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THE EFFECT OF TRANSDUCER IMPEDANCE ON DYNAMIC MEASUREMENTS

by I. P. VATZ *
Propulsion and Vehicle Engineering Laboratory

* *Brown Engineering Company*

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George C. Marshall Space Flight Center, Huntsville, Alabama

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THE EFFECT OF TRANSDUCER IMPEDANCE ON DYNAMIC MEASUREMENTS

By

I. P. Vatz*

George C. Marshall Space Flight Center

Huntsville, Alabama

ABSTRACT

There are inherent limitations in the use of vibration measurement transducers because of the effective mechanical impedance added to the mechanical system by the transducers themselves. A method of calculating the error thus imposed is presented. Equations are derived that give the useable frequency band based on a given allowable maximum error for accelerometers and impedance heads.

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I. P. Vatz

PROPULSION AND VEHICLE ENGINEERING LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

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THE EFFECT OF TRANSDUCER IMPEDANCE ON DYNAMIC MEASUREMENTS

SUMMARY

Equations that determine practical upper limit cutoff frequency for dynamic measurements are derived based on the effect of transducer impedance on measured values.

The following is a summary of the derived equations:

Accelerometer or impedance head attachment to:

A. Plates (Eq. 18 and 19)

$$\omega_x \approx \frac{N \epsilon_\nu M}{2 \pi M_2}$$

$$\omega_x \approx \frac{4N (E \rho h^4)^{1/2}}{\sqrt{3} M_2}$$

B. Rectangular Bar (Eq. 28)

$$\omega_x \approx \left(\frac{N M}{M_2 \ell}\right)^2 C_p h$$

C. Beam (Eq. 31)

$$\omega_x \approx \frac{8 N^2 \rho^2 S^2 C_p h}{M_2^2}$$

D. Lumped Mass and Impedance Head (Eq. 45)

$$\omega_x \approx \left(\frac{N E D}{M}\right)^{1/2}$$

E. Lumped Mass and Accelerometer (Eq. 53)

$$\omega_x \approx \left(\frac{N E^2}{\rho M_2 a \pi}\right)^{1/2}$$

Nomenclature

ω_X = Upper limit cutoff frequency

N = Allowable error - eg 1/10

ϵ_ν = Frequency difference between modes ($\Delta\omega$)

M = Total mass of the test specimen

M_2 = Mass of the transducer or effective mass of an impedance head

E = Young's modulus

h = Plate thickness or bar thickness

ℓ = Bar length

C_p = Wave propagation velocity in the material

S = Cross sectional area of a beam

r = Radius of gyration

ρ = Mass density of the material

a = Acceleration of the accelerometer

SECTION I. INTRODUCTION

There are inherent limitations to the use of vibration measurement transducers such as force gages, accelerometers, and impedance heads because of the effect of the transducer impedance on the response of the test specimen.

The effective weight of the transducer can be reduced to inconsequential values by the proper application of transducer mass to modal mass ratios or by electronic cancellation of the transducer effect on response signal. This report deals with only the selection of suitable transducers and mounting methods as a means of preserving proper readout.

The material in this report is based, in part, on Wilcoxon Research Technical Bulletin No. 1, [1] August, 1963, which stimulated thought on the subject.

SECTION II. THE EFFECT OF TRANSDUCER IMPEDANCE ON MECHANICAL SYSTEM RESPONSE

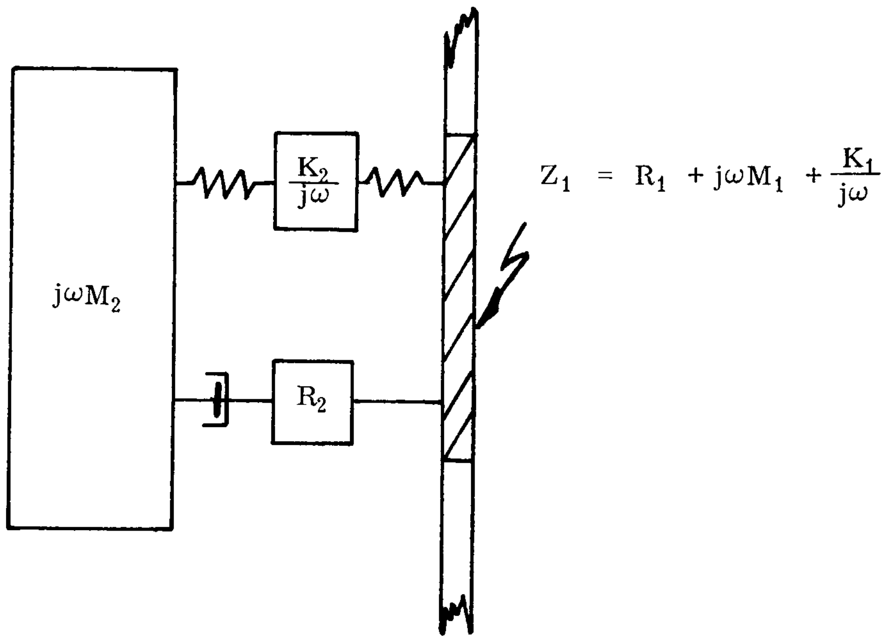


FIGURE 1. TRANSDUCER ATTACHMENT IMPEDANCE DIAGRAM

A transducer, when assumed a lumped mass (M_2), is attached to the test structure by a spring-damper device. Such a system is depicted in Figure 1. The attachment location lumped impedance (Z_1) is made up of the sum of the total modes of vibration effective at that location. Therefore R_1, M_1, and K_1 are the generalized constants that will define the various effective modes. Conversely, M_2, R_2, and K_2 are constants defining a single degree of freedom subsystem of one mode. Figure 1 represents the general case of transducer attachment. The response equations are written more conveniently from a force-node diagram of the above system as shown in Figure 2. Solution of the force-node equations will lead to the analytical evaluations of transducer attachments.

