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THERMAL RADIATION FROM THE IONOSPHERE

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ABSTRACT

The ionosphere is considered as a dissipative medium in which the random thermal motions of the charged particles act as a source of thermal radiation. Attention has been focused on the electrons colliding with ions and neutral particles in the ionosphere. A method of analysis has been developed with the aid of the Maxwell and Langevin equations based on a linear, macroscopic, fluctuating electromagnetic field theory. The spectral density of the random-current source function is derived in terms of the conductivity tensor of the ionosphere.

The ionosphere is divided into a large number of incremental volume elements, each containing an ionized medium which represents an anisotropic elementary radiating system, characterized by the spectral density of the source function. The radiation characteristic of the radiating system observed at a point located outside of the source region is obtained with the aid of the potential functions which relate the thermal electromagnetic fields at the observation point to their source function. It is observed that when the frequency of radiation is such that $Y^2 + Z^2 = 3$, where $Y = \omega_b/\omega$ and $Z = \nu/\omega$, the radiating system loses its anisotropic feature. ω_b and ν are the cyclotron and collision frequencies of the electrons, respectively. Based on the superposition principle, general expressions have been derived for $W(f,V_s)$, the thermal noise power generated per unit bandwidth from any given source region V_s of the ionosphere, and for $P_o(f,V_s)$, the available thermal noise power per unit bandwidth at a receiving antenna. These expressions are valid for most regions of interest in the ionosphere where the electron collision process plays a major role in the thermal radiation and are not limited in frequency range.

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I. INTRODUCTION

It is well known that because the ionosphere acts as an absorber of radio waves, it can also act as an emitter of thermal radio noise. It has been conclusively demonstrated by various workers[1,2,3,4] that the thermal emission from the D-region can, under favorable conditions, be observed with a dipole antenna. For example, Pawsey et al.[1] have identified and measured the thermal radiation from the ionosphere in the vicinity of 2 mc/sec in the temperate latitude.

It appears that usually the thermal radiation has been neglected because its level is exceedingly low as illustrated by Pawsey et al. and it does not constitute an appreciable source of interference in radio communication. However, the noise radiated from a plasma (e.g., the ionosphere) is not necessarily a detrimental effect in all cases, as it is in communication, since if the spectral distribution of the emitted energy is characteristic of the plasma properties, a measurement of radiation provides specific information on the plasma. For example, knowledge of the radiated power gives a measure of the electron temperature in the plasma and this has been used as a powerful diagnostic technique.

For a steady-state plasma a macroscopic radiative transfer concept can be applied without detailed knowledge of the atomic processes and the emission spectrum can be determined from the electromagnetic wave absorption, transmission and reflection properties of the plasma. These determinations are, in general, complicated by the nonuniformity and geometrical configuration of the emitting plasma.

In view of the fact that a current survey of the literature shows that no detailed information with regard to the mechanism of generation of ionospheric thermal radiation is available, it is the purpose of the present report to study this phenomenon in the hope that it will serve as a useful step toward an understanding of the fundamental process.

It is well known that the thermal radiation from dissipative bodies is due to the random thermal motion of the charges in the body. If the body is at a uniform temperature, one approach that may be used for studying radiation may be called the integral approach. The body as a whole is considered to be nonradiating and the power that is absorbed from its surroundings, which is assumed to be at the temperature of the body, can be computed. This power is set equal to the power radiated by the body. In this approach no attempt is made to determine the noise current fluctuations that are the cause of the thermal radiation. In those cases in which the temperature of the body is nonuniform this approach fails.

Another approach, which may be called the "Nyquist source treatment"[5,6,7], focuses attention upon the sources of the radiation and determines their relevant statistical properties. Once these are known, the determination of the radiation is conceptually a simple problem, although mathematical difficulties usually arise. A step toward the determination of the current fluctuation in a linear, dissipative medium has been taken by Rytov[5]. He considered ordinary conducting homogeneous media and showed that by postulating some correlations for the current fluctuation in such a media a correct description of the thermal electromagnetic field can be obtained. Haus[6] has been able to determine the correlations of the current fluctuation in all uniform linear dissipative media. Vanwormhoudt and Haus[7] have generalized these results to nonuniform media.

In the present study, the "Nyquist source treatment" is adopted and the ionosphere is considered as an anisotropic dissipative medium in which the random thermal motions of the charged particles act as a source of the thermal radiation. It is further postulated that in the ionosphere a linear constitutive local relation exists between the driven a-c conduction current density $\vec{J}_d$ and an applied a-c electric field intensity $\vec{E}$ of the form

$$\vec{J}_d\,(\omega,\vec{r}) = \overleftrightarrow{\sigma}\,(\omega,\vec{r}) \cdot \vec{E}\,(\omega,\vec{r}) \quad , \tag{1}$$

where $\overleftrightarrow{\sigma}$ is the conductivity tensor of the ionosphere, and a function of the angular frequency ω and position variable $\vec{r}$ which characterize the medium under consideration. A small-signal analysis is made throughout the present paper.

II. DERIVATION OF THE CONDUCTIVITY TENSOR

For a macroscopic analysis the Langevin equation can be used effectively to describe the motion of an electron, and it can be expressed as follows:

$$m\,\frac{\partial \vec{v}}{\partial t} + m\,\nu\,\vec{v} = e\,[\vec{E} + \vec{v} \times \vec{B}] \quad , \tag{2}$$

where $\vec{B}(\vec{r})$ is the static geomagnetic field, $\nu(r)$ is the average electronic collision frequency with ions and neutral particles, e, m and $\vec{v}$ are the electronic charge taken as a negative value, mass and velocity respectively.

On the other hand the convection current density $\vec{J}$ is related to the velocity $\vec{v}$ by

$$\vec{J} = N_o e \vec{v} , \tag{3}$$

where $N_o (\vec{r})$ is the electron number density.

Assuming the time harmonic variation $e^{j\omega t}$ for the quantities of interest, upon elimination of $\vec{v}$ from Eqs. 2 and 3 the following relationship is established:

$$U \vec{J} + j (\vec{J} \times \vec{Y}) = - j\omega \epsilon_o X \vec{E} , \tag{4}$$

where

$$X = \frac{\omega_p^2}{\omega^2} , \qquad \omega_p^2 = \frac{N_o e^2}{m \epsilon_o} ,$$

$$\vec{Y} = \frac{e\vec{B}}{m\omega} , \qquad Y = \frac{\omega_b}{\omega} = \frac{-e |\vec{B}|}{m\omega} ,$$

$$Z = \frac{\nu}{\omega} , \qquad U = 1 - jZ , \tag{5}$$

in which ω_p and ω_b are the plasma and gyrofrequencies of the electrons, respectively, and ϵ_o is the dielectric constant of vacuum.

