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# Reliable Time Multiplexing by Replacement

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## ABSTRACT

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This Report describes a switching array for time multiplexing using active redundancy at the function level and passive redundancy at the component level to increase reliability. It has been divided into two parts: a description of the multiplexer, and a computer verification of its behavior. Analysis of the multiplexer is based on failure modes instead of failure probabilities. The computer simulation verifies that failure localization occurs, and that it extends the useful life of the multiplexer.

Author

# I. INTRODUCTION

This Report describes a redundant time multiplexer using a new electromechanical switch. It has been written with a dual purpose: to explore a concept to obtain reliable time multiplexing with the simplification of using electromechanical instead of solid state switching, and to exploit the properties of the switch itself. The switch is a miniature magnetic latching device with all the properties of a relay except the wear-out phenomenon. Switches may be cascaded as a result of the short circuit–open circuit signal path. Isolation is maintained between the signal path and the switching pulses.

The Report has been divided into two parts: a description of the multiplexer, and a computer verification of its behavior. Since the electromechanical switch is not immediately available, analysis of the multiplexer is based on failure modes instead of failure probabilities. Relative component probabilities are used in the computer simulation to verify that failure localization occurs, and that it extends the maximum useful life of the multiplexer.

# II. BEHAVIOR

### *A. The Multiplexer*

The time multiplexing function can be decomposed into two functions: the selection of a pulse from a clock train to control a switch, and operation of the switch to sample an input. This is shown diagrammatically in Fig. 1.

It is evident that a solid state shift register is most appropriate for routing pulses with electromechanical switches to sample the inputs. The reliability of the multiplexer is increased by increasing the reliability of each of its subfunctions appropriate to that subfunction. Since the shift register is solid state, replacement of the whole shift register is more practical than component

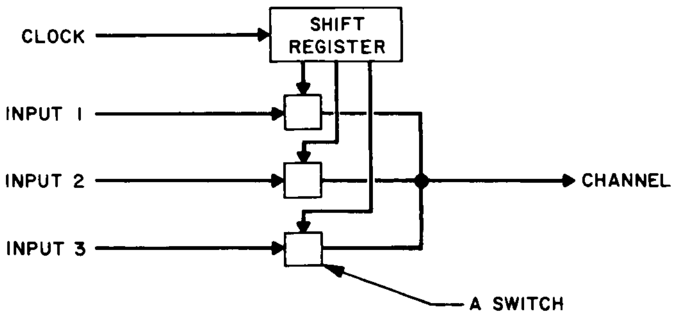



**Fig. 1. Time multiplexer**

replacement upon failure. The reliability of the shift register may be increased by accepted redundancy techniques, and therefore the reliability of the multiplexer depends on correcting failures in the sampling switches. The remainder of this Report is concerned with implementing a redundant switching function.

The multiplexer with standby components is represented in Fig. 2. Some terms that will be used to describe the switching matrix are cell, vector, and array. *Cell* $A$ contains one switch for sampling an input and one switch to call a replacement $B$ when sampling ceases because of a failure. $A$ and the exact duplicates $B$ and $C$ comprise a *vector*. Each vector may be thought of as a reliable switch associated with one input to the multiplexer. An *array* is the assemblage of $mn$ cells to multiplex $m$ inputs with 1 cell and $n - 1$ successive replacements per input. The vectors are independent if there is no coupling by the shift register, for there are no vector–vector connections in the array.

## B. The Vector and Replacement

Each cell contains an input signal path and a control pulse path. The control pulses received from the shift register not only multiplex the input signal but are used to effect replacement of a faulted cell. Replacement is an ordered procedure based on the following conventions:

1) The leftmost cell capable of switching is called the *active* cell.
2) There is at most one active cell at a time.
3) When the active cell fails, it passes the control pulses to a right neighbor—the next cell capable of switching. This cell becomes the active cell.
4) If there is no right neighbor capable of switching when the active cell fails, the vector enters a *search* mode.
5) The search mode selects all cells within the vector *capable of switching* and the leftmost of these becomes the active cell.

The physical meaning of *capable of switching*, of *active cell*, and of *search* mode will become clear upon examining the circuit of a cell.

As examples of replacement, consider the sequence of events associated with a vector in Table 1.

The signal path passes serially through each of the cells. The path is a short circuit through all cells except the active cell which, by definition, is open or short to accomplish switching. When the active cell fails, it short-circuits the signal path through itself and another cell becomes active and resumes switching. During the search mode, while there is no active cell, the input signal is temporarily shorted to the channel.

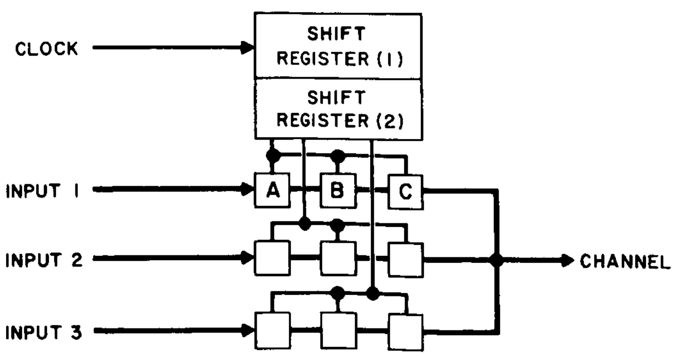



**Fig. 2. Redundant time multiplexer**

**Table 1. Three examples of replacement**

| | Cells | | | | Comments |
|---|---|---|---|---|---|
| | ① | ② | ③ | ④ | |
| | $1_A$ | 1 | 0 | 1 | ③ is a permanent failure |
| TIME ↓ | 0 | 1 | 0 | 1 | ① fails permanently |
| | 0 | $1_A$ | 0 | 1 | ② becomes the active cell |
| | 0 | 0 | 0 | 1 | ② has a (temporary) failure |
| | 0 | 0 | 0 | $1_A$ | ④ becomes the active cell |
| | 0 | 0 | 0 | 0 | ④ fails (permanently); |
| | 0 | 0 | 0 | 0 | search for an active cell |
| | 0 | 1 | 0 | 0 | ② is capable of switching |
| | 0 | $1_A$ | 0 | 0 | ② becomes the active cell |

$A$ — the active cell
1 — capable of switching
0 — not capable of switching

### C. *The Cell and Failure Detection*

The basic responsibility of a cell is to perform the switching function, which may be represented by the state diagram in Fig. 3. The + and − pulses from the shift register switch the cell between 0 (*ON*) and the 1 (*OFF*) states. When a failure occurs the state diagram degenerates, as shown in Fig. 4. Since it is not possible to distinguish the failure states 1 or 0 from the *OFF* or *ON* states, two more states (one switch) are needed. Failure detection is accomplished by associating the control pulses + and − with the switching states 1 and 0 respectively. When a cell in the 11 state receives a + pulse it naturally passes to the 01 state. But if the cell in the 11 state receives a − pulse it passes to a failure state 10, as shown in Fig. 5.

#### 1. State Diagram

The inconsistency between an input pulse and the present state causing transfer to a failure state will be due to one of the following:

1) Initial conditions
2) A control pulse failure
3) A temporary cell failure
4) A permanent cell failure

To guard against 4), the same course of action is followed for 1), 2), 3), and 4); the cell passes control to another cell. If 1), 2), or 3) has occurred, the cell is permitted to resume switching as the active cell after a search. The complete state diagram is shown in Fig. 6. The shaded states represent short-circuit signal paths through the cell. The reason for the 10–00 transition *from* 10 *to* 00 is not obvious, but is important. Therefore a slight digression will be made to reveal the mapping between this state diagram and two switches, whereupon the significance of the transition will become apparent.

Consider two switches in the four possible states shown in Fig. 7 and compare these with the adjoining state diagram. The following relations become obvious:

$S_1 = 0 \longleftrightarrow$ right half of A

$S_1 = 1 \longleftrightarrow$ left half of A

$S_2 = 0 \longleftrightarrow$ lower half of B

$S_2 = 1 \longleftrightarrow$ upper half of B

Then the transitions $11 \rightarrow 01$ and $10 \rightarrow 00$ involve the same switch $S_1$, but with $S_2$ in the 1 and 0 states respectively.

Therefore the 10 to 00 transition shown in Fig. 6 is an attempt to use $S_1$ after it may have previously failed to

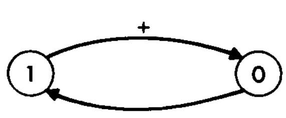



**Fig. 3. Switching function state diagram**

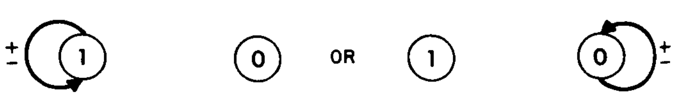



**Fig. 4. Switching function failure**

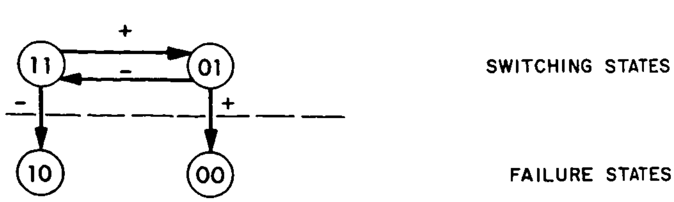



**Fig. 5. Switching function with failure states**

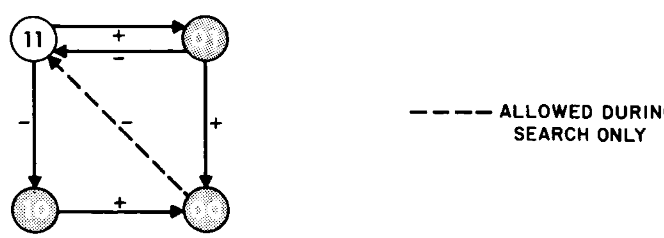



**Fig. 6. Cell state diagram**

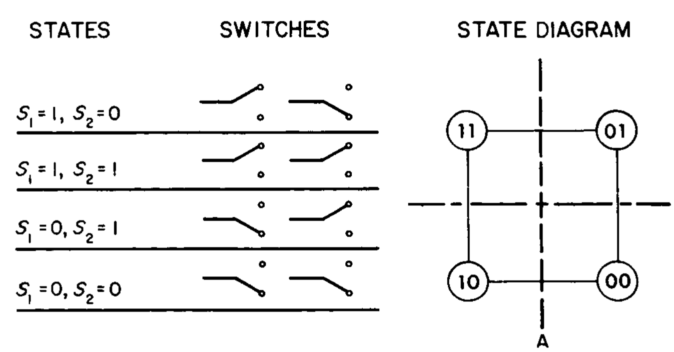
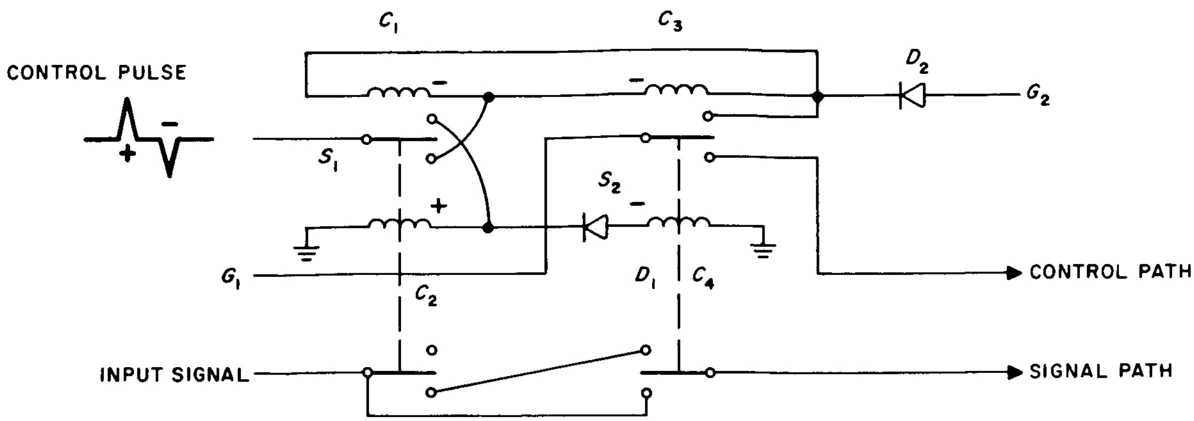
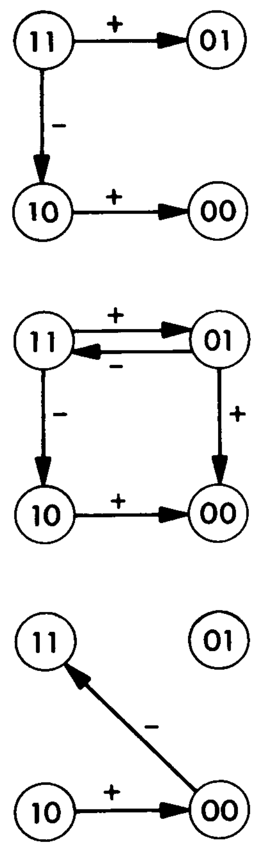



