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TORQUE REQUIREMENTS FOR ORIENTATION OF A SOLAR BRAYTON SYSTEM IN EARTH ORBIT

by Donald L. Sowers

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SUMMARY

Several torques that must be considered in the orientation control of a solar Brayton power system are analyzed. The torques are divided into three groups: external, internal reaction, and torque necessary to compensate for variations in angular velocity. For each group of torques, general equations are derived and numerical values of the torques are calculated for a typical 10-kilowatt solar Brayton system in a 300-nautical-mile ecliptic orbit.

Of the external torques, gravitational and aerodynamic on the collector are the largest. They are sinusoidal and have maximums of 2.5×10^{-3} and 1.65×10^{-3} foot-pound, respectively. Torque due to solar radiation on the collector is several orders of magnitude smaller.

The internal reaction torque, which is caused by accelerations of the turbomachinery, is the largest torque considered. A torque as large as 0.82 foot-pound is due to acceleration of the turboalternator during a typical alternator load transient. The internal torque, however, does not misorient the collector, but rather rotates it about its axis, provided the turbomachinery axes are alined parallel to the collector axis of symmetry. If the turbomachinery axes are alined perpendicular to the collector axis, torques of 4.3×10^{-2} and 2.0×10^{-2} foot-pound for the turbocompressor and turboalternators, respectively, are required to keep the collector alined within $1/4^{\circ}$.

Torque necessary to compensate for variations in angular velocity is shown to be negligible. This torque compensates for angular acceleration with respect to the Sun by applying an equal angular acceleration to the power system about its center of gravity.

INTRODUCTION

Studies are being made by NASA to determine the feasibility of developing a solar Brayton system for space electric power generation. This system uses an all-gas thermodynamic cycle with the Sun as the energy source. A large paraboloidal collector reflects the Sun's rays onto an absorber located at the focal point. Heat is transferred from the absorber to the working fluid (inert gas). The gas expands through a turbine which drives an alternator. The gas is then cooled, compressed, and returned to the absorber. Further discussions of Brayton cycle systems are given in references 1 and 2.

A major requirement of this system is the proper alinement of the collector with respect to the Sun. Studies have indicated that the collector must be alined within approximately $1/4^{\circ}$ to insure effective operation of the power system. In order to determine the size and type of attitude control system required to keep the solar collector alined, the magnitude and direction of the disturbing torques must be known. This report presents an analysis of torques encountered by a solar Brayton system in Earth orbit.

The torques have been divided into three groups: external torque, internal reaction torque, and torque necessary to adjust for variations in angular velocity. External torque is caused by external forces acting on the power system, such as gravity, atmospheric drag, or light pressure. Internal reaction torque is caused by the acceleration of moving parts in the power system. The third torque is that torque necessary to change the angular velocity of the collector about its center of gravity. As the collector revolves about the Earth and Sun, its angular velocity with respect to the Sun is constantly changing. The collector, therefore, must be rotated about its center of gravity at the same rate of change to keep it in proper orientation with the Sun.

Torques caused by the connecting apparatus between the power system and the space vehicle have not been considered. No consideration has been given to the space vehicle to which the power system would be attached.

General equations are derived for each of the torques, and then specific dimensions of a typical 10-kilowatt solar Brayton system are used to obtain values of the torques. In this analysis, it is assumed that the solar collector and the power system are rigidly attached. For the numerical calculations the solar Brayton system has a 30-foot-diameter collector and a radiator 30 feet in diameter and 5 feet long. The system weighs 1500 pounds and is in a 300-nautical-mile ecliptic orbit.

SYMBOLS

A area

a semimajor axis of ellipse

b semiminor axis of ellipse

C_D drag coefficient

c distance from center of Earth to center of power system

d half length of cylinder

e eccentricity of ellipse

F	force
g	gravitational constant at Earth's surface
h	radius of cylinder
I	moment of inertia
$\bar{i}, \bar{j}, \bar{k}$	unit vectors along the rectangular coordinate axes
K	pressure constant
k	constant
L	distance from edge of radiator to origin of coordinate system
l	distance from origin of coordinate system to center of gravity
M	total mass
m	mass
N	normal to surface
n	distance from Earth to Sun
P	pressure from diffuse reflection of air
p	angular speed of turbine
R	radius of Earth
r, θ, Z	cylindrical coordinates
r_1	distance from elemental mass to center of Earth
S	solar radiation pressure constant at Earth's orbit
S_r	molecular speed ratio
s	surface
T	temperature
T_i	ambient temperature
T_r	surface temperature
t	time
U	velocity of Earth with respect to power system

V	velocity
V_o	velocity of power system about Earth
V_{ot}	tangential velocity component of $\bar{V}_o$
V_s	velocity of power system about Sun
V_{st}	tangential velocity component of $\bar{V}_s$
v	volume
x,y,z	set of rectangular coordinate axes
α	angular acceleration
β	misorientation angle
γ	angle between velocity vector and Z-axis
δ	density
η	angle between radiation vector and normal to surface
ρ	magnitude of position vector
τ	torque
φ	angle of collector axis with line through center of Earth
χ	angle defining orbit position
ψ	angle between velocity vector and normal to surface
Ω	angular velocity of body with respect to Earth
ω	angular velocity of body with respect to Sun

Superscript:

$-$	denotes vector

EXTERNAL TORQUE

There are several types of external torque. Of these, three types are dominant and will be considered in detail: gravitational torque, aerodynamic torque, and solar radiation torque.

There are some other small external torques that should be mentioned but which are not calculated. These torques arise from the Earth's magnetic field, cosmic ray bombardment, and the gravitational fields of other celestial bodies. The reaction of the power system to the Earth's magnetic field depends on the electrical circuitry which has not yet been specified. By proper design this torque can be made small. The latter two torques, cosmic ray bombardment and gravitational torque of other celestial bodies, are of the order of 10^{-9} times that due to solar radiation and gravitational torque from the Earth (ref. 3).

Meteoroid bombardment, another external torque, is not calculated in this report.

Gravitational Torque

A body in orbit about the Earth experiences a force due to gravity and centrifugal force. If the body's energy is dissipated, these forces aline the axis with least moment of inertia with the center of the Earth.

The torque acting on a body about its center of gravity is

$$\overline{\tau} = \int_V \overline{\rho} \times \overline{dF} \tag{1}$$

where $\overline{\rho}$ is the vector from the center of gravity of the body to the elemental mass where the differential force $\overline{dF}$ acts. The force acting on an elemental mass due to gravity is $g\left(R^2/r_1^2\right)dm$, where R is the radius of the Earth, dm is the elemental mass, and r_1 is the distance from the elemental mass to the center of the Earth. The centrifugal force acting on the elemental mass is $r_1\Omega^2\ dm$, where Ω is the angular velocity of the body around the Earth. The magnitude of the combined force acting on an elemental volume is

$$dF = \left(g\frac{R^2}{r_1^2} - r_1\Omega^2\right)dm \tag{2}$$

and is in the direction of $-\overline{r_1}$ (that is, a vector from the elemental mass to the center of the Earth).

