R CHART CONTROL LIMITS BASED ON A SMALL NUMBER OF SUBGROUPS

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1. Introduction

The Shewhart control chart for the range R is a valuable tool for controlling the variability of a production process. Unfortunately, sufficiently accurate control limits cannot be established by present methods until a large number of subgroups have been inspected. This has prevented the valid use of the R chart during the crucial initiation of a new process, during the start-up of a process just brought into statistical control again, or for a process whose total output is not sufficiently large.

The author [9] recently proposed a new method for setting statistically sound $\bar{X}$ chart control limits based on a small number of subgroups. The general objective of this paper is to adapt the same type of method to the R chart. Thus, after evaluating the reliability of the conventional R chart control limits, this paper presents a new method for setting these limits so that they can be used reliably regardless of how few subgroups have been inspected.

2. Current Practice in Setting $\bar{X}$ Chart Control Limits

As is described in various books such as [1], [3], [4], and [13], the R chart is based upon the measurement of a single measurable quality characteristic of sample items drawn from the production process. The observations are grouped into small samples (called subgroups), commonly of size five, where each subgroup is as homogeneous as possible. The statistic plotted on the chart is the range R, which is the difference between the largest and smallest measurements within the subgroup. Given $\overline{R}$, the average of the subgroup ranges, the control limits are set at $D_3\overline{R}$ and $D_4\overline{R}$, where D_3 and D_4 are appropriate constants. If the R for a subgroup falls outside these limits, it is concluded that the variability of the process probably has changed, i.e., the process probably has fallen "out of statistical control," so that corrective action may be required.

The statistical theory behind the R chart may be summarized as follows. Assume that the process is in statistical control so that the observations are drawn from the same probability distribution, which is assumed to be a normal distribution. Therefore, the range for the respective subgroups also has some common probability distribution with a mean $\overline{R}'$ and standard deviation σ'_R. Let the random variable R be the range of a subgroup. Under these conditions, if the subgroup size is five, the probability that R will be within the interval $\overline{R}' \pm 3\sigma'_R$ is 0.9954, so that only about one out of every 220 observed values of R will be outside this interval. Therefore, when an observed value of R does fall outside, one likely explanation is that the assumption that the process is in statistical control is not justified. The important conclusion is that, if $\overline{R}'$ and σ'_R could be determined, then $\overline{R}' \pm 3\sigma'_R$ would be appropriate control limits. Furthermore, these limits probably should not be made wider since this would decrease the sensitivity of the chart to changes in the variability of the process.[1]

In practice, $D_3\bar{R}$ and $D_4\bar{R}$ commonly are used as estimates of $\bar{R}' - 3\sigma_R'$ and $\bar{R}' + 3\sigma_R'$, respectively.[2] When the subgroup size is five, $D_3 = 0$ and $D_4 = 2.115$. Unfortunately, these are not very accurate estimates unless the number of subgroups is quite large. One prevalent recommendation is that the control limits should be based on at least 25 subgroups, although this number is sometimes reduced to about ten in practice. Thus, when only a small number of subgroups have been observed, $D_3\bar{R}$ and $D_4\bar{R}$ may differ greatly from $\bar{R}' - 3\sigma_R'$ and $\bar{R}' + 3\sigma_R'$, respectively. One consequence of this would be that a relatively large proportion (much more than 0.46%) of future values of R may fall outside the control limits even when the process is in statistical control. In short, without a sufficient number of subgroups, setting the control limits at $D_3\bar{R}$ and $D_4\bar{R}$ provides a very unreliable basis for indicating when the process has gone out of statistical control.

3. Evaluation of Conventional R Chart Control Limits

Although it has been recognized that a fairly large number of subgroups is required for setting sufficiently accurate R chart control limits by conventional methods, precise information for deciding just how many are needed has not been available previously. The recommendation

[1] See Scheffé [12] for a statistical analysis of the operating characteristics of the R chart.

[2] The one exception is that, if $\bar{R}' - 3\sigma_R' < 0$, then D_3 is set equal to zero instead.

that at least 25 subgroups be used is only a rule of thumb. Although it is claimed that, with fewer subgroups, the probability of a Type I error (i.e., the probability that a new subgroup will indicate that the process has gone out of statistical control even though it has not) is much too large, precise supporting data have not been given. However, by applying the theory described in Section 6, it is now possible to obtain the actual probability of a Type I error, thereby giving the precise probability that a subgroup will indicate erroneously that trouble exists. These probabilities are reported in Table I for the case where the subgroup size is five.

