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PRESSURE FLUCTUATIONS IN TURBULENT BOUNDARY LAYERS

by M. V. Lowson

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Huntsville, Ala.

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DEFINITION OF SYMBOLS

Symbol	Definition
A	arbitrary forcing function in equation (26).
e_{ij}	strain tensor
$O()$	order of magnitude
p	local pressure
p'	fluctuating component of pressure
p_{rms}	root mean square value of fluctuating pressure
p_{ij}	aerodynamic stress tensor
P	mean local pressure
r	distance of source from field point at wall
Re_θ	Reynolds number based on momentum thickness $= \frac{U_o \theta}{\nu}$
t	time
$u_i (i = 1, 2, 3)$	local fluctuating component of velocity in Cartesian tensor notation
u, v, w	also used in Cartesian components of fluctuating velocity $u_i = (u, v, w)$
U_o	local free stream velocity just outside boundary layer
U_τ	shear velocity $= \tau_o / \rho_o$
$U_i (i = 1, 2, 3)$	local mean velocity component in Cartesian tensor notation
U, V, W	also used as Cartesian components of mean velocity
$v_i (i = 1, 2, 3)$	local component of velocity in Cartesian tensor notation $v_i = U_i + u_i$

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
$\underset{\sim}{x}$	vector referring to field point position
$x_i (i = 1, 2, 3)$	Components of field point in Cartesian tensor notation
$\underset{\sim}{y}$	vector referring to source point position
$y_i (i = 1, 2, 3)$	components of source point in Cartesian tensor notation
x, y, z	also used as Cartesian coordinates with x in the free stream direction and y normal to the wall.
δ	boundary layer thickness
δ^*	boundary layer displacement thickness
δ_{ij}	Kronecker delta = 1, i = j; = 0 i $\neq$ j
θ	boundary layer momentum thickness
μ	viscosity
ν	kinematic viscosity = μ/ρ_o
ρ	local density
ρ_o	free stream density
σ	volume element in volume integral
τ_o	wall shear stress
ω	frequency Hz
$\overline{(\quad)}$	mean value of
$\lvert\ \rvert$	absolute value of vector quantity
$(\underset{\sim}{\ })$	vector quantity

PRESSURE FLUCTUATIONS IN TURBULENT BOUNDARY LAYERS

SUMMARY

The equations governing the pressure fluctuations in a turbulent boundary layer are derived following the known methods for incompressible flow. The attached turbulent boundary layer is discussed, and the hypothesis is advanced that the major cause of wall pressure fluctuations is the intermittent eruption of the laminar sublayer observed during flow visualization experiments. This hypothesis agrees with all the known features of boundary layer pressure fluctuations and offers a good physical framework for the understanding of their actions.

The separated turbulent boundary layer is also discussed. Compared to the attached boundary layer, the equations already derived show how additional sources of pressure fluctuations may be present in the separated boundary layer. First, the "turbulence-turbulence" interaction terms may be important, and second, it is shown how additional pressure fluctuations can arise from the interaction of the turbulence with the gradients of velocity in the free stream direction. The exact magnitude of these terms will not be known until more detailed experimental information becomes available. Finally, it is shown how the conventional turbulence-mean shear interaction gives rise to pressure patterns which are convected at only a fraction of the free stream velocity. Since this conclusion contradicts the presently available supersonic experimental results, additional mechanisms of pressure fluctuation may be occurring in supersonic separated flow.

SECTION I. INTRODUCTION

The first work on the estimation of the pressure fluctuations within turbulence was due to Heisenberg [Ref. 1] and Batchelor [Ref. 2]. Their work referred to the highly idealized model of homogeneous (the same from point to point), isotropic (having no preferred orientation) turbulence. Even for this simple case, considerable manipulation and approximation were required to derive a result. It was found that the root mean square value of the pressure fluctuations in homogeneous isotropic turbulence was given by

$$p_{rms} = 0.58\, \rho_o \overline{u^2}, \tag{1}$$

where ρ_o is the density and ú is the instantaneous velocity fluctuation. Note that the magnitude of the velocity fluctuation will be the same for any direction in isotropic turbulence.

Uberoi [Ref. 3] made similar calculations using more detailed experimental results. These calculations gave an average value of about

$$p_{rms} = 0.7 \rho_o \overline{u^2} . \tag{2}$$

All these calculations have relied on estimating the correlation patterns of the turbulent velocity fluctuations, and relating these to the pressure fluctuations via the Navier-Stokes equations and all have referred to incompressible flows. Although it is possible, in principle, to extend these calculations to include the effects of compressibility, this would be a highly complicated task, particularly because of the need to consider retarded time differences within the flow.

The results of equations (1) and (2) refer to the pressure fluctuations produced by turbulent interactions alone. This source of pressure fluctuations is the "turbulence-turbulence" contribution. However, the majority of real flows will contain some mean shear, which has a pronounced effect on the pressure fluctuations which occur. This second contribution from the "turbulence-mean shear" interaction was first investigated by Kraichnan [Ref. 4]. He extended this work [Ref. 5] to cover wall pressure fluctuations in the turbulent boundary layer. Kraichnan's work has been reviewed and extended by Lilley, and Lilley and Hodgson [Ref. 6 and 7].

Kraichnan's original calculations in Reference 5 indicated that the ratio of the turbulence-turbulence contribution to that of the turbulence-mean shear was 1:32. Hodgson has also made calculations [Ref. 8] which give a ratio of 1:20. It would thus appear that it is the turbulence-mean shear interaction which gives the major contribution to the wall pressure fluctuations, and this result has been generally accepted. However, Corcos [Ref. 9] has reported calculations which find the ratio to be only 1:1.6, so that the question cannot yet be considered as completely resolved.

In the following sections the equations for the boundary layer pressure fluctuations will first be derived following the known methods, and the various approximations required discussed. The contribution of both the turbulence-turbulence and the turbulence-mean shear interactions is shown. The attached turbulent boundary layer is discussed in detail and a new hypothesis is advanced offering a physical basis for the understanding of the pressure fluctuations. The equations are then applied to the separated boundary layer, and it is shown how some of the approximations made for the attached boundary layer are no longer valid, so that new sources of pressure fluctuation may be expected in the separated turbulent boundary layer.

SECTION II. EQUATIONS FOR THE PRESSURE FLUCTUATIONS

Derivation of the Differential Equation

In this section the basic equations describing the pressure fluctuations are derived. This derivation is an amalgam of the methods given in References 4 through 9. The analysis begins from the exact equations of aerodynamics which incorporate the effects of both viscosity and compressibility, and uses tensor notation with the summation convention.

The equation for the conservation of mass may be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho v_i) = 0 , \tag{3}$$

and the equation for conservation of momentum as

$$\frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} (\rho v_i v_j) + \frac{\partial}{\partial x_j} (p_{ij}) = 0 . \tag{4}$$

Both experiment and theory agree that simple non-relaxing fluids show a linear dependence of viscous shear on velocity gradient; the aerodynamic stress tensor p_{ij} can therefore be written as

$$p_{ij} = (p + \frac{2}{3} \mu e_{kk})\delta_{ij} - 2 \mu e_{ij} , \tag{5}$$

where p is the local static pressure and

$$e_{ij} = \frac{1}{2} \left\{ \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right\}$$

is the "strain" tensor. Also,

$$e_{kk} = \frac{\partial v_k}{\partial x_k} .$$

The derivation of equation (5) is given in detail by Jeffries [Ref. 10]. Differentiating equation [5] and rearranging gives

$$\frac{\partial p_{ij}}{\partial x_j} = \frac{\partial p}{\partial x_i} - \mu \left\{ \frac{1}{3} \frac{\partial}{\partial x_i}\left(\frac{\partial v_j}{\partial x_j}\right) + \frac{\partial^2 v_i}{\partial x_j^2} \right\} . \tag{6}$$

