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A MORE EXACT THEORY OF
BACKSCATTERING FROM STATISTICALLY
ROUGH SURFACES

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Donald E. Barrick

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Department of ELECTRICAL ENGINEERING



THE OHIO STATE UNIVERSITY
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THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION
COLUMBUS, OHIO 43212

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Grant Number	NsG-213-61
Investigation of	Theoretical and Experimental Analysis of the Electromagnetic Scattering and Radiative Properties of Terrain, with Emphasis on Lunar-Like Surfaces
Subject of Report	A More Exact Theory of Backscattering from Statistically Rough Surfaces
Submitted by	Donald E. Barrick Antenna Laboratory Department of Electrical Engineering
Date	31 August 1965

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ABSTRACT

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This investigation has as its objectives improving and extending of the theory of backscattering from rough surfaces with an eye towards ascertaining a more satisfactory relationship between the statistics of the backscattered electromagnetic wave and the statistics of the rough surface. Towards this end contributions are made in the following areas.

The approximations involved in the use of the Helmholtz integral for the backscattered field are critically examined. The starting point of this analysis is the Chu-Stratton vector integral equation for the field at a point in space in terms of the time-harmonic electromagnetic vector fields at the scattering surface and in terms of the free space three dimensional Green's function, since no approximations are involved in this equation. (It is derived directly from Maxwell's equations. Then each of the approximations commonly made in the reduction of this vector integral equation to the familiar Helmholtz integral is separately considered. Several of the approximations made previously are avoided here; approximations that cannot be avoided are examined in detail so that their ranges of validity will be understood.

In particular, the application of the integral formulation to non-closed or finite scattering surfaces is considered. It is shown that when considering a perfectly conducting surface, the neglect of terms in the integrand involving surface slopes contributes large errors to the backscattered field at angles greater than 20° away from normal. By proper choice of the coordinate system it is shown that it is possible to properly include the effect of these extra terms without resorting to the use of the cumbersome Karhunen-Loeve expansion and the inherent complexities of the resulting mathematics. Results compare with measurements far better than any heretofore published solutions at higher backscattering angles.

The physical optics approximation is examined; this approximation permits one to write the total fields at the surface in terms of the incident fields and scattered fields, which are related to the incident fields by constants (reflection coefficients). The ranges of this high frequency approximation are determined. The usual far field approximations are re-evaluated and these requirements are relaxed when applied to rough surfaces.

Author

Backscattering from non-perfectly conducting rough surfaces is treated using the local Fresnel reflection coefficients to relate the scattered fields at every point on the surface to the incident fields. The resultant integral for the backscattered field written in terms of these Fresnel coefficients is remarkably simple and to date has not appeared in the literature. In this case, depolarization of the incident wave is found to take place and for the first time the relationship between the depolarized backscattered component and the surface properties may be specified.

Closed form solutions for the backscattering cross section for a class of rough surfaces are obtained for several statistical models. These results are compared with measured data from the lunar surface; besides prediction of reasonable lunar surface parameters, the comparison as a function of incidence angle is accurate to a degree hitherto unknown. Polarization sensitive backscattering cross sections for perfectly and non-perfectly conducting rough surfaces are derived for two statistical models both functionally and in curve form for linear and circular polarization states.

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CHAPTER I INTRODUCTION

In recent years much interest has been shown in the scattering of electromagnetic waves from rough surfaces, judging from the amount of published material. Particular emphasis has been given to the prediction of surface properties from a study of the waves scattered from the surface, with less emphasis on the converse problem, i. e., prediction of the scattered field knowing the surface properties. Reasons why such studies are being made are obvious: many surfaces are inaccessible to man at the present time (planetary surfaces) and are so distant as to preclude accurate optical observations; other surfaces present difficulties in obtaining direct measurement of the surface properties (e. g., sea surface). Distance in the case of planetary surfaces precludes accurate optical observations because of limitations in resolution.

The types of surface properties which appear feasible to obtain by measurements of their scattering properties are such things as statistical or overall geometrical properties (e. g., rms height, slope, the surface height probability density functions, correlation function, etc.) as well as electro-chemical properties (permittivity, permeability,

conductivity, etc.), and even possibly properties of geological interest (types of material, presence of sublayers, composition, etc.). People involved in transmission and reception of radio waves have known for years that all of the above properties of the earth's surface affect the transmission of signals, due to scattering by the surface.

From the studies already made, one can say that no one has arrived at a satisfactory theoretical relationship, which is either exactly or even approximately correct, between the scattering properties and the surface properties. Many claim that rms surface slope can be determined more or less satisfactorily from backscattering measurements of a scattered CW signal by varying the angle of incidence, but even here there is much variation between results because of the assumption of different statistical models for the surface (e. g. , Daniels, Reference 19). Predictions of rms surface slopes for the moon by various investigators, for example, range from about 5° to 20° . No one even claims to be able to predict any of the other surface properties at the present time.

There are indeed some who feel that there will never be any satisfactory correspondence between theoretically predicted and true surface properties, and who, therefore, believe that the best that can be expected is a comparison of the scattering characteristics of an unknown surface with the scattering characteristics of many sample surfaces on the earth whose surface properties have already been measured, in the hope

of matching the scattering characteristics and thus determining the properties of the unknown surface. This seems to be the philosophy of the many investigators who are presently making numerous measurements of radar return from natural surfaces such as terrain, the sea surface, vegetation, etc. However, from the various scattering tests conducted on known surfaces, it is apparent that there is no one-to-one correspondence between scattering characteristics obtainable by any of the tests presently proposed and the properties of the actual surface giving such characteristics. It does not appear, therefore, fruitless to this author to continue searching for a better understanding of the scattering characteristics with a view to a better explanation of the surface properties in terms of them.

In theoretical studies of scattering from rough surfaces, there seem to be two avenues of approach. One approach attempts to approximate the rough surface by a surface having a mathematically deterministic form, or by a surface made up of many such mathematically deterministic forms; this type of surface will be called a geometrical model. Rayleigh, for instance, studied scattering from one-dimensional, periodically rough surfaces and solved the problem for normal incidence as early as 1895[41] . Others have extended his treatment[30, 31, 32, 33, and 34] . A good discussion and summary of studies made on periodically rough surfaces may be found in Chapter 4 of Reference 1. Others have studied surfaces made up of many individual, mathematically

describable scatterers, such as cylinders[35] , bosses[36] , randomly spaced half planes[37] , a facet surface generated by a Markov chain[38] , and a surface of brilliant points[39] . A summary of most of these works is given in Chapter 6 of Reference 1.

The second avenue of approach defines the height of the surface above a given mean reference level as a random variable, and then assumes a certain statistical model of the surface (i. e. , assumes a given form for the probability density functions and correlation functions). Aside from Rice's[17] treatment of a slightly rough surface (which employs a perturbation method, attempting to satisfy the boundary conditions and Maxwell's equations after expanding the fields in a series), practically all of these methods begin with an approximate form for the scattered field based upon either the Kirchhoff-Helmholtz scalar integral equation or the Stratton-Chu vector integral equation. Invariably, all of these methods then apply some form of the physical optics approximation in order to relate the scattered field on the surface to the incident field at the surface. Isakovich[13] seemed to be the first to employ successfully this approach, and most later work appears to be only a slight modification of his treatment. After making the above mentioned two approximations, many more assumptions and approximations are made (e. g. , a perfectly conducting surface, no shadowing, no multiple scattering, Gaussian joint probability density function, certain form of surface height correlation function), and finally a result is obtained for

the scattered field characteristics which agrees with experimental measurements rather poorly in all cases, and at best offers questionable agreement for angles of incidence within only a few degrees from normal. Many authors* in recent years have chosen to attempt to improve the solution (i. e. , to obtain a better "fit" between theoretically predicted and experimentally measured results) by searching for a magical, heretofore undiscovered, surface height probability density function or correlation function which will give a better fit, and have thus obscured the true reasons for disagreement and led readers to think that the problem reduces to that of happening upon the true statistical distributions of the surface. The author believes that this is indeed a part of the problem of improving the solution, but a minor part at present; the particular surface statistics chosen are no more important than the initial form of the integral chosen and the approximations made to simplify it and reduce it to the physical optics solution. That the initial form of the scattering integral (sometimes referred to as the Helmholtz, Kirchhoff, or Huygen's integral) is a subject of debate is attested to in the comparison of the work by Hughes[22] and Beckmann[23] ; Beckmann discusses differences between his and Hughes results and attributes these differences

*Fung and Moore[40] , for example, choose a surface height correlation coefficient involving six arbitrary constants, and then choose the constants so that predicted results match measured results. See also Daniels[42] for a similar treatment.

to the assumption of different forms of the scattering integral as the starting point.

In this work, therefore, the basic scattering surface integrals in their exact form are considered first, and then all the approximations commonly made to obtain a solution are critically reviewed. Many of the most questionable approximations made in past studies have been avoided here. The author includes a term present in the original surface integral equations which is omitted by other investigators; this term is negligible near normal incidence but contributes significantly to the solution at more than about 15° from normal incidence. The restriction to a perfectly conducting surface is removed and a solution is obtained involving the parallel and perpendicular local surface reflection coefficients. By employing the vector form of the surface integral equation (the Stratton-Chu equation), information concerning polarization and depolarization is obtained in the solution; to the author's knowledge, the polarization scattering characteristics have not previously been investigated in this manner. All of these restrictions are removed and relatively simple closed form solutions are obtained at the expense of specializing initially to the case of backscattering rather than the more general case of bistatic scattering. However, in almost all of the experimental work done in the past and contemplated in the future, and especially that involving planetary surfaces, only backscattering measurements have been made.

It is shown how in general it is possible to predict averages of the surface height and slope, and even find the surface height correlation function, without initially assuming a certain form for the surface height joint probability density function. To obtain numerical results for comparison with previous work and experimentally measured data, several forms or models for the probability density function and correlation function are used and compared.

CHAPTER II SCATTERING PROBLEM FORMULATION

A. Discussion of the Commonly Made Approximations

In scattering from rough surfaces, many approximations are generally necessary in order to solve the integrals involved. However, it seems desirable to examine these approximations in order to ascertain the relative significance and validity of each of them and their effect upon the final results. Some of the approximations are stated in the literature, some are implied in the literature, and others are not generally even recognized as approximations. Such an examination is undertaken in this section, beginning with the initial integral equations and proceeding all the way to the final physical optics form of the integral.

1. Initial simplification of the integral equations

A true starting point to the theory of scattering from rough surfaces is the Stratton-Chu integral equations (Ref. 3, pp. 466-467).

These are

$$(2.1a) \quad \overline{E}(P') = -\frac{1}{4\pi} \int_S [j\omega\mu (\hat{n} \times \overline{H})\phi - (\hat{n} \times \overline{E}) \times \nabla\phi + (\hat{n} \cdot \overline{E}) \nabla\phi] da$$

$$(2.1b) \quad \bar{H}(P') = + \frac{1}{4\pi} \int_S [j\omega\epsilon (\hat{n} \times \bar{E})\phi - (\hat{n} \times \bar{H}) \times \nabla\phi - (\hat{n} \cdot \bar{H}) \nabla\phi] da$$

The above equations represent the fields at the observation point P' ; it is assumed that the space in which the field is of interest is source free, i. e., that all charge and current distributions are confined only to the surface of integration. In this case, the volume integrals present in the initial form of the Stratton-Chu equations vanish, since there are no space charges or currents in the volume. All fields are assumed to be harmonic in time, and the waves incident upon the surface will later be assumed to be plane of the form $\bar{H}^i = \bar{H}_0^i e^{j(\bar{k} \cdot \bar{\rho} - \omega t)}$; therefore the appropriate free space Green's function has the form $\phi = e^{jkR}/R$, where $R = |\bar{P} - \bar{P}'|$, $\bar{P}$ being a point on the surface, $\hat{n}$ is the unit normal to the surface at point P . The surface of integration is considered to be closed, and portions of it may lie at infinity. The field vectors $\bar{E}$ and $\bar{H}$ are assumed to be continuous over the entire surface. In general, there may be more than one closed surface to consider, but in the case at hand, only one surface is considered. The two above equations are vector integral equations and remain such unless a method of relating the reflected or scattered field at the surface to the incident field permits the total fields at the surface to be written in terms of the incident field; in general this is not possible at all wavelengths, but in limited ranges of wavelength certain approximations are permissible,

as will be seen later. If such a relationship can be established between incident and scattered fields at the surface, the integral equation reduces to a definite integral, which in most cases is still quite complicated, both for many deterministic surfaces and for randomly rough surfaces.

As discussed on pp. 468-470 of Stratton[3] and pp. 107-109 of Silver[4] , the surface integrals (2. 1a) and (2. 1b) can be transformed into the following forms when the surface is closed.

$$(2. 2a) \quad \bar{E}(P') = - \frac{1}{4\pi} \int_S \left(\bar{E} \frac{\partial \phi}{\partial n} - \phi \frac{\partial \bar{E}}{\partial n} \right) da$$

$$(2. 2b) \quad \bar{H}(P') = - \frac{1}{4\pi} \int_S \left(\bar{H} \frac{\partial \phi}{\partial n} - \phi \frac{\partial \bar{H}}{\partial n} \right) da \quad .$$

Here, n refers to distance along the normal to the surface. For rectangular coordinates, the above vector equations separate into three scalar equations each of the form

$$(2. 3) \quad u_k(P') = - \frac{1}{4\pi} \int_S \left(u_k \frac{\partial \phi}{\partial n} - \phi \frac{\partial u_k}{\partial n} \right) da \quad .$$

u_k refers to the x, y , or z component of either $\bar{E}$ or $\bar{H}$. Most authors use (2. 3) as the starting point for scattering from rough surfaces, either overlooking the fact that it is only valid for a closed surface or assuming that little error is introduced by using (2. 3) for non-closed surfaces. Since all scattering surfaces of interest here are of finite dimensions, especially when viewed from the far field, and since in general one must consider only a part of the closed surface of the

scattering body (because of the difficulty in determining the fields at the surface on the shadowed side of the object and near the shadow boundary), this question deserves some attention.

It is pointed out on pp. 468-470 of Stratton[3] and pp. 164-166 of Silver[4] that when the surface under consideration is not closed or when the surface field vectors are not continuous at all points of the surface, then extra terms are introduced into (2.1a) and (2.1b) which account for the discontinuity of the fields at the edge of the surface or at the shadow boundary (thus these terms can be thought of as due to line charges present at the edge where the fields are discontinuous); these terms must be present in order for (2.1a) and (2.1b) to satisfy Maxwell's equations. A detailed proof of this statement can be found in Ref. 5 (pp. 43-60) and Ref. 6 (pp. 102-123). Equations (2.1a) and (2.1b) then become

$$(2.4a) \quad \begin{aligned} \bar{\mathbf{E}}(\mathbf{P}') = & -\frac{1}{4\pi} \int_{S_1} [j\omega\mu (\hat{\mathbf{n}} \times \bar{\mathbf{H}})\phi + (\hat{\mathbf{n}} \times \bar{\mathbf{E}}) \times \nabla\phi \\ & + (\hat{\mathbf{n}} \cdot \bar{\mathbf{E}})\nabla\phi] da - \frac{1}{j\omega\epsilon} \cdot \frac{1}{4\pi} \oint_C \nabla\phi \bar{\mathbf{H}} \cdot \overline{ds} \end{aligned}$$

$$(2.4b) \quad \begin{aligned} \bar{\mathbf{H}}(\mathbf{P}') = & +\frac{1}{4\pi} \int_{S_1} [j\omega\epsilon (\hat{\mathbf{n}} \times \bar{\mathbf{E}})\phi - (\hat{\mathbf{n}} \times \bar{\mathbf{H}}) \times \nabla\phi \\ & - (\hat{\mathbf{n}} \cdot \bar{\mathbf{H}})\nabla\phi] da + \frac{1}{j\omega\mu} \cdot \frac{1}{4\pi} \oint_C \nabla\phi \bar{\mathbf{E}} \cdot \overline{ds} \quad . \end{aligned}$$

Without loss of generality the scattering surface can be assumed to be a rough plate of local thickness, t , as shown in Fig. 1. If it is desired to know the scattered fields at P' in terms of the fields over the illuminated surface S_1 , then (2.4a) and (2.4b) must be used. The additional term is the integral around the contour denoting the shadow boundary of the surface, and $\overline{ds}$ is the vector representing an element of length on this contour; the field on the shadowed side is set equal to zero in this derivation, and hence the contour integral accounts for the discontinuity between the fields on the illuminated side of the shadow boundary and the hypothetical zero value of these fields on the shadowed side.

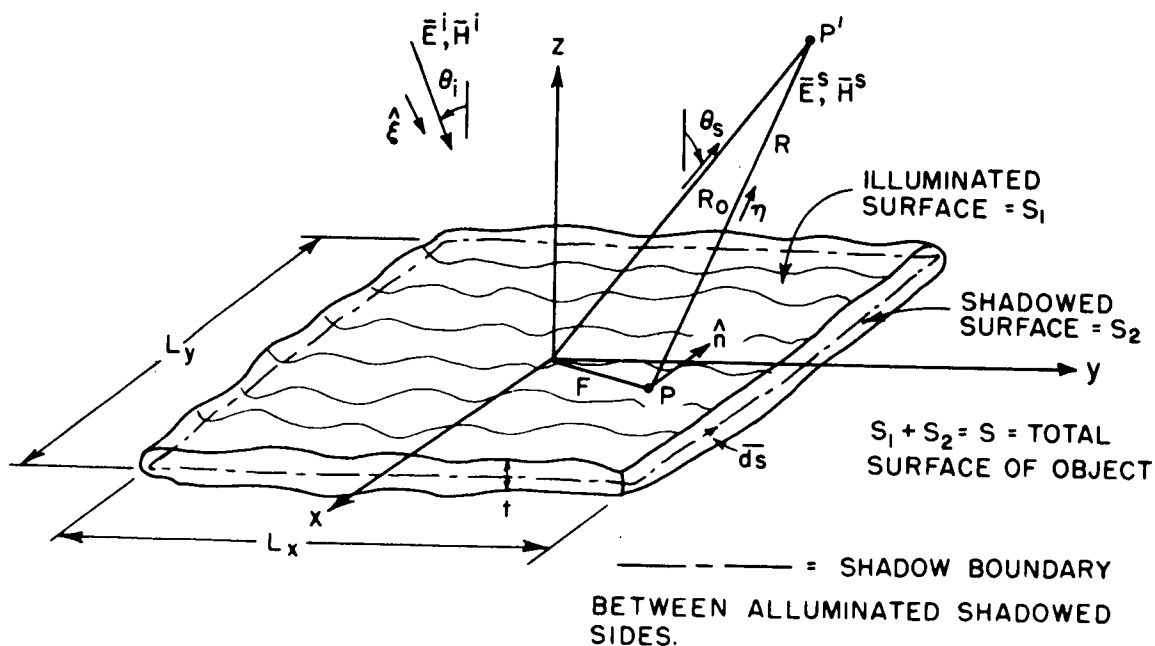


Fig. 1--General planar scattering surface.
located in x-y plane.

The thickness of the plate is immaterial so long as the effect of the discontinuity in the surface fields at the shadow boundary is taken into account by the line integral term, and it may even be allowed to shrink to zero if convenient.

It should be noted that the surface integrals of (2.4a) and (2.4b) no longer transform to those of (2.2a) and (2.2b). In the case where the surface of integration is not closed, as in (2.4a) and (2.4b), an additional term arises in the transformation to the form of (2.2a) and (2.2b), in addition to the already existing line integral of (2.4a) and (2.4b). The result of the transformation is given on pp. 469 of Stratton and is repeated here.

$$(2.5a) \quad \overline{E}(P') = -\frac{1}{4\pi} \int_{S_1} \left(\overline{E} \frac{\partial \phi}{\partial n} - \phi \frac{\partial \overline{E}}{\partial n} \right) da - \frac{1}{j\omega\epsilon} \cdot \frac{1}{4\pi} \oint_C \nabla \phi \cdot \overline{H} \cdot \overline{ds} \\ + \frac{1}{4\pi} \oint_C \phi \overline{E} \times \overline{ds}$$

$$(2.5b) \quad \overline{H}(P') = -\frac{1}{4\pi} \int_{S_1} \left(\overline{H} \frac{\partial \phi}{\partial n} - \phi \frac{\partial \overline{H}}{\partial n} \right) da + \frac{1}{j\omega\mu} \cdot \frac{1}{4\pi} \oint_C \nabla \phi \cdot \overline{E} \cdot \overline{ds} \\ + \frac{1}{4\pi} \oint_C \phi \overline{H} \times \overline{ds} \quad .$$

It should be noted that the above equations do not separate easily into scalar equations in each of the Cartesian field components as did (2.2a) and (2.2b). In general, therefore, (2.5a) and (2.5b) are more difficult to use than (2.4a) and (2.4b); the integrals (2.4a) and

(2.4b) will be used throughout this work. It is possible that in certain limited cases the last two terms of (2.5a) and (2.5b) are negligible: for example, Silver[Ref. 4, p. 165] shows that these terms are proportional to $\sin \theta_s$, the scattering angle with respect to normal incidence; the first term of (2.5a) and (2.5b) is therefore proportional to $\cos \theta_s$. At $\theta_s = 45^\circ$ it is evident that the relative magnitudes of the surface integrals and the last two terms are of the same order; thus error in the predicted scattered fields may become as great as 100% at $\theta_s = 45^\circ$ if these latter terms are dropped. It is not surprising that predicted results for backscattering thus far begin to deviate significantly from their measured values at about 20° incidence angle when Eq. (2.3) is used as a starting point.

Beckmann[Ref. 1, p. 180] makes the statement, "The Helmholtz integral" (i. e. , Eq. (2.3) here) "is easily applied to surfaces that are not closed, e. g. , in our application to a rough surface in Chapter III, one may regard the rough surface S as a part of any closed surface S', of which only S has a non-vanishing reflection coefficient; the remaining integral over S-S' is then easily shown to vanish so that there is no need to introduce the surface S' . " The above statement, although it sounds logical, appears to be an erroneous simplification. In the first place, the fields at the surface when passing from S to S' will suffer a discontinuity (e. g. , consider as the extreme case that S is perfectly conducting and S' is free space); this violates

the requirement that the fields be everywhere continuous over the closed surface, which is recognized as a necessary requirement by Beckmann (p. 178). Secondly, although it is true that the integration over a surface S' with zero reflection coefficients can be shown to vanish, the remaining integral over a non-closed surface giving a field component at P' does not satisfy Maxwell's equation. This simplification is not made by Beckmann alone; many in the literature either make similar statements or use as starting points results of others who have made such assumptions (e. g. , Davies, Daniels, Ament, etc.).

The Helmholtz integral (2.3) suffers from other inadequacies also. It yields little information about depolarization due to surface roughness. Consider as an example backscattering, where the incident wave is polarized in one of the three rectangular directions, e. g. , $H_y^i = H_z^i = 0$, $H_x^i \neq 0$. Then $u_k = H_x$ in (2.3), but it is difficult to see that there can be any solution for H_y or H_z at P' from (2.3), due to the absence of an incident field in these directions. Yet depolarization is both an experimental fact in this case and a seemingly reasonable phenomenon. As will be seen later, depolarization can be studied and predicted quantitatively if (2.4a) and (2.4b) are used as a starting point.

2. The far field approximations

In most applications, the scattered field of interest is far removed from the radiating or scattering object; however, to avoid possible ambiguities in definition of the far-zone or far field, the following approximations are stated here and considered in this section.

$$(a) \quad R \gg D,$$

$$(b) \quad R \gg \lambda,$$

$$(c) \quad R > \frac{2D^2}{\lambda}.$$

Here, R is the distance between any point on the scattering surface and the field point and D is the largest dimension of the surface.

The first approximation merely means that the distance between field point and scatterer is large in terms of the size of the scatterer, or in terms of the maximum dimension of the illuminated region of the scattering body. All of the cases to be considered in this analysis meet this requirement.

The second approximation that distance between object and scatterer is very large compared to wavelength will also be satisfied in all cases under consideration in this analysis.

The third approximation is the most commonly used definition of the far-zone region. It can be shown, however, that this requirement is not necessary in the case at hand where the scattering surface

is rough; it can therefore be ignored.* This should allay the skepticism of many who have undoubtedly noted or read that the earth is in the "near field" of the moon (the entire lunar body being considered the scatterer) at centimeter wavelengths when criterion (c) is used to determine far and near field regions. Therefore, all results derived in this analysis can be applied to the lunar surface, since it meets requirements (a) and (b).

Approximation (b) permits terms in R^{-1} to be neglected compared with terms in λ^{-1} ; such terms arise when one takes the gradient of the Green's function, ϕ , as indicated in (2.4a) and (2.4b). Approximation (a) permits one to consider that lines from all points on the scattering object to the field point are essentially parallel. Both of these assumptions are now used to simplify (2.4a) and (2.4b). Only the results are stated here; the reader interested in their proof can find the details in Appendix A.**

* Demonstration of the validity of this statement is given in Appendix B for the case of the Gaussian surface statistical model. The reader is advised to read through the analysis of this statistical model in the main body before attempting to read Appendix B however.

** The development here is similar to that of Silver [Ref. 4, pp. 80-90] with the exception that he considers only a perfect conducting surface. Also, Silver employs a plane wave representation of

$\vec{H}^i = \vec{H}^{io} e^{j(\omega t - \vec{k} \cdot \vec{P})}$ and therefore a Green's function $\phi = e^{-jkR}/R$; this gives rise to differences in signs of some of the terms in the results. The time dependence used here agrees with that of Stratton.

First of all, the line integrals in (2.4a) and (2.4b) can be converted to surface integrals using Stoke's theorem.

$$(2.6a) \quad \oint \nabla \phi \cdot \overline{\mathbf{H}} \cdot \overline{ds} = \int_{S_1} \{ -[(\hat{\mathbf{n}} \times \overline{\mathbf{H}}) \cdot \nabla] \nabla \phi - j\omega\epsilon (\hat{\mathbf{n}} \cdot \overline{\mathbf{E}}) \nabla \phi \} da$$

$$(2.6b) \quad \oint \nabla \phi \cdot \overline{\mathbf{E}} \cdot \overline{ds} = \int_{S_1} \{ -[(\hat{\mathbf{n}} \times \overline{\mathbf{E}}) \cdot \nabla] \nabla \phi + j\omega\mu (\hat{\mathbf{n}} \cdot \overline{\mathbf{H}}) \nabla \phi \} da$$

When the line integrals in (2.4a) and (2.4b) are replaced by their equivalent surface integrals according to the above two equations, the last term of each of the above two cancel with the last terms of the original surface integrals to yield

$$(2.7a) \quad \overline{\mathbf{E}}(\mathbf{P}') = \frac{1}{j\omega\epsilon} \cdot \frac{1}{4\pi} \int_{S_1} \{ [(\hat{\mathbf{n}} \times \overline{\mathbf{H}}) \cdot \nabla] \nabla + k^2(\hat{\mathbf{n}} \times \overline{\mathbf{H}}) - j\omega\epsilon (\hat{\mathbf{n}} \times \overline{\mathbf{E}}) \times \nabla \} \phi da$$

$$(2.7b) \quad \overline{\mathbf{H}}(\mathbf{P}') = -\frac{1}{j\omega\mu} \cdot \frac{1}{4\pi} \int_{S_1} \{ [(\hat{\mathbf{n}} \times \overline{\mathbf{E}}) \cdot \nabla] \nabla + k^2(\hat{\mathbf{n}} \times \overline{\mathbf{E}}) + j\omega\mu (\hat{\mathbf{n}} \times \overline{\mathbf{H}}) \times \nabla \} \phi da$$

where $k^2 = \omega^2 \mu \epsilon$.

Referring to Fig. 1 and defining $\hat{\eta}$ to be the unit vector in the direction of $\mathbf{P}'$ from the surface point $\mathbf{P}$, the following results are stated here:

$$(2.8a) \quad \hat{\eta} \equiv \frac{\overline{\mathbf{R}}}{R} ;$$

$$(2.8b) \quad \nabla R = -\hat{\eta}, \text{ where } \nabla \text{ is the gradient with respect to the surface point } P.$$

$$(2.8c) \quad \nabla \phi = \left(\frac{1}{R} - jk \right) \phi \hat{\eta}$$

$$(2.8d) \quad \begin{aligned} (\hat{n} \times \overline{H}) \cdot \nabla \nabla \phi &= \frac{d^2 \phi}{dR^2} [(\hat{n} \times \overline{H}) \cdot \hat{\eta}] \hat{\eta} \\ &= \left[\frac{2}{R^2} - \frac{2jk}{R} - k^2 \right] \phi [(\hat{n} \times \overline{H}) \cdot \hat{\eta}] \hat{\eta} \\ (\hat{n} \times \overline{E}) \cdot \nabla \nabla \phi &= \frac{d^2 \phi}{dR^2} [(\hat{n} \times \overline{E}) \cdot \hat{\eta}] \hat{\eta} \\ &= \left[\frac{2}{R^2} - \frac{2jk}{R} - k^2 \right] \phi [(\hat{n} \times \overline{E}) \cdot \hat{\eta}] \hat{\eta} . \end{aligned}$$

Using (2.8) in (2.7a) and (2.7b) and regrouping, one finds

$$(2.9a) \quad \begin{aligned} \overline{E}(P') &= \frac{1}{j\omega\epsilon} \cdot \frac{1}{4\pi} \int_{S_1} \{ k^2 [(\hat{n} \times \overline{H}) \cdot \hat{\eta} \cdot (\hat{n} \times \overline{H}) \hat{\eta}] \\ &\quad - k\omega\epsilon [(\hat{n} \times \overline{E}) \times \hat{\eta}] \} \phi \, da + \overline{O}\left(\frac{1}{R}\right) \end{aligned}$$

$$(2.9b) \quad \begin{aligned} \overline{H}(P') &= -\frac{1}{j\omega\mu} \cdot \frac{1}{4\pi} \int_{S_1} \{ k^2 [(\hat{n} \times \overline{E}) \cdot \hat{\eta} \cdot (\hat{n} \times \overline{E}) \hat{\eta}] \\ &\quad + k\omega\mu [(\hat{n} \times \overline{H}) \times \hat{\eta}] \} \phi \, da + \overline{O}\left(\frac{1}{R}\right) . \end{aligned}$$

The last term in each of the above equations represents all terms of the integrand which approach zero as $R \rightarrow \infty$ at least as fast as $1/R^2$; these terms are **therefore** neglected according to approximation (b). It may also be noticed that the term $\hat{\eta} \cdot (\hat{n} \times \overline{H}) \hat{\eta}$ removes the

component of $\hat{n} \times \vec{H}$ in the direction, $\hat{\eta}$, of wave travel from the surface to the field point, P' ; this meets the requirement that in the far field the E and H components be perpendicular to the direction of propagation only.

If the dimensions of the scattering surface are small compared to R , then the Green's Function can be approximated as

$$(2.10a) \quad \phi \simeq \frac{e^{jkR}}{R_0},$$

where $R_0 \equiv \text{constant} = \text{distance from the origin, which is located somewhere in the surface.}$

If R and R_0 are very large compared to any surface dimension (approximation (a)); then they are approximately parallel; assuming they are parallel, one can state that

$$(2.10b) \quad R \simeq R_0 - \hat{\eta} \cdot \vec{r}^*$$

in the far field. $\vec{r}$, as shown in Fig. 1 is the vector from the origin to the point P on the surface. Therefore, the Green's function becomes

$$(2.10c) \quad \phi \simeq \frac{e^{jkR_0 - jk\hat{\eta} \cdot \vec{r}}}{R_0}.$$

* The additional quadratic phase distortion term which is included in (2.10b) when approximation (c) is not satisfied is omitted here, since its effect on the desired answer is negligible as shown in Appendix B.

When (2.10c) is substituted into (2.9a) and (2.9b) and the results simplified, one obtains,

$$(2.11a) \quad \overline{E}(P') = - \frac{j\omega\mu e^{jkR_o}}{4\pi R_o} \int_{S_1} \{ (\hat{n} \times \overline{H}) - \hat{\eta} \cdot (\hat{n} \times \overline{H}) \hat{\eta} - Y (\hat{n} \times \overline{E}) \times \hat{\eta} \} e^{-jk\hat{\eta} \cdot \overline{r}} da ,$$

$$(2.11b) \quad \overline{H}(P') = \frac{j\omega\epsilon e^{jkR_o}}{4\pi R_o} \int_{S_1} \{ (\hat{n} \times \overline{E}) - \hat{\eta} \cdot (\hat{n} \times \overline{E}) \hat{\eta} + Z (\hat{n} \times \overline{H}) \times \hat{\eta} \} e^{-jk\hat{\eta} \cdot \overline{r}} da ,$$

where $Z \equiv \sqrt{\frac{\mu}{\epsilon}} \equiv \frac{1}{Y} .$

It will be stated again that the first two approximations made in this section to obtain (2.11a) and (2.11b) are valid for the class of problems being considered. Therefore, up to this point, no approximations have been made which will seriously affect the accuracy of the results in the far field.

3. The assumption of a perfectly conducting surface

Most authors at one point or another make the assumption that the scattering surface may be considered perfectly conducting. Intuitively and logically, one would feel that this assumption seriously restricts the class of surfaces for which the results are meaningful. Whether one can attempt to use the results for a perfectly conducting surface to predict the scattering properties of the lunar surface is

indeed questionable. The application of this restriction does, of course, simplify the solution of the integral equations considerably. For a perfect conductor, it is true that at the surface $\hat{n} \times \overline{E} = \overline{O}$. Therefore, equations (2.11a) and (2.11b) will reduce to

$$(2.12a) \quad \overline{E}(P') = - \frac{j\omega\mu e^{jkR_o}}{4\pi R_o} \int_{S_1} [(\hat{n} \times \overline{H}) - \hat{\eta} \cdot (\hat{n} \times \overline{H}) \hat{\eta}] e^{-jk\hat{\eta} \cdot \overline{r}} da$$

$$(2.12b) \quad \overline{H}(P') = \frac{jk e^{jkR_o}}{4\pi R_o} \int_{S_1} [(\hat{n} \times \overline{H}) \times \hat{\eta}] e^{-jk\hat{\eta} \cdot \overline{r}} da$$

Equation (2.12b) is the simpler of the two and is often seen in the literature as the starting point for most rough surface problems. This equation could have been derived much more simply for the perfectly conducting surface directly from (2.4b), since $\hat{n} \cdot \overline{H}$ is zero at the surface of a perfect conductor and $\overline{E} \cdot \overline{ds}$ is zero along the edge of a perfect conductor.

The author will not attempt to discuss the merits of the perfectly conducting surface assumption at this point; results will be found for both the perfectly conducting surface and for the non-perfectly conducting surface in a later section and the reader can make the comparison.

4. The physical optics approximation

The physical optics approximation is made by everyone thus far in the literature who takes the surface integral approach to the scattering problem. The term "physical optics", unfortunately, is used by different people to mean different things. This author will use physical optics to denote the approximation which attempts to express the fields reflected from the surface at a local point P in terms of a constant times the incident fields at that point on the surface; this constant, or reflection coefficient, is a function of the local angle of incidence, the direction of polarization of the incident wave with respect to the local plane of incidence, and finally the electrical properties of the surface. In order to express the reflected field at the surface in this manner, it is necessary to assume that the reflected field near the surface at every point of the surface is a plane wave. It is this assumption which places serious restrictions on the classes of surfaces under consideration. As most authors state, and correctly so, this assumption is valid only as the ratio of wavelength to local radius of curvature of the surface shrinks to zero at every point of the surface. When the surface is perfectly conducting, this assumption may be written as $\hat{n} \times \vec{H}^i = \hat{n} \times \vec{H}^r = \frac{1}{2} \hat{n} \times \vec{H}$ at the surface; another way of stating the same result is in terms of the surface current density, $\vec{J}$;

$$(2.13) \quad \vec{J} = \hat{n} \times \vec{H} = 2\hat{n} \times \vec{H}^i$$

at the surface. This equation is easily verified if the reflected wave, $\bar{H}^r$, is assumed to be plane, since $\hat{n} \times (\bar{E}^i + \bar{E}^r) \equiv \bar{O}$ is always true at a perfect conductor and for plane waves, $\bar{H}^r$ and $\bar{H}^i$ are related to $\bar{E}^r$ and $\bar{E}^i$ respectively by the same constant.

When the surface is not perfectly conducting, the reflected waves are related to the incident wave in the physical optics approximation by the following equations, derived by Stratton [Ref. 3, pp. 492-510],

$$(2.14a) \quad \bar{E}_\perp^r = \frac{\mu_s k_o \cos \theta_i - \mu_o \sqrt{k_s^2 - k_o^2 \sin^2 \theta_i}}{\mu_s k_o \cos \theta_i + \mu_o \sqrt{k_s^2 - k_o^2 \sin^2 \theta_i}} \bar{E}_\perp^i \equiv \rho_\perp \bar{E}_\perp^i,$$

for the component of the incident $\bar{E}^i$ field perpendicular to the local plane of incidence, and

$$(2.14b) \quad \hat{n}_r \times \bar{E}_\parallel^r = \frac{\mu_o k_s^2 \cos \theta_i - \mu_s k_o \sqrt{k_s^2 - k_o^2 \sin^2 \theta_i}}{\mu_o k_s^2 \cos \theta_i + \mu_s k_o \sqrt{k_s^2 - k_o^2 \sin^2 \theta_i}} \hat{n}_i \times \bar{E}_\parallel^i \\ \equiv \rho_\parallel \hat{n}_i \times \bar{E}_\parallel^i,$$

for the component of the incident $\bar{E}^i$ field which lies in the local plane of incidence.

The unit vector $\hat{n}_i$ points along the direction of propagation of the incident wave, while the unit vector $\hat{n}_r$ points in the direction of specular reflection from the surface at the surface point in question. Here it is assumed that the region containing the field point P' is free space, having $\mu = \mu_o$, $\epsilon = \epsilon_o$, and $k = k_o = \omega \sqrt{\mu_o \epsilon_o}$; the surface has

$\mu = \mu_s$ and $k_s = \sqrt{\omega^2 \epsilon_s \mu_s + j\omega\sigma_s \mu_s}$. Therefore $\rho_{\perp}$ and $\rho_{\parallel}$ can in general be complex, especially when the surface is not a perfect dielectric where $\sigma_s = 0$.

Thus, when using the physical optics approximations of (2.13), (2.14a), or (2.14b), the total E and H fields at the surface can be expressed entirely in terms of the incident fields, and the integral equations (2.11a) and (2.11b) reduce to definite integrals. Up to the present time, the actual integral equations (2.11a) and (2.11b) have never been solved for the case where the surface height is represented by a random variable; only after the use of the physical optics approximation and the reduction to a definite integral has anyone ever solved the problem. Hence, where the physical optics approximation is valid, solutions for the scattered fields and power have been obtained which are none the less still somewhat at variance with measured data due to the other assumptions made in their solutions. The question to be raised at this point is the following: how much less than unity must the ratio of wavelength to surface radius of curvature be in order for the physical optics approximation to be valid. Many authors intuitively state that when wavelength is of the same order of magnitude of surface radius of curvature, physical optics begins to break down. In order to see more precisely and quantitatively at what ratio physical optics begins to break down, plots of surface current induced on a perfectly conducting infinitely long circular cylinder by incident plane

waves are made as a function of $ka = 2\pi a/\lambda$, where a = radius of cylinder (see Figs. 2 and 3). Two cases are considered: (1) magnetic field polarized perpendicular to the cylinder axis, producing only an axial surface current, and (2) magnetic field polarized parallel to the cylinder axis, producing only a circumferential surface current. The surface current magnitudes are normalized with respect to the physical optics prediction and the surface current phase angles are subtracted from those predicted from physical optics to give the phase difference between physical optics and the exact solution. The exact currents are found from Harrington's formulas 5-109 and 5-116[7]; these are in the form of series in Hankel functions. The currents predicted by physical optics are computed from Eq. (2.13). It should be noted that the physical optics approximation also implies that the magnitude of all currents on the shadowed side of the cylinder will be zero (the current phase angle on the shadowed side is therefore meaningless and is not plotted).