A homogeneous right circular cylinder, as shown in figure 1, is considered as a model for the power system. The radius of the cylinder is h and the length is 2d. By inspection of figure 1, the vector $-\overline{r_1}$ is

$$-\overline{r_1} = (Z \sin \varphi - r \cos \theta \cos \varphi)\overline{i} - r \sin \theta\, \overline{j} - (c + Z \cos \varphi + r \cos \theta \sin \varphi)\overline{k} \tag{3}$$

where r, θ, and Z are cylindrical coordinates of the elemental mass, c is the distance from the center of gravity to the center of the Earth, φ is the angle between the axis of the cylinder and the vector $\overline{c}$, and $\overline{i}$, $\overline{j}$, and $\overline{k}$

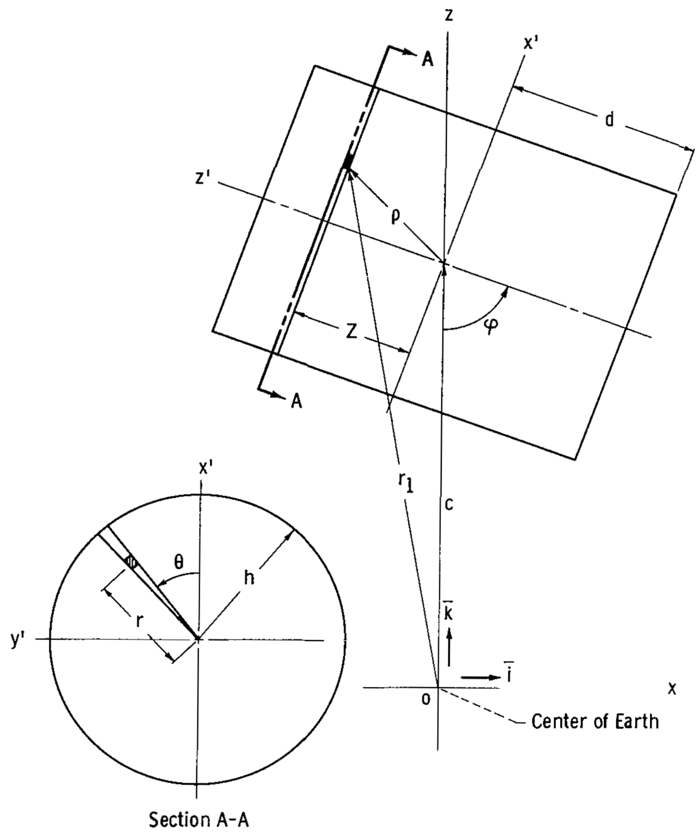

Figure 1. - Model of power system (right circular cylinder).

are unit vectors along the x, y, and z axes, respectively.

When equations (2) and (3) are used, the differential force can be written in vector form as follows:

$$\overline{dF} = \left(g \frac{R^2}{r_1^3} - \Omega^2\right) dm \Big[(Z \sin \varphi - r \cos \theta \cos \varphi)\bar{i} - r \sin \theta\, \bar{j} - (c + Z \cos \varphi + r \cos \theta \sin \varphi)\bar{k}\Big] \tag{4}$$

The vector $\bar{\rho}$ from the center of gravity of the cylinder to the elemental mass is

$$\bar{\rho} = (-Z \sin \varphi + r \cos \theta \cos \varphi)\bar{i} + r \sin \theta\, \bar{j} + (Z \cos \varphi + r \cos \theta \sin \varphi)\bar{k} \tag{5}$$

Substituting equations (4) and (5) into (1) yields

$$\bar{\tau} = \int_v \left(g \frac{R^2}{r_1^3} - \Omega^2\right)\left[-cr \sin\theta\, \bar{i} + c(-Z \sin\varphi + r \cos\theta \cos\varphi)\bar{j}\right]dm \qquad (6)$$

The mass of an elemental volume dm is $\delta r\, dr\, d\theta\, dZ$, where δ is density. Substituting this expression in equation (6) gives

$$\bar{\tau} = c\delta g R^2 \int_v \frac{-r^2 \sin\theta}{r_1^3}\, dr\, d\theta\, dZ\, \bar{i}$$

$$+ c\delta\Omega^2 \int_v (r^2 \sin\theta\, \bar{i} + Zr \sin\varphi\, \bar{j} - r^2 \cos\theta \cos\varphi\, \bar{j})dr\, d\theta\, dZ$$

$$+ c\delta g R^2 \int_v \frac{-Zr \sin\varphi + r^2 \cos\theta \cos\varphi}{r_1^3}\, dr\, d\theta\, dZ\, \bar{j} \qquad (7)$$

where the range of integration is $0 \leq r \leq h$, $0 \leq \theta \leq 2\pi$, $-d \leq Z \leq d$. The distance r_1 can be expressed as a function of c, r, Z, θ, and φ:

$$r_1 = \sqrt{c^2 + Z^2 + r^2 + 2cZ \cos\varphi + 2cr \sin\varphi \cos\theta}$$

The first integral in equation (7) is an odd function of θ; therefore, when integrating over θ from $-\pi$ to π, the integral is zero. The second integral also is zero; this can be seen by integrating Z from $-d$ to d and θ from 0 to 2π. Therefore, the total gravitational torque acting on the body is given by

$$\bar{\tau} = c\delta g R^2 \left[\sin\varphi \int_v \frac{-Zr\, dr\, d\theta\, dZ}{\left(c^2 + Z^2 + r^2 + 2cZ \cos\varphi + 2cr \sin\varphi \cos\theta\right)^{3/2}}\right.$$

$$\left. + \cos\varphi \int_v \frac{r^2 \cos\theta\, dr\, d\theta\, dZ}{\left(c^2 + Z^2 + r^2 + 2cZ \cos\varphi + 2cr \sin\varphi \cos\theta\right)^{3/2}}\right]\bar{j} \qquad (8)$$

The integrals of equation (8) are evaluated in the appendix. The integration yields the following expression for the gravitational torque:

$$\bar{\tau} = \frac{1}{2} Mg \frac{R^2}{c^3}\left(d^2 - \frac{3}{4} h^2\right)(\sin 2\varphi)\bar{j} \tag{9}$$

where M is the total mass of the cylinder.

It can be seen from equation (9) that the gravitational torque is zero if the moment of inertia is the same about all axes ($d^2 = 3/4\ h^2$). Also it can be seen that the gravitational torque is periodic. The period is one-half the orbit period with the angle φ directly related to the angle defining orbit position. The maximum magnitude of the torque occurs four times in each orbit.

In order to estimate the magnitude of the torque for the power system assumed, the following dimensions are used in equation (9): $h = 15$ feet, $d = 9$ feet, and $M = 1500$ pounds. When these values are substituted into the torque equation, the gravitational torque about the center of gravity of the power system in a 300-nautical-mile ecliptic Earth orbit is $-2.45\times10^{-3}(\sin 2\varphi)\bar{j}$ foot-pound.

Aerodynamic Torque

At altitudes up to 500 miles, air density is great enough to produce a significant force on a moving body. When the net force does not act through the center of gravity, a torque is produced. The aerodynamic torque consists of two parts - torque due to the incidence of the air molecules and secondly, torque due to the reflection of the air molecules. In this analysis the air molecules are assumed to be diffusely reflected.

The torque on a surface is

$$\bar{\tau} = \int_s \bar{\rho} \times \overline{dF} \tag{10}$$

where $\bar{\rho}$ is the vector from the center of gravity to the point where the differential force $\overline{dF}$ acts. The component of the differential force due to the incidence of the air is

$$\overline{dF}_{\text{incidence}} = C_D \frac{1}{2} \delta U^2 \cos\psi\ dA\ (-\sin\gamma\ \bar{i} - \cos\gamma\ \bar{k}) \tag{11}$$

where C_D is the drag coefficient, δ is the air density, U is the velocity of the air with respect to the body, ψ is the angle between the velocity vector and the normal to the surface, γ is the angle between the velocity vector and the z-axis, and $\bar{i}$, $\bar{j}$, and $\bar{k}$ are unit vectors along the x, y, and z axes, respectively (fig. 2). For this analysis, C_D is assumed to be 2.

