NASA TN D-2795

A STUDY OF LUNAR LANDING SITES AND ASSOCIATED STAY TIMES

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

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A study utilizing the results of a patched conic approximation was conducted to determine the possible landing sites and associated stay times on the lunar surface that are compatible with the lunar-orbit-rendezvous technique, that is, descent from and return to a vehicle orbiting the moon. Particular emphasis is placed on landing sites that allow a return to the established lunar orbit at all times during the exploration period. Three different landing and take-off maneuvers are considered as well as the effect of variations of the energy and inclination of the earth-to-moon transfer trajectory. Consideration is given to maneuvers that permit the most flexibility in selection of landing sites as well as maneuvers that allow the longest stay time on the lunar surface for a given total plane-change capability.

The results of this study indicate that the landing sites and associated stay times are strongly dependent on the inclination of the established lunar orbit as well as the specific landing and take-off maneuvers that are employed. If the inclination of the established lunar orbit is constrained to equal the sum of the landing-site latitude φ and the plane-change capability δ, much versatility exists in the selection of possible landing sites on the lunar surface and in the corresponding stay times. Employment of a plane-change capability during the take-off maneuver allows extended exploration periods and provides an inherent safety factor for the mission. For this case, the landing vehicle can return to the established lunar orbit at any time during the exploration period.

AUTHOR

INTRODUCTION

Current plans for manned lunar exploration are based on utilization of the lunar-orbit-rendezvous (LOR) technique. This technique requires the establishment of a close lunar satellite orbit prior to the initiation of the lunar landing operation. The landing, take-off, and rendezvous maneuvers are performed by a small spacecraft which is detached from a larger orbiting vehicle. This small spacecraft is referred to as a lunar lander or a lunar exploration module. The larger vehicle which remains in the established lunar orbit is called the command module.

Since the earth-moon transfer trajectory for the early Apollo missions will probably be a circumlunar fail-safe type of a trajectory, the lunar satellite orbit will of necessity have a low inclination to the lunar equatorial plane and therefore the lunar exploration module will land in the low-latitude regions of the lunar surface. (See ref. 1.) Later missions, however, will probably not be subject to this constraint, but will be designed to explore specific regions elsewhere on the lunar surface. Therefore, it is of interest to determine the lunar landing sites which are accessible within the constraints of the lunar-orbit-rendezvous technique. In addition, it is desirable to ascertain how long the exploration module can stay on the lunar surface at any desired landing site. Some of the factors that affect the stay time are location of the landing site with respect to the lunar orbit and plane-change capability which is a direct function of the onboard fuel limitations.

The relationship between the injection conditions for the earth-moon transfer trajectory and the geometrical characteristics of lunar orbits that can be established from a given transfer trajectory is discussed in reference 2. Reference 3 gives the results of a preliminary analysis of the possible lunar landing sites and their associated stay times on the lunar surface.

The purpose of this report is to present the results of a more comprehensive analysis of the lunar landing site and stay-time problem based on the lunar-orbit-rendezvous technique. The results of a patched-conic technique is utilized to relate the characteristics of the earth-moon transfer trajectory to the characteristics of the lunar orbits that are established. Consideration is given to three different landing and take-off procedures, various plane-change capabilities, and the effect of inclination of the earth-moon transfer trajectory on the lunar landing sites and their associated stay times.

SYMBOLS

i — inclination of lunar orbit plane to lunar equator (positive as shown in fig. 1), deg

i_{min} — minimum inclination orbit that can be established, $i_{min} = \eta$, deg

i_o — inclination of earth-moon transfer trajectory plane to earth-moon plane, deg

k — arc of spherical triangle (see sketch (4))

$\left(\frac{V}{V_p}\right)_o$ — ratio of injection velocity to parabolic velocity at earth-injection altitude

T_s — time required to go from lunar sphere of influence to lunar-orbit altitude, days

ΔT — specified stay time, days

β	spherical angle (see sketch (4))
δ	magnitude of plane-change capability, that is, offset, deg
δ_i	initial or landing offset, deg
δ_f	final or take-off offset, deg
δ_t	total offset, deg
γ	arc of spherical triangle, deg (see sketch (2))
η	latitude of entry point on sphere of influence
Ω	longitude of node of lunar orbit plane when vehicle is at moon's sphere of influence, measured from earth-moon line, deg
Ω_2	longitude of node of lunar orbit plane when vehicle is at orbit-establishment altitude, measured from earth-moon line, deg
λ_3	longitude of westernmost lunar landing site, deg (point A, fig. 3)
λ_4	longitude of point B (fig. 3), deg
λ_5	landing-site longitude for a given stay time on lunar surface, deg
λ^*	longitude of inplane landing sites associated with an unlimited stay time, deg
$\dot{\omega}$	moon's angular rate (13.2° per day)
φ	parallel of latitude on lunar surface measured positive from lunar equator north, deg
φ^*	maximum latitude of landing site for an unlimited stay time on lunar surface, deg
φ_1	inplane landing-site latitude, $i - \delta$, deg (point A, sketch (1))
φ_2	inplane landing-site latitude, $i - \frac{\delta}{2}$, deg (point B, sketch (1))
φ_3	inplane landing-site latitude, $i - \frac{\delta}{4}$, deg (point C, sketch (1))
φ_4	inplane landing-site latitude, i, deg (point D, sketch (1))
φ_1'	inplane landing-site latitude, $i - \delta$, deg (point E, sketch (1))

φ_2'	inplane landing-site latitude, $i - \frac{\delta}{2}$, deg (point F, sketch (1))
φ_3'	inplane landing-site latitude, $i - \frac{\delta}{4}$, deg (point G, sketch (1))
ρ	arc of spherical triangle (see sketch (2))
τ	stay time or exploration period on lunar surface, days
θ	angle between points A and B, projected on lunar surface, deg
θ_1	arc of spherical triangle, deg (see sketch (3))
θ_2	arc of spherical triangle, deg (see sketch (3))
ξ	longitude of entry point on lunar sphere of influence, deg

PRELIMINARY CONSIDERATIONS

It is assumed that the lunar equator and the earth-moon plane are coincident (they actually differ by approximately 6.7°) and the moon is in a circular orbit about the earth at its mean distance (384,560 km). It is also assumed that the landing maneuver is initiated immediately after the lunar orbit is established, and the effect of lunar oblateness, which causes a slight regression of the lunar orbit node line, is negligible.

Lunar-Orbit Characteristics

The orbit inclination i and nodal position Ω are the orbit characteristics that define the spatial orientation of an established lunar orbit as illustrated in figure 1. In view of the prohibitive expense involved in making large orbital plane changes, it is expected that the exploration module will land nearly in the orbital plane of the command vehicle. With this consideration in mind, the landing sites which can be achieved form a band on the surface of the moon directly beneath the lunar orbit, also illustrated in figure 1. The width of this band is determined by the amount of onboard fuel available for making small orbital plane changes during the landing maneuver. It can be seen that the location of the possible landing sites are strongly dependent on the geometric characteristics (inclination i and ascending node Ω) of the lunar orbits that are established from a typical earth-moon transfer trajectory.

Reference 2, which is based on a "patched conic" approximation, has shown that the orbital planes of lunar orbits that can be established from a given earth-moon transit trajectory have approximately a common point of intersection. This common point on the lunar sphere of influence defines a point with longitude ξ and latitude η which is often called the "normal entry" point. It is shown in reference 1 that the coordinates of the so-called normal entry point are related to the injection conditions of the transfer trajectory by a set of algebraic equations. Figure 2 presents typical values of ξ and η for a range of injection velocity ratios $(V/V_p)_o$ and transfer trajectory inclinations i_o. It should be noted that the flight time from the earth to the moon is excessively long for the lower velocity ratios and payload considerations usually limit the higher velocities. Hence, for manned flight, only the middle region of figure 2 is practical; thus, the entry point is restricted to a relatively small region on the sphere of influence.

The entry-point concept provides a relationship between the longitude ξ and latitude η of the entry point, the lunar orbit inclination i, and the orbit nodal position Ω at the sphere of influence, which is derived in reference 2 as

$$\Omega = \xi + \sin^{-1} \frac{\tan \eta}{\tan i} \qquad (i \geqq \eta) \qquad (1)$$

Since ξ and η are fixed for a given transfer trajectory, it is clear that Ω and i are not independent parameters, and once one is selected, the other is defined. It should be noted that equation (1) defines a minimum inclination orbit $(i \equiv \eta)$ that can be established for a given entry point, and that it is possible to establish a lunar orbit that will pass over any point on the lunar surface by the proper selection of the orbit inclination i or orbit nodal position Ω.

Lunar Landing and Take-Off Procedures

There are a number of possible procedures for effecting the lunar landing and take-off phases of the mission. Consideration has been given to three basic landing and take-off procedures which are as follows: (1) inplane landing and take-off, (2) inplane landing and offset capability during take-off, and (3) offset capability during landing and take-off.

The offset capability is defined as a plane-change capability of magnitude δ. The magnitude of the offset δ is usually restricted by propulsion considerations, that is, the amount of onboard fuel available for making plane changes during the lunar landing and take-off operations. The use of an offset capability and some of the fuel requirements for earth rendezvous maneuvers have been demonstrated in reference 4. Landings at any point on the lunar surface are possible, without regard to the specific landing maneuver employed, through the establishment of lunar orbits that pass over the desired landing sites. However, the exploration time allowed on the lunar surface is dependent on both the landing and take-off maneuvers that are employed. The geometry of the lunar landing and take-off procedures is shown in figure 3. For the case of inplane landing and take-off, the descent from the lunar orbit is made at point C with the return to orbit at point D. For the inplane landing and offset capability take-off, the descent from orbit is made at point C and the return to orbit is initiated at point A'. For the case of offset capability during both landing and take-off, the descent to the lunar surface is made at point A and the return to orbit is again initiated at point A'. Since the lunar orbit is fixed in space and the moon rotates at an angular rate of approximately 13.2° per day, there exists a relative motion between a given point on the lunar surface and points on the established lunar orbit. This relative motion, coupled with the type of landing and take-off operation employed, defines the length of the exploration period on the lunar surface. The three landing and take-off maneuvers mentioned above are discussed more fully in the following paragraphs.

Inplane landing and take-off.- The inplane landing and take-off maneuver ($\delta_i = 0^{\circ}$; $\delta_f = 0^{\circ}$) is conceptually the simplest method for performing the lunar landing and take-off operations. The primary requirement on the lunar orbit for the inplane maneuver is that the orbit ground track must pass over the desired landing site so that the inplane landing maneuver can be accomplished.

The main advantage of the inplane method is that the total impulse required to perform this operation is a minimum as compared with the other landing and take-off maneuvers. However, this method imposes restrictions on the lunar landing sites as well as their associated stay times. That is, if the latitude of the landing site is specified, then the longitude of the landing site is restricted to the point at which the orbit ground track crosses the selected lunar latitude (point C or D, fig. 3). Also restrictions are imposed on the mission design in that once a landing site is specified, the inclination is restricted so that the orbit ground track intersects the selected landing site. Once the exploration module has landed on the lunar surface, two return possibilities exist. The first possibility is an immediate return to the lunar orbit; thus no finite exploration period is allowed. The succeeding possibilities give finite periods during which the lunar surface can be explored. These exploration periods are determined by the time elapsed between successive crossings of the landing-site latitude and the ground track of the established lunar orbit. For the inplane operation, a return to the established lunar orbit cannot be made unless the landing-site location intersects the lunar orbit plane (point D or C, fig. 3). Therefore, the exploration periods associated with inplane landing and take-off maneuvers are really mandatory wait periods during which an inplane return to the lunar orbit cannot be accomplished.

The advantage of minimum total impulse for the inplane landing and take-off operation is overshadowed by the constraints that are imposed on the selection of lunar landing sites and the mandatory wait periods before returning to the established lunar orbit. These constraints can be relaxed and in some cases removed entirely if consideration is given to allowing the capability of small plane changes during the take-off phase of the lunar operation.

Inplane landing and offset capability during take-off.- The constraint requiring a take-off from the lunar surface at the precise time that the lunar orbit track and the lunar landing site intersect can be removed by considering a small offset ($\delta_f = \delta$), corresponding to a plane-change capability, on the return-to-orbit phase of the lunar mission. This offset represents a practical approach to the return maneuver in that it provides a take-off safety margin or lunar launch window. The offset or plane-change capability is obtained from an onboard fuel allotment in the exploration module. The geometry of this landing and take-off operation is also presented in figure 3. The landing maneuver is an inplane descent terminating at point C on the lunar surface. The apparent motion between the landing site and the established lunar orbit causes the exploration module to move to the right along the parallel of latitude of the landing site at a rate of 13.2° per day. With an offset capability of δ during the lunar take-off phase, the exploration module can return to the established lunar orbit anytime that the landing site is not more than δ out of the lunar orbit plane. If the exploration module lands at point C (fig. 3), there are two regions in which it can return to the lunar orbit within the offset capability δ. These regions are along the landing-site latitude from points C to E and from points F to B as illustrated in figure 3. It should also be noted that the exploration module can land anywhere in the orbit plane and stay for at least a few hours. This stay time is associated with the time that it takes for the landing site to move δ out of the orbit plane. That is, if the landing were made at point D (fig. 3), the vehicle would have to return to the lunar orbit at point B, which is δ out of the orbit plane; thus, a relatively short stay time on the lunar surface as compared with an inplane landing at point C would be allowed. For the lunar orbits that have inclinations equal to the sum of latitude and offset capability ($i = \varphi + \delta$), the exploration module has the capability of returning to the established lunar orbit anytime during the exploration period (from C to B) as shown in figure 3. This maneuver allows a longer stay time on the lunar surface than maneuvers employing an arbitrary orbit inclination. This condition exists because any point on the latitude φ within the region from C to B is never more than δ from the lunar orbit.

