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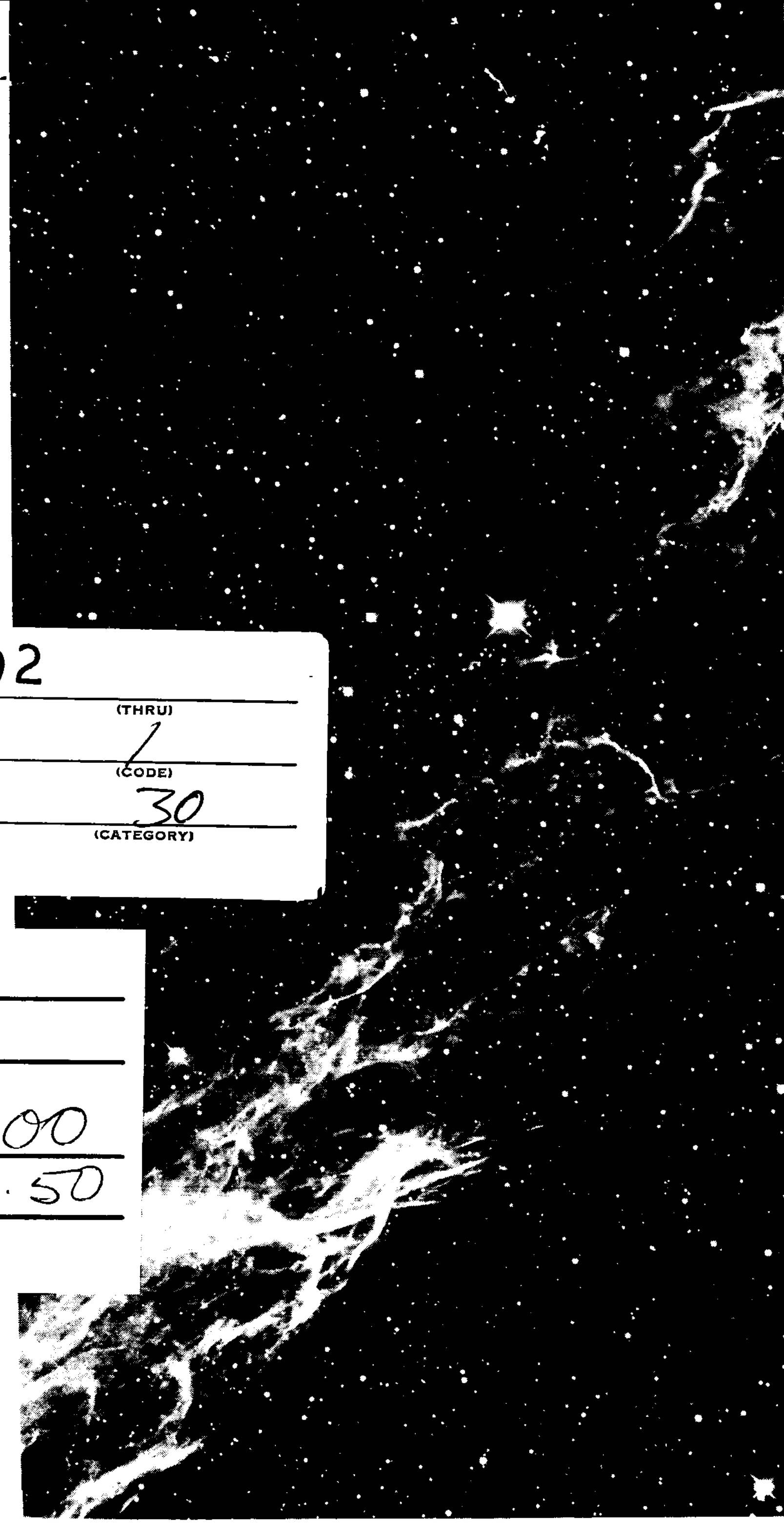

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Report No. T-14

LOW-THRUST TRAJECTORY AND PAYLOAD ANALYSIS FOR SOLAR SYSTEM EXPLORATION UTILIZING THE ACCESSIBLE REGIONS METHOD

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LOW-THRUST TRAJECTORY AND PAYLOAD ANALYSIS FOR SOLAR SYSTEM EXPLORATION UTILIZING THE ACCESSIBLE REGIONS METHOD

by

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FRONTISPIECE

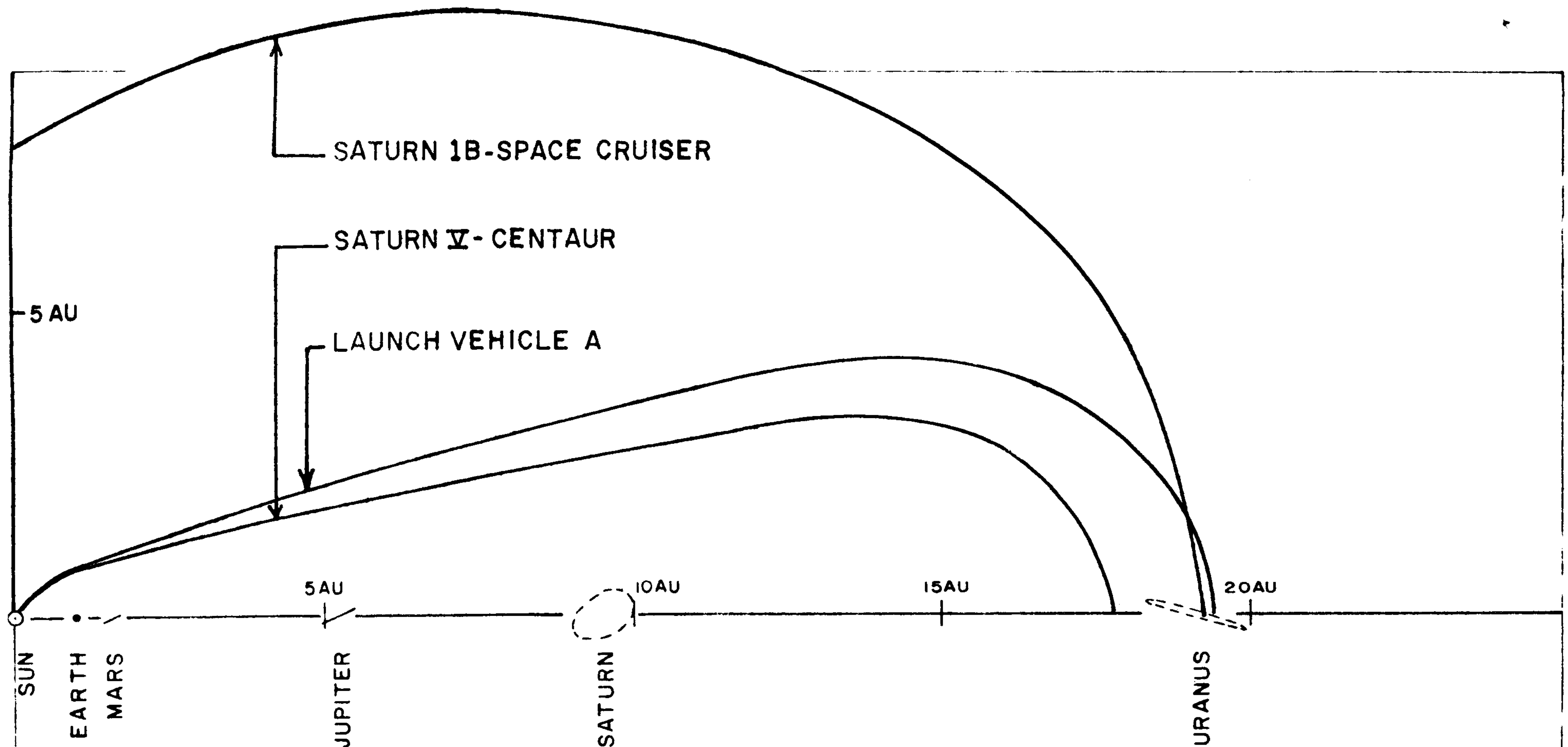
An accessible regions comparison of the Saturn 1B-Space Cruiser (low thrust) vehicle with ballistic launch vehicles for a 500 lb experimental and communications payload and a maximum trip time of 1100 days.

(1) Space Cruiser Parameters:

• Initial weight in Earth orbit	20,000 lbs
• Power rating	500 kwe
• Power plant specific weight	20 lb/kw
• Specific impulse	12,000 sec
• Estimated weight of structure tankage and G&C	3500 lb

(2) Launch Vehicle A is a theoretical 4 stage vehicle with the first stage fueled with O_2/H_2 and the upper 3 stages fueled with $O_2/H_2/Be$. Initial weight is about 1 million lbs.

(3) For the ballistic spacecraft, the total weight assumed is 1300 lbs including shroud and adapter. Of this weight, 700 lbs are allocated to structure, power and G&C.



AN ACCESSIBLE REGIONS COMPARISON OF THE SATURN 1B-SPACE CRUISER (THRUSTED) VEHICLE WITH BALLISTIC LAUNCH VEHICLES FOR A 500 LBS. EXPERIMENTAL AND COMMUNICATIONS PAYLOAD AND MAXIMUM TRIP TIME OF 1100 DAYS.

THE FIGURE BACKGROUND IS THE PLANE P_N NORMAL TO THE ECLIPTIC PLANE, WITH THE PROJECTIONS OF THE INCLUDED PLANETARY ORBITS SHOWN: DRAWN TO SCALE

25AU SCALE

SUMMARY

The accessible regions method provides a simple and graphic means of delineating generalized trajectory energy requirements for solar system exploration, and, by relating these requirements to specific vehicle systems, also provides a convenient graphic assessment of payload capabilities. This report applies the accessible regions method to low-thrust fly-by missions throughout the solar system. By analogy to the familiar "ΔV" associated with ballistic flight, the quantity "J" (defined as the time integral of thrust acceleration squared) is used herein to link trajectory energy requirements and vehicle system characteristics for low-thrust flight.

The principal idea of this concept of data presentation lies in showing the maximum region in the solar system which can be reached with a given value of J. This is accomplished by assuming that a mission can always be launched when Earth is in the proper longitudinal position to minimize the J required to reach a given target position in a given time of flight. The elimination of the Earth's position from the computation

effectively reduces the three dimensional solar system to a two-dimensional model. Basic results are presented in the form of J contours plotted on a two-dimensional spatial background of the solar system. Every potential target position has a corresponding point in this plane given by its heliocentric distance and latitude.

J contours for 100, 300 and 1000 day flight times and for both variable thrust and constant thrust modes of propulsion are presented. The variable thrust contours may be considered a "reference solution" which gives the theoretical upper performance bound for power-limited vehicles. The performance degradation due to the more realistic constant thrust mode of operation is then indicated by comparison. In addition to the J contours, this report illustrates the accessible regions method in terms of the payload capability of a conceptual nuclear-electric Space Cruiser*.

The accessible regions method of data presentation may be used to compare the exploration capability of ballistic and thrusted vehicles. The frontispiece is such a comparison between a Saturn 1B-Space Cruiser and several ballistic launch vehicles. Each contour shown represents the region accessible with a 500 lb. experimental and communications payload in a maximum trip time of 1100 days. For the Space Cruiser, the "useful" payload figure is the gross payload delivered to the

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* The Space Cruiser is a JPL design concept having an initial weight in Earth orbit of 20,000 lbs. and a 500 kwe nuclear turboelectric power plant.

target less 13,500 lbs allocated to the power plant, structure, tankage, and guidance and control. For the ballistic spacecraft, "useful" payload was arrived at by subtracting an estimated 700 lbs of structure, power, and guidance and control from the spacecraft weight at injection.

The Space Cruiser is able to deliver the 500 lb payload to the region in the ecliptic plane extending from the Sun out to Uranus (19.5 AU), and to all angles out of the ecliptic including an "over-the-Sun" flight at a distance of 7.5 AU. For the trip time limitation of 1100 days, the region accessible to the Saturn V-Centaur launch vehicle extends from the near vicinity of the Sun to a point, just short of Uranus (only direct ballistic trajectories are considered here). Out-of-the ecliptic flights are limited to about 46° latitude and 3.1 AU vertical displacement. The high energy launch vehicle A (theoretical) can extend this region somewhat, but still does not match the performance of the electric Space Cruiser for out-of-the ecliptic missions. The more rounded shape of the Space Cruiser contour reflects the relative ease of achieving plane changes with continuous thrusting vehicles. It should be mentioned that the performance advantage of the thrusted upper stage is mainly characteristic of very high-energy missions and long flight times. For less ambitious missions (e.g., fly-bys of the nearer planets), this potential advantage would decrease and may even disappear. Also, it is noted that the use of planetary bodies for gravity assist would tend to fill in the performance gaps between the direct ballistic and thrusted modes of flight.