TABLE I. Probability of a Type I Error

Table gives the probability that, when the process is in statistical control, the range for a randomly selected subgroup will fall outside the conventional control limits, 0 and 2.115 $\bar{\bar{R}}$, based on m subgroups of size five.

m	Probability
1	0.093
5	0.0176
10	0.0102
15	0.0081
20	0.0072
25	0.0066
50	0.0056
100	0.0050
∞	0.0046

When evaluating the results given in Table I, keep in mind that the assumed probability is approximately 0.0046. In other words, the theory behind the use of conventional R charts is based on assumptions which would imply that, on the average, only about one out of every 220 subgroups will indicate trouble erroneously. By contrast, Table I indicates that, for example, an average of slightly more than one out of every 100 subgroups will indicate trouble erroneously if control limits are set after ten subgroups.

No general conclusions can be drawn from Table I regarding the number of subgroups that should be inspected before setting conventional control limits. However, it does provide information for deciding each individual case on the basis of the need for early control and the cost of looking for trouble when none exists.

4. Proposed Method of Setting R Chart Control Limits

A new method now will be proposed for setting R chart control limits that can be used reliably regardless of how few subgroups have been inspected. This method amounts to replacing D_3 and D_4 by more appropriate constants, hereafter denoted as D_3^* and D_4^*, respectively. D_3^* and D_4^* would be chosen in such a way that the control limits, $D_3^*\bar{R}$ and $D_4^*\bar{R}$, would give the desired probability of a Type I error. In particular, each constant would be chosen so as to give the desired probability that a new subgroup range will fall outside the corresponding control limit when the process actually is in statistical control. In order to achieve this probability, the number of subgroups upon which the control limits are based determines the value of each constant. The values of D_3^* and D_4^* are given in Tables II and III, respectively, for many different numbers of subgroups and for several different probabilities when each subgroup size is five.[3] To facilitate interpolation, these values are also plotted in Figures 1 and 2. The procedure for deriving these values is described in Section 6.

TABLE II. Factor for Lower Control Limit of R Chart

When Subgroup Size is Five

Factor D_3^* such that $\mathrm{Prob}\{R \leq D_3^* \bar{R}\} = \alpha$, where R is the range of a future subgroup of size five and $\bar{R}$ is the average range of m subgroups each of size five, and where all observations are drawn from the same normal distribution.

m \ α	0.001	0.005	0.010	0.025	0.050
1	0.1340	0.2037	0.2454	0.3170	0.3893
2	0.1444	0.2189	0.2631	0.3381	0.4128
3	0.1485	0.2248	0.2700	0.3464	0.4216
4	0.1507	0.2280	0.2736	0.3507	0.4265
5	0.1520	0.2300	0.2759	0.3535	0.4296
6	0.1529	0.2305	0.2766	0.3543	0.4319
7	0.1536	0.2323	0.2782	0.3567	0.4332
8	0.1542	0.2331	0.2795	0.3578	0.4343
9	0.1546	0.2337	0.2802	0.3585	0.4353
10	0.1549	0.2342	0.2808	0.3592	0.4359
15	0.1559	0.2355	0.2825	0.3612	0.4381
20	0.1564	0.2363	0.2833	0.3622	0.4392
25	0.1567	0.2368	0.2834	0.3628	0.4399
50	0.1574	0.2376	0.2849	0.3640	0.4414
100	0.1576	0.2381	0.2852	0.3647	0.4421
∞	0.1580	0.2386	0.2859	0.3653	0.4428

[3] Five has been chosen as the subgroup size because it is commonly used in practice. However, it should be noted that Grubbs and Weaver [5] have found that subgroup sizes of seven or eight are the best from a statistical viewpoint, although five is only slightly less desirable.

