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THE SYNTHESIS OF A LAMINATED PLATE FOR HIGH TEMPERATURE APPLICATION

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by

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ENGINEERING

SYNTHESIS

GROUP

ABSTRACT

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Synthesis has been defined as the rational directed evolution of a system configuration which, in terms of a defined criterion, efficiently performs a set of specified functional purposes. This work presents the application of the synthesis idea to a system with a thermoelastic technology. The system is a three layered plate; the outer two layers of ceramic material for thermal protection and the third layer of metal for structural purposes. There are six design parameters; the density and the depth of each layer. The behavior constraints are the temperatures at the surface and the two interfaces and the stresses at the upper and lower boundaries of the third layer. Side constraints are provided on the six design parameters.

The merit function is the weight per unit surface area of the plate. A steepest-descent alternate step synthesis method is used. Results of three example syntheses are included with a discussion of a possible pseudo-design parameter. The results indicate that a thermo-elastic system may be successfully synthesized.

Author

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SYMBOLS

d_i	depth of i^{th} layer in inches
d_i^U	upper limit on depth
d_i^L	lower limit on depth
i	layer subscript
j	node subscript
k	thermal conductivity
q	interface subscript
s	Stephan-Boltzman Constant = $3.34 \times 10^{-15} \frac{\text{Btu}}{\text{in}^2\text{sec}°\text{R}^4}$
t	time variable
t_i	dimensionless depth
Δt	time increment
u	displacement in x direction
v	displacement in y direction
w	displacement in z direction
$\tilde{x}'$	new design point
$\tilde{x}_o$	occupied design point
Δz	space increment
E	elastic modulus
N_x	force in x direction per unit length
N_y	force in y direction per unit length
N_{xy}	shear force per unit length
Q(t)	heat load in Btu/in^{2}sec
R	response matrix

T	temperature variable
T'	temperature at time $t + \Delta t$
TIF	interface temperature
$T_{i_{max}}$	maximum allowable temperature
T_0	initial temperature
T_s	surface temperature
W	weight
α	coefficient of linear thermal expansion
ε	emissivity
ε_x	strain in x direction
ε_y	strain in y direction
λ	fixed distance of travel
λ'	variable distance of travel
ν	Poisson's ratio
σ_x	stress in x direction
σ_y	stress in y direction
σ_1	stress at upper surface of layer 3
σ_2	stress at lower surface of layer 3
σ_{yp}	yield stress
ρ_i	density of i^{th} layer
ρ_i^U	upper limit on density
ρ_i^L	lower limit on density
τ_{xy}	shear stress

ϕ	dimensionless weight
$\tilde{\phi}$	direction cosines
ω_i	dimensionless density

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CHAPTER I

FORMULATION AND SOLUTION OUTLINE

1.1 Introduction

This work is part of the effort being made to study the application of structural synthesis ideas to a wide variety of problems with different governing technologies.

Previous studies have been made of problems with technologies from the areas of structural mechanics(1), dynamics(2) and aeroelasticity(3).

Thermoelasticity, the governing technology for this problem, embraces the theory of the flow of heat and the theory of strains and stresses due to the flow of heat.

The mathematical model chosen for study is shown in Figure 1. The structure is a laminated plate of three layers, square in shape, and of arbitrary dimensions. It is assumed to be part of a similar but much larger structure.

The loading to which the plate is subjected consists of a series of time dependent heat pulses applied at the surface. Radiation cooling is provided at the surface and the lower boundary is insulated.

Layers one and two are assumed to be composed of high temperature resistant ceramic materials of variable porosity, expressible in terms of the density, and of relatively high and low thermal conductivity respectively. The first two layers are assumed to be constructed in such a manner that each possesses an effective modulus of elasticity low enough to reduce the induced thermal stresses and any influence on the stiffness of the third or structural layer to a negligible level. One way of providing the effective low modulus of elasticity is to construct the layers in a cellular or honeycomb form with spaces or plastic material between the cells to provide stress relief. The thermal properties of beryllium oxide and aluminum oxide are used to represent the properties of layers one and two respectively.

The third layer is a metallic structural plate to which the interpolated materials concept is applied. The thermal and mechanical properties of this layer are assumed to be functions of the density at a given temperature.

This type of heat resistant structure is "passive" in the sense that it depends on radiation cooling and heat capacity to absorb heat loads of high intensity and relatively short duration[4].

Changes in the thermal and mechanical properties of the materials composing the three layers due to changes in temperature and density are considered. These relationships are shown in Figures 9 through 17. The applicable equations are listed in Appendix C.

1.2 Thermal Analysis

The heat flow in the structure is assumed to be one-dimensional and is taken to be positive in the direction of the applied heat pulse. The space and time dependent temperature response within the structure is found by solving the one-dimensional heat flow equation:

$$\frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z}\right) = \rho c \frac{\partial T}{\partial t}$$

The boundary condition at the surface expresses the fact that the heat absorbed by the structure is equal to the difference between the applied heat and the radiated heat. This condition is:

$$Q(t) - s \varepsilon (T_s^{4} - T_o^{4}) = - k \frac{\partial T}{\partial z}$$

The lower boundary condition is:

$$\frac{\partial T}{\partial z} = 0$$

expressing the fact that no heat flows through the insulated surface.

The resistance to heat flow between layers is assumed to be zero therefore the boundary conditions for the interface between the i^{th} and $i+1^{st}$ layers are:

$$T_i = T_{i+1}$$

and

$$k_i \left(\frac{\partial T}{\partial z}\right)_i = k_{i+1} \left(\frac{\partial T}{\partial z}\right)_{i+1}$$

These equations mean that the temperatures of the two layers must be equal at the interface and that the heat flowing out of the

i^{th} layer must equal the heat flowing into the $i+1^{st}$ layer.

The consideration of the variation of the thermal properties of the materials with temperature necessitates solving the heat flow equation by numerical means. A finite difference technique is used and is discussed in detail in Appendix A.

1.3 The Elastic Analysis

Thermal stresses are assumed to be induced only in the third layer and the thin plate theory is used for analysis. The layer is subjected to two sets of edge boundary conditions. In case 1 the midplane of the layer is allowed to expand freely but the midplane deflection is kept equal to zero by appropriate bending moments applied at the edges. This case represents a condition of low stress in the material. Case 2 represents the condition of high stress. In this case the midplane is neither allowed to expand nor deflect by appropriate inplane forces and bending moments applied at the edges. Possible buckling of the plate in this case is not considered. The elastic analysis is discussed further in Appendix B.

1.4 The Synthesis

The design parameters, the variables of the system which must be assigned to completely define a design, are the three densities ρ_1, ρ_2, ρ_3 and the three depths d_1, d_2 and d_3. Specification of the depths fixes the geometry of the structure. Specification of the densities defines the porosities of the materials composing layers one and two and the type of material to be used for layer three. These six variables are constrained only by upper and lower bounds on the values of each. These bounds are called side constraints and the values used in this problem are listed in the Results and Discussion section.

The maximum temperature in each layer, always occuring at the surface and at the interfaces, and the maximum stress, which occurs at the upper or lower surface of the third layer, are the measures of the response of the structure to a given heat input. These variables are termed the behavior variables and are constrained by upper limits.

The design space approach is used in the synthesis of the structure. The space is imagined as being formed by a set of six mutually orthogonal axes with each design parameter represented linearly along an axis. A design point is then described by a six dimensional vector in the space.

The points comprising the design space may be divided into acceptable points and unacceptable points. The acceptable designs are those designs which do not violate either side constraints or behavior constraints. The unacceptable designs then are those which do violate one or more of these constraints.

It is impossible to separate off regions of the space which contain unacceptable designs by explicit functions of the design parameters. (Behavior functions). This is because of the nature of the numerical approach taken in the solution of the heat flow equation.

The object of the synthesis then is to find by some automatic process that acceptable point or group of acceptable points which causes the value of the merit function associated with the system to assume either a maximum or a minimum value.

The merit function is an expression involving the design parameters and is a measure of how much better one acceptable design is than another.

The merit function for this problem is the expression for the weight of the structure per unit area of surface:

$$W = \rho_1 d_1 + \rho_2 d_2 + \rho_3 d_3 .$$

The minimum of this function is to be sought by the synthesis method.

The technique used to achieve this minimization is a steepest descent alternate step method which is discussed in Appendix D.

CHAPTER II

RESULTS AND DISCUSSION

2.1 Results

Three cases are selected as examples of the synthesis process. Case 1 and 2 represent low and high stress conditions in layer three respectively. Case 3 is simply case 1 with the insulated lower boundary replaced by a constant temperature heat sink.

The thermal loading to which the structure is subjected in Cases one and two is a set of two heat pulses each of 100 second duration.

The first pulse is defined by the following equations:

$$t < 0 \text{ seconds} \qquad Q(t) = 0 \text{ Btu/in}^2\text{sec.}$$
$$0 \leq t \leq 100 \qquad Q(t) = 2 - 0.02\,t$$
$$t > 100 \qquad Q(t) = 0$$

This is a triangular pulse with a maximum ordinate at $t = 0$ equal to 2 Btu/in^{2}sec.

The second heat pulse is described as follows:

$$t < 0 \text{ seconds} \qquad Q(t) = 0 \text{ Btu/in}^2\text{sec.}$$
$$0 \leq t \leq 100 \qquad Q(t) = 1$$
$$t > 100 \qquad Q(t) = 0$$

This is a rectangular pulse with a value of 1 Btu/in^{2}sec.

For Case 3 the duration of each load condition is shortened to 60 seconds to speed the analysis. Thus load condition 1 becomes:

$$t < 0 \text{ seconds} \qquad Q(t) = 0 \text{ Btu/in}^2\text{sec.}$$

$$0 \le t \le 60 \qquad Q(t) = 2 - t/30$$

$$t > 60 \qquad Q(t) = 0$$

and load condition 2 is:

$$t < 0 \text{ seconds} \qquad Q(t) = 0 \text{ Btu/in}^2\text{sec.}$$

$$0 \le t \le 60 \qquad Q(t) = 1$$

$$t > 60 \qquad Q(t) = 0$$

Two design paths are presented for Cases 1 and 2 and one path for Case 3. The weight reduction as a function of time for each design path is shown for each case in Figures 2, 5 and 8 respectively.

The designs presented for comparison for Case 1 are those designs obtained after approximately 3000 seconds running time. The designs for Case 2 are those reached after approximately one hour of running time; and the design for Case 3 is that reached after approximately 5000 seconds. The computing time required to reach a minimum weight is due to the length of time necessary to complete a design analysis, approximately 30 to 40 seconds.

The results for Case 1 are:

DESIGN PATH 1

Initial Design		Final Design	
ρ_1 = 0.1 #/in^3	d_1 = 2.0 in.	ρ_1 = 0.1053 #/in^3	d_1 = 0.617 in
ρ_2 = 0.1	d_2 = 2.0	ρ_2 = 0.0749	d_2 = 1.186
ρ_3 = 0.2835	d_3 = 1.0	ρ_3 = 0.0729	d_3 = 1.048
Weight = 0.6843 lb/in^2		Weight = 0.2302 lb/in^2	

DESIGN PATH 2

Initial Design		Final Design	
ρ_1 = 0.1048 #/in^3	d_1 = 1.702 in.	ρ_1 = 0.1 #/in^3	d_1 = 0.66 in.
ρ_2 = 0.1091	d_2 = 1.863	ρ_2 = 0.0743	d_2 = 1.666
ρ_3 = 0.0772	d_3 = 0.781	ρ_3 = 0.0664	d_3 = 0.615
Weight = 0.4419 lb/in^2		Weight = 0.2306 lb/in^2	

The results for Case 2 are:

DESIGN PATH 1

Initial Design		Final Design	
$\rho_1 = 0.08$ #/in^3	$d_1 = 2.0$ in.	$\rho_1 = 0.1$ #/in^3	$d_1 = 0.656$ in.
$\rho_2 = 0.12$	$d_2 = 2.9$	$\rho_2 = 0.0742$	$d_2 = 2.016$
$\rho_3 = 0.14$	$d_3 = 1.0$	$\rho_3 = 0.1626$	$d_3 = 0.501$
Weight = 0.6582 lb/in^2		Weight = 0.2967 lb/in^2	

DESIGN PATH 2

Initial Design		Final Design	
$\rho_1 = 0.1$ #/in	$d_1 = 2.5$ in.	$\rho_1 = 0.1068$ #/in	$d_1 = 0.588$
$\rho_2 = 0.1$	$d_2 = 2.5$	$\rho_2 = 0.0793$	$d_2 = 1.569$
$\rho_3 = 0.2$	$d_3 = 0.75$	$\rho_3 = 0.1638$	$d_3 = 0.646$
Weight = 0.6509 lb/in^2		Weight = 0.2930 lb/in^2	

The results for Case 3 are:

Initial Design		Final Design	
$\rho_1 = 0.1\ \#/in^3$	$d_1 = 0.75$ in.	$\rho_1 = 0.0804\ \#/in^3$	$d_1 = 0.254$
$\rho_2 = 0.1$	$d_2 = 0.75$	$\rho_2 = 0.138$	$d_2 = 0.251$
$\rho_3 = 0.1$	$d_3 = 0.75$	$\rho_3 = 0.0636$	$d_3 = 0.502$
Weight = $0.2255\ lb/in^2$		Weight = $0.0869\ lb/in^2$	

The emissivity at the surface for all cases is set at 0.5 and the maximum allowable temperature in each layer is 4000°R, 3000°R and 1000°R respectively. The stress limits for layer 3 are shown in Figure 17.

The upper and lower limits on the design parameters for all cases are:

$$\rho_1^L = 0.0665\ \#/in^3 \qquad \rho_1^U = 0.108\ \#/in^3$$

$$\rho_2^L = 0.0741 \qquad \rho_2^U = 0.1445$$

$$\rho_3^L = 0.0631 \qquad \rho_3^U = 0.2835$$

$$d_1^L = 0.25 \text{ in.} \qquad d_1^U = 3.0 \text{ in.}$$

$$d_2^L = 0.25 \qquad d_2^U = 3.0$$

$$d_3^L = 0.5 \qquad d_3^U = 2.0$$

The limits on the densities and temperatures are controlled by the limits used in the material property data. The bounds on the depths are arbitrary.

2.2 Discussion

2.2.1 Case 3

Case 3 is included to test the program by showing that the best thermal structure with a heat sink boundary condition is no structure.

That the structure has a tendency to disappear for this condition is shown by the fact that d_1, d_2, d_3 and ρ_3 are essentially at their lower limits.

The temperatures in the structure are at a maximum at the surface and at the first interface for load condition 1 and therefore the design is "on" a behavior constraint. Any further reduction of ρ_1 and ρ_2, by decreasing the conductivity, would result in a violation of the temperature constraint.

It is not known why ρ_1 and ρ_2 assume a low and high value respectively. This is opposite to the behavior of the designs in Cases 1 and 2 which are discussed next. The reversal of relative values may indicate the fact that relative minima exist although this idea is not examined further.

Obviously by providing some cooling at the lower boundary the weight of the thermal structure can be reduced but a weight penalty may be paid in providing the cooling mechanism.

The weight reduction as a function of time for Case 3 is shown in Figure 8.

2.2.2 Cases 1 and 2

These cases are the ones of greatest interest. The weight reduction as a function of time for each case is shown in Figures 2 and 5 respectively. In Case 1, the initial designs are of considerably different weight and yet converge to two final designs of nearly equal weight, the difference being 0.17%. Design path 2 for Case 1 was allowed to run to 7000 seconds. Only a 0.4% weight improvement was realized over the design at 3000 seconds.

For Case 2 the paths, initially at essentially the same weight, diverge during the synthesis and reconverge after a period of approximately one hour. The weight difference for the two final designs in Case 2 is 1.2%.

Two design paths are run for each case to attempt to reach the same final design. That this is not accomplished is obvious by examining the values of the design parameters for each design path.

For both cases the density design parameters show more similarity than the depth parameters indicating a lack of sensitivity of the response of the structure to its geometry.

For the two final designs in each case, the density of layer 1 is fairly close to its upper limit resulting in a relatively high conductivity for this layer; for layer 2, the density is near the lower limit resulting in a relatively low thermal conductivity.

The density of layer 3 for both runs in Case 1 lies in the magnesium range reflecting the fact that little high temperature strength is required for the lowest stress case.

In Case 2, the density of layer 3 for both runs lies in the titanium range. This material posses sufficient high temperature strength and a low coefficient of linear thermal expansion. These properties combine to provide a reduction of the thermal stress and an avoidance of the stress constraint. possesses

Temperature constraints are the only active ones for both cases. Although the stress constraints were active during the synthesis for Case 2 the final designs for both runs are not bound by these constraints.

The weight of the designs in Case 2 is higher than that in Case 1 due to the use of the higher density metal in layer 3.

For the designs of Case 1 the temperature responses at the surface and the two interfaces for load conditions 1 and 2 are shown in Figures 3 and 4.

The temperature response at the surface and the first interface is essentially identical for both designs in both load conditions throughout the time of analysis and is shown only up to the maximum value. The curves are separated on the drawing for clarity. The greater thickness of layer 2 in design 2 causes a delay in the time at which the maximum temperature in layer 3 is reached. The temperature constraints which act on the system are the maximum temperature of the third layer, reached in load condition 1, and the maximum temperature of the first layer, reached in load condition 2.

For Case 2 the temperature responses are shown in Figures 6 and 7. Again the response at the surface and at the first interface is essentially identical throughout the time of analysis for both designs for both load conditions. In this case, the maximum temperature response in the third layer of design 1 for load condition 1 is delayed by the greater thickness of layer 2. The same temperature constraints active in Case 1 are also active in Case 2.

Although the weights of the two runs for each case are essentially the same, the designs are not identical. The differences are explainable in terms of the heat stored in each design as a function of time.

In Cases 1 and 2 the heat stored in any system at any time, t, is given by:

$$Q_{stored} = \int_0^t Q(t)\,dt - s\varepsilon \int_0^t [T_s^4 - T_o^4]\,dt$$

Since s and ε are assumed constant they are removed from the integral sign. This equation is valid for either load condition 1 or 2. In both cases the greatest amount of heat is transmitted to the structure during load condition 1 due to the lower surface temperatures.

The surface temperature response of the two designs in Case 1 is the same. The response is also the same for the two designs in Case 2.

Thus, from the heat storage equation it is seen that designs with the same surface temperature response contain the same amount of heat energy at any time t. From this point of view, the designs in Case 1 are the same and those in Case 2 are the same. The ability to store heat energy is termed a pseudo-design parameter.

It is impossible to express this heat storage ability in an analytical fashion due to the non-linearity of the problem and the numerical approach used in its solution.

CHAPTER III

CONCLUSIONS AND RECOMMENDATIONS

3.1 Conclusions

This work has shown that synthesis ideas may be applied successfully to a system with a thermoelastic governing technology.

The system which was investigated was a three-layered plate subjected to heat pulses at the surface. The problem was to design the plate for minimum weight such that maximum allowable temperatures and thermal stresses were not exceeded.

Attempts to double-point designs were not successful. However, from the point of view of the weight and a pseudo-design parameter, the heat storage ability, the designs were shown to be essentially identical.

It cannot be said that the synthesis technique leads to an absolute minimum weight design. It can be said that the synthesis program results in a design improvement although some confidence in the ability of the program to reach a minimum in this problem is felt from the fact that double-pointing resulted in designs of essentially the same weight.

The analysis portion of the program is general in that it may be used to solve any one dimensional heat flow problem as long as the thermal properties are known.

The synthesis program is restricted to a problem of this type where the merit function is not "pathological" (is continuous in value and slope) and does not have zero or negative gradient components.

3.2 Recommendations

Originally all three layers were assumed to sustain thermal stresses. It was found in early analyses that the stresses in the brittle ceramic layers were too high, greatly exceeding the rupture strength of the materials. A lack of a suitable ceramic failure criterion and a knowledge of the fact that structures of this type have been built and subjected to very high temperatures led to the assumption of low effective modulus of elasticity for the ceramic materials.

It would be interesting to include the ceramic layers as an integral part of the load bearing structure if a failure criterion and modulus of elasticity data were available.

It would also be interesting to include a temperature dependent emissivity at the surface and a maximum temperature for layer 3 which would depend on the material used.