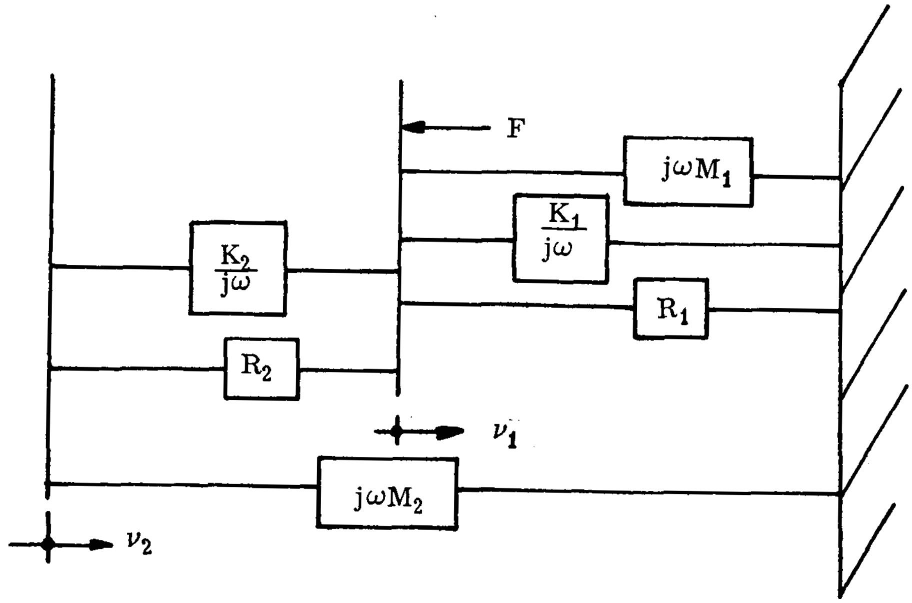


FIGURE 2. FORCE NODE DIAGRAM

The force node equations are:

$$F = \left[j\omega M_1 + \frac{K_1 + K_2}{j\omega} + R_1 + R_2 \right] \nu_1 - \left[\frac{K_2}{j\omega} + R_2 \right] \nu_2 \qquad (1)$$

$$O = -\left[\frac{K_2}{j\omega} + R_2 \right] \nu_1 + \left[\frac{K_2}{j\omega} + R_2 + j\omega M_2 \right] \nu_2 \qquad (2)$$

$$F = \left[j\omega M_1 + R_1 + \frac{K_1}{j\omega} \right] \nu_1 + j\omega M_2 \nu_2 \qquad (3) = (\text{Eq. } 1+2)$$

If the attachment (K_2) is very stiff so that the ν_2 is approximately equal to ν_1 then Equation 3 reduces to:

$$F = \left[j\omega \left(M_1 + M_2 \right) + \frac{K_1}{j\omega} + R_1 \right] \nu_1 \qquad (4)$$

The attachment point impedance then becomes

$$\frac{F}{\nu_1} = j\omega \left(M_1 + M_2 \right) + \frac{K_1}{j\omega} + R_1 \qquad (4A)$$

where $M_2 \ll M_1$, Equation 4 will reduce to $F \approx (j\omega M_1 + \frac{K_1}{j\omega} + R_1)\nu_1$ and it becomes apparent the transducer has little effect on the measured acceleration. On the other hand, when M_2 is $\not\ll M_1$, the transducer will affect the measured response. Where necessary, steps must be taken to correct for the transducer effect on measured data.

There is another approach which will increase comprehension of Equation 4A.

The transducer (accelerometer) is primarily made up of a spring and a mass in mechanical series. This type of system has an impedance as follows:

$$Z_2 = \frac{-j}{\frac{\omega}{K_2} - \frac{1}{\omega M_2}} \qquad (5)$$

At frequencies below anti-resonance the system is mass controlled. Since transducers (piezoelectric) are used below suspension resonance of the accelerometer mass the impedance may be simplified over the operational frequency range, to

$$Z_2 = j\omega M_2.$$

Equation 4A also shows that the accelerometer adds only mass to the measurement point impedance.

Equation 4 may be rewritten

$$Z_0 = \frac{F}{\nu_0} = j\omega\left(M_1 + M_2\right) + \frac{K_1}{j\omega} + R_1 \qquad (6)$$

where Z_0 is the combined impedance relating F to ν_0 and ν_0 is the attachment point velocity after the addition of the transducer. Then

$$\frac{\nu_1}{\nu_0} = \frac{F}{Z_1} \Big/ \frac{F}{Z_0} = \frac{Z_0}{Z_1} = \frac{j\omega\left(M_1 + M_2\right) + \frac{K_1}{j\omega} + R_1}{j\omega M_1 + \frac{K_1}{j\omega} + R_1} \qquad (7)$$

Here ν_1 represents the velocity at the attachment point before the transducer is attached and

$$\nu_1 = \nu_0 \frac{j\omega\left(M_1 + M_2\right) + \frac{K_1}{j\omega} + R_1}{j\omega M_1 + \frac{K_1}{j\omega} + R_1} \qquad (8)$$

It should be noted in Equation 2 that if $\frac{K_2}{j\omega}$ is not a predominant value, ν_2 will be different from ν_1 and the relationship equivalent to Equation 8 would be the complicated equation of

$$\nu_1 = \nu_0 \frac{\left(j\omega M_1 + \frac{K_1}{j\omega} + R_1\right)\left(j\omega M_2 + \frac{K_2}{j\omega} + R_2\right)}{\left(j\omega M_1 + \frac{K_1 + K_2}{j\omega} + R_1 + R_2\right)\left(j\omega M_2 + \frac{K_2}{j\omega} + R_2\right) - \left(\frac{K_2}{j\omega} \quad R_2\right)^2} \tag{9}$$

Since we are seeking to optimize the fractional part of Equation 9 to unity, a flexible attachment of the transducer is highly undesirable.

It was originally stated that

$$Z_1 = R_1 + j\omega M_1 + \frac{K_1}{j\omega}$$

and when $\nu_1 = \nu_2$ at frequencies below accelerometer resonance

$$Z_2 = j\omega M_2$$

then Equation 8 can be reduced to

$$\nu_1 = \nu_0 \frac{Z_1 + Z_2}{Z_1} \tag{10}$$

At low frequencies K_1 of Equation 8 will be the most significant value and the fraction will be equivalent to unity. At the first resonance the fraction will become R_1/R_1 and again equivalent to unity but the resonant frequency will be influenced by M_2. At higher frequencies the fraction will become equivalent to $\frac{Z_1 + j\omega M_2}{Z_1}$.

SECTION III. FREQUENCY LIMIT FOR ACCELEROMETER ATTACHMENT TO PLATES

When the structure is in the form of a plate and the frequencies are such that

$$\omega \geq \epsilon_\nu Q \tag{11}$$

where ϵ_ν is the angular frequency difference between modes and Q is the resonance quality factor. Then the attachment point impedance approaches:

$$Z \approx \frac{2\,\epsilon_\nu M_\nu \; (A)}{\pi} \qquad [2]\ [3] \tag{13}$$

where M_ν (A) is the modal mass of the nearest mode. For edge supported plates at high frequencies M_ν may be approximated by 1/4 M where M is the total mass of the plate.