TABLE III. Factor for Upper Control Limit of R Chart

When Subgroup Size is Five

Factor D_4^* such that $\text{Prob}\{R \geq D_4^*\bar{R}\} = \alpha$, where R is the range of a future subgroup of size five and $\bar{R}$ is the average range of m subgroups each of size five, and where all observations are drawn from the same normal distribution.

m \ α	0.050	0.025	0.010	0.005	0.001
1	2.600	3.215	4.197	5.098	7.940
2	2.068	2.387	2.833	3.194	4.140
3	1.919	2.169	2.501	2.758	3.386
4	1.850	2.071	2.357	2.573	3.082
5	1.810	2.014	2.274	2.468	2.915
6	1.784	1.977	2.221	2.400	2.810
7	1.765	1.951	2.184	2.354	2.739
8	1.751	1.931	2.157	2.320	2.685
9	1.741	1.917	2.137	2.294	2.646
10	1.732	1.906	2.121	2.274	2.615
15	1.707	1.871	2.071	2.214	2.525
20	1.695	1.854	2.048	2.185	2.482
25	1.688	1.845	2.034	2.168	2.456
50	1.673	1.824	2.006	2.134	2.407
100	1.666	1.814	1.993	2.117	2.382
∞	1.659	1.804	1.979	2.101	2.358

When choosing α, one's natural inclination might be to select an extremely small value. However, this would cause the control limits to become very wide. It is desirable that the control limits be relatively narrow so that the control chart will be sensitive to changes in the variability of the process. Therefore, α should be made no smaller than is required to give an adequate degree of reliability. The objective when choosing α is to obtain a proper balance between the smallness of α and the sensitivity of the control chart, taking into account the costs involved.

