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TECHNICAL MEMORANDUM NO. 40

DISCUSSION OF TELEMETRY PERFORMANCE STANDARDS

Leonard S. Schwartz

June 1964

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND
CONTRACT NAS 5-3508

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ABSTRACT

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Standards for operation of telemetry systems are presented. It is shown that suitable performance standards are given in terms of error and rejection probabilities. The trade-off between these probabilities with signal-to-noise ratio as parameter is discussed, and a method of testing different equipments for a given policy or of testing different policies on the same equipment is outlined. Possible application of these ideas on the S-3 vehicle data is examined.

Author

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DISCUSSION OF TELEMETRY PERFORMANCE STANDARDS

1. INTRODUCTION

In looking for standards of operation for telemetry or communication systems, we must recognize two things: first, the nature of the standards must be appropriate to the operational performance required of the system, and second, having agreed on the standards, the specified performance indices are at best subjective. In any case, they can be assigned only by operational people. For example, in tracking radar, operational performance is in terms of the tracking error, which determines the kill probability, the quantity of ultimate interest. Should the kill probability be 50 percent, 75 percent or 90 percent? The specified kill probability will be the result of an estimate of the damage that attacking planes or missiles can inflict on friendly targets. If 25 percent of the attacking force is presumed to penetrate the defenses, the estimates of possible inflicted damage may be unacceptably high. It will then be necessary to attempt to increase the kill probability, but notice that the subjective aspect enters in the statement of what is considered an unacceptable loss.

In communication systems the operational performance is given in terms of error and rejection probabilities, but the assigned values of these probabilities depend ultimately on their relative subjective importance. For example, a maximum allowable error probability of one percent, reflects the belief that an average of more than one error in a hundred has bad consequences. The subjective element here is evident.

A possible operational performance measure for communication or telemetry systems is information rate or per-unit equivocation. It will be recalled that the error-free information rate I is given by

$$I = H(x) - H(x|y)$$

where H(x) is the transmitted information in bits per symbol and $H(x|y)$ is the per-symbol equivocation. A measure of the information-rate efficiency of a system is obtained by normalizing I with respect to the transmitted rate H(x). Thus, the normalized error-free rate is

$$I_{nor} = \frac{I}{H(x)} = 1 - \frac{H(x|y)}{H(x)}$$

The ratio $H(x|y)/H(x)$ is the normalized equivocation, called the per-unit equivocation and designated by the symbol E. Thus

$$I_{nor} = 1 - E$$

Hence, I_{nor} is maximized by minimizing E, and either one of these parameters can serve as a performance measure when information rate is the criterion of performance. The objection to E and therefore I_{nor} as a performance measure is that they are computed from the set of transition probabilities, all of which must be known before the calculations can be made, and their employment is strictly valid only for infinitely long messages.

Question has arisen concerning the possibility of using the quantity T_t as a performance measure. T_t is the ratio of the total number of digits transmitted to the total number accepted. It is given by

$$T_t = \frac{1}{1 - U}$$

where U is the rejection probability. Since T_t and U are functionally related, either parameter may be used as a performance measure; but since probability of error is clearly one measure that should be used, it is reasonable that both measures should be probabilities.

2. CRITERIA OF PERFORMANCE

Error probability and rejection probability are regarded as reasonable performance measures, but signal-to-noise ratio is not. The reason is that signal-to-noise ratio is not the final objective of communications or telemetry. The final objective is performance with minimum error and rejection probabilities. Actually, in practical cases we necessarily and, in fact, reasonably settle for less. The reasonable objective is performance with maximum acceptable error and minimum rejection probability. Now this result is achieved by means of a certain signal-to-noise ratio and a certain threshold setting or decision system. If we change the threshold setting or the decision system, for the same signal-to-noise ratio, the error probability, the rejection probability, or both will be different. For this reason, signal-to-noise ratio should not be regarded as a performance measure. It is, however, the means by which two systems may be compared. Thus, if two systems perform with the same error and rejection probabilities for different signal-to-noise ratios, the better system is the one requiring the smaller signal-to-noise ratio.