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1. INTRODUCTION

The application of electric propulsion systems to upper stage space vehicles is generally thought to offer a high performance potential for carrying out a long-range, comprehensive plan of solar system exploration. Considerable effort is now underway toward the development of lightweight and reliable advanced propulsion systems which are necessary if this potential is to be realized. Concurrent with this developmental effort there has been an increased activity in mission related studies including analysis of low-thrust trajectories and payload capabilities. These studies have been mainly concerned with missions to particular target bodies in the solar system such as Venus, Mars and Jupiter. Although these planets are undoubtedly of interest for early exploratory missions, there are numerous other celestial bodies and regions of the solar system which present interesting targets for future scientific investigations.

This report presents results of a trajectory/payload capability study which adopts this broader view of solar system exploration.

For long-range mission planning purposes it is useful to have available a convenient graphic means of determining the trajectory energy requirements for flights throughout the solar system. The accessible regions method provides such a means, and, by relating these requirements to particular vehicle system characteristics, also provides a graphic assessment of payload capabilities. The fundamental idea behind the accessible regions method is to portray the three-dimensional solar system in a two-dimensional model. This is accomplished by assuming that a mission can always be launched when Earth is in the proper longitudinal position to minimize the energy requirement to reach a given target in a given time of flight. Results are presented graphically as performance contours (e.g., trajectory energy requirements or payload) plotted on a two-dimensional spatial background of the solar system.

The accessible regions method was developed for a study of ballistic fly-by flights throughout the solar system (Narin 1964). This report extends the method to low-thrust fly-by missions.

2. PARAMETERS OF ADVANCED PROPULSION SYSTEMS

In any analysis which attempts to relate trajectory requirements and vehicle performance in a quantitative manner, it is necessary to have a suitable "link" between the trajectory kinematics and the vehicle propulsion characteristics. In the case of ballistic interplanetary flight, which is associated with high-thrust launch vehicles, trajectory energy requirements are usually expressed in terms of an impulsive velocity increment - the familiar "ΔV". Here, ΔV is strictly a trajectory-dependent parameter and provides the necessary link between kinematics and propulsion. In the case of thrusted interplanetary flight, which is associated with low-thrust upper stage vehicles, ΔV no longer serves as a useful linking parameter. Thrust acceleration cannot be considered impulsive, but rather, takes place over a significant portion of the trajectory. Furthermore, the two classes of vehicle propulsion do not necessarily operate under the same constraints.

The kinematic performance parameter now in common usage for low-thrust trajectory analysis is designated by the symbol J (Melbourne 1961). This is defined as

$$J = \int_{0}^{T_f} a^2(t)\, dt \tag{1}$$

where T_f is the specified time of flight and $a(t)$ is the thrust acceleration associated with the mission trajectory. The above integral is somewhat analogous to ΔV in that it provides the necessary link between trajectory energy requirements and vehicle performance.

Unlike chemical propulsion systems, wherein performance is limited in part by the energy content of the propellant, electric propulsion systems are power-limited, i.e., limited in performance by the characteristics of the separate power plant needed to generate kinetic energy of the propellant. These characteristics include the electric power rating, weight, and conversion efficiency of the power plant. It is generally assumed that the power plant will be operated at its maximum power rating during periods of electric thrustor operation. This constant power constraint reflects upon the necessary regulation of propellant flow rate ($\dot{M}$) and jet velocity (V_j); or, alternatively, thrust (F) and specific impulse (I_{SP}). The quantitative relations are (in the MKS system of units)

$$2P_j = 2\eta P_e = \dot{M} V_j^2 \qquad (2)$$

$$2P_j = FV_j = (F I_{SP}) g_o \qquad (3)$$

where P_j is the kinetic power in the exhaust jet, P_e is the electric power rating, and η is a factor combining power conversion efficiency and power utilization. Since a = F/M, equations (1), (2) and (3) combine, after integration, to yield the characteristic mass equation for power-limited flight (constant η is assumed)

$$\frac{1}{M(T_f)} = \frac{1}{M(o)} + \frac{J}{2\eta P_e} \qquad (4)$$

The significance of J now becomes clear as does its analogy to ΔV. For a given initial mass and power rating of the vehicle, it is seen that the value of J required to accomplish a given mission determines the final mass of the vehicle.

Furthermore, it is observed that minimizing J is equivalent to maximizing the final mass (or minimizing the propellant requirement). It can readily be shown that this conclusion is also true for maximizing payload.

Following current practice, we assume that the vehicle mass is simply allocated into propellant (M_p), power plant (M_{pp}), and net payload (M_{pl}), i.e.,

$$M(t) = M_p(t) + M_{pp} + M_{pl} \tag{5}$$

If the initial propellant load is assumed to be just that amount needed to accomplish the mission (propellant depletion at $t = T_f$), then the final vehicle mass is

$$M(T_f) = M_{pp} + M_{pl} = M(o) - M_p(o) \tag{6}$$

In clarification of the above definitions, $M(T_f)$ could be thought of as the gross payload delivered to the target. For nuclear-electric power plants, M_{pp} would consist of the entire propulsion system less tankage and propellant, e.g., nuclear reactor system, radiation protection shield, heat rejection radiators, turbo-electric generating system, power conditioning, and electric thrustors. Net payload, M_{pl}, might typically be broken down into structure, tankage, guidance and control equipment, and scientific experiments including communication equipment.

Now, defining the specific mass of the power plant as

$$\alpha = \frac{M_{pp}}{P_e} \tag{7}$$

equation (4) may be rewritten in terms of the net payload

$$\frac{M_{pl}}{M_o} = \frac{M_{pp}}{M_o} \left[\frac{1}{\frac{M_{pp}}{M_o} + \frac{\alpha J}{2\eta}} - 1 \right] \tag{8}$$

This expression clearly shows the desirability for having low values of both α and J. The allocation of power plant mass also has an effect on the maximum payload that can be obtained. This, however, is a question of vehicle design which is beyond the scope of this study. The point to be made here is that, for a given vehicle system (real or hypothetical) and values of J corresponding to particular missions of interest, equation (8) provides a convenient means of estimating payload capability.

Figure 1 illustrates a payload vs J curve for a conceptual design of a nuclear-electric "Space Cruiser" developed in a JPL study (Beale et al. 1963). The Space Cruiser concept, considered to be feasible within the near future state of the art, would be capable of probing a significant portion of the solar system with reasonable mission times. System parameters chosen for this example are

$$M_o = 20{,}000 \text{ lb.}$$
$$P_e = 500 \text{ kw}$$
$$\eta = 0.8$$
$$\alpha = 20 \text{ lb/kw}$$

Figure 1 shows the characteristic decrease of net payload as the mission difficulty increases, i.e., increasing values of J.

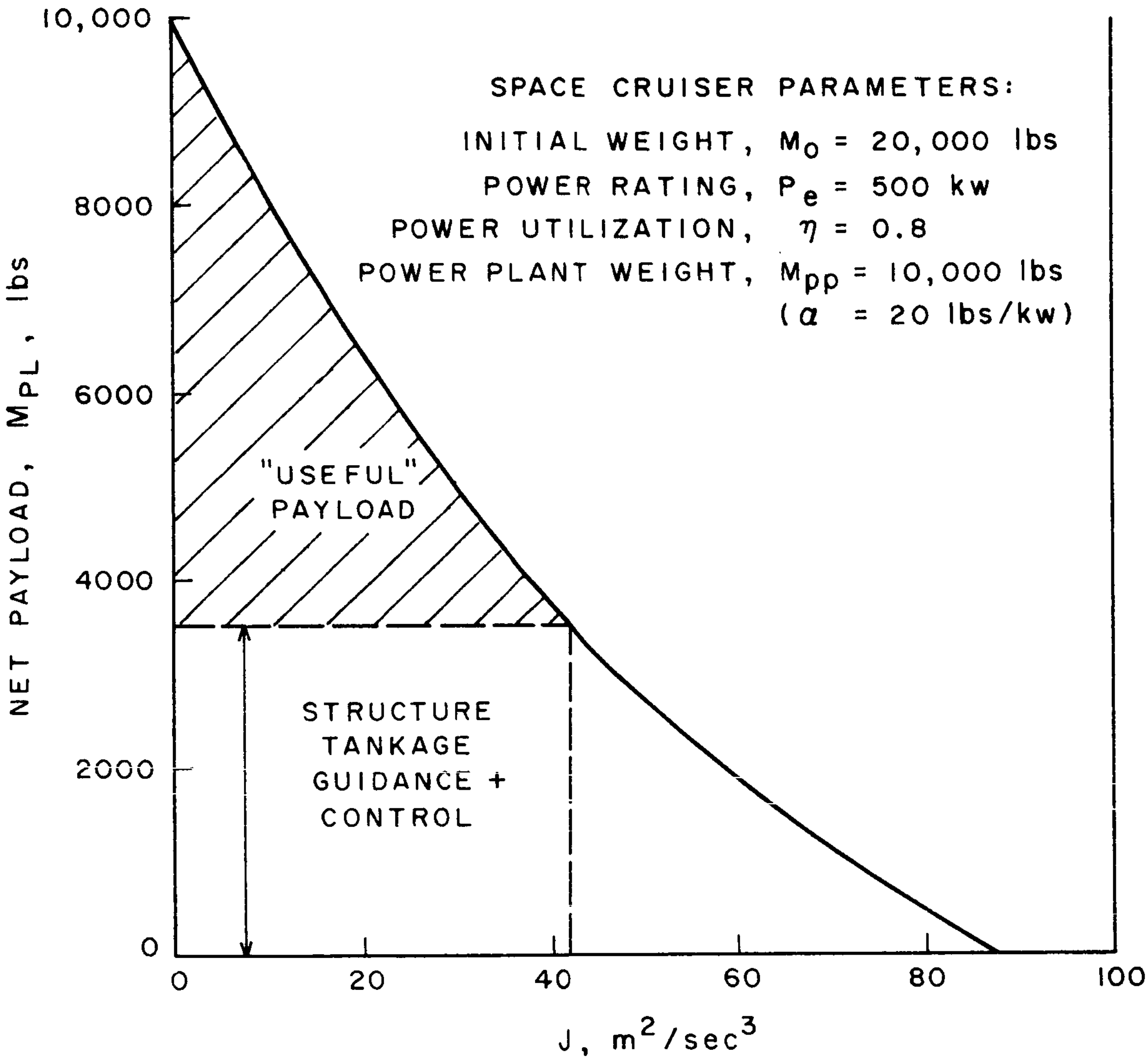


FIGURE I PAYLOAD VS J CURVE FOR ELECTRIC SPACE CRUISER

It is observed that payload vanishes altogether for J requirements greater than 88 m^2/sec^3. However, if one wishes to assess the "useful" payload capability of the Space Cruiser, the practical limitation placed on J is about 40 m^2/sec^3. This limit is arrived at by subtracting from the net payload an estimated weight of 3500 lb. allocated to structure, tankage, and guidance and control equipment. Thus, for example, a mission that can be accomplished with a J requirement of 34 m^2/sec^3 would allow about 1000 lbs. for scientific and communications equipment.

It is to be emphasized that the above example was presented mainly to illustrate the meaning of J in relation to payload. Although this example has practical significance, it is realized that other vehicle system configurations would change the particular numbers given above.

Although it may have been tacitly implied that J depends only upon the specified trajectory conditions (target coordinates and flight time), this is not generally true. Unlike ballistic flight, wherein ΔV is strictly trajectory-dependent, the J requirements for thrusted flight are related to the mode of propulsion system operation. Two different modes of electric thrustor operation that are usually considered are

1. Variable thrust
2. Constant thrust

In the variable thrust mode it is assumed that any thrust level and specific impulse can be obtained consistent with the constant power constraint, i.e., F and I_{SP} may be varied arbitrarily provided their product remains constant. Accordingly, in obtaining trajectory solutions for the variable thrust mode, a(t) is varied in an optimum manner in order to minimize the J requirement. In this case, J is not a function of the propulsion system parameters but rather depends only upon the specified trajectory conditions. Variable thrust, then, is the least restrictive mode of propulsion and, hence, yields the best possible performance in a theoretical sense.