Differentiating equation (4) with respect to x_i (equivalent to taking the divergence) and using equation (6) gives

$$\frac{\partial}{\partial t} \frac{\partial(\rho v_i)}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j}(\rho v_i v_j) + \frac{\partial^2 p}{\partial x_i^2} - \mu \left\{ \frac{1}{3} \frac{\partial^2}{\partial x_i^2} \frac{\partial v_j}{\partial x_j} + \frac{\partial^2}{\partial x_j^2} \frac{\partial v_i}{\partial x_i} \right\} = 0 . \tag{7}$$

Using equation (3) on the first term of equation (7) and interchanging the "dummy" suffixes in the last term gives

$$\frac{-\partial^2 \rho}{\partial t^2} + \frac{\partial^2}{\partial x_i \partial x_j} (\rho v_i v_j) + \frac{\partial^2 p}{\partial x_j^2} - \frac{4\mu}{3} \left\{ \frac{\partial^2}{\partial x_j^2} \frac{\partial v_i}{\partial x_i} \right\} = 0 . \tag{8}$$

Equation (8) is still exact within the limitations of equations (5). However, to proceed further, it will be necessary to approximate, and from here on the fluid will be assumed to be incompressible. With the assumption of incompressibility, equation (3) becomes

$$\frac{\partial v_i}{\partial x_i} = 0 . \tag{9}$$

When this result is applied to equation (8), the last term vanishes. It can be seen, therefore, that the viscosity has no direct effect on the pressure, and that even in a compressible flow the effect of viscosity will only enter via the effect of compressibility. Thus, the last term in equation (8) may be ignored in all practical cases. This conclusion for local pressure fluctuations parallels that of Lighthill for sound radiation. Lighthill [Ref. 11] showed that the inertia terms gave the leading contribution to the sound radiation, and in the present case, the inertia terms are again the most important. For an incompressible flow, the first term in equation (8) is also zero, although this term will certainly require consideration in any treatment of a compressible flow. However, this first term will be ignored henceforth. Thus, for incompressible flows equation (8) becomes

$$\frac{\partial^2}{\partial x_i \partial x_j} (\rho_o v_i v_j) + \frac{\partial^2 p}{\partial x_i^2} = 0 . \tag{10}$$

This is the equation from which the pressure fluctuations may be calculated.

Now,

$$\frac{\partial^2(\rho_o v_i v_j)}{\partial x_i \partial x_j} = \rho_o \frac{\partial}{\partial x_i}\left\{ v_i \frac{\partial v_j}{\partial x_j} + v_j \frac{\partial v_i}{\partial x_j} \right\}. \tag{11}$$

Using (9), the first term in equation (11) may be seen to be zero, leaving

$$\frac{\partial^2(\rho_o v_i v_j)}{\partial x_i \partial x_j} = \rho_o \left\{ \frac{\partial v_j}{\partial x_i} \frac{\partial v_i}{\partial x_j} + v_j \frac{\partial}{\partial x_j}\left(\frac{\partial v_i}{\partial x_i}\right) \right\}. \tag{12}$$

Using equation (9) again, the second term in equation (12) is zero, so that equations (10) and (12) show

$$\nabla^2 p = - \rho_o \frac{\partial v_j}{\partial x_i} \frac{\partial v_i}{\partial x_j} . \tag{13}$$

If the variables are now put equal to the sum of their mean and fluctuating parts

$$\left.\begin{aligned} v_i &= U_i + u_i \\ v_j &= U_j + u_j \\ p &= P + p' \end{aligned}\right\} , \tag{14}$$

then the relationships for the mean and fluctuating parts may be established. Putting relations (14) in equation (9) and taking means gives

$$\frac{\partial U_i}{\partial x_i} = 0 , \tag{15}$$

since the mean of the fluctuating quantities is zero. Subtracting equation (15) from equation (9) shows also

$$\frac{\partial u_i}{\partial x_i} = 0 . \tag{16}$$

Now putting relations (14) in equation (13) yields

$$\nabla^2 P + \nabla^2 p' = -\rho_o \left\{ \frac{\partial U_j}{\partial x_i} \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \frac{\partial U_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j} \right\} , \tag{17}$$

and taking means yields

$$\nabla^2 P = -\rho_o \left\{ \frac{\partial U_j}{\partial x_i} \frac{\partial U_i}{\partial x_j} + \overline{\frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j}} \right\} . \tag{18}$$

Subtracting equation (18) from equation (17), and noting the identity of the two middle terms of the right side of equation (17), which differ only in their "dummy" subscripts, gives

$$\nabla^2 p' = -\rho_o \left\{ 2 \frac{\partial U_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \left(\frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} - \overline{\frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}} \right) \right\} . \tag{19}$$

Note also that, by virtue of equation (16),

$$\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} = \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} , \tag{20}$$

so that an alternative form of equation (19) is

$$\nabla^2 p' = -\rho_o \left\{ 2 \frac{\partial U_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \left(\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} - \frac{\partial^2 \overline{u_i u_j}}{\partial x_i \partial x_j} \right) \right\} . \tag{21}$$

The right-hand sides of equations (19) and (21) have been split into two parts. The first term depends on both the mean shear $\partial U_i / \partial x_j$ and the turbulence intensity u_j, whereas the second part of the expression depends only on turbulence intensities. These two parts correspond to the turbulence-mean shear and turbulence-turbulence contributions, respectively.

Application to the Turbulent Boundary Layer

Suppose now that equation (21) is applied to the case of the two-dimensional turbulent boundary. Ignore, for the time being, the nine component contribution of the turbulence-turbulence term. The contribution of the turbulence-mean shear term may be written, in ordinary Cartesian coordinates with x in the free stream direction and y normal to the wall, as

$$\nabla^2 p' = -2\rho_o \left\{ \frac{\partial U}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial V}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial U}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial V}{\partial x}\frac{\partial u}{\partial y} \right\}, \tag{22}$$

where U, V are the components of mean velocity and u, v are the fluctuating components of velocity in the x and y directions, respectively.

The orders of magnitude, O( ), of these terms can be calculated following the classic boundary layer approximation technique.

Equation (15) shows

$$O\left(\frac{U}{x}\right) + O\left(\frac{V}{\delta}\right) = 0$$

so that

$$V = O\left(\frac{\delta U}{x}\right), \tag{23}$$

where δ is the thickness of the boundary layer. Using equation (23) in equation (22) gives the terms as

$$\nabla^2 p' = -2\rho_o\, O(f) \left\{ O\left(\frac{U}{x}\right) + O\left(\frac{U}{x}\right) + O\left(\frac{U}{\delta}\right) + O\left(\frac{\delta U}{x^2}\right) \right\}, \tag{24}$$

where the orders of magnitude of each of the fluctuating terms, O(f), have been taken as equal. In the attached boundary layer, the data of Klebanoff [Ref. 12], reproduced in Figure 2, indicate that the u fluctuation is about 50 percent larger than the v fluctuation, with the magnitude of the lateral fluctuations falling between that of the u and v fluctuations. However, the supposition of the equality of O(f) cannot be confirmed or denied for the case of a separated boundary layer since no experimental evidence is presently available. For the separated case, the supposition can only be regarded as a reasonable *a priori* assumption.

When the boundary layer thickness (δ) is small, it is clear from equation (24) that the term is third dominant. Thus, for the attached boundary layer equation (22) may be written

$$\nabla^2 p' = -2\rho_o \frac{\partial U}{\partial y} \frac{\partial v}{\partial x} . \tag{25}$$

This equation has been the foundation of a number of attempts to predict attached boundary layer pressure fluctuations [Ref. 5 through 9]. However, for the separated case, these arguments are not valid. A discussion of the separated boundary layer is given in Section 4, but first of all the formal solution to these equations will be written.

