NASA TECHNICAL MEMORANDUM

NASA TM X-53351

October 21, 1965

NASA TM X-53351

AN ENVIRONMENTAL MODEL FOR VAN ALLEN BELT PROTONS

By William T. Roberts
Aero-Astrodynamics Laboratory

NASA

George C. Marshall Space Flight Center, Huntsville, Alabama

NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER

TECHNICAL MEMORANDUM X-53351

AN ENVIRONMENTAL MODEL FOR VAN ALLEN BELT PROTONS

By

William T. Roberts

ABSTRACT

/3126

The majority of the manned and unmanned missions in space for at least the next ten years will be composed mainly of earth orbiting satellites and space stations. Although there are many environmental aspects of space which must be explored and elaborated upon before the successful launching of such missions, few are so critical as the radiation environment which will be encountered.

The radiation environment for earth orbiting vehicles will consist primarily of (1) Van Allen particles, (2) solar flare particles, and (3) galactic cosmic particles. Of these three, the Van Allen belt particles present the greatest hazard.

This report summarizes the knowledge of the energy spectrum and distribution of the Van Allen proton belt as it is known today. A computer program has been developed for calculating Van Allen particles incident upon vehicles in earth orbits. Based upon the 1963 FLUX program of McIlwain, we find that a vehicle in a highly elliptic orbit may encounter as many as 1.1×10^8 protons/cm^2/orbit with 40 Mev $\leq F \leq$ 110 Mev. In the magnetic equatorial plane, the flux of protons with $E \geq .5$ Mev may be as high as 2.2×10^7 protons/cm^2/sec.

Author

NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER

TECHNICAL MEMORANDUM X-53351

October 21, 1965

AN ENVIRONMENTAL MODEL FOR VAN ALLEN BELT PROTONS

By

William T. Roberts

SPACE ENVIRONMENT GROUP
AEROSPACE ENVIRONMENT OFFICE
AERO-ASTRODYNAMICS LABORATORY

TECHNICAL MEMORANDUM X-53351

AN ENVIRONMENTAL MODEL FOR VAN ALLEN BELT PROTONS

SUMMARY

The majority of the manned and unmanned missions in space for at least the next ten years will be composed mainly of earth orbiting satellites and space stations. Although there are many environmental aspects of space which must be explored and elaborated upon before the successful launching of such missions, few are so critical as the radiation environment which will be encountered.

The radiation environment for earth orbiting vehicles will consist primarily of (1) Van Allen particles, (2) solar flare particles, and (3) galactic cosmic particles. Of these three, the Van Allen belt particles present the greatest hazard.

This report summarizes the knowledge of the energy spectrum and distribution of the Van Allen proton belt as it is known today. A computer program has been developed for calculating Van Allen particles incident upon vehicles in earth orbits. Based upon the 1963 FLUX program of McIlwain, we find that a vehicle in a highly elliptic orbit may encounter as many as 1.1×10^8 protons/cm^2/orbit with 40 Mev $\leq$ F $\geq$ 110 Mev. In the magnetic equatorial plane, the flux of protons with E $\geq$ .5 Mev may be as high as 2.2×10^7 protons/cm^2/sec.

I. INTRODUCTION

Before 1958 the Van Allen radiation zones were not known to exist, and the only forms of radiation expected outside of the earth's atmosphere were the low flux, high energy, cosmic rays and occasional solar flares. Although little information was available about solar flares, they were known to be composed of high energy particles and to correlate temporally with gigantic eruptions on the photospheric surface of the sun.

The first American satellite, Explorer I, carried a Geiger counter for the purpose of obtaining further information on the flux of cosmic ray particles. The experiment worked well until the satellite reached about a thousand kilometers altitude, at which point the counting rate suddenly decreased. The experimenters (Van Allen, McIlwain, Ludwig, and Ray) were suspicious of what was happening, but did not publicize their suspicions until after some Explorer III data were examined. When the same phenomenon was observed on Explorer III, Van Allen announced (Ref. 38) that the apparent decrease in the Geiger counter was due to saturation by a high flux of particles.

These particles, which are trapped by the magnetic field of the earth, present a radiation problem which must be overcome before extended earth orbital missions of manned spacecraft can be performed. In addition, the environment to which unmanned satellites are exposed may decrease the lifetime of on-board experiments, power supply, telemetry, etc. Thus, extensive environmental studies of the radiation zones should be conducted before the development of sophisticated long-lifetime satellites.

This report summarizes the known facts and suggests a possible environmental model for Van Allen belt protons.

II. MOTION OF CHARGE PARTICLES IN THE MAGNETOSPHERE

The motion of charged particles trapped in the earth's vicinity by the geomagnetic field is influenced by this field. Perturbations caused by an electric field imposed by the earth's equatorial ring current also affect the orbital motions of these charged particles.

We may consider a charged particle moving in this magnetosphere as being subjected to the geomagnetic field, $\vec{B}$, and a variable field, $\vec{E}$, due to the high altitude ($\sim 6\,R_e$) ring current.(R_e = geocentric earth radii). The force, $\vec{F}$, exerted on the particle moving with velocity, $\vec{v}$, is

$$\vec{F} = q\ (\vec{E} + \vec{v} \times \vec{B}),$$

where q is the total charge of the particle. We will here consider only the effects of the geomagnetic field on the particle. The force, $\vec{F}$, is normal to the plane formed by $\vec{v}$ and $\vec{B}$; thus, the particle is induced, assuming that no other forces are present, to move in a circle with no loss in kinetic velocity.

If we consider the portion of $\vec{v}$ normal to the magnetic field as $v_\perp$, we may say that

$$\frac{mv_\perp^2}{r} = qv_\perp\ B,$$

where r is the Larmor radius, the radius of the circle in which the particle moves, or

$$r = \frac{mv_\perp}{q\ B} .$$

The Larmor frequency, ω, is then

$$\omega = \frac{v_\perp}{r} = \frac{q\ B}{m} .$$

The particle is now in space moving in a circle which is dependent upon the magnetic field strength, the particle's charge, mass, and velocity normal to the magnetic field. When the particle begins to orbit, the center of the orbit is located on a particular "line of force" of the magnetic field. This specific line of force then remains the center of gyration or "guiding center" of the particle, unless the particle is subjected to some external force. The orbit of particles trapped in the geomagnetic field is subjected to perturbations caused by the presence of the ring current, as well as inhomogeneities and slow variations of the field itself.

The component of velocity, $v_{\parallel}$, which is parallel to the magnetic field must also be considered. This component of the particle's velocity causes the particle to oscillate between "mirror points" in the northern and southern hemispheres. The particles are reflected at these points of equal magnetic field strength. When the particle is at the magnetic equator of the earth, it is at the point of minimum field strength on that line of force. As it moves either north or south along that line of force, it encounters a stronger field which in turn exerts a force, $F_{\parallel}$, on the particle, which is opposed to $v_{\parallel}$.

This force is dependent upon the magnetic moment, μ, of the particle, where

$$\mu = \frac{r^2\ q\ \omega}{2}$$

or

$$\mu = \frac{\frac{1}{2}\ mv_\perp^2}{B}$$

Finally, the retarding force on a particle moving along a line of force into a region of more intense magnetic field is given by

$$F_{\parallel} = - \mu (\nabla B),$$

where the (∇B) denotes the gradient of the magnetic field which is in the direction of $\vec{B}$. It should be remembered that this same force is exerted on a particle leaving a region of more intense magnetic field, in which case it is then an accelerating force.

