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THE RESPONSE
OF MAGNETIC INSTRUMENTS
TO EARTHQUAKE WAVES

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Abstract: Part I.   The response of standard magnetographs to earthquake

waves is a seismograph effect.   These instruments react as ultra-low-sensi-

tivity seismic recorders.   With little or no additional effort some magnetic

observatories could be used as supplementary seismic stations at the times

of great earthquakes.   The effects of the Alaskan earthquake of March 28, 1964,

are studied in detail in this paper.   A number of similar cases are known.

Part II.   Even if the response of standard magnetographs is definitely

of a mechanical nature it is likely that the earthquake waves do generate

real magnetic waves.   Some experimental evidence (from Bergen Park, Colorado)

of the existence of such waves is presented.   Mechanisms for generation of

the observed waves are discussed.   It is shown that piezomagnetic oscillations

in magnetic rock or induced currents in a region with enhanced conductivity

may offer an explanation.   These effects are associated with anomalous

conditions in the crust.   It seems evident that the properties of the

"average crust" cannot explain the observed magnetic variations.

INTRODUCTION

Since the early days of continuous photographic recording of the geomagnetic field the response of magnetographs using suspended magnets to earthquakes has attracted the interest of many observers. Opinions have differed regarding the "reality" of these effects. Some have ascribed them to electric or magnetic oscillations generated by the earthquake, others to mechanical vibrations of the magnetic instruments.

Davison (1905) reported on thirteen magnetic observatories recording disturbances from the Riviera earthquake of February 23, 1887.

While Bauer was in charge of the geomagnetic work of the U. S. Coast and Geodetic Survey he arranged for the installation of primitive seismographs at four magnetic observatories. He thus provided means for direct comparison of the two types of earthquake record. The results were discussed by Burbank (1905) and Bauer (1906). These investigations showed that some earthquakes were recorded on the magnetographs but not on the seismographs, and vice versa. The onset times of the magnetic disturbances generally agreed roughly with the times of arrival of the seismic waves, when both kinds of waves were observed.

Mascart (1887) believed that the variations recorded by the magnetograph were real, i.e. caused by variations in the magnetic field rather than by mechanical vibrations. He tried to explain them by means of electric currents generated by the earthquake.

Moureaux and Mascart (1889) attempted to decide between the mechanical and magnetic theory by using a second variometer where the suspended magnet

had been replaced by a copper bar. At the time of subsequent earthquakes the magnet oscillated but the copper bar did not. This result was thought to preclude a mechanical explanation of the phenomenon.

The response considered as a mechanical effect was discussed tentatively by Davison (1885). He confined himself to compass-needles. The case of suspended systems (unifilar and bifilar) was attacked mathematically by Liznar (1895) and, independently, by Reid (1914). A summary of their argumentation will be given later.

Chapman (1930) reported two incidents when ships' compasses were acting queerly in connection with earthquakes. The observed variations appear to have been to one side rather than of oscillatory type. Since the ships were moving, the explanation might be that they passed over spatial anomalies which have nothing to do with the earthquakes. Without checking this the observations are inconclusive.

The present writer proposes an explanation of the instrument response to seismic waves which differs somewhat from that of the earlier workers. It is shown that a magnetic torque is introduced by the pendulum swing of the suspended magnets. The magnetographs may in fact in some cases be used as quantitative seismographs of ultra-low sensitivity.

Even if the response of standard magnetographs is definitely of a mechanical nature, it is likely that the earthquake waves do generate real magnetic waves. The expected amplitudes of these are calculated for three simple models, the piezomagnetic model, the field gradient model, and

the induction model. It is found that these effects are usually too small to be observed against the background of geomagnetic noise. There may however be exceptional cases when "seismomagnetic waves" can actually be observed. Some experimental evidence (from Bergen Park, Colorado) of the existence of such waves is presented.

I. Seismograph effect

1. Observed disturbances

Fig. 1 shows typical variations following a great earthquake, in this case the Alaskan earthquake of March 28, 1964. They were obtained at the Swedish magnetic stations, Kiruna and Enköping. The primary field sensors of these instruments (modified Grenet pulsation recorders) are suspended magnets. This is a common feature of most of the magnetic standard instruments around the world. The Alaskan earthquake has been recorded in a similar way at a large number of the geomagnetic observatories.

The sensitivity of the Kiruna and Enköping magnetographs is unusually high for instruments with suspended magnet. This is due to special construction features; see Voelker (1963). The vertical lines in Fig. 1 indicate minutes and hours. The sensitivity of the records depends on the period of the variations. For a given period it is however nearly the same at both stations for any one component. The largest amplitude seen is that for Kiruna Z at about 04h04m (U.T.). It is about 3γ. It should be noted that the disturbances start with an oscillation of short period (2-4 sec), followed by an oscillation of longer period (12-30 sec).

The basic data for the Alaskan earthquake are as follows:
Origin time: March 28, 1964, 03h36m10s U.T. Epicenter: 61.1°N, 147.8°W.
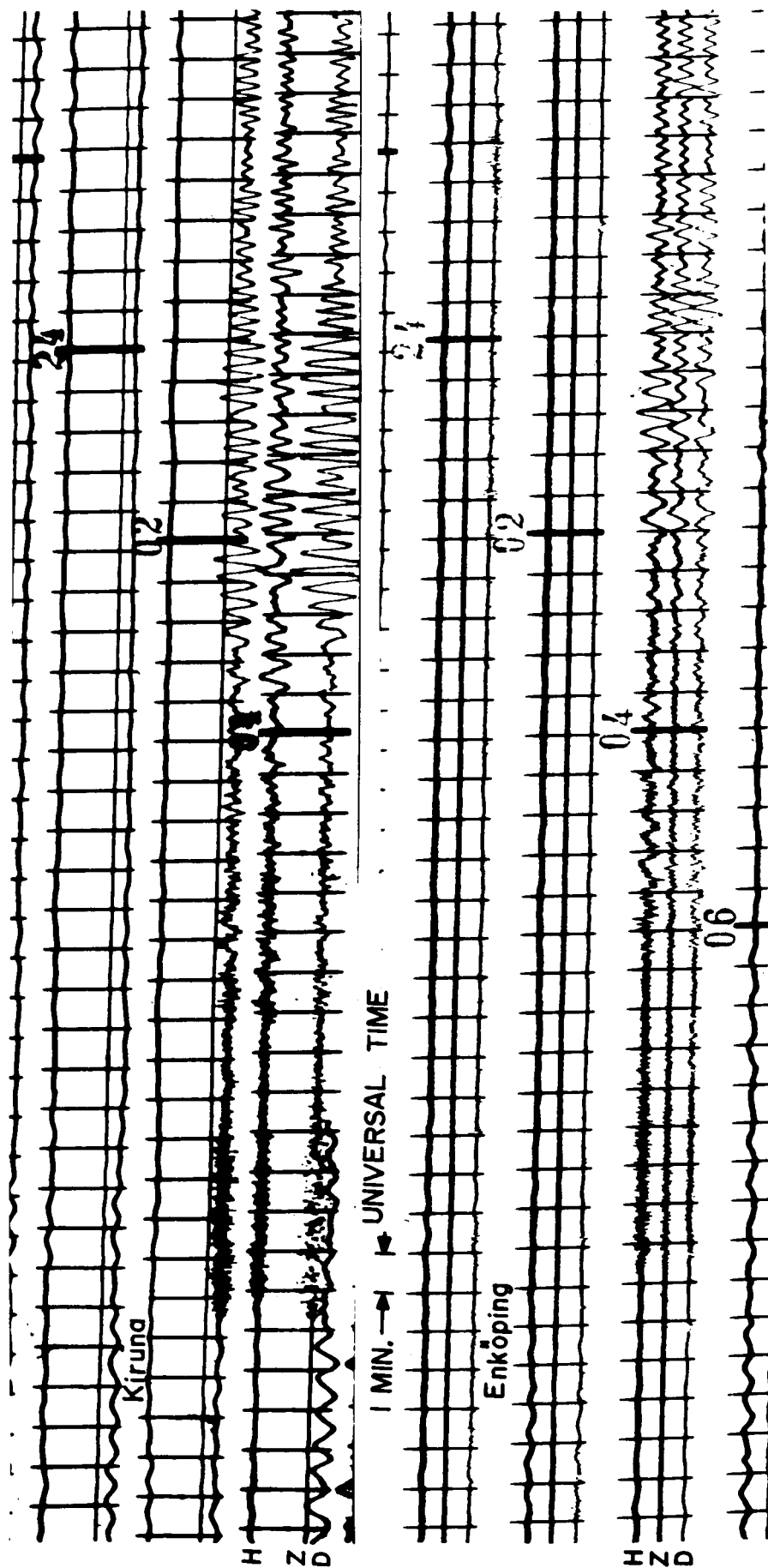
Figure 1. Earthquake vibrations recorded at Enköping and Kiruna in Sweden (modified Grenet pulsation recorders), March 28, 1964.

Depth: about 20 km. Magnitude: 8.5 .

The starting times of the magnetic effects as found on the records are: for Kiruna 03h45m12s,   for Enköping 03h46m18s. These times have been compared with the times of arrival of the seismic p waves at the adjacent seismic stations at Kiruna and Uppsala; see Båth (1965). With due corrections for differences in position we find that the times of the magnetic and the seismic disturbances agree within a few seconds. This applies also to a number of details in the two kinds of records, e.g. the occurrence of long periodic waves more than 15 minutes after the start of the first oscillations and the phase relations between the horizontal and vertical components of the Rayleigh waves. Similar results have been confirmed by several other observatories.

Fig. 2 shows an example of the same disturbance as recorded on the standard rapid-run magnetograph at Tucson, Arizona (U.S. Coast and Geodetic Survey).

2. Nature of response

In order to get an independent check of the magnetic effects described above, magnetic records derived from other types of equipment were inspected. Induction magnetometers with coil sensors are operated in Kiruna (U. S. Bureau of Standards type) and Uppsala (Benioff type). Neither of these showed any magnetic effect. On the contrary, they indicated unusually quiet conditions. Only one explanation of this seems tenable: the magnetic standard equipment with suspended magnets responds as a seismograph to this kind of disturbance.
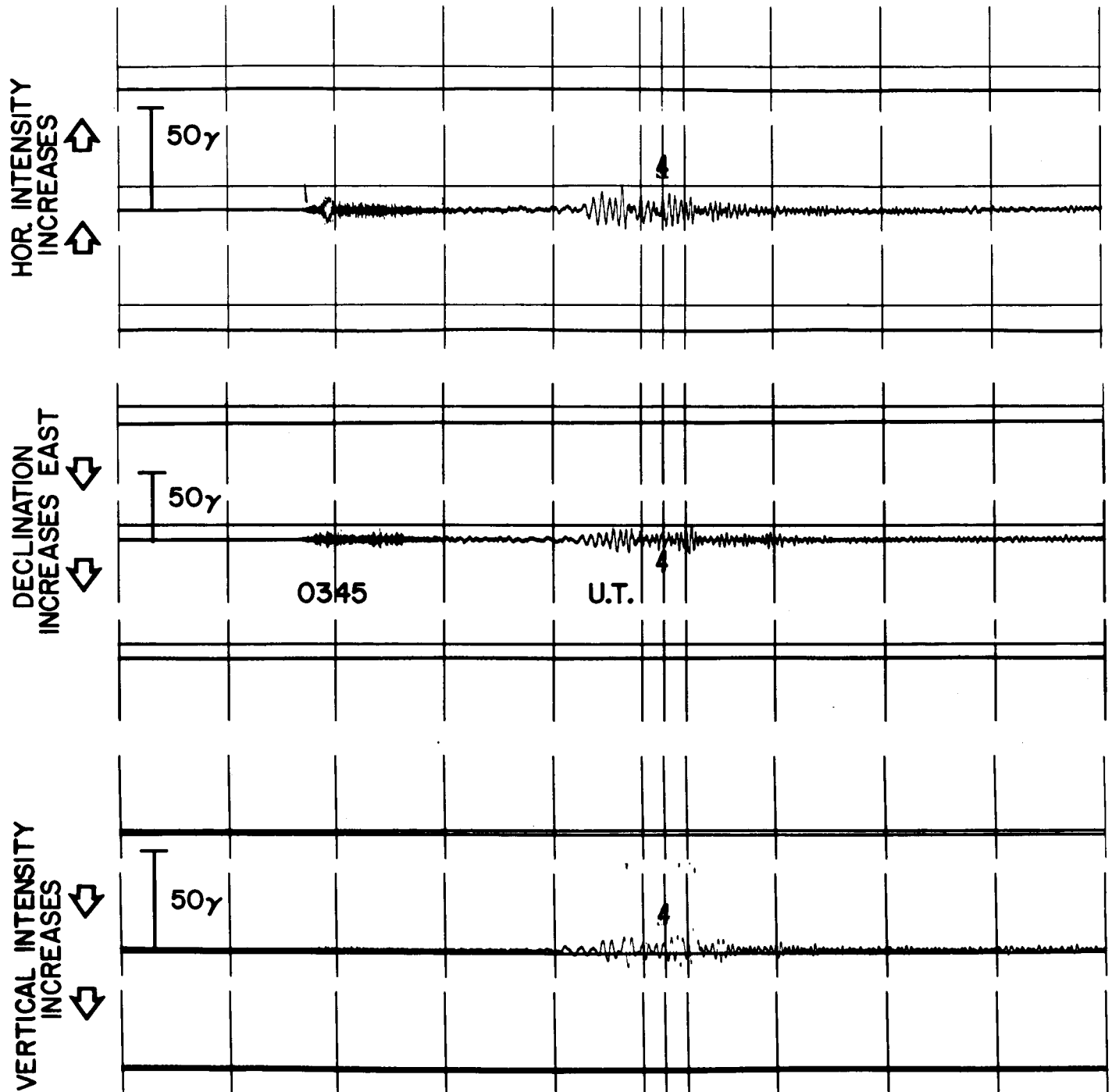
TUCSON MAGNETIC OBSERVATORY

