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by Kenneth J. Baumeister and Thomas D. Hamill

Lewis Research Center
Cleveland, Ohio

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SUMMARY

The mass evaporation rates and heat transfer coefficients are determined theoretically for liquid drops that are supported by their own superheated vapor over a flat hot plate.

The liquid drops are assumed to have a flat disk geometry with a uniform vapor gap beneath the drop and a saturated vapor cover. The assumptions are made that the bottom of the drop is at the saturation temperature and that evaporation takes place uniformly beneath the drop.

For steady-state laminar incompressible flow, assuming constant properties, the Navier-Stokes equations with inertia terms neglected, the continuity equation, and the energy equation are solved simultaneously to obtain the heat-transfer coefficient.

$$h_i = 0.68(k^3\lambda_i^* g\rho_\ell\rho_v/\Delta T\mu L_e)^{1/4}$$

For heat transfer to the drop

$$\lambda_\ell^* = \lambda\left[1 + (7C_p\,\Delta T/20\lambda)\right]^{-3}$$

For heat transfer from the plate

$$\lambda_p^* = \lambda\left[1 + (7C_p\,\Delta T/20\lambda)\right]^5 \Big/ \left[1 + (1/5)(C_p\,\Delta T/\lambda)\right]^4$$

where h is the heat-transfer coefficient, k is the thermal conductivity of the vapor, ρ is the density, μ is the absolute viscosity, L_e is the equivalent geometry factor, and λ is the heat of vaporization.

These closed form results agree with the value of the heat-transfer coefficient previously obtained by a computer solution of the exact Navier-Stokes equations. The earlier work was in excellent agreement with experiments. The conclusion is drawn that inertia terms in the Navier-Stokes equation are negligible relative to the viscous terms.

INTRODUCTION

Investigations of heat transfer to liquid drops that are supported by their own superheated vapor were begun as far back as 1756 when Leidenfrost (ref. 1) described the phenomenon of film boiling. At present, references 2 and 3 indicate considerable interest in the subject of drop vaporization in the fields of internal combustion engines and metallurgy.

Dimensional and semiempirical correlations for the evaporation of liquid drops in film boiling on a flat plate are presented in references 4 and 5. Reference 6 presents a solution of the exact Navier-Stokes equation and the energy equation; however, a grapical approach was used to combine the energy and momentum equations. Although this approach gives a physical insight into the coupling between the momentum and energy equations, the relation of the heat-transfer coefficient to the basic fluid properties is obscured.

Reference 7 shows how the computer solution to the exact Navier-Stokes equation and energy equation can be combined to yield a closed form solution for the heat-transfer coefficient h. The resulting form of the heat-transfer coefficient is similar in nature to that derived by Bromley (ref. 8) for heat transfer in stable film boiling. Bromley's analysis is based on the solution of the momentum equations in which the inertia terms have been assumed negligible.

The similarity between the results of references 7 and 8 led to the observation that a closed-form solution of the governing equations without the use of computer solutions could be obtained by neglecting inertia effects (creeping flow).

Solution of the Navier-Stokes equations neglecting inertia terms is obtained, and the results are compared to the computer solutions, which had considered the inertia effects.

SYMBOLS

A	area, ft^2
a	constant of proportionality, sec^{-1}
C	constant
C_p	specific heat of vapor at constant pressure, Btu/(lb mass)($^{\circ}$F)
$\bar{e}_z$	unit vector in upward z-direction
F	force, lb force
f	transformation variable, ft/sec

G	axial pressure variable, sq ft
g	acceleration of gravity, ft/sec^2
g_c	dimensional conversion factor, 32.1739 $(ft)(lb_{mass})/(lb_{force})(sec^2)$
h	heat-transfer coefficient, $Btu/(hr)(ft^2)(^oF)$
k	thermal conductivity of vapor, $Btu/(hr)(ft)(^oF)$
L_e	equivalent geometry factor, ft (see eq. (31))
ℓ	average calculated drop thickness, ft
M	mass of drop, lb mass
P	static pressure, lb $force/ft^2$
P_o	environmental pressure, lb $force/ft^2$
Pr	Prandtl number
q	rate of heat flow, $Btu/(hr)(ft^2)$
r	radius, ft
r_o	maximum radius of drop, ft
T	temperature, ^oF
ΔT	$T_p - T_{sat}$, ^oF
t	time, sec
u	radial velocity, ft/sec
V	drop volume, ft^3
W	velocity, ft/sec
w	axial velocity, ft/sec
z	axial coordinate, ft
α	thermal diffusivity of vapor, $\alpha \equiv k/\rho_v C_p$, ft^2/sec
δ	gap thickness, ft
η	z/δ
λ	heat of vaporization, Btu/lb mass
λ^*	modified heat of vaporization, Btu/lb mass
μ	absolute viscosity of vapor, lb mass/(ft)(sec)
ν	kinematic viscosity of vapor, ft^2/sec

ξ dummy variable, ft

ρ density, lb mass/ft^3

τ dummy variable, ft

Subscripts:

f film

i index

ℓ liquid

p plate

sat evaluated at saturation condition

v vapor

z axial coordinate

δ evaluated at lower surface of drop

Superscripts:

' derivative with respect to independent variable

— vector

BASIC MODEL AND EQUATIONS

The model used in this study applies to large drops for which a flat disk model (fig. 1), can be used. For example, references 5 to 7 indicate that for water drops in the volume range 0.05 to 1 cubic centimeter, the analytical model based on a flat disk geometry reasonably satisfies the physical situation.