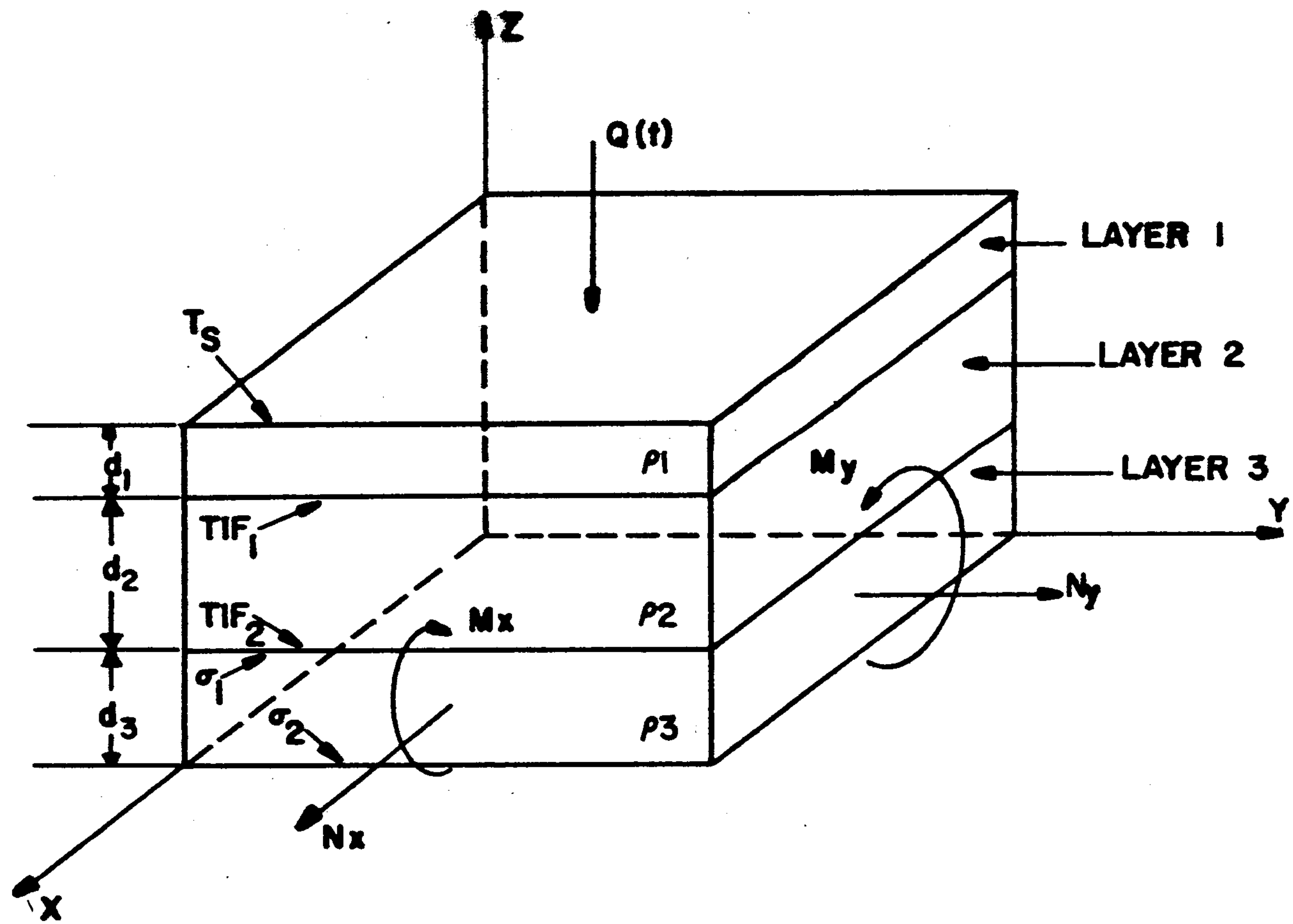

FIGURE I MATHEMATICAL MODEL

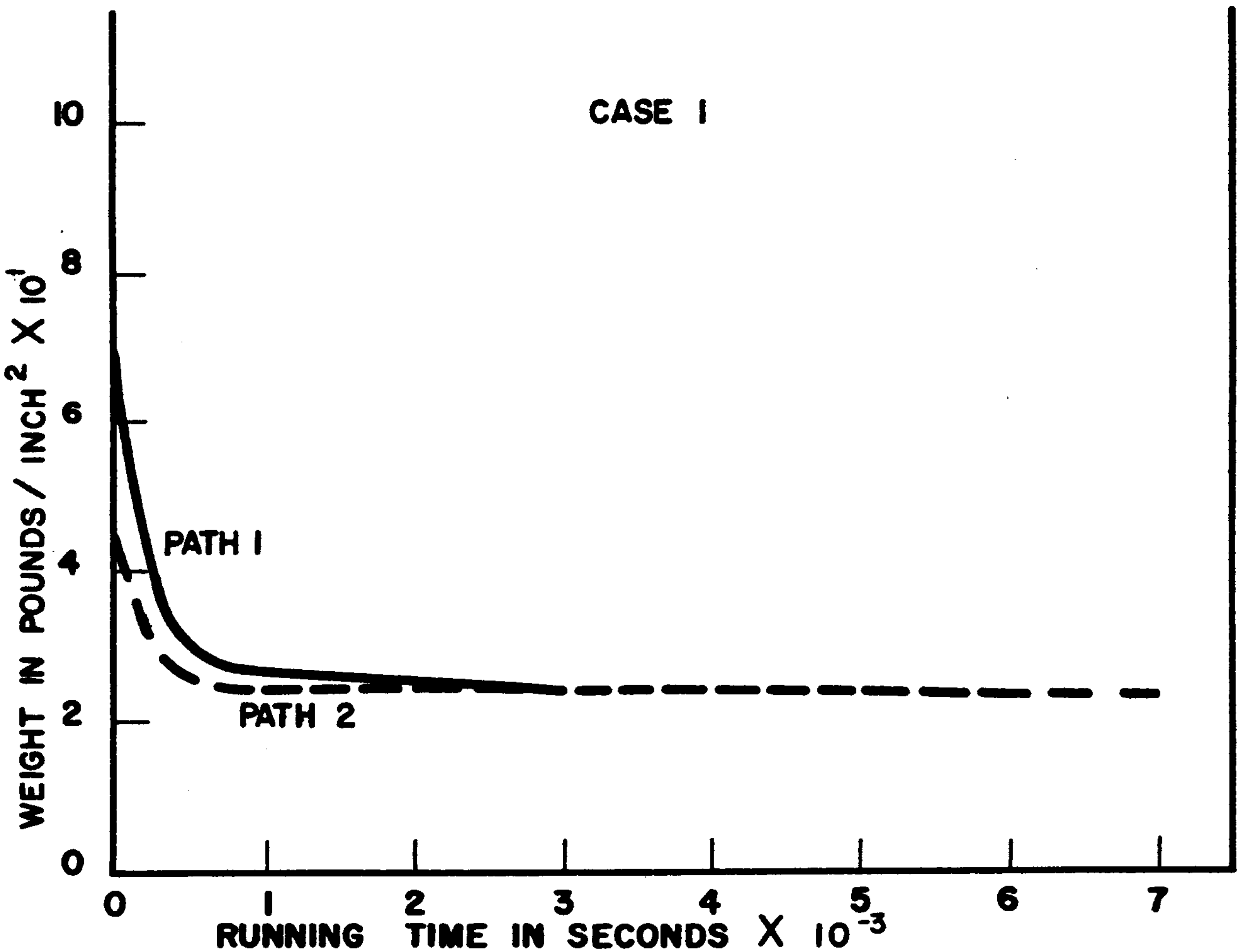

FIGURE 2 WEIGHT VS. RUNNING TIME

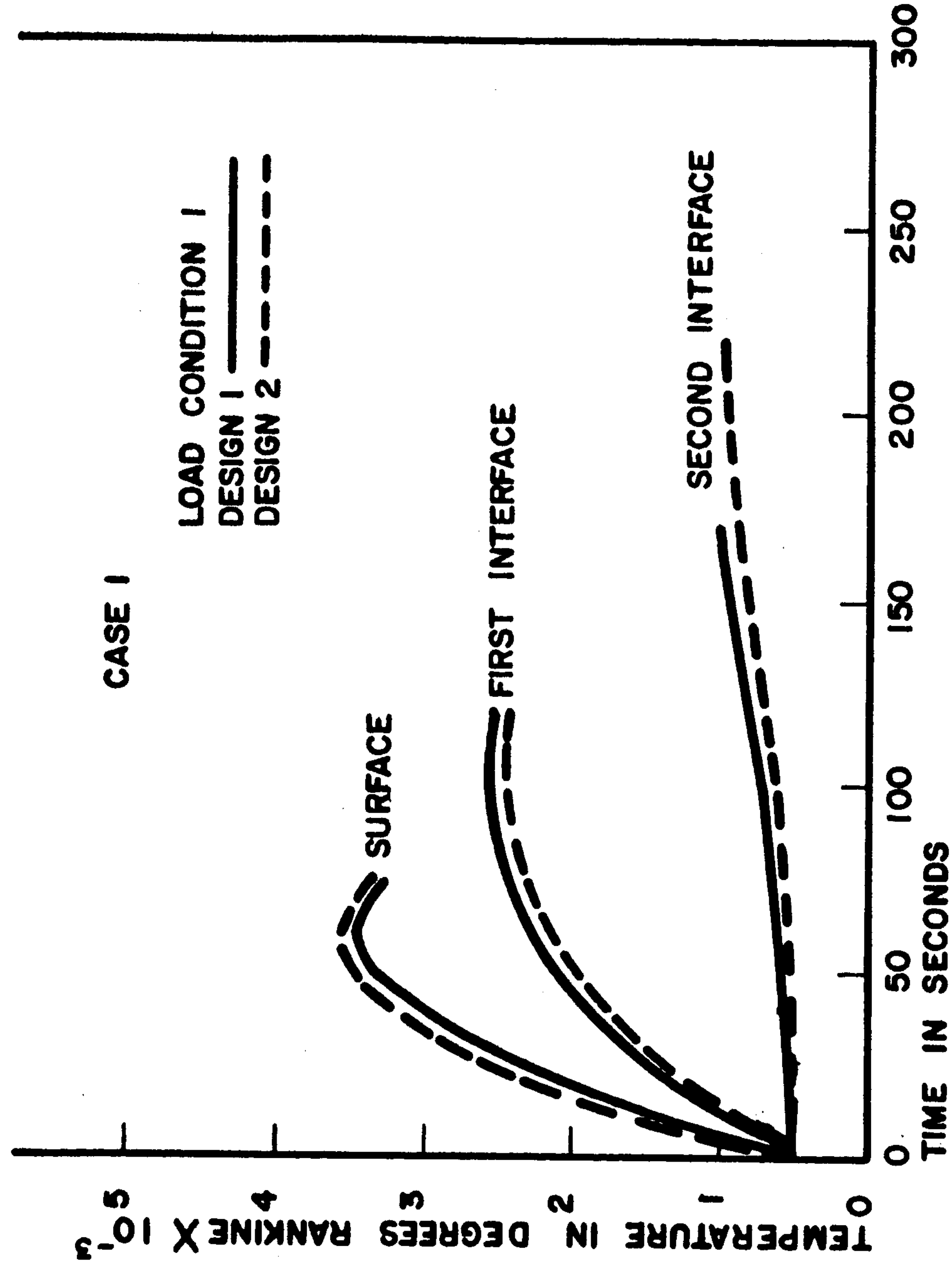

FIGURE 3 TEMPERATURE VS. TIME

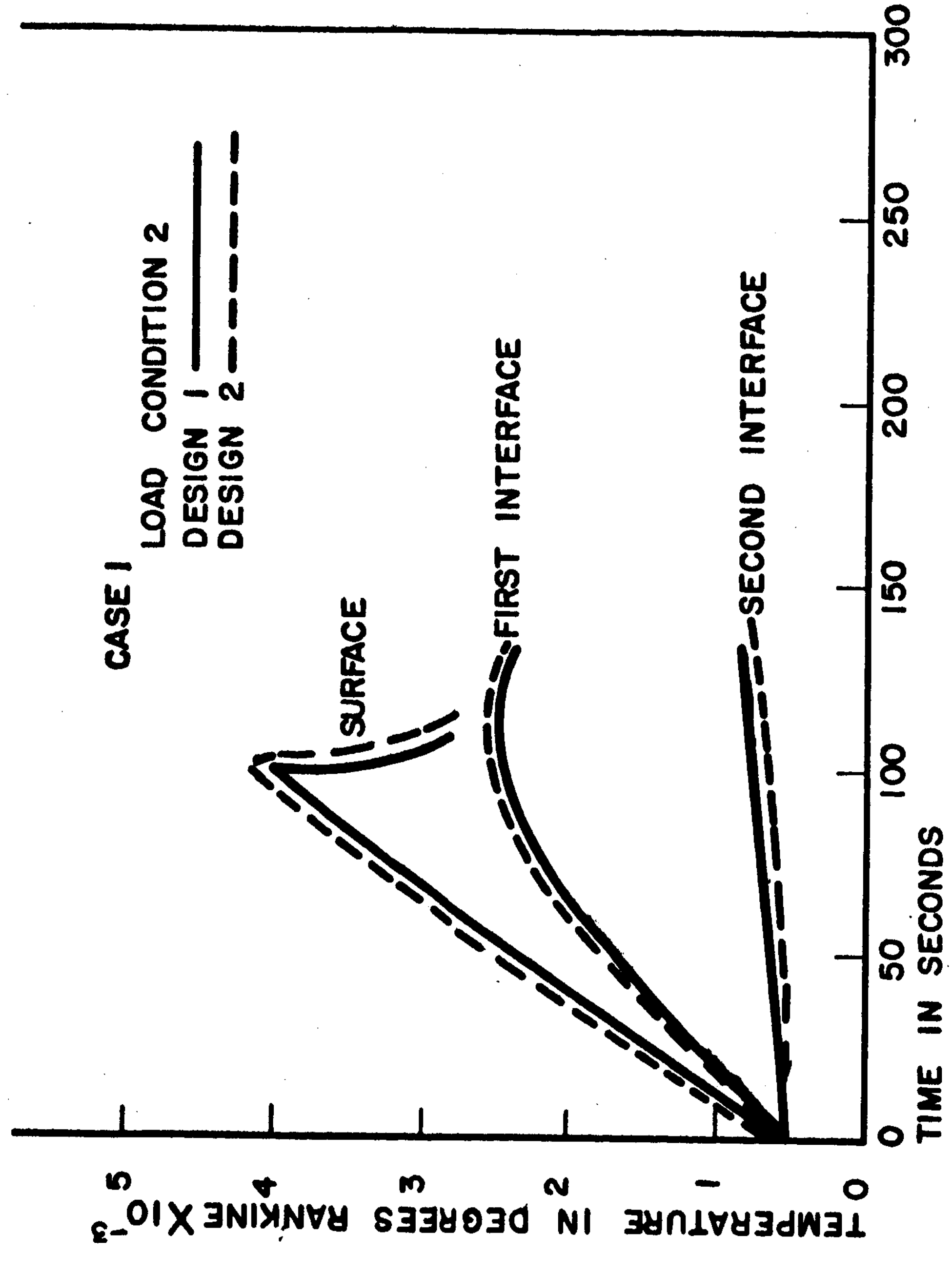

FIGURE 4 TEMPERATURE VS. TIME

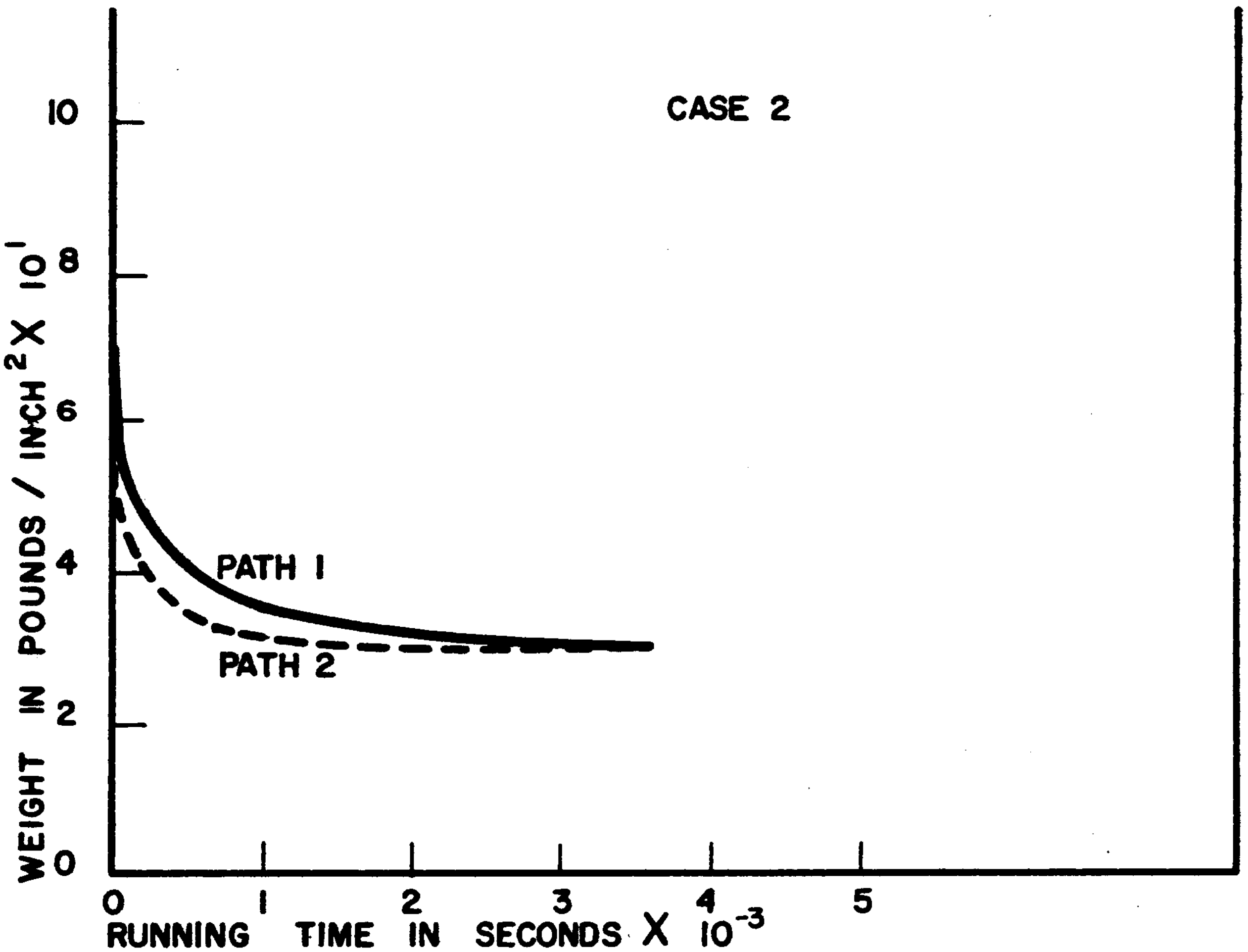

FIGURE 5 WEIGHT VS. RUNNING TIME

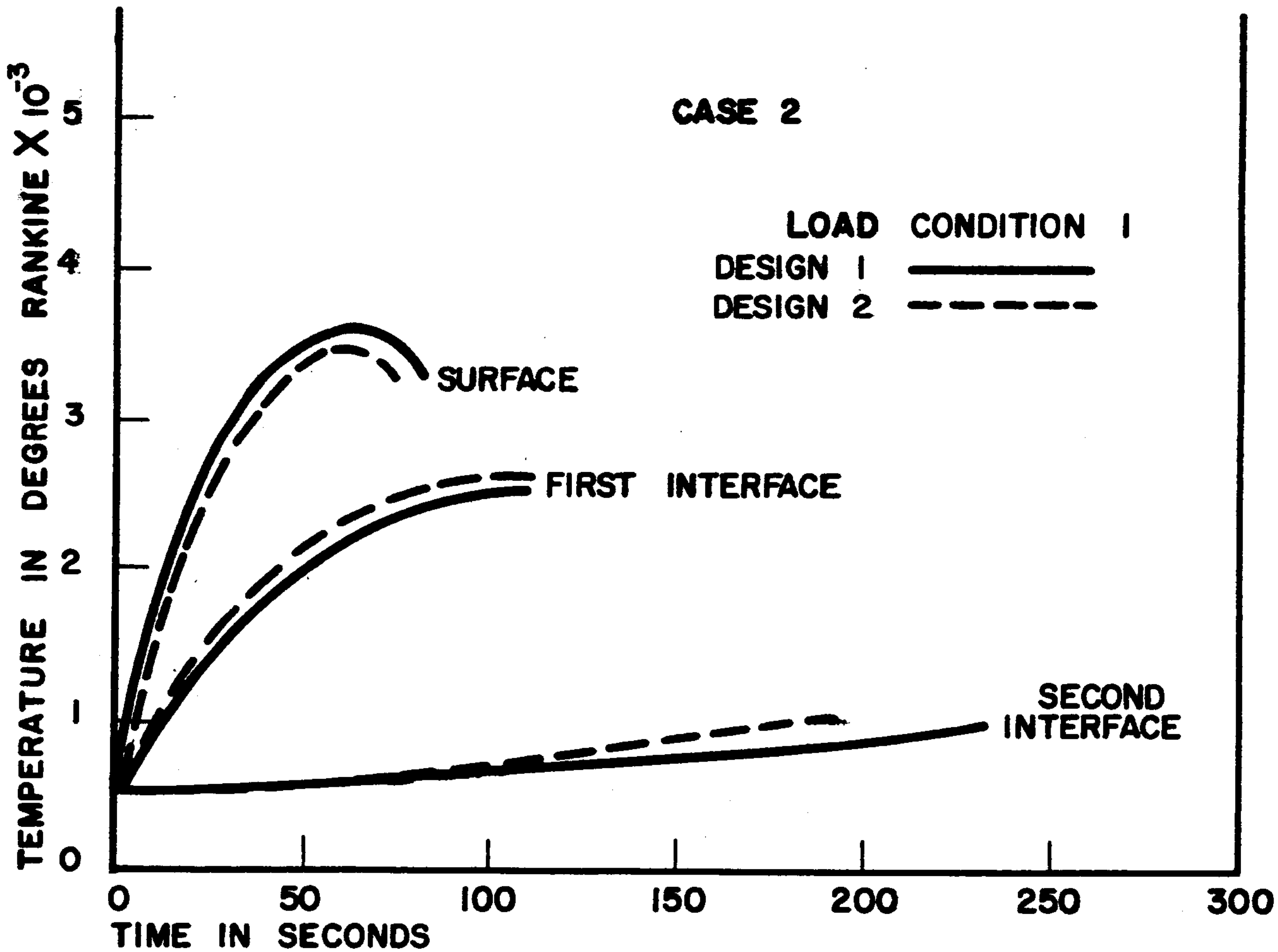

FIGURE 6 TEMPERATURE VS. TIME

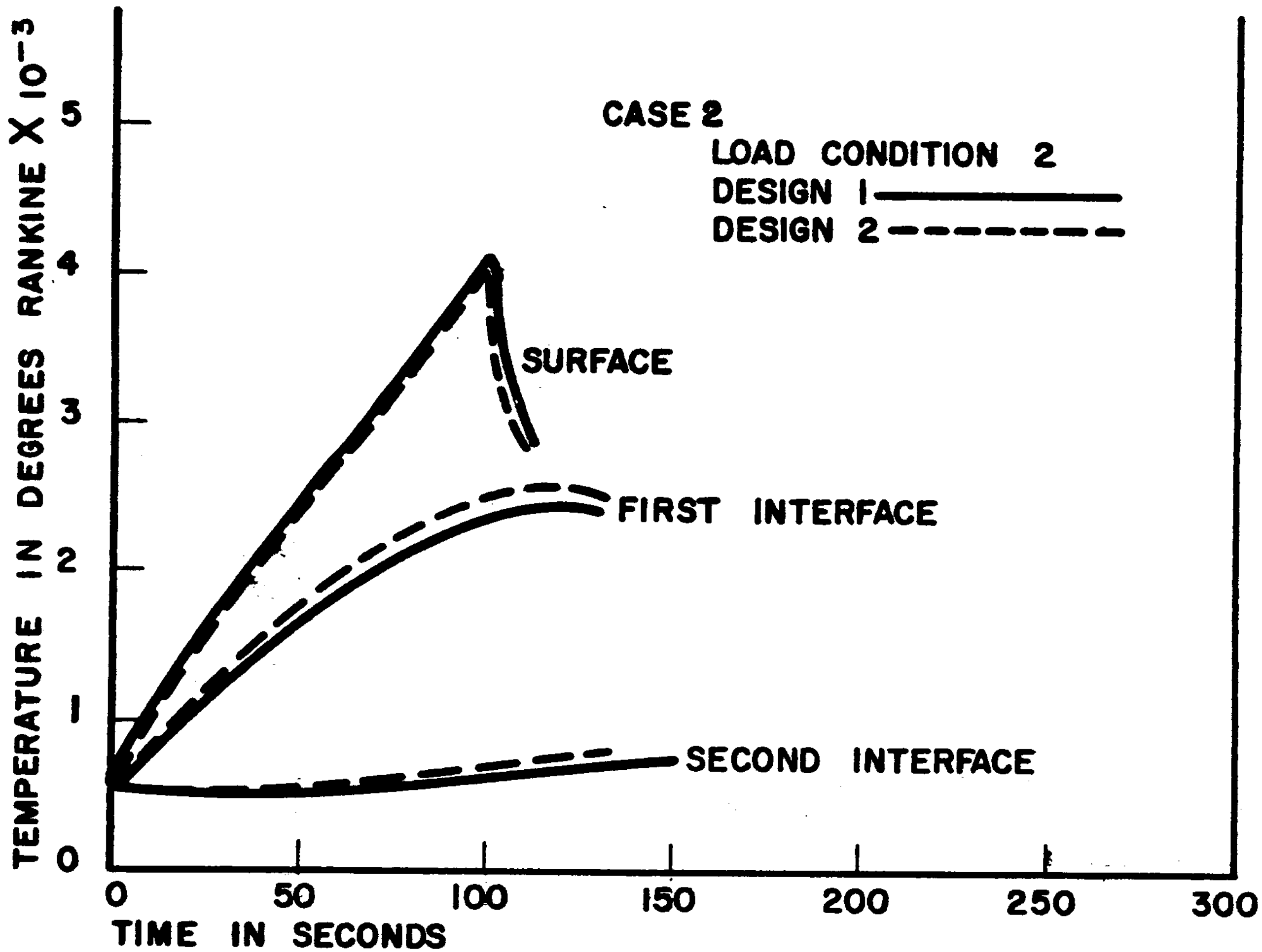

FIGURE 7 TEMPERATURE VS. TIME

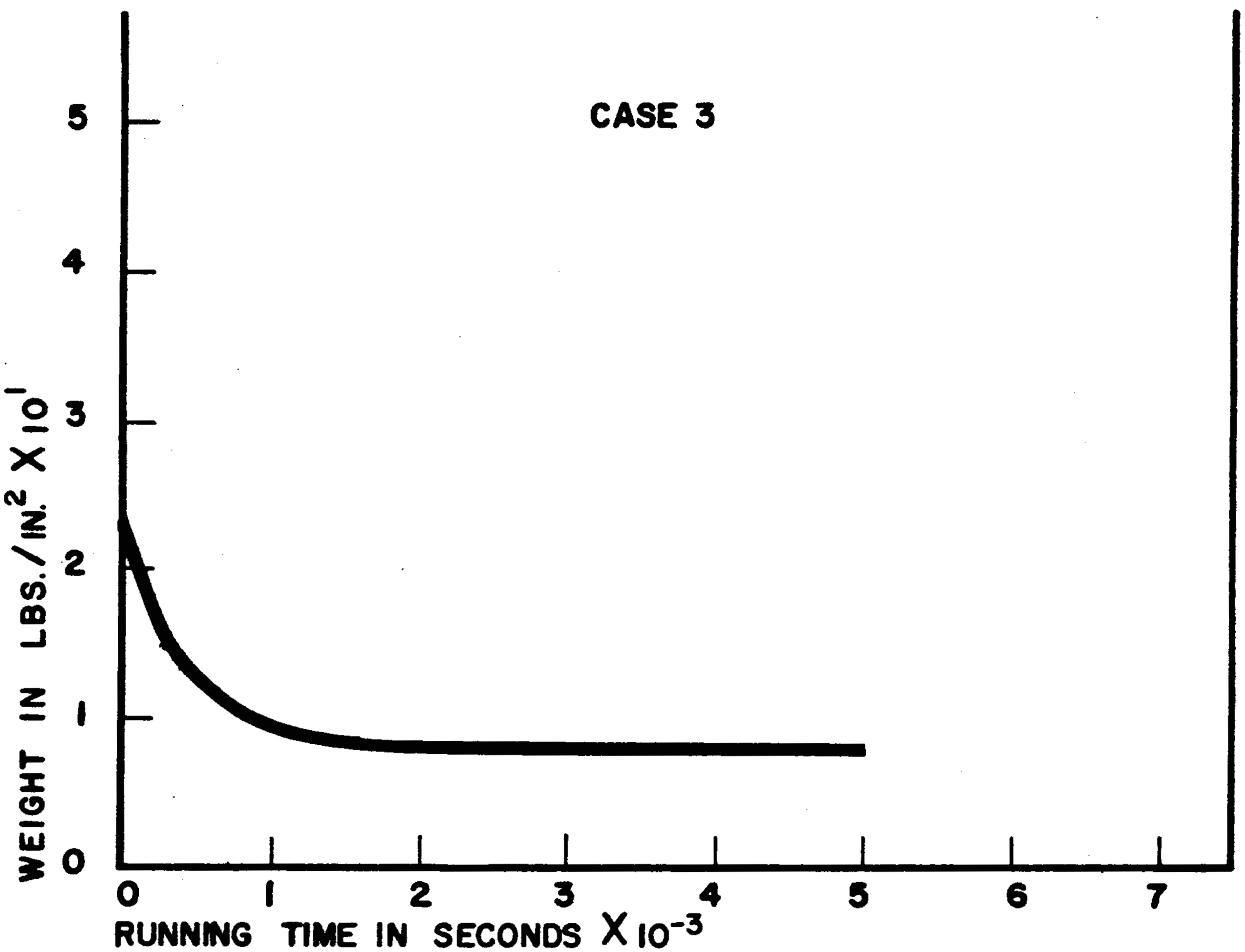

FIGURE 8 WEIGHT VS. TIME

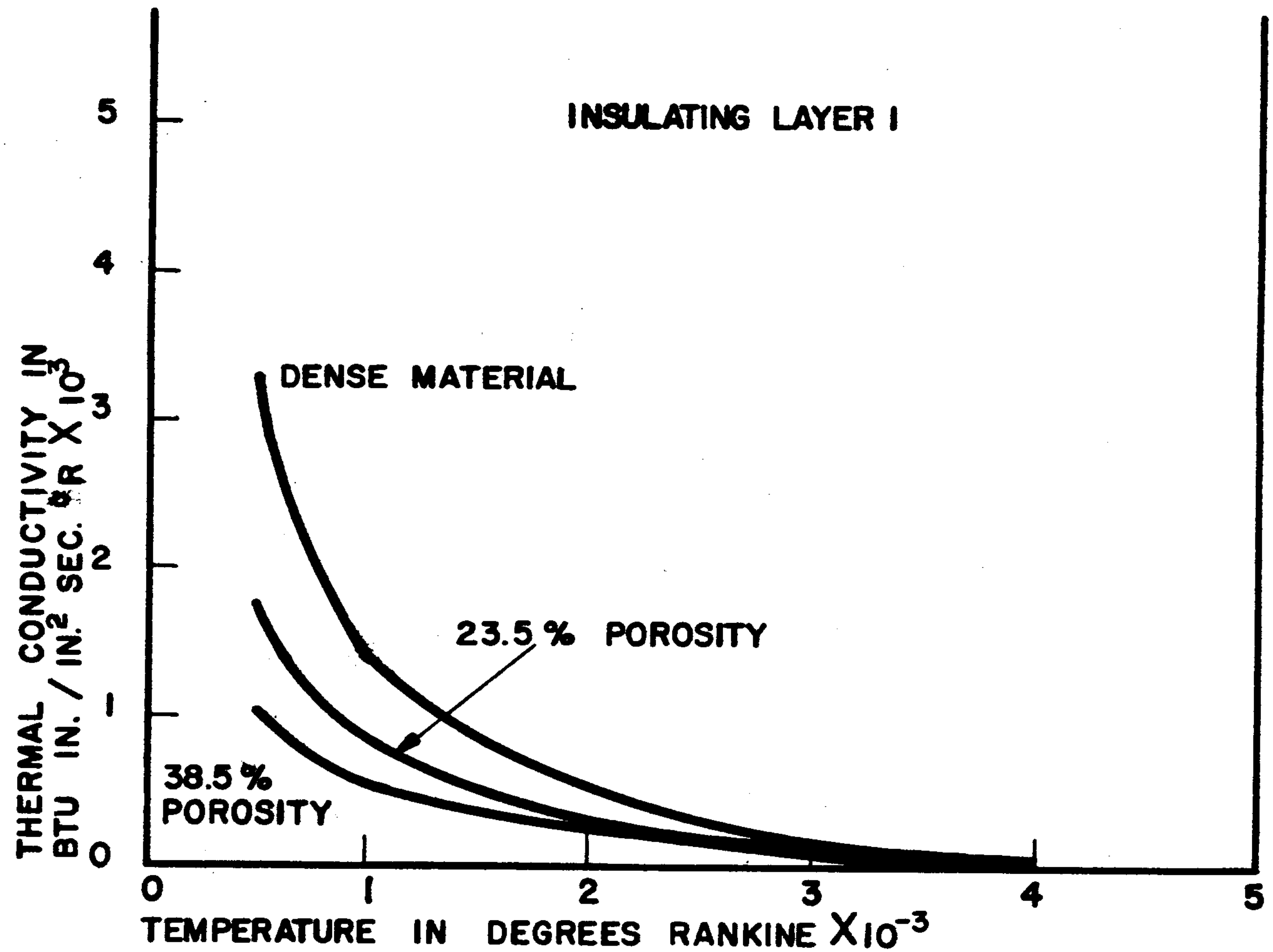

FIGURE 9 CONDUCTIVITY VS. TEMPERATURE

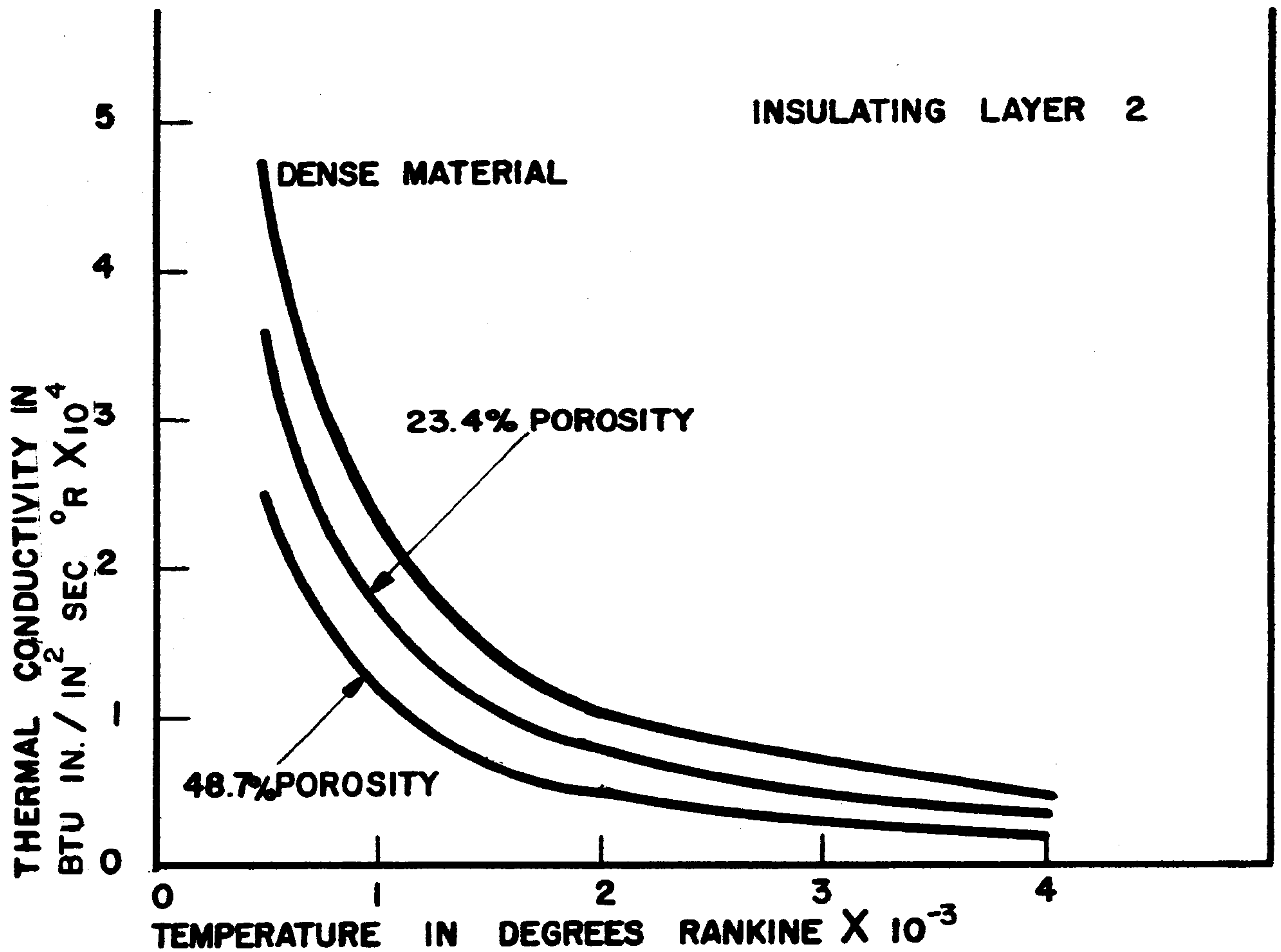

FIGURE 10 CONDUCTIVITY VS. TEMPERATURE

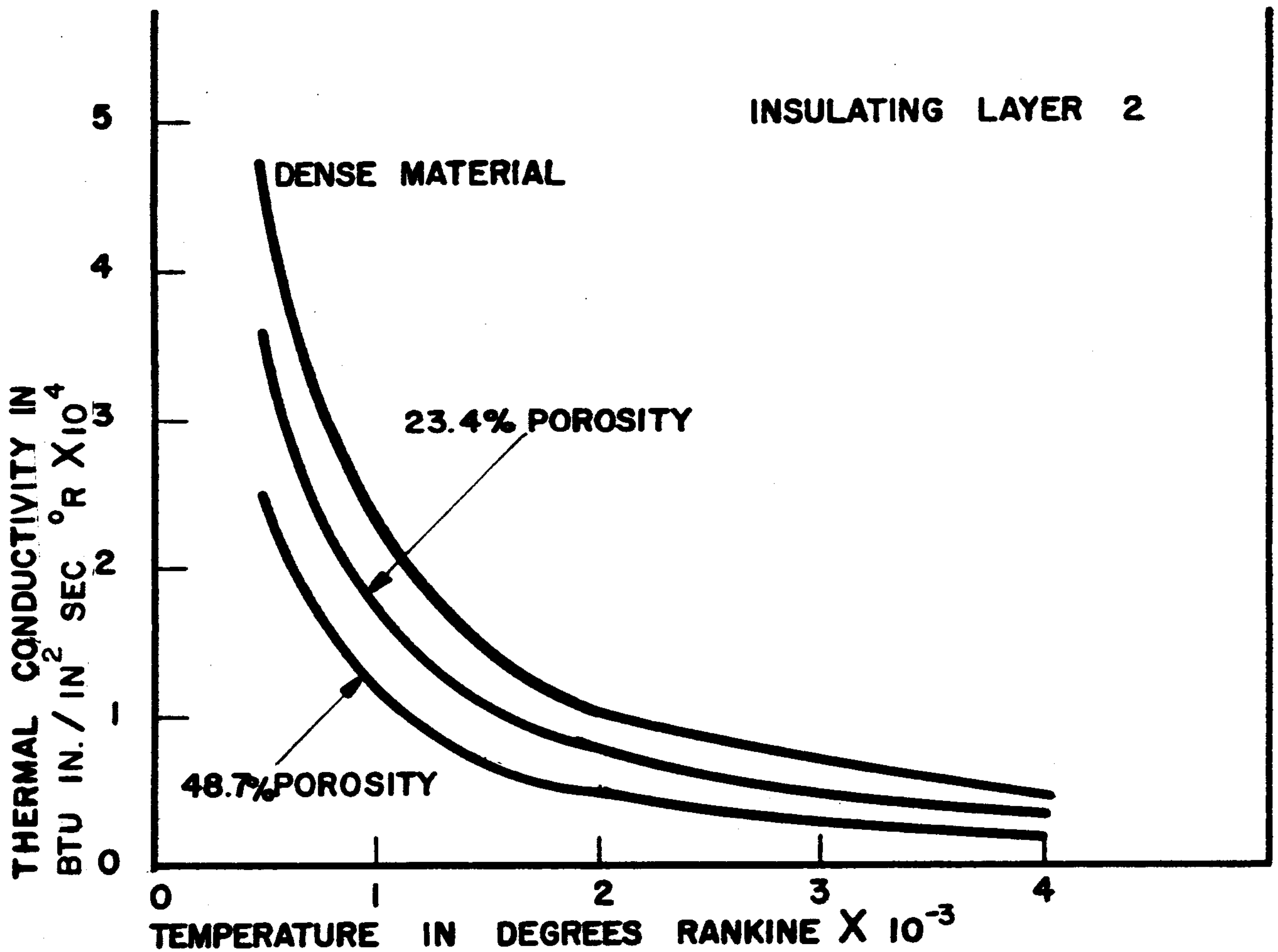

FIGURE 10 CONDUCTIVITY VS. TEMPERATURE

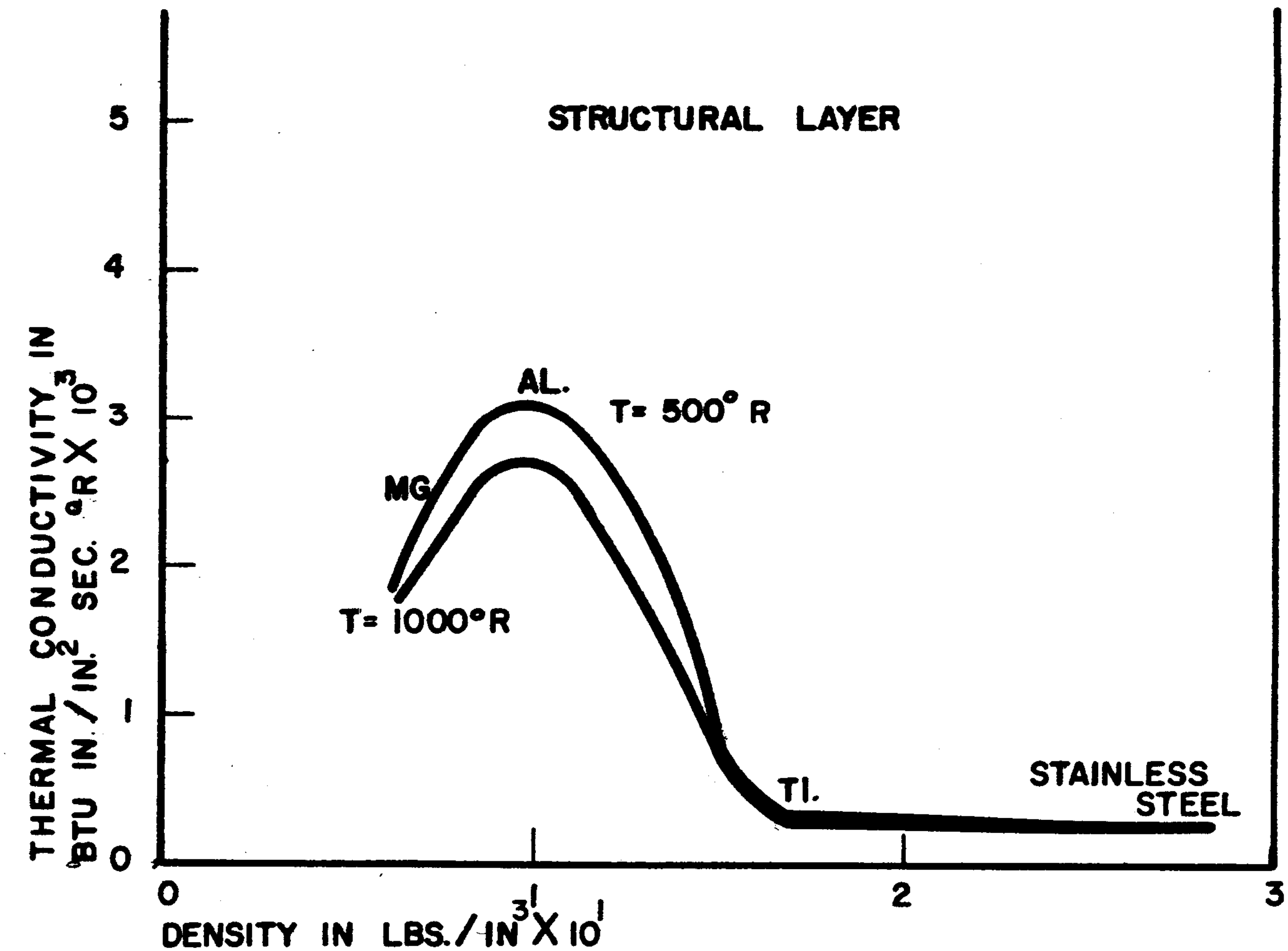

FIGURE II CONDUCTIVITY VS. DENSITY

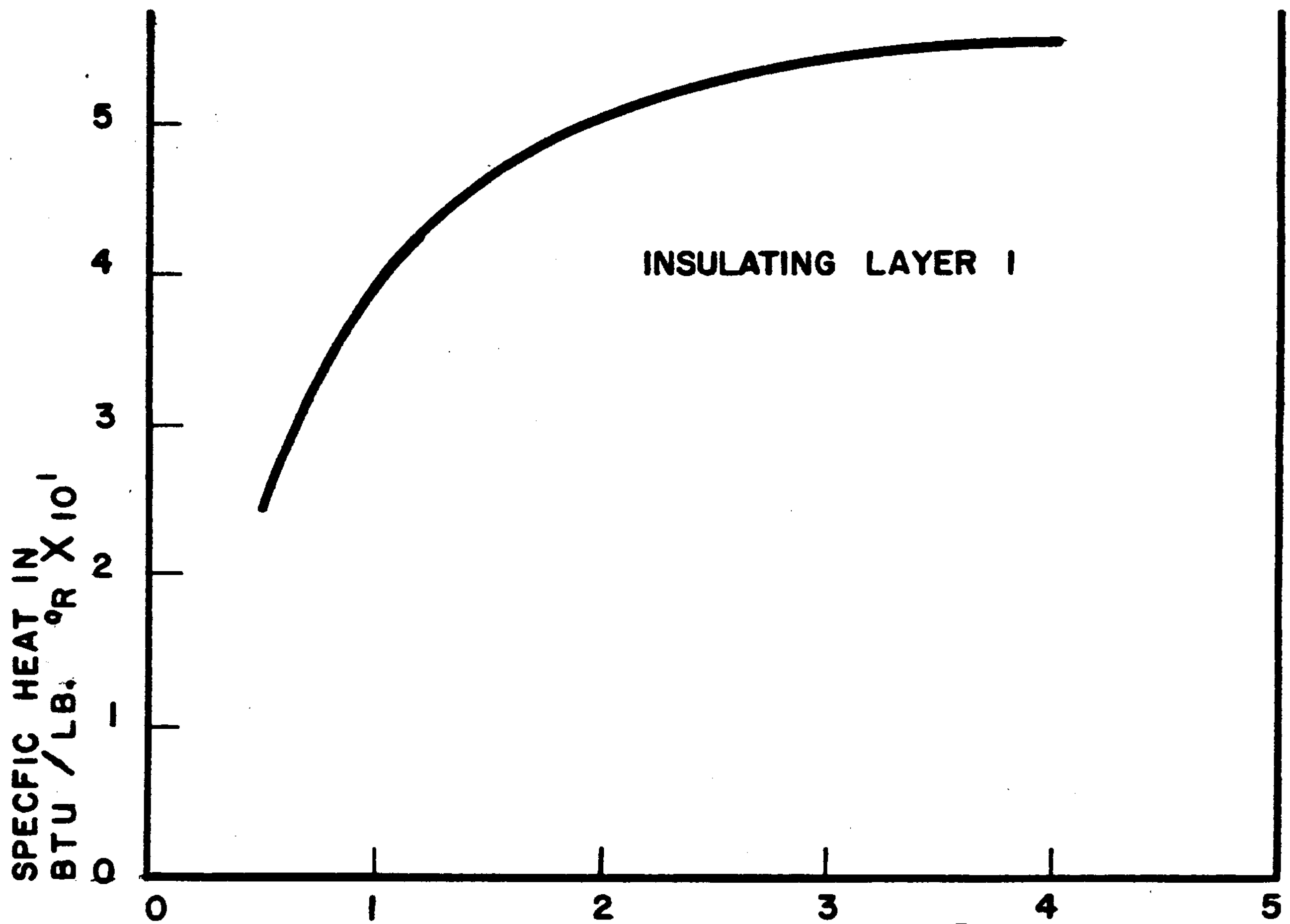

FIGURE 12 SPECIFIC HEAT VS. TEMPERATURE

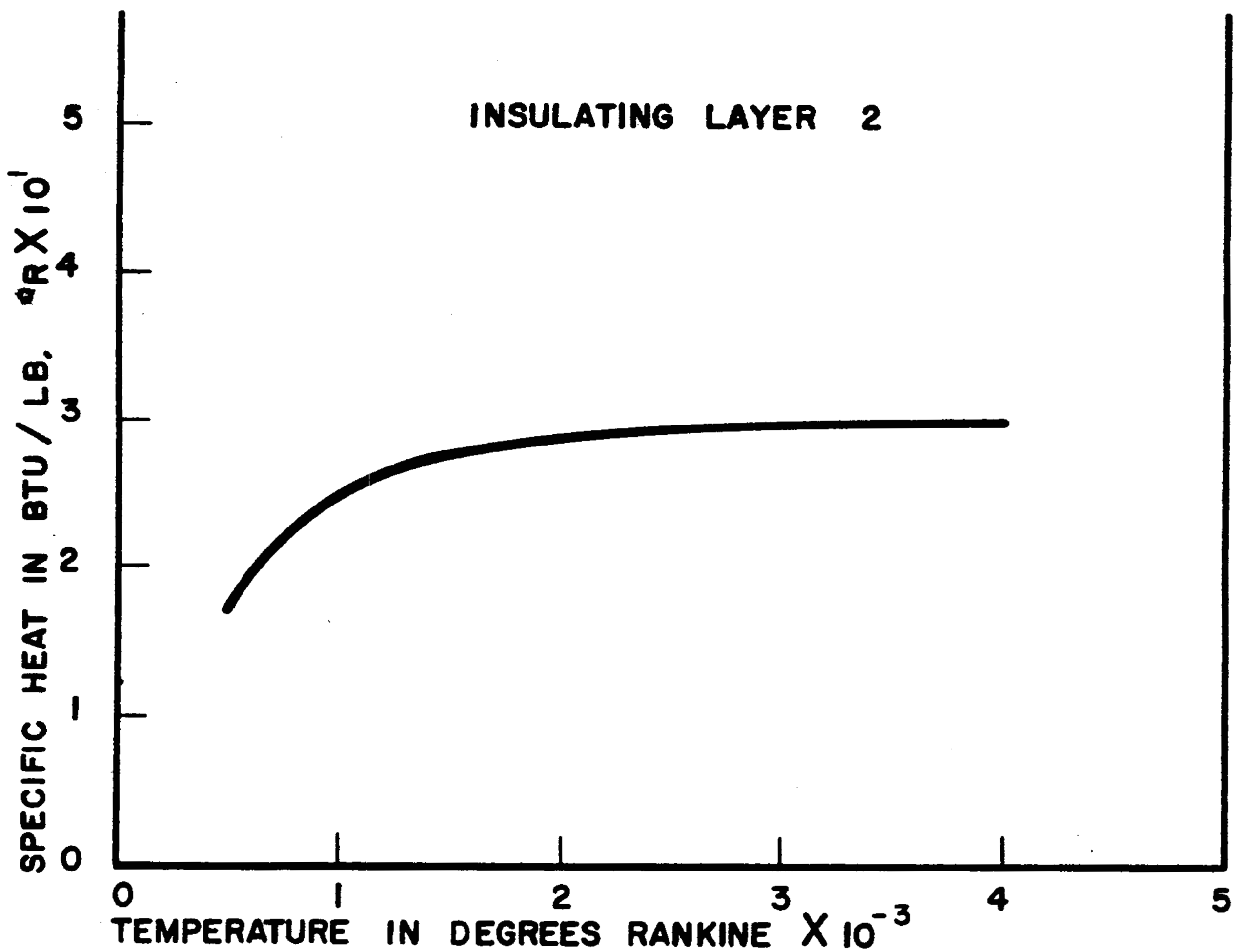

FIGURE 13 SPECIFIC HEAT VS. TEMPERATURE