These particles will be subjected to drifting motions due to external forces exerted by electric fields, the gravitational field, and inhomogeneities in the geomagnetic field. We will cover these very briefly.

If the particle moves in the presence of an electric field, $\vec{E}$, with a component perpendicular to $\vec{B}$ then a new velocity, $\vec{W}$, is defined such that

$$\vec{v} = \vec{W} + \frac{\vec{E} \times \vec{B}}{B^2} .$$

Since $\vec{E}$ and $\vec{B}$ are assumed constant,

$$m \frac{d\vec{W}}{dt} = q \vec{W} \times \vec{B}.$$

The drift velocity then results from the $\frac{\vec{E} \times \vec{B}}{B^2}$ term. In other words, the drift velocity, W_d, may be found by

$$W_d = \frac{E_\perp}{B} .$$

In the same respect, the gravitational force on a particle having a component $mg_\perp$ perpendicular to $\vec{B}$ yields a drift velocity

$$W_d = \frac{g_\perp}{\omega} .$$

The variations in the magnetic field which the particle encounters will also cause a third drift velocity which takes the form,

$$W_d = \frac{1}{\omega R} (\tfrac{1}{2} W_\perp^2 + W_\parallel^2)$$

where R is the radius of curvature of the geomagnetic line of force.

III. MAGNETOSPHERE

Since particles are trapped in the geomagnetic field, perhaps some elaboration on the earth's magnetosphere is necessary. The earth's magnetic field forms a definite "magnetic pocket" around us which has become known as the magnetosphere. The boundary of the magnetosphere is determined by the solar wind which "blows" radially outward from the sun. At a certain point above the earth, the geomagnetic field strength equals the kinetic energy of the solar wind. In that region, there is a turbulent, chaotic, intermingling of magnetic lines of force and energetic particles, neither of which is dominant. This region is known as the magnetospheric boundary or transition zone. In the magnetosphere, the geomagnetic field controls, while outside the magnetosphere, the solar wind is the dominant energy mechanism.

The inner boundary of the transition zone, which is called the magnetopause, occurs at about 10 R_e on the sunward side of the earth, while the outer boundary of the transition zone is in the form of a collisionless shock wave at about 3-4 R_e beyond the magnetopause.

Figure 1 illustrates the field strength F, as well as its spatial coordinates, θ, and ϕ (also see Reference 34). As would be expected, the shock wave outer boundary is most clearly seen by a smoothing out of F, with the direction components still variable, though less so.

The magnetopause extends around the earth in the apparent form of an elongated teardrop with the tail pointing directly away from the sun. The shock wave, however, appears as a shroud and shows no tendency toward convergence. Figure 2 illustrates the recent interpretation of the IMP-I data.

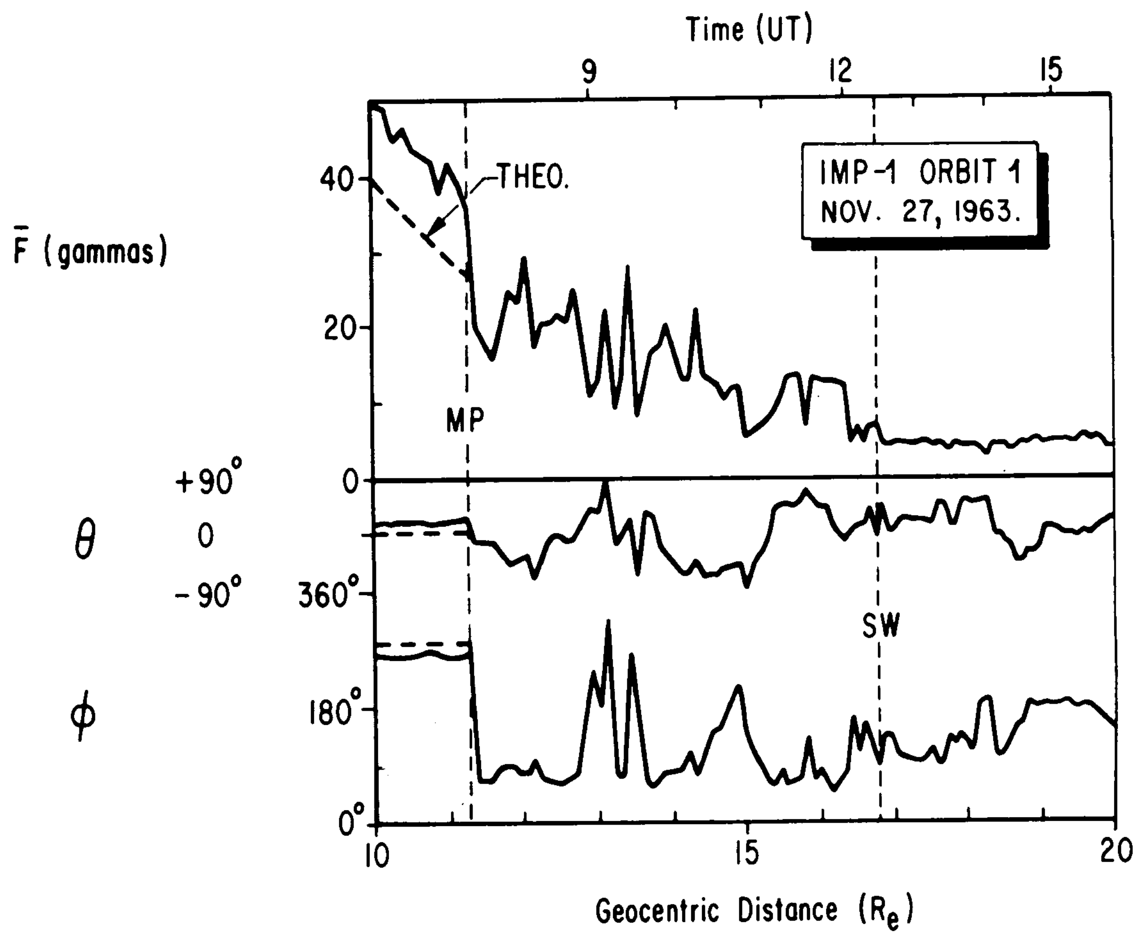

FIG. 1. OBSERVED MAGNETIC FIELD RESULTS ON OUTBOUND ORBIT #1 BY THE IMP-1 SATELLITE, NOV. 27, 1963. CLEARLY EVIDENT ARE THE MAGNETOSPHERE BOUNDARY AT 11.3 R_e AND THE COLLISIONLESS SHOCK WAVE AT 16.8 R_e.