The component of the differential force due to the diffuse reflection of air molecules is given by

$$\overline{dF}_{\text{Reflection}} = P\ dA\ \frac{\overline{N}}{|\overline{N}|} \tag{12}$$

where P is pressure and N is the normal to the surface. Above 100 miles altitude, the air density is sufficiently low so that collisions between molecules after reflection can be neglected. The value of the pressure P is found in reference 4 to be

$$P = \frac{\sqrt{\pi}}{2}\sqrt{\frac{T_r}{T_i}}\ \frac{\delta U^2}{S_r} \cos \psi$$

where S_r is the molecular speed ratio, T_r is the surface temperature, and T_i is the ambient temperature. For a given orbit where the altitude is a constant, let $P = K \cos \psi$ where K is a constant and

$$K = \frac{\sqrt{\pi}}{2}\sqrt{\frac{T_r}{T_i}}\ \frac{\delta U^2}{S_r}$$

The total differential force then, adding equations (11) and (12), is

$$\overline{dF} = \left[\delta U^2(-\sin \gamma\ \overline{i} - \cos \gamma\ \overline{k}) + K\, \frac{\overline{N}}{|\overline{N}|}\right] \cos \psi\ dA \tag{13}$$

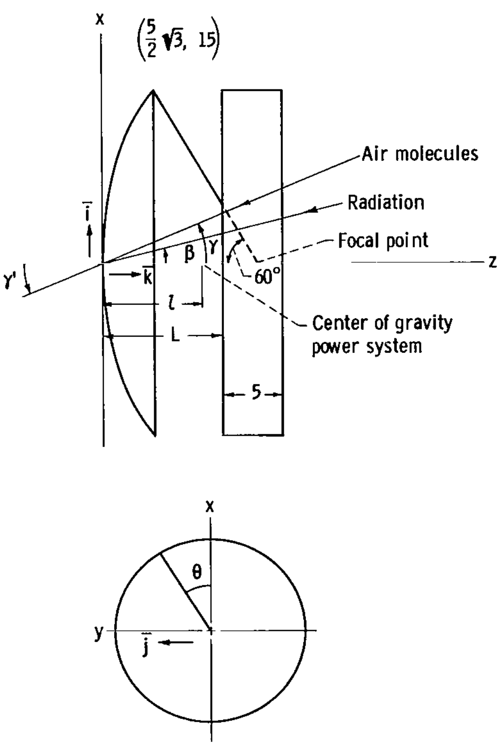

Figure 2. - Collector and radiator.

The power system has two components, the collector and the radiator, that contribute most of the surface area. Only these components, therefore, are considered in this analysis.

Collector. - The collector is shown schematically in figure 2. It is a paraboloid with axes x, y, and z fixed in the collector. The angle between a line joining the rim and the focal point and the z-axis is 60°. The radius of the collector is 15 feet, and since the slope of the surface at the rim in the x, z-plane is $\sqrt{3}$, the equation of the surface is

$$x^2 + y^2 - 30\sqrt{3}z = 0$$

The normal to the surface is the gradient of the surface. Therefore, for this collector the normal to the surface is the vector

$$\bar{N} = x\bar{i} + y\bar{j} - 15\sqrt{3}\,\bar{k} \tag{14}$$

where $\bar{i}$, $\bar{j}$, and $\bar{k}$ are unit vectors along the x, y, and z axes, respectively.

If the angle γ (eq. (13)) is the angle between the incident air molecules and the z-axis, parallel to the x,z-plane, then the unit vector representing the direction of the incident air is $-(\sin\gamma)\bar{i} - (\cos\gamma)\bar{k}$. When $0^\circ \leq \gamma \leq 60^\circ$, the surface striking the air molecules is concave. This means that some of the reflected molecules will strike the surface at other points. However, the concavity of the collector is small enough that this effect can be neglected. The cosine ψ is

$$\cos\psi = \frac{(x\bar{i} + y\bar{j} - 15\sqrt{3}\,\bar{k})\cdot(-\sin\gamma\,\bar{i} - \cos\gamma\,\bar{k})}{\sqrt{x^2 + y^2 + 675}}$$

$$= \frac{-x\sin\gamma + 15\sqrt{3}\cos\gamma}{\sqrt{x^2 + y^2 + 675}}$$

Expressed in cylindrical coordinates

$$\cos\psi = \frac{-r\cos\theta\sin\gamma + 15\sqrt{3}\cos\gamma}{\sqrt{r^2 + 675}} \tag{15}$$

An elemental area is

$$dA = \frac{r}{15\sqrt{3}}\sqrt{r^2 + 675}\,dr\,d\theta \tag{16}$$

Substituting equations (14), (15), and (16) into equation (13) yields

$$dF = \left[\delta U^2(-\sin\gamma\,\bar{i} - \cos\gamma\,\bar{k}) + \frac{K}{\sqrt{r^2 + 675}}(r\cos\theta\,\bar{i} + r\sin\theta\,\bar{j} - 15\sqrt{3}\,\bar{k})\right]$$

$$\left(-\frac{r^2}{15\sqrt{3}}\cos\theta\sin\gamma + r\cos\gamma\right)dr\,d\theta \tag{17}$$

The vector $\bar{\rho}$ of equation (10) is defined as the vector from the center of gravity of the power system to each elemental area. The center of gravity of the power system is assumed to lie along the axis of the collector and is located at $z = l$. Therefore,

$$\bar{\rho} = r\cos\theta\,\bar{i} + r\sin\theta\,\bar{j} + \left(\frac{r^2}{30\sqrt{3}} - l\right)\bar{k} \tag{18}$$

Substituting equations (17) and (18) into equation (10) yields

$$\bar{\tau} = \int_S \left\{ \left[-\delta U^2 r \cos\gamma \sin\theta - \frac{Kr \sin\theta}{\sqrt{r^2 + 675}} \left(15\sqrt{3} - l + \frac{r^2}{30\sqrt{3}} \right) \right] \bar{i} \right.$$

$$+ \left[\delta U^2 \left(r \cos\theta \cos\gamma - \frac{r^2 \sin\gamma}{30\sqrt{3}} + l \sin\gamma \right) \right.$$

$$\left. + \frac{Kr \cos\theta}{\sqrt{r^2 + 675}} \left(15\sqrt{3} - l + \frac{r^2}{30\sqrt{3}} \right) \right] \bar{j}$$

$$\left. + \left(\delta U^2 r \sin\theta \sin\gamma \right) \bar{k} \right\} \left(-\frac{r^2}{15\sqrt{3}} \cos\theta \sin\gamma + r \cos\gamma \right) d\theta \, dr$$

where the range of integration is $0 \leq r \leq 15$ and $0 \leq \theta \leq 2\pi$. Integrating θ from 0 to 2π (noticing that only terms containing $\cos^2\theta$ alone or which are independent of θ will not vanish) yields

$$\bar{\tau} = \frac{\pi}{15\sqrt{3}} \sin\gamma \int_0^{15} \left[\delta U^2 \cos\gamma \left(30\sqrt{3}\, lr - 2r^3 \right) \right.$$

$$\left. - K \frac{r^3}{\sqrt{r^2 + 675}} \left(15\sqrt{3} - l + \frac{r^2}{30\sqrt{3}} \right) \right] dr \, \bar{j}$$

and when integration is performed over r, the aerodynamic torque exerted on the collector is given by

$$\bar{\tau} = \frac{\pi}{\sqrt{3}} \left[\delta U^2 \, 112.5 \left(\sqrt{3}\, l - \frac{15}{2} \right) \sin 2\gamma + \frac{K}{15} \left(442\, l - 12\ 721 \right) \sin\gamma \right] \bar{j} \tag{19}$$

For a body in a 300-nautical-mile circular orbit, U is 4.71 miles per second, the air density is about 1.5×10^{-15} slugs per cubic foot (ref. 5), and K is 2.02×10^{-7} pounds per square foot. Using these values in equation (19) and assuming the center of gravity is at $l = 10$ feet, yield the aerodynamic torque

$$\bar{\tau} = (1.86 \times 10^{-3} \sin 2\gamma - 2.03 \times 10^{-4} \sin\gamma)\bar{j} \text{ ft-lb} \tag{20}$$

It is noted that this expression is valid only for $-60^\circ \leq \gamma \leq 60^\circ$. When $60^\circ < \gamma < 90^\circ$, the air molecules are striking partly the front and back of the collector, and when $90^\circ < \gamma < 120^\circ$, the air molecules strike part of the back of the collector. It is difficult to get an exact solution for these ranges of γ. When $120^\circ \leq \gamma \leq 240^\circ$, the air molecules are striking the entire back of the collector, and the formula for the torque is the negative of the torque derived previously for $-60^\circ \leq \gamma \leq 60^\circ$ (eq. (20)); γ is therefore replaced by γ', which is measured from the -z-axis (as shown in fig. 2, p. 9). When $\gamma = 90^\circ$, exactly half of the collector is bombarded by the air molecules; therefore, a calculation for this special case will give some insight into the case for $60^\circ < \gamma < 120^\circ$.