Although a larger total impulse is required for this method than that previously discussed, the landing site remains limited to the point at which the orbit track crosses the selected lunar landing latitude. The restrictions on the landing site can be eased by considering a small offset capability during the lunar landing maneuver.

Offset capability during landing and take-off.- An initial offset or plane-change capability during the landing maneuver removes the fixed-landing-site restrictions by allowing the exploration module to land in a region on either side of the orbit ground track as shown in figure 1. For illustrative purposes, consider the initial and final offsets to be equal ($\delta_i = \delta_f = \delta$) and an arbitrary inclination of the established lunar orbit. It can be seen in figure 3 that the exploration vehicle can land anywhere within the regions between A and E or F and B and can return to the established lunar orbit anytime that the exploration module is in these regions. If the inclination and latitude of the established lunar orbit are constrained as before by the relation $i = \varphi + \delta$, while the exploration module is between points A and B (fig. 3), it will never be more than δ out of the orbit plane and the capability of returning to the established lunar orbit will exist at all times during this exploration period. The maximum range of lunar landing sites and stay times for this maneuver is obtained by utilizing the full plane-change capability during both the landing and take-off phases, that is, by landing δ out of the orbit plane (point A, fig. 3) and taking off when the vehicle is again δ out of the orbit plane (point B, fig. 3). The ability to land at any point between A and B (fig. 3) can be realized at the possible expense of decreasing the stay time. This condition is possible because at no point between A and B is the landing-site latitude more than δ out of the established lunar orbit plane.

ANALYSIS AND DISCUSSION

The accessible lunar landing sites and their associated exploration periods are strongly dependent on the latitude of the landing site as well as on the offset capabilities allowed during the landing and take-off phases of the mission. It has been shown that landings can be made at any point on the lunar surface by simply establishing an orbit that passes over the desired landing site. If small out-of-plane maneuvers are allowed during the landing and take-off phases, the landing vehicle can explore the lunar surface for periods of up to a few hours. However, this paper is primarily concerned with landing sites at which a vehicle can land, stay on the lunar surface for periods from 1 day to 2 weeks or more, and have the capability of returning to the established orbit at any time during the exploration period.

The landing sites and stay times for each of the three landing and take-off maneuvers discussed in the previous section have been determined. The data obtained for the examples given in this paper are for a median energy-transfer trajectory $\left(\frac{V}{V_p}\right)_o = 0.994$ and for transfer trajectory inclinations of 0^o and 30^o. (See fig. 2.)

Equations for Analysis

The various landing and take-off procedures are considered by assuming values of the initial and final offset capabilities that correspond to the specific landing and take-off maneuvers desired. The general equations for determining the lunar landing sites and their corresponding stay times are derived in the appendix. The longitude of the landing sites and take-off points for a specified orbit inclination i and lunar latitude φ is given by

$$\lambda_3 = \Omega_2 - \theta_1 \tag{2}$$

and

$$\lambda_4 = \Omega_2 - 180^o + \theta_2 \tag{3}$$

respectively. The exploration time, in days, is given by

$$\tau = \frac{\theta}{\dot{\omega}} = \frac{1}{13.2}\left(180^o - \sin^{-1}\frac{\sin\varphi\cos i - \sin\delta_i}{\cos\varphi\sin i} - \sin^{-1}\frac{\sin\varphi\cos i - \sin\delta_f}{\cos\varphi\sin i}\right) \tag{4}$$

• A relation between the geometric characteristics Ω, i of the lunar orbit and the entry points ξ, η on the sphere of influence is given by equation (1). The corresponding relationship between the transfer trajectory and the landing sites on the lunar surface can be seen by considering figure 2. Once the energy and inclination of the transfer trajectory are specified, a unique value for the entry point is defined. It is through this entry point that the planes of all the lunar orbits that are established from a given transfer trajectory must pass. Hence, if the orbit inclination is specified, the nodal position is defined and the landing sites can be obtained from equation (2).

Inplane Landing and Take-Off Maneuver

The inplane landing and take-off maneuver is the most restricted case because both the landing and take-off maneuvers are performed in the orbital plane, that is, the initial and final offsets δ_i, δ_f are both zero. The equations for θ_1 and θ_2 (derived in the appendix) are reduced to comply with the inplane maneuvers by setting $\delta_i = 0^\circ$ and $\delta_f = 0^\circ$; thus,

$$\theta_1 = \theta_2 = \sin^{-1} \frac{\tan \varphi}{\tan i} \qquad (\varphi \leqq i) \qquad (5)$$

The stay time on the lunar surface, which is defined by

$$\tau = \frac{\theta}{13.2} = \frac{1}{13.2}\left(180^\circ - \theta_1 - \theta_2\right)$$

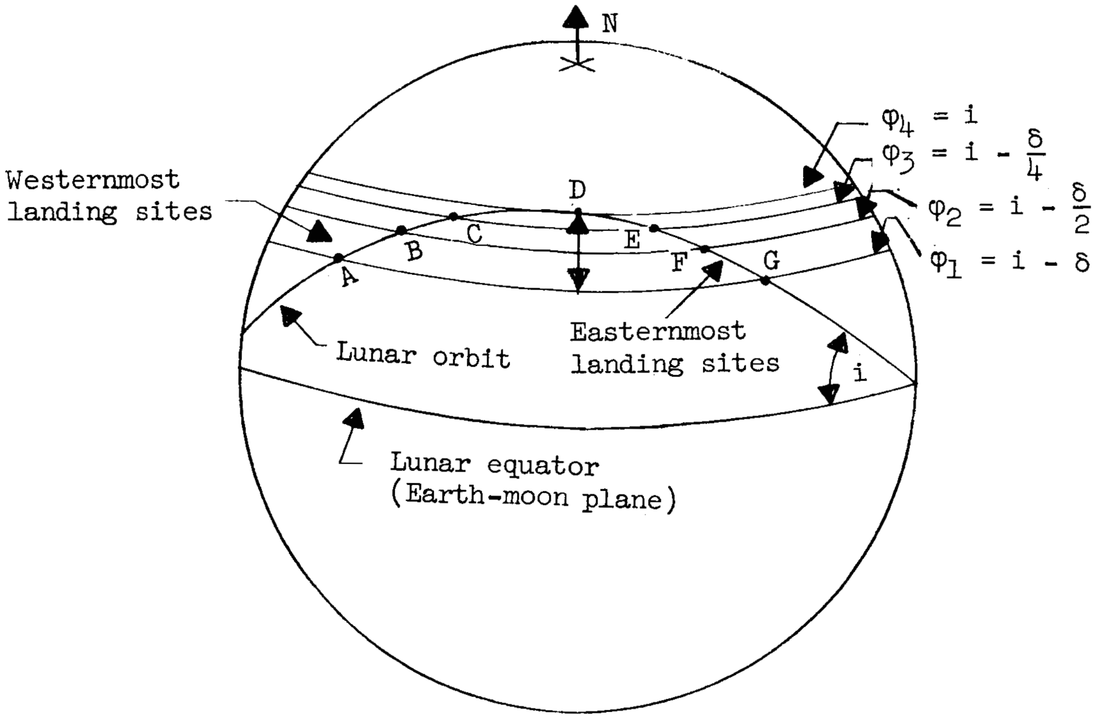
becomes

$$\tau = \frac{1}{13.2}\left(180^\circ - 2 \sin^{-1} \frac{\tan \varphi}{\tan i}\right) \qquad (6)$$

Equations (2) and (3) for the longitude of the landing and take-off points remain unchanged. However, the appropriate values of θ_1 and θ_2 (eq. (5)) must be used. The stay time, which is given by equation (6), is really a period of mandatory wait time on the lunar surface. This wait period is the time elapsed from the lunar landing until the lunar orbit plane again intersects the landing-site location and the inplane take-off can be initiated. Because of the fixed and mandatory stay times imposed by this maneuver, it is not considered desirable for manned landing and is not discussed further.

Inplane Landing With Offset Capability During Take-Off

Although the landing maneuver is restricted to the plane of the lunar orbit, provisions for a plane-change capability during the take-off maneuver gives considerable flexibility in the choice of landing sites on the lunar surface. The geometry of this maneuver is shown in sketch (1).



Sketch (1).- Possible landing sites for an inplane landing and offset capability during take-off.

As discussed in the previous section, landings can be made anywhere in the lunar orbit ground track. However, with an offset capability during take-off, the criterion is that the vehicle land at a point on the surface such that it has a return-to-orbit capability at all times during the exploration period. That is, for an offset of δ, the landing site cannot be more than δ out of the orbit plane before a return to orbit must be initiated.

Sketch (1) shows the range of landing-site latitudes and stay times that are possible, for a given orbit inclination, within the return-to-orbit restrictions. Point D (sketch (1)) is the maximum landing-site latitude, whereas points A and G show the lowest landing-site latitudes that can be attained for a given orbit inclination. However, the stay time associated with landings at point D and the other eastern landing sites (E, F, and G) are relatively short as compared with the stay times for the western landing sites (A, B, and C). This difference occurs because the eastern landing sites are associated only with the time required for the landing site to move δ out of the orbit plane. The stay times for the western landing sites are associated with the time required for the landing sites to move from their landing longitudes A, B, and C past their respective eastern landing longitudes E, F, and G until the vehicle is δ out of the established orbit plane. This type of maneuver allows the vehicle on the landing-site latitude to move out of the established orbit plane not more than δ (point D) back into the orbit plane, and then out of the orbit plane again. At the time the landing site moves δ out of the orbit plane, the return to orbit must be initiated because at this point the angle between the landing site and orbit plane is increasing and any further motion would not allow a return to orbit within the offset capabilities. The relation that gives the latitude range of possible western landing sites for a given orbit inclination is

$$(i - \delta) \leqq \delta \leqq i \tag{7}$$

The longitude of these respective landing sites is obtained from equation (2). The westernmost landing site, which allows the longest stay time, is given when $\varphi = i - \delta$. Increasing the magnitude of the offset δ allows a greater range of landing sites as well as longer exploration periods for a given orbit inclination.

Figure 4 illustrates the landing sites and maximum stay times that are associated with inplane landing maneuvers and take-off offset capabilities of 2.5°, 5.0°, and 10.0°, respectively. Figures 4(a), 4(b), and 4(c) are for a transfer trajectory inclination of 0° and figures 4(d), 4(e), and 4(f) are for a transfer trajectory inclination of 30°. Four representative landing-site latitudes $(\varphi_1, \varphi_2, \varphi_3, \text{ and } \varphi_4)$ are selected for a given lunar orbit inclination with latitude-inclination relations of $\varphi_1 = i - \delta$, $\varphi_2 = i - \frac{\delta}{2}$, $\varphi_3 = i - \frac{\delta}{4}$, and $\varphi_4 = i$. For a given orbit inclination with the inplane landing maneuver, it is possible to land along the φ_1 curve, or in the region to the right of the φ_1 curve, and have a return-to-orbit capability at any time during the exploration period. The corresponding stay time, in days, is given by the numbered curves in figure 4. The φ_4 line in figure 4 shows landing sites that are associated with the maximum latitude at which a vehicle can land with a given orbit inclination. In figures 4(a), 4(b), and 4(c), the φ_4 curve appears as a great circle at a longitude of approximately -45°. This meridian is a result of the entry point being in the earth-moon plane (that is, for $i_o = 0$, $\eta = 0$). Therefore, the orbit nodal position does not vary with inclination, and the longitude of the φ_4 landing sites, which are 90° from the nodal longitude, is fixed for any inclination lunar orbit. The latitude-inclination curves $(\varphi_1, \varphi_2, \text{ and } \varphi_3)$ to the west of the φ_4 curve in figure 4 are associated with the longer stay times (points A, B, and C, sketch (1)), and the curves $(\varphi_1', \varphi_2', \text{ and } \varphi_3')$ to the east of the φ_4 curve are for shorter stay times at the same respective latitudes (points E, F, and G, sketch (1)).

The use of figure 4 is illustrated by the following examples taken from figure 4(c) in which $\delta_f = \delta = 10^\circ$:

(1) If the inclination of an established lunar orbit is 30°, the maximum latitude of landing is along φ_4 $(\varphi_4 = i)$ at a longitude of -45° and the vehicle can stay at most 4 days. If the landing was made at φ_2, which is at a latitude of 25° $\left(\varphi = i - \frac{\delta}{2}\right)$ and longitude of -8.5°, the stay time would be approximately 7.5 days. The lowest latitude at which the vehicle can land and

have an extended exploration period is on the φ_1 curve at 20° latitude $(\varphi_1 = i - \delta)$ and 6.0° longitude. The stay time for this landing site is at least 9 days.