**Fig. 7. Switch-state diagram mapping**




| INPUTS | | STATE | | NEXT | |
|---|---|---|---|---|---|
| $G_1$ | $G_2$ | $S_1$ | $S_2$ | + | − |
| 0 | 0 | 0 | 0 | 00 | 00 |
| 0 | 0 | 0 | 1 | 01 | 01 |
| 0 | 0 | 1 | 0 | 00 | 10 |
| 0 | 0 | 1 | 1 | 01 | 10 |
| 1 | 0 | 0 | 0 | 00 | 00 |
| 1 | 0 | 0 | 1 | 00 | 11 |
| 1 | 0 | 1 | 0 | 00 | 10 |
| 1 | 0 | 1 | 1 | 01 | 10 |
| 1 | 1 | 0 | 0 | 00 | 11 |
| 1 | 1 | 0 | 1 | Ø | Ø |
| 1 | 1 | 1 | 0 | 00 | 10 |
| 1 | 1 | 1 | 1 | Ø | Ø |

STATE DIAGRAM




MODE

INITIAL CONDITION
$G_1 = 0$

ACTIVE CELL WHEN:
$G_1 = 1$ AND $S_2 = 1$

SEARCH WHEN:
$G_1 = 1$, $S_2 = 0$ AND $G_2 = 1$

$G_1$, $G_2$ = 1 GROUND POTENTIAL
= 0 NO VOLTAGE APPLIED
Ø = CONDITION DOESN'T EXIST

**Fig. 8. Cell circuit, transition tables, and state diagram**

complete the 11 to 01 transition. If the failure is permanent, the cell remains in state 10 and is not considered *capable of switching* in later search modes. Otherwise the cell moves to and remains in state 00, and awaits its turn to be the active cell. During the search mode, the 00 to 11 transition is completed.

The 10–00–11 transition is preferred to a 10–11 transition, for the latter increases the possibility of remaining permanently in 11. This would be caused by a failure $S_2 = 1$ and would lead to permanently decoupling the input signal when another cell may be capable of switching. The 01 to 00 failure transition is not followed by a 00 to 10 transition, for it is not needed. If the cell fails in the 01 state, it is not possible to pass from 00 to 11 during the search, and thus the signal path through the cell is not jeopardized.

### 2. Circuit

Figure 8 shows the circuit of a cell, the transition tables, and the state diagrams. The tables and diagrams are partitioned by mode—initial condition, active cell, and search—according to the inputs to the cell. The result portrays the cell as three sequential machines, but this is only because the inputs form three sets. The circuit uses two double-pole switches; the poles are paired by the dotted lines. The polarity of pulse which draws the switch to the coil is indicated for each coil. The opposite pulse repels the switch. The switches are capable of setting themselves, i.e., $S_1 = 1 \xrightarrow{+} S_1 = 0$ via $C_2$.

## D. *Multiplexing*

This portion of the Report will consider the definition of the active cell, the solution of initial conditions, unfouling, power consumption, and checkout of a vector before usage. To simplify the discussion the system will be assumed free of failures. The second half of the Report includes a failure analysis as part of the verification.

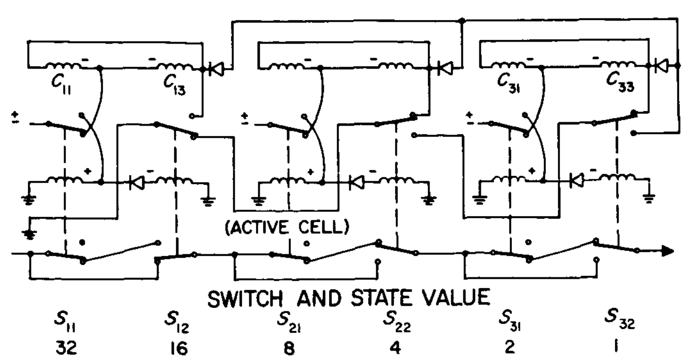



**Fig. 9. Vector with three cells**

From Fig. 9, a vector with three cells, the active cell is uniquely defined by being the leftmost cell with $S_2 = 1$. Any cell to the left has $S_2 = 0$ and therefore decouples $C_1$ and $C_3$ from ground. Any cell to the right has its coils $C_1$ and $C_3$ decoupled by $S_2$ of the active cell. The definition of one active cell limits power consumption to that necessary to perform the switching function or replacement. In addition it helps insure the availability of other cells by not "using" them.

### 1. Initial Conditions

The solution of initial conditions and unfouling is obtained in at most two passes through the array. This consists of three steps:

1) The control portion of each cell seeks to short-circuit the cell signal path.
2) Each vector defines an active cell according to the states of its cells.
3) All vectors are synced by the shift register to short-circuit only one active cell signal path at a time.

The state diagram in Fig. 10 demonstrates that a vector will begin switching from any state and with either control pulse. The switches in three cells have been weighted in powers of 2 to enumerate the $2^6$ vector states: from left to right, 32, 16, 8, 4, 2, and 1. State 4 (000100) corresponds to $S_{22} = 1$ (up) and all other switches down. With a + control pulse the vector passes to state 0 as part of the failure mechanism. A − control pulse leads to state 12 from state 4 as part of the active cell switching. With cells numbered from the left, the diagram is organized as follows:

| States | Active Cell | Active Cell Signal Path |
|---|---|---|
| 16–31 | 1 | Short |
| 48–63 | 1 | Open |
| 4–7, 36–39 | 2 | Short |
| 12–15, 44–47 | 2 | Open |
| 1, 9, 33, 41 | 3 | Short |
| 3, 11, 35, 43 | 3 | Open |

### 2. Stable and Transient States

States 0, 2, 8, 10, 32, 34, 40, and 42 belong to the search mode with a signal path short through the vector. Within

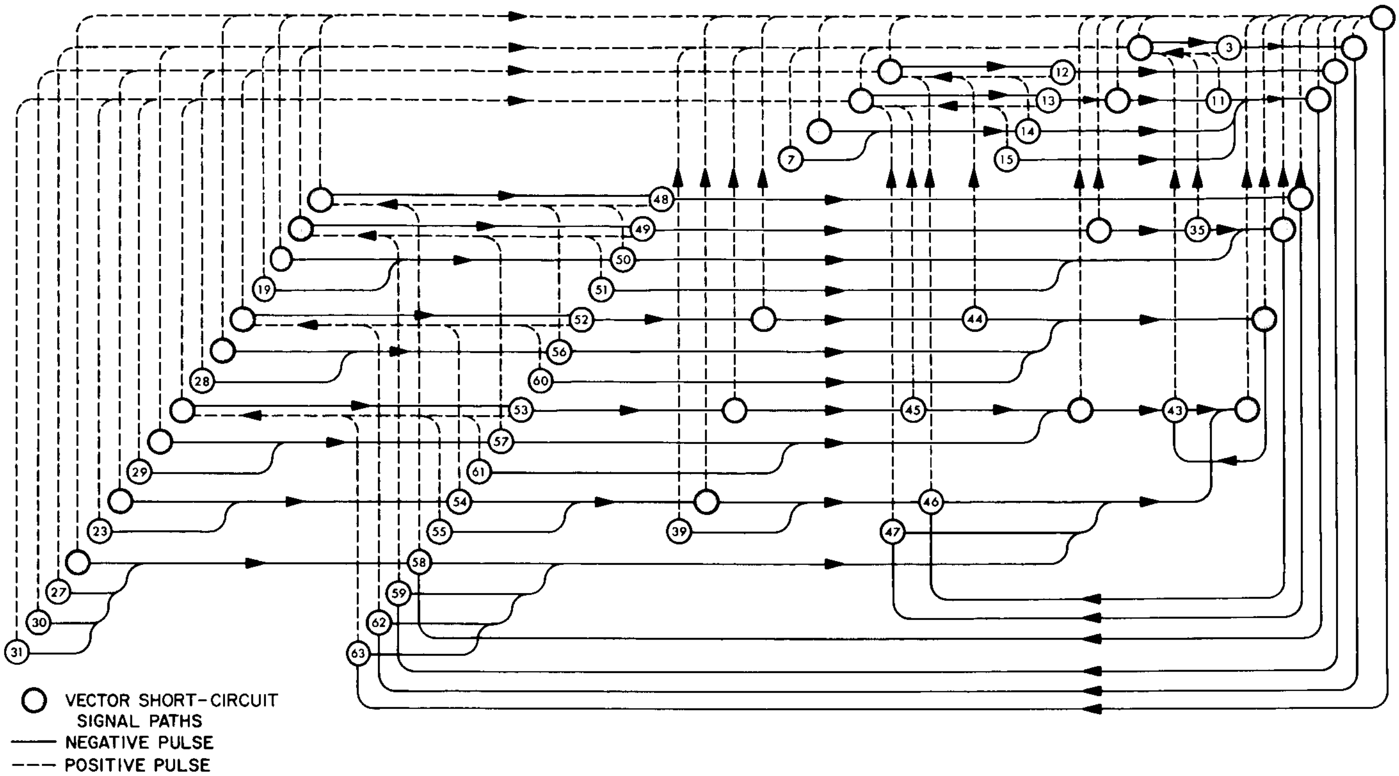



**Fig. 10. State diagram for a vector with three cells**

each set of states are principal switching states which perform the switching function for the vector. These are stable states of the vector, whereas the states associated with replacement and search are transient. In Table 2 a list of the principal switching states with their binary equivalents reveals the stable switch configurations.

With failures other pairs of states perform the switching function. For example, suppose $S_{21} = 1$ ($S_1$ of cell 2) is a permanent failure and there are no other failures.

**Table 2. Principal switching states**

| States | | Signal path short | | | Signal path open | | |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 00 | 00 | $\boxed{01}$ | 00 | 00 | $\boxed{11}$ |
| 4 | 12 | 00 | $\boxed{01}$ | 00 | 00 | $\boxed{11}$ | 00 |
| 5 | 13 | 00 | $\boxed{01}$ | 01 | 00 | $\boxed{11}$ | 01 |
| 16 | 48 | $\boxed{01}$ | 00 | 00 | $\boxed{11}$ | 00 | 00 |
| 17 | 49 | $\boxed{01}$ | 00 | 01 | $\boxed{11}$ | 00 | 01 |
| 20 | 52 | $\boxed{01}$ | 01 | 00 | $\boxed{11}$ | 01 | 00 |
| 21 | 53 | $\boxed{01}$ | 01 | 01 | $\boxed{11}$ | 01 | 01 |

Then the stable switching states are obtained by adding the numerical value of $S_{21} = 1$ to the numbers assigned the stable states above. Since the weight of $S_{21}$ is 8, the stable pairs are

| | | | | |
|---|---|---|---|---|
| 1 | 3 | +8 = | 9 | 11 |
| 16 | 48 | +8 = | 24 | 56 |
| 17 | 49 | +8 = | 25 | 57 |

The effect of this failure on the principal switching states may be observed in Fig. 10. The switching pairs (4, 12), (5, 13), (20, 52), and (21, 53) are not transformed to allowed switching states because $S_{21} = 1$ and $S_{22} = 1$ open the signal path.

The transient events of replacement with search are abstracted from Fig. 10 in Fig. 11. This diagram reveals that replacement always proceeds to the right between search modes. With the enumeration of the principal switching states and abstraction of replacement, Fig. 10 should now reveal that a vector will perform the switching function within two passes through the array. This is equivalent to two applications of +, − to the vector. It is evident that there are disconnected states and the

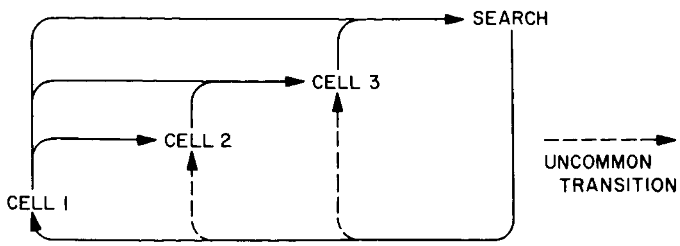



**Fig. 11. Replacement**

vector may cycle through a subset of the transient states only when there are permanent component failures.

### 3. Checkout

Most redundant systems, including all forms of quadded logic, do not permit a checkout of the components before use. That is, there is no way to detect individual component failures from the system output. However, since a vector is based on active replacement, the state of most of the components may be identified with states of the sequential machine of Fig. 10. By cycling through certain combinations of states, the components necessary for these transitions are demonstrated to be operational. The determination of optimal input sequences to analyze a sequential machine is a classical problem, and only an example will be given.