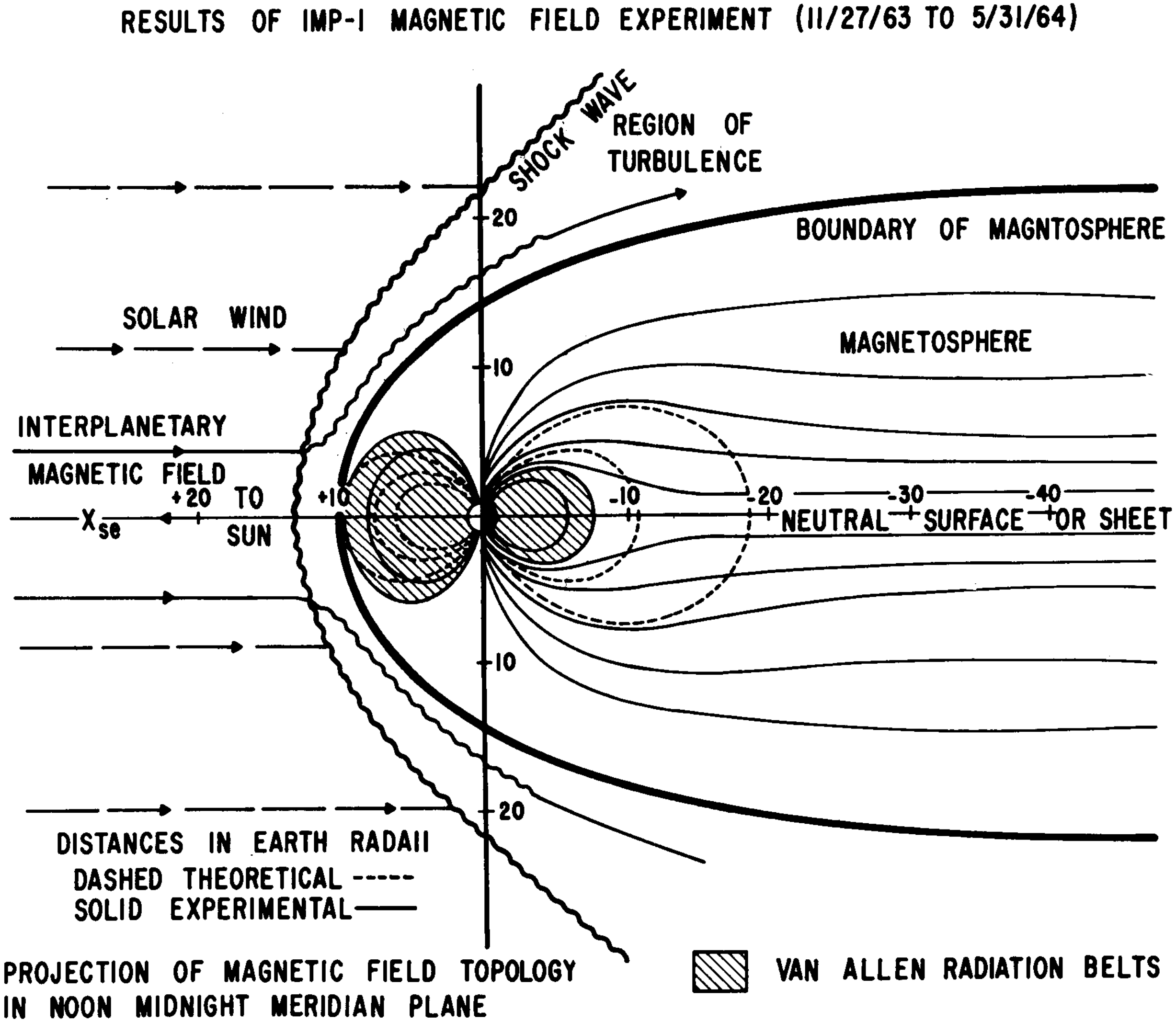

FIG. 2. SUMMARY ILLUSTRATION OF THE INTERPRETATION OF THE IMP-I DATA PERPENDICULAR TO THE PLANE OF THE ECLIPTIC ILLUSTRATRATING STRONG DAY-NIGHT ASYMMETRY IN THE RADIATION BELTS AND THE DEVELOPMENT OF THE EXTENDED MAGNETIC TAIL OF THE EARTH.

The neutral sheet extending toward the magnetic "tail" is an area in which a magnetic neutral surface exists. This region appears to contain an excess of particles so that it was previously thought to be a third radiation zone. At present, however, the feeling is that these excess particles are necessary so that their combined magnetic moments may balance the geomagnetic field. It has been suggested that this region could be the source of the high energy auroral particles. Perturbations occurring in the field would produce a plasma pinch effect which could accelerate the low energy particles in the neutral sheet to very high velocities.

The configuration of "earth, radiation belt, and magnetosphere" is very much like that of a typical comet with the earth representing the nucleus, the Van Allen belts representing the coma, and the magnetosphere representing the comet's tail.

The IMP-I data show no indication of convergence of the magnetospheric boundary even though the satellite apogee extended out to 200,000 kilometers. Thus, it is presumed that the magnetospheric tail extends out well beyond the orbit of the moon.

IV. THE B-L COORDINATE SYSTEM

Perhaps the best way to explain the B-L coordinate system for trapped ionized particles is to take a naive approach. One normally thinks of a magnetic field in terms of the so-called "lines of force." Each "line of force" is an indication of the direction of the magnetic field along it, and the field strength is determined by the compactness of these lines of force. The following sketch (Figure 3) shows this concept for a simple dipole magnetic field.

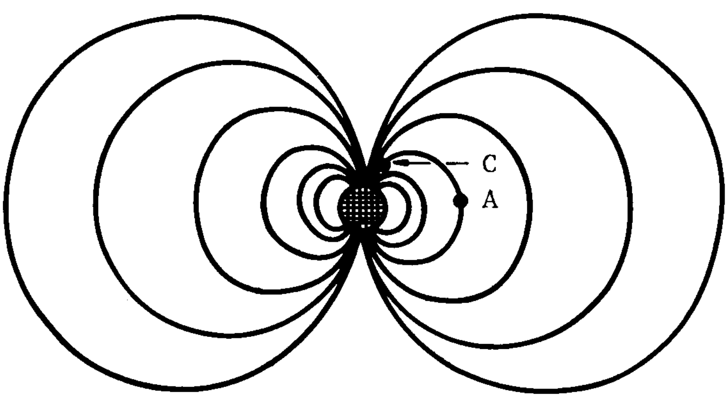

FIGURE 3

If this dipole is assumed to be enclosed in a sphere, then the earth's magnetic field could be roughly approximated in this way. If points A and C are two points along the same line of force, which may be designated as L, the magnetic field strength, B, is much more intense at C than at A. If a coordinate system is constructed based upon the field strength and lines of force, the resulting system could be used to specify equal points in any field.

B is relatively easy to specify. It is simply the field strength at any point in question. But how is L defined? For simplicity, let us define L as the radial distance to the line of force in question, measured in the magnetic equatorial plane. Thus, in Figure 3 the L coordinate for point A is 2R, where R is the radius of the sphere enclosing the magnetic dipole. The L coordinate for point C is also 2R, and here one will notice that point C does not lie at 2R radial distance from the center of the sphere. This discrepancy is accounted for by the different value of B at point C as compared to point A.

It is interesting to realize that many satellites in near-circular orbits which carry experiments to determine the Van Allen radiation belt environment use the term L as an indication of the geomagnetic latitude. As can be seen from Figure 3, this is an acceptable procedure.

Although this has been a very crude approximation, it is suitable in terms of the function of the B and L coordinate system. Actually, though B is as defined in the previous discussion, the geomagnetic field is not adequately represented by a dipole field. In fact, a deviation of 3 per cent of the calculated field at a point from the actual field may result in an error of more than an order of magnitude between the calculated flux and the measured flux. For this reason, the earth's magnetic field is usually determined from a spherical harmonic expansion, which gives an adequate representation of the geomagnetic field.

