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## OPTIMUM TARGET ORIENTATION IN NUCLEAR REACTION EXPERIMENTS

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SUMMARY

The conditions are examined under which the energy spread in outgoing particles from a nuclear reaction due to finite target thickness is minimized. Physical restrictions on the angle of incidence and emission are explicitly considered, and it is shown that the minimum in energy spread does not always occur at the limits. Targets are considered both in transmission and in reflection. A FORTRAN IV program embodying these results is described. The program determines the optimum target angle for each of a sequence of angles of observation and calculates the energy spread.

## INTRODUCTION

In the measurement of high resolution spectra of charged particles from nuclear reactions, it is of interest not only to minimize all energy spreads (raw beam, kinematic effects, and target thickness effects) contributed by the various experimental arrangements but to have a knowledge of how these energy spreads vary over the spectrum. This report is concerned with the effects of finite target thickness. The existence of a target of finite thickness can introduce a spread into the energies of the outgoing particles because the reactions can occur at any depth within the target and both the incident and the outgoing particles lose some energy in traversing the target. In the general case these particles lose energy at different rates. Cohen (ref. 1) has considered this matter and shown that it is possible to reduce this energy spread to zero by choosing the angle with respect to the incident beam at which the target is mounted. Rosenblatt (ref. 2) and Naquib and McDaniels (ref. 3) have pointed out that Cohen ignored the effect on the outgoing particle energy of the change in the energy of the incident particle, and they gave expressions for the proper angle that include this contribution.

As part of an effort to produce a family of computer programs that would be useful in the design and analysis of experiments this matter was considered again. The previous discussions considered the target to be in transmission only; that is, the incident and


Figure 1. - Physical arrangement for target in transmission.
outgoing particles pass through opposite faces of the target. For completeness, the case when the target is in reflection, that is, the incident and outgoing particles pass through the same face of the target, is also considered.

## TARGET IN TRANSMISSION

Throughout this discussion the incident particle will be numbered 1, the target particle 2, the observed particle 3, and the residual particle 4. Let $\varphi$ be the angle that the normal to the target makes with the beam direction and let $\theta$ be the angle at which the outgoing particle is observed. The sign of $\varphi$ shall be positive when the target normal and the direction of observation are on the same side of the beam line. The target has a thickness $t$, and $x$ is the normal distance into the target to the point where the reaction occurs as shown in figure 1. Then, as shown in reference 2, the expression for the difference in energy of the outgoing particle from that resulting from the same reaction in an ideally thin target is given by

$$
\begin{equation*}
\Delta \mathrm{E}_{3}=\frac{\mathrm{dE}_{3}}{\mathrm{dx}} \frac{\mathrm{t}-\mathrm{x}}{\cos (\theta-\varphi)}+\left(\frac{\partial \mathrm{E}_{3}}{\partial \mathrm{E}_{1}}\right)_{\theta} \frac{\mathrm{dE}}{1} \mathrm{dx} \frac{\mathrm{x}}{\cos \varphi} \tag{1}
\end{equation*}
$$

which can be separated into a constant term and one dependent on $x$ ( $E^{\prime} S$ are energies). The constant term does not contribute to the spread in outgoing energies for reactions that occur at different $x$. The term dependent on $x$ is

$$
\begin{equation*}
\mathrm{x}\left[\left(\frac{\partial \mathrm{E}_{3}}{\partial \mathrm{E}_{1}}\right)_{\theta} \frac{\mathrm{dE}_{1}}{\mathrm{dx}} \frac{1}{\cos \varphi}-\frac{\mathrm{dE}}{3} \mathrm{dx} \frac{1}{\cos (\varphi-\theta)}\right] \tag{2}
\end{equation*}
$$

The maximum difference in outgoing energies ( $\delta \mathrm{E}_{3}$ ) occurs between events that take place on the front surface $(x=0)$ and those that take place on the back surface $(x=t)$ and is given by

$$
\begin{equation*}
\delta \mathrm{E}_{3}=\mathrm{t} \frac{\mathrm{dE}}{1}\left(\frac{\partial \mathrm{E}_{3}}{\mathrm{dx}}\left(\frac{1}{\partial \mathrm{E}_{1}}\right)_{\theta}\left[\frac{1}{\cos \varphi}-\frac{\mathrm{F}}{\cos (\theta-\varphi)}\right]\right. \tag{3}
\end{equation*}
$$

where

$$
F \equiv \frac{\frac{\mathrm{dE}_{3}}{\mathrm{dx}}}{\frac{\mathrm{dE}_{1}}{\mathrm{dx}}\left(\frac{\partial \mathrm{E}_{3}}{\partial \mathrm{E}_{1}}\right)_{\theta}}
$$

It is possible to make $\delta \mathrm{E}_{3}$ equal to zero by choosing $\varphi$ such that

$$
\begin{equation*}
\mathbf{F} \cos \varphi=\cos (\theta-\varphi) \tag{4}
\end{equation*}
$$

However, for large values of $\mathbf{F}$ and/or small values of $\theta$, this expression yields nonphysical values of $\varphi$.

In any experimental arrangement there are physical limits to the angle that either the incident or observed particles can make with the normal to the target, either because of the design of the target mount or because of errors due to target flatness and nonuniformity. Thus for one reason or another there are maximum values for $\varphi$ and $\theta-\varphi$, and when the $\varphi$ given by equation (4) is greater than this value $\Phi$, then the problem becomes a minimization one. The naive conclusion that the minimum occurs at this physical maximum angle $\Phi$ is not correct for all values of $F$.