Assume that a vector is failure-free and it is desired to demonstrate that all cells may perform the switching function. An input sequence of $n + 1$ positive pulses, where $n$ is the number of cells, will leave the vector in the 0 state regardless of its initial state. Then the sequence $-$, $+$ followed by $n$ applications of $-$, $+$, $+$ will cycle $S_1$ of each cell and set $S_2 = 0$. By observing the output of the vector signal path, it is possible to determine if the desired transitions have actually occurred. At the end of the sequence a $+$ pulse will not open-circuit the signal path until a $-$ pulse sets all the cells.

# III. VERIFICATION

## *A. Failure Analysis*

Essentially to this point the switching array has been assumed failure-free, to describe its behavior. Now, since a vector is expected to accept failures and continue switching, the behavior with failures will be analyzed. Failures may be considered temporary or permanent. Since failure detection is based on comparing the state of a cell with the polarity of the control pulse, temporary failures cannot be distinguished from a control pulse failure (e.g., $+, -, +, +, -$). Therefore, temporary failures are functionally the same as control pulse failures; when the vector remains sufficiently undegraded, unfouling corrects for the failure. The real problem then is the interaction of permanent failures.

A permanent failure is defined as the continuous inability of a component to perform its normal function. To determine the *logical effect* of failures arising from the electrical interconnections, the failures are assumed to be catastrophic. Switches fail to transfer and coils and diodes become open or short circuits. This assumption of catastrophic behavior is the only tractable way to proceed, for a finer analysis would exceed the 69,984 states that already exist for a *single cell.*

### 1. Cell Failures

The primary objective of a cell is to cycle $S_1$ until it fails, then transfer $S_2$ to obtain a replacement. There are three components necessary for the successful cycling of $S_1$: coils 1 and 2 and the switch itself. When one of these fails, the cycling ceases. For instance, $S_1$ may fail in the 1 state because of a switch failure, an open circuit in coil 2, or a short circuit in coil 2.

Coils 3 and 4 and diode 1 detect the inability to cycle $S_1$. Diode 2 during the search mode permits a cell to be reconsidered for active switching. At all other times it provides cell-to-cell isolation. Obviously the components $C_3$, $C_4$, $D_1$, and $S_2$ may fail, as do $C_1$, $C_2$, and $S_1$. To permit replacement, probabilities favor the components $C_3$, $C_4$, $D_1$, and $S_2$. This is accomplished by paralleling two coils to form $C_3$ and forming a redundant quad for

$D_1$ and $D_2$. The coil $C_4$ is not used (unless $D_1$ develops a short) and $S_2$ is not transferred until $C_1$, $C_2$, or $S_1$ fails.

Although failures may develop in the $C_3$, $C_4$, $D_1$, $S_2$ components before they appear in $C_1$, $C_2$, $S_1$, only a few combinations prevent calling a replacement. These failures are listed in Table 3.

**Table 3. Failure combinations which prevent replacement**

| Failure 1 | Failure 2 | Comments |
|---|---|---|
| $S_2 = 1$ | Any $C_1$, $C_2$, $S_1$ failure | Uncommon since $S_2$ is rarely switched |
| $C_3$ open | $S_1 = 0$ or $C_1$ open | Unlikely since both $C_3$ coils must fail open |
| $D_1$ open or short | $S_1 = 1$ or $C_2$ open | Unlikely since $D_1$ is a redundant quad |
| $C_4$ open | $S_1 = 1$ or $C_2$ open | Difficult to achieve for $C_4$ is rarely pulsed |

The occurrence of coil shorts does not appear in the table, for in almost every case a coil short prevents replacement. The double occurrence of paralleling, $C_1$–$C_3$ and $C_2$–$D_1C_4$, necessitates that the physical construction of the coils prevent short circuits. Without this restriction the first short circuit will prevent replacement and terminate the usefulness of the array with high probability.

The diode $D_1$ is necessary to prevent the failure $S_1 = 1$ from cycling $S_2$ by supplying bipolar pulses to $C_4$. Other intercell failure combinations exist but either are included in the previous table or do not prevent replacement.

During the search mode an attempt is made to set $S_2 = 1$ for each cell. Since this event is completely the opposite of seeking a replacement, the failure combinations are different. If any of the failures $S_1 = 1$, $S_2 = 0$, or $C_3$ open exist, the cell is passed over during the search mode. Otherwise $S_2 \rightarrow 1$ and switching and/or replacement occurs according to the failures accumulated by the vector. The inability to set $S_2 = 1$ for any cell is not in itself catastrophic to the vector. A vector failure is defined as the inability to locate a stable active cell or the permanent short-circuiting of the input-channel signal path.

**2. Isolation Breakdown**

The diode $D_2$ isolates each cell from the rest of the vector except during the search mode. The dependence of this isolation on the condition of $D_2$ may be understood from the two relations:

1) Assuming no $D_2$ diode failure in any cell, ground is present in only one cell at the node where $C_1$, $C_3$, and $D_2$ intersect. This is the active cell.

2) In a cell with ground at this node, only a negative pulse can lead to successful switching. A positive pulse may transfer $S_2$ ($1 \rightarrow 0$) but it cannot set $S_1$ ($0 \rightarrow 1$).

Consider three cells symbolized by $X$, $A$, and $Y$ in Fig. 12. $A$ is the active cell, and $X$ and $Y$ are any two remaining cells. $X$ and $Y$ may be located anywhere with respect to $A$, or only one of the two may exist. It will be shown that isolation is destroyed only when the diode $D_2$ of $A$ is a short circuit.

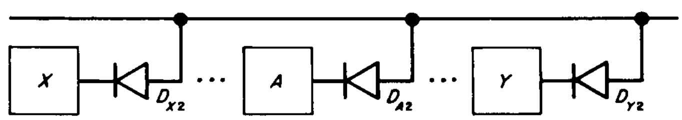



**Fig. 12. Three cells**

From 1) above, it is evident that for any $X$ and $Y$ a short circuit of $D_{X2}$ cannot lead to switching in $Y$, and similarly for $D_{Y2}$ and $X$. Cell $X$ is not grounded at coils $C_1$ and $C_3$, nor cell $Y$; $S_{X2}$ has passed ground to $A$ and $S_{A2}$ has denied ground to $Y$. Therefore neither $X$ nor $Y$ can pass ground to the other.

Since $A$ possesses ground, only it can pass ground along the feedback path to either $X$ or $Y$. From 2) above, meaningful switching can begin in either $X$ or $Y$ only with a negative pulse. This means that when $D_{A2}$ is a short circuit, switching may take place in another cell. For instance, a negative pulse may pass from $X$, through $D_{X2}$, through the shorted $D_{A2}$ to ground. This pulse sets $S_{X1}$ ($0 \rightarrow 1$) and a positive pulse through $C_{X2}$ will reset $S_{X1}$ ($1 \rightarrow 0$). When $D_{A2}$ short-circuits, $X$ may become the active cell if $S_{X2}$ transfers ($0 \rightarrow 1$). When an $X$ does not exist, $Y$ cannot become the active cell because $S_{A2}$ does not transfer ($1 \rightarrow 0$). $Y$ may cycle $S_{Y1}$ synchronously (but in phase), with $A$ depending upon its failures.

Cell–cell isolation is not meaningful during the search mode. With ground along the feedback path ($S_{i2} = 0$, $i = 1, 2, \ldots, n$), a $D_{i2}$ short circuit in cell $i$

cannot affect any cell $j$. Isolation between cells then may be destroyed in only $(n - 1)$ ways between the active cell and each of the remaining $(n - 1)$ cells, instead of the anticipated $\binom{n}{2}$ ways between any pair of cells.

### 3. Unstable Replacement

Typically a vector may have absorbed enough failures that it cycles among the cells without finding an active cell. This assumes that none of the failures is $S_2 = 1$, for this would prevent a search. The periodicity of this cycling may be from 1 to $(n + 1)$ pairs of input pulses $(+, -)$. That is, from one (e.g., $S_2 = 0 \rightarrow 1 \rightarrow 0$, etc.) to $n$ cells may contribute to the cycle. The latter would be of the form:

$$S_{12} = 0, S_{22} = 0, \ldots, S_{n2} = 0$$

$$S_{ij} = \text{switch } j \text{ of cell } i$$

1 $\quad S_{12} = 1, S_{22} = 1, \ldots, S_{n2} = 1$

2 $\quad S_{12} = 0, S_{22} = 1, \ldots, S_{n2} = 1$

3 $\quad S_{12} = 0, S_{22} = 0, \ldots, S_{n2} = 1$

.

.

.

$n + 1 \quad S_{12} = 0, S_{22} = 0, \ldots, S_{n2} = 0$

During a search the input signal is shorted to the channel; therefore a vector periodically searching for an active cell will degrade the channel. With additional failures this periodic searching may cease, as all switching ceases.

## B. Computer Simulation

The computer program is basically a Monte Carlo simulation. A representation of the switching array is stored in the computer and inputs are applied to perform multiplexing. Switching in each cell is a three-step process:

1) Determine a path for the control signal.

2) Determine the state of operational components along this path for this control pulse–i.e., possibly introduce failures.

3) Change component states as a result of switching and/or failures.

The main program and subroutine for introducing failures are outlined in Fig. 13.

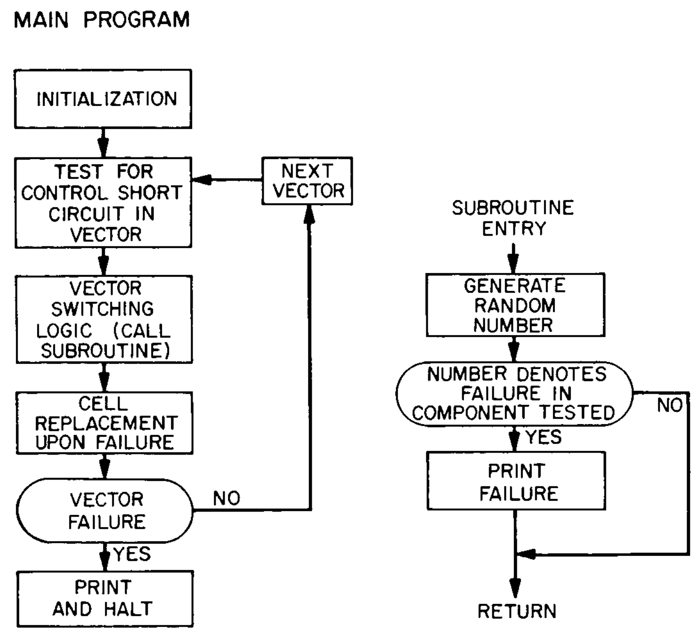



**Fig. 13. Computer simulation program**

The random number generator is based on the familiar congruence method

$$X_{i+1} = X_i \, (A + B) \bmod C$$

A suitable choice of $A$, $B$, and $C$ for the decimal IBM 1620 computer is $A = 10^5$, $B = 3$, and $C = 10^{10}$. To avoid degenerate sequences, the initial random number $X_0$ contained no multiples of 2 or 5. Only the high order 6 digits of each 10-digit random number were used to introduce failures. The period $p$ of the sequence is defined by

$$X_0 = X_0 \, (10^5 + 3)^p \bmod 10^{10}$$

and is about $10^9$ for suitable $X_0$. A computer program verified the properties of interest over 10,000 samples for several choices of $X_0$.

Since switching may include simultaneous events and the simulation is a serial procedure, there must be a tradeoff between program complexity and simulation realism. The convention adopted was to permit instantaneous sympathetic failures between components sharing an electromechanical relation. For instance, a coil failure may prevent switching at the occurrence of the failure. Instantaneous sympathetic failures are not allowed for components sharing an electrical relation.

That is, a diode short-circuiting a coil will not affect coil performance at the occurrence of failure. All succeeding attempts to use the coil will be met by the short circuit, however.

In the program there is also provision for introducing probabilities resulting from the physics of switch failure. This permits a switch to have a favored sympathetic failure behavior.

The running time of the computer simulation was adjusted to approximately 2 hours per trial by probability scaling. The mean life of a switch was set at $10^3$ cycles and the other components were scaled accordingly. The program contains provisions for using Weibull, exponential, normal, or uniform probability distributions as a function of component utilization to introduce failures. This paper concludes with the results of 90 trials which demonstrate the interaction of coil and diode failure probabilities. A typical computer printout for one trial is given in the Appendix.

## C. *Computer Results*

The cell-oriented multiplexer has been designed about the following probability hierarchy:

| | | | | | | |
|---|---|---|---|---|---|---|
| coil short<br>diode short<br>pulse failure | < | diode<br>open | < | coil<br>open | < | switch<br>failure |

To verify performance under this hierarchy, 9 sets of failure probabilities were run on the computer. These consisted of all combinations of 3 diode and 3 coil probabilities. The number and kind of failures were tabulated for 10 trials per set to determine optimum relations for maximum multiplexer life and failure absorption. The sets of failure probabilities are given in Table 4.