The following assumptions are made in developing the analytical model:

(1) Heat transfer and evaporation from the upper surface are considered negligible in comparison to rates of transport beneath the drop.

(2) Radiation is neglected.

(3) Physical observation indicates that the gap thickness is a function of radial position; however, to make the problem mathematically tractable, a uniform gap thickness δ is assumed.

(4) In a similar manner, the geometric shape of the drop is approximated by a flat disk with constant thickness ℓ, where ℓ is defined by the equation

$$\ell = \frac{V}{\pi r_o^2} \tag{1}$$

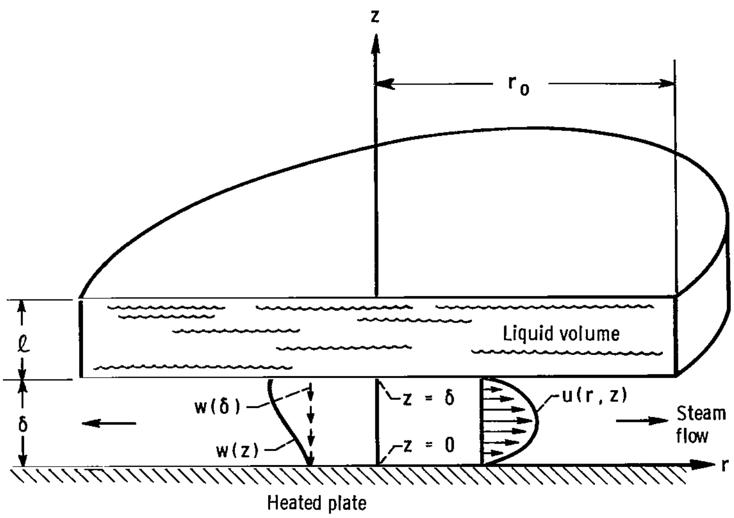

Figure 1. - Schematic model of evaporation of flat disk.

The relation between V and r_o

$$V = V(r_o) \tag{2}$$

was found numerically by balancing the gravitational force against the surface tension force (ref. 6).

(5) Because of a low calculated Reynolds number (ref. 6), the flow is assumed laminer and incompressible with negligible energy dissipation. In addition, the body force of the vapor in the momentum equation has been neglected.

(6) The inertia terms in the Navier-Stokes equation are neglected. Justification for this assumption is given in the main text of the report, as well as in appendix A.

(7) In the energy equation, it is assumed that

$$u \frac{\partial T}{\partial r} << w \frac{\partial T}{\partial z}; \; \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) << \frac{\partial^2 T}{\partial z^2} \tag{3}$$

(8) The velocity and temperature profiles at any instant are assumed to be in steady state.

(9) The bottom of the drop is at the saturation temperature.

(10) Evaporation takes place uniformly beneath the drop.

(11) The properties of the flow field are evaluated at the film temperature, defined as

$$T_f = \frac{T_p + T_{sat}}{2} \tag{4}$$

Consequently, for this case of axisymmetric flow, the governing equations are as follows:

(1) Momentum,

$$0 = -\frac{g_c}{\rho_v}\frac{\partial P}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{5}$$

$$0 = -\frac{g_c}{\rho_v}\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) \tag{6}$$

(2) Continuity,

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

(3) Energy,

$$w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2} \tag{8}$$

for the boundary conditions

$$z = 0 \qquad u = 0 \qquad w = 0 \qquad T = T_p \tag{9}$$

$$z = \delta \qquad u = 0 \qquad w = w(\delta) \qquad T = T_{sat} \tag{10}$$

$$r = 0 \qquad u = 0 \tag{11}$$

$$r = r_o \qquad z = 0 \qquad P = P_o \tag{12}$$

(4) Static-force balance (neglecting reactive force, see appendix B),

$$\int_0^{r_o} P(r, \delta) 2\pi r \, dr = V\rho_\ell \frac{g}{g_c} + \pi r_o^2 P_o \tag{13}$$

(5) Interface energy balance,

$$- \rho_v \lambda w(\delta) = - k \left. \frac{\partial T}{\partial z} \right|_{z=\delta} \tag{14}$$

The static-force balance has a physical interpretation; in that the weight of the drop is assumed equal to the force resulting from the pressure difference between the upper and lower surface of the drop. Mathematically, equations (13) and (14) are necessary to make the problem determinate, since the values of δ and $w(\delta)$ in boundary condition (eq. (10)) are unknown.

The solution involving the above set of differential equations and boundary conditions will now be performed. Figure 2 presents a solution flow chart of the problem. This chart may be used as an aid in understanding the analysis now presented.