From the plots it may be seen that by employing physical optics when the radius of curvature is equal to wavelength, the percentage error in both current magnitudes at the leading edge (normal incidence) is about 1.3%, while the phase error for both currents is about 4.3° . However, at the same radius the respective errors are greater further on around the cylinder; for example at 60° from the leading edge, the percentage error in current magnitudes is as high as 15% and the phase

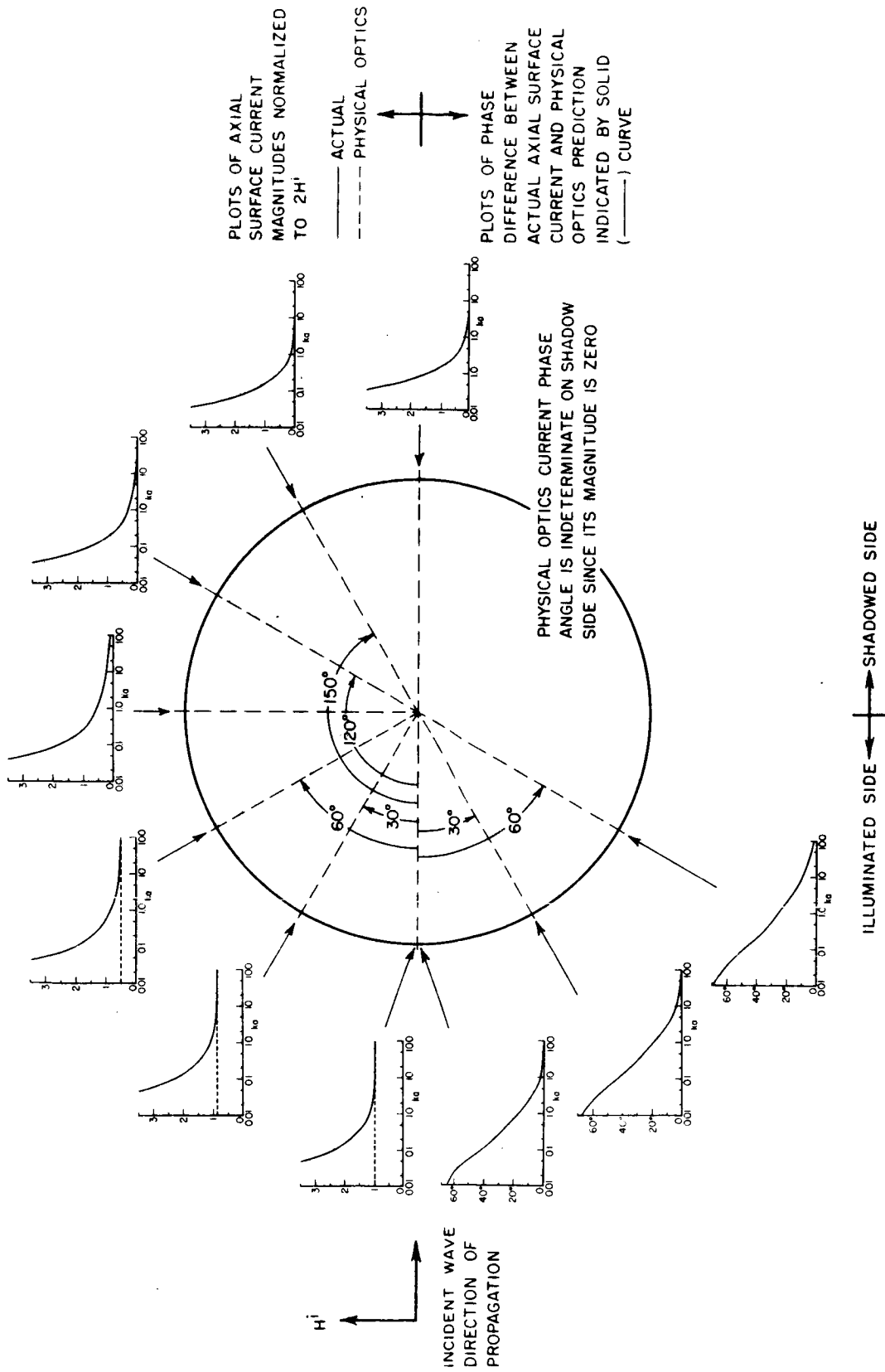


Fig. 2--Axial surface currents on a perfectly conducting circular cylinder.

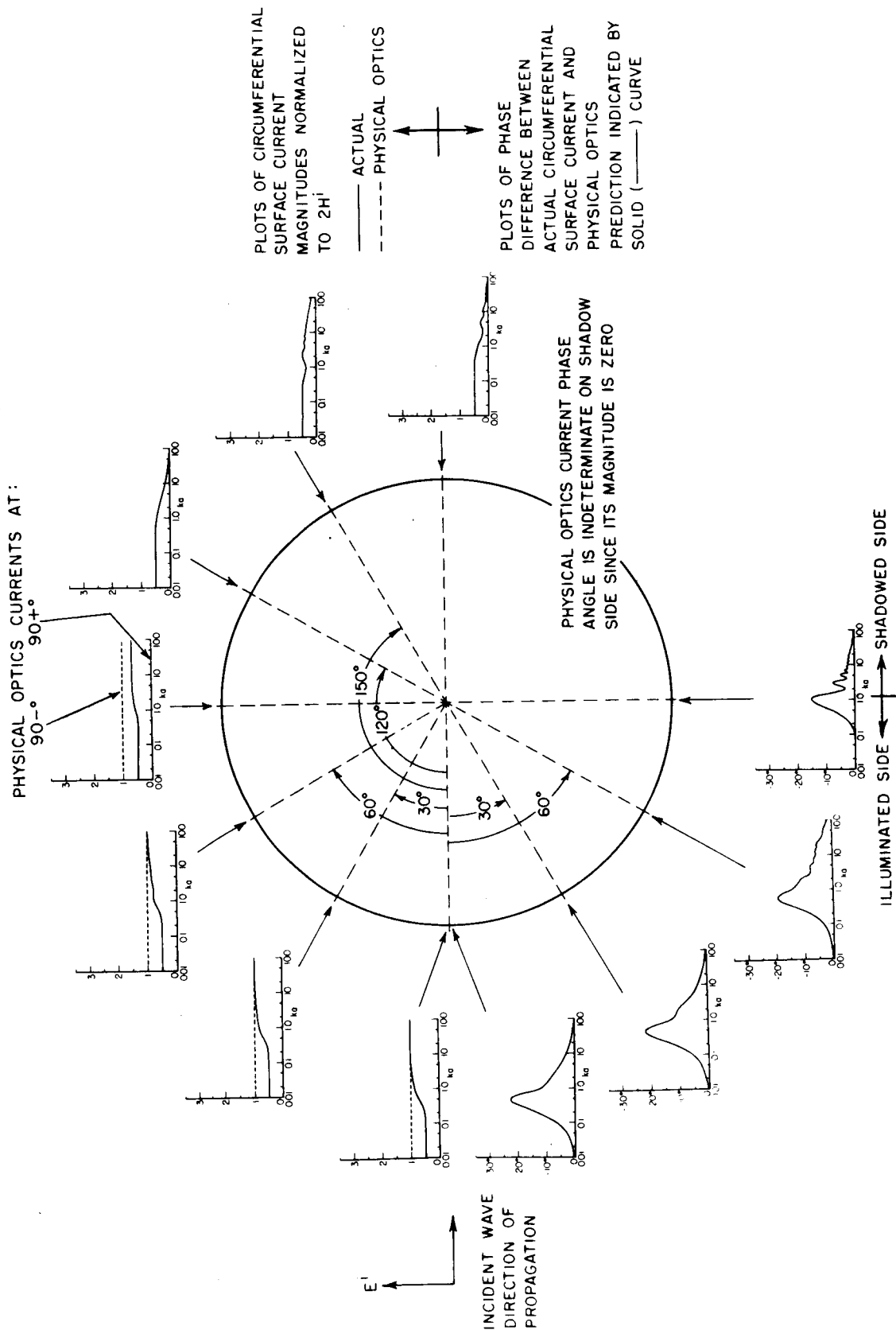


Fig. 3--Circumferential surface currents on a perfectly conducting circular cylinder.

error for the axial current is 13° while that for the circumferential current is 7° . Thus it can be seen that the greater the incidence angle away from normal, the smaller must be the wavelength in order to obtain the same degree of accuracy. It can be stated with fair certainty that if the radius of curvature at all points on the surface is no less than three times the wavelength, the physical optics approximation is quite good for incidence angles of less than 75° , producing about 10% error at 75° . One need not be unduly concerned about the failure of physical optics at quite large local incidence angles; these areas have less effect on the scattered field than the portions of the surface at which the waves are nearly normally incident. This can be seen from consideration of the principle of stationary phase when it is applied to reflection or radiation from a smooth surface containing curved regions.

It is instructive and more meaningful before passing to study the actual backscattered power from such a cylinder produced by the physical optics currents and compare it with the actual backscattered power. These results for both directions of polarization are shown in Fig. 4. Again, the effect of the physical optics approximation is negligible for $a > \lambda$ (i. e., $ka > 2\pi$), and at $a = \lambda$, the error is about 5% at the most. However, for $a < \lambda$, it can be seen that the true and physical optics backscattered fields differ significantly and even

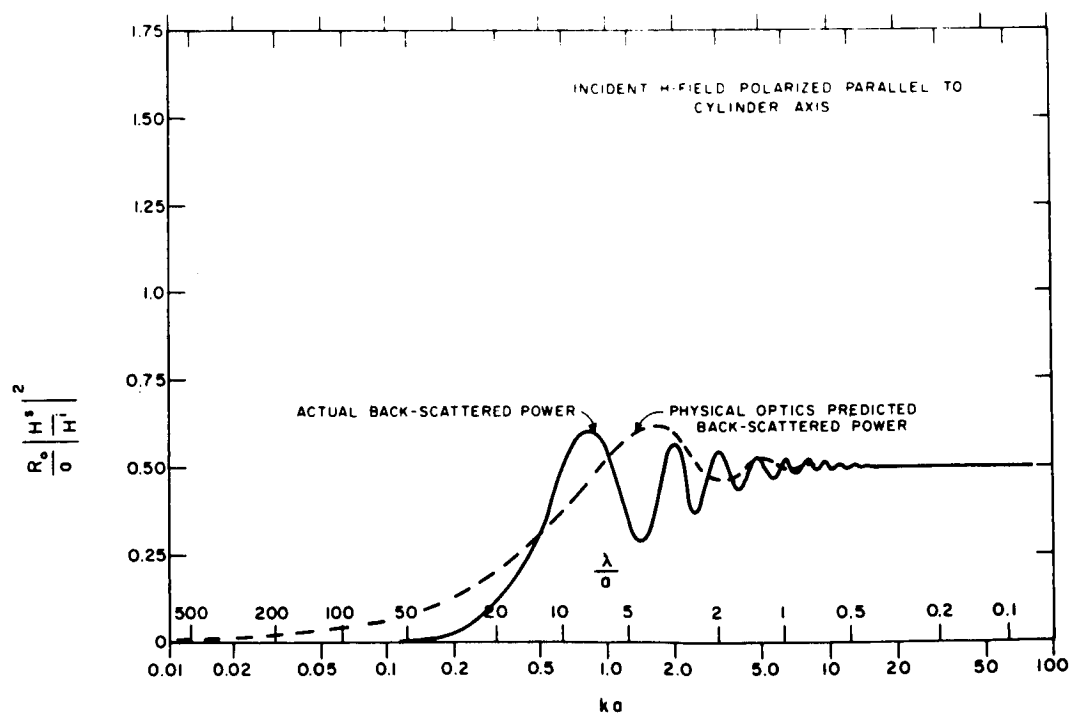
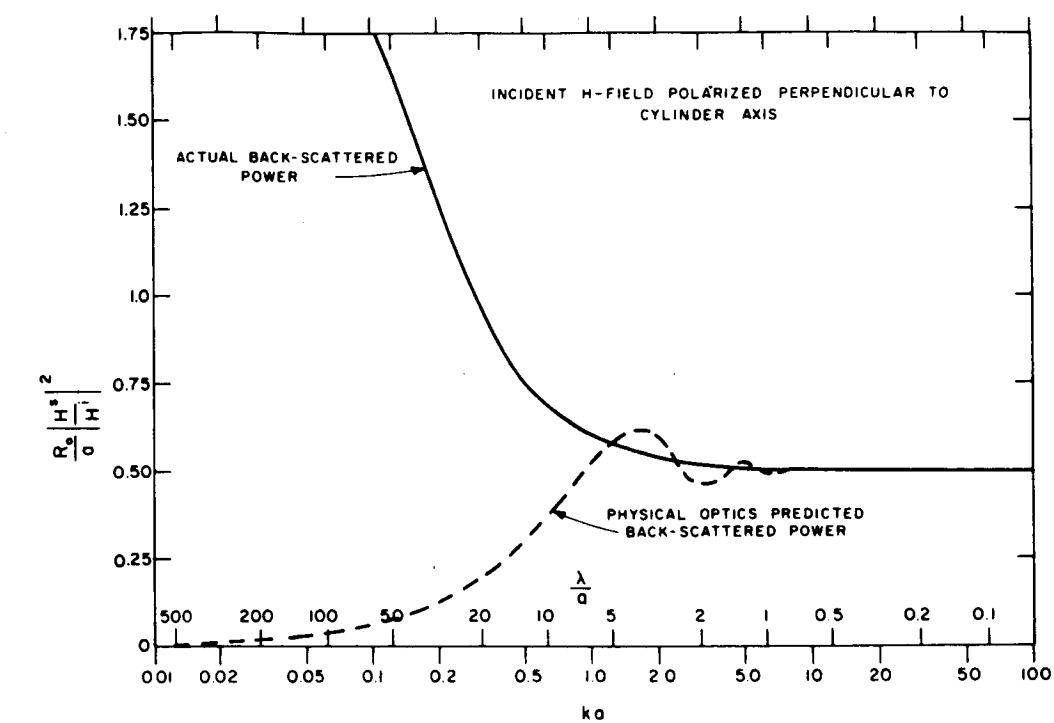


Fig. 4--Backscattered power from a perfectly conducting circular cylinder.

diverge radically for the case where the H field is polarized perpendicular to the cylinder axis, the direction in which the radius of curvature is infinite.

When one uses the physical optics approximation as defined in this section, he necessarily commits himself to at least three other approximations. They are as follows:

- (a) There are no sharp points or edges on the surface. This is implied when one assumes physical optics, since a sharp point or an edge may be considered a case where the surface has zero radius of curvature; such points must necessarily be excluded if the wavelength is to be everywhere less than the radius of curvature.
- (b) There is no shadowing of one part of the surface by another part. As seen previously, physical optics predicts zero surface current density in such areas since there is no directly incident field. Such is the case only as wavelength becomes very small, as seen from looking at the current density on the shadowed side of the cylinder when the ratio of wavelength to radius becomes very small.
- (c) There are no multiple reflections of waves between various points on the surface. Again, since physical optics says that the reflected field at every point is proportional to the incident field at that point only (and not to the reflected

field at another point of the surface), there is no provision in the definition for multiple scattering. Certainly the effect of multiple scattering will be greatest for highly reflecting surfaces (i. e., perfect conductors, perfect dielectrics with a high permittivity). Both multiple reflecting and shadowing become more pronounced as angle of incidence increases and as local surface slopes become quite large.

Throughout this work, the physical optics approximation as defined in this section will be used. It must be understood therefore that when the scattering characteristics for a surface are theoretically predicted after using the physical optics approximation, and when those predicted results differ significantly from measured results, one must seriously question whether the surface analyzed belongs to that class of surfaces which can be analyzed using physical optics; stated another way, it can be said that if results predicted by using physical optics significantly fail to agree with measured data, it is quite probable that there are many points on the surface which have a radius of curvature much smaller than wavelength.

5. Further simplification for a perfectly conducting surface

Assuming that the physical optics approximation has been employed, the scattered magnetic field at P' is obtained by substituting (2.13) into (2.12b):

$$(2.15) \quad \bar{H}(P') = - \frac{jk e^{jkR_o}}{2\pi R_o} \hat{\eta} \times \int_{S_1} (\hat{n} \times \bar{H}^i) e^{-jk\hat{\eta} \cdot \bar{r}} da$$

Here, $\hat{\eta}$ is removed from the integrand since it is nearly constant in direction over the surface, in accordance with the far field approximations. If one specializes to rectangular coordinates at this point, the rough random surface can be written as a function of x and y as follows:

$$\zeta \equiv \zeta(x, y) \equiv \text{height of surface above } z = 0 \text{ plane.}$$

In this coordinate system, the normal to the surface, $\hat{n}$ and an element of area on the surface have the forms

$$\hat{n} = \frac{-\frac{\partial \zeta}{\partial x} \hat{x} - \frac{\partial \zeta}{\partial y} \hat{y} + \hat{z}}{\sqrt{\left(\frac{\partial \zeta}{\partial x}\right)^2 + \left(\frac{\partial \zeta}{\partial y}\right)^2 + 1}},$$

$$da = dx dy \sqrt{\left(\frac{\partial \zeta}{\partial x}\right)^2 + \left(\frac{\partial \zeta}{\partial y}\right)^2 + 1}.$$

Therefore (2.15) can be written

$$(2.16) \quad \bar{H}(P') = - \frac{jk e^{jkR_o}}{2\pi R_o} \hat{\eta} \times \iint_{S_1} \left[-\frac{\partial \zeta}{\partial x} \hat{x} \times \bar{H}^i - \frac{\partial \zeta}{\partial y} \hat{y} \times \bar{H}^i + \hat{z} \times \bar{H}^i \right] e^{-jk\hat{\eta} \cdot \bar{r}} dx dy$$

This expression is exact insofar as physical optics is applicable to the surface under consideration. However, the appearance of the terms involving surface slopes causes considerable complication; since

surface height, ζ , is a random variable, its derivatives, or the x and y slopes, are also random variables. The solution of (2.16) in the form above is attempted by Hoffman[8] and Fung[9] by application of the Karhunen-Loeve theorem to expand ζ in a series of orthogonal functions. Hoffman more or less outlines the solution without showing or plotting any results, while Fung goes further and reduces the solution for power to a quite complicated series. Hagfors[16] attributes Fung's complex form for the solution to a mathematical mistake; at any rate, it is extremely difficult to see from Fung's results any simple relationship between the surface parameters and the average scattered power.

The majority who investigate the problem of scattering from rough surfaces simply choose to drop the first two terms of the integrand, evidently to eliminate excess complications. This approximation is quite costly as far as maintaining the validity of the results; at small angles of incidence and scattering away from normal, the error introduced by this approximation is a minimum, since $\hat{\eta}$ is close to $\hat{z}$ and $\bar{H}^i$ consists almost entirely of x and y components. With this simplification, the integral becomes

$$(2.17) \quad \bar{H}(P') = -\frac{jk e^{jkR_o}}{2\pi R_o} \hat{\eta} \times \left[\hat{z} \times \int \int_{S_1} \bar{H}^i e^{-jk \hat{\eta} \cdot \bar{r}} dx dy \right].$$

If, however, the surface slopes can be significant (e. g. , as high as 20°), and if the angles of incidence and scattering are greater than about 20° , then (2. 17) may possibly deviate more than 50% from the correct values; this conclusion is confirmed by comparison of the solutions of (2. 17) with measured data from the moon for angles of incidence greater than 20° .

Equation (2. 17) can also be obtained in scalar form if one begins with the Helmholtz equation, (2. 3), where $\mu_k(P') = H_k(P')$; one assumes for a perfect conductor that $\frac{\partial H_k}{\partial n} \sim 0$ at the surface (which is a poor approximation both for large incidence angles away from normal and for surface slopes appreciably different from zero). Then if one approximates $\frac{\partial \phi}{\partial n}$ as $\frac{\partial \phi}{\partial n} = \nabla \phi \cdot \hat{n} = \left(\frac{1}{R} - jk\right) \phi_{\hat{\eta}} \cdot \hat{n} \simeq -\frac{jk e^{jkR}}{R_0} \hat{\eta} \cdot \hat{z}$, in the far field limit, and if one lets the total H field H_k at the surface be equal to twice the incident field, H_k^i , and if one approximates the surface element as $da \simeq dx dy$, then one has

$$(2. 18) \quad H_k(P') = \frac{jk e^{jkR_0}}{2\pi R_0} \hat{\eta} \cdot \hat{z} \iint_{S_1} H_k^i e^{-jk \hat{\eta} \cdot \vec{r}} dx dy .$$

This, for example, is the form of the scattered field that Hagfors[2] uses in his analysis of the lunar surface.

This author believes that (2. 17) and (2. 18) will lead to results which are questionable at any other than normal incidence and for surfaces having other than very small slopes.

Stated another way, the Helmholtz form (or Huygen's principle), as expressed in (2.17) and (2.18), can predict quite meaningful results for radiation and diffraction patterns of smooth, deterministic objects or antennas, since the field of interest occurs near normal incidence (e.g., the entire diffraction pattern from a flat plate lies within a couple of degrees from the normal to the plate); however, these same Helmholtz equations become meaningless for application to problems involving very rough random surfaces at scattering angles several degrees away from the mean normal to the surface, $\hat{z}$. For backscattering from a perfect conducting rough surface, (2.16) will be used in this work; the results are not unduly complicated when one specializes immediately to the case of backscattering. These results will then be compared with the results obtained from the use of (2.17) or to the interpretation of experimentally obtained results from the lunar surface.

6. The assumption of a given statistical model

As will be shown later, the scattered power from a rough surface can be related to the characteristic function of the joint probability density function of the random surface height; this characteristic function, theoretically obtainable by measurements, contains all the statistical information about the surface that is normally desired (e.g., mean square surface height, surface height correlation function, higher

moments of surface height, mean square surface slope, etc.). However, employing this more general form for the scattered power, it is not possible to see or predict how the scattered power will vary with the various statistical parameters of the surface, such as mean square surface height, surface slope, etc. For this reason, it is often considered both necessary and convenient to assume that the surface height obeys a given probability distribution, use the corresponding characteristic function where necessary, and thus relate the scattered power to the statistical parameters of the distribution assumed; proceeding in this fashion, one is usually forced to choose a definite form for the surface height correlation function also. The surface height joint probability density function normally chosen is the Gaussian, both because of its simplicity of form and because of its frequent application to physical problems. The surface height correlation functions usually chosen are either the simple exponential or the Gaussian (i. e., either $e^{-|\rho|/a}$ or $e^{-\rho^2/a^2}$, where ρ is the separation between two points on the surface, and a is the correlation length). Obviously, when one chooses such form for the probability density function and correlation function one is making an assumption about the very surface whose statistical properties he does not know and is trying to determine.

Many in the literature employ the Gaussian form of the JPDF (joint probability density function) because of its convergence at large argument, because of the simplicity of the form of its Fourier transform (i. e. , its characteristic function), and not least, because of its frequent occurrence in many natural processes upon application of the central limit theorem of statistics. However, the use of the Gaussian JPDF is likewise criticized in the literature because of poor agreement between measured and predicted curves (e. g. , Hughes[11] , and others). The form of the surface correlation function is likewise discussed and various correlation function models are criticized (Hagfors[10]); the most frequency models employed are the Gaussian and the exponential correlation function.

It is undoubtedly true that a poor choice of such a statistical model for the surface JPDF or correlation function will result in less than perfect agreement with measured backscattered power from the lunar surface. However, this author will show that the choice of a given form for a surface statistical model will not give predictions for backscattered which are in serious error. To do this, two different forms for the surface JPDF and two forms for the correlation function will be chosen; it will be seen that the differences in predicted backscattered power will not differ by more than 6 db.

Thus the assumption of a given surface statistical model cannot be entirely responsible for the extremely poor agreement of predicted power with observed backscattered power at angles greater than 30° ; thus far predicted values have been low by as much as 50 db at back-scattering angles of more than 45° . The reason again is due to the assumption discussed in subsection 5 of this section, viz., the improper use of the Helmholtz or Huygen's integral for the scattered field.

It is also shown later that the choice of the exponential correlation function leads to contradictions in the prediction of the statistical parameters of the surface. These points are all discussed in Chapter III where statistical models are used.

The approximations discussed in this section represent basic steps regularly employed in the solution of the problem of predicting backscatter from rough surfaces using the integral equation formulation as a starting point. There are several mathematical assumptions made later in the analysis; these assumptions however appear to be minor and intrinsic to the class of surfaces under consideration, and therefore are generally accepted without undue controversy. They will be mentioned and discussed as they are encountered. There are several additional approximations needed to solve the problem of scattering from a non-perfectly conducting surface, and they will be discussed under that section.

B. Backscattering from a Perfectly Conducting Rough Surface

In this section two solutions for scattered power from a rough surface will be derived; the mean configuration for the rough surface is a plane, taken to be the $z = 0$ plane in the derivation of the first solution (see Fig. 1). For convenience, a rectangular surface will be chosen with dimensions $L_x \times L_y$, and the coordinate origin is located at the middle of the surface. Only that class of surfaces will be analyzed here whose roughness is isotropic (i. e. , there is no preferred direction to the roughness). The class of surfaces to be considered is restricted to "very rough" surfaces, that is, surfaces whose rms height is much larger than wavelength; also, since the physical optics approximation is to be used throughout this work, the radii of curvature at every point of the surface are much larger than wavelength.

In obtaining the first solution, (2. 17) will be used as the starting point; this equation was arrived at after approximating the normal vector to the surface by z and omitting terms containing surface slope. The solution obtained in this manner is not new. It was found by Isakovich[12] , repeated by Beckmann and Spizzichino[1] and others. It is repeated here merely for comparison with the second solution. It was derived first for the more general case of bistatic scattering, but may be specialized to backscattering at any point.

The second solution is obtained by using the exact equation (2.16) as the starting point; it is restricted immediately to backscattering and is valid only for backscattering. By orienting the surface properly initially (i. e., by a transformation of coordinates), the terms involving the surface slopes can be made to vanish and a simple closed form solution of (2.16) is found without employing the approximation used to obtain (2.17).

1. Solution of (2.17)

As shown in Fig. 1, let $\hat{\xi}$ be the unit vector pointing in the direction of incidence and $\hat{\eta}$ the unit vector pointing in the scattering direction of interest. Therefore, the incident field at the surface is given by:

$$\vec{H}^i = \vec{H}_0^i e^{jk \hat{\xi} \cdot \vec{r}},$$

where $\vec{r} = x \hat{x} + y \hat{y} + \zeta(x, y) \hat{z}$ at the surface.

Therefore, (3.27) becomes:

$$\vec{H}(P') = -\frac{jk e^{jk R_0}}{2\pi R_0} \hat{\eta} \times (\hat{z} \times \vec{H}_0^i) \iint_D e^{jk(\hat{\xi} - \hat{\eta}) \cdot \vec{r}} dx dy;$$

$$D = \begin{cases} -L_x/2 < x < +L_x/2 \\ -L_y/2 < y < L_y/2 \end{cases}.$$

Define: $\bar{\gamma} \equiv \hat{\xi} - \hat{\eta}$,

$$\therefore \bar{H}(P') = - \frac{jk e^{jkR_0}}{2\pi R_0} \hat{\eta} \times (\hat{z} \times \bar{H}_0^i)$$

$$\iint_D e^{jk \gamma_x x + jk \gamma_y y + jk \gamma_z \zeta(x, y)} dx dy .$$

The polarization of the incident wave is not specified, but is left in general form. It may be either horizontal or vertical or any combination of the two. But it is evident that once the direction of polarization of $\bar{H}_0^i$ is chosen, the direction of polarization of the scattered field at P' is fixed and is not a function of the actual roughness of the surface. Therefore, let H^s denote the component of the scattered field along the polarization direction produced by $\bar{H}_0^i$. Then

$$H^s = - \frac{jk e^{jk R_0} \bar{H}_0^i}{2\pi R_0} \iint_D e^{jk \gamma_x x + jk \gamma_y y + jk \gamma_z \zeta} dx dy .$$

To be more general, consider the product of the scattered fields produced at two different frequencies whose wave numbers are

$$k_1 = \omega_1 \sqrt{\mu_0 \epsilon_0} \text{ and } k_2 = \omega_2 \sqrt{\mu_0 \epsilon_0} .$$

$$(2.19) \quad \therefore H_1^s H_2^{s*} = \frac{k_1 k_2 e^{j(k_1 - k_2) R_0} H_{01}^i H_{02}^{i*}}{4\pi^2 R_0^2}$$

$$\iiint_D e^{jk_1 \gamma_x x - jk_2 \gamma_x x' + jk_1 \gamma_y y - jk_2 \gamma_y y' + jk_1 \gamma_z \zeta(x, y) - jk_2 \gamma_z \zeta'(x', y')} dx dy dx' dy' .$$

When $k_1 = k_2$, the above equation merely reduces to the power scattered from the surface at point P' . Taking the average of (2.19) with respect to the random variables ζ and ζ' , one has

$$\langle H_1^s H_2^{s*} \rangle = \frac{k_1 k_2 e^{j(k_1 - k_2) R_0} H_{01}^i H_{02}^{i*}}{4\pi^2 R_0^2}$$

$$\iiint_D e^{jk_1 \gamma_x x - jk_2 \gamma_x x' + jk_1 \gamma_y y - jk_2 \gamma_y y'} M_{\zeta \zeta'}(jk_1 \gamma_z, -jk_2 \gamma_z; \rho) dx dy dx' dy' ,$$

where $M_{\zeta \zeta'}(jk_1 \gamma_z, -jk_2 \gamma_z; \rho) \equiv \langle e^{jk_1 \gamma_z \zeta - jk_2 \gamma_z \zeta'} \rangle \equiv$ joint characteristic function of the random variables ζ and ζ' .*

It should be noted that the joint characteristic function is in general a function of the surface autocorrelation function, which in turn is a function of the separation, ρ , between the two surface points at x, y and x', y' for an isotropic surface (i. e., $\rho = \sqrt{(x-x')^2 + (y-y')^2}$).

* The brackets $\langle \rangle$ denote ensemble averages over an ensemble of rough surfaces. The joint characteristic function is formally defined and discussed in the first subsection of the following section.

Define: $\Delta k \equiv k_1 - k_2$,

$$\tau_x = x - x' ,$$

$$\tau_y = y - y' .$$

$$\therefore \langle H_1^s H_2^{s*} \rangle = \frac{k_1 k_2 e^{j\Delta k R_0} H_{01}^i H_{02}^{i*}}{4\pi^2 R_0^2}$$

$$\int_{-L_x/2}^{L_x/2} \int_{-L_y/2}^{L_y/2} \left\{ \int_{-L_x/2-x'}^{L_x/2-x'} \int_{-L_y/2-y'}^{L_y/2-y'} e^{jk_1 \gamma_x \tau_x + jk_1 \gamma_y \tau_y} \right. \\ \left. M_{\zeta\zeta'}(jk_1 \gamma_z - jk_2 \gamma_z; \rho) d\tau_x d\tau_y \right\} e^{j\Delta k \gamma_x x' + j\Delta k \gamma_y y'} dx' dy' .$$

The true covariance of the two scattered fields is given as

$$\text{Cov}[H_1^s H_2^{s*}] = \langle H_1^s H_2^{s*} \rangle - \langle H_1^s \rangle \langle H_2^{s*} \rangle ;$$

however, the average fields $\langle H_1^s \rangle$ and $\langle H_2^{s*} \rangle$ from a very rough surface are always zero. Therefore, $\text{Cov}[H_1^s H_2^{s*}] = \langle H_1^s H_2^{s*} \rangle$.

However, for the sake of completeness, the average scattered field product can be included to give for the covariance

$$(2.20) \quad \text{Cov}[H_1^s H_2^{s*}] = \frac{k_1 k_2 e^{j\Delta k R_0} H_{01}^i H_{02}^{i*}}{4\pi^2 R_0^2} \int_{-L_x/2}^{L_x/2} \int_{-L_y/2}^{L_y/2} \\ \left\{ \int_{-L_x/2-x'}^{L_x/2-x'} \int_{-L_y/2-y'}^{L_y/2-y'} e^{jk_1 \gamma_x \tau_x + jk_1 \gamma_y \tau_y} [M_{\zeta\zeta'}(jk_1 \gamma_z, -jk_2 \gamma_z; \rho) - \right. \\ \left. -M_{\zeta\zeta'}(jk_1 \gamma_z, 0, \infty) M_{\zeta\zeta'}(0, -jk_2 \gamma_z; \infty)] d\tau_x d\tau_y \right\} e^{j\Delta k \gamma_x x' + j\Delta k \gamma_y y'} dx' dy'$$

where

$$M_{\zeta\zeta'}(jk_1\gamma_z, 0, \infty) \equiv M_{\zeta}(jk_1\gamma_z) \equiv \langle e^{jk_1\gamma_z\zeta} \rangle$$

$$M_{\zeta\zeta'}(0, -jk_2\gamma_z; \infty) \equiv M_{\zeta'}(-jk_2\gamma_z) \equiv \langle e^{-jk_2\gamma_z\zeta'} \rangle$$

It is assumed that the dimensions of the surface (L_x and L_y) are much larger than either the surface roughness or the surface correlation length. Therefore, as $\rho = \sqrt{\tau_x^2 + \tau_y^2}$ becomes large, but is still much less than L_x or L_y , the random variables ζ and ζ' representing the surface heights at two points separated by ρ become uncorrelated; the surface correlation function reduces to zero as ρ becomes large, and

$$\lim_{\rho \rightarrow \infty} M_{\zeta\zeta'}(jk_1\gamma_z, -jk_2\gamma_z; \rho) \rightarrow M_{\zeta\zeta'}(jk_1\gamma_z, 0; \infty) \\ M_{\zeta\zeta'}(0, -jk_2\gamma_z; \infty),$$

and the integrand of (2.20) vanishes. In this case the limits on the integral in τ_x and τ_y can be replaced by $\pm \infty$, since the distance, ρ , over which the integrand is different from zero is much less than L_x or L_y . Then the integration over x' and y' can be performed to yield

$$\text{Cov}[H_1^s H_2^{s*}] = \frac{k_1 k_2 e^{j\Delta k R_0} L_x L_y H_{01}^i H_{02}^{i*}}{4\pi^2 R_0^2}$$

$$\left(\frac{\sin\left(\Delta k \gamma_x \frac{L_x}{2}\right)}{\Delta k \gamma_x \frac{L_x}{2}} \right) \left(\frac{\sin\left(\Delta k \gamma_y \frac{L_y}{2}\right)}{\Delta k \gamma_y \frac{L_y}{2}} \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{jk_1 \gamma_x \tau_x + jk_1 \gamma_y \tau_y}$$

$$[M_{\zeta\zeta'}(jk_1 \gamma_z, -jk_2 \gamma_z; \rho) - M_{\zeta\zeta'}(jk_1 \gamma_z, 0; \infty) M_{\zeta\zeta'}(0, -jk_2 \gamma_z; \infty)] d\tau_x d\tau_y$$

Now, make the change of variable $\tau_x = \rho \cos \alpha$, $\tau_y = \rho \sin \alpha$; since the expression in brackets is independent of α , the integration over α can be performed.

$$\text{Cov}[H_1^s H_2^{s*}] = \frac{k_1 k_2 e^{j\Delta k R_0} L_x L_y H_{01}^i H_{02}^{i*}}{4\pi^2 R_0^2}$$

$$\left(\frac{\sin\left(\Delta k \gamma_x \frac{L_x}{2}\right)}{\Delta k \gamma_x \frac{L_x}{2}} \right) \left(\frac{\sin\left(\Delta k \gamma_y \frac{L_y}{2}\right)}{\Delta k \gamma_y \frac{L_y}{2}} \right) \int_0^{\infty} \int_0^{2\pi} e^{jk_1 \rho (\gamma_x \cos \alpha + \gamma_y \sin \alpha)} d\alpha d\rho$$

$$[M_{\zeta\zeta'}(jk_1 \gamma_z, -jk_2 \gamma_z; \rho) - M_{\zeta\zeta'}(jk_1 \gamma_z, 0; \infty) M_{\zeta\zeta'}(0, -jk_2 \gamma_z; \infty)] d\rho$$

$$(2.21) \quad \therefore \text{Cov}[H_1^s H_2^{s*}] = \frac{k_1 k_2 e^{j\Delta k R_0} L_x L_y H_{01}^i H_{02}^{i*}}{2\pi R_0^2}$$

$$\left(\frac{\sin\left(\Delta k \gamma_x \frac{L_x}{2}\right)}{\Delta k \gamma_x \frac{L_x}{2}} \right) \left(\frac{\sin\left(\Delta k \gamma_y \frac{L_y}{2}\right)}{\Delta k \gamma_y \frac{L_y}{2}} \right) \int_0^{\infty} \rho J_0(k_1 \sqrt{\gamma_x^2 + \gamma_y^2} \rho) d\rho$$

$$[M_{\zeta\zeta'}(jk_1 \gamma_z, -jk_2 \gamma_z; \rho) - M_{\zeta\zeta'}(jk_1 \gamma_z, 0; \rho) M_{\zeta\zeta'}(0, -jk_2 \gamma_z; \rho)] d\rho$$

In the above expression one finds the familiar diffraction factors,

$$\frac{\sin\left(\Delta k \gamma_x \frac{L_x}{2}\right)}{\Delta k \gamma_x \frac{L_x}{2}}, \text{ which occur because of the frequency difference and}$$

the tilt of the surface; these factors become unity for specular reflection ($\gamma_x = \gamma_y = 0$) and in the case where the frequency difference is zero ($\Delta k = 0$).

At any time, the second term of (2.21) may be dropped since it represents the average of the scattered field and is zero for a very rough surface. If $k_1 = k_2$, ($\Delta k \rightarrow 0$), then (2.21) represents the variance of the scattered field and is the same as the average scattered power, and it simplifies to

$$(2.22) \quad \text{Var}[H_1^s H_1^{s*}] = \frac{k^2 L_x L_y |H_0^i|^2}{2\pi R_0^2} \int_0^\infty \rho J_0(k\sqrt{\gamma_x^2 + \gamma_y^2} \rho) \\ [M_{\zeta\zeta'}(jk\gamma_z, -jk\gamma_z; \rho) - M_{\zeta\zeta'}(jk\gamma_z, 0; \rho) M_{\zeta\zeta'}(0, -jk\gamma_z; \rho)] d\rho.$$

In the case of backscattering where θ is the angle of incidence with respect to the z axis,

$$\gamma_z = 2 \cos \theta \\ \sqrt{\gamma_x^2 + \gamma_y^2} = 2 \sin \theta.$$

Then for backscattering, (2.22) becomes

$$(2.23) \quad \therefore \text{Var}[H_1^s H_1^{s*}] = \frac{k^2 L_x L_y |H_0^i|^2}{2\pi R_0^2} \int_0^\infty \rho J_0(2k \sin \theta \rho) [M_{\zeta\zeta_1}(j2k \cos \theta, -j2k \cos \theta; \rho) - M_{\zeta\zeta_1}(j2k \cos \theta, 0; \rho) M_{\zeta\zeta_1}(0, -j2k \cos \theta; \rho)] d\rho.$$

One can see that (2.21), (2.22), and (2.23) cannot be simplified further until one assumes a definite form for the characteristic functions. They will be left in this form until the next chapter, where they will be discussed further and the integral will be evaluated for various characteristic functions and surface autocorrelation functions.