In a three-dimensional space any vector $\vec{F}$ can be written as

$$\vec{F} = \sum_{\alpha=1}^{3} \vec{u}_\alpha F_\alpha , \tag{6}$$

where the unit vectors $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$ form a complete orthogonal set of basic vectors for the space and F_α is the component of the vector in the direction of $\vec{u}_\alpha$. At the same time, the vector $\vec{F}$ may also be expressed as a column matrix, denoted by $\underline{F}$ as

$$\underline{F} = \{F_\alpha\} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} , \qquad \alpha = 1, 2, 3 . \tag{7}$$

The vector Eq. 4 may be conveniently expressed in the following matrix form:

$$\underline{\underline{\gamma}}\,\underline{J} = \underline{E} \tag{8}$$

or equivalently in tensor notation as

$$\overleftrightarrow{\gamma} \cdot \vec{J} = \vec{E} \ , \tag{9}$$

where the resistivity matrix $\underline{\underline{\gamma}}$ is defined as

$$\gamma = \{\gamma_{\alpha\beta}\} \ , \qquad \alpha, \beta = 1, 2, 3 \ , \tag{10}$$

with its elements being given by

$$\gamma_{11} = \gamma_{22} = \gamma_{33} = \frac{jU}{\omega\,\epsilon_o\,X} \ ,$$

$$\gamma_{12} = -\gamma_{21} = \frac{-Y_3}{\omega\,\epsilon_o\,X} \ ,$$

$$\gamma_{13} = -\gamma_{31} = \frac{-Y_2}{\omega\,\epsilon_o\,X}$$

and

$$\gamma_{23} = -\gamma_{32} = \frac{-Y_1}{\omega\,\epsilon_o\,X} \tag{11}$$

and with its determinant $|\underline{\underline{\gamma}}|$ given by

$$|\underline{\underline{\gamma}}| = \frac{jU}{(\omega\,\epsilon_o\,X)^3}\,[Y^2 - U^2] \tag{12}$$

in which

$$Y^2 = Y_1^2 + Y_2^2 + Y_3^2 \ . \tag{13}$$

In view of the fact that $|\underline{\underline{\gamma}}|$ can be zero only for a special situation where $\nu = 0$ and $\omega = \omega_H$ occur simultaneously. Since $\nu = 0$ is not of interest to the present study, $|\nu|$ can be considered to possess an inverse, which is denoted by $\underline{\underline{\sigma}}$ and is referred to as the "conductivity matrix", i.e.,

$$\underline{\underline{\sigma}}\,\underline{\underline{\gamma}} = \underline{\underline{I}} , \qquad (14)$$

where $\underline{\underline{I}}$ is the unit matrix. Consequently, from Eq. 8 and Eq. 14, $\underline{J}$ can be expressed in terms of $\underline{E}$ explicitly as

$$\underline{J} = \underline{\underline{\sigma}}\,\underline{E} \qquad (15)$$

or in a tensor notation as

$$\vec{J} = \overset{\sqcup}{\sigma} \cdot \vec{E} . \qquad (16)$$

It should be noted that the components of the tensors $\overset{\sqcup}{\sigma}$ and $\overset{\sqcup}{\gamma}$ do depend upon the particular choice of the coordinate system. In the present report, the spherical coordinate system is employed because it appears to be the most convenient one to use for the configuration of the region under investigation.

It is well known that the geomagnetic field $\overline{B}$ can be approximated by a dipole field which is induced by a uniformly magnetized spherical earth, and may be expressed[8] as

$$\overline{B} = \frac{Ma^3}{3} \nabla \left(\frac{\cos \theta}{r^2} \right) , \qquad (17)$$

where the space variables r and θ denote, respectively, the radial and polar angular coordinates of the geomagnetic spherical coordinate system with its origin located at the center of the earth, and the constants M and a are the magnetization and the radius of the earth respectively.

If the unit vectors $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$ are taken as the basic vectors in the spherical coordinate system (r, θ, φ), then the indices 1, 2 and 3 can be made to correspond to the coordinate variables r, θ and φ respectively. Thus F_1, F_2 and F_3 represent the r-, θ- and φ-components of the vector $\vec{F}$.

By adopting the model of the geomagnetic field described by Eq. 17 it is not difficult to see that the components of the vector $\vec{Y}$, defined in Eq. 5, are given by

$$Y_1 = 2G\cos\theta \ , \quad Y_2 = G\sin\theta \text{ and } Y_3 = 0 \ , \tag{18}$$

where

$$G = \left(\frac{-e}{\omega m}\right)\frac{M}{3}\left(\frac{a}{r}\right)^3 \tag{19}$$

and the components of $\underline{\underline{\gamma}}$ and $\underline{\underline{\sigma}}$ are given as follows:

$$\gamma_{11} = \gamma_{22} = \gamma_{33} = \frac{jU}{\omega\,\epsilon_o\,X} \ ,$$

$$\gamma_{12} = -\gamma_{21} = 0 \ ,$$

$$\gamma_{13} = -\gamma_{31} = \frac{G\sin\theta}{\omega\,\epsilon_o\,X} \ ,$$

$$\gamma_{23} = -\gamma_{32} = -\frac{2G\cos\theta}{\omega\,\epsilon_o\,X} \tag{20}$$

and

$$\sigma_{\alpha\beta} = DC_{\alpha\beta} \ , \qquad \alpha,\beta = 1,\,2,\,3 \ , \tag{21}$$

where

$$D = j\omega\epsilon_o\,\frac{X}{U(U^2 - Y^2)} \tag{22}$$

and

$$Y^2 = G^2(1 + 3\cos^2\theta) \tag{23}$$

with

$$C_{11} = (Z^2 - 1 + 4G^2 \cos^2 \theta) + j2Z$$

$$C_{22} = (Z^2 - 1 + G^2 \sin^2 \theta) + j2Z$$

$$C_{33} = (Z^2 - 1) + j2Z$$

$$C_{12} = C_{21} = 2G^2 \sin \theta \cos \theta$$

$$C_{13} = - C_{31} = - (Z + j)\, G \sin \theta$$

$$C_{23} = - C_{32} = 2(Z + j)\, G \cos \theta \quad . \tag{24}$$

III. NOISE POWER RADIATED FROM THE IONOSPHERE

A body with a nonuniform temperature distribution is not in thermodynamic equilibrium. However, in those cases in which the distribution function of charge carriers deviates only slightly from the equilibrium distribution (so as to produce heat and current flow), and this includes all cases for which a temperature can be reasonably defined, it would be expected that the radiated noise power could still be computed as the superposition of the noise power radiated from the various volume elements of the body. In this case each element at a particular temperature radiates the same noise power it would radiate at equilibrium at the same temperature. Such an analysis calls for an approach to the fluctuation problem that considers each differential volume element separately as an absorber and emitter of noise power. It calls for the introduction of a source term into Maxwell's equations analogous to the source term of the Langevin equation in the theory of Brownian motion.