The differential equations (5) to (8) contain two independent and four dependent variables. The boundary conditions specified by equations (9) to (14) do not make the problem determinate at this time. For example, additional boundary conditions on w are required at the radial boundaries; however, as seen in figure 2, a similarity analysis is

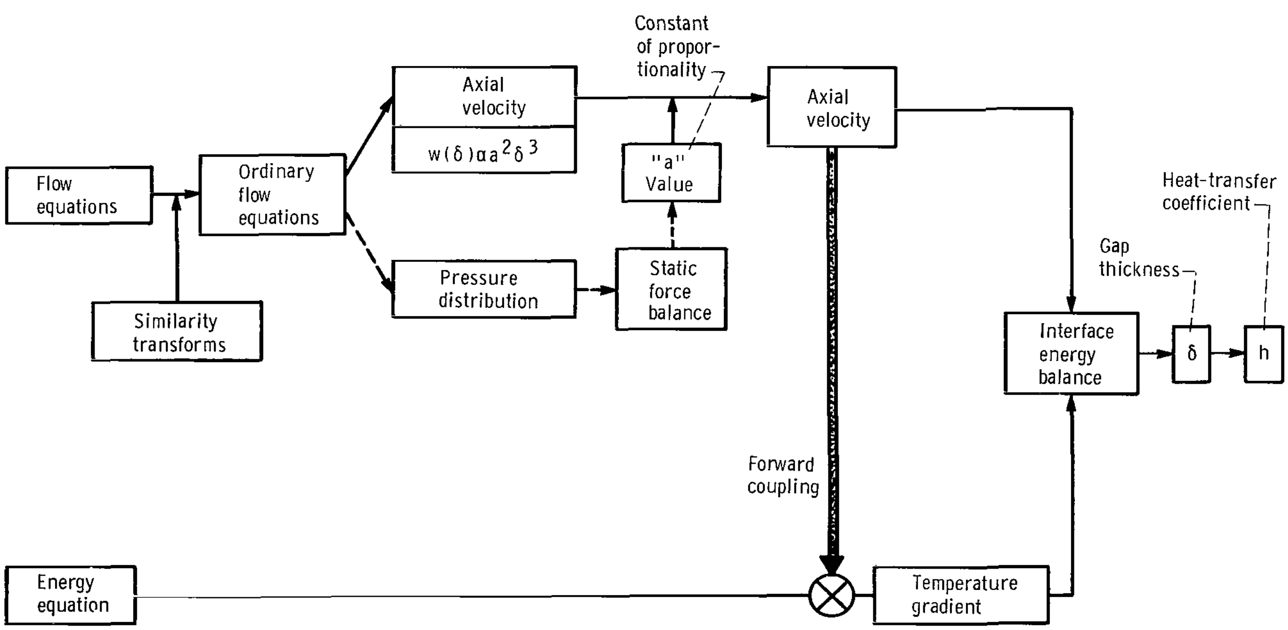

Figure 2. - Solution flow chart.

used in the solution. The similarity transforms will reduce the order of the equation and will satisfy the boundary conditions so that the problem becomes determinant.

METHOD OF SOLUTION

Momentum Equations

By assuming constant fluid properties, the interaction of the equations of motion with the energy equation ceases, and the velocity field no longer depends on temperature. A similarity transform is now used that reduces the partial differential equations (5) to (7) into a set of ordinary differential equations. Since the governing equations and boundary conditions between the three-dimensional stagnation flow problem and this problem are similar, the transforms for the three-dimensional stagnation flow problem are used (ref. 9):

$$w = -2f(z) \tag{15}$$

$$u = rf'(z) \tag{16}$$

$$P = \frac{1}{2} \frac{a^2 \rho_v}{g_c} \left[r_o^2 - r^2 + G(z)\right] \tag{17}$$

These transformations satisfy the requirements of reducing the partial differential equations in two variables into an ordinary equation in one variable and of satisfying the radial boundary condition of equation (11). In addition, these transformations reduce the functional dependency of the dependent variables on the independent variables in such a manner that the problem is now determinant. For example, w is no longer a function of r and thereby requires no specification at the radial boundaries.

Substituting equations (15) and (16) into equations (5) to (7) results in the ordinary differential equations

$$f''' = -\frac{a^2}{\nu} \tag{18}$$

and

$$G' = \frac{-4\nu}{a^2} f'' \tag{19}$$

Solving equation (18) and applying boundary conditions (9) and (10) result in

$$f = +\frac{1}{6}\frac{a^2}{\nu}\left(\frac{3}{2}\delta z^2 - z^3\right) \tag{20}$$

But expressing f in terms of w by equation (15) yields

$$w = -\frac{1}{3}\frac{a^2}{\nu}\left(\frac{3}{2}\delta z^2 - z^3\right) \tag{21}$$

Therefore, at the bottom surface of the drop

$$w(\delta) = -\frac{1}{6}\frac{a^2}{\nu}\delta^3 \tag{22}$$

Thus, the axial velocity at the surface of the drop is directly proportional to the cube of the gap thickness. Physically, for a given pressure distribution under the drop less flow is required to keep the same pressure level for smaller values of δ, as prescribed by equation (15). The constant a is now determined from conditions of static equilibrium.