Since the air molecules are striking the back of the collector only, the normal to the surface is

$$\bar{N} = -r \cos \theta \, \bar{i} - r \sin \theta \, \bar{j} + 15\sqrt{3} \, \bar{k}$$

The cosine ψ is

$$\cos \psi = \frac{\bar{N} \cdot (-\bar{i})}{|\bar{N}|} = \frac{r \cos \theta}{\sqrt{r^2 + 675}}$$

Substituting these values for $\bar{N}$ and $\cos \psi$ into equation (13) yields the differential force

$$\overline{dF} = \frac{1}{15\sqrt{3}} \left[-\delta U^2 \bar{i} + \frac{K}{\sqrt{r^2 + 675}} \left(-r \cos \theta \, \bar{i} - r \sin \theta \, \bar{j} + 15\sqrt{3} \, \bar{k} \right) \right] r^2 \cos \theta \, dr \, d\theta \tag{21}$$

Substituting equations (18) and (21) into equation (10) yields

$$\bar{\tau} = \frac{1}{15\sqrt{3}} \int_s \left\{ \left[\frac{Kr \sin \theta}{\sqrt{r^2 + 675}} \left(15\sqrt{3} + \frac{r^2}{30\sqrt{3}} - l \right) \right] \bar{i} + \left[-\delta U^2 \left(\frac{r^2}{30\sqrt{3}} - l \right) - \frac{Kr \cos \theta}{\sqrt{r^2 + 675}} \left(15\sqrt{3} + \frac{r^2}{30\sqrt{3}} - l \right) \right] \bar{j} + \left(\delta U^2 r \sin \theta \right) \bar{k} \right\} r^2 \cos \theta \, dr \, d\theta$$

The range of integration in this case is $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $0 \leq r \leq 15$.

Integrating over θ from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$ yields

$$\bar{\tau} = \frac{1}{15\sqrt{3}} \int_0^{15} \left[-2\delta U^2 \left(\frac{r^4}{30\sqrt{3}} - l r^2\right) - \frac{\pi K}{2} \frac{r^3}{\sqrt{r^2 + 675}} \left(15\sqrt{3} + \frac{r^2}{30\sqrt{3}} - l\right)\right] dr\, \bar{j}$$

$$= \left[25\sqrt{3}\ \delta U^2 (2l - 3\sqrt{3}) + \frac{\pi K}{30\sqrt{3}} (442\, l - 12\ 721)\right] \bar{j} \tag{22}$$

If the center of gravity of the power system is at $l = 10$ feet, the torque in equation (22) is

$$\bar{\tau} = 0.50 \times 10^{-3}\ \bar{j}\ \text{ft-lb} \qquad \gamma = 90^{\circ} \tag{23}$$

It can be seen from equations (20) and (23) that the aerodynamic torque is cyclic. If the center of gravity is at $l = 10$ feet, the torque is positive for $0 < \gamma < 180^{\circ}$ and negative for $180^{\circ} < \gamma < 360^{\circ}$. The torque is zero for $\gamma = 0^{\circ}$ and 180°. The maximum occurs near $\gamma = 45^{\circ}$ and is 1.65×10^{-3} foot-pound.

Radiator. - The second structure to consider for aerodynamic torque is the radiator. It is located as shown in figure 2 (p. 9). The model for the radiator is a circular hollow cylinder 30 feet in diameter, 5 feet long, and located L feet from the origin.

The external surface is considered first. The air strikes the part $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ when the range of γ is $0 \leq \gamma \leq 180^{\circ}$. Since the surface is convex, the reflecting air does not reimpinge on the surface. The total differential force acting on an elemental area is given in equation (13). The normal to the surface is

$$\bar{N} = -r \cos\theta\, \bar{i} - r \sin\theta\, \bar{j} \tag{24}$$

The velocity of the air with respect to the body is

$$\bar{U} = U(-\sin\gamma\, \bar{i} - \cos\gamma\, \bar{k})$$

The cosine of the angle between these two vectors is

$$\cos\psi = \frac{\bar{U} \cdot \bar{N}}{|U||N|}$$

$$\cos\psi = \cos\theta \sin\gamma \tag{25}$$

An elemental area is

$$dA = r\, d\theta\, dz \tag{26}$$

Substituting equations (24), (25), and (26) into equation (13) yields

$$dF = \left[\delta U^2(-\sin\gamma\,\bar{i} - \cos\gamma\,\bar{k}) + K(-\cos\theta\,\bar{i} - \sin\theta\,\bar{j})\right](\cos\theta\sin\gamma)r\,d\theta\,dz \qquad (27)$$

The position vector from the center of gravity is

$$\bar{\rho} = r\cos\theta\,\bar{i} + r\sin\theta\,\bar{j} + (z - l)\bar{k} \qquad (28)$$

Substituting equations (27) and (28) into equation (10) yields

$$\bar{\tau} = \int_s \Big\{\left[-\delta U^2 r\cos\gamma\sin\theta + (z - l)K\sin\theta\right]\bar{i}$$

$$+ \left[-(z - l)\delta U^2\sin\gamma - (z - l)K\cos\theta + r\delta U^2\cos\gamma\cos\theta\right]\bar{j}$$

$$+ r\delta U^2\sin\gamma\sin\theta\,\bar{k}\Big\}(\cos\theta\sin\gamma)r\,d\theta\,dz$$

where the range of integration is $L \leq z \leq L + 5$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. When the integral is evaluated, the torque exerted on the external surface is given by

$$\bar{\tau} = \left\{\left[5(L - l) + \frac{25}{2}\right](-2\delta U^2\sin\gamma - \frac{\pi}{2}K) + 5\frac{\pi}{2}r\delta U^2\cos\gamma\right\}r\sin\gamma\,\bar{j}$$

$$(0 \leq \gamma \leq \pi) \qquad (29)$$

A portion of the interior of the radiator is exposed to the air except at $\gamma = 0$ and $\gamma = \pi/2$. The interior surface will be analyzed in two parts: first where $0 \leq \gamma \leq 80.5^\circ$ and $99.5^\circ \leq \gamma \leq 180^\circ$, and secondly where $80.5^\circ \leq \gamma \leq 99.5^\circ$. In the first range of γ, the air will be assumed to strike half of the interior surface; that is, $\pi/2 \leq \theta \leq 3\pi/2$. The error will be less than 20 percent with this assumption. For the second range of γ, the air strikes a projected area in the form of an ellipse with a major axis 30 feet and minor axis of 30 cot γ feet. Torque due to reflected air is not calculated because it is smaller than incident torque, and since the interior surface is concave, a large portion of the air will be reimpinging on the radiator surface. The differential force acting on an elemental area is

$$\overline{dF} = \delta U^2(-\sin\gamma\,\bar{i} - \cos\gamma\,\bar{k})(-\cos\theta\sin\gamma)r\,d\theta\,dz \qquad (30)$$

Substituting equations (28) and (30) into equation (10) gives

$$\bar{\tau} = \int_L^{L+5}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \Big\{-r\sin\theta\cos\gamma\,\bar{i} + \left[-(z - l)\sin\gamma + r\cos\theta\cos\gamma\right]\bar{j}$$

$$+ r\sin\gamma\sin\theta\,\bar{k}\Big\}\delta U^2(-\cos\theta\sin\gamma)r\,d\theta\,dz \qquad (31)$$

Evaluting the integral in equation (31) gives

$$\bar{\tau} = -\delta U^2 r \sin\gamma\left\{\left[10(L - l) + 25\right]\sin\gamma + \frac{5}{2} r\pi \cos\gamma\right\}\bar{j} \tag{32}$$

where $0 \leq \gamma \leq 80.5^{\circ}$ and $99.5^{\circ} \leq \gamma \leq 180^{\circ}$.