Although Maxwell's equations and the constitutive relation are sufficient to solve most electromagnetic problems, they are insufficient for noise studies. The current density derived from the constitutive relation represents only the current driven by the electromagnetic fields. Besides this driven current, the current density fluctuation caused by the random motion of the charge must be considered. This can be taken into account by introducing into Maxwell's equations a random driving current density distribution which is independent of the electromagnetic fields, i.e.,

$$\nabla \times \vec{e} = -\mu_o \frac{\partial \vec{h}}{\partial t} \tag{25}$$

and

$$\nabla \times \vec{h} = \epsilon_o \frac{\partial \vec{e}}{\partial t} + \vec{i} \quad , \tag{26}$$

where $\vec{e}$ and $\vec{h}$ are the time-dependent electric and magnetic fields, respectively, and $\vec{i}$ is the current density, μ_o is the permeability of vacuo. The current density $\vec{i}$ in Eq. 26 consists of two parts. First of all there is the "driven" component $\vec{i}_d$ that is produced by the electric field $\vec{e}$ and is related to $\vec{e}$ by Eq. 15. The spontaneous noise fluctuations of the field at thermal equilibrium can be taken into account by another current component of $\vec{i}$ in Eq. 26, the source current density $K(t,\vec{r})$, a statistical quantity which is a stationary function of time.

3.1 The Dyadic Spectral Density of Current Source Functions

In the study of problems involving radiation of noise power, it is convenient to introduce Fourier transformations in time of all field quantities in Eqs. 25 and 26. In the present case all random time functions are stationary and, strictly speaking, they do no possess Fourier transformations. However, this difficulty may be overcome by constructing a periodic substitute function[5,6] according to the definition

$$\vec{F}(t, \vec{r}, T) = \vec{F}(t, \vec{r}) \quad , \qquad \text{for } -\frac{T}{2} < t < \frac{T}{2}$$

and

$$\vec{F}(t + nT, \vec{r}, T) = \vec{F}(t, \vec{r}, T) \quad . \tag{27}$$

These substitute functions have Fourier transformations of the form

$$\vec{F}(\omega, \vec{r}, T) = \frac{1}{T} \int_{-(T/2)}^{T/2} \vec{F}(t, \vec{r}, T)\, e^{-j\omega t}\, dt \quad . \tag{28}$$

In the limit as $T \to \infty$, the substitute functions are indistinguishable from their originals. The spectral density of any noise process can be obtained directly from the ensemble average of products of these Fourier components. Thus, the dyadic spectral density of $\vec{F}$ is given by

$$\overset{\leftrightarrow}{S}_{\vec{F}}(\omega, \vec{r}, \vec{r}\,') = \lim_{T \to \infty} \frac{T}{2\pi} \langle \vec{F}(\omega, \vec{r}, T)\, \vec{F}^*(\omega, \vec{r}\,', T) \rangle_{avg} \quad , \tag{29}$$

where the symbol * denotes the complex conjugate.

It should be noted that the spectral analysis of the periodic substitute function leads to a discrete spectrum extending over negative, as well as positive, frequencies. With lines at frequency interval $\Delta f = (1/T)$ the expression

$$\langle 2\vec{F}(\omega, \vec{r}, T)\, \vec{F}^*(\omega, \vec{r}\,', T) \rangle_{avg} = 4\pi \Delta f\, \overset{\leftrightarrow}{S}_{\vec{F}}(\omega, \vec{r}, \vec{r}\,') \tag{30}$$

may be identified in the limit of large T as "the mean-square fluctuation of $\vec{F}$ in the frequency interval Δf". Furthermore, for a stationary time function $\vec{F}$, (see Reference 9),

$$\frac{T}{2\pi} \langle \vec{F}(\omega, \vec{r}, T)\, \vec{F}^*(\omega', \vec{r}, T) \rangle_{avg} = 0 \quad , \qquad \omega \neq \omega' \quad . \tag{31}$$

Precisely this kind of treatment must be kept in mind in applying the formal expansion of the Fourier integral and using the spectral amplitude densities in the study of electromagnetic fluctuation, on which the present report is based.

As a matter of convenience, for a particular physical variable, the lower case letter is used for the stationary time function and for its periodic substitute function, while the capital letter is used for its Fourier transform, in the following discussion. It is obvious, from Eqs. 25 and 26, with the aid of Eq. 16 that the Fourier amplitudes of the periodic substitute functions are related in the following manner:

$$\nabla \times \vec{E} = -j\omega\mu\, \vec{H} \tag{32}$$

and

$$\nabla \times \vec{H} = j\omega\, \epsilon_o\, \vec{E} + \overleftrightarrow{\sigma} \cdot \vec{E} + \vec{K} \quad . \tag{33}$$

Suppose that a region of the ionosphere under study is divided into a large number of sufficiently small elementary volume elements such that within each one of these elementary volumes the medium may reasonably be assumed to be uniform at a certain temperature T_o. Strictly speaking these elementary volume elements should be made to approach zero. On the other hand, they have to be kept large enough to contain a large number of charge carriers in order that statistical arguments may be applied. A tensor-conductivity description of the medium as given by Eq. 16 is possible only because the current in an elementary volume depends upon the electric field in the same volume, but not upon its derivatives, that is, upon the value of the electric field in the neighboring elementary volumes. In view of this fact, it is quite reasonable to expect that the source currents caused by the random motion of the charge carriers in two neighboring elementary volumes are uncorrelated. In other words, if $\vec{r}$ and $\vec{r}\,'$ denote the points belonging to two different elementary volumes, then $\vec{K}(\omega, \vec{r})$ and $\vec{K}(\omega, \vec{r}\,')$ are not correlated and the dyadic spectral density of $\vec{K}$ has the form

$$\overleftrightarrow{S}_{\vec{K}}(\omega, \vec{r}, \vec{r}\,') = \delta(\vec{r} - \vec{r}\,')\, \overleftrightarrow{\psi}(\omega, r) \quad , \tag{34}$$

where $\delta(\vec{r} - \vec{r}\,')$ is the usual Dirac delta function.

