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POSSIBLE OCCURRENCE OF BOUNDARY SHOCK WAVES

by E. Dale Martin

Ames Research Center
Moffett Field, Calif.

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SUMMARY

The possible occurrence of a viscous region near a surface from which there is rapid efflux of gas, accompanied by large heat transfer, is postulated and investigated theoretically. Such a viscous region, denoted as a boundary shock wave, may occur in the case of a large high-speed meteor entering the earth's atmosphere. The conditions across a boundary shock wave and its structure are calculated from the appropriate macroscopic equations reduced to closed-form expressions under the restriction of a perfect gas flowing at constant total enthalpy.

INTRODUCTION

The purposes of this report are to introduce the concept and to postulate the occurrence of boundary shock waves. A boundary shock wave may be defined as a thin region of viscous flow adjacent to a surface from which gas is flowing at a very high rate with large heat transfer. The flow in this region is dominated by heat conduction and viscous compressive stresses whose rapid decay causes rapid variations of the flow properties over a small distance. A detailed discussion of the causes of boundary shock waves is given in the section following the Introduction.

If a boundary shock wave can occur as postulated, the distinct region of viscous flow would be possible only with high influx Reynolds number and with a source of high heat transfer (such as intense radiation being absorbed) at the wall. This high Reynolds number theory is the opposite limiting case to the more usual mass injection or vapor ablation at a rate small enough that the influx does not appreciably alter the boundary-layer character of an external flow along the wall. In the latter case of mass influx at a small rate (and small influx Reynolds number), the injected flow is entirely viscous, whereas injected flow at high influx Reynolds number must be essentially inviscid a short distance from the surface. The boundary shock wave is therefore a "quick-transition region" belonging to the class of asymptotic phenomena discussed by Friedrichs (ref. 1). The boundary shock is analogous both to a boundary layer, in which the flow is along a wall, and to a shock wave, in which the flow is normal to the thin viscous region (see fig. 1; notation is defined in appendix A).

The boundary shock phenomenon may be important in understanding the flow over a certain class of meteoric fireballs. Large stone meteors that enter the earth's atmosphere at very high velocity have a high rate of mass transfer from the surface by vapor ablation as a result of the intense radiative heating of the surface by the extremely high temperature air behind the shock wave (see refs. 2 to 5). When the ablation rate is so large that the distance, L, from the forward molten surface of the meteor to the interface between the air and the vaporized meteoric material is much greater than the "viscous length," l_v, then $Re_b \equiv L/l_v$ is large and a boundary shock wave is postulated to be present in the vapor flow, with inviscid flow just outside it. (See fig. 2, which represents the flow near the stagnation point of a blunt body.)

A boundary shock wave may also occur in the flow of a gas out of a porous wall if there is a strong source of heat, such as radiation being absorbed at the surface and conducted back into the wall and gas in the pores. The general theory to be developed below will apply to that case if the flow can be laminar, not turbulent.

The boundary shock wave will be analyzed here from the macroscopic, or continuum, point of view. The thermodynamic and flow variables must satisfy the classical macroscopic conservation laws, which will be approximated by using the Navier-Stokes equations of one-dimensional flow. The results in the general theory will be limited by the applicability of the approximate linear, i.e., first order, transport laws (the stress-strain rate relationship and the heat-conduction law) of the Navier-Stokes formulation. If the gradients become too steep, so that the gas is too far out of mechanical equilibrium, these transport laws will not give a very good approximation, and this must be kept in mind. It would be desirable to have a comparable microscopic analysis for comparison, but the state of the art in nonequilibrium kinetic theory for flow near surfaces may not at present be sophisticated enough to permit such an analysis. The one-dimensional macroscopic equations to be used here to study the flow through the boundary shock wave are the same as those that have been used successfully in the study of shock-wave structure. Professor Liepmann and his coworkers (ref. 6) have shown that the structure of most shock waves, especially in the downstream portion, is described very well by the Navier-Stokes equations. The structure of a boundary shock wave corresponds in many cases to a downstream portion of a simple-shock-wave solution. (Note then that the boundary shock may be entirely subsonic.)

Calculation of the flow through a boundary shock will be based largely on the wealth of existing literature (see refs. 6 to 27) on shock waves and their structure. The relationships between the states and flow variables on both sides of a shock wave have analogous counterparts in relationships across a boundary shock which depend on the heat-transfer coefficient at the boundary as well as on the thermodynamic states (to be shown).

The sections which follow in this report will include: (1) a discussion of the causes of boundary shock waves; (2) development of the general theory for one-dimensional, laminar, steady flow through a boundary shock in a perfect gas with longitudinal Prandtl number equal to unity, including explicit expressions for the jump conditions, the heat-conduction coefficient at the wall, and the structure, or variation of properties through the boundary shock; and (3) an approximate special application of the general theory to the special case of very rapid vapor ablation from a blunt body, with the properties of a stone meteor, entering the earth's atmosphere at high speed.

CAUSES OF BOUNDARY SHOCK WAVES

Consider a gas flowing out of a nearly plane wall (fig. 1(c)) by either blowing through pores in the wall or vapor ablation, with radiative heat flux, $-q_r$, being absorbed at the wall (to the left of b) in both cases. Let the influx Reynolds number, Re_b, be large enough that most of the flow within length L is inviscid. Since there is no source of viscous effects to the right of b, any viscous effects must originate at the wall (to the left of b). To understand the origin of the viscous effects in b to e, we must examine the mechanism of mass transfer within the region from s to b.

Consider first the case of blowing through very small pores. At b the velocity has become essentially perpendicular to the wall. Assume that $-q_r$ is absorbed within a very short distance inside the surface s and is conducted back through the wall and into and back through the gas in the holes (to the left in fig. 1(c)) at a high rate. In a one-dimensional flow the viscous-compressive stress and heat-conduction flux are, respectively,

$$\tau = \tilde{\mu}\ du/dx \text{ and } q_c = -k\ dT/dx \tag{1}$$

so that

$$\tau/q_c = -\widetilde{Pr}(du/c_p\ dT) \tag{2}$$

where

$$\widetilde{Pr} \equiv \tilde{\mu}c_p/k \tag{3}$$

and

$$\tilde{\mu} = \lambda + 2\mu = (4/3)\mu + (1/3)\kappa \tag{4}$$

and where $\widetilde{Pr}$ is the "longitudinal Prandtl number," μ is the shear-viscosity coefficient, λ is the second viscosity coefficient, κ is the bulk viscosity coefficient, k is the coefficient of thermal conductivity, c_p is the specific heat at constant pressure, u is the velocity, and T is the temperature. In the effective one-dimensional flow just inside the holes, the large heat flux, $-q_c$, to the left is accompanied by an effective viscous-compressive stress τ. Unless $\widetilde{Pr}$ is zero or infinite, heat conduction in a compressible gas is in general always accompanied by a viscous stress. Recall also from kinetic theory (e.g., for a monatomic gas, see ref. 6, p. 1317) that the thermodynamic pressure is $p = (1/3)\rho(\overline{c_1^2} + 2\overline{c_2^2})$ and $p - \tau = \rho\overline{c_1^2}$ (where $u + c_1$ and c_2 are the components of molecular velocity in the x direction and its normal, respectively, and $\overline{(\ )}$ is the average weighted by the distribution function, so that $\tau = (2/3)\rho(\overline{c_2^2} - \overline{c_1^2})$. The viscous stress τ is different from zero if the averages of the different components of the random velocity are not equal. They evidently are not equal in the presence of a large heat-conduction flux, $q_c = \rho\overline{c^2c_1}$, which itself is different from zero because of the asymmetry of c_1. (Here $c^2 = c_1^2 + 2c_2^2$.) The asymmetry of c_1 in general makes its average squared value different from $\overline{c_2^2}$.

Furthermore, the viscous stress just inside the holes cannot end abruptly, but must decay within a finite distance that is determined by the equations of motion. Therefore the viscous stress, and an accompanying heat flux, exist also at b. Thus, if there is a significant amount of heat absorbed at the wall, viscous effects will be present at b, and the region of their decay is the boundary shock wave.

In the case of rapid vapor ablation, consider only a material that vaporizes from a liquid surface and has no chemical reactions after the vaporization. Assume the radiation is absorbed and the molten material is vaporized within a few molecular diameters at s. The high heat conduction to the left within that region is developing the conditions that will produce a viscous-compressive stress in the vapor state: $\tau = (2/3)\rho(\overline{c_2^2} - \overline{c_1^2})$ (for a monatomic gas). Note that under mild conditions (small heat transfer, low-speed vaporization), the vaporization takes place reversibly, and the vapor is considered to be in equilibrium at the end of the phase change. Under more extreme conditions (high rate of radiation absorption and heat conduction just inside the surface, producing the high rate of vaporization) one expects mechanical (e.g., translational) nonequilibrium to be significant in the vaporization process. Translational nonequilibrium is described macroscopically by the concept of <u>viscosity</u>. A large rate of heat transfer (to the left in fig. 1 or fig. 3) at s and within ϵ is accompanied by a time lag in passing the translational energy from the axial mode (x direction) on to the degrees of freedom of the motion in the directions normal to the x direction. The lag makes $\overline{c_1^2} > \overline{c_2^2}$; that is, because of the lag, the molecules with motion mostly normal to the x direction are not moving (on the average) quite as rapidly as those in the x direction (relative to the mass velocity u), and are therefore tending not to break their intermolecular bonds as rapidly. The viscous stress τ is a measure of this lag. (For polyatomic molecules there will also be a lag associated with the rotational degrees of freedom, also affecting τ.)

The stress τ develops rapidly during the vaporization because, in a pure liquid state, the molecules are effectively in contact and there is probably very little lag in transferring the energy to modes normal to the axial motion; but the lag develops as the molecules acquire the additional energy that will free them from the constraints of the intermolecular forces, and as they begin to separate from each other. By the time the phase transition is completed (at b), the viscous stress τ can be represented by $\tau = (2/3)\rho(\overline{c_2^2} - \overline{c_1^2}) = \tilde{\mu}\, du/dx$. This stress is accompanied by viscous dissipation of energy and an associated heat-conduction flux which, in the vapor state, can be represented by $q_c = \rho\overline{c^2 c_1} = -k\, dT/dx$.

In the above discussion, from the microscopic (molecular) point of view we see that there is a tendency, which is measured by τ, for the vaporization process to be retarded. This has the effect of a <u>compression</u> opposing the <u>expansion</u> of the phase change, so $\tau < 0$. Refer to figure 3, on which the sketch and curves are idealized and qualitative.* Let the mass velocity derivative, du/dx, be separated into two terms (effects of two contributions),

$$\frac{du}{dx} = \left(\frac{du}{dx}\right)_{\text{phase}} + \left(\frac{du}{dx}\right)_{\text{visc}}$$

where

$(du/dx)_{\text{phase}}$	the contribution representing the tendency of the molecules at a station x to break their intermolecular bonds and separate from each other
$(du/dx)_{\text{visc}}$	the retarding effect of viscosity on the velocity change (effect of mechanical, or translational, nonequilibrium); or a resistance to the separation of the molecules (associated with τ)

At b, the term $(du/dx)_{\text{phase}}$ has vanished and the term $(du/dx)_{\text{visc}}$ has the value appropriate to a gas, and is given by $\tau/\tilde{\mu}$. The effect of $(du/dx)_{\text{visc}}$ between s and b is a downward shift from the curve of $(du/dx)_{\text{phase}}$ to give the resulting mass velocity derivative, du/dx. In order for the vaporization process to be retarded by viscosity, τ must be negative, so that the velocity derivative (proportional to acceleration), du/dx, during the phase change is decreased by the viscosity. This macroscopic reasoning is therefore completely in line with the results of reasoning on the molecular level, above.

The viscous effects that develop in the vaporization process cannot end abruptly, but must decay within a distance determined by the conservation equations. That is, although the intermolecular bonds are assumed to be overcome very quickly, the lag in communicating the translational energy to the lateral degrees of freedom may take much more time to accommodate. The region in which these mechanical nonequilibrium effects relax (i.e., viscous effects decay) with distance from their source is the boundary shock wave.

It may be noted that, in order for τ_b to be negative as described above, the density must undergo a slight "undershoot" at the end of the vaporization process, and the mass velocity u undergoes a slight "overshoot." Thus, near the end of the vaporization process, the curve of velocity versus distance (fig. 3) has a maximum point. This overshoot in mass velocity is actually exceedingly small, and the slope of the velocity curve at $x = x_b = 0 + \epsilon$ is extremely small in comparison to, say, $\Delta u/\epsilon$ (slope of dashed line on curve of u vs x). The overshoot in velocity may be explained qualitatively by referring to figure 3. From figure 3 one can see the overshoot in u, or the maximum point on the u curve, as corresponding to the

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*On figure 3 the curve of velocity, for example, is highly distorted. The change of u (or ρ) during the phase change, from s to b, is several orders of magnitude greater than the change from b to e. Also the distance from b to e is several orders of magnitude greater than the distance from s to b.

point where du/dx goes through zero. It is the point where $(du/dx)_{phase}$ has come back down to the relatively very small value equal to $-(du/dx)_{visc}$ before vanishing.

The arguments that have been presented in this section are not intended as proof that the postulated viscous effects will occur. Although one does not expect that the viscous effects will have vanished at b, absolute certainty as to whether or not τ_b will be different from zero in a given case, so that a boundary shock wave will occur, would depend on either adequate experimental verification or on a rigorous analysis by nonequilibrium kinetic theory of liquids and gases for the phase-transition region. Only such an analysis could supply an exact value for τ_b.

The values of the stress τ_b and the heat conduction q_{c_b} would be determined by solving simultaneously all the equations pertinent to a specific problem, as is done in the example calculation in the last section of the analysis. However, the general theory of the boundary shock can be worked out in terms of τ_b and q_{c_b}, without specific application, as in the following section.

GENERAL THEORY OF THE PLANE LAMINAR BOUNDARY SHOCK WAVE IN A PERFECT GAS WITH $\widetilde{Pr} = 1$

The general theory of the laminar boundary shock wave is formulated independently of the details of the mass-transfer process under the assumptions: (1) the flow is close enough to mechanical equilibrium that the Navier-Stokes equations give a valid approximation to the description of the macroscopic thermodynamic and flow variables; and (2) the geometrical structure of the surface is of small enough detail that the velocity vector can be considered to be essentially one-dimensional (normal to the wall) in the main portion of the region of interest. This is equivalent to the assumption $\epsilon \ll \delta$ in the previous section. Then the conditions across the boundary shock wave and the structure can be calculated from the Navier-Stokes equations and depend only on the values of the macroscopic parameters at the vapor boundary b.