Then

$$Z = \frac{2\,\epsilon_\nu M}{4\pi} = \frac{\epsilon_\nu M}{2\pi} \tag{14}$$

L. Cremer is quoted in [1] as the source of a similar equation where

$$Z_1 = \frac{4}{\sqrt{3}}\,(E\rho\, d^4)^{1/2} \tag{15}$$

where E is Young's Modulus, ρ is the material mass density and d is the plate thickness.

By using Equation 10 the error $(\nu_0 - \nu_1)$ can be equated to impedance and to N, the fraction of the velocity allowable as readout error.

$$\nu_1 - \nu_0 = -\frac{\nu_1 Z_1}{Z_1 + Z_2} + \nu_1 = N\nu_1 \tag{16}$$

or by dividing through by ν_1 we get

$$Z_2 = N\,(Z_1 + Z_2) \tag{17}$$

at high frequencies $|Z_2| \approx \omega M_2$ and Z_1 is as in Equation 13 then

$$\omega_X M_2 = N\left(\frac{\epsilon_\nu M}{2\pi} + \omega_X M_2\right)$$

$$\omega_X M_2\,(1-N) = \frac{N\epsilon_\nu M}{2\pi}$$

Solve for ω_X

$$\omega_X = \frac{N\epsilon_\nu M}{2\pi\,(1-N)M_2} \approx \frac{N\epsilon_\nu M}{2\pi M_2} \tag{18}$$

Where ω_X is the upper limit of the useable frequency band.

Reference [1] uses Equation 15 to arrive at:

$$\omega_x M_2 = N \frac{4}{\sqrt{3}} \quad (E \rho h^4)^{1/2}$$

$$\omega_x = \frac{4N}{\sqrt{3}} \frac{(E\rho h^4)}{M_2}^{1/2} \tag{19}$$

Both Equations 14 and 15 are dependent on rather ideal conditions of damping and modal frequency distribution. Therefore Equations 17 and 19 are subjected to the same limitations. To be rationally correct, ω_x should be evaluated as an order of magnitude rather than a precise frequency. The derivation of Equation 15 has not been examined but Equation 13 can be worked to something of a reasonable equivalent as follows:

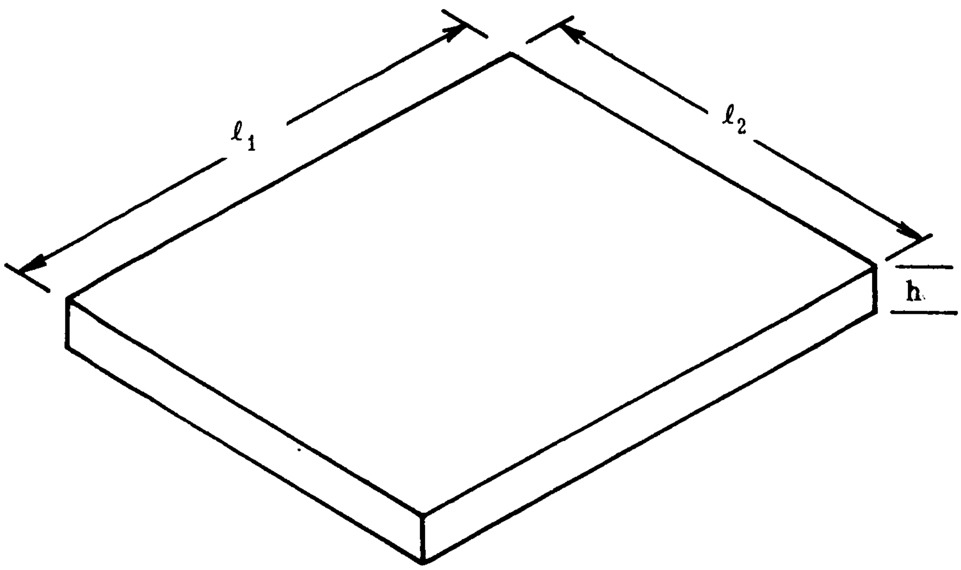


FIGURE 3. PLATE DIMENSIONS

$$Z = \frac{2}{\pi} \quad \epsilon_\nu M_\nu \quad (A) = \frac{8}{\pi^2} \quad (\omega_{ap} \ \omega_{bp})^{1/2} \quad M \tag{13}$$

where $\omega_{M,n} = \omega_{ap} M^2 + \omega_{bp} \eta^2$

and M, n are modal numbers.

Skudrzyk expands Equation 13 for a simply supported rectangular plate or any plate at high frequencies to:

$$Z_1 = \frac{8}{\pi^2} \left(\frac{C_p h \qquad \pi^2}{[12(1-\eta^2)]^{1/2} \ell_1 \ell_2} \right) M \qquad [3]$$

$$Z_1 = \frac{8}{3\pi^2} \left(\frac{C_p h \qquad \pi^2}{\ell_1 \ \ell_2} \right) M$$

$$Z_1 = \frac{8}{3} \left[\left(\frac{E}{\rho} \right)^{1/2} \frac{h}{\ell_1 \ell_2} \right] M$$

$$Z_1 = \frac{8}{3} \left(\frac{E}{\rho} \right)^{1/2} \frac{h\rho\,(h\ell_1\ell_2)}{\ell_1\ell_2}$$

$$Z_1 = \frac{8}{3} \left(E\rho h^4 \right)^{1/2} \tag{13A}$$

Which is the practical equivalent of

$$Z_1 = \frac{4}{\sqrt{3}} \left(E\rho h^4 \right)^{1/2} \tag{15}$$

SECTION IV. FREQUENCY LIMIT FOR ACCELEROMETER ATTACHMENT TO A RECTANGULAR BAR

The impedance (Z_1) at high frequencies will approach

$$Z_1 = \frac{\epsilon_\nu M_\nu \,(A)}{\pi} (1 + j) \qquad [3] \tag{20}$$

$$|Z_1| = \frac{\sqrt{2}\, M_\nu \,(A)\, \epsilon_\nu}{\pi} \tag{21}$$

For a bar (Fig. 4)