Integrating equation (19) and substituting in the first derivative from equation (20) result in

$$G = -2\left(\delta z - z^2\right) + C \tag{23}$$

Substituting into equation (17) gives the pressure distribution as

$$P = \frac{1}{2}a^2\frac{\rho_v}{g_c}\left[r_o^2 - r^2 - 2\left(\delta z - z^2\right) + C\right] \tag{24}$$

Solving for C using boundary condition (12) and substituting the value of C into equation (24) result in

$$P = \frac{1}{2}a^2\frac{\rho_v}{g_c}\left[r_o^2 - r^2 - 2\left(\delta z - z^2\right)\right] + P_o \tag{25}$$

At the bottom surface of the drop

$$P(r, \delta) = \frac{1}{2} a^2 \frac{\rho_v}{g_c}\left(r_o^2 - r^2\right) + P_o \tag{26}$$

Substituting the above relation into equation (13) and solving for a^2 result in

$$a^2 = \frac{4}{\pi} V\left(\frac{\rho_\ell}{\rho_v}\right)\frac{g}{r_o^4} \tag{27}$$

Therefore, the final expression for pressure is

$$P(r, z) = P_o + \frac{2}{\pi}\frac{V\rho_\ell}{r_o^4}\frac{g}{g_c}\left(r_o^2 - r^2\right) - \frac{4}{\pi}\frac{V\rho_\ell}{r_o^4}\frac{g}{g_c}\left(\delta z - z^2\right) \tag{28}$$

When equation (27) is substituted into equation (22), the axial velocity at the surface becomes

$$w(\delta) = -\frac{2}{3\pi} V \frac{\rho_\ell}{\rho_v}\frac{g}{r_o^4 \nu}\delta^3 \tag{29}$$

Note that for the flat disk geometry used in this model

$$V = \pi r_o^2 \ell \tag{30}$$

Defining

$$L_e = \frac{r_o^4}{V} = \frac{V}{\pi^2 \ell^2} \tag{31}$$

yields

$$w(\delta) = -\frac{2}{3\pi}\frac{\rho_\ell}{\rho_v}\frac{g}{\nu L_e}\delta^3 \tag{32}$$

Thus, for the drop to remain in equilibrium, the magnitude of the axial velocity leaving the surface of the drop is proportional to the cube of gap thickness.

Energy Equation

Since w is a known function of z, the energy equation can now be solved directly. Equation (8) can be rewritten in the form

$$\frac{d}{dz} \ln \frac{dT}{dz} = \frac{w}{\alpha} \tag{33}$$

Integrating the above equation and applying the boundary condition at $z = 0$ (eq. (9)) result in

$$T - T_p = C_2 \int_0^z d\xi \exp \int_0^\xi \frac{w(\tau)}{\alpha} d\tau \tag{34}$$

At $z = \delta$,

$$T_{sat} - T_p = C_2 \int_0^\delta d\xi \exp \int_0^\xi \frac{w(\tau)}{\alpha} d\tau \tag{35}$$

Therefore, dividing equation (34) by equation (35) yields

$$\frac{T - T_p}{T_{sat} - T_p} = \frac{\int_0^z d\xi \exp \int_0^\xi \frac{w(\tau)}{\alpha} d\tau}{\int_0^\delta d\xi \exp \int_0^\xi \frac{w(\tau)}{\alpha} d\tau} \tag{36}$$

Substituting the value of w from equation (21) into equation (36) gives for the first integration

$$\exp \int_0^\xi \frac{w(\tau)}{\alpha} d\tau = \exp - \frac{a^2}{6\nu\alpha}\left(\delta\xi^3 - \frac{\xi^4}{2}\right) \tag{37}$$

Since $\xi \leq \delta$

$$\left| \int_0^{\xi} \frac{w(\tau)}{\alpha} dT \right| \leq \left| \frac{a^2 \delta^4}{12 \nu \alpha} \right| \tag{38}$$

As a general numerical example, for a 1-cubic centimeter water drop on a flat plate at 600° F (appendix A, and noting that $\alpha = \nu/\mathrm{Pr}$)

$$\frac{a^2 \delta^4}{12 \nu \alpha} = 0.056 \tag{39}$$

Therefore, the exponential may be expanded and the second and higher order terms neglected. Thus,

$$\exp \int_0^{\xi} \frac{w(\tau)}{\alpha} d\tau \cong 1 - \frac{a^2}{6 \nu \alpha}\left(\delta z^3 - \frac{\xi^4}{2}\right) \tag{40}$$

Substituting equation (40) into equation (36) and continuing the integration result in

$$\frac{T - T_p}{T_{sat} - T_p} = \frac{z + \frac{a^2}{\alpha \nu}\left(\frac{z^5}{60} - \frac{\delta z^4}{24}\right)}{\delta\left(1 - \frac{a^2}{\alpha \nu} \frac{1}{40} \delta^4\right)} \tag{41}$$

The temperature gradient of any point z is found by a simple differentiation of equation (41) resulting in

$$\frac{dT}{dz} = (T_{sat} - T_p) \frac{1 + \frac{a^2}{\alpha \nu}\left(\frac{5}{60} z^4 - \frac{4 \delta z^3}{24}\right)}{\delta\left(1 - \frac{1}{40} \frac{a^2}{\alpha \nu} \delta^4\right)} \tag{42}$$