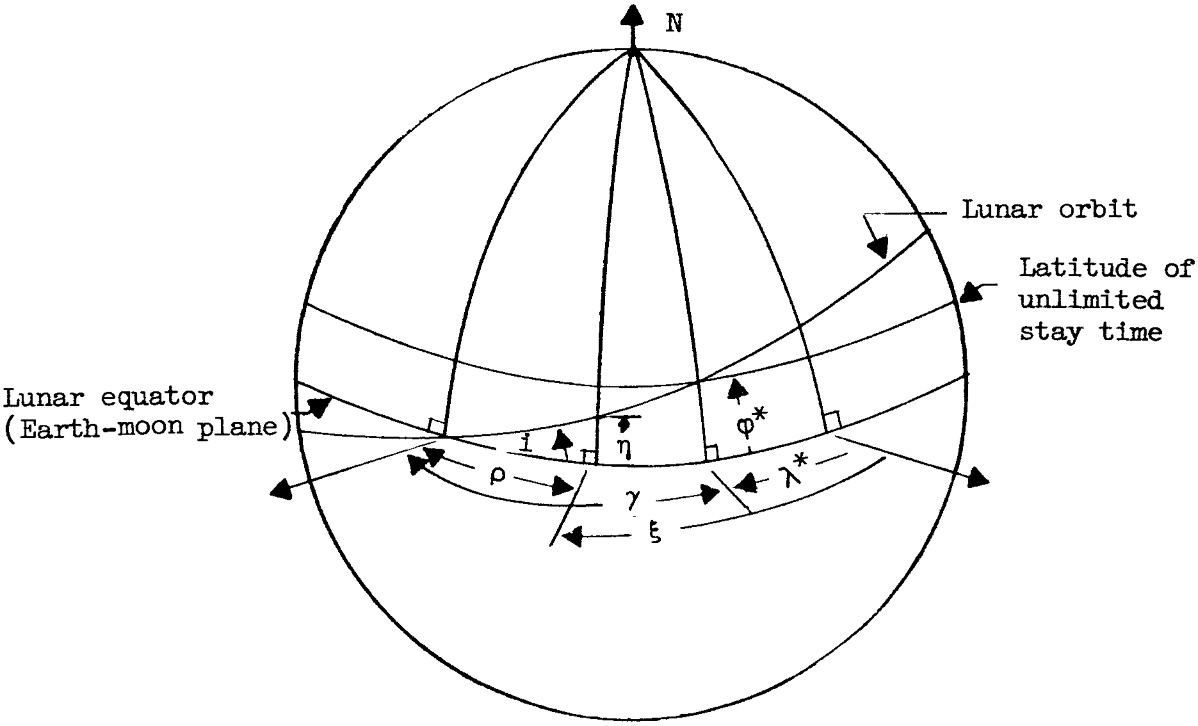
(2) If a particular landing site is selected first, the problem then is to determine the orbit inclination required to achieve this landing site. For example, suppose a landing site is selected on the φ_3 curve $\left(\varphi_3 = i - \frac{\delta}{4}\right)$, at the intersection of the 7-day stay time curve (-17.5° longitude and 22° latitude). The orbit inclination that is required to land at the selected point is 24.5° $\left(\text{that is, } i = \varphi_3 + \frac{\delta}{4}\right)$. Thus, it is clear that a wide selection of landing sites and corresponding stay times is available by judiciously selecting the orbit inclination and the landing-site latitude.

A comparison of figures 4(a), 4(b), and 4(c) with figures 4(d), 4(e), and 4(f), respectively, shows the effect of a variation in the inclination i_o of the transfer trajectory on the landing sites and their corresponding stay times. Since the transfer trajectory is in the earth-moon plane $(i_o = 0)$ for figures 4(a), 4(b), and 4(c), the landing sites in the northern hemisphere corresponding to positive inclination orbits are symmetric with landing sites in the southern hemisphere which correspond to negative inclination orbits. The associated stay times are the same for both cases. The asymmetry of figures 4(d), 4(e), and 4(f) are a result of the transfer trajectory's inclination to the earth-moon plane (that is, $i_o = 30°$; $\eta = 4.58°$). In general, it can be seen that for a positive value of i_o, the landing sites move farther westward in the northern hemisphere, whereas the landing sites move eastward in the southern hemisphere. For negative inclinations i_o of the transfer trajectory, the landing sites are reflected about the lunar equator.

Unlimited Stay Times for an Inplane Landing and Offset Take-Off

In general, there are three areas on the lunar surface where a vehicle can land in the plane of the established lunar orbit, stay for an unlimited period of time, and have a return-to-orbit capability at all times during the exploration period. These areas encompass both lunar poles and the lunar equator. Since this is a special case of the inplane landing and offset take-off maneuver, only the unlimited stay times in the equatorial region are considered.

It is possible to land anywhere along the lunar-orbit ground track and stay for an unlimited period of time with a return-to-orbit capability at any time if the inclination of the established lunar orbit is less than or equal to $\frac{\delta}{2}$. If the inclination of the established lunar orbit is between $\frac{\delta}{2}$ and δ, then unlimited stay times exist for landing sites along portions of the orbit ground track. It is these landing sites that define the boundaries of the unlimited stay-time regions for a given entry point latitude. The geometry of this problem is presented in sketch (2) projected on the lunar surface.



Sketch (2).- Geometry of the unlimited stay-time regions for an inplane landing and offset take-off maneuver.

The latitude at which a vehicle can land in the orbit plane and have an unlimited stay time is

$$\varphi^* = \delta - i \qquad (\eta \leqq i \leqq \delta) \qquad (8)$$

With the offset capability δ specified, equation (8) gives the maximum latitude at which a vehicle can land and stay indefinitely for various inclination lunar orbits. Once the maximum latitude is determined, it is necessary to determine the longitude of these unlimited stay-time landing sites. This value is determined by solving the two spherical triangles shown in sketch (2) for ρ and γ. From the four parts formula of reference 5, expressions are obtained in terms of the orbit inclination, entry point, and landing-site latitude.

$$\rho = \sin^{-1} \frac{\tan \eta}{\tan i} \qquad (\eta \leqq i) \qquad (9)$$

and

$$\gamma = \sin^{-1} \frac{\tan \varphi^*}{\tan i} \qquad (\varphi \leqq i) \qquad (10)$$

The longitude of the landing site for an unlimited stay time is

$$\lambda^* = \xi + \rho - \gamma \tag{11}$$

In order to generate these regions of unlimited stay time, it is necessary to consider both positive and negative inclination lunar orbits for a given entry point latitude. In addition, each orbit inclination has landing sites associated with both northern (positive φ^*) and southern (negative φ^*) latitudes. This problem requires solution of equations (9), (10), and (11) for each landing-site latitude.

The unlimited stay-time regions illustrated in figure 5 are for a take-off offset capability of 5°. The curves were generated by varying the inclination of the lunar orbit between i_{min} and δ where i_{min} is the minimum inclination orbit that can be established and is equal to the latitude of the entry point η on the lunar sphere of influence. (See eq. (1).) The data for figure 5 were obtained by letting $i_{min}(\eta)$ vary from 0° to δ. This procedure is essentially a variation of the inclination of the earth-moon transfer trajectory from $i_o = 0^\circ$ to approximately $i_o = 32.5^\circ$ as can be seen in figure 2. The unlimited stay-time regions are bounded by latitudes of $\pm\frac{\delta}{2}$. Clearly, if an inplane landing was made at a latitude greater than $\frac{\delta}{2}$, then at some time the landing site would be more than δ out of the orbit plane, at which time a return to orbit could not be initiated. As i_{min} increases from 0° to δ, the unlimited stay-time region (fig. 5) reduces from a continuous region that encompasses the lunar equator to a pair of points on the equator at longitudes $\pm 90^\circ$ away from the entry points.

Offset Capability During Landing and Take-Off

A greater flexibility in the choice of landing sites is achieved if consideration is given to utilizing an offset capability during both the landing and the take-off maneuver. The offset capability during the landing maneuver removes the restriction of landing in the orbital plane and provides a landing-site region on either side of the orbit ground track. (See fig. 1.) The width of the landing-site region depends on the magnitude of the landing offset and the stay time is a function of the latitude of the landing site. With the landing and take-off offsets both equal (that is, $\delta_i = \delta_f = \delta$), and the requirement that a return-to-orbit capability exist at all times during the exploration period, the landing sites and their stay times are readily determined. These landing sites and their associated stay times are illustrated in figure 6 for values of the offset δ equal to 2.5° and 5.0°, and for transfer trajectory inclinations of 0° and 30°, respectively. The border between the stippled region and the unshaded region, in figure 6, defines the westernmost landing sites that are possible for landings that are initiated very soon after establishment of the lunar orbit. The stay times associated with these landing sites are a maximum; hence they provide the longest exploration periods for a given lunar latitude. The lunar-orbit inclination is varied from η to 90°

and the latitude for these maximum-stay-time landing sites is such that $\varphi = i - \delta$.

If the magnitude and direction of the landing offset is properly varied, the exploration vehicle will have the capability of landing at any longitude between λ_3 and λ_4 for the selected lunar latitude. Under these conditions, the longitude of the landing sites is determined from the following expression:

$$\lambda_5 = \lambda_4 + 13.2\Delta T \tag{12}$$

where ΔT is the specified stay time in days (not necessarily the maximum stay time). The contour curves in figure 6 show these landing sites and their associated stay times on the lunar surface.

The hatched areas that encompass the lunar equator are the regions in which the exploration vehicle can land anywhere, stay on the lunar surface for an unlimited period of time, and never be more than δ out of the established lunar orbit plane. The width of these unlimited stay-time regions is dependent on the inclination i of the established lunar orbit and is expressed as

$$\varphi^* = \pm(\delta - i) \qquad (\eta \leqq i \leqq \delta) \tag{13}$$

The width of these unlimited stay-time regions is a maximum when the orbit inclination i is a minimum, that is, when $i = \eta$. Figures 6(a) and 6(b) are for a transfer trajectory inclination of 0° (entry-point latitude η of 0°), and figures 6(c) and 6(d) are for a transfer trajectory inclination of 30° (entry-point latitude of 4.58°). Therefore, the minimum orbit inclination associated with figures 6(a) and 6(b) is $i_{min} = 0^\circ$ and with figures 6(c) and 6(d) is $i_{min} = 4.58^\circ$. As the inclination of the established lunar orbit increases from η to δ, the unlimited stay region reduces to a line coincident with the lunar equator. It can be seen from equation (13) that in order to have an unlimited stay-time region, the magnitude of the offset δ must be greater than or equal to the latitude of the entry point η; therefore, if the offset capability is 2.5° and the latitude of the entry point is 4.58° (fig. 6(c)), it is clear that equation (13) is not valid and there is no equatorial region of unlimited stay time.

Comparison of Procedures

As discussed previously, the maneuver that employs small plane-change capabilities δ of equal magnitude during both the landing and take-off phases allows the most flexibility in the selection of landing sites on the lunar surface. It is also of interest to determine the maneuver that allows a longer stay time on the lunar surface for a given total offset capability δ_t. Consideration is given to two basic maneuvers which utilize a total offset of $\delta_t = 2\delta$. These maneuvers are (1) equal offset capabilities of δ during both the landing and take-off phases, and (2) an inplane landing maneuver with the total offset δ_t employed during take-off. A comparison of figures 6(b) and 4(c) shows that, in general, for a given latitude the exploration period is longer if the total offset capability is used during take-off (fig. 4(c)) rather than dividing the total offset capability equally between the landing and take-off maneuver (fig. 6(b)). The increased stay-time capability can be seen by considering a specific landing-site latitude and comparing the stay times for each maneuver.

Other Considerations

The effect of variations in the energy of the transfer trajectory can be seen in figure 2. As the transfer-trajectory energy decreases, the longitude of the entry point increases, but there is very little change in the latitude of the entry point. Therefore, a variation in the energy of the transfer trajectory essentially causes a rotation of the possible landing sites about the lunar poles. The largest variation in longitude (fig. 2) is approximately 9° for a change in trajectory energy $\left(V/V_p\right)_o$ of 0.001.

If a delay or wait period in orbit is allowed, the effect on the landing sites is a westward precession of the sites relative to the lunar orbit. This precession takes place at a rate of 13.2° for each day delay in orbit and is due to the rotation of the moon on its polar axis. There is no effect on the length of the exploration period on the lunar surface due to this delay. Therefore, a landing at any point on the lunar surface is possible by selecting the proper in-orbit wait prior to initiating the landing maneuver.

Other variations in the possible landing sites on the lunar surface exist as a result of the assumptions made in this paper that the lunar equator is always in the earth-moon plane and the moon is in a circular orbit about the earth. However, throughout a lunar month the inclination of the lunar equator to the earth-moon plane varies from 0° to $\pm 6.7^{\circ}$ in latitude. Also the elliptic orbit (eccentricity of 0.0549) of the moon about the earth causes a variation in the mean longitude of the moon of approximately $\pm 8.0^{\circ}$. Due to these librations, a landing site on the lunar surface as presented herein may shift at most $\pm 6.7^{\circ}$ in latitude and $\pm 8.0^{\circ}$ in longitude in a month.

If stay times on the order of 1 lunar day are desired, consideration should be given to a regression of the node line due to the oblateness of the moon. This regression causes an apparent westward motion of a point on the lunar surface, relative to the lunar orbit, at a rate equal to the combined lunar rotation and nodal regression rates. This effect reduces the allowable stay time on the lunar surface.

CONCLUDING REMARKS

A study was made to determine the possible lunar landing sites and associated stay times that are compatible with the lunar orbit rendezvous technique. The results of a patched conic technique were utilized to provide a useful relation between the entry point on the sphere of influence and the geometrical characteristics of the established lunar orbits.

• The results of this study indicate that the landing sites and associated stay times are strongly dependent on the inclination of the established lunar orbit as well as the specific landing and take-off maneuvers employed. The inplane landing maneuver is very restrictive in that the landing sites must be in the orbit ground track. A constraint on the inclination i and latitude φ such as $i = \varphi + \delta$, where δ is the offset capability, provides the mission with the capability of a return to orbit at any time during the exploration period.