For the constant thrust mode of operation, specific impulse is held constant and acceleration must obey the relationship

$$a(t) = \frac{a_o}{1 - \frac{a_o}{g_o I_{SP}} t} \qquad \underline{\text{constant thrust}} \tag{9}$$

Furthermore, the initial acceleration, a_o, is constrained by the parameters of the vehicle system design, e.g.,

$$a_o = \frac{2\eta}{g_o I_{SP}} \cdot \frac{P_e}{M_o} = \frac{2\eta}{g_o I_{SP} \alpha} \cdot \frac{M_{pp}}{M_o} \tag{10}$$

As a result of the above constraints, $J = \int a^2 \, dt$ must be functionally dependent upon the vehicle system parameters in addition to the specified trajectory conditions. For a particular mission, however, it is possible to minimize this dependence by an appropriate choice of specific impulse and by inserting a coast period in the trajectory design. When this is done, J can be reduced to a value which approaches the optimum variable thrust solution (about 15-20 percent higher).

Because of the additional parameters to be considered, trajectory and payload analysis is much more difficult for the constant thrust mode. However, from a practical standpoint, it is necessary to treat this case since constant level thrustors are more representative of realistic electric propulsion systems. The accessible regions method will be illustrated for both the variable and constant thrust modes of propulsion.

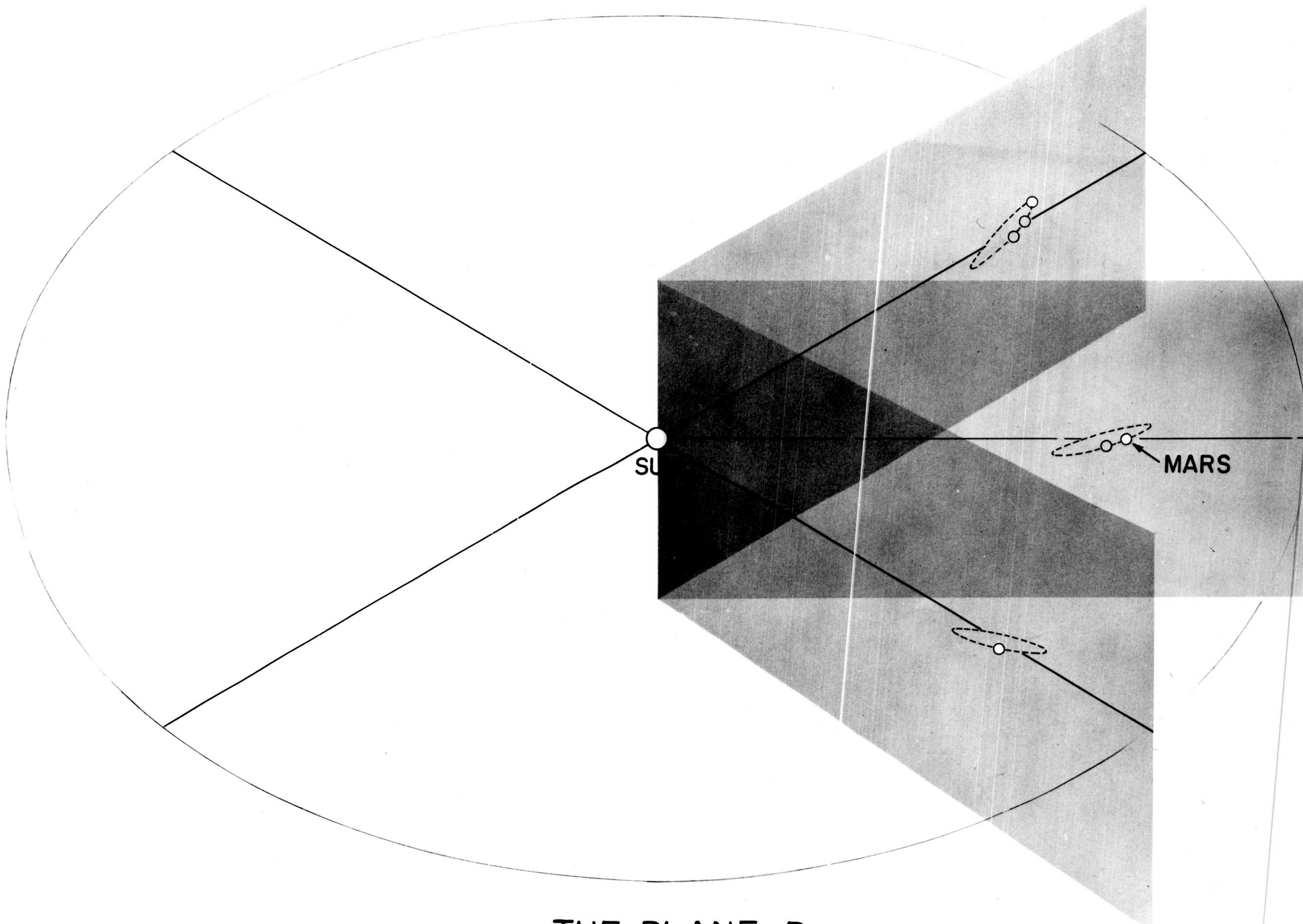
3. ACCESSIBLE REGIONS METHOD

The basic idea of the accessible regions method has been discussed in the introductory remarks. A more detailed description of this method as it applies to thrusted trajectory analysis will now be given.

Consider some arbitrary target point in the solar system as specified by its heliocentric distance (R), latitude (β), and longitude (λ). Suppose, now, that we specify a time of flight for an Earth-launched space vehicle to intercept the target. As the Earth moves around the Sun, there is some longitudinal position from which the specified flight may be launched such that the trajectory energy requirement is a minimum, i.e., a minimum value of J. Suppose we choose to launch the flight from this optimum longitude in order to achieve maximum performance. Hence, the target longitude may be eliminated from consideration, and the three-dimensional solar system is effectively reduced to a two-dimensional model. Of course, information regarding the date of launch is not directly available from this method. However, for long-range planning purposes the exact date of launch is not of immediate importance since every position of Earth in its orbit is attained once per year.

The accessible regions method of data presentation involves plotting contours of equal-valued performance parameters (e.g., J or payload) on a fictitious plane P_N, normal to the ecliptic plane. P_N intersects the ecliptic plane at the Sun and is free to rotate about the Sun (this is essentially a side view of the solar system). Each contour represents the maximum boundary of the spatial region accessible to a space vehicle having a particular performance parameter and a specified flight time.

Figure 2 is a perspective drawing of the plane P_N, showing the path which the planet Mars would trace if the plane were to move (in longitude) with Mars as the planet revolves about the Sun. Figure 3 is a diagram of the plane on a 5 AU scale. The horizontal width of the planetary orbital projections is a measure of the orbital eccentricity, while the vertical height is a measure of the orbital inclination. Every potential target position has a corresponding point in this plane as represented by the radial distance from the Sun (R) and the latitude measured from the ecliptic plane (β). A typical J contour is shown.



THE PLANE P_N

The method used to construct the J contours may be explained with reference to the basic geometry shown in Figure 3. Along a given β direction, a series of R values are selected, and the minimum J values required to reach these points from an arbitrary Earth position in a given flight time are computed. (Earth is assumed to have a circular orbit in the ecliptic plane for the purpose of this calculation.) Note that computed flight paths are fly-by or intercept trajectories - no constraint on terminal velocity is specified. From the J values so obtained, a graphical plot such as Figure 4 is constructed. In plotting a contour for a given value of J, the corresponding (R, β) coordinates are graphically ascertained. Interpolation is usually necessary to obtain a sufficient number of points to accurately define the contour. This is accomplished by making up plots of J versus β for constant parameter R.

The basic data for construction of the accessible regions contours were obtained from numerical integration solutions of optimum (minimum J) trajectories. These solutions were implemented by the JPL Low-Thrust Trajectory Optimization Code (Richardson 1963). This computer program, written in FAP language, is based on a variational calculus approach to the optimization problem. It should be noted that the computer program was utilized only to obtain solutions of the heliocentric portion of the flight, that is, beginning at the Earth-escape condition. Hence, the basic J contours to be presented represent this portion of flight only. The Earth-escape phase of the

