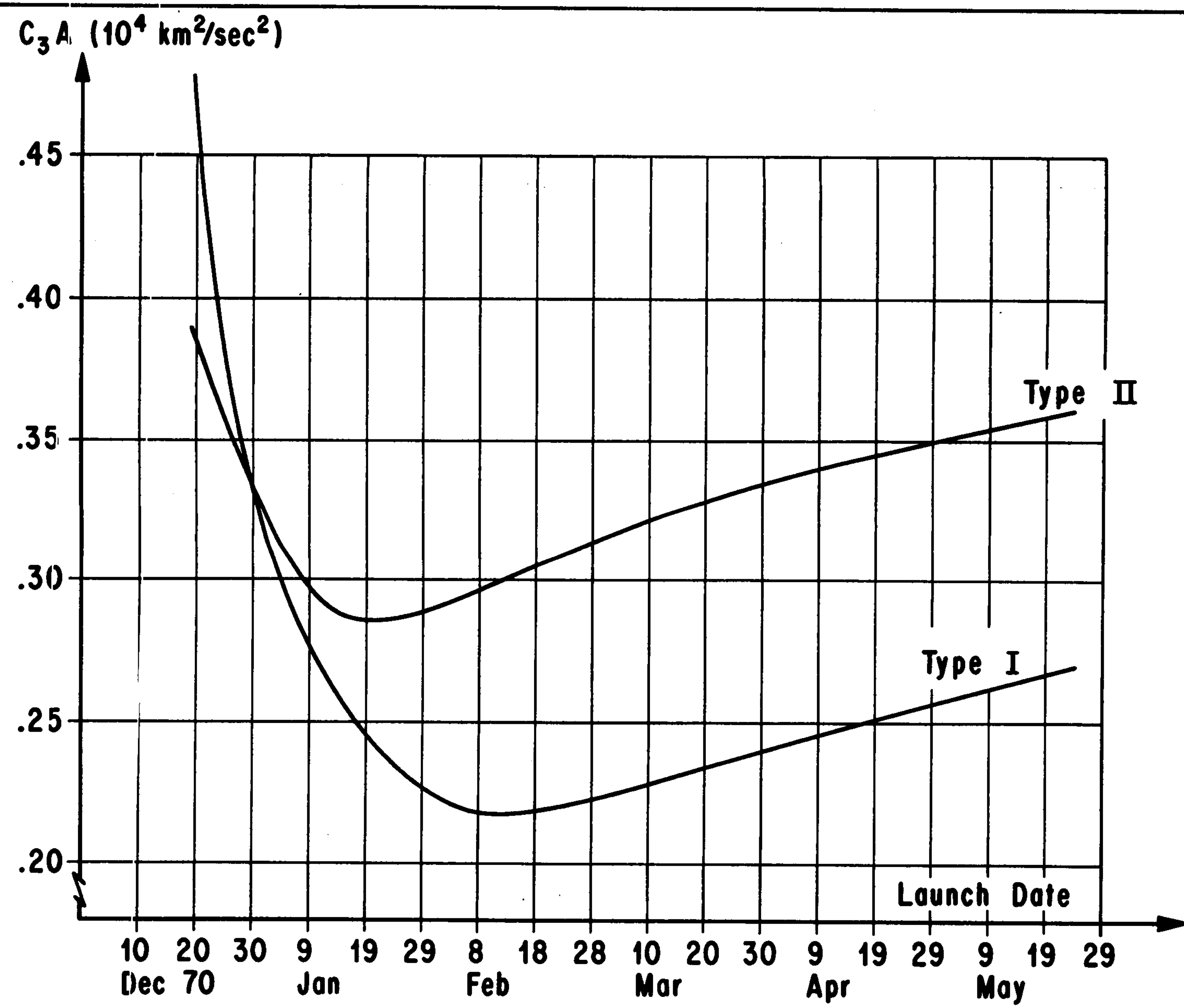


FIG. 11. ARRIVAL ENERGY VERSUS LAUNCH DATE
MINIMUM ENERGY EARTH TO JUPITER TRANSFER

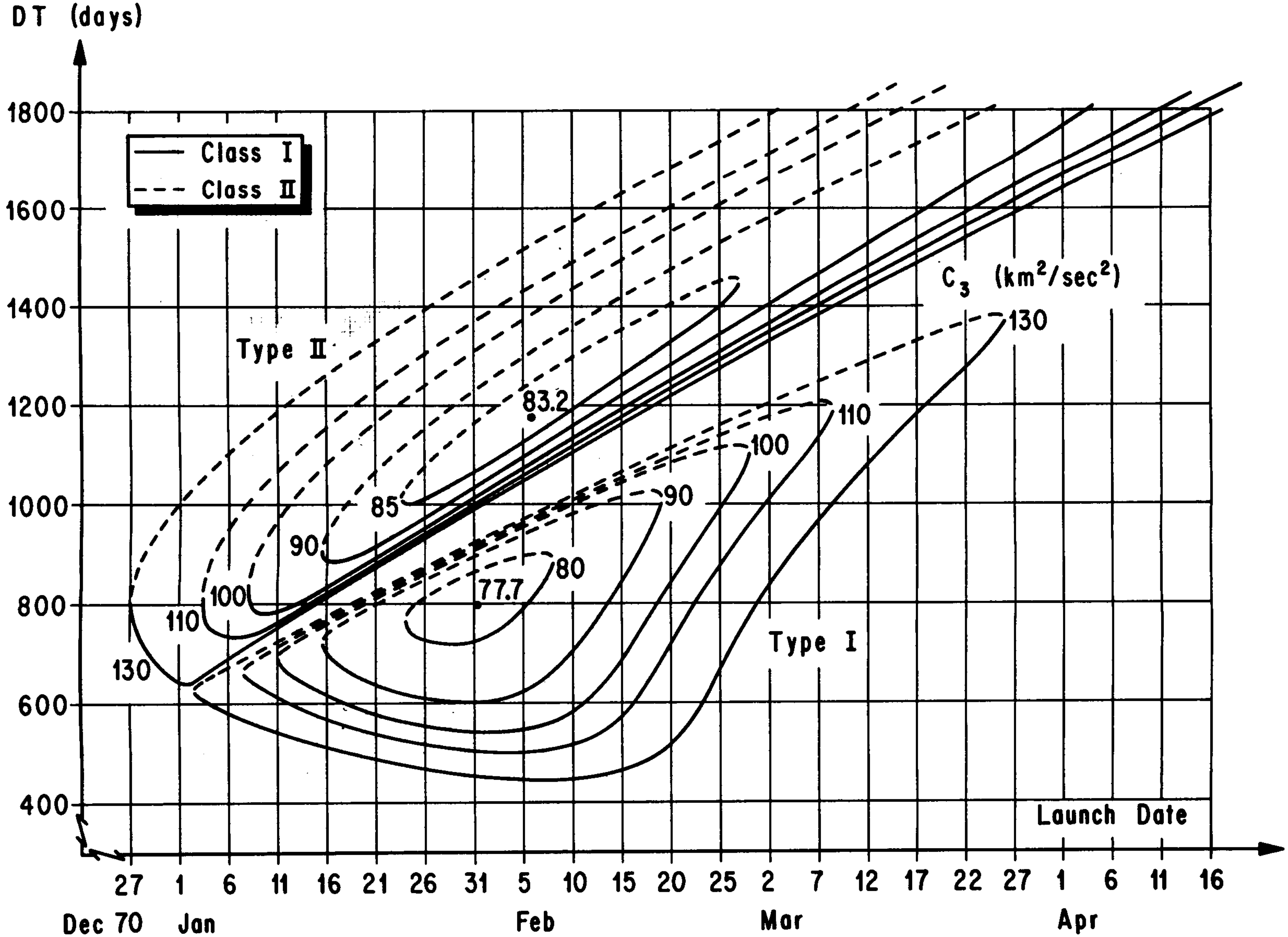