The term L is a function of B and the integral invariant, I. If we denote the "mirror points" as M_n and M_s for the northern and southern hemispheres, respectively, and ds as a segment of path length along the line of force connecting these "mirror points," then

$$I = \oint_{M_s}^{M_n} v_{\parallel} \, ds \; .$$

This forms a shell along which particles are constrained to move. This shell contains particles with a given energy spectrum. Particles of higher energy levels will have their "mirror points" at lower altitudes within this shell, assuming equal pitch angles. The pitch angle, α, may be defined by

$$\alpha = \text{arc cos } \frac{v_{\parallel}}{v_t} ,$$

where v_t is the particle's total velocity and $v_{\parallel}$ is that component of velocity parallel to the magnetic field in the geomagnetic equatorial plane.

V. SOURCES OF CHARGED PARTICLES

One of the main problems which remain to be solved in explaining the Van Allen radiation belts is the question of the source of these high energy particles. The two main sources of particles apparently are the decay of neutrons caused by cosmic ray interactions with the atmosphere, and the "leaking in" of solar particles at the poles and diffusion through the magnetospheric boundary. The neutron source has been well established as a contributor of particles in the inner Van Allen belt.

Cosmic rays are incident at the earth at about a rate of 2.5 particles $(cm^2 \text{ sec})^{-1}$. These particles range in energies from a few Mev to about 10^9 Bev. In the process of stopping these high energy particles, an average of about seven secondary neutrons are produced per cosmic ray particle. These neutrons then decay into protons and electrons which are in turn captured by the geomagnetic field.

Solar particles are suspected of entering the magnetosphere at the polar regions, where the lines of force offer the least resistance. When these particles enter as a high energy plasma from a solar flare, the ring current is strongly enhanced. The precise way in which these particles are dispersed throughout the magnetosphere once they have entered is a subject of controversy.

There is also the possibility that solar particles diffuse into the magnetosphere. Since there is always a solar plasma (in the form of the solar wind) surrounding the magnetosphere, particles could enter the radiation zones in this manner. The problem arises when one tries to explain how these particles with energies of a few hundred electron volts (in the solar wind) are accelerated into the Mev energy range. One might suspect that the acceleration of particles could take place if there were a rapid change in B, say during magnetic storms. A large variation in B would produce a pinch effect which in turn could accelerate particles to high velocities. There is also a theory that particles may be accelerated by an electromagnetic field exerting a force on the charged particles (Reference 39).

VI. THE VAN ALLEN BELT MODEL

The Van Allen belts consist mainly of protons and electrons trapped in the magnetic field of the earth. In the present report, we ignore the electron component of the Van Allen belts and deal only with the proton belt. This belt extends from about 1.1 R_e out to about 8 R_e with peak flux at $\sim$ 1.5 R_e measured at the geomagnetic equator. The proton component according to flux and energy spectra of the particles is described.

The question of what should be used as a threshold value for the energy spectrum is one which is not easily answered. By moving the energy threshold up and down, one is able to behold a uniquely changing picture of the radiation zones in space. This is due to the fact that regions of high flux, low energy particles will be overlooked if the energy threshold is too high, whereas regions of high energy, low flux particles might be obscured by setting the energy threshold too low. The energy spectrum is a complicated function when the energy threshold is low also. Thus, for manned space flight, we usually consider only protons with $E \geq 40$ Mev. For unmanned satellites, the problem is even more complicated. Since many experiments and supporting equipment may be exposed directly to the flux of particles incident upon them, it is necessary that we have a reliable radiation environment for consideration in designing the vehicles. Obviously, then, we need to study the entire energy spectrum of trapped particles.

This problem would be challenging enough even if the trapped particles were temporally stable. However, this is far from true, especially in the case of outer zone electrons. These particles undergo spatial as well as temporal variations in flux by as much as an order of magnitude in a matter of a few hours, and for the coup de grâce, even the energy spectrum at a particular point in space has been observed to change radically in a short time.

It appears then that, if we are to construct a model of low energy, high flux particles in the radiation zones, it should be very simple. Then as our knowledge of the causative effects increases, we may update our model accordingly.

It would seem extremely inadvisable at this time to attempt to construct a complex model based upon our present concepts of the Van Allen radiation zones and their variations. Some of the problems which must be answered first are (1) how do trapped particles react to changes in the earth's ring current, to changes in solar activity, and as the sun-earth-particle angle varies longitudinally, (2) how does the earth's magnetic field react to these same factors, and (3) to what other, if any, perturbing forces are trapped particles subject?

Let us now elaborate somewhat on possible preliminary models of the Van Allen radiation belt.

One of the primary experimental problems in measuring the particles incident upon a counter in space is that of defining the type of particle which causes a particular counter to count. For instance, some frequently used counters on board experimental satellites are listed below (Table I) with the particular types of radiation to which they are sensitive.

TABLE I

Type of Detector	Shielding	Particles to which Sensitive	
Anton GM 302	265 mg/cm^2 Mg + 400 mg/cm^2 steel	Protons	($E \geq 23$ Mev)
		Electrons	($E \geq 1.6$ Mev)
Anton 213 (Directional)	1.2 mg/cm^2 mica	Protons	($E \geq 500$ Kev)
		Electrons	($E \geq 40$ Kev)
	48 mg/cm^2 Al	Protons	($E \geq 4.5$ Mev)
		Electrons	($E \geq 230$ Kev)
	1.2 mg/cm^2 mica plus sweeping magnet	Protons	($E \geq 500$ Kev)
		Electrons	($E \geq 200$ Kev)

If a large array of detectors could be carried on board a satellite with a highly eccentric orbit and a reasonably long lifetime, a broad picture of the Van Allen belts could be obtained.

The Van Allen inner zone is composed of protons with peak intensity at about 3600 kilometers above the geomagnetic equator. The inner zone is fairly stable. This is to say that the flux of particles, as well as the energy spectrum, is constant over a long time period. However, one might suspect that eventually a long-term variation will show up in the inner zone, resulting from two effects. First, the cosmic ray flux incident upon the earth's upper atmosphere is about a factor of two higher at minimum solar activity. Therefore, more albedo neutrons will be available to the magnetic field for decay and subsequent capture of charged particles (see Section V). In addition, at least in the lower part of the inner radiation belt, the atmosphere may be less dense (Ref. 25) during minimum solar activity, thus giving the particles longer lifetimes. The lifetimes of trapped particles depend upon their mean free paths, which in turn are inversely dependent upon the average density of the medium through which the particles pass.

A good method of measuring the extent of the variation in atmospheric density at high altitudes might be to monitor the change in the decay rate of inner zone particles. If the decay rate increases drastically as solar activity increases, the increase could probably be attributed to an increase of atmospheric density over the particle's length. This speculation is based upon the change in decay rates of artificial radiation zones noted by Hess (Ref. 25).

The inner zone protons were at one time thought to be fairly localized below about 2 R_e; however, it now appears that low energy protons will be found all the way out to the magnetospheric boundary on the sunward side of the earth.