As we may wish to specify different operational values of error and rejection probabilities for different applications, a trade-off curve between them will be useful. Such a curve is obtained by adjusting the decision threshold for a given signal-to-noise ratio as parameter. A high setting of the decision threshold results in low error probability and high rejection probability; conversely, a low setting of the decision threshold results in high error probability and low rejection probability. If the signal-to-noise ratio is changed, a new trade-off curve results. Hence, a family of curves is described for different signal-to-noise ratios, as shown in Fig. 1. The hatched rectangle is the area of acceptability, with dimensions determined by specified maximum allowable error and rejection probabilities. The

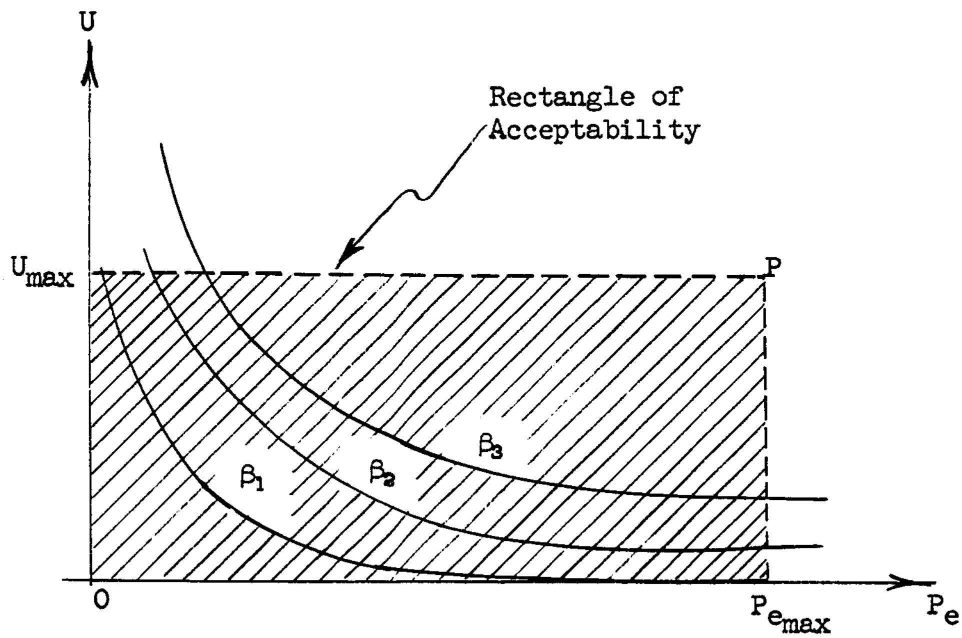

Fig. 1 - Trade-Off Curves for Rejection Probability U Versus Error Probability P_e. The β's Represent Signal-to-Noise Ratios in Order of Decreasing Magnitude.

corner point labelled P is the point of maximum acceptable error and rejection probabilities. It is a unique point and, therefore, one logically to be used in a performance test. By adjusting the decision threshold and the signal-to-noise ratio, one can always reach the corner point.

3. TEST PROCEDURES

To devise meaningful test procedures, we must consider the goal of a data reduction system. It is to maximize the amount of useful data that can be recovered from the data storage tapes. Useful data are those data that meet some criterion of acceptability, specifically, some minimum reliability, which means that the error probability cannot exceed a specified maximum. According to this point of view the object in devising test procedures is twofold: (1) To devise test formats that will permit us to compare different equipments for a given decision policy and (2) To devise test formats that will enable us to compare the effectiveness of different policies on a given equipment. Let us examine each of these objectives in turn.

3.1 Comparison of Equipments for the Same Decision Policy

A basic step, common to all testing procedures, is to prepare simulated data tapes for a variety of signal-to-noise ratios, including one obtained for the noise-free condition, to be used as a reference for the others. In accordance with the signal-to-noise ratio, errors and rejections occur and the corresponding probabilities can be estimated by reference to the noise-free data. Before the testing procedure can properly begin, we must assume that the maximum acceptable values of the error and rejection probabilities have been specified. This defines the corner point P in Fig. 1, the point at which the test begins. Each equipment to be evaluated can be brought to the corner point by suitable adjustment of signal-to-noise ratio and decision threshold. Now suppose that for each equipment we increase the signal-to-noise ratio in steps and at each step readjust the decision threshold to minimize the rejection probability, subject to the requirement that $P_e \leq P_{emax}$. With each step of increasing signal-to-noise ratio we increasingly penetrate the rectangle of acceptability, but ideally we would like to proceed along the edge of the rectangle from the corner point P to P_{emax}.