Figure 2. Earthquake vibrations recorded at Tucson, Arizona, March 28, 1964.

This result is not so obvious as might be supposed. Except at locations which are close to the epicenter, the earth's movements are virtually parallel translations. Recorded magnetic variations imply a rotary motion of the suspended magnet. It is not immediately clear how the earthquake tremor could produce such a motion.

3.   The response of H and D variometers

Fig. 3 illustrates schematically the performance of the suspended magnet of a D (declination) variometer being subject to a horizontal vibration. For simplicity, we confine ourselves to a sinusoidal motion of amplitude $\underline{a}$ perpendicular to the local magnetic meridian. (Extension to a more general motion and to the H variometer can easily be made).

The earth tremor makes itself felt as vibrations of the torsion head T where the fiber is attached. It causes the magnet to deviate from its unperturbed position 0 and start pendular oscillations. The magnet has the mass $\underline{m}$; L is the length of the fiber, or rather the distance between T and the center of mass (C) of the magnet system (including mirror etc.).

The Y-axis represents the undisturbed position of the system at the time $\underline{t}=0$ when the tremor begins. A horizontal force $\underline{f}$ then starts to act on the upper end of the fiber. At the time $\underline{t}$ the magnet (C) has reached a position $(\underline{x};\underline{y})$. T has then the position $(\xi;L)$. The difference in horizontal displacement between T and 0 is denoted by $x' = \xi - x$.

For the following short time interval $\underline{dt}$ we have the energy equation

$$f(dx+dx') = m\frac{d^2 x}{dt^2} dx + m(\frac{d^2 y}{dt^2}+g) dy \tag{1}$$
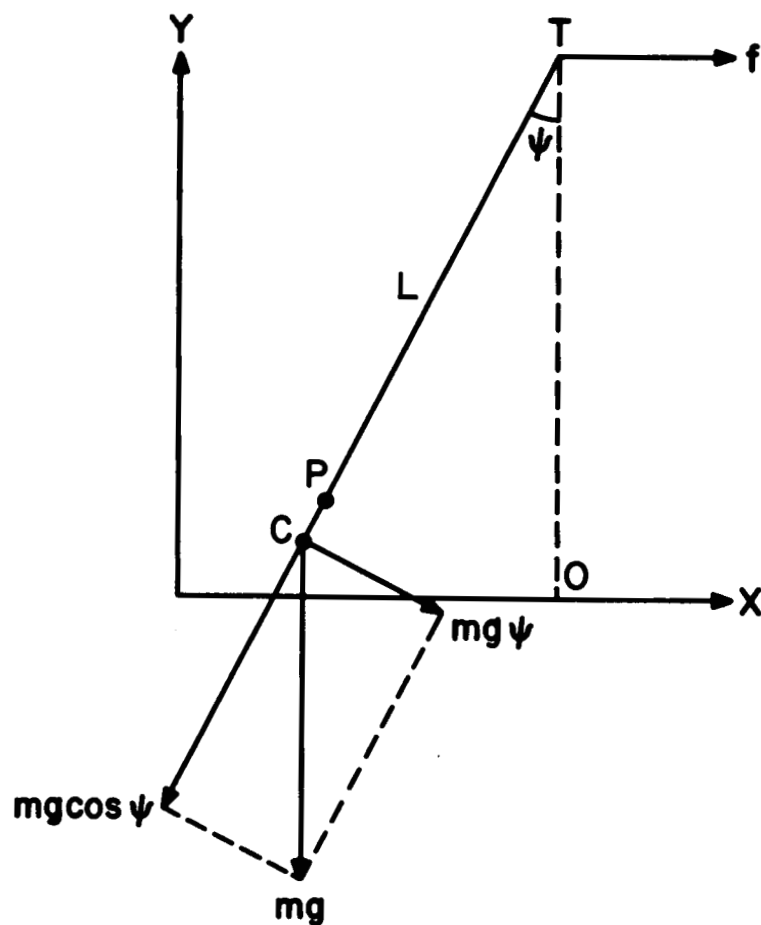
Figure 3.   The magnetic system of the D variometer as seen from magnetic
            South.

Assuming a slow motion we get $f = m\dfrac{d^2 x}{dt^2}$

and
$$f dx' = mg dy$$

The last two equations give for small oscillations

$$f = mg\frac{dy}{dx'} = mg\frac{x'}{L}$$

and
$$\frac{d^2 x}{dt^2} = g\frac{x'}{L} \tag{2}$$

If we assume a horizontal displacement in the magnetic East-West direction of the form

$$\xi = \underline{a}\ \sin\omega t$$

we get
$$\frac{d^2 \xi}{dt^2} = \frac{d^2 x}{dt^2} + \frac{d^2 x'}{dt^2} = -\omega^2 a \sin\omega t \tag{3}$$

Inserting this in (2) we get

$$\frac{d^2 x'}{dt^2} + g\frac{x'}{L} + \omega^2 a \sin\omega t = 0 \tag{4}$$

In deriving this equation for the pendulum swing of the magnet we have neglected the damping and the rotation of the magnet. This is justified for the small, slow variations we are studying here. For a more general approach, see Appendix I.

Equation (4) can be written

$$\frac{d^2 \psi}{d^2} + \omega_p^2 \psi + \frac{\omega^2 a}{L}\ \sin\omega t = 0 \tag{5}$$

where ω_p is the natural <u>pendulum</u> frequency of the magnet system.

The solution contains a natural vibration term. This dies out due to the damping which although small, is of course always present (air friction, eddy currents, etc.). We also get a   steady   forced vibration

$$\psi = - \frac{a\omega^2}{L(\omega_p^2 - \omega^2)} \sin(\omega t - \delta)$$

where δ is the phase angle.   For $\omega_p^2 \gg \omega^2$ we get the amplitude

$$\psi_o = \frac{a\omega^2}{L\omega_p^2} = \frac{aT_p^2}{LT^2}$$

Since

$$T_p = 2\pi\sqrt{\frac{L}{g}}$$

this can be written

$$\psi_o = \frac{4\pi^2 a}{gT^2}$$

It is obvious that, due to the pendulum swing, the geomagnetic Z field will have a component $Z\psi$ which is perpendicular to the plane through the magnet and its axis of rotation, defined by the fiber. $Z\psi$ introduces a torque which for small values of ω, brings the magnet to turn until it reaches an equilibrium angle θ defined by $H\theta = Z\psi$ where H is the horizontal field. (We are here neglecting the torsion of the fiber).

The magnetographs will record an apparent magnetic vibration in the D component (regardless of sign and phase)

$$\Delta D = \frac{4\pi^2 Za}{gT^2} \sin\omega t \qquad\qquad (6)$$

For our purpose we can with sufficient accuracy use g = 980 cm sec^{-2}
for the whole earth. Then (6) may be written

$$\Delta D_o = \frac{\Delta D}{\sin\omega t} = 4.0.10^{-2} \frac{Za}{T^2} \qquad\qquad (7)$$

It is easy to show (see Appendix 2) that we will get exactly the
same equation for ΔH if $\underline{a}$ signifies the amplitude of an earth tremor in
the direction of the local magnetic meridian. The results also apply to
X and Y variometers.

It is thus seen that the recorded magnetic trace will show a vibration
with the same period as the earthquake waves. Its amplitude is determined
by the period and the amplitude of the earth movements, and the local magnetic
field. For actual evaluation the magnetic scale value must also be known.

Equation (6) is valid as long as T is long compared with the natural
periods of the pendulum (T_p) and of the magnet (T_M). This condition is
generally fullfilled for the long periodic seismic surface waves. For the
short periodic waves the expressions get somewhat more complicated. See
Appendix 1.

4. Reid's approach

Reid (1914) has also attacked the problem of the response of suspended
magnets to earth vibrations. He has somewhat extended an earlier investi-
gation by Liznar (1895). An outline of Reid's argument will be given here.

For the system at rest the point of support (P) where the fiber is

attached to the magnet system is generally not in the same vertical line
as the center of mass (C) of the magnet system. When the system performs
pendular oscillations the tension of the fiber may be resolved in a vertical
and a horizontal part. Reid assumes that the latter part will tend to turn
the magnet about C. Since the tension equals the weight of the magnet system
the torque would be (leaving out second order effects)

$$mg\Psi c \tag{8}$$

where c is the (horizontal) distance between P and C.

Reid further assumes that the distance c can be determined by equating
the opposing torques from the gravitation and the magnetic Z field. When
the magnet is horizontal this gives

$$c = \frac{ZM}{mg} \tag{9}$$

where M is the magnetic moment.

On the basis of the previous assumptions Reid derives the formulas for
free oscillations (damped and undamped) and also for forced oscillations due
to earth tremor. He restricts himself to the cases when the periods of the
earthquake waves closely agree with the period of the pendular or the magnetic
oscillation.

It is doubtful, however, if the reasoning of Liznar and Reid is tenable
in the case of a suspended magnet. The only force, except Z, having a com-
ponent perpendicular to the plane through the magnetic axis and the axis of

rotation (assumed to coincide with the fiber) and thus capable of producing a rotation, is the gravity force mg. But since mg acts at the center of gravity no rotation occurs. Also, the expression for $\underline{c}$ overlooks the stiffness of the (quartz) fiber which is probably important.

Solving Reid's equations for the long periodic case (cf Section 5) we once more get the result (6), in spite of the different mechanisms assumed. If both mechanisms were effective simultaneously they would in fact give twice the effect (6). As will be seen in later sections most observations seem to agree better with the deviation ΔD than with $2 \Delta D$. Since the magnetic torque mechanism introduced in this paper appears to be an inevitable consequence of the pendulum swings it is probably the only effective mechanism.

It is possible to conceive such deviations from the types of oscillations assumed here that Reid's approach would be at least partly applicable. One such deviation is that the axis of rotation does not coincide with that of the fiber. However, this alternative seems unlikely for the small oscillations considered here.