Figure 4 (curve M) shows the flux versus L values of McIlwain (Ref. 1) for particles of energies greater than 40 Mev. The hump at 2.2 R_e is due to the artificial radiation particles, and the area from 3 R_e out results from estimates of upper limits of protons. Curve R is a similar, but hypothetical, curve constructed on the basis of experimental data gleaned from several sources (Ref. 1, 3, 6, 7, 8, 11, 13, 35). The second curve, however, assumes a low energy threshold level of 500 Kev for protons. Curve R does not take into account the secondary proton peak shown on curve M, and includes an exponential interpolation of flux with height.

Again, it should be pointed out that in the construction of these graphs no attempt was made to account for temporal variations. In the same sense, the spurious anomalous regions will not be considered.

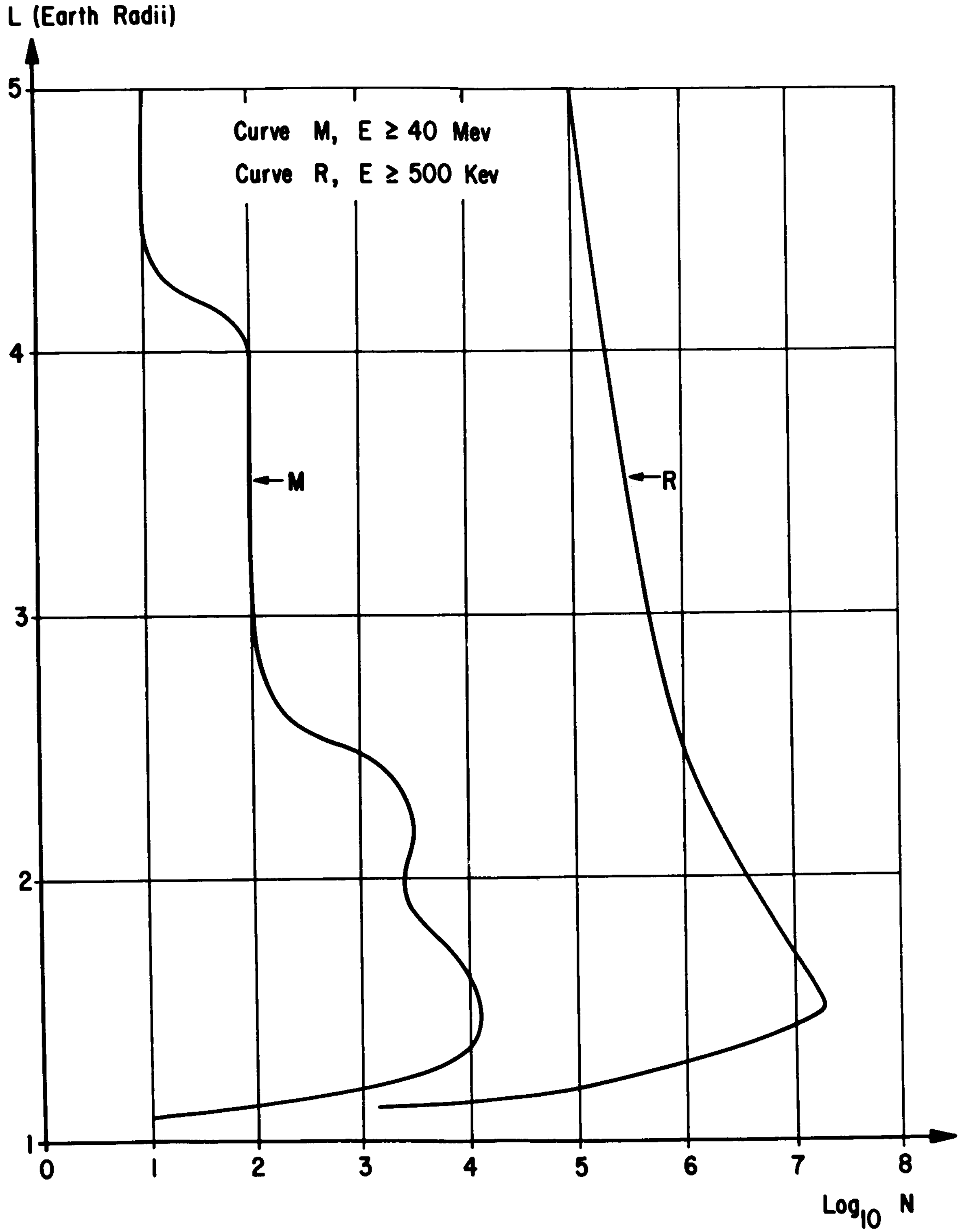

FIG. 4. FLUX OF PROTONS IN THE GEOMAGNETIC EQUATORIAL PLANE

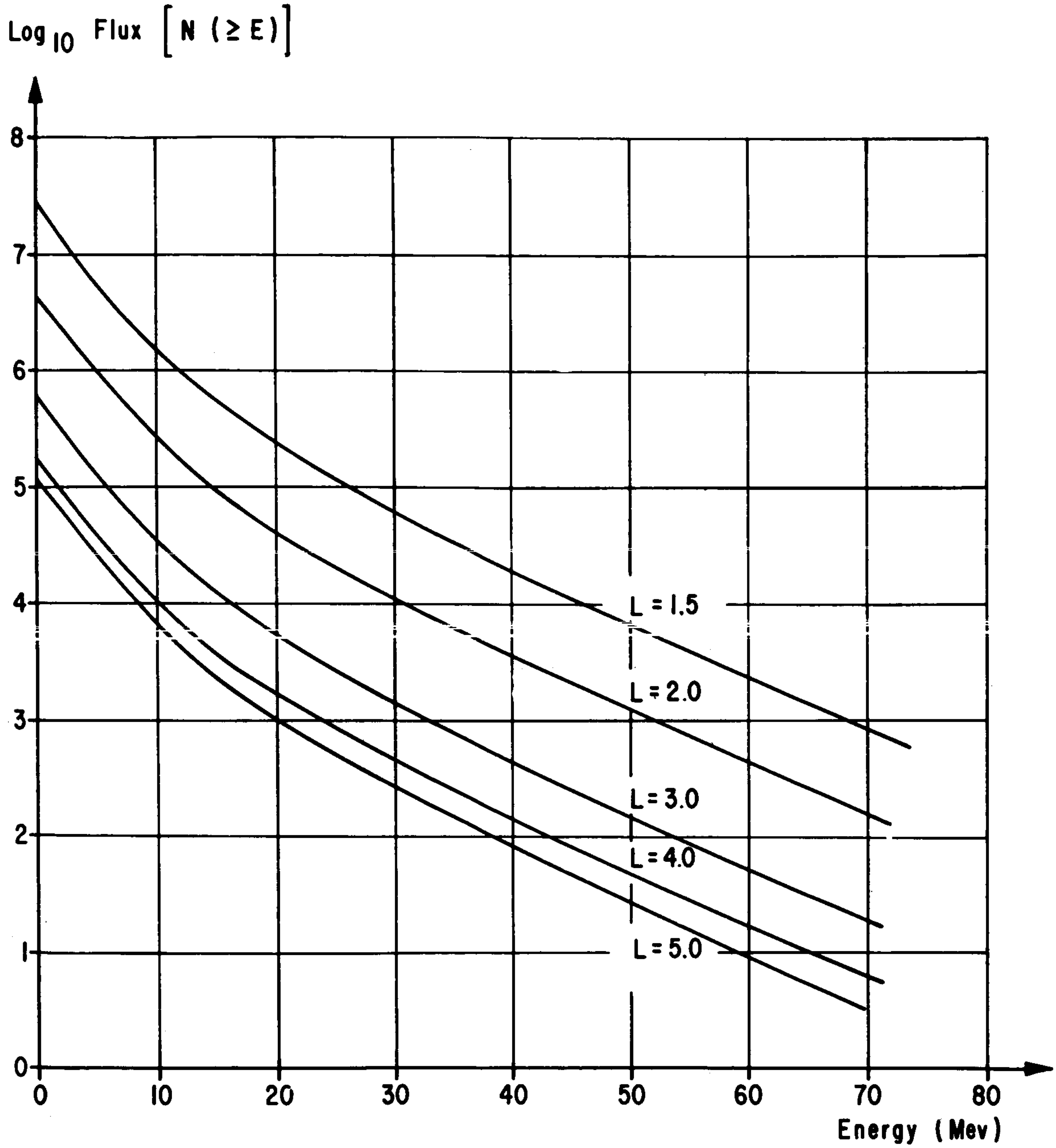

FIG. 5. ENERGY – FLUX SPECTRUM FOR PROTONS AT VARIOUS L VALUES IN THE GEOMAGNETIC EQUATORIAL PLANE

Figure 5 illustrates the flux-energy curve we have assigned to these flux curves found in Figure 4. A family of curves was constructed which seemed to be best indicated by the various data sources mentioned in conjunction with the construction of Figure 4. The energy spectrum has been found to vary spatially as well as temporally, but no attempt is made here to account for this variation.

A least squares polynomial fit to these curves yields a method of finding the precise value of N ($\geq$ E) for any given E and L value specified. The data used here are again restricted to the magnetic equatorial plane. This method, programmed by Miss Linda Flenker, is found in Appendix A.

We see from our previous discussion that the proton belt is subject to the geomagnetic field. At the nadir, this field is much changed from the sunward side of the earth. Thus, the proton belt will not extend to the magnetopause. Instead, there will be a sharp cutoff of protons beyond about 5 R_e. This is due to the distortions of the field by the elongation of the magnetosphere through the action of the solar wind (see Section III). Because of this distortion, the particles will not move along lines of force defined by the integral invariant, but instead will be subject to perturbations caused by the stretched lines.

In this region of the magnetosphere, the field seems to react strongly to such things as solar flares, enhanced ring currents, etc. The effect is that these perturbations serve to keep the magnetosphere beyond about 5-8 R_e relatively clear of trapped protons on the side of the earth away from the sun.