FIG. 12. FLIGHT TIME VERSUS LAUNCH DATE
EARTH TO JUPITER TRANSFER

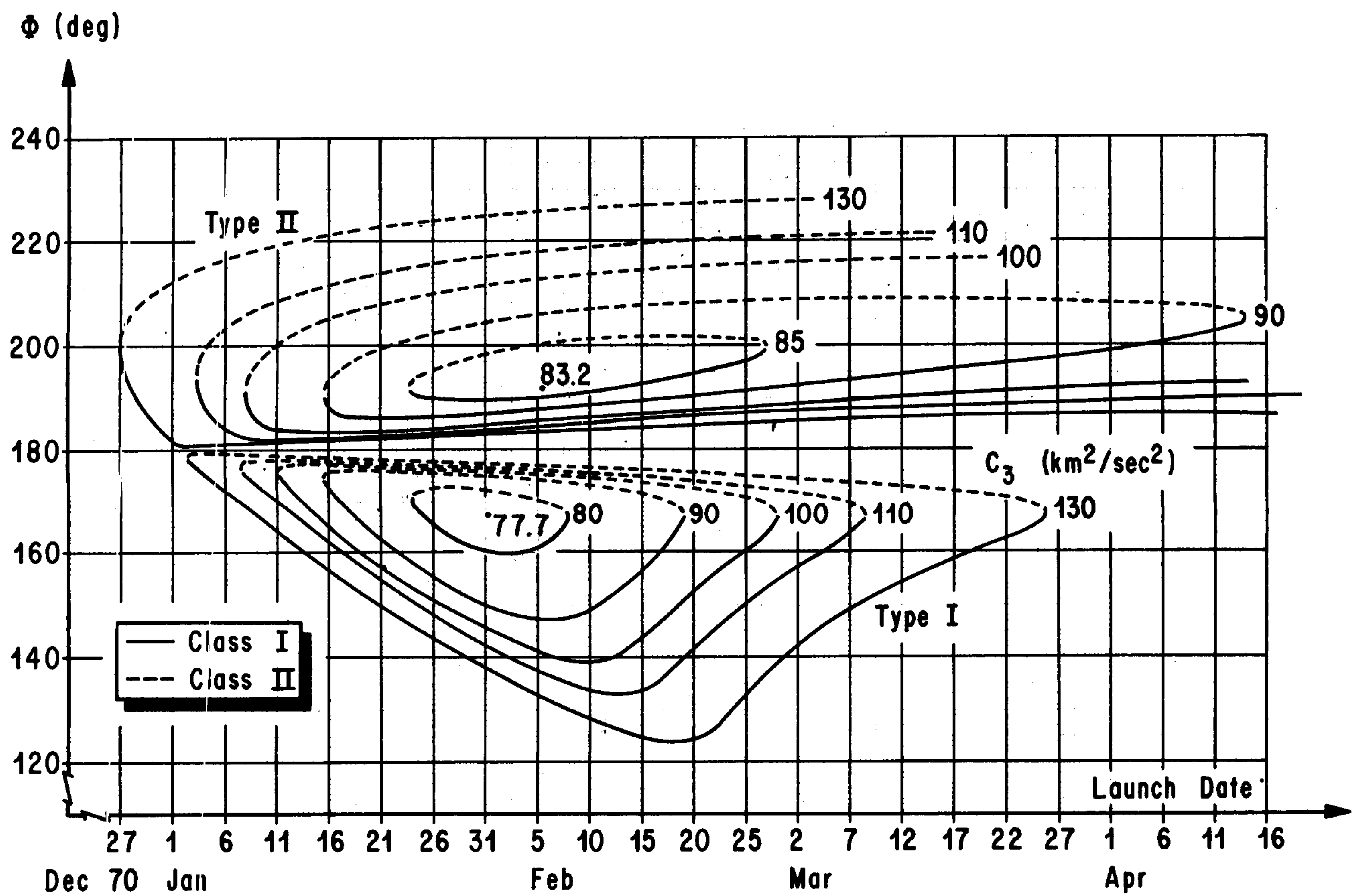


FIG. 13. CENTRAL ANGLE VERSUS LAUNCH DATE EARTH TO JUPITER TRANSFER