In all sets the probability of a switch failure is $0.5 \times 10^{-3}$ and the probability of a pulse failure is $0.010 \times 10^{-3}$. All probabilities were chosen to:

1) Conform with the probability hierarchy
2) Limit computer simulation time
3) Demonstrate cell behavior

They are not meant to represent current physical component probabilities. In addition, to simplify conclusions about cell performance, the probability distributions are uniform for this simulation.

**Table 4. Failure probability sets**

| Decreasing Diode Probabilities ↑ | | |
|---|---|---|
| **Set 1** | **Set 4** | **Set 7** |
| C3O $= 0.020 \times 10^{-3}$<br>C3S $= 0.050 \times 10^{-3}$<br>CO $= 0.400 \times 10^{-3}$<br>CS $= 0.025 \times 10^{-3}$<br>*DO* $= 0.025 \times 10^{-3}$<br>*DS* $= 0.010 \times 10^{-3}$ | $0.015 \times 10^{-3}$<br>$0.030 \times 10^{-3}$<br>$0.200 \times 10^{-3}$<br>$0.015 \times 10^{-3}$ | $0.010 \times 10^{-3}$<br>$0.020 \times 10^{-3}$<br>$0.100 \times 10^{-3}$<br>$0.010 \times 10^{-3}$ |
| **Set 2** | **Set 5** | **Set 8** |
| *DO* $= 0.050 \times 10^{-3}$<br>*DS* $= 0.015 \times 10^{-3}$ | (The matrix is filled from the row and column entries) | |
| **Set 3** | **Set 6** | **Set 9** |
| *DO* $= 0.100 \times 10^{-3}$<br>*DS* $= 0.025 \times 10^{-3}$ | → Decreasing Coil Probabilities | |
| *D* — Diode<br>C — Coils 1, 2, 4<br>C3 — Coil 3 (two coils paralleled) | O — Open circuit<br>*S* — Short circuit | |

### 1. Failure List

The results of the computer simulation are given in Table 5. The average number of failures is listed by component for each set. *NB* represents the average number of passes through an array of 10 vectors before the multiplexer fails. The average number of permanent failures occurring within a trial is given by *FAIL*.

By noting trends in the data, it is possible to reach certain conclusions with the limited statistics available. These conclusions are divided into three categories: Constraint correlates component failures with component failure probabilities, Behavior demonstrates the interdependence of failures, and Result compares array performance by set in terms of *NB* and *FAIL*.

a. Constraint

1) Coil failures are directly proportional to the probability of a coil failure. The number of coil failures decreases comparing sets 1, 4, 7; sets 2, 5, 8; and sets 3, 6, 9. The number of coil failures is relatively constant comparing sets 1, 2, 3; sets 4, 5, 6; and sets 7, 8, 9.

2) Similar statements can be made about diode failures with respect to diode failure probabilities.

**Table 5. Simulation failures**

| Set 1 | | | | Set 4 | | Set 7 | |
|---|---|---|---|---|---|---|---|
| C1 short | 0.3 | S1 down | 2.0 | 0.1 | 5.3 | 0.1 | 6.8 |
| C1 open | 2.0 | S1 up | 3.6 | 1.6 | 4.7 | 0.9 | 5.5 |
| C2 short | 0.2 | S2 down | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 |
| C2 open | 3.0 | S2 up | 0.0 | 3.0 | 0.0 | 1.2 | 0.0 |
| C3 short | 0.2 | | | 0.1 | | 0.1 | |
| C3 open | 0.1 | | | 0.0 | | 0.0 | |
| C4 short | 0.0 | | | 0.1 | | 0.0 | |
| C4 open | 0.6 | | | 0.5 | | 0.3 | |
| D1 short | 0.0 | | | 0.0 | | 0.1 | |
| D1 open | 0.2 | | | 0.3 | | 0.6 | |
| D2 short | 0.3 | *NB* | 525.5 | 0.4 | 887.5 | 0.4 | 957.0 |
| D2 open | 0.3 | *FAIL* | 11.8 | 0.7 | 16.8 | 0.3 | 16.5 |
| **Set 2** | | | | **Set 5** | | **Set 8** | |
| C1 short | 0.0 | S1 down | 2.5 | 0.1 | 3.6 | 0.1 | 6.3 |
| C1 open | 3.3 | S1 up | 3.5 | 2.5 | 3.7 | 1.1 | 7.8 |
| C2 short | 0.4 | S2 down | 0.0 | 0.2 | 0.0 | 0.1 | 0.1 |
| C2 open | 4.0 | S2 up | 0.0 | 2.3 | 0.0 | 1.6 | 0.1 |
| C3 short | 0.4 | | | 0.0 | | 0.0 | |
| C3 open | 0.1 | | | 0.3 | | 0.0 | |
| C4 short | 0.0 | | | 0.0 | | 0.0 | |
| C4 open | 1.4 | | | 0.6 | | 0.4 | |
| D1 short | 0.2 | | | 0.5 | | 0.1 | |
| D1 open | 0.8 | | | 1.0 | | 1.3 | |
| D2 short | 0.8 | *NB* | 688.9 | 0.6 | 882.4 | 0.8 | 1134.2 |
| D2 open | 0.9 | *FAIL* | 18.5 | 1.8 | 17.4 | 2.5 | 22.3 |
| **Set 3** | | | | **Set 6** | | **Set 9** | |
| C1 short | 0.2 | S1 down | 1.6 | 0.2 | 3.2 | 0.1 | 4.4 |
| C1 open | 1.9 | S1 up | 1.9 | 1.5 | 2.9 | 0.7 | 3.9 |
| C2 short | 0.1 | S2 down | 0.0 | 0.0 | 0.1 | 0.2 | 0.0 |
| C2 open | 3.8 | S2 up | 0.0 | 1.8 | 0.0 | 1.3 | 0.0 |
| C3 short | 0.5 | | | 0.2 | | 0.2 | |
| C3 open | 0.1 | | | 0.0 | | 0.1 | |
| C4 short | 0.0 | | | 0.0 | | 0.0 | |
| C4 open | 0.5 | | | 0.3 | | 0.3 | |
| D1 short | 0.1 | | | 0.3 | | 0.2 | |
| D1 open | 1.5 | | | 0.9 | | 0.7 | |
| D2 short | 0.7 | *NB* | 479.2 | 1.3 | 737.0 | 0.3 | 853.9 |
| D2 open | 1.4 | *FAIL* | 14.3 | 2.7 | 15.5 | 2.3 | 14.9 |

b. Behavior

1) There is a slight indication that the number of diode failures increases as the number of coil failures decreases in sets 2, 5, and 8. This may be attributed to the fact that diodes usually appear in series with a coil; when the coil fails open, the diode is no longer used. The converse does not occur because coil failures are more probable than diode failures in the probability hierarchy.

2) The number of switch failures increases when moving from any set to set 8, although the probability of a switch failure is constant for all sets.

c. Result

1) The average number of successful passes through the array, *NB*, increases when moving from any set to set 8.

2) In a similar manner the average number of total failures, *FAIL*, is a maximum for set 8.

Statements a. 1) and a. 2) reveal that *sufficient data (10 trials per set) have been taken to indicate the characteristics of the array.* Statement b. 2) demonstrates that *the cell concept permits failure localization by controlling the number of ineffectual switching failures.* Statements c. 1) and c. 2) verify the array's expected response to the probability hierarchy by demonstrating that *the higher degree of failure localization in set 8 has led to both maximum failure absorption and maximum usable life for the array.*

### 2. Failure Interaction

Intuitively it would be expected that set 7 with the smallest coil and diode failure probabilities would yield superior cell performance. There are instances, however, when component failures extend the usefulness of a vector by preventing a degraded cell from being called as a replacement. For instance an open-circuited diode $D_2$ can prevent a cell from being used as a replacement when it might set $S_2$ permanently and prevent replacement thereafter. Although some examples of intercell failures are given, they do not sufficiently describe vector behavior. Events arising from intracell failures are very interesting and some examples from trials 1–90 will be analyzed. These will demonstrate that the interaction, not just the occurrence, of failures controls the replacement of a faulted cell.

One of the most common failures is sketched in Fig. 14. This prevents $B$ from becoming a replacement cell except immediately after a search. A variation on this same failure is shown in Fig. 15. The failure of $D_{B2}$ has led to a left replacement by establishing what looked like a search to cell $A$. When the two cells are reversed, the case of two cells cycling together is obtained in Fig. 16. The simultaneous switching of $S_{A1}$ and $S_{B2}$ does not destroy the switching property of the vector, for 01 and 00

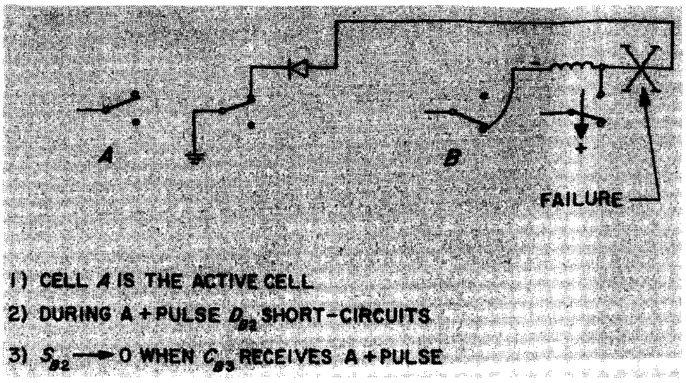



Fig. 14. Feedback diode failure (1)

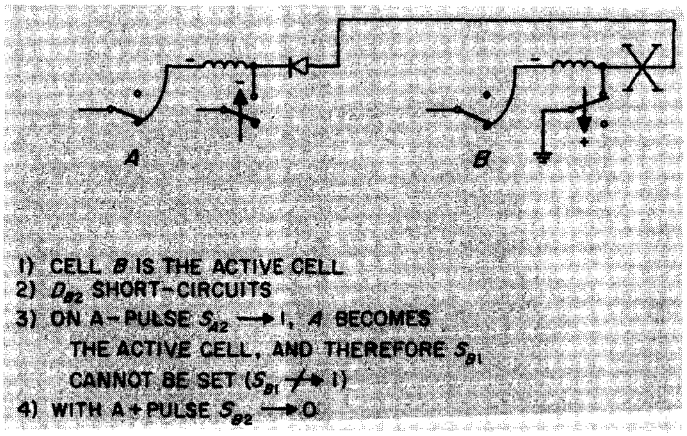



Fig. 15. Feedback diode failure (2)

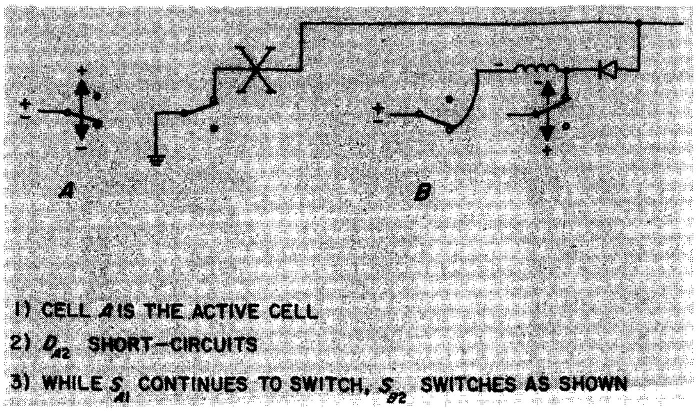



Fig. 16. Feedback diode failure (3)

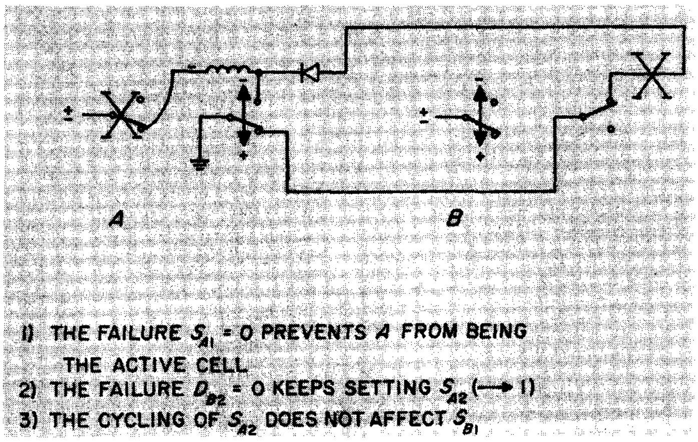



Fig. 17. Feedback diode failure (4)

are both short circuits through *B*. Another case of multiple switching may be obtained with one additional failure, as shown in Fig. 17. Again the signal path is not compromised for 01 and 00 are both short circuits through *A*.

The failures shown in Figs. 14–17 do not prevent a vector from correctly switching an input. In most cases the vector fails because failures in the active cell prevent replacement while the signal path is closed. These failures were listed in Section III. A. 1.

# IV. CONCLUSIONS

This Report has completely described a switching array for time multiplexing using active redundancy at the function level and passive redundancy at the component level to increase reliability. The system has been shown to begin sequencing properly from any initial state, assuming certain critical failures have not occurred. Similarly the array is capable of unfouling after temporary failures. Power consumption is directly proportional to the number of inputs sampled and is minimized by denying power to spares. The array also permits checking the functional redundancy by comparing input pulse and output signal sequences.

