THE UNIVERSITY OF ALABAMA

# SEMI-ANNUAL REPORT

TO THE NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

ON NsG-381 IN SUPPORT OF RESEARCH

IN THE AEROSPACE PHYSICAL SCIENCES



UNIVERSITY, ALABAMA

November 1, 1965

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# A. INTRODUCTION

## 1.0 GENERAL

This report covers the progress during the six-month period March 1 - August 31, 1965 of research and related activities at the University of Alabama made possible through, or assisted by, the National Aeronautics and Space Administration's sustaining Grant NsG-381.

This grant has been in effect since March 1, 1963. Initially, it provided general research support for the University of Alabama Research Institute in Huntsville, Alabama. Support of the Research Institute has been at an annual level of $300,000. Funds available have been used to build staff, purchase equipment, employ consultants, and for administration. A particular and important use of the funds is to support a staff member in the initial phase of a project, thus enabling him to develop a proposal for a contract or grant to support his research. While the major focus of the efforts of the Research Institute has been research (particularly that of interest to NASA), the existence of the Research Institute has been a significant and positive force in the development of a large graduate instructional program in Huntsville. Progress of the efforts at the Research Institute were detailed in four previous semi-annual reports on NsG-381.

Beginning with the summer of 1964, additional general research

support was provided to Main-Campus departments of mathematics, electrical engineering, chemistry, and physics. Progress under this initial support was reported in the fourth semi-annual report.

Funds available have been used to support individual faculty members, assisting graduate students, and for purchase of equipment and supplies in these departments for research projects of interest to NASA and relevant to the departments' instructional programs. The level of support on the Main Campus is $75,000 annually.

## 2.0 ORGANIZATION OF THE REPORT

This report consists of this introductory section; a Main-Campus section, which includes individual progress reports made by each faculty member directing a study; a section on the Research Institute, which includes a description of the status of development of staff and facilities, progress and events in related educational programs, summary descriptions of research conducted under NsG-381, and detailed Research Institute expenditure reports; and a section that is a consolidated expenditure report for both the Main Campus and the Research Institute.

## B. REPORT OF ACTIVITIES ON THE MAIN CAMPUS

### 1.0 GENERAL

During this reporting period, ongoing studies, which were previously reported on, in the fields of chemistry, mathematics, and electrical engineering, were continued. Two new studies in the field of physics were initiated by Professor Bartlett, the restricted 3-body problem and the structure and behavior of thin (passive) films.

One study, the analysis of combined nonlinearities conducted by Professor Lueg, progressed sufficiently to permit Professor Lueg to submit a proposal for specific project support for his work. In the event that the proposal is accepted, Professor Lueg will complete his work under NsG Grant 381 during December, 1965 or January, 1966, thus permitting a new investigator to be supported.

The sections which immediately follow contain progress reports by project directors on the ongoing and two new studies, as well as descriptions of the new studies which were begun in September, 1965, one in electrical engineering (solid state electronics) and the other in mathematics (techniques of solutions of boundary-value problems for differential equations).

It should be noted that the expenditures for the Main Campus are shown in a consolidated expenditure report in Section D of this report.

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2.1 Progress Report on Chemistry, Dr. Donald F. Smith

Progress Report on Research in Determination of Thermodynamic Functions of Inorganic Solids and the Use of Fused Salts as a Medium for the Preparation of Some Hygroscopic Inorganic Materials Under NASA Grant NsG-381 for period March 1--August 31, 1965

Earlier work concerning the synthesis of hygroscopic inorganic chlorides was concerned mainly with chlorinations of oxides with aluminum chloride to produce the corresponding chlorides. The solvent media for the reactions were fused salt eutectric mixtures: LiCl-NaCl-KCl (m. p. 320°C) and $AlCl_3$-NaCl (m. p. 120°C).

More recent work has been devoted to chlorinations in which the element whose chloride is being sought is contained in the anion of the starting material. In the reactions to be tabulated below the $AlCl_3$-NaCl eutectic was used. It possesses two advantages over the use of the LiCl-NaCl-KCl eutectic: It melts at a much lower temperature and does not contain the very hygroscopic lithium chloride.

1. $PbSnO_4 + AlCl_3 - NaCl \rightarrow SnCl_4$
2. $CaTiO_3 + AlCl_3 - NaCl \rightarrow TiCl_4$
3. $CaSO_4 + PCl_5 + AlCl_3 - NaCl \rightarrow POCl_3 + SOCl_2$
4. $Na_2SeO_4 + PCl_5 + AlCl_3 - NaCl \rightarrow$ No $SeOCl_2$
5. $Na_2SeO_3 + PCl_5 + AlCl_3 - NaCl \rightarrow$ No $SeOCl_2$

Chlorinations similar to reaction three are known in which sulfuric acid is reacted with $PCl_5$ to produce $POCl_3$ and $SOCl_2$ or

$SO_2Cl_2$, depending upon the amount of $PCl_5$ used. The use of a solid sulfate eliminates moisture contamination that would be present in sulfuric acid. Sodium sulfate has previously been chlorinated in a similar manner to produce the same products.

Since selenium is just below sulfur in the periodic chart, it was logical to see if a selenate would chlorinate to produce the correspondingly similar $SeOCl_2$. However, as indicated in equation four, no $SeOCl_2$ was formed. The last equation, with a selenite as a starting material, also yields no $SeOCl_2$. Sulfites can be chlorinated with $PCl_5$ to produce $SOCl_2$.

A second copper block calorimeter has been constructed and a Pt-Rh round furnace was purchased. This setup will enable us to extend our temperature range, i.e., 1,000 to 1,750°K in which range the heat capacities of $Ca(ClO_3)$, $Ca(BrO_3)_2$, $Ca(IO_3)_3$, $Ba(ClO_3)_2$, $Ba(BrO_3)_2$, $Ba(IO_3)_2$, $BaI_2$, $SrBr_2$, $Sr(BrO_3)_2$, $Sr(IO_3)_3$, and $SrF_2$, $SrI_2$ will be measured. The calorimeter constructed earlier will be used to measure their respective heat capacities from 298°K to 1,000°K.

The temperature rise in the copper block will be measured by a platinum resistance thermometer recently calibrated by the Bureau of Standard. Measurement will be made in connection with a Type G-2 Leeds and Northrup recording Mueller Bridge. The temperature of the furnace will be controlled by a Honeywell electro-o-volt control unit installed this summer.

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2.2 Progress Report on Electrical Engineering, Dr. C. A. Gibson

Progress Report on Implementation of Time-Optimal Control of Plants Characterized by Predominant Coulomb Friction and by Predominant Viscous Friction under NASA Grant NsG-381 for period of March 1--August 31, 1965

Work on bang-bang control systems has been reported by a number of researchers, e.g. [1] - [4] , and Pontryagin's Maximum Principle [5] also has been applied to this problem in optimization. The Maximum Principle uses the costate to give general indications of the properties of the control signal required to yield optimal performance.

The purpose of this investigation was to apply the Maximum Principle to specific control systems, obtain information as to the nature of the time-optimal control signal, and then implement this control strategy by practical and reliable means. Work on the implementation is being carried out at the present time.

In small DC control motors brush friction usually overshadows the other friction forces. This brush friction opposes the velocity but is fairly independent of the magnitude of the velocity, thus it may be classified as coulomb friction. An analysis of a system characterized by coulomb friction is carried out below.

A constant armature current field-controlled DC motor has the following equation of motion when the field time constant is negligible in comparison to the mechanical times involved:

$$\ddot{\theta}(t) = \frac{K}{J} v_f(t) - \frac{D}{J} \frac{\dot{\theta}(t)}{|\dot{\theta}(t)|} \qquad (1)$$

where $\theta$ is the angular position of the shaft (and $\dot{\theta} \triangleq d\theta/dt$, etc.), J is the moment of inertia, D is the magnitude of the friction force, $v_f$ is the field voltage (control signal), and K is a motor constant. It is desired to find the $v_f$ that will move the motor in the minimum possible time from some initial state of position and velocity to a new state having a different position and zero velocity.

Making the substitutions A = K/J and B = D/J and defining the state variables

$$x_1(t) \triangleq \theta(t)$$

$$x_2(t) \triangleq \dot{\theta}(t)$$

Equation (1) can be written in the form

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = A\, v_f(t) - B \frac{x_2(t)}{|x_2(t)|} \qquad (2)$$

A constraint is placed on the control signal such that $|v_f(t)| \leq M$. The cost functional is given by

$$S = \int_0^T 1 \cdot dt \qquad (3)$$

where T is the control interval and is, of course, not fixed beforehand. The cost functional may be included as an additional state variable $x_3(t)$ and the state Equations (2) become

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = A\, v_f(t) - B\, x_2(t)/|x_2(t)| \qquad (4)$$

$$\dot{x}_3(t) = 1$$

The Hamiltonian H for equation (4) is

$$H = \sum_{i=1}^{3} p_i(t)\dot{x}_i(t) = p_1(t)x_2(t) + p_2(t)\left[A v_f(t) - B \frac{x_2(t)}{|x_2(t)|}\right] + p_3(t) \tag{5}$$

where the $p_i(t)$ are the costate variables, which are determined from the differential relations

$$\dot{p}_i(t) = - \frac{\partial H}{\partial x_i(t)} , \qquad i = 1,2,3 \tag{6}$$

According to the Maximum Principle, the necessary condition that the cost functional be minimized is that H be maximized with respect to $v_f(t)$ for all time in the control interval. From Equation (5) it is seen that only one term involves $v_f(t)$. Thus, it is evident that H is maximized when the product $p_2(t)\, A v_f(t)$ is at its maximum value, which means that

$$v_f(t) = M \text{ sgn } \left[p_2(t)\right] \tag{7}$$

Equation (7) shows that if $p_2(t)$ were known the time-optimal control signal would be known as a function of time.

For the system under study Equation (6) yields

$$\begin{aligned} \dot{p}_1(t) &= 0 \\ \dot{p}_2(t) &= - p_1(t) + p_2(t)\, B \frac{\partial(\text{sgn } [x_2(t)])}{\partial x_2(t)} \\ \dot{p}_3(t) &= 0 \end{aligned} \tag{8}$$

where $\text{sgn } [x_2(t)] \triangleq x_2(t)/|x_2(t)|$ . It is immediately evident that both $p_1(t)$ and $p_3(t)$ are constants. Let $p_1(t) = \lambda_1$. The second term on the right hand side of the equation of $p_2(t)$ is zero except at $x_2(t) = 0$ since $\text{sgn } [x_2(t)]$ is either +1 or -1 with a point of discontinuity

at $x_2(t) = 0$. If there is no change in sign of $x_2(t)$ during the control interval, the equation of $p_2(t)$ is

$$p_2(t) = -\lambda_1 t + \lambda_2 \tag{9}$$

where $\lambda_2$ is a constant. However, if the initial conditions are such that $x_2(t)$ will change sign as the system moves to the final state, it will be necessary to break the solution for $p_2(t)$ into two parts with boundary conditions determined from the initial plant state, the zero velocity state, and the final plant state. It is found that from any initial plant state there will be at most one change in sign of $p_2(t)$. Thus, the optimal control strategy requires that $v_f(t)$ be at its maximum magnitude (M) at all times and that there be at most only one reversal of $v_f(t)$.

With this condition in mind, this second order system was simulated on the Analog computer and the phase plane plots were obtained and are shown in Figure 1. From this plot the switching boundary AOB was easily determined. An object of this study was to arrive at an equation by which the instant of switching could be determined. This instant will definitely be the function of the system initial conditions. A set of equations were derived to that effect. For example, the following equation was derived by which the elapsed time can be determined when the system initial conditions are such that we are in the first quadrant of the phase plane.

$$t_s = \frac{x_2(0)}{AM + B} + \frac{\sqrt{x_2^2(0) + 2(AM + B)\, x_1(0)}}{2B} \tag{10}$$

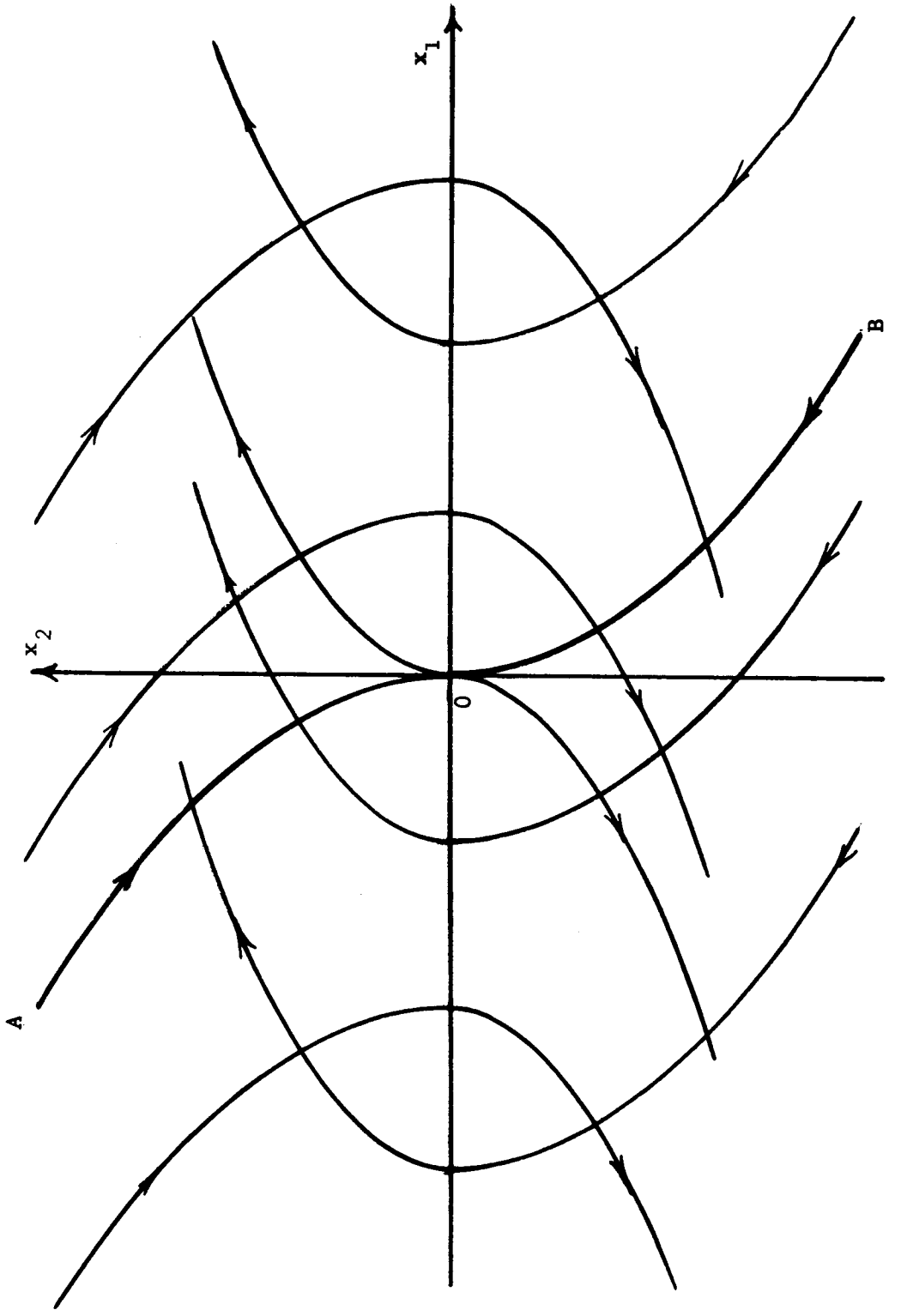


Figure 1. Phase Plane Plot for System Having Coulomb Friction

In order to effect the optimal control it is necessary to monitor the plant state and compare the state to the switching boundary.

A similar analysis was made for an AC control motor, which was described as a second-order system having viscous friction. The time-optimal control of the AC system has the same general properties as that for the DC motor. The control winding voltage should be at its largest allowable magnitude at all time in order to move the motor from one rest state to a new rest state in the minimum possible time. Further, only one phase reversal of the control winding voltage is required.

Work is being done to develop a simple time-optimal controller for the AC control motor. A block diagram of the system is shown in Figure 2.

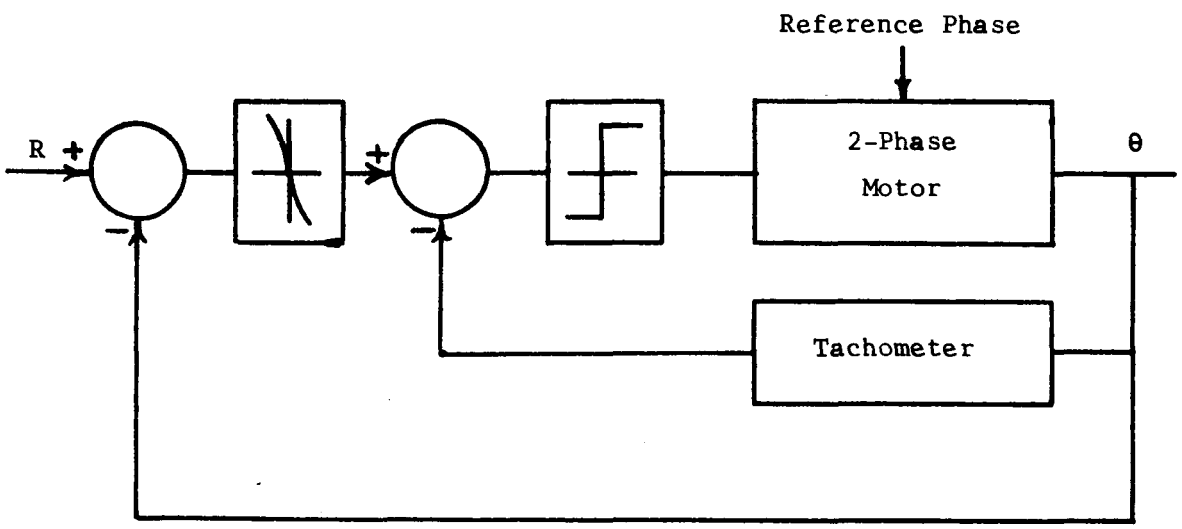


Figure 2. Block diagram of the Time-optimal Control System

Power is supplied to the control motor from a two phase source. The reference phase is connected directly to one phase of the source and the control phase is excited by the second phase. Power to the control phase is switched by gate controlled AC switches so that a 180 degree phase

reversal in the control phase may be obtained.

The nonlinear switching boundary is simulated by a network of resistors, diodes and zeener diodes. The same general control scheme can be used for systems with widely divergent characteristics, but it is necessary to tailor the simulated switching boundary to the particular system.

A modified flip-flop has been tested as a driver for the gates of the AC switches. The circuit operates with the gate in series with the flip-flop transistors. Due to the cutoff time of the switches there has been difficulty in insuring that only one AC switch is on at any given time. Variations in the design of the gate driver are being carried out to achieve reliable operation.

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## 2.3 Progress Report on Electrical Engineering, Dr. R. E. Lueg

Progress Report on Research in Analysis of Control Systems Containing Multiple Nonlinearities under NASA Grant NsG-381 for period March 1--August 31, 1965

During the course of study under NASA Grant NsG-381, many possible avenues of investigation were uncovered, but most of the problems encountered seemed too specialized or limited in application to be of any real interest. One area which did seem to have much merit was the problem concerning control systems containing multiple nonlinearities. In an effort to become familiar with control systems with multiple nonlinearities, a particular problem was simulated on the analog computer. Although this particular problem was solved in considerable detail, many more problems need to be simulated to complete the desired investigation.

The primary purpose of this project was to develop analytical techniques which would be useful to the control designer who dealt with systems containing multiple nonlinear elements. It was felt that the describing function analysis method would be most useful to the engineer since it is a relatively easy and straightforward technique to apply. It was anticipated that certain standard criteria or curves would be developed as an engineering aid in analyzing systems with multiple nonlinearities; and, although progress was made in this area, much remains to be done.

The research supported by NsG-381 has enabled the author to prepare a proposal to another agency for support of an investigation of how best to handle multiple nonlinearities. If the proposed investigation is supported, the work under it, together with preliminary work accomplished under NsG-381, should result in significantly useful material for the servo designer. In addition, a graduate student should complete his dissertation requirement for the Ph. D. in this area.

## The Problem

It might be appropriately stated that all servo systems contain multiple nonlinearities even though many servo problems are solved as though the entire systems were linear or contained only one dominant nonlinearity. The linearization of a predominately nonlinear problem is often accomplished by employing the very powerful mathematical technique of perturbation or the method of small motions. The describing function technique which is used in this report is essentially a method which linearizes the nonlinear element by substituting a real frequency linear transfer function for the nonlinear element. The problem analysis is then carried out with the aid of Nyquist, Bode, or Nichols plots.