The exact equations for conservation of mass, momentum, and energy in one-dimensional steady flow in a nonaccelerating coordinate system are:

$$\frac{d}{dx}(\rho u) = 0 \tag{5}$$

$$\frac{d}{dx}(\rho u^2) = \frac{df}{dx} \tag{6}$$

$$\rho u \frac{d}{dx}\left(e + \frac{1}{2} u^2\right) = \frac{d}{dx}(-q + uf) \tag{7}$$

where x is the distance to the right of the boundary b, e is the internal energy per unit mass, and f is the sum of the "surface forces" in the x direction on an element of mass. In the gas,

$$f = -p + \tau \tag{8}$$

It is often convenient to discuss the equations in terms of the specific enthalpy,

$$h = e + p/\rho \tag{9}$$

rather than the internal energy. The equations of state for a thermally and calorically perfect gas are

$$p = \rho RT, \ h = c_p T, \ e = c_v T \tag{10}$$

where R, c_p, and c_v are constants related by

$$c_p/c_v = \gamma, \ c_p - c_v = R \tag{11}$$

If heat transfer by conduction and radiation are considered, we can write for the heat flux in the x direction (positive to the right):

$$q = q_c + q_r \tag{12}$$

where q_r is the radiative heat flux. Although radiation is included, it will be assumed that the gas is transparent to radiation and that radiation emitted by the gas in the boundary shock region can be neglected. Thus, all radiation is assumed to be emitted or absorbed outside the region from b to e, and q_r is therefore constant across the boundary shock. In the Navier-Stokes formulation the viscous stress and the conduction heat flux are given by equations (1).

The boundary conditions to be used with equations (5), (6), and (7) are:

$$\text{at } x = x_b = 0^+: \quad u = u_b, \ T = T_b, \ q_c = q_{c_b} \tag{13}$$

$$\text{as } x \to \infty: \quad \frac{du}{dx} \to 0, \ \frac{dT}{dx} \to 0 \tag{14}$$

For simplicity, in the following sections, it will be assumed that $\widetilde{\Pr}$ (eq. (3)) is equal to unity. This assumption is not overly restrictive since, as Liepmann and his coworkers point out, $\widetilde{\Pr} \approx 1$ for most real gases (ref. 6). The result, as has been shown for shock waves and as will be shown in a later section for the boundary shock (eq. (57)), is that

$$q_c = \tau u, \ (\widetilde{\Pr} = 1) \tag{15}$$

at every point in the boundary shock. With this result, it will be convenient to present further results in terms of

$$C_{h_c} \equiv \frac{-q_{c_b}}{(1/2)\rho_b u_b{}^3} = \frac{-\tau_b}{(1/2)\rho_b u_b{}^2} \tag{16}$$

Conditions Across a Boundary Shock for $\widetilde{\Pr} = 1$

The conditions across a boundary shock wave are analogous to the conditions across an ordinary shock wave, including the Rankine-Hugoniot relation, the Prandtl relation, and other normal-shock relations (see, e.g., ref. 10, pp. 57-64) and are obtained in much the same manner.

Integration of equations (5), (6), and (7) and use of the remainder of equations (1) through (15) give

$$\rho_b u_b = \rho_e u_e \tag{17a}$$

$$p_b + \rho_b u_b{}^2 - \tau_b = p_e + \rho_e u_e{}^2 \tag{17b}$$

$$\frac{1}{2} u_b{}^2 + \frac{\gamma}{\gamma-1} \frac{p_b}{\rho_b} = \frac{1}{2} u_e{}^2 + \frac{\gamma}{\gamma-1} \frac{p_e}{\rho_e} \tag{17c}$$

where subscript e denotes a value as $x \to \infty$, that is, outside the boundary shock. Note that the assumption of $\widetilde{\Pr} = 1$ has eliminated τ and q from equation (17c), so that the total enthalpy, $h + (1/2)u^2$, is constant.

To derive a relation analogous to the Rankine-Hugoniot relation, combine (17a) and (17b) to obtain

$$\frac{p_b}{\rho_b u_b} - \frac{p_e}{\rho_e u_e} = u_e - u_b + \frac{\tau_b}{\rho_b u_b} \tag{18}$$

Then multiply (18) by $u_e + u_b$ to obtain an expression for $u_e^2 - u_b^2$, which may be equated to another expression from (17c) with the resulting relation analogous to the "Hugoniot relation" (cf. ref. 22, p. 6):

$$(p_e - p_b)\left(\frac{1}{\rho_e} + \frac{1}{\rho_b}\right) + \frac{\tau_b}{\rho_b}\left(1 + \frac{\rho_b}{\rho_e}\right) = \frac{2\gamma}{\gamma-1}\left(\frac{p_e}{\rho_e} - \frac{p_b}{\rho_b}\right) \tag{19}$$

This can be rearranged into a form analogous to what Liepmann and Roshko call the "Rankine-Hugoniot relation" (ref. 10, p. 64):

$$\frac{\rho_e}{\rho_b} = \frac{1 + \frac{\gamma+1}{\gamma-1}\frac{p_e}{p_b} - \frac{\tau_b}{p_b}}{\frac{\gamma+1}{\gamma-1} + \frac{p_e}{p_b} + \frac{\tau_b}{p_b}} \tag{20}$$

We may plot p_e/p_b versus ρ_b/ρ_e, which depends on $(-\tau_b/p_b) = (1/2)\gamma M_b^2 C_{h_c}$ (see fig. 4). Later remarks are given concerning positive and negative values of this parameter. We may call this plot a "Boundary-Shock Hugoniot Diagram," by analogy to the Hugoniot Diagram for a simple shock wave.

Next is developed a relation for boundary shock waves which is analogous to (and, in the case $\tau_b \rightarrow 0$, reduces to) the Prandtl relation for shock waves. Denoting the speed of sound as

$$a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT} \tag{21}$$

we can write equation (18) as

$$u_b - u_e = \frac{a_e^2}{\gamma u_e} - \frac{a_b^2}{\gamma u_b} + \frac{\tau_b}{\rho_b u_b} \tag{22}$$

and (17c) as:

$$\frac{u_b^2}{2} + \frac{a_b^2}{\gamma-1} = \frac{u_e^2}{2} + \frac{a_e^2}{\gamma-1} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right) a*^2 \equiv C \tag{23}$$

(Equation (23) may be regarded as the definition of a^* and the constant C. In cases where $u_b \geq a_b$, a^* is the actual value of a in the boundary shock wave where $u = a$.) Note from equation (23) that

$$a_b^2 = (\gamma - 1)C - \frac{\gamma-1}{2} u_b^2; \; a_e^2 = (\gamma - 1)C - \frac{\gamma-1}{2} u_e^2$$

which may be substituted into (22) to obtain

$$u_b - u_e = a^{*2}\left(\frac{1}{u_e} - \frac{1}{u_b}\right) + \frac{2\gamma}{\gamma+1}\left(\frac{\tau_b}{\rho_b u_b}\right) \tag{24a}$$

Equation (24a) may be further manipulated to give finally

$$u_e u_b = a^{*2} + \frac{2\gamma}{\gamma+1}\left(\frac{\tau_b}{\rho_e - \rho_b}\right) \tag{24b}$$

For the case of the simple shock wave ($\tau_b = 0$ and the "boundary" $\rightarrow -\infty$), the Prandtl relation, which determines that flow through a shock wave must go from either supersonic to subsonic or vice versa (see ref. 10, p. 57), is recovered from equation (24b). For the case of the boundary shock, such a restriction is not imposed.

Other useful relationships across the boundary shock can now be developed as follows: Equations (23) and (24b) give two expressions for $(a^*/u_b)^2$ which, when equated, give an equation that can be rearranged into the quadratic form

$$\left(\frac{\rho_b}{\rho_e}\right)^2 - 2B\left(\frac{\rho_b}{\rho_e}\right) + A = 0 \tag{25}$$

where

$$\left.\begin{aligned} 2B &\equiv \frac{2\gamma}{\gamma+1}\left(1 + \frac{1}{\gamma M_b^2} + \frac{1}{2} C_{h_c}\right) \\ A &\equiv \frac{2\gamma}{\gamma+1}\left(\frac{\gamma-1}{2\gamma} + \frac{1}{\gamma M_b^2}\right) \\ M_b &\equiv \frac{u_b}{a_b} \end{aligned}\right\} \tag{26}$$

The solution is

$$\frac{\rho_b}{\rho_e} = B \pm \sqrt{B^2 - A} \tag{27}$$

which is thus determined for given γ, M_b, and C_{h_c} (or given A and B). To find the pressure ratio, combine equations (17a) and (17b) into the form

$$\frac{p_e}{p_b} = 1 + \gamma M_b{}^2 \left(1 - \frac{\rho_b}{\rho_e} + \frac{1}{2} C_{h_c}\right) \tag{28}$$

The temperature ratio is obtained from the thermal equation of state in (10) and the energy relation (17c) as

$$\frac{T_e}{T_b} = 1 + \frac{\gamma-1}{2} M_b{}^2 \left[1 - \left(\frac{\rho_b}{\rho_e}\right)^2\right] \tag{29}$$

The Mach number downstream of the boundary shock, M_e, can be found by writing equation (17b) in the form

$$\frac{1}{\gamma M_b{}^2} + 1 - \frac{\tau_b}{\rho_b u_b{}^2} = \frac{u_e}{u_b}\left(\frac{1}{\gamma M_e{}^2} + 1\right)$$

from which

$$\gamma M_e{}^2 = \left[\frac{\rho_e}{\rho_b}\left(1 + \frac{1}{\gamma M_b{}^2} + \frac{1}{2} C_{h_c}\right) - 1\right]^{-1} \tag{30a}$$

As an alternative, one could also use

$$M_e{}^2 = \frac{u_e{}^2}{\gamma R T_e} \, \frac{\gamma R T_b M_b{}^2}{u_b{}^2}$$

into which equations (29) and (17a) may be substituted to give

$$M_e^2 = \frac{\left(\frac{\rho_b}{\rho_e}\right)^2 M_b^2}{1 + \frac{\gamma-1}{2} M_b^2\left[1 - \left(\frac{\rho_b}{\rho_e}\right)^2\right]} \qquad (30b)$$

where ρ_b/ρ_e is given by (27).

The conditions prescribed by the equations in this section are possible only if the associated entropy production is positive; thus the second law of thermodynamics must supplement the conservation equations, (17). It can be expressed as

$$\Delta_i s \equiv \Delta s - \Delta_e s = s_e - s_b - \frac{q_{c_b}}{\rho_b u_b T_b} \geq 0 \qquad (31a)$$

where s is the specific entropy, $\Delta_i s$ is the entropy production per unit mass due to the irreversible processes (cf. refs. 28 and 29) within the boundary shock wave, and $\Delta_e s$ is the change in entropy due to heat transfer (external process) across the boundary shock; or

$$\frac{\Delta_i s}{R} = \frac{s_e - s_b}{R} + \frac{1}{2} \gamma M_b^2 C_{h_c} \geq 0 \qquad (31b)$$

The state variable, entropy, can be obtained for a perfect gas from

$$s - s_{ref} = c_p \, ln \frac{T}{T_{ref}} - R \, ln \frac{p}{p_{ref}} \qquad (32)$$

With use of s_b as the reference value and s_e as the value of interest, equation (32) leads to

$$\frac{\gamma-1}{R} (s_e - s_b) = ln\left[\frac{p_e}{p_b} \left(\frac{\rho_b}{\rho_e}\right)^\gamma\right] \qquad (33)$$

Finally, equations (31) and (33) give as the required condition

$$\frac{\gamma-1}{R} \Delta_i s = ln\left[\frac{p_e}{p_b} \left(\frac{\rho_b}{\rho_e}\right)^\gamma\right] + \frac{1}{2} \gamma(\gamma - 1) M_b^2 C_{h_c} \geq 0 \qquad (34)$$

An interesting point here is that the dimensionless number on which the curves on the boundary-shock Hugoniot diagram (fig. 4) depend (in addition to γ) is just the ratio of the change of specific entropy of an element of mass due to transport of entropy (i.e., exclusive of entropy production within the mass element) across the boundary shock, to the gas constant R, that is, (cf. eqs. (31))

$$\frac{-\tau_b}{p_b} = \frac{1}{2} \gamma M_b^2 C_{h_c} = - \frac{\Delta_e s}{R} \tag{35}$$

For given γ and C_{h_c} the quantities $[(\gamma - 1)/R]\Delta_i s$, $[(\gamma - 1)/R](s_e - s_b)$, ρ_e/ρ_b, p_e/p_b, T_e/T_b, and M_e can now be calculated and plotted versus M_b using equations (25) through (34). The results for $C_{h_c} \geq 0$ are shown in figures 5 through 10. The possibility of cases for which $C_{h_c} < 0$ will be discussed later. Although the results shown on the figures were computed for $\gamma = 5/3$, they are qualitatively the same for other values of γ and are easily computed from equations given herein.

It is important to note first of all that the density ratio given by equation (27) and the other quantities given in figures 5 through 10 are double-valued because the quadratic equation (25) has two roots. The second law of thermodynamics (in the form of eq. (34)) determines that only the minus sign in (27) is possible since, as seen in figure 4, the roots corresponding to the plus sign, which are shown as dashed lines, are characterized by negative entropy production. As a matter of interest, the total specific entropy change is shown in figure 6 to point out that it is negative in some cases where the entropy production is positive as required by the second law of thermodynamics. Figure 6 is presented simply to illustrate this fact. It is interesting to compare figures 5 and 6. The difference between the total entropy change and the entropy production is, of course, the nondissipative part of the entropy change, that is, the transport of entropy due to heat conduction (see. eq. (35)).

From figures 7 through 10 we see that the case for which $C_{h_c} = 0$ is obviously singular. For the limiting case $C_{h_c} = 0$ there is no boundary shock. If the efflux is subsonic ($M_b < 1$) and $C_{h_c} \to 0$, the boundary shock simply vanishes. If the efflux is supersonic ($M_b > 1$) and $C_{h_c} \to 0$, the results correspond to a simple shock wave with no boundary, that is, the boundary shock becomes a detached simple shock wave. (This latter case, the detached shock wave, is known to have been observed experimentally within the gas ejected supersonically from a porous blunt body (ref. 30). This has been referred to as "shocking down" of the injected gas.)