It is useful to make a change in variables at this point. Let $\varphi=\theta / 2+\delta$ so that

$$
\begin{equation*}
\delta \mathrm{E}_{3}=\mathrm{A}\left[\frac{1}{\cos \left(\frac{\theta}{2}+\delta\right)}-\frac{\mathrm{F}}{\cos \left(\frac{\theta}{2}-\delta\right)}\right] \tag{5}
\end{equation*}
$$

where

$$
\mathrm{A}=\mathrm{t} \frac{\mathrm{EE}_{1}}{\mathrm{dx}}\left(\frac{\partial \mathrm{E}_{3}}{\partial \mathrm{E}_{1}}\right)_{\theta}
$$

Equation (5) is used for $F>1$; for $F<1$, it is convenient to write $\delta \mathrm{E}_{3}$ as

$$
\begin{equation*}
\delta E_{3}=B\left[\frac{G}{\cos \left(\frac{\theta}{2}+\delta\right)}-\frac{1}{\cos \left(\frac{\theta}{2}-\delta\right)}\right] \tag{6}
\end{equation*}
$$

where $G=1 / F$ and $B=t \mathrm{dE}_{3} / \mathrm{dx}$. The behavior of equation (6) as a one parameter family of curves on $G$ is the same for negative (positive) values of $\delta$ as the behavior of


Figure 2. $-\delta E_{3} / \mathrm{A}$ as function of $\delta$ for $\theta=10^{\circ}$.


Figure 3. $-\delta E_{3} / \mathrm{A}$ as function of $\delta$ for $\theta=20^{\circ}$.
equation (5) on $F$ for positive (negative) values of $\delta$. The following discussion is therefore limited to values of $F>1$.

Figure 2 shows the results of calculations of $\delta \mathrm{E}_{3} / \mathrm{A}$ for $\theta=10^{\circ}$ and several values of $F$. It is assumed that the physical design of the target holder sets $\Phi=60^{\circ}$ so that $\delta$ is limited to $55^{\circ}$. It is seen that as $F$ increases from 1.5 to 1.8 the minimum of $\left|\delta \mathrm{E}_{3} / \mathrm{A}\right|$ shifts from the physical maximum angle to a smaller angle.

Figure 3 shows a similar plot for $\theta=20^{\circ}$. The same type of behavior is seen with the inflection point now occurring for $F \sim 2.8$. As $\theta$ increases, the inflection point moves out slowly from a $\delta$ value of $35^{\circ}$; thus, for $\theta$ in excess of some $\Theta\left(44^{\circ}\right.$ for the case of $\Phi=60^{\circ}$, see eq. (11)), even the inflection point is in the nonphysical region. Thus for $\theta<\Theta$ there are three regions for $F$ : for values of $F$ less than some $F^{\prime}$, $\delta \mathrm{E}_{3}$ can be made zero; for intermediate values, the minimum value of $\delta \mathrm{E}_{3}$ is obtained at the physical maximum angle; and at values larger than some $F^{\prime \prime}$, the minimum value of $\delta \mathrm{E}_{3}$ is found at a smaller angle. For $\theta>\Theta$, there are only the first two of the previous regions. Because of a preference for small angles of incidence or emission the optimum point is chosen at the internal minimum as soon as it develops.

The value of $F^{\prime}$ is obtained by setting $\theta / 2+\delta$ equal to $\Phi$ :

$$
\mathbf{F}^{\prime}=\frac{\cos (\theta-\Phi)}{\cos \Phi}
$$

To obtain $\mathrm{F}^{\prime \prime}$, the point of inflection is determined. In expanded form equation (5) is written as
$\delta \mathrm{E}_{3}=\mathrm{A}\left(\cos \delta \cos \frac{\theta}{2}+\sin \delta \sin \frac{\theta}{2}-\mathrm{F} \cos \delta \cos \frac{\theta}{2}+\mathrm{F} \sin \delta \sin \frac{\theta}{2}\right)$

$$
\begin{equation*}
\times\left[\left(\cos \delta \cos \frac{\theta}{2}+\sin \delta \sin \frac{\theta}{2}\right)\left(\cos \delta \cos \frac{\theta}{2}-\sin \delta \sin \frac{\theta}{2}\right)\right]^{-1}=A \times \frac{C}{D} \tag{7}
\end{equation*}
$$

The partial derivative of $\delta \mathrm{E}_{3}$ with respect to $\delta$ is found to be

$$
\begin{align*}
& \frac{\partial\left(\delta \mathrm{E}_{3}\right)}{\partial \delta}=\left[(1+\mathrm{F}) \cos \delta \sin \frac{\theta}{2}\left(\cos ^{2} \delta \cos ^{2} \frac{\theta}{2}-\sin ^{2} \delta \sin ^{2} \frac{\theta}{2}+2 \sin ^{2} \delta\right)\right. \\
&\left.-(1-\mathrm{F}) \sin \delta \cos \frac{\theta}{2}\left(\cos ^{2} \delta \cos ^{2} \frac{\theta}{2}-\sin ^{2} \delta \sin ^{2} \frac{\theta}{2}-2 \cos ^{2} \delta\right)\right] / \mathrm{D}^{2} \tag{8}
\end{align*}
$$