That this is the desired course is seen with the aid of a diagram such as Fig. 2, illustrating an artificial satellite in a highly eccentric orbit about the earth. When the satellite is close to the earth, the signal-to-noise ratio is large; when it is at

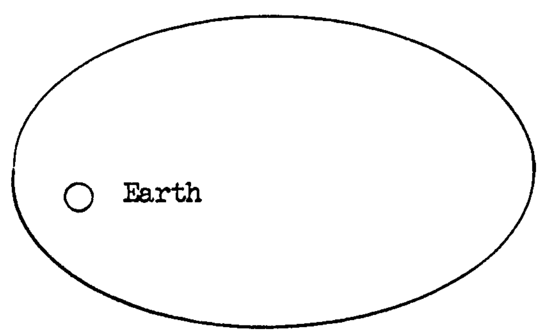

Fig. 2

maximum range, the signal-to-noise ratio is small. In comparing one data reduction system with another, we would like to know which of the two is capable of reducing the larger percentage of data with specified reliability. Ideally, we would of course like to recover all the data with specified reliability. This would be achieved most nearly by going along the edge of the rectangle of acceptability between P and P_{emax}.

Hence, in the light of Fig. 2, if we were to plot rejection probability U as a function of signal-to-noise ratio, the parameter of the curve would be a given equipment. The goal is clearly to select the equipment for which the area under the curve is less.

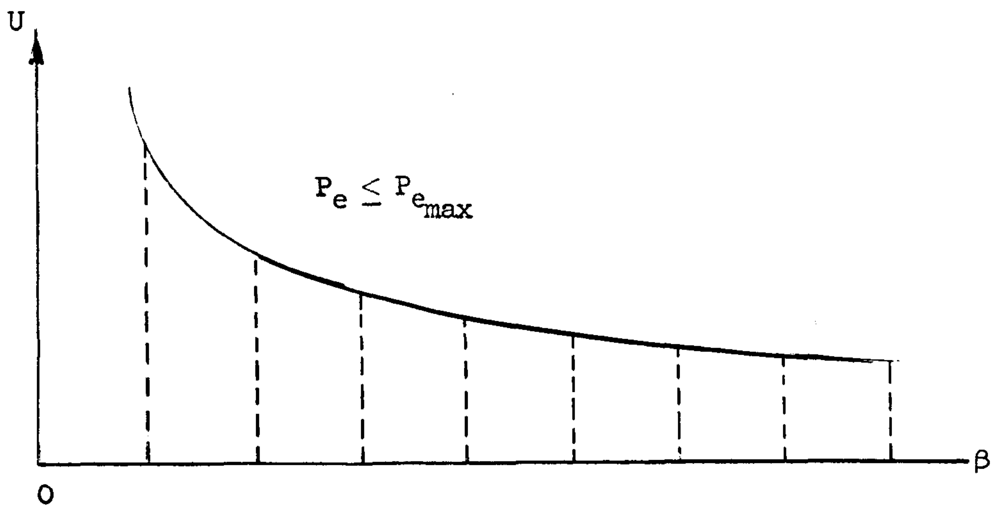

Fig. 3 - Rejection Probability Versus Signal-to-Noise Ratio for Given Equipment

3.2 Comparison of Decision Policies on the Same Equipment

The trade-off performance measure of Fig. 1 is useful not only for comparing the performance of different systems with the same decision policy but also for evaluating the performance of a given system with different policies. Consider, for example, PFM channel supercommutation in which a correct datum is not obtained unless a number of PFM channels are correctly received, as in the S-3 data. In the experiment, gamma ray counts are recorded. When the number of counts goes up or remains the same on successive readings, the data are accepted as correct. When it goes down, the data are considered in error. When decisions are withheld because of internal threshold settings, 9's are printed. The established decision policy is to reject the data from one or more channels on either side of a series of 9's or incorrect data points. This is a decision policy designed to minimize the error probability; but as a consequence the rejection rate is likely to be too high. In comparing different decision policies on the same equipment we proceed for each policy as we did above for each equipment, so that now the curve corresponding to Fig. 3, would have a given policy as parameter, and the object would be to choose the policy that would minimize the area under the curve.

We close with a word on the determination of error probability. Error probability is the weighted sum of the miss and false alarm probabilities. In the established method of processing S-3 data, the procedure of recognizing an error only when the count goes down is misleading because account is not taken of errors that may occur when the count increases as a result of noise. If a test tape of simulated data free of noise is used as a reference, so that the error and rejection probabilities can be correctly estimated for varying threshold settings and signal-to-noise ratios, we cas establish the correct family of trade-off curves for any rejection policy. The estimates of error and rejection probabilities can be obtained from the observed data by means of formulae (as in New York University Technical Memo No. 31), knowing the sample size.

A final point is that the test tape format of the kind just described will permit the effects of noise with the synchronizing waveform and with the data bits to be separately investigated, so that appropriate decision thresholds can be established for each separately.