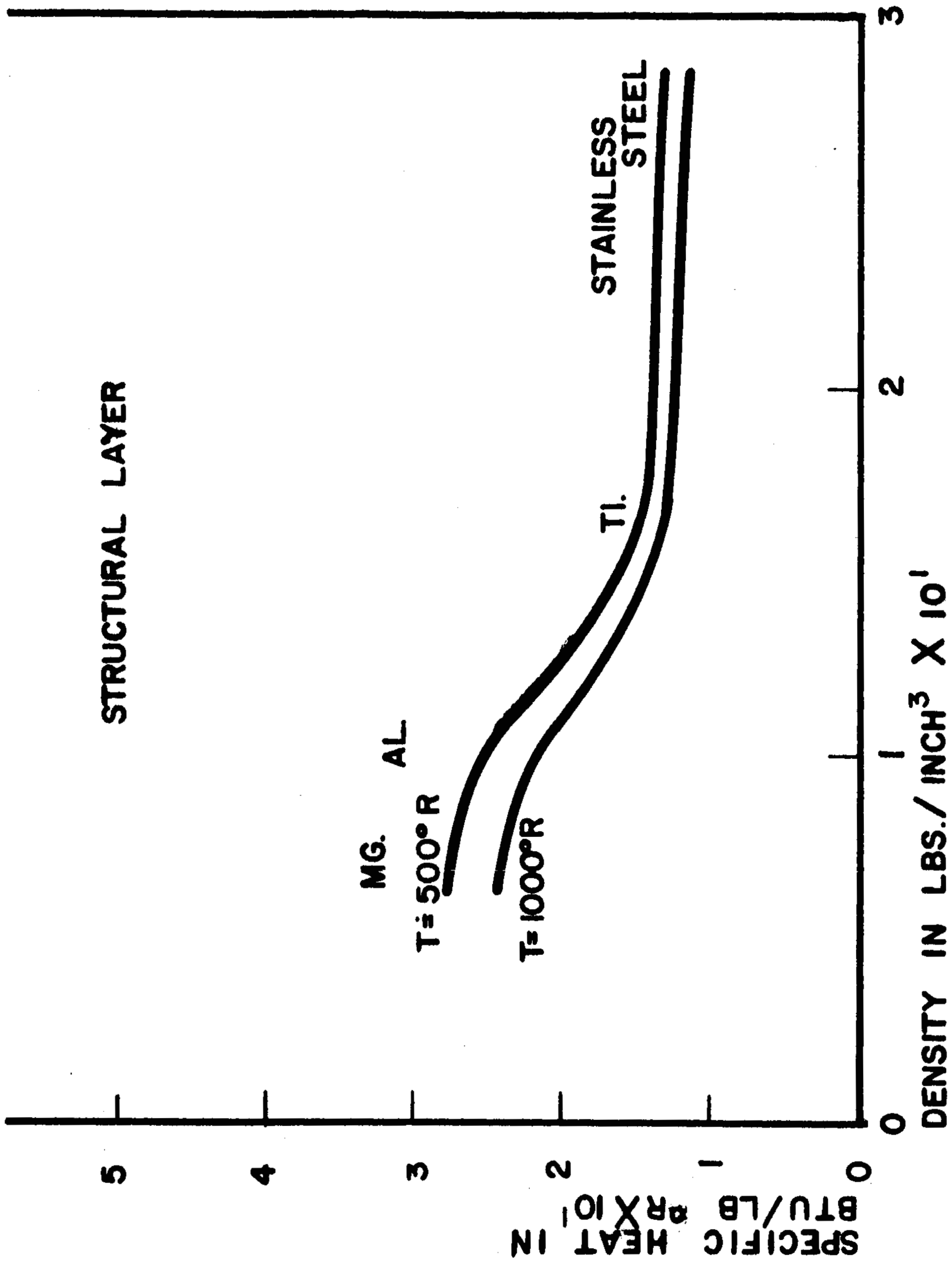

FIGURE 14 SPECIFIC HEAT VS. DENSITY

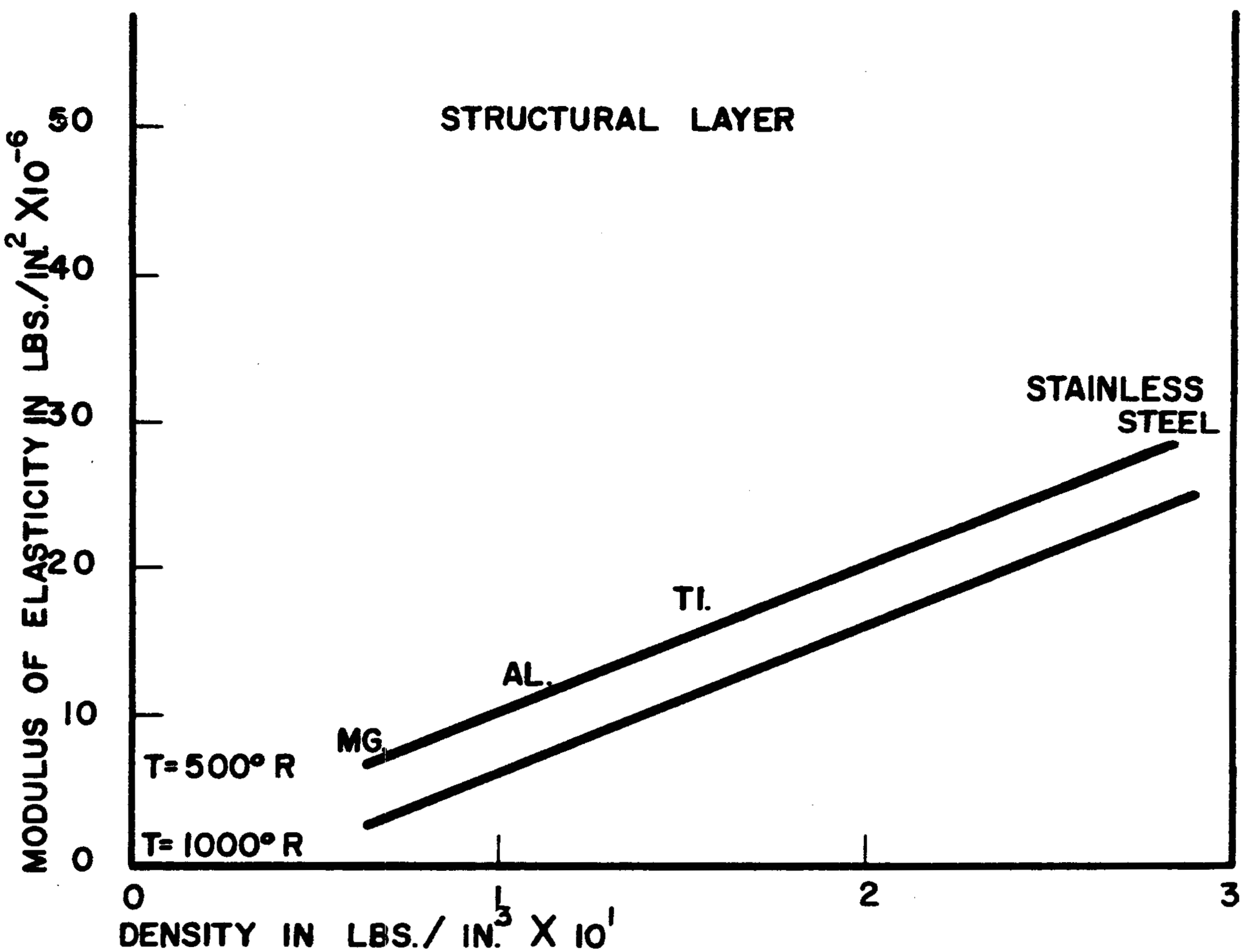

FIGURE 15 MODULUS OF ELASTICITY VS. DENSITY

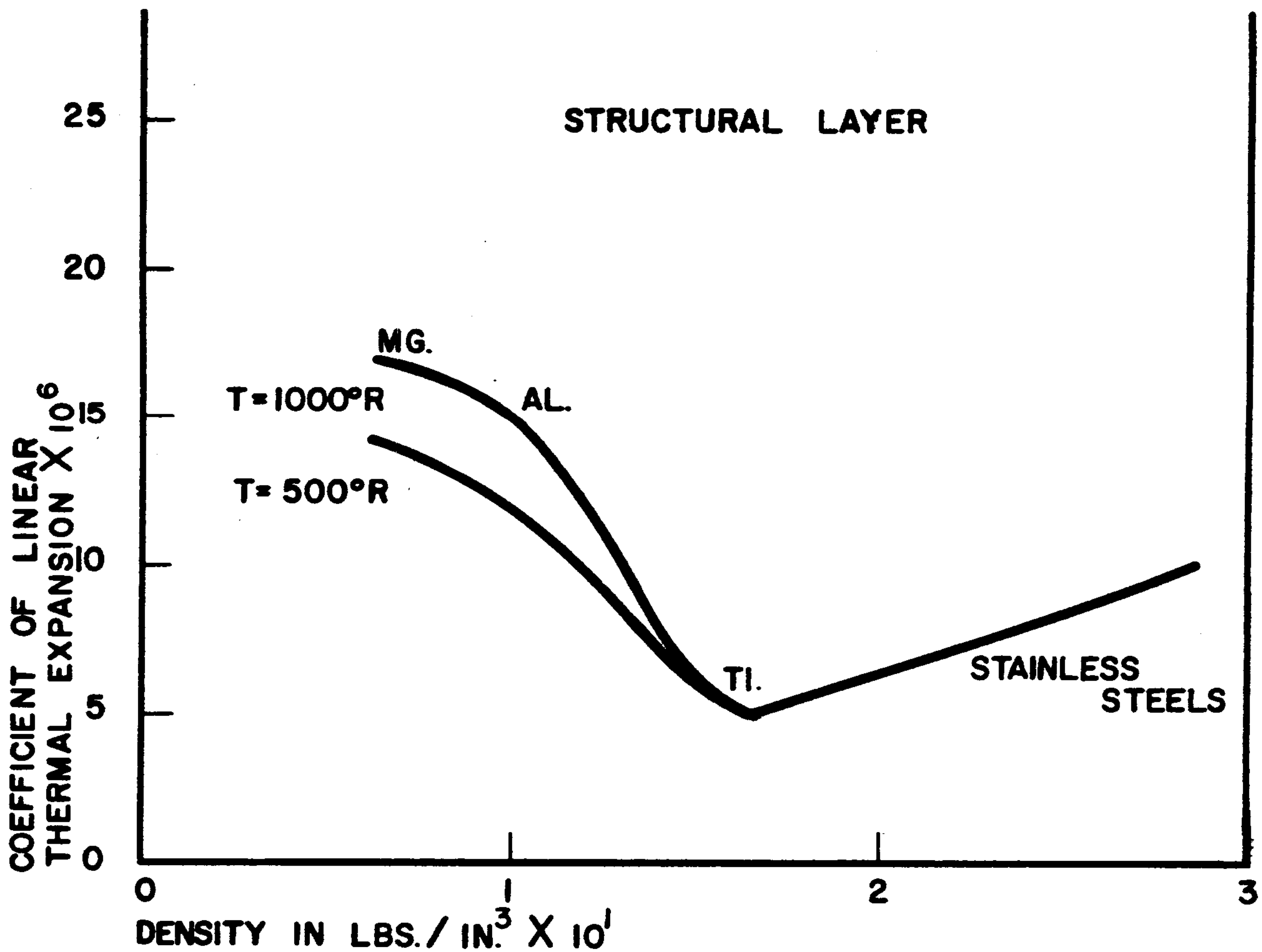

FIGURE 16 THERMAL EXPANSION VS. DENSITY

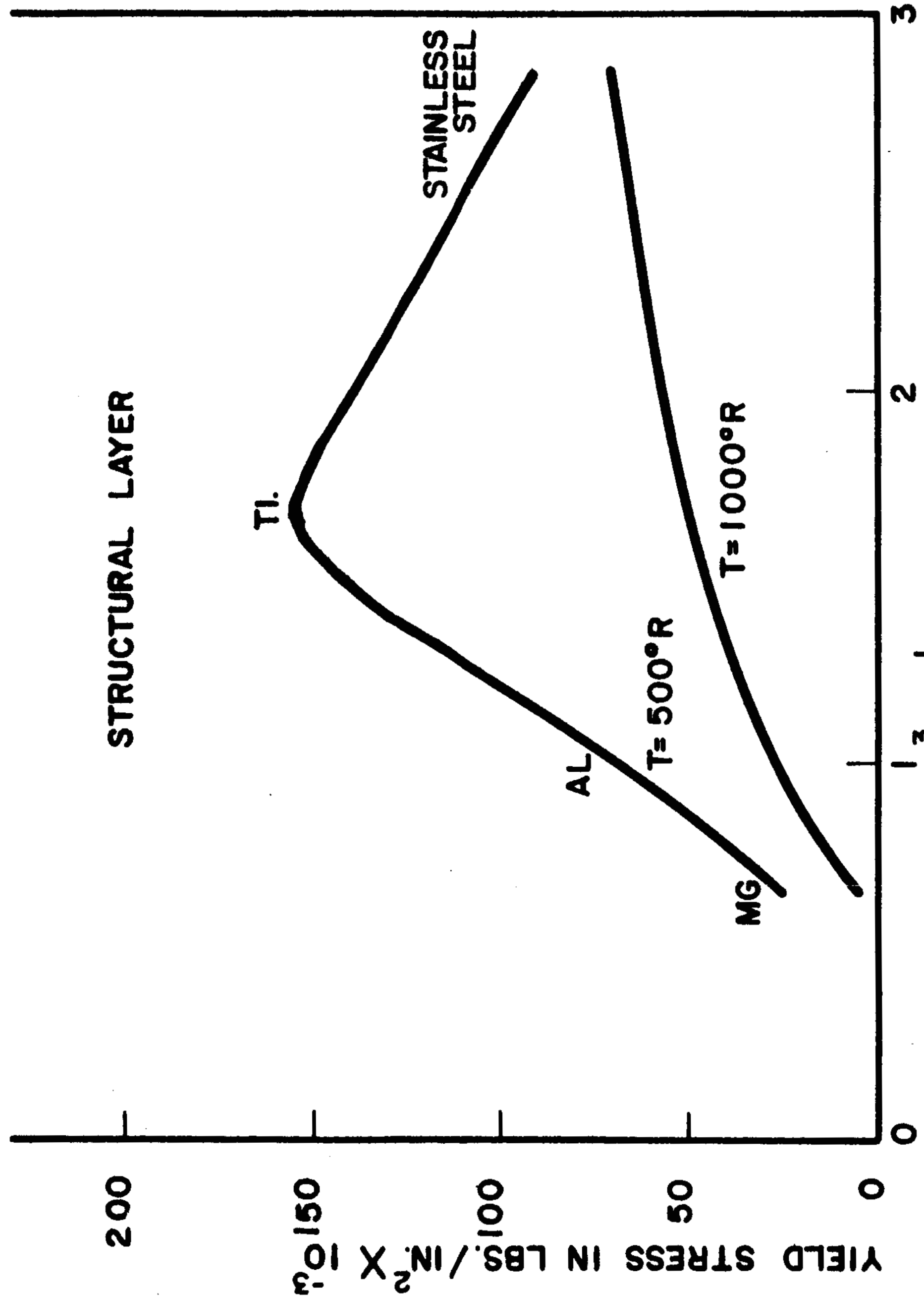

FIGURE 17 YIELD STRESS VS. DENSITY

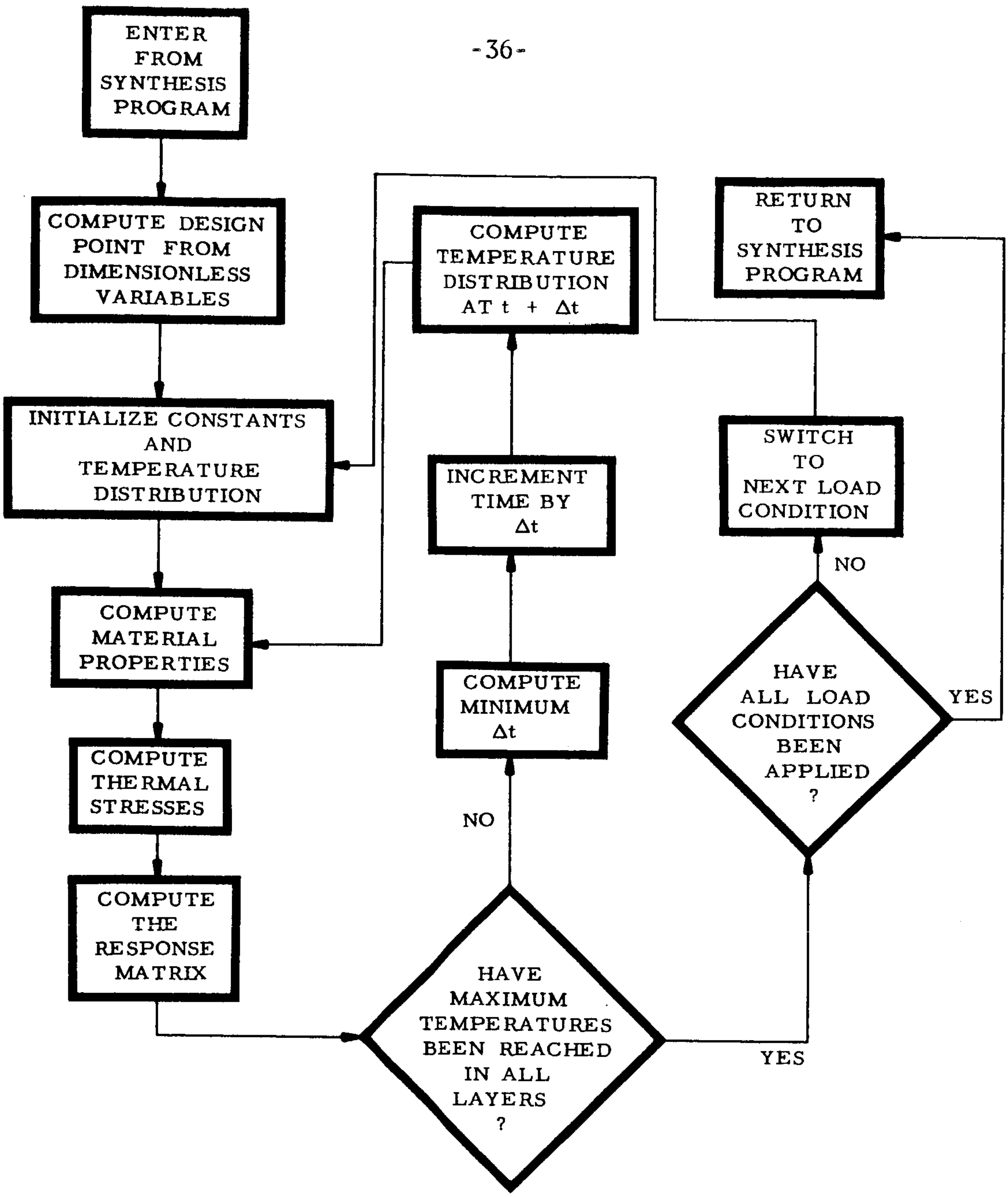

FIGURE 18 ANALYSIS FLOW CHART

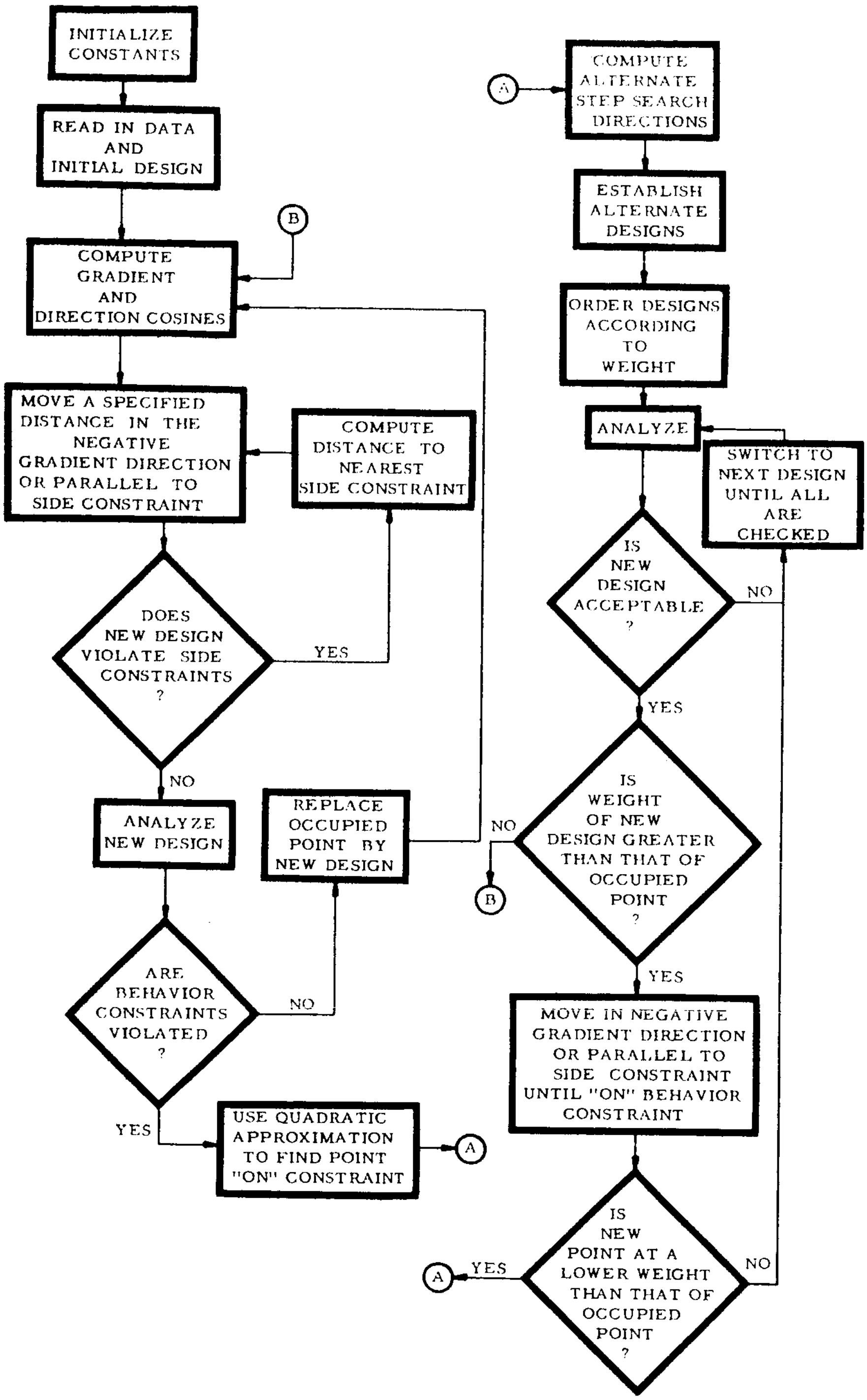

FIGURE 19 SYNTHESIS FLOW CHART

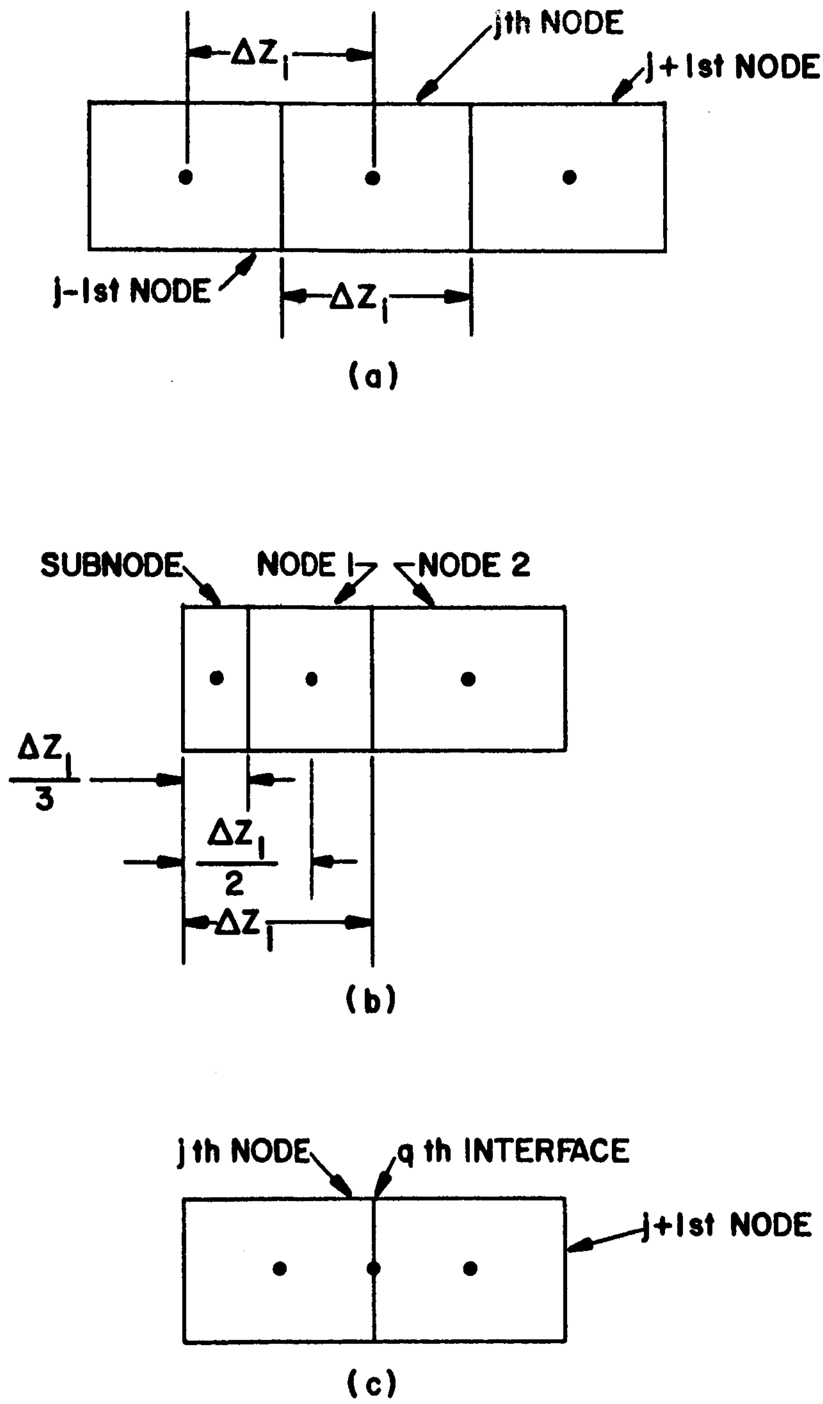

FIGURE 20 FINITE DIFFERENCE ANALYSIS

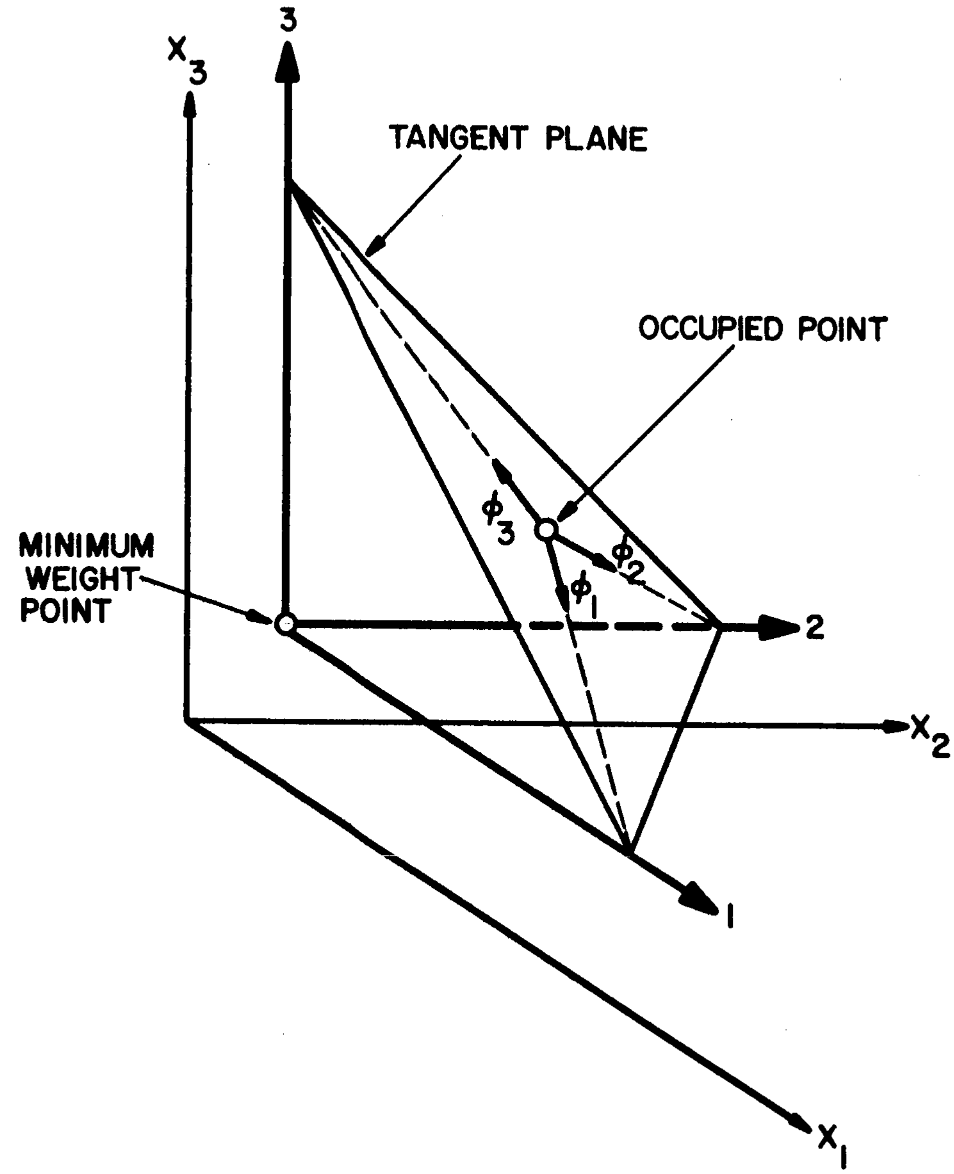

FIGURE 21 ALTERNATE STEP

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Appendix A

THERMAL ANALYSIS

An explicit type of finite difference solution to the heat flow equation is used.[5] Each layer is divided into n_i sub-layers or nodes of thickness $\Delta z_i = di/n_i$ and the material properties of each node are assumed to be constant at time t. It is also assumed that the temperature at the center of any node at time $t + \Delta t$ is dependent on the temperature of the node, the temperatures of adjacent nodes, and the material and geometric properties at time t.

A heat balance equation is written equating the net heat flow into a node to the heat stored in the node during a time interval Δt. This equation may then be solved for the tempera-ture of the node in question at time $t + \Delta t$.

For the j^{th} node of the i^{th} layer, shown in Figure 20-a, the heat balance equation is:

$$\left[\frac{k_{j-1} + k_j}{2}\right] \cdot [T_{j-1} - T_j] \cdot \frac{\Delta t}{\Delta z_i} - \left[\frac{k_j + k_{j+1}}{2}\right] \cdot [T_j - T_{j+1}] \cdot \frac{\Delta t}{\Delta z_i} =$$

$$\rho_i \, c_j \, \Delta z_i (T_j' - T_j)$$

This equation may be solved for the temperature of the j^{th} node at time $t + \Delta t$:

$$T_j' = T_j + \frac{\Delta t}{2\rho_i c_j \Delta z_i^2} [k_{j-1} (T_{j-1} - T_j)$$

$$+ k_j (T_{j-1} - 2T_j + T_{j+1}) - k_{j+1} (T_j - T_{j+1})]$$

This equation is valid for all interior points of the i^{th} layer. The average values of the conductivities of the adjacent nodes are used to provide a better approximation.