If $c \neq 0$ for a suspended magnet a mechanical torque is introduced by the moment of inertia of the fiber considered as rigid. In the practical case however, this gives an amplitude several orders of magnitude smaller than that of ΔD in (6).

As pointed out by Reid the H and D variometers would be sensitive to vertical earth-motions of period T only if Tp is commensurate with T. Thus we need not consider this effect here.

5. The compass needle

The expression (9) is exactly valid in the case of a compass needle pivoting about a vertical pin. It is therefore of a certain interest to study the apparent magnetic variation due to (9).

The torque due to the earth tremor causes the magnet to seek a new position of equilibrium defined by

$$m \frac{d^2 \xi}{dt^2} \, c = MH \, \theta$$

Here we have assumed that $c(d^2\theta/dt^2) << d^2\xi/dt^2$ and $T_M << T$. $H\theta$ is equivalent to an EW disturbance ΔD. Inserting (3) and (9) in this expression we get

$$\Delta D = \frac{4\pi^2 \, Za}{gT^2} \, \sin \omega t$$

This is the expression (6). Thus a suspended magnet and a compass needle respond in the same way. The result also indicates why Reid's approach gives the same response.

6. Numerical values

For $Z=0.47$ gauss (Stockholm area) equation (7) yields

$$\Delta D_O = 1.88 \times 10^3 \frac{a}{T^2} \, \gamma \qquad (10)$$

According to Båth (1965) the E component of the M waves recorded at Uppsala after the Alaskan earthquake had the amplitude $\underline{a} = 0.226$ cm and the period $T=22$ sec. Inserting this in (10) gives $\Delta D = 0.88\gamma$.

The actual maximum values of ΔD_O observed at the two adjacent magnetic

observatories Lovö and Enköping are respectively, 0.8γ and 0.9γ. The period

is 22 sec. The agreement is good considering that the recorded magnetic

variations have amplitudes of only about 1 mm or less.

Turning to the H component at Lovö we find that the amplitude of the

simultaneous H variations is 1.0γ. The corresponding period is about 22 sec.

According to (10) this gives a displacement amplitude in the NS direction

$$a_N = \frac{\Delta H_0 T^2}{1.88 \cdot 10^3} = 0.26 \, cm$$

Considering the geographical position of the station this would roughly

agree with the amplitude of the horizontal component of the Rayleigh waves.

The corresponding vertical component would be about 0.44 cm, assuming the

ratio 1.7 between the vertical and the horizontal displacements.

7.   Z variometer

The conventional Z variometer consists of a horizontal magnet rotating

about a horizontal axis defined by knife edges or fibers. In this case an

earthquake does not initaite a pendulum swing. The response is wholly dependent

on the separation between the pivot axis and the center of mass.

This separation can be resolved into a part $\underline{c}$ parallel to the magnetic

axis, and a part $\underline{h}$ perpendicular to the axis (see Fig. 4). The former part

is rigorously determined by (9). The part $\underline{h}$ decides the sensitivity of the

variometer. For $\underline{h} = 0$ the magnet becomes unstable if directed in magnetic

EW.

Fig. 4 pictures an EW-oriented Z magnet (of mass $\underline{m}$ and moment of inertia

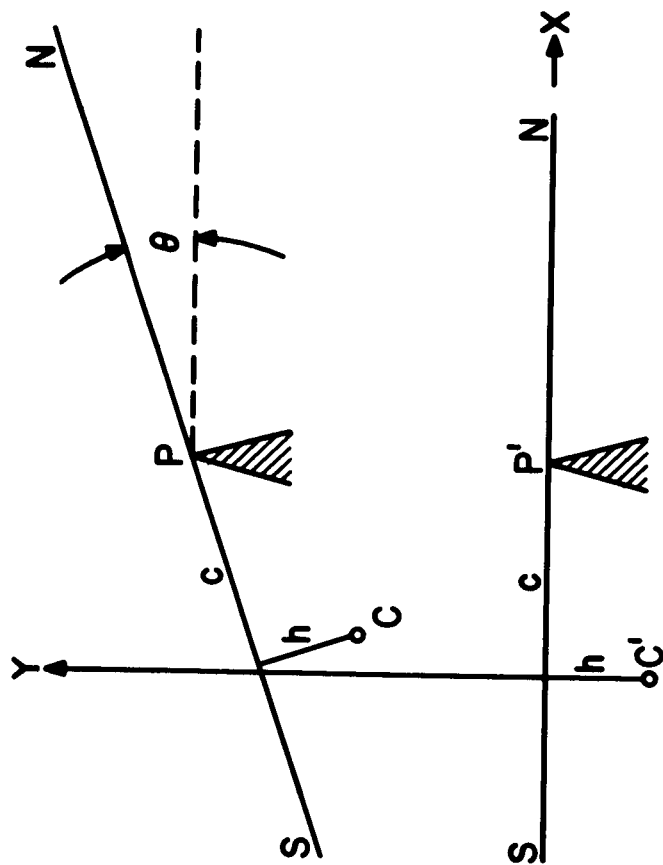I) in two positions. Due to an earth-tremor the pivot-point $\mathbf{P}$ is displaced

Figure 4. A Z magnet in EW direction, seen from the south, in two different tremor positions.

vertically a distance ξ. The center of mass C moves the distance $\underline{y}$ and

the magnet turns a small angle θ. This gives the connection

$$\xi = y + c\theta = a \sin\omega t$$

If $\underline{f}$ is the vertical force acting at P we get (neglecting damping)

$$f(dy + cd\theta) = m\frac{d^2 y}{dt^2} dy + mgh\theta d\theta + I\frac{d^2\theta}{dt^2} d\theta$$

After separation of translational and rotational terms these expressions

give

$$I\frac{d^2\theta}{dt^2} + mgh\theta = -mca\omega^2 \sin\omega t$$

The stationary solution is for small values (regardless of sign and phase)

$$\theta = \frac{mca}{I} \frac{T_M^2}{T^2} \sin\omega t \qquad (11)$$

where

$$T_M^2 = 4\pi^2 \frac{I}{mgh}$$

It may be shown that a horizontal earth tremor $a\sin\omega t$ along the axis of the

magnet would give the same expressions for θ but with $\underline{c}$ replaced by $\underline{h}$.

The discussion can be extended to the case of a Z magnet of arbitrary

orientation. This is done by adding the term $MH\theta\cos\alpha$ to the left side of

the differential equation (α = magnetic azimuth). The result (11) remains

formally the same, with

$$T_M^2 = 4\pi^2 \frac{I}{mgh + MH\cos\alpha} \qquad (12)$$

For a qualitative discussion of the performance of the Z balance under

resonance conditions, see Reid (1914).

8. Numerical check of Z

The apparent magnetic deviation corresponding to the vertical motion is

$$\Delta Z_v = S_z \theta \qquad (13)$$

where S_z is the magnetic scale-value defined by

$$S_z = \frac{mgh + MH\cos\alpha}{M} \qquad \frac{gauss}{radian} \qquad (14)$$

The expressions (9), (11), (12), (13) and (14) now yield

$$\Delta Z_v = \frac{4\pi^2 aZ}{gT^2} \sin\omega t \qquad (15)$$

This is exactly the same expression as that derived for the D and H magnetometers and the magnetic compass. For a horizontal earth motion in the direction of the magnet we get

$$\Delta Z_h = \frac{h}{c} \frac{4\pi^2 aZ}{gt^2} \sin\omega t \qquad (15')$$

When - as is often the case - $h \ll c$, ΔZ_h may be neglected in comparison with ΔZ_v. Otherwise, the Z trace displays a combined vertical and horizontal motion.

At the Lovo Observatory (Z = 0.47 gauss) the ratio h:c is roughly 0.25. The following values were observed at one phase of the earthquake:

$$\Delta Z = 1.4\gamma, \quad T = 22 \text{ sec.}$$

Inserting in (15) we get

$$a = 0.36 \text{ cm}$$

This is to be compared with the value 0.44 cm obtained in Section 5 as a rough estimate from the observed horizontal amplitude of the Rayleigh waves.

9. Magnetographs as supplementary seismographs

When looking for world wide seismic data associated with the Alaskan earthquake of March 28, 1964, the present writer could not avoid observing the great number of cases when the seismographs (short periodic and long periodic) failed to record properly the large surface waves studied here. Either the seismograms displayed a confusion of traces impossible to disentangle, or the traces disappeared altogether.

In most cases use has therefore been made of the nomogram for the maximum ground amplitude as a function of earthquake magnitude and distance from source, compiled by Gutenberg and Richter (1956). It pertains to the combined horizontal components for surface waves of 20-second period.

The general impression is that for earthquakes like the Alaskan event a net of ultra-low-sensitivity seismographs would be needed. The existing strong-motion instruments (like those on the American west coast) do not provide a satisfactory coverage.

In this situation the rapid-run standard recorders of the geomagnetic observatories may in some cases be useful as a supplement to the seismic network. The magnetographs should be of value within a range of about 30° - 80° from the source. As may be seen from the response equations derived

for long-periodic seismic waves they can be made quantitative through the determination of a few instrumental constants (most of which have to be known anyhow). With some additional work (including determination of the damping constant) a "magnification curve" could be derived for periods down to about 5 sec. The time resolution of the magnetic records is, however, generally not sufficient for studies of the short periodic waves.

Evidently, the magnetic records could be used for this purpose only at times when the magnetic conditions are reasonably quiet. The Alaskan earthquake for example occurred in a very quiet period. Especially in years of sunspot minima such events would not be rare.

The method should have unrestricted application to the horizontal wave components. Z-records could be used for evaluation of vertical movements when $c \gg h$, which is often the case (an important exception being when the north seeking end of the magnet points in a southerly direction).

The phase angles (neglected in the previous discussion) are easily determined from the instrument constants. In the case $T \gg T_p$ and $T \gg T_m$ we have $\delta = 0$ for all components. The relation between the direction of the earth movement and the recorded deviation has to be established for each individual instrument.

Since the amplitudes of the seismic ΔD, ΔH, and ΔZ are proportional to Z, the observations would be confined to those portions of the earth where Z is sufficiently large. The position of the border-lines is determined by the scale-value of the instruments. In Huancayo where Z is only about 1000γ, no seismic effect from the Alaskan earthquake could be observed.

The combined horizontal ground amplitude was about 2.3 mm and would have been observed at stations on medium high north or south latitudes. Questions such as the positions of the observatories, ground conditions etc., must of course also be contemplated. A list of observatories operating magnetic recorders suitable as supplementary seismographs could perhaps be compiled through appropriate bodies within the IUGG.

The instrument theory presented above specifically pertains to conventional variometers. It is probably also applicable to pulsation recorders of the Grenet-Voelker type (Fig. 1). In these the primary field sensors are magnets, the rotation of which induces currents in a surrounding coil. Although the pure pendulum swing of the H or D magnets would produce currents, the rotation is probably the main contributor. This is confirmed, at least for periods above 20 sec, by comparision with adjacent LaCour instruments. A chain of pulsation recorders has recently been established in Europe.

10. Suggested experiments

For an experimental control of the response of magnetic instruments to vibrations, a movable support would be needed. It should perform smooth sinusoidal movements (horizontal or vertical)with an amplitude of, say, 1cm and periods approximately within the range of 10-50 sec. The performance of the magnet could be studied either by means of a split photocell recorder, or simply by observations with an autocollimator mounted on the instrument.

For horizontal variometers it would probably be sufficient to make calibrations for each type of suspended system (Ruska, LaCour, etc.), the variations being small.

By making the experiment inside a Helmholtz coil the effect of Z on
the response of the horizontal variometers could be checked.

Deviations from the theoretical amplitude formulae might be expected
in cases when the lateral rigidity of the fiber is important in comparison
with the mass of the magnet. The likelihood of this should generally be
greater for H than for D variometers.

Comparisons between magnetograms and seismograms (Lovö - Uppsala)
indicate that the differences between theoretical and experimental response
might be small, in many cases probably insignificant.