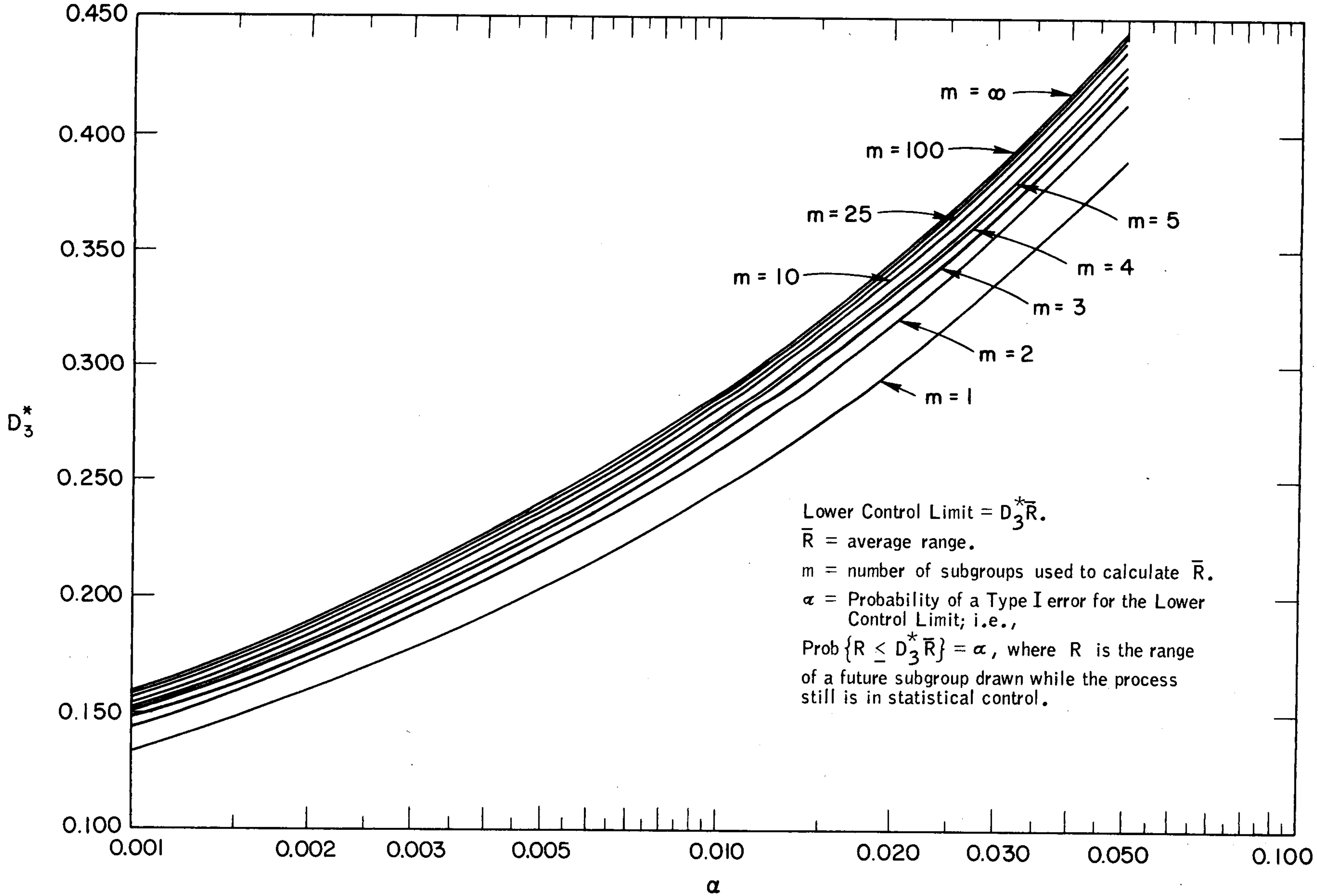

Figure 1. Factor for Lower Control Limit of R Chart When Subgroup Size is Five.

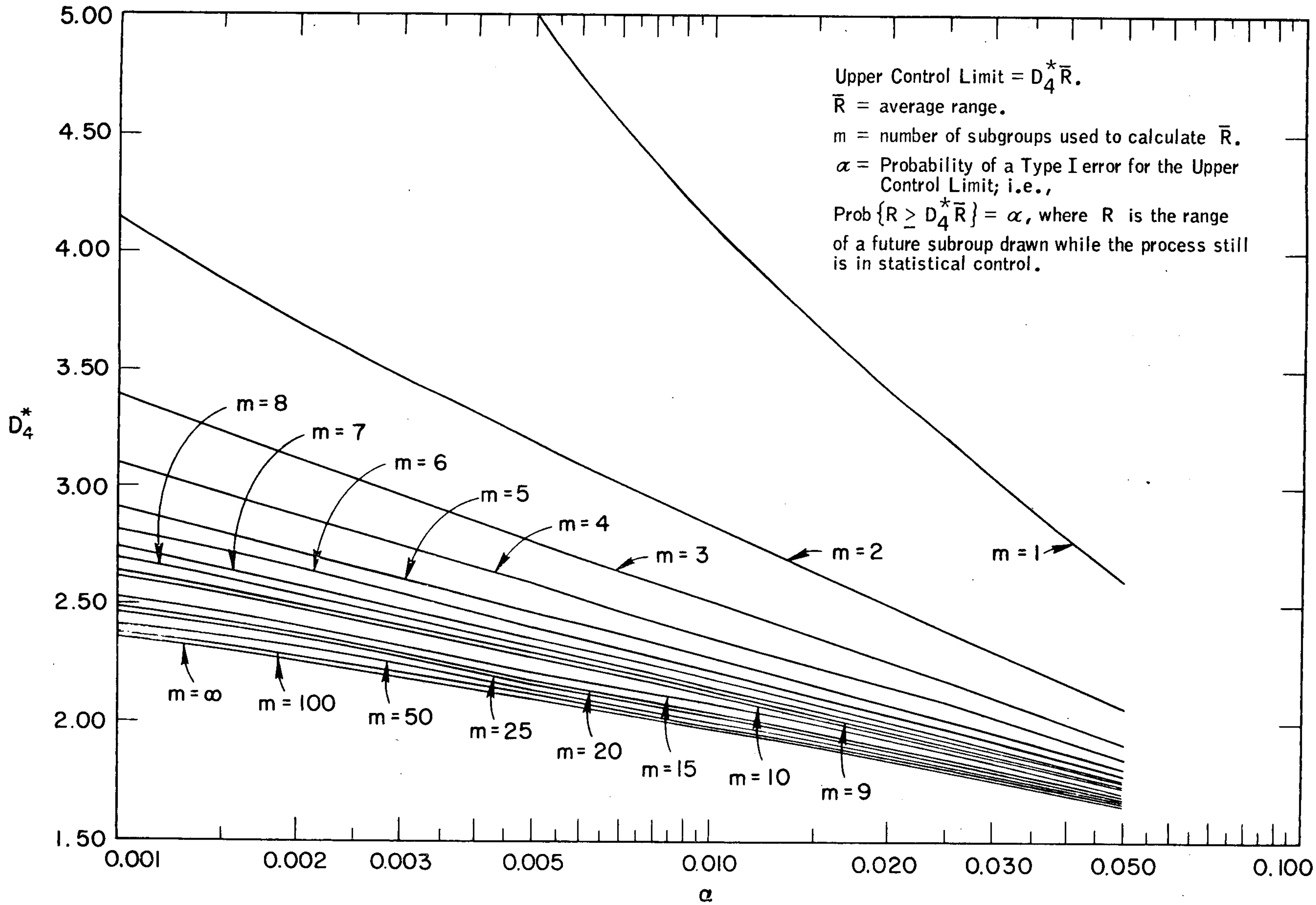

Figure 2. Factor for Upper Control Limit of R Chart When Subgroup Size is Five.