Electrons in this region do appear but their occurrence is, at the present time, unpredictable. The electron component of the radiation belt will be the subject of a later report.

APPENDIX A

METHOD OF CURVE FITTING BY LEAST SQUARES POLYNOMIALS

This method gives a single equation representation of a family of curves (i.e., a means of obtaining a function in terms of more than one independent variable).

Given: E (Energy in Mev); L; N (Flux)

To Find: $N = f(E, L)$

First,

$L = 1.5, \ln N_1 = a_{10} + a_{11}E + a_{12}E^2 + a_{13}E^3 + a_{14}E^4$

$L = 2.0, \ln N_2 = a_{20} + a_{21}E + a_{22}E^2 + a_{23}E^3 + a_{24}E^4$

$L = 3.0, \ln N_3 = a_{30} + a_{31}E + a_{32}E^2 + a_{33}E^3 + a_{34}E^4$

$L = 4.0, \ln N_4 = a_{40} + a_{41}E + a_{42}E^2 + a_{43}E^3 + a_{44}E^4$

$L = 5.0, \ln N_5 + a_{50} + a_{51}E + a_{52}E^2 + a_{53}E^3 + a_{54}E^4$

A fourth degree curve was used since higher degrees increased the accuracy only slightly. The errors incurred in the fourth degree curve fit are in the hundredths decimal place.

This curve fit gives the coefficients (a_0, a_1, ... a_4) for which the equation

$$\ln N = a_0 + a_1E + a_2E^2 + a_3E^3 + a_4E^4$$

is satisfied. In this first step ln N is obtained as a function of E, because the coefficients (a_0, a_1, ... a_4) are a function of E.

Second,

a_0's $= A_0 + A_1L + A_2L^2 + A_3L^3$

a_1's $= B_0 + B_1L + B_2L^2 + B_3L^3$

a_2's $= C_0 + C_1L + C_2L^2 + C_3L^3$

a_3's $= D_0 + D_1L + D_2L^2 + D_3L^3$

a_4's $= F_0 + F_1L + F_2L^2 + F_3L^3$

A third-degree polynomial was fit in this case with errors occurring in the hundredths decimal place.

These final coefficients (A, B, C, D, F) (see Table II) are a function of both E and L; and are obtained by fitting the coefficients (a_{10}, a_{11}, ... a_{54}) horizontally with respect to L.

Third,

$$\ln N = (A_0 + A_1L + A_2L^2 + A_3L^3)$$
$$+ (B_0 + B_1L + B_2L^2 + B_3L^3)$$
$$+ (C_0 + C_1L + C_2L^2 + C_3L^3)$$
$$+ (D_0 + D_1L + D_2L^2 + D_3L^3)$$
$$+ (F_0 + F_1L + F_2L^2 + F_3L^3) = X$$

TABLE II

A_0 = +.249547875 + 02	A_1 = -.694389747 + 01	A_2 = +.125157848 + 01
	A_3 = -.785188961 - 01	
B_0 = -.295686833 - 00	B_1 = -.513692717 - 01	B_2 = -.224835894 - 03
	B_3 = -.126209083 - 02	
C_0 = +.574196303 - 02	C_1 = +.13114399 - 02	C_2 = -.224835894 - 03
	C_3 = +.288122124 - 05	
D_0 = -.816146665 - 04	D_1 = -.963839773 - 05	D_2 = -.144865679 - 05
	D_3 = +.585676357 - 06	
F_0 = +.462310907 - 06	F_1 = -.255271946 - 07	F_2 = +.392723857 - 07
	F_3 = -.698480307 - 08	

where

$$\ln N = f(E, L)$$

to obtain N

$$N = e^{f(E,L)} = e^{X}$$

Input values:

Each input value consists of an E value and its corresponding L value.

To run this program the programmer needs the equation given in step 3, the coefficient values for this equation (the coefficients are constants for all runs), and a set of E and L values.