The second range of γ will now be considered ($80.5^{\circ} \leq \gamma \leq 99.5^{\circ}$). For this range of γ the range of variable z in integral (31) is changed to $L + 5 + 2r \cot\gamma \cos\theta \leq z \leq L + 5$ for $80.5^{\circ} \leq \gamma \leq 90^{\circ}$, and $L \leq z \leq L + 2r \cos\theta \cot\gamma$ for $90^{\circ} \leq \gamma \leq 99.5^{\circ}$. Evaluating integral (31) with this new range yields

$$\bar{\tau} = -\delta U^2 \pi r^2 (L - l + 5)\sin\gamma \cos\gamma\, \bar{j} \qquad 80.5^{\circ} \leq \gamma \leq 90^{\circ} \tag{33}$$

$$\bar{\tau} = \delta U^2 \pi r^2 (L - l)\sin\gamma \cos\gamma\, \bar{j} \qquad 90^{\circ} \leq \gamma \leq 99.5^{\circ} \tag{34}$$

If the radiator is located such that $L = 10$ feet, $l = 10$ feet, and $r = 15$ feet, the total torque on it in a 300-nautical-mile orbit is (by adding eq. (29) to eqs. (32), (33), and (34), respectively)

$$\bar{\tau} = \left(-6.98\times10^{-4} \sin^2\gamma - 0.60\times10^{-4} \sin\gamma\right)\bar{j} \text{ ft-lb}$$

$$0 \leq \gamma \leq 80.5^{\circ} \quad \text{and} \quad 99.5^{\circ} \leq \gamma \leq 180^{\circ} \tag{35a}$$

$$\bar{\tau} = \left(-3.49\times10^{-4} \sin^2\gamma - 16.5\times10^{-4} \sin\gamma \cos\gamma - 0.60\times10^{-4} \sin\gamma\right)\bar{j} \text{ ft-lb}$$

$$80.5^{\circ} \leq \gamma \leq 90^{\circ} \tag{35b}$$

$$\bar{\tau} = \left(-3.49\times10^{-4} \sin^2\gamma + 16.5\times10^{-4} \sin\gamma \cos\gamma - 0.60\times10^{-4} \sin\gamma\right)\bar{j} \text{ ft-lb}$$

$$90^{\circ} \leq \gamma \leq 99.5^{\circ} \tag{35c}$$

From the symmetry of the radiator it can be seen that the following relations exist for the range of γ, $180^{\circ} \leq \gamma \leq 360^{\circ}$:

$$\bar{\tau} = \left(6.98\times10^{-4} \sin^2\gamma - 0.60\times10^{-4} \sin\gamma\right)\bar{j} \text{ ft-lb}$$

$$180^{\circ} \leq \gamma \leq 260.5^{\circ} \quad \text{and} \quad 279.5^{\circ} \leq \gamma \leq 360^{\circ} \tag{35d}$$

$$\bar{\tau} = \left(3.49\times10^{-4} \sin^2\gamma + 16.5\times10^{-4} \sin\gamma \cos\gamma - 0.60\times10^{-4} \sin\gamma\right)\bar{j} \text{ ft-lb}$$

$$260.5^{\circ} \leq \gamma \leq 270^{\circ} \tag{35e}$$

$$\bar{\tau} = \left(3.49\times10^{-4} \sin^2\gamma + 16.5\times10^{-4} \sin\gamma \cos\gamma - 0.60\times10^{-4} \sin\gamma\right)\bar{j} \text{ ft-lb}$$

$$270^{\circ} \leq \gamma \leq 279.5^{\circ} \tag{35f}$$

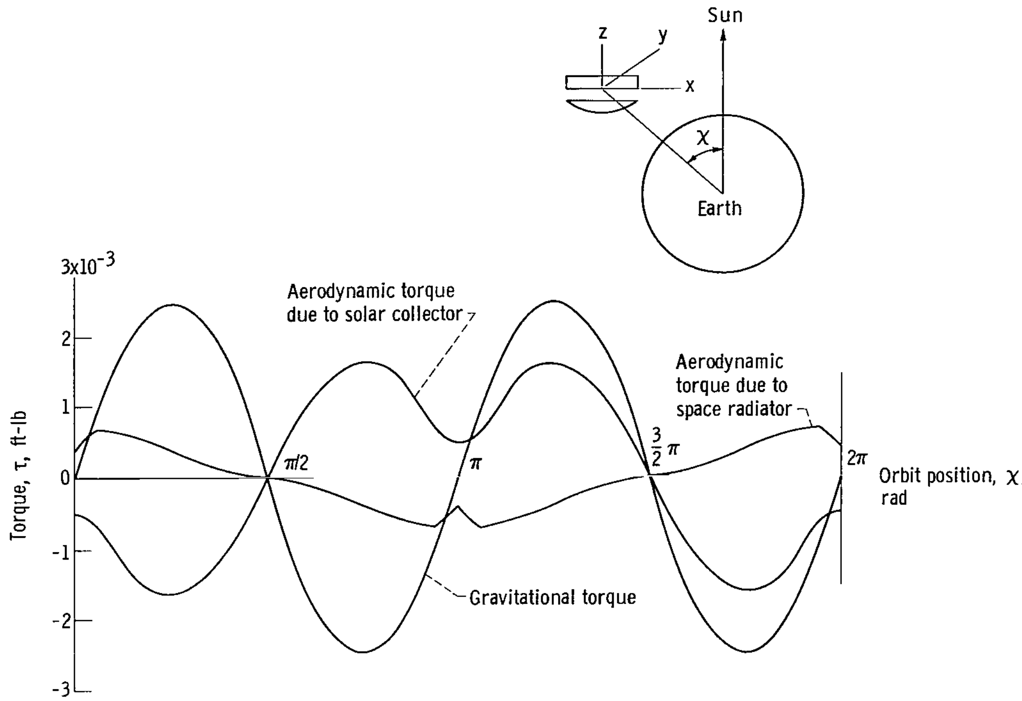

Figure 3. - Comparison of external torques against orbit position.

This torque can be expressed as a function of orbit position χ if the collector is properly oriented with respect to the Sun. The angle χ is related to γ as follows:

$$\chi = 90^{\circ} - \gamma \qquad 0 \leq \chi \leq 180^{\circ}$$

$$\chi = 270^{\circ} - \gamma \qquad \begin{cases} 180^{\circ} \leq \chi \leq 360^{\circ} \text{ for collector} \\ 0 \leq \chi \leq 360^{\circ} \text{ for radiator} \end{cases}$$

The relation between aerodynamic torque and orbit position for the collector and radiator is shown in figure 3. For comparison, the previously discussed gravitational torque is also shown as a function of orbit position χ where

$$\chi = 360^{\circ} - \varphi$$

Solar Radiation Torque

When a stream of radiation falls on a body, a pressure is exerted on that body. If the radiation makes an angle η with the normal to the surface, then the pressure on the surface is $S \cos^2\eta$, where S is the radiation pressure. The $\cos^2\eta$ term occurs because the components of both the force and the area on which the force acts must be considered. If the radiation is specularly reflected, the pressure is doubled since the change in momentum of the radiation is doubled.

The solar radiation torque is calculated for the solar collector and for the absorber. On the collector the radiation pressure is $2S \cos^2\eta$ assuming 100-percent reflection. On the absorber, the pressure is $S \cos^2\eta$ assuming 100-percent absorption. The pressure constant S includes light pressure, which is 9.4×10^{-8} pound per square foot, and solar wind pressure, which is 3.5×10^{-11} pound per square foot (ref. 6). Since solar wind pressure is negligible, the value of S used is 9.4×10^{-8} pound per square foot.

Collector. - When a paraboloidal collector is not alined with the Sun, a larger amount of pressure will be exerted on the segments that are more nearly normal to the Sun's radiation. A torque is produced by this unequal pressure.