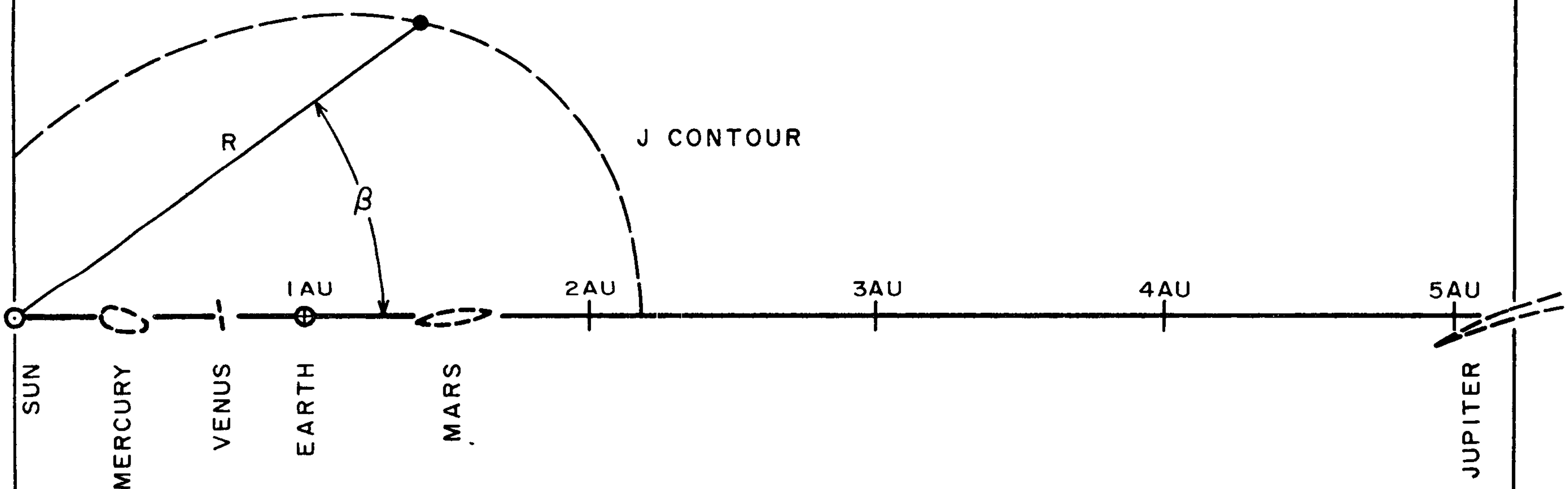


FIGURE 3 THE PLANE P_N, SHOWING BASIC GEOMETRY OF THE ACCESSIBLE REGIONS METHOD, 5AU SCALE

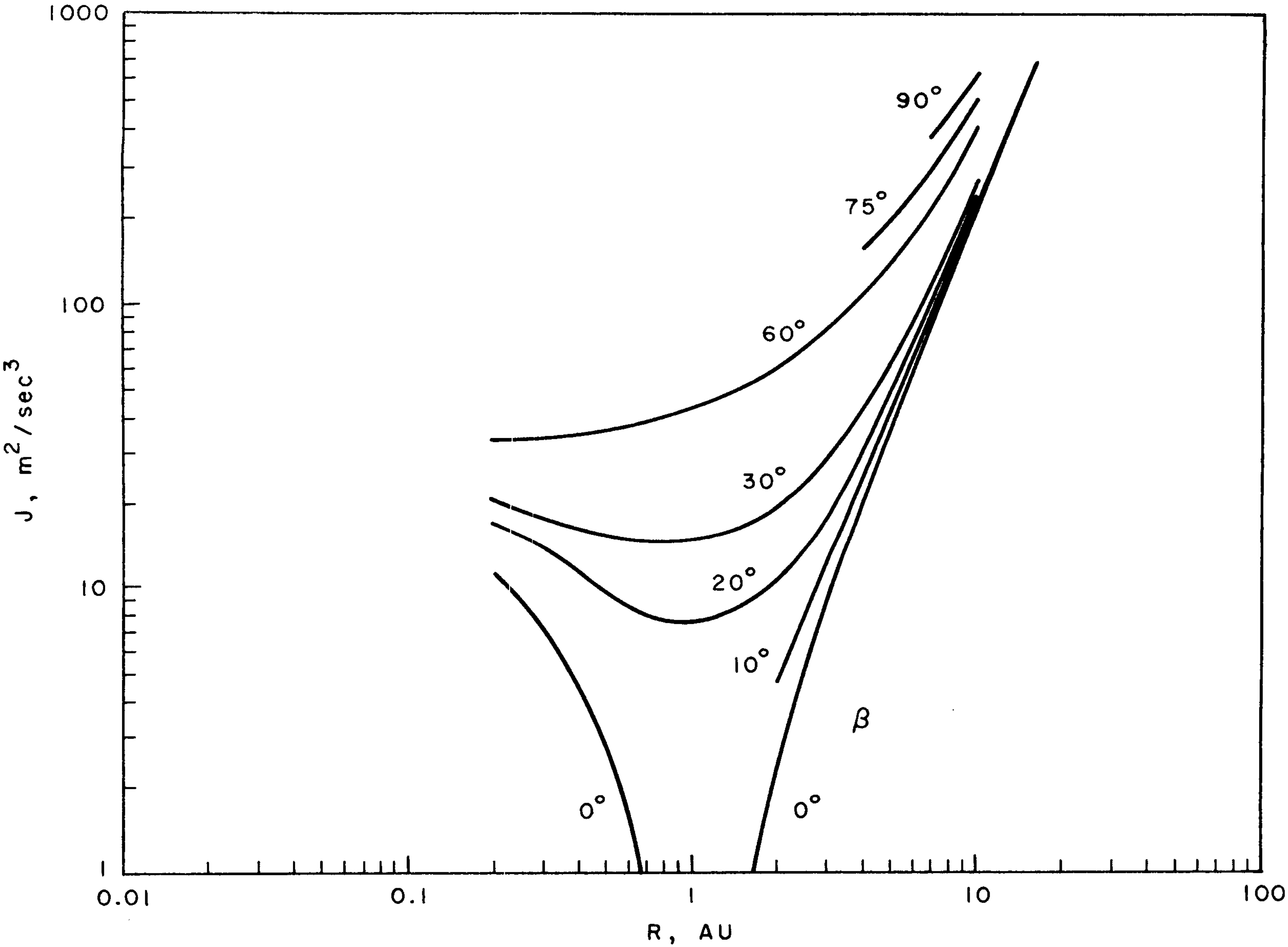


FIGURE 4 MINIMUM J REQUIREMENTS FOR 300 DAY FLY-BY TRAJECTORIES TO TARGET (R, β), VARIABLE THRUST MODE

mission may be treated as a separate problem since the J requirements for this phase are independent of the specified target coordinates. Inclusion of the Earth-escape phase of flight will be considered in a later section on payload contours.

4. J CONTOURS

4.1 Variable Thrust Mode

Figures 5 through 7 are accessible regions contours for minimum J flights of 100, 300, and 1000 days, respectively. Regions of the solar system out to 50 AU are covered. These contours are for the variable thrust mode of propulsion, and, hence, may be considered a "reference solution" which gives the theoretical upper performance bound for power-limited vehicles. As previously discussed, a practical limitation imposed upon J by the present state of the art of advanced propulsion technology is about 40-50 m^2/sec^3. It would be well to keep this limitation in mind in reading the accessible regions plots; however, to be complete, J values out to 200 m^2/sec^3 are shown.

Several types of information are readily obtained from these graphs. The minimum value of J required to reach a given point in the solar system in a given time of flight may be quickly estimated by visual interpolation. Conversely, the maximum spatial region accessible to a vehicle having a given J capability is immediately indicated.

Consider, for example, the 300 day contours shown in Figure 6. With a J of 20 m^2/sec^3 all regions in the ecliptic plane from the near vicinity of the Sun out to 4 AU may be