$$\epsilon_\nu M_\nu = 2M\,(\omega \;\; \omega ap)^{1/2} \qquad [3] \qquad (22)$$

where

$$\omega ap = \frac{C_p h \qquad \pi^2}{\left[12\;(1-\eta^2)\right]^{1/2}\,\ell^2} \qquad [3] \qquad (23)$$

and C_p = Velocity of sound in the bar material

h = Thickness of the bar

η = Poisson's ratio

ℓ = Length of the bar

M = Total mass of the bar

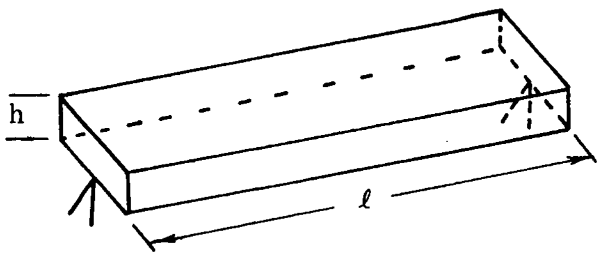


FIGURE 4. SIMPLY SUPPORTED BAR

Substituting 23 into 22

$$\epsilon_\nu M_\nu = \frac{2\pi\,M\omega^{1/2}}{\ell}\;(C_p h)^{1/2}\left(\frac{1}{12\;(1-\eta)}\right)^{1/4} \qquad (24)$$

and then substituting 24 into 21

$$\left| Z_1 \right| = \frac{2\pi\sqrt{2}\ M\omega^{1/2}}{\pi}(\mathrm{Cph})^{1/2} \left(\frac{1}{12\ (1-\eta)}\right)^{1/4}$$

$$\left| Z_1 \right| \approx \frac{\sqrt{2}\ (\mathrm{Cph})^{1/2}\ M}{\ell\ (1-\eta)^{1/4}}\ \omega^{1/2} \tag{25}$$

$$\left| Z_1 \right| \approx \frac{M\ (\mathrm{Cp\ h})^{1/2}\sqrt{2}}{\ell}\ \omega^{1/2} \tag{26}$$

or loosely $\left| Z_1 \right| \approx \dfrac{M(\mathrm{Cp\ h})^{1/2}}{\ell}\ \omega^{1/2}$

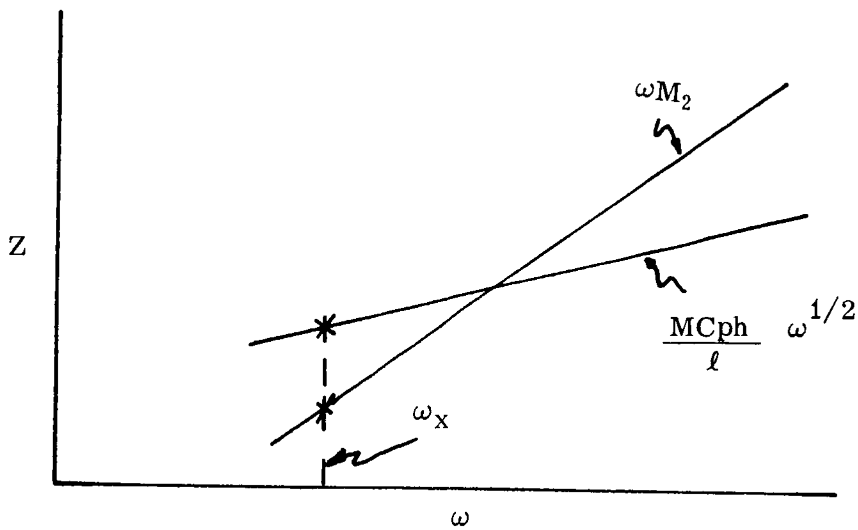


FIGURE 5. TRANSDUCER MASS EFFECT COMPARED TO THE IMPEDANCE OF A BAR

Substituting into Equation 17 we get:

$$\omega_x M_2 \approx \frac{NM\ (\mathrm{Cph}\ \omega_x)^{1/2}}{\ell} \tag{27}$$

$$\omega_x \approx \left(\frac{N\ M}{M_2 \ell}\right)^2 \mathrm{Cph} \tag{28}$$

where M_2 is the mass of the accelerometer.

Equation 28 has limitations imposed by the restrictions of Equation 21. Here it is assumed that $|Z_1|$ is the lower envelope of the impedance resonances and represents the locus of points where $\omega \doteq \omega_\nu$ (the natural frequencies of the bar). For a damped system this assumption will be accurate. For a high Q system the value of Equation 28 is questionable. ω_x, even in the most ideal situations must be considered qualitative rather than quantitative. Figure 5 is descriptive of the transducer effect ωM_2.

SECTION V. FREQUENCY LIMIT FOR ACCELEROMETER ATTACHMENT TO A BEAM

Reference [1] defines the impedance of beams at high frequencies as approaching

$$Z_1 = 4 \sqrt{\pi f} \quad (EI)^{1/4} \quad M_e^{3/4} \; e^{\frac{j\pi}{2}} \tag{29}$$

This reduces to

$$Z_1 = 4(\pi Cprf)^{1/2} \rho S \tag{30}$$

where f = Frequency, cps

E = Young's Modulus

I = Moment of inertia

M_e = Mass per unit of length

Cp = Velocity of propagation of sound in the material

r = Radius of gyration

ρ = Mass density of the material

S = Cross-sectional area of the beam

To keep ωM_2 at less than a required ratio, ω_X is equated as:

$$M_2 \, \omega_X = \frac{4N}{\sqrt{2}} \, \omega_X^{\,1/2} \, (Cpr)^{1/2} \, \rho \, S$$

$$\omega_X = \frac{8 \, N^2 \, \rho^2 \, S^2 \, Cp \, r}{M_2^2} \tag{31}$$

The derivation of Equations 29 and 30 have not been examined, therefore, the limitations can not be fully defined. A comparison of Equation 26 with Equation 30 is as follows:

$$\left| Z_1 \right| \approx \frac{\sqrt{2} \;\; M \, (Cp \, h \, \omega)^{1/2}}{\ell} \tag{26}$$