The maneuver utilizing offset on both landing and take-off proves to be most versatile in that it allows a wide selection of sites at which a vehicle can land, stay for an extended period of time, and be able to return to the established orbit at any time during the exploration period. In addition, for some limited locations in the equatorial regions it is possible to land, stay for an unlimited period of time, and have a return-to-orbit capability at all times during the exploration period.

If the inplane landing is coupled with an offset capability during the take-off maneuver, more flexibility is possible in the selection of landing sites and the corresponding stay times on the lunar surface. A maneuver that uses an inplane landing and utilizes the total offset capability during take-off, in general, allows longer stay times on the lunar surface than a maneuver that employs equal offsets during both the landing and take-off phases of the mission.

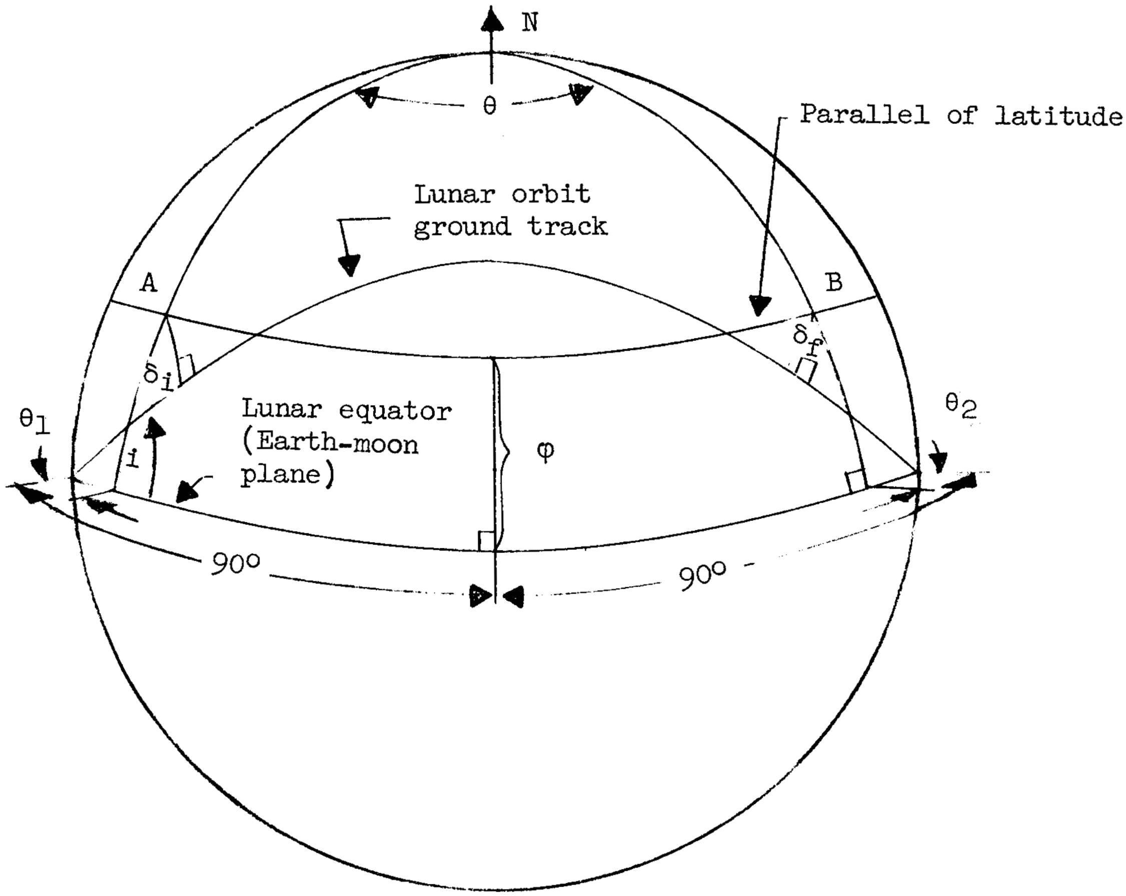
Maneuvers that have a plane-change capability during take-off provide unlimited stay times on the lunar surface for both the inplane and offset landing maneuvers. The inplane landing sites are still restricted to points that intersect the orbit ground track; however, the offset landing maneuver allows landings anywhere in these regions with unlimited stay time.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 26, 1965.

APPENDIX

EQUATIONS FOR DETERMINING THE POSSIBLE LUNAR LANDING SITES AND THEIR ASSOCIATED STAY TIMES ON THE LUNAR SURFACE

The geometrical aspects of the lunar landing site problems are shown in sketch (3) as projections on the lunar surface:



Sketch (3).- Geometry of the offset landing and take-off problem.

The angle θ between points A and B (sketch (3)) is dependent on both the landing and take-off offsets (or plane-change capabilities). Sketch (3) shows that

$$\theta = 180^{\circ} - \theta_1 - \theta_2 \tag{A1}$$

Since the moon rotates at a constant angular rate ($\dot{\omega} = 13.2^{\circ}$ per day), the maximum stay time, in days, on the lunar surface is obtained from equation (A1), that is

$$\tau = \frac{\theta}{\dot{\omega}}$$

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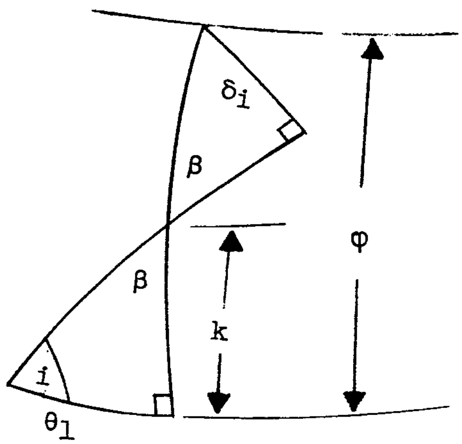
By applying the basic formulas of spherical trigonometry (ref. 5) to sketch (4), expressions for θ_1 and θ_2 are developed:



Sketch (4).- Angular relationships.

$$\sin \beta = \frac{\sin \delta_i}{\sin(\varphi - k)} \tag{A2}$$

and

$$\sin \beta = \frac{\sin i \sin \theta_1}{\sin k} \tag{A3}$$

also

$$\sin \theta_1 = \frac{\tan k}{\tan i} \tag{A4}$$

Eliminating β and k yields

$$\theta_1 = \sin^{-1} \frac{\sin \varphi \cos i - \sin \delta_i}{\cos \varphi \sin i} \tag{A5}$$

Similarly,

$$\theta_2 = \sin^{-1} \frac{\sin \varphi \cos i - \sin \delta_f}{\cos \varphi \sin i} \tag{A6}$$

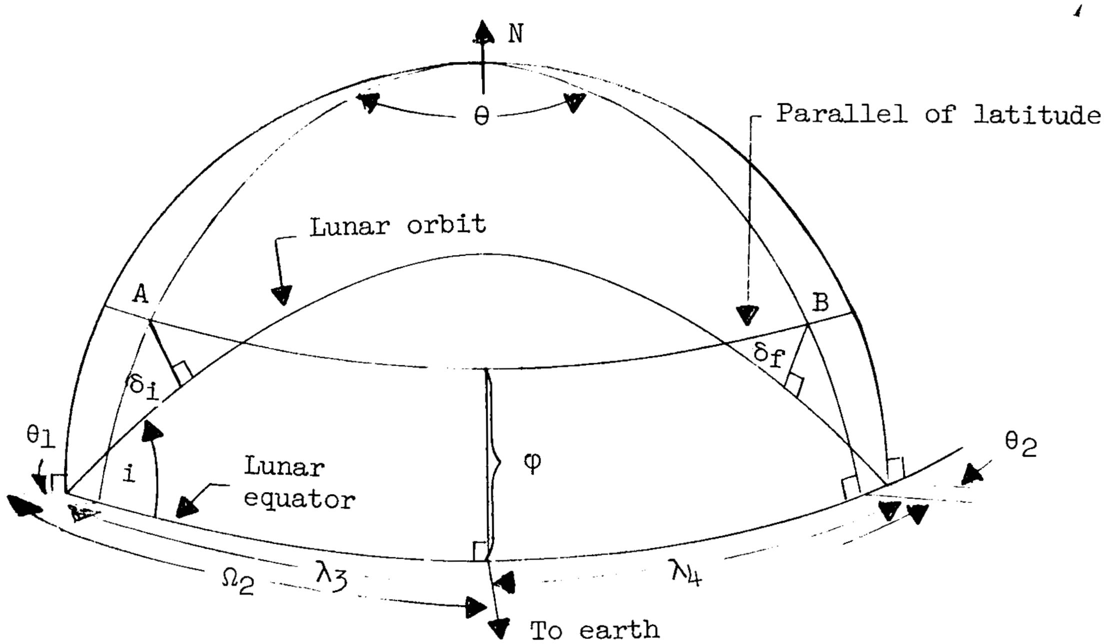
Substituting equations (A5) and (A6) into equation (A1) gives

$$\theta = 180^\circ - \sin^{-1} \frac{\sin \varphi \cos i - \sin \delta_i}{\cos \varphi \sin i} - \sin^{-1} \frac{\sin \varphi \cos i - \sin \delta_f}{\cos \varphi \sin i} \tag{A7}$$

The maximum stay time, in days, is

$$\tau = \frac{\theta}{\dot{\omega}} = \frac{1}{13.2}\left(180^\circ - \sin^{-1} \frac{\sin \varphi \cos i - \sin \delta_i}{\cos \varphi \sin i} - \sin^{-1} \frac{\sin \varphi \cos i - \sin \delta_f}{\cos \varphi \sin i}\right) \tag{A8}$$

The longitude of the landing sites and take-off points are obtained by considering the following sketch of the lunar surface (sketch (5)):



Sketch (5).- Lunar surface.

Equation (1) defines the longitude of the orbit node position at the sphere of influence. However, since the lunar orbit is not established at the sphere of influence, it is necessary to account for the motion of the line of nodes from the time the vehicle passes through the sphere of influence until the lunar orbit is established at the desired lunar altitude. The time T_s required for a vehicle to go from the sphere of influence to the orbit-establishment altitude is dependent on both the energy and inclination of the transfer trajectory. Since the angular rate of the moon is assumed to be constant (13.2° per day), the nodal position will move at this rate; therefore, equation (1) becomes

$$\Omega_2 = (\xi + 13.2T_s) + \sin^{-1}\frac{\tan \eta}{\tan i} \qquad (\eta \leqq i) \qquad \text{(A9)}$$

which defines the longitude of the orbit-node position at the orbit-establishment altitude. This study utilized values of T_s of 0.519 day for $i_o = 0^\circ$ and T_s of 0.506 day for $i_o = 30^\circ$. The values of T_s used in this paper were obtained from unpublished data; however, the T_s for $i_o = 0^\circ$ compares very favorably with those values presented in reference 6.

The longitude of the westernmost landing-site point A is

$$\lambda_3 = \Omega_2 - \theta_1 \qquad \text{(A10)}$$

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The longitude of point B is

$$\lambda_4 = \Omega_2 - 180^\circ + \theta_2 \tag{A11}$$

Sketch (5) shows that the maximum angular distance θ between the landing and take-off points on the lunar surface (eq. (A1)) is also

$$\theta = 180^\circ - \theta_1 - \theta_2 = \lambda_3 - \lambda_4 \tag{A12}$$

Equations (A1) to (A12) are for the general case of arbitrary plane-change capabilities δ_i and δ_f during the landing and take-off maneuvers. Specific cases can be obtained by selecting values of δ_i and δ_f that correspond to the type of landing and take-off maneuvers desired, the lunar-orbit inclination i, and the latitude of the desired lunar landing site φ.