Arguments in the paper and the results of computer simulation have shown that the most important design constraint is low probability for a short circuit in coils and diodes. Short-circuited coils lead directly to vector failures. Diode short circuits break down cell independence and lead to undesirable usage. The other important constraint is low probability of a diode open circuit. All feedback diodes are used equally by the bipolar pulses and thus feedback diodes failing open prevent cell use after a search. Failure detection diodes, if open, can prevent replacement. Low probability of a control pulse failure is desirable to prevent replacement of an operational cell.

The proposed multiplexer would consume approximately the same power as present multiplexers. The low power consumption that is being obtained with MOS circuits, however, cannot be matched by the envisioned implementation of this multiplexer. This appears to be the penalty for obtaining a high degree of failure localization and thereby fault correction with this switch.

Initially it was demonstrated that the array is a collection of independent reliable switches. Therefore it is not unnatural to use the vector—an integral unit—as a reliable switch. There are several spacecraft applications, such as power switching, which use electromechanical switches where a vector would increase the probability of successful operation.

# APPENDIX A

## Computer Printout

The computer printout is divided into three parts covering the initial conditions, dynamic behavior, and final conditions. In Figs. A-1–A-6 components are designated by ($A$, $B$, $C$) where $A$ and $B$ are respectively the vector and cell coordinates and $C$ is the component type. The convention for listing components is:

| | |
|---|---|
| 1, 2 | $S_1, S_2$ |
| 3, 4, 5, 6 | $C_1, C_2, C_3, C_4$ |
| 7, 8 | $D_1, D_2$ |

The entries ($A$, $B$, 9) are used to store information about the vector, such as the active cell number. The columns of numbers denote the component state and the number of times each component has been used. The states are defined as follows:

| | | |
|---|---|---|
| 2–3 | $S_1, S_2$ | switching states |
| 0–1 | $S_1, S_2$ | failure states |
| 0 | coil | short circuit |
| 1 | and | operational |
| 2 | diode | open |

Initially all of the components are operational and none has been used previously. The switching states in the various cells have not been initialized; this is to demonstrate that multiplexing can begin successfully without presetting the switches.

INITIAL MULTIPLEXING ARRAY
COMPONENT STATE AND COUNT

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ( 1,1,1) | 3. | 0. | ( 1,2,1) | 2. | 0. | ( 1,3,1) | 2. | 0. | ( 1,4,1) | 2. | 0. |
| ( 1,1,2) | 3. | 0. | ( 1,2,2) | 3. | 0. | ( 1,3,2) | 3. | 0. | ( 1,4,2) | 3. | 0. |
| ( 1,1,3) | 1. | 0. | ( 1,2,3) | 1. | 0. | ( 1,3,3) | 1. | 0. | ( 1,4,3) | 1. | 0. |
| ( 1,1,4) | 1. | 0. | ( 1,2,4) | 1. | 0. | ( 1,3,4) | 1. | 0. | ( 1,4,4) | 1. | 0. |
| ( 1,1,5) | 1. | 0. | ( 1,2,5) | 1. | 0. | ( 1,3,5) | 1. | 0. | ( 1,4,5) | 1. | 0. |
| ( 1,1,6) | 1. | 0. | ( 1,2,6) | 1. | 0. | ( 1,3,6) | 1. | 0. | ( 1,4,6) | 1. | 0. |
| ( 1,1,7) | 1. | 0. | ( 1,2,7) | 1. | 0. | ( 1,3,7) | 1. | 0. | ( 1,4,7) | 1. | 0. |
| ( 1,1,8) | 1. | 0. | ( 1,2,8) | 1. | 0. | ( 1,3,8) | 1. | 0. | ( 1,4,8) | 1. | 0. |
| ( 1,1,9) | 1. | 0. | ( 1,2,9) | 0. | 0. | ( 1,3,9) | 0. | 0. | ( 1,4,9) | 0. | 0. |
| ( 2,1,1) | 2. | 0. | ( 2,2,1) | 3. | 0. | ( 2,3,1) | 2. | 0. | ( 2,4,1) | 2. | 0. |
| ( 2,1,2) | 2. | 0. | ( 2,2,2) | 2. | 0. | ( 2,3,2) | 3. | 0. | ( 2,4,2) | 3. | 0. |
| ( 2,1,3) | 1. | 0. | ( 2,2,3) | 1. | 0. | ( 2,3,3) | 1. | 0. | ( 2,4,3) | 1. | 0. |
| ( 2,1,4) | 1. | 0. | ( 2,2,4) | 1. | 0. | ( 2,3,4) | 1. | 0. | ( 2,4,4) | 1. | 0. |
| ( 2,1,5) | 1. | 0. | ( 2,2,5) | 1. | 0. | ( 2,3,5) | 1. | 0. | ( 2,4,5) | 1. | 0. |
| ( 2,1,6) | 1. | 0. | ( 2,2,6) | 1. | 0. | ( 2,3,6) | 1. | 0. | ( 2,4,6) | 1. | 0. |
| ( 2,1,7) | 1. | 0. | ( 2,2,7) | 1. | 0. | ( 2,3,7) | 1. | 0. | ( 2,4,7) | 1. | 0. |
| ( 2,1,8) | 1. | 0. | ( 2,2,8) | 1. | 0. | ( 2,3,8) | 1. | 0. | ( 2,4,8) | 1. | 0. |
| ( 2,1,9) | 3. | 0. | ( 2,2,9) | 0. | 0. | ( 2,3,9) | 0. | 0. | ( 2,4,9) | 0. | 0. |
| ( 3,1,1) | 2. | 0. | ( 3,2,1) | 3. | 0. | ( 3,3,1) | 2. | 0. | ( 3,4,1) | 2. | 0. |
| ( 3,1,2) | 3. | 0. | ( 3,2,2) | 2. | 0. | ( 3,3,2) | 2. | 0. | ( 3,4,2) | 3. | 0. |
| ( 3,1,3) | 1. | 0. | ( 3,2,3) | 1. | 0. | ( 3,3,3) | 1. | 0. | ( 3,4,3) | 1. | 0. |
| ( 3,1,4) | 1. | 0. | ( 3,2,4) | 1. | 0. | ( 3,3,4) | 1. | 0. | ( 3,4,4) | 1. | 0. |
| ( 3,1,5) | 1. | 0. | ( 3,2,5) | 1. | 0. | ( 3,3,5) | 1. | 0. | ( 3,4,5) | 1. | 0. |
| ( 3,1,6) | 1. | 0. | ( 3,2,6) | 1. | 0. | ( 3,3,6) | 1. | 0. | ( 3,4,6) | 1. | 0. |
| ( 3,1,7) | 1. | 0. | ( 3,2,7) | 1. | 0. | ( 3,3,7) | 1. | 0. | ( 3,4,7) | 1. | 0. |
| ( 3,1,8) | 1. | 0. | ( 3,2,8) | 1. | 0. | ( 3,3,8) | 1. | 0. | ( 3,4,8) | 1. | 0. |
| ( 3,1,9) | 1. | 0. | ( 3,2,9) | 0. | 0. | ( 3,3,9) | 0. | 0. | ( 3,4,9) | 0. | 0. |
| ( 4,1,1) | 2. | 0. | ( 4,2,1) | 3. | 0. | ( 4,3,1) | 2. | 0. | ( 4,4,1) | 3. | 0. |
| ( 4,1,2) | 2. | 0. | ( 4,2,2) | 3. | 0. | ( 4,3,2) | 3. | 0. | ( 4,4,2) | 2. | 0. |
| ( 4,1,3) | 1. | 0. | ( 4,2,3) | 1. | 0. | ( 4,3,3) | 1. | 0. | ( 4,4,3) | 1. | 0. |
| ( 4,1,4) | 1. | 0. | ( 4,2,4) | 1. | 0. | ( 4,3,4) | 1. | 0. | ( 4,4,4) | 1. | 0. |
| ( 4,1,5) | 1. | 0. | ( 4,2,5) | 1. | 0. | ( 4,3,5) | 1. | 0. | ( 4,4,5) | 1. | 0. |
| ( 4,1,6) | 1. | 0. | ( 4,2,6) | 1. | 0. | ( 4,3,6) | 1. | 0. | ( 4,4,6) | 1. | 0. |
| ( 4,1,7) | 1. | 0. | ( 4,2,7) | 1. | 0. | ( 4,3,7) | 1. | 0. | ( 4,4,7) | 1. | 0. |
| ( 4,1,8) | 1. | 0. | ( 4,2,8) | 1. | 0. | ( 4,3,8) | 1. | 0. | ( 4,4,8) | 1. | 0. |
| ( 4,1,9) | 2. | 0. | ( 4,2,9) | 0. | 0. | ( 4,3,9) | 0. | 0. | ( 4,4,9) | 0. | 0. |
| ( 5,1,1) | 2. | 0. | ( 5,2,1) | 2. | 0. | ( 5,3,1) | 2. | 0. | ( 5,4,1) | 2. | 0. |
| ( 5,1,2) | 2. | 0. | ( 5,2,2) | 3. | 0. | ( 5,3,2) | 3. | 0. | ( 5,4,2) | 3. | 0. |
| ( 5,1,3) | 1. | 0. | ( 5,2,3) | 1. | 0. | ( 5,3,3) | 1. | 0. | ( 5,4,3) | 1. | 0. |
| ( 5,1,4) | 1. | 0. | ( 5,2,4) | 1. | 0. | ( 5,3,4) | 1. | 0. | ( 5,4,4) | 1. | 0. |
| ( 5,1,5) | 1. | 0. | ( 5,2,5) | 1. | 0. | ( 5,3,5) | 1. | 0. | ( 5,4,5) | 1. | 0. |
| ( 5,1,6) | 1. | 0. | ( 5,2,6) | 1. | 0. | ( 5,3,6) | 1. | 0. | ( 5,4,6) | 1. | 0. |
| ( 5,1,7) | 1. | 0. | ( 5,2,7) | 1. | 0. | ( 5,3,7) | 1. | 0. | ( 5,4,7) | 1. | 0. |
| ( 5,1,8) | 1. | 0. | ( 5,2,8) | 1. | 0. | ( 5,3,8) | 1. | 0. | ( 5,4,8) | 1. | 0. |
| ( 5,1,9) | 2. | 0. | ( 5,2,9) | 0. | 0. | ( 5,3,9) | 0. | 0. | ( 5,4,9) | 0. | 0. |
| ( 6,1,1) | 2. | 0. | ( 6,2,1) | 3. | 0. | ( 6,3,1) | 3. | 0. | ( 6,4,1) | 3. | 0. |