Another important point to consider in calculating the possible conditions existing across a boundary shock wave is that the density ratio given by equation (27) and, consequently, the pressure and temperature ratios and the Mach number M_e are physically possible only if

$$B^2 \geq A \tag{36a}$$

or

$$C_{h_c} \geq \sqrt{4\left(\frac{\gamma+1}{\gamma}\right)\left(\frac{\gamma-1}{\gamma} + \frac{2}{\gamma M_b{}^2}\right)} - \left(2 + \frac{2}{\gamma M_b{}^2}\right) \tag{36b}$$

For $\gamma = 5/3$, the curve of the minimum possible C_{h_c} for ρ_b/ρ_e to be real for specified M_b is shown in figure 11. Note that this consideration also does not rule out negative values of C_{h_c} (except at $M_b = 1$), but does limit their magnitude.

Results for positive C_{h_c} have been discussed above, for which the heat is conducted back through the outflowing fluid, in the same direction as the heat conduction occurs in the physical situation existing in a simple shock wave. We may now also consider the possibility of negative values of C_{h_c}, for which heat conduction is in the opposite direction to that in a simple shock wave, that is, in the same direction as the gas flow. As an example calculations were made for $C_{h_c} = -0.1$. The density and pressure ratios and the entropy production are shown in figure 12. As seen from figure 12, the Mach number at the boundary is limited to values less than 0.76 or greater than 1.57 for this case. Both roots in equation (27) are shown in figure 12, and, as seen, the branches of the curves corresponding to the two roots are connected at $M_b = 0.76$ and 1.57 since they have common values there. The entropy production for the branch corresponding to the plus sign in equation (27) is always negative, and is negative for part of the branch corresponding to the minus sign. (Negative-entropy-production portions of the curves on fig. 12 are characterized by dashed lines.) But there do exist "low-speed" and "high-speed" portions for which $\Delta_i s > 0$. For the low M_b portion $\Delta_i s/R$ for the branch corresponding to the minus sign in (27) is so small as to be imperceptible on figure 7 for $C_{h_c} = -0.1$, but it is positive near $M_b = 0$.

No claim is made here regarding the physical existence of the cases for negative C_{h_c} for the following reason. As pointed out by Hayes (ref. 22), existence of a gasdynamic discontinuity (which the boundary shock is for $Re_b = \infty$) depends on the stability of the phenomenon, both in the large and in the small, in addition to satisfaction of the conservation laws of mass, momentum, and energy, the equations of state, and the second law of thermodynamics. The cases with positive C_{h_c} correspond to simple shock-wave solutions which are known to be physically realistic, that is, to be stable and exist in reality. Although the cases with negative C_{h_c} do not correspond to solutions known to be physically realistic, and hence have not been proven to be stable, the possibility of their existence is not ruled out. In a given application, the actual value of C_{h_c} would be determined from appropriate equations, as, for example, is done in a later section in the application to rapid vapor ablation. It should be pointed out here that, as $C_{h_c} \to 0$ with $M_b > 1$, the boundary shock detaches. Thus, at least for $M_b > 1$, one may not expect the boundary shock wave to exist for $C_{h_c} < 0$ in some applications, since required jumps in the flow properties may be accommodated by a detached shock.

Equations are presented above for calculating conditions across a boundary shock wave for given values of γ, M_b, and C_{h_c}. However, in a given problem, for example, one discussed in a later section, one may wish to calculate the conditions for given γ, M_b, and p_e/p_b. The following equations are presented for convenience in that case. An expression for $[1 + (1/2)C_{h_c}]$ from equation (28) may be substituted into (25) and (26) to obtain

$$\left(\frac{\rho_b}{\rho_e}\right)^2 + 2B'\left(\frac{\rho_b}{\rho_e}\right) - A' = 0 \tag{37}$$

where

$$\left.\begin{aligned} 2B' &= \frac{2}{(\gamma-1)M_b^2}\,\frac{p_e}{p_b} \\ A' &= 1 + \frac{2}{(\gamma-1)M_b^2} \end{aligned}\right\} \tag{38}$$

which has the solution

$$\frac{\rho_b}{\rho_e} = -\,B' + \sqrt{(B')^2 + A'} \tag{39a}$$

(where only the positive square root is chosen in order that ρ_b/ρ_e be positive, since B' is always positive). In many cases $(B')^2$ may be much greater than A, so that it may be difficult to obtain an accurate number from (39a). In those cases the root of (37) is computed more easily from

$$\frac{\rho_b}{\rho_e} = \frac{A'}{B' + \sqrt{(B')^2 + A'}} \tag{39b}$$

Calculation of C_{h_c}, T_e/T_b, and M_e then follows from equations (28), (29), and (30).

In later work it will also be found convenient to make calculations for given values of γ, M_b, and M_e. For that purpose the following equations are given. Equation (30b) can be solved for $(\rho_b/\rho_e)^2$ as

$$\left(\frac{\rho_b}{\rho_e}\right)^2 = \frac{M_e^2}{M_b^2} \frac{1 + \frac{\gamma-1}{2} M_b^2}{1 + \frac{\gamma-1}{2} M_e^2} \tag{40}$$

Equation (29) becomes

$$\frac{T_e}{T_b} = 1 + \frac{\gamma-1}{2M_b^2} \frac{M_b^2 - M_e^2}{1 + \frac{\gamma-1}{2} M_e^2} \tag{41}$$

Since the density ratio is known from (40), the pressure ratio can be computed from

$$\frac{p_e}{p_b} = \frac{\rho_e u_e^2}{\gamma M_e^2} \frac{\gamma M_b^2}{\rho_b u_b^2} = \frac{\rho_b}{\rho_e} \frac{M_b^2}{M_e^2} \tag{42a}$$

$$= \frac{M_b}{M_e} \sqrt{\frac{1 + \frac{\gamma-1}{2} M_b^2}{1 + \frac{\gamma-1}{2} M_e^2}} \tag{42b}$$

and $(1/2)C_{h_c}$ can be found from equation (28), with use of (40) and (42a), as

$$\frac{1}{2} C_{h_c} = \frac{\rho_b}{\rho_e}\left(1 + \frac{1}{\gamma M_e^2}\right) - \left(1 + \frac{1}{\gamma M_b^2}\right)$$

$$= \frac{M_e}{M_b} \sqrt{\frac{1 + \frac{\gamma-1}{2} M_b^2}{1 + \frac{\gamma-1}{2} M_e^2}} \left(1 + \frac{1}{\gamma M_e^2}\right) - \left(1 + \frac{1}{\gamma M_b^2}\right) \tag{43}$$

Structure of a Plane Boundary Shock

In order to calculate the variation of the flow variables through the boundary shock wave, one may begin with the integrated forms of equations (5), (6), and (7) and with the boundary conditions (13) and (14). (Conditions (13) are actually just definitions of the symbols for the boundary values. To these could be added: $\rho = \rho_b$, $\tau = \tau_b$, $p = p_b$, etc., at $x = x_b = 0^+$.) With use of the appropriate auxiliary expressions, including the equations of state, (10), the differential equations and boundary conditions may be put in terms of only two variables, u and T. Then it is convenient to make the equations dimensionless by defining and substituting the dependent variables:

$$\bar{u} \equiv u/u_b, \quad \bar{T} \equiv T/T_b \tag{44}$$

and a new independent variable

$$\xi \equiv \rho_b u_b \int_0^x \frac{dx}{\tilde{\mu}} \tag{45}$$

With the assumption $\widetilde{Pr}$ = const., $\tilde{\mu}$ is proportional to k, since c_p is constant, but the variation of $\tilde{\mu}$ need not be further specified if the results are left in terms of the independent variable ξ. We now have the equations:

$$\frac{d\bar{u}}{d\xi} - \bar{u} - \frac{1}{\gamma M_b{}^2}\frac{\bar{T}}{\bar{u}} = \frac{\tau_b}{\rho_b u_b{}^2} - 1 - \frac{1}{\gamma M_b{}^2} \tag{46}$$

$$\frac{d}{d\xi}\left[\frac{1}{\widetilde{Pr}}\frac{\bar{T}}{(\gamma-1)M_b{}^2} + \frac{1}{2}\bar{u}^2\right] - \left[\frac{\bar{T}}{(\gamma-1)M_b{}^2} + \frac{1}{2}\bar{u}^2\right] = -\left[\frac{1}{(\gamma-1)M_b{}^2} + \frac{1}{2}\right] + \left(\frac{-q_{c_b}}{\rho_b u_b{}^3} + \frac{\tau_b}{\rho_b u_b{}^2}\right) \tag{47}$$

and the boundary conditions

$$\xi \to \infty: \quad \frac{d\bar{T}}{d\xi} \to 0, \quad \frac{d\bar{u}}{d\xi} \to 0 \tag{48}$$

In the special case considered here for which $\widetilde{Pr} = 1$, it is convenient to define

$$\bar{H} \equiv \frac{c_p T + \frac{1}{2}u^2}{u_b{}^2} \equiv \frac{\bar{T}}{(\gamma-1)M_b{}^2} + \frac{1}{2}\bar{u}^2 \tag{49}$$

Note, then, that

$$\frac{d\bar{H}}{d\xi} \equiv \frac{k\frac{dT}{dx}}{\rho_b u_b{}^3} + \frac{u\tilde{\mu}\frac{du}{dx}}{\rho_b u_b{}^3} \equiv \frac{-q_c + u\tau}{\rho_b u_b{}^3} \tag{50}$$

Further define:

$$\theta \equiv \overline{H} - \overline{H}_b \tag{51}$$

so that (47) is simply

$$\frac{d\theta}{d\xi} - \theta = \left(\frac{d\theta}{d\xi}\right)_b \tag{52}$$

where, by definition,

$$\text{at } \xi = 0, \ \theta = 0 \tag{53}$$

and where, from the boundary condition (48),

$$\text{as } \xi \to \infty, \ \frac{d\theta}{d\xi} \to 0 \tag{54}$$

The only possible solution is, of course,

$$\theta \equiv 0 \equiv \frac{d\theta}{d\xi} = \frac{d\overline{H}}{d\xi} \tag{55}$$

Hence, equations (49), (50), and (51) give

$$\overline{T} = 1 + \frac{\gamma-1}{2} M_b{}^2 \left(1 - \overline{u}^2\right) \tag{56}$$

and

$$q_c = u\tau \tag{57}$$

from which also equation (16) stems. Equation (46) then becomes,

$$\frac{d\overline{u}}{d\xi} = a\overline{u} - b + \frac{c}{\overline{u}} \tag{58}$$

where, by definition,

$$\text{at } \xi = 0, \bar{u} = 1 \tag{59}$$

and the boundary condition is:

$$\xi \rightarrow \infty, \frac{d\bar{u}}{d\xi} \rightarrow 0 \tag{60}$$

and where

$$a = \frac{\gamma+1}{2\gamma}, \; b = 1 + \frac{1}{\gamma M_b^2} + \frac{1}{2} C_{h_c}, \; c = \frac{\gamma-1}{2\gamma} + \frac{1}{\gamma M_b^2} \tag{61}$$

(There should be no confusion between this constant, a, and the speed of sound, a, used earlier.) The solution for $d^2 > 0$ (see eq. (36a)), where

$$d = \sqrt{b^2 - 4ac} = 2a\sqrt{B^2 - A} \tag{62}$$

is

$$e^{-2a\xi} = \left(\frac{a - b + c}{a\bar{u}^2 - b\bar{u} + c}\right)\left(\frac{2a\bar{u} - b + d}{2a\bar{u} - b - d} \cdot \frac{2a - b - d}{2a - b + d}\right)^{b/d} \tag{63}$$

from which, given γ, M_b, and C_{h_c}, one can calculate $\bar{u}$ versus ξ (or, given just the two parameters $b/a = 2B$ and $c/a = A$, one can find $\bar{u}$ versus $a\xi$; compare (25) and (58) for the case $\bar{u} = \bar{u}_e$, $d\bar{u}/d\xi = 0$). Then equation (56) gives $\bar{T}$ versus ξ, and finally one can find

and

$$\left.\begin{aligned} \bar{\rho} &\equiv \frac{\rho}{\rho_b} = \frac{1}{\bar{u}} \\ \bar{p} &\equiv \frac{p}{p_b} = \bar{\rho}\bar{T} \end{aligned}\right\} \tag{64}$$

versus ξ. Results for a typical case ($\gamma = 5/3$, $M_b = 1$, $C_{h_c} = 1/2$) are shown plotted in figure 13.

The variation of ξ with x, and hence of the flow variables with x, is easily found if $\tilde{\mu}$ is given as a function of T. This variation has been discussed by many authors in connection with the shock-wave problem. The variation of $\tilde{\mu}$ as a gas property, for which reference can be made to the literature, is not considered here, as the intent was simply to show how the boundary-shock-wave structure could be calculated, in addition to obtaining the result (57) needed in above developments.

APPROXIMATE SPECIAL APPLICATION OF THE GENERAL THEORY TO RAPID VAPOR ABLATION

The purpose of this section is to make a calculation of the boundary shock wave near the forward face of a blunt body having the properties of a stone meteor entering the earth's atmosphere under conditions for which the boundary-shock phenomenon might be expected to occur.

In an effort to eliminate all unnecessary complications, a number of idealizations and approximations are used in the present application. Consider the same assumptions used in the development of the theory above. The applicability of the properties of a perfect gas with $\tilde{Pr} = 1$ in the vapor from a stone meteor will be discussed later. The assumption of steady flow is here one of quasi-steady flow, for which time derivatives are neglected in comparison to space derivatives, with forces in the body, or wall mass, replaced by accelerations of the body. The ablating surface is considered to be receding back into the liquid material with constant relative speed u_s, so that in the coordinate system of figure 1(c), the surface $x = 0$ is stationary and the wall material is moving toward the surface with constant velocity u_s. Assume that very little heat radiation is emitted or absorbed within the short distance to the right of b in figure 1(c), but that all radiation is absorbed at the surface, within the very small distance ϵ in figure 1(c). Assume also that no chemical or phase changes take place to the right of x_b and that only vapor is blown off.