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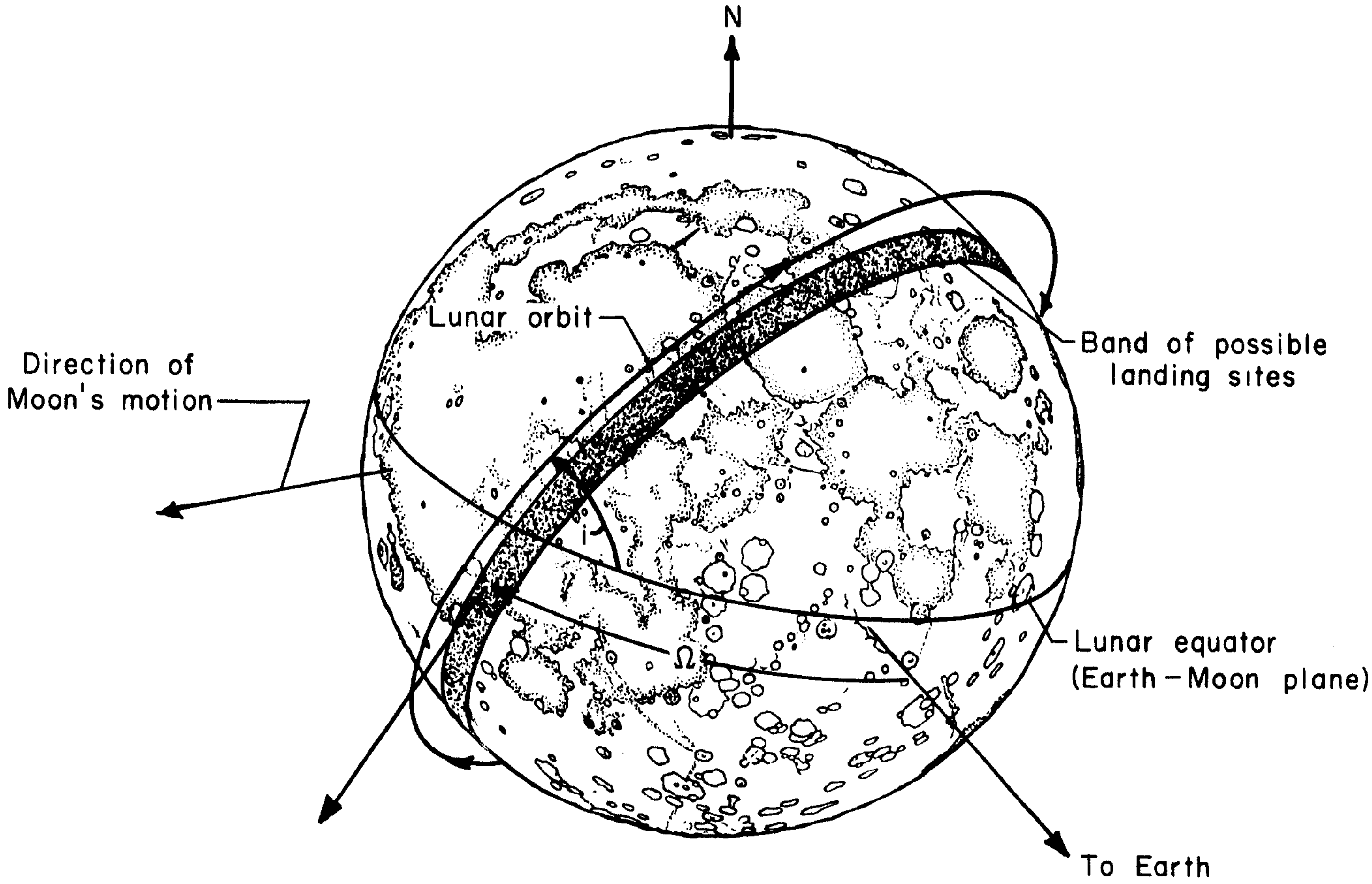


Figure 1.- Location of possible lunar landing sites.

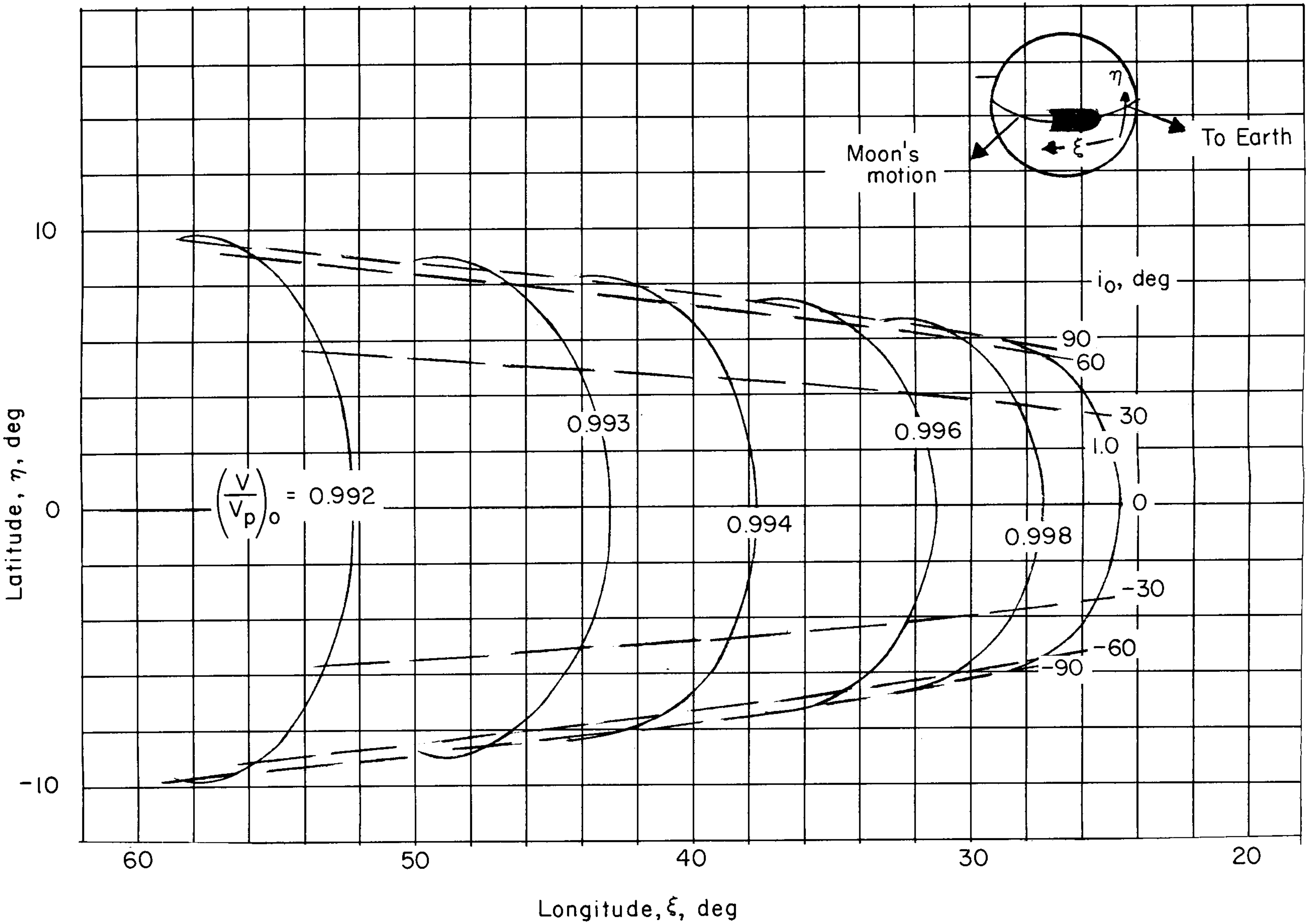


Figure 2.- Location of normal entry points on sphere of influence.

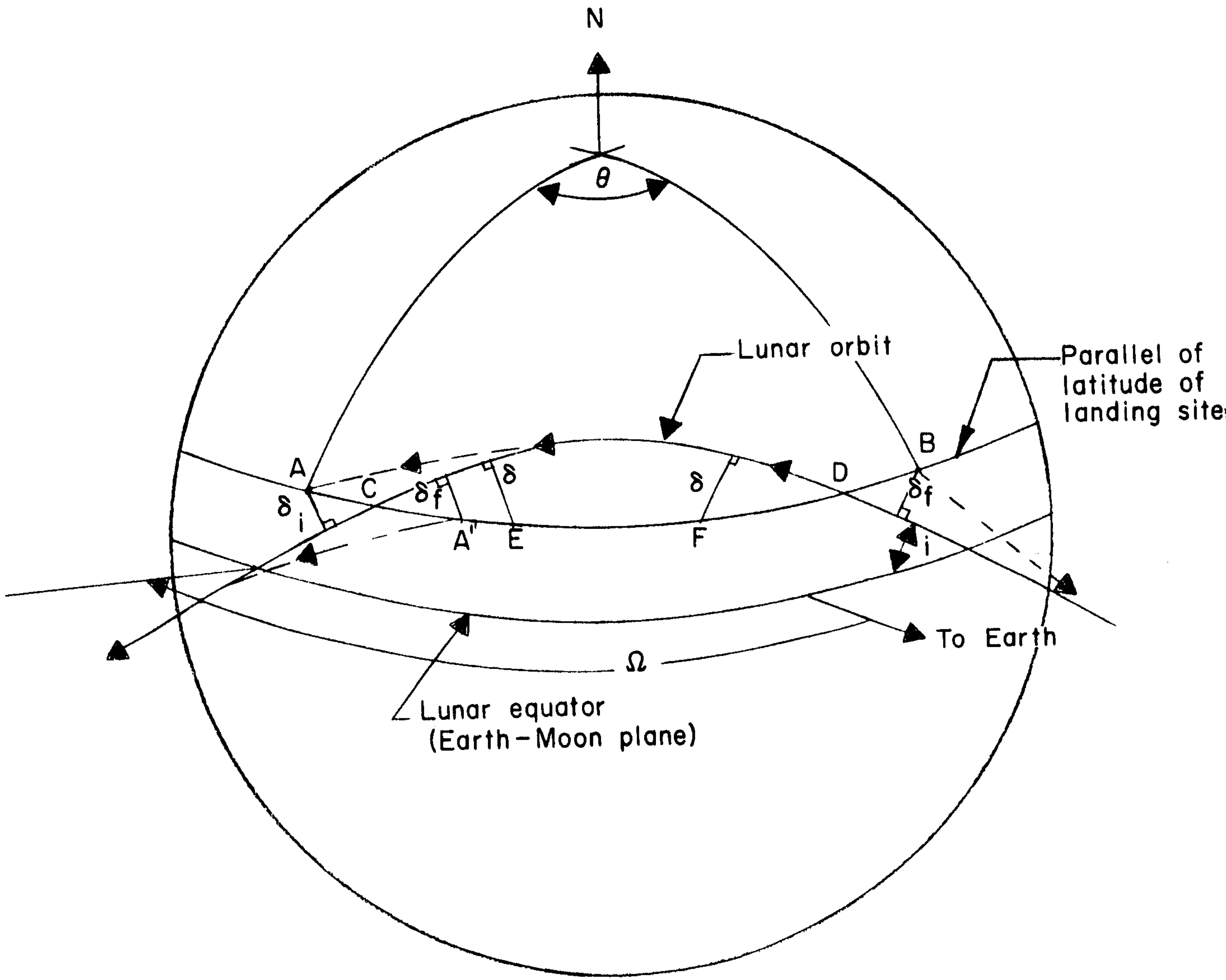
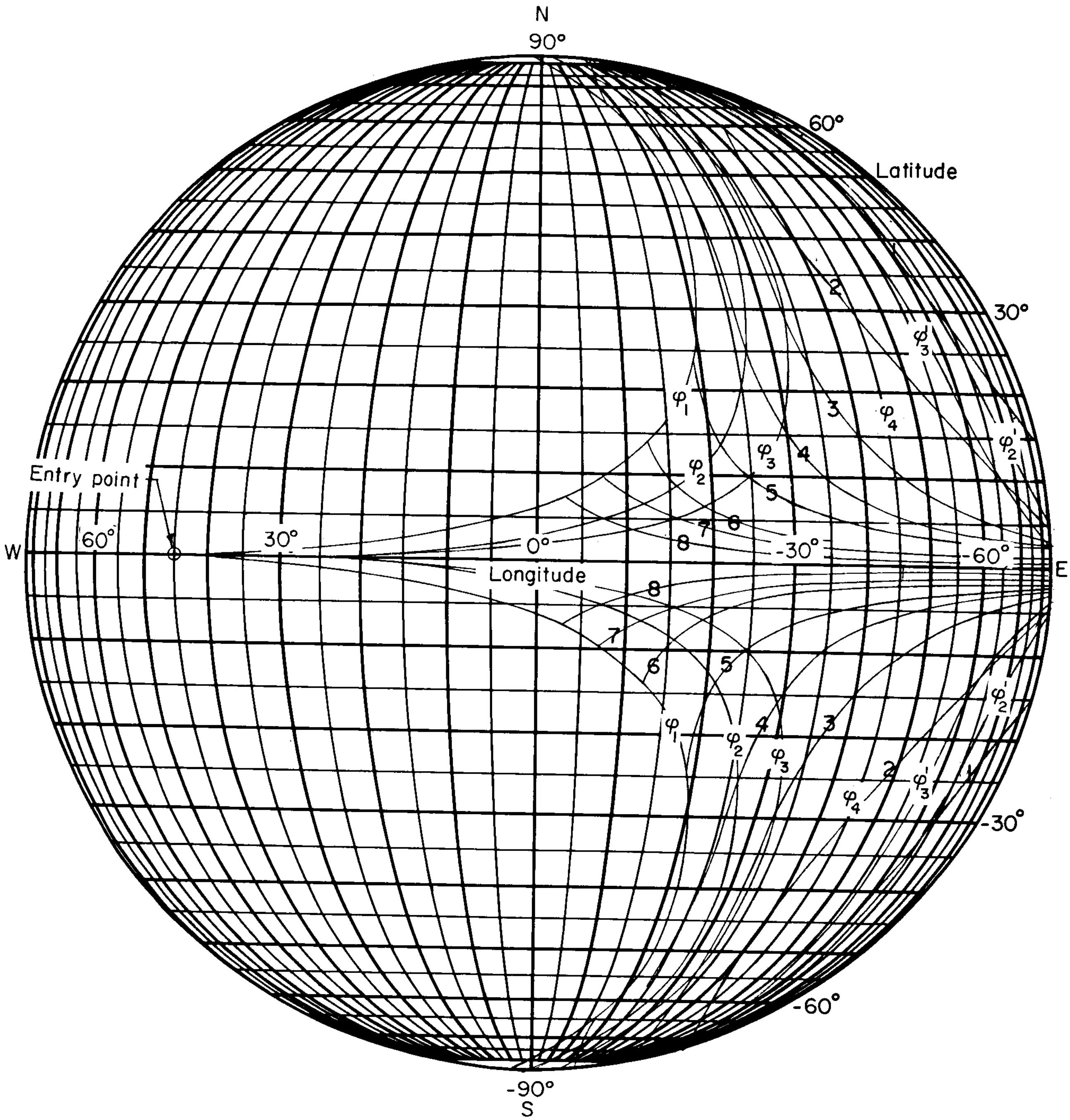
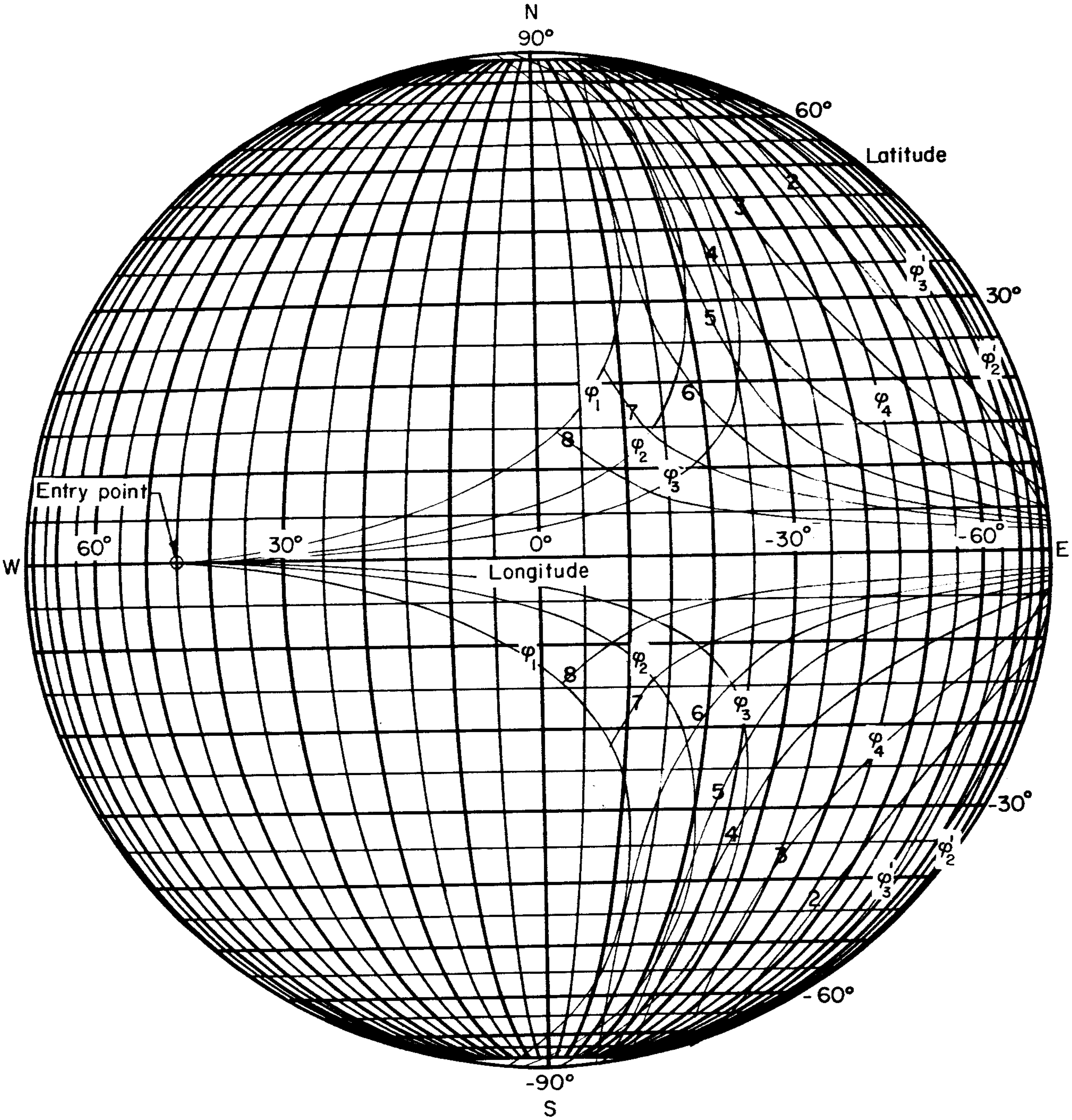