Fig. A-1

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ( 6,1,2) | 3. | 0. | ( 6,2,2) | 2. | 0. | ( 6,3,2) | 3. | 0. | ( 6,4,2) | 3. | 0. |
| ( 6,1,3) | 1. | 0. | ( 6,2,3) | 1. | 0. | ( 6,3,3) | 1. | 0. | ( 6,4,3) | 1. | 0. |
| ( 6,1,4) | 1. | 0. | ( 6,2,4) | 1. | 0. | ( 6,3,4) | 1. | 0. | ( 6,4,4) | 1. | 0. |
| ( 6,1,5) | 1. | 0. | ( 6,2,5) | 1. | 0. | ( 6,3,5) | 1. | 0. | ( 6,4,5) | 1. | 0. |
| ( 6,1,6) | 1. | 0. | ( 6,2,6) | 1. | 0. | ( 6,3,6) | 1. | 0. | ( 6,4,6) | 1. | 0. |
| ( 6,1,7) | 1. | 0. | ( 6,2,7) | 1. | 0. | ( 6,3,7) | 1. | 0. | ( 6,4,7) | 1. | 0. |
| ( 6,1,8) | 1. | 0. | ( 6,2,8) | 1. | 0. | ( 6,3,8) | 1. | 0. | ( 6,4,8) | 1. | 0. |
| ( 6,1,9) | 1. | 0. | ( 6,2,9) | 0. | 0. | ( 6,3,9) | 0. | 0. | ( 6,4,9) | 0. | 0. |
| | | | | | | | | | | | |
| ( 7,1,1) | 3. | 0. | ( 7,2,1) | 2. | 0. | ( 7,3,1) | 2. | 0. | ( 7,4,1) | 3. | 0. |
| ( 7,1,2) | 2. | 0. | ( 7,2,2) | 2. | 0. | ( 7,3,2) | 2. | 0. | ( 7,4,2) | 2. | 0. |
| ( 7,1,3) | 1. | 0. | ( 7,2,3) | 1. | 0. | ( 7,3,3) | 1. | 0. | ( 7,4,3) | 1. | 0. |
| ( 7,1,4) | 1. | 0. | ( 7,2,4) | 1. | 0. | ( 7,3,4) | 1. | 0. | ( 7,4,4) | 1. | 0. |
| ( 7,1,5) | 1. | 0. | ( 7,2,5) | 1. | 0. | ( 7,3,5) | 1. | 0. | ( 7,4,5) | 1. | 0. |
| ( 7,1,6) | 1. | 0. | ( 7,2,6) | 1. | 0. | ( 7,3,6) | 1. | 0. | ( 7,4,6) | 1. | 0. |
| ( 7,1,7) | 1. | 0. | ( 7,2,7) | 1. | 0. | ( 7,3,7) | 1. | 0. | ( 7,4,7) | 1. | 0. |
| ( 7,1,8) | 1. | 0. | ( 7,2,8) | 1. | 0. | ( 7,3,8) | 1. | 0. | ( 7,4,8) | 1. | 0. |
| ( 7,1,9) | 0. | 0. | ( 7,2,9) | 0. | 0. | ( 7,3,9) | 0. | 0. | ( 7,4,9) | 0. | 0. |
| | | | | | | | | | | | |
| ( 8,1,1) | 2. | 0. | ( 8,2,1) | 2. | 0. | ( 8,3,1) | 2. | 0. | ( 8,4,1) | 2. | 0. |
| ( 8,1,2) | 3. | 0. | ( 8,2,2) | 2. | 0. | ( 8,3,2) | 3. | 0. | ( 8,4,2) | 2. | 0. |
| ( 8,1,3) | 1. | 0. | ( 8,2,3) | 1. | 0. | ( 8,3,3) | 1. | 0. | ( 8,4,3) | 1. | 0. |
| ( 8,1,4) | 1. | 0. | ( 8,2,4) | 1. | 0. | ( 8,3,4) | 1. | 0. | ( 8,4,4) | 1. | 0. |
| ( 8,1,5) | 1. | 0. | ( 8,2,5) | 1. | 0. | ( 8,3,5) | 1. | 0. | ( 8,4,5) | 1. | 0. |
| ( 8,1,6) | 1. | 0. | ( 8,2,6) | 1. | 0. | ( 8,3,6) | 1. | 0. | ( 8,4,6) | 1. | 0. |
| ( 8,1,7) | 1. | 0. | ( 8,2,7) | 1. | 0. | ( 8,3,7) | 1. | 0. | ( 8,4,7) | 1. | 0. |
| ( 8,1,8) | 1. | 0. | ( 8,2,8) | 1. | 0. | ( 8,3,8) | 1. | 0. | ( 8,4,8) | 1. | 0. |
| ( 8,1,9) | 1. | 0. | ( 8,2,9) | 0. | 0. | ( 8,3,9) | 0. | 0. | ( 8,4,9) | 0. | 0. |
| | | | | | | | | | | | |
| ( 9,1,1) | 2. | 0. | ( 9,2,1) | 2. | 0. | ( 9,3,1) | 3. | 0. | ( 9,4,1) | 2. | 0. |
| ( 9,1,2) | 3. | 0. | ( 9,2,2) | 3. | 0. | ( 9,3,2) | 3. | 0. | ( 9,4,2) | 2. | 0. |
| ( 9,1,3) | 1. | 0. | ( 9,2,3) | 1. | 0. | ( 9,3,3) | 1. | 0. | ( 9,4,3) | 1. | 0. |
| ( 9,1,4) | 1. | 0. | ( 9,2,4) | 1. | 0. | ( 9,3,4) | 1. | 0. | ( 9,4,4) | 1. | 0. |
| ( 9,1,5) | 1. | 0. | ( 9,2,5) | 1. | 0. | ( 9,3,5) | 1. | 0. | ( 9,4,5) | 1. | 0. |
| ( 9,1,6) | 1. | 0. | ( 9,2,6) | 1. | 0. | ( 9,3,6) | 1. | 0. | ( 9,4,6) | 1. | 0. |
| ( 9,1,7) | 1. | 0. | ( 9,2,7) | 1. | 0. | ( 9,3,7) | 1. | 0. | ( 9,4,7) | 1. | 0. |
| ( 9,1,8) | 1. | 0. | ( 9,2,8) | 1. | 0. | ( 9,3,8) | 1. | 0. | ( 9,4,8) | 1. | 0. |
| ( 9,1,9) | 1. | 0. | ( 9,2,9) | 0. | 0. | ( 9,3,9) | 0. | 0. | ( 9,4,9) | 0. | 0. |
| | | | | | | | | | | | |
| (10,1,1) | 2. | 0. | (10,2,1) | 3. | 0. | (10,3,1) | 2. | 0. | (10,4,1) | 2. | 0. |
| (10,1,2) | 2. | 0. | (10,2,2) | 3. | 0. | (10,3,2) | 2. | 0. | (10,4,2) | 3. | 0. |
| (10,1,3) | 1. | 0. | (10,2,3) | 1. | 0. | (10,3,3) | 1. | 0. | (10,4,3) | 1. | 0. |
| (10,1,4) | 1. | 0. | (10,2,4) | 1. | 0. | (10,3,4) | 1. | 0. | (10,4,4) | 1. | 0. |
| (10,1,5) | 1. | 0. | (10,2,5) | 1. | 0. | (10,3,5) | 1. | 0. | (10,4,5) | 1. | 0. |
| (10,1,6) | 1. | 0. | (10,2,6) | 1. | 0. | (10,3,6) | 1. | 0. | (10,4,6) | 1. | 0. |
| (10,1,7) | 1. | 0. | (10,2,7) | 1. | 0. | (10,3,7) | 1. | 0. | (10,4,7) | 1. | 0. |
| (10,1,8) | 1. | 0. | (10,2,8) | 1. | 0. | (10,3,8) | 1. | 0. | (10,4,8) | 1. | 0. |
| (10,1,9) | 2. | 0. | (10,2,9) | 0. | 0. | (10,3,9) | 0. | 0. | (10,4,9) | 0. | 0. |

**Fig. A-2**

```
  COMPONENT FAILURE                                   CELL STATUS - INPUTS 1 TO 10                                     ARRAY
NR = 7159485143   NCO = 100   NCS = 10   NCO3 = 10   NCS3 = 20   NDO = 50   NDS = 15   NP = 10   NS = 500

                                 40123
                                        40123
                                                             34012
                                                                    34012
                                                                           12340
                                                                                  30124
                                                                                         23014

  166., ( 8,3,1) ZERO                                                                                                   84
                                                                                  01234
    1., ( 8,2,2) ZERO                                                                                                   85
                                                                                  13402
  399., ( 7,1,1) ONE                                                                                                   200
                                                                           23401
                                                                           34012
  309., ( 7,1,7) OPEN                                                                                                  254
  514., ( 1,1,1) ONE                                                                                                   257
                           23401
                           34012
  549., (10,2,1) ZERO                                                                                                  275
                                                                                                 40123
  595., ( 4,2,1) ZERO                                                                                                  298
                                                 30124
  338., ( 2,4,7) OPEN                                                                                                  339
  849., ( 3,4,1) ZERO                                                                                                  425
                                        01234
                                        12340
  465., ( 5,4,8) OPEN                                                                                                  465
 1019., ( 6,3,1) ONE                                                                                                   510
                                                                    40123
                                                                    01234
                                                                    12403
   90., ( 6,1,1) ONE                                                                                                   556
                                                                    24013
                                                                    40123
  476., ( 3,1,1) ZERO                                                                                                  664
                                        23401
  807., ( 4,3,1) ONE                                                                                                   702
                                                 01234
                                                 12403
 1223., ( 1,3,1) ZERO                                                                                                  870
                           40123
```

Fig. A-3

```
1590., ( 8,1,1) ONE                                                          880
                                                    34012
                                                    40123
1762., ( 9,2,1) ZERO                                                         882
                                                           30124
PULSE FAILURE (10,+)                                                         970
                                                                 01234
                                                                 12340
 991., ( 8,4,8) OPEN                                                         990
  52., (10,1,1) ZERO                                                         997
                                                                 23401
                                                                 34012
 901., ( 6,4,1) ONE                                                         1005
                                          01234
                                          20134
 555., ( 6,3,6) OPEN                                                        1064
 564., ( 1,4,1) ZERO                                                        1153
                    01234
                    23401
 270., ( 1,2,1) ONE                                                         1289
                    34012
                    40123
                    01234
                    34012
                    40123
                    01234
                    34012
                    40123
                    01234
                    34012
```

**Fig. A-4**

In Figs. A-3 and A-4 the failures and cell replacements are tabulated as a function of time. At the top of Fig. A-3 is the random number which began the trial and the set probabilities. The failures are listed on the left by count, component, and failure state. The failure states are

| | | |
|---|---|---|
| *ONE* | $S_i = 1$ | $i = 1, 2$ |
| *ZERO* | $S_i = 0$ | $i = 1, 2$ |
| *OPEN* | $C_i = 2$ or $D_j = 2$ | $i = 1, 2, 3, 4; j = 1, 2$ |
| *SHORT* | $C_i = 0$ or $D_j = 0$ | $i = 1, 2, 3, 4; j = 1, 2$ |

The time of failure is given by the number of the pass through the multiplexer. This number is located on the far right side of Figs. A-3 and A-4. The five-digit numbers scattered about the figures denote cell replacement. The convention is as follows:

*ABCDE*

| | |
|---|---|
| $A = 0$ | denotes the search mode; the leftmost cell from $B, C, D, E$ capable of switching will become the active cell. |
| $A > 0$<br>$B, C, D$ or $E = 0$ | $A$ is the active cell and the cells left of zero are capable of switching; those right of zero must pass through a search before becoming the active cell. |

Two examples are:

| | |
|---|---|
| 01234 | the search mode |
| 23014 | 2 is the active cell.<br>3 is capable of switching.<br>1 and 4 cannot become active cells before a search. |

The columns across the paper denote the inputs 1–10 from left to right. Thus all failures effecting cell replacement in one vector may be read by scanning the appropriate column. In some cases the solution of initial conditions requires replacement. Therefore the entries corresponding to inputs 2, 3, 5, 6, 7, 8, and 9 at the top of Fig. A-3 denote a change of active cell from the initial multiplexer without listing a failure. Also there are failures which do not interfere with cell switching and therefore do not lead to cell replacement.