At $z = \delta$

$$\left.\frac{dT}{dz}\right|_{z=\delta} = \frac{(T_{sat} - T_p)}{\delta}\left(\frac{1 - \frac{a^2}{12\alpha\nu}\delta^4}{1 - \frac{1}{40}\frac{a^2}{\alpha\nu}\delta^4}\right) \tag{43}$$

or at $z = 0$

$$\left.\frac{dT}{dz}\right|_{z=0} = \frac{T_{sat} - T_p}{\delta}\left(\frac{1}{1 - \frac{1}{40}\frac{a^2}{\alpha\nu}\delta^4}\right) \tag{44}$$

The above expression for the temperature gradient will be used in the determination of the heat-transfer coefficients in the next section.

Heat-Transfer Coefficient To The Drop

An energy balance on the liquid drop results in

$$-\rho_v w(\delta)\lambda = -k\left.\frac{dT}{dz}\right|_{z=\delta} \tag{45}$$

which is the interface energy balance of equation (14).

Substituting the value of w from equation (22) and the temperature gradient from equation (43) results in

$$\frac{1}{6}\rho_v\frac{a^2}{\nu}\lambda\delta^3 = -\frac{k}{\delta}(T_{sat} - T_p)\left(\frac{1 - \frac{a^2}{12\alpha\nu}\delta^4}{1 - \frac{1}{40}\frac{a^2}{\alpha\nu}\delta^4}\right) \tag{46}$$

When it is noted that $\frac{a^2\delta^4}{12\nu\alpha}$ is small compared with 1, expansion by the binominal theorem gives

$$\frac{1 - \frac{a^2}{12\alpha\nu}\delta^4}{1 - \frac{1}{40}\frac{a^2}{\alpha\nu}\delta^4} \simeq 1 - \frac{7}{120}\frac{a^2}{\alpha\nu}\delta^4 \tag{47}$$

as an approximation when second and higher order terms are neglected.

Solving equation (46) for δ results in

$$\delta = \left\{\frac{k(T_p - T_{sat})}{\rho_v \frac{a^2}{\nu}\frac{\lambda}{6}\left[1 + \frac{7}{20}\frac{(T_p - T_{sat})k}{\rho_v \lambda \alpha}\right]}\right\}^{1/4} \tag{48}$$

Substituting the value of a^2 from equation (27) yields

$$\delta = \left\{\frac{3\pi k(T_p - T_{sat})\mu r_o^4}{2\rho_v \rho_\ell V g \lambda \left[1 + \frac{7}{20}\frac{C_p(T_p - T_{sat})}{\lambda}\right]}\right\}^{1/4} \tag{49}$$

The heat-transfer coefficient to the drop is defined as

$$q_\ell = -\rho_v \lambda w(\delta) = h_\ell (T_p - T_{sat}) = h_\ell \, \Delta T \tag{50}$$

Therefore,

$$h_\ell = \frac{-\rho_v \lambda w(\delta)}{\Delta T} \tag{51}$$

Substituting the value of w from equation (22), the value of δ from equation (49) and rearranging terms result in a heat-transfer coefficient to the drop of the form

$$h_\ell = 0.68\left(\frac{k^3 \rho_v \rho_\ell g \lambda_\ell^*}{\mu \, \Delta T L_e}\right)^{1/4} \tag{52}$$

where the modified latent heat of vaporization λ_ℓ^* is defined as

$$\lambda_\ell^* = \frac{\lambda}{\left(1 + \frac{7}{20}\frac{C_p \Delta T}{\lambda}\right)^3} \tag{53}$$

Heat-Transfer Coefficient From Plate

The heat-transfer coefficient from the plate is defined by the following equation

$$h_p \Delta T = -k \left.\frac{\partial T}{\partial z}\right|_{z=0} \tag{54}$$

Substituting in the value of the temperature gradient given by equation (44), using equation (49) in the first term of equation (44), and rearranging terms give

$$h_p = \left[\frac{k^3 \rho_v a^2 \lambda \left(1 + \frac{7}{20}\frac{C_p \Delta T}{\lambda}\right)}{6\,\Delta T \nu}\right]^{1/4} \frac{1}{\left(1 - \frac{1}{40}\frac{a^2}{\alpha\nu}\delta^4\right)} \tag{55}$$

Substituting the values of a^2 (eq. (27)) and L_e (eq. (31)) into the above equation yields

$$h_p = 0.68\left(\frac{k^3 \rho_v \rho_\ell g \lambda_p^*}{\mu\,\Delta T L_e}\right)^{1/4} \tag{56}$$

where

$$\lambda_p^* = \frac{\lambda\left(1 + \frac{7}{20}\frac{C_p \Delta T}{\lambda}\right)}{\left(1 - \frac{1}{40}\frac{a^2}{\alpha\nu}\delta^4\right)^4} \tag{57}$$

But substituting into equation (57) the value of δ from equation (49), and the value of a^2 from equation (27), and clearing fractions yield

$$\lambda_p^* = \frac{\lambda\left(1 + \frac{7}{20}\frac{C_p\,\Delta T}{\lambda}\right)\left(1 + \frac{7}{20}\frac{C_p\,\Delta T}{\lambda}\right)^4}{\left(1 + \frac{1}{5}\frac{C_p\,\Delta T}{\lambda}\right)^4} \tag{58}$$

or

$$\lambda_p^* = \frac{\lambda\left(1 + \frac{7}{20}\frac{C_p\,\Delta T}{\lambda}\right)^5}{\left(1 + \frac{1}{5}\frac{C_p\,\Delta T}{\lambda}\right)^4} \tag{59}$$

The difference between h_ℓ and h_p results from sensible heating effects in the superheated vapor.