Figure 3.- Geometry of lunar landing and take-off procedures.



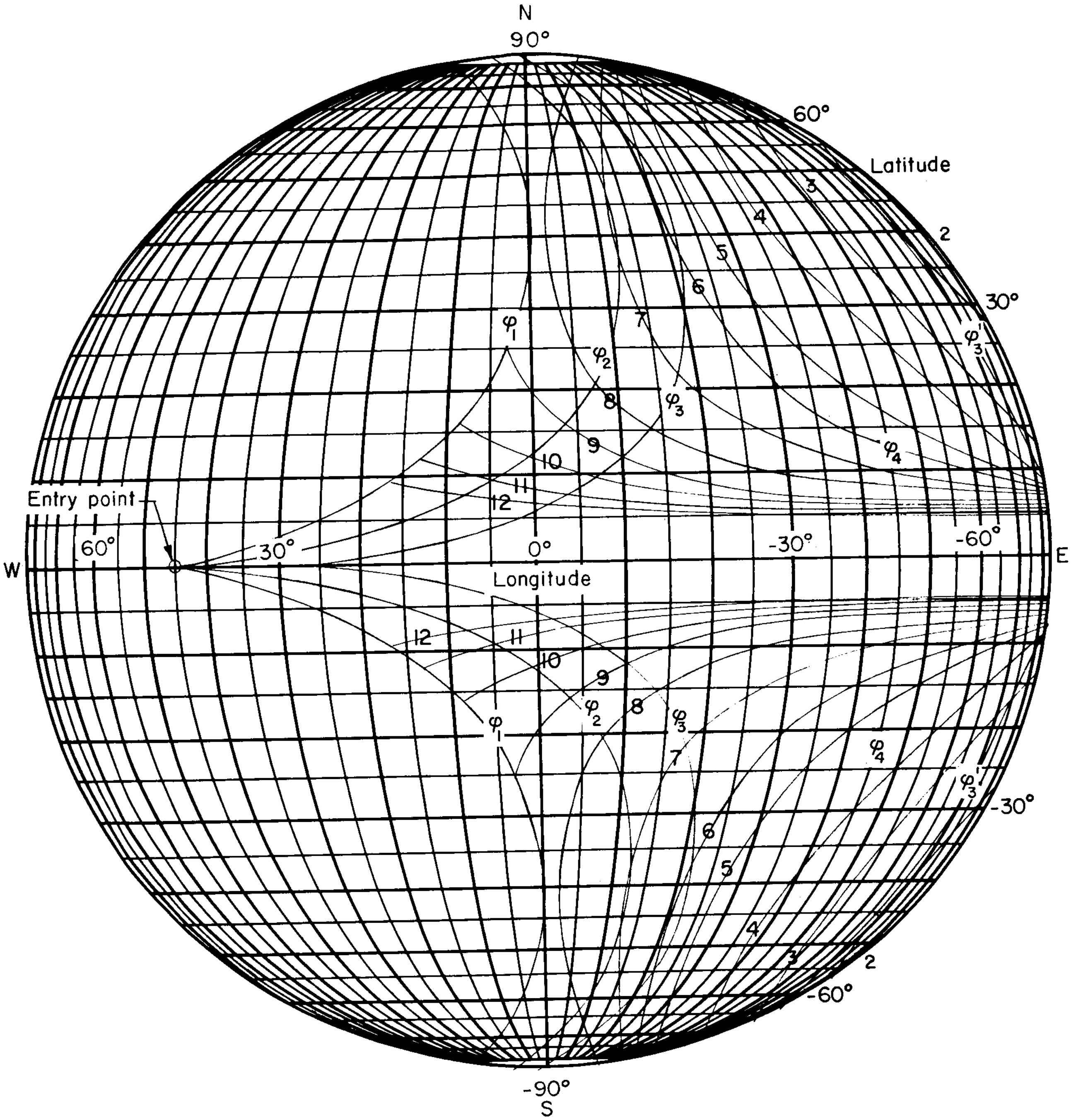
(a) $i_o = 0^\circ$; $\delta_i = 0^\circ$; $\delta_f = 2.5^\circ$.

Figure 4.- Landing sites and stay times on lunar surface for an inplane landing and offset take-off maneuver.



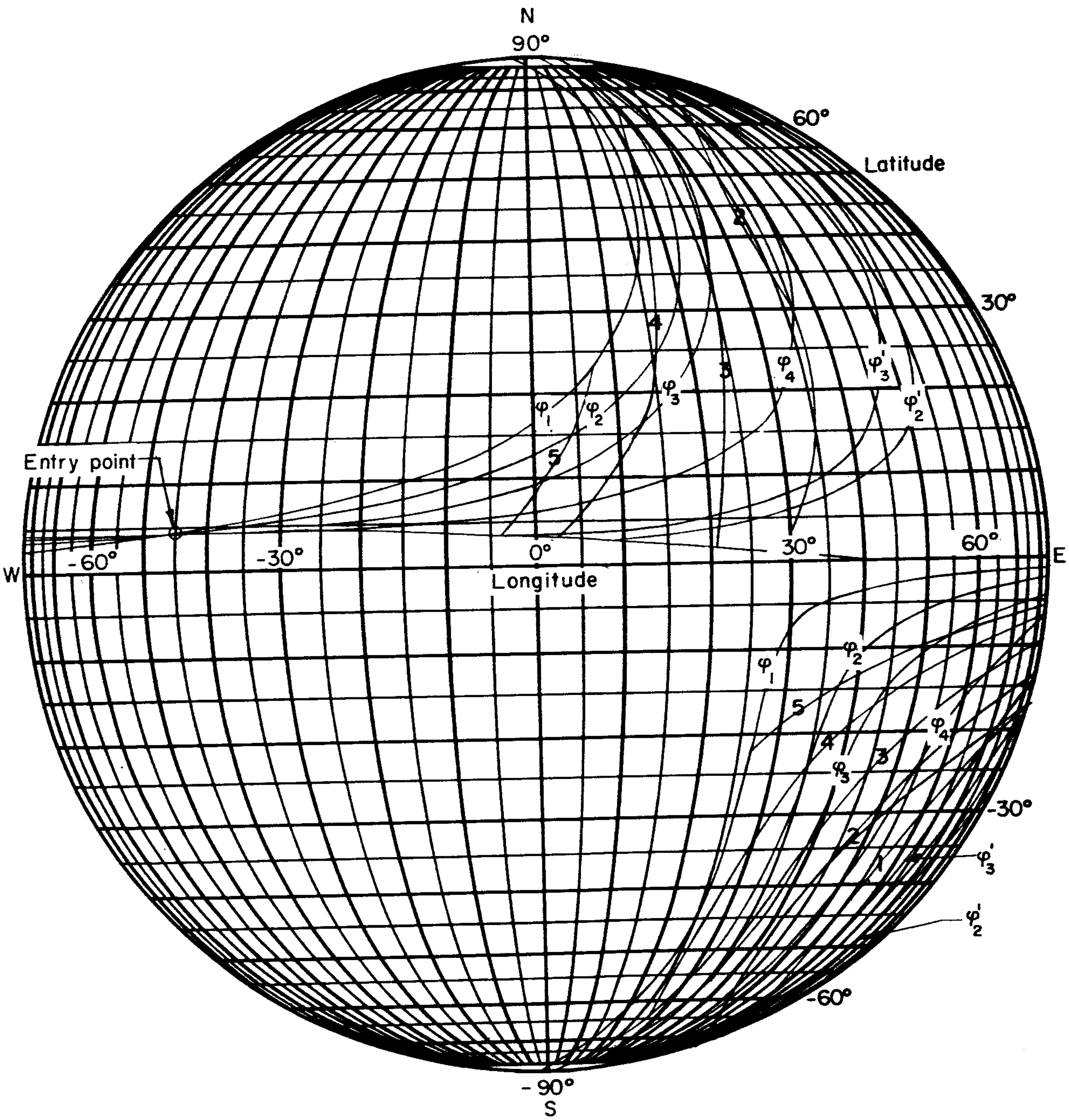
(b) $i_o = 0^o$; $\delta_i = 0^o$; $\delta_f = 5.0^o$.

Figure 4.- Continued.