Setting equation (8) equal to zero yields

$$
\begin{equation*}
\tan \delta=\frac{\frac{1+F}{1-F} \tan \frac{\theta}{2}}{1-\frac{2}{\cos ^{2} \frac{\theta}{2}+\sin ^{2} \delta}} \tag{9}
\end{equation*}
$$

as a condition for an extremum.
For this extremum to be an inflection point, the value of the second derivative must be zero. The first derivative is of the form $X / Y$ and since we are interested in the value of the second derivature at an extremum, the nonzero part of the second derivative is $(\partial \mathrm{X} / \partial \delta) / \mathrm{Y}$. The first derivative is given schematically by

$$
\frac{\partial \delta \mathrm{E}_{3}}{\partial \delta}=\frac{\mathrm{D} \frac{\partial \mathrm{C}}{\partial \delta}-\mathrm{C} \frac{\partial \mathrm{D}}{\partial \delta}}{\mathrm{D}^{2}}
$$

so that the nonzero part of the second derivative is


Now $C$ contains $\delta$ only in first-power trigonometric functions so that $\partial^{2} C / \partial \delta^{2}=-C$, and in addition, C is nonzero in our range of interest. Thus the value of $\delta$ for which the extremum is an inflection point is obtained from

$$
\mathrm{D}+\frac{\partial^{2} \mathrm{D}}{\partial \delta^{2}}=0
$$

This is found to give

$$
\begin{equation*}
\tan ^{2} \delta=\frac{1+\sin ^{2} \frac{\theta}{2}}{1+\cos ^{2} \frac{\theta}{2}} \tag{10}
\end{equation*}
$$



Figure 4. - Physical arrangement for target in reflection.


Figure 5. $-\delta \mathrm{E}_{3} / \mathrm{A}$ as function of $\delta$ for $0=110^{\circ}$.

The $\mathrm{F}^{\prime \prime}$ is determined from equation (9) when the $\delta$ value obtained from equation (10) is used. The $\Theta$ is defined by

$$
\begin{equation*}
\tan ^{2}\left(\Phi-\frac{\Theta}{2}\right)=\frac{1+\sin ^{2} \frac{\Theta}{2}}{1+\cos ^{2} \frac{\Theta}{2}} \tag{11}
\end{equation*}
$$

Similar expressions are obtained from equation (6) when $F<1$.

## TARGET IN REFLECTION

In some cases it becomes necessary to observe with the target in reflection. The physical arrangement is shown in figure 4. For this definition of angles, $\delta \mathrm{E}_{3}$ is again given by equation (3). Let

$$
\varphi=-\left[\frac{1}{2}\left(180^{\circ}-\theta\right)+\delta\right]=\frac{\theta}{2}-\delta+90^{\circ}
$$

Then

$$
\begin{equation*}
\delta \mathrm{E}_{3}=\mathrm{A}\left[\frac{1}{\sin \left(\frac{\theta}{2}-\delta\right)}+\frac{\mathrm{F}}{\sin \left(\frac{\theta}{2}+\delta\right)}\right] \tag{12}
\end{equation*}
$$

Physical arguments indicate that the normal to the target should lie between the incident beam and the detector, being closer to whichever beam consists of the particles with the larger value of $\mathrm{dE} / \mathrm{dx}$. Figure 5 shows $\delta \mathrm{E}_{3} / \mathrm{A}$ from equation (12) plotted for $\theta=110^{\circ}$ and several values of $F$. The improvement in $\delta \mathrm{E}_{3}$ made by choosing the minimum point rather than the half-angle position decreases as $\theta$ increases from $90^{\circ}$ to $180^{\circ}$. For $F=5$, it amounts to 6.5 percent at $110^{\circ}$ and 0.15 percent at $160^{\circ}$. Thus, in general, the choice of $\delta=0$ is quite acceptable. For completeness, the value of $\delta$ at the minimum of $\left|\delta \mathrm{E}_{3} / \mathrm{A}\right|$ is given by

$$
\begin{equation*}
\tan \delta=\left(\frac{1-F}{1+F} \cot \frac{\theta}{2}\right)\left(1-\frac{2}{\cos ^{2} \frac{\theta}{2}+\cos ^{2} \delta}\right) \tag{13}
\end{equation*}
$$

## PROGRAM DESCRIPTION

The program consists of a main program ANGOPT plus with several subroutines; one of which, ANGMIN, performs the minimization described previously. At any one detector angle a range of excitation energies in the residual nucleus is viewed, and the target angle can be optimized for only one value within this range. Further, it is often the case that there is more than one region where optimum resolution is needed. Therefore, it was decided to calculate the optimum target angle for two values of excitation energy $\mathrm{E}^{*}$ and, in addition, to calculate the energy spread for these two angles at several other excitation energies.


Figure 6. - Operational block diagram of ANGOPT.

An operational block diagram of ANGOPT is shown in figure 6. The major complexity is concerned with the logic used to determine whether the target is in transmission or reflection. The evaluation of $\partial E_{3} / \partial \mathbf{E}_{1}$ is performed by using equation (5) of reference 2 , which is in convenient form for machine calculation. For reflection, the choice of $\delta=0$ is taken so that in this case ANGMIN is bypassed and $\varphi$ set to be $-(180-\theta) / 2$. Listings of all the programs are given in appen$\operatorname{dix} \mathrm{A}$.