As Figures 1 and 2 show, when m is increased, D_3^* increases and D_4^* decreases. Therefore, the control limits tend to become tighter when the number of subgroups used to calculate $\bar{R}$ is increased. This might suggest that one should recompute the control limits after each new subgroup. However, this would be undesirable, both from the standpoint of the effort involved and the psychological impact on the workers affected by the control chart. A reasonable compromise might be, for example, to compute new control limits when $m = 5$, 10, 25, and 100, and possibly when a subgroup range falls very close to a control limit that has not been revised recently. Another alternative is to follow this procedure only until about 25 subgroups have been observed, and then revert to the conventional method for setting control limits described in Section 2.

If a new subgroup range should fall outside the control limits, the interpretation is the same as with conventional limits as described in Section 2. A slightly different situation is one in which a subgroup range fell inside the control limits when this range was observed but then it lies outside the new limits when they are revised at a later time. The interpretation would be that more complete information has indicated that the process probably was, and may still be, out of statistical control, even though there had not been sufficient evidence at the time the subgroup was observed to make this inference.

It should be emphasized that the value of α given here for particular values of D_3^* and D_4^* applies only to new subgroups, and not to the old subgroups upon which the current control limits are based. The underlying theory just does not apply when the subgroup range being compared to the control limits also was used in calculating these limits. For this case, all that is known is that the true value of α is less than the value of α given here. This implies that, when an old subgroup falls outside the new control limits, this is even stronger evidence than for a new subgroup that the process was out of statistical control when it was observed. Thus, the method described in this section can be applied retroactively to old subgroups, although it is somewhat more reliable (i.e., lower probability of a Type I error) and somewhat less sensitive to a change in the variability of the process than for new subgroups.

Since control limits are intended to describe the process when it is in statistical control, if any old subgroup ranges lie outside the current control limits, the limits should be recalculated without these ranges in $\overline{R}$. As a result, one should use the number of subgroups "in-control," and not the total number of subgroups observed, for the value of m.

It is particularly important that the process be investigated to ensure that it was in statistical control when the original subgroups were drawn for setting the initial control limits. As discussed above, one way of doing this is to check whether the ranges for these subgroups lie inside these limits or not. Then, given that the control limits are based only upon subgroups drawn when the process was in statistical control, the method described in this section can be applied to new subgroups with exactness.

5. Example

Consider a hypothetical manufacturing firm which decided to initiate control charts for $\overline{X}$ and R on a certain troublesome quality characteristic of one of its products. Using a subgroup size of five, the first 12 subgroups yielded the ranges given in Table IV. Since it was desired to begin the control charts as quickly as possible, the methods described in [9] and in this paper were used for setting the control limits for the $\overline{X}$ chart and the R chart, respectively. The method for the $\overline{X}$ chart already has been illustrated in [9], so no further attention will be given to it here. With respect to the R chart, it was decided to use $\alpha = 0.001$ for the lower control limit and $\alpha = 0.005$ for the upper limit in order to obtain a proper balance between the reliability and the sensitivity of the chart. This led to the control limits given in Table IV for the indicated values of m.

All points on the $\overline{X}$ chart happened to fall inside control limits. However, as Table IV indicates, this was not the case with the R chart. After inspecting the fourth subgroup, it was noticed that the range for this subgroup was considerably larger than those for the first three. To check whether this could have been just a chance occurrence, the control limits then were calculated on the basis of the first three subgroups. The fact that the range for the fourth subgroup fell above the resulting upper control limit confirmed that this range was larger than it normally would have been if the process had been in statistical control. A subsequent investigation revealed the source of the trouble, and corrective action was taken. After two additional subgroups, the control limits were recalculated on the basis of the five "in-control" subgroups obtained thus far. The process continued without any further indication of trouble until the eighth subgroup, whose range fell above the upper control limit. An investigation again discovered an assignable cause for the increase in the variability of the process, and this difficulty was rectified immediately. Subsequent subgroups indicated that the process apparently remained in statistical control thereafter. The control limits were recalculated after the twelfth subgroup on the basis of the ten "in-control" subgroups. Additional revisions were made after 25 and 100 such subgroups.