Consider the basic diagram shown in Figure 1. The linear portion of the system is designated G and the nonlinear N. The usual assumptions which are made to justify the use of the describing function method are:

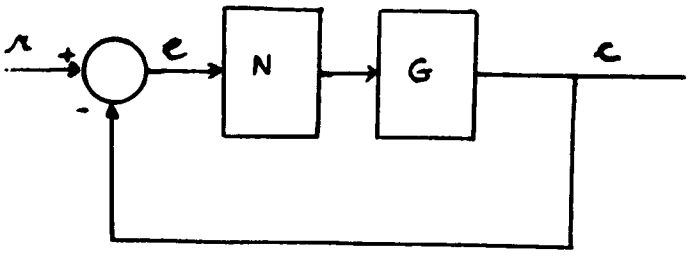


Figure 1. Basic Nonlinear System

1. One nonlinear element in the system which can be frequency, $\omega$, and/or amplitude, $a$, dependent
2. Nonlinear element characteristics are time invariant
3. G is a sufficiently good low pass filter to attenuate all of the harmonics generated by N
4. N generates no subharmonics

Under these assumptions, which are valid for many practical problems, the problem analysis is carried out by solving the characteristic equation $1 + GN = 0$ using any of the real-frequency domain plots. This is accomplished by plotting $G(j\omega)$ and $-\frac{1}{N(a, j\omega)}$ and noting the intersection, if any, of the resultant curves. If the curves intersect, then possible sustained oscillations (limit cycles) occur in the system performance; furthermore, indication of the over-all transient time-domain performance can also be deduced from these plots.

As indicated above, the nonlinear quantity can be both a function of the signal amplitude and the frequency. Many nonlinear elements yield only amplitude dependent describing functions which make the analysis considerably simpler than if the describing function is also frequency dependent.

In general, if the system contains multiple nonlinearities, two different analysis situations can occur as depicted in Figure 2, which shows two nonlinear elements either together or separated by a linear function. For the situation depicted in Figure 2a, it is best to combine both $N_1$ and $N_2$ into one nonlinear function before deriving the describing function. If the describing functions for $N_1$ and $N_2$ are computed individually and the system performance then analyzed, considerable error can usually be expected in the result. For the situation shown in Figure 2b, one has several choices in performing the analysis. If $G_1$ and $G_2$ are reasonably good low pass filters, then it is possible to derive the describing functions for the individual nonlinearities $N_1$ and $N_2$ and carry out the analysis by writing the characteristic equation in the form $G_1(j\omega)G_2(j\omega) = -\frac{1}{N_1(\alpha_1)N_2(\alpha_2)}$ and plotting $G_1 \cdot G_2$ and $-\frac{1}{N_1 \cdot N_2}$. The relation between the amplitude factors $\alpha_1$ and $\alpha_2$ can be easily determined if the nonlinearities are assumed to be piecewise linear. Like the basic system the intersection of these two curves indicates limit cycle oscillation.

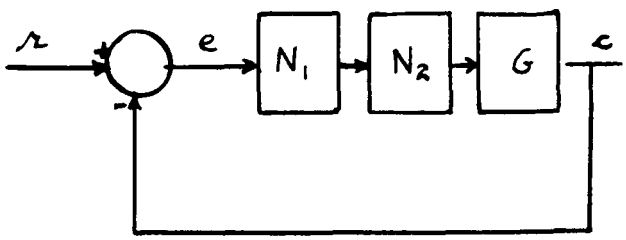


Fig. 2a. Two adjacent nonlinearities.

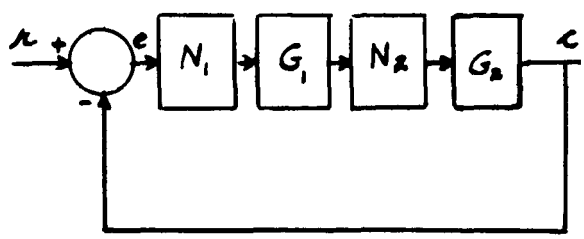


Fig. 2b. Two nonlinearities separated by a linear function.

Another approach is to combine the nonlinear and linear terms in an appropriate fashion. For instance, suppose $G_1$ in Figure 2b gives insufficient filtering for $N_1$ to be considered isolated from $N_2$. In this case $N_1$, $G_1$, and $N_2$ could be combined as an amplitude and frequency dependent describing function and $G_2(j\omega)$ and $-\frac{1}{N_1(a_1)G_1(j\omega)N_2(a_2)}$ could be plotted and examined for possible intersection. Since it is not desirable to have to work with frequency as well as amplitude dependent nonlinearities, it is worthwhile for the designer to have some figure or figures of merit to judge when and how to best combine multiple nonlinearities in systems.

An analog computer simulation of a typical servo valve problem, which is shown in block diagram form in Figure 3, illustrates a system with multiple nonlinearities. The first block in the forward loop represents the transfer function of an active circuit corrective network, the second and third blocks represent the servo valve amplifier which has sharp saturation characteristics, the fourth block represents the servo valve dynamics, the fifth block the valve orifice flow saturation, the sixth valve hysteresis, and the sixth the integration effect

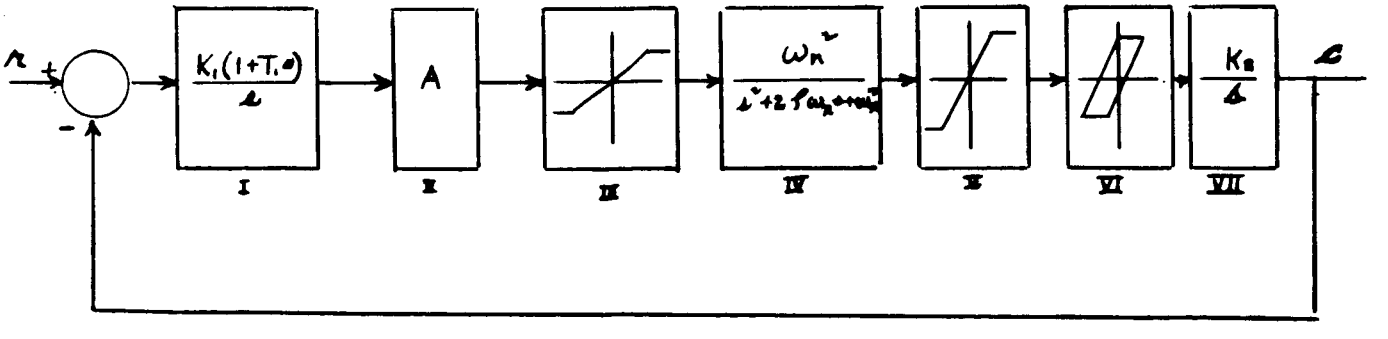


Figure 3. Block Diagram of a Servo Valve.

of the valve piston. Unity feedback is assumed. The computer results showed that neither the amplifier nor the valve flow saturations affected the limit cycle and that only the valve hysteresis contributed to the "small motion" limit cycles. Of course, both the amplifier and valve saturations did affect the initial transient "large motion" performance.

Although a great amount of information was obtained from the analog computer runs, the fact that the limit cycles were not a function of the two saturations did not make this particular problem a good example to explain the effects of multiple nonlinearities. A describing function analysis using hysteresis as the only nonlinearity in the system yielded answers which showed close correlation to those given by the computer. Since the computer runs and the describing function analysis yielded no new or startling information, the results are not included as a part of this report.

During the latter part of the study, several interesting problem areas developed but none have been brought to a complete solution. As mentioned earlier in this report, these problem areas and others are planned to be studied further under a proposed project. Briefly, the central problem is to consider two nonlinearities separated by a linear transfer function of the form

$$G(s) = \frac{K(1+T_1 s)\,(1+T_2 s)\,\text{---}\,(1+T_m s)}{s^N(1+T_a s)\,(1+T_b s)\,\text{--}\,(1+T_n s)}$$

Several different combinations of nonlinearities along with several types of linear transfer functions are currently being studied. Criteria

are being developed to ascertain the relation of the linear parameters of G(s) to the nonlinear parameters of the two separated nonlinearities. The various linear time constants and the order of the denominator to the numerator polynominal of G(s) will be compared to the nonlinear characteristics such as backlash width, slope changes, etc. The net result should be a set of criteria whereby the engineer can better judge when two or more nonlinearities can indeed be considered sufficiently isolated so that independent describing functions can be used instead of deriving an over-all describing function for the combination of linear and nonlinear elements.

## Conclusions

The development of criteria which will aid the designer in analyzing systems containing multiple nonlinearities is still in progress. If the proposal to another agency (mentioned earlier) is accepted, this work will be continued for that agency. Analog computer runs and describing function analyses in some systems with multiple nonlinearities have been completed, but as yet no general patterns or criteria have appeared as being significant enough to be included as a part of this report.

The author wishes to express his gratitude for the support under NsG-381 and fully expects to see the initial work performed under auspices of this grant bear fruit in the future.

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## 2.4 Description of New Research in Electrical Engineering, Dr. W. E. Webb

New Research in Solid State Electronics under
NASA Grant NsG-381, initiated September, 1965

It is well known that the application of homogeneous mechanical pressure affects the electrical properties of semiconductors, and the pressure dependence of the conductivity of many common semiconductors has been investigated. Less is known, however, about the effect of pressure on semiconductor junction devices. Previous studies of the pressure dependence of semiconductor characteristics have yielded valuable information concerning the energy band structure of these materials. It is expected that similar investigations of junctions will lead to a better understanding of the basic phenomenon underlying the operation of these devices. Recent work indicates that these effects may be of considerable practical, as well as theoretical, importance. It has been suggested that mechanical strain may alter the energy level structure of a junction device sufficiently to cause electrical failure and that this strain induced failure may be one of the principal factors limiting the lifetime of solid state electronic devices. Other possible applications are acoustoelectric amplifiers and junction type pressure or acoustic transducers. There also exists the possibility of producing an acoustically modulated laser.

### Research Study

A program of basic research on the effect of mechanical pressure

on semiconductor junctions will be undertaken. The initial phase of this research will consist of the fabrication of experimental junctions and measurement of the pressure dependence of their current-voltage relations, junction capacitance, charge storage, and recovery time. It is anticipated that a number of semiconductors will be used for these junctions and that both conventional and Easki type junctions will be studied. Of particular interest will be junctions in which one or both semiconductors are also piezoelectric. Initially, inhomogeneous pressure will be used.

The research outlined above will later be extended to homogeneous pressure and to the effect of acoustical waves on the junction. Since the proposed research is to be a continuing program of basic research on the mechanisms of solid state junction devices, it is presumed that the project director will be allowed considerable discretion as to the exact materials to be studied and experiments to be performed.

The Electrical Engineering Department has recently established a Solid State Electronics Laboratory. Approximately $23,000 of University funds has been spent, and an additional $15,000 has been budgeted for equipment and furniture. Also, a proposal is being submitted to another agency for $15,000 for equipment. This laboratory now has facilities for semiconductor material preparation and junction fabrication, and the basic electronic equipment needed to perform the first phase of the proposed research.

University of Alabama
University, Alabama
November 1, 1965

## 2.5 Progress Report on Mathematics, Dr. R. L. Plunkett

Progress Report on Optimization Techniques as Applied to Space Problems under NASA Grant NsG-381 for period Match 1--August 31, 1965

The following progress on subject study was made during this reporting period:

1. A detailed exposition of the proof of the "multiplier rule," as given in Bliss, "Lectures on the Calculus of Variations," was prepared and presented to a seminar over a period of several weeks. Though the multiplier rule is a foundation stone for almost all space trajectory analysis, few practitioners have ever been through the very difficult proof. For this reason, the exposition done is being prepared as an expository paper. No decision has been made on how widely it should be disseminated.

2. A paper entitled "The Function Concept" has been prepared and will be submitted for publication. This is also expository in nature and will be suitable for publication only in a journal concerned with the teaching of mathematics. The paper consists of an amplification of the content of a talk presented to the NSF High School Teacher Institute during the summer.

3. Detailed contributions were made to the supervision of an interesting thesis (written by R. E. Burns of NASA, Marshall) concerned with finding an analytic solution to guidance for an optimal fixed end-point space trajectory for the case of a throttleable engine.

4. The work of Russian mathematicians ("Pontrjagin Maximum Principle") and applications to the problems of optimal control theory are under investigation in conjunction with a graduate student who hopes to write a dissertation in this area. This field is very disorganized and, consequently, requires slow and careful study.

5. Study has begun on the contributions of algebraic topology to calculus of variations. This new approach has so far produced only the most elementary results but is of such power as to lead one to expect that more can be obtained from it.

6. Incidental progress has been made as follows:

(a) A topological problem from the American Mathematical Monthly was solved and the solution submitted to the journal.

(b) An apparent solution to an important problem of Klee's concerning linear topological spaces has been found and is under current study for possible flaws. If this solution holds up, it will provide the subject matter for two research papers.

University of Alabama
University, Alabama
November 1, 1965

2.6 Description of New Research in Mathematics, Dr. A. K. Bose

New Research on Techniques of Solutions of Boundary-Value Problems for Differential Equations as Applied to Space Problems under NASA Grant NsG-381, initiated in September, 1965

## Background

In case of boundary value problems of partial differential equations, for a given domain D, it is almost always necessary, under various hypotheses about the geometry of the boundary of the domain D and the behavior of the coefficients of the differential equation, to prove the existence, uniqueness, and continuous dependence on boundary data of the solution of the boundary value problems.

Although the question of existence is by all means the hardest to settle, the question of uniqueness is by no means easy. The main tool used today to prove the uniqueness is the so-called "Maximum-Principles" of the solutions of a certain class of differential equations. The uniqueness of the solution of the boundary value problem comes as a direct corollary to these "Maximum-Principles."

These "Maximum-Principles" have so far only been established for some special cases of elliptic linear and nonlinear equations (E. Hopf) and for a very special case of parabolic equations (L. Nirenberg).

## Research Study

Research on the above problem will be conducted. The objective will be to extend these "Maximum-Principles" to more general types

of elliptic and parabolic equations, with special emphasis on the quasi-linear equations. It is expected that a weaker form of the present "Maximum-Principle" can be extended to a large class of equations.

Other problems also will be investigated with regard to whether or not these "Maximum-Principles" can be replaced by some stronger properties. For instance, it has been proved recently (A. Friedman, W. Littman, A. K. Bose) that solutions of certain classes of elliptic equations possess a type of mean-value property similar to that of harmonic functions and the "Maximum-Principle" for these equations comes as a direct corollary to this mean-value property. Furthermore, for a special subclass of these classes of equations, the unique solution to the boundary value problem for circular regions can be obtained by Poisson type integral. It is expected, therefore, that further investigation of these mean-value properties will lead to interesting properties of the solutions, and hence to the solution of the boundary value problem.

University of Alabama
University, Alabama
November 1, 1965

2.7 Progress Report on Physics, Dr. James H. Bartlett

A. The Restricted 3-Body Problem

During this reporting period, Dr. Bartlett continued a study of the invariant domains of the differential equation $(d^2x/dt^2) + px^3 = 0$. He was assisted by a graduate student at the University of Illinois. Work on this equation and extension to the restricted 3-body problem will continue as previously proposed.

B. The Structure and Behavior of Thin (Passive) Films

During this reporting period, the equipment for this research was transferred from the University of Illinois to the University of Alabama, and Mr. Hudson, a graduate assistant, began experimentation with it.

During the summer, Mr. Donald DeSmet, who will join Dr. Bartlett at the University of Alabama on February 1, 1966, worked with Professor J. L. Ord of the University of Waterloo, Ontario, Canada, and there had considerable success applying the ellipsometer to thin film measurements. Unexpected results were obtained with passive iron, and were reported at the Buffalo meeting of the Electrochemical Society. An abstract of a report on this work appeared in the October 14, 1965 Journal of the Electrochemical Society, and a report is being prepared

for submission as an article. Mr. DeSmet will continue to collaborate closely with Professor Ord after he joins the staff here.

Dr. Bartlett has found that the results with iron are similar to those with tantalum. This latter metal has received much theoretical attention, but the theories so far advanced are inadequate to explain the experiments. Further experimental study is contemplated to resolve this difficulty.

C. REPORT OF ACTIVITIES AT THE UNIVERSITY OF ALABAMA RESEARCH INSTITUTE

## 1.0 STATUS OF DEVELOPMENT OF THE UNIVERSITY OF ALABAMA RESEARCH INSTITUTE AND THE HUNTSVILLE INSTRUCTIONAL PROGRAM

### 1.1 Staff of the Research Institute

Table 1 depicts the staff level, semi-annually and by category, for the 3-year period August 31, 1962--August 31, 1965. It should be noted that for this reporting period, the over-all staff increased from 61 to 69.

Also, five additional members were added on September 1, 1965, which brought the professional research staff to 30, 21 of whom hold the Ph.D. In addition to Research Institute staff, 13 graduate students and one post-doctoral student made use of Research Institute facilities during the reporting period. Of these, four were employees of NASA, seven were employees of the U. S. Army, and three were employees of industries supporting space and missile work in Huntsville.

With the exception of four members, the total professorial staff of the University of Alabama in Huntsville in the physical sciences and engineering is engaged in research activities at the Research Institute. This high percentage of participation emphasizes the degree which the research and instructional programs complement each other in support of the academic program in Huntsville.

A most important use for the NsG-381 funds is to support new personnel while they develop ideas and do sufficient preliminary research to enable them to submit sound proposals for specific contract or grant support. Among those who have received support from NsG-381 for the first time during the last six months are Dr. J. J. Brainerd, Associate Professor of Aerospace Engineering; Dr. J. C. Chang, Research Associate; Dr. D. R. Jeng, Assistant Professor of Mechanical Engineering; Dr. T. Oden, Associate Professor of Engineering Mechanics; and Dr. G. Wempner, Professor of Engineering Mechanics. Of these, Drs. Brainerd, Oden and Wempner have submitted formal proposals to NASA Headquarters, NASA's George C. Marshall Space Flight Center, and the National Science Foundation, respectively (See Table 5).

Dr. Chang has a preliminary proposal concerning research on parametric amplifiers in the hands of a possible Federal sponsor. Dr. Jeng, in collaboration with Dr. W. K. Kubitza, Professor of Engineering Mechanics, has been preparing a proposal to another Federal agency to investigate the thermal resistance across a mechanical contact.

Senior people of the Research Institute who will be receiving support for the first time during the next six-month period are Dr. E. V. Grohse, Professor of Chemical Engineering; Dr. A. F. Marcantonio, Associate Professor of Chemistry; Dr. E. V. Wilms, Assistant Professor of Engineering Mechanics; and Mr. R. D. Wood, Assistant Research Professor of Aerospace Engineering.

## 1.2 Facilities

### 1.2.1 Research Institute General Development

During the reporting period, fair progress has been made in eliminating the imbalance between theoretical and experimental research created by the fact that the establishment of the Research Institute and acquisition of a staff with a theoretical capability preceded the original building by two years. The Institute now has ten laboratories sufficiently staffed, equipped, and instrumented to do some experimental work in the areas of surface physics, communications, microwave, antenna, instrumentation, gyro evaluation, analytic instruments, structural mechanics, optics, and analog computation and control sciences. Of these, the surface physics laboratory is sufficiently equipped to be considered in a steady-state, the microwave and communications laboratories have adequate equipment and instrumentation for their immediate needs, and the others are in a state of continuous, orderly build-up.

The following changes which occurred during the reporting period in the activity and status of Research Institute laboratories are worth noting:

Structural Mechanics Laboratory. Progress in equipping this laboratory is described in Appendix III, p. 18.

Gyro Evaluation Laboratory. Dr. J. C. Dowdle, who was previously doing research on dielectrics on NsG-381, is presently developing

this laboratory (partly through support of the Marshall Space Flight Center). While neither Dr. Dowdle, nor any other personnel recruited primarily to work on gyro research, are expected to be funded by NsG-381 in the immediate future, some support from this grant will be required to build laboratory capability for gyro evaluations.

Organic Chemistry. A laboratory originally reserved for energy exchange, containing basic services and furnishings but without equipment for specified tasks, is being converted for use by Dr. Marcantonio as an organic chemistry laboratory. Energy exchange studies contemplated for this laboratory are being conducted in the optics laboratory under a contract with the Army Missile Command.

Instrumentation Research. The original proposal to NASA which resulted in the NsG-381 grant included an environmental sensors laboratory. It was to be essentially an instrumentation research and development laboratory which would design and build instruments to sense the environment in which a vehicle may be operating, either in the upper atmosphere or in space. However, to date, the instrumentation research function has developed along a slightly different line. Largely under contracts with the MSFC, Mr. V. M. McCarty, as principal investigator, has developed and built instrumentation to record the earth tremors generated by the static testing of large booster rockets. This instrumentation is being extended so that it will serve simultaneously as part of the U. S. Coast and Geodetic Survey seismic detection network.

Control Sciences Laboratory. A significant addition, purchased with $80,000.00 of State bond funds, is a new Applied Dynamics Model AD-80 hybrid analog computer facility. NsG-381 funds have assisted in the installation and checkout of the hybrid computer facility and in the purchase of some peripheral equipment. This facility fills a serious gap in the computational capability of the Research Institute. The computer will be used primarily for research, but it is also available for instructional use in courses on analog computation and on control theory.

Computer Sciences Laboratory. During this reporting period, an additional drum was added to the Univac 1107 computer, increasing its storage capacity to 2,359,296 words of drum and 65,536 words of ferrite core.