Formal Solution of the Equations

The equations for the pressure fluctuations, from the original of equation (10) through its successive forms of equations (13), (19), (21), (22), and (25), are all of the Poisson type. The solution to this equation is well known. If a typical equation is written as

$$\nabla^2 p = A(\underset{\sim}{x}, t) , \tag{26}$$

then the solution in the absence of any boundary is

$$p(\underset{\sim}{x}, t) = \frac{1}{4\pi} \int_V \frac{A(\underset{\sim}{y}, t)}{|\underset{\sim}{x} - \underset{\sim}{y}|} \, d\sigma(\underset{\sim}{y}) . \tag{27}$$

Here $\underset{\sim}{x}$ is a vector describing the field point and $\underset{\sim}{y}$ is the "dummy variable" of integration. There is an analogy between equation (27) and a solution to the wave equation which would be an identical integral, but evaluated at retarded time. This similarity has recently been used by Paterson [Ref. 13] in an attempt to produce a simpler mathematical and physical model for sound generation by turbulence. However, in the present case, we are necessarily interested in the evaluation of the integral at a wall. Here we follow Kraichnan [Ref. 5] and include the effects of the wall through a mirror flow model of the turbulence field, so that the solution to equation (26) in the presence of a wall becomes

$$p(\underset{\sim}{x}, t) = \frac{1}{4\pi} \int_{y_2 > 0} \{A(\underset{\sim}{y}, t)\} \left(\frac{1}{|\underset{\sim}{x} - \underset{\sim}{y}|} + \frac{1}{|\underset{\sim}{x} - \underset{\sim}{y}^*|} \right) d\sigma(\underset{\sim}{y}) , \tag{28}$$

where $\underset{\sim}{y}^*$ gives the value of $\underset{\sim}{y}$ reflected in the wall; i.e.,

$$y_1^* = y_1$$

$$y_2^* = -y_2$$

$$y_3^* = y_3$$

so that when the field point $\underset{\sim}{x}$ is actually on the wall, $\underset{\sim}{x} - \underset{\sim}{y} = \underset{\sim}{x} - \underset{\sim}{y}^*$, and equation (28) becomes

$$p(\underset{\sim}{x}, t) = \frac{1}{2\pi} \int_{y_2 > 0} \frac{A(\underset{\sim}{y}, t)}{|\underset{\sim}{x} - \underset{\sim}{y}|} \, d\sigma(\underset{\sim}{y}) . \tag{29}$$

The presence of the wall has caused pressure doubling. Before considering equation (29) in more detail, it is useful to return to equation (25) and apply simple similarity arguments to give the leading parameters governing the pressure fluctuations.

SECTION III. THE ATTACHED TURBULENT BOUNDARY LAYER

Similarity Arguments

It has been found that the turbulent boundary layer has a velocity profile which may be described, at least near the wall (Fig. 1), by a one parameter family of profiles, the well known logarithmic velocity distribution due to von Karman. This "law of the wall" is [Ref. 14]

$$\frac{U}{U_\tau} = \frac{1}{k} \log \left(\frac{y U_\tau}{\nu} \right) + A , \tag{30}$$

where k and A are experimentally determined constants. U_τ is the "shear velocity" defined by

$$U_\tau^2 = \tau_o / \rho_o , \tag{31}$$

where τ_o is the wall shear stress.

Now equation (25) showed the pressure fluctuations to be dependent on the mean shear through the flow; this may be found from the velocity profile of equation (30) as

$$\frac{\partial U}{\partial y} = \frac{U_\tau}{ky} . \tag{32}$$

In addition, the intensity of the fluctuating velocities through the boundary layer is proportional to the momentum transfer through the boundary layer, and thus

$$v = \alpha U_\tau , \tag{33}$$

where α is a constant of order unity. Using equations (32) and (33) in equation (25) shows that

$$\nabla^2 p' \sim -\rho_o U_\tau^2 . \tag{34}$$

The constant of proportionality in the relationship will depend on the typical eddy size and shape, i.e., on the correlation patterns within the flow.

Equation (34) establishes an approximate proportionality between pressure fluctuation and wall shear stress. It has been found experimentally that this proportionality gives a satisfactory description of the variations of pressure fluctuation in different attached turbulent boundary layers. Lilley has performed detailed calculations [Ref. 6] and finds

$$p_{rms} = 3.1 \tau_o . \tag{35}$$

The constant in this equation is found experimentally to be a slowly varying function of Reynolds number. The results from a number of experiments are reported by Bull [Ref. 15] and generally lie within the range

$$2 < p_{rms}/\tau_o < 4 .$$

This simple relationship between pressure fluctuation and local shear is the result of the one-parameter family of profiles that can be applied to the equilibrium attached turbulent boundary layer. It must be emphasized that any departure from such conditions will negate the conclusions reached above. For instance, local wall roughness or free stream turbulence will affect the results. In particular, there can be no reason to apply these results to the very different velocity distributions occurring in a separated flow.

Details of the Pressure Patterns

The arguments advanced above, although successful, have not given any detailed information on the pressure fluctuations. To obtain this information the formal solution to equation (25) will be written using equation (29). The pressure fluctuation at the wall is thus

$$p = \frac{-1}{2\pi} \int_{y>0} \frac{2\rho_o}{r} \frac{\partial U}{\partial y} \frac{\partial v}{\partial x} d\sigma . \tag{36}$$

Equation (36) has been written in terms of Cartesian coordinates x, y, z, and r is the distance from the field point at the wall. A complete discussion of equation (36) would require a consideration of the correlation patterns for the velocity fluctuations. However, this refinement will not be considered in this report, and the discussion will be limited to a consideration of the local interactions between the mean shear and the turbulence.

To draw some conclusions from equation (36), the data of Klebanoff have been analyzed in detail. These data give the most complete information on velocity fluctuations presently available, and are thus particularly suitable for the determination of the pressure producing mechanisms within the boundary layer. Klebanoff did not make any measurements of the pressure fluctuation field, but this is of no particular disadvantage in the present case, as a direct comparison of magnitudes is not required.

Figures 1, 2, 3a, and 3b are from Klebanoff's report [Ref. 12] and show the mean boundary layer profile, the velocity fluctuations, and the "dissipation derivatives," respectively. These dissipation derivatives are of particular interest since they also represent many of the pressure producing terms of equation (19). The term of particular interest is that of $\partial v/\partial x$, as this is the term which interacts with the mean shear in equation (36). But this term is one of the smaller derivatives plotted, and it seems that the turbulence-turbulence interactions, for instance, of $\frac{\partial u}{\partial x}\ \frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial z}\ \frac{\partial w}{\partial x}$, could be important in some cases. The mean shear for this boundary layer has been calculated and is shown in Figure 4. This figure has been derived by graphical differentiation of Figure 1, and therefore may not be accurate, although it should show the main effects of interest.

Multiplication of $\partial v/\partial x$ from Figures 3a and 3b with $\partial U/\partial y$ from Figure 4 gives a measure of the contribution of each part of the boundary layer to the overall pressure fluctuation, and the result is plotted in Figure 5. Clearly, a major source of the pressure fluctuation arises in the part of the boundary layer closest to the wall, since both the mean shear and the velocity fluctuation are increasing rapidly in this region. It should be remembered, however, that correlation areas could have a marked effect on this conclusion. In Figure 5 the pressure contribution is plotted against local mean velocity, as it is thought that this allows a more meaningful interpretation to be made. Contributions from the part of the boundary layer below $0.5\ U_o$ cannot be determined since $\partial v/\partial x$ is not given there. Measurement of the v component of fluctuating velocity requires the use of a cross-wire probe in the hot-wire anemometer, and the physical dimensions of the device preclude measurements near the wall. However, in this "laminar sublayer" region, the velocity fluctuations (particularly, $\partial v/\partial x$) may be expected to be small. Therefore, the contribution to the pressure fluctuation from well within this region will also be small.