At the surface the radiation boundary condition is approximated by assuming that the surface temperature is the temperature at the center of a subnode of depth $\Delta z_1/3$.
This may be seen in Figure 20-b. A heat balance equation for the subnode gives:

$$Q \Delta t - s\varepsilon \Delta t [T_s^4 - T_o^4] - 3 k_1 \frac{\Delta t [T_s - T_1]}{\Delta z_1} =$$

$$\rho_1 c_1 \Delta z_1 (T_s' - T_s)$$

This may be solved for the surface temperature at time $t + \Delta t$:

$$T_s' = T_s + \frac{\Delta t}{\rho_1 c_1 \Delta z_1^2} [Q \Delta z_1 - s\varepsilon \Delta z_1 (T_s^4 - T_o^4)$$

$$- 3 k_1 (T_s - T_1)]$$

The heat balance equation for the first node becomes:

$$Q\,\Delta t - s\varepsilon\,\Delta t\,[T_s^4 - T_o^4] - [\frac{k_1 + k_2}{2}]\,[T_1 - T_2] \cdot \frac{\Delta t}{\Delta z_1} =$$

$$\rho_1 c_1\,\Delta z_1\,(T_1' - T_1)$$

The temperature of the first node at time $t + \Delta t$ is:

$$T_1' = T_1 + \frac{\Delta t}{2\rho_1 c_1\,\Delta z_1^2}\,[2\,\Delta z_1\,(Q - s\varepsilon\,(T_s^4 - T_o^4)$$

$$- (k_1 + k_2)\,(T_1 - T_2)\,]$$

The interface temperature at the q^{th} interface between the i^{th} and $i + 1^{st}$ layer, see Figure 20-c, is found by writing the heat flow equation for the nodes adjacent to the interface:

$$\frac{2\,k_j\,[T_j - TIF_q]}{\Delta z_i} = \frac{2\,k_{j+1}\,(TIF_q - T_{j+1})}{\Delta z_{i+1}}$$

Therefore the interface temperature is:

$$TIF_q = \frac{k_j\,T_j + \frac{\Delta z_i}{\Delta z_{i+1}}\,[k_{j+1}\,T_{j+1}]}{k_j + \frac{\Delta z_i}{\Delta z_{i+1}}\,k_{j+1}}$$

where j denotes the interface node of the i^{th} layer. The first interface is taken to be between layers 1 and 2 and the second interface is between layers 2 and 3.

A heat balance equation for the j^{th} node on the q^{th} interface is:

$$\left[\frac{k_{j-1}+k_j}{2}\right] \cdot [T_{j-1}-T_j] \frac{\Delta t}{\Delta z_i} - 2\, k_j\, [T_j - TIF_q] \cdot \frac{\Delta t}{\Delta z_i} =$$

$$\rho_i c_j\, \Delta z_i\, (T_j' - T_j)$$

The temperature at the interface node at time $t + \Delta t$ is:

$$T_j' = T_j + \frac{\Delta t}{2\rho_i c_j\, \Delta z_i^2} [T_{j-1}\, (k_j + k_{j-1})$$

$$- T_j (k_{j-1} + 5k_j) + 4\, k_j\, TIF_q]$$

where j denotes the interface node of the i^{th} layer, and in a similar manner it is found that:

$$T_j' = T_j + \frac{\Delta t}{2\rho_{i+1}\, c_j\, \Delta z_{i+1}^2} [T_{j+1} (k_j + k_{j+1}) - T_j (k_{j+1} + 5\, k_j)$$

$$+ 4\, k_j\, TIF_q]$$

where j denotes the first node in the $i+1^{st}$ layer.

The temperature equation for the last node in the third layer is:

$$T_j' = T_j + \frac{\Delta t}{2\rho_3 c_j\, \Delta z_3^2} [(k_{j-1} + k_j)\, (T_{j-1} - T_j)]$$

The reason for setting up approximating equations for the surface and interface temperatures is that the maximum temperatures occur at these points and they are therefore of greatest interest.

The temperature response of the structure is then represented by dividing the maximum temperature in a layer by the maximum allowable temperature for the layer. Thus there are three values which must be checked for each load condition to see if a temperature constaint has been violated. The procedure in determining the temperature response is as follows: starting from an initial temperature distribution at $t = 0$, the material properties are evaluated at each node and the temperatures at each node, the surface, and the interfaces are calculated for time $t + \Delta t$. The material properties are then recalculated based on the new temperature distribution and the time is incremented once more. This process is repeated until a maximum temperature is reached in each layer. Care is taken to observe the stability relationship between Δt and Δz which is:

$$\Delta t < \frac{1}{2} \frac{\rho \; c \; \Delta z^2}{k}$$

The number of nodes for each layer is fixed at the start of the program and Δt must be less than the smallest value calculated from this relationship.

The emissivity is arbitrarily set at 0.5 for the cases discussed in this work, however the program is flexible enough to include any variation of this quantity with temperature, etc. The initial temperature throughout the structure is chosen as 500°R, and the maximum allowable temperatures for each layer are 4000°R, 3000°R and 1000°R respectively. These limits are somewhat arbitrarily chosen but are mainly controlled by the material property data.

The computer program is presented in Appendix E.

Appendix B

ELASTIC ANALYSIS

The thin plate theory is used in the stress analysis of the structural layer the material of which is assumed to be homogeneous, and isotropic.

For a coordinate system with origin at the midplane of the third layer the strain displacement relations are:[6]

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2}$$

and $$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x\,\partial y}$$

Since the deflection of the midplane is zero:

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial x\,\partial y} = 0$$

Therefore the strain does not explicitly depend on z.

The stress-strain temperature relations are:[7]

$$\sigma_x = \frac{E}{1-\nu^2}\left[\varepsilon_x + \nu\varepsilon_y\right] - \frac{E\alpha\ \Delta T}{1-\nu}$$

$$\sigma_y = \frac{E}{1-\nu^2}\left[\varepsilon_y + \nu\varepsilon_x\right] - \frac{E\alpha\ \Delta T}{1-\nu}$$

$$\tau_{xy} = \frac{E}{2(1-\nu)}(\gamma_{xy})$$

where $\Delta T = T - T_o$

For Case 1, $N_{xy} = N_x = N_y = 0$ and the applicable equilibrium equations are:

$$N_x = \int_z \sigma_x \, dz = 0 \qquad N_y = \int_z \sigma_y \, dx = 0 \qquad N_{xy} = \int_z \tau_{xy} \, dz = 0$$

Substituting the stress-strain relations into the equilibrium equation gives:

$$\varepsilon_x \int_z \frac{E}{1-\nu^2} dz + \varepsilon_y \int_z \frac{\nu E}{1-\nu^2} dz - \int_z \frac{E\alpha \quad \Delta T}{1-\nu} dz = 0$$

$$\varepsilon_y \int_z \frac{E}{1-\nu^2} dz + \varepsilon_x \int_z \frac{\nu E}{1-\nu^2} dz - \int_z \frac{E\alpha \quad \Delta T}{1-\nu} dz = 0$$

and

$$\gamma_{xy} \int_z \frac{E}{2(1-\nu)} dz = 0$$

The latter equation implies that $\gamma_{xy} = 0$ and that therefore the x and y directions are the principal stress directions. From the first two equations the conclusion is reached that $\varepsilon_x = \varepsilon_y$ and that therefore $\sigma_x = \sigma_y = \sigma$.

The quantity ε_x is given by:

$$\varepsilon_x = \frac{\int_z E\alpha\ \Delta T\ dz}{\int_z E\ dz} \qquad \text{(B-1)}$$

These integrals are evaluated numerically in the analysis program. The value for σ at the j^{th} node of the third layer is:

$$\sigma_j = \frac{E_j}{1 - \nu_3} (\varepsilon_x - \alpha_j\ \Delta T_j) \qquad \text{(B-2)}$$

Poisson's ratio is assumed to remain a constant for layer three.

For Case 2 from the stress-strain relations the stress in the layer is simply:

$$\sigma_j = - \frac{E_j\ \alpha_j\ \Delta T_j}{1 - \nu_3} \qquad \text{(B-3)}$$

where $\varepsilon_x = \varepsilon_y = 0$.

Since a biaxial state of stress exists the von Mises criterion is used to define failure of the material due to stress. This relationship is:

$$\frac{\sigma^2}{\sigma_{yp}^2} \leq 1 \qquad \text{(B-4)}$$

The maximum stress may occur at either the upper or lower boundary of the third layer. Therefore two points must be checked to see if a stress constraint violation has occurred.

The computer program is presented in Appendix E.

Appendix C

THERMAL AND MECHANICAL PROPERTIES

For layers 1 and 2 the density is used as an independent variable in describing the material properties. The relationship between porosity and density is:

$$\text{porosity} = [1 - \frac{\rho}{\rho U}]$$

Layer 1 - Beryllium Oxide

Conductivity Equations - Figure 9

for ρ = 0.108 lbs/in^3 (dense material)[10]

$$k = \frac{186 \times 10^{-2}}{T} - 43.4 \times 10^{-5} \quad \frac{\text{Btu in}}{\text{in}^2 \text{sec}^\circ\text{R}} \qquad \text{(C1)}$$

for ρ = 0.0826 (23.5% porosity)

$$k = \frac{93.8 \times 10^{-2}}{T} - 15.05 \times 10^{-5} \qquad \text{(C2)}$$

for ρ = 0.0665 (38.5% porosity)

$$k = \frac{54.2 \times 10^{-2}}{T} - 4.17 \times 10^{-5} \qquad \text{(C3)}$$

For values of the density which lie between the above values the conductivity is found by linear interpolation.

Specific Heat Equation - Figure 12[10]

for $500°R \leq T \leq 2000°R$

$$C = -1.2 \times 10^{-7}T^2 + 0.478 \times 10^{-3}T + 0.03 \quad \frac{Btu}{lb°R} \qquad (C4)$$

for $T > 2000°R$

$$C = 6.5 \times 10^{-5}\, T + 0.38 \qquad (C5)$$

<u>Layer 2 - Aluminum Oxide:</u>

Conductivity Equation - Figure 10

for $\rho = 0.1445$ lbs/in^3 (dense material)[10]

$$k = \frac{24.3 \times 10^{-2}}{T} - 1.422 \times 10^{-5} \quad \frac{Btu\ in}{in^2 sec°R} \qquad (C6)$$

for $\rho = 0.110$ (23.4% porosity)

$$k = \frac{18.7 \times 10^{-2}}{T} - 1.222 \times 10^{-5} \qquad (C7)$$

for $\rho = 0.0741$ (48.7% porosity)

$$k = \frac{13.8 \times 10^{-2}}{T} - 2.2 \times 10^{-5} \qquad (C8)$$

A linear interpolation is used to find values of k for densities which lie between the values given above.

Specific Heat Equation - Figure 12[10]

for $500°R \leq T \leq 2000°R$

$$C = -8 \times 10^{-8} T^2 + 0.28 \times 10^{-3} T + 0.05 \quad \frac{Btu}{lb°R} \tag{C9}$$

for $T > 2000°R$

$$C = 2.5 \times 10^{-5} T + 0.24 \tag{C10}$$

<u>Layer 3</u>

Conductivity Equation - Figure 11[10]

for $0.0631 \leq \rho \leq 0.166$ lbs/in^3

at $T = 500°R$

$$k = -7.25 \times 10^{-1} \rho^2 + 1.5 \times 10^{-1} \rho - 4.79 \times 10^{-3} \quad \frac{Btu\ in}{in^2 sec°R} \tag{C11}$$

at $T = 1000°R$

$$k = -5.94 \times 10^{-1} \rho^2 + 1.22 \times 10^{-1} \rho - 3.54 \times 10^{-3} \tag{C12}$$

for $0.166 < \rho \leq 0.2835$

at $T = 500°R$

$$k = -9.11 \times 10^{-4} \rho + 45.3 \times 10^{-5} \tag{C13}$$

and at $T = 1000°R$

$$k = 4.26 \times 10^{-5} \rho + 24.35 \times 10^{-5} \tag{C14}$$

for values of the temperature between 500°R and 1000°R k is found by linear interpolation.

Specific Heat Equations - Figure 14[10]

for all values of the density at 500°R

$$C = 0.180 + 0.0667 \tanh [26.11 (0.118-\rho)] \frac{Btu}{lb°R} \qquad (C15)$$

Corrections due to temperature are:

for $0.0631 \leq \rho \leq 0.0978$

$$\Delta C = 7 \times 10^{-5} T - 0.035 \qquad (C16)$$

for $0.0978 < \rho \leq 0.166$

$$\Delta C = (-0.289 \rho + 0.063) \cdot (\frac{T}{500} - 1) \qquad (C17)$$

and for $0.166 < \rho \leq 0.2835$

$$\Delta C = (-0.0113 \rho + 0.0172) \cdot (\frac{T}{500} - 1) \qquad (C18)$$

These corrections are added to the value found for a given density at T = 500°R.

Modulus of Elasticity - Figure 15[7,8,9]

at T = 500°R

$$E = 97.4 \rho \times 10^6 + 0.286 \times 10^6 \qquad (C19)$$

for T = 1000°R

$$E = 102 \rho \times 10^6 - 3.92 \times 10^6 \qquad (C20)$$

A linear interpolation is used to find E for 500°R < T < 1000°R.

Thermal Expansion - Figure 16[10]

for $0.0631 \leq \rho \leq 0.166$

for T = 500°R

$$\alpha = -4.63 \times 10^{-4} \rho^2 + 2.0 \times 10^{-5} \rho + 14.4 \times 10^{-6} \qquad (C21)$$

for T = 1000°R

$$\alpha = -1.016 \times 10^{-3} \rho^2 + 11.7 \times 10^{-5} \rho + 13.4 \times 10^{-6} \qquad (C22)$$

A linear interpolation is used to find α for $500°R < T < 1000°R$.

for $0.166 < \rho \leq 0.2835$

$$\alpha = 42.8 \times 10^{-6} \rho - 2.2 \times 10^{-6} \qquad (C23)$$

Yield Stress - Figure 17[7,8,9]

for T = 500°R

$$\sigma_{yp} = (-79.7 \times 10^3)(\cos (18.6 (\rho - 0.03))) + 9 \times 10^4 \frac{\text{lbs}}{\text{in}^2} \qquad (C24)$$

for T = 1000°R

$$\sigma_{yp} = -1.26 \times 10^6 \rho^2 + 7.39 \times 10^5 \rho - 3.76 \times 10^4 \qquad (C25)$$

A linear interpolation is used to find σ_{yp} for $500°R < T < 1000°R$.

The data is for use in illustrative examples. Improvements and refinements in material property data could be inserted into the program with relative ease.

Appendix D

SYNTHESIS

The technique used is a steepest-descent alternate-step method in which the alternate step is made in the hyper-plane tangent to the weight surface at a particular point.

The merit function is non-dimensionalized and the variables scaled by dividing both sides by the product of a reference density and depth, $\rho_R\, d_R$. The dimensionless merit function is:

$$\phi = \omega_1\, t_1 + \omega_2\, t_2 + \omega_3\, t_3$$

The variables are scaled so that the design parameters and gradient components with respect to the design parameters are of the same order of magnitude. In this problem $d_R = 1.0$ in. and $\rho_R = 0.1$ lb/in^3.

The response of the structure is expressed by a response matrix:

$$[R] = \begin{bmatrix} R_{11} & \cdots & R_{1N} \\ \vdots & & \\ R_{51} & \cdots & R_{5N} \end{bmatrix}$$

where the row subscript corresponds to the behavior function examined and the column subscript denotes the load condition. For example for load condition one the elements of the response matrix are:

$$R_{11} = \frac{TS_{max}}{T_{1max}}$$

$$R_{21} = \frac{TIF_{1\ max}}{T_{2\ max}}$$

$$R_{31} = \frac{TIF_{2\ max}}{T_{3\ max}}$$

$$R_{41} = \frac{\sigma^2}{\sigma_{yp}^2} \quad \text{at upper boundary of layer three}$$

$$R_{51} = \frac{\sigma^2}{\sigma_{yp}^2} \quad \text{at lower boundary of layer three.}$$

Thus whenever an element of the response matrix exceeds the value 1 a behavior constraint is violated and the particular design is unacceptable.

The merit function is thought of as forming a hyper-surface in the design space. The gradient to this surface is:

$$\nabla\phi = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

From this the direction cosines of the gradient, $\tilde{\phi}$, may be found.

The synthesis is initiated by starting from an acceptable design point and moving a specified distance in the negative gradient direction. This procedure provides the maximum weight reduction and is expressed by:

$$\tilde{x}' = \tilde{x}_o - \lambda \tilde{\phi}$$

In this problem the value of λ is set at 0.3. This value is simply the result of experimentation and gives reasonable changes in the values of the design parameters.

The new design is checked for violations of side and behavior constraints and if there are none $\tilde{x}_o$ is replaced by $\tilde{x}'$ and a similar move is made. No acceleration of the move ~~if~~ is provided since the dimensions of the space are sucn that only one or two moves of this type are necessary to cause constraint violation. If violation of one or more side constraints occurs the distance to the nearest side constraint, λ', is computed and a new move is made to the side constraint:

$$\tilde{x}' = \tilde{x}_o - \lambda' \tilde{\phi}$$

The constrained point is then checked for behavior constraint violations. If there are none a move parallel to the side constraints is made. This is done by moving in the negative gradient direction and then equating the violated constraints to their lower limits. Moves are then made in the new direction until side or behavior constraint violation occurs.

When a behavior constraint is violated a quadratic approximation is used to find a point that lies "on" a behavior constraint, i.e., a design for which the maximum value of any element of the response matrix is one. The maximum response is assumed to vary quadratically as a function of distance from the last acceptable point to the point of violation. Using the maximum values of the response matrix for each of these points and one halfway between as data a quadratic function is set up. The distance from the acceptable point to the desired point "on" a behavior constraint is then computed. This method worked very well and convergence usually took place within one or two cycles.

Once a point is found "on" a behavior constraint, the alternate step is made. This is accomplished by moving a specified distance, chosen as 0.5 in this problem, in the direction given by the following procedure: Six unit vectors are found in the directions of the points of intersection of the tangent hyper-plane with axes parallel to the coordinate axes and passing through the minimum weight point. This is done by starting with the equation for the hyper-plane which is:

$$[(\tilde{x}' - \tilde{x}_0), \tilde{\phi}] = 0$$

where $\tilde{x}'$ in this case represents a point on the hyper-plane, setting all the design parameter values on their lower limits, and solving for the point of intersection with each axis. The process is illustrated in Figure 21. In this three dimensional case X_1, X_2 and X_3 correspond to the design parameter axes; ϕ_1, ϕ_2 and ϕ_3 are the alternate step search directions from the occupied point. Axes 1, 2 and 3 are parallel to X_1, X_2 and X_3 respectively.

The six unit vectors are then used to generate other search directions. This is done by taking all possible vector-sum combinations of the six vectors. For example the six vectors are summed one at a time, two at a time, three at a time, etc. This process results in a total of 63 vectors which are all made of unit length. Moves are then made the specified distance in the plus and minus direction of each resulting in a total of 126 different moves.

Many of these moves may be prohibited if the occupied point lies on a side constraint. If a side constraint is encountered after moving the specified distance the distance to the constraint is computed and a new move is made one-half this distance in order to place the point in a supposedly free region.

The 126 new designs are ordered according to merit and checked starting with the lowest weight design. If a new acceptable design with a weight lower than that of the occupied point is found, this design is taken as a new starting point and the entire synthesis process begins again with a move in the negative gradient direction.

Acceptable designs of weight higher than that of the occupied point are checked but these moves are not accepted unless a valid design is found at a lower weight than that of the occupied point after moving in the negative gradient direction from the higher weight design.

If the problem is unsuccessful in finding a new acceptable design after checking 126 nearby alternate designs the occupied point is assumed to be the minimum.

A fixed number of search directions is chosen because it is felt that there is no advantage in taking a random approach to the problem due to the length of time, 30 to 40 seconds, needed to complete each design check.

The program is outlined in Appendix E.

Appendix E

COMPUTER PROGRAM

The computer program was written in the Algol 58 (Balgol) compiler for the Univac 1107 Digital Computer. Included in this appendix are a list of program symbols and a listing of the entire program.

The analysis section was set up as a procedure or independent sub-program. This made it possible to enter and leave the analysis routine at any point in the synthesis program.

Flow charts for the program are shown in Figures 18 and 19.