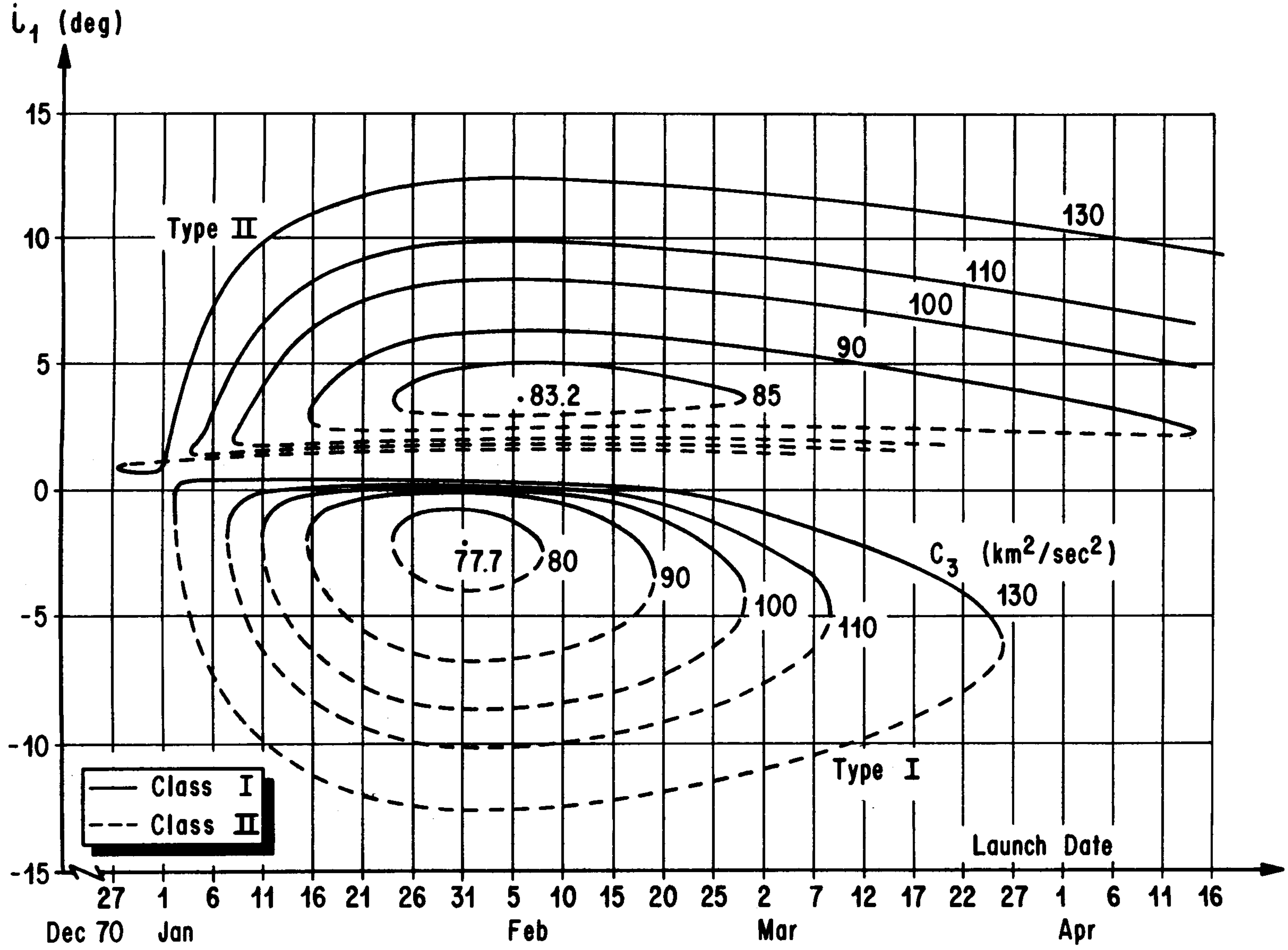


FIG. 14. INCLINATION OF TRANSFER ORBIT VERSUS LAUNCH DATE EARTH TO JUPITER TRANSFER