The total value, exclusive of the Univac 1107 computer, itself valued at over $3,000,000, of the equipment on hand in all of the laboratories of the Research Institute is:

VALUE OF EQUIPMENT, INSTRUMENTATION, AND FURNISHINGS
OF THE UNIVERSITY OF ALABAMA RESEARCH INSTITUTE AS OF AUGUST 31, 1965

| | University Owned | U. S. Gov't Owned; on Loan to UARI | Total |
|---|---|---|---|
| Laboratory Furniture | $ 68,477.36 | $ - | $ 68,477.36 |
| Moveable Instrumentation and Equipment | 401,648.93 | 232,065.59 | 633,714.52 |
| Office Furniture and Furnishings | 63,993.53 | 1,520.93 | 65,514.46 |
| Office Equipment (Includes Hand Calculators) | 17,057.63 | 1,390.84 | 18,448.47 |
| Tools, Machine Shop Equipment, Building Maintenance, Transportation, Storage, and Miscellaneous | 56,319.22 | 18,478.69 | 74,797.91 |
| Totals | $607,496.67 | $253,456.05 | $860,952.72 |

1.3 Significant Progress and Changes in the Related Instructional Programs

During the reporting period, the following significant progress and changes occurred:

a. Bids were accepted on the 57,666 square foot graduate instruction building, costing over $1,000,000. This building is located near the Research Institute.

b. Capital funds of approximately $1,000,000 became available for additional construction. Planning for this construction is underway.

c. In the Fall Quarter, 1965, there were 637 graduate students enrolled in forty-three courses. This compares to 721 graduate students enrolled in fall, 1964. It is believed the larger number in 1964 represents a surge in response to the establishment of the resident graduate degree program. Among the total of 1,177 students taking graduate courses in 1964-65, 1,006 were admitted to the graduate school, 882 expressed an intent to obtain a master's degree, and another 140 already have a master's degree and are working toward the Ph.D. degree. Nine students completed a master's degree with all work done in Huntsville, and 11 other students completed advanced degrees after taking part of their work in Huntsville.

d. A total of 438 full-time students enrolled in the new, four-year undergraduate program which was begun in the fall of 1964. Students may major in English, history, mathematics and physics.

e. The number of special short courses offered in such subjects as vacuum technology, cryogenics, and nuclear engineering, increased substantially during the reporting period.

1.4 Seminars and Lectures by Visiting Speakers at the Research Institute

During this reporting period, the Research Institute has continued to bring to the Huntsville Campus outstanding lecturers who contribute significantly to the enrichment of the research activities by providing additional stimulus to interdisciplinary activities. Eight lectures or seminars were given during the period. A number of personnel from the George C. Marshall Space Flight Center, the U. S. Army Missile Command, and aerospace and missile-related industrial contractors in the area attended these meetings. Listed in Appendix I are the speakers, their affiliation, title of talk, and a short abstract of the presentation.

1.5 Research Funding of the Research Institute

Figure 1 shows a chart of expenditures represented by direct costs of grants and contracts plus charges to the Research Institute administrative

budget. The total monthly charges for the last 5 months averaged $61,932. A $300,000 per year rate of expenditure on NsG-381 amounts to an average of $25,000 per month.

The Research Institute is continually trying to broaden its areas of research so that several agencies will want to sponsor certain of its research efforts. As of August 31, 1965 and as indicated in Table 2, the Research Institute had in force grants and contracts with a total value of $1,033,257.50 Those received in the present reporting period, either new or funded extensions of existing contracts and grants, are marked with an asterisk. The unspent amount of these contracts, and grants other than NsG-381, as of August 31, 1965, was $365,194.73. At the current spending rate, this amount plus NsG-381 funds would provide for less than 12 months of operations. Therefore, it is of paramount importance for the growth and stability of the Research Institute that additional sources of funds be cultivated, and major efforts are being expended in this direction. The results of these efforts are reflected in the fact that proposals totaling $1,006,866,26 for research programs, mostly of 2 and 3 years length, had been submitted as of August 31, 1965, to NASA Headquarters with a view toward the interests of the Electronics Research Center, to NASA, George C. Marshall Space Flight Center, the Advenced Research Projects Agency of the Department of Defense, and to the National Science Foundation. A listing is given in Table 3. Additionally, 2 other proposals are being prepared for submission.

1.6 Publications and Lectures by the Staff of the Research Institute

The productivity of the staff and the significance of the research results can, in part, be measured by the number and kinds of publications, the presentations of papers at conferences, and the issuance of research reports which summarize the findings and make the material available to the scientific

and engineering community. During the six-month period, members of the Research Institute staff have had 25 publications appear or papers presented, 19 others accepted for publication or presentation, and 10 submitted which are still under active consideration for acceptance. Also, six Research Institute Technical Reports have been issued. Publications are listed in Appendix II irrespective of source of support; those which received support from NsG-381 are marked with an asterisk. Copies of the latter have previously been sent to NASA Headquarters in accordance with terms of the grant. The listing in Appendix II gives the author, title, date or status of publication, and a short abstract. The abstracts are concise summaries of research, much of which had direct support of this grant.

## 2.0 Research Activities of the Research Institute

### 2.1 Summary of Activities during the Period March 1, 1965, to August 31, 1965.-

The 27 topics studied during this period which had NsG-381 support, at least in part, are distributed in the major disciplines as follows:

Mathematics - 3

Physics, including Chemical Physics and Optics - 12

Control Sciences and Hybrid Computation - 2

Electromagnetics, including Communications - 4

Fluid Mechanics - 3

Structural Mechanics - 3

The topics are listed below with the Principal Investigator(s) shown and grouped by major disciplines. Further explanations of the research efforts are given in Appendix III, Present Research Efforts of the Staff of UARI during the Period March 31, 1965, to August 31, 1965.

#### 2.1.1 Mathematics

Mathematical Analysis and Computer Sciences.- Dr. R. L. Causey, Associate Professor of Mathematics, has studied mathematical analysis, information retrieval, and has continued the development of the Computer Sciences Laboratory. Page 2, of Appendix III.

Topological Dynamics and Related Areas: Dr. H. S. Chu, Professor of Mathematics, has been doing research in the field of topological dynamics. Comments on five papers by Dr. Chu are given. Page 3, of Appendix III.

System Analysis. Mr. C. F. Chen, Research Associate states several results of his studies of system theory. Page 4, Appendix III.

#### 2.1.2 Physics, Including Chemical Physics and Optics

Investigations in Theoretical Atomic Physics - Dr. W. R. Garrett, Assistant Professor of Physics, has studied the first three topics listed, and Dr. R. A. Mann, Associate Professor of Physics, the fourth. Page 5 , Appendix III.

1) Low Energy Electron Scattering

2) Ion-Photon Processes

3) Atomic Polarizabilities

4) Slow Electron Inelastic Scattering

Theoretical Studies of Nuclear and Atomic Structure.

Dr. C. P. Bhalla, Associate Professor of Physics has been concerned with the following activities. Page 6 , Appendix III.

1) Nuclear Beta Decay

2) Nuclear Structure Calculations

3) Relativistic Atomic Studies

4) Electron Shielding

4) Graduate Students Supervision

Surface Physics and Related Areas. Dr. R. A. Mann is continuing the study of field emission microscopy. Related research sponsored by NASA, MSFC, is studying a method of producing ultra clean surfaces, and the behavior of these surfaces when brought into contact. Page 7 , Appendix III.

Coherent Light. Dr. R. A. Mann is doing a theoretical study of beam divergence in lasers. Page 7 , Appendix III.

Chemical Physics. Interaction of electromagnetic energy with matter. Dr. W. F. Arendale, Professor of Chemistry and Assistant Director of the Research Institute, has continued to develop laboratories for the study of electromagnetic energy in the visible and near-visible region of the spectrum with matter. Two related projects funded by U. S. Army Missile Command investigate (a) the absorption and reflected light characteristics

from selected targets and (b) nonlinear effects related to Raman scattering. Page 8 , Appendix III.

### 2.1.3 Control Sciences and Hybrid Computation

Dr. C. D. Johnson, Associate Professor of Electrical Engineering, has been engaged in two topics in control sciences:

1) Control theory: The study of necessary conditions for "singular solutions" in optimal control problems.

2) Development of a hybrid analog computer facility: The facility described in section 2.2.2 is being used to investigate "machine solution" techniques for two point boundary value problems, and for studies of asymptotic stability of nonlinear dynamical systems. Page 8 , Appendix III.

### 2.1.4 Electromagnetics, Including Communication

Wave Propagation in Groove Guides. Dr. N. F. Audeh, Associate Professor of Electrical Engineering, and Mr. H. Y. Yee, Research Associate, have developed a method to determine the cutoff frequencies of groove guides. Page 9 , Appendix III.

Scattering Problems in Electromagnetic Propagation. Mr. Yee has developed a point-matching numerical method to calculate the electromagnetic scattering properties of a conducting cylinder of arbitrary cross section. Page 10, Appendix III.

Communications. Dr. R. J. Polge, Associate Professor of Electrical Engineering, is working in the area of communication. Presently, he is conducting both theoretical and experimental research in the detection of coded pulse type signals in the presence of noise or jamming signals. The research was first supported by NsG-381, then by an Army grant. Because the Army grant is expiring, this research in adaptive detection will

continue on NsG-381 until another contract is secured. Efforts are being made to acquaint personnel of NASA's Electronic Research Center with the merits of the adaptive detection scheme developed as a result of the initial NsG-381 funding. Page 11, Appendix III.

Evaluation of Magnetic Influences on Gyros. Dr. J. C. Dowdle, Professor of Electrical Engineering, is studying magnetic field effects on the stability of gyros and is developing a laboratory to evaluate gyro performance under a static spatial environment. Page 12, Appendix III.

2.1.5 Fluid Mechanics, Gas Dynamics, and Heat Transfer

Unsteady Heat Transfer in Laminar Boundary Layers. Dr. D. R. Jeng, Assistant Professor of Mechanical Engineering, who taught full time in the area of heat transfer and thermodynamics during the period of January to June 1965, joined the research program on a full-time basis from July 1 to August 31, 1965. During this time, he conducted research in the fields of unsteady heat transfer in the boundary layers, and transpiration cooling. It is expected that he will contribute half of his time to research in the future. Page 13, Appendix III.

Transpiration Cooling in a Magneto-Hydrodynamic Three-Dimensional Stagnation Flow. Dr. Jeng has made an analysis of the steady three-dimensional stagnation flow where blowing and a magnetic field act simultaneously. Computer solutions for a range of parameters have been obtained. Page 14, Appendix III.

Hypersonic High-Temperative Gas Flow. Dr. Hermann, Professor of Physics and Aerospace Engineering and Director, Research Institute, and since August, 1965, Dr. J. J. Brainerd, Associate Professor of Aerospace Engineering, have conducted and supervised theoretical studies of hypersonic high-temperature gas flow including nonequilibrium oxygen dissociation around both blunt and pointed bodies. A numerical procedure

using the method of integral relations had been employed in the past. A variation of a finite difference scheme will be developed for use in cases where it has advantages, or where the method of integral relations cannot be applied. Page 14 , Appendix III.

2.1.6 Structural Mechanics

Numerical Analysis of Nonlinear Structures. Dr. J. T. Oden, Associate Professor of Engineering Mechanics, has undertaken a study of the nonlinear behavior of complex structures. The over-all objective is to develop a general numerical formulation of geometrically nonlinear problems in solid mechanics. Page 15 , Appendix III.

Research in Nonlinear Analysis of Shell Structures. Dr. G. A. Wempner, Professor of Engineering Mechanics, has initiated work on two related topics.

1) Nonlinear analysis of thin shells by a method of incremental loading.

2) Theories of composite shells

The first study is aimed at a general method for the solution of nonlinear problems of thin shells or shell-like structures. The method is based on the notion that any smooth function is linear in a small range of its variables. The second effort is a basic study and a development of the governing equations for sandwich shells. Page 17 , Appendix III.

Development of the Structural Mechanics Laboratory. Dr. W. K. Kubitza, Professor of Engineering Mechanics, has devoted considerable amount of his efforts, during the past six months, into planning, equipping, and developing the Structural Mechanics Laboratory. His research involved determination of stresses by means of the Moire apparatus he has designed and built. Page 18, Appendix III.

## 2.2 Programs Planned for the Next Six Months.

2.2.1 Continued Laboratory Development. It is expected that the equipping of laboratories will continue during the next period as the Research Institute develops its experimental capabilities. In particular, the Organic Chemistry Laboratory should be operational, the Gyro Evaluation Laboratory will expand, and the Hybrid Analog Computer Facility should be thoroughly checked out by the end of the period. Quite likely, a Heat Transfer Laboratory will be activated during the next period.

Mr. R. D. Wood, Assistant Research Professor of Aerospace Engineering, who joined the Research Institute in August, 1965, will make feasibility and preliminary design studies of components suitable for the high-temperature gasdynamics laboratory, the space environmental laboratory, and the plasma technology laboratory.

2.2.2. Continuation of Current Research Programs. It is planned to continue research outlined in Section 3.1 in the major desciplines as follows:

Mathematics

Mathematical Analysis and Digital Computer Sciences

Topological Dynamics and Related Areas

Physics, Including Chemical Physics and Optics

Theoretical Atomic Physics

Theoretical Nuclear and Atomic Structure

Surface Physics

Coherent Light

Chemical Physics

Control Sciences

Analog and Hybrid Computational Methods

Electromagnetics, Including Communication

Scattering Problems in Electromagnetic Propagation

Communications

Fluid Mechanics, Gas Dynamics, and Heat Transfer

Hypersonic High-Temperature Flow

Structural Mechanics

Numerical Analysis of Nonlinear Structures

Research in Nonlinear Analysis of Shell Structures

Development of Structural Mechanics Laboratory

2.2.3. New Research Programs. Although research will be continued in the topics mentioned in the preceding section, a major effort will be will be made to start new topics. A brief description of each topic is given in Appendix IV, "New Research Programs Planned at UARI for the Period September 1, 1965 to February 28, 1966."

Physics, Including Chemical Physics and Optics

Theoretical Relativistic Physics. Drs. Garrett and Bhalla of the theoretical physics group will devote a portion of their time to relativistic physics, relativistic atomic studies, and X ray calculations. Page 1, Appendix IV.

Chemistry

Organic Chemistry. Dr. A. F. Marcantonio, Associate Professor of Chemistry, plans investigations in the field of organic photochemistry and pyrolytic chemistry, involving the fate of short lived, energetic, and reactive intermediates generated by the treatment of organic molecules with various sources of ultraviolet and visible light and heat. Page 2, Appendix IV.

Electromagnetic, Including Communication

Traveling Wave Parametric Amplifier. Dr. J. C. Chang, Research Associate, plans both a theoretical and experimental investigation of traveling wave parametric amplifiers with nonlinear inductive and capacitive coupling reactances. Page 3, Appendix IV.

Wave Propagation in Waveguides of Arbitrary Cross-sections.

Dr. N. F. Audeh and Mr. H. Y. Yee have developed a point-matching method to solve problems of wave guides of arbitrary cross-section. They plan to use the method for hollow-piped wave guides and for composite wave guides. Along with the theoretical investigations, wave guides will be designed, constructed, and tested. Page 4, Appendix IV.

Fluid Mechanics, Gas Dynamics, and Heat Transfer

Energy and Mass Transfer. Dr. E. W. Grohse, Professor of Chemical Engineering, previously a member of the Department of Chemical Engineering at the Tuscaloosa Campus, joined the faculty of the University of Alabama in Huntsville on September 1. During the coming six months, Dr. Grohse will initially concentrate on developing research projects, probably in the areas of energy and mass transfer, which, it is hoped, can be successfully proposed for funding as contracts or specific grants. It is also expected that Dr. Grohse will develop courses in chemical and/or nuclear engineering aimed at fulfilling the specific requirements and interests of students attending the Huntsville Campus. He is tentatively scheduled to participate in the short course, "Nuclear Power Reactors," scheduled to be given during late October and/or November, 1965. Page 6, Appendix IV.

Thermal Contact Resistance in a Vacuum Environment

Dr. D. R. Jeng, Assistant Professor of Mechanical Engineering, plans an experimental and analytical study of thermal contact resistance in a vacuum environment. Tool for the experimental phase is an electrolytic tank to simulate the geometrical aspects of the thermal constriction resistance. Dr. W. K. Kubitza will participate in this research in the definition of typical actual surface contact areas and surface deformation. Page 6, Appendix IV.

Structural Mechanics

Stress Wave Propagation. Dr. E. V. Wilms, Assistant Professor of Engineering Mechanics, will study theoretically the propagation of stress waves in elastic materials. Page 7, Appendix IV.

## 2,2.4 Projected Funding for September 1, 1965 to February 28, 1966

The research funding from NsG-381 at the Research Institute for the next six-month period is projected to be distributed in the various areas as follows:

| | |
|---|---|
| Laboratory Development | $ 24,000 |
| Mathematics, Including Digital Computer Sciences | 5,000 |
| Physics, Including Chemical Physics and Optics | 14,000 |
| Chemistry | 7,000 |
| Electromagnetics, Including Communication | 24,000 |
| Fluid Mechanics, Gas Dynamics, Heat Transfer | 38,000 |
| Structural Mechanics | 13,000 |
| | $125,000 |
| Overhead 20 per cent | 25,000 |
| | $150,000 |

## 3.0 Financial Report

3.1 Expenditures March 1, 1965 to August 31, 1965. The financial report of funds spent from this grant is attached as Section D, a Consolidated Expenditure Report, which also sets forth in detail Main-Campus and Research Institute expenditures. The latter, during the six-month period, were:

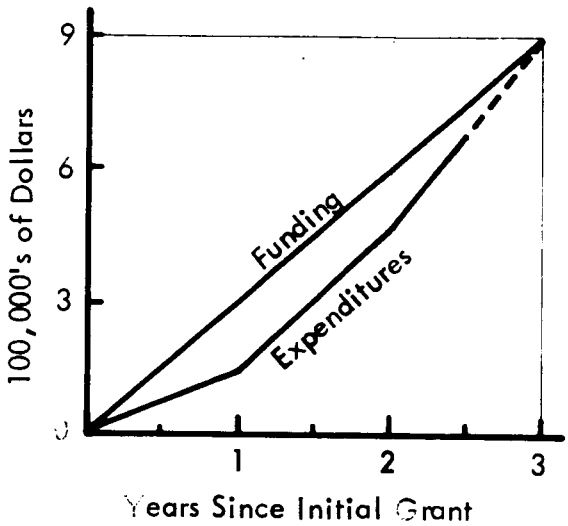
| | |
|---|---|
| Salaries | $ 77,961.03 |
| Operating Expenses | 16,371.18 |
| Equipment | 62,217.33 |
| | 156,549.54 |
| Indirect Cost(20/25%) | 36,893.58 |
| | $ 193,443.12 |

This rate of expenditure (in excess of the funded rate of $150,000 for a six-month period) is made possible by the initial lag in spending during the first grant year and cannot be sustained in the absence of a higher level of funding.

The following table shows the steady increase in the rate of utilization of grant funds and the fact that cumulative expenditures are within almost $50,000 of the maximum funds available ($750,000 for 2½ year) as of the end of the period:

| | |
|---|---|
| March 1, 1963 - February 28, 1964 | $152,818.18 |
| March 1, 1964 - February 28, 1965 | 331,761.55 |
| March 1, 1965 - August 31, 1965 | 193,443.12 |
| Encumbrances for equipment on order (including indirect cost) | 18,175.18 |
| Total | $696,198.03 |

The sketch to the right shows the cumulative funding and expenditures of NsG-381 at the Research Institute since the beginning of the grant, March 1, 1963. The initial lag in the spending rate was due mainly to the slow process of recruiting people, generally on the professional level, capable of acting as Principal Investigators.




The spending rate has exceeded the funding rate since the end of the first year. The excess funds carried forward from previous periods will have been utilized by the end of the current grant year.