Output:

On print out sheet

Name		Value
T	Corresponds to	L
S	Corresponds to	E
U	Corresponds to	ln N
EXPU	Corresponds to	N

```
T   0.149099999E 01
S0  0.170899916E  2  S1  -0.344228519E-00  S2  0.721296621E-02  S3  -0.973550832E-04  S4  0.493309271E-06
S5
S   0.500000000E 0[illegible]  U   0.169196683E 02  EXP U  0.222904378E 08
S   0.499099999E 01  U   0.155373120E 02  EXP U  0.559459790E 07
S   0.100000000E 02  U   0.142765809E 02  EXP U  0.158577048E 07
S   0.149099999E 02  U   0.132458814E 02  EXP U  0.565735268E 06
S   0.199099999E 02  U   0.123906963E 02  EXP U  0.240553171E 06
S   0.249099999E 02  U   0.116639081E 02  EXP U  0.116297657E 06
S   0.300000000E 02  U   0.110257988E 02  EXP U  0.614389212E 05
S   0.350000000E 02  U   0.104440500E 02  EXP U  0.343394463E 05
S   0.399099999E 02  U   0.989374311E 01  EXP U  0.198060576E 05
S   0.449099999E 02  U   0.935735912E 01  EXP U  0.115837268E 05
S   0.499099999E 02  U   0.882477864E 01  EXP U  0.680068516E 04
S   0.549099999E 02  U   0.829328195E 01  EXP U  0.399693044E 04
S   0.600000000E 02  U   0.776754889E 01  EXP U  0.236267302E 04
S   0.649099999E 02  U   0.725965914E 01  EXP U  0.142177179E 04
S   0.699099999E 02  U   0.678909164E 01  EXP U  0.888106478E 03
```

T 0.199999999E 01

S0 0.154451553E 02 S1 -0.350261164E-00 S2 0.748854900E-02 S3 -0.102000678E-03 S4 0.518467635E-06

S5

S	0.500000000E 00	U	0.152718840E 02	EXP U	0.429037247E 07
S	0.499999999E 01	U	0.138686371E 02	EXP U	0.105456311E 07
S	0.100000000E 02	U	0.125945824E 02	EXP U	0.294956306E 06
S	0.149999999E 02	U	0.115581564E 02	EXP U	0.104626949E 06
S	0.199999999E 02	U	0.107023009E 02	EXP U	0.444580038E 05
S	0.249999999E 02	U	0.997773508E 01	EXP U	0.215414076E 05
S	0.300000000E 02	U	0.934295489E 01	EXP U	0.114180976E 05
S	0.350000000E 02	U	0.876423345E 01	EXP U	0.640115328E 04
S	0.399999999E 02	U	0.821562083E 01	EXP U	0.369827150E 04
S	0.449999999E 02	U	0.767894414E 01	EXP U	0.216233245E 04
S	0.499999999E 02	U	0.714380744E 01	EXP U	0.126624036E 04
S	0.549999999E 02	U	0.660759198E 01	EXP U	0.740697258E 03
S	0.600000000E 02	U	0.607545580E 01	EXP U	0.435047750E 03
S	0.649999999E 02	U	0.556033414E 01	EXP U	0.259909669E 03
S	0.699999999E 02	U	0.508293907E 01	EXP U	0.161247277E 03

I 0.300000000E 01

SO 0.135672912E 2 S1 -0.352783986E-00 S2 0.773055263E-02 S3 -0.107754509E-03 S4 0.559591109E-06

S5

S	0.500000000E 0	0	0.130928183E 02	EXP 0	0.485443293E 06
S	0.499099999E 01	0	0.116835154E 02	EXP 0	0.118600437E 06
S	0.100000000E 02	0	0.104103479E 02	EXP 0	0.332014208E 05
S	0.149099999E 02	0	0.937956350E 01	EXP 0	0.118438439E 05
S	0.199099999E 02	0	0.853133095E 01	EXP 0	0.507119087E 04
S	0.249099999E 02	0	0.781421292E 01	EXP 0	0.247553774E 04
S	0.300000000E 02	0	0.718516594E 01	EXP 0	0.131970621E 04
S	0.350000000E 02	0	0.660954037E 01	EXP 0	0.742141630E 03
S	0.399099999E 02	0	0.606108050E 01	EXP 0	0.428838548E 03
S	0.449099999E 02	0	0.552192444E 01	EXP 0	0.250115907E 03
S	0.499099999E 02	0	0.498260409E 01	EXP 0	0.145853704E 03
S	0.549099999E 02	0	0.444204545E 01	EXP 0	0.849485219E 02
S	0.600000000E 02	0	0.390756813E 01	EXP 0	0.497777518E 02
S	0.649099999E 02	0	0.339488574E 01	EXP 0	0.298112472E 02
S	0.699099999E 02	0	0.292810572E 01	EXP 0	0.186921087E 02

T 0.399o99999E 01

S0 0.127792438E 2 S1 -0.348893978E-00 S2 0.757474948E-02 S3 -0.105863479E-03 S4 0.553532901E-06 S5

S	0.500n00000E 00	U	0.120066773E 02	EXP U	0.163845207E 06
S	0.499o99999E 01	U	0.106112557E 02	EXP U	0.405891046E 05
S	0.100n00000E 02	U	0.934745064E 01	EXP U	0.114695461E 05
S	0.149o99999E 02	U	0.832088556E 01	EXP U	0.410879700E 04
S	0.199o99999E 02	U	0.747292038E 01	EXP U	0.175973630E 04
S	0.249o99999E 02	U	0.675321794E 01	EXP U	0.856811499E 03
S	0.300n00000E 02	U	0.611974414E 01	EXP U	0.454748325E 03
S	0.350n00000E 02	U	0.553876777E 01	EXP U	0.258364371E 03
S	0.399o99999E 02	U	0.498486074E 01	EXP U	0.146183215E 03
S	0.449o99999E 02	U	0.444089784E 01	EXP U	0.848510911E 02
S	0.499o99999E 02	U	0.389805695E 01	EXP U	0.493065210E 02
S	0.549o99999E 02	U	0.335581887E 01	EXP U	0.286690706E 02
S	0.600n00000E 02	U	0.282196744E 01	EXP U	0.168098906E 02
S	0.649o99999E 02	U	0.231258942E 01	EXP U	0.101005454E 02
S	0.699o99999E 02	U	0.185207477E 01	EXP U	0.637302038E 01

14. Dessler, A. J. and B. J. O'Brien, "Penetrating Particle Radiation," Satellite Environment Handbook, Stanford University Press, Stanford, California, 1965.

15. Vernov, S. N., et al., "Investigation of the Van Allen Belts in the Region of the Brazilian Magnetic Anomaly at Altitudes of 235-345 km," Kosmicheskiue Isledovaniya, Vol. II, No. 3, 1964.

16. Spitzer, Lyman, Physics of Fully Ionized Gases, Interscience Publishers (John Wiley and Sons), New York, 1962.

17. Hess, Wilmot N., "Earth's Radiation Environment," Space/Aeronautics, Vol. 42, No. 6, November 1964.

18. Lingenfelter, R. E. and E. J. Flamm, "Solar Neutrons and the Earth's Radiation Belts," Science, Vol. 144, April 17, 1964.

19. Rosser, W. G. V., "Changes in the Structure of the Outer Radiation Zone Associated with the Magnetic Storm of September 30, 1961," Journal of Geophysical Research, Vol. 68, No. 10, May 15, 1963.

20. Vernov, S. N., et al., "Investigation of the Earth's Radiation Belts at the Altitudes of 200-400 km," COSPAR Meeting and International Space Science Symposium, Florence, Italy, May 8-20, 1964.

21. Hones, Edward W. Jr., "Motions of Charged Particles Trapped in the Earth's Magnetosphere," Journal of Geophysical Research, Vol. 68, No. 5, March 1, 1963.

22. Roberts, W. T., "Space Radiations: A Compilation and Discussion," NASA TM X-53018, George C. Marshall Space Flight Center, Huntsville Alabama, March 5, 1964.

23. McIlwain, Carl E., "The Radiation Belts, Natural and Artificial" Science, Vol. 142, October 18, 1963

24. Tverskoi, B. A., "On the Nature of the Earth's Radiation Belts," COSPAR Meeting, and International Space Science Symposium, Florence, Italy, May 8-20, 1964.