Figure 5 is particularly interesting since it shows a major contribution to the pressure fluctuation to come from parts of the flow moving with velocities of around

$0.6\ U_0$ or less. This contrasts with the generally quoted result that the pressure field in the boundary layer is dominated by components moving at $0.8\ U_0$.

Figure 6, taken from Reference 15, by Bull, offers a solution to this dilemma. This figure shows the apparent velocity of convection between two transducers, and has been determined from the time delay required for optimum correlation between the signals received from each transducer. At large transducer spacings, the apparent convection velocity of the overall signal is indeed about $0.8\ U_0$, but Figure 6 shows how this velocity approaches $0.53\ U_0$ at zero spacing. This second figure is in much better agreement with the arguments advanced above.

This variation in apparent convection velocity with transducer spacing has been reported by a number of authors [Ref. 16, 17, 18] and is usually explained as being the result of the variation in the distance for which various components of the flow are coherent. It seems reasonable to suppose that the high frequency small wave length components of the flow will lose their coherence in a much shorter distance than the low frequency large wave length components. In addition, it is not unreasonable to suppose that the small wave length components originate from the slower moving fluid near the wall, while the large wave length components are emitted from higher mean velocity regions further out from the wall. These two arguments in combination provide an explanation for the variation in convection velocity with spacing. The relatively rapid loss of coherence of the high frequency terms will result in the correlation at large spacings being dominated by the faster moving, lower frequency pressure patterns. Additional experimental evidence supporting these arguments is presented in References 15, 16, and 18. However, some authors have not drawn the inescapable conclusion that local pressure fluctuations are dominated by pressure producing phenomena near the wall, with a typical convection speed of near $0.6\ U_0$ or less. This agrees nicely with the arguments from the analysis of the data of Klebanoff put forward earlier.

The additional experimental evidence supporting these conclusions comes mainly from correlation measurements in narrow frequency bands. For this, the transducer signals are passed through narrow band filters before correlation. Reasonable consistency is achieved for high frequency components in the various experimental investigations, but there is less agreement on the exact effects of the low frequency parts. In this discussion the "high" and "low" frequencies may be regarded as being roughly those with $\omega\,\delta^*/U_0$ greater and less than unity, respectively. Bull [Ref. 15] concludes that the high frequency components lose coherence after convection for about four wave lengths while the low frequency components lose coherence in a way which is not a function of wave length. Willmarth and Woolridge [Ref. 16] conclude the loss of coherence extends over four to six wave lengths for both high and low frequencies. Both sets of results show how the high frequency components are convected at a slow speed, around $0.6\ U_0$, and the investigations also agree that the convection velocity of the low frequency components is near $0.8\ U_0$ at large transducer spacings. However, the exact effect of the low frequency components at small spacings is not clear. Bull's results for this case seem to lack internal consistency, and Willmarth and Woolridge's results do not admit a simple interpretation. This problem is considered further below.

Possible Flow Mechanisms

Although the effects of eddy coherence must be of practical significance, the "loss of coherence" hypothesis discussed in the last section is not the only mechanism by which the pressure fluctuation phenomena may be interpreted. In this section an alternative hypothesis is presented which provides a good explanation for many of the phenomena observed. Although the arguments are not conclusive, they offer an improved physical understanding of the causes of wall pressure fluctuations.

Suppose that the local pressure fluctuations at any time were dominated by the effects of a single eddy structure. Then the variation of convection velocity shown in Figure 6 could be simply the result of the outward movement of this eddy as it moved downstream. This outward movement would result in its attaining a greater convection velocity as it moved downstream. With this idea, Figure 6 may be used directly to predict a typical eddy locus from the known boundary layer velocity profile (Fig. 7).

The locus represents a weighted statistical average of all the eddies which have passed the transducer position. The weighting is provided by the pressure generation effects of each eddy, which may be expected to vary with distance from the wall. Thus, care should be taken in drawing conclusions from this figure. However, Figure 7 gives rise to some interesting speculations about the generation of pressure fluctuations in the turbulent boundary layer. It will be recalled that the arguments of the last section showed how a major source of pressure fluctuation, particularly of the high frequency part of the fluctuation, was located near the edge of the laminar sublayer where the convection velocities are near $0.6\ U_o$ (Fig. 1).

In 1956, Einstein and Li [Ref. 19] showed how the laminar sublayer suffered from an intermittent disintegration. This conclusion was supported by the work of Grant [Ref. 20] and the mechanism has recently been the subject of a thorough investigation by Runstadler, Kline and Reynolds [Ref. 21]. It is found that the laminar sublayer convolutes itself into eddy structures which erupt away from the wall. Runstadler, Kline and Reynolds describe this as "the ejection of momentum deficient fluid from the wall" and Grant sees the mechanism as a "stress-relieving motion." This powerful mechanism is at work near to the wall, in just the region where the highest contributions to the pressure fluctuations may be expected. It is not unreasonable to suppose, therefore, that a major fraction of the pressure fluctuations at the wall is a direct result of this intermittent eruption of the laminar sublayer.

This hypothesis implies that the pressure-producing eddy is expanding away from the wall as it passes downstream, as was concluded above from the convection velocity measurements (Fig. 7). Runstadler, Kline and Reynolds [Ref. 21] have made visual observations of the eddies resulting from the disintegration of the laminar sublayer and found that the generation is random in space and time. However, they were able to define typical paths taken by an eddy after generation as a statistical average of the many individual paths observed. This eddy locus is plotted on Figure 7 and may be compared with that predicted from the convection velocity measurements. The agreement is amazingly good, and provides powerful evidence in support of the hypothesis. The loss in agreement at downstream stations may be readily understood as being the result of the loss of coherence of the wall eddies. This would result in the convection velocities derived from correlation measurements reflecting a larger proportion of the more coherent higher speed eddies farther from the wall. But for eddies near to the wall, the two plots are almost identical, and one can again infer that the eddies near the wall dominate the local pressure fluctuation phenomena.

However, Figure 7 cannot be regarded as providing conclusive proof of the present hypothesis. As has already been mentioned, the locus derived from the fluctuating pressure convection velocity measurements represents some sort of weighted average. In addition, the variation of convection velocity used in the derivation on Figure 7 (from Ref. 15) is not identical to that found in other investigations, [Ref. 17 and 18]. There is also a difference in Reynolds number between the two cases of Figure 7. Bull's work was accomplished at $Re_\theta \sim 20,000$, whereas the work of Runstadler et al. had $Re_\theta \simeq 2,000$. Thus, the agreement between the two cases of Figure 7 could be entirely fortuitous. Nevertheless, physical intuition suggests that if such powerful "ejection" or "eruption" is taking place near to the wall, then it must be a major factor in producing the pressure fluctuations observed.

The writer feels that this mechanism probably provides all of the high frequency part and much of the low frequency part of the pressure fluctuations, and thus implies local convection velocities of $0.6\ U_o$ or less for these fluctuations. The remaining part of the low frequency fluctuation probably arises much farther from the wall. It is interesting to speculate that this remaining part may occur through an identical eruption of the turbulent layer out into the free stream, as part of the known intermittent processes in turbulent flow. The relationship between the eruption of the laminar sublayer and the intermittency observed in the outer parts of the turbulent boundary layer has been pointed out in References 20 and 21 and this second phenomenon could well be a source of low frequency pressure fluctuations with a convection speed near $0.8\ U_o$. It is difficult, at this stage, to determine the absolute values of the two postulated sources of pressure fluctuation. Examination of the magnitudes of the space time correlation curves of Reference 13 suggests that the effect of the inner region through sublayer eruption is at least five times stronger than the effects due to the outer region.

A possible mechanism producing pressure fluctuations, both through sublayer eruption and in the outer intermittent region, is eddy acceleration. Eddy accelerations have been shown to be a source of noise generation in References 22 and 23. It seems probable that these could become important sources of pressure fluctuation in high subsonic or supersonic boundary layers.