## D. CONSOLIDATED EXPENDITURE REPORT

UNIVERSITY OF ALABAMA
Semi-Annual Statement of Expenditures and Encumbrances
For Period March 1, 1965 through August 31, 1965
NASA Research Grant NsG-381
and Supplement No. 1

Consolidated Report of Expenditures and Encumbrances

| | | |
|---|---|---|
| Salaries | | |
| Professional | $ 71,744.85 | |
| Student | 14,922.87 | |
| Supporting Services | 6,460.94 | $ 93,128.66 |
| Supplies | | 7,631.82 |
| Travel and Communication | | 5,229.61 |
| General Expense | | 5,693.27 |
| Equipment | | 62,983.01 |
| Total Expenditures | | $174,666.37 |
| Indirect Costs: 25% of Direct Costs ($111,673.37) | | 27,918.34 |
| Indirect Costs: 20% of Direct Costs ($ 62,993.00) | | 12,598.60 |
| | | $215,183.31 |
| Plus Encumbrances | | |
| Equipment on Order | | 15,145.98 |
| Indirect Costs: 20% on Encumbered Items | | 3,029.20 |
| Total Expenditures and Encumbrances | | $233,358.49 |

UNIVERSITY OF ALABAMA
Semi-Annual Statement of Expenditures and Encumbrances
For Period March 1, 1965 through August 31, 1965
NASA Research Grant NsG-381
and Supplement No. 1
(Continued)

Expenditures listed below were made on the Tuscaloosa Campus during this period

| | |
|---|---|
| **Chemistry - Tuscaloosa Campus** | |
| Salaries, Professional | $ 2,883.30 |
| Supplies | 453.37 |
| Equipment | 230.68 |
| | $ 3,567.35 |
| Indirect Costs: 20% of Direct Costs | 713.47 |
| | $ 4,280.82 |
| **Physics - Tuscaloosa Campus** | |
| Salaries, Professional | $ 3,125.00 |
| Supplies | 1,502.09 |
| Travel Expense | 128.25 |
| Equipment | 535.00 |
| | $ 5,290.34 |
| Indirect Costs: 20% of Direct Costs | 1,058.07 |
| | $ 6,348.41 |
| **Mathematics - Tuscaloosa Campus** | |
| Salaries, Professional | $ 4,776.00 |
| Indirect Costs: 20% of Direct Costs | 955.20 |
| | $ 5,731.20 |
| **Electrical Engineering - Tuscaloosa Campus** | |
| Salaries, Professional | $ 4,383.33 |
| Supplies | 75.81 |
| General Expense | 24.00 |
| | $ 4,483.14 |
| Indirect Costs: 20% of Direct Costs | 896.62 |
| | $ 5,379.76 |

UNIVERSITY OF ALABAMA
Semi-Annual Statement of Expenditures and Encumbrances
For Period March 1, 1965 through August 31, 1965
NASA Research Grant NsG-381
and Supplement No. 1
(Continued)

Summary of Expenditures made
on the Tuscaloosa Campus

| | |
|---|---|
| Salaries, Professional | $ 15,167.63 |
| Supplies | 2,031.27 |
| Travel Expense | 128.25 |
| General Expense | 24.00 |
| Equipment | 765.68 |
| Total Expenditures | $ 18,116.83 |
| Indirect Costs: 20% of Direct Costs | 3,623.36 |
| | $ 21,740.19 |

UNIVERSITY OF ALABAMA
Semi-Annual Statement of Expenditures and Encumbrances
For Period March 1, 1965 through August 31, 1965
NASA Research Grant NsG-381
and Supplement No. 1
(Continued)

Expenditures and Encumbrances listed below were made at the Research Institute in Huntsville

| | | |
|---|---|---|
| Salaries | | |
| Professional | $ 56,577.22 | |
| Student | 14,922.87 | |
| Supporting Services | 6,460.94 | $ 77,961.03 |
| Supplies | | 5,600.55 |
| Travel and Communication | | 5,101.36 |
| General Expense | | 5,669.27 |
| Equipment | | 62,217.33 |
| Total Expenditures | | $156,549.54 |
| Indirect Costs: 25% of Direct Costs ($111,673.37) | | 27,918.34 |
| Indirect Costs: 20% of Direct Costs ($ 44,876.17) | | 8,975.24 |
| | | $193,443.12 |
| Plus Encumbrances | | |
| Equipment on Order | | 15,145.98 |
| Indirect Costs: 20% of Encumbered Items | | 3,029.20 |
| | | $211,618.30 |

I certify that this statement of expenditures and encumbrances is correct and in accordance with the terms of the grant.

Luther C. Callahan
Comptroller

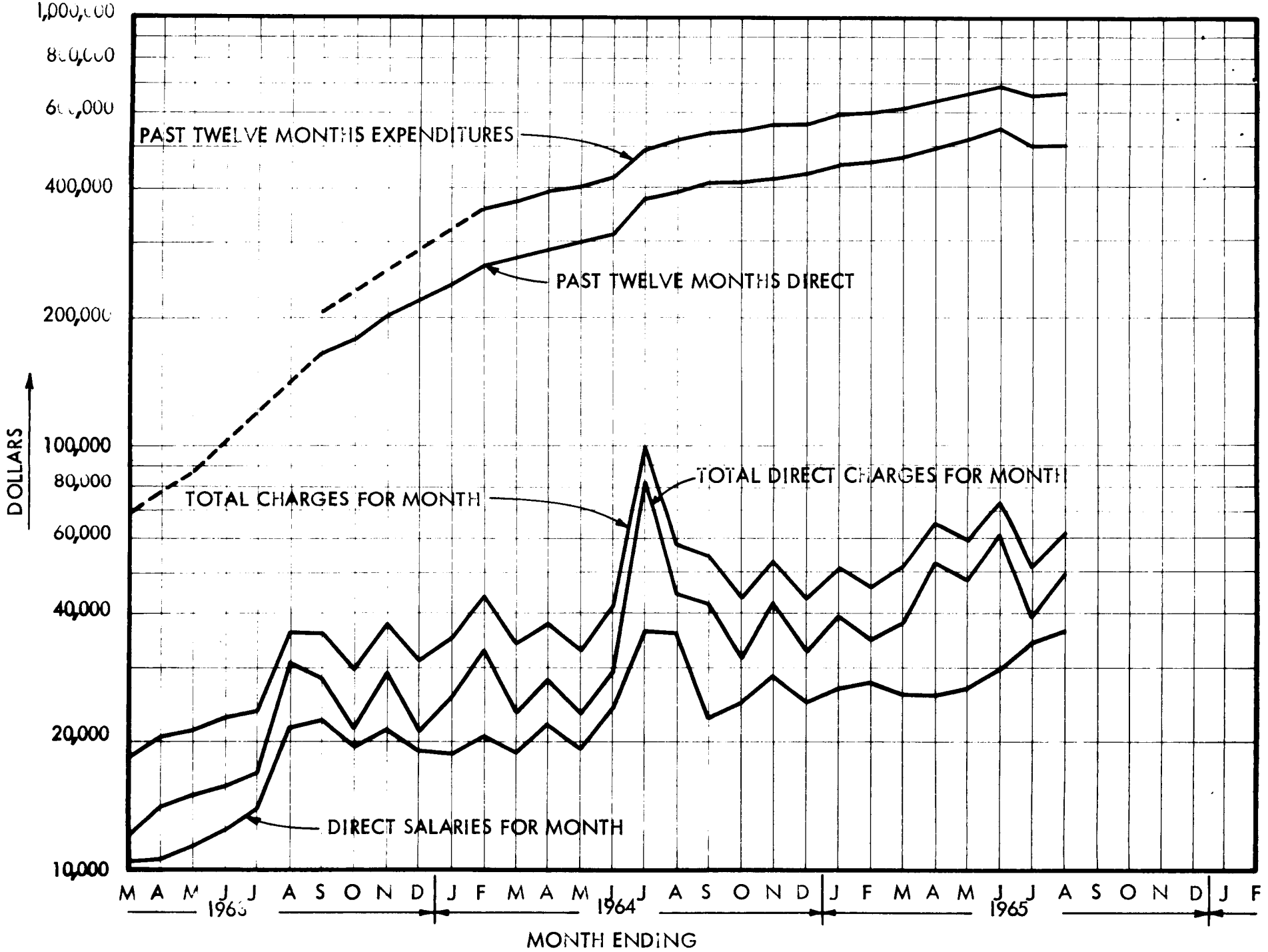



FIG. 1. GRAPH OF EXPENDITURES OF THE RESEARCH INSTITUTE

TABLE 1

RESEARCH INSTITUTE PERSONNEL STATISTICS

A — Number of Permanent Staff, Including Part Time

| | 8/31/62 | 2/29/63 | 8/31/63 | 2/29/64 | 8/31/64 | 2/28/65 | 8/31/65 |
|---|---|---|---|---|---|---|---|
| Academic | 2 | 6 | 9 | 15 | 16 | 15 | 19 |
| Research Associates and Professional Researchers | 0 | 0 | 4* | 5* | 6** | 9 | 9 |
| Plant Maintenance and Operations | 0 | 0 | 0 | 0 | 6 | 7 | 7 |
| Administration | 4 | 6 | 8 | 13 | 12 | 13 | 13 |
| Technical Service | 2 | 2 | 2 | 2 | 4 | 4 | 6 |
| Research Assistants (Graduate) | 1 | 2 | 4 | 7 | 8 | 10 | 11 |
| Undergraduate Assistants | 0 | 0 | 2 | 2 | 2 | 3 | 4 |
| Totals | 9 | 16 | 29 | 44 | 53 | 61 | 69 |

* Excludes 1 person jointly funded with the Univac Division of Sperry Rand Corporation.

** Excludes 10 persons jointly funded with Univac.

B — Number of Summer Students

| | 1963 | 1964 |
|---|---|---|
| Research Assistants (Graduate) | 1 | 3 |
| Undergraduate Assistants | 6 | 10 |
| Totals | 7 | 13 |

Table 2
University of Alabama Research Institute
Contracts and Grants as of August 31, 1965

| Title | Contract | Principal Investigator | Expiration | Amount |
|---|---|---|---|---|
| NASA | | | | |
| *Research in the Aerospace Physical Sciences | NsG-381 | Dr. R. Hermann | 3-1-68 | $ 300,000.00 |
| *Research & Development of Instrumentation Used in the Experimental Study of Cold Welding in Ultra High Vacuum as a Junction of Surface Contamination | NAS8-20159 | Dr. R. A. Mann | 6-3-66 | 34,421.00 |
| *Measurement of Earth Tremors Produced as a Result of Large Rocket Firings | NAS8-11202 | Mr. V. McCarty | 6-22-66 | 39,917.00 |
| *Development of High-Accuracy Long-Life Gyros | NAS8-20117 | Dr. J. C. Dowdle | 7-29-66 | 56,122.00 |
| Parameter Optimization in Vehicle-Borne Tracking Systems | NAS8-5411 | Dr. G. D. Cole | 11-8-65 | 47,162.00 |
| *Research of the Determination of Cesium Distribution in Porous Tungsten | NAS8-20171 | Dr. R. A. Mann | 6-28-66 | 49,921.00 |
| Interactions of Slow Electrons with Light Gas Atoms | NAS8-11249 | Dr. R. A. Mann | 9-27-65 | 9,735.00 |
| Investigation of Surface Ionization on Metallic Surfaces | NAS8-2585 | Dr. R. A. Mann | 10-15-65 | 40,086.50 |
| *Research on Topological Dynamics and Related Fields | NAS8-1646 | Mr. C. F. Chen** | 3-3-66 | 23,940.00 |
| Optimal-Adaptive Control Theory | NAS8-11231 | Dr. Johnson | 2-12-66 | 17,323.00 |

(continued on next page)

Table 2 (Contd.)
University of Alabama Research Institute
Contracts and Grants as of August 31, 1965

| Title | Contract | Principal Investigator | Expiration | Amount |
|---|---|---|---|---|
| AMC | | | | |
| Support Research Guidance & Control, Task A--Detection and Information Processing with Memory-Type Decision Circuits | DA-AMC-01-021-64-G1 | Dr. R. J. Polge | 9-30-65 | $ 61,424.00 |
| Support Research Guidance & Control, Task B--Wave Propagation Through Plasma with Emphasis on Quantum-Physics Method | DA-AMC-01-021-64-G1 | Dr. Ray Garrett | 9-30-65 | 42,430.00 |
| Basic Research in the Mathematical Theory of Automatic Control | DA-01-009-AMC-165(Z) | Dr. C. D. Johnson | 1-30-66 | 35,299.00 |
| A Theoretical Study of the Scattering of Coherent Light | DA-01-009-AMC-164(Z) | Dr. R. A. Mann | 9-30-65 | 54,354.00 |
| Numerical Analysis of Methods for Computing Three-Dimensional Supersonic Flow | DA-01-021-AMC-11901(Z) | Dr. R. L. Causey | 5-4-66 | 43,080.00 |
| *Basic Research in the Field of Inviscid High Temperature Hypersonic Flow of Air Past Pointed Bodies of Revolution | DA-01-021-AMC-12039(Z) | Dr. Rudolf Hermann | 10-1-65 | 20,768.00 |
| *Raman Scattering in Gases or Liquids | DA-01-021-AMC-12802(Z) | Dr. W. Arendale | 6-28-66 | 52,250.00 |
| *Study of Reflected Energy from Laser Illuminated Targets | DA-01-021-AMC-12250(Z) | Dr. R. A. Mann<br>Project Coordinator:<br>Dr. W. Arendale | 6-10-66 | 105,025.00 |
| Total Contracts and Grants | | | | $1,033,257.50 |

*Indicates the grant, contract or funded extension of an existing contract or grant, was received during the period of March 1, 1965 - August 31, 1965.
**Recommended successor to Dr. Hsin Chu

Table 3
University of Alabama Research Institute
Proposals pending as of August 31, 1965

| Submitted to | Name of Proposal | Principal Investigator | Period of Proposal | Amount |
|---|---|---|---|---|
| NASA Headquarters: | The Effect of Plasma Polarization and Conductivity on Microwave Propagation | Dr. W. R. Garrett | 3-year program | $118,429.64 |
| | Scattering of Electromagnetic Waves by Dielectric-Coated Conducting Bodies | Mr. H. Y. Yee | 2-year program | 104,677.50 |
| | Electromagnetic Wave Propagation in Groove Guides | Dr. N. H. Audeh | 2-year program | 149,979.74 |
| | A Study of Hypersonic Aerodynamics of Re-Entry Vehicles Affecting Electromagnetic Wave Propagation | Dr. Jerome J. Brainerd<br>Dr. R. L. Causey<br>Project Coordinator:<br>Dr. Rudolf Hermann | 3-year program | 226,500.80 |
| NASA Marshall Space Flight Center: | Numerical Analysis of Non-Linear Structures | Dr. J. T. Oden | 1-year program | 70,371.28 |
| | Research on the Determination of Single Sideband Generation and Demodulation | Dr. R. J. Polge | 1-year program | 40,951.12 |
| National Science Foundation: | Nonlinear Analysis of Thin Shells by a Method of Incremental Loading | Dr. G. Wempner | 2-year program | 99,559.18 |
| | Internal Conversion Coefficients for M-Shells | Dr. C. P. Bhalla | 2-year program | 146,397.00 |
| Advanced Research Projects Agency | Grant for Purchase to Improve the Graduate Program in Optics at the University of Alabama in Huntsville | Dr. W. F. Arendale | | 50,000.00 |
| | | TOTAL | | $1,006,866.26 |

## APPENDIX I

### University of Alabama Research Institute Seminars and Lectures

Dr. Theodor Buchhold, General Electric Company, Schenectady, New York. Lecture given March 9, 1965 with 23 persons in attendance.

Problems in the Field of Applied Superconductivity

The lecture included a discussion of superconductive gyros as used in inertial guidance systems. Research and development done in high magnetic field coils and magnetic flux pumping were also discussed.

Dr. K. P. Chopra, Space Physics Laboratory, Melpar, Inc., Falls Church, Virginia. Lecture given March 18, 1965 with 26 persons in attendance.

Plasmadynamics of Objects Moving in Ionized Space

The interactions of objects moving in ionized space with the space environment (including radiation) were described. The establishment of an electric charge through several processes, including accretion of charged particles from the space environment and electron emission (thermionic and/or photoelectric) were discussed. The nature of the charged particle cloud around the space object was described in terms of the profiles of the electric field and the densities of the charged particles. The generation of perturbations in the space plasma by these moving objects and the drag on the space object were discussed. Other applications of such studies in space science were pointed out.

Dr. Adli S. Kana'an, Department of Chemistry, Rice University, Houston, Texas. Lecture given April 14, 1965 with 10 persons in attendance.

Spectroscopic Investigations of Some Halides in Plasma Generating Devices

The behavior of some halides of carbon ($CF_4$, $CCl_4$, $CF_3Cl$ and Teflon), silicon ($SiF_4$ and $SiCl_4$), and boron ($BF_3$) in plasma jet reactors have been investigated by emission spectroscopy. The emission spectra from these compounds have been identified and arise from excited diatomic molecules SiF, SiO, BO, $C_2$, and CN and excited atoms of Si, B, and C. On the basis of the observed spectra and energetic and thermodynamic arguments it is concluded that dissociation, excitation and/or ionization processes requiring 25-30 ev are predominant in the plasma reactors under the experimental conditions of these studies.

(List of Seminars and Lectures, contd.)

Dr. Arnold F. Marcantonio, Research Associate, Tulane University, New Orleans, Louisiana. Lecture given on April 29, 1965, with 18 in attendance.

Nitrene Chemistry

Reactions of certain nitrenes (electron-deficient, short-lived nitrogen intermediates) with alkanes and compounds containing various functional groups have been studied. The nitrenes were formed by the decomposition of hydrazoic acid, azidoformates, sulfonyl azides, and arylazides. The rates of decomposition of these azides in various solvents have been investigated. The chemical reactivity of these intermediates, generated photochemically and thermally was compared. The effect of structure on the nature of the energy state--singlet and triplet--of the reacting nitrene was discussed.

Dr. K. F. Knoche, Department of Mechanical Engineering, Heat Transfer Laboratory, University of Minnesota, Minneapolis, Minnesota. Lecture given May 17, 1965 with 18 persons in attendance.

Enthalpy-Density Charts for Flow and Combustion Processes

Problems of high temperature gas flows can be studied conveniently using thermodynamic charts, especially Mollier's enthalpy-entropy diagram. In order to avoid additional work to contruct Fanno-and Rayleigh-lines another graphical representation seems to be useful in the investigation of simple flow processes. This diagram, originally proposed by Spalding, has enthalpy and density as coordinates. One-dimensional channel flows, normal, oblique and reflected shocks may be easily investigated. The full scale diagrams have been calculated assuming thermodynamic equilibrium in a temperature range up to 100,000°K. The general considerations being valid also for non-equilibrium cases, special non-equilibrium flows, like frozen flow or detonation processes may be solved graphically too.

Dr. George R. White, Manager, Optical-Electronics Division, Electro-Optical Systems, Inc., Pasadena, California. Lecture given May 24, 1965 with 54 persons attending.

Laser Atmospheric Probes

The interaction of laser radiation with the gaseous and aerosol constituents of the atmosphere offers promise for powerful new atmospheric probing techniques. Analytical work leading towards possible optical determination of pressure, temperature, density, water vapor concentration, $CO_2$ concentration, and wind velocity was discussed. Experimental evidence indicating that laser radiation is less strongly backscattered by aerosol than incoherent light was presented.

(List of Seminars and Lectures, contd.)

Dr. Edward W. Grohse, Professor of Chemical Engineering, University of Alabama, Tuscaloosa, Alabama. Lecture given July 19, 1965, with 17 in attendance.

Fluidization in Recovery of Uranium Fuel

The application of fluidized beds to the recovery of uranium from nuclear fuel elements was described. Experience gained while working on this project at Brookhaven National Laboratories during the summers of 1962 and 1963 was related with illustrations.

Dr. T. F. Bridgland, Jr., Associate Professor of Mathematics, University of South Carolina, Columbia, South Carolina. Lecture given August 6, 1965, with 37 attending.

The Existence of Optimal Feed-Back Controls

The mathematical formulation of a general optimal feed-back control problem, and the application of methods, originally devised by Caratheodory for treatment of classical problems of variational calculus, were discussed. A theorem for existence and uniqueness of extremal arcs was presented.

## APPENDIX II

Publications of University of Alabama Research Institute Staff Members

(All publications are listed; those receiving support from NsG-381 are indicated by an asterisk)

A. Papers published during March 1, 1965, to August 31, 1965:

*N. F. Audeh and H. Y. Yee, "On Dielectric Lenses," Proc. IEEE, Vol. 53, No. 4, April 1965, p. 391.

Abstract: When horns are used for launching electromagnetic waves in parallel plane guides, fringing is inevitably introduced due to the horn walls. A dielectric lens may be used to correct this difficulty. The shape of the lens is determined by the fact that the electrical distance traveled through the air plus that traveled through the lens is constant.

*C. P. Bhalla, "New Calculations of $\beta_4$ (M-Shell) in Tellurium," Proceedings of the International Conference on Internal Conversion Process, Academic Press, 1965.

Abstract: Numerical results for the M-shell internal conversion coefficients for the M4 transition in $Te^{121m}$ (81.78 KeV) are presented. Atomic screening effects (non-relativistic Hartree-Fock self consistent potential) and the finite nuclear size effects are included accurately in the present calculations. The calculated eigenvalues are found to be in good agreement with the experimental electron binding energies. The experimental data compares favorably well with the new theoretical results within the experimental and the theoretical errors. The basis of the calculations are summarized.

C. P. Bhalla, "Positron Shape-Factor of $Co^{56}$ ($4^+ \rightarrow 4^+$)," Nuclear Physics 67, 353 (1965).

Abstract: Experimental beta-shape factor of $Co^{56}$ ($4^+ \rightarrow 4^+$), as reported to be represented by (1 + b/W) by Hamilton, Langer, and Smith, has been analyzed by the inclusion of the second-order effects. Within the framework of the V-A theory, several excellent fits to the experimental positon shape-factor are obtained; thus, the need for a large P interaction and/or for the Uhlenbeck-Konopinski gradient coupling is unnecessary.

(List of Papers Published, contd.)

*C. P. Bhalla, "Theoretical Internal Conversion Coefficients (Magnetic Multipoles) for Z = 52," Handbook of Conference on Nuclear and Particle Physics, University of Liverpool, England, 1965.

Abstract: New calculations of internal conversion coefficients for the 81.78 KeV transition in Tellurium have been completed, in addition to various magnetic multipoles for k = 0.10 ($mc^2$). The present calculations are based on the following realistic physical model. Atomic screening effects both for the continuum and the bound states are included by the self-consistent Hartree-Fock treatment and the finite nuclear size effects are also incorporated. In the numerical integration of the Dirac equation, a large number (6001) of mesh points were used in order to eliminate appreciable cumulative errors. This numerical integration was performed (by using the Hartree-Fock potential) for the continuum radial functions assuring that the appropriate normalization factors have an accuracy of one part in a million. Specific results for the 81.78 KeV transitions are compared below.

| | Experiment | Present Calculations |
|---|---|---|
| $\beta_4(K)$ | [692.3 ± 14] | 692.3 |
| $\beta_4(L_I)$ | 310 ± 16 | 280.1 |
| $\beta_4(L_{II})$ | 67 ± 5 | 64.5 |
| $\beta_4(L_{III})$ | 518 ± 16 | 521.2 |

where the experimental value for K-shell is normalized to the theoretical value. Our theoretical results for the M-shells show an excellent agreement. The ratio of the experimental and our calculated values of $\Sigma L_i/\Sigma M_i = 1.2 \pm 0.08$ (to be compared with a value of $3.95 \pm 0.08$ when Rose's unscreened M-shell values are used). Numerical results for the matrix elements and for the other magnetic multipoles are presented.

