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SUMMARY

In a magnetic plasma compression experiment, a single-turn coil is subjected to magnetic pressure pulses. The energy developing the magnetic pressure pulses is stored in a 1-megajoule capacitor bank and delivered to the coil in a damped ringing electrical discharge. The kinetic pressures which can be developed in the coil depend on the coil's ability to contain these pressure pulses and that, in turn, is determined by the coil's mechanical design. Previously, all mechanical designs were based on calculations performed on the assumption that the peak current (and hence magnetic pressure) existed in the coil at all times during the discharge. This approach has a self-contained but unknown safety factor. In actuality, the coil undergoes a dynamic response throughout the discharge and is able to contain higher peak magnetic pressures than those predicted by static stress calculations because of an inertia term which is included in the dynamic response but is neglected in the static response. The dynamic response has been mathematically developed and the results have been compared with experimental data from a scaled-down model of the larger experiment.

INTRODUCTION

In many plasma experiments strong magnetic fields are generated for short times by the discharge of a capacitor bank through a single-turn coil. Often the discharge damps out long before the end of the first half-cycle of the mechanical oscillation of the coil. It was therefore thought that it might be quite possible for such coils to withstand much higher dynamic loads under these conditions than would be indicated by static stress calculations.

Dynamic stress calculations were therefore carried out for a thin single-turn coil which is much longer than its radius. In addition to the length requirement, it was assumed that the amplitude of the mechanical oscillation is always much smaller than the radius of the coil and that the pressure amplitude is not affected by the mechanical oscillations. These assumptions are satisfied in this experiment. The results apply to a range of mechanical oscillation amplitudes with stresses ranging from zero to the elastic limit of the material. The elastic limit in turn determines what is herein defined as the threshold pressure.

The predictions based upon the dynamic stress calculations were used in an experiment under conditions which are similar to those in a 1-megajoule magnetic compression experiment which is currently being designed and built.

The results could have consequences for the maximum attainable magnetic pressures in coils and, according to the relation (see ref. 1, p. 53)

$$p_M = \frac{B^2}{2\mu_o} = NkT = p_{kin}$$

it may then become possible to obtain higher temperatures and densities in the laboratory than were previously expected. (The formula given herein is the mks equivalent of the formula given in the cgs system in the reference.) This higher pressure capability could be important for the production of certain spectra of very highly ionized elements (mostly in the soft X-ray region) under controlled corona-like conditions as well as for the production of thermonuclear laboratory plasmas.

SYMBOLS

The rationalized mksAK system of units is used herein.

A_1,B_1,C_1	constants used in general solution
B	magnetic flux density
E	modulus of elasticity
k	Boltzmann constant
l	length of coil
N	particle density
p	pressure
$r(0)$	original inside radius of coil
S	yield strength
T	temperature
t	time
u	displacement

α	damping constant
β, β_1, β_2	constants
γ	coupling constant
δ	skin depth
ϵ	strain
η	constant
θ_o, θ_1	phase constants
μ	permeability of coil material
μ_o	vacuum permeability
ρ	mass density
σ	conductivity of coil material
τ	thickness of coil
ω_o	natural angular frequency of coil
ω_1	angular frequency of electrical discharge

Subscripts:

dyn	dynamic threshold
e	elastic
kin	kinetic
M	magnetic
max	maximum
s	static threshold

Dots over symbols indicate derivatives with respect to time.

EQUATION OF MOTION

The coil is considered to be a cylindrical ring with inside radius $r(0)$, thickness τ, and length l. If ρ is the mass density of the material, E the modulus of elasticity, and u the displacement defined by

$$u = r(t) - r(0)$$

in terms of the values of r at times t and 0, the equation of motion can be derived from a Lagrangian with the following terms (ref. 2):

kinetic energy	$\pi\rho\tau l r(0)\dot{u}^2$
stress potential	$-\pi E\tau l \dfrac{u^2}{r(0)}$
driving potential	$2\pi p l r(0) u$

where p is an internal driving pressure. The coil radius as a function of time $r(t)$ may be replaced by the coil inside radius $r(0)$ in the driving term if the condition

$$u \ll r(0) \tag{1a}$$

or

$$p \ll \frac{E\tau}{r(0)} \tag{1b}$$

is fulfilled. It is convenient to express the pressure as $p = p_{max}f(t)$, where $f(t)$ is dimensionless and p_{max} is independent of time. If, finally, the mechanical frequency ω_0 and the coupling constant γ (an acceleration) are introduced by