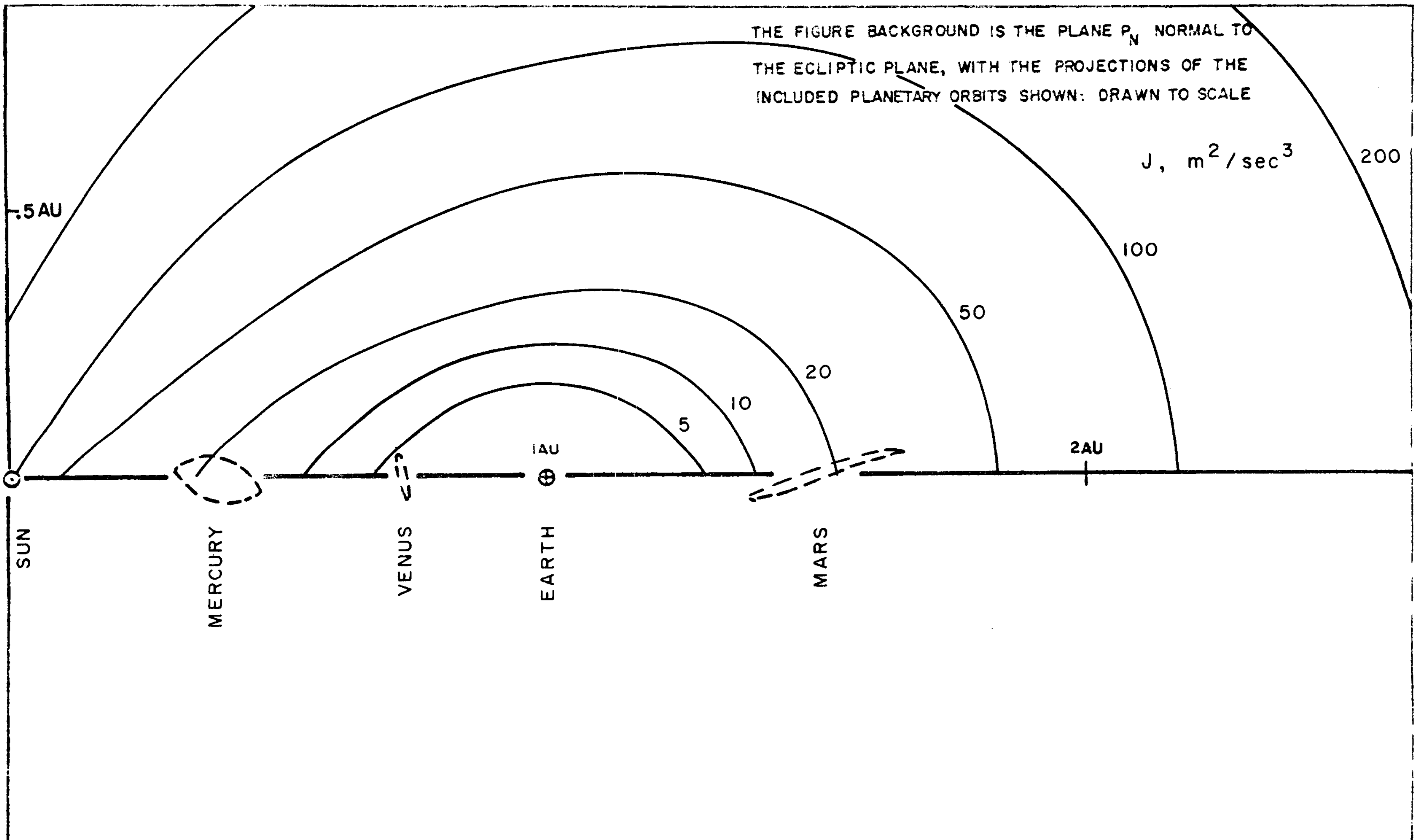


FIGURE 5 J CONTOURS FOR 100 DAY FLY-BY TRAJECTORIES, VARIABLE THRUST MODE

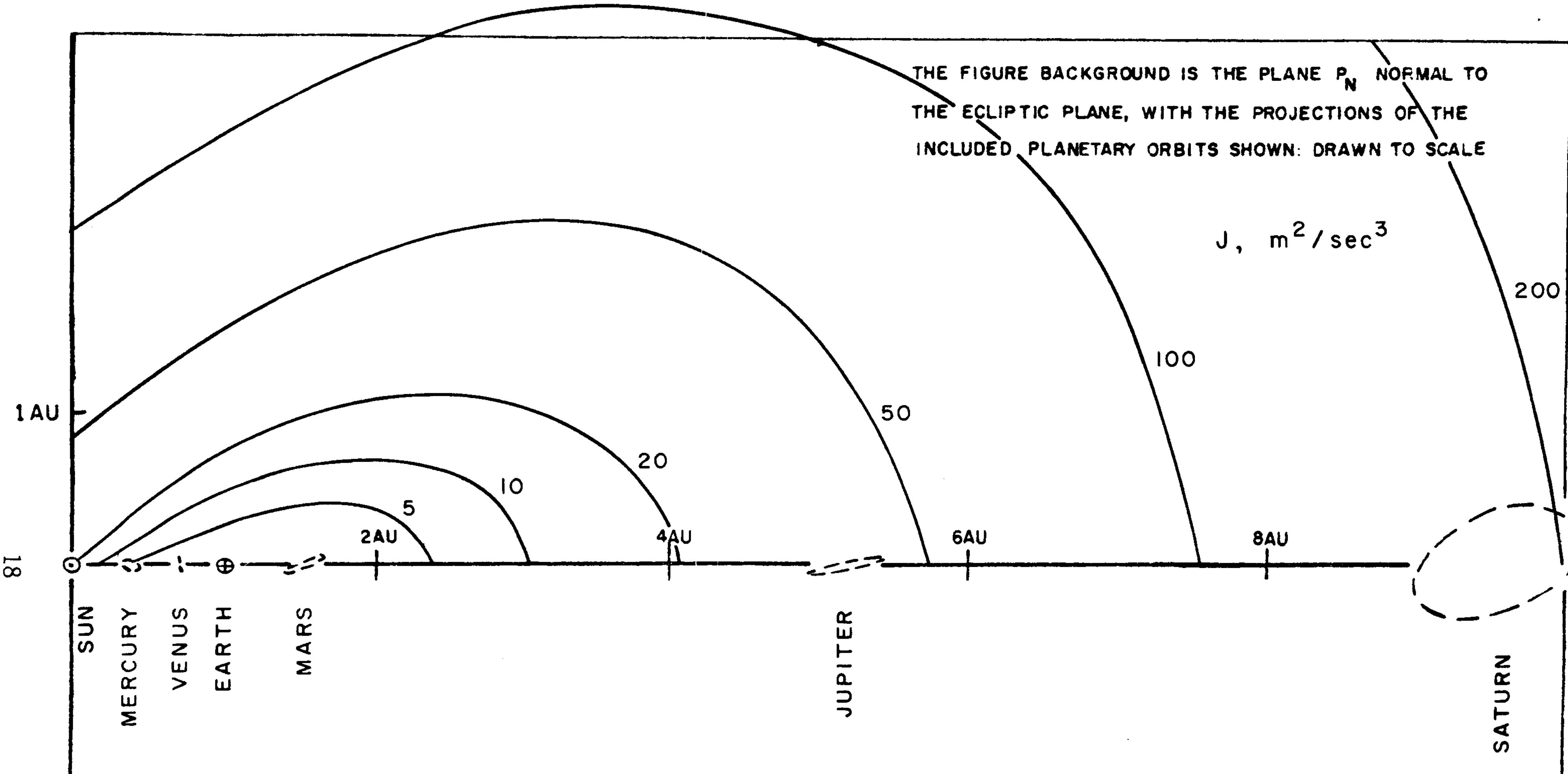


FIGURE 6 J CONTOURS FOR 300 DAY FLY-BY TRAJECTORIES, VARIABLE THRUST MODE

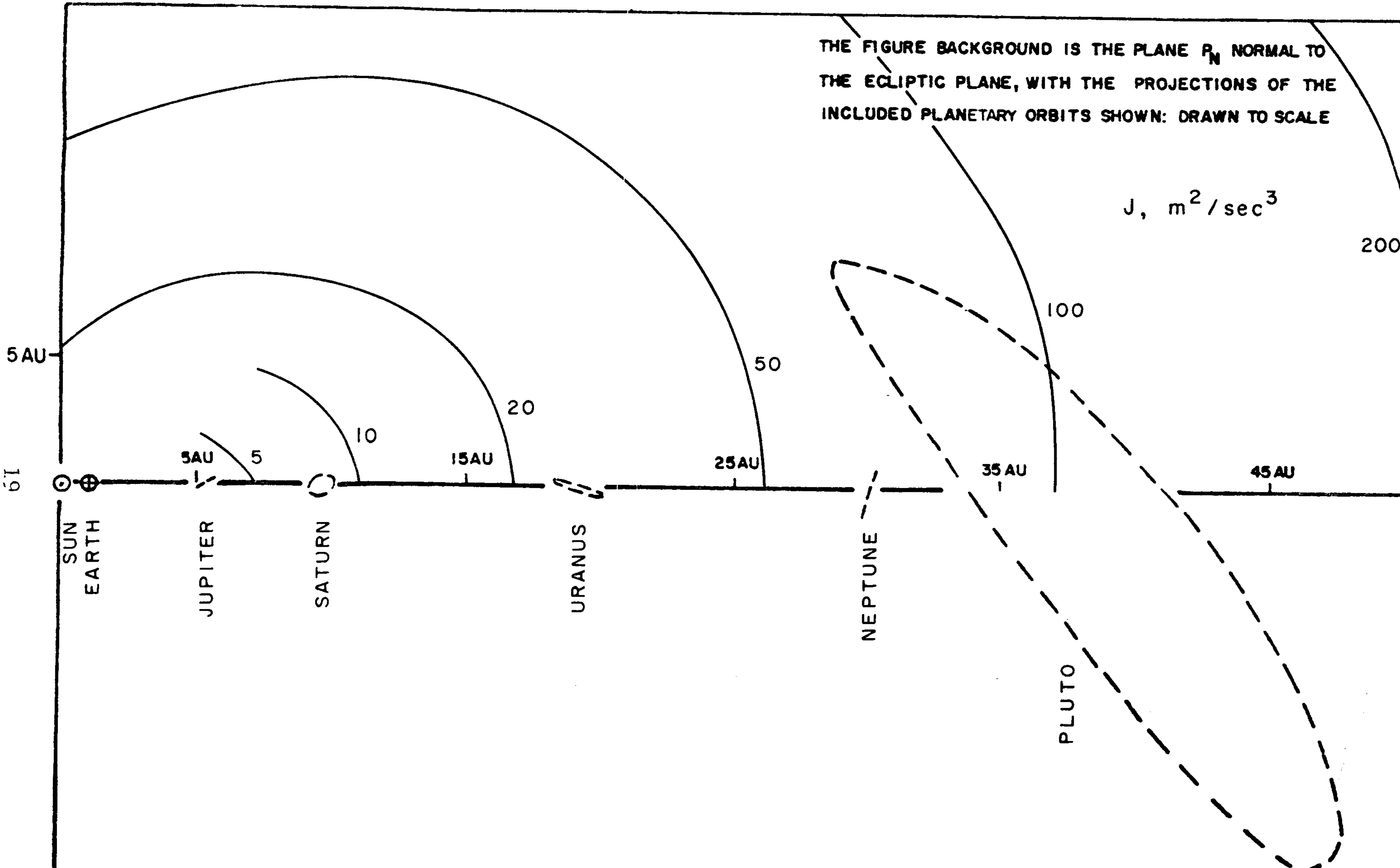


FIGURE 7 J CONTOURS FOR 1000 DAY FLY-BY TRAJECTORIES, VARIABLE THRUST MODE

explored. For this same value of J, the maximum attainable height above the ecliptic plane is about 1.1 AU, and the maximum latitude is about 41°. Increasing J to 50 m^2/sec^3 allows an "over-the-Sun" flight at a distance of 0.8 AU, and in-plane flights slightly beyond the orbit of Jupiter.

Since the projections of the planetary orbits are included in the figure, one can determine the range of J required to intercept a given planet in each synodic period. For fast flights of 100 days, a J capability of 9.5-30 m^2/sec^3 is needed to intercept Mars at any point in its orbit, while Mercury flights can be made with J ranging from about 15 to 30 m^2/sec^3. Venus can always be reached for a J as small as 5 m^2/sec^3. For a rather long flight of 1000 days, fly-by missions to Saturn may be made with a capability of 8-9 m^2/sec^3. However, Pluto can never be reached in this flight time unless J is greater than about 70 m^2/sec^3.

One of the most significant characteristics of the performance contours for thrusted flight is their steep slope at the ecliptic plane for flights beyond Earth. In contrast, ΔV contours for ballistic flight exhibit a rather shallow slope. This characteristic reflects the relative ease of achieving plane changes with continuous thrusting vehicles which is not afforded ballistic vehicles.

Returning to the point of practical limits placed on J values, it is seen from the accessible regions contours that some of the more ambitious missions, particularly to the outer planets, may not be possible within the time of flight assumed. This underlined qualification is an important point since a trade-off between J and T_f is always available. Figure 8 illustrates such a trade-off for ecliptic plane flights to several planetary distances. In general, the longer the time allowed to reach a given target, the smaller the J requirement. One notes, however, that there is a point of diminishing return, that is, the J vs T_f characteristic exhibits a leveling off beyond some flight time. This point generally occurs when the heliocentric travel angle of the trajectory becomes quite large (about 270°). For a J limitation of 40 m^2/sec, it is seen that the minimum flight times to Jupiter and Saturn are about 300 and 490 days, respectively.

4.2 Constant Thrust Mode

The accessible regions contours for the constant thrust mode of propulsion are obtained by an approximation method which is both simple and surprisingly accurate. Although the digital computer program employed in this study is perfectly capable of handling this mode of propulsion in an exact manner, the desire to minimize solution cost led to the consideration of this method which may be termed "characteristic length correlation" (Zola 1964). The approximate J values so derived were checked against the exact values obtained from computer solutions. In general, the comparison showed agreement to within several percent. The basis for this method and the pertinent correlation formulas are presented in Appendix A along with numerical data validating the accuracy of the method.

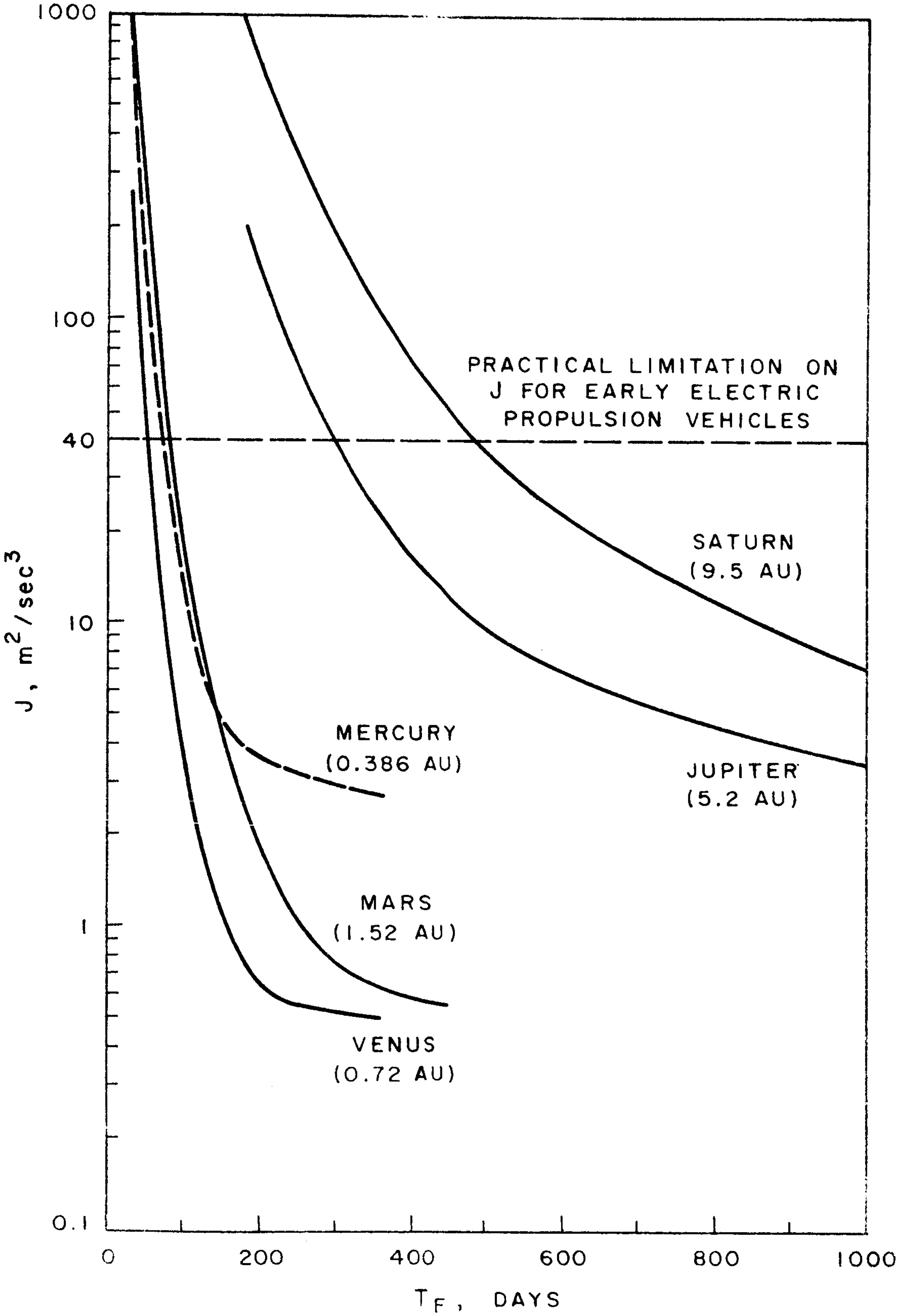


FIGURE 8 EFFECT OF FLIGHT TIME ON J REQUIREMENTS FOR PLANETARY FLY-BY, VARIABLE THRUST MODE

Figures 9 through 11 show accessible regions contours for constant thrust flights of 100, 300, and 1000 days. To facilitate a comparison, the variable thrust contours are also plotted in these figures. For a given flight time, the constant thrust contours depend upon the propulsion system parameters a_o and I_{SP}. The following parameter values are chosen for this example.

T_f, days	a_o, m/sec^2	I_{SP}, sec
100	3×10^{-3}	3000
300	1.5×10^{-3}	6000
1000	0.75×10^{-3}	12000

It should be noted that an optimum combination of a_o and I_{SP} could be found for each mission (T_f, R, β). The choice above represents a reasonable, but not optimum, set of parameters.

The performance loss due to the constant thrust constraint is apparent when one compares the variable and constant thrust contours. This loss is reflected by the smaller region accessible to constant thrust vehicles for a given J capability and flight time. Or, alternatively, the constant thrust vehicle can explore the same region only if the J capability is increased or the flight time extended.

In the case of constant thrust propulsion, there exists a maximum value of J which depends on T_f, a_o, and I_{SP} (see equations (1) and (9)). This theoretical upper limit occurs when thrust is required continuously throughout the flight duration - most constant thrust flights include a coast period