SYNTHESIS PROGRAM SYMBOLS

PU	upper limit on density
PL	lower limit on density
DU	upper limit on depth
DL	lower limit on depth
DP	design parameter (dimensionless)
DPU	upper limit
DPL	lower limit
DPO	initial design
DPP	new design
PSI	direction cosine
PHI	direction cosine
R	response matrix
RP	auxillary matrix
R_o	auxillary matrix
DPA	auxillary matrix
N	number of nodes
P_o	initial design density
D_o	initial design depth
DPW	auxillary matrix
DP1	auxillary matrix
DP2	auxillary matrix
O	auxillary matrix
DPT1	auxillary matrix
DPT2	auxillary matrix

Z1	auxillary matrix
DPS	auxillary matrix
WT	weight
CHI	direction cosine
I	layer subscript
J	node subscript
U	auxillary subscript
V	auxillary subscript
NQ	number of load conditions
G	number of nodes in layer 1
H	number of nodes in layers 1 and 2
M	total number of nodes
CR	load condition label
TEST	output of check procedure
NRM	number of elements in response matrix
F	auxillary variable
SC	auxillary variable
SIG	auxillary variable
CH	auxillary variable
K	auxillary variable
ANALYSIS	analysis procedure
QUAD	quadratic approximation procedure
CHECK	design test procedure
E	tolerance
L	distance of travel

ELL	auxillary variable
LN	auxillary variable
RMX	maximum response
RELMIN	auxillary variable
DIP	auxillary variable
MU	distance of travel