$$Z_1 = 4 \, (\pi \, Cp \, r \, f)^{1/2} \, \rho \, S \tag{30}$$

$$Z_1 = \frac{(16 \pi \, Cp \, r \, f)^{1/2} \, M}{\ell} \tag{30A}$$

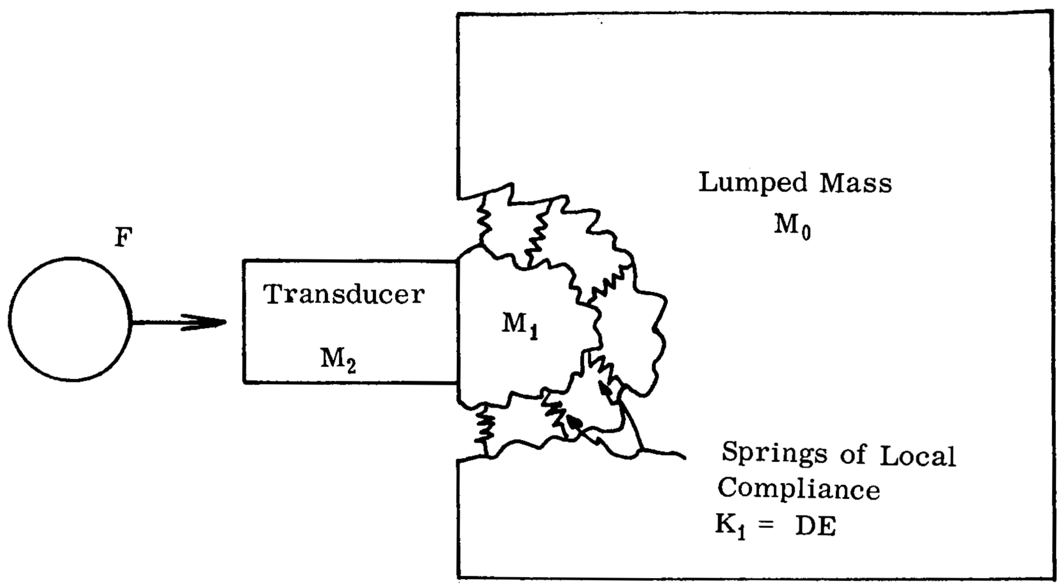
$$Z_1 = \frac{(8 \, Cp \, r \, \omega)^{1/2} \, M}{\ell} \tag{30B}$$

Since $r = .289 \, h \approx 1/4 \, h$ for a rectangle, Equation 30B can be reduced to the equivalent of Equation 26. It follows that the equations for bars and beams are interchangeable.

SECTION VI. FREQUENCY LIMIT FOR AN IMPEDANCE HEAD ATTACHMENT TO LUMPED MASSES

When an accelerometer or impedance head is attached to a structure that may be considered a lumped mass, the transducer will be influenced at high frequencies by the local compliance.

Transferring Figure 6 to a force node diagram, it results in Figure 7 after assuming M_2 is $<<$ than M_1 and that the total lumped mass (M) is equal to $M_0 \neq M_1$.



$M_2 \langle\langle M_1 \langle\langle M_0$

FIGURE 6. TRANSDUCER ATTACHED TO A LUMPED MASS SYSTEM

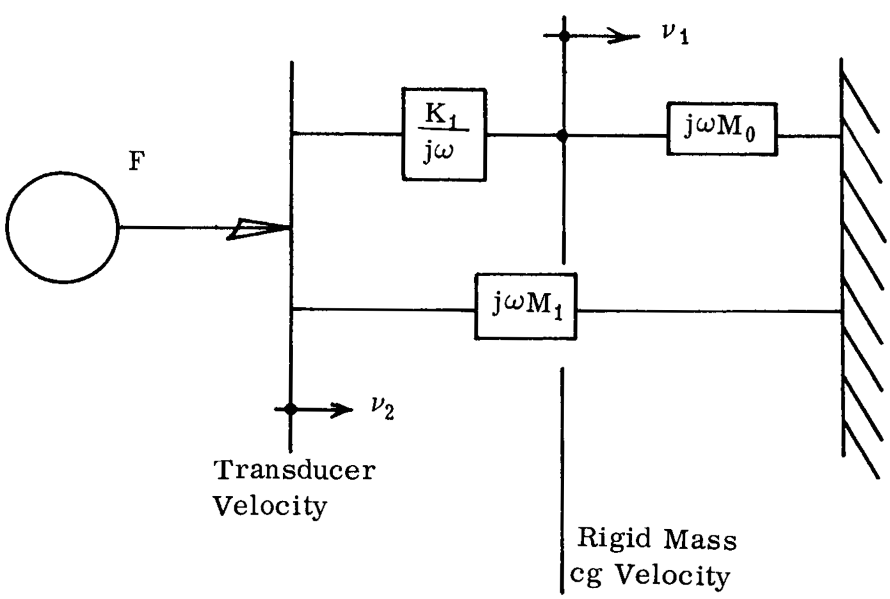


FIGURE 7. FORCE NODE DIAGRAM OF THE RIGID MASS SYSTEM EXCITED THROUGH THE TRANSDUCER

The force node equations are:

$$O = \left(\frac{K_1}{j\omega} + j\omega M_0\right) \nu_1 - \left(\frac{K_1}{j\omega}\right) \nu_2 \qquad (32)$$

$$F = -\left(\frac{K_1}{j\omega}\right) \nu_1 + \left(\frac{K_1}{j\omega} + j\omega M_1\right) \nu_2 \qquad (33)$$

$$F = j\omega M_0 \nu_1 + j\omega M_1 \nu_2 \qquad (34) = (\text{Eq. } 32 \not+ 33)$$

Solving Equation 32 for ν_1 we get

$$\nu_1 = \frac{\frac{K_1}{j\omega} (\nu_2)}{\frac{K_1}{j\omega} + j\omega M_0} \qquad (35)$$

and substituting into Equation (34) we get $Z_2 = \frac{F}{\nu_2}$ and

$$Z_2 = \frac{\frac{j\omega M_0 K_1}{j\omega}}{\frac{K_1}{j\omega} + j\omega M_0} + j\omega M_1$$

$$Z_2 = \frac{M_0 K_1 + M_1 K_1 - \omega^2 M_0 M_1}{\frac{K_1}{j\omega} + j\omega M_0}$$

$$Z_2 = \frac{j\omega M_0 K_1 + j\omega M_1 K_1 - j\omega^3 M_0 M_1}{K_1 - \omega^2 M_0} \qquad (36)$$

Since $M_1 \ll M_0$ and at very low frequencies $K \gg M_0$ then Z_2 can be reduced to:

$$Z_2 = \frac{j\omega M_0 K_1}{K_1} = j\omega M_0 \qquad (37)$$

and the impedance head will read the actual impedance of the lumped mass.