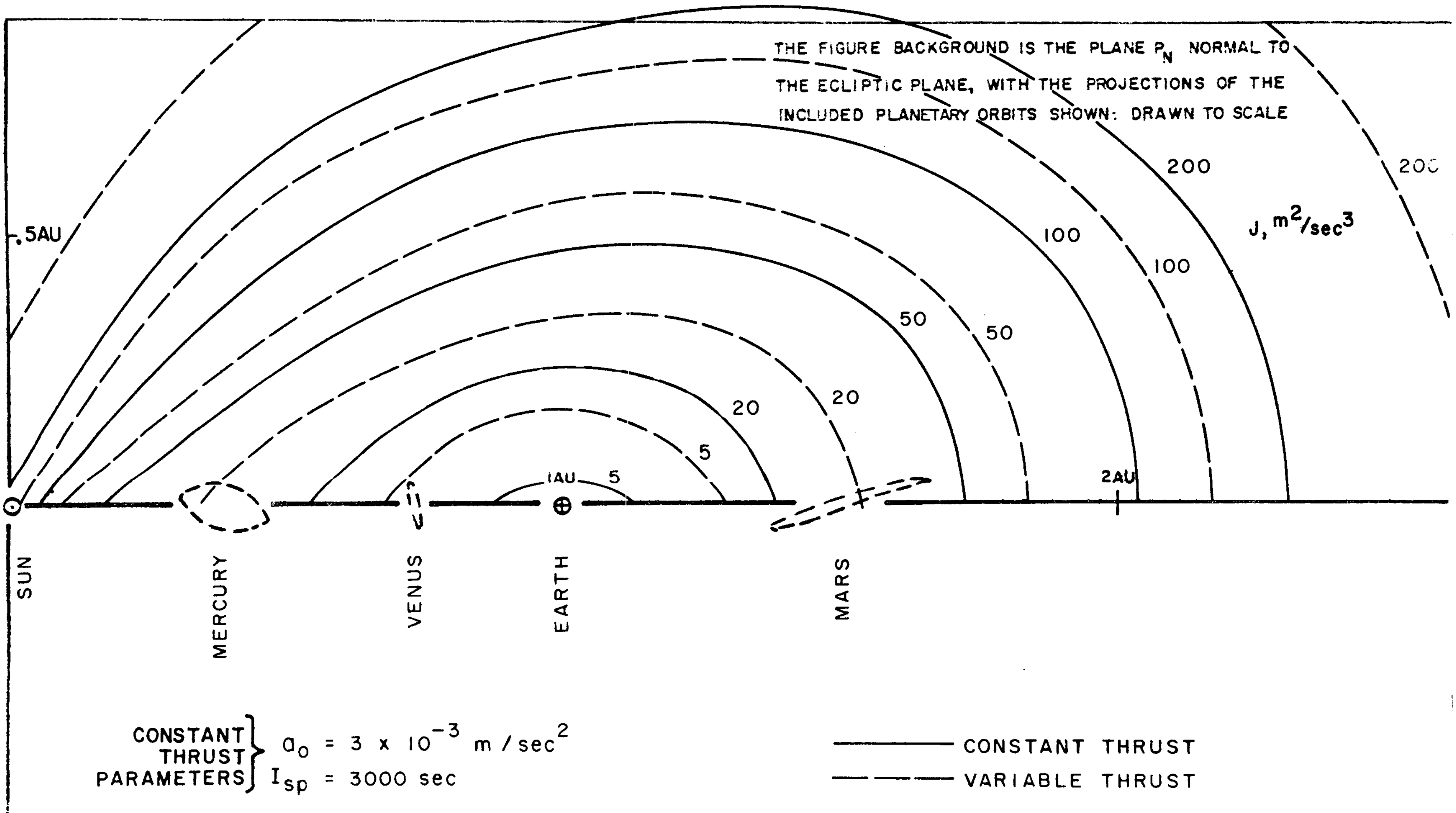


FIGURE 9 COMPARISON OF CONSTANT THRUST AND VARIABLE THRUST J CONTOURS FOR A FLIGHT TIME OF 100 DAYS

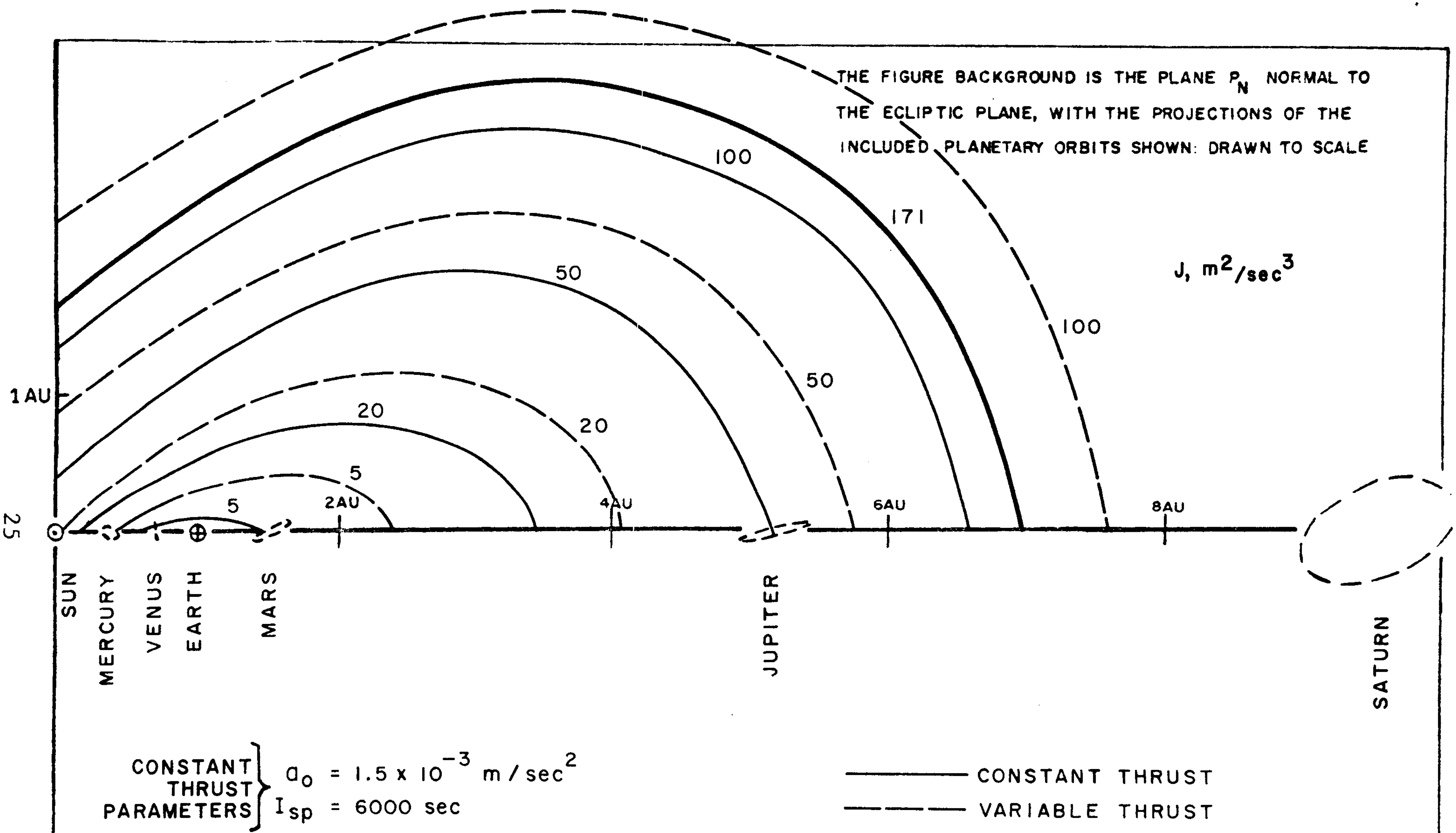


FIGURE 10 COMPARISON OF CONSTANT THRUST AND VARIABLE THRUST J CONTOURS FOR A FLIGHT TIME OF 300 DAYS

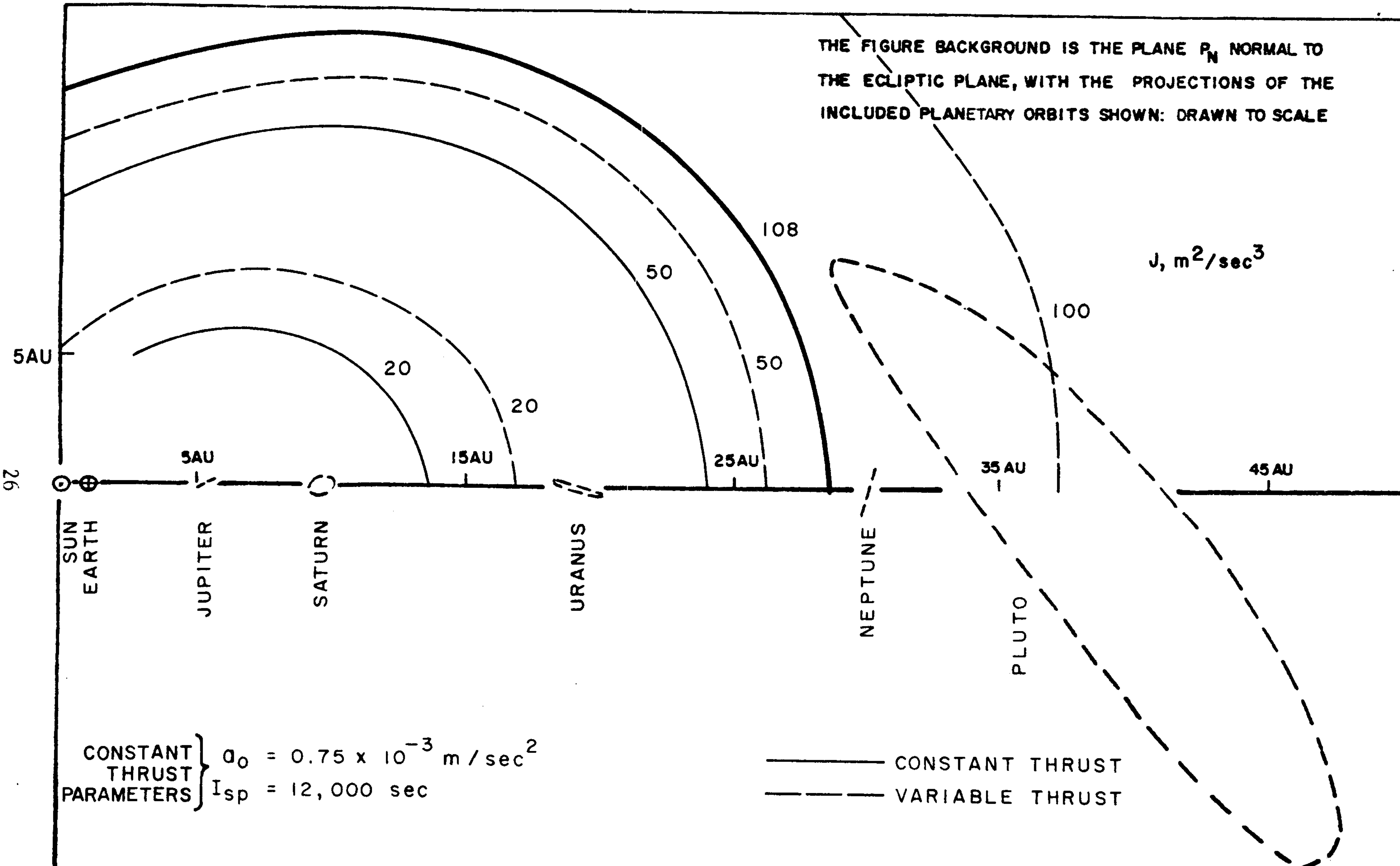


FIGURE 11 COMPARISON OF CONSTANT THRUST AND VARIABLE THRUST J CONTOURS FOR A FLIGHT TIME OF 1000 DAYS

of varying length. For the particular parameters of this example, the maximum values of J are 646, 171, and 108 m^2/sec^3 corresponding to flight times of 100, 300, and 1000 days, respectively. This limitation of J is reflected by the maximum spatial region accessible to a constant thrust vehicle. Thus, for example, Figure 10 shows that 300 day flights with the assumed propulsion parameters cannot reach beyond 6.9 AU. Of course, increasing a_o would increase this limit.

5. PAYLOAD CONTOURS FOR CONCEPTUAL SPACE CRUISER

Section 2 of this report discussed the relationship between trajectory energy requirements, vehicle system parameters, and payload capability. This section will illustrate the accessible regions method in terms of payload. The conceptual nuclear-electric space cruiser discussed in Section 2 will be considered for the purpose of this illustration To reiterate, the basic vehicle system parameters are an initial weight of 20,000 lb. a power plant rating of 500 kw, a power conversion and utilization factor of 0.8, and a power plant weight of 10,000 lbs. (α = 20 lb/kw).

It is assumed that the space cruiser begins its mission from a 200 N. mile circular satellite orbit about Earth. Hence, the space cruiser may be considered as the upper stage of a chemical launch vehicle such as the Saturn 1B. The mission proceeds in two phases: (1) Earth escape phase, and (2) heliocentric transfer phase. Accordingly, the total mission energy requirement in terms of J is given by

$$J = J_E + J_H$$

The method of obtaining J_H has already been discussed. For the purpose of this example, values of J_E are obtained from analytical formulas which give excellent agreement with the values of J_E obtained by numerical integration of the escape trajectory (Melbourne 1961). A tangential, constant thrust program is assumed for the escape phase. For the heliocentric phase of the mission, a constant thrust program with optimum thrust direction and coast period is assumed. The thrust acceleration is matched at the boundary of the escape and heliocentric phases. Each phase is considered a separate "two-body problem" with the Earth and Sun as successive central gravitational bodies.

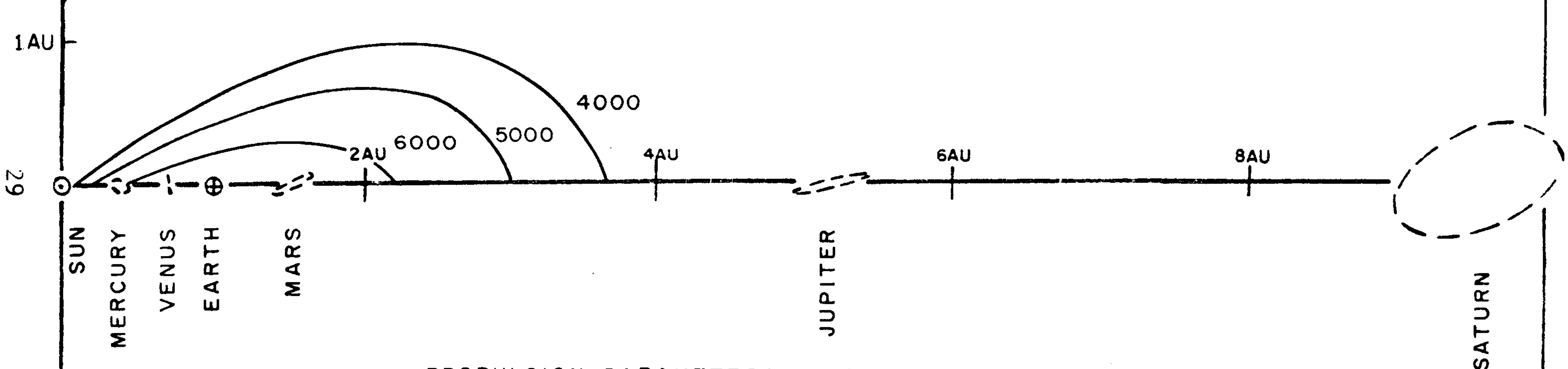
Two heliocentric flight times are considered, namely, 300 and 1000 days. A different specific impulse was chosen for each of these flight times, namely, 6000 and 12000 seconds. The following table lists the initial acceleration, escape time, and J_E which result.