2. Solution of (2.16)

If one searches for a transformation which will cause the terms in (2.16) containing the surface slopes to become zero, one finds that for the case of backscattering, if he chooses his field point P' such that it lies on the z axis, those terms involving the slopes become zero. (See Fig. 5.) For convenience, allow the mean surface plane to be rotated about the x_1 axis by an angle θ so that the mean angle of incidence is θ . (In the more general case, the surface may be rotated about both axes by arbitrary angles; the basic form of the answer is still unchanged, however.) The coordinate origin is still located at the center of the surface.

$$\begin{aligned} \text{In this case, } \hat{\eta} = z_1, \text{ and } \bar{H}^i &= (H_x^i \hat{x}_1 + H_y^i \hat{y}_1) e^{-jkz_1} \\ &= \bar{H}_0^i e^{-jkz_1}. \end{aligned}$$

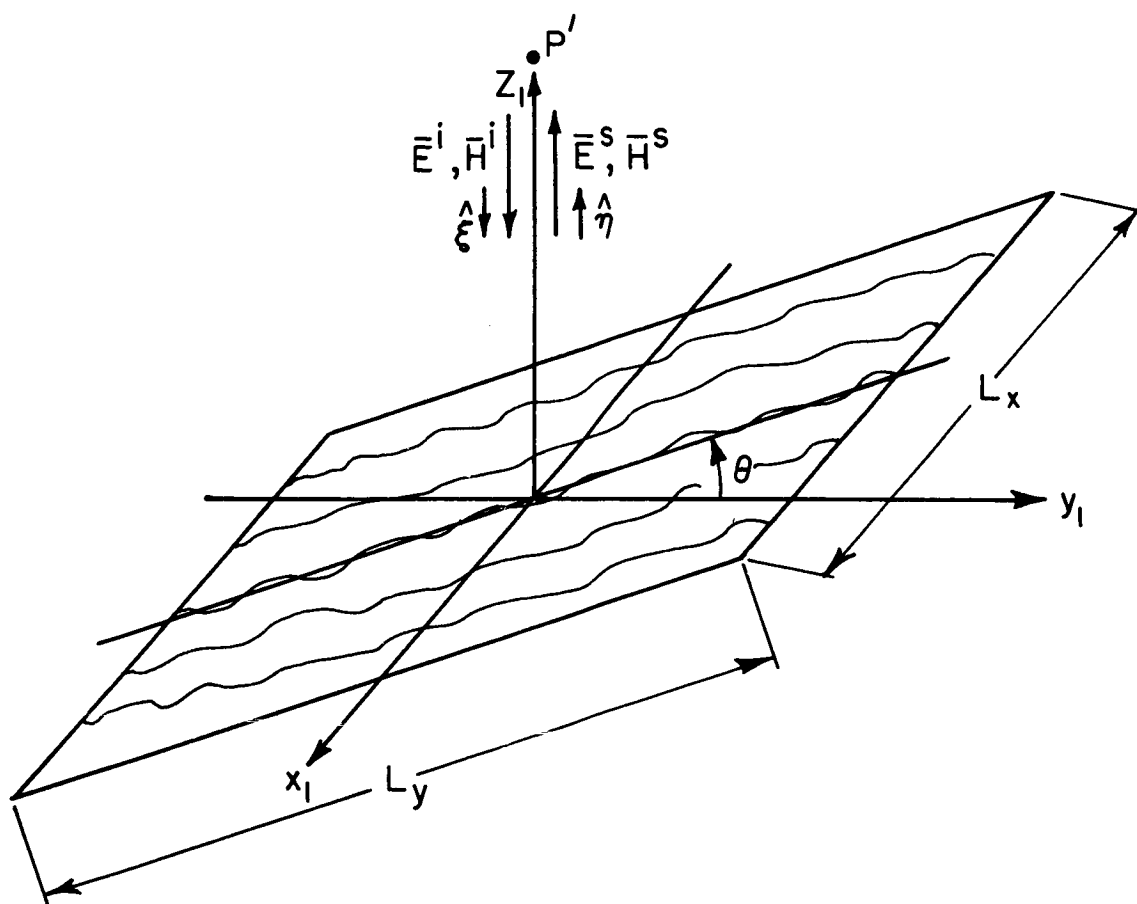


Fig. 5. Planar scattering surface tilted about x axis.

The integrand of (2.16) therefore becomes

$$\begin{aligned} \hat{\eta} \times \left[-\frac{\partial \zeta_1}{\partial x_1} \hat{x}_1 \times \bar{H}^i - \frac{\partial \zeta_1}{\partial y_1} \hat{y}_1 \times \bar{H}_1^i + \hat{z}_1 \times \bar{H}^i \right] \\ = -(H_{x_1}^i \hat{x}_1 + H_{y_1}^i \hat{y}_1) e^{-jk\zeta_1}, \end{aligned}$$

at the surface. Therefore, the integral becomes

$$(2.24) \quad \overline{H}(P') = \frac{jk e^{jkR_0}}{2\pi R_0} \overline{H}_0^i \iint_{D_1} e^{-j2k\zeta_1} dx_1 dy_1 ,$$

$$\text{where } D_1 = \begin{cases} -L_x/2 < x_1 < L_x/2 \\ -L_y \cos \theta/2 < y_1 < L_y \cos \theta/2 \end{cases} .$$

It should be noted that the polarization of the incident wave is unchanged after scattering by the perfect conducting rough surface. Such was also the case in the solution of (2.17), as seen previously, for the case of backscattering. Therefore, one can conclude that when one uses the physical optics approximation to solve for backscattering from any perfectly conducting surface, there will not be any depolarization of the incident wave. Conversely, one can say that if depolarization is observed in measuring the backscattered return from any surface, either the surface scale structure and radii of curvature are such that the physical optics approximation is invalid or the surface is not a perfect conductor. It will be seen later that depolarization does take place when the surface is not a perfect conductor; this depolarization is dependent upon the surface features and roughness.

Now consider that the surface orientation of Fig. 5 was obtained by rotating the surface of Fig. 1 by angle θ about the x_1 axis. Then the height of the surface of Fig. 5 (call it $\zeta_1(x_1, y_1)$) can be written in terms of the height of the surface of Fig. 1 (call it $\zeta(x, y)$) as follows:

$$(2.25a) \quad y_1 = y \cos \theta - \zeta(x, y) \sin \theta$$

$$(2.25b) \quad \zeta_1(x_1, y_1) = y \sin \theta + \zeta(x, y) \cos \theta .$$

This is merely the transformation for the point on the surface y, ζ after a rotation of the coordinate system about the x axis by the angle θ to the new position y_1, ζ_1 . Upon solving (2.25a) for y and substituting into (2.25b), one has

$$\zeta_1(x_1, y_1) = \frac{\sin \theta}{\cos \theta} y_1 + \frac{\sin^2 \theta}{\cos \theta} \zeta(x, y) + \cos \theta \zeta(x, y)$$

$$\therefore \zeta_1(x, y) = \frac{\sin \theta}{\cos \theta} y_1 + \cos \theta \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta} \right) \zeta(x, y) .$$

$$\text{Define: } m \equiv \frac{\sin \theta}{\cos \theta} = \tan \theta = \text{mean surface slope}$$

$$n = \cos \theta \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta} \right) = \cos \theta (1 + \tan^2 \theta) = \cos \theta \cdot \sec^2 \theta$$

$$= \sec \theta$$

$$\therefore \zeta_1(x_1, y_1) = m y_1 + n \zeta(x, y) .$$

It should be noted that it is actually $\zeta(x, y)$ that is the random variable as originally defined, and that $\zeta_1(x_1, y_1)$ is merely a new random variable defined in terms of the old one. Whereas the separation between two points in the original system was

$\rho = \sqrt{(x-x')^2 + (y-y')^2}$, the separation between two points in the new system is given by

$$\rho = \sqrt{(x_1 - x_1')^2 + \sec^2 \theta (y_1 - y_1')^2} = \sqrt{(x_1 - x_1')^2 + n^2 (y_1 - y_1')^2} .$$

Therefore, the product of the component of the scattered field in the direction of the incident polarization at frequency ω_1 and the component at frequency ω_2 is given by

$$H_1^s H_2^{s*} = \frac{k_1 k_2 e^{j(k_1 - k_2) R_0} H_{01}^i H_{02}^{i*}}{4\pi^2 R_0^2}$$

$$\iiint\limits_{D_1} e^{-j2k_1 \zeta_1 + j2k_2 \zeta_1'} dx_1 dy_1 dx_1' dy_1' = \frac{k_1 k_2 e^{j\Delta k R_0} H_{01}^i H_{02}^{i*}}{4\pi^2 R_0^2}$$

$$\iiint\limits_{D_1} e^{-j2k_1 m y_1 + j2k_2 m y_1' - j2k_1 n \zeta + j2k_2 n \zeta'} dx_1 dy_1 dx_1' dy_1' .$$

Averaging with respect to ζ and ζ' and making the change of variables $\tau_x = x_1 - x_1'$, $\tau_y = \sec \theta (y_1 - y_1') = n(y_1 - y_1')$, one obtains

$$\langle H_1^s H_2^{s*} \rangle = \frac{k_1 k_2 e^{j\Delta k R_0} H_{01}^i H_{02}^{i*}}{4\pi^2 R_0^2 n} \int_{-L_x/2}^{L_x/2} \int_{-L_y \cos \theta/2}^{L_y \cos \theta/2} e^{j2k_1 \frac{m}{n} \tau_y} \left\{ \int_{-L_x/2 - x_1'}^{L_x/2 - x_1'} \int_{-L_y/2 - ny_1'}^{L_y/2 - ny_1'} e^{j2k_1 \frac{m}{n} \tau_y} M_{\zeta \zeta'}(-j2k_1 n; +j2k_2 n; \rho) d\tau_x d\tau_y \right\} e^{j2\Delta k m y_1'} dx_1' dy_1' .$$

As before, the more general case of the covariance of the scattered fields will be considered. The product of the average scattered fields are added; they are zero in the case of a very rough surface. Also, the limits of the integrals in τ_x and τ_y can be allowed to approach infinity since it will be assumed that L_x and L_y are much larger than

the correlation length of the surface. Then the integration over x_1' and y_1' can be performed.*

$$\begin{aligned} \text{Cov}[H_1^s H_2^{s*}] &= \langle H_1^s H_2^{s*} \rangle - \langle H_1^s \rangle \langle H_2^{s*} \rangle \\ &= \frac{k_1 k_2 e^{j\Delta k R_0} H_{01}^i H_{02}^{i*} L_x L_y}{4\pi^2 R_0^2 n^2} \left(\frac{\sin(\Delta k m \cos \theta L_y)}{\Delta k m \cos \theta L_y} \right) \\ &\quad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \{ e^{j2k_1 \frac{m}{n} \tau_y} [M_{\zeta\zeta'}(j2k_1 n, -j2k_2 n; \rho) - M_{\zeta\zeta'}(j2k_1 n, 0, \infty) \\ &\quad M_{\zeta\zeta'}(0, -j2k_2 n; \infty)] \} d\tau_x d\tau_y \end{aligned}$$

Upon making the change of variable to ρ and α , where

$$\rho = \sqrt{\tau_x^2 + \tau_y^2} = \sqrt{(x-x')^2 + n^2(y-y')^2}, \text{ and noting that } m \cos \theta = m/n = \sin \theta, \text{ one has}$$

$$\begin{aligned} (2.26) \quad \text{Cov}[H_1^s H_2^{s*}] &= \frac{k_1 k_2 e^{j\Delta k R_0} L_x L_y H_{01}^i H_{02}^{i*}}{2\pi R_0^2 n^2} \left(\frac{\sin(\Delta k \sin \theta L_y)}{\Delta k \sin \theta L_y} \right) \\ &\quad \int_0^{\infty} \rho J_0(2k_1 \sin \theta \rho) [M_{\zeta\zeta'}(j2k_1 n, -j2k_2 n; \rho) - M_{\zeta\zeta'}(j2k_1 n, 0; \rho) \\ &\quad M_{\zeta\zeta'}(0, -j2k_2 n; \rho)] d\rho \end{aligned}$$

When the two frequencies are equal, (2.26) merely becomes the variance of the scattered field and is also the average scattered power when the surface is very rough.

* The method used here exactly parallels that of the preceding subsection; for the sake of brevity, many of the steps are omitted this second time.

$$(2.27) \quad \text{Var}[H_1^s H_1^{s*}] = \frac{k^2 L_x L_y |H_0^i|^2}{2\pi R_0^2 n^2} \int_0^\infty \rho J_0(2k \sin \theta \rho) \\ [M_{\zeta\zeta_1}(j2kn, -j2kn; \rho) - M_{\zeta\zeta_1}(j2kn, 0; \rho) M_{\zeta\zeta_1}(0, -j2kn; \rho)] d\rho.$$

When comparing the forms of (2.24) and the more precise equation (2.27), and remembering that $n = 1/\cos \theta$, one notes that (2.27) has a factor $\cos^2 \theta$ multiplying it whereas (2.24) does not; also, $\cos \theta$ in the characteristic functions of (2.24) is replaced by $1/\cos \theta$ in (2.27). These changes are relatively insignificant near normal incidence (i. e., for θ near zero), but at angles considerably different from normal, the two equations would appear to differ quite significantly. This in part explains the failure of (2.17) and hence (2.24) to accurately, or even approximately, predict back-scattered power at more than about 20° away from normal incidence.

C. Statistical Relationships and Averages

In this section, various statistical quantities of rough surfaces are defined and useful inter-relationships between these quantities are established. Theoretical methods for measuring these statistical functions are discussed.

1. Definitions: [14, 15]

Let $W(\zeta, \zeta')$ be the joint probability density function (JPDF) of two random variables, ζ and ζ' . Then an average of a function $f(\zeta, \zeta')$ is defined as follows:

$$\langle f(\zeta, \zeta') \rangle \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\zeta, \zeta') W(\zeta, \zeta') d\zeta d\zeta' .$$

One average of particular interest is the joint characteristic function of ζ and ζ' ; it is merely the Fourier transform of $W(\zeta, \zeta')$ and is defined as

$$(2.28) \quad M_{\zeta\zeta'}(ju, jv) = \langle e^{ju\zeta + jv\zeta'} \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(\zeta, \zeta') e^{ju\zeta + jv\zeta'} d\zeta d\zeta' .$$

It is assumed, of course, that the inverse transform exists (and is therefore unique) relating $W(\zeta, \zeta')$ to $M_{\zeta\zeta'}(ju, jv)$.

Since ζ and ζ' are random variables, there also exist single probability density functions, $W_s(\zeta)$ and $W_s(\zeta')$ (which need not be the same function). Averages involving either ζ or ζ' may then be defined alternatively in terms of the appropriate SPDF.

$$\langle g(\zeta) \rangle \equiv \int_{-\infty}^{\infty} g(\zeta) W_s(\zeta) d\zeta \quad ; \quad \langle h(\zeta') \rangle \equiv \int_{-\infty}^{\infty} h(\zeta') W_s(\zeta') d\zeta' .$$

Single characteristic functions of ζ and ζ' are defined accordingly;

$$M_{\zeta}(ju) \equiv \langle e^{ju\zeta} \rangle = \int_{-\infty}^{\infty} e^{ju\zeta} W_s(\zeta) d\zeta ; \quad M_{\zeta'}(jv) = \langle e^{jv\zeta'} \rangle = \int_{-\infty}^{\infty} e^{jv\zeta'} W_s(\zeta') d\zeta' .$$

Two other averages are of interest, namely the average of ζ (or ζ') and the average of ζ^2 (or ζ'^2).

$$\langle \zeta \rangle = \int_{-\infty}^{\infty} \zeta W_s(\zeta) d\zeta \quad , \quad \langle \zeta^2 \rangle = \int_{-\infty}^{\infty} \zeta^2 W_s(\zeta) d\zeta \equiv \sigma^2 .$$

In general, there can exist some correlation between ζ and ζ' ; this degree of correlation is contained in the correlation coefficient, R , which is defined as follows,*

$$\langle \zeta \zeta' \rangle = \iint_{-\infty}^{\infty} \zeta \zeta' W(\zeta, \zeta') d\zeta d\zeta' \equiv \sigma^2 R .$$

The correlation coefficient has the range $0 \leq |R| \leq 1$. The lower limit $R = 0$ means that ζ and ζ' are completely uncorrelated, and therefore independent, from each other; the upper limit means that ζ and ζ' are completely correlated such that $\zeta = \zeta'$ at all times.

It is true, though not heretofore stated, that $W(\zeta, \zeta')$, $M_{\zeta\zeta'}(ju, jv)$, and any average involving both ζ and ζ' e. g., $f(\zeta, \zeta')$, are dependent upon the degree of correlation between ζ and ζ' and therefore are implied functions of R .

There exists a relationship between the SPDF of ζ and ζ' and their JPDP; this relationship is stated here,

$$W_s(\zeta) = \lim_{R \rightarrow 0} \int_{-\infty}^{\infty} W(\zeta, \zeta') d\zeta' .$$

* It is assumed from this point on that ζ and ζ' obey the same SPDF. Therefore, $\langle \zeta \rangle = \langle \zeta' \rangle$, $\langle \zeta^2 \rangle = \langle \zeta'^2 \rangle = \sigma^2$, $M_{\zeta}(ju) = M_{\zeta'}(ju)$, etc.

The single characteristic function is therefore given by

$$M_{\zeta}(ju) = \int_{-\infty}^{\infty} W_s(\zeta) e^{ju\zeta} d\zeta = \lim_{R \rightarrow 0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(\zeta, \zeta') e^{ju\zeta + j0 \cdot \zeta'} d\zeta d\zeta' = \lim_{R \rightarrow 0} M_{\zeta\zeta'}(ju, 0) .$$

The above relationship was employed in the last section.

Using Fourier transform theory, the averages of ζ , ζ^2 , and $\zeta\zeta'$ can be expressed alternatively as

$$(2.29a) \quad \langle \zeta \rangle = \int_{-\infty}^{\infty} \zeta W_s(\zeta) d\zeta = \lim_{R \rightarrow 0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \zeta W(\zeta, \zeta') d\zeta d\zeta' = \lim_{R \rightarrow 0} \left[-j \frac{\partial M_{\zeta\zeta'}(ju, jv)}{\partial u} \right]_{u=v=0} = -j \frac{dM_{\zeta}(ju)}{du} \Big|_{u=0}$$

$$(2.29b) \quad \langle \zeta^2 \rangle = \int_{-\infty}^{\infty} \zeta^2 W_s(\zeta) d\zeta = \lim_{R \rightarrow 0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \zeta^2 W(\zeta, \zeta') d\zeta d\zeta' = \lim_{R \rightarrow 0} \left[-\frac{\partial^2 M_{\zeta\zeta'}(ju, jv)}{\partial u^2} \right]_{u=v=0} = -\frac{d^2 M_{\zeta}(ju)}{du^2} \Big|_{u=0} = \sigma^2$$

$$(2.29c) \quad \langle \zeta\zeta' \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \zeta\zeta' W(\zeta, \zeta') d\zeta d\zeta' = -\frac{\partial^2 M_{\zeta\zeta'}(ju, jv)}{\partial u \partial v} \Big|_{u=v=0} =$$

$$\sigma^2 R .$$

In this study, ζ and ζ' represent the surface heights at two different points (x, y) and (x', y') on a randomly rough surface. In this case ζ is a function of x and y and ζ' is a function of x' and y' . Also, physical reasoning demands that $W(\zeta, \zeta')$ must be symmetric on ζ and

ζ' since they are both points on the same surface, and by the same argument, $W_s(\zeta) = W_s(\zeta')$ for $\zeta = \zeta'$ at all values of ζ . The correlation coefficient for the surface heights, R , will therefore be a function only of the separation between the two points (x, y) and (x', y') , such that $R = R[\sqrt{(x-x')^2 + (y-y')^2}] = R(\rho)$, where $\rho \equiv \sqrt{(x-x')^2 + (y-y')^2}$; this is true for an isotropically rough surface.

It is known from spectral analysis that since the R is a function of the separation between the points (x, y) and (x', y') , then the surface covariance function, $\langle \zeta \zeta' \rangle = \sigma^2 R[(x-x'), (y-y')]$, is the Fourier transform of the "power density spectrum" of the surface, and conversely, such that*

$$(2.30) \quad \frac{1}{4} P(p, q) \equiv \sigma^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(\tau_x, \tau_y) e^{jp\tau_x + jq\tau_y} d\tau_x d\tau_y,$$

where $\tau_x = x - x'$, $\tau_y = y - y'$, and therefore $\rho = \sqrt{\tau_x^2 + \tau_y^2}$.

In the class of surfaces under consideration in which the surface roughness is isotropic, $R = R[\sqrt{\tau_x^2 + \tau_y^2}] = R(\rho)$, and therefore $P(p, q) = P[\sqrt{p^2 + q^2}]$. In this case of polar symmetry the Fourier transformation (called the polar Fourier transform) reduces to

$$(2.31a) \quad P(\xi) = 8\pi\sigma^2 \int_0^{\infty} \rho R(\rho) J_0(\xi\rho) d\rho,$$

* The factor $\frac{1}{4}$ is included in the definition in this case in keeping with the notation of similar analyses of Ott[16] and Rice[17].

where $\xi = \sqrt{p^2 + q^2}$, and

$$(2.31b) \quad R(\rho) = \frac{1}{8\pi} \int_0^{\infty} \xi P(\xi) J_0(\rho\xi) d\xi.$$

In the case of randomly rough surfaces, it is evident that $R(0) = 1$ and $R(\infty) = 0$, since these two cases represent the extremes of ζ and ζ' both at the same point and at infinitely separated points. Therefore it is physically reasonable that the polar Fourier transform pair (2.31) really exists. Then substitution of $\rho = 0$ in (2.31b) gives

$$(2.32) \quad \sigma^2 = \sigma^2 R(0) = \frac{1}{8\pi} \int_0^{\infty} \xi P(\xi) J_0(0\xi) d\xi, \text{ or}$$

$$\sigma^2 = \frac{1}{8\pi} \int_0^{\infty} \xi P(\xi) d\xi.$$

The maximum slope of the surface squared at the point (x, y) is given by $|\nabla\zeta_{xy}|^2 = \left(\frac{\partial\zeta}{\partial x}\right)^2 + \left(\frac{\partial\zeta}{\partial y}\right)^2$,* where $\frac{\partial\zeta}{\partial x}$ and $\frac{\partial\zeta}{\partial y}$ are the components of slope in the x and y direction at (x, y) . It is shown by Ott[16] and can be seen from the differentiation formulas of Fourier transform theory and use of Parseval's theorem that

* $\nabla_{xy}\zeta$ denotes the gradient of ζ with only the x and y components considered. $\nabla\zeta$ defines the normal vector to the surface.

$$(2.33a) \quad S_x^2 \equiv \left\langle \left(\frac{\partial \zeta}{\partial x} \right)^2 \right\rangle = \frac{1}{16\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p^2 P(p, q) dp dq, \text{ and}$$

$$(2.33b) \quad S_y^2 \equiv \left\langle \left(\frac{\partial \zeta}{\partial y} \right)^2 \right\rangle = \frac{1}{16\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} q^2 P(p, q) dp dq,$$

so that for an isotropically rough surface, $S_x^2 = S_y^2$, and

$$(2.34a) \quad S^2 \equiv \left\langle |\nabla \zeta_{xy}|^2 \right\rangle = \left\langle \left(\frac{\partial \zeta}{\partial x} \right)^2 \right\rangle + \left\langle \left(\frac{\partial \zeta}{\partial y} \right)^2 \right\rangle = 2S_x^2 = 2S_y^2, \text{ and}$$

$$(2.34b) \quad S^2 = \frac{1}{8\pi} \int_0^{\infty} \xi^3 P(\xi) d\xi.$$

Since $P(\xi)$ and $\sigma^2 R(\rho)$ have been defined to be polar Fourier transforms of each other according to (2.31a) and (2.31b), and since $\sigma^2 R(\rho)$ represents the covariance function of the surface height, it seems reasonable to search for a covariance function $S^2 R_s(\rho)$ of the absolute surface slope which is the polar Fourier transform of $\xi^2 P(\xi)$, since (2.32) and (2.34b) are similar in form. This covariance function can be found by taking the polar Fourier transform of $\xi^2 P(\xi)$, if the transform exists. However one must not fall into the trap of saying that $S^2 R_s(\rho) = -\sigma^2 R''(\rho)$; such a statement would be true if $P(\xi)$ and $\sigma^2 R(\rho)$ were normal Fourier transforms of each other instead of polar transforms. The next disastrous step made by many in the literature[21,23] is to say then that $S^2 = S^2 R_s(0) = -\sigma^2 R''(0)$. It is this step which has caused an ambiguity of a factor of 2 for the case of a Gaussian surface height correlation coefficient in the result for the mean square slope.

The use of (2.31a) to find $P(\xi)$ for a given surface correlation coefficient, $R(\rho)$, followed by the use of (2.34b) will always yield the correct result for the mean square surface slope, S^2 , if these integrals exist. An alternative method of finding S^2 in terms of the derivatives of the correlation coefficient will be given later.

2. Averages obtainable from
(2.21) and (2.26)

Equation (2.21) is rewritten here as follows (the same analysis can be applied identically to (2.26) since the integrands are identical in general form):

$$(2.35) \quad \text{Cov}[H_1^S H_2^{S*}] = \frac{k_1 k_2 e^{j\Delta k R_0} L_x L_y H_{01}^i H_{02}^i}{2\pi R_0^2} \left(\frac{\sin\left(\Delta k \gamma_x \frac{L_x}{2}\right)}{\Delta k \gamma_x \frac{L_x}{2}} \right) \left(\frac{\sin\left(\Delta k \gamma_y \frac{L_y}{2}\right)}{\Delta k \gamma_y \frac{L_y}{2}} \right) F(ju, jv, w),$$

where $u \equiv k_1 \gamma_z$, $v \equiv -k_2 \gamma_z$, $w \equiv k_1 \sqrt{\gamma_x^2 + \gamma_y^2}$, and therefore

$$F(ju, jv, w) \equiv \int_0^\infty \rho J_0(w\rho) [M_{\zeta\zeta'}(ju, jv; \rho) - M_{\zeta\zeta'}(ju, 0; \infty) M_{\zeta\zeta'}(0, jv; \infty)] d\rho.$$

It is assumed here that $\text{Cov}[H_1^S H_2^{S*}]$ is a measurable function for a given surface such that $F(ju, jv, w)$ can be determined at a sufficient number of points of the independent variables u, v , and w to assume that it is known everywhere. From $F(ju, jv, w)$, the surface correlation coefficient $R(\rho)$ can be determined as follows:

$$\left. \frac{\partial^2 F(ju, jv, w)}{\partial u \partial v} \right|_{u=v=0} = \int_0^\infty \rho J_0(\rho w) \left[\left. \frac{\partial^2 M_{\zeta\zeta'}(ju, jv; \rho)}{\partial u \partial v} \right|_{u=v=0} - \left. \frac{\partial M_{\zeta\zeta'}(ju, 0; \infty)}{\partial u} \right|_{u=0} \cdot \left. \frac{\partial M_{\zeta\zeta'}(0, jv; \infty)}{\partial v} \right|_{v=0} \right] d\rho.$$

But from equations (2.29), the derivatives of the characteristic function can be rewritten;

$$\left. \frac{\partial^2 M_{\zeta\zeta'}(ju, jv; \rho)}{\partial u \partial v} \right|_{u=v=0} = -\sigma^2 R(\rho)$$

$$\left. \frac{\partial M_{\zeta\zeta'}(ju, 0; \infty)}{\partial u} \right|_{u=0} = \left. \frac{\partial M_{\zeta\zeta'}(0, jv; \infty)}{\partial v} \right|_{v=0} = j\langle \zeta \rangle = j\langle \zeta' \rangle = 0$$

for the coordinate system chosen here.

$$\therefore \left. \frac{\partial^2 F(ju, jv, w)}{\partial u \partial v} \right|_{u=v=0} = -\sigma^2 \int_0^\infty \rho J_0(w\rho) R(\rho) d\rho.$$

But using equation (2.31a), the above equation can be rewritten in terms of the surface power density spectrum:

$$(2.36) \quad P(w) = -8\pi \left. \frac{\partial^2 F(ju, jv, w)}{\partial u \partial v} \right|_{u=v=0}.$$

This equation suggests a theoretically simple way of determining the power density of the rough surface from measurement of the covariance of the scattered fields at two frequencies and different angles. There is no assumption of a statistical model of any type involved. From the surface power density spectrum, one can use equations (2.31b), (2.32), and (2.34) to find the surface correlation

coefficient, the mean square height of the surface, and the mean square slope of the surface.

$$(2.37) \quad \sigma^2 R(\rho) = - \int_0^\infty \xi J_0(\rho \xi) \left. \frac{\partial^2 F(ju, jv, \xi)}{\partial u \partial v} \right|_{u=v=0} d\xi,$$

$$(2.38) \quad \sigma^2 = - \int_0^\infty \xi \left. \frac{\partial^2 F(ju, jv, \xi)}{\partial u \partial v} \right|_{u=v=0} d\xi,$$

$$S^2 = - \int_0^\infty \xi^2 \left. \frac{\partial^2 F(ju, jv, \xi)}{\partial u \partial v} \right|_{u=v=0} d\xi.$$

Not only can the above averages be obtained in this manner, but by taking the polar Fourier transform of $F(ju, jv, w)$ with respect to w (in this case, do not include the second term of the integrand due to the average of, the scattered field. Consider only the average of $H_1^S H_2^{S*}$ and not their covariance.), one obtains the joint characteristic function of the surface heights, ζ and ζ' , i. e., $M_{\zeta \zeta'}(ju, jv; \rho)$. This of course uniquely determines the JPDF of ζ and ζ' . Thus all the second order surface statistical functions and averages needed or desired can be determined from $F(ju, jv, w)$.

At this point, unfortunately, a word of caution and uncertainty must be inserted. Differentiation of the function $F(ju, jv, w)$ at zero assumes that one has obtained values for F near u and $v = 0$. This means that measurements have been made for $k_1 \gamma_Z$ and $k_2 \gamma_Z$ very small, i. e., wavelengths at least as large as all surface scale

dimensions; however, it is precisely in this region that the use of the physical optics approximation is questionable, and since F was obtained by using the physical optics approximation, its validity in this region is at least subject to questioning.

3. Representation of surface by several random variables

In many applications where it is necessary to employ statistical models for the random surface height, ζ , it might prove more useful and informative to treat the surface height as a sum of two or more random variables, i. e., $\zeta = \zeta_1 + \zeta_2 + \zeta_3 + \dots$, where each consecutive random variable may describe a portion of the surface roughness having a definite roughness magnitude scale and caused by a separate random process. (This seems especially applicable to planetary surfaces such as the lunar surface, where it is certain that there is a small-scale surface roughness superimposed upon the large-scale roughness of the mountains and craters.) If each random variable making up the total surface height is generated by a different random process, then they are all statistically independent from one another and each may possess its own JPDF and correlation coefficient. In this case the total surface height joint characteristic function to be used in (2. 23) and (2. 27) is

$$M_{\zeta\zeta'}(ju, jv; \rho) = M_{\zeta_1\zeta_1'}(ju, jv; \rho) M_{\zeta_2\zeta_2'}(ju, jv; \rho)$$

$$M_{\zeta_3\zeta_3'}(ju, jv; \rho) \dots$$

However it must be pointed out that the very use of (2.23) or (2.27) assumes that the physical optics approximation is valid for the surface in question; this restricts the minimum dimensions of the smallest scale random variable, ζ_i , that can be analyzed in this manner, since the minimum radii of curvature of ζ_i must be larger than wavelength.

It is not necessary, however, to attempt to include surface roughness of a scale much less than wavelength in dimensions (e. g. , roughness of a scale so fine that it effects only optical wave scattering), since such roughness appears essentially smooth to larger wavelengths. However, it is the surface random roughness term ζ_j , which has scale dimensions in the same order of magnitude as wavelength, (i. e. , $\lambda/4 < \zeta_j < 20\lambda$) [20] in which (2.23) and (2.27) are not completely correct due to failure of physical optics; it is for this type of roughness that predicted backscattered power will differ from measured power. Roughness of this scale will therefore cause the backscattered power to exhibit a wavelength dependence. Thus one might attempt to interpret Pettingill's measured results to predict that since backscattered power exhibits a barely detectable change between measurements at 7.84 meters and 68 cm, then there is little surface scale roughness having dimensions in this range. However, since there is an appreciable variation in results between the 68 cm and 3.6 cm

measurements, there is very likely a surface roughness having dimensions with an order of magnitude of 5-10 cm.

From observation of the largest scale roughness of the lunar surface from a study of Hayn's optical profiles, it is fair to assume that vertical points are quite improbable. However, when one considers likely small scale structure which might be present (such as gravel, boulders, rocks, etc.), vertical points may be quite common. The presence of vertical points affects the form of the surface correlation function, as will be shown later.

One should be cautious about using this multiple random variable approach to predict and compare backscattered power with measured results, since this approach will introduce more functions and constants which can all be manipulated to force predicted results to compare with measured data; the true validity of final results are thus obscured. Such caution seems especially appropriate since there has been such poor agreement over a wide range thus far for even the largest scale roughness when it is treated as the only random variable present. For this largest scale roughness, physical optics is certainly applicable. Therefore such a multiple random variable approach is deferred in this work and better results are sought first for the largest scale roughness treated as a single random variable, ζ .

4. Surface slope statistics of the planar rough surface

It will later be seen in dealing with nonperfectly conducting surfaces that certain averages and relationships involving the surface slopes are needed. Therefore, many of these necessary relationships are derived in this subsection.

Since the surface height, ζ , is a random variable, then the directional surface slopes $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ are also random variables; they are not equal in general at an arbitrary point on the surface, but they are statistically independent from each other, and when the surface is oriented as shown in Fig. 1 (i. e., so that the mean surface plane is identically the x-y plane), both $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ must obey the same distributions, since the position of the x and y axes are arbitrary and the surface roughness is isotropic. There is obviously some relationship between the statistics of ζ , surface height, and the surface slopes, $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$. This relationship is in general not simple to determine. It will be assumed that ζ and either surface slope $\left(\frac{\partial \zeta}{\partial x} \text{ or } \frac{\partial \zeta}{\partial y}\right)$ are statistically independent at a point on the surface; therefore averages of quantities such as $f(\zeta) g\left(\frac{\partial \zeta}{\partial x}\right) h\left(\frac{\partial \zeta}{\partial y}\right)$ can be written $\langle f(\zeta) \rangle \langle g\left(\frac{\partial \zeta}{\partial x}\right) \rangle \langle h\left(\frac{\partial \zeta}{\partial y}\right) \rangle$. This assumption seems physically justified. The fact that absolute or maximum slope square at a point is given by $|\nabla_{xy}\zeta|^2 = \left(\frac{\partial \zeta}{\partial x}\right)^2 + \left(\frac{\partial \zeta}{\partial y}\right)^2$ will be employed here, as shown in subsection 1 of this section.

There is probably only one satisfactory way of relating the statistics of $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ to those of ζ ; this is through a Karhunen-Loeve expansion of ζ into a series of products involving mutually orthogonal functions $\phi_n(x, y)$ and uncorrelated random variables v_n with zero means and unity variances. Then one can compute averages involving $\frac{\partial \zeta}{\partial x}$ by merely differentiating $\phi_n(x, y)$ in the series with respect to x . Such an expansion is always theoretically possible, but unfortunately is involved and in most cases it is not possible to find $\phi_n(x, y)$ in a closed form, since they are solutions to an integral equation which has the surface correlation coefficient as the kernel [14] .

However, if the surface height ζ has a Gaussian JPDF and if (2. 33a) and (2. 33b) exist, then the derivatives $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ also have a Gaussian JPDF. The variance and correlation coefficient of this Gaussian JPDF for $\frac{\partial \zeta}{\partial x}$ or $\frac{\partial \zeta}{\partial y}$ are determined in the following paragraph. In this case it should be noted that the absolute surface slope, $|\nabla_{xy}\zeta| = \sqrt{\left(\frac{\partial \zeta}{\partial x}\right)^2 + \left(\frac{\partial \zeta}{\partial y}\right)^2}$ will have a Rayleigh distribution with a non-zero mean, whereas $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ have zero means.

Regardless of whether the JPDF of $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ is Gaussian or even whether it can be determined at all, certain averages of these surface slopes can be found from the correlation coefficient $R(\rho)$ of surface height ζ ; $S_x^2 = \left\langle \left(\frac{\partial \zeta}{\partial x}\right)^2 \right\rangle$ and $S_y^2 = \left\langle \left(\frac{\partial \zeta}{\partial y}\right)^2 \right\rangle$ have already been determined from (2. 33a) and (2. 33b). It should be noted that since

P and R are Fourier transform pairs, Fourier transform theory yields the following transform pairs (if they exist):

Rectangular Form

Polar Form

$$P(\sqrt{p^2 + q^2}) \longleftrightarrow \sigma^2 R(\sqrt{\tau_x^2 + \tau_y^2})$$

$$P(\xi) \longleftrightarrow \sigma^2 R(\rho)$$

$$p^2 P(\sqrt{p^2 + q^2}) \longleftrightarrow \sigma^2 \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_x^2}$$

$$q^2 P(\sqrt{p^2 + q^2}) \longleftrightarrow \sigma^2 \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_y^2}$$

and since

$$\langle \zeta^2 \rangle = \frac{1}{16\pi^2} \iint_{-\infty}^{\infty} P(\sqrt{p^2 + q^2}) dp dq = \frac{1}{8\pi} \int_0^{\infty} \xi P(\xi) d\xi,$$

$$\begin{aligned} \left\langle \left(\frac{\partial \zeta}{\partial x} \right)^2 \right\rangle &= - \frac{1}{16\pi^2} \iint_{-\infty}^{\infty} p^2 P(\sqrt{p^2 + q^2}) dp dq \\ &= - \frac{1}{16\pi^2} \iint_{-\infty}^{\infty} q^2 P(\sqrt{p^2 + q^2}) dp dq = \left\langle \left(\frac{\partial \zeta}{\partial y} \right)^2 \right\rangle, \end{aligned}$$

then just as $\sigma^2 R(\sqrt{\tau_x^2 + \tau_y^2})$ is the correlation function of ζ , so in like manner $-\sigma^2 \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_x^2}$ and $-\sigma^2 \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_y^2}$ are the correlation functions of $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ respectively. Thus the correlation coefficient of these directional surface slopes is given as

$$\begin{aligned} R_{S_x}(\sqrt{\tau_x^2 + \tau_y^2}) &\equiv - \frac{\sigma^2}{S_x^2} \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_x^2}, \text{ and } - \frac{\sigma^2 \partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{S_y^2 \partial \tau_y^2} \\ &\equiv R_{S_y}(\sqrt{\tau_x^2 + \tau_y^2}) \text{ for } \frac{\partial \zeta}{\partial x} \text{ and } \frac{\partial \zeta}{\partial y}. \end{aligned}$$

Since $S_x^2 R_{S_x}(\sqrt{\tau_x^2 + \tau_y^2}) = -\sigma^2 \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_x^2}$ and $S_y^2 R_{S_y}(\sqrt{\tau_x^2 + \tau_y^2}) = -\sigma^2 \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_y^2}$, the total mean square slope of the surface S^2 can be found as follows:

$$(2.39) \quad S^2 = S_x^2 + S_y^2 = S_x^2 R_{S_x} \Big|_{\tau_x = \tau_y = 0} + S_y^2 R_{S_y} \Big|_{\tau_x = \tau_y = 0}$$

$$\therefore S^2 = -\sigma^2 \left[\frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_x^2} \Big|_{\tau_x = \tau_y = 0} + \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_y^2} \Big|_{\tau_x = \tau_y = 0} \right]$$

It should be noted, however, that the above equation is not the same as $S^2 = -\sigma^2 \frac{d^2 R(\rho)}{d\rho^2} \Big|_{\rho=0}$, which is the equation used by many in the literature as the definition of S^2 .

One word of caution is in order. Before one attempts to use (2.39) to find the mean square slope of a surface, he must be sure that $-\sigma^2 \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_x^2}$ is truly the Fourier transform of $p^2 P(\sqrt{p^2 + q^2})$; from a basic existence theorem of Fourier transform theory, they are Fourier transform pairs only so long as the integral

$$(2.40) \quad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |p^2 P(\sqrt{p^2 + q^2})|^2 dp dq$$

exists and is finite. Therefore it is impossible to define the mean square slope from (2.39) for a surface when the above integral diverges.*

If averages of higher derivatives of the surface are needed, they may be found by induction from (2.33a) and (2.33b). For example

$$\begin{aligned} \left\langle \left(\frac{\partial^2 \zeta}{\partial x^2} \right)^2 \right\rangle &= \frac{1}{16 \pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p^4 P(\sqrt{p^2 + q^2}) dp dq \\ &= \sigma^2 \left. \frac{\partial^4 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_x^4} \right|_{\tau_x = \tau_y = 0}, \end{aligned}$$

$$\begin{aligned} \left\langle \left(\frac{\partial^2 \zeta}{\partial x \partial y} \right)^2 \right\rangle &= \frac{1}{16 \pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p^2 q^2 P(\sqrt{p^2 + q^2}) dp dq \\ &= \sigma^2 \left. \frac{\partial^4 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_x^2 \partial \tau_y^2} \right|_{\tau_x = \tau_y = 0}, \end{aligned}$$

etc.