SECTION IV. PRESSURE FLUCTUATIONS IN A SEPARATED TURBULENT FLOW

Equation (25) has been the starting point for most of the attempts to predict boundary layer pressure fluctuations. However, it cannot be applied to the present problem of the separated turbulent boundary layer. First, it is by no means obvious that the "turbulence-turbulence" contribution to the pressure (the second part of equation 19) can be neglected. In a separated flow, high values of turbulence intensity are to be expected, and these could contribute substantially to the wall pressure fluctuations. However, until definitive measurements have been made in the separated boundary layer, it will be difficult to make any final statement on the importance of this contribution. Clearly, the turbulence intensities and correlation areas must be expected to be considerably higher than in the attached boundary layer. There is a general similarity between the velocity profile of a separated turbulent boundary layer (Fig. 8) and that of a jet. Little work has been done on the pressure fluctuations in a jet, but there has been much interest in the parallel problem of the noise radiation. For this case investigations have shown [Ref. 24] that the turbulence-mean shear interaction is dominant. In the absence of other information, it may be assumed, for the present, that the same dominance is reflected for the case of pressure fluctuations in separated flow.

This leaves the turbulence -mean shear interaction of equation (22) as the leading source of pressure fluctuations. However, this does not necessarily imply that its approximate form given in equation (25) is also valid. Equation (25) has been derived using the order of magnitude arguments presented in equation (24). While these cause no dissent for the case of an attached turbulent boundary layer, for a separated boundary layer, δ may not be small compared to x. If the separation is taken to occur at an angle of 17 degrees, [Ref. 25] then the ratio of δ to x will be of the order 0.3. The contribution of the first two terms of equation (22) could thus be equal in magnitude to that of the third term. This will clearly result in an increase in pressure fluctuation and could have a major modifying effect on the correlation and frequency patterns encountered.

The two regions which may be expected to contribute most of these effects are the parts of the flow with maximum positive and negative velocity, where the $\partial U/\partial x$ terms will be highest (Fig. 8). However, little can be said of the exact effects, since no information on the structure of the separated turbulent boundary layer is presently available. A further complication in the theoretical treatment of these flows is that it may not be valid to assume homogeneity in the turbulence field. It is hoped that the projected experiments at Wyle Laboratories will help to show the true relative importance of the various terms in equation (22) for the separated case.

It is possible to make some general statements about the effect on the pressure fluctuations of the third mean shear term in equation (22) ($\partial U/\partial y$). Figure 8 shows a conjectural velocity distribution for the turbulent separated boundary layer. The mean shear becomes large at two separate points in the flow. The first is near the wall. Here it may be anticipated that the law of the wall will apply, and that the pressure fluctuations from this part of the flow will be proportional to the wall shear stress as in equation (34). This contribution must be expected to be substantially lower than in the attached boundary layer since the velocity to which this shear is reacting is the velocity of the reverse flow, which is only a fraction of that of the free stream. The mechanism producing the pressure fluctuations from this region may again be that put forward in Section III, i.e., due to the direct influence of the intermittent eruption of the laminar sublayer.

The second region of high shear is the central part near the "dividing streamline" of the separated flow. The contribution of this region may be large. A separated flow grows at an angle perhaps ten times that of an attached flow, and this factor of ten is a measure of the total momentum transfer that is taking place through the flow. In the attached boundary layer, this transfer is a result of only the wall shear stress, but in the separated boundary layer, it is the interaction of the forward and reverse flows which gives rise to most of the momentum transfer terms. Momentum transfer in this second high shear region may thus be expected to be an order of magnitude larger than that near the wall in the attached boundary layer case, with proportionate effects on the wall pressure fluctuation.

Lilley and Hodgson [Ref. 7] have investigated a similar flow with two regions of high shear using a wall jet. They also found the major contribution to the pressure fluctuations to come from this outer region. A particular feature of this outer region in the present separated flow case is that it is moving at a small fraction of the free stream velocity. The pressure fluctuation patterns resulting from this region would therefore be expected to have a low convection speed. In addition, contributions may be expected from the region of high negative velocity through the first two terms of equation (22), and this will again result in the prediction of a low overall convection velocity.

This conclusion is in opposition to the experimental work of Kistler [Ref. 26] for supersonic separated flows. Kistler's work shows a convection velocity of about 0.6 of the local stream velocity. (Note that in his report the lower free stream velocity behind the separation shock was not accounted for in the analysis.) At this stage, it is difficult to state where this discrepancy arises. It could be due to the effect of eddies in the outer part of the boundary layer entering via the first two terms in equation (22). If this is the case, it will become apparent in the current low speed separated flow experiments planned at Wyle. On the other hand, the discrepancy could be due to effects associated with the supersonic flow, for example, the eddy acceleration effects discussed at the end of Section III. It is also possible that Kistler's result is due to moving shocks within the separation region. It is clear that the resolution of this discrepancy must form a central part of any attempt to explain the pressure fluctuations in supersonic separated flows.

SECTION V. CONCLUSIONS

The equations governing the generation of pressure fluctuations in a boundary layer have been derived and the various approximations made have been noted. The most important approximation is the assumption of incompressibility. The equations have been written in a form which allows identification of the contributions of the "turbulence - mean shear" and "turbulence - turbulence" to the pressure fluctuation. Both of these terms can be significant, but the more important effect is usually the turbulence - mean shear interaction. For the pressure fluctuations in a two-dimensional boundary layer, the equation takes the form

$$\nabla^2 p' = -2 \rho_o \left(\frac{\partial U}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial V}{\partial y} \frac{\partial v}{\partial y} + \frac{\partial U}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial V}{\partial x} \frac{\partial u}{\partial x} \right). \qquad (22)$$

For the attached boundary layer, the most important term will be the third, and the data of Klebanoff [Ref. 12] have been analyzed to show the approximate contributions of the various parts of the boundary layer. The analysis shows that the major source of the pressure fluctuations lies near the edge of the laminar sublayer, where eddy convection velocities may be expected to be about $0.6\, U_o$ or less. It is shown how a proper interpretation of the results of space-time pressure correlation measurements gives the same conclusion. The frequently quoted result of convection velocities near $0.8\, U_o$ refers to the convection speed of the coherent low frequency eddies which dominate the correlations at large spacings, but only represent a small part of local pressure fluctuation.

It is suggested that the major part of the local wall pressure fluctuations could arise from the intermittent eruption of the laminar sublayer observed in flow visualization experiments. This seems physically likely, and the typical eddy path implied by the pressure fluctuation measurements following this hypothesis is found to agree with a typical eddy path actually observed in experiment, although this agreement may be coincidental. A small proportion of the pressure fluctuation, confined to the low frequency region, is probably due to other parts of the boundary layer, the inner part of the intermittent region, for example. A description of the pressure fluctuations as being mainly the result of laminar sublayer eruption agrees with all the known experimental facts. It provides a good model for a physical understanding of the forces at work, and should lead to improved analytical models to describe the pressure fluctuation phenomena.

The pressure fluctuations in a separated turbulent boundary layer have been discussed in the light of equation (22). It was shown how all the terms in this equation may produce a significant contribution to the pressure fluctuations observed. It was also shown that the third term in equation (22) could be expected to yield pressure patterns which were convected at near zero velocity. The other terms would not be expected to alter this conclusion, but it is radically different from the experimental result of a convection speed near 0.6 of the local free stream velocity recorded by Kistler [Ref. 26]. This paradox was not resolved, but is thought to represent a fundamental problem in the analysis of the pressure fluctuations in separated flow. The possible modifying effects of the turbulence - turbulence interaction terms were noted, but again no firm conclusions can be drawn until more experimental evidence is available.