$$\omega_0^2 = \frac{E}{\rho r^2(0)} \tag{2}$$

and

$$\gamma = \frac{p_{max}}{2\rho\tau}$$

respectively, the equation of motion becomes

$$\ddot{u} + \omega_0^2 u = 2\gamma f(t) \tag{3}$$

If, in particular, the pressure is generated by a damped (damping constant $\alpha/2$) oscillatory discharge with frequency ω_1 through the coil, then $f(t)$ takes the form

$$f(t) = e^{-\alpha t}\sin^2\omega_1 t \tag{4}$$

Also, if the condition

$$\mu\sigma \gg \frac{1}{\omega_1\tau^2}$$

is fulfilled, that is, if the skin depth δ is much smaller than the coil thickness τ (μ and σ being the permeability and conductivity of the coil material, respectively), then

$$p_M = \frac{B^2}{2\mu_o}$$

If $\delta \to \tau$, part of the magnetic flux could penetrate the coil material and variations of p_M would result along the coil. The additional requirement $l \gg 2r(0)$ could be eliminated in the experiment.

EXPLICIT SOLUTION FOR SINGLE-TURN COILS

Equation (3) in conjunction with equation (4) can be solved by the Laplace transform method (ref. 3). With the definition

$$\beta^2 = \alpha^2 + \omega_o^2 + 4\omega_1^2$$

and the initial conditions $u(0) = \dot{u}(0) = 0$, the following solution is obtained:

$$u(t) = \frac{\gamma}{\alpha^2 + \omega_o^2}\left\{e^{-\alpha t} - \cos\omega_o t + \frac{\alpha}{\omega_o}\sin\omega_o t\right\}$$

$$+ \frac{\gamma}{\beta^4 - 16\omega_o^2\omega_1^2}\left\{e^{-\alpha t}\left[4\alpha\omega_1\sin 2\omega_1 t - \left(\beta^2 - 8\omega_1^2\right)\cos 2\omega_1 t\right]\right.$$

$$\left. + \left(\beta^2 - 8\omega_1^2\right)\cos\omega_o t - \frac{\alpha}{\omega_o}\beta^2\sin\omega_o t\right\} \tag{5a}$$

By combining terms, equation (5a) may be brought into the form

$$u(t) = \gamma\left[A_1 e^{-\alpha t} + B_1 e^{-\alpha t}\sin\left(2\omega_1 t + \delta_1\right) + C_1 \sin\left(\omega_0 t + \delta_0\right)\right] \tag{5b}$$

where

$$A_1 = \frac{1}{\alpha^2 + \omega_0^{\,2}}$$

$$B_1 = \frac{1}{\beta_1^{\,2}}$$

$$C_1 = \left[\left(\frac{\beta_2^{\,2}}{\beta_1^{\,4}} + A_1\right)^2 + \frac{\alpha^2}{\omega_0^{\,2}}\left(\frac{\beta^2}{\beta_1^{\,4}} - A_1\right)^2\right]^{1/2}$$

$$\beta_1^{\,4} = \beta^4 - 16\omega_0^{\,2}\omega_1^{\,2}$$

$$\beta_2^{\,2} = 8\omega_1^{\,2} - \beta^2$$

$$\theta_0 = \tan^{-1}\left[\frac{\omega_0}{\alpha}\,\frac{\dfrac{\beta_2^{\,2}}{\beta_1^{\,4}} + A_1}{\dfrac{\beta^2}{\beta_1^{\,4}} - A_1}\right]$$

and

$$\theta_1 = \tan^{-1}\frac{\beta_2^{\,2}}{4\alpha\omega_1}$$

Equation (5a) is in a slightly more manageable form than equation (5b). For example, it is easy to see that $u(t)$ is initially zero. If α is small compared with both ω_0 and ω_1, equation (5b) reduces to

$$u(t) = \frac{\gamma}{\omega_o^2\left(\omega_o^2 - 4\omega_1^2\right)}\left[\left(\omega_o^2 - 4\omega_1^2\right)e^{-\alpha t} - \omega_o^2 e^{-\alpha t}\cos 2\omega_1 t + 4\omega_1^2\cos \omega_o t\right] \qquad (6)$$

The threshold pressure can now be defined as that pressure which is just large enough to cause a permanent coil deformation. This threshold pressure is achieved if the elastic limit S is reached in the coil material (ref. 4). The static threshold pressure p_s is related to the elastic limit by

$$p_s = \frac{\tau}{r(0)\left[1 + \frac{u}{r(0)}\right]} S \approx \frac{\tau}{r(0)} S$$

The strain ϵ_e which corresponds to the elastic limit is related to the threshold pressure by

$$\epsilon_e \equiv \frac{u_e}{r(0)} \approx \frac{p_s r(0)}{\tau E} = \frac{\epsilon}{E} \qquad (7)$$