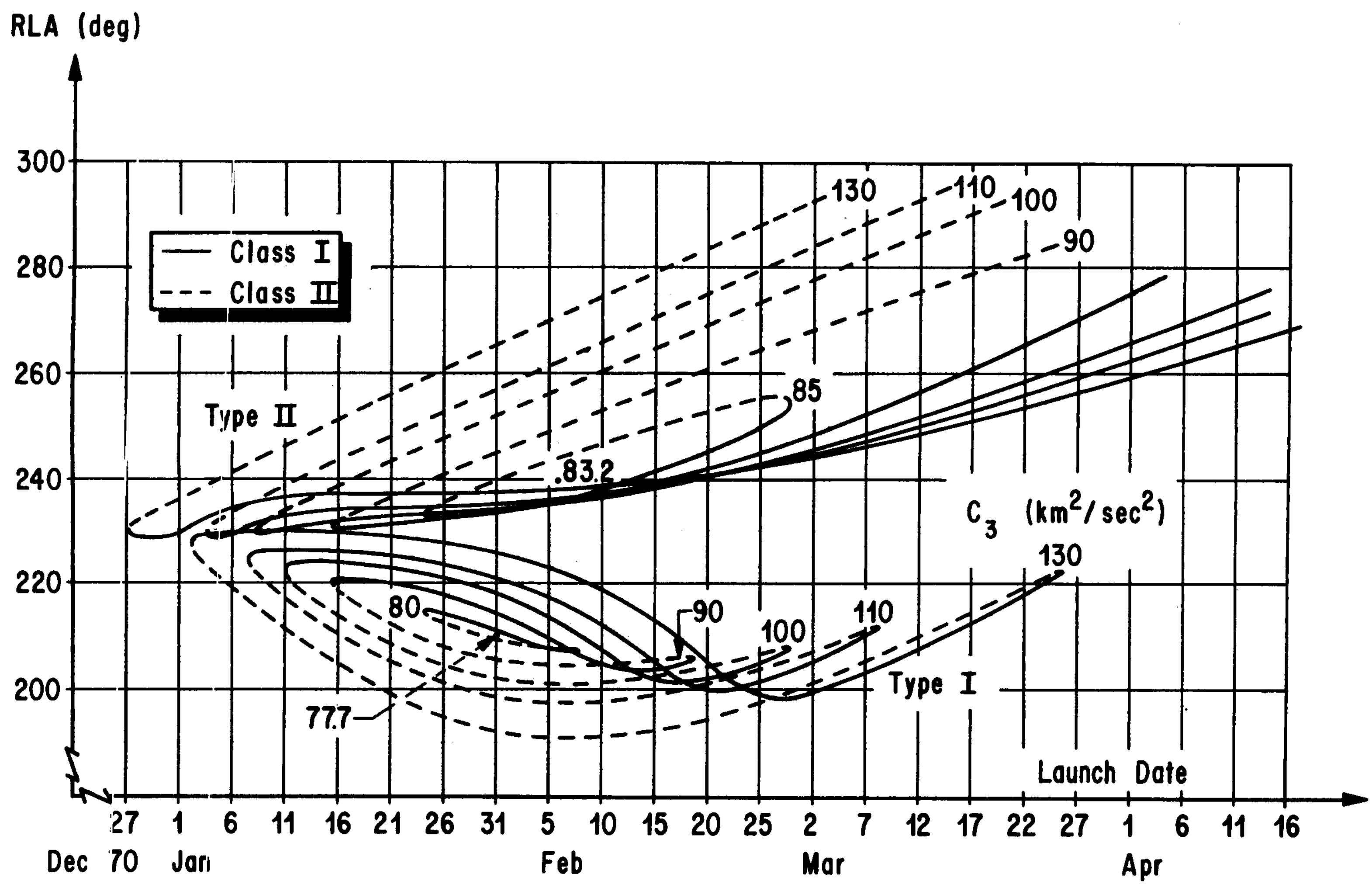


FIG. 15. RIGHT ASCENSION OF LAUNCH ASYMPTOTE VERSUS LAUNCH DATE EARTH TO JUPITER TRANSFER

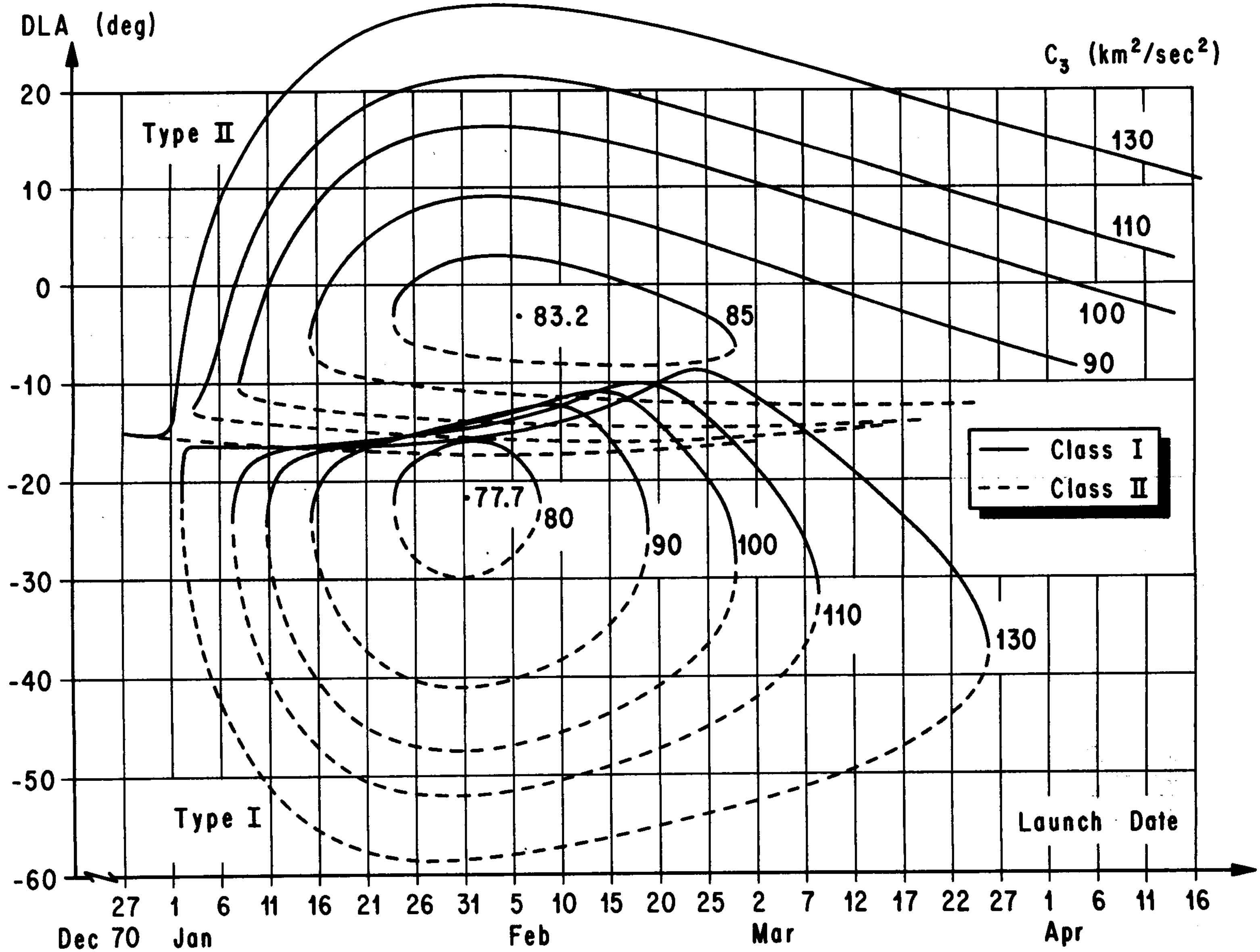