## Program Input and Options

The input data cards are described in table I, and the input for four test cases is shown in figure 7. The mean ionization potential ABI can be obtained from reference 4.

The program can be used to calculate either in transmission, reflection, or both by suitable choices of the angles. The values of the angles have the following restrictions:

TABLE I. - DATA INPUT DESCRIPTION

| Card | Quality | Format | Input |
| :---: | :---: | :---: | :---: |
| 1 | TITLE | 2A6 | Element name of target |
|  | M1 | F8. 0 | Incident particle mass, amu |
|  | M2 |  | Target particle mass, amu |
|  | M3 |  | Observed particle mass, amu |
|  | M4 |  | Residual particle mass, amu |
|  | Z1 |  | Incident particle charge |
|  | E*(1) |  | Excitation energy 1, MeV |
|  | E*(2) |  | Excitation energy 2, MeV |
| 2 | EO |  | Incident energy, MeV |
|  | ABS <br> ABZ |  | Absorber thickness, $\mathrm{mg} \mathrm{cm}^{-2}$ Absorber $\mathbf{Z}$ |
|  | ABZ |  |  |
|  | ABI |  | Mean ionization potential, eV |
|  | THETAO |  | Initial angle, deg |
|  | THETAT |  | Maximum angle in transmission, deg |
|  | DTHETA |  | Angle increment, deg |
|  | THETAR |  | Lowest angle in reflection, deg |
|  | THETAM | $\dagger$ | Maximum angle, deg |



Figure 7. - Facsimile of input for test data.
(1) For transmission only:

THETAO $>1^{\circ}$
THETAM = THETAT
(2) For reflection only:

THETAO $=$ THETAR $>1^{\circ}$
(3) For transmission and reflection:

THETAO $>1^{\circ}$
THETAR $<$ THETAT
THETAT < THETAM

## Program Output

The output for the input data of figure 7 is shown in appendix B. The heading of the output gives the important description of the physical situation, that is, target in transmission or reflection, target material, incident and observed particles named, and the two values of excitation energies for which $\varphi$ is optimized. In addition, the input masses, the incident energy, and the absorber thickness are written out.

For each detector angle two target angles are given, one for each of the two input values of $\mathrm{E}^{*}$. Then for each detector angle the spread in energy for the input $\mathrm{E}^{*}$ and for $E^{*}=0,5,10,15$, and 20 million electron volts is given.

## Subprograms

ANGMIN function. - This subroutine calculates the value of $\varphi$ to give the minimum value of energy spread subject to the constraints discussed. It is entered with a value of $F$ and $\theta$ and returns $\theta / 2+\delta$ to the calling program. It is written for $\Phi=60^{\circ}$ and $\Theta=44^{\circ}$. Equation (9) is solved in an iterative manner with the first choice of $\delta$ taken as

$$
\delta=\tan ^{-1}\left(\frac{F+1}{F-1} \tan \frac{\theta}{2}\right)
$$

Iteration is stopped when the change in $\delta$ is less than 0.001 radian.
ELOSS subroutine. - This subroutine calculates values of $\delta \mathrm{E}_{3}$ from equation (3) for the input value of $\mathrm{E}^{*}$ and for $\mathrm{E}^{*}$ values of $0,5,10,15$, and 20 million electron volts by using the angle that is optimum for the input $\mathrm{E}^{*}$. If the sequence of $\mathrm{E}^{*}$ values at a given angle leads to a nonphysical situation, then the energy spread for that and subse-
quent $\mathrm{E}^{*}$ values is set to zero.
FDEDX function. - This subroutine evaluates the Bethe equation for the rate of energy loss without consideration of shell correction:

$$
-\frac{\mathrm{dE}}{\mathrm{dx}}=\left(3.071 \times 10^{-4}\right) \frac{\mathrm{z}^{2} \mathrm{Z}}{\beta^{2} \mathrm{~A}}\left[\ln \left(\frac{1.022 \times 10^{\mathrm{o}}}{1-\beta^{2}}\right) \frac{\beta^{2}}{\mathrm{I}}-\beta^{2}\right]
$$

where $z$ is the charge of the particle, $Z$ is the atomic number of the stopping material, A is the atomic weight of the stopping material, I is the mean ionization potential of the stopping material in electron volts, and $\beta$ is the velocity of the particle in units of the velocity of light.

EVALUE function. - The nonrelativistic form of the energy equation is evaluated:

$$
\mathrm{E}_{3}=\left[\mathrm{B} \cos \theta \pm\left(\mathrm{A}-\mathrm{B}^{2} \sin ^{2} \theta\right)^{1 / 2}\right]^{2}
$$

where

$$
\begin{gathered}
A=\frac{M_{4}\left[E_{1}\left(1-\frac{M_{1}}{M_{3}+M_{4}}\right)+Q\right]}{M_{3}+M_{4}} \\
B=\frac{\left(M_{3} M_{1} E_{1}\right)^{1 / 2}}{M_{3}+M_{4}}
\end{gathered}
$$

The M's are the masses, the E's are energies, and $Q$ is the difference between the total kinetic energy before and after the collision. This subroutine is due to J. B. Ball of reference 5 .