5. Surface slopes of a rotated planar rough surface

In Fig. 5, the rough surface no longer lies in the x-y plane but is rotated by an angle θ about the x axis from its position in Fig. 1.

* This point is apparently overlooked by many in the literature (e. g., Beckman[21]) who attempt to use the incorrect equation $S^2 = -\sigma^2 R''(0)$ to determine mean square slope for $R(\rho) = e^{-|\rho|/a}$, i. e., when the surface has an exponential surface height correlation coefficient; in that particular case, (2.40) diverges. This requirement on the existence of these higher derivative transforms is discussed in (8.1-1) of Ref. 15.

Hence the new surface slopes in the new x_1 and y_1 directions, $\partial\zeta_1/\partial x_1$ and $\partial\zeta_1/\partial y_1$ will not be the same as those for the original coordinate system of Fig. 1, i. e., $\partial\zeta/\partial x$ and $\partial\zeta/\partial y$; nor is the problem merely one of differentiating equations (2.25). The new random variables $\partial\zeta_1/\partial x_1$ and $\partial\zeta_1/\partial y_1$ will be related to the old random variables $\partial\zeta/\partial x$ and $\partial\zeta/\partial y$ in this subsection.

A normal vector (not necessarily a unit vector) to the random surface ζ_i represented in any coordinate system x_i, y_i, z_i may be defined $\bar{N}_i = -\frac{\partial\zeta_i}{\partial x_i} \hat{x}_i - \frac{\partial\zeta_i}{\partial y_i} \hat{y}_i + \hat{z}_i$; the components of $\hat{x}_i$ and $\hat{y}_i$ define the surface slopes along the x_i and y_i directions. In the original system of Fig. 1 this normal is $\bar{N} = -\frac{\partial\zeta}{\partial x} \hat{x} - \frac{\partial\zeta}{\partial y} \hat{y} + \hat{z}$; in the new rotated coordinate system the new normal is $\bar{N}_1 = -\frac{\partial\zeta_1}{\partial x_1} \hat{x}_1 - \frac{\partial\zeta_1}{\partial y_1} \hat{y}_1 + \hat{z}_1$. The lengths of $\bar{N}$ and $\bar{N}_1$, when defined in this manner are not necessarily the same. The relationship between the old unit vectors $\hat{x}, \hat{y}$, and $\hat{z}$ and the unit vectors $\hat{x}_1, \hat{y}_1$ and $\hat{z}_1$ in the rotated system is simply obtained.

$$\begin{aligned}\hat{x} &= \hat{x}_1 \\ \hat{y} &= \cos \theta \hat{y}_1 + \sin \theta \hat{z}_1 \\ \hat{z} &= -\sin \theta \hat{y}_1 + \cos \theta \hat{z}_1\end{aligned}$$

Substituting the above relationships into $\bar{N}$, one obtains $\bar{N}$ in the new system

$$\bar{N} = \frac{\partial \zeta}{\partial x} \hat{x}_1 - \frac{\partial \zeta}{\partial y} (\cos \theta \hat{y}_1 + \sin \theta \hat{z}_1) + (-\sin \theta \hat{y}_1 + \cos \theta \hat{z}_1)$$

or

$$\bar{N} = -\frac{\partial \zeta}{\partial x} \hat{x}_1 - \left(\frac{\partial \zeta}{\partial y} \cos \theta + \sin \theta \right) \hat{y}_1 + \left(\cos \theta - \frac{\partial \zeta}{\partial y} \sin \theta \right) \hat{z}_1 .$$

The difference in length between $\bar{N}$ and $\bar{N}_1$ is obvious; $\bar{N}_1$ has unity component in the $\hat{z}_1$ direction. Therefore, to make $\bar{N}$, as expressed in the x_1, y_1, z_1 system equal in length to $\bar{N}_1$, divide by the component of $\hat{z}_1$.

$$\begin{aligned} \frac{\bar{N}}{\cos \theta - \frac{\partial \zeta}{\partial y} \sin \theta} &= -\frac{\frac{\partial \zeta}{\partial x}}{\cos \theta - \frac{\partial \zeta}{\partial y} \sin \theta} \hat{x}_1 - \frac{\sin \theta + \frac{\partial \zeta}{\partial y} \cos \theta}{\cos \theta - \frac{\partial \zeta}{\partial y} \sin \theta} \hat{y}_1 \\ &\quad + \hat{z}_1 \\ &= \bar{N}_1 = -\frac{\partial \zeta_1}{\partial x_1} \hat{x}_1 - \frac{\partial \zeta_1}{\partial y_1} \hat{y}_1 + \hat{z}_1 . \end{aligned}$$

The relationship between the new slopes $\partial \zeta_1 / \partial x_1$, $\partial \zeta_1 / \partial y_1$ and the original surface slopes is now evident from equating components of $\hat{x}_1$ and $\hat{y}_1$.

$$(2.41a) \quad \frac{\partial \zeta_1}{\partial x_1} = \frac{\frac{\partial \zeta}{\partial x}}{\cos \theta - \frac{\partial \zeta}{\partial y} \sin \theta}$$

$$(2.41b) \quad \frac{\partial \zeta_1}{\partial y_1} = \frac{\sin \theta + \frac{\partial \zeta}{\partial y} \cos \theta}{\cos \theta - \frac{\partial \zeta}{\partial y} \sin \theta} .$$

These relationships are exact, but unfortunately it is difficult to deduce the statistics of the two new random slopes $\partial\zeta_1/\partial x_1$ and $\partial\zeta_1/\partial y_1$ in terms of those of the original slopes because the original slopes appear in the denominator. However, if the original surface slopes $\partial\zeta/\partial x$ and $\partial\zeta/\partial y$ are relatively small and if the angle of incidence (away from normal), θ , is not too large, then the second term of the denominators is small compared with the first, so that

$$\cos \theta - \frac{\partial\zeta}{\partial y} \sin \theta \approx \cos \theta;$$

in this case

$$(2.42a) \quad \frac{\partial\zeta_1}{\partial x_1} \approx \sec \theta \frac{\partial\zeta}{\partial x}$$

$$(2.42b) \quad \frac{\partial\zeta_1}{\partial y_1} \approx \tan \theta + \frac{\partial\zeta}{\partial y}.$$

As an example, for the original surface slopes $\partial\zeta/\partial y$ no greater than 15° , i. e., $\partial\zeta/\partial y < 0.268$, then angle of incidence θ can be as great as 75° before the second term of the denominator is as large as the first term. Even at such extreme angles, taking averages of the new slopes in terms of the old by using (2.41a) and (2.41b), although yielding the exact relationships, is unduly complicated, so that use of (2.42a) and (2.42b) seems warranted for the sake of simplicity as long as one remembers that this approximation is a source of error near grazing.

Using (2.42a) and (2.42b), one can find averages of $\partial \zeta_1 / \partial x_1$ and $\partial \zeta_1 / \partial y_1$ in terms of averages of $\partial \zeta / \partial x$ and $\partial \zeta / \partial y$.

$$(2.43a) \quad \left\langle \frac{\partial \zeta_1}{\partial x_1} \right\rangle = \left\langle \sec \theta \frac{\partial \zeta}{\partial x} \right\rangle = 0$$

$$(2.43b) \quad \left\langle \frac{\partial \zeta_1}{\partial y_1} \right\rangle = \left\langle \tan \theta + \frac{\partial \zeta}{\partial y} \right\rangle = \tan \theta$$

$$(2.43c) \quad \left\langle \left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \right\rangle = \left\langle \sec^2 \theta \left(\frac{\partial \zeta}{\partial x} \right)^2 \right\rangle = \sec^2 \theta \left\langle \left(\frac{\partial \zeta}{\partial x} \right)^2 \right\rangle = \sec^2 \theta \cdot S_x^2$$

$$(2.43d) \quad \left\langle \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right\rangle = \left\langle \left(\tan \theta + \frac{\partial \zeta}{\partial y} \right)^2 \right\rangle = \tan^2 \theta + \left\langle \left(\frac{\partial \zeta}{\partial y} \right)^2 \right\rangle = \tan^2 \theta + S_y^2$$

$$(2.43e) \quad \left\langle \frac{\partial \zeta_1}{\partial x_1} \cdot \frac{\partial \zeta'_1}{\partial x'_1} \right\rangle = \sec^2 \theta \left\langle \frac{\partial \zeta}{\partial x} \frac{\partial \zeta'}{\partial x'} \right\rangle = \sec^2 \theta S_x^2 R_{S_x}$$

$$(2.43f) \quad \left\langle \frac{\partial \zeta_1}{\partial x_1} \cdot \frac{\partial \zeta'_1}{\partial y'_1} \right\rangle = \left\langle \frac{\partial \zeta'_1}{\partial x'_1} \cdot \frac{\partial \zeta_1}{\partial y_1} \right\rangle = \sec \theta \tan \theta \left\langle \frac{\partial \zeta}{\partial x} \right\rangle \\ + \sec \theta \left\langle \frac{\partial \zeta}{\partial x} \frac{\partial \zeta'}{\partial y'} \right\rangle = 0$$

$$(2.43g) \quad \left\langle \frac{\partial \zeta_1}{\partial y_1} \cdot \frac{\partial \zeta'_1}{\partial y'_1} \right\rangle = \tan^2 \theta + \tan \theta \left\langle \frac{\partial \zeta}{\partial y} \right\rangle + \tan \theta \left\langle \frac{\partial \zeta'}{\partial y'} \right\rangle \\ + \left\langle \frac{\partial \zeta}{\partial y} \frac{\partial \zeta'}{\partial y'} \right\rangle = \tan^2 \theta + S_y^2 R_{S_y},$$

where R_{S_x} and R_{S_y} denote the correlation coefficients for $\partial \zeta / \partial x$ and $\partial \zeta / \partial y$. Equation (27b) indicates an average surface slope of $\tan \theta$ in the y_1 direction as one would expect. Equation (2.43c) shows that the mean square slope in the x_1 direction is changed by a factor $\sec^2 \theta$, while (2.43d) shows that mean square slope in the y_1 direction has a constant factor $\tan^2 \theta$ added to it. Equation (2.43e) expresses the covariance of surface slopes in the x_1 direction at two different points

as a function of the covariance of the slope in the original x direction multiplied by $\sec^2\theta$. Equation (2.43f) shows that slopes in the x_1 and y_1 directions are uncorrelated just as the original slopes in the x and y directions were assumed uncorrelated. Equation (2.43g) shows that a constant $\tan^2\theta$ must be added to covariance of the original surface slopes in the y direction to give the covariance of the slopes in the y_1 direction.

One point to be noted from equations (2.43a-g) is the fact that $\partial\zeta_1/\partial x_1$ and $\partial\zeta_1/\partial y_1$ each no longer obey the same statistical distributions nor have the same averages as did $\partial\zeta/\partial x$ and $\partial\zeta/\partial y$ (i. e., $\langle \frac{\partial\zeta}{\partial x} \rangle = \langle \frac{\partial\zeta}{\partial y} \rangle$, $\langle \left(\frac{\partial\zeta}{\partial x}\right)^2 \rangle = \langle \left(\frac{\partial\zeta}{\partial y}\right)^2 \rangle$, $R_{S_x} = R_{S_y}$, etc.). The reason is apparent since the rough surface in the new coordinate system x_1, y_1, z_1 is no longer isotropic.

D. Backscattering from a Non-Perfectly Conducting Surface

1. Introduction and initial formulation

In this section, rough surfaces which are not perfectly conducting are examined, and the reflected field at every point on the surface is related to the incident field at that point by reflection coefficients as given in (2.14a) and (2.14b); these reflection coefficients apply when the physical optics approximation is valid, i. e., the surface radii of curvature at all points are large enough in terms of wavelength so that

one may replace the surface at a given point by an infinite tangent plane to the surface at that point. Therefore, since the incident waves are planar, the reflected waves from a given point are planar and leave the surface in the specular direction only. The total fields at the surface can then be found in terms of the incident field and scattering coefficients and substituted into (2.11a) to find the scattered E-field.

The problem one is faced with here is that of finding the local plane of incidence at a point P on the surface (see Fig. 5) so that he can then resolve the incident E-field into components parallel and perpendicular to this local plane of incidence; after these components are found, their products with their respective reflection coefficients then gives the reflected fields in these two directions. The local plane of incidence is the plane containing the incident wave direction, $\hat{n}_i$ (which is constant over the surface such that $\hat{n}_i = -\hat{z}$ here), and the normal to the surface, $\hat{n}$, at P. The direction of this normal, $\hat{n}$, varies from point to point on the surface and therefore the local plane of incidence varies from point to point. However, by specializing immediately to the case of backscattering and by choosing the coordinate system of Fig. 5 and Fig. 6 with the incident and scattering directions along the z_1 axis, a convenient integral will be obtained for the scattered field. Because the author has never seen this integral in the literature to date for the backscattered field from a

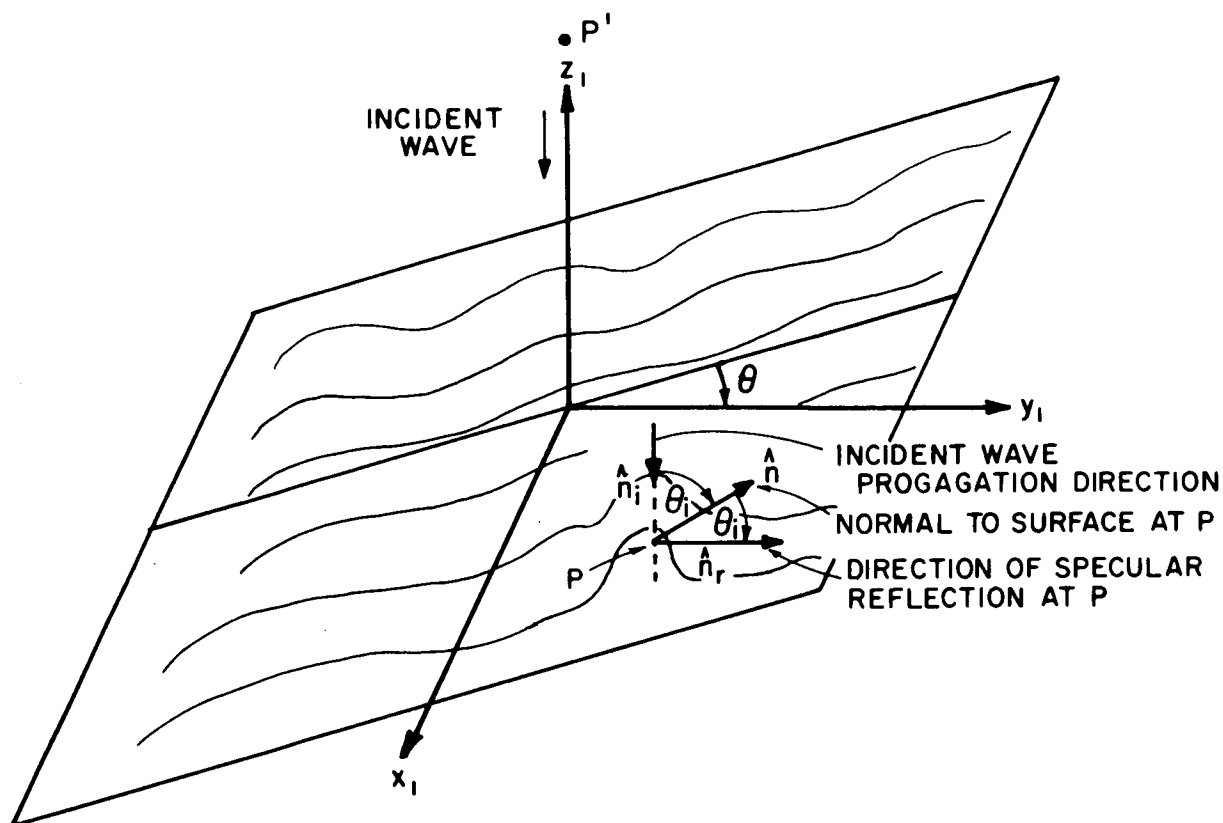


Fig. 6--Planar scattering surface showing the local normal and local scattering angles.

non-perfect conducting surface, all the steps and algebra involved in its derivation will be shown in this section.

2. The total E and H fields at the surface

In this subsection, the total E and H fields at the surface will be found at a surface point P in terms of the components of the E field perpendicular and parallel to the local plane of incidence at P and the local parallel and perpendicular reflection coefficients.

- a. The incident E and H fields at the surface

The unit normal to the plane of incidence is defined as

$$\hat{s} = \frac{1}{\sin \theta_i} \hat{n} \times \hat{n}_i = \frac{1}{\sin \theta_i} \hat{z}_1 \times \hat{n}. \quad \text{The following relationship exists}$$

between the local incidence angle, θ_i , the unit normal $\hat{n}$ at P, and the unit vectors in the directions of incidence and specular reflection at P,

$$\text{i. e., } \hat{n}_i \text{ and } \hat{n}_r: -\hat{n} \cdot \hat{n}_i = \hat{n} \cdot \hat{z}_1 = \cos \theta_i = \hat{n} \cdot \hat{n}_r. \quad \text{The unit vector in}$$

the local plane of incidence but perpendicular to $\hat{z}_1$ ($-\hat{z}_1$ being the direction of propagation of the incident wave) is given by

$$\hat{s} \times \hat{z}_1 = \frac{1}{\sin \theta_i} [(\hat{z}_1 \times \hat{n}) \times \hat{z}_1] = \frac{1}{\sin \theta_i} (\hat{n} - \cos \theta_i \hat{z}_1). \quad \text{Thus}$$

it is into components along $\hat{s}$ and $\hat{s} \times \hat{z}_1$ that the incident E field will

be resolved. First it will be assumed that the incident E-field is

polarized in the $\hat{x}_1$ direction (i. e., horizontal polarization when the

surface is tilted as shown in Fig. 5). Therefore the incident E field

is given by $\vec{E}^i = \hat{x} E_h^i e^{-jkz_1}$, where the time factor $e^{-j\omega t}$ is omitted.

Then the components of the E field along $\hat{s}$ and $\hat{s} \times \hat{z}_1$ are given by

$$(i) \quad \vec{E}_\perp^i = \hat{s} (\vec{E}^i \cdot \hat{s}) = - \frac{1}{\sin^2 \theta_i} (\hat{z}_1 \times \hat{n}) (\hat{n} \cdot \hat{y}_1) E_h^i e^{-jkz_1},$$

$$\text{where } \hat{s} \cdot \hat{x}_1 = - \frac{1}{\sin \theta} \hat{n} \cdot \hat{y}_1, \text{ and}$$

$$(ii) \quad \vec{E}_\parallel^i = (\hat{s} \times \hat{z}_1) [\vec{E}^i \cdot (\hat{s} \times \hat{z}_1)]$$

$$= \frac{1}{\sin^2 \theta_i} (\hat{n} - \cos \theta_i \hat{z}_1) (\hat{n} \cdot \hat{x}_1) E_h^i e^{-jkz_1}.$$

Since the incident waves are planar, the H field components corresponding to $\bar{E}_\perp^i$ and $\bar{E}_\parallel^i$ are related to them by constants, such that

$$\bar{H}_\perp^i = \sqrt{\frac{\epsilon_0}{\mu_0}} \hat{n}_i \times \bar{E}_\perp^i \text{ and } \bar{H}_\parallel^i = \sqrt{\frac{\epsilon_0}{\mu_0}} \hat{n}_i \times \bar{E}_\parallel^i \text{ (} \hat{n}_i = -\hat{z} \text{ here); therefore,}$$

$$(iii) \quad \bar{H}_\perp^i = -\frac{1}{\sin^2 \theta_i} \sqrt{\frac{\epsilon_0}{\mu_0}} (\hat{n} - \cos \theta_i \hat{z}_1) (\hat{n} \cdot \hat{y}_1) E_h^i e^{-jkz_1}$$

$$(iv) \quad \bar{H}_\parallel^i = -\frac{1}{\sin^2 \theta_i} \sqrt{\frac{\epsilon_0}{\mu_0}} (\hat{z}_1 \times \hat{n}) (\hat{n} \cdot \hat{x}_1) E_h^i e^{-jkz_1}.$$

(Note that $\bar{H}_\perp^i$, corresponding to $\bar{E}_\perp^i$, is not parallel to $\bar{E}_\perp^i$, but is parallel to $\bar{E}_\parallel^i$ and therefore lies in the local plane of incidence.

$\bar{H}_\parallel^i$ by the same token lies perpendicular to the plane of incidence.)

b. The reflected E and H fields at the surface

Having resolved the incident E and H fields at the surface into components parallel and perpendicular to the local plane of incidence, one can use the parallel and perpendicular reflection coefficients to find the reflected fields at the surface at point P. Thus, according to (2.14a) and (2.14b),

$$\bar{E}_\perp^r = \rho_\perp \bar{E}_\perp^i \text{ and } \hat{n}_r \times \bar{E}_\parallel^r = \rho_\parallel \hat{n}_i \times \bar{E}_\parallel^i.$$

By virtue of the fact that the incident and reflected waves at the surface are both considered planar, this last equation can be written as

$$\bar{H}_\parallel^r = \rho_\parallel \bar{H}_\parallel^i. \text{ Therefore,}$$

$$(i) \quad \bar{E}_\perp^r = -\frac{1}{\sin^2 \theta_i} \rho_\perp (\hat{z}_1 \times \hat{n})(\hat{n} \cdot \hat{y}_1) E_h^i e^{jk\hat{n}_r \cdot \bar{r}}$$

and

$$(ii) \quad \bar{H}_\parallel^r = -\frac{1}{\sin^2 \theta} \rho_\parallel \sqrt{\frac{\epsilon_o}{\mu_o}} (\hat{z}_1 \times \hat{n})(\hat{n} \cdot \hat{x}_1) E_h^i e^{jk\hat{n}_r \cdot \bar{r}}.$$

From the above two equations, the remaining two components of the reflected field are given by the following relationships:

$$\bar{H}_\perp^r = \sqrt{\frac{\epsilon_o}{\mu_o}} \hat{n}_r \times \bar{E}_\perp^r \text{ and } \bar{E}_\parallel^r = -\sqrt{\frac{\mu_o}{\epsilon_o}} \hat{n}_r \times \bar{H}_\parallel^r. \text{ Thus,}$$

$$(iii) \quad \bar{H}_\perp^r = -\frac{1}{\sin^2 \theta} \rho_\perp \sqrt{\frac{\epsilon_o}{\mu_o}} (\cos \theta_i \hat{z}_1 - \cos 2\theta_i \hat{n})(\hat{n} \cdot \hat{y}_1) E_h^i e^{jk\hat{n}_r \cdot \bar{r}},$$

$$(iv) \quad \bar{E}_\parallel^r = \frac{1}{\sin^2 \theta} \rho_\parallel (\cos \theta_i \hat{z}_1 - \cos 2\theta_i \hat{n})(\hat{n} \cdot \hat{x}_1) E_h^i e^{jk\hat{n}_r \cdot \bar{r}}. *$$

c. The total E and H fields at the surface

On the surface at point $\bar{P}$ the phase angles of the incident and reflected fields become identical, i. e., $-jkz_1 = jk\hat{n}_r \cdot \bar{r} = -jk\zeta_1(x_1, y_1)$, where $\zeta_1(x_1, y_1)$ is the height of the surface at point $P(x_1, y_1)$.

The total E and H fields at the surface are given by the following

* Use is made here of the fact that $\hat{n}_r \times (\hat{z}_1 \times \hat{n}) = [(\hat{n}_r \cdot \hat{n})\hat{z}_1 - (\hat{n}_r \cdot \hat{z}_1)\hat{n}]$, and the fact that $\hat{n}_r \cdot \hat{n} = \cos \theta$ and $\hat{n}_r \cdot \hat{z}_1 = \cos 2\theta$ as can be seen from Fig. 5.

$$\bar{\mathbf{E}} = \bar{\mathbf{E}}_{\perp}^i + \bar{\mathbf{E}}_{\parallel}^i + \bar{\mathbf{E}}_{\perp}^r + \bar{\mathbf{E}}_{\parallel}^r$$

$$(2.44a) \quad \therefore \bar{\mathbf{E}} = \frac{1}{\sin^2 \theta_i} \left\{ -(1 + \rho_{\perp}) (\hat{\mathbf{z}}_1 \times \hat{\mathbf{n}}) (\hat{\mathbf{n}} \cdot \hat{\mathbf{y}}_1) \right. \\ \left. + (1 - \rho_{\parallel} \cos 2\theta_i) \hat{\mathbf{n}} (\hat{\mathbf{n}} \cdot \hat{\mathbf{x}}_1) - \cos \theta_i (1 - \rho_{\parallel}) \right. \\ \left. \hat{\mathbf{z}}_1 (\hat{\mathbf{n}} \cdot \hat{\mathbf{x}}_1) \right\} E_h^i e^{-jk\zeta_1}$$

$$\bar{\mathbf{H}} = \bar{\mathbf{H}}_{\perp}^i + \bar{\mathbf{H}}_{\parallel}^i + \bar{\mathbf{H}}_{\perp}^r + \bar{\mathbf{H}}_{\parallel}^r$$

$$(2.44b) \quad \therefore \bar{\mathbf{H}} = -\frac{1}{\sin^2 \theta} \sqrt{\frac{\epsilon_0}{\mu_0}} \left\{ (1 + \rho_{\parallel}) (\hat{\mathbf{z}}_1 \times \hat{\mathbf{n}}) (\hat{\mathbf{n}} \cdot \hat{\mathbf{x}}_1) \right. \\ \left. + (1 - \rho_{\parallel} \cos 2\theta_i) \hat{\mathbf{n}} (\hat{\mathbf{n}} \cdot \hat{\mathbf{y}}_1) - \cos \theta_i (1 - \rho_{\perp}) \right. \\ \left. \hat{\mathbf{z}}_1 (\hat{\mathbf{n}} \cdot \hat{\mathbf{y}}_1) \right\} E_h^i e^{-jk\zeta_1} .$$

3. Simplification of the integral for the scattered E field

Equation (2.11a) will be used to give the scattered E field at field point P' along the $\mathbf{z}_1$ axis. The unit vector $\hat{\boldsymbol{\eta}}$ pointing in the scattering direction of interest is therefore identically $\hat{\mathbf{z}}_1$. The integrand of (2.11a) is

$$\bar{\mathbf{I}} = \left\{ (\hat{\mathbf{n}} \times \bar{\mathbf{H}}) - \hat{\boldsymbol{\eta}} \cdot (\hat{\mathbf{n}} \times \bar{\mathbf{H}}) \hat{\boldsymbol{\eta}} - \sqrt{\frac{\epsilon_0}{\mu_0}} (\hat{\mathbf{n}} \times \bar{\mathbf{E}}) \times \hat{\boldsymbol{\eta}} \right\} e^{-jk\hat{\boldsymbol{\eta}} \cdot \bar{\mathbf{r}}},$$

where $\bar{\mathbf{E}}$ and $\bar{\mathbf{H}}$ are given by (2.44a) and (2.44b). The phase factor $jk\hat{\boldsymbol{\eta}} \cdot \bar{\mathbf{r}}$ becomes $jk\zeta_1$ in this case. Using (2.44a) and (2.44b) and combining terms, the integrand becomes

$$\begin{aligned} \bar{I} = \frac{2 \cos \theta_i}{\sin^2 \theta_i} \{ \rho_{\parallel} \hat{n} (\hat{n} \cdot \hat{x}_1) - \rho_{\parallel} \cos \theta_i \hat{z}_1 (\hat{n} \cdot \hat{x}_1) \\ - \rho_{\perp} (\hat{n} \times \hat{z}_1) (\hat{n} \cdot \hat{y}_1) \} E_h^i e^{-j2k\zeta_1} \end{aligned}$$

At this point, the quantity $\bar{I} da$ in the integral can be further simplified by replacing $\hat{n}$, $\cos \theta_i$, and $\sin \theta_i$ by their equivalent expressions in terms of the surface slopes $\partial \zeta_1 / \partial x_1$ and $\partial \zeta_1 / \partial y_1$.

These relationships are given by

$$\hat{n} = \frac{-\frac{\partial \zeta_1}{\partial x_1} \hat{x}_1 - \frac{\partial \zeta_1}{\partial y_1} \hat{y}_1 + \hat{z}_1}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + 1 \right]^{1/2}},$$

$$\cos \theta_i = \hat{n} \cdot \hat{z}_1 = \frac{1}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + 1 \right]^{1/2}},$$

$$\therefore \sin^2 \theta_i = 1 - \cos^2 \theta_i = \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2}{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + 1},$$

$$\hat{n} \cdot \hat{x}_1 = \frac{-\frac{\partial \zeta_1}{\partial x_1}}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + 1 \right]^{1/2}};$$

$$\hat{n} \cdot \hat{y}_1 = \frac{-\frac{\partial \zeta_1}{\partial y_1}}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + 1 \right]^{1/2}}$$

$$\hat{n} \times \hat{z}_1 = \frac{\frac{\partial \zeta_1}{\partial x_1} \hat{y}_1 - \frac{\partial \zeta_1}{\partial y_1} \hat{x}_1}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + 1 \right]^{1/2}} ;$$

$$da = dx_1 dy_1 \left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + 1 \right]^{1/2} .$$

Substituting the above quantities into $\bar{I}da$, one arrives at

$$\therefore \bar{I}da = \frac{2 \left\{ \left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \rho_{||} - \left(\frac{\partial \zeta_1}{\partial y_1} \right) \rho_{\perp} \right] \hat{x} + \frac{\partial \zeta_1}{\partial x_1} \frac{\partial \zeta_1}{\partial y_1} (\rho_{||} + \rho_{\perp}) \hat{y} \right\} E_h^i e^{-j2k\zeta_1}}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]} .$$

The scattered field becomes

$$(2.45) \quad \bar{E}^s(P') = \frac{-jke^{jkR_o}}{2\pi R_o} E_h^i \hat{x}_1 \iint_D \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \rho_{||} - \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \rho_{\perp}}{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2} e^{-j2k\zeta_1} dx_1 dy_1$$

$$- \frac{jke^{jkR_o}}{2\pi R_o} E_h^i \hat{y}_1 \iint_D \frac{(\rho_{||} + \rho_{\perp}) \frac{\partial \zeta_1}{\partial x_1} \frac{\partial \zeta_1}{\partial y_1}}{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2} e^{-j2k\zeta_1} dx_1 dy_1 .$$

In like manner, the scattered field can be computed when the incident field is polarized in the y_1 direction (i. e., vertically polarized incident field when the surface is tilted in the direction shown in Fig. 5) such that $\bar{E}^i = E_v^i \hat{y}_1 e^{-jkz_1}$. The scattered field in this case is

$$(2.46) \quad \bar{E}^s(P') = \frac{-jke^{jkR_0}}{2\pi R_0} E_v^i \hat{x}_1$$

$$\iint_D \frac{(\rho_{||} + \rho_{\perp}) \frac{\partial \zeta_1}{\partial x_1} \frac{\partial \zeta_1}{\partial y_1}}{\left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1}\right)^2} e^{-j2k\zeta_1} dx_1 dy_1$$

$$- \frac{jke^{jkR_0}}{2\pi R_0} E_v^i \hat{y}_1 \iint_D \frac{\left(\frac{\partial \zeta_1}{\partial y_1}\right)^2 \rho_{||} - \left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 \rho_{\perp}}{\left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1}\right)^2} e^{-j2k\zeta_1} dx_1 dy_1 .$$

Therefore the case of an incident wave polarized in any arbitrary direction (in the x_1 - y_1 plane of course) can be treated as a linear combination of (2.45) and (2.46).

There are many facts of interest about (2.45) and (2.46) to be pointed out before leaving this subsection. For one thing, one observes a depolarized component in the backscattered field; if for example the incident wave was polarized in the x_1 direction, then the second integral of (2.45) gives the depolarized component in the y_1 direction. It was pointed out previously that in the case of backscattering from a perfectly conducting surface, the physical optics method did not show any depolarization. However in this case of the non-perfectly conducting surface, at last it has been proved that indeed a depolarization does take place in the backscattering direction, contrary to the statements of several in the literature (e. g., Beckmann [Ref. 1, p. 158] and Hagfors [Ref. 10, p. 3779], and a method has been found to

calculate this depolarized component.* However, it should be pointed out that there are two mechanisms which in practice account for depolarization. One is multiple scattering on the surface, which has not been considered here; this can and does exist on any practically rough surface and plays an increasingly greater role for surfaces having very large surface slopes and at larger angles of incidence (i. e., closer to grazing). The other mechanism producing depolarization, which is the only one considered here, is a result of the sensitivity of the reflection process to the polarization direction of the incident wave with respect to the local plane of incidence at every point on the surface; this mechanism gives rise to two (in general) different local reflection coefficients, $\rho_{\parallel}$ and $\rho_{\perp}$, at every point on the surface.

It is true that (2.45) and (2.46) satisfy Maxwell's equations and the radiation conditions at infinity. Being true far-field quantities, they exhibit no backscattered field components in the direction of propagation, $+z_1$.

In order to convince oneself of the validity of (2.45) (and (2.46)), consider the special case of a perfectly conducting surface; then

$\rho_{\perp} = -\rho_{\parallel} = 1$. Thus (2.45) becomes

$$\bar{E}^s(P') = \frac{jk e^{jkR_0}}{2\pi R_0} E_h^i \hat{x}_1 \iint_D e^{-j2k\zeta_1} dx_1 dy_1,$$

* This depolarization has of course been measured consistently and known to exist from experimentation for years.

which is identically the same as (2.24) except that H is replaced by E . The depolarized component vanishes in this case as it should.

Write the field scattered from an object as follows,

$$(2.47a) \quad \bar{E}^s(P') = a_{hh} \frac{E_h^i}{4\pi R_o} \hat{x}_1 + a_{hv} \frac{E_h^i}{4\pi R_o} \hat{y}_1$$

$$(2.47b) \quad \bar{E}^s(P') = a_{vh} \frac{E_v^i}{4\pi R_o} \hat{x}_1 + a_{vv} \frac{E_v^i}{4\pi R_o} \hat{y}_1,$$

where the a_{ij} are the scattering matrix elements of the object or scattering surface. Equations (2.45) and (2.46) therefore define the scattering matrix element a_{ij} for any object (see Ref. 24). For back-scattering, reciprocity considerations demand that $a_{hv} = a_{vh}$. This requirement is not violated by the results of (2.45) and (2.46), as can be seen from inspection. In general, however, $a_{hh} \neq a_{vv}$; (2.45) and (2.46) show that indeed these quantities are not equal.

The backscattering cross sections are defined as

$$\begin{aligned} \sigma_{hh} &= |a_{hh}|^2 = \frac{4\pi R_o^2 |E_h^s|^2}{|E_h^i|^2} \\ \sigma_{hv} &= \sigma_{vh} = |a_{vh}|^2 = |a_{hv}|^2 = \frac{4\pi R_o^2 |E_h^s|^2}{|E_v^i|^2} = \frac{4\pi R_o^2 |E_v^s|^2}{|E_h^i|^2}, \\ \sigma_{vv} &= |a_{vv}|^2 = \frac{4\pi R_o^2 |E_v^s|^2}{|E_v^i|^2}. \end{aligned}$$

Thus, for example,

$$\sigma_{hh} = \frac{k^2}{\pi} \iiint \left\{ \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \rho_{\parallel} - \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \rho_{\perp}}{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2} \right\} \\ \left\{ \frac{\left(\frac{\partial \zeta'_1}{\partial x'_1} \right)^2 \rho_{\parallel}^{*'} - \left(\frac{\partial \zeta'_1}{\partial y'_1} \right)^2 \rho_{\perp}^{*'}}{\left(\frac{\partial \zeta'_1}{\partial x'_1} \right)^2 + \left(\frac{\partial \zeta'_1}{\partial y'_1} \right)^2} \right\} e^{-jk(\zeta_1 - \zeta'_1)} dx_1 dy_1 \cdot dx'_1 dy'_1 .$$

The primed quantities are the values of these respective quantities at surface point (x'_1, y'_1, ζ'_1) , while the unprimed quantities are their values at surface point (x_1, y_1, ζ_1) . The other backscattering cross sections are found in the same manner as σ_{hh} .

4. Backscattered field in the case of circular polarization

In many cases of practical interest, circularly polarized waves are employed instead of linear. In the case of backscattering from a perfectly conducting surface, right circular incident waves produce a completely polarized scattered field which is left circular. However, in the case of the non-perfectly conducting surface under consideration here, there will be a depolarized scattered right circular component; this component was observed and measured by Evans and Pettengill[20] in the case of backscattering from the moon.

A circularly polarized wave incident upon the scattering surface of Fig. 5 in the $-z_1$ direction is given as follows:

$$(2.48a) \quad \bar{E}_R^i = E_R^i \left(\frac{\hat{x}_1 + j\hat{y}_1}{\sqrt{2}} \right) e^{-jkz_1} ; \bar{E}_L^i = E_L^i \left(\frac{\hat{x}_1 - j\hat{y}_1}{\sqrt{2}} \right) e^{-jkz_1} .$$

The subscripts R and L stand for right and left circularly polarized waves.

When equations (2.48) are substituted into (2.45) and (2.46) and the total scattered fields are computed, they have the form

$\vec{E}^s = E_x^s \hat{x}_1 + E_y^s \hat{y}_1$. A scattered field in this form can be resolved into circular components by the following relationships:

$$(2.48b) \quad E_R^s = \frac{E_x^s + jE_y^s}{2} \quad ; \quad E_L^s = \frac{E_x^s - jE_y^s}{2} .$$

When relations (2.48a) and (2.48b) are used in conjunction with (2.45) and (2.46), the following results are obtained.

a. For right circular incident polarization

$$(2.49a) \quad E_R^s = E_R^i \cdot \frac{1}{2} \left(\frac{-jk e^{jkR_0}}{2\pi R_0} \right) \iint_D (\rho_{\parallel} + \rho_{\perp}) \left\{ \frac{\left(\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 - \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + j2 \frac{\partial \zeta_1}{\partial x_1} \frac{\partial \zeta_1}{\partial y_1} \right)}{\left(\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right)} \right\} e^{-j2k\zeta_1} dx_1 dy_1 ,$$

$$(2.49b) \quad E_L^s = E_R^i \cdot \frac{1}{2} \left(\frac{-jk e^{jkR_0}}{2\pi R_0} \right) \iint_D (\rho_{\parallel} - \rho_{\perp}) e^{-j2k\zeta_1} dx_1 dy_1 .$$

Define $E_R^s = \frac{1}{\sqrt{4\pi} R_0} a_{RR} E_R^i$ and $E_L^s = \frac{1}{\sqrt{4\pi} R_0} a_{LR} E_R^i$.

b. For left circular incident polarization

$$(2.49c) \quad E_R^s = E_L^i \cdot \frac{1}{2} \left(\frac{-jk e^{jkR_0}}{2\pi R_0} \right) \iint_D (\rho_{\parallel} - \rho_{\perp}) e^{-j2k\zeta_1} dx_1 dy_1 ,$$

$$(2.49d) \quad E_L^s = E_L^i \cdot \frac{1}{2} \left(\frac{-jk e^{jkR_0}}{2\pi R_0} \right) \iint_D (\rho_{\parallel} + \rho_{\perp}) \left\{ \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 - \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 - j2 \frac{\partial \zeta_1}{\partial x_1} \frac{\partial \zeta_1}{\partial y_1}}{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2} \right\} e^{-jk\zeta_1} dx_1 dy_1 .$$

Likewise, define $E_R^s = \frac{1}{\sqrt{4\pi R_0}} a_{RL} E_L^i$ and $E_L^s = \frac{1}{\sqrt{4\pi R_0}} a_{LL} E_L^i$.

It is apparent that the backscattering cross sections $|a_{RL}|^2$ and $|a_{LR}|^2$ representing the polarized backscattered components are identical since $a_{RL} = a_{LR}$; reciprocity requirements are again satisfied by the backscattered fields. However it should be noted that $a_{LL} \neq a_{RR}$; the difference between a_{LL} and a_{RR} is small and lies only in the phase angle of the bracketed term in the integrands of (2.49a) and (2.49d) involving the surface slopes (these phase angles are complex conjugates).