*C. P. Bhalla, "Theoretical Results on Internal Conversion Coefficients of M4 Transitions in $Te^{121m}$," Paper No. 34, International Conference on Internal Conversion Process, May 1965.

Abstract: A large number of the ICC measurements in the M-shell incidate a significant discrepancy (by factors as large as three) between the available calculations and the experimental data. The only detailed published calculations for the M-shell are those

of Rose where the effects arising from the atomic screening and the finite nuclear size were neglected. The finite nuclear size effects are not expected to contribute significantly. The screening effects, however, cannot be ignored. For a few cases, the bound state and the continuum radial functions have been calculated by solving the Dirac equation for a self-consistent Hartree-Fock (nonrelativistic) potential. To obtain a quantitative estimate of these effects, new calculations are near completion for the M4 transition in $Te^{121m}$ (81.78 KeV). Numerical results are presented.

R. L. Causey, "Some Mathematical Problems Arising in Information Retrieval," Redstone Scientific Information Report, RSIC-423, June 1965.

Abstract: This report is the result of a study concerning possible applications of mathematical reasoning to the improvement of computer retrieval systems for library use.

The principle objectives of the study were: (1) to investigate in a generalized manner certain aspects of computer retrieval systems which affect the total elapsed time between a request for information and receipt of an answer from the computer; (2) prove that certain programming techniques involve minimum computer time at least with respect to other techniques which might have been used; (3) perform such studies without assuming computer characteristics peculier to a given machine, that is, assume computer characteristics (type of memory, logical organization, etc.) which are variable or which are typical and common to most of the medium to large scale digital computers available today. The data to be retrieved was not considered a variable. Methods for obtaining a defined bibliographic listing and file organization were subject to choice.

*C. F. Chen, "Accurate Determination of Complex Root Transfer Functions from Frequency Response Data," 1965 Joint Automatic Control Conference Record, June 1965, pp. 467-472.

Abstract: This paper presents a method for determining the transfer functions coefficients of a system from its frequency response. The method is based on Bush's transfer function decomposition technique, and Chen-Shen's response conversion formula. It involves no prejudgement and, therefore, has no cumulative error.

*C. F. Chen, "A New Rule for Finding the Breakaway Points of a System with Complex Roots," IEEE Transactions on Automatic Control, Vol. AC-10, No. 3, July 1965.

Abstract: This note attempts to develop a new rule for finding breakaway points if a system involves complex roots. The first part is to show how to reduce a complex-root system into a real-root system graphically. The second part is to develop an iterative procedure to find the breakaway points of a real-root system.

(List of Papers Published, contd.)

*C. F. Chen, "Obtaining System Performance Measures from Routh's Algorithm," Proceedings of the IEEE, Vol. 53, No. 6, June 1965.

Abstract: A Kalman-Bertram Liapunov function is obtained by using the first column of Routh array only. Park's and Schwarz' formulations are simplified.

Hsin Chu, "A Note on Compact Transformation Groups with a Fixed End Point," Proceedings of the American Mathematical Society.

Abstract: Professor A. D. Wallace proved the following: "Let T be a cyclic transformation group of a Peano continuum X leaving fixed an end point, then T has another fixed point." Professor H. C. Wang arrived at the same result by assuming that T is compact and X is an arcwise connected Hausdorff space. In this note, under the same assumption as Wang's we prove that T has countably many fixed points. In fact, we prove the following: THEOREM: Let (X, T, $\pi$) be a transformation group where X is an arcwise connected Hausdroff space. Let A be a closed T-invariant set B, $B \cap A = \phi$, by a point. If there is such a closed set B, then T has at least two distinct fixed points, one of them contained in A. If, furthermore, every orbit, under T, is closed, then T has countably many fixed points.

*Hsin Chu and P. A. Lucas, "An Elementary Problem on Numbers," Pentagon Journal of Mathematics, Spring Issue, pp. 85-91.

Abstract: In this note we consider the following problem: "Let $T_o$ be an unknown number of objects such that no object can be divided into a fractional part. If P/r parts of $T_o$ plus s/r of one object are removed from $T_o$, where $0 < s < r$ and $0 < p < r$, the remainder, $T_1$, has no fraction. If the process is continued n times so that P/r parts of $T_k$, plus s/r of one object are removed from $T_k$, leaving a remainder, $T_{k+1}$ ($k = 0, 1, 2, \ldots, n-1$), which has no fractional parts, the last remainder, $T_n$, will be zero. Can one determine how many objects there were in the beginning." The answer is quite elegant and simple, namely:

(a) If $sr^i$ is not divisible by $(r-p)^{i+1}$ for all $i = 0, 1, 2, \ldots, n-1$, the problem has no solution.

(b) If $sr^i$ is divisible by $(r-p)^{i+1}$ for all $i = 0, 1, \ldots, n-1$, then

$$T_{(o,n)} = \frac{s}{r-p}\left[\frac{\left(\frac{r}{r-p}\right)^n - 1}{\frac{r}{r-p} - 1}\right] .$$

Appendix II

(List of Papers Published, contd.)

Hsin Chu, "Fixed Points in a Transformation Group," Pacific Journal of Mathematics.

Abstract: Professor A. D. Wallace proved the following: "Let $(X, Z, \pi)$ be a transformation group, where Z = the discrete additive group of all integers. If X is a Peano continuum with a fixed end point under Z, then Z has another fixed point." An interesting question has been raised by Wallace: "Can one reach the same conclusion about either compact groups or abelian groups"? In the case of compact groups, Professor H. C. Wang answered the question in the affirmative. We also give an affirmative answer to the question in the case of abelian groups when the abelian group is of the type either $R^n \cdot K$ or $Z \cdot R^n \cdot K$ where $R^n$ is a vector group of dimension n and k is a compact abelian group. Actually, we also cover the case of non-abelian groups. The same conclusion can be reached if the group G is one of the following types: (1) It contains a subgroup $R^n$ such that $G/R^n$ is compact or (2) It contains a subgroup $R^n$ such that $G (Z \cdot R^n)$ is compact.

*R. Douglas Archer (University of New South Wales) and Rudolf Hermann, "Supersonic and Hypersonic Flow of an Ideal Gas Around an Elliptic Nose," AIAA Journal, Vol. 3, No. 5, pp. 987-988, May 1965.

Abstract: A solution is given for the steady flow field of a perfect gas around a two-dimensional elliptic cylinder in an inviscid supersonic free stream. The direct method of Dorodnitsyn and Belotserkovskii has been applied to the inviscid equations of the mixed subsonic-supersonic flow in the shock layer for the general case of a blunt body of any convex shape having continuous curvature. Results of first order theory are presented for a wide range of elliptic shapes and Mach numbers. An explicit form of the equations for infinite Mach number was obtained suitable for numerical calculations. A comparison of the first order theory and experiment in a hypersonic wind tunnel has been made at a Mach number of 6.8 for the case of an elliptic section in which the ratio of major to minor axis is 5, showing very satisfactory agreement.

(List of Papers Published, contd.)

B. T. Chao (University of Illinois) and D. R. Jeng, "Unsteady Stagnation Point Heat Transfer," Transactions of ASME, Journal of Heat Transfer, Series C, pp. 221-230, May 1965.

Abstract: An analysis is presented for the unsteady laminar, forced-convention heat transfer at a two-dimensional and axi-symmetrical front stagnation due to an arbitrarily prescribed wall temperature or heat flux variation. The flow is incompressible and steady. The procedure begins with a consideration of the thermal boundary-layer response caused by either a step change in surface temperature or heat flux. Two appropriate asymptotic solutions, valid for small and large times, respectively, are found and satisfactorily joined for Prandtl numbers ranging from 0.01 to 100. The key to the small time solution is the transformation of the energy equation in the Laplace transform plane to an ordinary differential equation with a large parameter. An essential feature of the large time solution is the use of Meksyn's transformation variable and the method of steepest descent in the evaluation of integrals.

It is found that, for both two-dimensional and axi-symmetrical stagnation, the time required for the thermal boundary layer to attain steady condition, for either a step change in surface temperature or heat flux, varies inversely with the free stream velocity and directly with 1/4 power of the Prandtl number of the fluid.

C. D. Johnson and W. M. Wonham (Brown University), "On a Problem of Letov in Optimal Control," ASME Transactions, Journal of Basic Engineering, 87, Ser. D: 81-89, March 1965.

Abstract: In a series of papers, A. M. Letov discussed an optimal regulator problem for a linear plant with bounded control variable and quadratic performance index. This problem was also discussed by Chang. Krasovskii and Letov observed later that the solution proposed in these papers may be correct only for special choices of the initial value of the state vector. In the present note, further aspects of the solution in the general case are described and three examples are given. The possible existence of a regime of unsaturated-nonlinear optimal control is demonstrated. The presence of this regime in the optimal control law was apparently overlooked in the papers mentioned above.

Appendix II

(List of Papers Published, contd.)

*F. H. Mitchell, Jr., "Microwave Propagation in Inhomogeneous Plasma," paper presented at First IEEE Annual Communications Convention in Boulder, Colorado, June 7-9, 1965.

Abstract: Microwave propagation in plasma sheaths is discussed in detail and several methods for studying the problem are described. Both homogeneous and inhomogeneous plasmas are considered and representative models are analyzed. The polarizability and conductivity of a plasma are examined and the electromagnetic fields computed for several cases of interest.

*F. H. Mitchell, Jr., and F. J. Tischer, "Radiation Through Plasma Adjoining a Conducting Surface," Transactions of IEEE Group on Antennas and Propagation (Communication), May 1965.

Abstract: A method for the study of electromagnetic wave propagation on inhomogeneous plasma media is described, where the solution is in the form of an integral recursion relation. The technique is most useful for weakly ionized plasmas. An appropriate example is worked out and the results discussed briefly.

J. T. Oden, "Bending of Corner-Supported Plates with Arbitrary Edge Conditions," Developments in Theoretical and Applied Mechanics, II, Pergamon Press, London, 1965.

Abstract: A general method for analyzing thin rectangular plates supported at their corners is presented. The small deflection theory is employed and solutions are given for plates subjected to a general system of loads and to arbitrary moments and forces distributed along their edges. Solutions are obtained in the form of trigonometric series and numerical results for special loading cases are included. Theoretical deflections for some loadings are compared with experimental values obtained from tests run on small-scale models.

Abrahim Lavi (Carnegie Tech) and R. J. Polge, "Induction Motor Speed Control with Static Inverter in the Rotor," IEEE Summer Power Meeting, Detroit, Michigan, June 1965.

Abstract: A speed regulating scheme using a wound rotor induction motor and a static synchronous inverter is investigated. The purpose of the inverter is to receive the slip power from the rotor and to deliver it to the a-c line.

The steady state and the transient performances of the system are studied and compared with experimental results. The dependence of system performances upon the design of the inverter and the commutating filter inductance is presented and design criteria are developed.

The resulting motor inverter system has a linear torque current relation independent of the speed. In this respect the system has the characteristic of a separately excited d-c motor.

(List of Papers Published, contd.)

G. A. Wempner, and J. L. Baylor (University of California), "General Theory of Sandwich Plates with Dissimilar Facings," Int. J. Solids Structures, Vol. 1, pp. 157-177, 1965.

Abstract: Equations are derived for the large deflections of sandwich plates with weak cores and presented in an invariant form. The general equations include the bending resistance of the facings and transverse extension of the core. Equations for buckling and for small deflections are obtained as special cases. An example illustrates the use of these equations to predict buckling loads.

*H. Y. Yee and N. F. Audeh, "Cutoff Frequencies of Waveguides With Arbitrary Cross-Sections," Proceedings of the IEEE, Vol. 53, No. 6, June 1965, pp. 637-638.

Abstract: The point matching method for solving the waveguide problems is useful when the method of separation of variables fails for waveguides with complex cross sections. By this method, the cutoff frequencies can be calculated, by matching the boundary conditions only at a finite number of points around the boundary of the waveguide.

*H. Y. Yee and F. J. Tischer, "On the Wave Propagation in Nonuniform Media," Proceedings of the IEEE, Vol. 53, p. 488, May 1965.

Abstract: The problem of wave propagation in nonuniform media is considered by a new approximate method. This method is applicable to permittivities of dielectric media with variation in only one spatial coordinate.

*H. Y. Yee, "Natural Resonant Frequencies of Microwave Dielectric Resonators," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-13, p. 256, March 1965.

Abstract: An approximate method is developed for determining the natural resonant frequencies of microwave dielectric resonators. This method is based on the approximate open-circuit boundary conditions at surfaces between air and the dielectric with high permittivity. Experimental results show good agreement with the theoretical calculations.

(List of Papers Published, contd.)

*H. Y. Yee, "An Approximate Method for the Computation of Scattering By Conducting Cylinders With Arbitrary Cross-Section, NASA Contractor Report, CR-282, August 1965.

Abstract: The Scattering of electromagnetic waves by conducting cylinders with arbitrary cross sections are computed by an approximated method called the point-matching method. The theory is confirmed by low frequency scattering and numerical examples. However, this method is not applicable to the scattering by an infinite strip. The low frequency scattering shows also that for a known scattered field the cross-section of the scatterer can be found.

H. Y. Yee, Approximate Methods for the Computation of Wave Propagation in Nonuniform Media," NASA Contractor Report, CR-283, August 1965.

Abstract: Applications of the collocational method to wave propagation through a nonuniform region with variation in only one spatial coordinate are shown for plane and cylindrical cases. Scattering and radiation, in the absence of and in the presence of a similar shaped conducting object are formulated. A simple example shows the accuracy of this method. In cases where the nonuniform region varies in more than one spatial coordinate, Green's function is applied to formulate an integral equation. Solutions of the integral equation can be obtained by an iterative method for small variations.

B. Papers accepted for publication during March 1, 1965, to August 31, 1965

*N. F. Audeh and H. Y. Yee, "Measurements of Cutoff Frequencies," to appear in IEEE Transactions on Microwave Theory and Techniques.

Abstract: The cutoff frequency of a waveguide of any cross sectional shape can be found by the resonance method. The waveguide under test is shorted on both ends thus a resonant cavity is formed. Energy is coupled in through an iris. From the knowledge of two resonant frequencies, the cutoff frequency of the waveguide is determined.

*C. P. Bhalla, "A Review of the Theory of Internal Conversion Process," Invited Talk at the Southeastern Section Meeting of the American Physical Society, 1965.

Abstract: Standard works of Rose in the U.S.A. and Sliv and Band in the U.S.S.R. are reviewed. Special emphasis is placed on the new information, which has been made possible because of the sophistication in experiments. The data low-energy transition appears to be not in agreement with either of these calculations. This is because of the sensitivity of theoretical results to the details of the atomic potential. The bases of our calculations are presented. New results for the M-subshells are found to be in excellent agreement with experimental data.

*C. P. Bhalla and Frances Yen, "Normalization Factors of Relativistic Continuum Functions for Screened Coulomb Potential," Southeastern Section Meeting of the American Physical Society, 1965.

Abstract: In the numerical integration of Dirac equation for a screened Coulomb potential, certain difficulties arise in obtaining the appropriate normalization. For example, $rf \rightarrow -[(W-1)/\pi p)]^{1/2} \sin(pr + \delta)$ is valid only when $r \rightarrow \infty$. It is not possible to consider such large distances in a numerical calculation. However, the WKB approximation may be useful in obtaining a relation for the amplitude in the asymptotic region to the amplitude at $\infty$. Pertinent numerical results are presented. An explanation of errors in numerical values in Fermi functions of earlier calculations is offered.

*C. P. Bhalla (co-authors C. L. Tolliver and W. R. Garrett), "Self-Consistent Relativistic Hartree-Fock Wave Functions," Southeastern Section Meeting of the American Physical Society, 1965.

(List of Papers Accepted, contd.)

Abstract: Self-consistent solutions of the Dirac equation for bound states are obtained in the Slater exchange approximation to the Hartree-Fock method. Finite nuclear size effects, which have been ignored in all earlier works, are included in the present calculations. Standard procedure of matching F/G is used and the degree of mismatch is taken to be less than one part in ten thousand. The sensitivity of radial functions and eigenvalues to the inclusion of these effects is presented. The relative contributions of the exchange term, finite size effects and relativistic corrections are discussed.

* C. F. Chen and C. Hsu, "The Determination of Root Loci Using Routh's Algorithm," to appear in Journal of Franklin Institute.

Abstract: This paper develops a new procedure simply using Routh's algorithm and coordinate transformation to determine the root loci of a feedback system. Since the method is algorithmic in nature, a digital computer can best be used to provide rapid solutions.

J. B. Kidd (Applied Physics Laboratory), T. E. Edgerton (Spaco), C. F. Chen, "Transfer Function Synthesis in Time Domain," to appear in Institute of Electrical and Electronic Engineers Transactions on Education.

Abstract: This paper presents a digital computer method for accurately determining the coefficients of a transfer function from its impulse response. The method consists of transforming higher order approximations of the various regions of the impulse response to a frequency dependent function by using a gate function technique and then minimizing the error between this function and a ratio of two polynomials.

Hsin Chu, "A Remark on Complete Controllability," to appear in SIAM Journal on Control.

Abstract: Consider a plant defined by

$$\dot{X} = AX + bU$$
$$Y = c \cdot X$$

where b and c are constant n-dimensional column vectors. The plant has an input U and an output Y. In this note, we show that the plant is always completely controllable (or completely observable) for every nonzero vector b (or c) if and only if $n = 2$ and the characteristic roots of A are complex. We also give a different and somewhat seemingly shorter proof of the following well-known result of Kalman's: The plant is completely controllable (or completely observable) for some b (or c) if and only if in the Jordan canonical form of A no two blocks are associated with the same eigenvalue.

W. R. Garrett (co-authors C. L. Tolliver and C. P. Bhalla), "Self-Consistent Relativistic Hartree-Fock Wave Functions," to be presented at the Southeastern Section Meeting of the American Physical Society, November 1-3, 1965, University of Virginia, Charlottesville, (see abstract above).

(List of Papers Accepted, contd.)

*W. R. Garrett, "Polarization and Exchange Effects in Slow Electron Scattering from Lithium and Sodium," Physical Review, Vol. 140, November 1965.

Abstract: Total elastic scattering cross sections have been calculated for low energy electrons incident on lithium and sodium. The effects of target distortion by the electric field of the incident particle and electron exchange between incident and bound electrons have been included through the "adiabatic exchange" approximation. In this method, the effect of the distortion induced in the target atom is computed through a polarization potential which is obtained by the method of polarized orbitals. The phase shifts and the total elastic scattering cross sections are obtained through the solution of a set of integrodifferential equations. The calculated polarization potential agrees asymptotically with experimental determinations of the dipole polarizability of Li and Na, and the total elastic scattering cross sections agree well with recent measurements over the entire experimental range. Results for both alkalis exhibit a small resonance at the experimental peak around 1 volt and a second peak at lower energies outside the present experimental range.

W. R. Garrett, "Atomic Polarizabilities in Ground and Excited States," to be presented at the Southeastern Section Meeting of the American Physical Society, November 1-3, 1965.

Abstract: Through an extension of the method due to Sternheimer, atomic dipole polarizabilities are computed for atomic oxygen and nitrogen in their ground state and in several excited states. In the present treatment the excited state wave functions are determined from a modified Hartree-Fock-Slater program whereby binding energies are made to agree with experimental values through variation of the Slater exchange term. The dipole polarizability in a given configuration is then determined through a first order perturbation calculation on the system in its excited state. Excellent agreement with experiment is obtained where available, thus establishing a reasonable degree of confidence in results for excited states where no experimental results exist. (Oxygen - $\alpha$ = .77 $\text{Å}^3$ present, $\alpha = .77 \pm .06$ $\text{Å}^3$ experimental; nitrogen - $\alpha$ = 1.08 $\text{Å}^3$ present, $\alpha = 1.13 \pm .06$ $\text{Å}^3$ experimental.) For excited states the atomic dipole polarizabilities become quite large, as one would expect, increasing by a factor of 100 or more for the first few excited states of oxygen and nitrogen.

(List of Papers Accepted, contd.)

*W. R. Garrett and H. T. Jackson, "The Photodetachment Cross Section for $O^-$," to be presented at the Eighteenth Annual Gaseous Electronics Conference in Minnesota, October 20-22, 1965.

Abstract: The theoretical photodetachment cross section for the negative atomic oxygen ion has been calculated for the three transitions: $O^-\,^2P$ to $O\,^3P$, $O\,^1D$, and $O\,^1S$. Results are compared with the experiments of Smith and Branscomb, Smith and Tisone giving excellent low energy agreement. Calculations differ from previous models of Klein and Brueckner, and Cooper and Martin by developing the polarization potential from first order perturbation theory in conjunction with the adiabatic approximation. A modified Hartree-Fock treatment utilizing the Slater approximation for exchange is used to compute the bound state radial functions for the neutral atom and negative ion. In addition, the scattering cross section for neutral oxygen, the polarizability, the attachment cross section and attachment coefficient for electron capture by neutral oxygen were also determined. The scattering cross section is compared with experimental data.