NAMES subroutine. - This subroutine identifies a light nuclear particle given the mass number and charge. It is a FORTRAN IV version of NAMER due to J. B. Ball of reference 5 .

## Lewis Research Center

National Aeronautics and Space Administration
Cleveland, Ohio, August 31, 1965.

## APPENDIX A

## PROGRAM LISTINGS

```
C
    DIMENSION DEL(2,6,400),TANG(2,400),ANGLAB(400),DELE(6),ESTAR(2),Q(
    12).TITLE(2)
10 READ (5,190) TITLE,Wl,W2,W3,W4,Zl,Z3,ESTAR(1),ESTAR(2),E0,ABS,ABZ
    1,ABA,ABI,THETAO,THETAT,DTHETA,THETAR,THETAM
    WRITE (6,191)
    IF (THETAO-THETAR) 15,20,20
15 WRITE (6,192)
    ASSIGN 35 TO MO
    GO TO 25
20 WRITE (6,193)
    ASSIGN 45 TD MD
25 MTEST=W1+0.002
    IZ=Z1
    CALL NAMES(MTEST,IZ,IA,IB)
    MTEST=W3+0.002
    IZ=Z3
    CALL NAMES(MTEST,IZ,IC,ID)
    WRITE (6,194) TITLE,IA,IB,IC,ID,ESTAR(1),ESTAR(2)
    WRITE (6,195) W1,W2,W3,W4,EO,ABS
    DEOXO=FDEDX(Z1,W1,EO,ABZ,ABA,ABI)
    KK=0
    DO 80 I=1,2
    ASSIGN 85 TO MA
    ASSIGN 10 TO MB
    J=1
    ANGLAB(J)=THETAO
    Q(I)=(W1+W2-W3-W4)*931.44-ESTAR(I)
30 E=EVALUE(W1,W3,W4,EO,Q(I),ANGLAB(J))
    DEDX=FDEDX(Z3,W3,E,ABZ,ABA,ABI)
    F=DEDX/(DEDXO*(E/EO)*(((W3+W4)*E+(W4-W1)*EO-W4*Q(I))/((W3+W4)*E+(W
    14-W1)*EO+W4*Q(1)1))
        GO TO MO,(35,45)
35 TANG(I,J)=ANGMIN(ANGLAB(J),F)
        CALL ELOSS(ANGLAB(J),TANG(I,J),EO,Q(I),ABS,ABZ,ABA,ABI,W1,W2,W3,W4
    1,Z3,DEDXO,DEDX,E,DELE)
        DO 40 MN=1,6
40 DEL (I,MN,J)=DELE(MN)
        J= J+1
        K=J-1
        ANGLAB(J)=ANGLAB(K)+DTHETA
        IF(THETAT-ANGLAB(J)) 55,30,30
45 TANG(I,J)=-{180.-ANGLAB(J))/2.
    CALL ELDSS(ANGLAB(J),TANG(I,J),EO,Q(I),ABS,ABZ,ABA,ABI,W1,W2,W3,W4
    1,Z3,DEDXO,DEDX,E,DELE)
    DO 50 MN=1,6
5CDEL(I,MN,J)=DELE (MN)
    J=J+1
    K=J-1
    ANGLAB(J)=ANGLAB(K)+DTHETA
    IF (THETAM-ANGLAB(J)) 65,30,30
5 5 ~ I F ~ ( T H E T A T - T H E T A M ) ~ 6 0 , 6 5 , 6 5 ,
60 KK=K
    ASSIGN 120 TO MA
    ASSIGN 125 TO MB
    ANGLAB(J)=THETAR
    ASSIGN 45 TD MD
    GO TO 30
65 IF(THETAO-THETAR) 70,75,75
70 ASSIGN 35 TO MD
    GO TO }8
```

```
    75 ASSIGN 45 TO MO
    80 CDNTINUE
    KL=K-KK
    M=0
    LSTOP=0
    MM=0
    GO TO MA, (85,120)
    85 ASSIGN 105 TO MC
    90 LSTART=LSTOP+1
    IF(K-13) 95,95,10
95 LSTOP=K+M+MM
    WRITE (6,196)(ANGLAB(LINE),(TANG(I,LINE),(DEL(I,JK,LINE),JK=1,6)
    1,I=1,2),LINE=LSTART,LSTOP)
    GO T0 MB,(10,125)
100 GO TO MC,(105,110)
105 LSTOP=13+M
    K=K-13
    MM=13
    ASSIGN 110 TO MC
    GO TO 115
110 LSTOP=LSTART+13
    K=K-14
    MM=MM+14
115 WRITE (6,196)(ANGLAB(LINE),(TANG(I,LINE),(DEL(I,JK,LINE),JK=1,6)
    1,I=1,2),LINE=LSTART,LSTOP)
    WRITE (6,197) TITLE,IA,IB,IC,ID,ESTAR(1),ESTAR(2)
    GO TO 90
120K=KK
    GO TO 85
125K=KL
    M=KK
    LSTOP=M
    ASSIGN 10 TO MB
    WRITE (6,191)
    WRITE (6,193)
    WRITE (6,194) TITLE,IA,IB,IC,ID,ESTAR(1),ESTAR(2)
    WRITE (6,195) Wl,W2,W3,W4,EO,ABS
    GO TO 85
190 FORMAT(4\times2A6,8F8.0/10F8.0)
191 FORMAT(53H1 OPTIMUM ANGLE FOR TARGET TD MINIMIZE ENERGY SPREAD )
192 FORMAT(1H+,54X22HTARGET IN TRANSMISSION)
193 FORMAT(lH+,54X2OHTARGET IN REFLECTION)
194 FORMAT(13HO TARGET IS ,2A6,25H INCIDENT PARTICLES ARE ,A6,A3,25
    1H, OBSERVED PARTICLES ARE,A6,A3,8H ESTAR1=,F7.3,8H ESTAR2=,F7.3)
195 FORMAT (5HO MI=F7. 5,5X3HM2=F8.4,5\times3HM3=F7.5,5\times3HM4=F3.4,5\times3HEO=F5.2
    1,3HMEV,5X4HABS =F5.2,8HMG/CM**2)
196 FORMAT (2HO ,5X8HDETECTOR,6X6HTARGET, 42X13HENERGY SPREAD/9X5HANGLE,
    18\times5HANGLE, 15 X7HOP T IMUM, 7X4HE* =0, 8X4HE* = 5, 8\times5HE* = 10, 7X5 HE* = 15,7\times5 HE