With very large Re_b, the region of possible rapid variations under consideration in the flow near the surface is very thin, so that consideration of one-dimensional flow out of a plane surface gives a valid approximation and conditions (14) are the appropriate first-order outer boundary conditions. The one-dimensional equations of motion which apply to the material regardless of its phase, and hence which can be used to relate conditions to the left of $x = 0$ (in the unvaporized state) to those in the flowing-vapor state are given by (5), (6), and (7), or in their integrated form:

$$\left.\begin{aligned} \rho u \equiv m &= \text{constant} \\ \rho u^2 - f &= \text{constant} \\ \rho u e + \frac{1}{2}\rho u^3 + q - uf &= \text{constant} \end{aligned}\right\} \tag{65}$$

Equations (1) through (15) apply for $x > 0$. The region from s to b in figure 1(c) is treated as being infinitely thin, so there are jump discontinuities in velocity and density at $x = 0$ due to the phase change. To the left of $x = 0$, the compressive stress is $-f$, the equation of state is

$$\rho = \rho_s = \text{constant}, \quad (x < 0) \tag{66}$$

and the heat flux is

$$q = q_c = -k \frac{dT}{dx}, \quad (x < 0) \tag{67}$$

where k is appropriate to the state of the material at the particular location x. With subscripts a, f, and s denoting values at the respective locations: arbitrary point inside the wall at the cold solid state (x_a), fusion interface (x_f), and surface just inside radiation-absorption and vaporization interface ($x_s = 0^-$), one can write for the internal energy in $x < 0$:

$$\left.\begin{aligned} e - e_a &= c_{sol}(T - T_a), \quad (x < x_f) \\ e - e_a &= c_{sol}(T_f - T_a) + L_f + c_{liq}(T - T_f), \quad (x_f < x \leq x_s = 0^-) \end{aligned}\right\} \tag{68}$$

where c_{sol} and c_{liq} are the specific heats in the solid and liquid, taken to be constant, and L_f is the latent heat of fusion.

For $x > 0$ the solution to equations (65) is given in the above section on the general theory of the boundary shock wave.

For $x < 0$ equations (65), (66), and (67) give

$$\left.\begin{aligned} \rho_a &= \rho_s \\ u_a &= u_s \\ f_a &= f_s \\ \rho_s u_s(e_a - e_s) &= k_a\left(\frac{dT}{dx}\right)_a - k_s\left(\frac{dT}{dx}\right)_s \end{aligned}\right\} \tag{69}$$

which simply determines the temperature distribution inside the wall and, hence, relates the heat conduction and temperature at the surface ($x = 0^-$) to the heat conduction and temperature at point a inside the wall. By definition, x_a is in the cold interior of the wall, where $T = T_a = \text{constant}$ and $dT/dx = 0$, so the term $k_a(dT/dx)_a$ in (69) is zero.

To relate the conditions across the surface at $x = 0$, equations (65) take the form:

$$\rho_b u_b = \rho_s u_s = m \tag{70a}$$

$$\rho_b u_b^2 + p_b - \tau_b = \rho_s u_s^2 - f_s \tag{70b}$$

$$\rho_b u_b e_b + \frac{1}{2}\rho_b u_b^3 + q_{c_b} + q_r + u_b(p_b - \tau_b) = \rho_s u_s e_s + \frac{1}{2}\rho_s u_s^3 - \left(k\frac{dT}{dx}\right)_s - u_s f_s \tag{70c}$$

Noting from equations (70a) and (70b) that

$$f_s = -\rho_b u_b(u_b - u_s) - p_b + \tau_b \tag{71}$$

and substituting this into (70c), we find

$$-q_{c_b} - q_r = \left(k\frac{dT}{dx}\right)_s + \rho_b u_b(e_b - e_s) + \frac{1}{2}\rho_b u_b(u_b - u_s)^2 + (p_b - \tau_b)(u_b - u_s) \tag{72}$$

The internal energy change across $x = 0$ is approximately

$$e_b - e_s = L_v - RT_b \tag{73}$$

for vaporization either at low rates or at high rates, where L_v is the latent heat of vaporization for low rates (see appendix B). It is convenient to define (using eqs. (68) and (73)):

$$\zeta_{ab} \equiv e_b - e_a + RT_b \tag{74a}$$

$$= c_{sol}(T_f - T_a) + L_f + c_{liq}(T_b - T_f) + L_v \tag{74b}$$

which is the heat that must be transferred to a unit mass to raise it from the cold state at temperature T_a to the vapor state at temperature T_b, including all phase changes. (Note that T_b has been taken to be equal to T_s; see appendix B.) Then the terms

$$\left(k\frac{dT}{dx}\right)_s + \rho_b u_b(e_b - e_s) \tag{75}$$

in equation (72) can be replaced, using (69), by

$$\rho_b u_b \left(\zeta_{ab} - RT_b\right) \tag{76}$$

Equation (72) is further simplified by using

$$u_b - u_s = u_b(1 - \rho_b/\rho_s) \approx u_b \tag{77}$$

to obtain

$$-q_r = \rho_b u_b \left(\zeta_{ab} + \frac{1}{2} u_b^2\right) + q_{c_b} - \tau_b u_b \tag{78}$$

For $\widetilde{Pr} = 1$, it was shown above that the solution for the vapor flow is possible only if $q_c = \tau u$ (eq. (57)), with which we have finally

$$-q_r = \rho_b u_b \left(\zeta_{ab} + \frac{1}{2} u_b^2\right) , \; (\widetilde{Pr} = 1) \tag{79}$$

where ζ_{ab}, given by equation (74b), is a linear function of T_b.

For given values of the incident radiative heat flux $(-q_r)$ and the external pressure (p_e), the above equations (including the equations for the boundary shock in the previous section) are sufficient to determine the boundary-shock solution in terms of two unknown quantities, say p_b and $\rho_b u_b$. Some approximate equations describing the vaporization process may be used to relate p_b and $\rho_b u_b$ to the other parameters of the problem, and thus to complete the determination, as described in the remaining paragraphs of this section.

The rate of vaporization, or the mass flux, depends on the velocities of the molecules leaving the surface during the phase change, and hence on the temperature at vaporization. For present purposes, let us accept the roughly approximate relation used by Öpik (ref. 31, p. 24):

$$(\rho u)_b = \frac{1}{2} \rho_b \overline{v}_x \tag{80a}$$

where ρ_b is the vapor density at saturation pressure and $(1/2)\overline{v}_x$ is the average component of molecular velocity in the +x direction. For typical meteoritic stone, this reduces (ref. 31, p. 161) to:

$$\left(\frac{\rho_b u_b}{1\ \text{gm cm}^{-2}\text{sec}^{-1}}\right) = 3.08 \times 10^{-4} \left(\frac{p_b}{1\ \text{dyne/cm}^2}\right)\left(\frac{1^\circ\ \text{K}}{T_b}\right)^{1/2} \tag{80b}$$

With use of the equation of state (10), equation (80b) gives directly

$$M_b = \frac{u_b}{\sqrt{\gamma R T_b}} = \sqrt{\frac{R}{\gamma}}\left(\frac{\rho_b u_b T_b{}^{1/2}}{p_b}\right) = 3.08 \times 10^{-4} \sqrt{\frac{R/\gamma}{1\ \text{cm}^2\text{sec}^{-2}(^\circ\text{K})^{-1}}} \tag{80c}$$

For meteoritic stone, Öpik gives (ref. 31, p. 160) values for the mean atomic weight as 23 and the mean molecular weight of vapors as 50. Hence, $R = R_u/(50\ \text{gm/mole}) = 1.66 \times 10^6\ \text{cm}^2/\text{sec}^2\ {}^\circ\text{K}$, where R_u is the universal gas constant. Since the mean molecular weight is about twice the mean atomic weight, we assume the gas mixture can be approximated as a perfect diatomic gas with $\gamma = 7/5$ and $c_p = \gamma R/(\gamma - 1) = (7/2)R = 5.8 \times 10^6\ \text{cm}^2/\text{sec}^2\ {}^\circ\text{K}$. (These values seem to be realistic, since c_p for the vapor is then about half the value of the liquid specific heat given by Öpik as 1.1×10^7. By way of comparison, the specific heat, c_p, of water vapor is about one-half the specific heat of liquid water.) The result from equation (80c) is then $M_b = 0.333$.

The vapor pressure, p_b, and the boiling temperature, T_b, may be related approximately by Clapeyron's equation:

$$\frac{p_b}{p_{ref}} = e^{\frac{L_v}{R}\left(\frac{1}{T_{ref}} - \frac{1}{T_b}\right)} \tag{81}$$

Clapeyron's equation is derived by assuming that the phase transition takes place reversibly at constant temperature and pressure while the entropy and specific volume (or density) change (ref. 32, p. 317; see also appendix B). Strictly speaking, therefore, Clapeyron's equation would apply only to vaporization at low rates. The use of Clapeyron's equation as a reasonable gross approximation relating p_b to T_b in the present application with higher vaporization rates is justified because of: (1) the exponential (rapidly varying) dependence of p_b on T_b (see appendix B), and (2) the fact that if one makes a gross intuitive modification of Clapeyron's equation to account for the viscous effects and the variation of pressure during the vaporization process, the results of the calculations are not significantly different from those obtained using Clapeyron's equation (see appendix B and numerical results given below).

For the application to the high-speed meteor composed of the typical meteoritic-stone material, Öpik (ref. 31, p. 161) gives the mean boiling point as 2960° K (at 760 mm Hg or 1.013×10^6 dynes/cm^2) which can be used as the reference value in equation (81). The value for R is estimated above, and L_v can be taken from Öpik's data (given below). For reasons that will become apparent, it is expedient to rewrite equation (79) as a relationship between q_r and T_b in terms of known quantities, using equation (81) in the form

$$\frac{p_b}{1\ \text{dyne/cm}^2} = \exp\left(26.13 - \frac{36{,}500\ {}^\circ\text{K}}{T_b}\right) \tag{82}$$

and the equations

$$\rho_b = p_b/RT_b \quad \text{and} \quad u_b = M_b\sqrt{\gamma RT_b} \tag{83}$$

The result is

$$\frac{-q_r}{1\ \text{erg cm}^{-2}\text{sec}^{-1}} = \exp\left(26.13 - \frac{36{,}500\ {}^\circ\text{K}}{T_b}\right)M_b\sqrt{\frac{\gamma RT_b}{1\ \text{cm}^2\text{sec}^{-2}}}\left[\frac{\zeta_{ab}(T_b)}{RT_b} + \frac{1}{2}\gamma M_b{}^2\right] \tag{84}$$

where $\zeta_{ab} = \zeta_{ab}(T_b)$ is given by equation (74b) and where, from Öpik (ref. 31, pp. 61 and 161), values of the required constants in (74b) for this case are:

$L_f = 2.65 \times 10^9$ erg/gm, $L_v = 6.05 \times 10^{10}$ erg/gm

$c_{sol} = 8.95 \times 10^6$ erg/gm °K, $c_{liq} = 1.1 \times 10^7$ erg/gm °K

$T_a = 200^\circ$ K $T_f = 1800^\circ$ K

If the radiative heat flux to the wall causing the ablation, $-q_r$, and the external pressure, p_e, are specified, then the solution of the equations for the boundary shock wave is determined and can be calculated by iteration on equation (84). Thus, a trial value of T_b may be chosen and iterated upon until the proper value for q_r is obtained. Then p_b is determined by equation (82), the ratio p_e/p_b can be computed, and equations (39), (28), (29), and (30) can then be used successively to find ρ_e/ρ_b, C_{hc}, T_e/T_b, and M_e.

The radiative heat flux, $-q_r$, is a function of conditions of a particular problem. It depends, for example, on a body radius, speed, and altitude. Representative values of $-q_r$ and p_e used are, respectively, 17,300 watts/cm^2 and 7.55×10^5 dynes/cm^2, found from a simplified approximate calculation of flow in the forward region of a spherical body of 4.62 m radius moving at 15 km/sec at an altitude of 60 km in the earth's atmosphere. The calculations as outlined herein, using the numbers given above corresponding to stone meteors, lead to: $M_b = 0.333$, $T_b = 2720^\circ$ K, $p_b = 3.345 \times 10^5$ dynes/cm^2,

$\rho_b = 7.41 \times 10^{-5}$ gm/cm^3, $u_b = 2.65 \times 10^4$ cm/sec, $p_e/p_b = 2.257$, $\rho_e/\rho_b = 2.215$, $M_e = 0.149$, and $C_{h_c} = 13.772$. If the "modified Clapeyron's equation" is used (appendix B), the above numbers are the same except for replacement of corresponding values by: $-q_r = 17{,}120$ watts/cm^2, $r_b = 4.58$ m, $T_b = 2{,}880^\circ$ K, $\rho_b = 7.00 \times 10^{-5}$ gm/cm^3, and $u_b = 2.73 \times 10^4$ cm/sec.

CONCLUDING REMARKS

An attempt has been made to present an introduction to a possible phenomenon defined herein as a boundary shock wave, a thin layer of gas through which viscous effects decay in very high-speed mass transfer accompanied by large heat transfer. The general theory of laminar flow through a plane boundary shock was developed. Results from the general theory include calculations of the conditions existing across a boundary shock, and of the internal boundary-shock structure, for given conditions at the boundary. The solution of the governing equations was reduced to closed-form expressions for the quantities of interest under the restrictions of a perfect gas flowing at constant total enthalpy. Numerical results were obtained and plotted for the case where the specific heat ratio, γ, is $5/3$.

This theory was then applied to rapid vapor ablation from a body with flight conditions appropriate to a possible meteoric fireball in the earth's atmosphere.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Sept. 22, 1965

APPENDIX A

PRINCIPAL NOTATION

A, B	constants defined by equations (26)
a	speed of sound (eq. (21))
a^*	critical speed of sound defined by equation (23)
C	constant defined by equation (23)
C_{h_c}	conduction heat-transfer coefficient, $\frac{-q_{c_b}}{\frac{1}{2}\rho_b u_b^3}$
c_1, c_2	components of random molecular velocity in the direction of x, and perpendicular to x, respectively
c_v, c_p	specific heats at constant volume and pressure, respectively
e	specific internal energy
f	stress, or "surface force" per unit area
$\overline{H}$	dimensionless total specific enthalpy (eq. (49))
h	specific enthalpy (eq. (9))
k	thermal conductivity
L	characteristic length, especially width of vapor region
L_f, L_v	latent heats of fusion and vaporization
l_v	viscous length, $\tilde{\mu}_b/\rho_b u_b$
M	Mach number, $\frac{u}{a}$
m	mass flux (eq. (65))
$\widetilde{Pr}$	Prandtl number based on $\tilde{\mu}$, $\frac{\tilde{\mu} c_p}{k}$
p	thermodynamic gas pressure
q	heat flux (to the right in fig. 1(c))
q_c	conduction heat flux (eq. (1))
q_r	radiative heat flux

R	gas constant (eq. (10))
Re_e, Re_1, Re_b	Reynolds numbers defined in figure 1
r_b	body radius
s	specific entropy
T	temperature
$\bar{T}$	dimensionless temperature (eqs. (44))
t	time
u	velocity in the x direction
$\bar{u}$	dimensionless velocity (eqs. (44))
x	space coordinate, positive to the right in figure 1(c)
$\Delta_e s$	change in specific entropy due to heat transfer (external process) across a boundary shock (eq. (35))
$\Delta_i s$	change in specific entropy due to irreversible (internal) processes, that is, entropy production, across a boundary shock (eqs. (31))
Δs	total change in specific entropy across a boundary shock, $s_e - s_b$ (eq. (33) for a perfect gas)
γ	ratio of specific heats, $\frac{c_p}{c_v}$
ζ_{ab}	heat that must be transferred to a unit mass of material to raise it from the cold state at temperature T_a to the gaseous state at temperature T_b (eqs. (74))
θ	defined by equation (51)
$\tilde{\mu}$	defined by equation (4)
ξ	dimensionless independent variable defined by equation (45)
ρ	mass density per unit volume
τ	defined by equation (1)

Subscripts

a	value inside the wall $(x < 0)$ where $k \frac{dT}{dx} \approx 0$
b	value at the boundary in the gas $(x = 0^+)$
e	value in the gas outside the boundary shock wave $(x \rightarrow \infty)$ (or outside the boundary layer in fig. 1)
s	value at the surface, inside the wall $(x = 0^-)$
1, 2	values upstream and downstream, respectively, of a simple shock wave

Superscript

*	condition where $u = a$

APPENDIX B

APPROXIMATE THERMODYNAMIC EQUATIONS FOR RAPID VAPORIZATION

Two different approximate formulations for the vaporization process are considered. The first is well known, but, for quantitative accuracy, its use might be limited to fairly low rates of vaporization. However, the results serve as a foundation for the proposed formulas applying to more rapid vaporization.