* A. A. Hayday, "On Axioms for Heterogeneous Continua," Zeitschrift fur angewandte Mathematik und Physik, Basel, Switerzerland.

Abstract: The paper deals with the derivation of general conservation equations for heterogeneous continua--substances consisting of several distinct, possibly interacting constituents. The analysis, resting for the most part on an admissible superposition of the diffusive motions of the postulated motion of the mixture is based on axioms in integral form from which the basic differential equations are deduced. The treatment is a rigorous, direct generalization of the classical theory for simple continua. The main results are compared with those of Truesdell and found to be in entire agreement.

*Rudolf Hermann and Jurgen Thoenes, "Hypersonic Flow of Air Past a Circular Cylinder with Non-Equilibrium Oxygen Dissociation Including Dissociation of the Free Stream," to be presented at the VI. European Aeronautical Congress, Munich, Germany, September 1-4, 1965.

Abstract: Hypersonic, chemically relaxing inviscid flow of air past a circular cylinder has been calculated using Dorodnitsyn's method of integral relations. Considered are the effects of non-equilibrium oxygen dissociation on the distribution of the flow variables in the subsonic and supersonic region of the shock layer. Also investigated is the influence of oxygen dissociation in the free stream on the shock detachment distance and the flow field in general.

(List of Papers Accepted, contd.)

Particular emphasis has been placed on the investigation of the flow along the stagnation streamline, which indicates the existence of an equilibrium region in front of the body. Its size depends strongly on the density, on the body size, and on the magnitude of the reaction rate constants. The numerical integration of the equations for the flow around the body does not readily yield the shock detachment distance on the stagnation streamline. An iteration procedure has to be used until a stagnation shock detachment distance is found such that the derivatives of all variables are continuous across the sonic line.

The results show that the shock wave, at fixed free stream composition, moves closer to the body with increasing free stream Mach number, that is with larger oxygen dissociation behind the shock. However, increasing oxygen dissociation in the free stream, at otherwise fixed free stream conditions, pushes the shock wave away from the body.

* Rudolf Hermann, "Hypersonic Aerodynamic Problems at Re-Entry of Space Vehicles," invited lecture at the IV. Astronautics Symposium, Goettingen, Germany, October 18-22, 1965.

Abstract: This survey paper first discusses briefly the properties and composition of the atmosphere and examines the flight corridors for earth re-entry of various type vehicles and missions. The real gas effects of molecular vibration, dissociation and ionization, and the effects of equilibrium ionization on re-entry communication are discussed. The non-equilibrium flow fields about the forward portions of a circular cylinder, a sphere, and a right circular cone were calculated, applying the integral method by Dorodnitsyn in one-strip approximation and using a simplified air model, valid in the thermodynamic regions from perfect gas through 99% oxygen dissociation. The results show that real gas effects are noticeable beginning at Mach number above 3 and are very pronounced at Mach number 14 in atmospheric flight. The strong effect of free-stream dissociation on many flow parameters is shown for the first time.

(List of Papers Accepted, contd.)

*J. H. Kallweit, "The Relationship Between the Viscosity and the DC Conductivity in PVC," Journal of Polymer Science.

Abstract: In general, it has been said that the dc-conductivity $\sigma$ is proportional to the inverse of the viscosity $\eta$. But the relation $\sigma \cdot \eta$ = constant has not been proved in the case of the system PVC/DOP. The experimental results can be described by $\sigma \cdot \eta^{(m/(m-1))}$ = constant if the plasticizer content is not too high. The ion mobility seems to depend on a local effective viscosity, which differs from that viscosity which is derived from retardation experiments.

*H. Y. Yee, "Scattering of Electromagnetic Waves by Circularly Dielectric-Coated Conducting Cylinders with Arbitrary Cross-Section," IEEE Transactions on Antennas and Propagation, Vol. AP-13, No. 5, p. 822, September 1965.

Abstract: An approximate solution of electromagnetic scattering properties of circularly dielectric coated conducting cylinder with arbitrary cross-section is discussed.

The boundary conditions at the circular dielectric surface is satisfied exactly while the intersurface satisfies the boundary conditions approximately through the application of the point-matching method. The incident wave can be either a plane wave or a line source at a finite distance. Similar analysis will lead to the solution of line sources inside the dielectric medium.

*H. Y. Yee, "Application of Point-Matching Method to Low-Frequency Scattering by Conducting Cylinders," IEEE Transactions on Antenna and Propagation, Vol. AP-13, No. 5, p. 818, September 1965.

Abstract: The electromagnetic scattering properties of conducting cylinder with arbitrary cross-section can be obtained easily by a numerical method, called point-matching. The validity of this technique is justified from a qualitative point of view, especially for low frequency approximation. From this justification, the applicability of the point-matching method to a given problem can be readily determined.

*H. Y. Yee and N. F. Audeh, "Uniform Waveguides with Arbitrary Cross-Section Considered by the Point-Matching Method," to appear in IEEE Transactions on Microwave Theory and Techniques.

Abstract: The point-matching method applies to the problem of wave propagation in many uniform waveguides of very general cross-sections. The boundary conditions are satisfied at a finite

number of points on the guide wall only. This method applies when the contour of the cross-section of the guide is a closed curve, the function of which is single-valued. The validity of the point-matching method is demonstrated qualitatively. Examples show that accurate values of cut-off wave numbers can be achieved easily.

*H. Y. Yee and N. F. Audeh, "Wave Propagation in Groove Guides," to be presented at the National Electronics Conference on October 25, 1965, in Chicago, Illinois.

Abstract: A groove guide consists of two parallel conducting plates with two grooves cut at the center in the longitudinal direction. The air filled deformed guide can be transformed into a parallel-plane guide filled with a nonuniform anisotropic dielectric. The posed boundary-value problem was solved by approximation techniques; the results were experimentally verified. The groove guide has the advantages of transporting most of the energy in the groove region, having a very low attenuation constant and, under certain conditions, propagating the dominant modes only.

C. Papers submitted for publication during March 1, 1965, to August 31, 1965

C. F. Chen, "A New Formula for Obtaining the Inverse Laplace Transformation in Terms of Laguerre Functions," submitted to IEEE 1966 International Convention.

Abstract: A new formula for performing the inverse Laplace transform in terms of Laguerre functions is established. Compared with Lanzcos, or Papoulis' method, the new formula requires much fewer calculations.

*C. F. Chen and Hsin Chu, "A Matrix for Evaluating Schwarz' Form," submitted to the Institute on Electrical and Electronic Engineers Transactions on Automatic Control.

Abstract: A matrix [P] is developed by using a phase variable form which can be easily converted into a Schwarz' form. Schwarz original numerical method is much simplified.

*C. F. Chen and Hsin Chu, "A Matrix for Evaluating Schwarz' Form, Part II," submitted to the Institute of Electrical and Electronic Engineers Transactions on Automatic Control.

Abstract: It is shown that the inverse of the conversion matrix can be obtained from Routh's Algorithm directly.

Hsin Chu, "A Note on the Method of Samuelson," submitted to Numerische Mathematik.

Abstract: In the note, he finds that the celebrated method of Samuelson (e.g., see P. A. Samuelson, "A Method of Determining Explicitly the Coefficients of the Characteristic Equation," Ann. Math. Stat., Vol. 13, pp. 424-429, 1942), is good only for a certain class of matrices. For example, if the degree of the minimal polynomial of a given matrix is less than the order of the matrix, then his method will fail to obtain the characteristic polynomial. In this note, an exact condition is established for matrices to which Samuelson's method can be applied.

Hsin Chu, "A Parallelepiped on Linear Dynamical Systems," submitted to SIAM Review.

Abstract: An n-dimensional parallelepiped of n vectors $X_1, X_2, \ldots, X_n$ in an n-dimensional real vector space $V_n(R)$ is a set which consists of all vectors of the form $a_1X_1 + a_2X_3 + \cdots + a_nX_n$ where $0 \leq a_i \leq 1$, $i = 1, 2, \ldots, n$. It is known that the volume of an n-dimensional

parallelepiped is equal to the determinant of $X_1, X_2, \ldots, X_n$, $D(X_1, X_2, \ldots, X_n)$. In this paper the volume, $W(x)$, of vectors, $X, AX, A^2X, \ldots, A^{n-1}X$, where $A = (a_{ij})$ is an nxn matrix over the field of real numbers is considered.

*A. A. Hayday and B. R. K. Choudary, "An Integral Method for the Solution of Boundary Layer Equations: Part 1 - Examples for Two-Dimensional Flows," submitted to ASME for the Winter Annual Conference 1965.

Abstract: The paper describes a generalization of the classical integral method for solving boundary layer equations. The method presented finds its main applications in the solution of flow problems governed by multiply coupled conservation equations. Basic parameters associated with the fluid (such as Prandtl number) may vary greatly. Intended for engineering applications, the method is kept simple. We illustrate its use and test its accuracy by considering a coupled equation system governing free convection from isothermal and non-isothermal vertical plates. The Prandtl number range covers both liquid metals and oils. Results based on the solution of the example problems are shown to be more accurate than those obtained by other approximate methods.

D. R. Jeng and B. T. Chao (University of Illinois), "Contribution to Unsteady Heat Transfer in Laminar Boundary Layers; Part I, Unsteady Heat Transfer in a Magneto-Hydrodynamic Three-Dimensional Stagnation-Point Flow with Simultaneous Suction or Blowing," submitted to the International Journal of Heat and Mass Transfer.

Abstract: In part I, consideration is given to the unsteady heat transfer in steady, incompressible, laminar boundary layer of an electrically conducting fluid at a three-dimensional front stagnation of regular surfaces under the action of a magnetic field normal to the wall with simultaneous suction or blowing. Expressions describing the thermal response behavior of the boundary layer due to either a step change in surface temperature or heat flux are given. Numerical results in graphical form are presented for Prandtl numbers ranging from 0.01 to 100 and in the absence of magnetic field and mass transfer. It is found that the time required for the thermal boundary layer to attain steadiness varies directly with 1/4 power of the Prandtl number and inverse with the product of 1/4 power of the major principal velocity gradient just outside the boundary layer and 3/4 power of the sum of the major and minor principal velocity gradients. Numerical results are also presented for magnetic-hydrodynamic flow of Prandtl

number of 0.01 and 0.7 at two-dimensional and axisymmetrical stagnation with simultaneous blowing. It is found that blowing and magnetic field interact in a rather complicated manner in increasing the response time. The influence due to blowing becomes greater with the larger Prandtl number but the converse is true for the effect of magnetic field on the response time.

D. R. Jeng and B. T. Chao (University of Illinois), "Contributions to Unsteady Heat Transfer in Laminar Boundary Layers; Part II, Unsteady Heat Transfer from a Rotating Disk and Cone with Suction or Blowing," submitted to International Journal of Heat and Mass Transfer.

Abstract: Part II concerns with an analysis for laminar boundary layers over rotating discs and cones including viscous dissipative effects and with boundary layer suction or blowing. Incompressible flow over a rotating disc is first examined. The solution for the two-dimensional stagnation obtained in Part I can be utilized after an appropriate change of the velocity function. For sufficiently strong suction, a solution is presented which is valid for the entire time-history of the non-steady heat transfer process due to a step change in wall temperature. It is found that, under such circumstances, the response time varies inversely with the Prandtl number, the angular speed and the second power of the suction parameter. Results of an analysis for the rotating cone show that the response time is $1/\sin\phi$ times that for a rotating disc for identical Prandtl number and angular speed of rotation and when the mass injection or suction velocities are in the ratio of $(\sin\phi)^{1/2}$, $2\phi$ being the opening angle of the cone. The foregoing statement is subject to the restriction that $\phi$ is sufficiently large and the suction or injection velocity is sufficiently small such that the flow retains the boundary layer character. Finally, it is shown that for the compressible flow of an ideal gas of constant Prandtl number and linear viscosity-temperature relation, the incompressible results may be directly applied provided that all properties are referred to the wall condition.

With the exception notes in Part II, the general procedure of analysis rests with finding two appropriate asymptotic solutions, respectively valid for small and larger times. The small time solution hinges on the transformation of the energy equation in the Laplace transform plane to an ordinary differential equation with a large parameter. The large time solution makes use of Meksyn's boundary layer variable and the method of steepest descent in the evaluation of integrals. It is demonstrated that the joining of the two solutions is satisfactory for all cases.

(List of Papers Submitted, contd.)

B. T. Chao (University of Illinois) and D. R. Jeng, "Unsteady Heat Transfer for Laminar Flow over a Flat Plate," submitted to Journal of Fluid Mechanics.

Abstract: The thermal response of a laminar boundary layer over a flat plate due to step temperature change is investigated. At a large distance away from the plate, the fluid has a constant velocity $U_\infty$ parallel to it. Incompressible flow is considered first and two asymptotic solutions for the wall flux variation, valid respectively for small and large times, are found. These solutions overlap and are well joined. For an ideal gas with constant Prandtl number whose viscosity varies linearly with temperature, the incompressible results may be directly applied provided that all properties in the heat transfer parameter are replaced by their wall values. It is demonstrated that, for both cases, the time required to obtain steadiness varies inversely with the free stream velocity and directly with the distance from the leading edge and with 1/3 power of the Prandtl number of the fluid.

C. D. Johnson, "A Note on Nonlinear Control Systems with One Nonlinear Element," submitted for publication.

Abstract: Several different "normal forms" have been used to study nonlinear control systems having one isolated nonlinearity. This note proposes a new normal form which offers certain advantages over those currently in use.

D. The interim and final reports on contracts and specific grants during March 1, 1965 to August 31, 1965

R. L. Causey, Quarterly Progress Report for period April 1, 1965 - June 30, 1965, "Numerical Analysis of Methods for Computing Three-Dimensional Supersonic Flow," DA-01-021-AMC-11901(Z).

Hsin Chu, Quarterly Technical Progress Reports for the periods December 1, 1964 - March 1, 1965; March 1, 1965 - June 1, 1965; June 1, 1965 - September 1, 1965; "Topological Dynamics," NAS8-1646.

G. D. Cole and J. C. Chang, Monthly Progress Reports for the periods March 1, 1965 - April 30, 1965; May 1 - 31, 1965; July 1-31, 1965; "Parameter Optimization," NAS8-5411.

G. D. Cole and J. C. Chang, Interim Report No. 6, Feburary 1, 1965 - June 30, 1965, "Parameter Optimization," NAS8-5411.

C. D. Johnson, Quarterly Report for the period January 1, 1965 - April 1, 1965, "Basic Research in the Mathematical Theory of Automatic Control," DA-01-009-AMC-165(Z).

C. D. Johnson, Monthly Progress Reports for March, April, May, June, July, "Study of Optimal and Adaptive Control Theory," NAS8-11231.

R. A. Mann, Quarterly Technical Report for the period December 15, 1964 - March 15, 1965, "Investigation of Surface Ionization on Metallic Surfaces," NAS8-2585.

R. A. Mann, Quarterly Progress Report for the period November 28, 1964 - February 28, 1965, "Interactions of Slow Electrons with Light Gas Atoms," NAS8-11249.

V. McCarty, Quarterly Technical Progress Reports for the periods December 22, 1964 - March 22, 1965; March 22, 1965 - May 22, 1965; May 22, 1965 - August 22, 1965, "Investigation of Earth Tremors During Large Rocket Firings," NAS8-11202.

W. F. Arendale, Quarterly Status Report No. 1, for the period May 11, 1965 - August 15, 1965, "Study of Reflected Energy from Laser Illuminated Targets," DA-01-021-AMC-12250(Z).

E. University of Alabama Research Institute Research Reports during March 1, 1965 to August 31, 1965

*W. R. Garrett, "Adiabatic Exchange Approximation in Electron Scattering from Multi-Electron Atoms," U.A.R.I. Research Report No. 19, August 1965.

*J. H. Kallweit, "The Relationship Between the Viscosity and the DC-Conductivity in Polyvinylchlorid," U.A.R.I. Research Report No. 20, March 1965.

*H. Y. Yee, "An Approximate Method for the Computation of Scattering by Conducting Cylinders with Arbitrary Cross-Section," U.A.R.I. Research Report No. 22, March 1965. Also identified as NASA Contract Report CR-282, August 1965.

*A. A. Hayday, "On Axioms for Heterogeneous Continua," U.A.R.I. Research Report No. 23, May 1965.

F. H. Mitchell, Jr., "Radiation Through Plasma Adjoining a Conducting Surface," U.A.R.I. Research Report No. 24, April 31, 1965.

Jurgen Thoenes, "Inviscid High Temperature Hypersonic Flow of Air Past Pointed Bodies of Revolution," U.A.R.I. Research Report No. 25, May 1965.

# APPENDIX III

## Present Research Efforts and Planned Continuations at the University of Alabama Research Institute During the Period March 1, 1965 to August 31, 1965

### Outline

1.0 MATHEMATICS

- 1.1 Mathematical Analysis and Digital Computer Sciences. Dr. Causey
- 1.2 Topological Dynamics and Related Areas. Dr. Chu
- 1.3 System Analysis. Mr. Chen

2.0 PHYSICS, INCLUDING CHEMICAL PHYSICS AND OPTICS

- 2.1 Investigations in Theoretical Atomic Physics
  - 2.1.1 Low Energy Electron Scattering. Dr. Garrett
  - 2.1.2 Ion-Photon Processes. Dr. Garrett
  - 2.1.3 Atomic Polarizabilities. Dr. Garrett
  - 2.1.4 Slow Electron Inelastic Scattering. Dr. Mann
- 2.2 Theoretical Studies of Nuclear and Atomic Structure. Dr. Bhalla
  - 2.2.1 Nuclear Beta Decay
  - 2.2.2 Nuclear Structure Calculations
  - 2.2.3 Relativistic Atomic Studies
  - 2.2.4 Electron Shielding
  - 2.2.5 Graduate Students Supervision
- 2.3 Surface Physics and Related Areas. Dr. Mann
- 2.4 Coherent Light. Dr. Mann
- 2.5 Chemical Physics. Interaction of Electromagnetic Energy with Matter. Dr. Arendale

3.0 CONTROL SCIENCES AND HYBRID COMPUTATION

- 3.1 Studies in Control Theory. Dr. Johnson
- 3.2 Development and Use of a Hybrid Analog Computer Facility. Dr. Johnson

4.0 ELECTROMAGNETICS, INCLUDING COMMUNICATION

- 4.1 Wave Propagation in Groove Guide. Dr. Audeh
- 4.2 Scattering Problems in Electromagnetic Propagation. Mr. Yee
- 4.3 Communication. Research on an Adaptive Device for Communications. Dr. Polge
- 4.4 Evaluation of Magnetic Influences on Gyros. Dr. Dowdle

5.0 FLUID MECHANICS, GAS DYNAMICS, AND HEAT TRANSFER

- 5.1 Unsteady Heat Transfer in Laminar Boundary Layers. Dr. Jeng
- 5.2 Transpiration Cooling in a Magneto-Hydrodynamic Three-Dimensional Stagnation Flow. Dr. Jeng
- 5.3 Hypersonic High-Temperature Gas Flow. Dr. Hermann, Dr. Brainerd

6.0 STRUCTURAL MECHANICS

- 6.1 Numerical Analysis of Nonlinear Structures. Dr. Oden
- 6.2 Research in Nonlinear Analysis of Shell Structures. Dr. Wempner
- 6.3 Development of the Structural Mechanics Laboratory. Dr. Kubitza

Summaries of research performed and plans for continuation where applicable are given below. Listed are the topics which received at least partial support of NsG-381. The topics, in lower case, are grouped under the major disciplines indicated by capital letters.

## 1.0 MATHEMATICS

### 1.1 Mathematical Analysis and Digital Computer Sciences

Dr. R. L. Causey, Associate Professor of Mathematics and Head, Computer Sciences Laboratory, is presently concerned with research in mathematical analysis and computer science. He is charged with the coordination and development of the University's computing activities utilizing the UNIVAC 1107 center located at the Research Institute. Time on the 1107 is available to any faculty member or staff member of the University of Alabama system. The Research Institute staff members have been the principal users, but there are also many users at the Tuscaloosa campus who enjoy direct communication with the 1107 via a UNIVAC 1004 and data-phone in the University's Computing Center at Tuscaloosa.

Research activities anticipated for the third grant year include study of (1) numerical methods for solving nonlinear ordinary and partial differential equations, (2) approximation problems in matrix spaces, and (3) computer methods for solving problems involving very large matrices.

Efforts will be made to obtain extensions of Dr. Causey's thesis[1]. Numerical experiments will be conducted on the UNIVAC 1107 in an attempt to find minimum sequences of approximating normal matrices. As a separate investigation, problems of mechanization of the solution of systems of linear algebraic equations with very large number of equations (more than 500) are anticipated.

Information retrieval from inverted files is a recurring problem which may involve a very considerable amount of computer time, simply due to a very large number of requests. A study of mathematical problems, applicable to several different computers, involved in computer-mechanized retrieval in the least amount of computer running time has been completed. The following report is now available from Redstone Scientific Information Center: R. L. Causey, "Some Mathematical Problems Arising in Information Retrieval from Inverted Files," Rep. No. RSIC-423.

Personnel assisting Dr. Causey in many of these activities have been Mr. Ronald Fischer and Mr. Samuel Ross, Numerical Analysts and members of the Computer Sciences Laboratory, who have had partial support from NsG-381. They have assisted other University staff in putting their problems on the UNIVAC 1107 Computer and have provided complete programming services in certain instances for University staff in Huntsville.