    2*=20/(1HO,2F13.2,10X6F12.4//20XF7.2,10X6F12.4))
197 FORMATI13HI TARGET IS ,2A6,25H INCIDENT PARTICLES ARE ,A6,A3,25
    1H, OBSERVED PARTICLES ARE,A6,A3,8H ESTAR1=,F7.3,8H ESTAR2=,F7.3)
    END
```

```
    FUNCTIDN ANGMIN(ANG,F)
    PHI=ANG/114.59156
    PHE=(ANG-60.)/57.29578
    PHO=ANG/57.29578
    IF(F-1.) 60,10,15
    10 ANGMIN=PHI*57.29578
    RETURN
    15 FF=COS(PHE)/COS(1.047197)
    IF(FF-F) 25,20,20
    20 PHI=ATAN((F-COS(PHO))/SIN(PHO))
    GO TO 10
    25 IF(ANG-40.) 35,30,30
    30 ANGMIN=60.
        RETURN
    35 DEL=ATAN(SQRT(11.+SIN(PHI)**2)/(1.+COS(PHI)**2)))
        A=(1.-(2./(CDS(PHI)**2+SIN(DEL)**2)))*(SIN(DEL)*COS(PHI))/(SIN(PHI
    1)*COS(DEL))
        FFF=(A-1.)/(A+1.)
        IF(FFF-F) 40,30,3
    4 0 ~ D O L = A T A N ( ( ( 1 . + F ) / ( F - 1 . ) ) * ( S I N ( P H I ) / C O S ( P H I ) ) )
    4 5 ~ D E L = A T A N ( ( 1 ( 1 \cdot + F ) / ( 1 \cdot - F ) ) * ( S I N ( P H I ) / C Q S ( P H I ) ) ) / ( 1 . - ( 2 . / ( C O S ( P H I ) * * * )
    12+SIN(DOL)**2))))
        DIL=DEL-DOL
        IF(DIL-0.001) 50,50,55
    50 ANGMIN=(PHI+DEL)*57.29578
    RETURN
    55 DOL=DEL
    GO TD 45
    60 G=1./F
    GG=CDS(PHE)/COS(1.047197)
    IF(GG-G) 70,65,65
    65 PHI=ATAN((I.-G*COS(PHO))/(G*SIN(PHO)))
    GO TO 10
    70 IF(ANG-40.) 80,75,75
    75 ANGMIN=ANG-60.
    RETURN
    80 DEL=ATAN(-SQRT((1*+SIN(PHI)**2)/(1.+COS(PHI)**2)))
        A=(1.-(2./(COS(DHI)**2+SIN(DEL)**2)))*(SIN(DEL)*COS(PHI))/(SIN(PHI
    1)*COS(DEL))
        GGG=(A+1.)/(A-1.)
        IF(GGG-G) 85,75,75
    85DOL=ATAN(((G+1.)/(G-1.))*(SIN(PHI)/COS(PHI)))
    90 DEL=ATAN((((G+1.)/(G-1.))*(SIN(PHI)/COS(PHI)))/(1.-(2./(COS(PHI)**
    12+SIN(DOL)**2)))।
    DIL=DOL-DEL
    IF(DIL-0.001) 95,95,100
95 ANGMIN=(PHI+DEL)*57.29578
    RETURN
100 DOL=DEL
    GO TO 90
    END
```

```
    SUBROUTINE ELOSS(ANG,TANG,EO,Q,ABS,ABZ,ABA,ABI,W1,W2,W3,W4,Z3,DEDX
    10,DEDX,E,DELE)
    DIMENSION DELE(6)
    BANG=(ANG-TANG)/57.29578
    CANG=TANG/57.29578
    DANG=ANG/57.29578
    JK=1
    A=(E/EO)*(((W3+W4)*E+(W4-Wl)*EO-W4*Q)/((W3+W4)*E+(W4-Wl)*EO+W4*Q))
    DELE(JK)=ABS*((DEDXO*A/COS(CANG))-(DEDX/COS(BANG)))
    P=(Wl+W2-W3-W4)*931.44
10 JK=JK+1
    EICOM=EO*(W3+W4-W1)/(W3+W4)
    V3SQ=2.O*W4*(EICOM+P)/(W3*(W3+W4))
    VVCM=SQRT(2.0*W1*EO)/(W3+W4)
    TEST=V3SQ-((VVCM*SIN(DANG))**2)
    IF(TEST) 20,15,15
15 F=EVALUE(Wl,W3,W4,EO,P,ANG)
    A=(F/EO)*(((W3+W4)*F+(W4-Wl)*EO-W4*P)/((W3+W4)*F+(W4-Wl)*E0+W4*P))
    DEDY=FDEDX(Z3,W3,F,ABZ,ABA,ABI)
    DELE(JK)=ABS*((DEDXO*A/COS(CANG))-(DEDY/COS(BANG)))
    P=P-5.
    IF(JK-6) 10,25,25
20 DELE (JK)=0.
    IF(JK-6) 10,25,25
25 RETURN
    END
    SUBROUTINE NAMES(IM,IZ,IW,IX)
    DATA R1,R2,R3,R4,R5,R6,R7,R8,R9,R10,R11/6HPROTON,6HS
        -6HDEUTER
    I,GHONS ,GHTRITIN,GHHELIUM,6H-3 ,GHALPHAS,GH ,GHNOT NA,GH
    2MED
    IF(IZ.EQ.1.AND.IM.EQ.1) GO TO 110
    IF(IZ.EQ.1.AND.IM.EQ.2) GO TO 111
    IF(IZ.EQ.1.AND.IM.EQ.3) GO TD 112
    IF(IZ.EQ.2.AND.IM.EQ.3) GO TO 113
    IF(IZ.EQ.2.AND.IM.EQ.4) GO TO 114
    IW=-IABS(+R10)
    IX=-IABS(+R11)
    RETURN
110 IW=-IABS(+R1)
    IX=-IABS(+R2)
    RETURN
111 IW=IABS(+R3)
    IX=-IABS(+R4)
    RETURN
112 IW=-IABS(+R5)
    IX=-IABS(+R2)
    RETURN
113 IW=IABS(+R6)
    IX=-IABS(+R7)
    RETURN
114 IW=IABS(+R8)
    IX=-IABS(+R9)
    RETURN
    END
```

```
FUNCTION FDED-X(PZ,PM,EINIT,ABSZ,ABSA,ABSI)
ARGI=(3.071E-4)*(PZ**2)*ABSZ/ABSA
ARG2=(1.022E6)/ABSI
SQBETA=1.0-(931.44/((EINIT/PM)+931.44))**2
FDEDX=(ARG1/SQBETA)*(ALOG(ARG2*SQBETA/(1.0-SQBETA))-SQBETA)
RETURN
END
```