Since the observed vibrations are due to a magnetic torque, the result
of the experiment of Moureaux and Mascart (mentioned in the "Introduction")
is to be expected. It supports the theory presented in this paper.

11. Worldwide observations

Table 1 lists the observed maximum horizontal displacements for the
Alaskan event as derived from the magnetic records of a number of observa-
tories. The observatories are arranged according to distance from the
epicenter (61.1°N, 147.8°W).

The table gives calculated amplitudes and periods for most of the
observatories that supplied microfilm copies of their rapid-run magneto-
grams to World Data Center A before March 1965. In some cases the quality
of the reproduction did not allow quantitative measurements. Magnetograms
from stations close to the epicenter, like College, Sitka, and Barrow could
not be used for determinations of M amplitudes because they were too disturbed.
College recorded minor tremors for several hours after the main earthquake.

Table 1

| Station	Lat.	Long.	Time U.T.	Dist. degrees	Seism. ΔHmm	T_H sec.	Seism. ΔDmm	T_D sec.	Nomogram
Tucson	32°15'	-110°50'	0358	39	14.8	19	11.3	18	12.0
Honolulu	21°19'	-158°00'	0357	42	6.7	10	7.0	10	9.0
Leirvogur	64°11'	-21°42'	0406	48	8.1	20	5.8	20	7.7
Tromsö	69°40'	18°57'	0406	49	7.0	21	0.9	22	5.3
Kiruna	67°50'	20°25'	0409	51	4.2	22	1.4	21	5.3
Nurmijärvi	60°31'	24°39'	0408	59	3.0	(22)	2.2		5.0
Lovö	59°21'	17°50'	0410	61	2.2	22	6.1	22	4.9
Rude Skov	55°51'	12°27'	0413	62	2.4	27	1.7	27	4.6
Witteveen	52°49'	6°40'	0416	64	4.4	22		20	4.2
Dourbes	50°06'	4°36'	0416	66	4.0	23	1.7	23	4.0
Guam	13°35'	144°52'	0417	68	2.4	17			4.0
Toledo	39°53'	-4°03'	0421	76	2.8	20			3.3
Port Moresby	-9°24'	147°09'	0426	86		.	2.5	22	2.7
Moca	3°21'	8°40'	0445	115	0.9	20			1.8

12. Response of other kinds of instruments

As a result of a world-wide request a number of observatories kindly sent copies of their recordings showing various degrees of response to the seismic waves. In some cases the records were accompanied by valuable comments. To present all the records here would be useless repetition. It will be sufficient to reproduce some results obtained at the Niemegk observatory (lat.52°04'N, long. 12°40'E) and kindly supplied by F̲anselau, G̲rafe, S̲chmidt (Private Communication, 1964).

Fig. 5a shows the effect on the observatory's insensitive system. This is more or less typical of the appearance on normal-speed magnetographs. The poor time resolution does not permit quantitative measurements.

Fib. 5b is the record of an induction magnetometer with three large pickup coils connected to galvanometers recording photographically. The coils are "air-coils" without cores. It is remarkable that this installation has recorded the earthquake disturbance, with onset time 0347 U.T. It is in all probability a "seismograph effect". It is possible that the galvanometers are responsible for this. If not, the vibrations must have caused some kind of distortion of the coils, changing the magnetic flux. As mentioned before the coil-instruments in Uppsala and Kiruna did not respond to the earthquake.
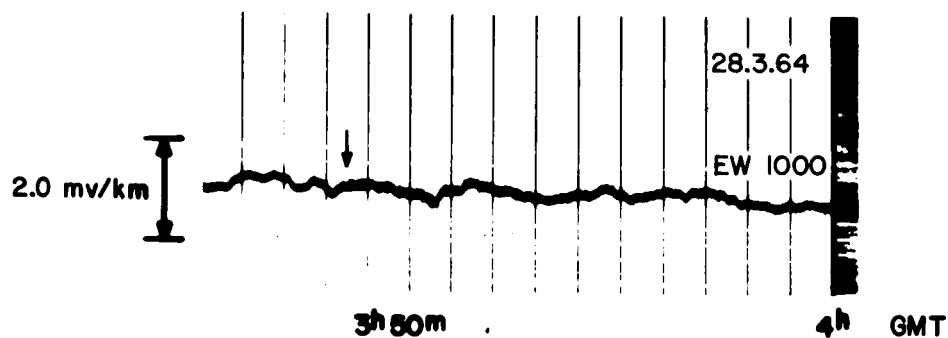
Fig. 5c shows the EW component of the Niemegk rapid-run earth-current recorder. Rapid vibrations, hardly discernible in reproduction, start at the time indicated by the arrow. Telluric effects of earthquakes will be discussed in the later part of this paper.
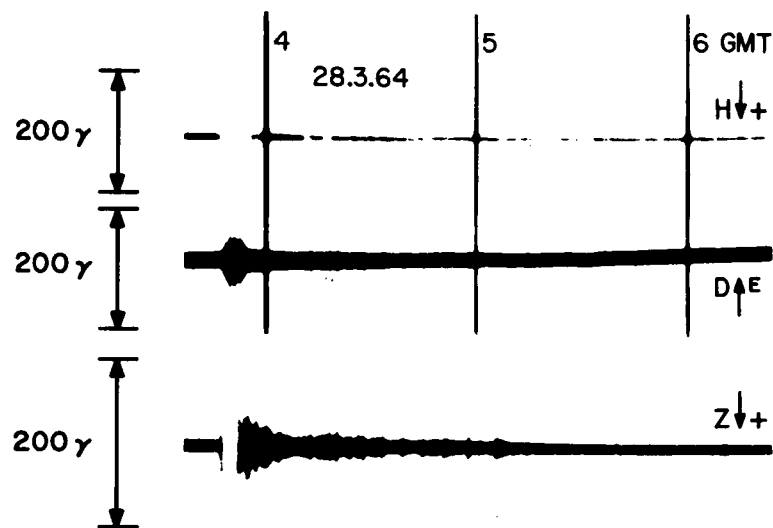
(a)
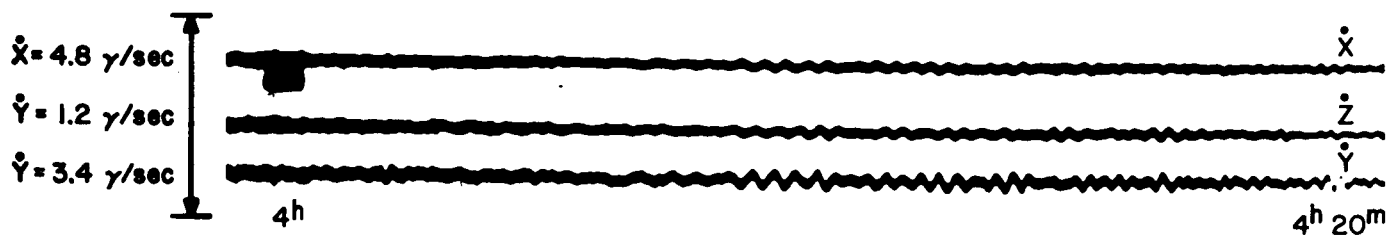
(b)

(c)

Figure 5.   Magnetic and electric records from Niemegk, Germany, March 28, 1964.

E. Selzer (Private Communication, 1964) reports of observations
at the French Observatory Chambon-la-Foret. This observatory operates
a 3-component coil magnetometer with mumetal cores ("bar-fluxmeters").
Oscillations appear practically only in the Z component, whereas normally
H and D are much more disturbed than Z. The Z variations are very similar
to the seismic disturbances. Selzer believes that the vertical bar acted
as a kind of gravity inclinometer, following the slight inclination of
the ground.

J. C. Noyes reports similar observations on the record of a coil-core
installation operated by Boeing Scientific Research Laboratories, Seattle,
Washington. So does J. E. Lokken of Pacific Naval Laboratory, Canada,
(Private Communications, 1965).

Even if the magnetic field sensor is insensitive to vibrations the
recording equipment (e.g. a pen recorder) may respond. At the Geophysical
Institute, College, Alaska, a helium magnetometer was in operation during
the earthquake. As may be seen on Fig. 6 it reacted violently to the
earth tremor. According to Mather (Private Communication, 1964) the
magnetometer recorder acted as a poor seismometer. Most of the disturbed
part of the record is therefore a seismograph effect. The content of real
magnetic effect is probably impossible to determine.

13. Conclusion

The response of the standard photographic magnetographs and many
other kinds of equipment is a seismograph effect. Any real magnetic
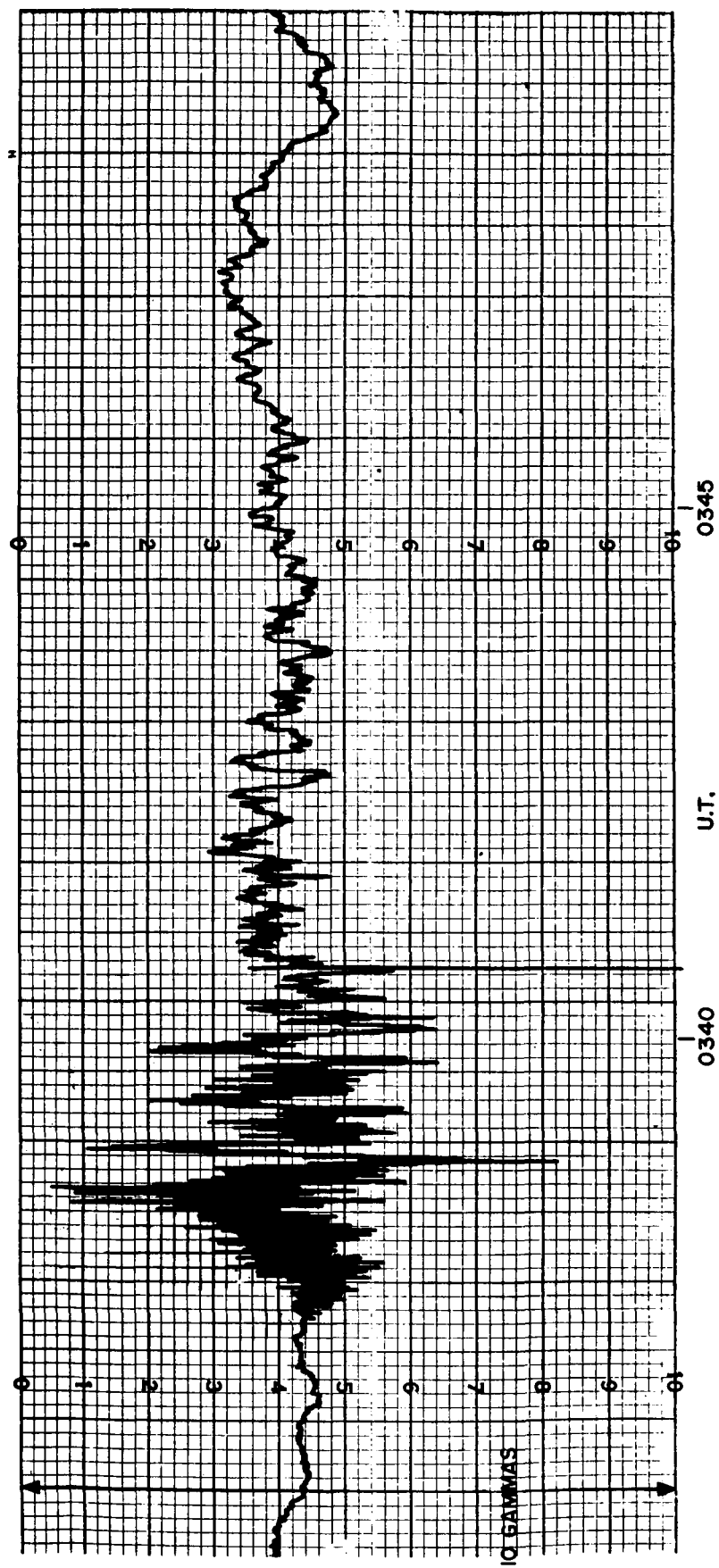variations associated with earthquakes are probably too small to be

Figure 6. Magnetic record (Helium Magnetometer) from College Alaska, March 28, 1964.

recorded by the standard instruments. The suspended magnetic system has obvious similarities with the seismic equipment and can in fact be used as an ultra-low-sensitivity seismic recorder. With little or no additional effort some magnetic observatories could operate as supplementary seismic stations at the times of great earthquakes, not coinciding with appreciable magnetic activity. One good example of this is the Alaskan earthquake of March 28, 1964.