The surprisingly simple form of (2.49b) and (2.49c) defining a_{RL} and a_{LR} should be noted; these relationships do not depend upon the surface slopes at all. In the case where the surface becomes a perfect conductor, $\rho_{\perp} = -\rho_{\parallel} = 1$; then (2.49b) and (2.49c) reduce

identically to the equation (2.24) which was derived initially for a perfectly conducting surface. In this case (2.49a) and (2.49d) vanish indicating no circular depolarization from a perfectly conducting surface.

5. Further simplification for rough surfaces

It should be noted that (2.47) and (2.48) as well as (2.49a-d) are valid for any scattering object in the case of backscattering by the physical optics method. The object or surface may be deterministic, such as an airplane, for which the surface $\zeta_1(x_1, y_1)$ has a known mathematical form. In this case the slopes $\partial\zeta_1/\partial x_1$ and $\partial\zeta_1/\partial y_1$ can be determined at every point on the surface and the reflection coefficients are also deterministic functions of the location on the surface. Thus in theory the backscattered fields can always be determined.

When the surface is rough, however, $\zeta_1(x_1, y_1)$ is not known deterministically; only its statistical properties may be known or obtainable. In this case the more complex form of (2.45), (2.46), and equations (2.48) pose some difficulty. However when one restricts the class of surfaces under consideration, several simplifications can be made. These simplifications are discussed here. As examples, consider the following averages.

$$\begin{aligned}
 (2.50a) \quad & \langle |E_R^s|^2 \rangle = \frac{1}{4\pi R_0^2} |E_R^i|^2 \langle |a_{RR}|^2 \rangle = |E_R^i|^2 \cdot \frac{k^2}{4\pi R_0^2} \cdot \frac{1}{4} \\
 & \iiint \langle (\rho_{||} + \rho_{\perp}) (\rho_{||}^{*'} + \rho_{\perp}^{*'}) \left\{ \frac{\left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 - \left(\frac{\partial \zeta_1}{\partial y_1}\right)^2 + j2 \frac{\partial \zeta_1}{\partial x_1} \frac{\partial \zeta_1}{\partial y_1}}{\left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1}\right)^2} \right\} \\
 & \left\{ \frac{\left(\frac{\partial \zeta_1'}{\partial x_1'}\right)^2 - \left(\frac{\partial \zeta_1'}{\partial y_1'}\right)^2 - j2 \frac{\partial \zeta_1'}{\partial x_1'} \frac{\partial \zeta_1'}{\partial y_1'}}{\left(\frac{\partial \zeta_1'}{\partial x_1'}\right)^2 + \left(\frac{\partial \zeta_1'}{\partial y_1'}\right)^2} \right\} e^{-j2k(\zeta_1 - \zeta_1')} \rangle dx_1 dy_1 dx_1' dy_1' ,
 \end{aligned}$$

$$\begin{aligned}
 (2.50b) \quad & \langle |E_L^s|^2 \rangle = \frac{1}{4\pi R_0^2} |E_R^i|^2 \langle |a_{LR}|^2 \rangle = |E_R^i|^2 \cdot \frac{k^2}{4\pi R_0^2} \cdot \frac{1}{4} \\
 & \iiint \langle (\rho_{||} - \rho_{\perp}) (\rho_{||}^{*'} - \rho_{\perp}^{*'}) e^{-j2k(\zeta_1 - \zeta_1')} \rangle dx_1 dy_1 dx_1' dy_1' .
 \end{aligned}$$

a. Simplification of the reflection coefficients

The reflection coefficients are functions of the electrical properties of the surface, i. e., μ_s , ϵ_s , and σ_s ; in general they are therefore complex. Not only that, but these electrical properties are functions of the position on the surface unless the surface is homogeneous. In this analysis it will be assumed either that the scattering surface is homogeneous (so that μ_s , ϵ_s , and σ_s are constants for the entire surface) or that to a first order of approximation the local values of these electrical properties may be replaced by their average values for the entire surface. This approximation is not a serious source of

error and will be employed here. Thus for the class of surfaces under consideration (rock, sand, fresh water, gravel, etc.) $\mu_s = \mu_0$, $\epsilon_s = \epsilon_r \cdot \epsilon_0 = \text{const}$ ($\epsilon_r > 2$ in general), and $\sigma_s = \text{constant}$ ($\sigma_s < 10^{-4}$ mho/meter in general). These surfaces also fall into the category considered by Stratton [Ref. 3, pp. 509-510], i. e., $\sigma_s / \omega^2 \epsilon_s^2 \ll 1$ (at radar frequencies). Therefore, although $\rho_{\parallel}$ and $\rho_{\perp}$ are complex due to the existence of σ_s , their imaginary parts are much less than the real parts in magnitude, so that they may be approximated with little error by pure real numbers.

The reflection coefficients are also functions of θ_i , the local angle of incidence (measured away from the normal to the surface); this angle θ_i at every point on the surface is a function of the local slopes $\partial \zeta_1 / \partial x_1$ and $\partial \zeta_1 / \partial y_1$ at that point. Therefore, $\rho_{\parallel}$ and $\rho_{\perp}$ are quite complicated functions of the surface slopes, and averaging them exactly becomes even more formidable. However, it should be observed that for angles of incidence less than about 60° , the quantities $|\rho_{\parallel} - \rho_{\perp}|^2$ and $|\rho_{\parallel} + \rho_{\perp}|^2$ appearing in (2.50a) and (2.50b) are constant (see Fig. 7). Hence in order to simplify matters, the local incidence angle θ_i will be replaced by θ , the mean incidence angle for the tilted surface. This substitution is completely valid for local surface slopes (due to roughness) $\partial \zeta / \partial x$ and $\partial \zeta / \partial y$ which are less than 20° everywhere and for the mean incidence angle less than 50° . The error introduced when θ is greater than 50° and approaches 90° (grazing)

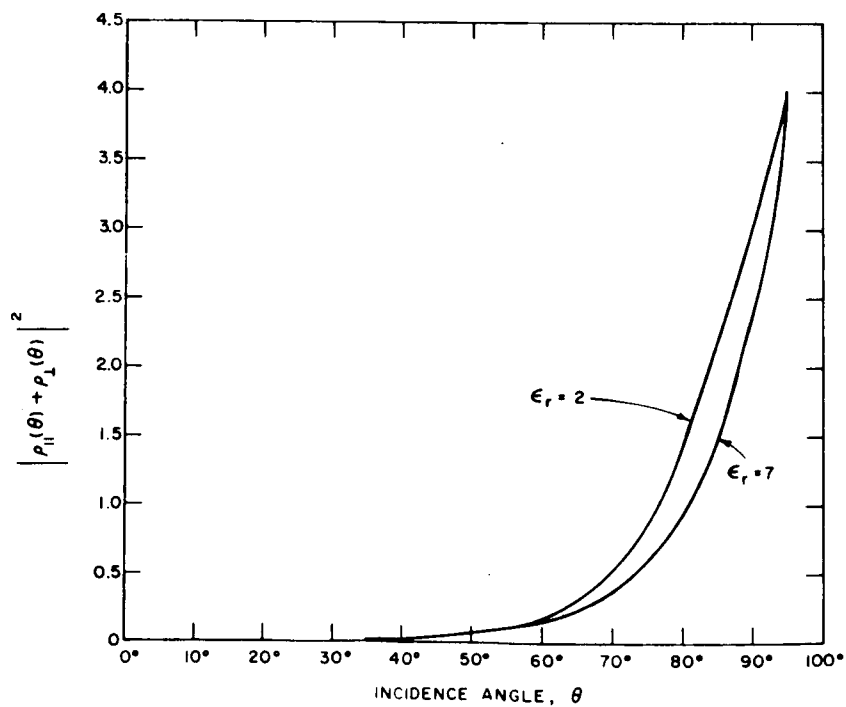
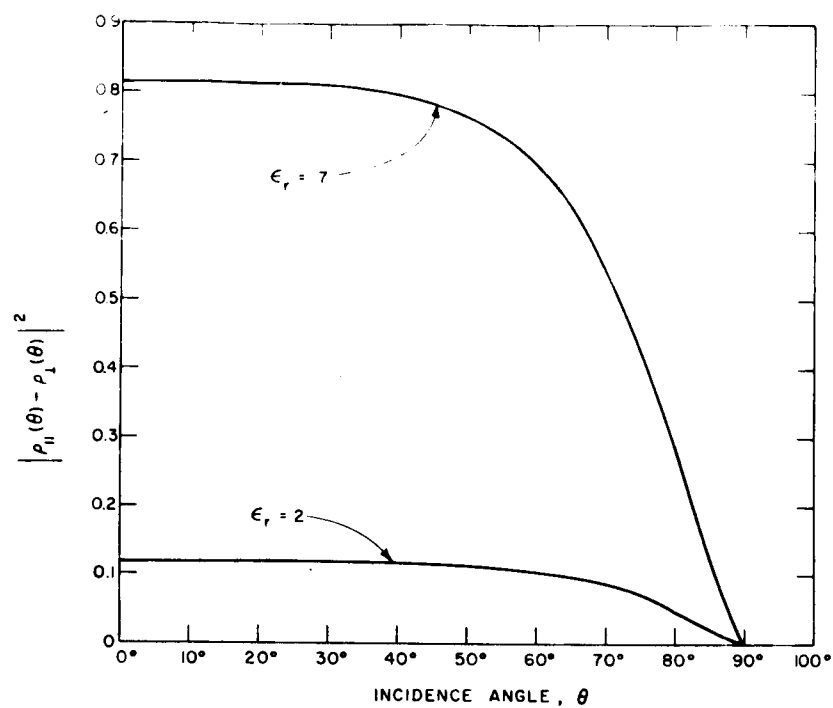


Fig. 7--Magnitudes squared of sum and difference of local reflection coefficients.

is not serious; this error was studied by Mathis[Ref. 25, p. 22] . Therefore, the reflection coefficients are treated as constants at a given mean angle of incidence, θ , and may be removed from the integrand.

b. Statistical independence
of surface slope factor

It has been discussed previously in Section C that factoring of averages such as $\langle f(\zeta_1) g\left(\frac{\partial \zeta_1}{\partial x_1}\right) h\left(\frac{\partial \zeta_1}{\partial y_1}\right) \rangle = \langle f(\zeta_1) \rangle \langle g\left(\frac{\partial \zeta_1}{\partial x_1}\right) \rangle \langle h\left(\frac{\partial \zeta_1}{\partial y_1}\right) \rangle$ is considered valid, i. e. , functions involving surface slope at a point on the surface are considered to be statistically independent from functions involving surface height at that point when statistical averaging is performed. This fact is not difficult to justify for completely random surfaces since there is no reason for any statistical correlation between the slope at any point and the height.* Therefore, equations (2. 50a) and (2. 50b) can be rewritten;

* Another reason to avoid any such assumed dependency relationship is the fact that such an assumption necessitates the selecting of another "model" for the correlation function between ζ_1 and $\partial \zeta_1 / \partial x_1$. Given enough such arbitrary functions and the accompanying constants associated with them, the answer can be made to assume almost any form the user wishes. How meaningful are such results?

$$(2.50c) \quad \langle |E_R^s|^2 \rangle = |E_R^i|^2 \cdot \frac{k^2}{4\pi^2 R_o^2} \cdot \frac{|\rho_{||} + \rho_{\perp}|^2}{4}$$

$$\iiint \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 - \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + j2 \frac{\partial \zeta_1}{\partial x_1} \frac{\partial \zeta_1}{\partial y_1}}{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 - \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2} \right\rangle$$

$$\left\{ \frac{\left(\frac{\partial \zeta'_1}{\partial x'_1} \right)^2 - \left(\frac{\partial \zeta'_1}{\partial y'_1} \right)^2 - j2 \frac{\partial \zeta'_1}{\partial x'_1} \frac{\partial \zeta'_1}{\partial y'_1}}{\left(\frac{\partial \zeta'_1}{\partial x'_1} \right)^2 + \left(\frac{\partial \zeta'_1}{\partial y'_1} \right)^2} \right\} \rangle e^{-j2k(\zeta_1 - \zeta'_1)} dx_1 dy_1 dx'_1 dy'_1 ,$$

$$(2.50d) \quad \langle |E_L^s|^2 \rangle = |E_R^i|^2 \cdot \frac{k^2}{4\pi^2 R_o^2} \cdot \frac{|\rho_{||} - \rho_{\perp}|^2}{4}$$

$$\iiint \langle e^{-j2k(\zeta_1 - \zeta'_1)} \rangle dx_1 dy_1 dx'_1 dy'_1 .$$

c. Correlation between the factors in braces

The quantities in braces involving the surface slopes at two different points on the surface (x_1, y_1, ζ_1) and (x'_1, y'_1, ζ'_1) are not uncorrelated. A method for finding the correlation coefficient between the slopes $\partial \zeta_1 / \partial x_1$ and $\partial \zeta'_1 / \partial x'_1$ at two different points was derived in Section C; this correlation coefficient is a function of the distance ρ between the two points on the surface. Therefore, the average of the surface slope factor may be written as

$$\langle \{ \quad \} \{ \quad \}'^* \rangle = X(R_{S_x}(\rho), R_{S_y}(\rho)),$$

where $R_{S_x}(\rho)$ and $R_{S_y}(\rho)$ are the correlation coefficients between the quantities $\partial\zeta/\partial x$ and $\partial\zeta'/\partial x'$ and $\partial\zeta/\partial y$ and $\partial\zeta'/\partial y'$ respectively. As $\rho \rightarrow 0$, $R_{S_x}(\rho) \rightarrow R_{S_y}(\rho) \rightarrow 1$, which means $\partial\zeta/\partial x = \partial\zeta'/\partial x'$ and $\partial\zeta/\partial y = \partial\zeta'/\partial y'$.

It can be shown that for the class of surfaces under consideration, i. e., very rough surfaces where $\sigma k \gg 1$ (σ = rms surface height) that the characteristic function $\langle e^{j2k(\zeta - \zeta')} \rangle = M_{\zeta\zeta'}(j2k, -j2k; \rho)$ in the integrand becomes vanishingly small for ρ slightly different from zero. This means that as ρ becomes different from zero, $M_{\zeta\zeta'}(j2k, -j2k; \rho)$ has become vanishingly small while $R_{S_x}(\rho)$ and $R_{S_y}(\rho)$ have not yet differed significantly from unity.* Hence for the non-zero range of the characteristic function, the surface slopes at the points (x_1, y_1, ζ_1) and (x'_1, y'_1, ζ'_1) may be considered to be completely correlated so that in this range, $\partial\zeta_1/\partial x_1 = \partial\zeta'_1/\partial x'_1$ and $\partial\zeta_1/\partial y_1 = \partial\zeta'_1/\partial y'_1$. Therefore the average of the quantities in brackets may be simplified and removed from the integrand since they are independent of ρ for the non-zero range of the integrand. Then (2.50c) becomes

* This fact is demonstrated in Appendix C for the case where the surface heights obey the Gaussian JPDF. The reader is advised to read through the next part on the application of the Gaussian JPDF before reading Appendix C.

$$\begin{aligned}
\langle |E_R^s|^2 \rangle &= |E_R^i|^2 \cdot \frac{k^2}{4\pi^2 R_o^2} \frac{|\rho_{\parallel} + \rho_{\perp}|^2}{4} \\
&\times \left| \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 - \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + j2 \frac{\partial \zeta_1}{\partial x_1} \frac{\partial \zeta_1}{\partial y_1}}{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2} \right|^2 > \iiint \langle e^{-j2k(\zeta_1 - \zeta'_1)} \rangle \\
&\quad dx_1 dy_1 dx'_1 dy'_1 \\
&= |E_R^i|^2 \cdot \frac{k^2}{4\pi^2 R_o^2} \frac{|\rho_{\parallel} + \rho_{\perp}|^2}{4} \times \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^4 + 2 \left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} >
\end{aligned}$$

$$\iiint \langle e^{-j2k(\zeta_1 - \zeta'_1)} \rangle dx_1 dy_1 dx'_1 dy'_1$$

$$(2.50e) \quad \therefore \langle |E_R^s|^2 \rangle = |E_R^i|^2 \cdot \frac{k^2}{4\pi^2 R_o^2} \frac{|\rho_{\parallel} + \rho_{\perp}|^2}{4}$$

$$\iiint \langle e^{-j2k(\zeta_1 - \zeta'_1)} \rangle dx_1 dy_1 dx'_1 dy'_1 \quad .$$

6. Average backscattering cross sections for rough surfaces

Using the simplifying assumptions stated in the previous subsection to evaluate the average backscattering cross sections in the same manner as was done for $|a_{RR}|^2$ and $|a_{LR}|^2$, one can find the remaining average cross sections defined by equations (2.45), (2.46), and (2.49).

Define: $Q \equiv \frac{k^2}{\pi} \iiint_D \langle e^{-j2k(\zeta_1 - \zeta'_1)} \rangle dx_1 dy_1 dx'_1 dy'_1$.

This integral has been simplified in section B subsection 2 as seen from (2.27) and the result is restated here.

$$Q = 2k^2 L_x L_y \cos^2 \theta \int_0^\infty \rho J_0(2k \sin \theta \rho) M_{\zeta \zeta'}(j2k \sec \theta, -j2k \sec \theta; \rho) d\rho .$$

Therefore, written in terms of Q , the average backscattering cross sections become

$$\begin{aligned} (2.51a) \quad \langle \sigma_{hh} \rangle &= \langle |a_{hh}|^2 \rangle = \frac{4\pi R_o^2 \langle |E_h^s|^2 \rangle}{|E_h^i|^2} \\ &= \left\{ | \rho_{||} |^2 \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle - 2\text{Re}(\rho_{||} \rho_{\perp}^*) \right. \\ &\quad \left. \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle + | \rho_{\perp} |^2 \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial y_1} \right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle \right\} \cdot Q , \end{aligned}$$

$$\begin{aligned} (2.51b) \quad \langle \sigma_{vv} \rangle &= \langle |a_{vv}|^2 \rangle = \frac{4\pi R_o^2 \langle |E_v^s|^2 \rangle}{|E_v^i|^2} \\ &= \left\{ | \rho_{||} |^2 \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial y_1} \right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle - 2\text{Re}(\rho_{||} \rho_{\perp}^*) \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle \right. \\ &\quad \left. + | \rho_{\perp} |^2 \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle \right\} \cdot Q , \end{aligned}$$

$$\begin{aligned}
 (2.51c) \quad & \langle \sigma_{hv} \rangle = \langle \sigma_{vh} \rangle = \langle |a_{hv}|^2 \rangle = \langle |a_{vh}|^2 \rangle = \frac{4\pi R_o^2 \langle |E_h^s|^2 \rangle}{|E_v^i|^2} \\
 & = \frac{4\pi R_o^2 \langle |E_v^s|^2 \rangle}{|E_h^i|^2} = \left\{ | \rho_{||} + \rho_{\perp} |^2 \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle \right\} \cdot Q,
 \end{aligned}$$

$$\begin{aligned}
 (2.52a) \quad & \langle \sigma_{RR} \rangle = \langle \sigma_{LL} \rangle = \langle |a_{RR}|^2 \rangle = \langle |a_{LL}|^2 \rangle \\
 & = \frac{4\pi R_o^2 \langle |E_R^s|^2 \rangle}{|E_R^i|^2} = \frac{4\pi R_o^2 \langle |E_L^s|^2 \rangle}{|E_L^i|^2} = \frac{1}{4} | \rho_{||} + \rho_{\perp} |^2 \cdot Q,
 \end{aligned}$$

$$\begin{aligned}
 (2.52b) \quad & \langle \sigma_{RL} \rangle = \langle \sigma_{LR} \rangle = \langle |a_{RL}|^2 \rangle = \langle |a_{LR}|^2 \rangle \\
 & = \frac{4\pi R_o^2 \langle |E_R^s|^2 \rangle}{|E_L^i|^2} = \frac{4\pi R_o^2 \langle |E_L^s|^2 \rangle}{|E_R^i|^2} = \frac{1}{4} | \rho_{||} - \rho_{\perp} |^2 \cdot Q.
 \end{aligned}$$

Another fact becomes apparent after averaging: $\langle \sigma_{RR} \rangle = \langle \sigma_{LL} \rangle$ for an isotropically rough surface, while in general $\langle \sigma_{hh} \rangle \neq \langle \sigma_{vv} \rangle$. This fact has been deduced by several purely from physical reasoning and consideration of the nature of circular polarization as compared with linear and the scattering properties of each.

Equations (2.51) and (2.52) show that once the solution for average backscattered power from a perfectly conducting surface has been determined, i. e., Q , this solution needs to be multiplied by various factors involving averages of surface slopes and the reflection (or Fresnel) coefficients to obtain the polarization sensitive solutions for average backscattered power from non-perfectly conducting surfaces.

CHAPTER III

PREDICTED BACKSCATTERED POWER USING STATISTICAL MODELS

A. Examination of Certain Statistical Models

Although it is possible in theory to obtain a knowledge of the surface statistics from measurements of waves reflected from the surface at many frequencies, as shown in the previous chapter (section C, subsection 2), it is evident that without such measurements, equations (2.24) and (2.27) left in terms of the joint characteristic function of the surface provide little insight into the behavior of backscattered power from the surface as a function of angle of incidence, mean square surface height, mean square surface slope, and frequency. It is for this reason of providing estimates and predictions of the functional behavior of backscattered power as a function of these various parameters that statistical models are employed.

In this section, two mathematical forms for the joint probability density function for the surface height will be considered in conjunction with two classes of surface correlation coefficients. Various facts about these statistical models are ascertained and discussed here.

1. The Gaussian correlation coefficient: $R(\rho) = e^{-\rho^2/a^2}$

The Gaussian correlation coefficient is often used in the literature because of its rapid convergence at infinity, its zero slope at the origin, and the likewise desirable qualities of its Fourier transform. The correlation length, is defined as "a", and its meaning is evident from Fig. 8. If one uses this form for the correlation coefficient, he can find the mean square slope of the surface from use of either (2.39) directly or from use of (2.31a) to find the surface "power density spectrum", $P(\xi)$, and then using (2.34b) to find the mean square slope; however, the latter integral must exist in order to be able to obtain a finite mean square slope.

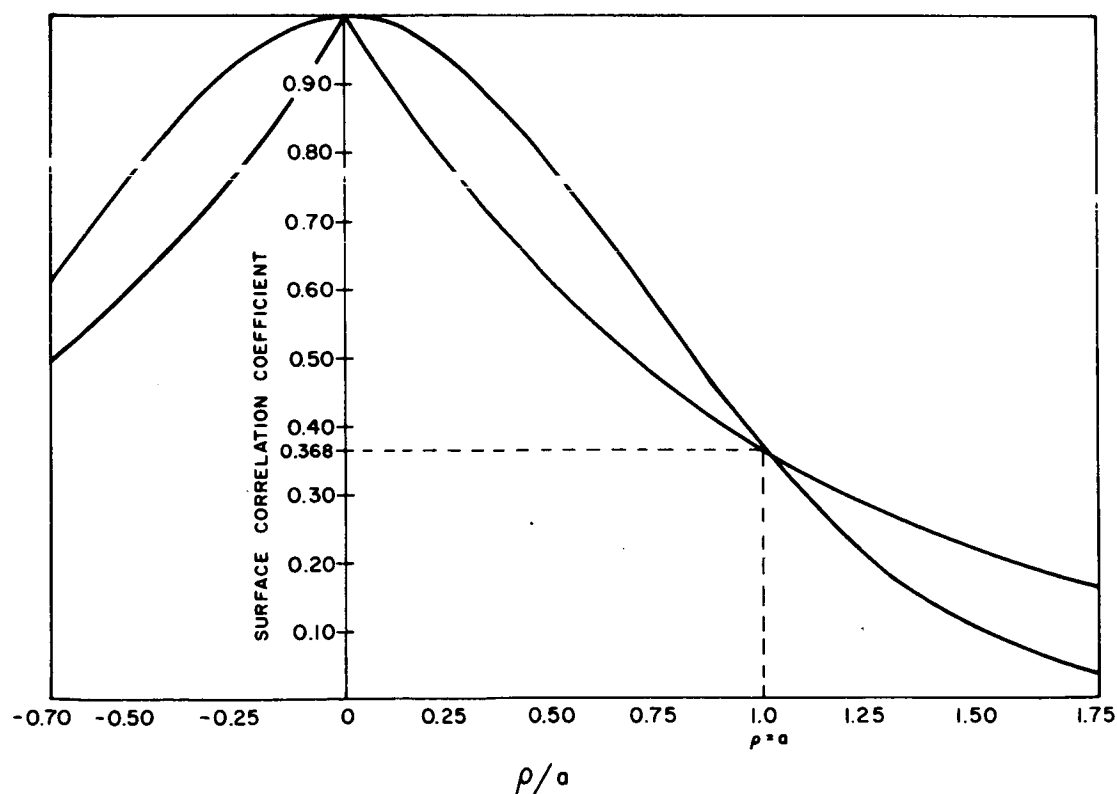


Fig. 8--Gaussian and exponential correlation coefficients.

The power spectrum of the surface having a Gaussian correlation coefficient, obtained from (2.31a), is

$$P(\xi) = 4\pi a^2 \sigma^2 e^{-a^2 \xi^2 / 4}.$$

Employing (2.34b), one finds that the mean square slope exists and is given by

$$(3.1) \quad S^2 = \frac{4\sigma^2}{a^2}.$$

This same result is obtained by using (2.39), i. e.,

$$S^2 = \frac{4\sigma^2}{a^2} = -\sigma^2 \left[\left. \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_x^2} \right|_{\substack{\tau_x=0 \\ \tau_y=0}} + \left. \frac{\partial^2 R(\sqrt{\tau_x^2 + \tau_y^2})}{\partial \tau_y^2} \right|_{\substack{\tau_x=0 \\ \tau_y=0}} \right],$$

where $\rho^2 = \tau_x^2 + \tau_y^2$. However, if one uses the incorrect formulation $S^2 = -\sigma^2 R''(0)$, one obtains $S^2 = \frac{2\sigma^2}{a^2}$; this erroneous value for slope is used by many such as Beckmann and Davies.

When the correlation coefficient is used later in the integration of the characteristic function, it becomes apparent that only the behavior of $R(\rho)$ near zero is important. In this region the Gaussian correlation coefficient is parabolic in shape as can be seen from the first two terms of its series expansion (i. e., $e^{-\rho^2/a^2} \simeq 1 - \frac{\rho^2}{a^2}$ for $\frac{\rho^2}{a^2} \ll 1$).

If one chooses a correlation coefficient of the form $\frac{1}{1 + \rho^2/a^2}$ or $\frac{1}{(1 + 2\rho^2/a^2)^{1/2}}$, (or in general $1 + \frac{\rho^2}{a^2} + C_2 \frac{\rho^4}{a^4} + C_3 \frac{\rho^6}{a^6} + \dots$)

and calculates the mean square slope, S^2 , it turns out to be the same as for the Gaussian correlation coefficient, i. e., $\frac{4\sigma^2}{a^2}$. This is due of course to the fact that the second term of their MacLaurin series is quadratic and identical; it is this second quadratic term which determines the mean square slope, as can be seen from (2.39). The rate of convergence of $R(\rho)$ to zero at large ρ is immaterial. Therefore, all correlation coefficients which are parabolic in behavior at the origin such that $R(\rho) = 1 - \frac{\rho^2}{a^2}$ for $\frac{\rho^2}{a^2} \ll 1$ yield identical results when applied to scattering from rough surfaces and will be classed together with the Gaussian correlation coefficient as belonging to the "Gaussian class" as far as this analysis is concerned.

2. The exponential correlation coefficient: $R(\rho) = e^{-|\rho|/a}$

Many authors favor the use of the exponential correlation coefficient, believing that it is more physically reasonable[18]. It is easily verified by use of (4-4a) that the power density spectrum of such a surface is $P(\xi) = \frac{8\pi\sigma^2a^2}{(1+a^2\xi^2)^{3/2}}$; using this function in (2.34b) to find the mean square slope, one notes that the integral diverges, indicating that the mean square slope is infinite in this case. This fact was noted by Daniels[19]. Such a surface has vertical points (at least one such point) where the slope is infinite.

It is also interesting to note that any correlation coefficient which exhibits linear behavior at the origin (i. e., $1 - \frac{|\rho|}{a}$ for $\frac{\rho}{a} < 1$) will yield an infinite mean square slope. This was also noted by Daniels. However, Beckmann[21] chooses to define a mean square slope in this case from the incorrect formula $S^2 = -\sigma^2 R''(0)$; since admittedly the second derivative does not exist at the origin, he chooses to define it as a limiting process $S^2 = -\sigma^2 \lim_{\epsilon \rightarrow 0} R''(\epsilon)$ and arrives at a value $S^2 = \frac{\sigma^2}{a^2}$. That such an analysis is faulty and superficial is quite evident.

This, however, does not mean that rough surfaces cannot in practice have exponential correlation coefficients exhibiting infinite mean square slopes (implying vertical points). It is quite likely that the small scale surface roughness of the lunar surface has such vertical points just as does the small scale structure of many points of the earth's surface (e. g., grass, houses, boulders, etc.)

It is proposed by Daniels, who shows an optical profile of the large scale lunar roughness made by Hayn and compares it with a time signal of a similar having a Gaussian correlation coefficient, that the large scale roughness is best described by a Gaussian class of correlation coefficients. This belief is shared also by this author. The exponential correlation function may still be employed when one wishes to consider classes of surfaces having many vertical

points in the largest scale structure. It will be employed in this paper in connection with the Gaussian JPDF to compute and compare with backscattered power from the lunar surface.

3. The Gaussian joint probability density function

The joint characteristic function for the Gaussian JPDF is well known and has the form

$$(3.2) \quad M_{\zeta\zeta'}(ju, jv; \rho) = e^{-\frac{\sigma^2}{2}(u^2 + 2uvR(\rho) + v^2)}.$$

Since the backscattered power is to be obtained from (4.4) and (4.9), the arguments of the characteristic function are equal to each other in magnitude in this case (i. e., $u = -v = k\gamma_z$ in the first case and $2kn$ in the second case); by definition of a very rough surface, $k\sigma \gg 1$.

Equation (3.2) becomes

$$(3.3) \quad M_{\zeta\zeta'}(ju, -ju; \rho) = e^{-\sigma^2 u^2 + \sigma^2 u^2 R(\rho)}.$$

It should be noted that in this case $M_{\zeta\zeta'}(ju, 0; \infty) = M_{\zeta\zeta'}(0, -ju; \infty) = e^{-\sigma^2 u^2/2} \simeq 0$, since $\sigma u \gg 1$. Therefore the second terms in the integrand of (2.22) and (2.27) may be neglected since they are very small.

Since $\sigma^2 u^2 \gg 1$, it should be noted that (3.3) is significantly different from zero only when $R(\rho)$ is very close to unity (at ρ very close to zero). Therefore, $R(\rho)$ may be approximated by the first two terms in its series.

$$(3.4a) \quad M_{\zeta\zeta'}(ju, -ju; \rho) = e^{-\sigma^2 u^2 \frac{\rho^2}{a^2}} \quad \text{for the Gaussian class of correlation coefficients}$$

$$(3.4b) \quad M_{\zeta\zeta'}(ju, -ju; \rho) = e^{-\sigma^2 u^2 \frac{\rho}{a}} \quad \text{for the exponential class of correlation coefficients and } \rho > 0.$$

These characteristic functions will be used in the next section to compute and compare the backscattered power from perfectly conducting surfaces.

4. A modified Bessel joint probability density function

In order to be able to compare the results obtained by using at least one other JPDF with those obtained from the Gaussian JPDF, the author sought such a function which possesses the same desirable qualities as the Gaussian (e.g., converges to zero at $\pm\infty$, is symmetrical about the mean, has zero slope at the mean, etc.). Ott[16] used a form of a modified Bessel function and showed that this function gave better agreement with measured power at angles of incidence near normal than did the Gaussian JPDF when both were used in conjunction with the Gaussian class of correlation coefficients to

predict the backscattered power from a perfectly conducting surface. The author has no physical reason or explanation why such a JPDF might be better suited to describe a rough surface than the Gaussian.

The form used by Ott involved a slight error or oversight in one of the correlation coefficients. The corrected form of the JPDF to be used here is written in terms of the conditional probability density function:

$$(3.5a) \quad W_c(\zeta' | \zeta) = \frac{\sqrt{3}}{\pi\sigma(1-R^2)^{1/2}} \left[\frac{\sqrt{3} |\zeta' - R\zeta|}{\sigma(1-R^2)^{1/2}} \right] K_1 \left[\frac{\sqrt{3} |\zeta' - R\zeta|}{\sigma(1-R^2)^{1/2}} \right] .$$

The above conditional probability density function expresses the probability of finding the random variable ζ' within an increment $d\zeta'$ when the value of ζ is already known and specified. The single probability density function of the above form is found by setting the correlation coefficient, R , equal to zero and interchanging ζ and ζ' .

$$(3.5b) \quad W_s(\zeta) = \frac{\sqrt{3}}{\pi\sigma} \left[\frac{\sqrt{3} |\zeta|}{\sigma} \right] K_1 \left[\frac{\sqrt{3} |\zeta|}{\sigma} \right] .$$

The JPDF is then defined in terms of these two density functions as follows:

$$(3.5c) \quad W(\zeta, \zeta') = W_s(\zeta) W_c(\zeta' | \zeta) \quad . \quad [\text{Refs. 14 and 15}]$$

The random variables ζ and ζ' , representing the surface height at different points, have zero means, variances σ^2 , and covariance function $\sigma^2 R$ when equations (3.5) define their JPDF. However it

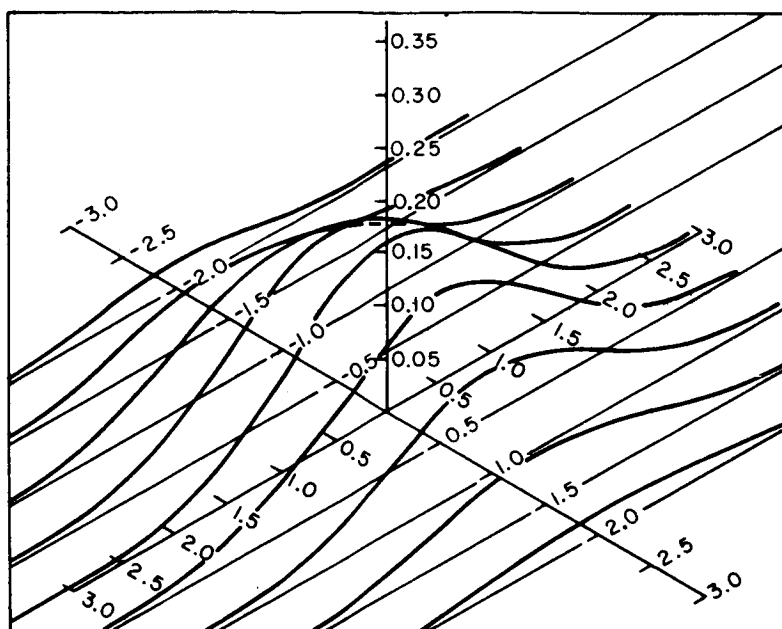
should be noted that the above JPDF is not symmetric in ζ and ζ' as it must be in order to describe a true physical surface. However, this lack of symmetry is not serious at values of R near unity and therefore this deficiency will be overlooked since no serious attempt is being made to justify this modified Bessel JPDF as having any real meaning, but merely for the sake of comparison.

In order to better see the behavior of this JPDF in comparison with the Gaussian, three dimensional plots were prepared for $W(\zeta, \zeta')$ as functions of ζ/σ and ζ'/σ at three values of the correlation coefficient, R (see Figs. 9, 10, 11).

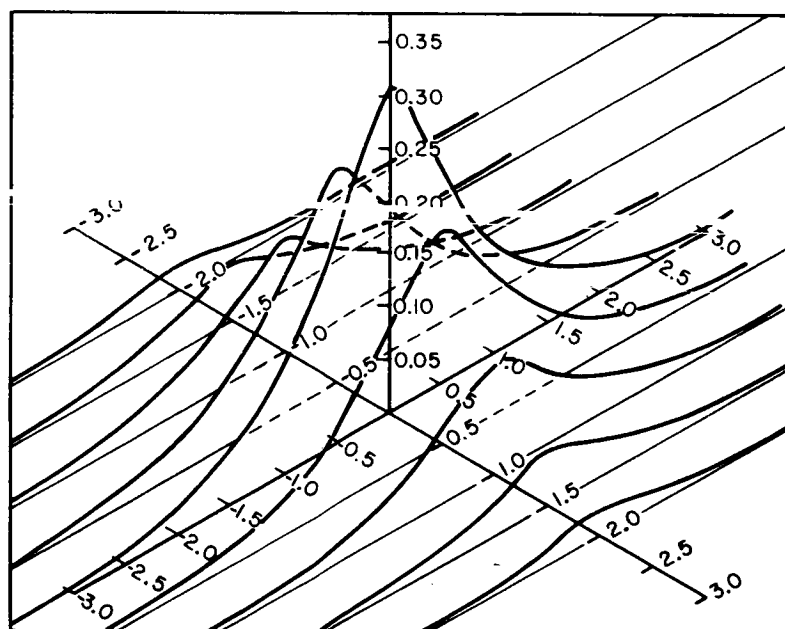
It can be seen that the lack of symmetry is not serious at $R = 0.98$ (only those values of R close to unity significantly affect the integral value). In general, the Bessel JPDF has higher and narrower peaks than the Gaussian, but decays more slowly at large ζ and ζ' (it is proportional to $\sqrt{|\zeta|} e^{-\sqrt{3}|\zeta|/\sigma}$ as $\zeta \rightarrow \infty$ whereas the Gaussian JPDF is proportional to $e^{-\zeta^2/2\sigma^2}$ as $\zeta \rightarrow \infty$). Both the Gaussian and Bessel JPDF degenerate to impulse functions along the $\zeta = \zeta'$ line as $R \rightarrow 1$ as they should, preserving unity volume under the surface.

The joint characteristic function for (3. 5) is derived in Appendix D and is merely stated here.

$$(3. 5d) \quad M_{\zeta\zeta'}(ju, jv; \rho) = \frac{1}{\left[1 + \frac{u^2 \sigma^2 (1-R^2)^{3/2}}{3} \right]} \cdot \frac{1}{\left[1 + \frac{\sigma^2}{3} (uR + v)^2 \right]^{3/2}} .$$



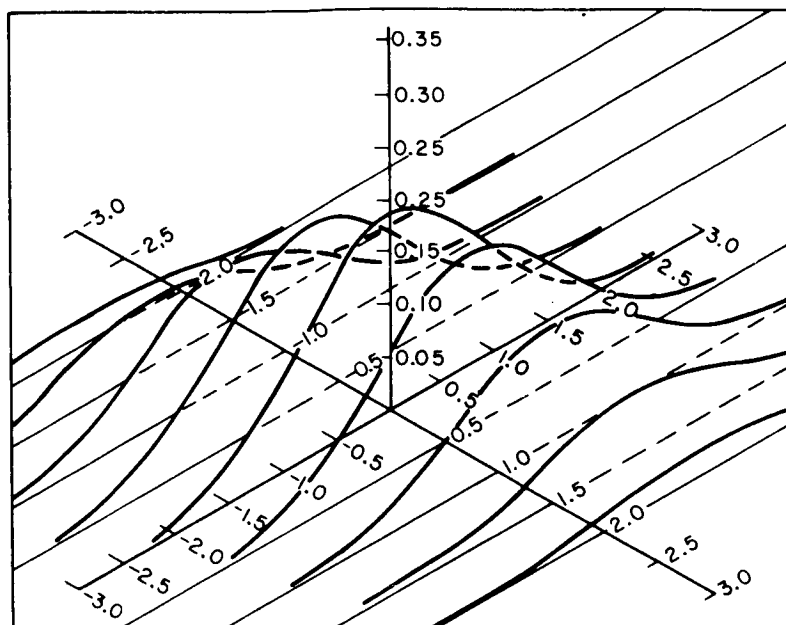
(a)



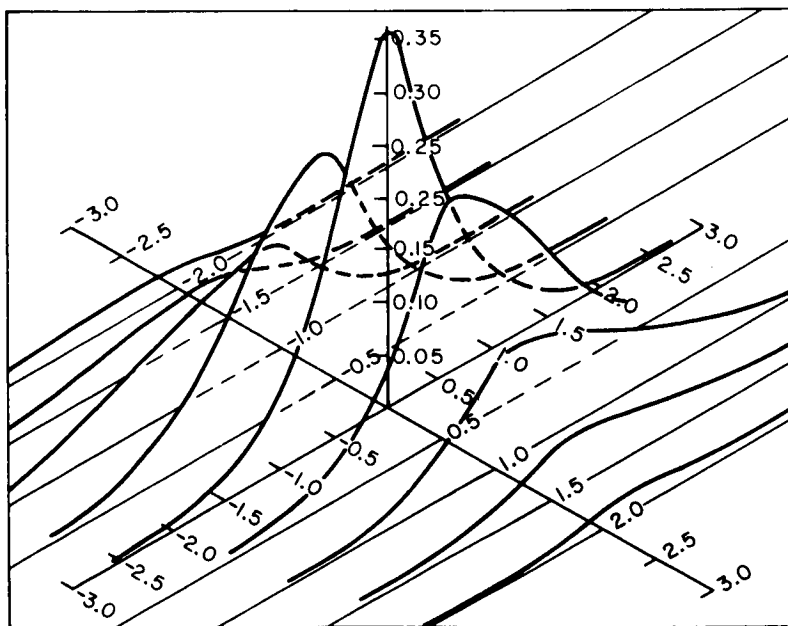
(b)

Fig. 9--Gaussian and Bessel joint probability density functions - Correlation coefficient = 0. 0.