FUNCTIDN EVALUE(W1,W3,W4,E1,Q,ANGLAB)
THETA=ANGLAB/57.29578
$E I C O M=E 1 *(W 3+W 4-W 1) /(W 3+W 4)$
$V 3 S Q=2.0 * W 4 *(E 1 C D M+Q) /(W 3 *(W 3+W 4))$
VVCM $=$ SQRT (2.0*WI*E1)/(W3+W4)
EVALUE $=0.5 * W 3 *((V V C M * C D S(T H E T A)+S Q R T(V 3 S Q-((V V C M * S I N(T H E T A)) * * 2))$
2**2)
RETURN
END

## APPENDIX B

## OPTIMUM ANGLE FOR TARGET TO MINIMIZE ENERGY SPREAD

Case I
optimum angle for target to minimize energy spread •target in transmission


## Case II

optimum angle for target to minimize energy spread target in reflection


Case III
OPTIMUM ANGLE FOR TARGET TO MINIMIZE ENERGY SPREAO TARGET IN TRANSMISSION
JARGET IS CALCIUM INCIDENT PARTICLES ARE ALPHAS , OBSERVED PARTICLES ARF ALPHAS ESTAZI= O. ESTAR2= 3. T3O

| $111=4.00260$ | H2 $=39.9651$ | $\mathrm{M} 3=4.00260 \mathrm{M}$ | M4 $=39.9551$ | $E J=41.60 \mathrm{MEV}$ | $A B S=$ | 1.50MG/CM**2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DETECTOR | TARGET |  |  | ENERGY | SPREAD |  |  |
| ANGLE | ANGLE | OPTIMUM | $4 \quad E *=0$ | $E *=5$ | E* $=10$ | $E *=15$ | $E *=20$ |
| 10.00 | 6.76 | 0. | 0. | -0.0193 | -0.0438 | -0.0761 | -0.1212 |
|  | 28.74 | -0.0000 | 0.0148 | -0.0056 | -0.0314 | -0.0654 | -0.1128 |
| 25.00 | 16.77 | 0. | 0. | -0.0199 | -0.0452 | -0.0786 | -0.1254 |
|  | 25.85 | -0.0000 | 0.0143 | -0.0054 | -0.0304 | $-0.0634$ | -0.1097 |
| 40.00 | 26.48 | -0. 0000 | -0.0000 | -0.0210 | -0.0478 | -0.0834 | -0. 2335 |
|  | 32.07 | $-0.0000$ | 0.0149 | -0.0057 | -0.0320 | -0.0668 | -0.1160 |
| 55.00 | 35.73 | -0. 0000 | $0 \quad-0.3000$ | -0.0228 | -0.0519 | -0.0909 | -0.1462 |
|  | 39.67 | -0.0000 | 0.0161 | -0.0062 | -0.0347 | -0.0728 | -0.1268 |
| 70.00 | 44.39 | -0.0000 | 0 -0.0000 | -0.0253 | -0.0579 | $-0.1018$ | -0.1646 |
|  | 47.36 | -0.0000 | 0.0 .0179 | -0.0068 | $-0.0387$ | -0.0815 | -0.1428 |
| 85.00 | 52.41 | -0.0000 | $0 \quad-0.2000$ | -0.0289 | $-0.0664$ | -0.1172 | -0.1907 |
| , | 54.69 | -0.0000 | 0.0204 | -0.0078 | -0.0444 | -0.0940 | -0.1656 |