(which can be derived from equation (3) under static conditions). Assume that under dynamic conditions also the coil deforms permanently when the strain exceeds ϵ_e; then in the limit of equation (6) the dynamic threshold pressure is given by

$$p_{dyn} = \frac{\tau}{r(0)} S\left(1 - \frac{\omega_o^2}{4\omega_1^2} + \frac{\alpha\pi}{2\omega_o}\right) \qquad \left(2\omega_1 \gg \omega_o \gg \alpha\right) \qquad (8)$$

This relation can be verified by expanding equation (5a) in powers of α. If only first-order terms are retained, one arrives at equation (6) with $(1 - \alpha t)$ in place of the exponentials. The maximum elongation occurs when $\omega_o t \approx \pi$. When $2\omega_1 \gg \omega_o$, the term with $\cos 2\omega_1 t$ will peak at least once near the peak of the third term; thus, $\cos 2\omega_1 t$ can be set equal to 1 to find the overall maximum value of u.

It is not surprising that for $\alpha = 0$ (undamped pressure, varying with $\sin^2\omega_1 t$) the dynamic and static threshold pressures are nearly equal in this limit. The average pressure for $\alpha = 0$ is $p_{max}/2$, and the coil oscillates about the position corresponding to this pressure with an amplitude which corresponds to $p_{max}/2$; thus, it reaches a displacement corresponding to p_{max} during every cycle. With mechanical damping, equation (8) should include an additional factor of approximately $1 + \pi\eta/\omega_o$ where $\eta \ll \omega_o$, $2\omega_1$ is a damping constant which would have come from an additional term $+2\eta\dot{u}$ in equation (4). In practice this correction is very small for most metals when $\eta \ll \omega_0$.

EXPERIMENT

A short coil was tested with a small capacitor bank under stress conditions that are similar to those which will be encountered in a planned 1-megajoule magnetic compression experiment. The experimental setup is shown in figure 1. The capacitor bank is connected to the coil by high-inductance busses and can be switched with a single triggered air gap. Most of the inductance is in these connections; only a few percent of the total inductance is in the coil. The most important parameters of the discharge circuit and the coil are given in table I; values obtained for the simulation experiment are compared, if possible, with those for the planned magnetic compression experiment.

In scaling down to lower energies, it was important to keep the coil radius the same; therefore, the length had to be reduced considerably. Ordinarily, this length reduction would mean a sizable increase in inductance, accompanied by an undesirable nonuniformity of the magnetic field. Both problems could be solved by inserting an aluminum slug into the short coil with a gap between the slug and the coil of only about 3 percent of the coil radius. The skin effect excludes the high-frequency electromagnetic field from the slug; this forces the field lines to be parallel to the coil and reduces the

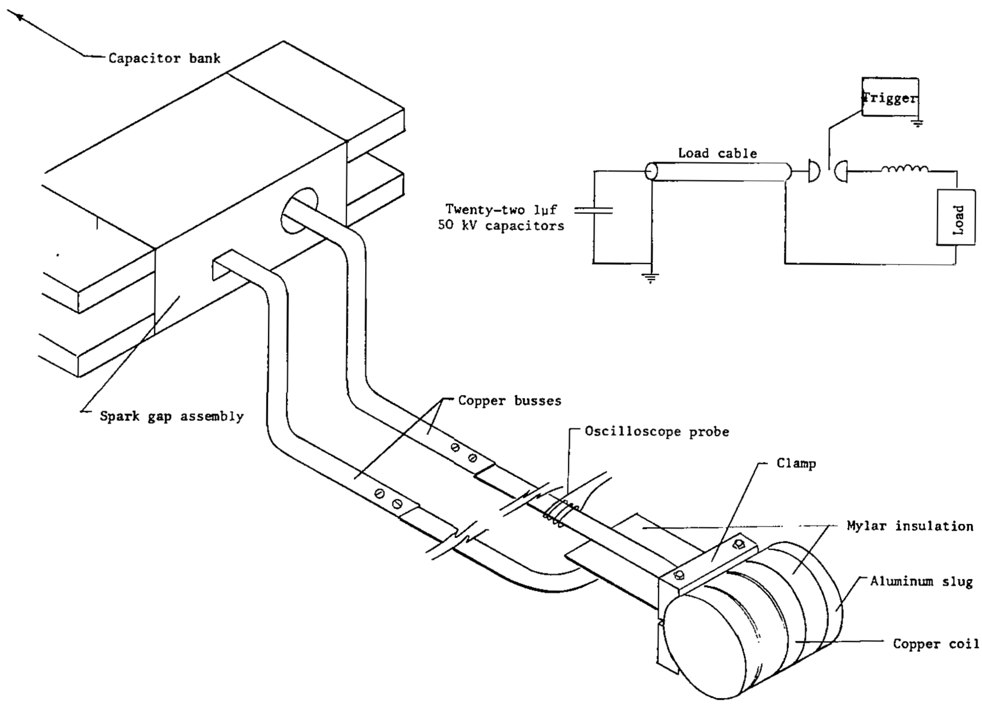

Figure 1.- Experimental setup.