## 1.2 Topological Dynamics and Related Areas

Dr. H. S. Chu, Professor of Mathematics, assisted by Miss P. A. Lucas, Research Assistant, has been doing research in the field of topological dynamics. Dr. Chu feels that the research is best summarized by the following papers:

I. A paper, "An Elementary Problem on Numbers", by Hsin Chu and P. A. Lucas, has been accepted for publication by Pentagon, and it will appear in the coming Spring issue.

II. A paper, "A Note on the Method of Samuelson," by Hsin Chu, is completed. In the note, he finds that the celebrated method of Samuelson, (e.g. see P. A. Samuelson, "A Method of Determining Explicitly the Coefficients of the Characteristic Equation," Ann. Math. Stat. Vol. 13, pp. 424-429, 1942 ) is good

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[1] Causey, Robert L., "On Closest Normal Matrices," Ph.D. Dissertation, Stanford University (May, 1964)

only for a certain class of matrices. For example, if the degree of the minimal polynomial of a given matrix is less than the order of the matrix, then his method will fail to obtain the characteristic polynomial. In this note, an exact condition is established for matrices to which Samuelson's method can be applied.

III. A paper, "A Method for Finding the Minimal Polynomial of a Square Matrix," by Hsin Chu is completed. One of the advantages of this method is that it can be performed by either a computer or a desk calculator.

IV. A paper, "A Remark on Complete Controllability," by Hsin Chu is completed. It is a geometric approach to this subject.

V. A paper, "A Method for Finding the Minimal Polynomial of a Vector," by Hsin Chu is completed. In this note, he gives a simple method for finding the minimal polynomial of an n-dimensional vector over a given field with respect to a linear operator A.

Dr. Chu has accepted a visiting professorship at the University of Maryland for one year beginning in September 1965. During Dr. Chu's absence, Mr. C. F. Chen plans to continue research in areas related to topological dynamics.

### 1.3 System Analysis

Mr. C. F. Chen, Research Associate, has achieved several significant results in his studies of system theory. First, the identification of a system with complex poles has been solved. The report was presented at the Fifth Joint Automatic Control Conference in New York. Secondly, Ralston's symmetrical Hurwitz-Routh criterion is rederived by establishing a new matrix which enables us to convert a phase variable form into a Schwarz form. Thirdly, a rule for the root locus method is developed. The report is published in IEEE Transactions on Automatic Control. Fourthly, a new formula for performing the inverse Laplace transform in terms of Laguerre functions is constructed. Compared with the old formulas, it needs less than half of the terms Lanczos uses.

Mr. Chen plans to extend the Laguerre polynomial technique for evaluating the transients of irrational transfer functions. Further possible applications of the new converting matrix for Schwarz form will be investigated. Schwarz form is not

only significant in the stability study of linear systems but also important in linear systems. It seems to Chen that a unified picture is shaping up. Finally, he is continuing to study the identification problem of nonlinear cases.

## 2.0 PHYSICS, INCLUDING CHEMICAL PHYSICS AND OPTICS

### 2.1 Investigations in Theoretical Atomic Physics

Dr. W. R. Garrett, Assistant Professor of Physics, and Dr. R. A. Mann, Associate Professor of Physics, have studied the topics as indicated.

#### 2.1.1 Low Energy Electron Scattering (Dr. Garrett)

Work is continuing on problems in low energy scattering theory. Two publications have resulted during the last twelve months from this work, and two of Dr. Garrett's graduate students are engaged in research in this area for theses (one M. S., one Ph.D.).

#### 2.1.2 Ion-Photon Processes (Dr. Garrett)

The efforts toward describing negative ion formation and photo-detachment cross sections are continuing. One paper has resulted from this work, and one graduate student, Mr. H. T. Jackson, has completed a thesis on research which was totally carried out here in the Research Institute in this area. Another paper is in preparation by Dr. Garrett on photoionization, and one other graduate student is working on this problem.

#### 2.1.3 Atomic Polarizabilities (Dr. Garrett)

Results have been obtained for atomic polarizabilities for oxygen and nitrogen in ground states and excited states. These results have important implications for problems in wave propagation and in certain atom-atom interactions in chemical physics problems. One paper has resulted from this work and will be presented at the Southeastern Section Meeting of the American Physical Society in November. One graduate student is presently working in this area.

#### 2.1.4 Slow Electron Inelastic Scattering (Dr. Mann)

Theoretical studies are being carried out on the inelastic scattering of electrons from light atoms and ions. These studies are based on the assumption of a

physically plausible model for the atom or ion and a numerical integration of the wave equations for the free particles. Mr. John Woo is working on his master's thesis in this area.

## 2.2 Theoretical Studies of Nuclear and Atomic Structure

Dr. C. P. Bhalla, Associate Professor of Physics, had initiated several research projects in theoretical nuclear and atomic structure. These are described, in brief, below.

### 2.2.1 Nuclear Beta Decay

Studies have been continued on the second-order effects in $\beta$-decay. So far, some of the anomalous beta-shape factors were satisfactorily explained. Further work is planned on the same lines as outlined in the following references: (1) C. P. Bhalla, "Second-Order Effects in Positron Spectrum of $Zr^{89}$," Phys. Rev. 132, 1177 (1963); (2) C. P. Bhalla, "Anomalous Beta Spectrum of $In^{114}$ ($1^+ \rightarrow 0^+$)," Physical Review 129, 2130 (1963); and (3) C. P. Bhalla, "Positron Shape Factor of $Co^{56}$ ($4^+ \rightarrow 4^+$)," Nuclear Physics 67, 353 (1965).

### 2.2.2 Nuclear Structure Calculations

Using the numerical methods developed by C. P. Bhalla, calculations are being performed for the low-lying nuclear states. These nuclear states are usually predicted by a theoretical model. Experimentally one measures the relative contribution of E2 and M1 transitions by a comparison with the theoretical values. Details of this work are described in (1) C. P. Bhalla, "New Calculations of $\beta_4$ (M-shells) for Tellurium," Proceedings of the International Conference on Internal Conversion Process, (Academic Press, 1965); (2) C. P. Bhalla, "Theoretical Internal Conversions Coefficients (Magnetic Multipoles) for $Z = 52$," Handbook of Conference on Nuclear and Particle Physics, University of Liverpool, England; (3) M. E. Rose, Internal Conversion Coefficients, (North-Holland, 1958).

### 2.2.3 Relativistic Atomic Studies

An investigation has been initiated for the calculation of binding energies for neutral atoms using a realistic atomic potential. Mr. C. L. Tolliver, a graduate

student, is participating in this program. The finite nuclear size effects are being studied as outlined by C. P. Bhalla and M. E. Rose, Phys. Rev. 128, 774 (1962).

2.2.4 Electron Shielding

Work on the calculation of continuum relativistic functions is continued. Mrs. Frances Yen, a graduate student, is also participating in these studies. The problem of obtaining the appropriate normalization of the continuum wave function is being studied at the present time. This arises from the fact that the asymptotic forms are strictly valid for $r \rightarrow \infty$, whereas the normalization factor can be obtained at a finite distance in the numerical intergration.

2.2.5 Graduate Students Supervision

During the period, supervision of the dissertation problems of four graduate students was continued. Three of the students are working toward a Ph.D. degree in Physics, and one graduate student is near completion of a thesis for M.S. degree in Physics.

2.3 Surface Physics and Related Areas

Dr. R. A. Mann is continuing research in the area of field emission microscopy. Field emission techniques are being used to study the migration of adsorbed cesium atoms on a tungsten substrate. Related research is being conducted in studies of ultra clean surfaces. The objective is to produce ultra clean surfaces by electron bombardment and to use the techniques of field emission to observe the removal of the adsorbed impurity atoms. The behavior of these clean surfaces will then be studied when such surfaces are brought into contact. Mr. E. L. Wilkinson, Senior Research Assistant is working in this area toward the Ph.D. degree.

2.4 Coherent Light

Dr. R. A. Mann is doing a theoretical study of lasers, the objective being to understand the observed beam divergence in terms of power level. For this purpose the structure of some of the different modes is being calculated.

Once the necessary calculations of the mode structure are complete this information will be used in rate equations to determine the power expected in each mode and from this the theoretical beam divergence. A Ph.D. candidate, Mr. W. L. Gamble, is being supervised on this topic.

### 2.5 Chemical Physics. Interaction of Electromagnetic Energy with Matter

Dr. W. F. Arendale, Professor of Chemistry and Assistant Director of the Research Institute has continued the effort during the six-month period, 1 March 1965 to 31 August 1965, to provide laboratories equipped for the study of the effects from the interaction of electromagnetic energy in the visible and near-visible region of the spectrum with matter. A laser capable of 8-10 joules in the non-Q-switched mode was placed in operation. Q-Switching provides 100 megawatt pulses of less than 10 nanoseconds duration. Ruby and neodynium rods are available. Two projects related to the work are funded by the U. S. Army Missile Command. One effort will investigate the characteristics of light absorbed by or reflected from selected targets. A Bausch and Lomb 2-meter spectograph has been installed so that it can be used in a convenient manner for the study of the spectral characteristics of the scattered radiation. The investigation of nonlinear effects related to Raman scattering is the subject of the second effort. Investigations of the chemical reactions taking place as the result of interactions with high energy light beams will form the third part of these studies. These studies are paced by the ability to obtain equipment and assimilate it into experimental set-ups.

## 3.0 CONTROL SCIENCES AND HYBRID COMPUTATION

Dr. C. D. Johnson, Associate Professor of Electrical Engineering and Head of the Control Science Laboratory has been concerned with the following two topics in the development of the Control Sciences Laboratory.

### 3.1 Studies in Control Theory

During the past six months, Dr. Johnson has continued his studies in the mathematical theory of automatic control. In particular, some results

have been obtained which provide new necessary conditions for the "singular solutions" which arise in certain optimal control problems. In addition, a technical device has been developed which permits the application of Popov's stability theorems to a (slightly) wider class of problems.

Presently, Dr. Johnson is studying a new class of variational problems in control theory and is developing a mathematical machinery for computing optimal controls for dynamical systems with "inputs".

### 3.2 Development and Use of a Hybrid Analog Computer Facility

During the reporting period, the Control Sciences Laboratory acquired a new Applied Dynamics Model AD-80 hybrid analog computer facility. This $80,000 facility, which was purchased through University (State of Alabama bond issue) funds, consists of two separate and independent computer consoles which can be slaved together to accommodate large scale problems.

This computer facility is presently being used to investigate "machine solution" techniques for two-point boudary value problems and domains of asymptotic stability for nonlinear dynamical systems.

During the past six months, Mr. Rolf Duerr, Research Assistant and M.S. candidate, has been primarily concerned with achieving operational status for the hybrid analog computer facility. In addition, he has studied analog and hybrid methods for solving two-point boundary value problems and has studied Zubov's method for determining domains of asymptotic stability by machine methods.

## 4.0 ELECTROMAGNETICS, INCLUDING COMMUNICATIONS

### 4.1 Wave Propagation in Groove Guides

Dr. N. F. Audeh, Associate Professor of Electrical Engineering, and Mr. H. Y. Yee, Research Associate, have continued the studies of wave propagation in groove guides which were discussed in detail in the Fourth Semi-Annual Report on NsG-381. More experimentations were performed for measuring the attenuation constant of the groove guide, using the familiar

resonance method. The cutoff frequency was determined by a method[1] which was developed by the investigators, the results of which agree closely with that calculated using the wavelength values.

A paper entitled, "Wave Propagation in Groove Guides," by H. Y. Yee and N. F. Audeh, has been prepared to be presented at the National Electronics Conference in Chicago on October 25, 1965.

It is proposed to continue the research in groove guides. In particular, the following tasks will constitute the major program:

1. Optimization of the size of the groove and the wall widths, in order to minimize the radiation loss;
2. Launching the electromagnetic waves in groove guides using dielectric lens,[2] grooved horn or a combination of both;
3. Behavior of groove guides at higher frequencies, by operating at frequency bands 18-26 GC and 26-40 GC, and eventually to millimeter waves.

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1. N. F. Audeh and H. Y. Yee, "Measurement of cutoff Frequencies," to appear in IEEE Trans. on Microwave Theory and Techniques, November 1965.
2. N. F. Audeh and H. Y. Yee, "On Dielectric Lenses," Proc. IEEE, Vol. 53, April 1965, page 391.

### 4.2 Scattering Problems in Electromagnetic Propagation

Mr. H. Y. Yee, Research Associate, has concentrated on the scattering problems in electromagnetic propagation during the past half year, with emphasis on numerical methods. The point-matching method has been applied successively to calculate the electromagnetic scattering properties of a conducting cylinder with arbitrary cross-section,[1,2] and of a circularly dielectric coated conducting cylinder with complicated cross-sections.[3] By means of this technique, the boundary conditions at the conducting surface are satisfied at a finite number of points. Justification of the validity of these conditions is shown qualitatively.[1,2] Numerical results are checked with known solutions or calculated by other approximate methods.

The above mentioned studies were limited to two-dimensional problems only. Efforts have been made on the extension to the three dimensional cases. It is proposed to continue the study of the three dimensional scattering problems with conducting scatterers or composite scatterers.

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1. H. Y. Yee, "Application of Point-Matching Method to Low Frequency Scattering by Conducting Cylinders," IEEE Trans. on Antennas and Propagation, Vol. AP-13, September 1965, page 818.
2. H. Y. Yee, "An Approximate Method for the Computation of Scattering by Conducting Cylinders with Arbitrary Cross-Sections," UARI Research Report No. 22, March 1965, or Contractor Report, NASA CR-282, August 1965.
3. H. Y. Yee, "Scattering of Electromagnetic Waves by Circularly Dielectric Coated Conducting Cylinders with Arbitrary Cross-Section," IEEE Trans. on Antennas and Propagation, Vol. AP-13, September 1965, page 822.


### 4.3 Communication. Research on an Adaptive Device for Communications

Dr. R. J. Polge, Associate Professor of Electrical Engineering, has been investigating an adaptive device as a signal detector in communications. The purpose of this investigation is to reduce the probability of error in the detection of pulse code modulated signals by use of an adaptive threshold device. The theoretical results are checked by simulating the adaptive threshold detection in the laboratory. In a pulse code modulated system, the signal is transmitted only during definite intervals of time, between those intervals only the noise and the residual voltage due to the previous sequence of pulses are present. The noise before the unknown signal is determined by sampling and the expected value of the noise during the detection can be evaluated by a correlation technique. In an adaptive scheme, the threshold is varied according to the expected value of the noise,resulting in a considerably smaller probability of error in the detection. When an integrator is used in front of the threshold detector, the threshold level is kept constant but the integrator can be made adaptive and integrates a corrected signal instead of the actual signal.

The following problems have been investigated and the results are part of the final report to the Army Missile Command:

1. A pulse code modulated signal mixed with RC noise is detected using a threshold device;
2. A pulse-code-modulated signal mixed with white normal noise is first filtered with a RC filter and then detected using a threshold device;
3. A pulse-code-modulated signal mixed with RC noise, is first filtered by an integrator and then detected using a threshold device;
4. A pulse-code-modulated signal mixed with RC noise is detected by a linear detector, then integrated and finally detected using a threshold device.

In each case the probability of error is reduced considerably by using an adaptive scheme.

The results are checked experimentally. Circuits have been built to perform all the logical operations (summer amplifiers, comparator, etc.). The errors in the detection are counted after comparing the detected noisy signal to the noiseless signal in a coincidence circuit.

Dr. Polge plans to continue research on adaptive schemes for the detection of pulse-code-modulated signals and to extend it in three main directions:

1. Application of adaptive detection on radar;
2. Use of nonlinear network;
3. Adaptive schemes for nonstationary perturbations.

4.4 Evaluation of Magnetic Influences on Gyros

Dr. Joseph C. Dowdle, Professor of Electrical Engineering, will serve as principal investigator and will be assisted by Dr. Robert L. Thurstone, Assistant Professor of Electrical Engineering, in a research study relative to magnetic field effects on gyros. Specifically, the study will be concerned with all significant sources of magnetic forces that cause drift rates on the Saturn V stabilization and guidance acceleration gyros. Support for the research will be derived on contract NAS8-20117.

Both, disturbing inputs to the gyro, either from the magnetic environment or from secondary influences produced by mechanical or electrical instability, give rise to spurious torques that result in drift of the gyro attitude reference. The degree to which drift is influenced by the various modifying and disturbing torques of magnetic origin will be evaluated. For the most part, only nonacceleration-sensitive influences will be treated. Procedures will be defined, developed and applied to experimentally identify these stray fields by direction and strength and their resulting drift rates. It will be considered that torques in the order of 0.1 dyne cm can result in precessional velocities (drift rates) of present accuracy magnitudes.

To experimentally complete the study as described, a laboratory for evaluation of gyro performance under a static spatial environment is being developed, partially under NsG-381 funding. Precise instrumentation of the gyro units is required in a controlled environment in order that identification be made between the spurious field effects and the electrical and magnetic inputs of the gyros. In general, the equipment used will be of the type to characterize open-loop measurements.

## 5.0 FLUID MECHANICS, GAS DYNAMICS AND HEAT TRANSFER

### 5.1 Unsteady Heat Transfer in Laminar Boundary Layers

In a continuation of research started while at the University of Illinois, Dr. D. R. Jeng, Assistant Professor of Mechanical Engineering, is investigating theoretically unsteady convective heat transfer in laminar boundary layers from a surface whose temperature or heat flux is a prescribed function of time. Special considerations are made on the three-dimensional stagnation flow, flow over a flat plate, rotating discs and cones covering wide ranges of Prandtl number. Both non-electrically and electrically conducting fluid are considered under the action of a uniform magnetic field normal to the wall with simultaneous suction or injection (See publication.)

He will continue the studies of the unsteady heat transfer problem for the case of time dependent free stream temperature and free convection problems.

5.2 Transpiration Cooling in a Magneto-Hydrodynamic Three-Dimensional Stagnation Flow

This is also a continuation of work done by Dr. Jeng while at the University of Illinois. He has made an analysis to determine the reduction in steady three-dimensional stagnation point heat transfer when blowing and a magnetic field act simultaneously. The equations governing the steady velocity field and temperature field for incompressible, constant-property, non-dissipative flow have been transformed to two simultaneous third order nonlinear ordinary differential equations. Solutions have been obtained for a range of parameters. The curvature parameter, defined as the ratio of curvatures along major and minor axes at the stagnation point, was varied from 0 to 1, the magnetic parameter from 0 to 2, the blowing parameter from 0 to 4, and the Prandtl number from 0.01 to 10.

5.3 Hypersonic High-Temperature Gas Flow

Dr. Rudolf Hermann, Professor of Physics and Aerospace Engineering and Director of the Research Institute; Dr. J. D. Brainerd, Associate Professor of Aerospace Engineering; Mr. K. O. Thompson, Research Associate; Mr. J. Thoenes, Senior Research Assistant; and Mr. M. Loh, Aeronautical Research Engineer; have been engaged in the studies of hypersonic flows made during the last six months. Dr. Brainerd joined the Research Institute at the end of July 1965.

During the past reporting period, the effort was concentrated on the calculation of hypersonic non-equilibrium flow of air past a circular cylinder, using Dorodnitsyn's method of integral relations and a simplified three component ($O_2$, O, $N_2$) air model. Considered were the effects of non-equilibrium oxygen dissociation on the distribution of the flow variables in the subsonic and supersonic region of the shock layer. Also investigated was the influence of free stream oxygen dissociation on the shock detachment distance and the flow field in general.

Many new results were obtained for free stream Mach numbers ranging from 3 to 14. It was found that the effect of free stream dissociation is to increase the

shock detachment distance. For small bodies, most of the shock layer was found to be in a frozen state; while for large bodies (body radius $\geq 0.1$ m), the flow equilibrates close behind the shock. The flow always reaches thermodynamic equilibrium at the stagnation point. It was observed that, in all cases, the surface pressure distribution is close to the one predicted by Modified Newtonian theory.

Presently, while more results for the cylinder are still obtained, the calculations are extended to the sphere. Future extensions of the project include more complicated body shapes, a larger number of species, as well as higher approximations of the integral method.

Simultaneously, hypersonic flow of air past pointed bodies of revolution was studied under sponsorship of the U.S. Army Missile Command, (Contracts DA-01-009-AMC-116(Z) and DA-01-021-AMC-12039(Z)). Calculations were performed for frozen, equilibrium, and non-equilibrium flow of a three component gas ($O_2$, O, $N_2$) past circular cones.

The development of new digital computer programs for calculating various problems in high-speed flow will be carried out by Dr. Brainerd. Variations of the Godunov and Lax-Wendroff finite difference schemes will be utilized. It is planned initially to program the method for the supersonic portions of flow fields in two dimensions and axial symmetry. A perfect gas will be assumed, but chemical reactions will be included later. In the future, it is planned to extend the program to include three dimensional flows, blunt body flows, and viscous flows. Certain unsteady problems may also be studied at a later time.

These programs will be important supplements to the Dorodnitsyn-Belotserkovskii integral technique presently in use.

In addition to the computer studies, a small amount of time will be devoted to an analytical study of the relationship between the maximum entropy and stagnation streamlines in asymmetric blunt body flows.

## 6.0 STRUCTURAL MECHANICS

### 6.1 Numerical Analysis of Nonlinear Structures

Dr. J. T. Oden, Associate Professor of Engineering Mechanics, has undertaken a study of the nonlinear behavior of complex structures using generalizations

of the finite element method. The over-all objective of the project is to develop a general numerical formulation of geometrically nonlinear problems in solid mechanics.