Case III (Concluded)
DPT IMUM AVGLE FOR TARGET TO MINIMIZE ENERGY SPREAD TARGET IN REFLECTION TARGET IS CALCIUM INCIDENT PARTICLES ARE ALPHAS , OBSERVED PARTICLES ARE ALPHAS ESTARI= O. ESTAR2= 3. 730

| M1 $=4.00260$ | M2 $=39.9651$ | $M 3=4.00250 \quad M 4=$ | 39.9551 | $E 3=41.60 \mathrm{MEV}$ | $A B S=1$ | 1.50MG/CM**2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DETECTOR | TARGE T |  |  | ENERGY | SPREAD | - |  |
| ANGLE | ANGLE | OP IIMUM | E* $=0$ | E*=5 | $E *=10$ | $E *=15$ | E* $=20$ |
| 85.00 | -47.50 | 0.5509 | 0.5539 | 0.5870 | 0.6338 | 0.6974 | 0.7893 |
|  | -47.50 | 0.5769 | 0.5539 | 0.5870 | 0.6338 | 0.6974 | 0.7893 |
| 100.00 | -40.00 | 0.4868 | 0.4858 | 0.5207 | 0.5649 | 0.6250 | 0.7125 |
|  | -40.00 | 0.5113 | 0.4868 | 0.5207 | 0.5649 | 0.6250 | 0.7125 |
| 115.00 | -32.50 | 0.4438 | 0.4438 | 0.4765 | 0.5192 | 0.5776 | 0.6630 |
|  | -32.50 | 0.4674 | 0.4438 | 0.4765 | 0.5192 | 0.5776 | 0.6630 |
| 130.00 | -25.00 | 0.4149 | 0.4149 | 0.4470 | 0.4890 | 0.5466 | 0.6310 |
|  | -25.00 | 0.4381 | 0.4149 | 0.4470 | 0.4890 | 0.5466 | 0.6310 |
| 145.00 | -17.50 | 0.3962 | 0.3962 | 0.4280 | 0.4696 | 0.5269 | 0.6111 |
|  | -17.50 | 0.4191 | 0.3962 | 0.4280 | 0.4696 | 0.5269 | 0.6111 |
| 160.00 | -10.00 | 0.3851 | 0.3851 | 0.4168 | 0.4583 | 0.5155 | 0.5998 |
|  | -10.00 | 0.4079 | 0.3851 | 0.4168 | 0.4583 | 0.5155 | 0.5998 |

Case IV
OPTIMUM ANGLE FOR TARGET TO MINIMIZE ENERGY SPREAD TARGET IN TRANSMISSION
TARGET IS CALCIUM INCIDENT PARTICLES ARE ALPHAS P OBSERVED PARTICLES ARE ALPHAS ESTARI= O. ESTARZ= 3. 73O



Case IV (continued)
TARGET IS CALCIUM INCIDENT PARTICLES ARE ALPHAS , OBSERVEDPARTICLES ARE ALPHAS ESTARI= O. ESTARZ= 3.730

| DETECTOR ANGLE | TARGET ANGLE | OPTIMUM | $E *=0$ | ENERGY | SPREAD |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | $E *=10$ | $E *=15$ | $E *=20$ |
| 112.00 | 60.00 | -0.0912 | -0.0912 | -0.1357 | -0.1939 | -0.2737 | -0.3907 |
|  | 60.00 | -0.1233 | -0.0912 | -0.1357 | -0.1939 | -0.2737 | -0.3907 |
| 116.00 | 60.00 | -0.1368 | -0.1368 | -0.1865 | -0.2517 | -0.3412 | -0.4726 |
|  | 60.00 | -0.1727 | -0.1358 | -0.1865 | -0.2517 | -0.3412 | -0.4726 |
| 120.00 | 60.00 | -0.1942 | -0.1942 | -0. 2506 | -0.3246 | -0.4264 | -0.5760 |
|  | 60.00 | -0.2349 | -0.1942 | -0.2506 | -0.3246 | -0.4264 | -0.5760 |

Case IV (Continued)
optimum angle far target to minimize energy spread target in reflection
target is calcium incident particles arf. alphas , observed particles are alphas estarl= 0. estarz= 3.730


Case IV (Continued)
TARGET IS CALCIUM INCIDENT PARTICLES ARE ALPHAS , OBSERVED PARTICLES ARE ALPHAS ESTARI= O. FSTARZ= $3.73 O$


Case IV (Concluded)
TARGET IS CALCIUM INCIDENT PARTICLES ARE ALPHAS - OBSERVEDPARTICLES ARE ALPHAS ESTARI= O. ESTARZ= 3. TS:


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