On the other hand, an elementary volume element may be considered as a linear network containing a noise source in thermal equilibrium and the technique developed in the theory of linear noise network[6,7], which makes use of the generalized Nyquist theorem, can be applied. Using the concept of a linear network, for example, Haus[6] has obtained a simple expression for $\overleftrightarrow{\psi}(\omega, \vec{r})$ as follows:

$$\overleftrightarrow{\psi}(\omega, \vec{r}) = \frac{kT_o(r)}{2\pi} [\overleftrightarrow{\sigma}(\omega, \vec{r}) + \overleftrightarrow{\sigma}^{\dagger}(\omega, \vec{r})] \quad , \tag{35}$$

where k is the Boltzmann constant and the symbol dagger ($\dagger$) indicates the complex-conjugate transpose of the conductivity matrix $\overleftrightarrow{\sigma}$. If the average volume density of thermal energy $\tau(r)$ in joules per m^3 is introduced, defined as the ratio of the amount of thermal energy generated within an elementary volume ΔV to the volume ΔV, then from Eqs. 34 and 35 one has

$$\overleftrightarrow{S}_K(\omega, \vec{r}) = \frac{\tau(r)}{2\pi} [\overleftrightarrow{\sigma}(\omega, \vec{r}) + \overleftrightarrow{\sigma}^{\dagger}(\omega, \vec{r})] \tag{36}$$

and from Eq. 30

$$<2\vec{K}(\omega, \vec{r})\ \vec{K}^*(\omega, \vec{r})>_{avg} = 2\Delta f\ \tau(r)\ [\overset{\leftrightarrow}{\sigma}(\omega, \vec{r}) + \overset{\leftrightarrow}{\sigma}{}^{\dagger}(\omega, \vec{r})] \quad , \tag{37}$$

which may be given alternatively in its component form with the aid of Eq. 24 as follows (see the appendix for the details):

$$<2 K_{\alpha}(\omega, r)\ K_{\beta}^*(\omega, r)>_{avg} = \tau(r)\ \Delta f\ L_{\alpha\beta}(\omega, r) \quad , \tag{38}$$

where

$$L_{\alpha\beta}(\omega, r) = 4\omega\,\epsilon_o \left(\frac{XZ}{1 + Z^2}\right) [\ell_{\alpha\beta} + jm_{\alpha\beta}] \quad ,$$

$$\alpha,\beta = 1, 2, 3 \quad , \tag{39}$$

with

$$\ell_{11} = \frac{1}{Q(Y,Z)} [(1 + Z^2)(1 + Z^2 + Y^2) + (Z^2 + Y^2 - 3)\ 4G^2 \cos^2\theta] \quad ,$$

$$\ell_{22} = \frac{1}{Q(Y,Z)} [(1 + Z^2)(1 + Z^2 + Y^2) + (Z^2 + Y^2 - 3)\ G^2 \sin^2\theta] \quad ,$$

$$\ell_{33} = \frac{1}{Q(Y,Z)} [(1 + Z^2)(1 + Z^2 + Y^2)] \quad ,$$

$$\ell_{12} = \ell_{21} = \frac{1}{Q(Y,Z)} [(Z^2 + Y^2 - 3)\ G^2 \sin 2\theta] \quad ,$$

$$\ell_{13} = \ell_{31} = \ell_{23} = \ell_{32} = 0 \quad ,$$

$$m_{11} = m_{22} = m_{33} = m_{12} = m_{21} = 0 \quad ,$$

$$m_{13} = -m_{31} = \frac{1}{Q(Y,Z)} [2(1 + Z^2)\ G \sin\theta] \quad ,$$

$$m_{23} = -m_{32} = \frac{-1}{Q(Y,Z)} [4(1 + Z^2)\ G \cos\theta]$$

and

$$Q(Y,Z) = (Y^2 + Z^2 - 1)^2 + 4Z^2 \quad . \tag{40}$$

It is observed that $Y = 0$ when $G = 0$. In this case, $\ell_{\alpha\beta} = 1$ if $\alpha = \beta$ and $\ell_{\alpha\beta} = 0$ if $\alpha \neq \beta$, while $m_{\alpha\beta}$ becomes zero regardless of whether $\alpha = \beta$ or $\alpha \neq \beta$.

the tensor $\{L_{\alpha\beta}\}$ appearing in Eq. 38 becomes a scalar and the medium becomes isotropic. This is perfectly reasonable since when G = 0 the geomagnetic field is completely absent.

It is also interesting to note that for the case

$$Y^2 + Z^2 = 3 \tag{41}$$

$\ell_{\alpha\beta}$ again becomes either equal to unity or to zero according to whether $\alpha = \beta$ or $\alpha \neq \beta$ and

$$m_{13} = - m_{31} = \frac{1}{2} G \sin \theta$$

and

$$m_{23} = - m_{32} = - G \cos \theta . \tag{42}$$

3.2 Time Average Thermal Noise Power Radiated

It is not difficult to see that from Eqs. 25 and 26 the following energy conservation relation can be obtained:

$$\nabla \cdot \vec{p} = - \vec{e} \cdot \vec{i} - \frac{\partial}{\partial t}\left(\frac{1}{2} \epsilon_o e^2 + \frac{1}{2} \mu_o h^2\right) , \tag{43}$$

which is the familiar "Poynting theorem", where $\vec{p} = \vec{e} \times \vec{h}$ is the Poynting vector representing power flow density. The first term on the right-hand side of Eq. 43 represents the joule heat losses per unit volume in the medium and the other term represents the rate of change of the stored electromagnetic energy density.

For the periodic field, the average time rate of change of stored energy is zero, so that

$$\overline{\nabla \cdot \vec{p}} = \nabla \cdot \overline{\vec{p}} = - \overline{\vec{e} \cdot \vec{i}} , \tag{44}$$

where the bar denotes the time average of the quantity. Using the fact that

$$\overline{\vec{e} \cdot \vec{i}} = \frac{1}{2} \text{Re } [\vec{E} \cdot \vec{I}^*]$$

and

$$\vec{p} = \frac{1}{2} \operatorname{Re} [\vec{E} \times \vec{H}^*] \qquad (45)$$

the total average power radiated from a system of a current is given by[10]

$$W = \oint_S \vec{p} \cdot d\vec{S} = -\frac{1}{2} \operatorname{Re} \int_V (\vec{E} \cdot \vec{I}^*) \, dV \quad . \qquad (46)$$

Thus radiation can be calculated either by integrating the normal component of the Poynting vector over a closed surface S including all sources or by integrating the power expended per unit volume over the current distribution. In the present discussion the latter approach is taken.