Thus, in this hypothetical example, the early diagnosis given by the method described in this paper led to a stable production process much sooner than would have been possible with conventional methods.

TABLE IV. Data for the Example

Subgroup Number	R	$\bar{R}$	m	D_3^*	D_4^*	LCL = $D_3^*\bar{R}$	UCL = $D_4^*\bar{R}$
1	17						
2	9						
3	13	13.0	3	0.1485	2.758	1.93	35.9
4	37						
5	12						
6	15	13.2	5	0.1520	2.468	2.01	32.6
7	19						
8	40						
9	12						
10	8						
11	21						
12	16	14.2	10	0.1549	2.274	2.20	32.3

6. Derivation of Results

Suppose that R is the range (largest observation minus smallest observation) in a sample of n observations from a normal population with mean μ and standard deviation σ. Suppose that m further independent samples (subgroups) of n have been drawn from the same population, and let the mean of these ranges be $\bar{R}$. The probability relationship of interest is

$$\text{Prob}\ \{R \leq K_\beta \bar{R}\} = \beta \ .$$

The problem in Section 3 was to find $\alpha = \beta$, given $n = 5$ and $K_\beta = D_4 = 2.115$. The problem in Section 4 was to find $D_3^* = K_\alpha$ and $D_4^* = K_{1-\alpha}$, given $n = 5$ and α.

It has been proposed by Patnaik [10] that $\bar{R}/\sigma$ is approximately distributed as $c\chi/\sqrt{\nu}$, where χ is a chi-variate with ν degrees of freedom and c is a scale factor. These constants, c and ν, depend on m and n and are obtained by equating the first two moments of $\bar{R}/\sigma$ to the first two moments of $c\chi/\sqrt{\nu}$. Resnikoff [11] has demonstrated for $n = 5$ that this is an excellent approximation even for small values of m. Therefore, using this s-approximation to the distribution of $\bar{R}$, it follows that $cR/\bar{R}$ will be distributed approximately as the ratio of (a) a range in a sample of n observations to (b) an independent root-mean-square estimate of σ based on ν degrees of freedom. By definition, this ratio is the studentized range, whose distribution has been tabulated extensively by Harter, Clemm, and Guthrie [8]; (also see

Harter [6] for a portion of these tables and for historical background on the studentized range). Hence, since

$$\text{Prob } \{R \leq K_\beta \bar{R}\} = \text{Prob } \{\frac{cR}{\bar{R}} \leq cK_\beta\} ,$$

the Patnaik approximation provides the means for deriving the desired results.

For $m \leq 5$, the values of c and ν were obtained from the table given by Patnaik [10]. For $m > 5$, the values of ν given by Duncan [2] were used, and the values of c were obtained to five significant digits from the expression,

$$c = 2.3259\left(1 + \frac{1}{4\nu} + \frac{1}{32\nu^2} - \frac{5}{128\nu^3}\right) ,$$

given by Patnaik [10]. The value of cK_β was obtained by using linear harmonic ν-wise interpolation (linear interpolation for $1/\nu$) in the table of percentage points of the studentized range given by Harter, Clemm, and Guthrie [8].

Ignoring the error introduced by using the Patnaik approximation, the values of D_3^* given in Table II for $m \geq 3$ are accurate in the fourth significant digit to within four places for $\alpha = 0.050$ and to within two places for $\alpha = 0.001, 0.005, 0.010, 0.025$. Under the same condition, the values of D_4^* given in Table III for $m \geq 3$ are accurate in the fourth significant digit to within four places for $\alpha = 0.001$, to within three places for $\alpha = 0.005, 0.010$, and to within two places for $\alpha = 0.025, 0.050$. It is difficult to ascertain how large an error is introduced by using the Patnaik approximation. However, by referring to the tables given by Harter and Clemm [7] and by Harter, Clemm, and Guthrie [8] and to the comparisons presented by Resnikoff [11], it appears that the values of D_3^* and D_4^* given for $m = 5$ may be in error by several places in the fourth significant digit (especially for small values of α) due to the use of the Patnaik approximation. However, the accuracy of this approximation improves rapidly as m increases.