- (a) Gaussian joint probability density function.
 (b) Bessel joint probability density function.



(a)

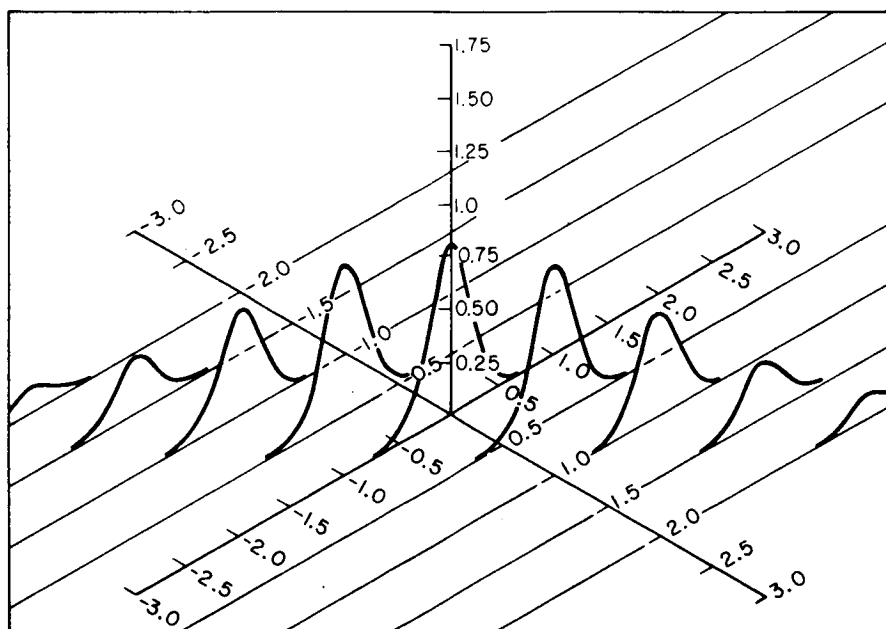


(b)

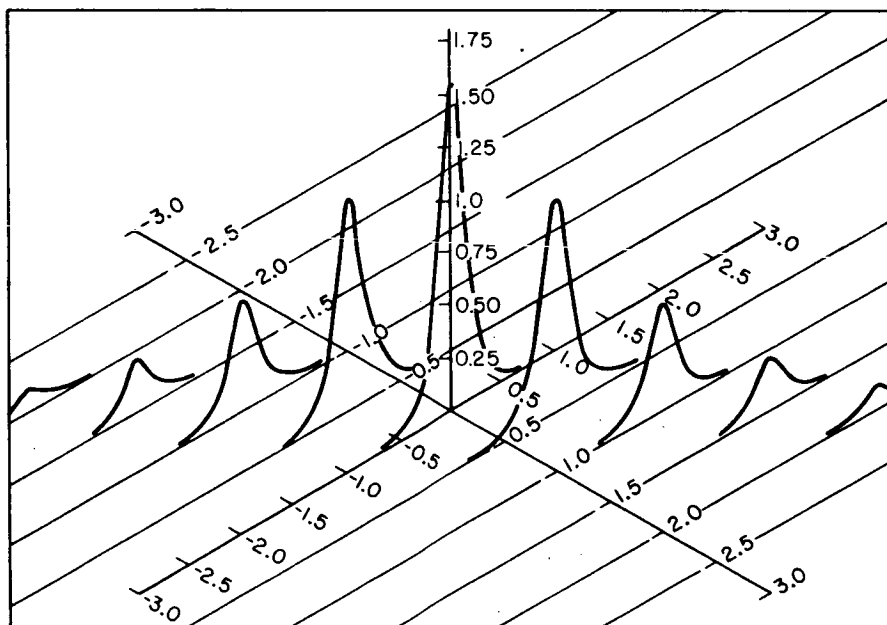
Fig. 10--Gaussian and Bessel joint probability density functions - Correlation coefficient = 0.5.

(a) Gaussian joint probability density function

(b) Bessel joint probability density function



(a)



(b)

Fig. 11--Gaussian and Bessel joint probability density functions - Correlation coefficient = 0.98.

(a) Gaussian joint probability density function

(b) Bessel joint probability density function

Note that when $R = 0$ (i. e. , when $\rho \rightarrow \infty$) and the random variables are uncorrelated, then

$$\begin{aligned}
 (3.5e) \quad M_{\zeta\zeta'}(ju, jv; \infty) &= \frac{1}{\left(1 + \frac{u^2 \sigma^2}{3}\right)^{3/2} \left(1 + \frac{v^2 \sigma^2}{3}\right)^{3/2}} \\
 &= M_{\zeta\zeta'}(ju, 0; \infty) M_{\zeta\zeta'}(0, jv; \infty) \quad .
 \end{aligned}$$

Thus when ζ and ζ' are uncorrelated, they are also statistically independent because their joint characteristic function factors into the product of the two single characteristic functions. From (3.5e) it may be noted that when $\sigma u, \sigma v \sim \sigma k \gg 1$, then (3.5e) is vanishingly small and the second term of the integrand is again negligible.

B. Backscattered Power from a Perfectly Conducting Surface

Using the joint characteristic functions for the statistical models discussed in the preceding section in equations (2.23) and (2.27), one can arrive at closed form solutions for backscattered power as a function of mean angle of incidence away from normal, θ , and other statistical parameters of the rough surface. It should be recalled that (2.23) was derived for a perfectly conducting isotropically rough surface using the standard Kirchhoff-Huygen integral approximation for the scattered field (which is valid only near normal incidence) while (2.27) was derived directly from the Chu-Stratton vector integral equation and should be accurate at all angles of incidence (the accuracy here

depends for the most part upon the validity of the physical optics approximation). The reason that the less accurate integral is carried along throughout the analysis of this section is to contrast the difference between solution obtained from it and the solution obtained from the more accurate integral (2.27) at larger angles of incidence. Both of these solutions are compared with Pettengill's measurements from the moon at 68 cm wavelength in order to point out the far superior agreement of the results of equation (2.27) over the previously derived solutions employing (2.23) or some modification of it.

The second term in the integrands of (2.23) is omitted here because it was shown in the preceding section that for all the statistical models it is much smaller than the first term since only very rough surfaces are considered here, where $k\sigma \gg 1$.

It was shown in the previous section dealing with non-perfectly conducting surfaces that the polarization sensitive backscattered power for all polarization combinations depends upon a factor, Q , which is the solution for a perfectly conducting surface, i. e., (2.23) or (2.27). Thus the results derived here employing statistical models for the perfectly conducting surface will be used later in connection with the non-perfectly conducting surface.

1. Solution employing the less accurate integral (2.23)

Neglecting the second term, this integral becomes

$$(3.6) \quad \langle |H_1^s|^2 \rangle = \frac{k^2 L_x L_y |H_0^i|^2}{2\pi R_0^2} \int_0^\infty \rho J_0(2k \sin \theta \rho) M_{\zeta\zeta_1}(j2k \cos \theta; -j2k \cos \theta; \rho) d\rho.$$

If $\langle \sigma_{pc} \rangle = \frac{4\pi R_0^2 \langle |H_1^s|^2 \rangle}{|H_0^i|^2}$ is the backscattering cross-section for a perfect conducting surface (irrespective of polarizations), then one can define an average back-scattering cross section per unit area of the scattering surface as $\gamma_{pc} = \frac{\langle \sigma_{pc} \rangle}{L_x L_y}$.

Writing (3.6) in terms of γ_{pc} , one obtains

$$(3.7) \quad \gamma_{pc} = 2k^2 \int_0^\infty \rho J_0(2k \sin \theta \rho) M_{\zeta\zeta_1}(j2k \cos \theta, -j2k \cos \theta; \rho) d\rho.$$

a. Gaussian JPDF, Gaussian correlation coefficient

For this statistical model, the joint characteristic function 102a derived in the previous section is substituted into 111 and the integral is evaluated (it is easily found in integral tables).

$$(3.8a) \quad \gamma_{pc} = \frac{1}{S^2} \cdot \frac{1}{\cos^2 \theta} e^{-\frac{1}{S^2} \tan^2 \theta},$$

where $S^2 = \frac{4\sigma^2}{a^2}$ is employed in the case of the Gaussian class of correlation coefficients, as was shown in the preceding section.

b. Gaussian JPDF, exponential correlation coefficient

Equation (3.4b) in this case is substituted into (3.7) and the integral is evaluated from the tables. The result is

$$(3.8b) \quad \gamma_{pc} = \frac{a^2}{8\sigma^4 k^2} \cdot \frac{1}{\cos^4 \theta} \cdot \frac{1}{\left(1 + \frac{a^2}{4\sigma^4 k^2} \frac{\sin^2 \theta}{\cos^4 \theta}\right)^{3/2}}.$$

Both (3.8a) and (3.8b) are not new and have appeared in various papers from time to time, sometimes with a difference of a factor of $\cos^2 \theta$ depending upon the form of the Kirchhoff-Helmholtz approximation initially chosen.

c. Bessel JPDF, Gaussian correlation coefficient

When one substitutes (3.5d) for the joint characteristic function into (3.7) and simplifies the form considerably (for the details of this simplification see Appendix D), one arrives at the result

$$(3.8c) \quad \gamma_{pc} = \frac{1}{S^2} \cdot \frac{3}{\cos^2 \theta} e^{-\frac{\sqrt{6}}{S} \tan \theta}.$$

This result is similar in form to that derived by Ott[16] but differs by a constant $\sqrt{2}$ wherever the rms slope, S, appears; this difference is believed due to the slightly incorrect form of the JPDF which he started with.

2. Solution employing the more accurate integral (2.27)

When one defines an average backscattering cross section per unit area in terms of (2.27), one arrives at the result

$$(3.9) \quad \gamma_{pc} = 2k^2 \cos^2 \theta \int_0^\infty \rho J_0(2k \sin \theta \rho) M_{\zeta\zeta'}(j2k \sec \theta, -j2k \sec \theta; \rho) d\rho = \frac{Q}{L_x L_y},$$

where $Q = \sigma_{pc}$ (the total backscattering cross section for a perfect conducting surface) was defined in the section dealing with non-perfectly conducting surfaces.

a. Gaussian JPDF, Gaussian correlation coefficient

Upon substitution of the joint characteristic function of (3.4a) into (3.9), the integral to be solved has the same form as before as a function of ρ ; the difference is that $\cos \theta$ in the argument is replaced by $\sec \theta$. The result is

$$(3.10a) \quad \gamma_{pc} = \frac{1}{S^2} \cos^4 \theta e^{-\frac{1}{S^2} \sin^2 \theta \cos^2 \theta}$$

b. Gaussian JPDF, exponential correlation coefficient

Employing (3.4b) in (3.9), one obtains the solution

$$(3.10b) \quad \gamma_{pc} = \frac{a^2}{8\sigma^4 k^2} \cos^6 \theta \cdot \frac{1}{\left(1 + \frac{a^2}{4\sigma^4 k^2} \sin^2 \theta \cos^4 \theta\right)^{3/2}}$$

c. Bessel JPDF, Gaussian correlation coefficient

Substituting (3.5d) into (3.9), simplifying, and performing the integration, one obtains

$$(3.10c) \quad \gamma_{pc} = \frac{1}{S^2} \cdot 3 \cos^4 \theta e^{-\frac{\sqrt{6}}{S} \sin \theta \cos \theta}.$$

3. Comparison and discussion of results

Differences between the results for backscattered power obtained from the less accurate integral and those from the more accurate integral are obvious.

a. Near normal incidence, i. e. , for θ near zero, $\cos \theta \simeq 1 \simeq \sec \theta$, $\tan \theta \simeq \sin \theta$, and the results of subsection 1 in all cases are identical with the results of subsection 2.

b. Further away from normal incidence, i. e. , for $\theta > 20^\circ$, the results obtained in subsection 1 in all cases decrease toward zero much more rapidly than the results of subsection 2; this rapid decrease of power with θ above 20° has always plagued the theoreticians who compare their results with measured data.

In order to present a graphical comparison, the results of both integrals are plotted as a function of θ and compared with Pettengill and Evans' measured power from the moon at 68 cm wavelength using circular polarization and a pulse length of 12 μ sec. (The form of polarization used to make the measurements is immaterial when the

results are to be compared with those from a perfectly conducting surface, since such a surface is blind to all polarization information.) The measured curve of cross-section per unit area is plotted first absolutely; this can be done by integrating the measured curve graphically as a function of incidence (Figs. 8 and 9 of Ref. 20) and multiplying the result by the proper constant in order to give a total cross-section of $0.074\pi A^2$, where A is the mean radius of the moon. This percentage cross section of 7.4% was measured by Fricker[26] and is confirmed by Pettengill[20] who obtains $7 \pm 1\%$. Thus the absolute level of cross section per unit area measured by Pettengill is plotted; however the theoretical results for a perfectly conducting surface are bound to be greater than the measured values from the non-perfectly conducting lunar surface, so that it is generally accepted that one need merely compare the curves in shape only, normalizing them in absolute value. This normalizing must be done for a perfectly conducting surface, but as will be seen later in the case of the solution for the non-perfectly conducting surface, the preservation of absolute values yields more information about the electrical properties of the surface. It is informative to note that in absolute levels, true solutions for the lunar surface and predicted values for the best fit perfectly conducting surface curve differ by about 12 db.

These curves are shown in Figs. 12-14. To save space, only the "best fit" curves are shown here; the remaining curves for back-scattered power from a perfectly conducting surface employing the more exact integral formulation are shown in Appendix E along with the curves of backscattered power from non-perfectly conducting surfaces for various types of polarization.

Several comparisons and observations can be made now from both the formulas for power and the predicted curves when contrasted with the experimental results.

a. Comparison of the different
integral formulations

It can be seen from comparison of the predicted results with the experimental that for all three statistical models, the second integral formulation offers far better agreement with experimentally measured data than the first, heretofore accepted, integral formulation. The first integral approach, as many others (e. g. , Pettengill[20]) have noted, differs by as much as 30-50 db from experimental results beyond about 40° from normal. Therefore, the less accurate integral formulation is to be discarded for the remainder of this analysis in favor of the more accurate formulation, and all discussion after this point will be concerned only with the results of the second, more accurate approach.

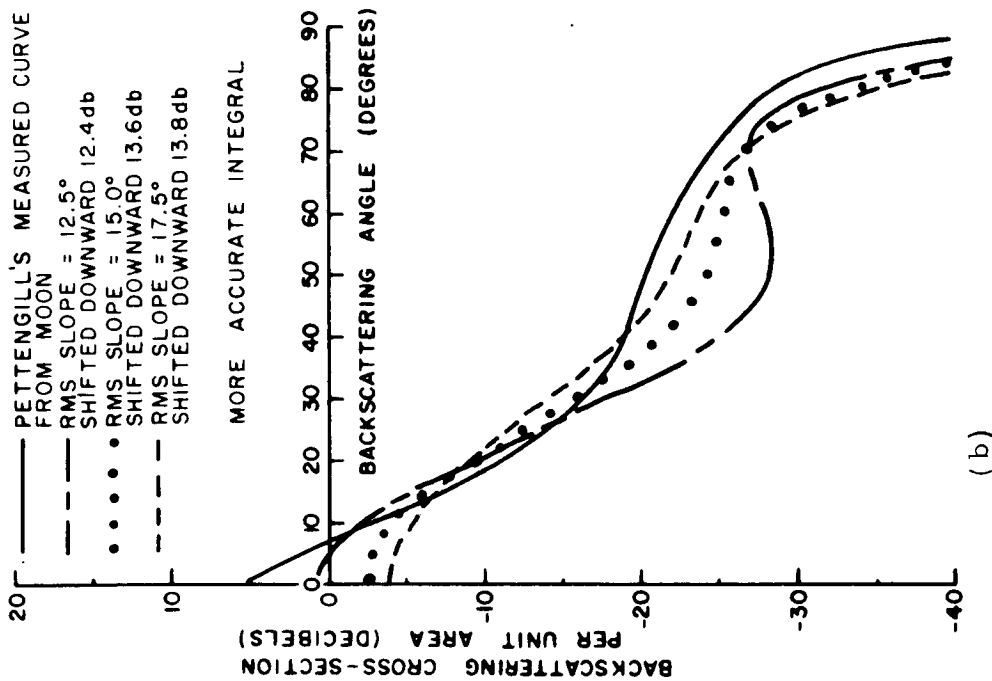
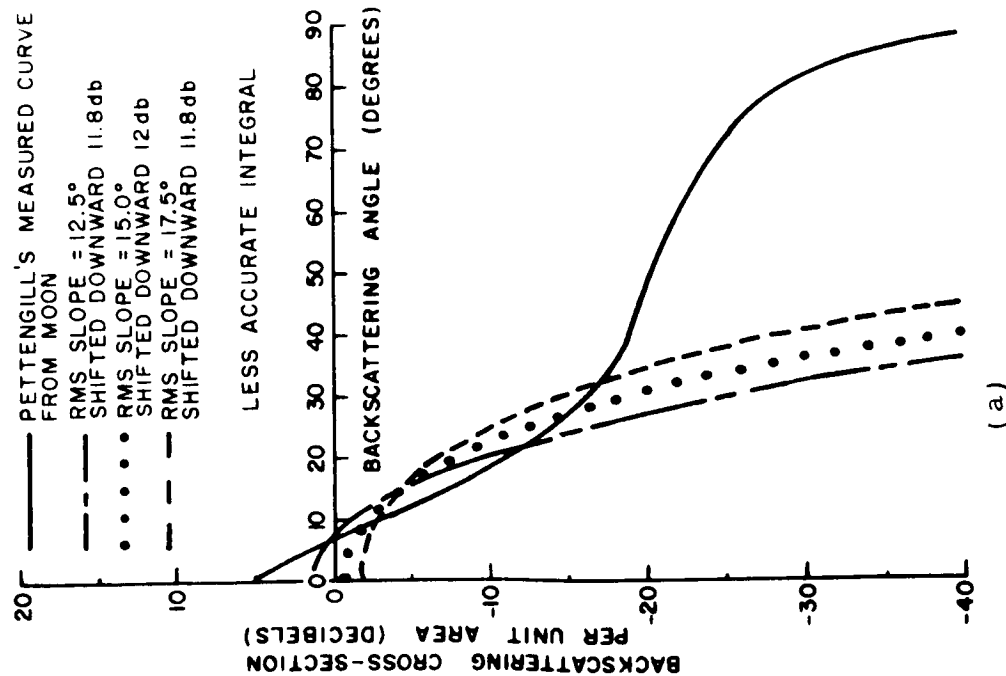


Fig. 12--"Best Fit" curves for backscattered power from a perfectly conducting surface - Comparison with measured power from the Moon. Gaussian JPDF and Gaussian correlation coefficient.

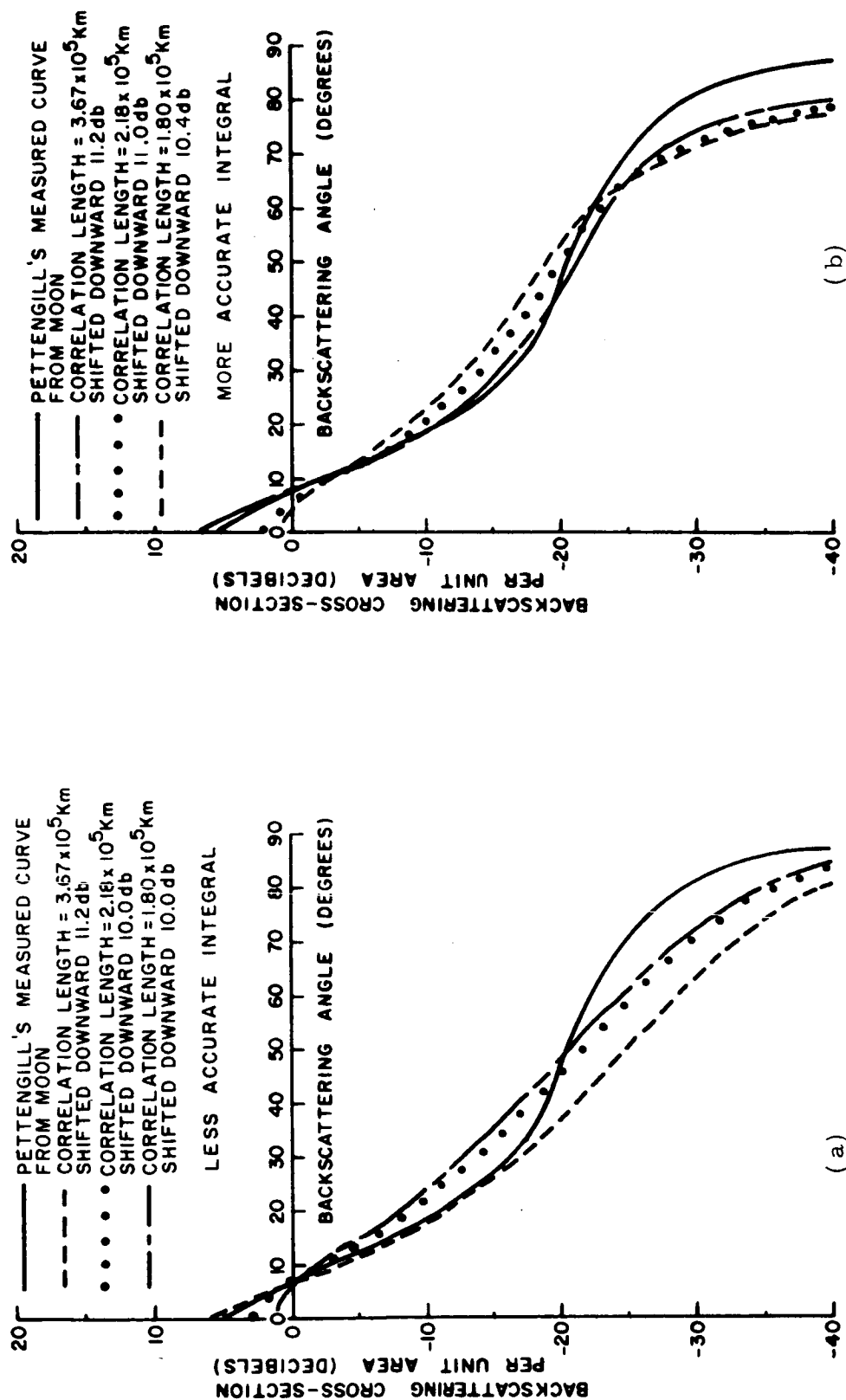


Fig. 13--"Best Fit" curves for backscattered power from a perfectly conducting surface - Comparison with measured power from the Moon. Gaussian JPDF and exponential correlation coefficient.

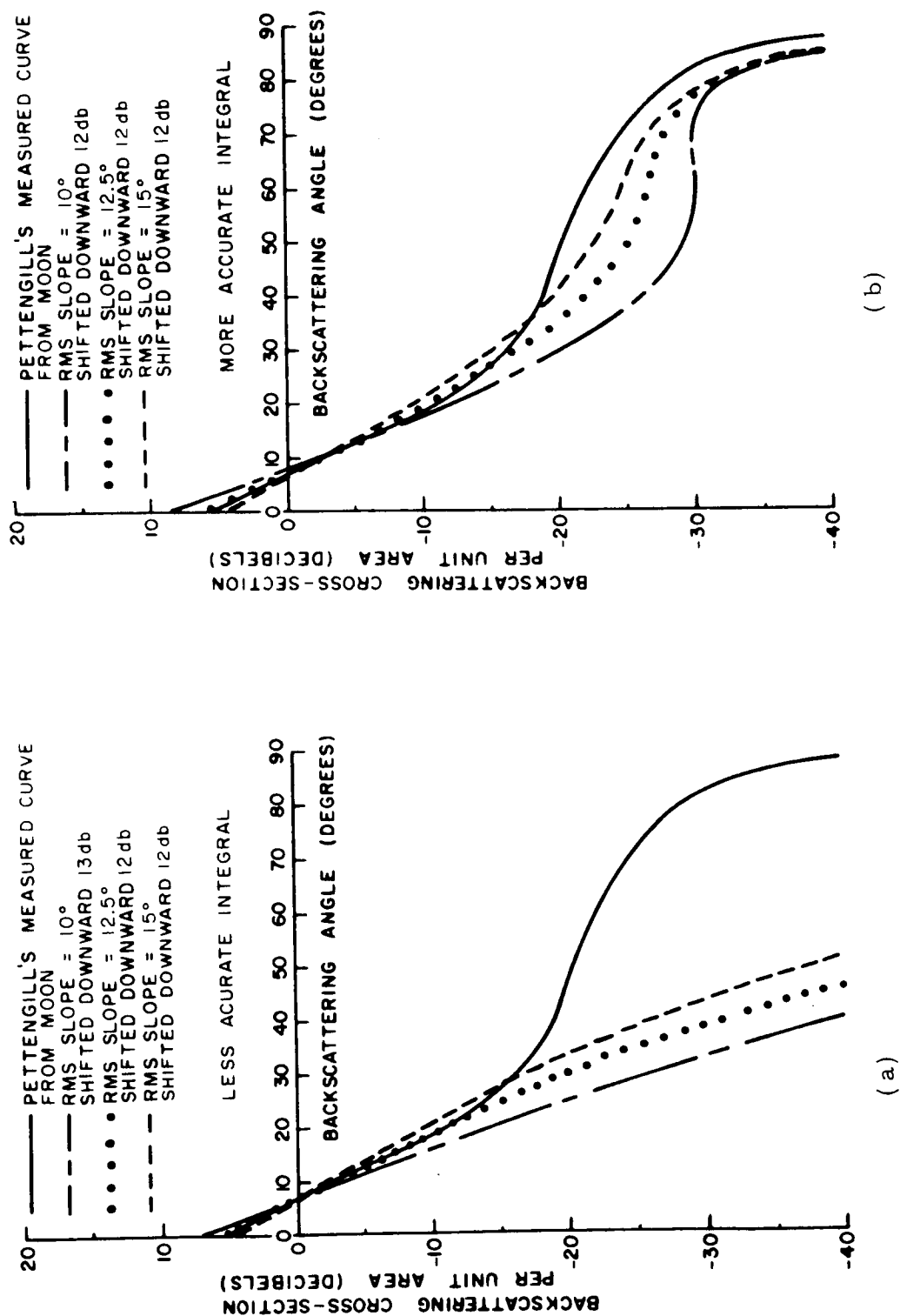


Fig. 14--"Best Fit" curves for backscattered power from a perfectly conducting surface - Comparison with measured power from the Moon. Bessel JPDF and Gaussian correlation coefficient.

- b. Comparison of results from the Gaussian and Bessel JPDF when used with the Gaussian correlation coefficient

The plots show that although the curve predicted by the Gaussian JPDF differs from the experimentally measured curve in the vicinity of normal incidence both in shape (the Gaussian curve is parabolic near the origin, whereas the measured curve seems linear) and in magnitude (as much difference as 4 db at normal incidence), still the two curves compare reasonably well all the way out to grazing, with no more than about 4 db difference at any point. The third model employing the Bessel JPDF does exhibit both the proper behavior near normal incidence and the same reasonably good fit at larger incidence angles.

From the literature the author has found nobody who tried a JPDF different from the Gaussian. Consequently, many have generally assumed that the improper parabolic behavior of the predicted back-scattered power near the origin is due to the choice of the Gaussian correlation coefficient, and not to the JPDF itself. It is implied by many that the Gaussian correlation coefficient can never yield the correct shape near normal. However, the Bessel JPDF used in conjunction with the Gaussian correlation coefficient does indeed exhibit the proper behavior at the origin, indicating that the form of the JPDF for the surface is certainly as important as the form of the correlation

coefficient in determining the shape of the curve for backscattered power.

The author does not wish to imply that he has or is even attempting to show that the Bessel JPDF is better than the Gaussian for describing the true lunar surface heights. The point that is to be stressed here is that both models compare reasonably well with experimental data in all regions. What is more important, the value of the mean square surface slope, $S^2 \equiv \tan^2 \alpha$, which produces the best fit for both models corresponds to $\alpha = \tan^{-1} S \approx 14^\circ$. This is in accordance with the results of others thus far in the literature who have predicted rms slope for the lunar surface roughness of between ten and twenty degrees.

In both of these models, the only statistical parameter of the rough surface upon which power depends is the mean square slope, S^2 , as can be seen from the equations (3.10a) and (3.10c). There is no dependence upon mean square surface height, σ^2 , or wavelength, $\lambda = 2\pi/k$, in either case. This is in agreement with the findings of others who use the Gaussian JPDF and Gaussian correlation coefficient.

c. Results using the exponential correlation coefficient

The most significant fact to be noted from the use of the model employing the Gaussian JPDF and exponential correlation coefficient for the largest (or overall) surface height random variable, ζ , of the lunar surface is that it leads to ridiculous results and must be discarded as having no meaningful significance. The curve for backscattered power offering the best fit with experimental measurement is obtained at a value of the constant $a^2/k^2\sigma^4 = 461.5$. Hayn's optical studies of the lunar surface[27] show that the root mean square height of the largest scale structure, σ , is approximately equal to 1.36 kilometers, a result which is physically reasonable for such a mountainous terrain. At $\lambda = 68$ cm, at which wavelength Pettengill's measurements were made, the correlation length using the best fit criterion turns out to be $a - \sigma^2 k \times \sqrt{461.5} = 367,000$ kilometers. This correlation length is greater than the distance to the moon, which is an absurdity besides the fact that it violates one of the previous stipulations that the correlation length is much less than any of the dimensions of the scattering surface.

Beckmann[21] notes this discrepancy also and attempts to improve this problem by treating the surface as a composite surface made up of several random variables, as discussed in Section C, subsection 3 of the preceding part of this analysis. Employing

the Gaussian JPDF and exponential correlation coefficient for each random variable, Beckmann finds a solution quite similar in form to equation (3.8b), except that wherever $a^2/4k^2\sigma^4$ appears, it is to be replaced by

$$\frac{1}{\left(\frac{k\sigma_1^2}{a_1} + \frac{k\sigma_2^2}{a_2} + \frac{k\sigma_3^2}{a_3} + \dots\right)^2} \equiv R,$$

where the subscripts refer to the different surface random variables, $\zeta_1, \zeta_2, \zeta_3, \dots$ which are employed to make up the rough surface. However, since in this case $R \approx 115$, and since σ_1 corresponding to the largest scale structure is still 1.36 Km, the addition of more terms representing more random variables in the denominator of the expression R only makes the correlation length, a_1 , of the largest scale roughness even larger.

The backscattered power using the exponential correlation coefficient is also wavelength dependent. This wavelength dependency will be discussed in more detail later.

As previously noted, many smaller scale surfaces in nature have an abundance of vertical points which arise also in the case of the exponential correlation coefficient. As an example, consider the one dimensionally rough surface shown in Fig. 15, where the process generating m , the interval between jumps, is a Poisson process, and the height, ζ , which the function may assume at any jump is also a random variable. This surface has been shown to have an exponential

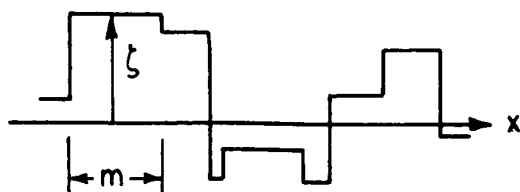


Fig. 15--Example of a random surface having an exponential correlation coefficient.

correlation coefficient. Such a surface has many vertical points, and therefore is in keeping with the general type of surfaces having the exponential class of correlation coefficients in that the mean square slope, S^2 , is indeterminate (or infinite). A finite portion of such a surface made up of pieces of flat planes gives scattered power which is wavelength dependent in the far field at any angle of incidence, as elementary physical optics theory shows; this wavelength dependency is also predicted by the statistical surface model having the exponential correlation coefficient, as evidenced from equation (3.8b) or (3.10b) for backscattered power.

The artificial surface discussed here in Fig. 15 was only introduced to illustrate one example of a surface possessing an exponential correlation coefficient. Surfaces which are smoothly curving and have no vertical points generally have Gaussian class (parabolic at origin) correlation coefficients. Daniels[19] shows examples of

smooth random signals having Gaussian class correlation coefficients; these he generated by passing white noise through a low-pass filter. The resulting signals look quite similar to optical profiles of the largest scale structure of the lunar surface. The point to be made before leaving this subject is that while many classes of surfaces can and do have exponential correlation coefficients, it appears to be extremely doubtful that the largest scale structure of the lunar surface has an exponential class of correlation coefficient. Therefore, this class of correlation coefficient will not be seriously considered in the remainder of this analysis when predicted results are to be compared with lunar measurements.

d. Wavelength dependency
of backscattered power

Both the Gaussian and Bessel JPDF when used with the Gaussian correlation coefficient yield a result for backscattered power which is wavelength independent; this is reasonable since practically all three-dimensional deterministic smoothly curving surfaces (that are of a non-focusing shape) analyzed thus far exhibit backscattering cross sections which are wavelength independent in the high frequency approximation and in the far field (e. g., spheres, ellipsoids, spheroids, specularly reflecting portions of a curved surface having two finite radii of curvature, etc.).

The model employing the exponential correlation coefficient yields a result for power which does depend upon wavelength; at normal incidence, it varies as the square of wavelength. Measured backscattered power from the lunar surface at various wavelengths seems to exhibit a wavelength dependency; power at 3.6 cm wavelength, as measured by Pettengill and Evans[20] is generally somewhat higher and the curve is slightly different in shape than the curve at 68 cm. Therefore, many people feel that the theoretical problem of backscattering from the lunar surface will never be solved completely until a solution is found which exhibits the proper wavelength dependency; in their quest, many immediately adopt the exponential correlation coefficient model, since it is the only statistical model exhibiting such a wavelength dependency. However, upon closer examination, one sees that if the exponential model really explained the wavelength dependency of the lunar surface correctly, the results at normal incidence for 3.6 cm wavelength should be $(68/3.6)^2 = 356$ times greater (or 25.5 db higher) than those at 68 cm. Such is far from being even approximately related to the measured results. Thus, the exponential correlation coefficient model, although it does exhibit wavelength dependency, cannot be seriously considered on this count alone since the dependency is not correct. When the other faults of the exponential correlation coefficient model (mentioned previously) are considered also, this model falls back into its previous disfavor.

In explanation of the observed wavelength dependency of the lunar surface, this author prefers that offered by Pettengill[20]; as wavelength decreases, more surface structure which has dimensions in the order of the magnitude of wavelength comes into play in the scattering process. At larger wavelengths, this surface structure appears smooth. Therefore, it is the effect of this "smallest visible" structure which gives the backscattered power its wavelength dependency; this small scale structure cannot be analyzed by physical optics and consequently the hope of including this small scale structure functionally in the solution seems dim. The correct wavelength dependency is thus a function of the various scale structures involved in the scattering process and its prediction for an unknown surface before measurement is impossible.

As wavelength decreases and smaller scale structure comes into play on the scattering process, the absolute mean square slope, S^2 , of the surface must increase since the smaller surface fluctuations are seen in the scattering process as well as the large scale fluctuations. Hence the Gaussian correlation coefficient models may be used if the mean square slope S^2 is allowed to change to account for all the visible roughness. Mean square slope, S^2 , as determined from comparison of predicted with measured backscattered power, must be interpreted as applying only to that surface roughness which is larger

in scale than the wavelength, and therefore is in a sense a function of wavelength.

C. Evaluation of Certain Averages Involving Surface Slopes

1. The probability density function of surface slopes

In Section D of the previous part it was shown that in order to find the backscattered power from non-perfectly conducting surfaces, certain averages involving the surface slopes must be determined. In particular, there are three averages which arise, as seen in equations (2.51):

$$\left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1}\right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1}\right)^2\right]^2} \right\rangle, \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 \left(\frac{\partial \zeta_1}{\partial y_1}\right)^2}{\left[\left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1}\right)^2\right]^2} \right\rangle, \text{ and}$$

$$\left\langle \frac{\left(\frac{\partial \zeta_1}{\partial y_1}\right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1}\right)^2\right]^2} \right\rangle, \frac{\partial \zeta_1}{\partial x_1} \text{ and } \frac{\partial \zeta_1}{\partial y_1}$$

are the slopes of the tilted surface and are to be expressed in terms of the slopes of the original

surface (i. e., $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$) which lies in the x-y plane. It was

shown in section C of the preceding part that the slopes $\partial \zeta / \partial x$ and

$\partial \zeta / \partial y$ are statistically independent of each other at every point. In

the same section it was shown that if the rms surface slopes (i. e.,

$$\tan^{-1} \sqrt{\left\langle \left(\frac{\partial \zeta}{\partial x}\right)^2 \right\rangle} = \tan^{-1} \sqrt{\left\langle \left(\frac{\partial \zeta}{\partial y}\right)^2 \right\rangle}) \text{ are less than about } 15^\circ \text{ and}$$

if the angle of tilt of the new surface about the x-axis (or angle of

incidence away from normal), θ , is less than 75° , then the new surface slopes are related to the original ones as follows:

$$(3.6a) \quad \frac{\partial \zeta_1}{\partial x_1} = \sec \theta \frac{\partial \zeta}{\partial x}$$

and

$$(3.6b) \quad \frac{\partial \zeta_1}{\partial y_1} = \tan \theta + \frac{\partial \zeta}{\partial y}.$$

Then if the statistics of the original random variables $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ are known, the statistics of the new random variables defined in (2.42a) and (2.42b) may be found. These two new random variables $\frac{\partial \zeta_1}{\partial x_1}$ and $\frac{\partial \zeta_1}{\partial y_1}$ will be statistically independent of each other since the original two surface slopes are independent.

It was mentioned in Section C of the preceding part that if the JPDF of the original surface height ζ is Gaussian and if the correlation coefficient of the surface heights meets the proper requirements on the continuity of its derivatives everywhere (these requirements are met in the case of the Gaussian correlation coefficient), then the surface slopes $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ are also Gaussian. In general, if the JPDF of surface height, ζ , is not Gaussian, the slopes need not be Gaussian. However since only the single probability density function of the slopes are required, i. e., $W_s\left(\frac{\partial \zeta}{\partial x}\right)$ and $W_s\left(\frac{\partial \zeta}{\partial y}\right)$, the requirements may be relaxed somewhat. (Requiring $\frac{\partial \zeta}{\partial x}$ to have a Gaussian JPDF is a more serious restriction than requiring it to have a Gaussian SPDF.) It will be assumed, therefore, that the

surface slopes $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ have a Gaussian SPDF regardless of the form of the original surface JPDF. Immediately, the mean square slopes of this SPDF are known from the derivations of the preceding part: $S_x^2 = S_y^2 = \langle \left(\frac{\partial \zeta}{\partial x} \right)^2 \rangle = \langle \left(\frac{\partial \zeta}{\partial y} \right)^2 \rangle = \frac{2\sigma^2}{a^2}$; and $\langle \frac{\partial \zeta}{\partial x} \rangle = \langle \frac{\partial \zeta}{\partial y} \rangle = 0$.