T_H, days	I_{SP}, sec	a_o, m/sec^2	T_E, days	J_E, m^2/sec^3
300	6000	1.5×10^{-3}	52	11.2
1000	12000	0.75×10^{-3}	107	5.5

Figure 12 shows accessible regions payload contours for the 352 day mission. The payload quantity here is the net payload, M_{pl}, delivered to the target, exclusive of the weight

THE FIGURE BACKGROUND IS THE PLANE P_N NORMAL TO THE ECLIPTIC PLANE, WITH THE PROJECTIONS OF THE INCLUDED PLANETARY ORBITS SHOWN: DRAWN TO SCALE

M_{PL}, INCLUDING 3500 lbs OF STRUCTURE AND G + C



PROPULSION PARAMETERS: CONSTANT THRUST MODE

$a_o = 1.5 \times 10^{-3}$ m / sec^2

$I_{sp} = 6000$ sec

VEHICLE PARAMETERS:

$M_o = 20{,}000$ lbs

$P_e = 500$ kw

$\eta = 0.8$

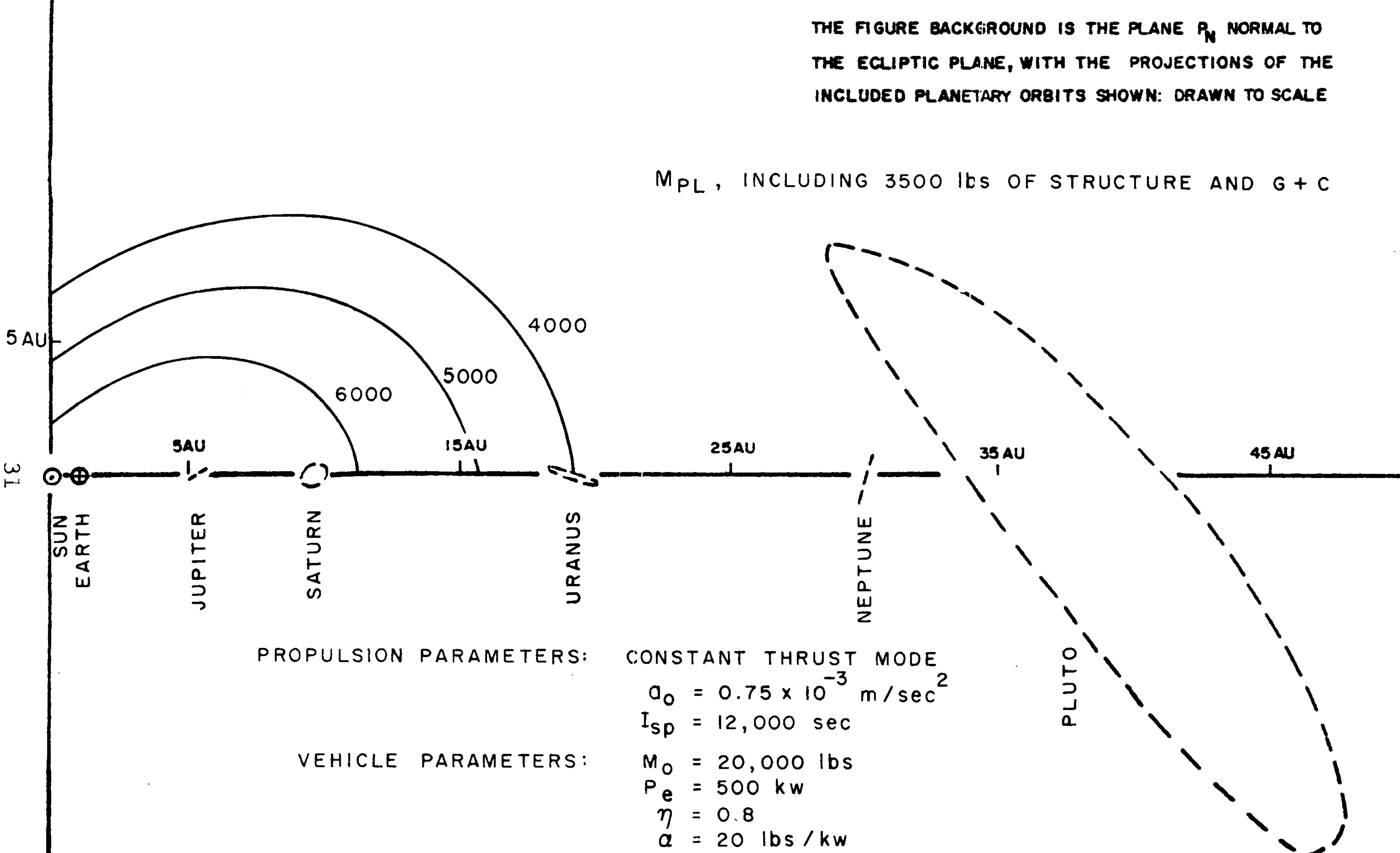
$\alpha = 20$ lbs / kw

FIGURE 12 PAYLOAD CONTOURS FOR CONCEPTUAL SPACE CRUISER WITH A TOTAL FLIGHT TIME OF 352 DAYS of the power plant. This payload figure, then, includes vehicle structure and tankage, guidance and control, communications, and scientific experiments. With a 4000 lb payload, the region of the solar system that may be explored extends from 0.12 AU to 3.6 AU in the ecliptic plane and to 1 AU normal to the ecliptic plane. This region includes the planets Mercury, Venus and Mars. The 6000 lb payload can be delivered to both Venus and Mars, but just short of Mercury.

Figure 13 illustrates the mission-payload capability for 1107 day missions. Here, one sees that a 6000 lb net payload can be delivered beyond Saturn and to about 2 AU "over-the-Sun". 4000 lbs can be delivered to Uranus over about half of its orbit.

Figures 14 and 15 show the effect of power plant mass on the region accessible to a Space Cruiser delivering a 4000 lb payload. It is recalled that the nominal specific mass of the power plant was assumed to be 20 lb/kw. If it were possible to reduce the specific mass to 15 lb/kw, the Jupiter mission could be accomplished in 352 days total trip time. A reduction to 10 lb/kw would extend the 1107 day mission to Neptune and almost to Pluto. If, however, the specific mass of the power plant were increased to 25 lb/kw, the region accessible to the Space Cruiser is significantly reduced.

Although the assumed configuration of the Space Cruiser is considered to be representative of the present or near future state of the art, this configuration is not necessarily optimal.



PROPULSION PARAMETERS: CONSTANT THRUST MODE

$a_o = 0.75 \times 10^{-3}$ m/sec^2

$I_{sp} = 12{,}000$ sec

VEHICLE PARAMETERS:

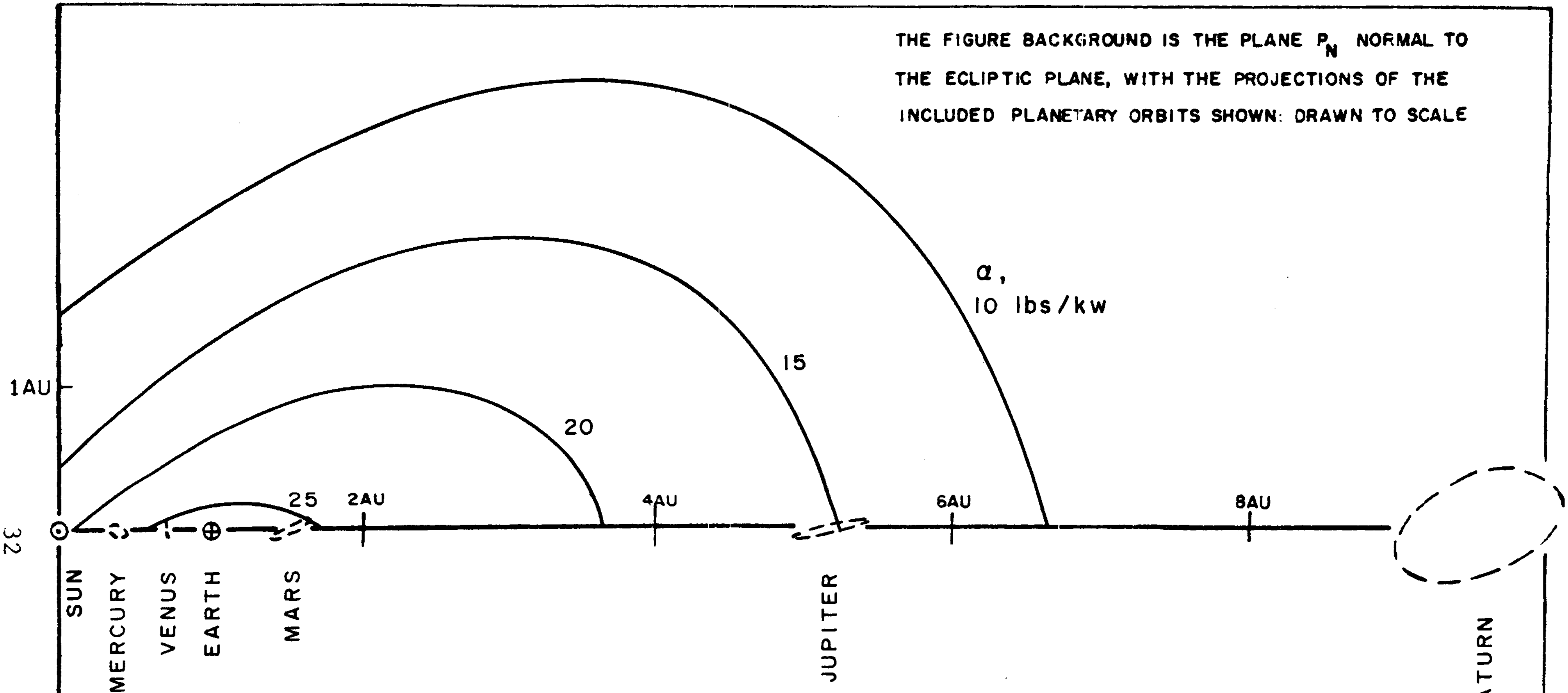
$M_o = 20{,}000$ lbs

$P_e = 500$ kw

$\eta = 0.8$

$\alpha = 20$ lbs/kw

FIGURE 13 PAYLOAD CONTOURS FOR CONCEPTUAL SPACE CRUISER WITH A TOTAL FLIGHT TIME OF 1107 DAYS



PROPULSION PARAMETERS: CONSTANT THRUST MODE

$a_o = 1.5 \times 10^{-3}$ m/sec^2

$I_{sp} = 6000$ sec

VEHICLE PARAMETERS:

$M_o = 20,000$ lbs

$P_e = 500$ kw

$\eta = 0.8$

FIGURE 14 EFFECT OF POWERPLANT SPECIFIC MASS ON REGION ACCESSIBLE TO A 4000 LB NET PAYLOAD IN 352 DAYS TOTAL FLIGHT TIME

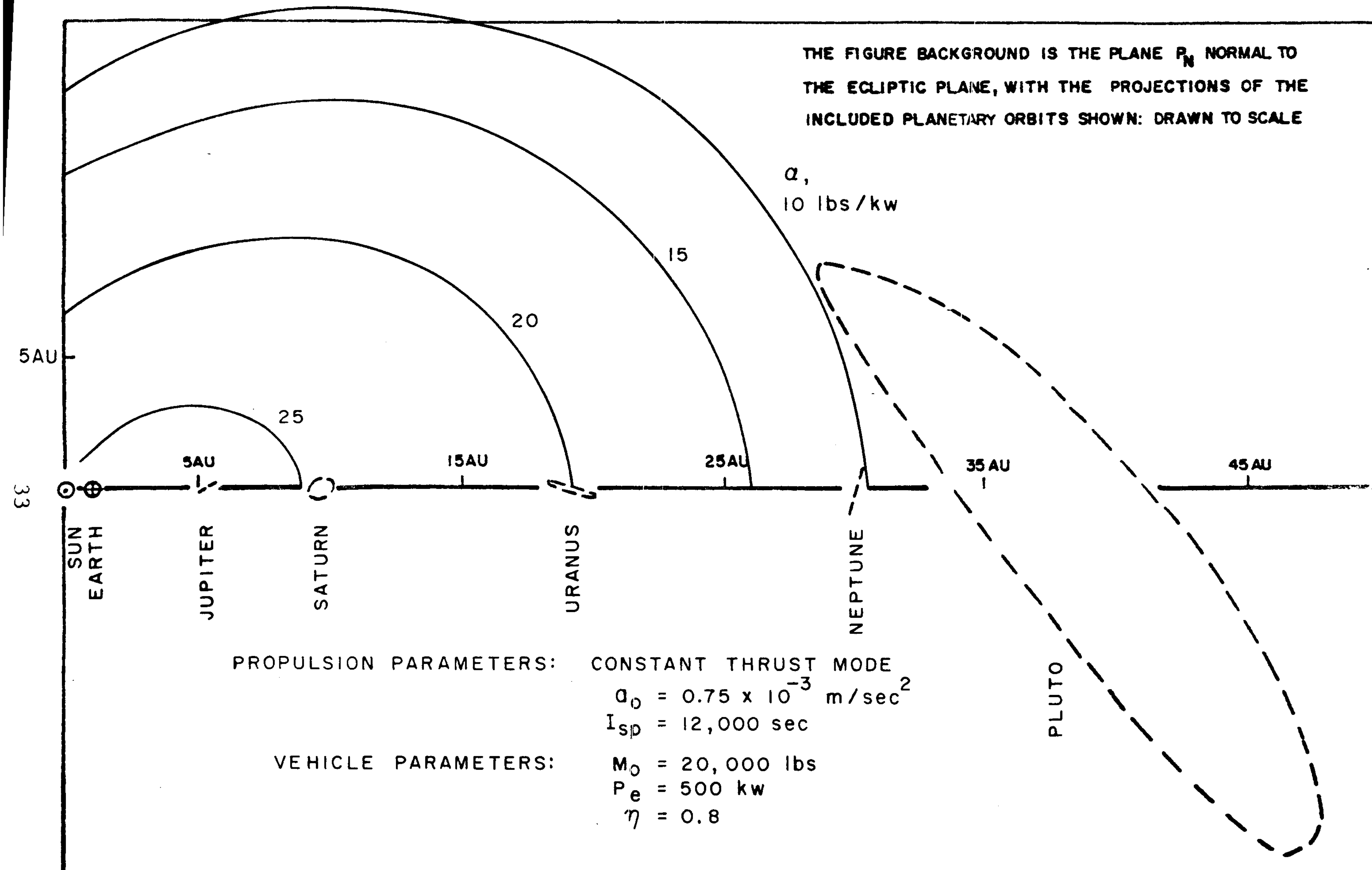


FIGURE 15 EFFECT OF POWERPLANT SPECIFIC MASS ON REGION ACCESSIBLE TO A 4000 LB NET PAYLOAD IN 1107 DAYS TOTAL FLIGHT TIME

For example, no attempt was made here to arrive at the best power level, specific impulse or trip time for any given mission objective. Hence, the above results, although providing useful information, should best be considered exemplary of the method of analysis and graphic presentation which was the prime objective of this report.

Appendix A

AN APPROXIMATION METHOD FOR DETERMINING CONSTANT-THRUST J REQUIREMENTS

The accessible regions contours are obtained from optimum trajectory solutions covering a wide range of target positions. Optimum trajectory analysis is characterized by a mixed two-point boundary value problem in which a set of initial conditions must be found in order to satisfy a set of specified terminal conditions. Since the correct initial conditions are not usually known in advance, numerical iteration procedures are required in order to find the solution. The addition of an iteration requirement to an already complex numerical integration procedure results in a very time-consuming and costly solution process. This situation is difficult for variable thrust solutions and becomes even worse in the case of constant thrust propulsion which involves the consideration of additional parameters. Although the digital computer program employed in this study is perfectly capable of handling this mode of propulsion in an exact manner, the desire to minimize solution cost has led to the consideration of a simple and surprisingly accurate method of approximation.

The use of a simple model to obtain approximate solutions to trajectory problems is certainly not a new idea and, in fact, has been the subject of numerous investigations reported in the literature. For the present study, an approximation method described in a recent paper (Zola 1964) has been adopted. The basis for this method can also be found in an earlier paper (Melbourne 1961).

The key idea of this method, which may be termed "characteristic length correlation", lies in the consideration of a trajectory length parameter which may be used to relate, or correlate, the trajectory energy requirements of various modes of propulsion. The underlying assumption here is that every fixed-time trajectory between two specified terminals has associated with it a characteristic length which is nearly invariant with the particular mode of propulsion employed. For each propulsion mode of interest, the relationship between the performance parameter J, the characteristic length L, and the flight time T_f is derived from a simple trajectory model; a rectilinear flight path in field-free space. Then, if an exact solution of J is available for any one propulsive mode, the values of J for other propulsive modes may be found directly from the correlation formulas.

In the first reference cited above (Zola 1964), the author suggests that ballistic trajectory solutions (infinite impulsive thrust) may serve as the reference mode of propulsion from which thrusted trajectory requirements can be derived. Since ballistic solutions are readily obtained, the computational reduction afforded by this approach is extremely large. In evaluating this approach for selective Earth-Mars flights, Zola found the approximation accuracy to be within 5-10 percent. However, when an attempt was made to apply the ballistic correlation method to the present problem, we quickly found it to be completely inadequate. This was particularly true for out-of-the-plane trajectories and for trajectories having a large heliocentric travel angle. This result was not entirely unexpected since ballistic trajectories are often far too different from low-thrust trajectories to be useful as a reference.

It was decided to use variable thrust trajectory solutions as a source of reference values of characteristic length. The variable thrust J requirements for any particular mission are obtained by exact numerical solutions using the JPL Low-Thrust Trajectory Optimization Code (Richardson 1963). The correlation formulas from which the constant thrust J requirements are obtained are given below. Although not discussed in the text of this report, the constant acceleration mode of propulsion is included here also.

We assume that the variable thrust J requirement for a particular fly-by mission of interest has been obtained. This requirement is denoted as J_{VT}. The characteristic length associated with this mission is then computed from

$$L^2 = \frac{J_{VT}\ T_f^{\ 3}}{3} \tag{A1}$$

Now define a_o as the initial acceleration (at the initiation of the heliocentric transfer) and t_p as the propulsion time ($t_p \leq T_f$). The correlation formulas for constant thrust and constant acceleration propulsion are as follows:

1. Constant Thrust

$$L = V_j \left(\frac{V_j}{a_o} - T_f\right) \ln \left(1 - \frac{a_o}{V_j} t_p\right) + V_j\ t_p \tag{A2}$$

$$J_{CT} = \frac{a_o^{\ 2}\ t_p}{1 - \frac{a_o}{V_j} t_p} \tag{A3}$$

The constraint $V_j/a_o > t_p$ must apply when determining t_p from (A2).

2. Constant Acceleration

$$t_p = T_f - \sqrt{T_f^{\ 2} - \frac{2L}{a_o}} \tag{A4}$$

$$J_{CA} = a_o^{\ 2}\ t_p \tag{A5}$$

where the constraint $a_o \geq \frac{2L}{T_f^{\ 2}}$ must apply. If the initial acceleration can be freely chosen to minimize J_{CA}, then

$$a_o^{\circ} = \frac{9}{4} \frac{L}{T_f^{\ 2}} \tag{A6}$$

$$t_p^{\circ} = 2/3\ T_f \tag{A7}$$

$$J_{CA}^{\circ} = 1.125 \; J_{VT} \tag{A8}$$

In constructing the accessible regions contours for constant thrust propulsion, formulas (A1) through (A3) are worked in reverse order. Thus, for chosen values of J_{CT}, a_o, and V_j, the parameter t_p is first computed, then L, and finally J_{VT}. The contour associated with this value of J_{VT} is then equivalent to the desired J_{CT} contour.

The numerical data presented in Tables A1 and A2 compare results of trajectory solutions predicted by the correlation formulas with the actual results obtained by the JPL Low-Thrust Trajectory Optimization Code. Both constant acceleration and constant thrust modes of propulsion are considered. In the former case, one notes that the predicted values of initial acceleration and propulsion time do not always agree closely with the actual values, however, the deviations combine in such a manner that the critical parameter J* is accurate to within several percent. The accuracy of the correlation formulas is somewhat less in the case of the constant thrust trajectories compared - the worst example shows a 13 percent discrepancy. In general, the approximate and exact results are in close enough agreement to substantiate the usefulness of the correlation formulas.

Table A1

ACCURACY VALIDATION OF APPROXIMATE[1] SOLUTION

OPTIMAL, CONSTANT THRUST ACCELERATION TRAJECTORIES

T_f days	β deg	R AU	a_o^* m/sec^2	a_o m/sec^2	$\frac{t_p^*}{T_f}$	$\frac{t_p}{T_f}$	J^* m^2/sec^3	J m^2/sec^3
300	0	8	2.48×10^{-3}	2.96×10^{-3}	0.667	0.610	135	138
300	0	6	1.71	2.08	0.667	0.591	64.1	66.3
300	0	4	0.998	1.23	0.667	0.567	21.8	22.3
300	0	2	0.350	0.413	0.667	0.596	2.68	2.64
300	0	0.8	0.112	0.124	0.667	0.680	0.278	0.271
300	30	8	3.26	3.33	0.667	0.644	184	185
300	30	6	2.43	2.46	0.667	0.654	102	102
300	30	4	1.69	1.63	0.667	0.700	49.4	48.2
300	30	1	0.961	0.817	0.667	0.890	16.0	15.4
300	30	0.845	0.961	0.810	0.667	0.900	16.0	15.3
300	30	0.694	0.965	0.812	0.667	0.913	16.1	15.6
300	30	0.540	0.989	0.824	0.667	0.922	16.9	16.2
300	30	0.387	1.02	0.855	0.667	0.931	18.0	17.7
300	30	0.234	1.11	0.917	0.667	0.940	21.4	20.5

1 Starred variables refer to solution obtained from correlation formulas.

Table A2

ACCURACY VALIDATION OF APPROXIMATE[1] SOLUTION
CONSTANT THRUST TRAJECTORIES

T_f days	a_o m/sec^2	I_{SP} sec	β deg	R AU	J* m^2/sec^3	J m^2/sec^3
300	2.79×10^{-3}	6000	0	8	168.1	168.5
300	2.79×10^{-3}	6000	0	6	85.9	86.5
300	2.79×10^{-3}	6000	0	4	42.5	40.4
300	1.5×10^{-3}	6000	0	6	72.1	83.2
300	1.5×10^{-3}	6000	0	4	27.4	25.7
300	1.5×10^{-3}	6000	0	2	7.54	7.51
300	1.5×10^{-3}	6000	30	4	54.6	53.1
300	1.5×10^{-3}	6000	30	3	37.4	39.0
100	3×10^{-3}	3000	0	1.7	46.8	44.0
100	3×10^{-3}	3000	0	1.4	20.2	19.0

1 Starred variable refers to solution obtained from correlation formulas.

Appendix B

NOMENCLATURE

a	thrust acceleration, m/sec^2
g_o	Earth surface gravity, 9.806 m/sec^2
I_{SP}	specific impulse, sec
J	integral of a^2 dt, m^2/sec^3
L	characteristic length, m
M	vehicle mass, kg
M_p	propellant mass, kg
M_{pp}	power plant mass, kg
M_{pl}	net payload mass, kg
P_j	kinetic jet (exhaust) power, watts
P_e	electrical power rating, watts
T_f	flight time, sec
t_p	propulsion time, sec
V_j	jet (exhaust) velocity, m/sec
α	specific mass of power plant, kg/watt
η	power conversion and utilization factor

REFERENCES

Beale, R. J., Speiser, E. W. and Womach, J. R. "The Electric Space Cruiser for High-Energy Missions", JPL-TR-32-404, 1963.

Melbourne, W. G. "Interplanetary Trajectories and Payload Capabilities of Advanced Propulsion Vehicles", JPL-TR-32-68, 1961.

Narin, F. "The Accessible Regions Method of Energy and Flight Time Analysis for One-Way Ballistic Interplanetary Missions", ASC/IITRI Report No. T-6, Contract No. NASr-65(06), 1964.

Richardson, D. E. "Optimum Thrust Programs for Power-Limited Propulsion Systems Programmed for the IBM 7090 Computer", JPL-EPD-179, 1963.

Zola, C. L. "Trajectory Methods in Mission Analysis for Low-Thrust Vehicles", NASA Lewis TP-20-63, 1964.