Appendix 1.

A more general derivation of the response expression of the D variometer will be given here.

In the energy equation (1) we may insert one term corresponding to the rotation of the magnet:

$$MZ\Psi\,d\theta$$

where M is the magnetic moment and $d\theta$ is the rotation which occurs while Ψ changes the amount $d\Psi$.

We may further insert the friction term

$$\alpha\frac{dx'}{dt}dx$$

Note that the friction is proportional to the relative velocity between the magnet and the damping media (air, conductors). The latter move with the velocity $d\xi/dt$ of the instrument. The velocity dependence is well confirmed experimentally for small velocities.

Assuming an earth movement $\xi = -a \sin\omega t$ this leads to the equation

$$\frac{d^2\Psi}{dt^2} + \frac{\alpha}{m}\frac{d\Psi}{dt} + \frac{g}{L}\left(1 + \frac{MZ}{mgL}\frac{d\theta}{d\Psi}\right)\Psi = \frac{\omega^2 a}{L}\sin\omega t$$

This corresponds to (5).

It can be shown that the expression

$$\frac{MZ}{mgL}\frac{d\theta}{d\Psi} \qquad\qquad (16)$$

is small. According to (9) it may be written (at least approximately)

$$\frac{c}{L}\frac{d\theta}{d\Psi}$$

For normal magnets $\underline{c}$ does not exceed about 0.01 of the length of the

magnet. If this is taken to be 1 cm and if L = 15 cm we get

$$\frac{c}{L} = 0.0007$$

The ratio $d\theta/d\Psi$ has in most places the order of magnitude 1 as can

be seen from the equilibrium equation

$$Z\Psi = H\theta$$

Hence, in all practical cases, the term (16) may be neglected in the

parenthesis in the equation above. This may be written

$$\frac{d^2\Psi}{dt^2} + \frac{\alpha}{m}\frac{d\Psi}{dt} + \omega_p^2\Psi = \frac{\omega^2 a}{L}\sin\omega t$$

This well-known equation for forced vibrations has for small damping the
stationary solution

$$\Psi = \frac{\omega^2 a \sin (\omega t - \delta_1)}{L[(\omega_p^2 - \omega^2)^2 + (\alpha/m)^2 \omega^2]^{\frac{1}{2}}} \tag{17}$$

The differential equation for the rotation of the magnet is

$$\frac{d^2\theta}{dt^2} + \frac{\beta}{I}\frac{d\theta}{dt} + \omega_M^2\theta = \frac{MZ}{I}\Psi$$

where β is the damping constant.

Inserting the Ψ-value from (17) this has the stationary solution

$$\theta = \frac{MZ \omega^2 a \sin(\omega t - \delta_1 - \delta_2)}{IL[(\omega_p^2 - \omega^2)^2 + (\alpha/m)^2\omega^2]^{\frac{1}{2}}[(\omega_M^2 - \omega^2) + (\beta/I)^2\omega^2]^{\frac{1}{2}}}$$

$$tg\ \delta_1 = \frac{\alpha\omega}{m(\omega_p^2 - \omega^2)}$$

$$tg\ \delta_2 = \frac{\beta\omega}{I(\omega_M^2 - \omega^2)}$$

For an ideal bar magnet, the damping factors α/m and β/I are equal.
In practical variometers this is not the case (mirrors etc.).

For small values of ω, α, and β we will once more get the expression (6).

Appendix 2.

Fig. 7 pictures the magnetic system of a H variometer as seen from
magnetic East. In the unpertubed position (a) the horizontal field H
produces a torque that balances the fiber torsion, thus keeping the magnet
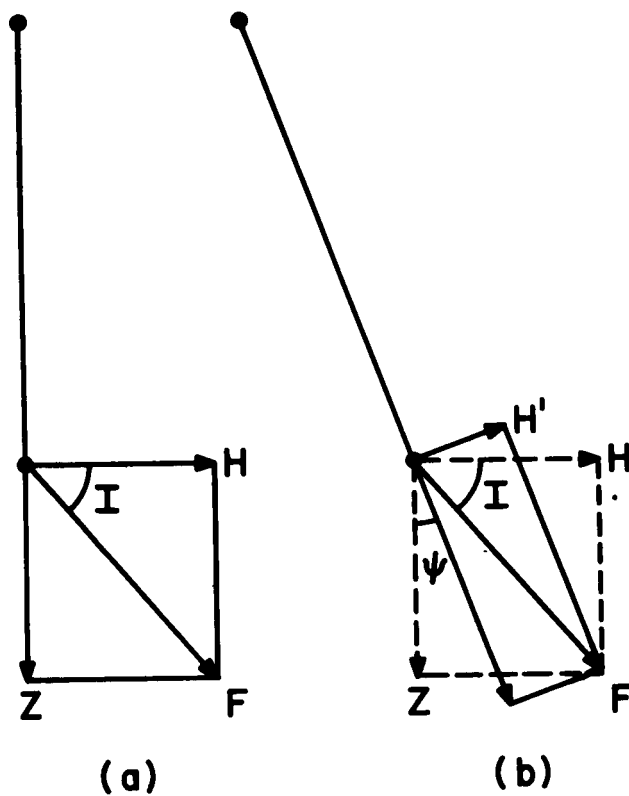
Figure 7. The magnetic system of the H variometer, in two positions, seen from the west.

in the EW direction.

In (b) the magnet performs a pendulum swing. The magnetic torque is now MH_1. The change in torque is

$$M\Delta H = M(H-H_1)$$

From Fig. 7 we get

$$\Delta H = H - H_1 = F[\cos I - \cos(I+\Psi)] =$$
$$= F(\cos I - \cos I \cos \Psi + \sin I \sin \Psi)$$

For small Ψ this gives

$$\Delta H \approx F\Psi \cdot \sin I = Z\Psi$$

The result implies that the response of the H variometer is governed by the same expressions as those derived for the D variometer.

II.   Magnetic effect.

1.   Inertia-free instruments

Obviously it is not safe to rely on magnetometers with moving parts,

galvanometers, pen and ink recorders etc. when trying to determine

whether earth tremors produce real magnetic variations.

With the advent of instruments based on nuclear or electronic

precession the situation has become more hopeful. An inquiry regarding

observed effects from the event of March 28, 1964, resulted in a handful

of recordings kindly made available by different agencies.

One of the records show magnetic variations that appear to be associated

with the earthquake. It is from the U. S. Geological Survey's Geoelectric

Observatory at Bergen Park, Colorado. The instruments are housed in the

Colorado School of Mines' Cecil H. Green Geophysical Observatory which is

designated as "GOL" in the world-wide seismic net. The geographic coordinates

are as follows

Lat. 39° 42'N
Long. 105° 22'W

The variations of the total field intensity are recorded by a meta-stable

helium vapor magnetometer, the data being stored on tape. It seems highly

improbable that this system in itself is sensitive to vibrations, especially

the slow vibrations studied here.

Regarding local sources of disturbance, magnetic bodies performing a

relative motion would have an effect. According to W.P. Hasbrouch (Private

Communication, 1965) there are no oscillating iron masses (e.g. the

pendulum of a seismograph) harmfully close to the sensing head of the

magnetometer. Metal structures in the vicinity, e.g. an aluminum

roof or a metallic instrument case, would carry induced currents. As

shown in a later section their magnetic effect is probably negligible

We can therefore be reasonably confident that the magnetic variations

recorded by this equipment reflect a geophysical event having its sources

outside man-made installations.

2.   Observed effect.

Fig. 8 shows a playback from the tape recorded at the Bergen Park

facility between U.T. 03h50m and 04h00m of March 28, 1964. The two upper

traces indicate telluric variation, the lower trace the output of the

meta-stable helium magnetometer. The labeling in the figure should be

sufficient for its present use.

The large variations of the magnetic record stand out very clearly in

comparison with the parts of the trace preceding and following the interval

covered by Fig. 8. They appear to agree closely in time with the arrival

of the large surface waves. Due to difficulties to interpret the seismograms

of the adjacent GOL station this had to be checked in an indirect way, by

using the rapid-run magnetograms of the magnetic observatory at Tucson

(Lat. 32°15'N Long. 110°50'W). These give a very clear picture of the

seismic M waves, with a starting time in Z of about 03h 55.5m; see Fig. 2.

When the difference in distance from the focus was corrected (the time

GEOELECTRIC OBSERVATORY

BERGEN PARK

NS

10
mv/km

TELLURICS

EW

10
mv/km

He—MAGNETOMETER

F

I GAMMA

0350   0355   0400
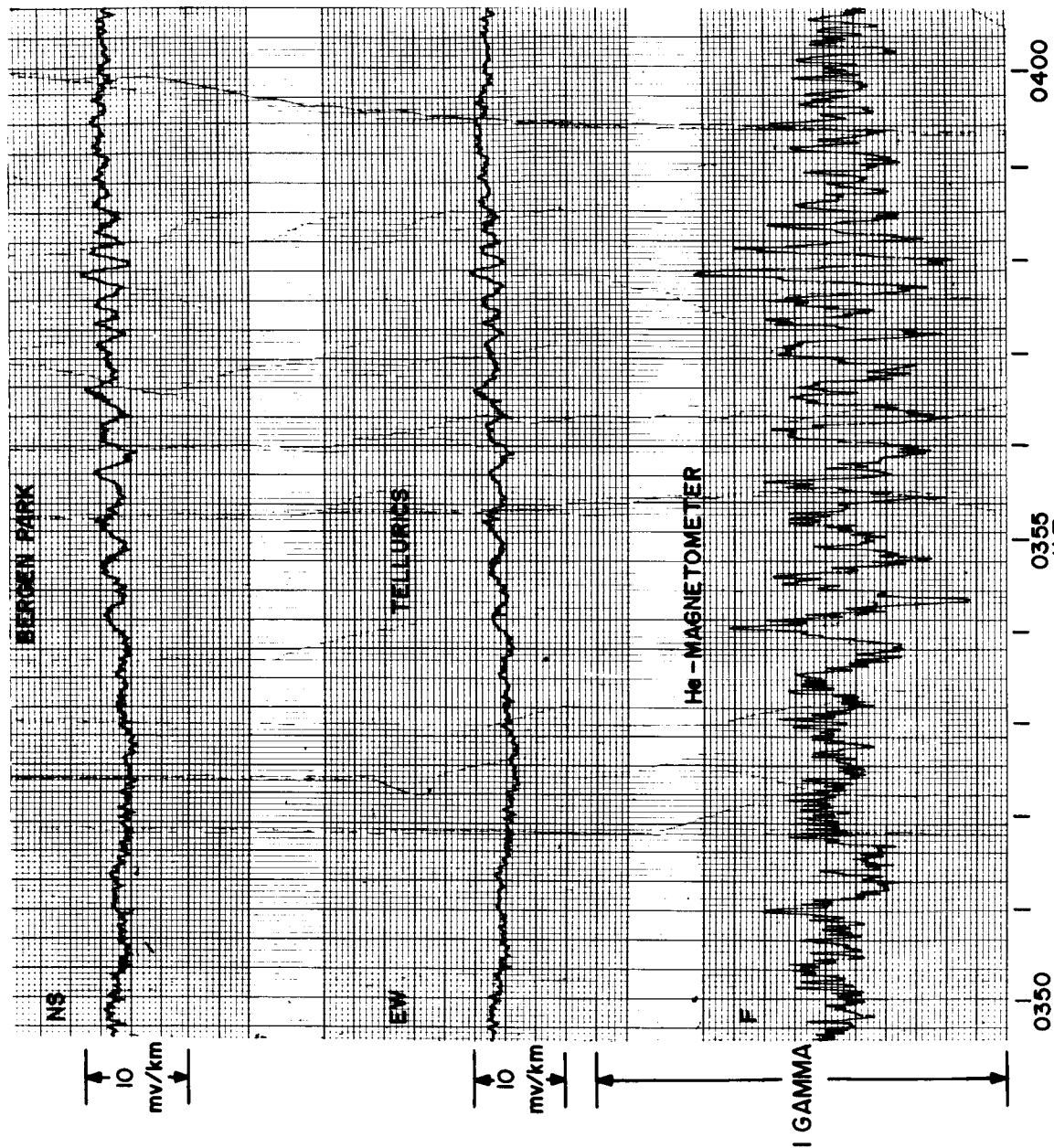   U.T.