Let the error = $Z_2 - j\omega M_0$ and let the allowable error = $Nj\omega M_0$, then

$$Z_2 - j\omega M_0 = Nj\omega M_0 \qquad (38)$$

$$\frac{j\omega M_0K_1 + j\omega M_1K_1 - j\omega^3M_0M_1}{K_1 - \omega^2M_0} - j\omega M_0 = Nj\omega M_0$$

$$\frac{j\omega M_0K_1}{j\omega M_0} + \frac{j\omega M_1K_1}{j\omega M_0} - \frac{j\omega^3M_0M_1}{j\omega M_0} - \frac{j\omega M_0K_1}{j\omega M_0} + \frac{j\omega^3M_0^2}{j\omega M_0}$$

$$= \frac{Nj\omega M_0K_1}{j\omega M_0} - \frac{Nj\omega^3M_0^2}{j\omega M_0}$$

$$\omega^2M_1 - \omega^2M_0 = -NK_1 + N\omega^2M_0$$

$$\omega^2\left[(1+N)\ M_0 - M_1\right] = NK_1$$

$$\omega = \left[\frac{NK_1}{(1+N)\ M_0 - M_1}\right]^{1/2} \qquad (39)$$

where N is small and $M_1 \ll M_0$

$$\omega_x \approx \left(\frac{NK_1}{M_0}\right)^{1/2} \qquad (40)$$

if N = 1 which is where the error = $j\omega M_0$ or where $Z_2 = 2\ j\omega M_0$

$$\omega_x \approx \left(\frac{K_1}{2M_0}\right)^{1/2} \qquad (41)$$

In order to put Equations 39 through 41 in a workable form, the following approximations are made:

$$M_1 \approx \frac{\pi\rho_1 FD}{E}\ * \qquad (42)$$

$$K_1 \approx ED \quad [4] \qquad (43)$$

Equation 42 should be accepted from a qualitative rather than a quantitative concept.

---

* See Appendix

ρ_1 = Mass density of the lumped mass

D = Diameter of the transducer attachment area

E = Young's Modulus of the lumped mass

F = Total driving force

Substituting 43 into 39

$$\omega_x = \sqrt{\frac{NED}{(1+N)\,M_0 - M_1}} \tag{44}$$

or where $M_0 = M \gg M_1$ and N is small

$$\omega_x \approx \sqrt{\frac{NED}{M}} \tag{45}$$

When driving a lumped mass through an impedance head the upper limit of reliable data is a function of Young's Modulus, the transducer attachment diameter and the quantity of the lumped mass. It would follow that for a given test sample the useable frequency range is controlled by the attachment diameter with a large attachment area as optimum. However when driving a sample subject to bending modes a large attachment area may influence the stiffness of the sample.

SECTION VII. FREQUENCY LIMIT FOR AN ACCELEROMETER ATTACHMENT TO A LUMPED MASS

When the transducer of Figure 6 is excited by only the motion of M_0 such as in the use of accelerometers, the force node diagram is represented in Figure 8.

The force node equations are:

$$F = \frac{K_1}{j\omega}\,\nu_1 - \frac{K_1}{j\omega}\,\nu_2 \tag{46}$$

$$O = -\frac{K_1}{j\omega}\,\nu_1 + \left(\frac{K_1}{j\omega} + j\omega M_1\right)\nu_2 \tag{47}$$

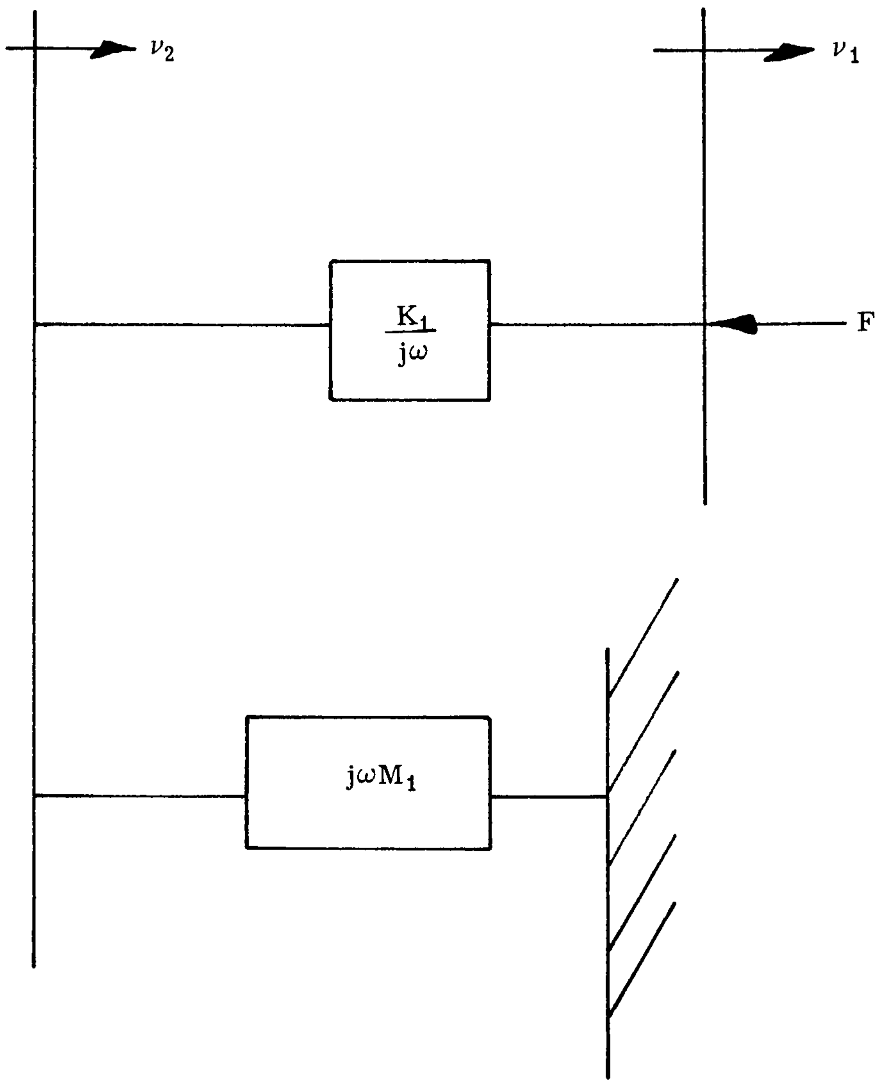


FIGURE 8. FORCE NODE DIAGRAM

$$\frac{\nu_1}{\nu_2} = \frac{\frac{K_1}{j\omega} + j\omega M_1}{\frac{K_1}{j\omega}} = 1 - \frac{\omega^2 M_1}{K_1} \tag{48}$$