FIG. 16. DECLINATION OF LAUNCH ASYMPTOTE VERSUS LAUNCH DATE EARTH TO JUPITER TRANSFER

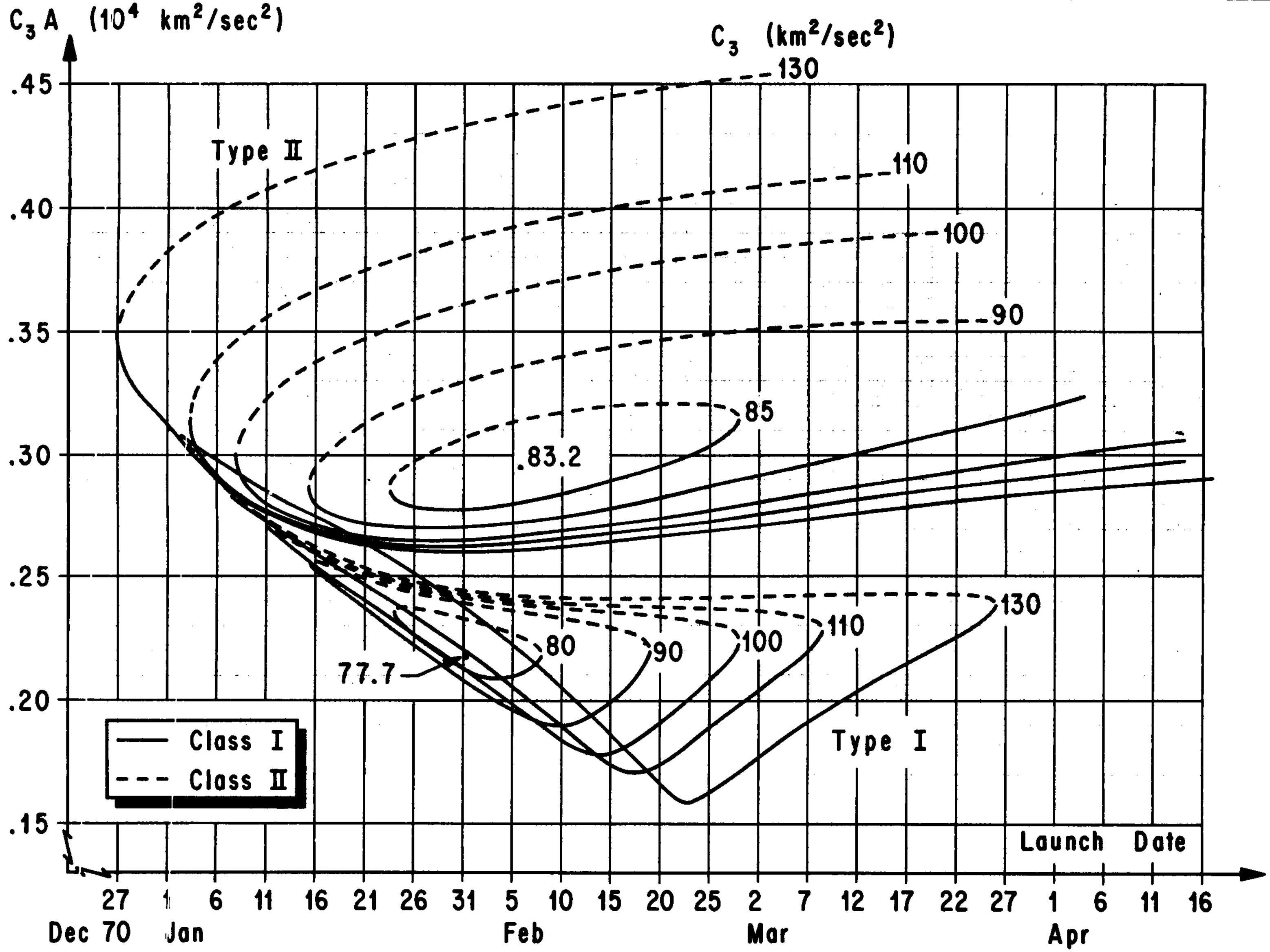
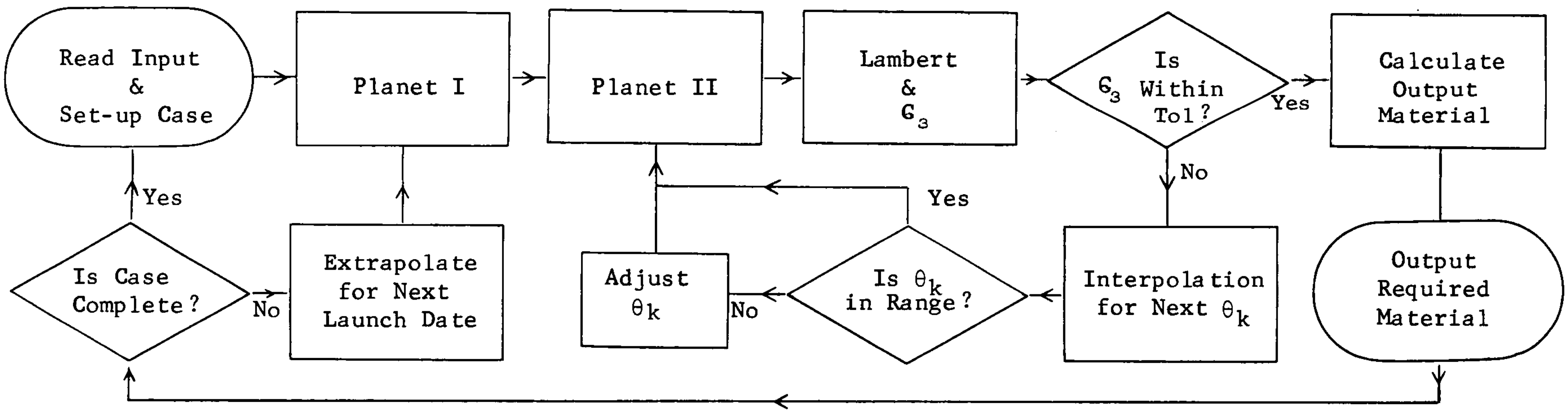