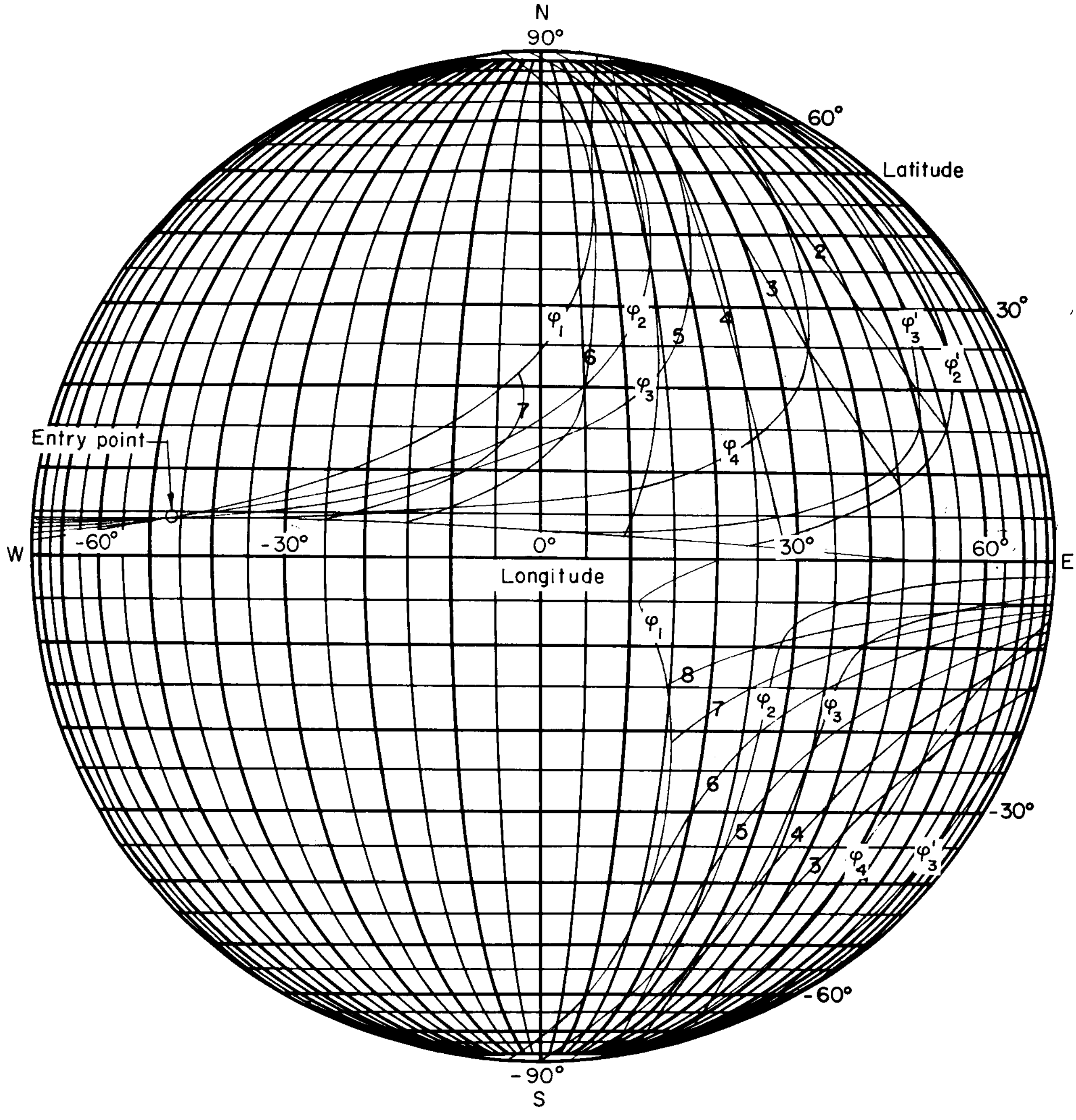
(c) $i_o = 0^o$; $\delta_i = 0^o$; $\delta_f = 10^o$.

Figure 4.- Continued.



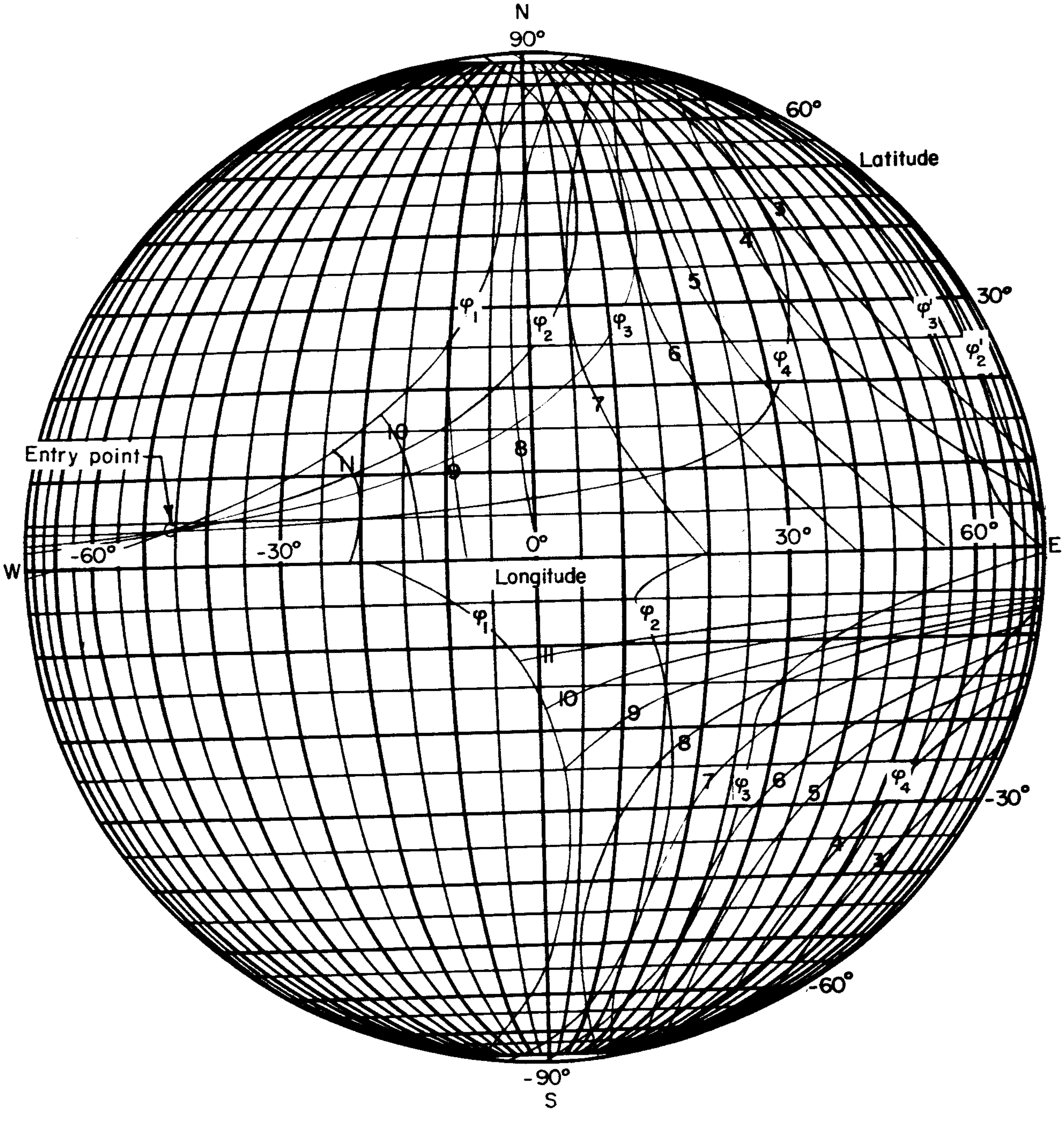
(d) $i_o = 30^o$; $\delta_i = 0^o$; $\delta_f = 2.5^o$.

Figure 4.- Continued.



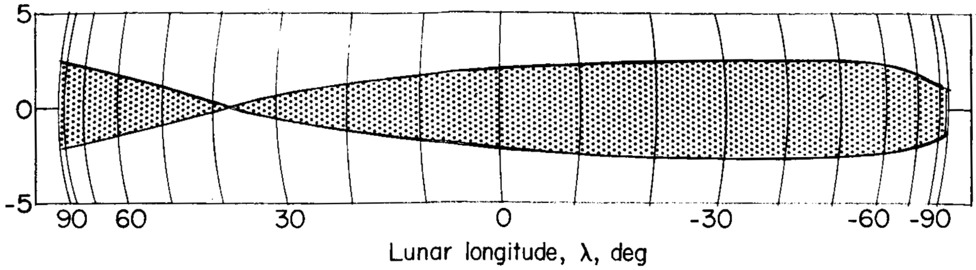
(e) $i_o = 30^o$; $\delta_i = 0^o$; $\delta_f = 5.0^o$.

Figure 4.- Continued.

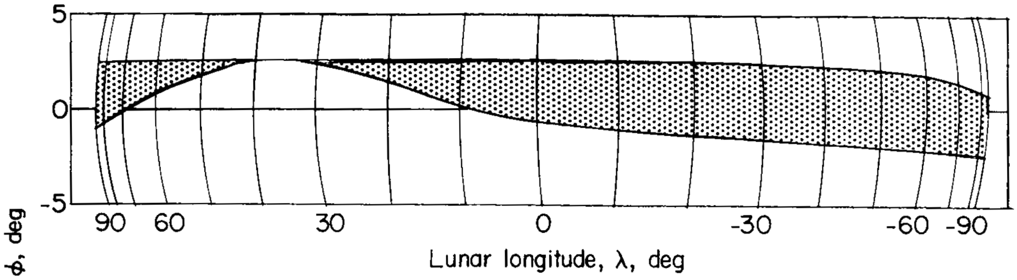


(f) $i_o = 30^\circ$; $\delta_i = 0^\circ$; $\delta_f = 10.0^\circ$.

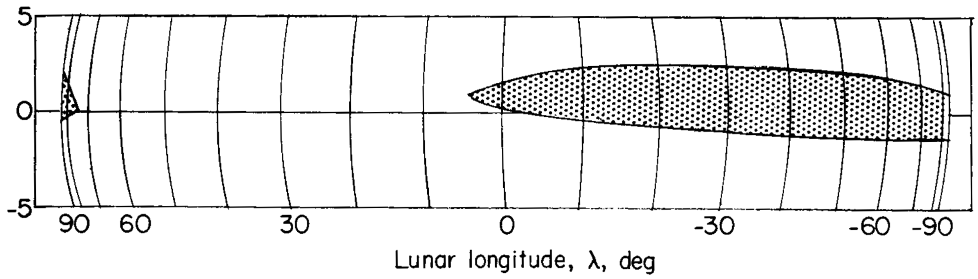
Figure 4.- Concluded.



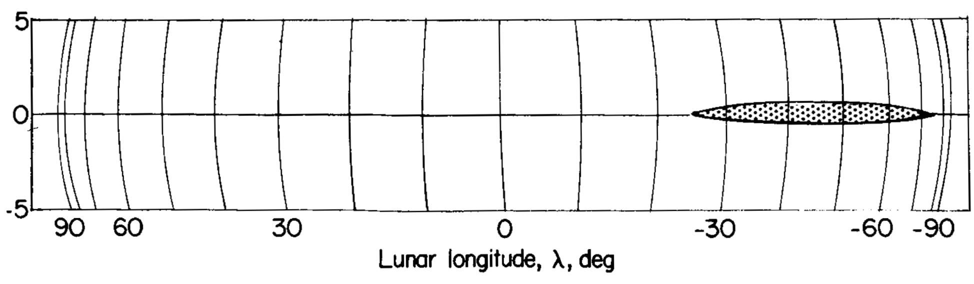
(a) $\eta = 0^{\circ}$.



(b) $\eta = 2.5^{\circ}$.

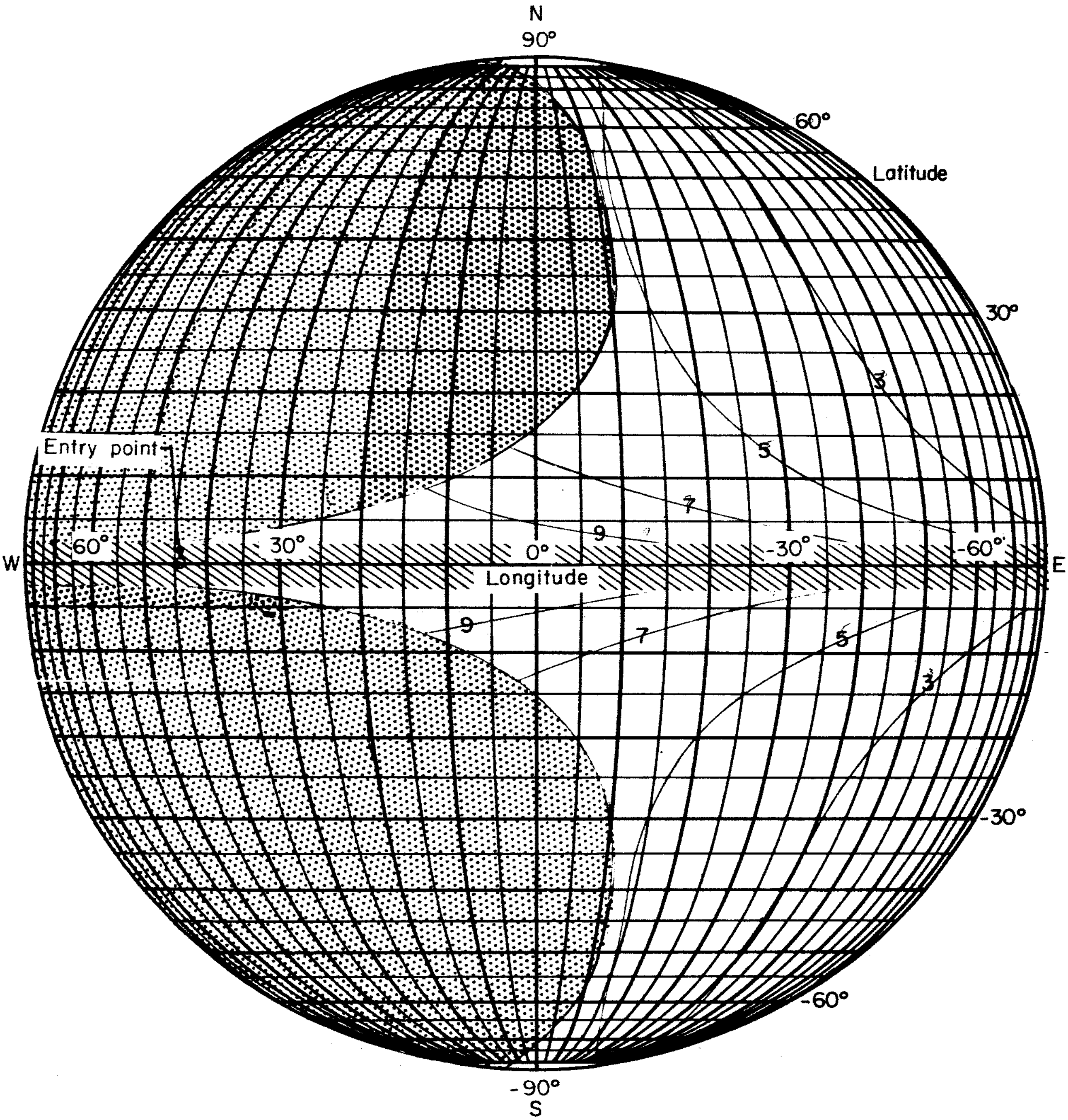


(c) $\eta = 3.0^{\circ}$.