FINAL MULTIPLEXING ARRAY
COMPONENT STATE AND COUNT

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ( 1,1,1) | 1. | 514. | ( 1,2,1) | 1. | 270. | ( 1,3,1) | 0. | 1223. | ( 1,4,1) | 0. | 564. |
| ( 1,1,2) | 2. | 1. | ( 1,2,2) | 2. | 3. | ( 1,3,2) | 3. | 8. | ( 1,4,2) | 3. | 8. |
| ( 1,1,3) | 1. | 257. | ( 1,2,3) | 1. | 136. | ( 1,3,3) | 1. | 620. | ( 1,4,3) | 1. | 294. |
| ( 1,1,4) | 1. | 2333. | ( 1,2,4) | 1. | 148. | ( 1,3,4) | 1. | 611. | ( 1,4,4) | 1. | 282. |
| ( 1,1,5) | 1. | 257. | ( 1,2,5) | 1. | 136. | ( 1,3,5) | 1. | 620. | ( 1,4,5) | 1. | 290. |
| ( 1,1,6) | 1. | 1038. | ( 1,2,6) | 1. | 7. | ( 1,3,6) | 1. | 0. | ( 1,4,6) | 1. | 0. |
| ( 1,1,7) | 1. | 2333. | ( 1,2,7) | 1. | 148. | ( 1,3,7) | 1. | 611. | ( 1,4,7) | 1. | 282. |
| ( 1,1,8) | 1. | 258. | ( 1,2,8) | 1. | 1289. | ( 1,3,8) | 1. | 1296. | ( 1,4,8) | 1. | 1299. |
| ( 1,1,9) | 3. | 1289. | ( 1,2,9) | 0. | 1. | ( 1,3,9) | 1. | 1. | ( 1,4,9) | 0. | 1. |
| ( 2,1,1) | 2. | 0. | ( 2,2,1) | 2. | 1. | ( 2,3,1) | 2. | 0. | ( 2,4,1) | 3. | 2587. |
| ( 2,1,2) | 2. | 0. | ( 2,2,2) | 2. | 0. | ( 2,3,2) | 2. | 1. | ( 2,4,2) | 3. | 0. |
| ( 2,1,3) | 1. | 0. | ( 2,2,3) | 1. | 0. | ( 2,3,3) | 1. | 1. | ( 2,4,3) | 1. | 1294. |
| ( 2,1,4) | 1. | 0. | ( 2,2,4) | 1. | 1. | ( 2,3,4) | 1. | 0. | ( 2,4,4) | 1. | 1293. |
| ( 2,1,5) | 1. | 0. | ( 2,2,5) | 1. | 0. | ( 2,3,5) | 1. | 1. | ( 2,4,5) | 1. | 1294. |
| ( 2,1,6) | 1. | 0. | ( 2,2,6) | 1. | 0. | ( 2,3,6) | 1. | 0. | ( 2,4,6) | 1. | 0. |
| ( 2,1,7) | 1. | 0. | ( 2,2,7) | 1. | 1. | ( 2,3,7) | 1. | 0. | ( 2,4,7) | 2. | 338. |
| ( 2,1,8) | 1. | 1294. | ( 2,2,8) | 1. | 1293. | ( 2,3,8) | 1. | 1293. | ( 2,4,8) | 1. | 1295. |
| ( 2,1,9) | 4. | 0. | ( 2,2,9) | 0. | 0. | ( 2,3,9) | 1. | 0. | ( 2,4,9) | 0. | 0. |
| ( 3,1,1) | 0. | 476. | ( 3,2,1) | 3. | 1262. | ( 3,3,1) | 2. | 2. | ( 3,4,1) | 0. | 849. |
| ( 3,1,2) | 2. | 3. | ( 3,2,2) | 3. | 1. | ( 3,3,2) | 3. | 1. | ( 3,4,2) | 3. | 2. |
| ( 3,1,3) | 1. | 241. | ( 3,2,3) | 1. | 631. | ( 3,3,3) | 1. | 1. | ( 3,4,3) | 1. | 427. |
| ( 3,1,4) | 1. | 238. | ( 3,2,4) | 1. | 631. | ( 3,3,4) | 1. | 1. | ( 3,4,4) | 1. | 424. |
| ( 3,1,5) | 1. | 240. | ( 3,2,5) | 1. | 631. | ( 3,3,5) | 1. | 1. | ( 3,4,5) | 1. | 427. |
| ( 3,1,6) | 1. | 0. | ( 3,2,6) | 1. | 0. | ( 3,3,6) | 1. | 0. | ( 3,4,6) | 1. | 0. |
| ( 3,1,7) | 1. | 238. | ( 3,2,7) | 1. | 631. | ( 3,3,7) | 1. | 1. | ( 3,4,7) | 1. | 424. |
| ( 3,1,8) | 1. | 1293. | ( 3,2,8) | 1. | 1294. | ( 3,3,8) | 1. | 1294. | ( 3,4,8) | 1. | 1295. |
| ( 3,1,9) | 2. | 664. | ( 3,2,9) | 0. | 0. | ( 3,3,9) | 1. | 0. | ( 3,4,9) | 0. | 0. |
| ( 4,1,1) | 3. | 1181. | ( 4,2,1) | 0. | 595. | ( 4,3,1) | 1. | 807. | ( 4,4,1) | 2. | 3. |
| ( 4,1,2) | 3. | 1. | ( 4,2,2) | 3. | 2. | ( 4,3,2) | 2. | 1. | ( 4,4,2) | 3. | 1. |
| ( 4,1,3) | 1. | 591. | ( 4,2,3) | 1. | 300. | ( 4,3,3) | 1. | 404. | ( 4,4,3) | 1. | 1. |
| ( 4,1,4) | 1. | 590. | ( 4,2,4) | 1. | 298. | ( 4,3,4) | 1. | 1587. | ( 4,4,4) | 1. | 2. |
| ( 4,1,5) | 1. | 591. | ( 4,2,5) | 1. | 299. | ( 4,3,5) | 1. | 404. | ( 4,4,5) | 1. | 1. |
| ( 4,1,6) | 1. | 0. | ( 4,2,6) | 1. | 0. | ( 4,3,6) | 1. | 592. | ( 4,4,6) | 1. | 0. |
| ( 4,1,7) | 1. | 590. | ( 4,2,7) | 1. | 298. | ( 4,3,7) | 1. | 1587. | ( 4,4,7) | 1. | 2. |
| ( 4,1,8) | 1. | 1295. | ( 4,2,8) | 1. | 1294. | ( 4,3,8) | 1. | 704. | ( 4,4,8) | 1. | 1293. |
| ( 4,1,9) | 1. | 703. | ( 4,2,9) | 0. | 0. | ( 4,3,9) | 1. | 0. | ( 4,4,9) | 0. | 0. |
| ( 5,1,1) | 2. | 0. | ( 5,2,1) | 2. | 0. | ( 5,3,1) | 3. | 2587. | ( 5,4,1) | 2. | 0. |
| ( 5,1,2) | 2. | 0. | ( 5,2,2) | 2. | 1. | ( 5,3,2) | 3. | 0. | ( 5,4,2) | 3. | 0. |
| ( 5,1,3) | 1. | 0. | ( 5,2,3) | 1. | 1. | ( 5,3,3) | 1. | 1294. | ( 5,4,3) | 1. | 0. |
| ( 5,1,4) | 1. | 0. | ( 5,2,4) | 1. | 0. | ( 5,3,4) | 1. | 1293. | ( 5,4,4) | 1. | 0. |
| ( 5,1,5) | 1. | 0. | ( 5,2,5) | 1. | 1. | ( 5,3,5) | 1. | 1294. | ( 5,4,5) | 1. | 0. |
| ( 5,1,6) | 1. | 0. | ( 5,2,6) | 1. | 0. | ( 5,3,6) | 1. | 0. | ( 5,4,6) | 1. | 0. |
| ( 5,1,7) | 1. | 0. | ( 5,2,7) | 1. | 0. | ( 5,3,7) | 1. | 1293. | ( 5,4,7) | 1. | 0. |
| ( 5,1,8) | 1. | 1294. | ( 5,2,8) | 1. | 1293. | ( 5,3,8) | 1. | 1295. | ( 5,4,8) | 2. | 465. |
| ( 5,1,9) | 3. | 0. | ( 5,2,9) | 0. | 0. | ( 5,3,9) | 1. | 0. | ( 5,4,9) | 0. | 0. |
| ( 6,1,1) | 1. | 90. | ( 6,2,1) | 3. | 578. | ( 6,3,1) | 1. | 1019. | ( 6,4,1) | 1. | 901. |

**Fig. A-5**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ( 6,1,2) | 2. | 3. | ( 6,2,2) | 3. | 3. | ( 6,3,2) | 2. | 1. | ( 6,4,2) | 2. | 3. |
| ( 6,1,3) | 1. | 46. | ( 6,2,3) | 1. | 290. | ( 6,3,3) | 1. | 509. | ( 6,4,3) | 1. | 451. |
| ( 6,1,4) | 1. | 1522. | ( 6,2,4) | 1. | 289. | ( 6,3,4) | 1. | 2079. | ( 6,4,4) | 1. | 1028. |
| ( 6,1,5) | 1. | 46. | ( 6,2,5) | 1. | 290. | ( 6,3,5) | 1. | 509. | ( 6,4,5) | 1. | 451. |
| ( 6,1,6) | 1. | 739. | ( 6,2,6) | 1. | 0. | ( 6,3,6) | 2. | 555. | ( 6,4,6) | 1. | 289. |
| ( 6,1,7) | 1. | 1522. | ( 6,2,7) | 1. | 289. | ( 6,3,7) | 1. | 1849. | ( 6,4,7) | 1. | 1028. |
| ( 6,1,8) | 1. | 556. | ( 6,2,8) | 1. | 1006. | ( 6,3,8) | 1. | 510. | ( 6,4,8) | 1. | 1005. |
| ( 6,1,9) | 2. | 1006. | ( 6,2,9) | 0. | 0. | ( 6,3,9) | 1. | 0. | ( 6,4,9) | 0. | 0. |
| ( 7,1,1) | 1. | 399. | ( 7,2,1) | 2. | 2. | ( 7,3,1) | 3. | 2189. | ( 7,4,1) | 2. | 3. |
| ( 7,1,2) | 2. | 2. | ( 7,2,2) | 2. | 2. | ( 7,3,2) | 3. | 1. | ( 7,4,2) | 3. | 1. |
| ( 7,1,3) | 1. | 199. | ( 7,2,3) | 1. | 2. | ( 7,3,3) | 1. | 1095. | ( 7,4,3) | 1. | 1. |
| ( 7,1,4) | 1. | 2369. | ( 7,2,4) | 1. | 1. | ( 7,3,4) | 1. | 1094. | ( 7,4,4) | 1. | 2. |
| ( 7,1,5) | 1. | 199. | ( 7,2,5) | 1. | 2. | ( 7,3,5) | 1. | 1095. | ( 7,4,5) | 1. | 1. |
| ( 7,1,6) | 1. | 55. | ( 7,2,6) | 1. | 0. | ( 7,3,6) | 1. | 0. | ( 7,4,6) | 1. | 0. |
| ( 7,1,7) | 2. | 399. | ( 7,2,7) | 1. | 1. | ( 7,3,7) | 1. | 1094. | ( 7,4,7) | 1. | 2. |
| ( 7,1,8) | 1. | 200. | ( 7,2,8) | 1. | 1295. | ( 7,3,8) | 1. | 1295. | ( 7,4,8) | 1. | 1293. |
| ( 7,1,9) | 3. | 200. | ( 7,2,9) | 0. | 0. | ( 7,3,9) | 1. | 0. | ( 7,4,9) | 0. | 0. |
| ( 8,1,1) | 1. | 1590. | ( 8,2,1) | 2. | 2. | ( 8,3,1) | 0. | 166. | ( 8,4,1) | 3. | 829. |
| ( 8,1,2) | 2. | 3. | ( 8,2,2) | 0. | 1. | ( 8,3,2) | 2. | 3. | ( 8,4,2) | 3. | 1. |
| ( 8,1,3) | 1. | 796. | ( 8,2,3) | 1. | 1. | ( 8,3,3) | 1. | 87. | ( 8,4,3) | 1. | 415. |
| ( 8,1,4) | 1. | 1624. | ( 8,2,4) | 1. | 1. | ( 8,3,4) | 1. | 83. | ( 8,4,4) | 1. | 414. |
| ( 8,1,5) | 1. | 796. | ( 8,2,5) | 1. | 1. | ( 8,3,5) | 1. | 86. | ( 8,4,5) | 1. | 415. |
| ( 8,1,6) | 1. | 415. | ( 8,2,6) | 1. | 0. | ( 8,3,6) | 1. | 0. | ( 8,4,6) | 1. | 0. |
| ( 8,1,7) | 1. | 1624. | ( 8,2,7) | 1. | 1. | ( 8,3,7) | 1. | 83. | ( 8,4,7) | 1. | 414. |
| ( 8,1,8) | 1. | 880. | ( 8,2,8) | 1. | 990. | ( 8,3,8) | 1. | 990. | ( 8,4,8) | 2. | 991. |
| ( 8,1,9) | 4. | 880. | ( 8,2,9) | 0. | 0. | ( 8,3,9) | 1. | 0. | ( 8,4,9) | 0. | 0. |
| ( 9,1,1) | 2. | 0. | ( 9,2,1) | 0. | 1762. | ( 9,3,1) | 3. | 824. | ( 9,4,1) | 2. | 0. |
| ( 9,1,2) | 2. | 1. | ( 9,2,2) | 2. | 1. | ( 9,3,2) | 3. | 0. | ( 9,4,2) | 2. | 0. |
| ( 9,1,3) | 1. | 1. | ( 9,2,3) | 1. | 883. | ( 9,3,3) | 1. | 412. | ( 9,4,3) | 1. | 0. |
| ( 9,1,4) | 1. | 0. | ( 9,2,4) | 1. | 881. | ( 9,3,4) | 1. | 412. | ( 9,4,4) | 1. | 0. |
| ( 9,1,5) | 1. | 1. | ( 9,2,5) | 1. | 882. | ( 9,3,5) | 1. | 412. | ( 9,4,5) | 1. | 0. |
| ( 9,1,6) | 1. | 0. | ( 9,2,6) | 1. | 0. | ( 9,3,6) | 1. | 0. | ( 9,4,6) | 1. | 0. |
| ( 9,1,7) | 1. | 0. | ( 9,2,7) | 1. | 881. | ( 9,3,7) | 1. | 412. | ( 9,4,7) | 1. | 0. |
| ( 9,1,8) | 1. | 1293. | ( 9,2,8) | 1. | 1294. | ( 9,3,8) | 1. | 1294. | ( 9,4,8) | 1. | 1294. |
| ( 9,1,9) | 3. | 882. | ( 9,2,9) | 0. | 0. | ( 9,3,9) | 1. | 0. | ( 9,4,9) | 0. | 0. |
| (10,1,1) | 0. | 52. | (10,2,1) | 0. | 549. | (10,3,1) | 3. | 593. | (10,4,1) | 2. | 1390. |
| (10,1,2) | 2. | 2. | (10,2,2) | 2. | 3. | (10,3,2) | 3. | 1. | (10,4,2) | 3. | 2. |
| (10,1,3) | 1. | 28. | (10,2,3) | 1. | 279. | (10,3,3) | 1. | 297. | (10,4,3) | 1. | 695. |
| (10,1,4) | 1. | 26. | (10,2,4) | 1. | 275. | (10,3,4) | 1. | 296. | (10,4,4) | 1. | 695. |
| (10,1,5) | 1. | 27. | (10,2,5) | 1. | 277. | (10,3,5) | 1. | 297. | (10,4,5) | 1. | 695. |
| (10,1,6) | 1. | 0. | (10,2,6) | 1. | 0. | (10,3,6) | 1. | 0. | (10,4,6) | 1. | 1. |
| (10,1,7) | 1. | 26. | (10,2,7) | 1. | 275. | (10,3,7) | 1. | 296. | (10,4,7) | 1. | 696. |
| (10,1,8) | 1. | 1293. | (10,2,8) | 1. | 1293. | (10,3,8) | 1. | 1294. | (10,4,8) | 1. | 1294. |
| (10,1,9) | 3. | 997. | (10,2,9) | 0. | 0. | (10,3,9) | 1. | 0. | (10,4,9) | 0. | 0. |

IP = 1 NB = 1295 NN = 221157

**Fig. A-6**

To summarize the above discussion three failures will be considered. On pass 85 $S_1$ of input 8, cell 3 fails in the zero state after switching 166 times. The vector goes into the search mode and, by coincidence, during the search $S_2$ of input 8, cell 2 fails in the zero state. (This was one of four $S_2$ failures in 90 trials.) After the search cell 1 is the active cell and cell 2 is obviously incapable of switching. The inability of cell 3 to become the active cell is concealed by its $S_1 = 0$ failure. After cell 1 fails, control will consider and immediately pass over cell 3 to make cell 4 the active cell. See the failure of (8, 1, 1) on pass 880 for verification of these remarks.