The technical approach under investigation is the so-called finite element method. Basically, the idea is to represent in a consistent and a general manner a continuous medium by a finite number of component parts called elements. Stress and/or displacement fields in these elements are selected so as to satisfy the equilibrium equations and/or the compatibility equations within each element. The displacements of boundary surfaces of an element are then approximated by kinematically consistent displacement vectors at a prescribed number of boundary points. Alternatively, surface tractions are replaced by a statically equivalent systems of generalized forces at prescribed boundary points. When the generalized forces corresponding to the boundary displacement vectors are evaluated (or vice-versa), the element can be fitted into a discrete model of continuous system of any shape with virtually any type of boundary conditions and loading. The technique has, in addition, the advantages that thermal effects, anisotropy, nonhomogenuity, and discontinuities can be easily accounted for and that natural frequencies and classical buckling loads can be evaluated. Further, if the displacement or stress fields satisfy certain continuity conditions, and if the number of elements is increased and the dimensions of each element are continuously decreased, the results obtained from the discrete model will approach those corresponding to the theoretically exact solutions for continuous systems.

This approach has been applied successfully to certain linear elasticity problems since 1958 and recently solutions to linear three-dimensional elasticity problems have been presented. Attempts to extend the ideas to nonlinear problems, however, have, with few exceptions, been very limited. Those published can be divided into two categories: (1) successive corrections of the linearized problem; and (2) formulation of the nonlinear problem which is then solved by successive approximations. The first approach permits little insight into the nonlinear phenomena and is often incapable of detecting characteristics of the behavior of the structure which are peculiar to the

nonlinear problem. The second approach has been attempted only recently, and the particular formulations were subject to severe limitations.

In the present study, formulations of the general geometrically nonlinear problem with the restriction of small strains is considered. Specific objectives of the project are as follows:

1. To formulate in general curvilinear coordinates the stiffness relations for an elastic, homogeneous, and isotropic continuum undergoing large displacements but small strains.
2. To adapt the relations of part 1 to a generalized finite element representation.
3. To establish convergence criteria for numerical procedures employing finite elements.
4. To demonstrate the application of the theory by solving representative numerical problems.

Thus far, several means for obtaining consistent elements for three-dimensional elastic bodies have been investigated. Special consideration has been given to certain nonlinear problems in plane elasticity and in the nonlinear theory of plates. A general formulation for bodies of arbitrary shape is under consideration.

6.2 Research in Nonlinear Analysis of Shell Structures

Dr. G. A. Wempner has initiated work on two related topics:

1. Nonlinear Analysis of Thin Shells by a Method of Incremental Loading.*

Thin shells or shell-like structures are the primary structural components of flight vehicles. Such shells are extremely sensitive to the presence of initial imperfections and/or extraneous loads which can cause buckling at much lower loads than those predicted by classical methods, viz. the computation of the eigenvalues of linear homogeneous equations. The unadvoidable feature of these buckling problems is the existence of nonlinearities which originate with the large rotations of elemental parts.

The current study is aimed at the development of a general method for the solution of nonlinear problems of this type. The method is based on the notion that any smooth function is linear in a small range of its variables. In particular,

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* Further details are contained in a (NSF) proposal by the same title.

any small increment of loading is accompanied by a linear response. Of course, the geometry of the structure is slightly altered by the incremental load, and the alteration changes the response to a subsequent loading. Indeed, it is the continuous change of geometry which accounts for the nonlinearity.

With the existing numerical procedures for linear equations and the use of a digital computer, the method under study has unlimited possibilities. Since the analysis follows the history of the deformations as they would occur in reality, it is possible to analyze structures composed of nonlinear materials and inelastic materials. Moreover, by tracing the history, the method should provide insight to the physical phenomena.

At the present time Drs. Wempner and Oden are using this method to study the effects of large prebuckling deformations on the buckling of a shell-type structure.

2. Theories of Composite Shells

Recent efforts to reduce weight in flight vehicles have drawn attention to sandwich shells and the means to analyze them. Unfortunately, it has been expedient to amend and apply established theories of one-layer shells. Often the results are questionable and unreliable. Then, too, new materials and methods of fabrication have led to composites which lie beyond the existing constitutive equations.

The current effort is a basic study and a development of the governing equations for sandwich shells. These include the effects of large deflections and apply to problems of structural instability, vibration and wave propagation.

## 6.3 Development of the Structural Mechanics Laboratory

Dr. W. K. Kubitza, Professor of Engineering Mechanics and Head of the Structural Mechanics Laboratory, has continued the planning, equipping and development of the Structural Mechanics Laboratory in the report period. Equipment already received and installed provides research capabilities for structures or structural elements subjected to static loading in the following areas:

1. Mechanical Strain and Deformation Measurement by means of a sufficient number of Dial Gages.

2. Electrical Strain Gage Technique using a combination of a portable 10-channel switch and balance unit.

3. Photostress (Photoelastic Coating Technique) to determine surface stresses in plane and curved structural elements. Stroboscopic light attachment together with the necessary photographic equipment permits strain measurements under cyclic dynamic conditions.

4. Brittle Coating. Available Stresscoat equipment will be particularly useful for the exploratory determination of tension stress levels, the direction of principal stresses, and for the qualitative detection of stress concentration in structures of complex shape.

5. Moire' Apparatus. Based on the Ligtenberg method an apparatus has been built in the laboratory which permits investigation of stresses in transversely loaded flat plates and grid systems of complex geometric configuration and stiffness characteristics. Provisions for changing parameters have been incorporated in this apparatus which allow variation of desired accuracy and adaptation to models of different size and loading.

6. Universal Testing Machine with 60,000 lb. capacity equipped with electronic recorder designed to produce stress-strain curves, constant rate of loading indicator and electronic strain rate indicator.

Delivery within four weeks is expected of the following equipment:

1. Huggenberger Mechanical Extensiometers, which because of their versatility in application and high recording accuracy will add greatly to the capability of determining surface stresses by mechanical means in structures of all types.

2. Diffused Light Transmission Polariscopes, to be used for studies of stresses in two and three-dimensional plastic models.

3. Moire' Grids of 500 Pitch Density, as a tool to determine stress fields in flat surfaces of structures in cases where Photostress or Brittle Coating cannot be used because of high temperature of inaccessibility.

In addition to the major equipment described above, tools and accessories have been purchased or built which will allow construction of models, check-out and calibration of instruments.

- After having established broad research capabilities for static loading which is presently to about 80 per cent operative, plans are being developed for conducting investigations of structures subjected to dynamic loading and of fatigue phenomena.

# APPENDIX IV

## New Research Programs Planned at the University of Alabama Research Institute for the Period September 1, 1965, to February 28, 1966

## OUTLINE

1.0 PHYSICS, INCLUDING CHEMICAL PHYSICS AND OPTICS

- 1.1 Theoretical Relativistic Physics. Drs. Garrett and Bhalla

2.0 CHEMISTRY

- 2.1 Organic Chemistry. Dr. Marcantonio

3.0 ELECTROMAGNETICS, INCLUDING COMMUNICATION

- 3.1 Traveling Wave Parametric Amplifiers. Dr. Chang
- 3.2 Wave Propagation in Wave Guides of Arbitrary Cross-Sections. Dr. Audeh and Mr. Yee

4.0 FLUID MECHANICS, GAS DYNAMICS AND HEAT TRANSFER

- 4.1 Research in Transfer of Energy and Mass. Dr. Grohse
- 4.2 Thermal Contact Resistance in a Vacuum Environment. Dr. Jeng
- 4.3 Stress Wave Propagation. Dr. Wilms

It is planned to do research in the following seven new topics under support of NsG-381 at the Research Institute during the next six months period. In five of the seven topics the Principal Investigator is a man who recently joined the Research Institute staff. The topics, shown in lower case, are grouped under the major disciplines indicated by capital letters.

## 1.0 PHYSICS, INCLUDING CHEMICAL PHYSICS AND OPTICS

### 1.1 Theoretical Relativistic Physics

Work will be continued in the theoretical physics group on the following topics: Dr. W. R. Garrett, Assistant Professor of Physics, along with two of his graduate students will allot a portion of his time to studies relating to relativistic physics, x-ray calculations, slow-electron scattering and photoelectric effect, described in Appendix III. Dr. C. P. Bhalla, Associate Professor of Physics, along with two of his graduate students will devote a portion of his time to investigations in the fields of nuclear beta decay, nuclear structure calculations, relativistic atomic studies and electron shielding, described in Appendix III. A total of 12 publications have appeared or are to appear in theoretical physics as outlined in Appendix II.

## 2.0 CHEMISTRY

### 2.1 Organic Chemistry

Dr. A. F. Marcantonio, Associate Professor of Chemistry, plans investigations in the fields of organic photochemistry and pyrolytic chemistry, involving the fate of short-lived, energetic, and reactive intermediates generated by the treatment of organic molecules with various sources of ultraviolet and visible light and heat. The theoretical and mechanistic aspects of these chemical reactions, along with applications to organic synthesis, are to be emphasized.

Of particular interest in this context is the reactive electron-deficient nitrogen intermediate known as the "nitrene" ($R\dot{N}\cdot$ or $R\ddot{N}$), which can exist in the triplet or singlet energy state. The nitrene species can be formed by the decomposition of organic azides ($R\bar{N}-\overset{+}{N}\equiv N \leftrightarrow R-N=\overset{+}{N}=\bar{N}$). Solution photolysis and pyrolysis of hydrazoic acid, azidoformates, sulfonyl azides, and aryl azides will be examined because these azides give rise to "rigid" nitrene systems which do not readily undergo the Curtius reaction or intramolecular rearrangements.

The reactions of HN intermediate, derived from hydrazoic acid ($HN_3$), with solvents containing various functional groups will be investigated in detail. These studies will include the photochemical decomposition of amine hydrazides ($\geqslant N\overset{+}{H}\overset{-}{N_3}$). The role of solvent, effects of concentration, the influence of environmental molecular structure on reactivity, the effect of photosensitizers and catalysts, and the role of the reaction phase will be included in these studies.

The nitrene species, HN, has been of special interest to the astrophysicist because its presence has been detected in comet heads and tails, and in the sun. The condensed reactive species $(HN)_n$ has been thought to contribute to some of the colors on Jupiter. It is also believed that HN is an intermediate in the Raschig synthesis of hydrazine from chloramine and ammonia. Oxidation of hydrazines in liquid rocket fuels may also involve the HN species.

Other rigid nitrene systems are to be examined, such as $\geqslant NSO_2N_3$, $\geqslant P(O)N_3$, $\geqslant BN_3$, $-(N_3)C=N-$, etc. Organic systems which may give rise to nitrenes (e.g., cyclic imines, chloramines, ylids containing nitrogen, etc.), excluding azides, will be studied and their chemistry compared with nitrenes derived from azides. Correlations

will be made between the chemistry of nitrenes and of electron-deficient reactive intermediates of carbon, oxygen, and sulfur.

## 3.0 ELECTROMAGNETICS, INCLUDING COMMUNICATION

### 3.1 Traveling Wave Parametric Amplifiers

Dr. J. C. Chang, Research Associate, plans both a theoretical and experimental investigation of traveling wave parametric amplifiers. He will be assisted by Mr. G. R. Keener, Electrical Enginner, in the experimental portions.

Parametric amplification is associated with frequency-mixing devices which utilize the properties of nonlinear or time-varying reactances. An essential and very desirable feature of parametric amplification is due to its nonlinear reactance which channels energy from an a.c. source to a useful load. Since the reactance does not contribute thermal noise to a circuit, the parametric amplifier can have excellent noise performance. In 1956, Manley and Rowe[1] published their famous paper concerning the general energy relations of some general properties of nonlinear elements. Since then, parametric amplification has gained special interest theoretically and experimentally among many people.[2,3] Most of them used backbiased semiconductor diodes as a nonlinear capacitive reactance for the single tuned parametric amplifiers. One limitation of the single tuned parametric amplifier is its restricted bandwidth. A second limitation of the single tuned parametric amplifier is its potentially unstable and bilateral gain characteristics. If the resonant structure of either a single or multiple tuned parametric amplifier is replaced by a suitable traveling wave propagating structure, it is possible to achieve a measure of unilateral gain with improved stability and bandwidth. All the experimental traveling wave parametric amplifiers have used semiconductor diodes as time-varying capacitors.[4,5]

Very little work has been done concerning the traveling wave parametric amplifier using time-varying inductors.[6] So far, nobody has done any work of investigating the possibilities of constructing a variable phase traveling wave parametric amplifier.

It is planned to do the following:

(1) Develop special circuits and techniques for the purpose of broadening the knowledge of the characteristics of traveling wave parametric amplifiers.

(2) Investigate theoretically and experimentally several possible variable phase traveling wave parametric amplifiers with nonlinear inductive and capacitive coupling

reactances. The object is to determine the characteristics of a variable phase traveling wave parametric amplifier which could replace presently used traveling tube amplifiers.


1. J. M. Manley, and H. E. Rowe, "Some General Properties of Nonlinear Elements. I. General Energy Relations," Proc. Inst. Radio Engrs. 44: 904-413, 1956.
2. H. Heffner, and K. Kotzebue, "Experimental Characteristics of a Microwave Parametric Amplifier Using Semiconductor Diodes," Proc. Inst. Radio Engrs. 46: 1301, 1958.
3. K. K. N. Chang, and S. Bloom, "Parametric Amplifier Using Lower Frequency Pumping," Proc. Inst. Radio Engrs. 45: 1383-1386, 1958.
4. V. V. Bell, and G. Wade, "Circuit Consideration in Traveling Wave Parametric Amplifiers," Inst. of Radio Engrs. WESCON Convention Record, 1959, Part 2: 75-83, 1959.
5. R. Mavaddnt, and F. J. Hyde, "Investigation of an Experimental Traveling Wave Parametric Amplifier," Proc. Instn. Elect. Engrs. 109B: 405-413, 1962.
6. J. C. Chang, "A Parametric Traveling Wave Amplifier Using Oriented Thin Film," Ph.D. Dissertation, Iowa State University, 1965.


### 3.2 Wave Propagation in Waveguides of Arbitrary Cross-Sections
(Study proposed by Dr. N. F. Audeh and Mr. H. Y. Yee)

Waveguides of certain shapes, rectangular, circular, elliptical, and parabolic have been studied extensively. The solutions of the wave equations for such configurations are relatively easy to obtain because of the separability of the equations in the cross-sectional coordinate systems. For waveguides with more complex cross-sections, the method of separation of variables fails. Consequently, approximate techniques must be utilized. Several authors[1-4] studied the properties of such waveguides using approximation and conformal mapping techniques.

Yee and Audeh[5] introduced the point-matching method to solve the problem of waveguides with arbitrary cross-sections. If the cross-sectional view forms a closed contour, the representative function of which is single-valued in the radial direction, then the wave function may be written as

$$\psi = \sum_{n=o}^{\infty} J_n(kr)\,[A_n \cos n\theta + B_n \sin n\theta]$$

where r and $\theta$ are the polar coordinates. $J_n$ is the nth-order Bessel function of the first kind, $A_n$ and $B_n$ are constants to be determined by the boundary conditions and k is the eigenvalue. In general, it is difficult to have the wave function satisfy the boundary conditions at every point around the closed contour. However, it is possible to make the wave function satisfy the boundary at finite number points N, if the series is assumed to converge uniformly and rapidly.

The point-matching method was applied to square and elliptical waveguides and excellent accuracy was easily attained. The degeneracies of wave modes can also be located. With the knowledge of the field expressions, the power transmitted, the attenuation constant, and etc., can be evaluated numerically.

It is proposed to apply the point-matching method to several waveguides with cross-sections of arbitrary shapes. Such guides are:

(1) Hollow-piped waveguides: triangular, trapezoidal, channel

(2) Composite waveguides: partially-filled with dielectric, dielectric-coated (Surface guide), and eccentric. Other guides will eventually arise. For each waveguide, the dimenstions will assume several different values; and their effect will be determined.

Along with the theoretical investigations, waveguides will be designed, constructed, and tested for cutoff frequencies, attenuations, and etc., in the Microwave Laboratory of the University of Alabama Research Institute.

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1. A. IA. Iashkin, "A Method of Approximate Calculation for Waveguides with Complicated Cross-Sectional Form," Radiotekhnika, USSR, Vol. 13, pp. 831-833, 1958.

2. A. Y. Ju and A. Ishimaru, "The Dominant Cut-Off Wavelength of a Lunar Line," IRE Trans. on MTT, Vol. MTT-9, No. 6, pp. 552-556, November 1961.

3. H. H. Meinke, K. P. Lange, and J. F. Ruger, "TE- and TM-Waves in Waveguide of Very General Cross-Section," Proc. of IEEE, Vol. 51, No. 11, pp. 1436-1443, November 1963.

4. F. J. Tischer and H. Y. Yee, "Waveguide with Arbitrary Cross-Section Considered by Conformal Mapping," UARI Research Report No. 12, January 1964, also "Parabolic and Elliptic Waveguides Considered by Conformal Mapping," UARI Research Report No. 16, May 1964, University of Alabama Research Institute, Huntsville, Alabama.

5. H. Y. Yee and N. F. Audeh, "Uniform Waveguides with Arbitrary Cross-Section Considered by the Point-Matching Method," to appear in the IEEE Trans. on MTT in November 1965 or January 1966.

## 4.0 FLUID MECHANICS, GAS DYNAMICS AND HEAT TRANSFER

### 4.1 Research in Transfer of Energy and Mass

Dr. E. W. Grohse, Professor of Chemical Engineering, plans initially to look into various problems of heat exchange (with or without simultaneous mass transfer) involved both in chemical and nuclear-propelled space vehicles, with the objective of developing research programs which can be successfully proposed for funding as contracts or specific grants. Advanced as well as "conventional" propulsion schemes will be considered. Tentative research areas to be considered include the following:

(1) Boiling heat transfer and two-phase flow (e.g., relating to the vaporization of liquid hydrogen in space vehicle fuel tanks, in graphite core nuclear rocket reactors, etc.)

(2) Simultaneous heat and mass transfer studies (e.g., ablation effects during hydrogen flow through graphite core nuclear rocket reactors, during combustion products gas flow through chemical rocket nozzles, etc.

(3) High temperature fluidized-bed nuclear reactor concept, reprocessing of nuclear fuels, etc.)

(4) Vaporization studies relating to liquid-core nuclear reactor concept.

(5) Application of X-ray and gamma ray attenuation measurement techniques (e.g., for the measurement of voids in boiling liquids and fluidized beds).

(6) Methods of enhancing heat and mass transfer (e.g., via vibration of transfer surfaces).

Dr. Grohse is also generally interested in research areas offering relatively broad opportunity for application of the discipline of chemical engineering, such as the areas of fuel cell research and water desalinization research.

### 4.2 Thermal Contact Resistance in a Vacuum Environment

Dr. D. R. Jeng, Assistant Professor of Mechanical Engineering, plans an experimental and analytic study of thermal contact resistance in a vacuum environment. The object of the research is to obtain an empirical formula to predict thermal conductance.

In the case of the actual surface contact, the shape of the contact spots is quite irregular; the shape of contact spots, a contact area, number of the contact spots and its distribution on the contact surface have a significant effect on the constriction resistance. Thus, an electrolytic tank will be constructed to provide a convenient means of investigating the geometrical aspects of contact on the thermal constriction resistance. With some particular surface assumed, the constriction resistance may be measured. Specifically, the influence of the distribution on the microscopic contact area, the size of this area and the length of the members on the resistance will be studied.

The theoretical study will be concerned with

i) possibility of analyzing the contact of dissimilar metals

ii) different geometrical configurations.

Another phase of the research will be concerned with the study of some actual contact areas and a determination of the number of the contact spots and their distribution. In cooperation with Dr. W. K. Kubitza, Professor of Engineering Mechanics, the surface contact area and surface deformation will be examined by photoelastic means and statistical methods.

4.3 Stress Wave Propagation

Dr. E. V. Wilms, Assistant Professor of Engineering Mechanics, will undertake a study in stress wave propagation. Approximate models to study the propagation of stress waves in elastic materials include the Bernoulli-Euler beam, and the Timoshenko beam. The Bernoulli-Euler beam neglects the effects of transverse shear and rotary inertia. These effects, which are included in the Timoshenko beam model, are particularly significant in impact problems and result in improved estimates of wave propagation speeds and stress distributions. Transform techniques are frequently used in the solution of Timoshenko beam problems. One of the chief difficulties is the inversion of the transformed solutions. Some of the same difficulties encountered in the solution of the Timoshenko beam arise in the equation governing the propagation of stress waves in a thermoelastically coupled material. Iteration

techniques have been successfully employed in the solution of transient thermoelastic problems for the half space.[1,2]

Similar perturbation methods will be applied to the solution of Timoshenko beam problems. Specific cases to be considered will be semi-infinite beams, initially undisturbed, with various time dependent loads applied at the free end.

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1. E. V. Wilms, "On Coupling Effects in Transient Thermoelastic Problems," Journal of Applied Mechanics, Vol. 31, No. 4, 1964, pp. 719-722.

2. A. I. Soler, M. A. Brull, "On the Solution to Transient Coupled Thermoelastic Problems by Perturbation Techniques," to appear in Journal of Applied Mechanics, A.S.M.E. Paper No. 65-APM-5.