ANALYSIS PROGRAM SYMBOLS

D	depth
P	density
Z	space variable
ALPHA	coefficient of thermal expansion
NU	Poisson's Ratio
T	temperature
TP	temperature at t + Δt
C	specific heat
K	thermal conductivity
YM	elastic modulus
YS	yield stress
SF	safety factor
TMAX	maximum temperature
TIF	interface temperature
TIFP	auxillary matrix
EXTENT	duration of heat pulse
RIM	auxillary matrix
SIF1	stress case 1
SIF2	stress case 2
DIP	auxillary matrix
T_o	initial system temperature

```
   RUN  13004,3,(300,500)
    BAL    99999
COMMENT SYNTHESIS OF LAMINATED HEAT SHIELD   $
ARRAY PU(3),PL(3),DU(3),DL(3),DP(6),DPU(6),DPL(6),DP0(6),
DPP(6),PS1(6),PHI(6),     R(80,5),RP(80,5),R0(80,5),
DPA(6),N(4),P0(3),D0(3),Y(6),DPW(6),DP1(6),DP2(6),
O(126), DPT1(6),DPT2(6) ,
Z1(70,6),DPS(150,6),WT(150),CHI(70,6)                             $
INTEGER   I,J,U,V,NQ,G,H,M,CR,N( ) ,TEST,NRM,T,F,SC,SIG,CH,K,O( ),
DIP                                                               $
PROCEDURE ANALYSIS (NQ,M,G,H,N( ),PR,DR,DPA( )$R(,),W)  $
BEGIN
ARRAY         D(3),P(3),Z(3),ALPHA(60),NU(3),  T(60),TP(60),
C(60),K(60),YM(60),STRES1(60),STRES2(60)    ,YS(60),SF(3) ,
TMAX(3),T1F(4),T1FP(4),EXTENT(5)   ,       RIM(80,5),
S1F1(6),S1F2(6),DTP(3)                                  $
INTEGER I,J,M,  N( ) , G,H,X,COUNT,V,CH,CR,NQ ,ME,NRM         $
COMMENT HEAT TRANSFER PROGRAM  TIME SPACE VAR      $
FLOATING  Z      $
FOR I=(1,1,3)$ BEGIN P(I) = ((PR).(DPA(I))) $
D(I)=((DR).(DPA(I+3)))$END$
NQ = 2 $
BOX1000..
NRM = 5 $
                              SF(3) = 1.0 $
                             NU(3)=0.285 $
Q0 = 0.5   $
                             E= 0.5 $ S=3.36**-15 $ DT=0.1$T0=500.0$
EXTENT(1) = 100.0   $ EXTENT(2) = 100.0 $
V=NRM  $
FOR I=(1,1,V)    $        BEGIN FOR J=(1,1,NQ)$BEGIN
R(I,J)=0.0$RIM(I,J)=0.0$END$END$
CR=0$
BOX600.. CR = CR + 1 $
IF CR GTR NQ $ GO TO BOX30$
FOR J=(1,1,4) $ T1F(J) = 500.0 $
                          FOR I=(1,1,3)$Z(I)=(D(I))/(N(I))   $
FOR J=(1,1,M)$BEGIN TP(J) = 0.0 $ T(J) = 500.0$ END $
LIMIT = 10.0**3 $
FOR J=(1,1,4)$T1FP(J)=500.0 $
TS = 500.0   $
TIME=0.0   $
TRACK = 1.0   $
TMAX(1)=4000.0$TMAX(2)=3000.0$TMAX(3)=1000.0$
BOX80..

COMMENT COEFF OF THERMAL EXPANSION PROGRAM      $
I=3$ J=H+1$
BOX31.., IF (P(I)) GEQ 0.166 $ GO TO BOX32 $
BOX36.. IF (T(J)) LEQ 1000.0 $ GO TO BOX33 $
IF (T(J)) LEQ 1500.0 $ GO TO BOX34 $
   GO TO BOX30    $
BOX34.. ALPHA(J)=(((1500.0-(T(J)))/500.0).(((-1.016**-3).
(P(I)).(P(I)))+((11.7**-5).(P(I)))+13.4**-6))
+((((T(J))-1000.0)/500.0).(((-1.836**-3).(P(I)).(P(I)))
+((2.78**-4).(P(I)))+9.35**-6))$
GO TO BOX35  $
BOX33.. ALPHA(J)=(((1000.0-(T(J)))/500.0).(((-4.63**-4).
(P(I)).(P(I)))+((2.0**-5).(P(I)))+14.4**-6))+((((T(J))-500.0
)/500.0).(((-1.016**-3).(P(I)).(P(I)))+((11.7**-5).(P(I)))
```

```
+13.4**-6)) $
BOX35.. IF J LSS M $ BEGIN J=J + 1 $ GO TO BOX36 $ END $
GO TO BOX75 $
BOX32.. FOR J=(H+1,1,M)$ALPHA(J)=((42.8**-6).(P(I)))-2.2**-6$
BOX75..
COMMENT END OF THERMAL EXPANSION PROGRAM $

COMMENT MODULUS OF ELASTICITY PROGRAM   $
J=H+1$      I=3$
BOX68.. IF T(J) LEQ 1500.0 $ GO TO BOX64 $
GO TO BOX30  $
BOX64.. IF T(J) LEQ 1000.0 $ GO TO BOX65 $
YM(J)=(((T(J)/500.0)-2.0).(((93.7**6).P(I))-5.23**6))
+((3.0-(T(J)/500.0)).(((102.0**6).P(I))-3.92**6)) $
GO TO BOX66 $
BOX65..
YM(J)=((((102.0**6).(P(I)))-3.92**6).(((T(J))/500.0)-1.0))
+ ((((97.4**6).(P(I)))+0.286**6).(2.0 - ((T(J))/500.0)))$
BOX66.. IF J GEQ M $ GO TO BOX67 $
J=J + 1 $ GO TO BOX68 $
BOX67..
COMMENT END OF MOD OF ELAST PROGRAM   $

COMMENT SPECIFIC HEAT PROGRAM   $
FOR J=(G+1,1,H) $
  BEGIN
     C(J) = ((-8.0**-8).(T(J)).(T(J)))+((0.28**-3).(T(J)))+0.05$
  END $
FOR J=(1,1,G)$
  BEGIN
    C(J)=((-1.2**-7).(T(J)).(T(J)))+((0.478**-3).(T(J)))+0.03$
  END $
Y=(26.1).(0.118 - (P(3))) $
CP=(0.180)+((0.0667).(((EXP(Y))-(1.0/EXP(Y)))/
((EXP(Y))+(1.0/EXP(Y))))) $
L1.. IF (P(3)) GEQ 0.0978 $ GO TO L2 $
FOR J = (H+1,1,M)$ C(J)=(CP)+((7.0**-5).(T(J)))-(0.035)$
GO TO L5 $
L2.. IF (P(3)) GEQ 0.166 $ GO TO L3 $
FOR J=(H+1,1,M) $ C(J)=CP + ((((-0.289).(P(3)))+
0.063).(((T(J))/500.0) - 1.0)) $
GO TO L5 $
L3..  GO TO L4       $
L4.. FOR J=(H+1,1,M)$ C(J) = CP + ((((-0.0113).(P(3)))+
0.0172).(((T(J))/500.0) - 1.0 )) $
L5..
COMMENT END OF SPECIFIC HEAT PROGRAM  $

COMMENT THERMAL CONDUCTIVITY PROGRAM  $
I=2$J=G+1$
IF (P(I)) GTR 0.1110 $ GO TO BOX50 $
FOR J=(G+1,1,H)$
               K(J)=((((P(I))-0.0741).(((18.7**-2)/(T(J)))
-1.222**-5))+((0.1110-(P(I))).(((13.8**-2)/(T(J)))
-2.200**-5)))/0.0369   $
GO TO BOX51 $
BOX50..
FOR J=(G+1,1,H) $
K(J)=((((P(I))-0.1110).(((24.3**-2)/(T(J)))-1.422**-5))
+((0.1445-(P(I))).(((18.7**-2)/(T(J)))-1.222**-5)))/0.0335 $
```

```
BOX51..
I=1 $J=1 $
IF (P(I)) GTR 0.0826 $ GO TO BOX52 $
FOR J=(1,1,G) $
                    K(J)=((((P(I))-0.0665).(((93.8**-2)/(T(J)))
-15.05**-5))+((0.0826-(P(I))).(((54.2**-2)/(T(J)))
-4.17**-5)))/0.0161   $
GO TO BOX53  $
BOX52..
FOR J=(1,1,G) $
                                K(J)=((((P(I))-0.0826).
(((186.0**-2)/(T(J)))-43.4**-5))+((0.108-(P(I))).
(((93.8**-2)/(T(J)))-15.05**-5)))/0.0254    $
BOX53.. I=3 $ J= M+1 $
IF (P(I)) LSS 0.166 $ GO TO BOX54 $
BOX56.. IF (T(J)) GTR 1000.0 $ GO TO BOX55 $
K(J)=((((T(J))/500.0)-1.0).(((4.26**-5).(P(I)))+24.35**-5))
+((2.0 - ((T(J))/500.0)).(((-9.11**-4).(P(I)))+45.3**-5)) $
IF J GEQ M $ GO TO BOX60 $
J=J+1 $ GO TO BOX56 $
BOX55.. K(J)=(((((T(J))/500.0)-2.0).(((6.3**-4).(P(I)))
+12.76**-5))+((3.0 - ((T(J))/500.0)).(((4.26**-5).(P(I)))
+24.35**-5 ))    $
IF J GEQ M $ GO TO BOX60 $
J = J + 1 $ GO TO BOX56 $
BOX54.. IF (T(J)) GTR 1000.0 $ GO TO BOX57 $
K(J)= (((2.0)-((T(J))/500.0)).(((-7.25**-1).(P(I)).(P(I)))+
((1.51**-1).(P(I)))-4.79**-3))+((((T(J))/500.0)-(1.0)).(
((-5.94**-1).(P(I)).(P(I)))+((1.22**-1).(P(I)))-3.535**-3))$
IF J GEQ M $ GO TO BOX60 $
J= J + 1 $ GO TO BOX54 $
BOX57.. K(J)=(((3.0)-((T(J))/ 500.0)).(((-5.94**-1).(P(I)).
(P(I)))+((1.22**-1).(P(I)))-3.535**-3))+((((T(J))/ 500.0)
-2.0).(((-5.15**-1).(P(I)).(P(I)))+((1.027**-1).(P(I)))
-2.71**-3)) $
IF J GEQ M $ GO TO BOX60 $
J=J+1 $ GO TO BOX54 $
BOX60..
COMMENT END OF THERMAL COND PROGRAM  $

COMMENT YIELD STRESS PROGRAM    $
I=3$ J=M + 1 $
BOX94.. IF T(J) LEQ 1500.0 $ GO TO BOX90 $
   GO TO BOX30    $
BOX90.. IF T(J) LEQ 1000.0 $ GO TO BOX91  $
YS(J)=(SF(I)).(((((T(J))/500.0)-2.0).(((2.385**5).(P(I)))
-18.0**3)) + ((3.0 - ((T(J))/500.0)).(((-1.26**6).
(P(I)).(P(I))) + ((7.39**5).(P(I)))       -37580.0))) $
GO TO BOX92 $
BOX91.. BETA = (18.6).(P(I) - 0.03 )     $
         YS(J)=(SF(I)).(((((T(J))/500.0)-1.0).
(((-1.26**6).(P(I)).(P(I)))+((7.39**5).(P(I)))
-37580.0))+((2.0 - ((T(J))/500.0)).
(((-79700.0).(COS (BETA)))+ 90000.0)))$
BOX92.. IF J GEQ M $ GO TO BOX93 $
J=M      $ GO TO BOX94 $
BOX93..
COMMENT END OF YIELD STRESS PROGRAM  $

COMMENT START OF STRESS PROGRAM   $
```

```
I=3$J=H+1$A=0.0$B=0.0$
A=A+((Z(I)/4.0).(((YM(J)).(ALPHA(J)).((TIF(3))+(T(J))-
((2.0).(T0)))) + ((YM(M)).(ALPHA(M)).((TIF(4))+(T(M))-
((2.0).(T0)))))) $
BOX20.. A=A + ((Z(I)/2.0).(((YM(J)).(ALPHA(J)).((T(J))-T0))
+((YM(J+1)).(ALPHA(J+1)).((T(J+1))-T0)))) $
IF J EQL (M-1)$GO TO BOX21 $
J=J+1$ GO TO BOX20$
BOX21.. J=H+1 $
BOX22..        B=B + ((YM(J)).(Z(I))) $
IF J EQL M $ GO TO BOX23$
J=J+1$ GO TO BOX22$
BOX23..        UX=A/B  $ X=3$J=H+1$
SIF1(1)=((YM(J)/(1.0-NU(I))).(UX-((ALPHA(J)).((TIF(X))-T0))))$
X=4$ J=M$
SIF1(2)=((YM(J)/(1.0-NU(I))).(UX-((ALPHA(J)).((TIF(4))-T0))))$
COMMENT END OF STRESS PROGRAM  $

COMMENT RESPONSE MATRIX PROGRAM  $
J=1$ V=1$ I=1$ U=1$
BOX201..
R(I,CR)=TIF(U)/TMAX(V)$
IF U GEQ 3 $ GO TO BOX200 $
U=U+1$V=V+1$I=I+1$ GO TO BOX201 $
BOX200.. J=H + 1 $ I= I + 1 $
R(I,CR)=((SIF1(1)).(SIF1(1)))/((YS(J)).(YS(J)))$
J=M$I=I+1$R(I,CR)=((SIF1(2)).(SIF1(2)))/((YS(J)).(YS(J)))$
V=NRM  $
FOR I=(1,1,V)$BEGIN FOR J=(1,1,NQ)$BEGIN IF R(I,J) GTR
RIM(I,J) $ BEGIN RIM(I,J)=R(I,J)$END$END$END$
IF  N(4) EQL 1 $
  BEGIN FOR I=(1,1,V)$
    BEGIN FOR J=(1,1,NQ) $
      BEGIN IF RIM(I,J) GEQ 1.01 $
        BEGIN N(4) = 0 $ GO TO BOX30$
      END $
        END $
    END $
  END $
COMMENT END OF RESPONSE MATRIX PROGRAM    $


COMMENT LIMIT PROGRAM  $
BOX207.. IF TIF(1) LSS TIFP(1)$GO TO BOX300 $
T1 = TIME $
BOX300.. IF TIF(2) LSS TIFP(2) $ GO TO BOX301 $
T2 = TIME $
BOX302.. FOR J=(1,1,4) $ TIFP(J) = TIF(J) $
GO TO BOX303 $
BOX301.. IF TIME GEQ EXTENT(CR)$GO TO BOX304$
GO TO BOX302 $
BOX304.. LIMIT=(((T2).(D(1)+D(2)))-(( T1).(D(2))))/D(1) $
BOX303.. IF TIME LSS LIMIT $ GO TO BOX500 $
GO TO BOX600 $
COMMENT END OF LIMIT PROGRAM  $
BOX500..

GO TO BOX420 $
BOX420..
COMMENT DELTA T PROGRAM    $
```

```
V=1$ I=1$ J=N(I)    $
BOX411.. DTP(V)=((0.9).(P(I)).(C(J)).(Z(I)).(Z(I)))/
((2.0).(K(J))) $
I=I + 1 $
IF J EQL M $ GO TO BOX410 $
J=J + N(I) $ V=V + 1 $ GO TO BOX411 $
BOX410.. DT=MIN(DTP(1),DTP(2),DTP(3)) $
IF DT GTR 2.0 $ DT = 2.0 $
COMMENT END OF DELTA T PROGRAM    $
TIME = TIME + DT    $

IF CR EQL 2 $ GO TO BOX402$
COMMENT START OF HEAT PULSE ONE   $
IF TIME LEQ 100.0 $ BEGIN Q=((2.0)-((TIME)/50.0))$GO TO BOX401$
END $
Q=0.0 $
GO TO BOX401 $
COMMENT END OF HEAT PULSE ONE   $
COMMENT HEAT PULSE TWO  $
BOX402..
IF TIME LEQ 100.0 $ BEGIN Q=1.0$ GO TO BOX401 $ END $
Q=0.0      $
COMMENT END OF HEAT PULSE TWO   $
BOX401..     GO TO BOX97$

COMMENT TEMP DIST PROGRAM   $
BOX97.. GO TO BOX8   $
BOX8.. I=1 $
X=0$J=1$
BOX9.. X=X + N(I) $
BOX3..F=DT/((2.0)(P(I))(C(J))(Z(I))(Z(I)))   $
IF J GTR 1 $ GO TO BOX2 $
TSP=TS +(((F).(((Q).(Z(I))) - ((S).(E).(Z(I)).(((TS).
(TS).(TS).(TS)) - ((T0).(T0).(T0).(T0))))
- ((3.0).(K(J)).(TS - (T(1)))))).(2.0))  $
TP(1)=(T(1)) + ((F).(((2.0).(Z(1)).(Q - ((S).(E).
(((TS  ).(TS  ).(TS  ).(TS  )) - ((T0).(T0).(T0).(T0)))))
-((K(1) + K(2)).(T(1) - T(2)))))   $
J=J+1$  GO TO BOX3$
BOX2.. IF J EQL M $ GO TO BOX6  $
IF J EQL X $ GO TO BOX4 $
TP(J)=T(J)+(F.(((K(J-1)).(T(J-1)-T(J)))
+((K(J)).(T(J-1)-((2.0)(T(J)))+T(J+1)))-((K(J+1)).
(T(J)-T(J+1)))))      $
J=J+1 $ GO TO BOX3 $
BOX4..TIM=((K(J)).(T(J))+(((Z(I)).(K(J+1)).(T(J+1)))/(Z(I+1))))
/((K(J))+(((Z(I)).(K(J+1)))/(Z(I+1))))     $
TIF(I+1) = TIM  $
TP(J)=(T(J))+((F).(((T(J-1)).((K(J))+(K(J-1))))-((T(J)).
((K(J-1))+((5.0).(K(J)))))+((4.0).(K(J)).(TIM))))    $
  J=J+1 $ I=I+1 $
     F=DT/((2.0)(P(I))(C(J))(Z(I))(Z(I)))   $
TP(J)=(T(J))+((F).(((T(J+1)).((K(J))+(K(J+1))))-((T(J)).
((K(J+1))+((5.0).(K(J))))) +((4.0).(K(J)).(TIM))))   $
J=J+1 $ GO TO BOX9 $
BOX6..
TP(J)=T(J) + ((F).(((K(J-1) + K(J)).(T(J-1) - T(J))))) $
BOX7..
TS = TSP    $
TIF(1) = TS  $
```

```
TIF(4) = TP(M) $
FOR J=(1,1,M)$T(J)=TP(J)$
COMMENT END OF TEMP DIST PROGRAM  $

GO TO BOX80  $
BOX30..

W=0.0$
FOR I=(1,1,3)$ W= W + (P(I).D(I)) $
WRITE ( $ $ PAR5) $
WRITE ($ $ WEIGHT , HEAVY)$
V= NRM   $
FOR I=(1,1,V)$BEGIN FOR J=(1,1,NQ)$BEGIN R(I,J)=RIM(I,J)END$END$
WRITE ( $ $ PAR4) $
WRITE ($ $ RESP,MAT)  $
OUTPUT RESP ( FOR I=(1,1,V)     $(FOR J=(1,1,NQ)$R(I,J))) $
FORMAT MAT (2(F14.3,B3),W6) $
OUTPUT  WEIGHT (W)$
FORMAT HEAVY ( 1(F14.4),W0)$
FORMAT PAR5 (*THE WEIGHT IN LBS PER SQ IN IS *,W0)$
FORMAT PAR4 ( * RESPONSE MATRIX * , W0 ) $
RETURN END ANALYSIS( )   $
PROCEDURE CHECK (NQ,M,R( , ) $ TEST,RMX)  $
BEGIN
INTEGER NQ,M,TEST,I,J,V ,NRM                       $
TEST = 1        $
NRM=5  $
E= 0.01  $
FOR I=(1,1,NRM)  $  BEGIN FOR J=(1,1,NQ)$ BEGIN IF R(I,J)
GTR ( 1.0 + E ) $    TEST = 0 $ END $ END $
RMX=MAX(R(1,1),R(1,2),R(2,1),R(2,2),R(3,1),R(3,2),R(4,1),
R(4,2),R(5,1),R(5,2))$
WRITE ( $ $ PAR12 ) $
WRITE ( $ $ PAR10,PAR11) $
OUTPUT PAR10 (RMX) $
FORMAT PAR11 ( 1 (F17.3),W4) $
FORMAT PAR12 (*THE MAXIMUM RESPONSE IS * , W0 )      $
RETURN END CHECK( )    $
PROCEDURE QUAD (S,Z,Y( )$MU) $
BEGIN
A=((Y(3)-Y(1))/((S.S)-(Z.S)))
-((S.(Y(2)-Y(1)))/(Z.((S.S)-(Z.S))))$
B=((Y(2)-Y(1))/Z) - (A.Z) $
C=(Y(1)) - 0.999   $
MVD=(B.B)-((4.0).A.C) $
MU=(-B + SQRT(MVD))/((2.0).A) $
RETURN END QUAD( )   $

RELMIN = 0 $

DIP = 0 $
CR=1 $
N(1) = 5 $  N(2) = 4 $ N(3) = 3 $
N(3) = 2 $
G=N(1) $ H=N(1) + N(2)   $
M=N(1)+N(2)+N(3)      $
D=0.5  $
E = 0.001 $
NRM=5$
L=0.3  $
```

```
P=0.8  $
N(4) = 0 $
N(1) = 3 $
GEN..
READ ( $ $ DATA )$
INPUT DATA (FOR I=(1,1,3)$(PO(I),DO(I))) $
COMMENT CALC OF DPO,DPU,DPL, AND CHECK ON INT DES          $
FOR I=(1,1,3)$BEGIN DPO(I)=PO(I)/PR $DPU(I)=PU(I)/PR$
     DPL(I)=PL(I)/PR $ END $
FOR I=(4,1,6)$BEGIN DPO(I)=DO(I-3)/DR$DPU(I)=DU(I-3)/DR$
DPL(I)=DL(I-3)/DR $END$
WRITE ( $ $ PAR35,PAR23) $
OUTPUT PAR35( FOR J=(1,1,6)$DPL(J)) $
WRITE ( $ $ PAR36,PAR23) $
OUTPUT PAR36(FOR J=(1,1,6)$DPU(J)) $
FOR J=(1,1,6)$DPO(J)=(ENTIRE((1000.0 ).(DPO(J))))/1000.0 $
FOR J=(1,1,6)$DPO(J)=DPU(J) + E $