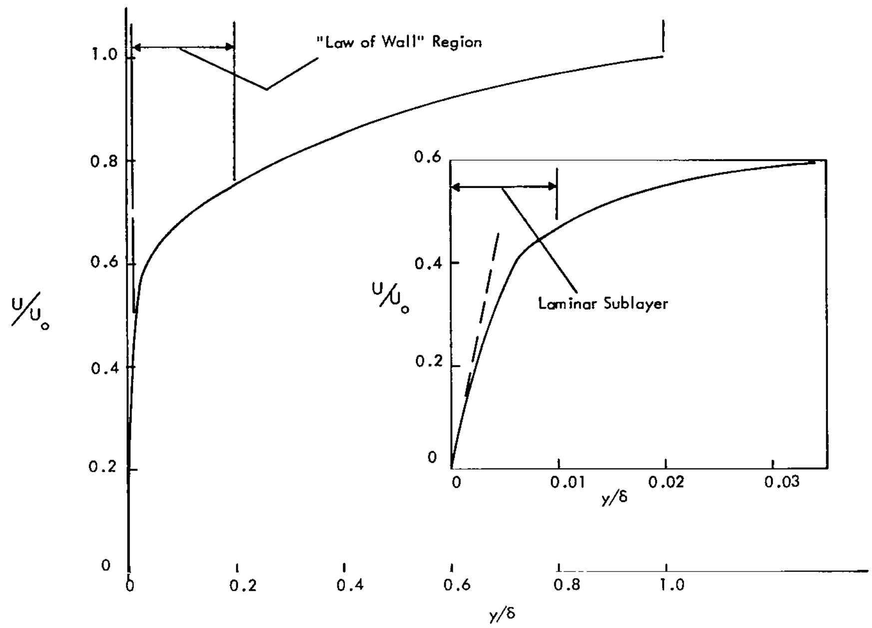

FIGURE 1. A MEAN VELOCITY PROFILE FOR THE ATTACHED TURBULENT BOUNDARY LAYER AFTER KLEBANOFF REFERENCE 12

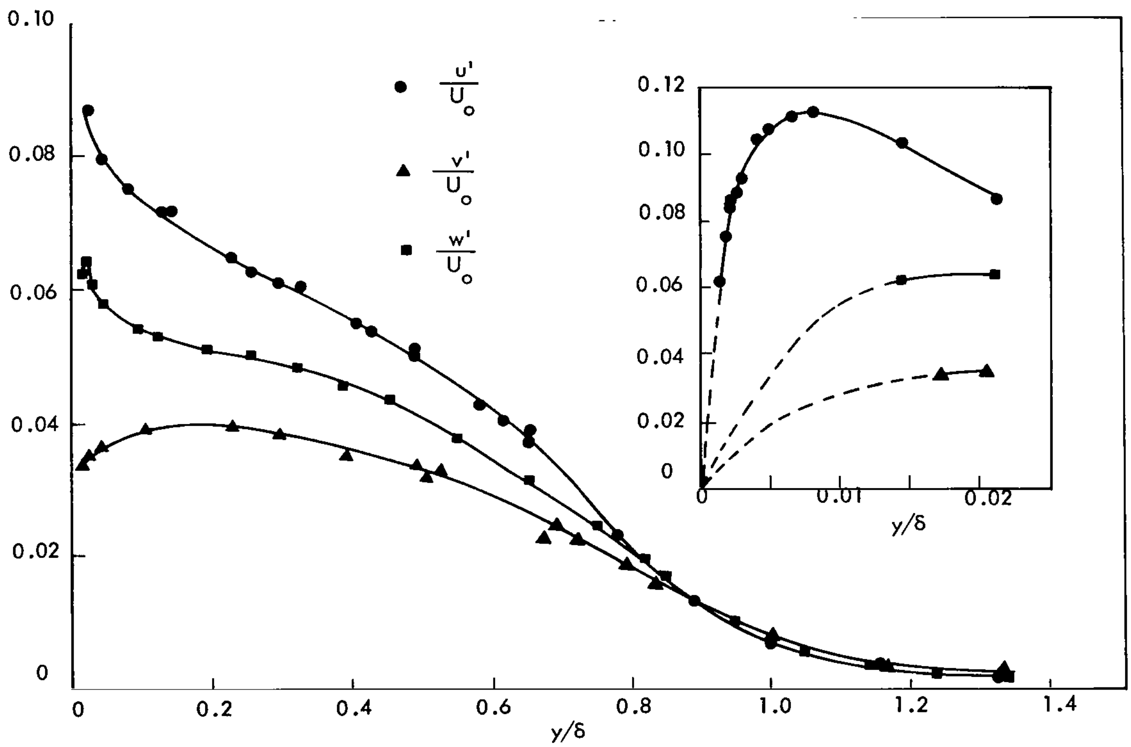

FIGURE 2. DISTRIBUTION OF TURBULENCE INTENSITIES FROM KLEBANOFF REFERENCE 12

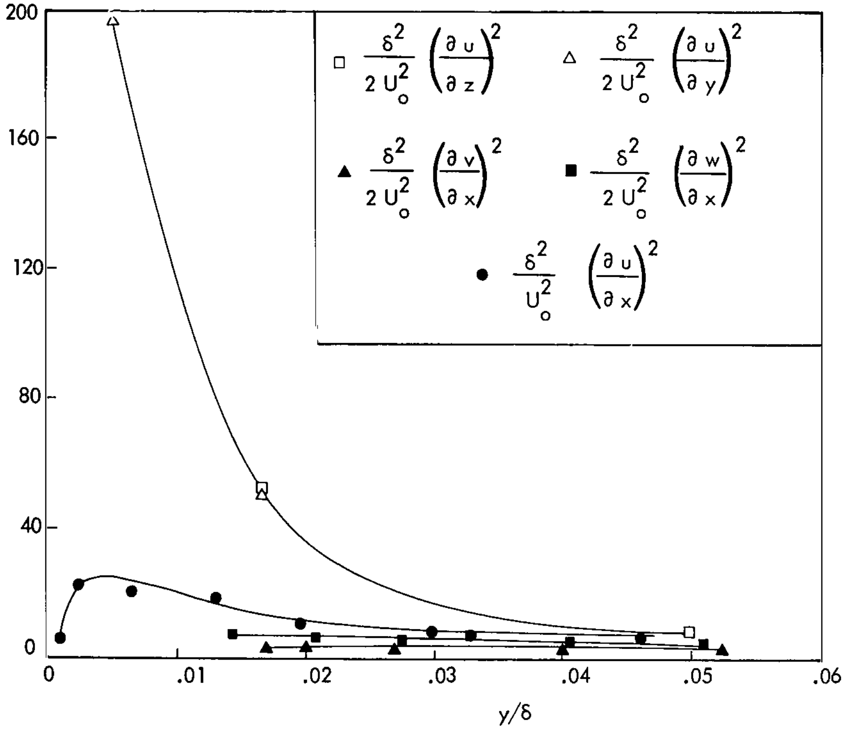

FIGURE 3a. DISTRIBUTION OF DISSIPATION DERIVATIVES NEAR THE WALL. FROM KLEBANOFF REFERENCE 12.