Equations (52) and (56) agree with the functional form of Bromley's equation (ref. 8) for film boiling heat transfer from a horizontal tube. The only significant difference is that the diameter of the tube is replaced by the newly defined geometry factor L_e.

COMPARISON TO COMPUTER SOLUTION OF THE EXACT NAVIER-STOKES EQUATION

References 6 and 7 present solutions to this problem by solving the exact Navier-Stokes equation. It is enlightening at this time to compare some of the results of this earlier work, which considered inertia effects, to the results of the present paper, which has neglected inertia effects.

It was observed in reference 6 that the radial velocity profile was for all practical purposes parabolic in shape. This agrees with the result of the present paper, since combining equation (16) and the first derivative of equation (20) yields

$$u = -\frac{1}{2}\frac{a^2 r}{\nu}\left(z^2 - \delta z\right) \tag{60}$$

or

$$u = -\frac{ra^2\delta^2}{8\nu}\left[\left(\frac{2z}{\delta} - 1\right)^2 - 1\right] \tag{61}$$

which for a fixed r, indicates that the velocity distribution is parabolic in shape.

The axial velocity at the surface of the drop, as given in reference 8, is

$$w = -0.172\,\frac{a^2}{\nu}\,\delta^3 \tag{62}$$

Likewise, this compares very closely with the results shown in equation (22). In fact, the difference in these two coefficients could be due mostly to computer inaccuracies rather than to the magnitude of the inertia terms: except for corrections on the modified latent heat of vaporization in equation (53) and (59), the expressions for the heat-transfer coefficients are equal to the results in reference 7 where inertia terms were considered.

Finally, a numerical calculation is presented in appendix A which shows that the radial and axial components of the inertia terms are negligibly small at the surface of the drop and at the plate; however, the inertia term for the axial component is significant in the center of the vapor gap because the viscous term goes to zero. Nevertheless, conditions at the center of the gap do not control the heat-transfer process going on at the wall and at the drop surface. Consequently, the inertia effects can be dropped in this analysis.

CONCLUDING REMARKS

In the evaporation process to a liquid drop, the heat-transfer coefficient is shown to be equal to

$$h_i = 0.68\left(\frac{k^3\lambda_i^* g\rho_\ell\rho_v}{\Delta T\;\mu L_e}\right)^{1/4}$$

For heat-transfer to the drop

$$\lambda_i^* = \lambda_\ell^* = \lambda\left(1 + \frac{7}{20}\,\frac{C_p\,\Delta T}{\lambda}\right)^{-3}$$

For heat-transfer from the plate

$$\lambda_i^* = \lambda_p^* = \frac{\lambda\left(1 + \frac{7}{20}\frac{C_p\,\Delta T}{\lambda}\right)^5}{\left(1 + \frac{1}{5}\frac{C_p\,\Delta T}{\lambda}\right)^4}$$

It is shown by a comparison with a numerical solution of the exact Navier-Stokes equations, that the inertia effects in the flow field are negligible. The analytic expressions for h_i are identical in form to the Bromley equation for film boiling from a vertical plate. Theoretical and experimental values of the heat-transfer rates are in good agreement as shown in reference 7.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, August 10, 1965

APPENDIX A

COMPARISON OF INERTIA TERMS WITH PRESSURE AND VISCOUS TERMS IN NAVIER-STOKES EQUATION

The validity of the creeping motion approximation can be checked by evaluating the acceleration terms in the momentum equations using the velocity functions (eqs. (15) and 16)).

If the acceleration terms are small compared with viscous or pressure terms, at the very least, the solutions are consistant with the assumptions under which they were derived.

Inertia effects in the radial direction are first considered. The ratio of viscous to inertia acceleration in the radial direction is given by

$$\frac{\nu \dfrac{\partial^2 u}{\partial z^2}}{\left(u \dfrac{\partial u}{\partial r} + w \dfrac{\partial u}{\partial z}\right)} = \frac{\nu^2}{a^2 \delta^4}\left[\frac{1}{4}\left(\eta - \eta^2\right)^2 - \frac{1}{6}(1 - 2\eta)\left(\frac{3}{2}\eta^2 - \eta^3\right)\right]^{-1} \tag{A1}$$

As a typical case, consider a 1-cubic-centimeter water drop on a plate at 600° F and atmospheric pressure. For this case (ref. 6)

$$a^2 = 4.5\times10^{6} \qquad \mathrm{sec}^{-2}$$

$$\nu = 4.0\times10^{-4} \qquad \mathrm{ft}^2/\mathrm{sec}$$

$$\delta = 4.0\times10^{-4} \qquad \mathrm{ft}$$

By using the preceding equations, the ratio expressed in equation (A1) has a minimum value at $\eta = 1$ of about 18, and goes to infinity at the wall. Thus, the viscous terms always dominate the inertia terms for the radial flow. A plot of the ratio (eq. (A1)) is shown in figure 3.