FIG. 17. ARRIVAL ENERGY VERSUS LAUNCH DATE EARTH TO JUPITER TRANSFER

FLOW FOR CONSTANT AND MINIMUM C_3



FLOW FOR STRAIGHT RUNS

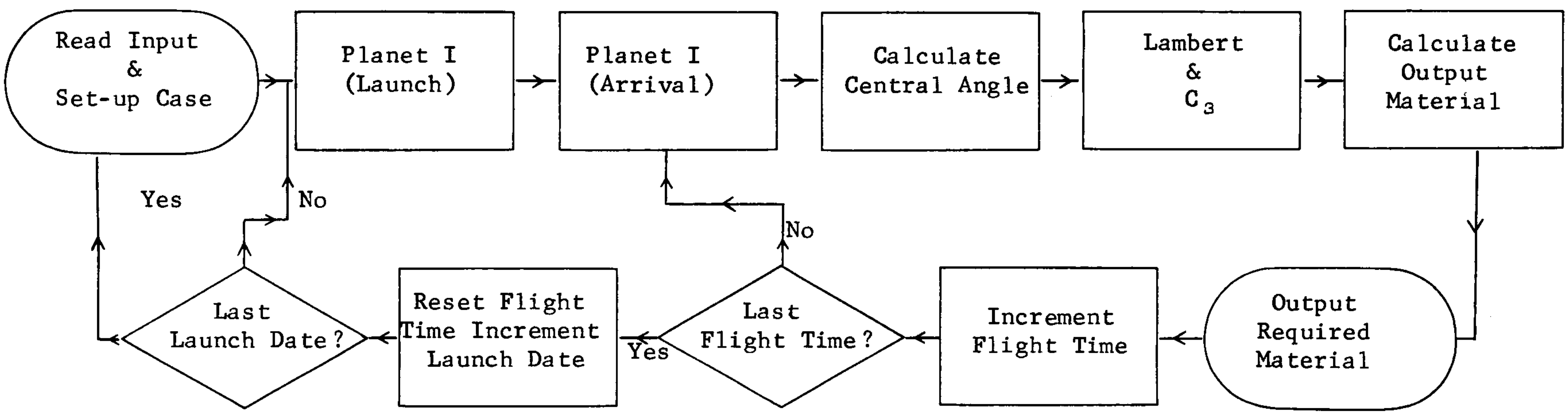


FIGURE A1

APPENDIX

This appendix is designed to give a mathematical treatment to the material presented in the main report. It is not designed as a complete description of the computer program. A user's guide to be published by the MSFC, Computation Laboratory will satisfy this requirement. The simplified flow diagram of Figure A1 shows the order used to perform the required calculations. The calculations performed in each block are listed in the descriptions of the subroutines which follow.

PLANET POSITION I

Using date and the mean conic elements of the planets, this routine calculates the position of the desired planet and the angular position at which the other planet will be at the transition between Type I and Type II trajectories [6].

$$M_j = n_j(T_j - t_{p_j}) \qquad 0 \leq M < 2\pi$$

E_j is calculated by iteration on the equation

$$M_j = E_j - e_j \sin E_j \qquad 0 \leq M < 2\pi$$

$$\theta_j = 2 \arctan \sqrt{\frac{1+e_j}{1-e_j}} \tan E_j/2 \qquad 0 \leq \theta_j < 2\pi$$

$$r_j = A_j \ (1-e_j \cos E_j)$$

$$\alpha_j = \theta_j - w_{j,k} \qquad 0 \leq \alpha < 2\pi$$

$$\theta_x = w_{k,j} + \arctan\left(\frac{\tan \alpha_j}{\cos i_{j,k}}\right) \qquad 0 \leq \theta_x < 2\pi$$

PLANET POSITION II

Using the true anomaly of the desired planet, the mean conic elements of the planets, and the date and position of the other planet, this routine calculates the radius and date of the desired planet, the angular separation of the two planets, and the flight time [6].

$$\alpha_k = \theta_k - w_{k,j} - \pi \qquad 0 \le \alpha_k < 2\pi$$

$$\Phi = \arccos\left(\cos\alpha_k \cos\alpha_j + \sin\alpha_k \sin\alpha_j \cos C_{j,k}\right)$$

$$0 \le \Phi < 2\pi$$

$$r_k = pp_k^2/\left(1+e_k \cos\theta_k\right)$$

$$T_k = \frac{1}{n_k}\left(\arcsin\left(\frac{r_k \sin\theta_k}{a_k\sqrt{1-e_k}}\right) - e_k \frac{r_k \sin\theta_k}{a_k\sqrt{1-e_k^2}}\right) + t_{p_k}$$

$$DT = T_k - T_j$$

LAMBERT CALCULATION

Using the positions of the two planets and flight time, this routine calculates the required transfer ellipse [2,6].

The chord from the launch to arrival point is

$$C = \sqrt{r_j^2 + r_k^2 - 2r_j r_k \cos\Phi}$$

$$s = (r_j + r_k + C)/2$$

$$E = -\frac{s}{2a'} \qquad -1 \le E \le 0$$

where a' is a first guess at the semi-major axis of the transfer ellipse.

$$K = 1 - \frac{C}{s}$$

Normalized time for a parabolic transfer is

$$t_a = \tfrac{2}{3}\left(1 - P\,K^{\frac{3}{2}}\right)$$

where $P = +1$ if $\Phi < 180^\circ$ and $P = -1$ if $\Phi > 180^\circ$.