The first law of thermodynamics for an element of mass undergoing a phase change is

$$T\,ds = de + p\,dv \tag{B1}$$

$$= dh - v\,dp \tag{B2}$$

$$= c_v\,dT + T\left(\frac{\partial p}{\partial T}\right)_v dv \tag{B3}$$

where $v \equiv 1/\rho$ is the volume per unit mass. Denote the <u>initial</u> state (liquid) by subscript i and the <u>final</u> state (vapor) by subscript f.

1. First approximate method.-

If the phase change takes place at a low rate, the process is nearly reversible, isothermal, and isobaric. The heat transferred reversibly to the element, per unit mass, or the latent heat of vaporization, is, from equation (B2),

$$L_v = T(s_f - s_i) = h_f - h_i \tag{B4a}$$

$$= e_f - e_i + p(v_f - v_i)$$

$$\approx e_f - e_i + pv_f$$

$$= e_f - e_i + RT \tag{B4b}$$

Since p and T are constant during the process,

$$p = p(T) \tag{B5}$$

so that

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{dp}{dT} \tag{B6}$$

Integration of equation (B3) for the process at constant T gives then

$$L_V = T \frac{dp}{dT} (v_f - v_i) \approx T \frac{dp}{dT} v_f \tag{B7}$$

Use of the perfect gas equation of state:

$$p_f v_f = RT_f \tag{B8}$$

in (B7) then gives

$$\frac{dp}{p} = \frac{L_V}{R} \frac{dT}{T^2} \tag{B9}$$

Equation (B9) integrates (for constant L_V) to give the well-known Clapeyron's equation for the pressure to be overcome by the molecules escaping the liquid (the saturation vapor pressure) as a function of the vaporization (boiling) temperature:

$$\frac{p_f}{p_{ref}} = e^{\frac{L_V}{R}\left(\frac{1}{T_{ref}} - \frac{1}{T_f}\right)} \tag{B10}$$

For a vaporization process rapid enough that it is not reversible and the pressure is not constant, Clapeyron's equation should still provide a reasonable first approximation for p_f as a function of T_f for the following reason. The variation of p_f with T_f is exponential, so the variation of p_f due to changes of p during the process and due to irreversibility will be negligible in comparison to the dependence on T if those effects are not large.

2. Second approximate method.-

For higher rates of vaporization the process is not reversible because of viscous dissipation, and not isobaric because of significant viscous stress, which makes the normal stress on a fluid element different from purely thermodynamic pressure. The following estimate of the relationships among the thermodynamic variables for the vaporization process is proposed as an improved approximation in that case.

The steady-flow momentum equation is

$$p + \rho u^2 - \tau = \text{constant} \tag{B11}$$

For the first approximation above it was implicitly assumed that both ρu^2 and $-\tau$ were negligible, so that p was constant during the process of the phase change. For this improved approximation, assume only that

$$\rho u^2 \ll p - \tau \tag{B12}$$

(with this condition remaining as a limitation on the use of the final results). Then, during the vaporization, we have, approximately,

$$p - \tau = p_f - \tau_f \tag{B13}$$

For rapid vaporization, the total heat supplied to a unit mass during vaporization could be denoted, using equation (B2), as

$$L_v^* \equiv T(s_f - s_i) = h_f - h_i - \int_{p_i}^{p_f} v \, dp \tag{B14}$$

Although L_v^* may be quite different from the value L_v for slow vaporization rates, equation (B4b) may still be retained as a valid approximate energy relation for high rates. The reasoning is as follows: In the process of vaporization from a liquid surface, the individual molecules escaping the intermolecular bonds (close-range forces) acquire a certain amount of potential energy, which, in the macroscopic sense, is thought of as increased internal energy of the fluid. This change in internal energy of the fluid, or the value of the potential energy added to the individual molecules, is assumed not to be dependent on how fast the interface between the vapor and the liquid recedes into the liquid, that is, on the rate of vaporization. Thus the same internal-energy change is required for rapid vaporization of the material as under conditions of low-speed vaporization at the same temperature. Since $\rho_f \ll \rho_i$, the enthalpy change is approximately

$$h_f - h_i = e_f - e_i + RT \tag{B15}$$

Thus, for the same temperature and same $(e_f - e_i)$, the enthalpy change is the same for high rates as for low rates; that is,

$$h_f - h_i = e_f - e_i + RT = L_v \tag{B16}$$

An additional consequence of the above arguments is that the temperature remains nearly constant during vaporization at a high rate even though the pressure is changing and viscous effects are important. Since $h_i = h_i(T)$ and $h_f = h_f(T)$ and since $h_f - h_i$ is the same for high rates as for low rates at a given initial enthalpy, h_i, and $T_f = T_i$ for low rates, it follows that $T_f = T_i$ for high rates of vaporization as well.

In order to understand more completely the relationship between L_v and L_v^* for rapid vaporization, consider the following. Equations (B14) and (B16), along with (B13), show that

$$L_v - L_v^* = \int_{p_i}^{p_f} v\,dp = \int_0^{\tau_f} v\,d\tau \tag{B17}$$

The quantity L_v^* is different from the latent heat for low rates, L_v, because of the variation of p during the process at high rates, or, equivalently, because of the significant variation of τ. The relationship between $\int_0^{\tau_f} v\,d\tau$ in equation (B17) and the irreversible heating due to viscous dissipation in the process can be found by first writing, for the heat addition, per unit mass, to the mass element considered in equations (B1) to (B3):

$$T\,ds = T\,d_e s + T\,d_i s \tag{B18}$$

where

$$T\,d_e s = \frac{-dq}{\rho u} = \frac{-dq_c - dq_r}{\rho u} \tag{B19}$$

is the heat transferred to the differential element per unit mass during the vaporization process,

$$q = q_r + q_c \tag{B20}$$

is the heat flux (positive in the direction of flow) composed of the flux of radiation being absorbed during the process and flux of heat conduction, and where

$$T\,d_i s = \frac{\tau\,du}{\rho u} \tag{B21}$$

is the heat added by viscous dissipation per unit mass of the differential element. Now, from equation (B17),

$$L_v - L_v^* = \int_0^{\tau_f} v\,d\tau = \int_0^{\tau_f} \frac{u\,d\tau}{\rho u} = \frac{\tau_f u_f}{\rho u} - \int_{u_i}^{u_f} \frac{\tau\,du}{\rho u} \tag{B22}$$

where ρu is constant for the vaporization process, by conservation of mass. With equation (B21), then,

$$L_V - L_V^* = \frac{\tau_f}{\rho_f} - \int_{(i)}^{(f)} T \, d_i s \tag{B23}$$

Thus the difference between L_V and L_V^* is due to the work term $\tau_f u_f/\rho u = \tau_f/\rho_f$ as well as to the irreversible heating by viscous dissipation. Even if the irreversible part of $T(s_f - s_i)$ in (B14) is much less than the reversible part (which is true because of large q_r being absorbed in the vaporization process), L_V^* can be significantly different from L_V because the reversible part of $T(s_f - s_i)$ is different due to the pressure change. However, as pointed out above, $h_f - h_i = L_V$ remains the same at a high vaporization rate as at a low rate.

The vapor pressure, p_f, in Clapeyron's equation, (B10), is interpreted as the "pressure to be overcome" by the molecules escaping the liquid. Since for rapid vaporization there is an additional stress, $-\tau_f$, to be overcome, let us assume that p_f in Clapeyron's equation can be replaced by $p_f - \tau_f$ to obtain a rough approximate relationship between the final total compressive stress of the process, $p_f - \tau_f$, and the temperature, $T_f = T_i$:

$$\frac{p_f - \tau_f}{p_{ref}} = e^{\frac{L_V}{R}\left(\frac{1}{T_{ref}} - \frac{1}{T_f}\right)} \tag{B24}$$

Recall from kinetic theory (for a monatomic material) that if the material is in equilibrium in the final state, as assumed in deriving Clapeyron's equation, then,

$$p_f = \frac{1}{3}\rho_f\left(\overline{c_1^2} + 2\overline{c_2^2}\right)_f = \left(\rho\overline{c_1^2}\right)_f$$

(see the section "Causes of Boundary Shock Waves" for terminology). Also from kinetic theory in the case of mechanical nonequilibrium (i.e., significant viscosity),

$$(p - \tau)_f = (\rho\overline{c_1^2})_f$$

Hence, if $(\rho\overline{c_1^2})_f$ is regarded as the quantity limiting the outflow of new vapor molecules (equal to the pressure in the equilibrium case), then it is natural to write as an approximation the above modified Clapeyron equation, (B24), for the nonequilibrium (viscous) case. Since the relationship between p_f and T_f now involves τ_f, for a given application τ_f would have to be determined from the coupling with the equations for the flow outside the immediate region of the phase change.

Although the intuitive result, equation (B24), was not derived rigorously, it is consistent in that, as $\tau_f \rightarrow 0$, Clapeyron's equation is recovered. The use of this "modified Clapeyron's equation," (B24), is justified by the fact that there must be some such relationship characterizing the properties of the material during the rapid vaporization, and the results obtained with its use must be assumed to be qualitatively, rather than quantitatively, correct. Also, equation (B24) is useful in determining when Clapeyron's equation, (B10), is valid quantitatively. If under given conditions both (B10) and (B24) give nearly the same quantitative results, it can be assumed that those results are accurate.

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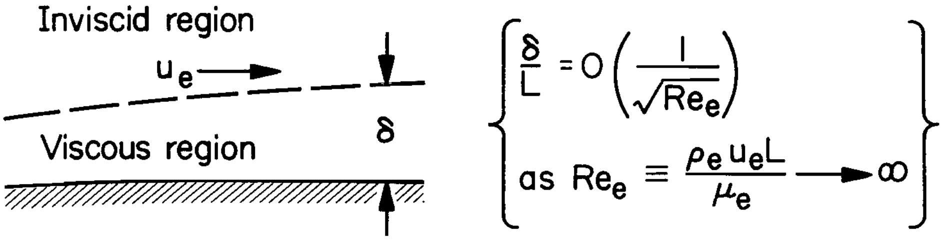
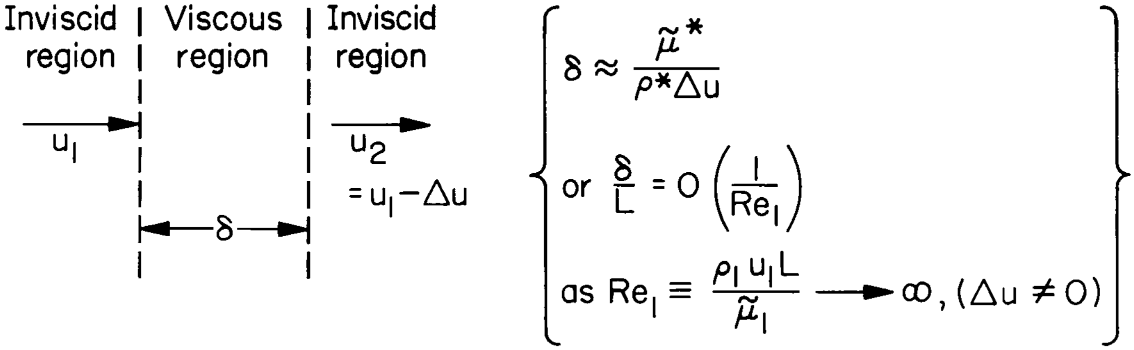
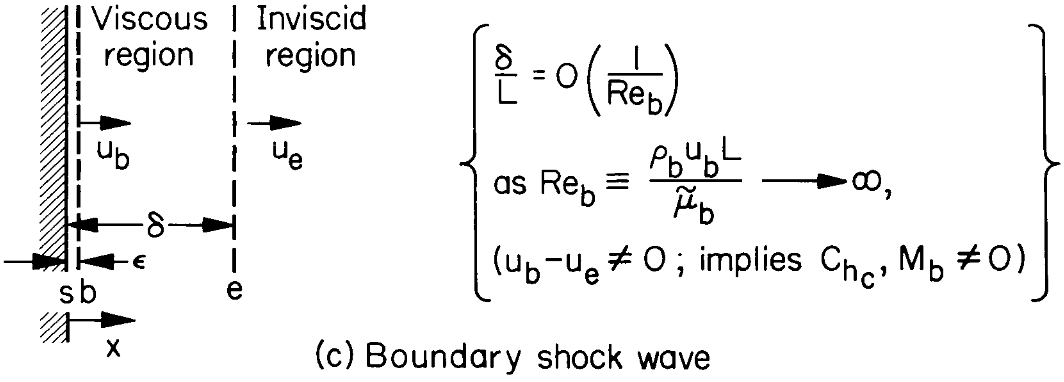


(a) Boundary layer



(b) Shock wave



(c) Boundary shock wave

Figure 1.- Analogy of boundary shock wave with boundary layer and shock wave.