Since ν_1 is the velocity that should be measured and ν_2 is the velocity that will be measured the error will be $\nu_2 - \nu_1$.

$$\nu_2 - \nu_1 = \frac{\nu_1}{1 - \frac{\omega^2 M_1}{K_1}} - \nu_1 \tag{49}$$

By setting the error equal to $N\nu_1$, we have

$$N\nu_1 = \frac{\nu_1}{1 - \frac{\omega^2 M_1}{K_1}} - \nu_1$$

$$N\left(1 - \frac{\omega_x^2 M_1}{K_1}\right) = 1 - \left(1 - \frac{\omega_x^2 M_1}{K_1}\right)$$

$$\omega_x = \left[\frac{NK_1}{(N+1)\, M_1}\right]^{1/2} \approx \left(\frac{NK_1}{M_1}\right)^{1/2} \tag{50}$$

Substituting Equations 42 and 43 into Equation 50 we get:

$$\omega_x = \left[\frac{NE^2}{(N+1)\,\rho F \pi}\right]^{1/2} \approx \left(\frac{NE^2}{\rho F \pi}\right)^{1/2} \tag{51}$$

F in this case is the total force induced on the surface of the sample by the transducer.

$$F \approx M_2 a \tag{52}$$

where a is the acceleration measured by the transducer and M_2 is the transducer mass. Then substituting 52 into 51

$$\omega_x \approx \left(\frac{NE^2}{\pi\, \rho\, M_2 a}\right)^{1/2} \tag{53}$$

Thus the useable frequency range of an accelerometer is nonlinear and will decrease with accelerometer weight and acceleration amplitude. Since E is in most cases a very large value the criteria of 53 is usually not important because ω_x is large enough to allow any practical measurements.

SECTION VIII. FREQUENCY LIMIT FOR AN IMPEDANCE HEAD ATTACHMENT TO PLATE AND BARS

The attachment of an impedance head to a Plate may be represented by the force node diagram of Figure 9.

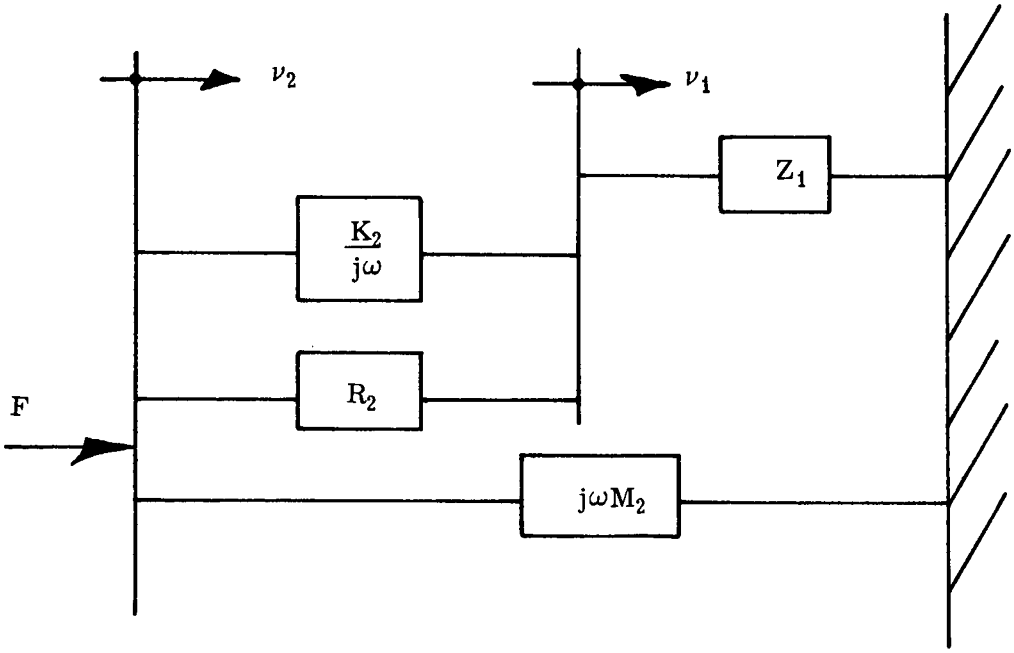


FIGURE 9. FORCE NODE DIAGRAM

The force node equations are:

$$O = (Z_1 + \frac{K_2}{j\omega} + R_2)\nu_1 - \left(\frac{K_2}{j\omega} + R_2\right)\nu_2 \tag{54}$$

$$F = -\left(\frac{K_2}{j\omega} + R_2\right)\nu_1 + \left(\frac{K_2}{j\omega} + R_2 + j\omega M_2\right)\nu_2 \tag{55}$$

$$F = Z_1\nu_1 + j\omega M_2\nu_2 \qquad (\text{Eq. } 54 + 55 = 56)$$

Then:
$$\nu_1 = \frac{\frac{K_2}{j\omega} + R_2}{Z_1 + \frac{K_2}{j\omega} + R_2}\nu_2$$

and
$$Z_2 = \frac{F}{\nu_2} = \frac{\left(\frac{K_2}{j\omega} + R_2\right) Z_1}{Z_1 + \frac{K_2}{j\omega} + R_2} + j\omega M_2 \tag{57}$$

Where Z_1 = The complex impedance of the attachment location or true driving point impedance

K_2 = Stiffness of the impedance head attachment

R_2 = Damping within the attachment

M_2 = Mass between the piezoelectric crystal and the attachment

Z_2 = Measured impedance

At low frequencies $\frac{K_2}{j\omega} \gg j\omega M_2$, if R_2 is small and $Z_1 \ll \left(\frac{K_2}{j\omega} + R_2\right)$ then Equation 57 reduces to

$$Z_2 = \frac{\left(\frac{K_2}{j\omega}\right) Z_1}{\left(\frac{K_2}{j\omega}\right)} + j\omega M_2$$

$$Z_2 = Z_1 + j\omega M_2 \tag{58}$$

At very high frequencies $\frac{K_2}{j\omega}$ approaches zero and

$$Z_2 \approx j\omega M_2 \tag{59}$$

or if R is a measurable quantity

$$Z_2 = \frac{R_2 Z_1}{R_2 + Z_1} + j\omega M_2 \tag{60}$$

The above analysis shows that at low frequencies the effective transducer mass (M_2) is always a factor for error. At high frequencies the transducer effective mass overrides other system impedances and becomes the only readout. Where the damping of the attachment is a significant value another source of error is present.