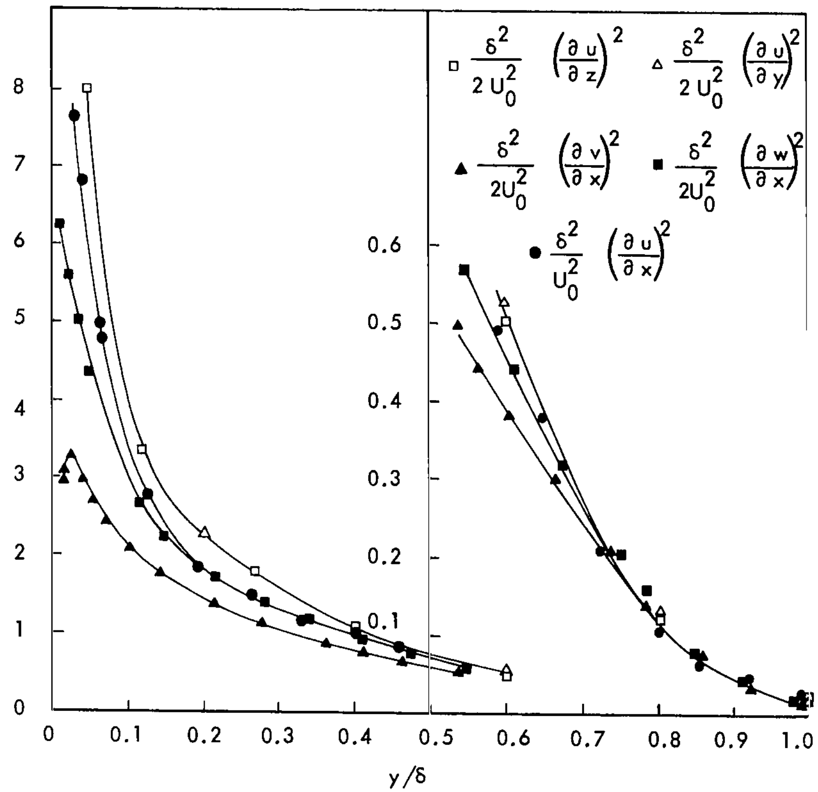

FIGURE 3b. DISTRIBUTION OF DISSIPATION DERIVATIVES AWAY FROM WALL. FROM KLEBANOFF REFERENCE 12.

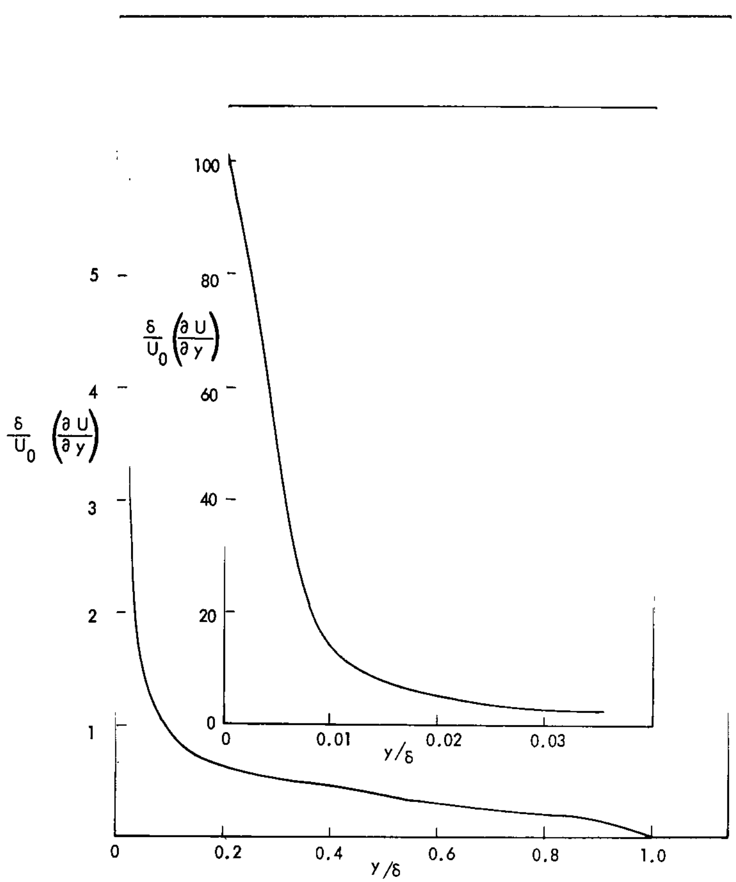

FIGURE 4. THE MEAN SHEAR GIVEN BY GRAPHICAL DIFFERENCE OF FIGURE 1

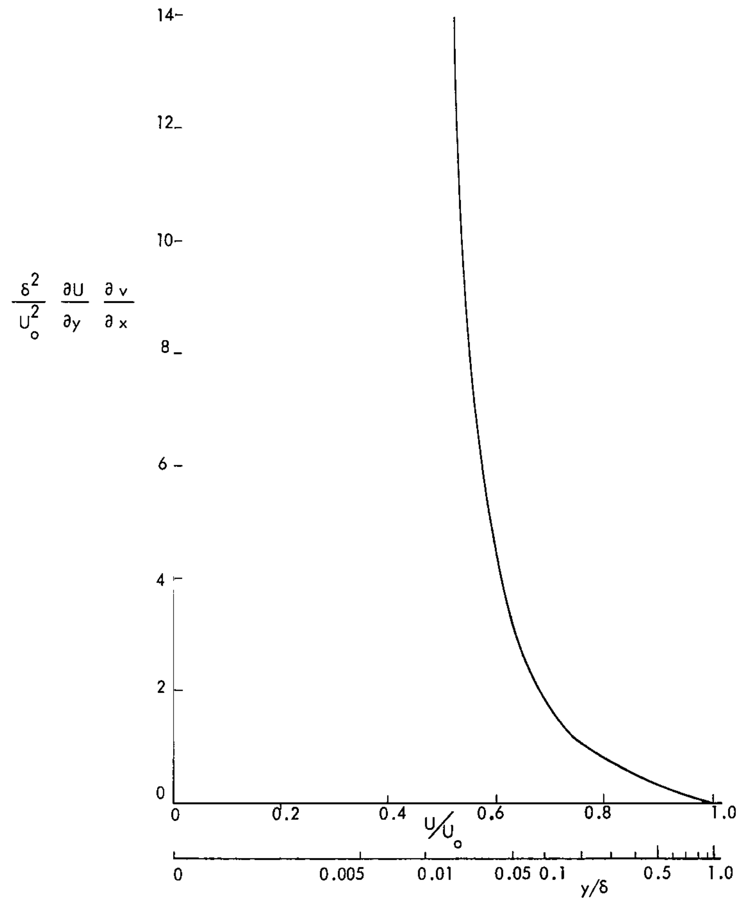

FIGURE 5. CONTRIBUTION TO THE PRESSURE FLUCTUATION FROM VARIOUS PARTS OF THE BOUNDARY LAYER ESTIMATED FROM FIGURES 3a, 3b, AND 4