Normalized required transfer time is

$$t = \frac{86400 \times \sqrt{2\mu}}{S^{\frac{3}{2}}} DT$$

and time for a minimum heliocentric energy transfer is

$$t_b = \frac{\pi}{2} - P(\arcsin \sqrt{K} - \sqrt{K(1-K)}\ ).$$

If $t < t_b$, $Q = +1$; if $t > t_b$, $Q = -1$. Then the desired E is found by iterating the solution on one of the following transcendental functions

$$t' = (-E)^{-\frac{3}{2}}\left(\frac{\pi}{2} + Q(-\frac{\pi}{2} + \arcsin \sqrt{-E} - \sqrt{E(1+E)}\ )\right.$$
$$\left. - P(\arcsin \sqrt{-E\ K} - \sqrt{E\ K(1+E\ K)}\ )\right) .$$

If $Q > 0$ and $E > -.2$, a series solution, obtained by expanding each term in a series and adding term by term, is used.

$$t' = \sum_{i=1}^{N} C_i(1 - P\ K^{\frac{3}{2}+i})E^{i-1} .$$

The first ten coefficients are

C_1 = .66666667	C_2 = -.2
C_3 = .10714286	C_4 = -.069444444
C_5 = .049715909	C_6 = -.037860577
C_7 = .030078125	C_8 = -.024643842
C_9 = .020671644	C_{10} = -.017663865.

The next guess at E is found by making a linear interpolation between $(-1, t_b)$ and (E, t'). Then a new value of t' is calculated and additional linear interpolations are made between the last two calculated points until t' is within the desired tolerance of t.

Then the elements of the transfer ellipse are calculated as follows

$$\frac{\eta_1}{2} = \arctan\left(Q\sqrt{(1+E)(s-r_k)/(1+EK)(s-r_j)}\right) \qquad -\frac{\pi}{2} \le \frac{\eta}{2} \le \frac{\pi}{2}$$

$$\frac{\xi_1}{2} = \arctan\left(P\sqrt{K(s-r_j)/(s-r_k)}\right) \qquad -\frac{\pi}{2} \le \frac{\xi}{2} \le \frac{\pi}{2}$$

$$\psi_1 = \frac{\pi}{2} - \frac{\xi_1}{2} + \frac{\eta_1}{2}$$

$$a = -\frac{s}{2E}$$

$$V_1 = \sqrt{\mu\left(\frac{2}{r_j} - \frac{1}{a}\right)}$$

$$pp = \left| V_1 r_j \sin\psi/\sqrt{\mu} \right|$$

$$e = \sqrt{1 - \frac{pp^2}{a}}$$

$$\theta_1 = \arctan\left(\frac{pp^2 \cot\psi}{r_j} \Big/ \left(\frac{pp^2}{r_j} - 1\right)\right) \qquad 0 \le \theta < 2\pi$$

$$i_1 = \sin^{-1}\left(\frac{\sin i_{j,k} \sin \alpha_k}{\sin \Phi}\right) \qquad -\frac{\pi}{2} \le i \le \frac{\pi}{2}$$

$$\dot{x} = V_1 \cos\psi$$

$$\dot{y} = V_1 \sin i_1 \sin\psi$$

$$\dot{z} = V_1 \cos i_1 \sin\psi$$

C_3 ROUTINE

Using the velocity of the probe and the mean conic elements of the planets, this routine calculates the velocity of the planet, the velocity of the probe with respect to the planet, and the planetocentric energy of the probe [6].

$$\dot{\theta}_j = \frac{\sqrt{\mu}\; pp_j}{r_j}$$

$$\dot{r}_j = \frac{\sqrt{\mu}\; e_j \sin \theta_j}{pp_j}$$

$$\dot{x}' = \dot{x} - \dot{r}_j$$

$$\dot{y}' = \dot{y} - \dot{\theta}_j$$

$$\dot{z}' = \dot{z}$$

$$C_3 = \dot{x}'^2 + \dot{y}'^2 + \dot{z}'^2$$

INTERPOLATION FOR CONSTANT C_3

This is not a subroutine but is incorporated in the main line of the program. The slope of C_3 vs θ_k is checked to be sure the interpolation is being made for the proper class trajectory. Then a linear interpolation between the last two points is made to determine the θ_k which will produce the desired C_3. If both values of C_3 used for the interpolation are above the desired value, the calculated change in θ_k is increased slightly to avoid creeping on the solution.

INTERPOLATION FOR MINIMUM C_3

This routine uses three points on the C_3 vs θ_k curve to interpolate for the θ_k which will produce the minimum point on the curve [7].

The three points used for interpolation are designated by (θ_{k_1}, C_{3_1}), (θ_{k_2}, C_{3_2}) and (θ_{k_3}, C_{3_3}), and are arranged such that $C_{3_1} \leq C_{3_2} \leq C_{3_3}$. Then to reduce scaling problems a transformation of coordinates is made such that the origin is at the first point.

$$x_1 = \theta_{k_1} - \theta_{k_1} = 0 \qquad y_1 = C_{3_1} - C_{3_1} = 0$$

$$x_2 = \theta_{k_2} - \theta_{k_1} \qquad y_2 = C_{3_2} - C_{3_1}$$

$$x_3 = \theta_{k_3} - \theta_{k_1} \qquad y_3 = C_{3_3} - C_{3_1}$$

The equation of a parabola through these three points is

$$y = \frac{x(x-x_3)}{x_2(x_2-x_3)} y_2 + \frac{x(x-x_2)}{x_3(x_3-x_2)} y_3$$

$$\frac{dy}{dx} = \frac{2x-x_3}{-x_2(x_2-x_3)} y_2 + \frac{2x-x_2}{x_3(x_3-x_2)} y_3 = 0.$$

Thus the minimum is at

$$x = \frac{x_2{}^2 y_3 - x_3{}^2 y_2}{2(x_2 y_3 - x_3 y_2)}$$

and

$$\theta_k = x + \theta_{k_1}.$$

EXTRAPOLATION ROUTINE USING LAGRANGE FORMULAS

Using the last M solutions to the problem, this routine extrapolates the first guess on θ_k and a for the solution to the problem at the next launch date [7].

The values of T_j, a, θ_k are saved in a table and designated as T_i, a_i, θ_{k_i}.

For the second point T_j is incremented and a is set equal to a_1 and θ_k is set equal to θ_{k_1}.