Therefore, the SPDF for surface slopes becomes

$$(3.11a) \quad W_s \left(\frac{\partial \zeta}{\partial x} \right) = \frac{1}{\sqrt{2\pi} \sqrt{\frac{2\sigma^2}{a^2}}} e^{-\frac{(\partial \zeta / \partial x)^2}{4\sigma^2/a^2}} = \frac{1}{\sqrt{2\pi} S_x} e^{-\frac{(\partial \zeta / \partial x)^2}{2 S_x^2}}$$

and

$$(3.11b) \quad W_s \left(\frac{\partial \zeta}{\partial y} \right) = \frac{1}{\sqrt{2\pi} \sqrt{\frac{2\sigma^2}{a^2}}} e^{-\frac{(\partial \zeta / \partial y)^2}{4\sigma^2/a^2}} = \frac{1}{\sqrt{2\pi} S_y} e^{-\frac{(\partial \zeta / \partial y)^2}{2 S_y^2}}.$$

Hence, a change in random variables to $\frac{\partial \zeta_1}{\partial x_1}$ and $\frac{\partial \zeta_1}{\partial y_1}$ merely results in a change of the means and variances in the above expressions.

$$(3.12a) \quad W_s \left(\frac{\partial \zeta_1}{\partial x_1} \right) = \frac{1}{\sqrt{2\pi} \sqrt{\frac{2\sigma^2 \sec^2 \theta}{a^2}}} e^{-\frac{(\partial \zeta_1 / \partial y_1)^2}{4\sigma^2 \sec^2 \theta / a^2}} \\ = \frac{1}{\sqrt{2\pi} S_x \sec \theta} e^{-\frac{(\partial \zeta_1 / \partial x_1)^2}{2 S_x^2 \sec^2 \theta}},$$

$$\begin{aligned}
 (3.12b) \quad W_s \left(\frac{\partial \zeta_1}{\partial y_1} \right) &= \frac{1}{\sqrt{2\pi} \sqrt{\frac{2\sigma^2}{a^2}}} \cdot e^{-\frac{(\partial \zeta_1 / \partial y_1 - \tan \theta)^2}{4\sigma^2/a^2}} \\
 &= \frac{1}{\sqrt{2\pi} S_y} \cdot e^{-\frac{(\partial \zeta_1 / \partial y_1 - \tan \theta)^2}{2 S_y^2}} .
 \end{aligned}$$

It is these new Gaussian probability density functions which must be used to evaluate the averages in equations (2.51).

2. An approximate method of
evaluation of the surface
slope averages

Evaluation of the average of the three rational functions in $\frac{\partial \zeta_1}{\partial x_1}$ and $\frac{\partial \zeta_1}{\partial y_1}$ appears difficult. One approximation which might be made is to average numerator and denominator separately; i. e.,

$$(3.13a) \quad \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle = \frac{\left\langle \left(\frac{\partial \zeta_1}{\partial x_1} \right)^4 \right\rangle}{\left\langle \left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2 \right\rangle} ,$$

$$(3.13b) \quad \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle = \frac{\left\langle \left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \right\rangle \left\langle \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right\rangle}{\left\langle \left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2 \right\rangle} , \text{ and}$$

$$(3.13c) \quad \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial y_1} \right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle = \frac{\left\langle \frac{\partial \zeta_1^4}{\partial y_1} \right\rangle}{\left\langle \left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2 \right\rangle} .$$

Such an averaging approximation is not valid in general since numerator and denominator are not independent, but it often yields results which are surprisingly close to correct. In this case, the results of such an approximation are extremely close to the actual values near normal incidence. * Although the exact averages are evaluated by computer in the next subsection, the above approximations permit the results to be written in a closed form in terms of the rough surface parameters, which gives better insight into the nature of the scattering process.

When numerator and denominator of the above equations are evaluated by simply integrating, the results are:

$$(3.14a) \quad \frac{\left\langle \left(\frac{\partial \zeta_1}{\partial x_1} \right)^4 \right\rangle}{\left\langle \left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2 \right\rangle} = \frac{\frac{3}{4} S^4 \sec^4 \theta}{S^4 \left(\frac{3}{4} \sec^4 \theta + \frac{1}{2} \sec^2 \theta + \frac{3}{4} \right) + S^2 \left(\sec^2 \theta \tan^2 \theta + 3 \tan^2 \theta \right) + \tan^4 \theta},$$

$$(3.14b) \quad \frac{\left\langle \left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \right\rangle \left\langle \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right\rangle}{\left\langle \left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2 \right\rangle} = \frac{\frac{1}{4} S^4 \sec^2 \theta + \frac{1}{2} S^2 \sec^2 \theta \tan^2 \theta}{S^4 \left(\frac{3}{4} \sec^2 \theta + \frac{1}{2} \sec^2 \theta + \frac{3}{4} \right) + S^2 \left(\sec^2 \theta \tan^2 \theta + 3 \tan^2 \theta \right) + \tan^4 \theta},$$

* These approximations exhibit the proper behavior all the way out to grazing incidence, but the error increases to as much as 40% at higher angles of incidence.

$$(3.14c) \quad \frac{\left\langle \left(\frac{\partial \zeta_1}{\partial y_1} \right)^4 \right\rangle}{\left\langle \left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2 \right\rangle} = \frac{\frac{3}{4} S^4 + 3 S^2 \tan^2 \theta + \tan^4 \theta}{S^4 \left(\frac{3}{4} \sec^4 \theta + \frac{1}{2} \sec^2 \theta + \frac{3}{4} \right) + S^2 (\sec^2 \theta \tan^2 \theta + 3 \tan^2 \theta) + \tan^4 \theta}.$$

Here the substitution has been made $S^2 = 2S_x^2 = 2S_y^2 = \frac{4\sigma^2}{a^2}$.

Although these approximations do not introduce considerable error, the exact averages have nevertheless been evaluated in the next subsections.

3. Determination of $W_s \left(\frac{\partial \zeta_1 / \partial x_1}{\partial \zeta_1 / \partial y_1} \right)$

The averages of interest may be rewritten:

$$(3.15a) \quad \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle = \left\langle \frac{w^4}{(w^2 + 1)^2} \right\rangle \equiv G_1,$$

$$(3.15b) \quad \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle = \left\langle \frac{w^2}{(w^2 + 1)^2} \right\rangle \equiv G_2,$$

$$(3.15c) \quad \left\langle \frac{\left(\frac{\partial \zeta_1}{\partial y_1} \right)^4}{\left[\left(\frac{\partial \zeta_1}{\partial x_1} \right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1} \right)^2 \right]^2} \right\rangle = \left\langle \frac{1}{(w^2 + 1)^2} \right\rangle \equiv G_3,$$

where $w \equiv \frac{\partial \zeta_1 / \partial x_1}{\partial \zeta_1 / \partial y_1}$

The problem at hand therefore is the determination of the SPDF of the newly defined random variable, w , by employing the SPDF's of the original variables $\frac{\partial \zeta_1}{\partial x_1}$ and $\frac{\partial \zeta_1}{\partial y_1}$. The following standard statistical process is used to determine this new SPDF.

a. Since $\frac{\partial \zeta_1}{\partial x_1}$ and $\frac{\partial \zeta_1}{\partial y_1}$ are independent, their joint probability density function may be written

$$W\left(\frac{\partial \zeta_1}{\partial x_1}, \frac{\partial \zeta_1}{\partial y_1}\right) = W_s\left(\frac{\partial \zeta_1}{\partial x_1}\right) W_s\left(\frac{\partial \zeta_1}{\partial y_1}\right) .$$

b. Two new variables, $v \equiv \frac{\partial \zeta_1}{\partial y_1}$ and $w \equiv \frac{\partial \zeta_1 / \partial x_1}{\partial \zeta_1 / \partial y_1}$ are defined; the new JPDP of v and w is found as follows;

$$W(v, w) = W\left(\frac{\partial \zeta_1}{\partial x_1}(v, w), \frac{\partial \zeta_1}{\partial y_1}(v, w)\right) \cdot |J| ,$$

where $|J|$ is the absolute value of the Jacobian of the two random variables. The domain of the new random variables must be determined from the domain of the original.

c. Finally, the SPDF of w is found by integrating on v , i. e. ,

$$W_s(w) = \int_{-\infty}^{\infty} W(v, w) dv .$$

The domain of $\frac{\partial \zeta_1}{\partial x_1}$ and $\frac{\partial \zeta_1}{\partial y_1}$ is A ,

$$A = \left\{ \left(\frac{\partial \zeta_1}{\partial x_1}, \frac{\partial \zeta_1}{\partial y_1} \right) ; -\infty < \frac{\partial \zeta_1}{\partial x_1}, \frac{\partial \zeta_1}{\partial y_1} < +\infty \right\} .$$

Therefore, $A \rightarrow B$, $B = \{ (v, w); -\infty < w, v < +\infty \}$, i. e., the mapping of A onto B is single valued where v and w have values between $\pm \infty$.

The Jacobian is given by

$$J = \begin{vmatrix} \frac{\partial}{\partial w} \left(\frac{\partial \zeta_1}{\partial x_1} \right) & \frac{\partial}{\partial v} \left(\frac{\partial \zeta_1}{\partial x_1} \right) \\ \frac{\partial}{\partial w} \left(\frac{\partial \zeta_1}{\partial y_1} \right) & \frac{\partial}{\partial v} \left(\frac{\partial \zeta_1}{\partial y_1} \right) \end{vmatrix},$$

$$\therefore |J| = |v|$$

$$\therefore W(v, w) = \frac{1}{\pi S^2 \sec \theta} e^{-\frac{w^2 v^2}{S^2 \sec^2 \theta} - \frac{(v - \tan \theta)^2}{S^2}} \cdot |v|;$$

$$-\infty < v, w < +\infty.$$

Then the single probability density function $W_s(w)$ is found by integration over v .

$$W_s(w) = \frac{1}{\pi S^2 \sec \theta} e^{-\frac{\tan^2 \theta}{S^2}} \int_{-\infty}^{\infty} |v| e^{-\frac{1}{S^2} (w^2 \cos^2 \theta + 1) v^2 + \frac{2 \tan^2 \theta}{S^2} v} dv$$

$$= \frac{1}{\pi S^2 \sec \theta} e^{-\frac{\tan^2 \theta}{S^2}} \left[-\int_{-\infty}^0 v e^{-av^2 + bv} dv + \int_0^{\infty} v e^{-av^2 + bv} dv \right]$$

$$= \frac{2}{\pi S^2 \sec \theta} e^{-\frac{\tan^2 \theta}{S^2}} \int_0^{\infty} v e^{-av^2} \cosh bv dv$$

$$(3.16) \quad \therefore W_s(w) = \frac{\tan \theta \sec^2 \theta}{\pi (w^2 + \sec^2 \theta)^{3/2}} \cdot e^{-\frac{\tan^2 \theta}{S^2}} \cdot \frac{w^2}{w^2 + \sec^2 \theta} \cdot \left[\frac{\frac{1}{S} \tan \theta \sec \theta}{(w^2 + \sec^2 \theta)^{1/2}} \right] + \frac{\sec \theta e^{-\frac{\tan^2 \theta}{S^2}}}{\pi (w^2 + \sec^2 \theta)}.$$

The function $\phi(x)$ denotes the error function and is defined

$$\phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (\text{evaluated from tables, Ref. 28}).$$

Hence, knowing $W_s(w)$, the averages in equations (3.15) can be evaluated in theory. In order to obtain an idea of the behavior of $W_s(w)$ for various values of the parameters θ and S , plots of this probability density function are shown in Fig. 16.

4. Exact evaluation of the surface slope averages

Theoretically, to find the averages G_1 , G_2 , and G_3 in equations (3.15), it is necessary to integrate the product of $W_s(w)$ and the three functions in w which are to be averaged. It does not appear possible to evaluate these integrals exactly from the tables because of the appearance of the error function in the integrand. The function $(w^2 + \sec^2 \theta)^{3/2}$ in the denominator precludes any simple evaluation by complex variable theory. Since there is no large parameter, asymptotic methods are of no avail. Therefore, the averages were integrated on a computer by numerically integrating the various

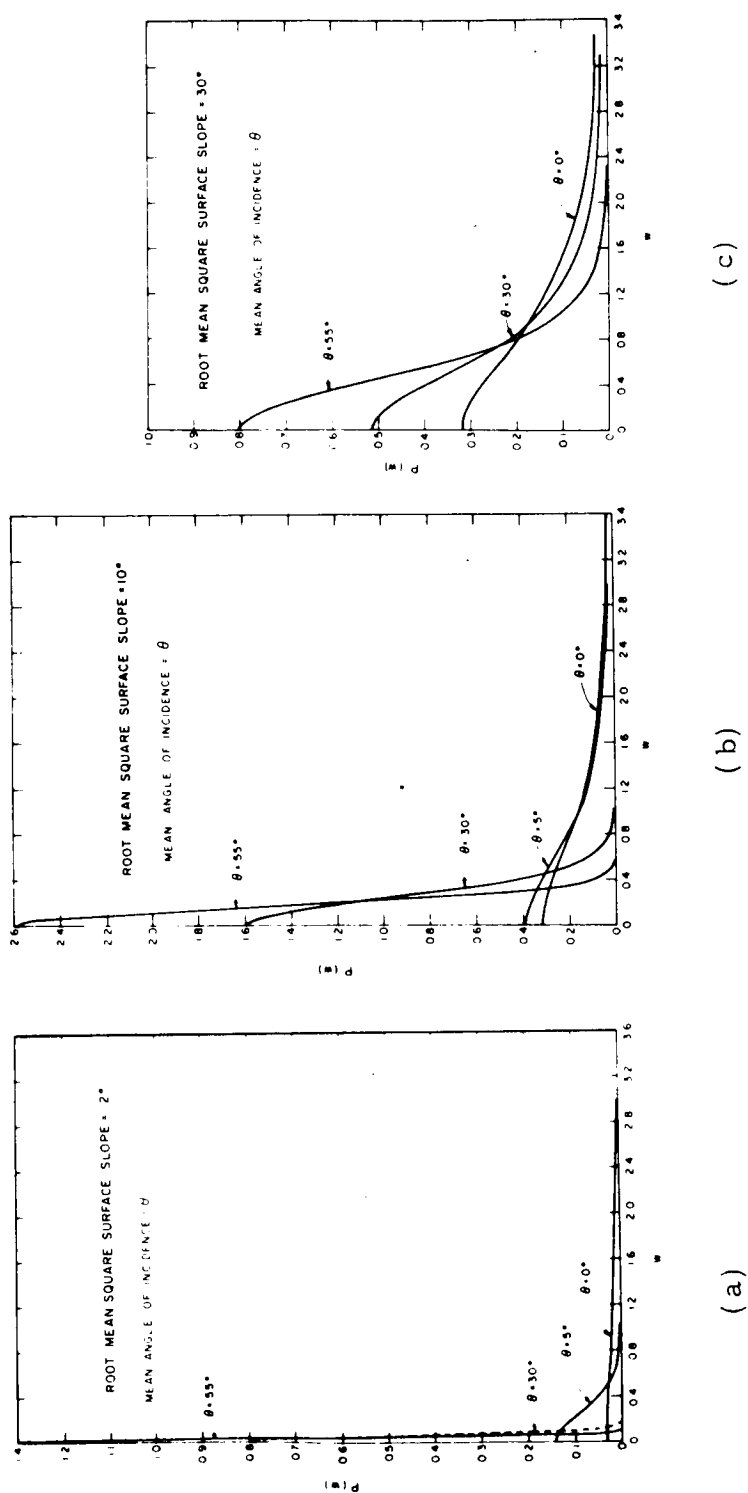


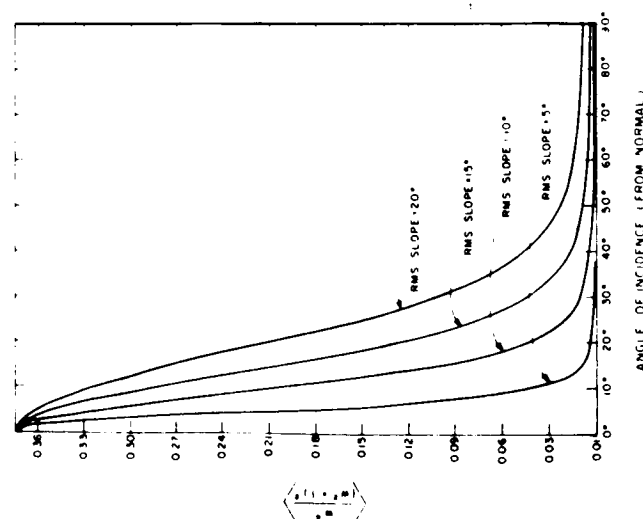
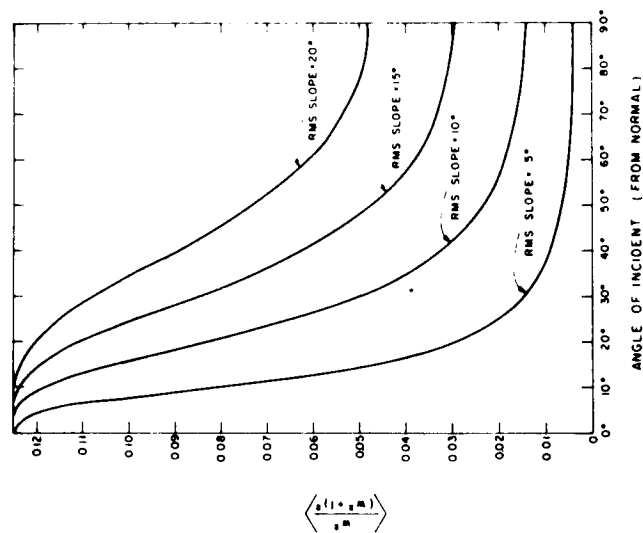
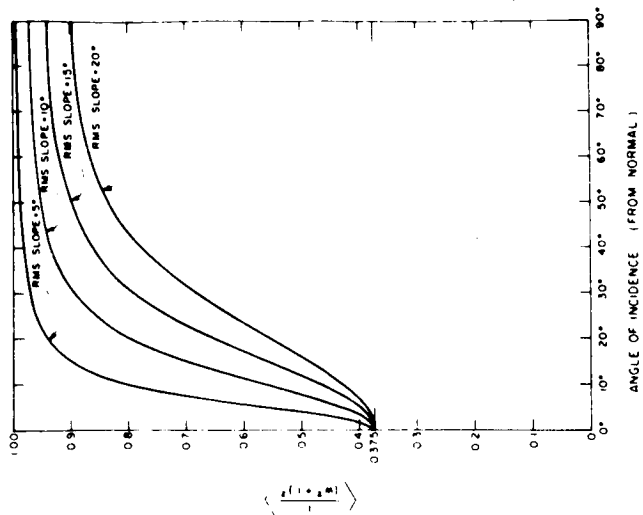
Fig. 16---Probability density function of ratio of x and y surface slopes.

functions. The error involved in all cases was less than one part in 10^4 . These averages are shown in Fig. 17; they will be used in the next section.

D. Backscattered Power from a Non-Perfectly Conducting Surface

1. Circular polarization and comparison with lunar surface data

Equations (2.52) give the four backscattering cross sections involved in circular polarization in terms of the reflection coefficients $\rho_{||}$ and $\rho_{\perp}$ and in terms of Q (the backscattering cross section for a perfectly conducting surface). The function to be used for Q therefore depends upon the statistical model chosen as the JPDF of the surface height; in this analysis, both the Gaussian and the Bessel JPDF are selected. The factors involving the reflection coefficients are functions of the mean angle of incidence, θ , the mean relative dielectric constant of the surface, ϵ_r , and the surface conductivity; these reflection coefficients are defined in equations (2.14a) and (2.14b). The various surface slope averages G_1 , G_2 , and G_3 evaluated in the previous section are not needed for circular polarization and therefore the assumptions made in the previous section concerning the statistical distribution of the surface slopes do not affect the accuracy of the solutions for circular polarization.



(c)

(b)

(a)

Fig. 17--Averages of rational functions of the surface slopes arising in the backscattering problem.

A dimensionless average backscattering cross section per unit area can be defined here as in the case of the perfectly conducting surface by dividing the respective cross sections by the mean area of the scattering surface; e. g. , $\gamma_{RR} = \langle \sigma_{RR} \rangle / L_x L_y$. Thus the various cross sections become

$$(3.17a) \quad \gamma_{RR} = \gamma_{LL} = \frac{1}{4} \left| \rho_{\parallel}(\epsilon_r, \sigma, \theta) + \rho_{\perp}(\epsilon_r, \sigma, \theta) \right|^2 \cdot$$

$$\left\{ \begin{array}{ll} \frac{1}{S^2} \cos^4 \theta e^{-\frac{1}{S^2} \sin^2 \theta \cos^2 \theta} & \text{for Gaussian JPDF} \\ \frac{3}{S^2} \cos^4 \theta e^{-\frac{6}{S} \sin \theta \cos \theta} & \text{for Bessel JPDF} \end{array} \right.$$

$$(3.17b) \quad \gamma_{RL} = \gamma_{LR} = \frac{1}{4} \left| \rho_{\parallel}(\epsilon_r, \sigma, \theta) - \rho_{\perp}(\epsilon_r, \sigma, \theta) \right|^2 \cdot$$

$$\left\{ \begin{array}{ll} \frac{1}{S^2} \cos^4 \theta e^{-\frac{1}{S^2} \sin^2 \theta \cos^2 \theta} & \text{for Gaussian JPDF} \\ \frac{3}{S^2} \cos^4 \theta e^{-\frac{6}{S} \sin \theta \cos \theta} & \text{for Bessel JPDF} \end{array} \right. .$$

For surfaces such as sand, rock, gravel, dirt, and fresh water, these reflection coefficients are almost completely insensitive to the surface conductivity and to the wavelength (so long as $\frac{\sigma_s^2}{\omega^2 \epsilon_s^2} \ll 1$). However, for the sake of generality, values $\sigma = 10^{-5}$ mhos/meter (typical for earth and rock) and $\lambda = 3$ centimeters were used.

Curves were plotted from equations (3. 17) at various values of the parameters ϵ_r and S ; these curves are shown in Appendix E. Curves representing the polarized components of backscattered power, γ_{RL} and γ_{LR} , are shown along with the curve for backscattered power from the perfectly conducting surface; for the depolarized components γ_{RR} and γ_{LL} there is no backscattered power from a perfectly conducting surface.

At small mean square surface slopes (i. e. , less than 10°), one will notice that the curves exhibit a non-zero minimum at some intermediate angle of incidence; beyond this angle, θ , the scattered power again increases to a maximum and then drops to zero at grazing incidence. This minimum has not been fully explained, but the author offers two reasons for its existence without proof:

(i) Both of the statistical models chosen for the JPDF do not really represent a physical surface such as the lunar surface. In the case of the perfectly conducting surface this minimum may not appear when the proper surface JPDF is used.

(ii) For the curves for the non-perfectly conducting surfaces, part of the reason lies in the Brewster angle effect. This effect will be seen more clearly later in the cases of linear polarization. For rough surfaces, this effect disappears as the surface becomes rougher, i. e., as S^2 increases.

Pettengill has obtained curves of both the polarized and depolarized backscattered power from the moon as a function of the angle of incidence using circular polarization. When these are converted to backscattering cross sections per unit area and plotted preserving magnitude information, the "best fit" curves may be selected from the appendix. This is done by matching curves both in form and absolute value; in this manner, both mean square slope, S^2 , and dielectric constant, ϵ_r of the lunar surface can be predicted. These best fit curves are shown in Figs. 18 and 19 for the Gaussian and Bessel JPDF when both are used with the Gaussian class of correlation coefficients.

The depolarized components are shown in each case also plotted along with Pettengill's measured depolarization curve. Pettengill's curve for the depolarized component must not be considered valid at angles of incidence, θ , of less than about 20° , because the isolation in his antennas as far as the ability to discriminate between left and right circular polarization is probably not more than 12-18 db. Common sense indicates that actual depolarized power should be a

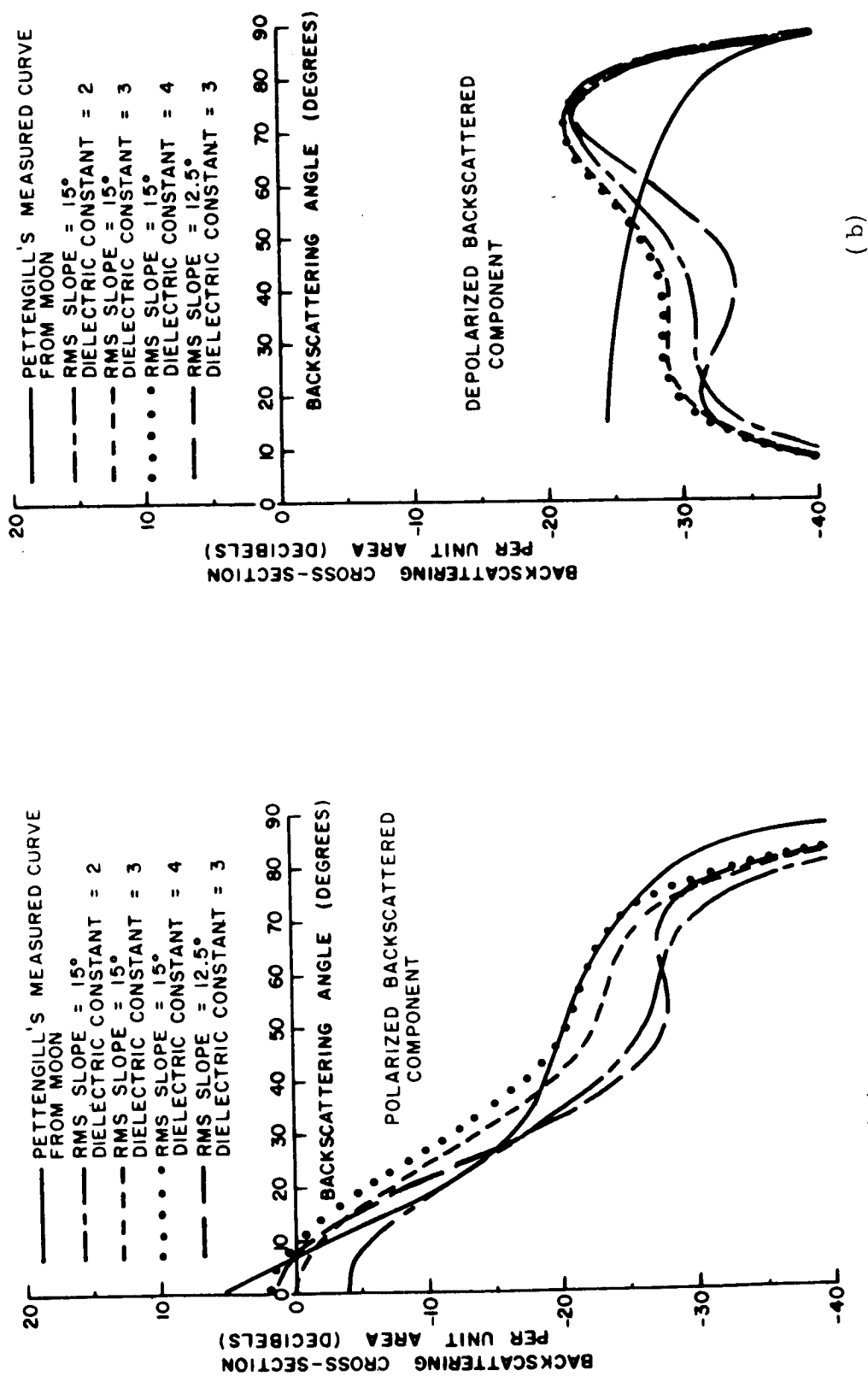
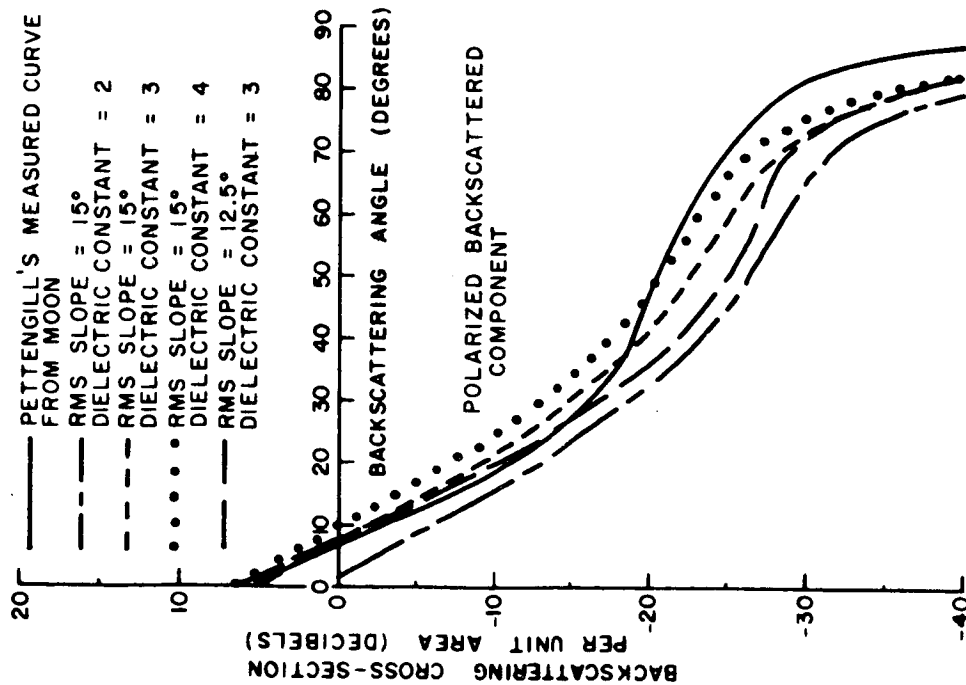
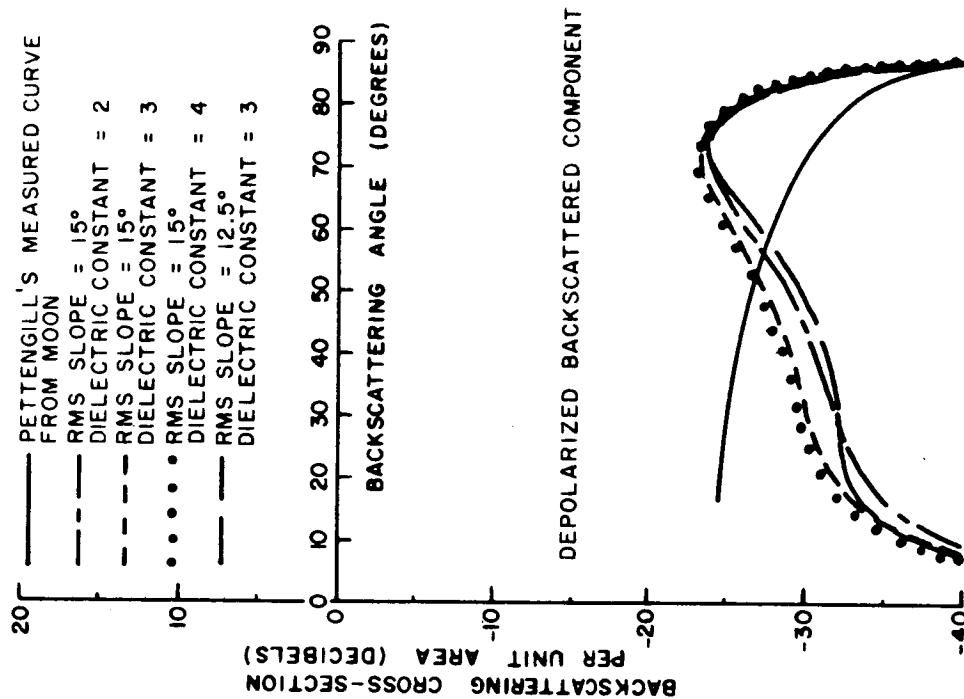


Fig. 18--"Best Fit" curves for backscattered power from a non-perfectly conducting surface using circular polarization-- Comparison with measured power from the Moon.
Gaussian JPDF model.



(a)



(b)

Fig. 19--"Best Fit" curves for backscattered power from a non-perfectly conducting surface using circular polarization-Comparison with measured power from the Moon.
Bessel JPDF model.

minimum, if not zero, at normal incidence; Pettengill's curve does not show this because of the lack of high isolation when the strong polarized component is present. At angles greater than 20° , the polarized and depolarized components are not greatly different and lack of isolation is no longer a problem.

Both the Gaussian and Bessel JPDF best fit curves show that the best value of overall dielectric constant for the lunar surface is slightly greater than 3 (viz., 3.1 - 3.3) and that the rms slope of the roughness seen at a wavelength of 68 cm is about 14° . This prediction for the dielectric constant is not greatly different from that of Pettengill[20] , who suggested 2.8.

2. Linear polarization

Equations (2.51) give the four backscattering cross sections involved in the backscattering of vertically and horizontally polarized waves in terms of the reflection coefficients $\rho_{||}$ and $\rho_{\perp}$, in terms of the averages involving surface slopes G_1, G_2 , and G_3 , and in terms of the backscattering cross section of a perfectly conducting surface, Q . Defined as dimensionless cross sections per unit area, these quantities are

$$\begin{aligned}
 (3.18a) \quad \gamma_{hh} = & \left[\left| \rho_{||}(\theta) \right|^2 G_1(\theta, S) - 2\operatorname{Re}(\rho_{||}(\theta) \rho_{\perp}^*(\theta)) G_2(\theta, S) + \right. \\
 & \left. + \left| \rho_{\perp}(\theta) \right|^2 G_3(\theta, S) \right] \times \begin{cases} \frac{1}{S^2} \cos^4 \theta e^{-\frac{1}{S^2} \sin^2 \theta \cos^2 \theta} & \text{for Gaussian JPDF} \\ \frac{3}{S^2} \cos^4 \theta e^{-\frac{\sqrt{6}}{S} \sin \theta \cos \theta} & \text{for Bessel JPDF} \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 (3.18b) \quad \gamma_{vv} = & \left[\left| \rho_{||}(\theta) \right|^2 G_3(\theta, S) - 2\operatorname{Re}(\rho_{||}(\theta) \rho_{\perp}^*(\theta)) G_2(\theta, S) + \right. \\
 & \left. + \left| \rho_{\perp}(\theta) \right|^2 G_1(\theta, S) \right] \times \begin{cases} \frac{1}{S^2} \cos^4 \theta e^{-\frac{1}{S^2} \sin^2 \theta \cos^2 \theta} & \text{for Gaussian JPDF} \\ \frac{3}{S^2} \cos^4 \theta e^{-\frac{\sqrt{6}}{S} \sin \theta \cos \theta} & \text{for Bessel JPDF} \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 (3.18c) \quad \gamma_{hv} = \gamma_{vh} = & \left[\left| \rho_{||}(\theta) + \rho_{\perp}(\theta) \right|^2 G_2(\theta, S) \right] \times \\
 & \times \begin{cases} \frac{1}{S^2} \cos^4 \theta e^{-\frac{1}{S^2} \sin^2 \theta \cos^2 \theta} & \text{for Gaussian JPDF} \\ \frac{3}{S^2} \cos^4 \theta e^{-\frac{\sqrt{6}}{S} \sin \theta \cos \theta} & \text{for Bessel JPDF} \end{cases}
 \end{aligned}$$

The curves for these cross sections are plotted in Appendix F at various values of the parameters ϵ_r and S .

The Brewster angle effect is quite evident from the curves of γ_{vv} when compared to those for γ_{hh} . The appearance of this effect may be explained from equations (3.18a) and (3.18b). It is the parallel reflection coefficient $\rho_{||}$ which causes this effect; the significance of the $\rho_{||}$ term is enhanced in (3.18b) by multiplication by the factor $G_3(\theta, S)$ which increases with respect to θ , whereas the significance of the $\rho_{||}$ term in (3.18a) is reduced by multiplication by the factor $G_1(\theta, S)$ which decreases with respect to θ . (The behavior of G_1 and G_3 is shown in Fig. 17.) However, as rms slope, S , increases, the magnitude of $G_3(\theta, S)$ multiplying $\rho_{||}$ in (3.18b) decreases, which in turn decreases the Brewster angle effect. As seen from the curves, the rougher the surface at a given wavelength when measured in terms of the rms surface slope S , the less pronounced is the Brewster angle effect. As the surface becomes smoother, the Brewster angle effect increases until it becomes identical to that for a plane surface. This effect is observed from experiment also.

Unfortunately, it is not possible to compare these cross sections for linear polarization with measured power from the lunar surface, because a radar pulse linearly polarized striking a certain section of the moon has both vertical and horizontal components, depending upon

what point of the illuminated annulus one is considering. However, there are measurements underway for power scattered from planar rough surfaces of terrain and other artificial rough surfaces on the earth, and when this measured data is ready it will be compared with the curves in the Appendix.

3. Scattering of linear polarization from the Moon

As mentioned previously, linear waves striking an hemispherical surface are vertically polarized at certain points of the illuminated annulus, (see Fig. 20), for example, at A, and horizontally polarized

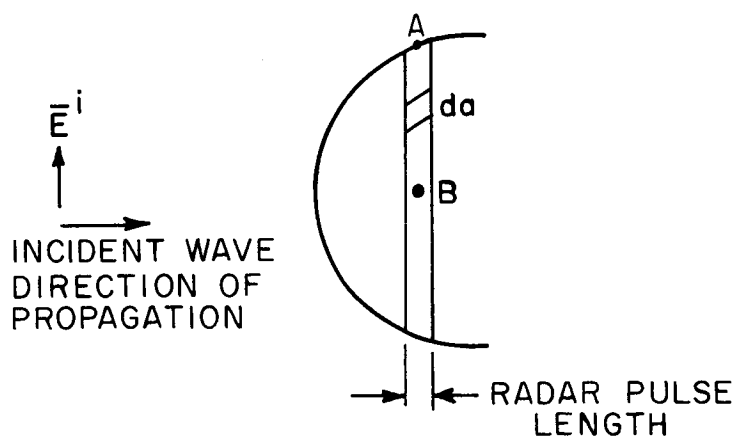


Fig. 20--Geometry of spherical scattering body illuminated by a radar pulse.

at other points such as B, while at the remaining points the waves are a combination of both vertical and horizontal. Thus an average cross section per annulus must be determined in terms of the

horizontal and vertical cross sections γ_{hh} , γ_{vv} , γ_{hv} by finding the return from an element of area da and then integrating the result over the entire annulus. This is done in Ref. 29, and the resulting backscattering cross sections per unit area for a given annulus (before averaging over many such annuli) are repeated here.

$$(3.19a) \quad \frac{\sigma_{\parallel}}{\text{Area}} = \frac{1}{8} \frac{1}{\text{Area}} [3\sigma_{hh} + 4\sigma_{hv} + 3\sigma_{vv} + (a_{hh} a_{vv}^* + a_{hh}^* a_{vv})]$$

$$(3.19b) \quad \frac{\sigma_{\perp}}{\text{Area}} = \frac{1}{8} \frac{1}{\text{Area}} [\sigma_{hh} + 4\sigma_{hv} + \sigma_{vv} - (a_{hh} a_{vv}^* + a_{hh}^* a_{vv})] \quad .$$

$\sigma_{\parallel}$ is the total backscattering cross section from a range ring (annulus) when power is transmitted and received with linearly polarizing antennas which are parallel in polarization direction to each other (representing the polarized return). The cross sections and scattering matrix elements σ_{hh} , σ_{hv} , σ_{vv} , a_{hh} , a_{vv} , are defined in Section D of the preceding part. $\sigma_{\perp}$ is the total cross section from a range ring when the linear transmitting and receiving antennae have their polarization directions perpendicular to each other (representing the depolarized return). The area involved is the area of such a range ring. Fortunately, such an averaging process over a range ring is not necessary when one is considering only circular polarization states, since, for example, right circular polarization remains right circular at every point on a range ring.