Going back to the force node equations, if K_2 is very stiff then $\nu_1 \approx \nu_2$. Equation 56 can be rewritten as

$$F = Z_1\nu_2 + j\omega M_2\nu_2 \tag{56A}$$

and the measured impedance is

$$Z_2 = \frac{F}{\nu_2} = Z_1 + j\omega M_2 \tag{58}$$

This is another way at reaching Equation 58.

The sample driving point impedance ($Z_1 = R_1 + j\omega M_1 + \frac{K_1}{j\omega}$) is explained in Section II as being multi-modal as applicable to the sample. For plates and a small driving location Figure 9 is representative of the measured impedance. The cutoff frequency here is determined by the impedance quantity and the error would be equal to ωM_2. Then

$$\omega_X M_2 = NZ_1 = \frac{N\epsilon_\nu M}{2\pi}$$

$$\omega_X = \frac{N\epsilon_\nu M}{2\pi M_2} \quad \text{which is Equation 18}$$

after the simplification of dropping $(1 - N)$.

SECTION IX. SELECTION OF A PRACTICAL VALUE OF ALLOWABLE ERROR IN TRANSDUCER MEASUREMENT

In the preceding sections of this paper the symbol "N" represented a maximum portion of the true measurement allowable as error due to the transducer attachment. This, of course, is dependent on the requirement of each measurement and upon the accuracy of the other components making up the measurement system. The equations are presented in such a way that the user may select any "N".

For general usage, N = 1/10 is a reasonable value. Most dynamic measurements are recorded in decibel scales. This error is $\pm$ 2 db which is consistent with most measurement systems. Where more accuracy is needed , N = 1/20 may be used. N = 1/20 is equivalent to about $\pm$ 1.4 db error. These decibels are based on the dynamic response of the system.

APPENDIX

Rough Approximation of the Mass in Vibration Resulting From a Small Dynamic Source Applied to the Surface of a Large Mass

The use of the quantity M_1 of Figure 6 is not important in determining critical frequency limitations due to transducer impedance. It does appear in Equation 44 where its value may be considered small enough to be inconsequential. It also appears in Equation 50, but here the resulting limiting frequency is beyond that of other limitations. A rigorous derivation here is not warranted. A rough approximation is presented in order to arrive at an order of magnitude.

The approximation is based on a description of Figure A-1. There is an obvious displacement of mass where the surface has been displaced. There also is an internal

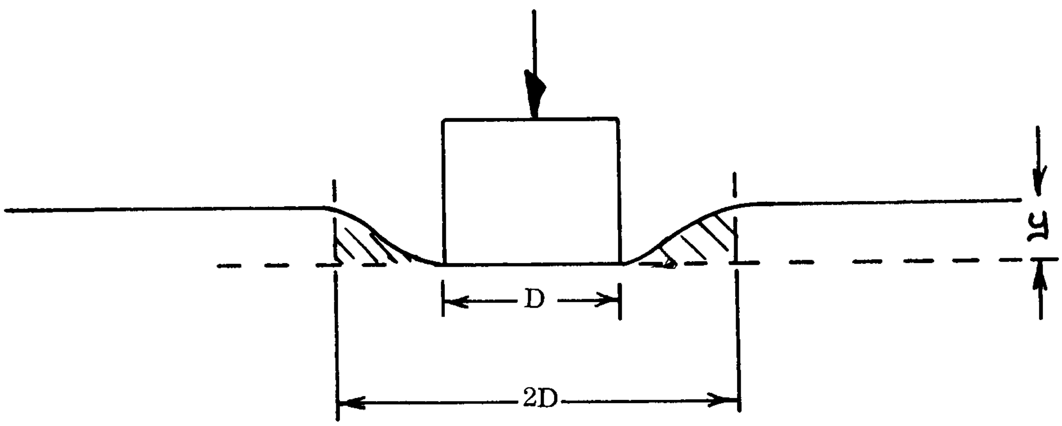


FIGURE A-1. DISPLACEMENT OF MASS

displacement of mass due to the compression of the material. This total displacement is assumed equivalent to $\rho \frac{(2d)^2}{4} \pi y$ with the shaded volume compensating for the internal displacement. Since $y = \frac{F}{DE}$

then $M_2 \approx \rho \frac{\pi D^2 F}{DE} \approx \frac{\pi \rho DF}{E}$

where:

ρ = Mass density

D = Diameter of the transducer attachment

F = Force applied through the transducer

E = Young's Modulus of the material of the lumped mass

REFERENCES

1. Wilcoxon Research Technical Bulletin No. 1, August 1, 1963.

2. "Vibration of a System with a Finite or an Infinite Number of Resonances," Skudrzyk, Eugen J., Journal of the Acoustical Society of America, Volume 30, Number 12, December, 1958.

3. "Theory of Noise and Vibration Insulation of a System with Many Resonances," Skudrzyk, Eugen J., Journal of the Acoustical Society of America, Volume 31, Number 1, January, 1959.

4. Timoshenko and J. N. Goodier, Theory of Elasticity, McGraw-Hill Book Company, Inc., 2nd Edition., Equation 214 and 215.

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THE EFFECT OF TRANSDUCER IMPEDANCE ON DYNAMIC MEASUREMENTS

By I. P. Vatz

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R. W. Schock

R. W. SCHOCK
Chief, Vibration Section

C. E. Green

CONCURRENCE: C. E. GREEN
Chief, Measurement and Evaluation Section

J. H. Farrow

J. H. FARROW
Chief, Vibration and Acoustics Branch

G A Kroll

G. A. KROLL
Chief, Structures Division

Emil A. Hellebrand

for F. B. CLINE
Director, Propulsion and Vehicle Engineering Laboratory

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Boeing / 5-7513
Mr. Siegel

Datacraft, Inc.
Mr. Johnson (2)

Michoud Assembly Facility
P.O. Box 29300
New Orleans, La. 70129
Attn: Mr. Sowers, CCSD/ Michoud