If two or more points are in the table T_j is incremented and the following formulas are used to calculate first guesses for a and θ_k,

$$N = \prod_{i=1}^{M} (T_j - T_i)$$

$$D_\ell = \prod_{i=1}^{M} (T_\ell - T_i) \qquad i \neq \ell \qquad \ell = 1, M$$

$$C_\ell = \frac{N}{D_\ell (T_\ell - T_j)} \qquad \ell = 1, M$$

$$\theta_k = \sum_{\ell=1}^{M} C_\ell \, \theta_{k_\ell}$$

$$a = \sum_{\ell=1}^{M} C_\ell \, a_\ell .$$

VELOCITY CALCULATION ROUTINE

Using the elliptical elements of the transfer trajectory, the mean conic elements of the planets and the position of the arrival planet, this routine calculates (1) velocity of the probe at arrival, (2) velocity of arrival planet at arrival, (3) inclination of transfer conic to arrival planet conic, (4) velocity of probe with respect to arrival planet and (5) energy of the planetocentric conic [6].

$$\dot{\theta}_k = \frac{\sqrt{\mu} \;\; pp_{\dot{k}}}{r_{\dot{k}}}$$

$$\dot{r}_k = \frac{\sqrt{\mu} \; e_k \sin \theta_k}{pp_{\dot{k}}}$$

$$i_2 - \sin i_{k,j} \sin \alpha_j / \sin \Phi$$

$$\dot{\theta}_2 = \sqrt{\mu} \quad pp/r_k$$

$$\dot{r}_2 = \sqrt{\mu} \quad e\left(\sin \theta_1\right)/pp$$

$$V_x = \dot{r}_2 - \dot{r}_k$$

$$V_y = \dot{\theta}_2 \cos i_2 - \dot{\theta}_k$$

$$V_z = \dot{\theta}_2 \sin i_2$$

$$C_3 A = V_x^2 + V_y^2 + V_z^2$$

SUBROUTINE ROTATE

This routine rotates a vector in a coordinate system where the x-axis is along r, the z-axis is perpendicular to the planet's orbital plane, and the y-axis completes a right-handed system into a coordinate system analogous to the earth's "Inertial Cartesian Equatorial System." That is, a coordinate system where the x-axis is in the direction of the "vernal equinox" (sun ascending node of the planet's equatorial plane), the z-axis is perpendicular to the planet's equatorial plane, and the y-axis completes a right handed system. After the rotation, the routine calculates the right ascension and declination for an "Inertial Spherical Equatorial System" [2].

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{pmatrix} \cos(\theta_i - \tau_i') & -\sin(\theta_i - \tau_i') & 0 \\ \sin(\theta_i - \tau_i')\cos ei_i & \cos(\theta_i - \tau_i')\cos ei_i & -\sin ei_i \\ \sin(\theta_i - \tau_i')\sin ei_i & \cos(\theta_i - \tau_i')\cos ei_i & \cos ei_i \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

RLA = arctan (y'/x')
(RAA)

DLA = arctan $(z'/\sqrt{x'^2 + y'^2})$

DISTRIBUTION

DIR
Dr. von Braun
Mr. Williams

DEP-T
Dr. Rees

EX
Mr. Maus

AST-S
Dr. Lange

R-DIR
Mr. Weidner
Dr. McCall

R-FP
Dr. Koelle
Mr. Huber
Dr. Ruppe

R-ASTR
Mrs. Neighbors
Lt. Bayer

R-COMP
Mr. Johnson (6)
Mr. Montgomery

R-RP (5)

LO-DF (5)

R-P&VE (20)

R-AERO
Dr. Geissler
Mr. Jean
Mr. Horn
Mr. Thomas
Mr. Tucker
Mr. Young
Mr. Noblitt
Mr. McDaniel
Mr. Lester

R-AERO (Cont'd)
Mr. Goldsby
Mr. Cremin
Mrs. Bauer
Mr. Clingman
Dr. Sperling
Mr. Jandebeur
Mr. Dahm
Dr. Speer
Mrs. McNair
Mr. Baker
Mr. Lovingood
Mr. Hart
Mr. Schwaniger (10)
Mr Dearman
Mr. Redus
Mr. Simmons
Mr. Powers
Mr. Lisle
Mr. Lechner
Mr. Ingram
Mr. Herring
Mr. Galloway
Mr. French
Mr. Causey
Lt. Bullock (15)

MS-IP

MS-IL (8)

I-RM-M

MS-H

CC-P

Scientific & Technical Information (25)
ATTN: NASA Representative (S-AK/RKT)
P.O. Box 33
College Park, Maryland 20740

APPROVAL NASA TMX-53319

CONIC SOLUTIONS TO THE INTERPLANETARY TRANSFER PROBLEM WITH CONSTANT OR MINIMUM INJECTION ENERGY

By Lamar E. Bullock

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

Clyde D. Baker

Clyde D. Baker, Chief
Astrodynamics & Guidance Theory Division

E. D. Geissler, Director
Aero-Astrodynamics Laboratory

DISTRIBUTION (CONT'D)

EXTERNAL

Auburn Research Foundation
Auburn University
Auburn, Alabama

Jet Propulsion Laboratory Library
4800 Oak Grove Drive
Pasadena 3, California

Space Flight Library
University of Kentucky
Lexington, Kentucky

University of Kentucky Library
University of Kentucky
Lexington, Kentucky

Michigan State University Library
Michigan State University
East Lansing, Michigan

Dr. Rudolf F. Hoelker
Space Guidance Laboratory
Electronics Research Center, NASA
575 Technology Square
Cambridge, Massachusetts - 02139

Mr. Nolan J. Braud
Space Guidance Laboratory
Electronics Research Center, NASA
575 Technology Square
Cambridge, Massachusetts - 02139

Mr. L. Hooper
Space Technology Laboratories
6001 Gulf Freeway
Houston, Texas

Mr. Gary P. Herring
Chrysler Corporation
Missile Division
Huntsville, Alabama