TABLE I.- DISCHARGE CIRCUIT AND COIL PARAMETERS

	Simulation experiment	Magnetic compression experiment
Capacitance, C, μF	22	5000
Voltage rating, V, kv	30	20
Angular frequency, ω_1, 1/sec	1.38×10^5	1.25×10^5
e-fold time, $2/\alpha$, sec	2.2×10^{-4}	----------
Coil radius, r(0), m	6.2×10^{-2}	6.7×10^{-2}
Coil length, l, m	1.46×10^{-2}	1.2
Coil thickness, τ, m	1.6×10^{-3}	3×10^{-2}
Coil material	Electrolytic copper	Beryllium copper
Modulus of elasticity, E, psi	1.7×10^7	1.9×10^7
Modulus of elasticity, E, N/m^2	1.2×10^{11}	1.3×10^{11}
Yield strength, S, psi	2.9×10^4	10^5
Yield strength, S, N/m^2	2×10^8	6.9×10^8
Density, ρ, kg/m^3	8.9×10^3	9×10^3
Angular mechanical frequency, ω_0, 1/sec	6.1×10^4	6.6×10^4

inductance considerably. The magnetic pressure in the gap could be controlled by varying the charging voltage of the capacitor bank.

In the first series of tests the general reaction of the coil to the pressure pulses was studied in an effort to determine the pressure range in which deformations could be expected. For the sake of safety, the first tests were made at bank voltages of 2.6, 3.5, and 4kV; in other tests the voltages were increased in increments of 1kV. (The full pressure as in the planned experiment would be reached if the initial bank voltage were 55kV.) No changes were detected up to a bank voltage of 18kV when a small permanent deformation was first recorded. This bank voltage corresponds to a peak magnetic pressure of 8.3×10^6 N/m^2 (1200 psi). A further increase in the permanent deformation was observed after tests at still higher pressures.

After about three tests at a particular pressure, the coil ceased to expand farther because of the increasing nonuniformity of the magnetic field in the gap, accompanied by a slight drop in pressure, and because of possible work-hardening effects in the coil material.

Ordinarily, work hardening does not take place unless the elastic limit has been exceeded, which in turn would imply some permanent deformation. In order to reduce the possible effects of work hardening prior to the first observed permanent deformation, a second series of tests was made with a new coil, starting at a bank voltage of 9kV and increasing the voltage in increments of 2kV. The first permanent deformation was detected at a bank voltage of 17kV, corresponding to a peak magnetic pressure of 7.6×10^6 N/m^2 (1100 psi). Averaged over various diameters, a permanent change of 1.5×10^{-4} m (0.006 in.) in diameter was measured. Again, further permanent deformations occurred in tests at higher peak pressures.

The displacement u of the coil oscillating according to equation (5) for values of ω_1 and ω_0 in this experiment is shown in figure 2. The pressure corresponds to a bank voltage of 17kV. For comparison, the strain computed from the approximate equation (6) is also given in the figure. If the same peak magnetic pressure (7.6×10^6 N/m^2) had been applied statically, the stress in the coil would have been

$$S = p_s \frac{r(0)}{\tau} = 7.6 \times 10^6 \times \frac{6 \times 10^{-2}}{1.6 \times 10^{-3}} = 285 \times 10^6 \text{ N/m}^2$$

From the static stress-strain characteristic of the coil material given in figure 3 it can be seen that this value is much greater than the breaking point of the material. Alternatively, it can be shown that the permanent strain (0.0012) sustained by the coil dynamically corresponds to a peak pressure of approximately 160×10^6 N/m^2 on the static curve. (The location of peak pressure and strain is at the intersection of the characteristic curve and a line parallel to its linear portion at a distance on the abscissa equal to the permanent strain.) This fact indicates that the inertial term has accounted for approximately* 44 percent $\frac{285 - 160}{285}$ of the peak dynamic stress.