When the quantities involved in (3. 19a) and (3. 19b) are substituted in and averages are made, the resulting average backscattering cross sections become

$$(3. 20a) \quad \gamma_{\parallel} = \frac{\langle \sigma_{\parallel} \rangle}{\text{Area}} = \frac{1}{8} [2 | \rho_{\parallel}(\theta) - \rho_{\perp}(\theta) |^2 + | \rho_{\parallel}(\theta) + \rho_{\perp}(\theta) |^2] \times$$

$$\begin{cases} \frac{1}{S^2} \cos^4 \theta e^{-\frac{1}{S^2} \sin^2 \theta \cos^2 \theta} & \text{for Gaussian JPDF} \\ \frac{3}{S^2} \cos^4 \theta e^{-\frac{\sqrt{6}}{S} \sin \theta \cos \theta} & \text{for Bessel JPDF} \end{cases}$$

$$(3. 20b) \quad \gamma_{\perp} = \frac{\langle \sigma_{\perp} \rangle}{\text{Area}} = \frac{1}{8} [| \rho_{\parallel}(\theta) + \rho_{\perp}(\theta) |^2]$$

$$\begin{cases} \frac{1}{S^2} \cos^4 \theta e^{-\frac{1}{S^2} \sin^2 \theta \cos^2 \theta} & \text{for Gaussian JPDF} \\ \frac{1}{S^2} \cos^4 \theta e^{-\frac{\sqrt{6}}{S} \sin \theta \cos \theta} & \text{for Bessel JPDF} \end{cases}$$

These average cross sections per range ring are plotted in Appendix G at various values of the parameters ϵ_r and S . It should be noted from the above equations that these cross sections, like those for circular polarization (per range ring also), are not functions of the averages involving slope, G_1 , G_2 , and G_3 . However, these cross sections are somewhat different in form from those for circular polarization; it is interesting to note that $\gamma_{\perp} = \frac{1}{2} \gamma_{RR}$. Many

attempt in the literature to compare results made with linear polarization directly with Pettengill's curves made using circular polarization. That such a comparison cannot be made is evident when comparing $\gamma_{\parallel}$ with γ_{RL} .

The author at the present time has no curves of backscattered power from the lunar surface using linear polarization to compare with those predicted in this subsection. In the near future, such measurements will begin at The Ohio State University Antenna Laboratory, and the measured data will be compared with the curves in Appendix G.

CHAPTER V CONCLUSIONS

In summary, this analysis offers the following original contributions to the theory of backscattering of electromagnetic waves from rough surfaces.

(a) The approximations generally implicit in the analyses of most investigators are critically reviewed here and the validity of each is discussed. Several of the serious restrictions and approximations are avoided in this analysis.

(b) It is found that the usual Kirchhoff-Huygens form of the physical optics integral suffers from the neglect of an important term in the analysis of scattering from perfectly conducting surfaces; this term is insignificant only near normal incidence. This term is included in the present analysis and resultant mathematical difficulties are avoided by a proper choice of the coordinate system. The results of this more exact formulation are quite simple mathematically and compare quite favorably with measured average backscattered power from rough surfaces, whereas former results were generally several degrees of magnitude in error at scattering angles more than 45° from normal.

(c) Backscattering from non-perfectly conducting surfaces is studied, and a simple integral form for the backscattered field from any scattering object is derived; this integral involves the local Fresnel parallel and perpendicular reflection coefficients and also the local surface slopes on the object. For rough surfaces, results for average backscattered power are obtained which permit one to determine the dielectric constant of a given scattering surface when compared with measured backscattered power from the surface. The predicted results are compared in particular with average backscattered power from the moon, and the comparison shows that the lunar surface has a relative dielectric constant of about 3.2 and a root mean square roughness slope of about 14° when compared to the measured return at 68 cm wavelength.

(d) In the analysis of backscattering from non-perfectly conducting surfaces, it is shown that depolarization does take place and an expression for the depolarized component is derived for both linear polarization states and circular polarization. The phenomenon of depolarization of backscattered waves is both intuitively logical and has been observed experimentally, but to date no theory of backscattering from perfectly conducting surfaces (or even non-perfectly conducting for that matter) employing the physical optics approximation has managed to yield any depolarization. The depolarized

component derived here conforms to one's expectations: it is smallest at normal incidence and at grazing incidence, and it increases in magnitude as surface roughness increases.

(e) Several statistical models are employed simultaneously and the results for average backscattered power are compared with measured results in each case. Thus one can readily see just how much error is introduced by each model. Surprisingly, both models for the surface joint probability density function give similar results (within about 6 db everywhere), whereas they are locally somewhat different in shape. The surface exponential correlation coefficient model, although it yields satisfactory curves for backscattered power, is shown to lead to contradictions and absurdities regarding surface roughness.

(f) Several facts about general rough surface statistics are brought out. Relationships for the root mean square surface slope derived from the surface correlation coefficient are examined in detail, and at least two misconceptions found in the literature in this respect are clarified. One involves the determination of the rms slope from the surface correlation coefficient and the other demonstrates that surfaces having exponential correlation coefficients all have infinite rms surface slopes.

The results of backscattering from rough surfaces has by no means been completely solved here. Only scattering from surfaces with roughness large in terms of wavelength has been considered here. The entire analysis has been restricted to backscattering rather than the more general case of bistatic scattering. The importance and role of the statistical model in the behavior of backscattered power has been only superficially examined here. Secondary effects such as multiple scattering by various points on the surface, shadowing, and treatment of inhomogeneous scattering bodies are omitted here. Hence, there is fertile ground in any of these areas for future investigation.

APPENDIX A

1. Proof of (3-10) and (3-11):

$$\oint_c \nabla \phi \cdot \bar{H} \cdot \bar{ds} = \sum_{\alpha=1}^3 \hat{e}_\alpha \oint_c \frac{\partial \phi}{\partial x_\alpha} \bar{H} \cdot \bar{ds}, \text{ where } \hat{e}_1 = \hat{x}, \hat{e}_2 = \hat{y}, \hat{e}_3 = \hat{z}.$$

Also, define $\bar{G}^\alpha = \frac{\partial \phi}{\partial x_\alpha} \bar{H}$. Application of Stokes theorem yields

$$\therefore \oint_c \nabla \phi \cdot \bar{F} \cdot \bar{ds} = \sum_{\alpha=1}^3 \hat{e}_\alpha \oint_c \bar{G}^\alpha \cdot \bar{ds} = \sum_{\alpha=1}^3 \hat{e}_\alpha \oint_{s_1} (\nabla \times \bar{G}^\alpha) \cdot \hat{n} da.$$

But: $\nabla \times \bar{G}^\alpha = \nabla \times \left(\frac{\partial \phi}{\partial x_\alpha} \bar{H} \right) = \left(\nabla \frac{\partial \phi}{\partial x_\alpha} \right) \times \bar{H} + \frac{\partial \phi}{\partial x_\alpha} \nabla \times \bar{H},$

The first term can be simplified by re-arranging;

$$\left[\left(\nabla \frac{\partial \phi}{\partial x_\alpha} \right) \times \bar{H} \right] \cdot \hat{n} = - (\hat{n} \times \bar{H}) \cdot \nabla \frac{\partial \phi}{\partial x_\alpha},$$

$$\therefore \sum_{\alpha=1}^3 \hat{e}_\alpha (\hat{n} \times \bar{H}) \cdot \nabla \frac{\partial \phi}{\partial x_\alpha} = (\hat{n} \times \bar{H}) \cdot \nabla \nabla \phi.$$

The second term can be simplified using Maxwell's equations;

$$\left(\frac{\partial \phi}{\partial x_\alpha} \nabla \times \bar{H} \right) \cdot \hat{n} = -j\omega \epsilon \frac{\partial \phi}{\partial x_\alpha} (\hat{n} \cdot \bar{E}).$$

$$\therefore \oint_c \nabla \phi \cdot \bar{H} \cdot \bar{ds} = - \int_{s_1} [(\hat{n} \times \bar{H}) \cdot \nabla \nabla \phi + j\omega \epsilon (\hat{n} \cdot \bar{E}) \nabla \phi] da$$

Proof of (3-11) is analagous to the proof of (3-10).

2. Proof of (3-14b):

$$R = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}.$$

$$\therefore \nabla R = - \frac{(x' - x) \hat{x} + (y' - y) \hat{y} + (z' - z) \hat{z}}{\sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}} = - \frac{\bar{R}}{R} = - \hat{\eta}.$$

3. Proof of (3-14c):

$$\nabla \phi = \frac{d\phi}{dR} \nabla R = \left(-\frac{1}{R} + jk \right) \frac{e^{jkr}}{R} (-\hat{\eta}) = \left(\frac{1}{R} - jk \right) \phi \hat{\eta}.$$

4. Proof of (3-14d):

$$(\hat{n} \times \bar{H}) \cdot \nabla \nabla \phi = (\hat{n} \times \bar{H}) \cdot \nabla \left[-\frac{d\phi}{dR} \hat{\eta} \right] = -(\hat{n} \times \bar{H}) \cdot \nabla R \left[\frac{d}{dR} \left(\frac{d\phi}{dR} \hat{\eta} \right) \right]$$

$$\frac{d}{dR} \left(\frac{d\phi}{dR} \hat{\eta} \right) = \frac{d^2 \phi}{dR^2} \hat{\eta} + \frac{d\phi}{dR} \frac{d\hat{\eta}}{dR}.$$

But if the field point P' is far from the surface compared with any dimension of the surface, then the unit vector, $\hat{\eta}$, is constant not only in magnitude, but is nearly constant in direction over the surface as a function of R for a fixed field point P' . Therefore, $d\hat{\eta}/dR \simeq 0$.

$$\therefore \frac{d}{dR} \left(\frac{d\phi}{dR} \hat{\eta} \right) \simeq \frac{d^2 \phi}{dR^2} \hat{\eta}, \quad (\hat{n} \times \bar{H}) \cdot \nabla R = -(\hat{n} \times \bar{H}) \cdot \hat{\eta}$$

$$\therefore (\hat{n} \times \bar{H}) \cdot \nabla \nabla \phi = \frac{d^2 \phi}{dR^2} [(\hat{n} \times \bar{H}) \cdot \hat{\eta}] \hat{\eta}.$$

The proof of the second equation of (3-14d) is identical.

APPENDIX B

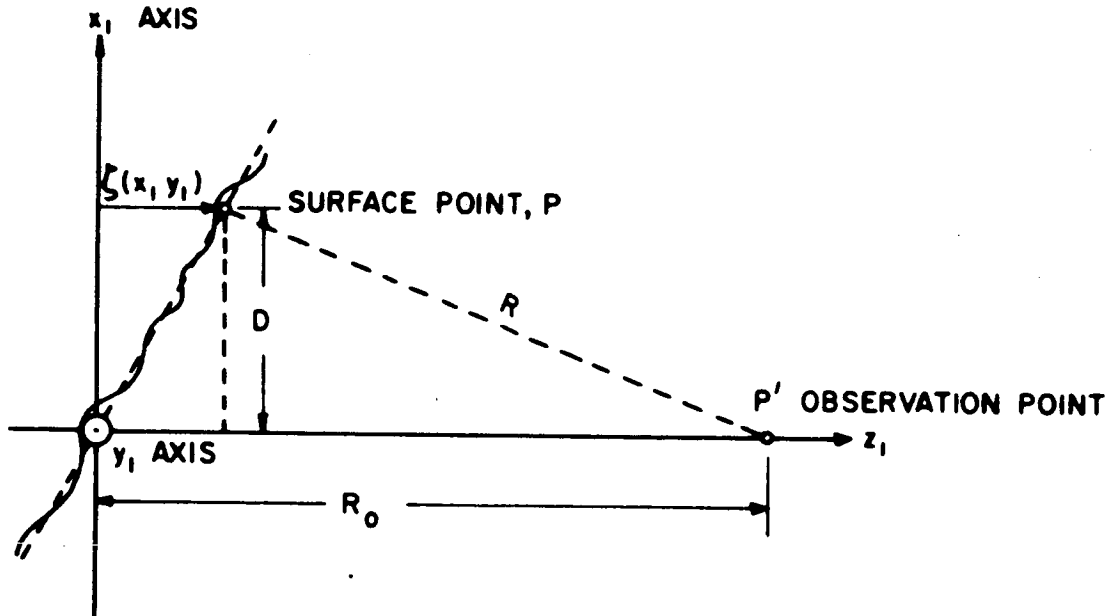


Fig. 25--Exaggerated view of the phase distortion introduced between different points on the surface.

In subsection 2 of Section A of Chapter II, the approximation was made that the line R is almost parallel to the z_1 axis when the observation point P' is far enough away so that the distance R is approximately given by

$$(A-1) \quad R \simeq R_0 - \xi_1 ;$$

this is to be used in the exponential $e^{jkR} = e^{jkR_0} \cdot e^{-jk\xi_1}$. It is well known from diffraction and antenna theory, however, that for a generally deterministic aperture or scatterer if $R_0 < 2D^2/\lambda$, then

approximation of R by equation (A-1) introduces a phase error in the exponential which is larger than $\pi/8$. It will be shown here that when a surface is randomly rough, the usual far field criterion $R_0 > 2D^2/\lambda$ need not be applied. The Gaussian JPDF model and Gaussian correlation coefficient will be used for this demonstration.

Let R be given by its more exact expression,

$$R = \sqrt{(R_0 - \zeta_1)^2 + D^2} \simeq R_0 - \zeta_1 + \frac{D^2}{2(R_0 - \zeta_1)} \simeq R_0 - \zeta_1 + \frac{D^2}{2R_0} ,$$

where $R_0 \gg D, \zeta_1$.

The additional term $D^2/2R_0$ then appears in the phase factor of (2.24); for the geometry shown, $D^2 = x_1^2 + y_1^2$. Thus the backscattered power is given by

$$\begin{aligned} |H^s|^2 &= \frac{k^2 |H^i|^2}{4\pi^2 R_0^2} \\ &\iiint\limits_{D_1} e^{-j2k\zeta_1 + j2k\zeta_1' + jk\frac{D_1^2}{2R_0}} dx_1 dy_1 dx_1' dy_1' \\ &= \frac{k^2 |H^i|^2}{4\pi R_0^2} \iiint\limits_{D_1} e^{-j2kn(y_1 - y_1') + j\frac{k}{2R_0} [(x_1^2 - x_1'^2) + (y_1^2 - y_1'^2)]} \\ &\quad e^{-j2kn\zeta_1 + j2kn\zeta_1'} dx_1 dy_1 dx_1' dy_1' . \end{aligned}$$

Upon making the substitution $x_1 = x_1' + \tau_x$, $y_1 = y_1' + \cos \theta \tau_y$, one arrives at a quadruple integral in terms of $x_1', y_1', \tau_x, \tau_y$. The upper and lower limits on the integral in τ_x are $\frac{L_x}{2} - x_1'$ and $-\frac{L_x}{2} - x_1'$,

while the upper and lower limits on the integral in τ_y are $\frac{L_y}{2} - \sec \theta y_1'$ and $-\frac{L_y}{2} - \sec \theta y_1'$; using the same reasoning as before upon averaging, these limits on τ_x and τ_y may be replaced by $\pm \infty$ because the integrand becomes vanishingly small when τ_x and τ_y become large due to the joint characteristic function. Therefore, the average power may be written

$$\begin{aligned} \langle |H^S|^2 \rangle = & \frac{k^2 |H^i|^2 \cos \theta}{4\pi^2 R_o^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M(-j2k \sec \theta, j2k \sec \theta; \rho) \\ & e^{-j2k \sin \theta \tau_y + j \frac{k}{2R_o} (\tau_x^2 + \cos^2 \theta \tau_y^2)} \times \\ & \left\{ \int_{-L_x/2}^{L_x/2} \int_{-L_y \cos \theta/2}^{L_y \cos \theta/2} e^{j \frac{k}{R_o} \tau_x x_1' + j \frac{k}{R_o} \cos \theta \tau_y y_1'} dx_1' dy_1' \right\} d\tau_x d\tau_y. \end{aligned}$$

The integration in braces over x_1' and y_1' may be carried out, yielding

$$\begin{aligned} \langle |H^S|^2 \rangle = & \frac{k^2 |H^i|^2 L_x L_y \cos^2 \theta}{4\pi^2 R_o^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\frac{\sin\left(\frac{kL_x}{2R_o} \tau_x\right)}{\left(\frac{kL_x}{2R_o} \tau_x\right)} \right] \\ & \left[\frac{\sin\left(\frac{kL_y}{2R_o} \cos^2 \theta \tau_y\right)}{\left(\frac{kL_y}{2R_o} \cos^2 \theta \tau_y\right)} \right] \cdot M_{\zeta\zeta}'(-j2k \sec \theta, j2k \sec \theta; \rho) \times \\ & e^{-j2k \sin \theta \tau_y + j \frac{k}{2R_o} (\tau_x^2 + \cos^2 \theta \tau_y^2)} d\tau_x d\tau_y. \end{aligned}$$

In the case of the Gaussian JPDF and Gaussian correlation coefficient, the joint characteristic function becomes

$$M_{\zeta\zeta}(-j2k \sec \theta, j2k \sec \theta; \rho) = e^{-4\sigma^2 k^2 \sec^2 \theta \left(1 - e^{-\frac{\rho^2}{a^2}}\right)}$$

$$\approx e^{-4\sigma^2 k^2 \sec^2 \theta \frac{\rho^2}{a^2}} ;$$

the second approximation is possible because $4\sigma^2 k^2 \sec^2 \theta \gg 1$ for the class of surfaces under consideration. Since $\rho^2 = \tau_x^2 + \tau_y^2$, one can easily see that the joint characteristic function goes to zero very rapidly for τ_x and τ_y even slightly different from zero due to the large constant $4\sigma^2 k^2 \sec^2 \theta / a^2$ in the exponent. However, in this range of small τ_x and τ_y , the factors in square brackets in the integrand are almost identically unity because of the small constants $kL_x/2R_0$ and $kL_y \cos^2 \theta / 2R_0$; it was assumed previously that L_x/R_0 , $L_y/R_0 \ll 1$. Therefore these terms in brackets have the nearly constant value $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ over the ranges of small τ_x and τ_y and may be replaced by their limiting value. The integral becomes

$$\langle |H^s|^2 \rangle = \frac{k^2 |H^i|^2 L_x L_y \cos^2 \theta}{4\pi^2 R_0^2}$$

$$\int_{-\infty}^{\infty} e^{-\left(4\sigma^2 k^2 \frac{\sec^2 \theta}{a^2} - j \frac{k}{2R_0}\right) \tau_x^2} d\tau_x$$

$$\int_{-\infty}^{\infty} e^{-\left(4\sigma^2 k^2 \frac{\sec^2 \theta}{a^2} - j \frac{k}{2R_0} \cos^2 \theta\right) \tau_y^2 - j2k \sin \theta \tau_y} d\tau_y .$$

When these integrals are evaluated from the tables, they give

$$\begin{aligned} \langle |H^S|^2 \rangle = & \frac{|H^i|^2 L_x L_y \cos^4 \theta}{4\pi R_o^2} \frac{a^2}{4\sigma^2} \cdot \frac{1}{\sqrt{1-j \frac{a^2 \cos^2 \theta}{8k R_o \sigma^2}}} \cdot \\ & \frac{1}{\sqrt{1-j \frac{a^2 \cos^4 \theta}{8k R_o \sigma^2}}} \cdot e^{-\frac{a^2}{4\sigma^2} \times \sin^2 \theta \cos^2 \theta / \left(1-j \frac{a^2 \cos^4 \theta}{8k R_o \sigma^2}\right)} \end{aligned}$$

However, this above equation reduces identically to (3.10a) when one replaces $S^2 = 4\sigma^2/a^2$ and converts to backscattering cross section per unit area if the imaginary term is very much less than unity wherever it appears. This stipulation requires that $a^2/8kR_o\sigma^2 \ll 1$, or

$$(A-2a) \quad \frac{8kR_o\sigma^2}{a^2} \gg 1.$$

Stated in terms of rms surface slope, this requirement becomes

$$(A-2b) \quad 2S^2 \cdot kR_o \gg 1.$$

This demand is nearly always satisfied, since kR_o is a very large number. However, one cannot expect that when mean square surface slope, S^2 , approaches zero in the limit that the resulting back-scattered power is valid for a smooth flat plate because of violation of the implied restriction imposed by (A-2). Nonetheless,

restriction (A-2) is much more easily satisfied in general than by requiring that $R_0 > \frac{2L_x^2}{\lambda}$, as, for instance, when one considers the moon as a scattering body.

APPENDIX C

It has been shown in subsections 4 and 5 of Section D of Chapter II that certain integrals of the following form are to be evaluated.

$$I = \iiint \int < f\left(\frac{\partial \zeta_1}{\partial x_1}, \frac{\partial \zeta_1}{\partial y_1}\right) f^*\left(\frac{\partial \zeta'_1}{\partial x'_1}, \frac{\partial \zeta'_1}{\partial y'_1}\right) > e^{-j2k(\zeta_1 - \zeta'_1)} dx_1 dy_1 dx'_1 dy'_1$$

The primed quantities are considered at the surface point (x'_1, y'_1) while the unprimed quantities are at point (x_1, y_1) . The functions $f\left(\frac{\partial \zeta_1}{\partial x_1}, \frac{\partial \zeta_1}{\partial y_1}\right)$ involve the surface slopes $\frac{\partial \zeta_1}{\partial x_1}$ and $\frac{\partial \zeta_1}{\partial y_1}$ which are independent from each other at any surface point; for example, the quantity f can represent the function

$$\frac{\frac{\partial \zeta_1}{\partial x_1} \frac{\partial \zeta_1}{\partial y_1}}{\left(\frac{\partial \zeta_1}{\partial x_1}\right)^2 + \left(\frac{\partial \zeta_1}{\partial y_1}\right)^2},$$

which appears in equations (2.45) and (2.46) for the depolarized scattered linear component.

The surface slopes $\frac{\partial \zeta_1}{\partial x_1}$ and $\frac{\partial \zeta_1}{\partial y_1}$ are functions of the original surface slopes $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ of the rough planar surface located in the x-y plane; the functional relationships between $\frac{\partial \zeta_1}{\partial x_1}$, $\frac{\partial \zeta_1}{\partial y_1}$ and $\frac{\partial \zeta}{\partial x}$, $\frac{\partial \zeta}{\partial y}$

are given in equations (2.41) and (2.42). Hence the function f can be considered a function of $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$, i.e.,

$$\left\langle f\left(\frac{\partial \zeta_1}{\partial x_1}, \frac{\partial \zeta_1}{\partial y_1}\right) f^*\left(\frac{\partial \zeta'_1}{\partial x'_1}, \frac{\partial \zeta'_1}{\partial y'_1}\right) \right\rangle = \left\langle g\left(\frac{\partial \zeta}{\partial x}, \frac{\partial \zeta}{\partial y}\right) g^*\left(\frac{\partial \zeta'}{\partial x'}, \frac{\partial \zeta'}{\partial y'}\right) \right\rangle,$$

where
$$g\left(\frac{\partial \zeta}{\partial x}, \frac{\partial \zeta}{\partial y}\right) \equiv f\left(\frac{\partial \zeta_1}{\partial x_1}, \frac{\partial \zeta_1}{\partial y_1}\right).$$

The second average in the integral of I is given by

$$(C-1) \quad \left\langle e^{-j2k(\zeta_1 - \zeta'_1)} \right\rangle = e^{j2k_1 \sin \theta (y - y')} \cdot M_{\zeta \zeta'}(-j2k \sec \theta, j2k \sec \theta; \rho),$$

following the analysis of subsection 2, Section B of Chapter II. In the case where the Gaussian JPDF and Gaussian correlation coefficient are used, this characteristic function becomes

$$(C-2) \quad M_{\zeta \zeta'} = e^{-4k^2 \sigma^2 \sec^2 \theta \frac{\rho^2}{a^2}},$$

where $\rho^2 = \tau_x^2 + \tau_y^2 \equiv$ distance along the mean surface plane between points (x_1, y_1) and x'_1, y'_1 .

The random variables $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta'}{\partial x'}$ (as well as $\frac{\partial \zeta}{\partial y}$ and $\frac{\partial \zeta'}{\partial y'}$) appearing in the averages of gg^* are correlated; the correlation coefficient between $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta'}{\partial x'}$ is given in subsection 4, Section C of Chapter II. The correlation function is

$$(C-3) \quad \left\langle \frac{\partial \zeta}{\partial x} \frac{\partial \zeta'}{\partial x'} \right\rangle = S_x^2 R_{S_x}(\sqrt{\tau_x^2 + \tau_y^2}) = -\sigma^2 \frac{\partial^2}{\partial \tau_x^2} R(\sqrt{\tau_x^2 + \tau_y^2}),$$

where $R(\sqrt{\tau_x^2 + \tau_y^2}) = e^{-(\tau_x^2 + \tau_y^2)/a^2} = e^{-\rho^2/a^2}$ = surface height correlation coefficient, $S_x^2 = 2\sigma^2/a^2$ = mean square x-direction slope, and $R_{S_x} = R_{S_y}$ = correlation coefficients for x and y direction slopes. From (C-3), these correlation coefficients of surface slopes are

$$(C-4a) \quad R_{S_x}(\sqrt{\tau_x^2 + \tau_y^2}) = -\frac{a^2}{2} \frac{\partial^2}{\partial \tau_x^2} R(\sqrt{\tau_x^2 + \tau_y^2}) = \left(1 - \frac{2\tau_x^2}{a^2}\right) e^{-\tau_x^2 + \tau_y^2/a^2},$$

$$(C-4b) \quad R_{S_y}(\sqrt{\tau_x^2 + \tau_y^2}) = \left(1 - \frac{2\tau_y^2}{a^2}\right) e^{-\tau_x^2 + \tau_y^2/a^2}.$$

Hence, since the second order probability density functions of $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta'}{\partial x}$ are given in terms of S_x^2 and R_{S_x} , then the averages of gg^* are also in terms of S_x^2 and R_{S_x} (and also S_y^2 and R_{S_y}). Therefore, write

$$(C-5) \quad \left\langle g\left(\frac{\partial \zeta}{\partial x}, \frac{\partial \zeta}{\partial y}\right) g^*\left(\frac{\partial \zeta'}{\partial x}, \frac{\partial \zeta'}{\partial y}\right) \right\rangle = h(S_x, R_{S_x}; S_y, R_{S_y}).$$

But it can be seen from (C-1) and (C-2) that the characteristic function in the integrand becomes vanishingly small for τ_x or τ_y slightly different from zero due to the large constant $4k^2\sigma^2\sec^2\theta/a^2$ in the exponential of (C-2), whereas in this range of very small τ_x and τ_y , R_{S_x} and R_{S_y} are still very close to unity, as can be seen from equations (C-4). Therefore, the average of the functions gg^* are essentially constant over the range of very small τ_x and τ_y in which the joint characteristic function (C-1) is non-zero; this constant

value is that where $\tau_x = \tau_y = 0$ and $R_{S_x} = R_{S_y} = 1$. Hence, averages of the form in (C-5) may be replaced by

$$\begin{aligned}
 \text{(C-6)} \quad & \left\langle g\left(\frac{\partial \zeta}{\partial x}, \frac{\partial \zeta}{\partial y}\right) g^*\left(\frac{\partial \zeta'}{\partial x'}, \frac{\partial \zeta'}{\partial y'}\right) \right\rangle = \left\langle \left| g\left(\frac{\partial \zeta}{\partial x}, \frac{\partial \zeta}{\partial y}\right) \right|^2 \right\rangle \\
 & = \left\langle \left| g\left(\frac{\partial \zeta'}{\partial x'}, \frac{\partial \zeta'}{\partial y'}\right) \right|^2 \right\rangle,
 \end{aligned}$$

i. e., the random variables $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta'}{\partial x'}$ are perfectly correlated in this range so that $\frac{\partial \zeta}{\partial x} = \frac{\partial \zeta'}{\partial x'}$. These averages as in (C-6) are no longer functions of τ_x or τ_y and may be removed from the integrand.

APPENDIX D

From equations (3.5), the statistical model for the JPDF to be used is

$$(D-1) \quad W(\zeta, \zeta'; \rho) = \frac{3}{\pi^2 \sigma^2 (1-R^2)^{1/2}} \left[\frac{\sqrt{3} |\zeta|}{\sigma} \right] \left[\frac{\sqrt{3} |\zeta' - R\zeta|}{\sigma (1-R^2)^{1/2}} \right] \\ K_1 \left[\frac{\sqrt{3} |\zeta|}{\sigma} \right] K_1 \left[\frac{\sqrt{3} |\zeta' - R\zeta|}{\sigma (1-R^2)^{1/2}} \right],$$

where ζ and ζ' are the two random variables representing the surface height at the points (x, y) , and (x', y') ; R is the correlation coefficient between ζ and ζ' , while the means of ζ and ζ' are both zero and the variances of both are σ^2 . K_1 is a first order modified Bessel function of the second kind. R is a function of $\rho = \sqrt{(x-x')^2 + (y-y')^2}$, i. e., separation between the two points.

A. Determination of the Joint Characteristic Function

The joint characteristic function is defined as

$$M_{\zeta \zeta'}(ju, jv; \rho) = \iint_{-\infty}^{\infty} e^{ju\zeta + jv\zeta'} W(\zeta, \zeta'; \rho) d\zeta d\zeta' \\ = \frac{3}{\pi^2 \sigma^2 (1-R^2)^{1/2}} \iint_{-\infty}^{\infty} e^{ju\zeta + jv\zeta'} \left[\frac{\sqrt{3} |\zeta|}{\sigma} \right] \left[\frac{\sqrt{3} |\zeta' - R\zeta|}{\sigma (1-R^2)^{1/2}} \right] K_1 \left[\frac{\sqrt{3} |\zeta|}{\sigma} \right] \\ K_1 \left[\frac{\sqrt{3} |\zeta' - R\zeta|}{\sigma (1-R^2)^{1/2}} \right] d\zeta d\zeta'.$$

Make the following change of variables

$$x \equiv \frac{\sqrt{3}}{\sigma} \zeta \quad ; \quad y \equiv \frac{\sqrt{3}}{\sigma(1-R^2)^{1/2}} [\zeta' - R\zeta] \quad .$$

Then the joint characteristic function becomes

$$\begin{aligned} M_{\zeta\zeta'}(ju, jv; \rho) &= \frac{1}{\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j \frac{u\sigma(1-R^2)^{1/2}}{\sqrt{3}} y + j \frac{u\sigma R}{\sqrt{3}} x + j \frac{u\sigma}{\sqrt{3}} x} \\ &\quad |x| |y| K_1(|x|) K_1(|y|) dx dy \\ &= \left\{ \frac{1}{\pi} \int_{-\infty}^{\infty} e^{jay} |y| K_1(|y|) dy \right\} \\ &= \left\{ \frac{1}{\pi} \int_{-\infty}^{\infty} e^{jbx} |x| K_1(|x|) dx \right\} \quad , \end{aligned}$$

where $a = \frac{u\sigma(1-R^2)^{1/2}}{\sqrt{3}}$ and $b = \frac{\sigma}{\sqrt{3}}(uR + v)$. The integrals in braces are identical in form and each is integrated as follows

$$\begin{aligned} \frac{1}{\pi} \int_{-\infty}^{\infty} e^{jay} |y| K_1(y) dy &= \frac{1}{\pi} \left[- \int_{-\infty}^0 e^{jay} y K_1(-y) dy \right. \\ &\quad \left. + \int_0^{\infty} e^{jay} y K_1(y) dy \right] \quad . \end{aligned}$$

$$\text{But } - \int_{-\infty}^0 e^{jay} y K_1(-y) dy = + \int_0^{\infty} e^{jay} y K_1(-y) dy = \int_0^{\infty} e^{-jay} y K_1(y) dy \quad ,$$

where the last integral was obtained by replacing y by $-y$. Thus the original integral becomes

$$\begin{aligned} \frac{1}{\pi} \int_{-\infty}^{\infty} e^{j a y} |y| K_1(|y|) dy &= \frac{1}{\pi} \int_0^{\infty} (e^{j a y} + e^{-j a y}) y K_1(y) dy \\ &= \frac{2}{\pi} \int_0^{\infty} y \cos a y K_1(y) dy. \end{aligned}$$

The last integral is evaluated in the tables [Ref. 28, p. 763, #12] :

$$\frac{2}{\pi} \int_0^{\infty} y \cos a y K_1(y) dy = \frac{1}{(1 + a^2)^{3/2}}.$$

Hence the joint characteristic function becomes

$$\begin{aligned} M_{\zeta \zeta'}(j u, j v; \rho) &= \frac{1}{(1 + a^2)^{3/2}} \cdot \frac{1}{(1 + b^2)^{3/2}} \\ \text{(D-2)} \quad \therefore M_{\zeta \zeta'}(j u, j v; \rho) &= \frac{1}{\left(1 + \frac{u^2 \sigma^2}{3} (1 - R^2)\right)^{3/2}} \cdot \frac{1}{\left(1 + \frac{\sigma^2}{3} (u R + v)^2\right)^{3/2}}. \end{aligned}$$

Several facts may be readily verified from this characteristic function:

$$\text{(i)} \quad 1 = \iint_{-\infty}^{\infty} W(\zeta, \zeta') d\zeta d\zeta' = M_{\zeta \zeta'}(j 0, j 0; \rho)$$

$$\text{(ii)} \quad 0 = \langle \zeta \rangle = \langle \zeta' \rangle = -j \left. \frac{\partial M_{\zeta \zeta'}}{\partial u} \right|_{u, v=0} = -j \left. \frac{\partial M_{\zeta \zeta'}}{\partial v} \right|_{u, v=0}$$

$$\text{(iii)} \quad \sigma^2 = \langle \zeta^2 \rangle = \langle \zeta'^2 \rangle = - \left. \frac{\partial^2 M_{\zeta \zeta'}}{\partial u^2} \right|_{u, v=0} = - \left. \frac{\partial^2 M_{\zeta \zeta'}}{\partial v^2} \right|_{u, v=0}$$

$$\text{(iv)} \quad \sigma^2 R = \langle \zeta \zeta' \rangle = - \left. \frac{\partial^2 M_{\zeta \zeta'}}{\partial u \partial v} \right|_{u, v=0}.$$

Thus the statistical model of (D-1) and (D-2) possesses all the necessary requirements of a true probability density function and characteristic function of a random surface height, except that $W(\zeta, \zeta'; \rho)$ is not symmetric in ζ and ζ' (and consequently $M_{\zeta\zeta'}(ju, jv; \rho)$ is not symmetric in u and v).

B. Evaluation of Integral (2.26)
and (2.27)

Employing the statistical model of (D-2) for the joint characteristic function, the expressions for the covariance between the scattered fields at two frequencies (given in (2.26) and (2.27); (2.26) is the same as (4.1) when the average backscattered field is zero) can be evaluated; by making the frequencies equal to each other, the result is the average backscattered power. Since both integrals are similar in form, only the derivation for (2.26) will be made here. The integral of this equation has the form

$$I \equiv \int_0^{\infty} \rho J_0(2k_1 \sin \theta \rho) M_{\zeta\zeta'}(-j2k_1 \sec \theta, +j2k_2 \sec \theta; \rho) d\rho,$$

where according to (D-2), the joint characteristic function has the form (defining $m \equiv \frac{2k_1 \sec \theta \sigma}{\sqrt{3}}$ and $n \equiv \frac{2k_2 \sec \theta \sigma}{\sqrt{3}}$

$$M_{\zeta\zeta'}(-j2k_1 \sec \theta, j2k_2 \sec \theta; \rho) = \frac{1}{\{[1 + m^2(1 - R^2)][1 + (-mR + n)^2]\}^{3/2}} \\ \equiv \frac{1}{D^{3/2}}$$

Upon expanding D, one has

$$D = 1 + m^2 + n^2 + m^2 n^2 (1 - R^2) + m^4 (R^2 - R^4) - 2mn(R + m^2(R - R^3))$$

But as correlation function R approaches zero, (i. e., for separation, ρ , very large), this function becomes $D = 1 + m^2 + n^2 + m^2 n^2$, which is a very large quantity because of the fact that $k_1 \sigma$ and $k_2 \sigma$ are very large. Hence $1/D^{3/2}$ is very small and the integrand is negligible. But for $R = 1$ (i. e., $\rho = 0$), then $D = 1 + m^2 + n^2 - 2mn = 1 + (m - n)^2$, which may be small for m and n approximately equal. However, since D becomes very large for R varying slightly from unity, the first two terms in the series expansion of $R = e^{-\rho^2/a^2}$ may be used (only the Gaussian class correlation coefficient model is considered here); therefore, in this range, $R = 1 - \frac{\rho^2}{a^2}$. The equation for D becomes,

$$D = 1 + (m - n)^2 + \frac{2}{a^2} [m^2(m - n)^2 + mn] \rho^2, \text{ or}$$

$$D \equiv \alpha^2 + \beta^2 \rho^2,$$

where $\alpha^2 = 1 + (m - n)^2$, $\beta^2 = \frac{2}{a^2} [m^2(m - n)^2 + mn]$.

Let $\gamma \equiv 2k_1 \sin \theta$. Therefore the integral I becomes

$$I = \frac{1}{\beta^3} \int_0^\infty \rho J_0(\gamma \rho) \frac{1}{(\rho^2 + \alpha^2/\beta^2)^{3/2}} d\rho = \frac{1}{\alpha \beta^2} e^{-\gamma \frac{\alpha}{\beta}}.$$

The value of this last integral was found in Ref. 28, p. 696, #6. 554-4.

Substituting the actual constants into α , and β and noting that

$k_2 = k_1 + \Delta k$, one obtains

$$\alpha = \sqrt{1 + \frac{4}{3} \sigma^2 \Delta k^2 \sec^2 \theta}$$

$$\beta = \frac{\sqrt{2}}{a} \sqrt{\frac{16}{9} \sigma^4 \sec^4 \theta k_1^2 \Delta k^2 + \frac{4}{3} \sigma^2 \sec^2 \theta k_1 (k_1 + \Delta k)} ;$$

however, since $\Delta k \ll k_1$, the expression for β becomes

$$\beta = \frac{\sqrt{2}}{a} \frac{2}{\sqrt{3}} \sigma \sec \theta k_1 \sqrt{1 + \frac{4}{3} \sigma^2 \Delta k^2 \sec^2 \theta} = 2 \sqrt{\frac{2}{3}} \frac{\sigma}{a} k_1 \sec \theta \alpha .$$

Therefore, the integral I becomes

$$I = \frac{3}{8} \frac{a^2}{\sigma^2 k_1^2} \cos^2 \theta \frac{1}{\left(1 + \frac{4}{3} \sigma^2 \Delta k^2 \sec^2 \theta\right)^{3/2}} e^{-\sqrt{\frac{3}{2}} \frac{a}{\sigma} \sin \theta \cos \theta} ;$$

substituting this value for I into the covariance of equation (2.26) and

employing the fact that $S^2 = \frac{4\sigma^2}{a^2}$, and $\frac{k_1}{k_2} = 1$ for $\Delta k \ll k_1$, one

obtains

$$(D-3) \quad \text{Cov}[H_1^s H_2^{s*}] = \frac{H_{01}^i H_{02}^i}{4\pi R_o^2} \frac{3}{S^2} L_x L_y e^{j\Delta k R_o} \\ \left(\frac{\sin(\Delta k \sin \theta L_y)}{\Delta k \sin \theta L_y} \right) \cos^4 \theta e^{-\frac{\sqrt{6}}{S} \sin \theta \cos \theta} \\ \frac{1}{\left[1 + \frac{4}{3} \sigma^2 \Delta k^2 \sec^2 \theta\right]^{3/2}}$$

The averaged backscattered power is obtained by setting $k_1 = k_2 = k$, $\Delta k = 0$ in the above equation; this gives

$$(D-4) \quad \text{Var}[H^S] = \langle |H^S|^2 \rangle = \frac{|H^i|^2 L_x L_y}{4\pi R_o^2} \gamma_{PC} =$$

$$\frac{|H^i|^2}{4\pi R_o^2} L_x L_y \cdot \frac{3}{S^2} \cos^4 \theta e^{-\frac{\sqrt{6}}{S} \sin \theta \cos \theta} .$$

Equation (D-3) is equation (4. 7) of the text while (D-4) is used to find (3. 10c).

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