The torque on the collector about the center of gravity is

$$\overline{\tau} = \int_S \overline{\rho} \times \overline{dF} \tag{10}$$

where $\overline{\tau}$, is the torque and $\overline{\rho}$ is the position vector from the center of gravity to the differential force $\overline{dF}$. The magnitude of the differential force is

$$dF = 2S \cos^2\eta \; dA \tag{36}$$

where S is the radiation pressure constant, η is the angle between the incident ray and the normal to the surface, and dA is an elemental area. Expressions for $\cos \eta$ and dA were given by equations (15) and (16), where γ is replaced by the misorientation angle β. Substituting these equations into equation (36) gives

$$dF = 2S \frac{\frac{r^3}{15\sqrt{3}} \sin^2\beta \cos^2\theta - 2r^2 \sin\beta \cos\beta \cos\theta + 15\sqrt{3}\, r \cos^2\beta}{\sqrt{r^2 + 675}} \, dr \, d\theta$$

The direction of the differential force is normal to the surface; the differential force vector being

$$\overline{dF} = \frac{\overline{N}}{|N|} dF$$

When the previous equation for $\overline{N}$ is used (eq. (14)), the differential force vector becomes

$$\overline{dF} = \left(r \cos\theta\,\overline{i} + r \sin\theta\,\overline{j} - 15\sqrt{3}\,\overline{k}\right) \frac{dF}{\sqrt{r^2 + 675}} \tag{37}$$

Substituting equation (37) into equation (10), along with the previous equation for $\overline{\rho}$ (eq. (18)), yields the torque expression

$$\bar{\tau} = \int_S \left(-\sin\,\theta\,\bar{i} + \cos\,\theta\,\bar{j}\right)\frac{r\left(\frac{r^2}{30\sqrt{3}} - l + 15\sqrt{3}\right)}{\sqrt{r^2 + 675}}\,dF$$

When integrating over θ, the torque becomes

$$\bar{\tau} = -2S\pi \int_0^{15} \frac{\sin\,\beta\,\cos\,\beta}{r^2 + 675}\left[\frac{r^5}{15\sqrt{3}} + \left(30\sqrt{3} - 2l\right)r^3\right]dr\,\bar{j}$$

When integrating over r, the torque equation is

$$\bar{\tau} = 2S\left(-1390 + 48.4\,l\right)\sin\,2\beta\,\bar{j}$$

If the value of $S = 9.4\times10^{-8}$ pound per square foot is used, the torque is

$$\bar{\tau} = \left(-2.61\times10^{-4} + 0.91\times10^{-5}\,l\right)\sin\,2\beta\,\bar{j}\ \text{ft-lb} \tag{38}$$

This is the solar radiation torque about an axis perpendicular to the x,z-plane and passing through the point $(0,0,l)$. It is noted that the torque expression is valid only for $0 \leq \beta \leq 30^\circ$, since for $\beta > 30^\circ$ part of the reflected rays strike the collector's surface again. However, if the collector is properly alined throughout the orbit, β is always small (less than $1/4^\circ$).

Equation (38) indicates that solar radiation torque is small even for large misorientation angles. For example, with the center of gravity at $l = 10$ feet and with misalinement of 5°, the solar radiation torque is still only 3.0×10^{-5} foot-pound, which is several orders of magnitude less than gravitational and aerodynamic torque.

Absorber. - When the collector is misorientated, the solar radiation on the absorber will cause a torque about the center of gravity of the power system. The torque is calculated for a flat plate located at the focal point of the collector and perpendicular to the axis of the collector, rather than for a hemispherical cavity as the absorber actually is.

In order to determine the torque, it is assumed that during misorientation the image is displaced from the axis of the collector by the distance $7.5\sqrt{3}\,\tan\,\beta$ feet on the focal plane. The image is assumed to keep its original size and original light intensity distribution. These assumptions will limit β to 1° or less. With these conditions the torque is the product of the net force on the focal plane at zero misorientation and the distance away from the collector axis, which is $7.5\sqrt{3}\,\tan\,\beta$ feet.

To find the net force, the force from a solar ray reflected from a differential area on the collector is multiplied by the cosine of the angle between the direction of the reflected ray and the normal to the focal plane. The cosine of this angle can be written as a function of the cylindrical coordinates of the collector. For a ray striking the collector at distance r from the collector axis, the cosine is $(675 - r^2)/(675 + r^2)$. The net force from solar rays striking the collector at r is

$$S2\pi r \frac{675 - r^2}{675 + r^2} dr$$

The total net force from the collector is

$$F = \int_0^{15} S2\pi r \frac{675 - r^2}{675 + r^2} dr$$

Solving for F and using the value $S = 9.4\times10^{-8}$ pound per square foot give

$$F = 4.82\times10^{-5} \text{ lb}$$

The torque then is the product of F and $7.5\sqrt{3} \tan \beta$:

$$\bar{\tau} = 6.27\times10^{-4} \tan \beta \, \bar{j} \tag{39}$$

The torque vector has the direction of the positive y-axis when β is as shown in figure 2 (p. 9).

The total torque exerted on the collector-absorber unit is the sum of the individual torques. If β is always small (less than 5°), $\sin \beta$ and $\tan \beta$ can be replaced by β. Therefore the sum of the torques, adding equations (38) and (39) (where $l = 10$ ft), becomes

$$\begin{aligned}\bar{\tau} &= \left(-3.40\times10^{-4} \beta + 6.27\times10^{-4} \beta\right)\bar{j} \\ &= 2.87\times10^{-4} \beta\bar{j}\end{aligned}$$

INTERNAL TORQUE

Internal torque is due to acceleration of the turbomachinery. For the system considered there are two such machines, a turboalternator and turbocompressor. Torques must be exerted on these machines to change their angular velocity. Equal and opposite torques are exerted on the power system. The torques are equal to the product of the moment of inertia of the rotating shaft and the angular acceleration of the shaft. The torques considered are only those which occur in transients from design point operation; torques in startup are not calculated.

The turboalternator will be considered first. If there is a load change on the alternator, there will be a transient in rotating speed before the speed control can reestablish the speed at the design value. Typically, the speed may vary as much as 1 percent from design in 1 second. The angular acceleration α, then, would be 12.57 radians per second^2 for a design speed of 12 000 rpm. A moment of inertia of 0.065 foot-pound second^2 is representative of a turbo-alternator sized for the collector and radiator used in the discussion of external torques. The torque, therefore, occuring during a typical alternator load change is 0.816 foot-pound.

The turbocompressor has a greater speed range; however, the changes in speed are slower. The torque will be calculated for a 10-percent change in speed from design in 60 seconds. For this unit, a design speed of 38 500 rpm and a moment of inertia of 0.003 foot-pound second^2 are representative. The turbocompressor torque, therefore, is 0.02 foot-pound.

The torque needed to turn the alternator and compressor and to overcome bearing friction is experienced only inside the turbine packages. The vehicle does not have a torque exerted on it unless the angular momentum of the turbine packages changes.

It should be noted that the internal torques are much larger than the external torques. Of more significance is the change in angular momentum of the turbomachinery. The maximum change in angular momentum from steady-state operation is 1.20 foot-pound second for the turbocompressor and 0.82 foot-pound second for the turboalternator.

However, if the turbomachinery axes are alined parallel to the axis of symmetry of the collector and radiator, the internal torque only causes the power system to rotate about the z-axis and does not cause misorientation. If the turbomachinery axes are not alined parallel to the axis of the collector, then internal torque does cause misorientation, and therefore, of prime interest, is the torque required to keep the collector alined within $1/4^\circ$.

Consider the turbomachinery axes to be alined perpendicular to the collector axis; during a speed change in the turbocompressor, the maximum change in angular velocity of the power system is 0.31×10^{-3} radian per second, which is the quotient of the change in angular momentum (1.21 ft-lb sec) and the moment of inertia of the power system about the y-axis, as shown in figure 1 (p. 6), with dimensions $l = 9$ feet, $h = 15$ feet, and $M = 1500$ pounds. The torque then is the product of the moment of inertia and the angular acceleration α, which is that acceleration necessary to change the angular velocity of the power system by 0.31×10^{-3} radian per second without misorienting more than $1/4^\circ$.