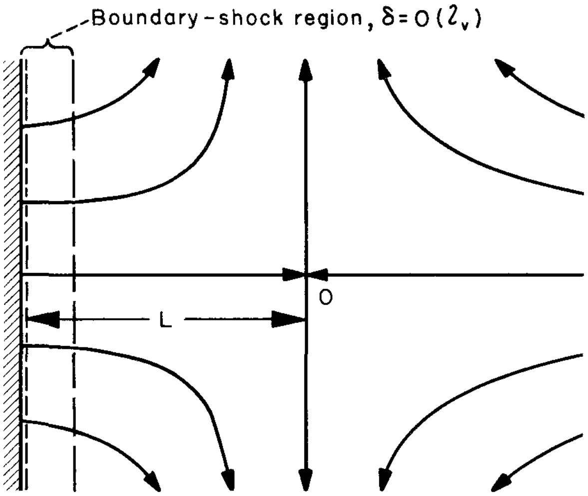


Figure 2.- Boundary shock wave in flow with stagnation point.

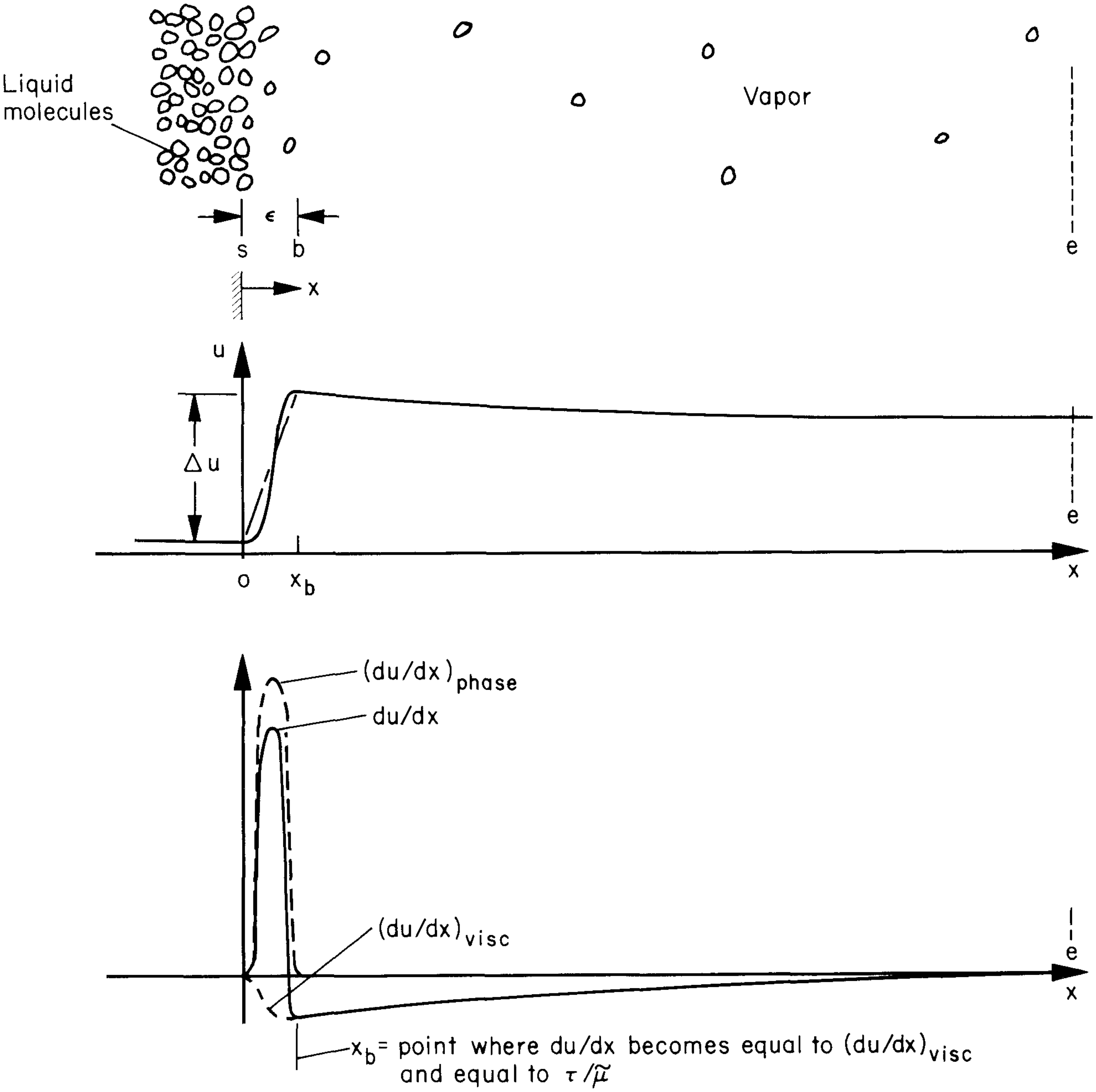


Figure 3.- Development of viscous stress during rapid vaporization.

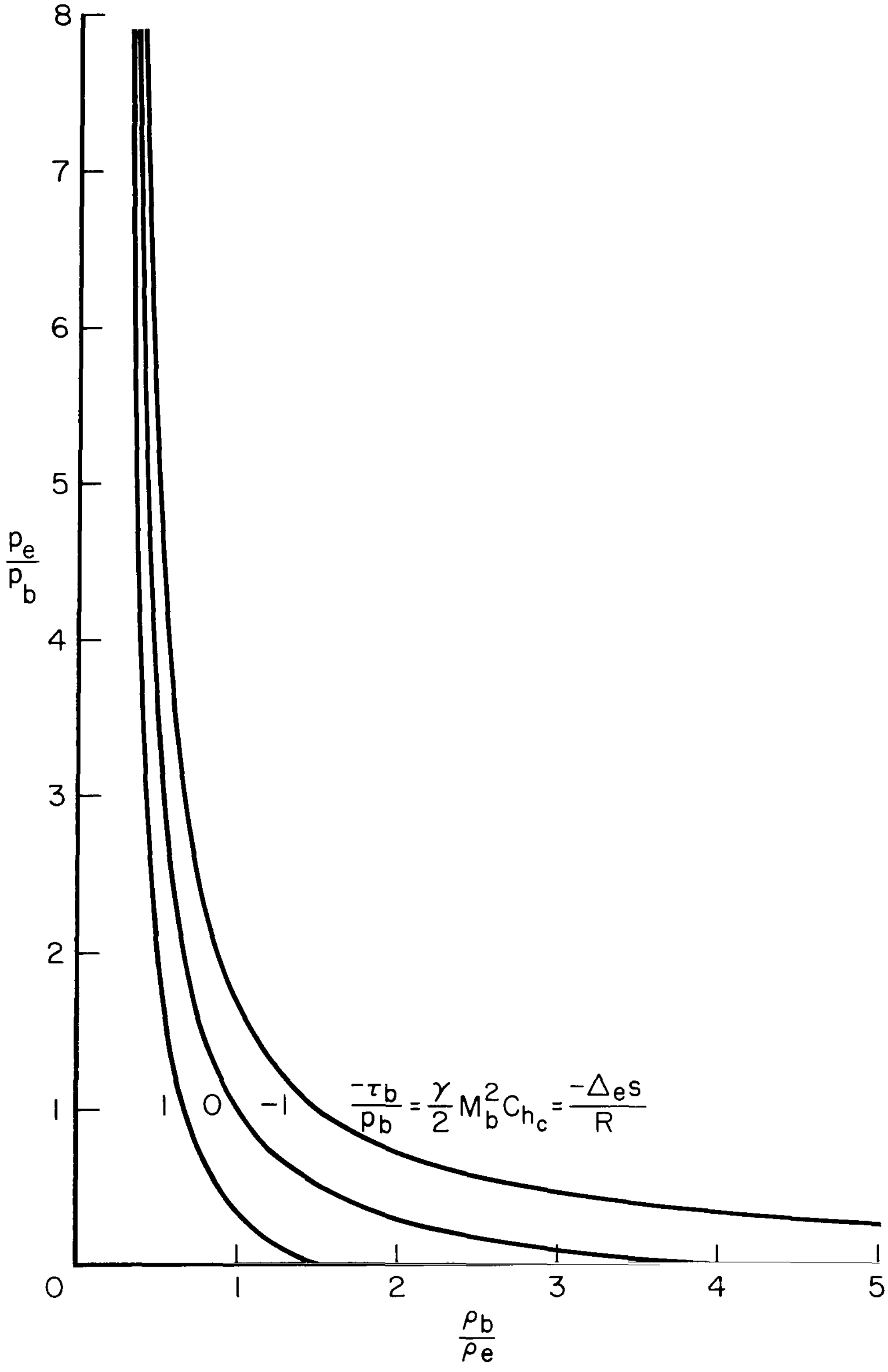


Figure 4.- Boundary-shock Hugoniot diagram for $\gamma = 5/3$, $\widetilde{Pr} = 1$.

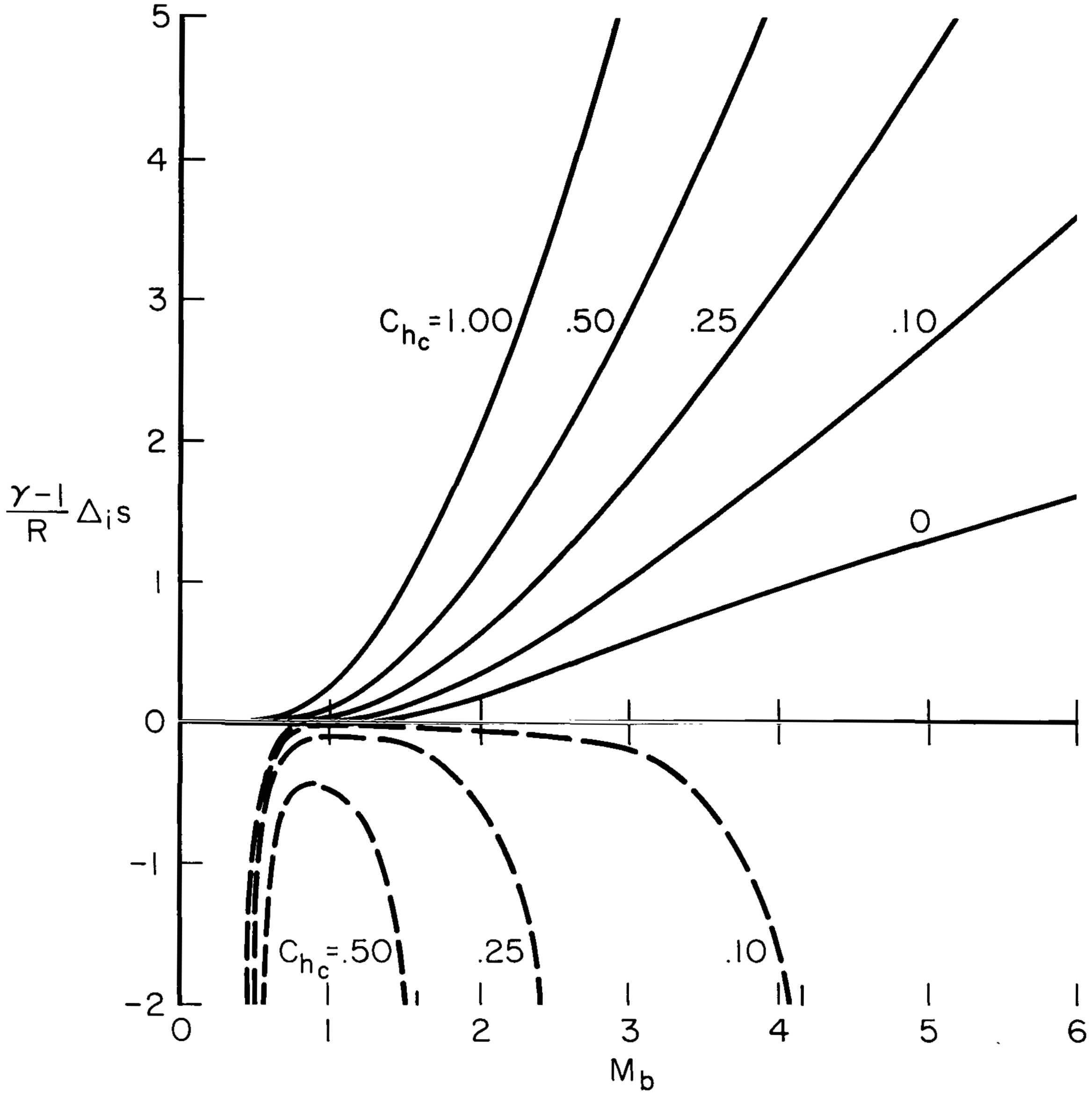


Figure 5.- Entropy production across a boundary shock ($\gamma = 5/3$, $\widetilde{Pr} = 1$).

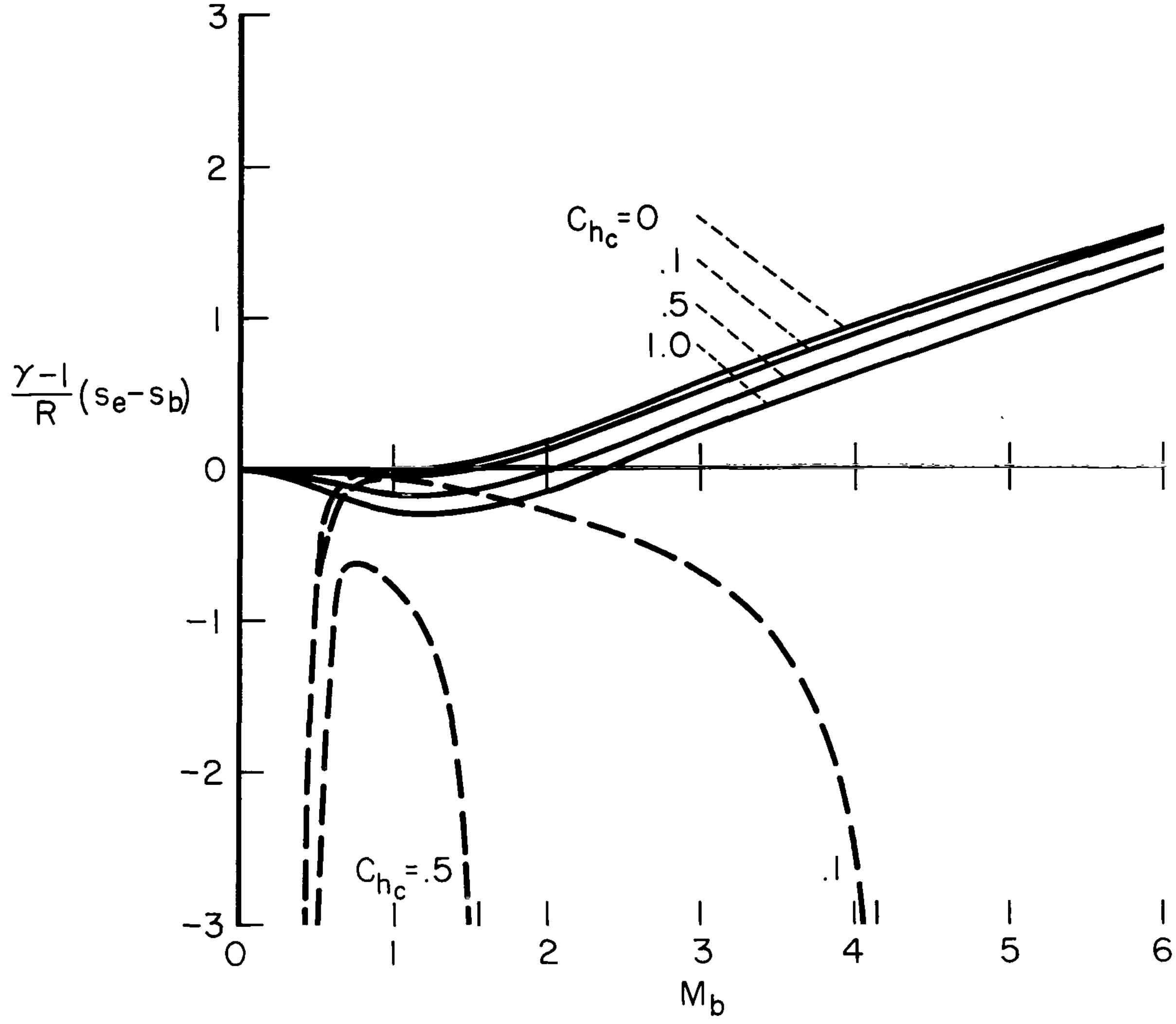


Figure 6.- Total entropy change across a boundary shock ($\gamma = 5/3$, $\widetilde{Pr} = 1$).