Finally, consider the axial component of momentum with inertia terms retained

$$w \frac{dw}{dz} = -\frac{g_c}{\rho}\frac{\partial P}{\partial z} + \nu \frac{d^2 w}{dz^2} \tag{A2}$$

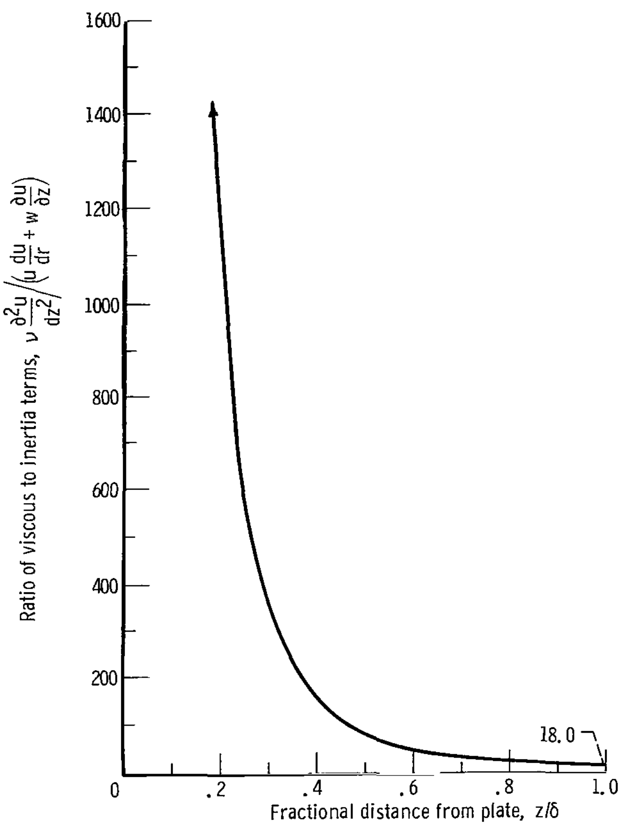

Figure 3. - Comparison of viscous with inertia terms in radial direction.

where $w = w(z)$ only.

It must be shown that

$$w \frac{dw}{dz} << \nu \frac{d^2w}{dz^2} \tag{A3}$$

Using the expression for $w(z)$ (eq. (21)) and defining $\eta = z/\delta$ yield

$$w \frac{dw}{dz} = \frac{a^4\delta^5}{3\nu^2}\left(\frac{3}{2}\eta^2 - \eta^3\right)\left(\eta - \eta^2\right) \tag{A4}$$

and

$$\nu \frac{d^2w}{dz^2} = - a^2\delta(1 - 2\eta) \tag{A5}$$

For the 1-cubic centimeter drop previously considered the preconstants of the η functions become

$$\frac{a^4\delta^5}{3\nu^2} = 430 \text{ ft/sec}^2 \tag{A6}$$

$$a^2\delta = 1800 \text{ ft/sec}^2 \tag{A7}$$

The inertial acceleration in the z-direction $w\, dw/dz$ goes through a maximum at about $\eta = 0.686$. The value of the inertial acceleration is computed to be 35.3 feet per second squared. At the same distance from the wall the viscous term is about 20 times greater than the inertia term; however, the results of this calculation can be misleading because there is a region near the center of the gap where the computed inertia terms are larger than the viscous terms. For example, at $z = \delta/2$ the viscous acceleration term is identically zero; whereas, the inertia term $w\, dw/dz$ is equal to 27 feet per second squared. A complete picture of the region where the inertia terms become appreciable is given in figure 4, where the ratio of viscous to inertia terms is plotted as a function of the distance from the plate. In the range $0.44 \leq z/\delta \leq 0.60$ the ratio of viscous to

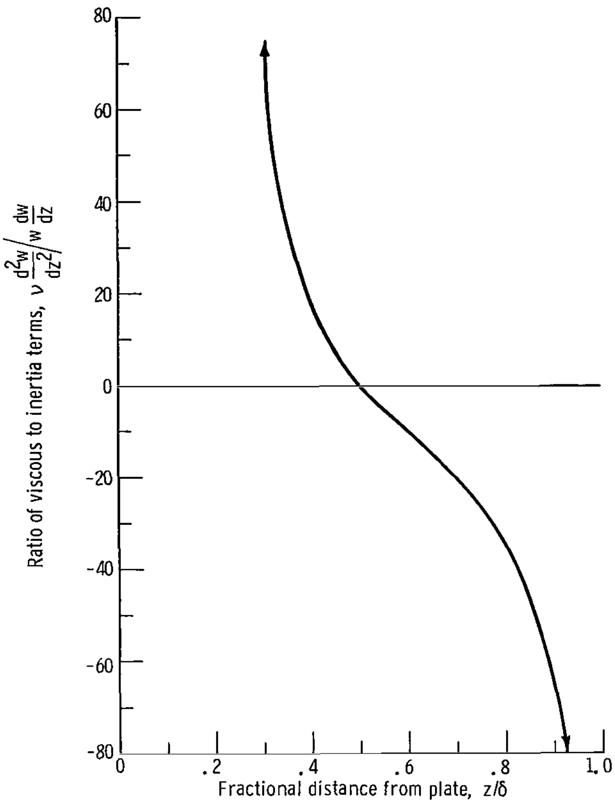

Figure 4. - Comparison of viscous with inertia terms in axial direction.

inertia terms becomes less than 10 in absolute magnitude. Outside this region the inertia forces are small compared with viscous forces.