Figure 8.   Magnetic and electric records from Bergen Park, Colorado,
            March 28, 1964.

difference is about 1.8 min) we get a starting time in Bergen Park of 03h 53.7m. This closely agrees with what is actually observed. As a further check, the periods of the waves (divided in three sub-groups) were determined and compared with the corresponding waves at Tucson (the Z component). The periods of the three groups, 33, 27, and 15 sec. agree at the two stations. They also agree with periods of the telluric waves in Fig. 8. These comparisons indicate that the telluric and magnetic variations observed at Bergen Park are associated with the seismic M waves. Periods and amplitudes for the Bergen Park variations are given in Table 2.

It is worth mentioning that the short period P and S waves arriving at 03h 42.8m (according to the GOL-seismograms), gave no appreciable effects on the Bergen Park records. The standard magnetographs of the magnetic observatories responded strongly also to these waves. The conclusion is that appreciable magnetic oscillations are only associated with the large surface waves.

3. Absence of effects.

The magnetic effect appears to be geographically confined. Records from Rubidium vapor magnetometers were obtained from the following places.

> U. S. Naval Air Development Center, Johnsville, Pennsylvania
>
> Lamont Geological Observatory of Columbia University;
> instrument location: Lebanon State Forest, New Jersey
>
> U. S. Department of Commerce, National Bureau of Standards;
> instrument location: Jicamarca Radar Observatory, Peru

None of these stations recorded an appreciable earthquake effect. Although their sensitivities are considerably lower than that of the

Table 2

Subgroups of variations observed
at Bergen Park

		I	II	III
Approx. time interval		0353.5-55.1	55.1-57.0	57.0-58.9
Average Period	F	33 sec.	27 sec.	15 sec.
	NS	34 sec.	27 sec.	15 sec.
	EW	32 sec.	27 sec.	15 sec.
Average Amplitude	F	0.2γ	0.2γ	0.2γ
	NS	1.5 mV/km	1.7 mV/km	1.7 mV/km
	EW	1.3 mV/km	1.5 mV/km	1.5 mV/km
Max. Amplitude	F	0.3γ	0.2γ	0.3γ
	NS	1.5 mV/km	1.7 mV/km	2.2 mV/km
	EW	1.3 mV/km	1.5 mV/km	1.9 mV/km

He-magnetometer at Bergen Park, a similar effect would have been visible above the noise. (The Jicamarca Station may be too far away from the source in Alaska.) A negative result is also reported by Radoski (Private Communication, 1965) for the Rubidium vapor magnetometer of Weston Observatory, Massachusetts (Boston College).

4. Theoretical models

There is evidently a close coincidence in time between the seismic M waves and the magnetic variations at Bergen Park. This makes it reasonable to assume that the magnetic waves have their sources fairly close to the observatory rather than, for example, at the focus of the earthquake.

4.1. Piezoelectric and piezomagnetic effects.

In principle the elastic deformations due to seismic waves might give rise to electric and magnetic polarizations (piezoelectrics and piezomagnetics). The strains and shears in the crust are of the order $2\pi a/\lambda$ (a = wave amplitude, λ = wavelength). For the large surface waves this quantity is about $4 \cdot 10^{-7}$.

Breiner (1964) has discussed the piezomagnetic effect in a spherical region at the time of a local earthquake. He also gives a number of references to reports of laboratory experiments pertaining to magnetic rocks under stress. On the basis of these results we may try to estimate the magnitude of the piezomagnetic effects associated with the seismic surface waves.

Within certain limits the magnetic susceptibility as well as the remanent magnetization of rocks decrease in the direction of an applied compressive stress (and to a lesser extent increase at right angles to the stress). For a change in stress, $\Delta\sigma$, these relationships can be expressed as

$$\Delta\chi = -S_\chi \; \Delta\sigma\chi_o$$

$$\Delta I = -S_I \; \Delta\sigma I_o$$

where $\Delta\chi$, χ_o and ΔI, I_o are the change and initial values of the susceptibility and remanent magnetization respectively. The stress sensitivities, S_χ and S_I, are approximately equal and reported to be $0.7 - 2\cdot10^{-4} \; cm^2/kg$.

For simplicity, let us consider a uniformly magnetized sphere beneath Bergen Park with a radius $\underline{r}$ of the order of some km. Suppose further that its center is situated at the depth $\underline{r}$. Its susceptibility χ_o is assumed to be $2\cdot10^{-3}$ e.m.u. This is in accordance with data indicating a magnetic anomaly of some 1000γ at Bergen Park.

The value of $\Delta\sigma$ is determined from $\Delta\sigma = E \; \Delta L/L$ where E is Young's modulus. According to numerical data given by Gutenberg (1959) $E = 10^6 \; kg/cm^2$ is a reasonable value for some common rocks. For the fractional elongation $\Delta L/L$ we may use the value 4.10^{-7}. This gives

$$\Delta\sigma = 0.4 \; kg \; cm^{-2}$$

The change in magnetization of the sphere is given by

$$\Delta M = \frac{4}{3} \; \pi r^3\Delta\chi$$

The alternating magnetic field on the surface of the magnetized

sphere has the maximum amplitude

$$\Delta B = \frac{2\Delta M}{r^3} = -2(4\pi/3) \; S\chi \; \chi_o \; \Delta \sigma$$

Inserting $S\chi = 10^{-4} \; cm^2 \; kg^{-1}$, $\chi_o = 2.10^{-3}$ gauss, $\Delta\sigma = 0.4 \; kg \cdot cm^{-2}$ we get

$$\Delta B = -0.06\gamma$$

A similar result would be obtained for a sphere with remanent magnetiza-

tion. The direction of ΔB is uncertain.

The observed variations in F are about 0.2γ. With some adjustments

of the parameters used above and a more realistic shape of the magnetized

body it does not appear unlikely that piezomagnetism could explain at least

part of the observed ΔF.

The piezoelectric effect depends on the reaction of certain crystals

to stress. In the literature piezoelectrics is discussed for such substan-

ces as quartz, tourmaline etc. The strains mentioned above when applied

to a quartz crystal would produce electric fields which are very large

from a geophysical point of view. Yet, in view of the polycrystalline

structure and chemical constitution of the crust it seems unlikely that

appreciable net effects could be produced in nature.

4.2. Magnetic field gradients

Due to the attenuation of the seismic amplitudes with depth a

relative motion exists between a magnetized body in the crust and the magnetic instrument on the surface.

For simplicity, let us assume a vertical dipole M buried at a depth z below the surface and capable of producing a disturbance field f_p on the surface, see Fig. 9. We have

$$f_p = \frac{2M}{z^3}$$

At a point Q on the surface with coordinates x and z the f from the dipole has the components

$$f_x = 3xz\,\frac{M}{r^5}$$

$$f_z = (2z^2 - x^2)\frac{M}{r^5}$$

whence

$$f = \sqrt{f_x^2 + f_z^2} = \frac{M\,(x^2 + 4z^2)^{\frac{1}{2}}}{(x^2 + z^2)^2}$$

Now the field gradients $\partial f/\partial x$, $\partial f/\partial z$ may be determined. It can be shown that

$$\left|\frac{\partial f}{\partial x}\right|_{max} \approx \frac{f_p}{z} \quad \text{for } x \approx 0.5z$$

and

$$\left|\frac{\partial f}{\partial z}\right|_{max} = \frac{3f_p}{z} \quad \text{for } x = 0.$$

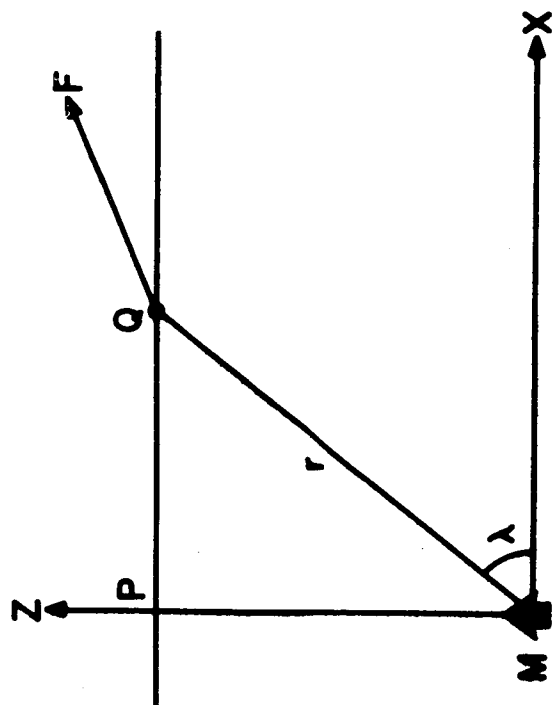If Δs is the displacement of the dipole relatively to a point on

Figure 9. A magnetic dipole beneath the earth's surface.

the crust, and Δf is the amplitude of the observed magnetic variation, we get for the vertical case

$$\Delta f = \frac{3f_p}{z} \Delta s$$

Let us put $f_p = 1000\gamma$. Data indicate that a magnetic anomaly of this magnitude may exist in the Bergen Park area. If we place our dipole at the depth $z = 10\text{km}$ where Δs can certainly not exceed 1 cm (see Stoneley and Hochstrasser, 1957, and Stoneley, 1958), we get

$$\Delta f < \frac{3 \cdot 10^3 \cdot 1}{10^6} = 3 \cdot 10^{-3}\gamma$$

This is only about 1:100 of the observed amplitude. Similar results are obtained for any reasonable set of values f_p, Δs, and z.

The movement of the magnetometer in a field gradient does not appear to be likely explanation of the observed variation.

4.3. The induction effect

4.3.1. Basic expressions

The earthquake displacement

$$s = \underline{a} \sin\omega t$$

corresponds to a ground velocity

$$v = \frac{ds}{dt} = \frac{2\pi a}{T} \cos \omega t$$

In a frame of reference moving with the ground we have

the electric field

$$\overline{E}' = \overline{E} + \overline{v} \times \overline{F}/c + m \frac{d\overline{v}}{dt/e}$$

In the present case the last term is negligible $\overline{E}$ is the electric

field observed from a frame at rest. If a conductor moves as a whole in

a magnetic field, without change of flux, a separation of charges occurs

which gives the electric field

$$\overline{E} = -\overline{v} \times \overline{F}/c$$

This makes $\overline{E}' = 0$. One example of this is the earth rotating in its

own magnetic field. In the case of earthquakes however, a deformation

takes place and closed current loops are set up, with $\overline{E}'$ as the effective

electric field. We get the current density

$$\overline{i} = \sigma (\overline{E} + \overline{v} \times \overline{F}/c)$$

In some applications the induced currents may be closed through

comparatively small resistances. In this case $\overline{E} \approx 0$ so that

$$\overline{i} = \sigma \overline{v} \times \overline{F}/c$$

This assumption is not essential for the subsequent discussion. A

simple electric circuit with some important features in common with the

geophysical case will be discussed in a later section.

4.3.2 <u>Induced current loops</u>.