Keeping in mind that the concern here is with the random current distribution and since the time average power radiated per unit volume, $w(\omega, \vec{r})$, is given by

$$w(\omega, \vec{r}) = \frac{1}{2} \operatorname{Re} [\vec{K}^* \cdot \vec{E}] \quad , \qquad \text{watts/m}^3 \quad , \qquad (47)$$

in which $\vec{K}$ is the cause and $\vec{E}$ is its effect, and with the aid of Eq. 9, $w(\omega, \vec{r})$ becomes

$$w(\omega, \vec{r}) = \frac{1}{2} \operatorname{Re} [\vec{K}^* \cdot (\overleftrightarrow{\gamma} \cdot \vec{K})] = \frac{1}{2} \operatorname{Re} [\underline{K}^+ \underline{\underline{\gamma}} \, \underline{K}] \quad . \qquad (48)$$

The substitution of Eq. 20 into Eq. 48 yields (see the appendix for the details)

$$w(\omega, r) = \frac{Z}{2\omega\epsilon_o X} [K_1 K_1^* + K_2 K_2^* + K_3 K_3^*] \quad . \qquad (49)$$

On the other hand, with the aid of Eqs. 38 and 39, the thermal noise power generated per unit volume per unit bandwidth, $w_o(f, r)$, may be given as

$$w_o(f, \vec{r}) = kT_o\left(\frac{Z^2}{1 + Z^2}\right)[\ell_{11} + \ell_{22} + \ell_{33}] \quad , \tag{50}$$

where ℓ_{11}, ℓ_{22} and ℓ_{33} are given in Eq. 40.

It is interesting to observe that w_o given in Eq. 50 does not depend explicitly upon the electron number density N_o since it does not contain the parameter X. Furthermore, the radiated thermal noise power consists of three terms; the term associated with ℓ_1, with ℓ_2 and with ℓ_3, each representing the contribution from the current fluctuation in three directions along the coordinate axes, namely, in the r-, θ- and φ-directions respectively. The fact that no term containing the parameter $\ell_{\alpha\beta}$, with $\alpha \neq \beta$, appears in Eq. 50 suggests that no correlation between the components of source currents K_α and K_β contribute to the thermal radiation. On the other hand, when $Y = 0$, $\ell_{11} = \ell_{22} = \ell_{33} = 1$, which suggests that the contributions from the three different directions are equal, and for a system with three degrees of freedom the thermal noise power radiated is proportional to $3kT_o$, which is reasonable.

Having obtained the expression for $w_o(f, \vec{r})$, the evaluation of the thermal noise power radiated from any region V_s of interest in the ionosphere is a relatively simple matter if the way in which the temperature T_o and the parameters Y and Z depend upon position is known.

For example, if the stratified model of the ionosphere[11] is considered, and it is assumed that the electron number density N_o, the electron temperature T_o and the collision frequency ν all depend only upon the altitude h, or $r = a + h$, but not upon the polar angle θ or azimuthal angle φ, then the parameters X and Z are functions of h, or of r, whereas the parameter Y, being defined in terms of the earth's magnetic field, in general depends upon the altitude h as well as the polar angle θ. Therefore, the time average thermal noise power $W(f, V_s)$

per unit bandwidth radiated from a region $V_s(r_o \leq r \leq r_1,\ \theta_o \leq \theta \leq \theta_1$ and $\varphi_o \leq \varphi \leq \varphi_1)$ can be given with the aid of Eqs. 40 and 50 as follows (see the appendix for the details):

$$W(f,\ V_s) = (\varphi_1 - \varphi_o) \int_{r_o}^{r_1} \left(\frac{kT_o Z^2 r^2}{1 + Z^2} \right) \zeta\ (Z,\ \theta_o,\ \theta_1)\ dr\ , \tag{51}$$

where

$$\zeta\ (Z,\ \theta_o,\ \theta_1) = \left[\cos\theta_o - \cos\theta_1 + 2(Z^2 + 1) \int_{u_1}^{u_o} \Theta\ (Z,\ u)\ du \right] \tag{52}$$

and

$$\Theta\ (Z,\ u) = \frac{au^2 + b}{cu^4 + du^2 + g} \tag{53}$$

with

$$u = \cos\theta\ , \quad a = 3G^2\ , \quad b = G^2 + Z^2 + 1\ ,$$
$$c = 9G^4\ , \quad d = 6G^2(G^2 + Z^2 - 1)\ ,$$
$$g = G^4 + 2G^2(Z^2 - 1) + (Z^2 + 1)^2\ . \tag{54}$$

IV. OBSERVATION OF THERMAL RADIATION FROM THE IONOSPHERE

The rigorous determination of the radiation intensity within the emitting region of the ionosphere must be based on the study of the electromagnetic wave propagation in an anisotropic absorbing medium, in which each volume element can act as an emitter as well as an absorber of the thermal radiation. However, this problem is not discussed in the present report.

Nevertheless, it is of interest and of a considerable practical importance to know about the characteristics of noise power received from the ionospheric thermal radiation at a detecting antenna located outside of the source region. For example[1,2,3,4], many experimental observations have been made with an antenna located on the surface of the earth; it may also be on the ground or on the sea.

In view of the fact that the relation of the radiation fields to their sources is most readily found in terms of potential functions, and since the information with regard to some statistical properties of the random source current function $\vec{K}$ is available from Section III, the retarded vector potential function is introduced here and expressed in complex form as $\vec{A}(\omega, x_\alpha)\, e^{j\omega t}$, with

$$\vec{A}(\omega, x_\alpha) = \frac{\mu_o}{4\pi} \int_{V_s} \frac{\vec{K}(\omega, x'_\alpha)\, e^{-jk_o R(x_\alpha, x'_\alpha)}}{R(x_\alpha, x'_\alpha)}\, dV' \quad , \tag{55}$$

where x_α and x'_α denote the coordinates of the observation point and the source point respectively, and $R(x_\alpha, x'_\alpha)$ is the distance between them. $V_s(x'_\alpha)$ is the volume of the source region under investigation and k_o is the wave number. In the present discussion x_α is taken in the air and x'_α is taken in the ionosphere.

It should be observed that Eq. 55 signifies superposition of the solutions of the inhomogeneous wave equation

$$\nabla^2\vec{A} + k_o^2\vec{A} = -\vec{K} \quad , \tag{56}$$

where $k_o^2 = \omega^2\mu_o\epsilon_o$ and corresponds to a source at the point x'_α given by $\vec{K} = C\delta(x_\alpha - x'_\alpha)$, with $\delta(x_\alpha - x'_\alpha)$ being the usual Dirac delta function. On the other hand the retarded scalar potential function $\Phi(\omega, x_\alpha)$ is related to the vector potential by[10]

$$\nabla \cdot A + j\omega\mu\epsilon\Phi = 0 \quad , \tag{57}$$

which expresses the idea of conservation of charge. It should be noted that Eq. 57 is valid in free space (air) whereas it is only an approximation in a region of the conducting medium in which $|\sigma/j\omega\epsilon_o| \ll 1$.