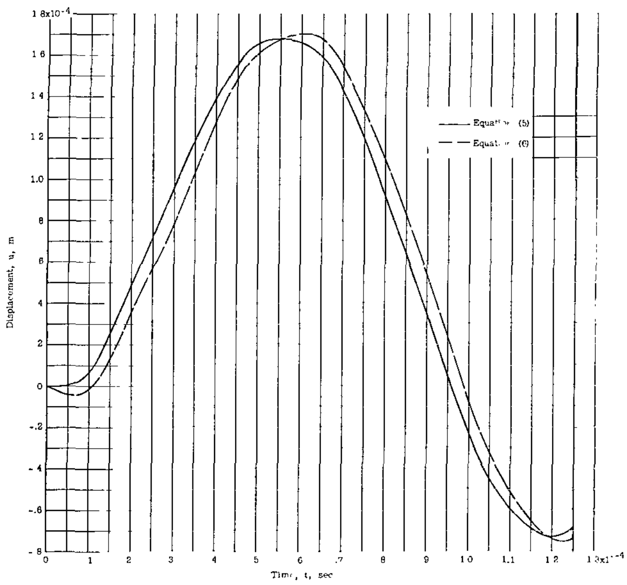

Figure 2.- Displacement as a function of time.

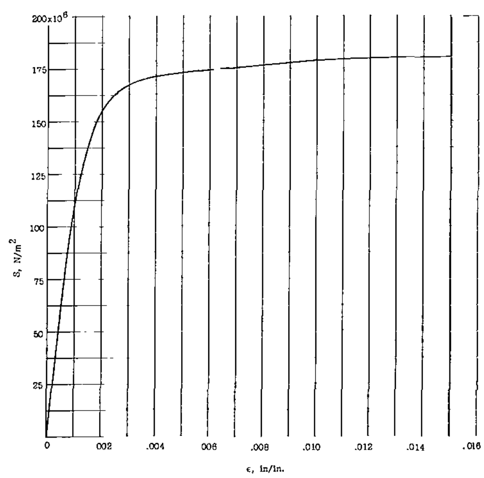

Figure 3.- Static stress-strain characteristic of coil material.

*Copper is stress-rate sensitive due to crystalline effects also. However, the good agreement herein between experiment and theory when only the inertial effect is used indicates that the crystalline effect is small.

The peak strain predicted by equation (5a) can be obtained from figure 2 by dividing the value of $u(t)$ by the coil radius. The maximum strain predicted by equation (5a) is 0.0027. The maximum strain observed is 0.0026. The maximum strain lies slightly in the nonlinear portion of the static characteristic of the material. (This is requisite for a permanent strain.) In the linearized theory presented herein, E should be calculated from a linear curve which best fits this nonlinear segment. (This fit insures that the same energy per unit volume will be delivered to the coil as would be delivered under the nonlinear segment of the characteristic.) The nonlinear characteristic (fig. 3) can be represented very accurately by

$$p = 1.48 \times 10^{11}\epsilon - 4.1 \times 10^{13}\epsilon^2 + 2.73 \times 10^{15}\epsilon^3 \qquad (0 \leqq \epsilon < 0.014)$$

The desired relation is

$$p = E\epsilon$$

When the area between the two curves is equated to zero, E is found to be 0.86×10^{11} N/m^2. This value of E is used to calculate the maximum strain from equations (5a) and (6).

The error in the electrical quantities is of the order of 5 percent, or about 10 percent for the magnetic pressure (because of the square dependence on the current). The errors in yield strength and modulus of elasticity are likely to be only 1 or a few percent each. Thus, the comparison of experiment and theory should show agreement within 12 percent or better.

According to equation (5a) the maximum strain during the test which first indicated that yield strength had been exceeded was within 4 percent of the maximum strain determined from the characteristic curve and permanent strain. The agreement is based on the "best fit" calculated value of E. (Operating slightly into the nonlinear region of the characteristic curve means E is a nonlinear function of ϵ.) Use of the best fit value of E provides a means of comparing experiment and theory. Normally, E from the linear portion of the characteristic curve would be used.

CONCLUDING REMARKS

Very good agreement between experimental and theoretical results is found for this experimental arrangement. The applicability of the simple relations to more general cases, such as thick coils, very high pulse frequencies, and long durations of the pressure pulse, remains to be tested. Within the framework of this formalism, it appears that the high frequency of oscillatory pressure pulses does result in significantly higher load capabilities in this experiment. Further, a short duration of the pressure pulses (i.e., a large damping factor) may lead to significantly higher load capabilities. Mechanical damping is negligible for this purpose for most metals unless the pulse lasts much longer than one period of the mechanical oscillation.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., September 14, 1965.

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