In the seismic wave field let us consider one velocity

ridge and the adjacent trough, both of width $\lambda/2$ (one half

of a wavelength). The ground velocities have different signs in the two

ranges. Neglecting the attenuation of the wave amplitude with distance

from focus we find the velocities $\bar{v}$ and $-\bar{v}$ symmetrically spaced about

the border line ($\bar{v} = 0$). Specifically, we find the amplitude values

$\bar{v}_o$ and $-\bar{v}_o$ a distance $\lambda/2$ apart. The same reasoning holds for the

induced currents $\bar{i}$. They flow parallel but in opposite directions on

the two sides of the borderline. Under certain conditions they form

closed circuits. This is illustrated in a qualitative way in Fig. 10.

An extreme case would be currents flowing concentrically around the

epicenter, without crossing the border-line. The current loops move

with the speed of propagation of the earthquake waves.

4.3.3 Calculated electric fields at Bergen Park

In order to study more closely what ground motions and induced

currents can be expected as an effect of the Alaskan earthquake at

Bergen Park, see Fig. 11. A corresponds to the epicenter (Anchorage),

B to the recording station at Bergen Park. The line AB which is the

direction of propagation of the seismic surface waves, forms an angle

of approximately 48° with the local magnetic meridian in B.

During passage of a Rayleigh wave the motion of a surface particle

is elliptic and retrograde, the major axis occurring in the vertical

direction with about 1.7 the amplitude of the displacement in the

direction of propagation. The Love waves are large transverse surface

waves.

In the following we use the notations L = Love wave,

R = Rayleigh wave, h = horizontal, v = vertical, and H, D, Z
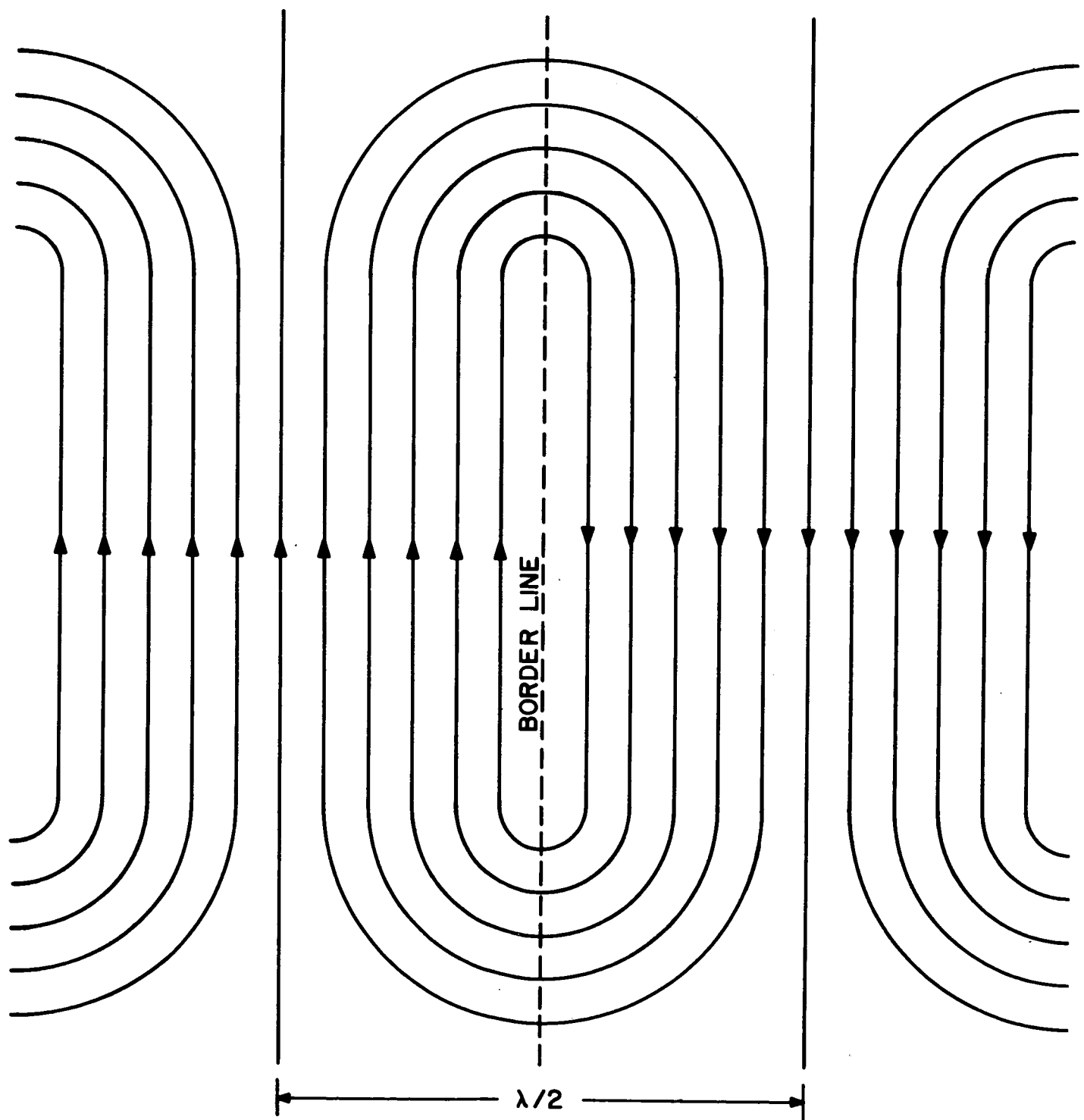
BORDER LINE
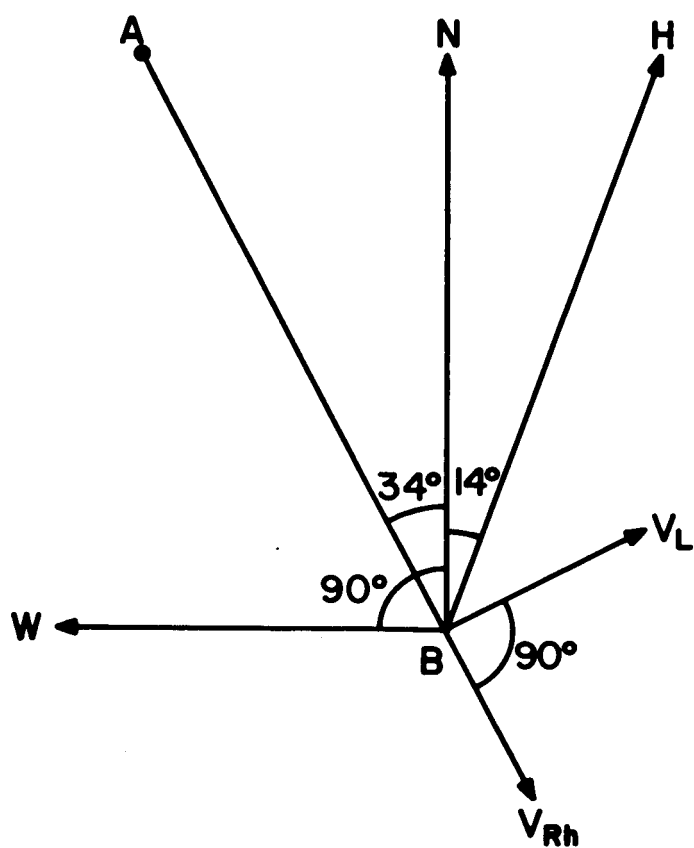
λ/2

Figure 10. Seismically induced current loops.

Figure 11. The seismomagnetic directions at Bergen Park.

for the geomagnetic components.

In Table 3 we have listed the amplitude values of the electric field components and their directions. The numerical values are based on the magnetic values H = 0.22 gauss, Z = 0.52 gauss, and the seismic peak velocities (corresponding to 20 sec period): $V_{Rh} = V_L = 0.4$ cm sec^{-1}, $V_{Rv} = 0.7$ cm sec^{-1}. The latter values are estimates from the Tucson rapid-run magnetograph (used as seismograph), since no useful seismograms were available for the Bergen Park area.

Table 3

Electric Field	Numerical value	Direction
$E_{Rh}H{\sim}V_{Rh}H\sin 48^o$	0.07 mV/km	vertical
$E_{Rh}Z{\sim}V_{Rh}Z$	0.21 mV/km	horiz. $\perp$ AB
$E_{LH}{\sim}V_L H\sin 42^o$	0.06 mV/km	vertical
$E_{LZ}{\sim}V_L Z$	0.21 mV/km	horiz. along AB
$E_{RvH}{\sim}V_{Rv}H$	0.15 mV/km	horiz. $\perp$ H

4.3.4 <u>Observed electric fields</u>.

The values and directions given in Table 3 indicate a maximal horizontal amplitude of about 0.34 mV/km. The values observed at the telluric system of Bergen Park are above 1 mV/km.

There may be some doubt whether a telluric line shunted to the earth and sharing its motion would record the induced electric field. As shown in the next section it is likely that it does.

The discrepancy between calculated and observed electric fields may or may not be significant. The ground amplitudes at Bergen Park can in

reality be somewhat larger than those used here. The calculations

pertain to sinusoidal ground vibrations. If the waveform came closer

to that of the square wave we would get E-spikes of larger amplitude.

But this wave type does not seem to agree with observations and has

also no support in seismic theory.

More important is that the model of the induced telluric currents

given here is too simplified. In a more realistic model various potential

gradients could add up to the observed value. This will be discussed later.

4.3.5. Equivalent circuit.

For a survey of the induction currents a simplified equivalent

circuit may be helpful. The circuit depicted in Fig. 12 is in some respects

similar to an induced current loop. It consists of $\underline{n}$ voltaic cells, each

with e.m.f. ϵ and inner resistance R_i. The total e.m.f. $n\epsilon$ corresponds

to the induced e.m.f. in the earth. The "outer resistance" of the circuit

is R_y.

Across one of the cells a galvanometer circuit G is connected. This

corresponds to the telluric line. Since it takes part in the earth's motion

it also contains an e.m.f. ϵ. Its resistance R_g is of the order 1 megohm.

Thus $R_g \gg nR_i + R_y$.

Kirchhoff's law gives the e.m.f. $I_G R_G$ indicated on the galvanometer

in the moving frame of reference

$$\epsilon_m = I_G R_G = - \frac{\epsilon n R_i}{n R_i + R_y}$$
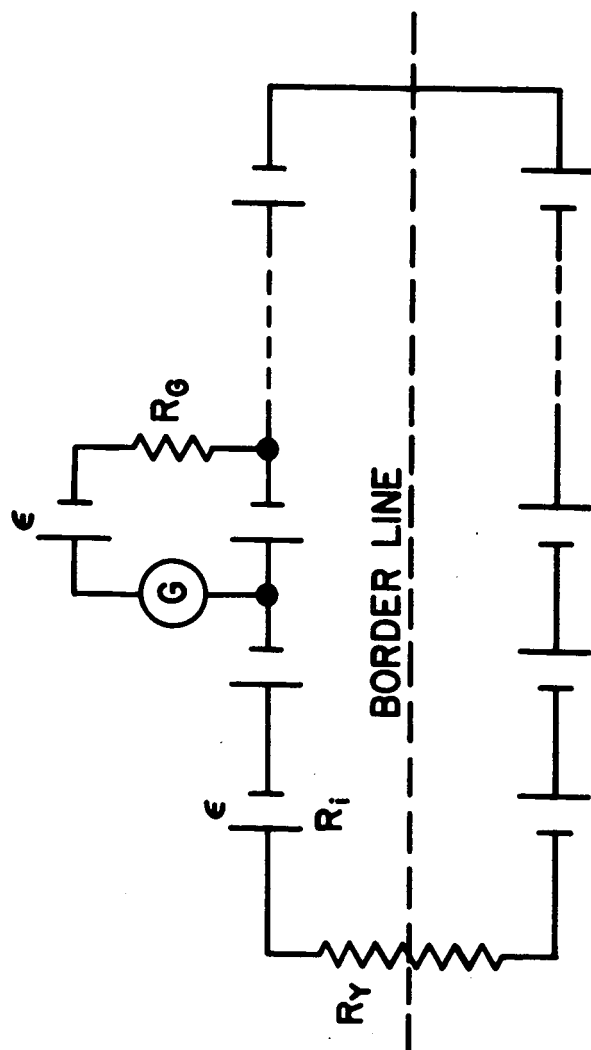
If $nR_i \gg R_y$ we get