On pass 970 a positive pulse is lost while multiplexing the tenth input. The cell interprets this as a failure, although not recognizing the origin, and is replaced after a search. It can be shown by examining the vector 10 component states in the final multiplexing array tabulation that the pulse failure has had no permanent effect.

The vector may successfully use cell 4 again for the active cell.

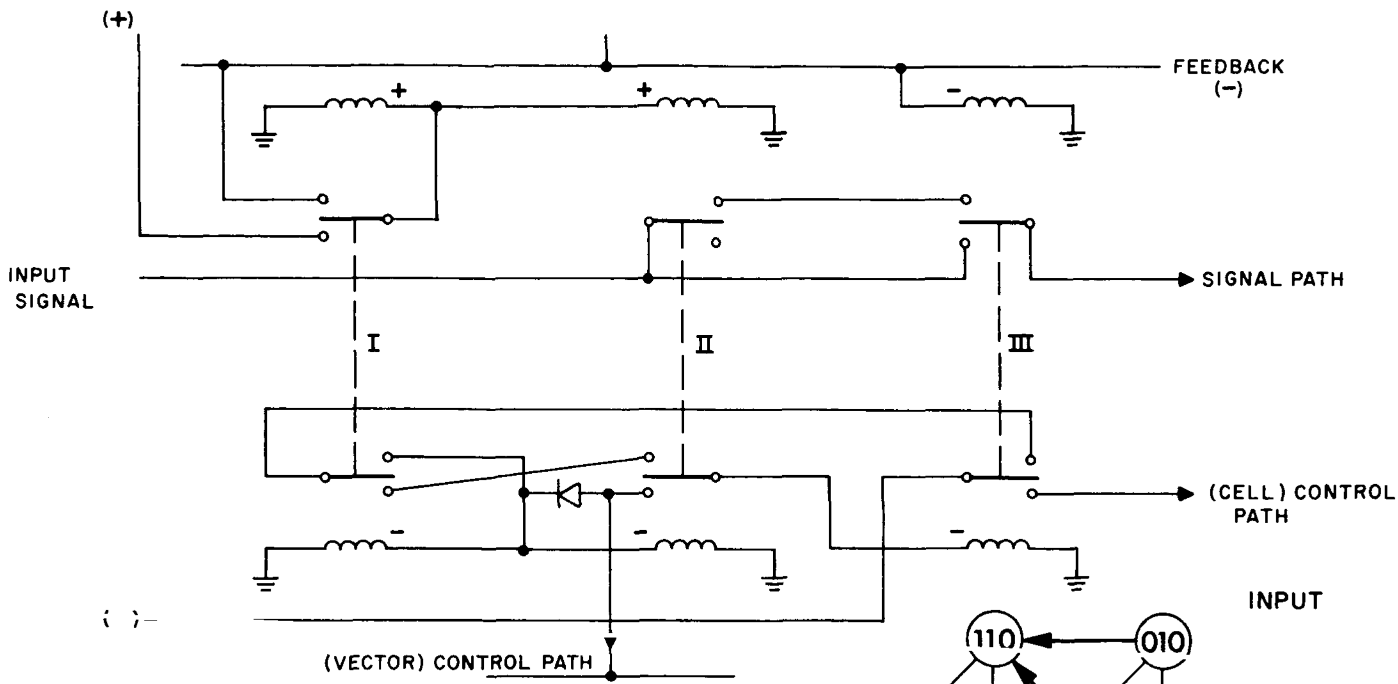
The last failure to be discussed occurred on pass 1289. The failure of $S_1$, (1, 2, 1), short-circuits the input to the channel and the failures (1, 1, 1) = 1, (1, 3, 1) = 0, and (1, 4, 1) = 0 prevent obtaining a stable active cell. After pass 1289 the vector cycles the cells with periodicity 3 looking for a replacement. The computer program terminates after detecting, checking, and double-checking the cyclic behavior.

The state of the multiplexing array immediately after the last failure is shown in Figs. A-5 and A-6. The final multiplexing array tabulations have been particularly valuable in determining failure dependencies by comparing component use counts. For instance, in the second vector, note that only cell 4 has been the active cell. The feedback diodes, however, are all used by the bipolar control pulses and several components in cells 2 and 3 have been used once in the solution of initial conditions. Notice also that the switch failure (7, 1, 1) = 1 on pass 200 has led to excessive use of the coil (7, 1, 4) and the diode failure (7, 1, 7) on pass 254. *NB* denotes the number of passes through the array and *NN* is the number of random numbers generated.

# APPENDIX B

## Cellular Multiplexer

To determine the complexity of switching and sequencing within the same cell, the network of Fig. B-1 was devised. Switches I and II are cycled together to provide sequencing and switching respectively. Switch III interprets the skewed states 10 or 01 of I and II as a failure and transfers to obtain a replacement. An array of 3 inputs with 2 cells per input is shown in Fig. B-2. The entire array is driven by a single bipolar clock train. The pulses are separated by polarity and applied as indicated.




| STATE | NEXT | | FEEDBACK |
|---|---|---|---|
| I II III | + | − | |
| 0 0 0 | 1 1 0 | 0 0 0 | 0 0 1 |
| 0 1 0 | 1 1 0 | 0 1 0 | 0 1 1 |
| 1 0 0 | 1 0 0 | 1 0 0 | 0 0 1 |
| 1 1 0 | 1 1 0 | 1 1 0 | 0 0 1 |
| 0 0 1 | 1 1 1 | 0 0 1 | 0 0 1 |
| 0 1 1 | 1 1 1 | 0 1 0 | 0 1 1 |
| 1 0 1 | 1 0 1 | 0 0 0 | 1 0 1 |
| 1 1 1 | 1 1 1 | 0 0 1 | 1 1 1 |

TO SIMPLIFY THE STATE DIAGRAMS THE ALLOWED TRANSITIONS ARE SHOWN BY INPUT INSTEAD OF MODE. THE SWITCHING FUNCTION IS CARRIED OUT BETWEEN STATES 001 AND 111

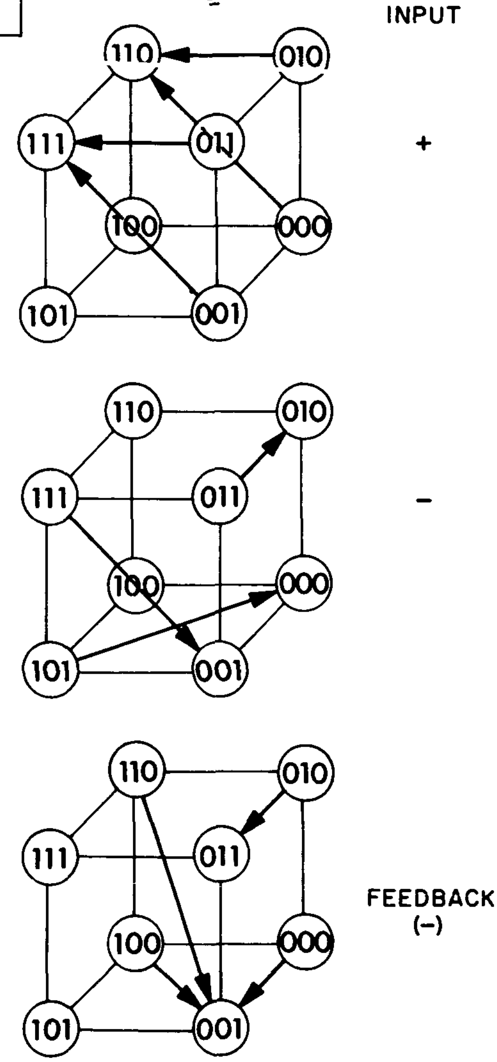



Fig. B-1

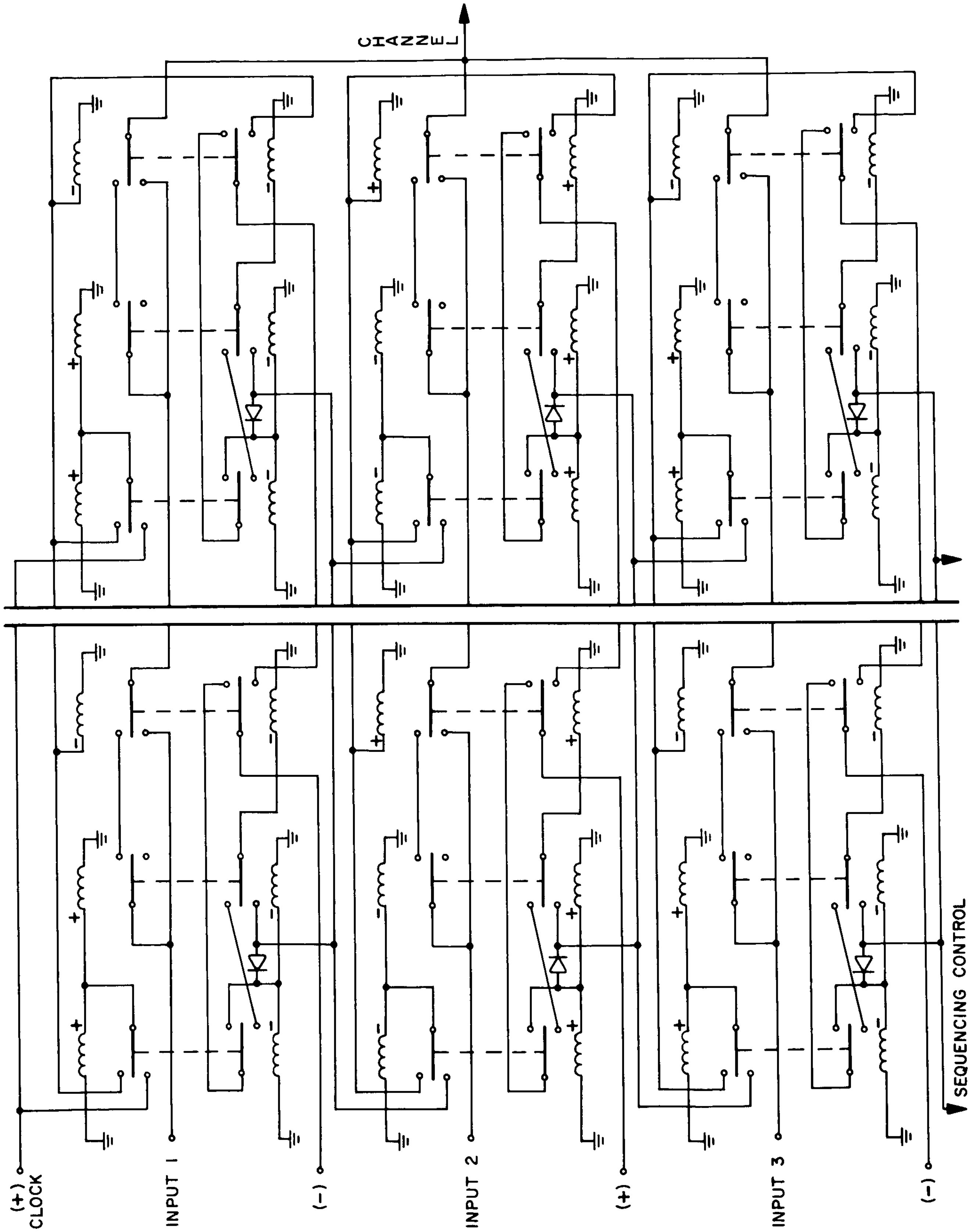



Fig. B-2

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