It is well known that the electromagnetic fields at an observation point x_α, taken in air, can be derived from these potential functions by

$$\vec{E} = -\nabla \Phi - j\omega\vec{A} \tag{58}$$

and

$$\vec{H} = \frac{1}{\mu_o} \nabla \times \vec{A} , \tag{59}$$

where the spatial differential operator ∇ should be understood as ∇_{x_α}, which only operates on the function of x_α. The utilization of potential functions is particularly convenient because space differentiation ∇, under the sign of the volume integration, does not touch $\vec{K}(\omega, x'_\alpha)$ and thereby the field intensities $\vec{E}$ and $\vec{H}$ in the same manner do not contain derivatives of $\vec{K}$.

The substitution of Eqs. 55 and 57 into Eqs. 58 and 59 gives the following expression (see the appendix for the details):

$$\vec{E} = \frac{1}{j\omega\epsilon} \frac{1}{4\pi} \int_{V_s} \left[\vec{K} \left\{ \frac{k_o^2}{R} - \frac{jk_o}{R^2} - \frac{1}{R^3} \right\} \right.$$

$$\left. - \vec{R}(\vec{K} \cdot \vec{R}) \left\{ \frac{k_o^2}{R^3} - \frac{j3k_o}{R^4} - \frac{3}{R^5} \right\} \right] e^{-jk_oR} \, dV' \tag{60}$$

and

$$\vec{H} = \frac{1}{4\pi} \int_{V_s} (\vec{K} \times \vec{R}) \left[\frac{jk_o}{R^2} + \frac{1}{R^3} \right] e^{-jk_oR} \, dV' , \tag{61}$$

where the vector $\vec{R}$ is directed from the source point x'_α to the observation point x_α, with its magnitude being the distance between these points.

It is of interest to note that in Eqs. 60 and 61, every volume element dV' of the medium gives the same kind of field as an electric dipole at the point of observation x_α and the terms which, with increasing R, decrease faster than 1/R correspond to the quasi-stationary field parts of the elementary dipoles, while the terms decreasing with increasing R as 1/R correspond to the radiation field.

If only a 1/R dependent radiation field is taken into account, the electric and magnetic fields may be written from Eqs. 60 and 61 as

$$\vec{E} = \frac{1}{j\omega\epsilon}\frac{1}{4\pi}\int_{V_s}[\vec{k} \times (\vec{K} \times \vec{k})]\,\frac{e^{-j\vec{k}\cdot\vec{R}}}{R}\,dV' \qquad (62)$$

and

$$\vec{H} = \frac{j}{4\pi}\int_{V_s}(\vec{K} \times \vec{k})\,\frac{e^{-j\vec{k}\cdot\vec{R}}}{R}\,dV' \qquad (63)$$

in which the propagation vector $\vec{k} = \vec{n}k_o$ is introduced and the unit vector $\vec{n}$ is defined as $\vec{R}/R$ so that $\vec{k}$ and $\vec{R}$ are in the same direction.

The electromagnetic fields given by Eqs. 62 and 63 can be considered as the random thermal electromagnetic fields since their source function $\vec{K}$ is a random, statistical quantity. The power flow density at the observation point x_α may be considered now, with the aid of the Poynting vector defined in Eq. 45.

The substitution of Eqs. 62 and 63 into Eq. 45 yields (see the appendix for the details)

$$\vec{p}(\omega,x_\alpha) = \frac{k_o\Delta f}{2\lambda^2}\int_{V_s}\vec{n}(x_\alpha,x'_\alpha)\left[\frac{kT_oXZ}{1+Z^2}\right]\frac{\Gamma(x_\alpha,x'_\alpha)}{R^2(x_\alpha,x'_\alpha)}\,dV' \quad , \qquad (64)$$

where λ is the free-space wavelength, Z, X, and T_o are functions of the source point coordinate x'_α and $\Gamma(x_\alpha,x'_\alpha)$ is defined by

$$\Gamma(x_\alpha, x'_\alpha) = (1 - n_1^2)\ell_{11} + (1 - n_2^2)\ell_{22} + (1 - n_3^2)\ell_{33} - 2n_1 n_2 \ell_{12} \tag{65}$$

in which n_1, n_2 and n_3 are the components of the unit vector $\vec{n}$ along r- , θ- and φ-coordinate axes, and ℓ_{11}, ℓ_{22}, ℓ_{33} and ℓ_{12} are given in Eq. 40.

It should be observed that Eq. 64 is based on the concept that the radiation intensity in any solid angle can be treated as energy, transferable in a bundle of plane, nonextinguishable waves whose normals are included in the solid angle. In a homogeneous isotropic medium the direction of the vector of energy flux coincides with the wave normal[5]. The unit vector $\vec{n}\,(x_\alpha, x'_\alpha)$ indicates the direction of propagation of the wave originating at the source point x'_α.

Having determined the time average Poynting vector $\bar{\vec{p}}(\omega, x_\alpha)$, the noise power received from the ionospheric thermal radiation at the receiving antenna can be obtained by taking a proper surface integral of $\bar{\vec{p}}(\omega, x_\alpha)$ over the aperture of the antenna A_o,

$$P(\omega) = \int_{A_o} \bar{\vec{p}}(\omega, x_\alpha) \cdot \vec{ds} \quad , \tag{66}$$

where $\vec{ds} = \vec{n}_o ds$, with $\vec{n}_o$ being a unit vector normal to the differential surface area ds.

Since $P(\omega)$, given by Eq. 66, is nothing but the available noise power at the receiving antenna in the frequency interval between f and f + Δf, with the aid of elementary antenna theory, the mean-square fluctuation of the induced noise voltage $< V^2 >$ on the antenna can be obtained from the following simple relationship:

$$P(\omega) = \frac{< V^2 >}{4R_r} \quad , \tag{67}$$

where R_r is the radiation resistance of the antenna. Equation 67 is based on the assumption that the lossless antenna is oriented for maximum response and the receiving system is designed in such a way that there is maximum power transfer from the receiving antenna to its terminal impedance.

On the other hand, the effective antenna temperature, T_{eff}, can also be determined by the Nyquist formula

$$P(\omega) = kT_{eff}\,\Delta f \qquad (68)$$

in which the receiving antenna is assumed to be in thermal equilibrium with its surroundings.

In practice, the antenna used for the measurement of ionospheric thermal radiation is characterized by its directivity D_o, by its effective area (or maximum effective aperture) A_e, or by its beam area B, which is the solid angle Ω_a through which all the power radiated would stream if the power per unit solid angle equaled the maximum value of radiation intensity over the beam area.