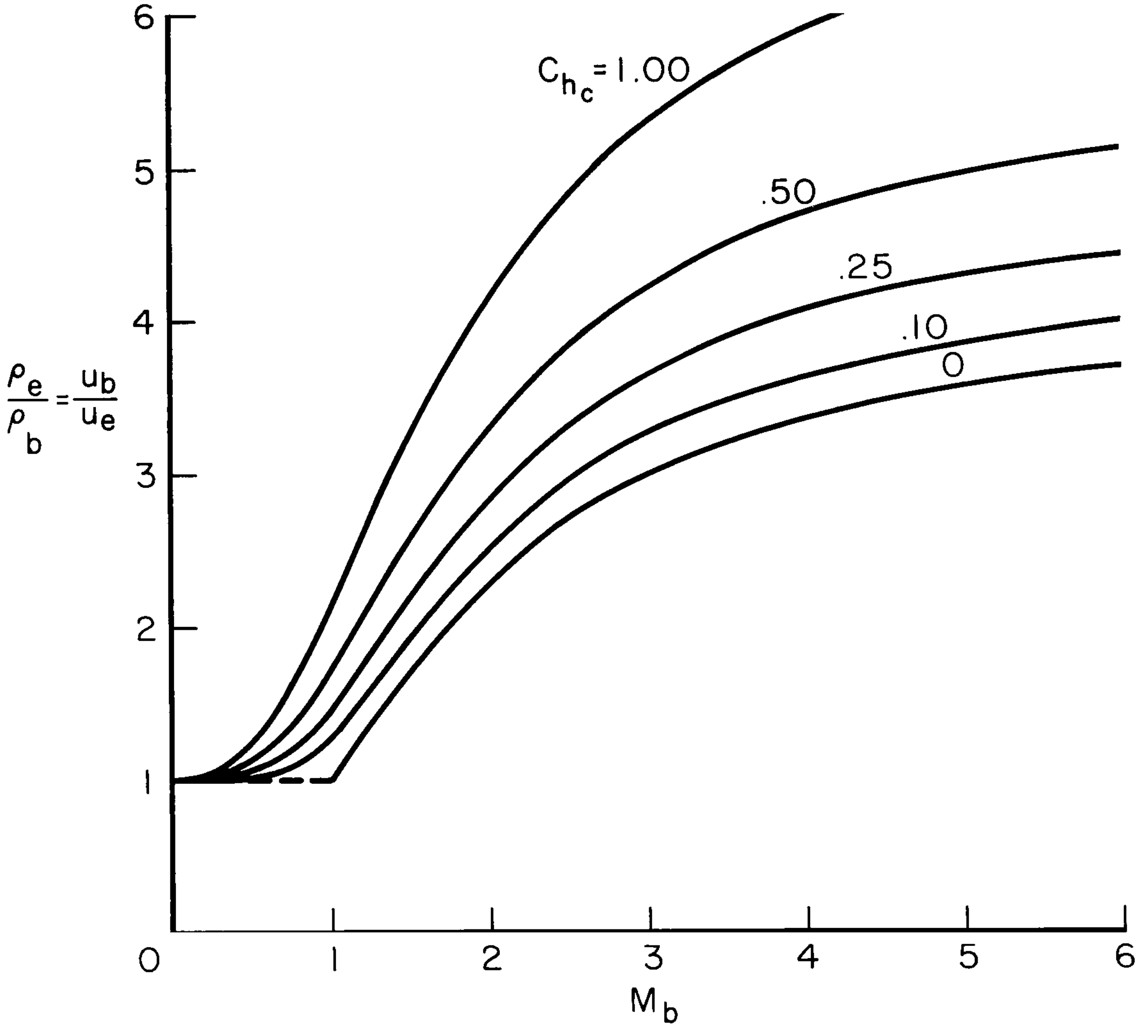


Figure 7.- Density ratio and velocity ratio across a boundary shock ($\gamma = 5/3$, $\widetilde{Pr} = 1$).

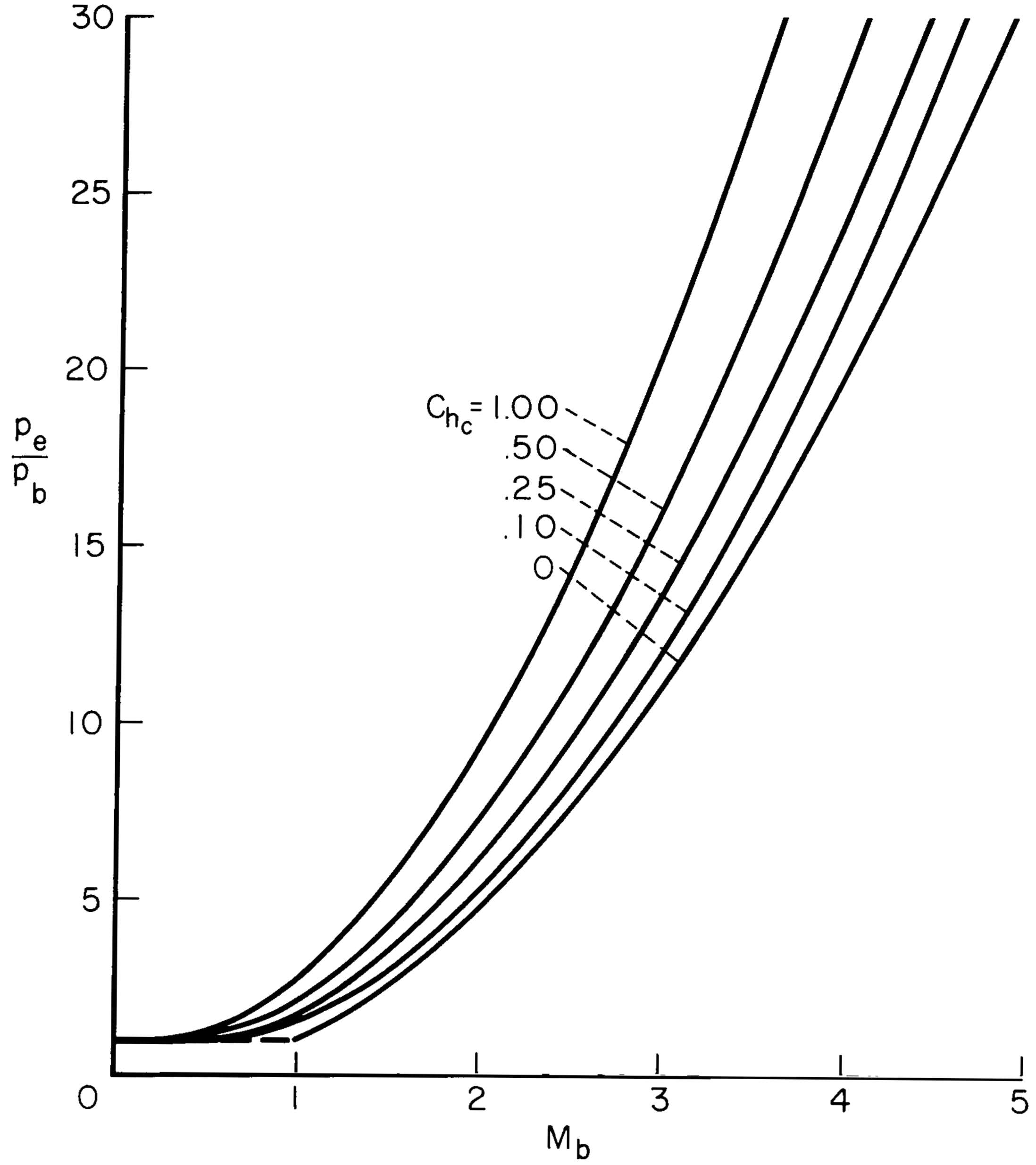


Figure 8.- Pressure ratio across a boundary shock $(\gamma = 5/3, \widetilde{Pr} = 1)$.

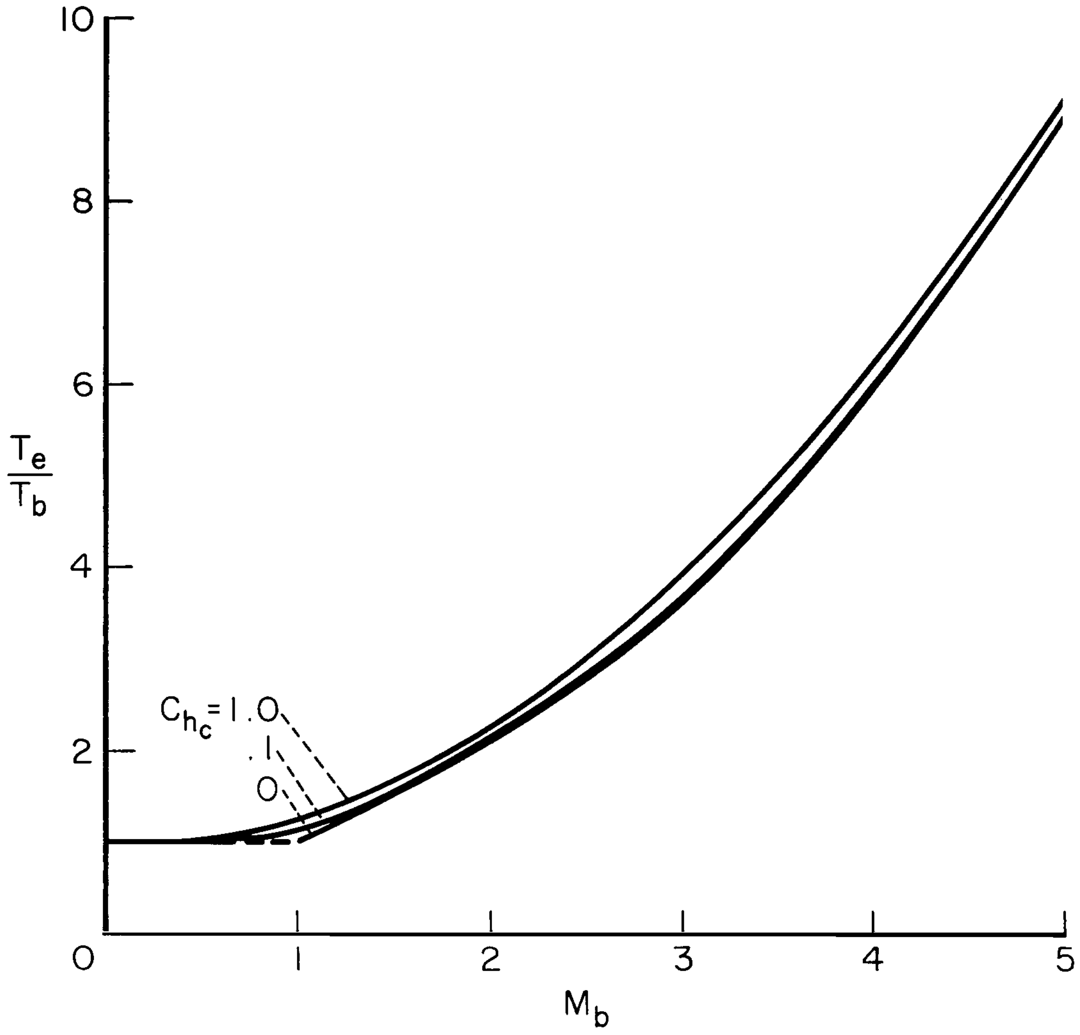


Figure 9.- Temperature ratio across a boundary shock $(\gamma = 5/3, \widetilde{Pr} = 1)$.