FOR I=(1,1,6)$DPA(I)=DPU(I)   $
WRITE ( $ $ PAR6) $
WRITE ($ $ DES,PAR1)          $
          ANALYSIS (NQ,M,G,H,N( ),PR,DR,DPA( )$R(,),W)   $
          CHECK (NQ,M,R( , ) $ TEST,RMX)   $

IF (RMX GEQ 0.999) AND (RMX LSS 1.01) $ BEGIN FOR J=(1,1,6)$
DPP(J) = DPU(J) $ GO TO L5 $ END $
COMMENT THIS SECTION COMPUTES  MOVE IN NEG GRAD DIR $
COMMENT INT DES OK BEGIN SYNTHESIS               $
L20.. T=1$ Y(T)=RMX$
L2..  ELL=0.0  $
FOR I=(1,1,6)$ELL=ELL + ((DPO(I)).(DPO(I))) $
LN = SQRT(ELL)$
FOR I=(1,1,3)$PHI(I)=(DPO(I + 3 ))/LN $
FOR I=(4,1,6)$PHI(I)=(DPO(I - 3 ))/LN  $
F=0 $ MU=L $ SC= 0 $
L3..
FOR  I=(1,1,6)$DPP(I)=DPO(I)-((MU).(PHI(I))) $
FOR I=(1,1,6)$DPP(I)=(ENTIRE((1000.0 ).(DPP(I))))/1000.0   $
FOR I=(1,1,6)$DPP(I) = DPP(I) + E $
FOR I=(1,1,6)$ DPA(I) = DPP(I) $
WRITE ( $ $ PAR2)      $
WRITE ( $ $ DES,PAR1)      $
FOR I=(1,1,6)$BEGIN IF DPP(I) LSS DPL(I)$
BEGIN SC=1$             MU=(DPO(I)-DPL(I))/(PHI(I))$ IF MU LSS E$
                       BEGIN FOR J=(1,1,6)$ DPP(J)=DPO(J)$
                       GO TO L5$
                      END$
                    GO TO L3$
                    END$
                ENDS
FOR I=(1,1,6)$DPA(I)=DPP(I)$
WRITE ( $ $ PAR3)      $
WRITE ( $ $ DES,PAR1)      $
N(4) = 0 $
          ANALYSIS (NQ,M,G,H,N( ),PR,DR,DPA( )$R(,),W)   $
          CHECK (NQ,M,R( , ) $ TEST,RMX)   $
IF (RMX GTR 0.999) AND (RMX LSS 1.01)$ GO TO L5 $
IF TEST EQL 1 $ BEGIN IF F EQL 1 $
     BEGIN IF (RMX GTR 0.999) AND (RMX LSS 1.01) $ GO TO L5 $
                    GO TO L4$ END $
IF F EQL 2 $ GO TO L5 $
```

```
IF SC EQL 1 $
BEGIN
 FOR J=(1,1,6)$DPO(J) = DPP(J) $
MU = L $
 FOR J=(1,1,6)$ DPP(J) = DPO(J) - ((MU).(PHI(J))) $
 FOR J=(1,1,6)$
     BEGIN
      IF DPP(J) LEQ (DPL(J) + E)$ DPP(J) = DPL(J) $
     END $
 ELL = 0.0 $
 FOR J=(1,1,6)$ ELL = ELL + (((DPP(J))-(DPO(J))).
                         ((DPP(J)) - (DPO(J)))) $
 LN = SQRT(ELL) $
 FOR J=(1,1,6)$ PHI(J) =((DPO(J)) -(DPP(J)))/LN $
 GO TO L3 $
END $
IF (RMX GEQ 0.999) AND (RMX LSS 1.01) $ GO TO L5 $
T=1 $
Y(T)=RMX$ FOR I=(1,1,6)$ DPO(I)=DPP(I)  $
GO TO L2 $ END $
L4.. X=RMX $
SC = 0 $
IF F EQL 2 $ BEGIN
IF Y(2) GTR 1.0 $ BEGIN
Y(3)=Y(2)$ Y(2)=X$ S=Z$ Z= MU $ GO TO L30 $ END $
S=MU $ Y(3) = X $ GO TO L30 $
                END$
IF F EQL 1 $ BEGIN
Y(2)=X$ F=2$       Z=MU$        S=(2.0).MU $
GO TO L30 $ END $
Y(3)=X$ F=1$ MU=MU/2.0$ GO TO L3$
L30..     QUAD (S,Z,Y( ) $ MU) $
GO TO L3$
COMMENT END OF NEG GRAD DIR     $
COMMENT THIS SECTION COMPUTES 42 NEW MOVES    $
L5..
IF RELMIN EQL 1 $
BEGIN
N(4) = 0  $
  W=0.0 $ FOR J=(1,1,3)$ W= W + (DPP(J).DPP(J+3)) $
  IF W GTR WP $
  BEGIN
    FOR J=(1,1,6)$ DPO(J) = (DPT1(J) + DPT2(J)) / 2.0 $

    FOR J=(1,1,6)$DPT2(J) = DPO(J) $
IF DIP EQL 1 $BEGIN WT(K) = 1000.0 $ CH = CH + 1 $ DIP = 0 $
                 IF CH EQL 126 $ GO TO L95 $ GO TO L90 $ END $
FOR J=(1,1,6)$DPO(J)=(ENTIRE((1000.0 ).(DPO(J))))/1000.0 $
FOR J=(1,1,6)$DPO(J)=DPO(J) + E $
FOR J=(1,1,6)$ DPA(J) = DPO(J) $
WRITE ( $ $ PAR6)      $
WRITE ( $ $ DES,PAR1)       $
           ANALYSIS (NJ,M,G,H,N( ),PR,DR,DPA( )$R(,),W)   $
           CHECK (NQ,M,R( , ) $ TEST,RMX)   $
DIP = 1 $
    GO TO L20$
    END $
  RELMIN=0 $
END $
     WP=0.0$FOR I=(1,1,3)$WP=WP + (DPP(I).DPP(I+3))$
```

```
WRITE ( $ $ PAR32) $
DW=WP $
WRITE ( $ $ PAR24,PAR25 ) $
ELL = 0.0        $
FOR I=(1,1,6)$ELL = ELL + ((DPP(I)).(DPP(I)))$
LN=SQRT(ELL)$
FOR I=(1,1,3)$PSI(I)=DPP(I+3) /LN$
FOR I=(4,1,6)$PSI(I)=DPP(I-3) /LN$
L80..
FOR I=(1,1,6)$
BEGIN
FOR J=(1,1,6)$ DP(J) = DPL(J) $
S1=0.0 $
FOR J=(1,1,6)$S1=S1 + (DPP(J).PSI(J)) $
S2=0.0 $
J=1$
L41.. IF J EQL I $ GO TO L40$
S2=S2 + (DP(J).PSI(J))$
L40.. J=J+1 $ IF J LEQ 6 $ GO TO L41$
DP(I) =  (S1      - S2)/(PSI(I)) $
ELL = 0.0 $
FOR J=(1,1,6)$ ELL = ELL + ((DP(J)-DPP(J)).(DP(J)-DPP(J)))$
LN=SQRT(ELL) $
FOR J=(1,1,6)$CHI(I,J)=(DP(J)-DPP(J))/LN $
END $
I=6$
FOR K=(1,1,5)$
BEGIN
     FOR J=(K+1,1,6)$
    BEGIN I=I+1$
    FOR U=(1,1,6)$
   Z1(I,U)=CHI(K,U)+CHI(J,U) $
    END $
END $
FOR I=(7,1,21)$
BEGIN ELL =0.0  $
        FOR J=(1,1,6)$
          ELL=ELL + (Z1(I,J).Z1(I,J)) $
LN=SQRT(ELL)$
      FOR J=(1,1,6)$
CHI(I,J)=Z1(I,J)/LN $
END $
I=21$
FOR K=(7,1,10),(12,1,14),(16,1,17),19 $
BEGIN
   IF K EQL 19 $ V=13 $
   IF K LEQ 17 $ V=11 $
   IF K LEQ 14 $      V=8 $
   IF K LEQ 10 $ V=4 $
    FOR J=(K-V,1,6)$
    BEGIN I=I+1$
    FOR U=(1,1,6)$
   Z1(I,U)=CHI(K,U)+CHI(J,U) $
    END $
END $
FOR I=(22,1,41)$
BEGIN ELL =0.0  $
        FOR J=(1,1,6)$
          ELL=ELL + (Z1(I,J).Z1(I,J)) $
LN=SQRT(ELL)$
```

```
      FOR J=(1,1,6)$
CHI(I,J)=Z1(I,J)/LN $
END $
I = 41 $
FOR K=(22,1,24),(26,1,27),29,(32,1,33),35,38 $
BEGIN
   IF K EQL 38 $ V= 32 $
   IF K EQL 35 $ V=29 $
   IF K LEQ 33 $ V=27 $
   IF K  EQL 29 $ V=23 $
   IF K LEQ 27 $ V=21 $
   IF K LEQ 24 $ V=18 $
    FOR J=(K-V,1,6)$
    BEGIN I=I+1$
    FOR U=(1,1,6)$
   Z1(I,U)=CHI(K,U)+CHI(J,U) $
    END $
END $
FOR I=(42,1,56)$
BEGIN ELL =0.0  $
        FOR J=(1,1,6)$
          ELL=ELL + (Z1(I,J).Z1(I,J)) $
LN=SQRT(ELL)$
      FOR J=(1,1,6)$
CHI(I,J)=Z1(I,J)/LN $
END $
I=56 $
FOR K=(42,1,43),45,48,52  $
BEGIN
   IF K EQL 52 $ V=46 $
   IF K EQL 48 $ V=42 $
   IF K EQL 45 $ V=39 $
   IF K LEQ 43 $ V=37 $
    FOR J=(K-V,1,6)$
    BEGIN I=I+1$
    FOR U=(1,1,6)$
   Z1(I,U)=CHI(K,U)+CHI(J,U) $
    END $
END $
FOR I=(57,1,62)$
BEGIN ELL =0.0  $
        FOR J=(1,1,6)$
          ELL=ELL + (Z1(I,J).Z1(I,J)) $
LN=SQRT(ELL)$
      FOR J=(1,1,6)$
CHI(I,J)=Z1(I,J)/LN $
END $
FOR U=(1,1,6)$Z1(63,U)=CHI(57,U) + CHI(6,U) $
I=63 $
BEGIN ELL =0.0  $
        FOR J=(1,1,6)$
          ELL=ELL + (Z1(I,J).Z1(I,J)) $
LN=SQRT(ELL)$
      FOR J=(1,1,6)$
CHI(I,J)=Z1(I,J)/LN $
END $
FOR J=(1,1,126)$O(J)=0 $
U=1$
FOR I=(1,1,63)$
BEGIN
```

```
MU=0 $
L52.. FOR J=(1,1,6)$DP1(J)=DPP(J)+(MU.CHI(I,J)) $
FOR J=(1,1,6)$DP1(J)=(ENTIRE((1000.0 ).(DP1(J))))/1000.0  $
FOR J=(1,1,6)$DP1(J)=DP1(J) + E $
FOR J=(1,1,6)$ BEGIN IF DP1(J) GTR DPU(J) $
     DP1(J) = DP1(J) - E $ END $
FOR J=(1,1,6)$BEGIN IF DP1(J) LSS DPL(J) $ BEGIN
MU=(DPL(J) - DPP(J))/(CHI(I,J))$IF MU LSS  E  $ BEGIN
FOR V=(1,1,6)$DPS(U,V)=DPP(V)$
O(U)=U $
U=U+1 $
                      GO TO L60 $ END $ GO TO L52 $ END $ END $
FOR J=(1,1,6)$BEGIN IF DP1(J) GTR DPU(J) $ BEGIN
MU=(DPU(J) - DPP(J))/(CHI(I,J))$IF MU LSS  E  $ BEGIN
FOR V=(1,1,6)$DPS(U,V)=DPP(V)$
O(U)=U $
U=U+1 $
                      GO TO L60 $ END $ GO TO L52 $ END $ END $
FOR J=(1,1,6)$ EITHER
IF DP1(J) LEQ (DPL(J) + E ) $
                              BEGIN  MU = MU/2.0 $
                             IF MU LSS (E/2.0)$ BEGIN FOR V=(1,1,6)$
                                                 DPS(U,V)=DPP(V)$
O(U)=U $
                                                   U=U+1 $
                                                 GO TO L60$
                                                 END $
                              GO TO L52$
                              END $
OR IF DP1(J) GEQ (DPU(J) - E ) $
                              BEGIN  MU = MU/2.0 $
                             IF MU LSS (E/2.0)$ BEGIN FOR V=(1,1,6)$
                                                 DPS(U,V)=DPP(V)$
O(U)=U $
                                                   U=U+1 $
                                                 GO TO L60$
                                                 END $
                              GO TO L52$
                              END $END$
FOR V=(1,1,6)$DPS(U,V)=DP1(V)$
U=U+1$
L60.. MU=0  $
L61.. FOR J = (1,1,6)$DP2(J) = DPP(J) - (MU.CHI(I,J)) $
FOR J=(1,1,6)$DP2(J)=(ENTIRE((1000.0 ).(DP2(J))))/1000.0 $
FOR J=(1,1,6)$DP2(J)=DP2(J) + E $
FOR J=(1,1,6)$ BEGIN  IF DP2(J) GTR DPU(J) $
     DP2(J) = DP2(J) - E $ END $
FOR  J=(1,1,6)$ BEGIN IF DP2(J) LSS DPL(J) $ BEGIN
MU = (DPP(J) - DPL(J))/(CHI(I,J))$IF MU LSS  E  $ BEGIN
FOR V=(1,1,6)$DPS(U,V)=DPP(V)$
O(U)=U $
U=U+1$ GO TO L97$
                               END $ GO TO L61 $ END $ END $
FOR  J=(1,1,6)$ BEGIN IF DP2(J) GTR DPU(J) $ BEGIN
MU = (DPP(J) - DPU(J))/(CHI(I,J))$IF MU LSS  E  $ BEGIN
FOR V=(1,1,6)$DPS(U,V)=DPP(V)$
O(U)=U $
U=U+1$ GO TO L97$
                               END $ GO TO L61 $ END $ END $
FOR J=(1,1,6)$ EITHER
```

```
IF DP2(J) LEQ (DPL(J) + E ) $
                                    BEGIN  MU = MU/2.0 $
                                   IF MU LSS (E/2.0)$ BEGIN FOR V=(1,1,6)$
                                                DPS(U,V)=DPP(V)$
O(U)=U $
                                                   U=U+1 $
                                                GO TO L97 $
                                                END $
                                    GO TO L61$
                                    END $
OR IF DP2(J) GEQ (DPU(J) - E ) $
                                    BEGIN  MU = MU/2.0 $
                                   IF MU LSS (E/2.0)$ BEGIN FOR V=(1,1,6)$
                                                DPS(U,V)=DPP(V)$
O(U) = U $
                                                   U=U+1 $
                                                GO TO L97 $
                                                END $
                                    GO TO L61$
                                    END $END$
FOR V=(1,1,6)$DPS(U,V)=DP2(V)$
U=U+1$
L97..
END $
COMMENT END OF NEW MOVE SECTION    $
COMMENT THIS CALCS WT MATRIX $
FOR I=(1,1,126)$ BEGIN WT(I) = 0.0 $
FOR J=(1,1,3)$WT(I)=WT(I) +(DPS(I,J).DPS(I,J+3))$END$
COMMENT END OF WT MATRIX CALC $
WRITE ( $ $ PAR26) $
WRITE ( $ $ PAR20,PAR21 ) $
COMMENT THIS CALC THE MIN WT  $
CH = 0 $
L90.. U=1 $
L91.. FOR I=(1,1,126)$BEGIN IF WT(U) GTR WT(I) $ BEGIN U=I$GO TO
L91 $ END $ END $
COMMENT END OF MIN WT CALC $
DW = WT(U) $
I=U $
WRITE ($ $ ITEM,PAR8) $
WRITE ( $ $ PAR28)$
WRITE ( $ $ PAR24,PAR25)$
IF U EQL O(U) $ BEGIN WT(U) = 1000.0 $ CH = CH + 1 $
IF CH EQL 126 $ GO TO L95 $ GO TO L90 $ END $
FOR J=(1,1,6)$ DPA(J) =DPS(U,J) $
N(4) = 1 $
          ANALYSIS (NQ,M,G,H,N( ),PR,DR,DPA( )$R(,),W)   $
          CHECK (NQ,M,R( , ) $ TEST,RMX)   $
IF TEST EQL 0, $ BEGIN WT(U)   = 1000.0 $
CH    = CH    + 1 $
IF CH    EQL 126 $ GO TO L95 $
GO TO L90 $ END $
IF WT(U) GTR WP $ BEGIN
                 FOR J=(1,1,6)$DPT1(J)=DPP(J)$
                 FOR J=(1,1,6)$DP0(J)=DPS(U,J) $
                 FOR J=(1,1,6)$DPT2(J) = DPS(U,J) $
IF (RMX GTR 0.999) AND (RMX LSS 1.01) $ BEGIN
K=0 $
L300.. FOR J=(1,1,6)$DP0(J) = (DPT1(J) + DPT2(J))/ 2.0 $
K=K+1 $ IF K EQL 2 $ GO TO L400$
```

```
    FOR J=(1,1,6)$ DPT2(J) = DPO(J) $
    IF K EQL 2 $ BEGIN
    L400..
                          WT(U) = 1000.0 $ CH = CH + 1 $
                IF CH EQL 126 $ GO TO L95 $ GO TO L90 $ END $
    L405..
    FOR J=(1,1,6)$DPO(J)=(ENTIRE((1000.0 ).(DPO(J))))/1000.0 $
    FOR J=(1,1,6)$DPO(J)=DPO(J) + E $
    FOR J=(1,1,6)$ DPA(J) = DPO(J) $
    WRITE ( $ $ PAR6)      $
    WRITE ( $ $ DES,PAR1)       $
    N(4) = 0 $
             ANALYSIS (NQ,M,G,H,N( ),PR,DR,DPA( )$R(,),W)   $
             CHECK (NQ,M,R( , ) $ TEST,RMX)   $
    IF (RMX GTR 0.999) AND (RMX LSS 1.01 ) $ GO TO L300 $
    IF TEST EQL 0 $ GO TO L300 $
    END $
    K=0 $
    RELMIN = 1 $ GO TO L20 $ END $
    IF RMX GEQ 0.999       $ BEGIN FOR J=(1,1,6)$DPP(J)=DPA(J)$
    D=0.5  $
    GO TO L5 $ END $
    FOR J=(1,1,6)$ DPO(J) = DPA(J) $
    GO TO L20 $
    L95..
    D=D/ 5.0  $
    DW = D $ WRITE ( $ $ PAR24, PAR25 ) $
                  IF D LEQ   E     $ GO TO L43 $
    GO TO L80 $
    OUTPUT PAR22 (FOR U=(1,1,126)$WT(U)) $
    OUTPUT PAR61 (FOR U=(1,1,63)$(FOR V=(1,1,6)$CHI(U,V))) $
    OUTPUT PAR20 (FOR U=(1,1,126)$(U,WT(U),(FOR V=(1,1,6)$DPS(U,V)))) $
    OUTPUT PAR24 (DW)   $
    FORMAT PAR25(1(F14.8),W0)$
    FORMAT HIB (*INITIAL DESIGN INHIBITED*,W4)$
    FORMAT PAR3 (*DES PARS AFTER CHECK ON DP CONSTRAINTS*,W0)$
    FORMAT PAR2 (*DES PARS AFTER MOVE IN NEG GRAD DIR*,W0)$
    OUTPUT DES ( FOR J=(1,1,6)$DPA(J))   $
    FORMAT PAR32 (* THE VALUE OF WP IS*,W0) $
    FORMAT PAR8 ( I3 , W4 )         $
    OUTPUT ITEM (I )           $
    FORMAT PAR23 (6(F14.8,B3),W0) $
    FORMAT PAR1 ( 1(F14.8,B3),W4)             $
    FORMAT PAR21 (I3,B3,1(F14.8,B5),6(F10.4,B3),W0) $
    FORMAT PAR26 ( * THE DESIGN PARAMETER MATRIX IS*,W0)$
    FORMAT PAR28 (*THE MINIMUM DIMENSIONLESS WEIGHT IS*,W0)$
    FORMAT PAR27 ( *THE DIMENSIONLESS WEIGHT MATRIX IS*,W0)$
    FORMAT PAR6 ( * THE DIMENSIONLESS DES PARS ARE * , W3 ) $
    FORMAT PAR29 ( I3,B2,6(F14.8,B3),W0) $
    OUTPUT MAIL (FOR J=(1,1,6)$DP1(J)) $
    OUTPUT JUNK ( MU ) $
    OUTPUT METI ( U ) $
    L43..
    FINISH $
FIN
```