It is well known from antenna theory that these parameters are related to each other in the simple manner[12]

$$A_e = \frac{\lambda^2}{4\pi} D_o \quad \text{and} \quad D_o = \frac{4\pi}{\Omega_a} \; . \qquad (69)$$

For example, if the receiving antenna is properly oriented for maximum response, the available noise power $P(\omega)$ can be given by

$$P(\omega) = A_e \bar{p}_o(\omega) \;, \qquad (70)$$

where $\bar{p}_o(\omega)$ is the time average Poynting vector at the position of the receiving antenna, and in the present discussion it must be given by Eq. 64.

In order to determine $\vec{\overline{p}}(\omega, x_\alpha)$ from Eq. 64, the source region V_s, which is determined by the beam area of the receiving antenna, must be specified and the integrand must be expressed as a function of conveniently chosen coordinate variables. Although the parameter $\ell_{\alpha\beta}$ was expressed in spherical coordinate variables (r, θ, φ) in the previous section it is not difficult to see that the integration can conveniently be introduced with respect to the solid angle, subtended at the observation point instead of carrying out the volume integration in a spherical coordinate system as in Eq. 64; this is illustrated in the following discussion.

Suppose that the observation point x_α is taken on the surface of the earth and the radial component of total noise power received by the antenna is sought. Then, from Eq. 64,

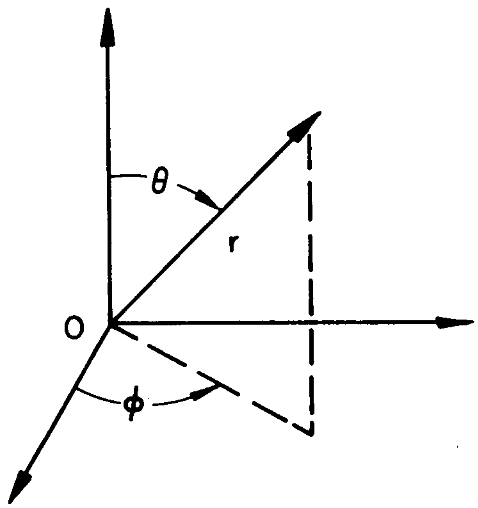
$$\begin{aligned}\overline{p}_r(\omega, x_\alpha) &= \vec{\overline{p}}(\omega, x_\alpha) \cdot \vec{u}_1(x_\alpha) \\ &= \frac{k_o \Delta f}{2\lambda^2} \int_{r_o}^{r_1} \int_{\Omega_o} (\vec{n} \cdot \vec{u}_1) \left[\frac{kT_o XZ}{1 + Z^2} \right] \frac{\Gamma}{R^2} \, r^2 d\Omega \, dr \quad , \end{aligned} \tag{71}$$

where $d\Omega = \sin\theta \, d\theta \, d\varphi$ is the differential solid angle subtended at the origin (the center of the earth) by a source located at x'_α and Ω_o is the solid angle subtended by the source region V_s at the origin.

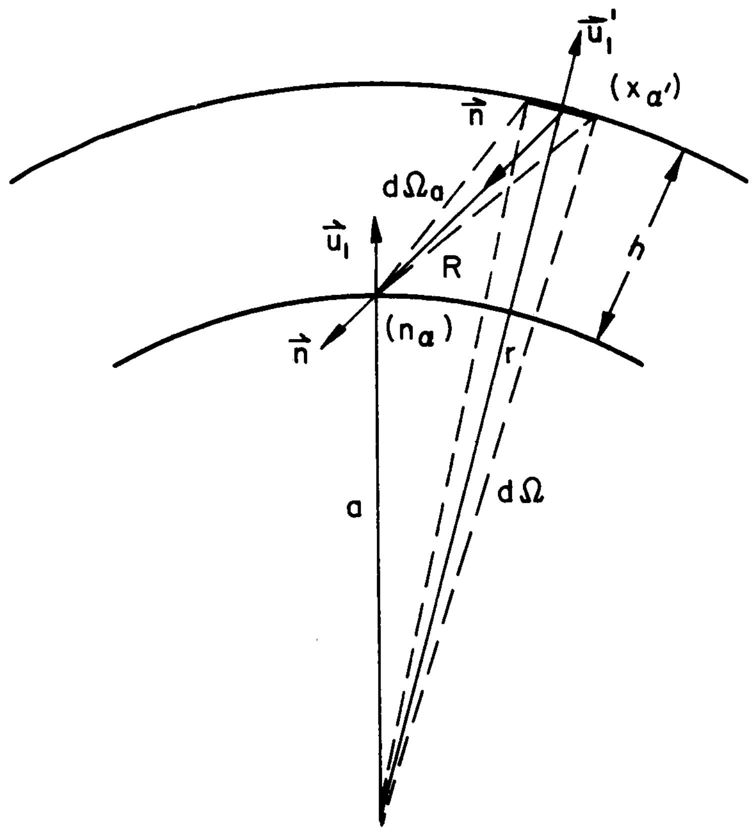
If $d\Omega_a$ is the differential solid angle subtended at the observation point x_α by the source located at x'_α, then it is not difficult to see that with the aid of Fig. 1,

$$\frac{R^2 d\Omega_a}{|(\vec{n} \cdot \vec{u}'_1)|} = r^2 d\Omega \quad , \tag{72}$$

where $\vec{u}'_1(x'_\alpha)$ is the radial unit vector at the source point. Consequently, Eq. 71 can be written as follows:



(a) Geomagnetic Spherical Coordinate System



(b) Geometrical Relation Between the Source Points x'_α and the Observation Points x_α

FIG. 1 COORDINATE SYSTEM AND DEFINITION OF VARIABLES.

$$\vec{p}_r(\omega,x_\alpha) = \frac{k_o \Delta f}{2\lambda^2} \int_{h_o}^{h_1} \int_{\Omega_a} y(x_\alpha, x'_\alpha) \left[\frac{kT_o XZ}{1 + Z}\right] \Gamma d\Omega_a dh \quad , \qquad (73)$$

where

$$y(x_\alpha, x'_\alpha) = \frac{\vec{n}(x_\alpha, x'_\alpha) \cdot \vec{u}_1(x_\alpha)}{|\vec{n}(x_\alpha, x'_\alpha) \cdot \vec{u}'_\alpha(x'_\alpha)|} = \frac{\cos \psi_o}{\cos \psi'_o} \qquad (74)$$

and Ω_a is the solid angle representing the beam area of the receiving antenna and $r = a + h$ is used in the derivation. The angles ψ_o and ψ'_o appearing in Eq. 74 are those between $\vec{n}$ and $\vec{u}_1$ and between $\vec{n}$ and $\vec{u}'_1$ respectively, and they are related geometrically as is shown in Fig. 1.