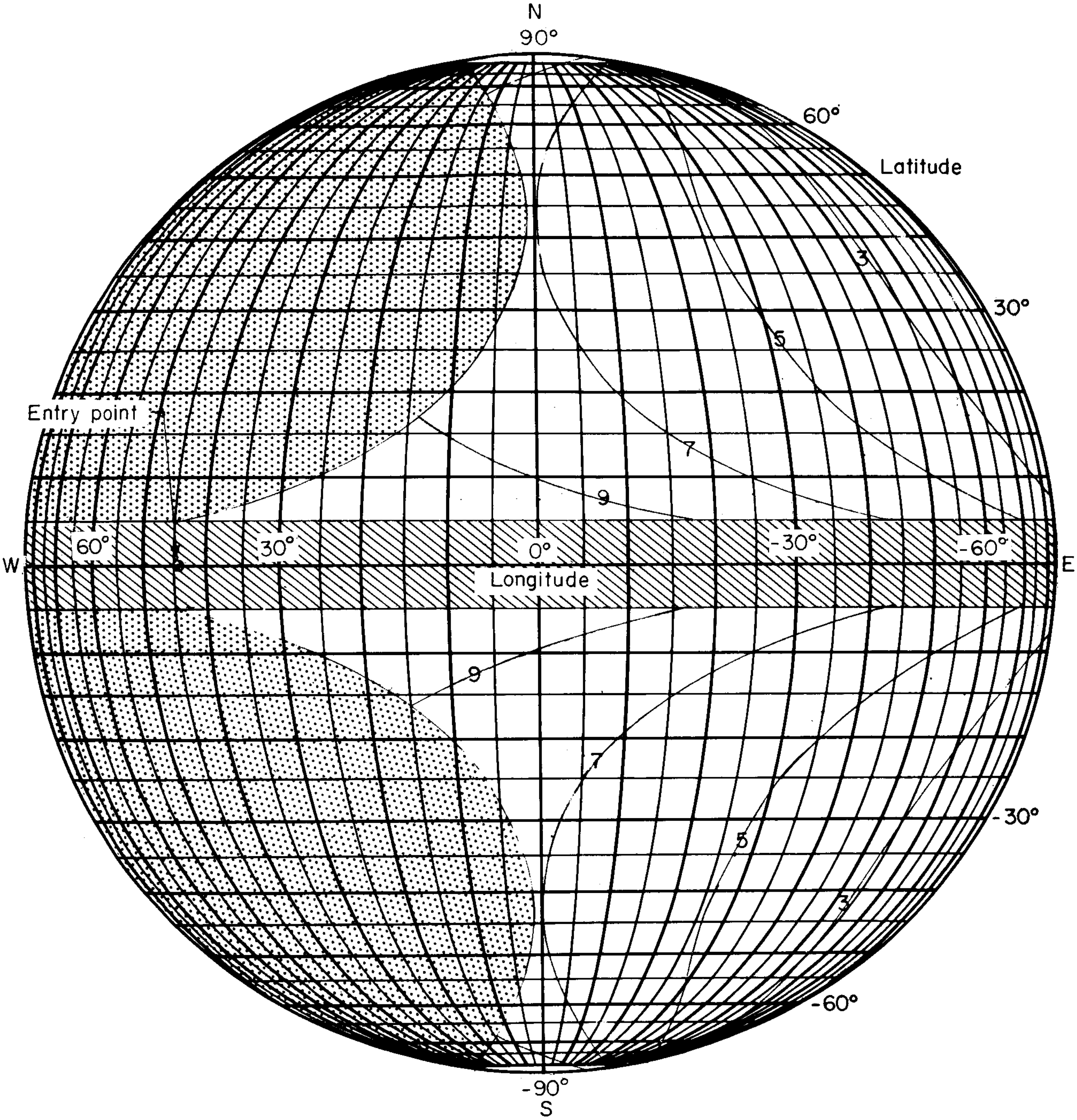
(d) $\eta = 4.58^{\circ}$.

Figure 5.- Equatorial regions of unlimited stay time for an inplane landing maneuver with plane-change capability on take-off. $\delta_i = 0^{\circ}$; $\delta_f = 5^{\circ}$.



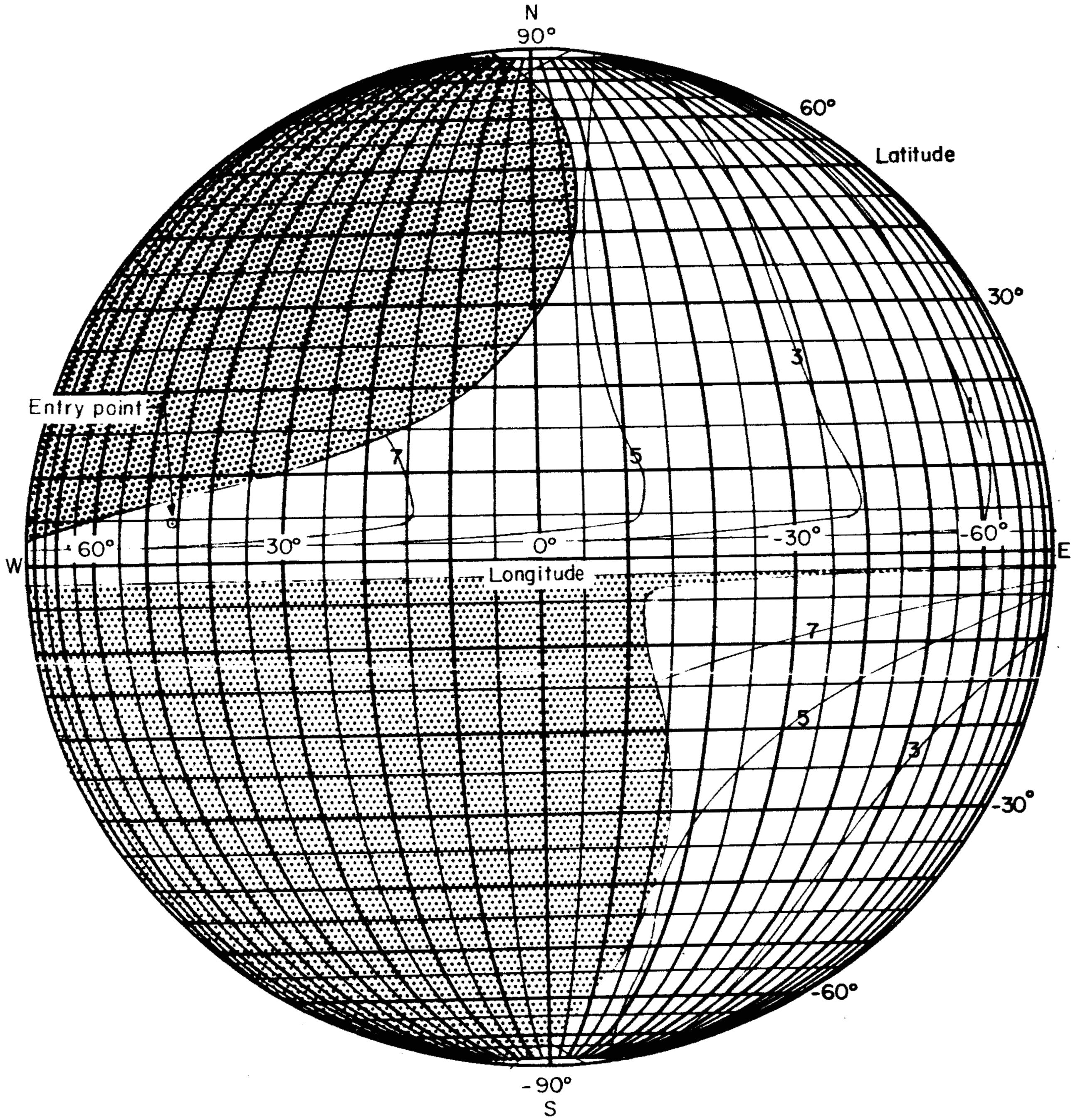
(a) $i_o = 0^o$; $\delta_i = \delta_f = 2.5^o$.

Figure 6.- Landing sites and stay times on lunar surface for an offset landing and take-off maneuver.



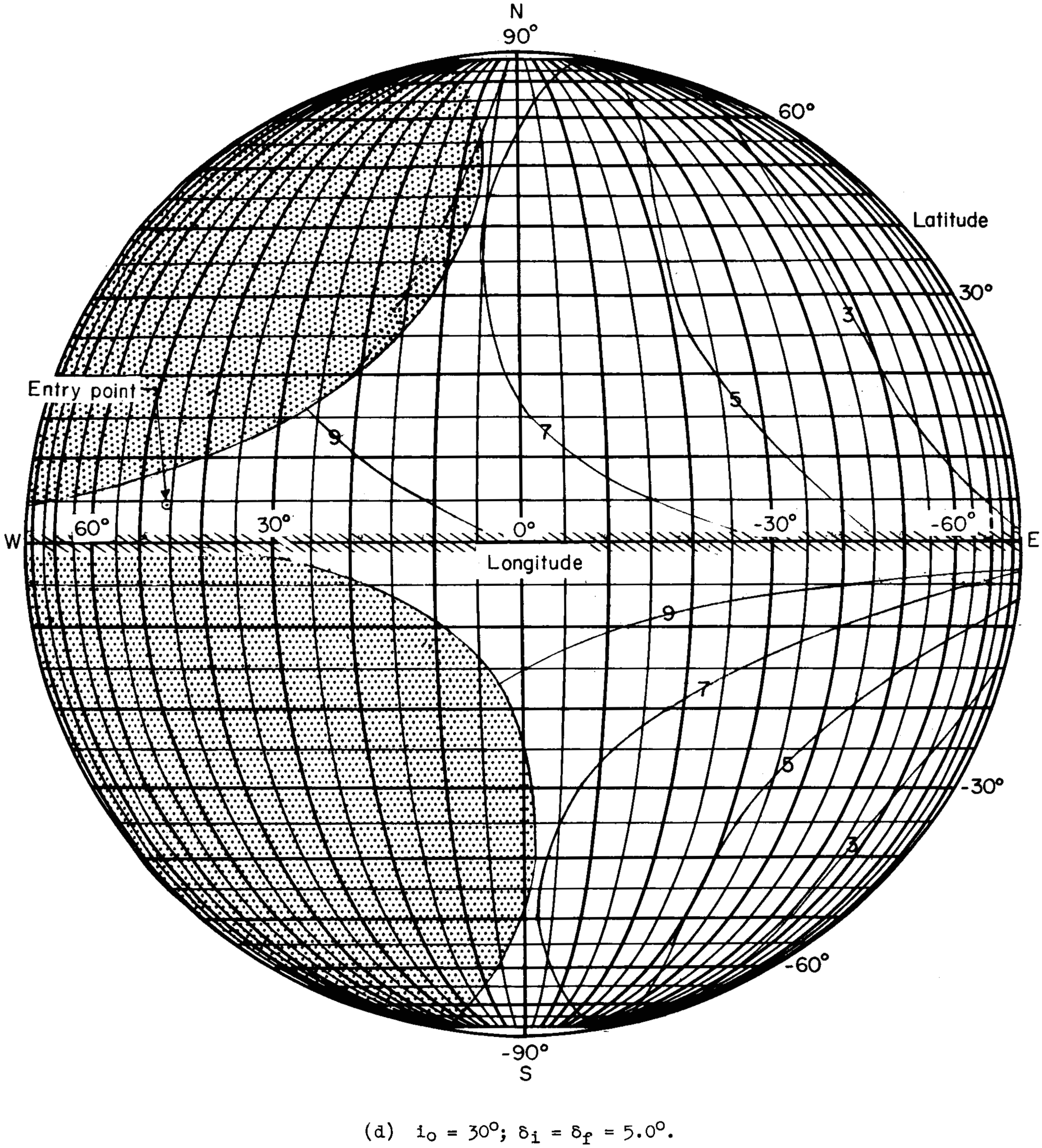
(b) $i_o = 0^o$; $\delta_i = \delta_f = 5.0^o$.

Figure 6.- Continued.



(c) $i_o = 30^{\circ}$; $\delta_i = \delta_f = 2.5^{\circ}$.

Figure 6.- Continued.



(d) $i_o = 30^\circ$; $\delta_i = \delta_f = 5.0^\circ$.

Figure 6.- Concluded.

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