Figure 12. Equivalent circuit.

$$\epsilon_{\text{in}} = -\epsilon$$

The telluric recorder thus measures the induced electric field. In a frame at rest we would get the following galvanometer reading

$$\epsilon_r = \epsilon - \frac{\epsilon n R_i}{n R_i + R_y} = \epsilon + \epsilon_m$$

with the special cases

$$\epsilon_r = 0 \quad \text{for } n R_i \gg R_y$$

$$\epsilon_r = \epsilon \quad \text{for } n R_i \ll R_y$$

The equation $\epsilon_m = \epsilon_r - \epsilon$ corresponds in some respects to

$$\overline{E}' = \overline{E} + \overline{v} \times \overline{F}/c$$

If the galvanometer were instead shunted over a fraction $1/m$ of R_y we would get a reading ϵ_y. Assuming that the telluric line has now a direction perpendicular to its earlier direction, it has no e.m.f. ϵ. We get

$$\epsilon_y = \frac{R_y I}{m} = \frac{n \epsilon R_y}{m(n R_i + R_y)}$$

For $R_y \gg n R_i$ and for a given length of the telluric line (say 1 km) this would correspond to an electric field expression

$$E_y = \frac{n}{m} \left| \overline{v} \times \overline{F}/c \right|$$

Under certain conditions we could easily have

$$E_y > \left| \, \overline{v} \ \times \ \overline{F}/c \, \right|$$

as indicated on the Bergen Park telluric recorder.

A large metal structure (e.g. an aluminum roof) close to the magneto-meter could in itself easily have a conductance comparable with that of a large portion of the crust. Large currents in this structure could modify or cancel the magnetic variations due to ground currents. In the practical case however the connections between the structure and the crust would involve a large resistance. Thus only a negligible fraction of the earth-currents would pass through the metal.

4.3.6. The telluric recorder

Some caution may be needed in the quantitative interpretation of telluric data. The standard telluric equipment contains moving parts (e.g. the suspended system of the galvanometers) and may be susceptible to vibrations. One example of this is the earth-current equipment in Niemegk. The recording system at Bergen Park is purely electronic and should be safe in this respect. It did not respond noticeably to the rapid vibrations preceding and following the M waves. As mentioned before suspended magnets responded to the whole spectrum of seismic vibrations. So did the telluric equipment in Niemegk.

During the hours preceding and following the earthquake the two telluric traces (NS and EW) at Bergen Park showed a curious parallelism most of the time. This may indicate some kind of instrumental "cross talk" and tends to cast some doubt on the quantitative reliability of the equipment at the time of the earthquake.

On the other hand: outside the earthquake effect the electric and magnetic

registrations at Bergen Park for long-periodic waves appear to be consistent with the main conductivity of the crust; cf Table 4.

The conclusion of the previous discussion is that we may have some confidence that the telluric recordings are quantitatively correct. For complete security independent observations are needed.

4.3.7 Induced currents in the "normal" crust.

According to Table 3 the two largest electric fields and E_{RhZ} and E_{LZ}. They are both horizontal. Only E_{RhZ} is parallel to the seismic wave front and thus capable of producing current loops of the kind shown in Fig. 10.

The distribution of i within the earth depends on the attenuation of the earthquake amplitude a with depth. The surface waves are guided waves being confined to a channel which is defined by the free surface and the bottom of the crust.

Stoneley and Hochstrasser (1957) and Stoneley (1958) have made theoretical studies of the attenuation of Rayleigh waves and Love waves. They consider a double surface layer of total thickness $s = T_1 + T_2$. If we identify this layer with the earth's crust ($s \approx 30$ km) we find that for the large surface waves ($\lambda = 3s$) the value of a decreases comparatively slowly within the crust. It seems safe, according to this model, to assume an average amplitude of about $0.7a$ within the uppermost 30 km of the earth.

To determine the order of magnitude of the induced field the following discussion should be sufficient. We consider an infinitely long slab of the crust directed parallel to the seismic wavefront. Its width is $\lambda/2$

and its thickness s is the thickness of the crust. For simplicity,
we assume that the slab as a whole performs horizontal vibrations (per-
pendicular to its length) of the average amplitude $0.7\ a_{Rh}/\sqrt{2}=0.5\ a_{Rh}$.
We get the induced current density.

$$i = \frac{\sigma\pi\ a_{Rh}Z}{cT} = i_0\ \cos\omega t$$

A direct current of density i_0 would give a magnetic field in the
center of the upper side of the slab with the horizontal component

$$\Delta H = 0.4\ i_0 \int_0^s \int_0^{\lambda/4} \frac{zdzdx}{x^2+z^2}$$

Here z is the vertical axis; the x-axis is perpendicular to the
wavefront. For finite values of λ we get

$$\Delta H < 0.2\pi i_0 s$$

With $\qquad V_{Rh}=0.4$ cm sec^{-1}; s=30 km;

$\sigma = 10^{-6}$ ohm^{-1} cm^{-1} we get

$$\Delta H < 10^{-4}\gamma$$

The σ value is representative for granite and dry sand. It also
agrees well with the value obtained for long-period pulsations according
to Table 4.

In reality i is an alternating current. The electric field in the
crust is determined by the seismic amplitudes rather than by the skin effect.
The magnetic variations generated at a certain depth decrease somewhat faster
than a magnetostatic field when propagated to the surface. The value of

the alternating ΔH at the surface is therefore certainly not greater than the value corresponding to direct currents.

The observed ΔF at Bergen Park is 0.2γ. ΔF is of course the resultant field of ΔH and ΔZ. Since ΔZ from the slab current is of the same magnitude as ΔH, the observed field is about 1000 times larger than the calculated field. It therefore appears completely unlikely that the vibration of the "normal" crust can explain the observed ΔF.

A small residue of the surface tremor may reach depths of some 700 km where the mantle conductivity is appreciable. Due to the attenuation of the electromagnetic waves it is unlikely that fields from such depths would be noticeable on the surface. According to Gaignard (1956) the "depth of penetration" ζ where the amplitude is reduced by 1/e is given by

$$\zeta = \frac{1}{2\pi}\sqrt{\frac{T}{10\sigma}} \qquad \text{(in km)}$$

For T = 20 sec. and σ = 10^{-3} ohm^{-1} cm^{-1} (a rough average for the uppermost 700 km) we get ζ=7.1 km.

4.3.8 Enhanced conductivity

To explain the observed ΔF at Bergen Park as an induction effect we must assume a local conductivity anomaly. If "normal" conductivity conditions had provided an explanation the effect would be universal. As mentioned earlier the effect was not observed at other stations operating inertia-free instruments.

The Bergen Park Observatory is situated in an area rich in minerals.

A conductivity anomaly close to the surface would therefore not appear

unlikely. It could of course also be located at greater depths where

the earth tremor was still appreciable. For example, Rikitake (1964)

assumes an anomaly at a depth of about 100 km beneath Japan to explain

observed magnetic variations.

For plane electromagnetic waves penetrating a homogeneous earth,

the following expression may be deduced from Maxwell's equations (Gaignard,

1956)

$$\rho = \sigma^{-1} = 20T \left(\frac{Ey}{Hx} \right)^2 \quad \text{ohm.cm}$$

H_x and E_y are the magnetic and electric fields observed on the

surface. In homogeneous soil the value of ρ would be independent of T.

If not, the dependence indicates that ρ as a function of depth is not

constant.

In the hours preceding and following the earthquake, pulsations

of magnetospheric origin were abundant on the Bergen Park records. The

present writer selected four events of different period, avoiding beat-type

effects. According to the normal magnetogram of the USC&GS's Boulder

Observatory the pulsation activity was almost completely limited to the

H component. Assuming this to be the case also in Bergen Park, H_x was

derived from the expression

$$H_x = \Delta H = (F/H)\Delta F$$

As E_y the telluric EW component was used. Since the magnetic

declination in this area is about $14°E$ no great error is introduced.

Average amplitudes over several cycles were scaled.

Table 4 summarizes the results of this investigation. The assumptions of plane waves and a horizontally stratified earth are oversimplifications, especially in a mountainous region as that of Bergen Park. Still, the results may indicate the order of magnitude and tendency of variation of the quantities involved.

Table 4

Date 1964	U.T.	H_x γ	E_y mV/km	T sec	ρ ohm.cm
March 28	0300	0.17	0.9	11	$0.062.10^5$
27	2325	1.50	19.5	45	$1.52.10^5$
28	0757	0.73	12.2	72	$4.0.10^5$
28	0834	0.87	14.7	133	$7.6.10^5$

If the results in Table 4 are reliable they indicate that with increasing period the resistivity approaches the value 10^6 ohm.cm assumed to be valid for the crust. For the more shallow waves of shorter periods, ρ decreases rapidly. This supports the assumption of a high conductivity region in the upper crust.

This is probably not simply a homogeneous layer of enhanced conductivity but a complicated structure with anisotropic properties. All kinds of induced current loops may be set up, horizontal as well as vertical. Since F in Bergen Park has a close resemblance to Z in Tucson (regarded as seismic) vertical movements probably play an important role.

Wide range induced currents might be closed through narrow horizontal

channels close to the surface. Effects of this kind could help to explain the observed value of the electric field at Bergen Park, by adding to the induced field the potential gradients associated with these currents.

The telluric lines have components perpendicular as well as parallel to the seismic wavefronts and should therefore respond to electric fields across these fronts. This would correspond to a galvanometer shunted over part of the resistance R_y in the simplified analog circuit considered earlier.

A region with average conductivity 10^{-2} ohm^{-1}cm^{-1} would have to be a few km thick to explain the observed effects. The usual relatively thin top layer of water-impregnated soil and rock alone is not sufficient.

As mentioned before, no noticeable magnetotelluric effect was observed simultaneously with the rapid waves preceding and following the M waves. A few tentative explanations might be suggested here.

(1) The effects are too small to be distinguished from ionospheric and instrumental noise.

(2) Due to the short wavelengths, effects of waves in opposite phases tend to cancel each other.

(3) The periods of the M waves correspond to an electromagnetic pass-band determined by the local conditions.

5. Conclusion

Of the mechanisms discussed here piezomagnetic oscillations and induced currents in a region with enhanced conductivity appear most likely to explain the observed electric and magnetic variations at Bergen Park. Clearly, many

more observations and a better knowledge of the local elastic, electric, and magnetic properties of the crust are needed for a quantitative treatment. Since the seismic waveform at Bergen Park is not accurately known it is difficult to judge which mechanism is most favored by the observed variations. It seems evident, theoretically as well as empirically, that the properties of the "average crust" cannot explain the observed magnetic variations at Bergen Park. This is probably also true for the electric variations.

Figure Captions

1. Earthquake vibrations recorded at Enköping and Kiruna in Sweden (modified Grenet pulsation recorders), March 28, 1964.

2. Earthquake vibrations recorded at Tucson, Arizona, March 28, 1964.

3. The magnetic system of the D variometer as seen from magnetic South.

4. A Z magnet in EW direction, seen from the south, in two different tremor positions.

5. Magnetic and electric records from Niemegk, Germany, March 28, 1964.

6. Magnetic record from College, Alaska, March 28, 1964.

7. The magnetic system of the H variometer, in two positions, seen from the west.

8. Magnetic and electric records from Bergen Park, Colorado, March 28, 1964.

9. A magnetic dipole beneath the earth's surface.

10. Seismically induced current loops.

11. The seismomagnetic directions at Bergen Park.

12. Equivalent circuit.

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