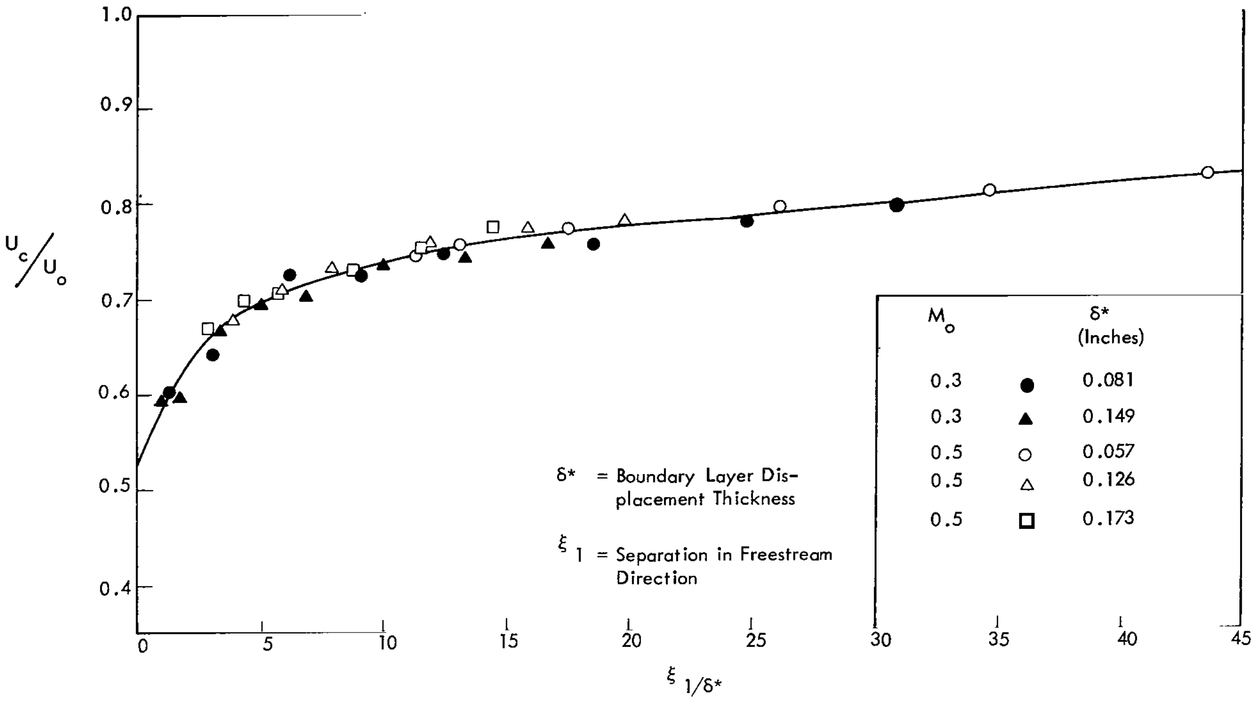

FIGURE 6. VARIATION OF CONVECTION VELOCITY WITH SPATIAL SEPARATION FROM BULL REFERENCE 15

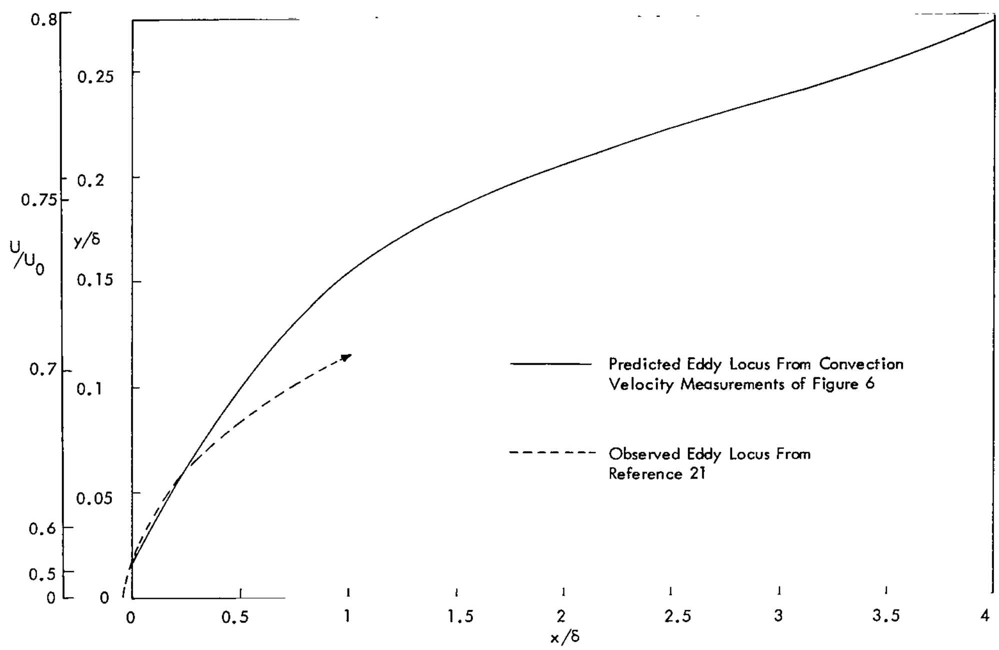

FIGURE 7. PREDICTED AND OBSERVED EDDY LOCI AFTER ERUPTION OF THE LAMINAR SUBLAYER

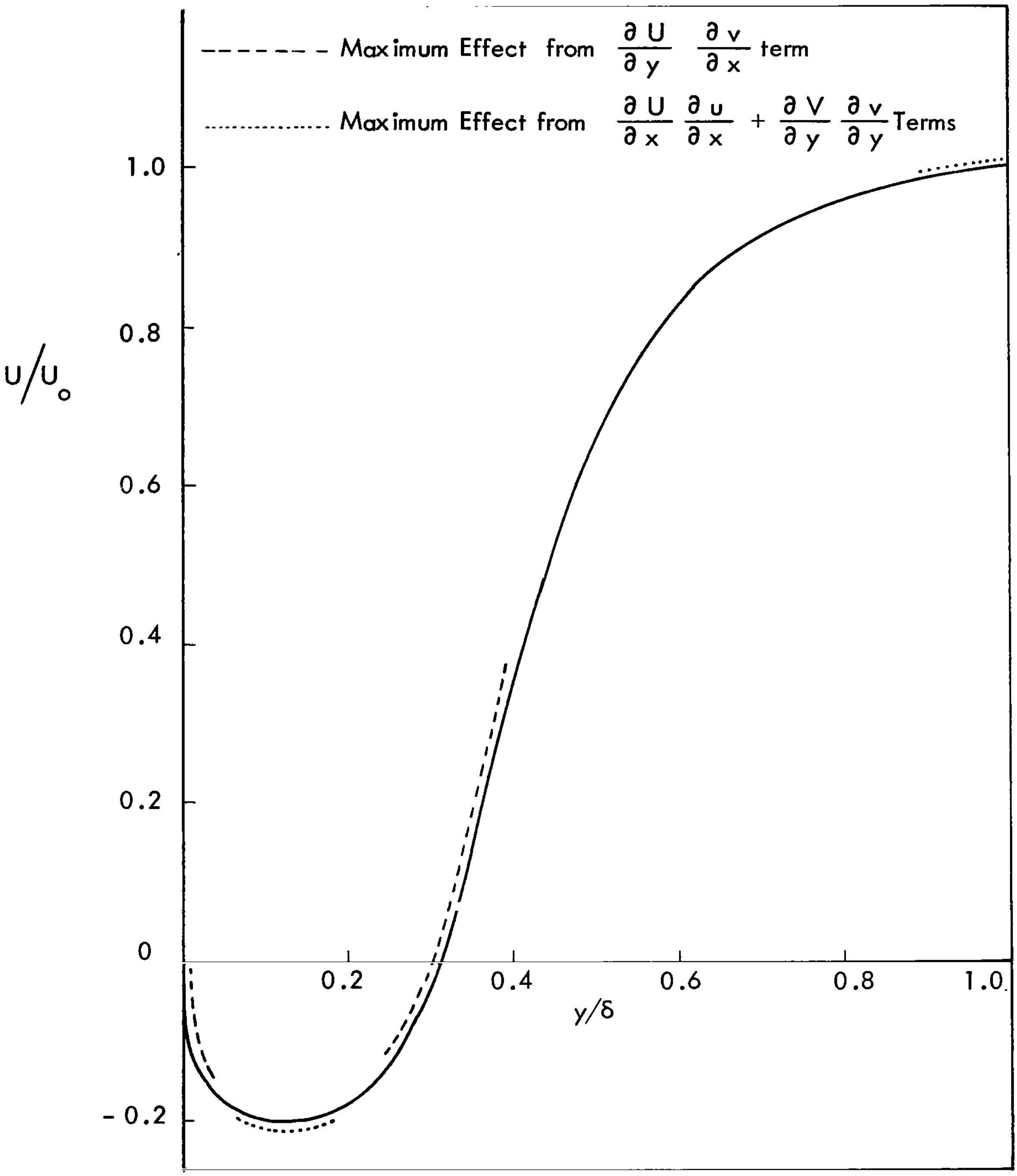

FIGURE 8. CONJECTURAL VELOCITY DISTRIBUTION FOR THE SEPARATED TURBULENT BOUNDARY LAYER SHOWING MAJOR REGIONS OF FLUCTUATING PRESSURE GENERATION

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