The region of inconsistency encountered in comparing viscous and acceleration terms in the axial direction might cast some doubt on the analytical solution, but it is felt that the only effect of the inertia terms is to distort the velocity profile somewhat in the center of the vapor film.

The inertia effects on the velocity, pressure, and temperature profiles in the vicinity of the wall or the vapor-liquid interface are small. The heat-transfer coefficient depends mostly on the thickness of the gap, which is determined by the pressure distribution at the liquid-vapor interface. Since the inertia effects produce only minor perturbations of the pressure field in the vicinity of the interface, the computed gap thickness and heat-transfer coefficient should not be affected by neglecting the inertia terms. In the problem of film boiling in a forced-convection boundary layer, a similar conclusion was reached in reference 10: inertia forces are not important in the vapor film for the range of parameters usually encountered in practice.

APPENDIX B

REACTIVE FORCE

In reference 11, the momentum theorem for a control volume V is given by

$$\sum \overline{F} = \frac{\partial}{\partial t} \iiint \frac{\rho_\ell \overline{W}_\ell}{g_c} dV + \oint\!\!\oint \frac{\rho_v}{g_c} (\overline{W}_v \cdot d\overline{A}) \overline{W}_v \tag{B1}$$

All the elements of the drop were assumed to move at the same velocity

$$\iiint \frac{\rho_\ell \overline{W}_\ell dV}{g_c} = \frac{\overline{W}_\ell M}{g_c} \tag{B2}$$

For this problem $\overline{W}_\ell$ and $\overline{W}_v$ are parallel to the z-axis and may be conveniently expressed as the product of a scalar and a unit vector $\overline{e}_z$ in the axial direction. Thus,

$$\overline{W}_v = W_v(\delta)\overline{e}_z \tag{B3}$$

$$\overline{W}_\ell = W_\ell \overline{e}_z \tag{B4}$$

$$d\overline{A} = -dA\overline{e}_z \tag{B5}$$

Therefore, the force balance in the axial direction yields

$$F_z = \frac{M}{g_c}\frac{\partial W_\ell}{\partial t} + \frac{W_\ell}{g_c}\frac{\partial M}{\partial t} - \oint\!\!\oint \frac{\rho_v}{g_c} W_v^2(\delta)dA \tag{B6}$$

If the gap thickness is assumed to remain approximately constant over a short period of time, W_ℓ and W_v can be considered constants. Thus,

$$\sum F_z = \frac{W_\ell}{g_c}\frac{\partial M}{\partial t} - \frac{\rho_v}{g_c} W_v^2(\delta)A \tag{B7}$$

But,

$$\frac{dM}{dt} = \rho_v A W_v(\delta) \tag{B8}$$

Therefore,

$$\sum F_z = \frac{1}{g_c} \frac{dM}{dt} \left[W_\ell - W_v(\delta) \right] \tag{B9}$$

If it is assumed that the drop moves uniformly downward to keep the gap thickness constant,

$$W_\ell = \frac{\frac{dM}{dt}}{\rho_\ell A} \tag{B10}$$

Therefore, substituting equations (B8) and (B10) into equation (B9) results in

$$\sum F_z = - \frac{1}{g_c \rho_v A} \left(\frac{dM}{dt} \right)^2 \left(1 - \frac{\rho_v}{\rho_\ell} \right) \tag{B11}$$

For regions sufficiently far from the critical point, the density ratio is negligible compared with 1; thus,

$$\sum F_z = - \frac{\rho_v A W_v^2(\delta)}{g_c} \tag{B12}$$

The forces exerted by the surroundings are a result of pressure and gravity. Thus

$$-V\rho_\ell \frac{g}{g_c} + \int_0^{r_o} P(r, \delta) 2\pi r \, dr - \pi r_o^2 P_o = - \frac{\rho_v A W_v^2(\delta)}{g_c} \tag{B13}$$

The term on the right side of equation (B13) is defined as the reactive force associated with the ejection of mass from the drop surface.

For a plate temperature of 600° F and a 0.5-cubic centimeter water drop, the reactive force is computed (ref. 6) to be 4.6×10^{-6} pound force; compared with the weight of the drop, 1060×10^{-6} pound force, the reactive force is not important. Thus.

$$\int_0^{r_o} P(r, \delta) 2\pi r \, dr = V\rho_\ell \frac{g}{g_c} + \pi r_o^2 P_o \tag{B14}$$

Equation (B14) is referred to as the static-force balance in the text (eq. (13)).

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