25. Hess, Wilmot N., "Energetic Particles in the Inner Van Allen Belt," NASA TN D-1749, Washington, D. C., July 1963.

26. Naumann, Robert J., "A Qualitative Determination of the Lower Extremities of the Radiation Belts," NASA MTP-M-PR-61-7, George C. Marshall Space Flight Center, Huntsville, Alabama, March 21, 1961.

27. Pieper, G. F., C. O. Bostrum, and A. J. Zmuda, "Trapped Protons in the South Atlantic Magnetic Anomaly, July through December 1961," Journal of Geophysical Research, Vol. 70, No. 9, May 1, 1965.

28. Freden, S. C., J. B. Blake, and G. A. Paulikas, "Spatial Variation of the Inner Zone Trapped Proton Spectrum," Journal of Geophysical Research, Vol. 70, No. 13, July 1, 1965.

29. Russak, S. and K. Richardson, "Proton Flux, Dosage and Damage Estimates in the Van Allen Belt," Second Symposium on Protection Against Radiation in Space, Gatlinburg, Tennessee, October 12-14, 1964.

30. Heckman, Harry H. and George H. Nakano, "East-West Asymmetry in the Flux of Mirroring Geomagnetically Trapped Protons," Journal of Geophysical Research, Vol. 68, No. 8., April 15, 1963.

31. Freden, Stanley C. and George A. Paulikas, "Trapped Protons at Low Altitudes in the South Atlantic Magnetic Anomaly," Journal of Geophysical Research, Vol. 69, No. 7, April 1, 1964.

32. Hess, Wilmot N., "The Artificial Radiation Belt Made on July 9, 1962," Journal of Geophysical Research, Vol. 68, No. 3, February 1, 1963.

33. Van Allen, J. A., L. A. Frank, and B. J. O'Brien, "Satellite Observations of the Artificial Radiation Belt of July 1962," Journal of Geophysical Research, Vol. 68, No. 3, February 1, 1963.

34. Ness, Norman F., et al., "A Summary of Results from the IMP-I Magnetic Field Experiment," NASA TM X-612-65-180, Goddard Space Flight Center, Greenbelt, Maryland, May 1965.

35. Le Galley, Donald P., Space Science, John Wiley and Sons, New York, 1963.

36. Smith, Robert E., Editor, "Space Environment Criteria Guidelines for Use in Space Vehicle Development," NASA TM X-53142, George C. Marshall Space Flight Center, Huntsville, Alabama, September 30, 1964.

37. "National Environment and Physical Standards for the Apollo Program," NASA Office of Manned Space Flight, M-D E 8020.008B, SE 015-001-1, Washington, D. C., April 1965.

38. Van Allen, James A., "First Public Lecture on the Discovery of the Geomagnetically-Trapped Radiation," Department of Physics and Astronomy, State University of Iowa, Iowa City, SUI 60-13, May 1958.

39. Boyer, J. M. and W. T. Roberts, "An Untra-Low Frequency Wave Forve Mechanism for the Ionosphere," NASA TM X-53147, George C. Marshall Space Flight Center, Huntsville, Alabama, October 5, 1964.

APPROVAL TM X-53351

AN ENVIRONMENTAL MODEL FOR VAN ALLEN BELT PROTONS

By William T. Roberts

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

ROBERT E. SMITH
Chief, Space Environment Group

WILLIAM W. VAUGHAN
Chief, Aerospace Environment Office

E. D. GEISSLER
Director, Aero-Astrodynamics Laboratory

DISTRIBUTION

DEP-T
Dr. Rees

AST-S

R-AS
Mr. Williams
Mr. Gradecak
Mr. Hamby
Mr. Ball
Mr. deFries (4)
Mr. J. A. Belew
Dr. Spears

R-S
Mr. Dannenberg

R-TS
Mr. Ludie Richards

R-ASTR
Dr. Haeussermann
Mr. Currie
Mr. Singley
Mr. J. Boehm

R-RP
Dr. Stuhlinger (5)
Dr. Shelton
Mr. Urban
Mr. Burrell
Mr. Dozier

MS-IP
MS-IPL (8)
CC-P
HME-P
MS-H

MS-T
Roy Bland (5)

R-AERO
Dr. Geissler
Mr. Horn (3)
Mr. Baker (2)
Mr. Scoggins
Mr. Dahm (4)

R-AERO
Mr. W. Vaughan (2)
Mr. McNair
Mr. Dalton
Mr. R. E. Smith
Mr. O. Vaughan
Mr. Clingman
Mr. Roberts (50)
Mr. Daniels

I-MO
Dr. Speer
Mr. Kurtz

NASA Headquarters
Federal Office Bldg. 6
Washington 25, D. C.
Attn: Technical Information Div. (2)

Sci. & Tech. Info. Facility (25)
P. O. Box 5700
Bethesda, Maryland
Attn: NASA Rep. (S-AK/RKT)

NASA
Office of Manned Space Flight
Federal Office Bldg. No. 6
Washington 25, D. C.
Attn: Director (2)

NASA
Office of Space Science and Applications
Federal Office Bldg. No. 6
Washington 25, D. C.
Attn: Dr. John E. Naugle
Mr. Henry J. Smith
Mr. Newell (2)

NASA
Manned Spacecraft Center
Houston 1, Texas
Attn: Director
Mr. R. Thompson
Mr. J. M. Eggleston (4)
Mr. J. L. Modisette

DISTRIBUTION

NASA Headquarters
Office of International Programs
Washington 25, D. C.

North American Aviation, Inc.
Thru: MSFC Saturn IV Project Office

Douglas Aircraft Corporation
Thru: MSFC Saturn IV Project Office

Office of Advanced Research and Technology
1520 H. Street, N. W.
Washington 25, D. C.
Attn: Director (2)

NASA
Goddard Space Flight Center
Greenbelt, Md.
Attn: Director
Dr. Frank B. McDonald
Dr. Norman F. Ness
Mrs. H. H. Malitson
Library (2)

AMC Research Library (2)
Bldg. 4488
Redstone Arsenal

NASA
Langley Research Center
Langley Field, Va.
Attn: Director
Mr. R. Crouch
Technical Library (2)

NASA
Ames Research Center
Moffett Field, Calif.
Attn: Director
Library (2)

NASA
Jet Propulsion Lab.
4800 Oak Grove Dr.
Pasadena, Calif.
Attn: Director
Mr. A. J. Beck
Technical Library (2)

DISTRIBUTION (Continued)

NASA
Lewis Research Center
Cleveland, Obio
Attn: Director
Technical Library (2)

Air Force Cambridge Research Labs.
L. G. Hanscom Field
Bedford, Mass.
Attn: Director
Technical Library (2)

Space Technology Laboratories, Inc.
One Space Park
Redando Beach, Calif.
Attn: Technical Library (2)

Boeing Aircraft Co.
Thru: MSFC Saturn V Project Office

R-P&VE

Mr. Cline	Mr. H. Paul (2)
Mr. Goerner (2)	Dr. W. Lucas (2)

NASA
Kennedy Space Center
Attn: Dr. Debus (3)
Dr. Knothe (2)

IO
Director
Dr. Mrazek

I-I/IB
Col. James

I-V
Dr. Rudolph

I-E-MGR
Mr. Belew