The misorientation angle β is

$$\beta = \frac{1}{2}\alpha t^2$$

The angular velocity $\dot{\beta}$ is

$$\dot{\beta} = \alpha t$$

Eliminating time t gives

$$\beta = \frac{1}{2}\frac{\dot{\beta}^2}{\alpha}$$

Solving for angular acceleration α yields

$$\alpha = \frac{1}{2}\frac{\dot{\beta}^2}{\beta}$$

The maximum misorientation angle allowed is

$$\beta = \frac{1}{4}^{\circ} = 0.00436 \text{ rad}$$

When $\beta = 1/4^{\circ}$, $\dot{\beta} = 0.31\times10^{-3}$ radian per second and then $\alpha = 1.10\times10^{-5}$ radian per second^2. The torque is

$$\tau = I\alpha = 4.3\times10^{-2} \text{ ft-lb}$$

Therefore, if a torque of 4.3×10^{-2} foot-pound is applied to the power system, the collector will not misorient more than $1/4^{\circ}$ as a result of a change of speed of the turbocompressor. The corresponding torque for the turboalternator is 2.0×10^{-2} foot-pound.

TORQUE COMPENSATING FOR VARIATIONS IN ANGULAR VELOCITY

When the power system is in orbit about the Earth, its angular velocity with respect to the Sun is constantly changing. In order to aline the collector with the Sun, the angular velocity of the power system about its center of gravity must be the same as the angular velocity of its center of gravity with respect to the Sun. Therefore, a torque must be applied to the power system to change its angular velocity and keep it alined with the Sun.

There are two effects to consider, the changing angular velocity of the power system as it orbits the Earth and the changing angular velocity of the power system as it orbits the Earth in orbit about the Sun. The maximum angle that the collector misorients due to an orbit about the Earth can be shown to be small. If the collector is assumed to have a fixed orientation in space, the maximum misorientation angle β is

$$\beta \approx \frac{2c}{n}$$

where $2c$ is the diameter of the orbit about the Earth and n is the distance from the Earth to the Sun. For a 300-nautical-mile ecliptic orbit, β is 20 seconds of arc.

For the solar Brayton power system this misorientation angle of 20 seconds is considerably less than the allowed misorientation angle and requires no correction. However, because there may be systems that require greater orientation

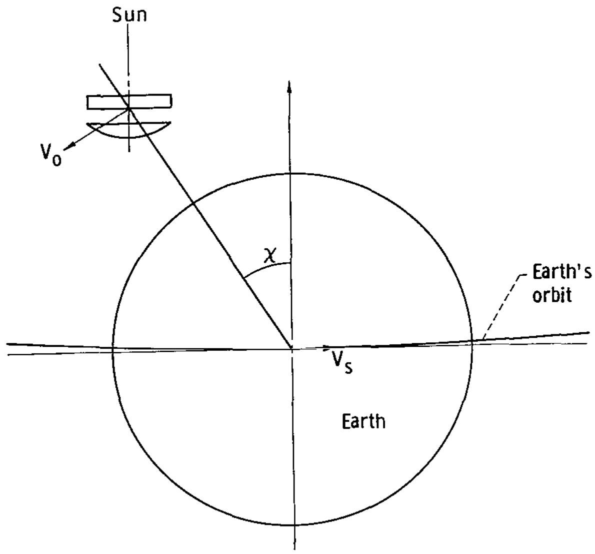

Figure 4. - Orbit of power system.

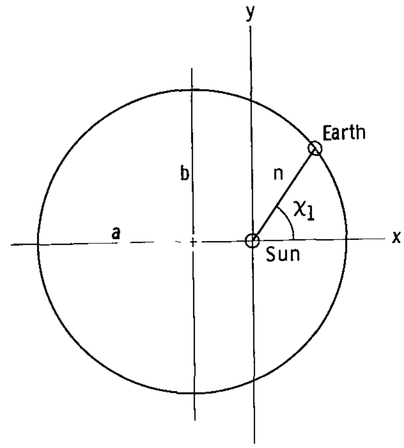

Figure 5. - Earth's orbit.

accuracy than 20 seconds, and because the variations in the angular velocity of the Earth orbiting the Sun cause the collector to misorient more than is allowable for the power system, a general discussion of these two effects of variations in angular velocity follows.

The velocity vector $\overline{V}$ of the power system in inertial space consists of two parts: the velocity vector $\overline{V}_o$ of the power system about the Earth and the velocity vector $\overline{V}_s$ of the Earth about the Sun (as shown in fig. 4). The angular velocity of the power system with respect to the Sun ω is the tangential component of the velocity divided by the distance from the Sun to the power system:

$$\omega = \frac{V_{ot}}{n} + \frac{V_{st}}{n} \tag{40}$$

where n is the distance from the Earth to the Sun. The tangential velocity component of $\overline{V}_o$ is $V_{ot} = -V_o \cos X$. Therefore, the angular velocity ω from equation (40) becomes

$$\omega = \frac{-V_o \cos X}{n} + \frac{V_{st}}{n} \tag{41}$$

where V_{st}/n is the angular velocity of the Earth with respect to the Sun. The angular velocity of the Earth with respect to the Sun (V_{st}/n) can be derived from Kepler's law, which states that the radius vector $\overline{n}$ sweeps out equal areas in equal time (see fig. 5 for definition of n and X_1). Therefore,

$$\frac{n^2}{2}\, dX_1 = k\, dt$$

$$\frac{dX_1}{dt} = \frac{2k}{n^2} \qquad (42)$$

where $k = \pi ab$ unit area per year. In this equation dX_1/dt is the angular velocity of the Earth with respect to the Sun which is equal to V_{st}/n.

In polar coordinates the equation of the Earth's orbit as shown in figure 5 is

$$\frac{(n \cos X_1 + ae)^2}{a^2} + \frac{n^2 \sin^2 X_1}{b^2} = 1$$

where e is the eccentricity of the ellipse. Solving for n gives

$$n = \frac{ab^2(1 - e \cos X_1)}{b^2 \cos^2 X_1 + a^2 \sin^2 X_1} \qquad (43)$$

Substituting equation (43) into equation (42) yields

$$\frac{dX_1}{dt} = \frac{2\pi}{(1 - e^2)^{3/2}} (1 + e \cos X_1)^2 \text{ rad/yr} \qquad (44)$$

When equation (44) is substituted into equation (41), the total angular velocity with respect to the Sun is

$$\omega = \frac{-V_o \cos X}{n} + \frac{2\pi}{(1 - e^2)^{3/2}} (1 + e \cos X_1)^2 \qquad (45)$$

If equation (45) is differentiated, the angular acceleration α is

$$\alpha = \dot{\omega} = \frac{V_o(\sin X)\dot{X}}{n} - \frac{8\pi^2 e}{(1 - e^2)^3} (1 + e \cos X_1)^3 \sin X_1$$

For a collector in a 300-nautical-mile ecliptic orbit, $V_o = 4.71$ miles per second, $\dot{X} = 0.00109$ radian per second, $e = 0.0167$, and $n = 93\,000\,000$ miles. Therefore, the angular acceleration with respect to the Sun is

$$\alpha = 5.52\times10^{-11} \sin X - 1.32\times10^{-15} (1 + e \cos X_1)^3 \sin X_1 \text{ rad/sec}^2 \qquad (46)$$

The torque needed to turn the power system has two components: torque required to turn the mass of the system (τ_1) and the gyroscopic torque required to turn the rotating turbomachinery masses (τ_2). The first component of torque is $\tau_1 = I\alpha$, where α is the angular acceleration about the center of gravity which is equal to the angular acceleration of the power system with respect to the Sun. The second term in the acceleration (eq. (46)) is negligible. The value of the moment of inertia I about the y-axis for the power system model shown in figure 1 (p. 6) is 3.9×10^3 foot-pound second^2, if $l = 9$ feet, $h = 15$ feet, and $M = 1500$ pounds. Therefore, the torque τ_1 is $2.15\times10^{-7} \sin X$ foot-pound.