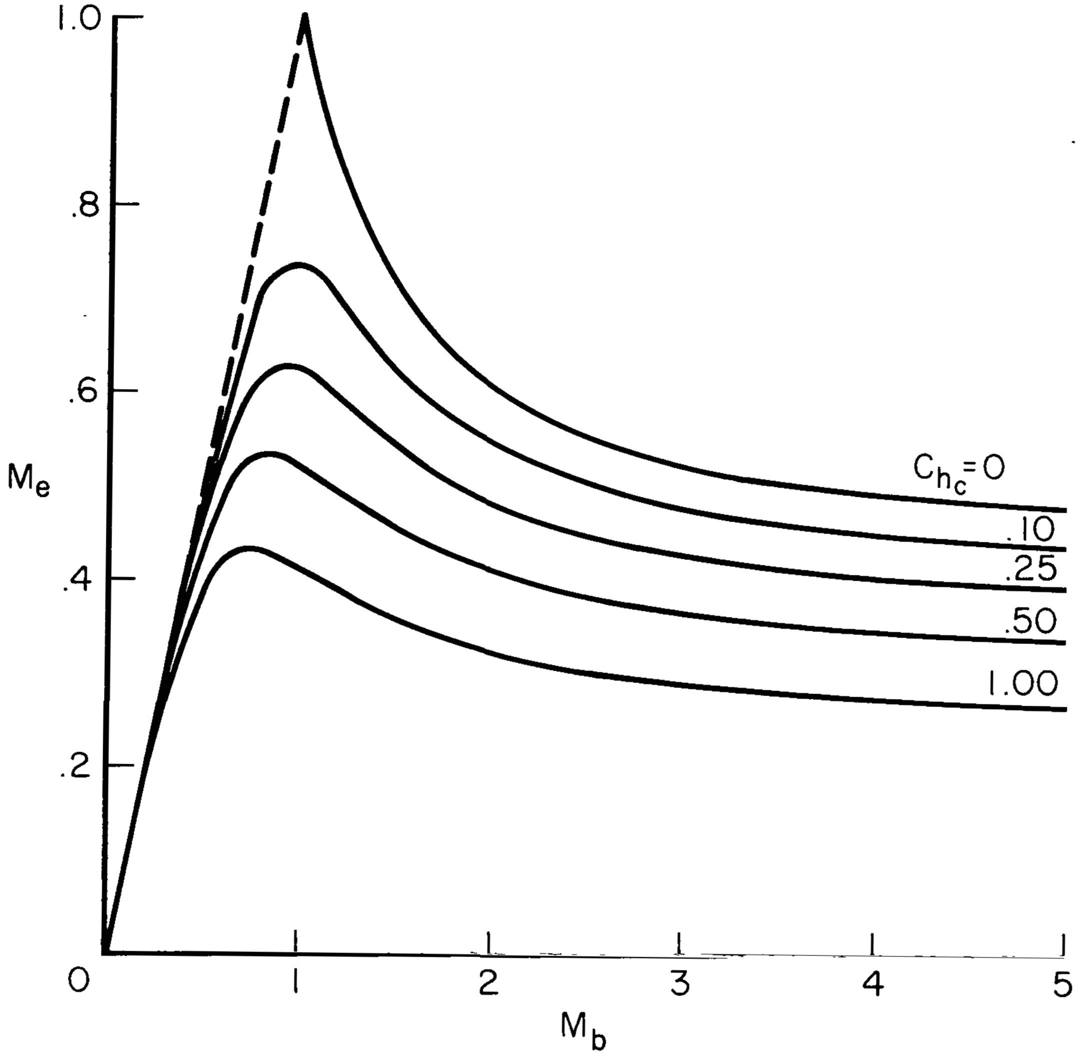


Figure 10.- Mach number downstream of a boundary shock ($\gamma = 5/3$, $\widetilde{Pr} = 1$).

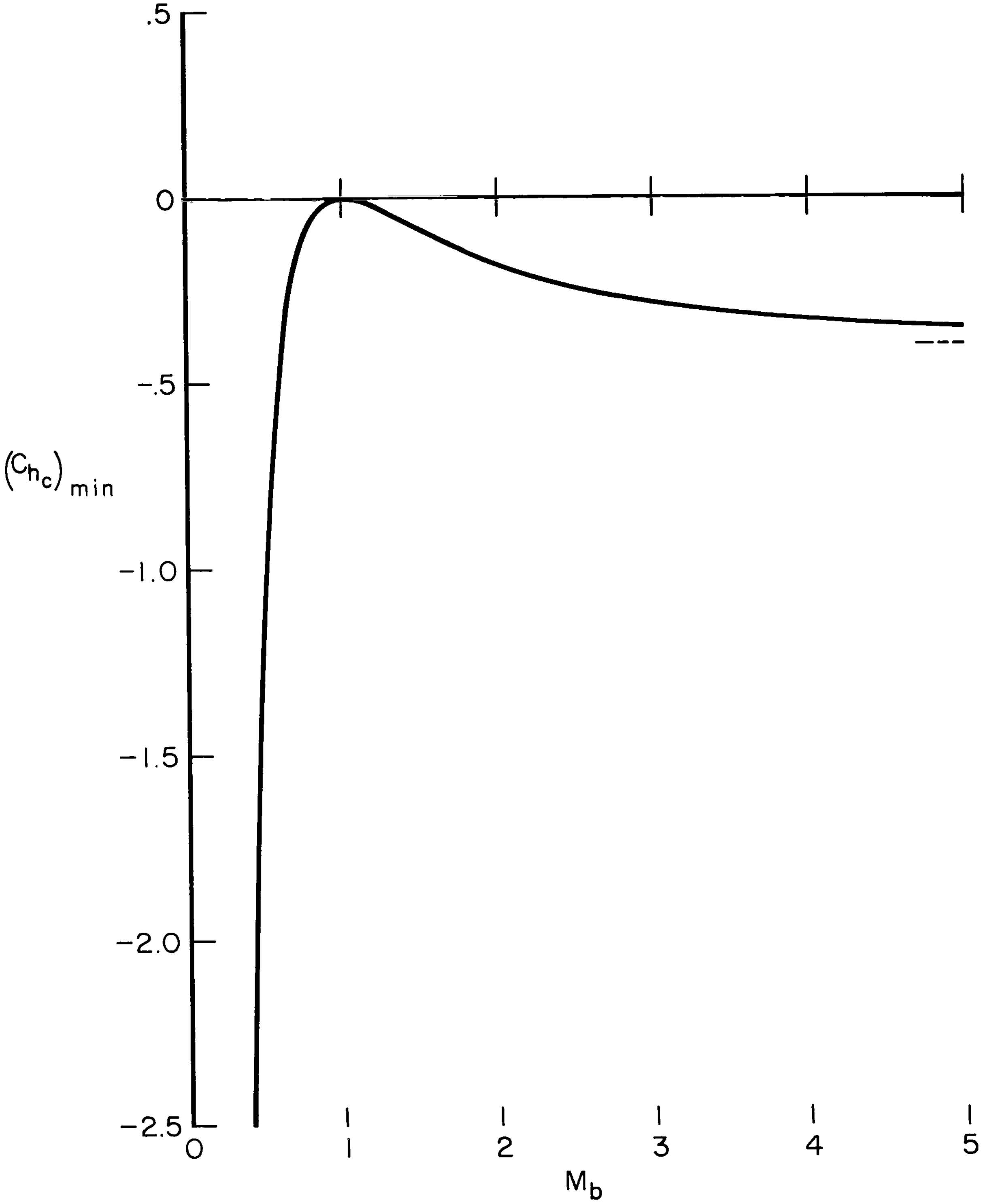


Figure 11.- Minimum C_{h_c} for real density ratio ($\gamma = 5/3$, $\widetilde{Pr} = 1$).

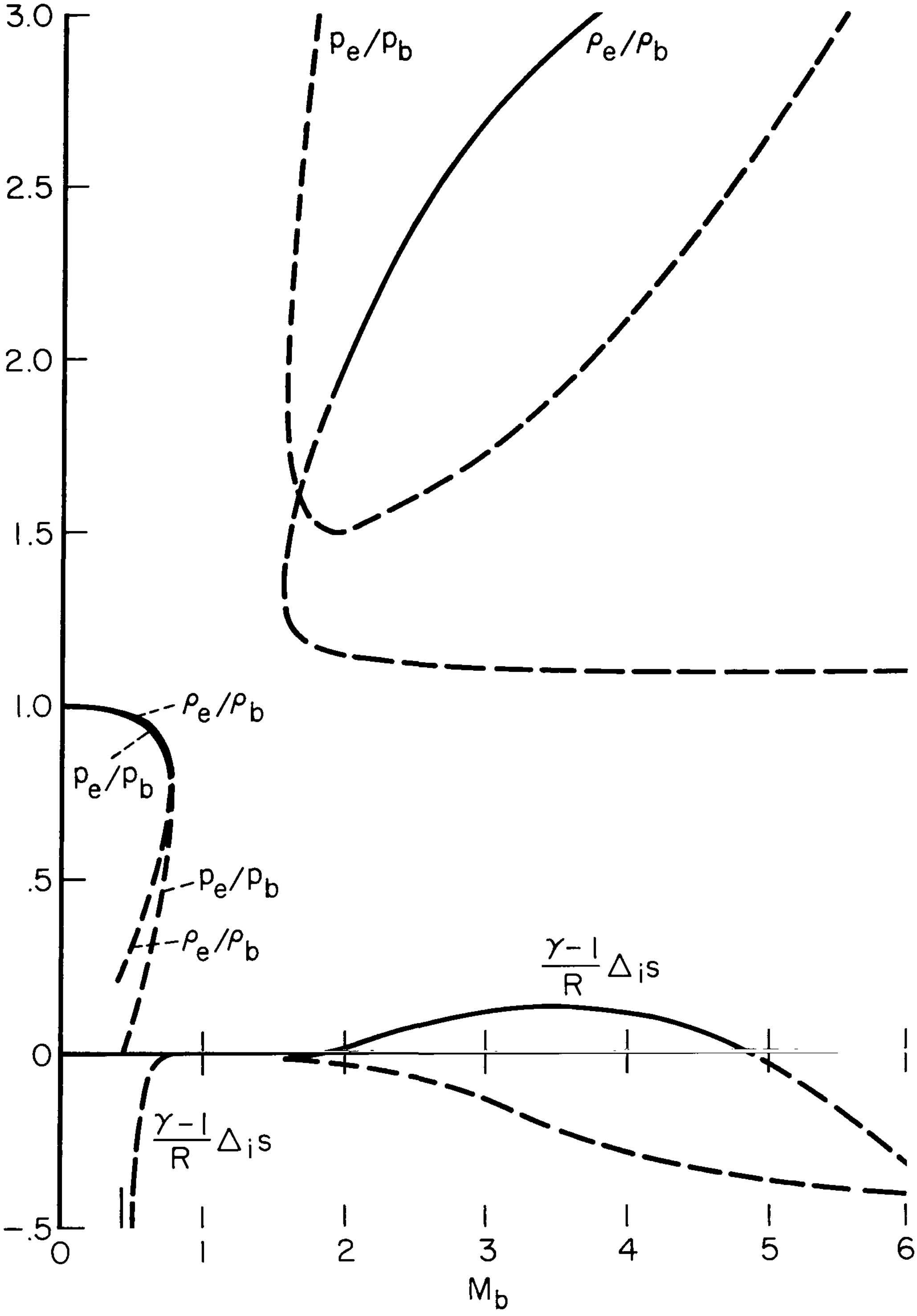


Figure 12.- Conditions across a boundary shock for $C_{h_c} = -0.1$ ($\gamma = 5/3$, $\widetilde{Pr} = 1$).

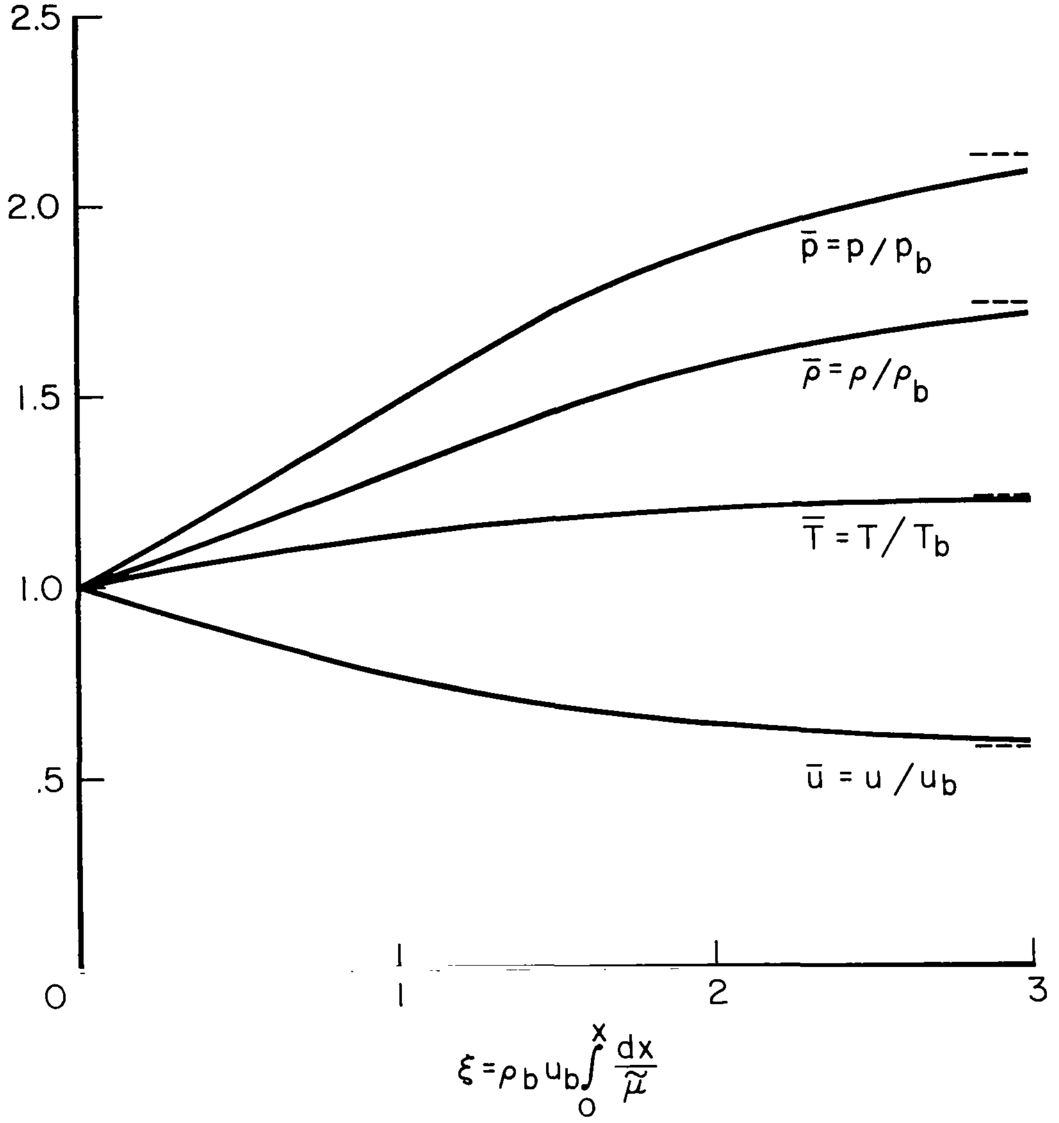


Figure 13.- Boundary-shock structure: variation of pressure, density, temperature, and velocity within a boundary shock for $\gamma = 5/3$, $M_b = 1$, $C_{h_c} = 1/2$, $\widetilde{Pr} = 1$.

3/22/85

C8

> *"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."*
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