7. Conclusions

As Table I indicates, conventional control limits for the R chart provide an unreliable basis for indicating when the process has gone out of statistical control unless these limits are based on a fairly large number of subgroups. However, by using the method presented here, one obtains statistically sound control limits regardless of how few subgroups have been inspected. Thus, it now is feasible to begin applying the R chart reliably much sooner than was possible before. The method is simple and the interpretation of results is essentially the same as with conventional control limits.

An analogous method for setting $\overline{X}$ chart control limits based on a small number of subgroups has been proposed previously in [9]. Therefore, since the R chart usually is used in conjunction with an $\overline{X}$ chart, it now is possible to set statistically sound control limits for both charts as soon as desired.

References

[1] Bowker, Albert H., and Lieberman, Gerald J., Engineering Statistics, Prentice-Hall, Englewood Cliffs, New Jersey, 1959.

[2] Duncan, Acheson J., "Design and Operation of a Double-Limit Variables Sampling Plan," Journal of the American Statistical Association, Vol. 53 (1958), pp. 543-550.

[3] Duncan, Acheson J., Quality Control and Industrial Statistics, Richard D. Irwin, Homewood, Illinois, 3rd edition, 1965.

[4] Grant, Eugene L., Statistical Quality Control, McGraw-Hill, New York, 3rd edition, 1965.

[5] Grubbs, Frank E., and Weaver, Chalmers L., "The Best Unbiased Estimate of Population Standard Deviation Based on Group Ranges," Journal of the American Statistical Association, Vol. 42 (1947), pp. 224-241.

[6] Harter, H. Leon, "Tables of Range and Studentized Range," Annals of Mathematical Statistics, Vol. 31 (1960), pp. 1122-1147.

[7] Harter, H. Leon, and Clemm, Donald S., "The Probability Integrals of the Range and of the Studentized Range — Probability Integral, Percentage Points, and Moments of the Range," Wright Air Development Center Technical Report 58-484, Vol. I (ASTIA Document No. AD 215024), April, 1959.

[8] Harter, H. Leon, Clemm, Donald S., and Guthrie, Eugene H., "The Probability Integrals of the Range and of the Studentized Range — Probability Integral and Percentage Points of the Studentized Range; Critical Values for Duncan's New Multiple Range Test," Wright Air Development Center Technical Report 58-484, Vol. II, October, 1959.

[9] Hillier, Frederick S., "$\bar{X}$ Chart Control Limits Based on a Small Number of Subgroups," Industrial Quality Control, Vol. 20, No. 8 (Feb., 1964), pp. 24-29.

[10] Patnaik, P. B., "The Use of Mean Range as an Estimator of Variance in Statistical Tests," Biometrika, Vol. 37 (1950), pp. 78-87.

[11] Resnikoff, George J., "The Distribution of the Average-Range for Subgroups of Five," Technical Report No. 15, Contract N6onr-25126, Applied Mathematics and Statistics Laboratories, Stanford University, Stanford, Calif., July 26, 1954.

[12] Scheffé, Henry, "Operating Characteristics of Average and Range Charts," Industrial Quality Control, Vol. 5, No. 6 (May, 1949), pp. 13-18.

[13] Shewhart, Walter A., with the editorial assistance of Deming, W. Edwards, Statistical Method from the Viewpoint of Quality Control, Graduate School, U.S. Department of Agriculture, Washington D.C., 1939.

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13. ABSTRACT

This report verifies that conventional control limits for the Shewhart control chart for range (R chart) provide an unreliable basis for indicating when the process has gone out of statistical control unless these limits are based on a fairly large number of subgroups.

A new method then is developed for using a small number of subgroups to set reliable control limits for the R chart. These limits have the property that they yield the desired probability of a Type I on each subsequent subgroup. Tables for calculating such limits are presented.

The author previously has proposed an analogous method for setting control limits for the $\overline{X}$ chart. Since the R chart usually is used in conjunction with the $\overline{X}$ chart, these results enable one to set statistically sound control limits for both charts as soon as desired. This permits the valid use of both charts during the crucial initiation of a new process, during the start-up of a process just brought into statistical control again, or for a process whose total output is small.

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14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.