The second (gyroscopic) component of torque is $\tau_2 = Ip\omega$ where I is the moment of inertia of the rotating mass and p is its angular speed. From equation (45), assuming e is zero, the angular velocity ω is

$$\omega = \left(-5.06 \cos X + 19.92\right)10^{-8} \text{ rad/sec}$$

The gyroscopic torque due to the turboalternator is

$$\tau_{2A} = \left(-4.13 \cos X + 16.27\right)10^{-6} \text{ ft-lb}$$

and that due to the turbocompressor is

$$\tau_{2C} = \left(-6.11 \cos X + 24.1\right)10^{-7} \text{ ft-lb}$$

If the turboalternator and the turbocompressor are rotating in the same direction, then the total gyroscopic torque is

$$\tau_2 = \left(-4.74 \cos X + 18.68\right)10^{-6} \text{ ft-lb}$$

The torques τ_1 and τ_2 are perpendicular to each other. The torque τ_1 is perpendicular to the ecliptic plane and τ_2 is in the ecliptic plane. Therefore, the torques must be added vectorially to get the total torque. When τ_1 and τ_2 are added vectorially, it can be seen that the maximum value of the total torque required to compensate for variations in angular velocity is less than 0.01 of either of the large external torques (aerodynamic or gravitational) and is negligible.

SUMMARY OF RESULTS

Three types of torques exerted on a solar Brayton system in orbit were studied. They were external torque, internal reaction torque, and torque compensating for variations in angular velocity.

Two external torques were found to be dominant, the gravitational and the aerodynamic torques. Both are periodic with gravitational torque having a maximum of 2.5×10^{-3} foot-pound and aerodynamic torque having a maximum of 1.65×10^{-3} foot-pound for the collector and 0.75×10^{-3} foot-pound for the radiator. These torques will be at their maximum four times in each orbit. The solar radiation torque is several orders of magnitude less than the above torques even for large misorientation angles.

The internal torques were the largest considered. The torque due to the acceleration of the turboalternator may be as high as 0.82 foot-pound during a typical alternator load change. The torque due to a typical acceleration of the turbocompressor may reach 0.02 foot-pound. These torques do not misorient the collector if the turbomachinery axes are alined parallel to the collector axis. If the turbomachinery axes are alined perpendicular to the collector axis, torques of 4.3×10^{-2} and 2.0×10^{-2} foot-pound for the turbocompressor and turboalternator, respectively, are required to keep the collector alined within $1/4^{\circ}$.

Torque compensating for variations in angular velocity was found to be less than 0.01 of either of the dominant external torques and therefore is negligible.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, August 27, 1965.

APPENDIX - EVALUATION OF GRAVITATIONAL TORQUE INTEGRALS

The first integral to evaluate from equation (8) is

$$\int_{-d}^{d}\int_{0}^{h}\int_{0}^{2\pi} \frac{-Zr\ d\theta\ dr\ dZ}{\left(c^2 + Z^2 + r^2 + 2cZ\cos\varphi + 2cr\sin\varphi\cos\theta\right)^{3/2}}$$

As θ varies from 0 to 2π the function varies from

$$\frac{-Zr}{\left(c^2 + Z^2 + r^2 + 2cZ\cos\varphi + 2cr\sin\varphi\right)^{3/2}}$$

to

$$\frac{-Zr}{\left(c^2 + Z^2 + r^2 + 2cZ\cos\varphi - 2cr\sin\varphi\right)^{3/2}}$$

The average value of the function is about

$$\frac{-Zr}{\left(c^2 + Z^2 + r^2 + 2cZ\cos\varphi\right)^{3/2}}$$

Therefore, integrating over θ, the integral becomes approximately

$$2\pi\int_{-d}^{d}\int_{0}^{h} \frac{-Zr\ dr\ dZ}{\left(c^2 + Z^2 + r^2 + 2cZ\cos\varphi\right)^{3/2}}$$

Integrating with respect to r yields

$$2\pi\int_{-d}^{d}\int_{0}^{h} \frac{-Zr\ dr\ dZ}{\left(c^2 + Z^2 + r^2 + 2cZ\cos\varphi\right)^{3/2}}$$

$$= 2\pi\int_{-d}^{d}\left[\frac{1}{\left(c^2 + Z^2 + r^2 + 2cZ\cos\varphi\right)^{1/2}}\right]_{0}^{h} Z\ dZ$$

Since h is negligible compared to c, $(c^2 + Z^2 + h^2 + 2cZ \cos \varphi)^{1/2}$ becomes

$$(c^2 + Z^2 + 2cZ \cos \varphi)^{1/2} = (c + Z \cos \varphi)$$

Then,

$$2\pi \int_{-d}^{d} \int_{0}^{h} \frac{-Zr\, dr\, dZ}{(c^2 + Z^2 + r^2 + 2cZ \cos \varphi)^{3/2}} \approx -\pi h^2 \int_{-d}^{d} \frac{Z\, dZ}{(c + Z \cos \varphi)^3}$$

Integrating with respect to Z gives

$$-\pi h^2 \int_{-d}^{d} \frac{Z\, dZ}{(c + Z \cos \varphi)^3} = - \frac{\pi h^2}{\cos^2 \varphi} \left[- \frac{1}{(c + Z \cos \varphi)} + \frac{c}{2(c + Z \cos \varphi)^2} \right]_{-d}^{d}$$

Since d is negligible compared to c, $(c^2 - d^2 \cos^2 \varphi)$ becomes c^2, then the first integral in equation (8) becomes

$$\frac{2\pi h^2\, d^3}{c^4} \cos \varphi$$

The second integral to evaluate from equation (8) is

$$\int_{-d}^{d} \int_{0}^{h} \int_{0}^{2\pi} \frac{r^2 \cos \theta\, d\theta\, dr\, dZ}{(c^2 + Z^2 + r^2 + 2cZ \cos \varphi + 2cr \sin \varphi \cos \theta)^{3/2}}$$

Since Z^2 and r^2 are negligible in comparison to c^2, the integral can be written as

$$2 \int_{-d}^{d} \int_{0}^{h} \int_{0}^{\pi/2} \left[\frac{1}{(c^2 + 2cZ \cos \varphi + 2cr \cos \theta \sin \varphi)^{3/2}} \right.$$

$$\left. - \frac{1}{(c^2 + 2cZ \cos \varphi - 2cr \cos \theta \sin \varphi)^{3/2}} \right] r^2 \cos \theta\, d\theta\, dr\, dZ$$

Expanding the two terms in the bracket by the binomial theorem and neglecting higher order terms enable the integral to be rewritten as follows:

$$-12 \int_{-d}^{d} \int_{0}^{h} \int_{0}^{\pi/2} \frac{r^3 \cos^2\theta}{c^4} \sin\varphi \, d\theta \, dr \, dZ$$

Integrating with respect to θ gives

$$-12 \int_{-d}^{d} \int_{0}^{h} \int_{0}^{\pi/2} \frac{r^3 \cos^2\theta}{c^4} \sin\varphi \, d\theta \, dr \, dZ$$

$$= \frac{-3\pi}{c^4} \sin\varphi \int_{-d}^{d} \int_{0}^{h} r^3 \, dr \, dZ$$

Integrating with respect to r gives

$$-3 \frac{\pi}{c^4} \sin\varphi \int_{-d}^{d} \int_{0}^{h} r^3 \, dr \, dZ = -\frac{3\pi}{4} \frac{h^4}{c^4} \sin\varphi \int_{-d}^{d} dZ$$

Integrating with respect to Z gives

$$-\frac{3\pi}{4} \frac{h^4}{c^4} \sin\varphi \int_{-d}^{d} dZ = -\frac{3\pi}{2} \frac{h^4}{c^4} d \sin\varphi$$

The torque in equation (8) becomes

$$\bar{\tau} = \delta g R^2 \pi \frac{h^2}{c^3} d\left(2 d^2 - \frac{3}{2} h^2\right) \sin\varphi \cos\varphi \, \bar{j}$$

The mass of the cylinder is

$$M = 2\pi h^2 d\delta$$

The torque, therefore, is

$$\bar{\tau} = \frac{1}{2} Mg \frac{R^2}{c^3} \left(d^2 - \frac{3}{4} h^2\right) \sin 2\varphi \, \bar{j} \qquad (9)$$

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