If an antenna of sufficiently small beam area is used for measurement, some approximation can be made in Eq. 73. That is to say, if Ω_a is sufficiently small, then the unit vector $\vec{n}(x_\alpha, x'_\alpha)$ may be considered as a constant vector within the solid angle Ω_a, and may be replaced by $\vec{n}(x_\alpha, q_\alpha)$, where q_α is the representative source point lying on the axis of Ω_a and the factor y given in Eq. 74 becomes independent of the source point x'_α also. Therefore, Eq. 73 becomes

$$\bar{p}(\omega,x_\alpha) = \frac{k_o \Delta f}{2\lambda^2} \Omega_a \bar{y} \int_{h_o}^{h_1} \left[\frac{kT_o XZ}{1 + Z^2}\right] \bar{\Gamma} \; dh \qquad (75)$$

and from Eq. 70, with the aid of Eqs. 69 and 75, the expression for the available noise power at the receiving antenna is

$$P_r(\omega) = \frac{k_o \Delta f}{2} \bar{y} \int_{h_o}^{h_1} \left[\frac{kT_o XZ}{1 + Z^2}\right] \bar{\Gamma} \, dh \quad , \qquad (76)$$

where

$$\bar{y} = \frac{\vec{\bar{n}} \cdot \vec{u}_1(x_\alpha)}{\vec{\bar{n}} \cdot \vec{u}'_1(q_\alpha)} \tag{77}$$

and

$$\bar{\Gamma} = (1 - \bar{n}_1^2)\ell_{11}(h) + (1 - \bar{n}_2^2)\ell_{22}(h)$$
$$+ (1 - \bar{n}_3^2)\ell_{33}(h) - 2\, n_1 n_2 \ell_{12}(h) \quad . \tag{78}$$

It is observed that for the case of a vertical incident measurement, $\vec{u}_1 = \vec{u}'_1$, $\bar{n}_1 = 1$ and $\bar{n}_2 = \bar{n}_3 = 0$, so that $\bar{\Gamma}_o = \ell_{22} + \ell_{33}$ and $\bar{y} = 1$. Consequently the available thermal noise power at the receiving antenna per unit bandwidth, $P_o(f)$, for the case of vertical incident measurement, may be given by

$$P_o(f) = \frac{\pi}{\lambda} \int_{h_o}^{h_1} \left[\frac{kT_o XZ}{1 + Z^2}\right] \bar{\Gamma}_o \, dh \quad . \tag{79}$$

It is interesting to note that for a special case Y = 0 (corresponding to the absence of a geomagnetic field), $\ell_{22} = \ell_{33} = 1$ and $\bar{\Gamma}_o = 2$. Furthermore, if $Z^2 \ll 1$, then Eq. 79 is reduced essentially to the same form as that used by many workers[1,2,3,4,13,14].

Having considered $w_o(f, x'_\alpha)$, the thermal noise power generated per unit volume per unit bandwidth, and the power flow density (Poynting vector) at the point of observation, the following natural question can now be asked: How efficiently is the generated thermal noise power in the ionosphere being converted into thermal electromagnetic wave energy, received at the detecting antenna on the ground?

One reasonable way to answer this question is as follows. Let p' be the power flow density observed at the observation point x_α,

$$\vec{p}_r(\omega, x_\alpha) = \frac{k_o \Delta f}{2\lambda^2} \int_{h_o}^{h_1} \int_{\Omega_a} y(x_\alpha, x_\alpha') \left[\frac{kT_o XZ}{1 + Z}\right] \Gamma d\Omega_a dh \quad , \tag{73}$$

where

$$y(x_\alpha, x_\alpha') = \frac{\vec{n}(x_\alpha, x_\alpha') \cdot \vec{u}_1(x_\alpha)}{|\vec{n}(x_\alpha, x_\alpha') \cdot \vec{u}_\alpha'(x_\alpha')|} = \frac{\cos \psi_o}{\cos \psi_o'} \tag{74}$$

and Ω_a is the solid angle representing the beam area of the receiving antenna and $r = a + h$ is used in the derivation. The angles ψ_o and ψ_o' appearing in Eq. 74 are those between $\vec{n}$ and $\vec{u}_1$ and between $\vec{n}$ and $\vec{u}_1'$ respectively, and they are related geometrically as is shown in Fig. 1.

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$$\bar{p}(\omega, x_\alpha) = \frac{k_o \Delta f}{2\lambda^2} \Omega_a \bar{y} \int_{h_o}^{h_1} \left[\frac{kT_o XZ}{1 + Z^2}\right] \bar{\Gamma} \; dh \tag{75}$$

and from Eq. 70, with the aid of Eqs. 69 and 75, the expression for the available noise power at the receiving antenna is

$$P_r(\omega) = \frac{k_o \Delta f}{2} \bar{y} \int_{h_o}^{h_1} \left[\frac{kT_o XZ}{1 + Z^2}\right] \bar{\Gamma} \, dh \quad , \tag{76}$$

where

$$\bar{y} = \frac{\vec{\bar{n}} \cdot \vec{u}_1(x_\alpha)}{\vec{\bar{n}} \cdot \vec{u}_1'(q_\alpha)} \tag{77}$$

and

$$\bar{\Gamma} = (1 - \bar{n}_1^2)\ell_{11}(h) + (1 - \bar{n}_2^2)\ell_{22}(h)$$
$$+ (1 - \bar{n}_3^2)\ell_{33}(h) - 2\, n_1 n_2 \ell_{12}(h) \quad . \tag{78}$$

It is observed that for the case of a vertical incident measurement, $\vec{u}_1 = \vec{u}_1'$, $\bar{n}_1 = 1$ and $\bar{n}_2 = \bar{n}_3 = 0$, so that $\bar{\Gamma}_o = \ell_{22} + \ell_{33}$ and $\bar{y} = 1$. Consequently the available thermal noise power at the receiving antenna per unit bandwidth, $P_o(f)$, for the case of vertical incident measurement, may be given by

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It is interesting to note that for a special case $Y = 0$ (corresponding to the absence of a geomagnetic field), $\ell_{22} = \ell_{33} = 1$ and $\bar{\Gamma}_o = 2$. Furthermore, if $Z^2 \ll 1$, then Eq. 79 is reduced essentially to the same form as that used by many workers[1,2,3,4,13,14].

Having considered $w_o(f, x_\alpha')$, the thermal noise power generated per unit volume per unit bandwidth, and the power flow density (Poynting vector) at the point of observation, the following natural question can now be asked: How efficiently is the generated thermal noise power in the ionosphere being converted into thermal electromagnetic wave energy, received at the detecting antenna on the ground?

One reasonable way to answer this question is as follows. Let p' be the power flow density observed at the observation point x_α, per unit bandwidth in the case of a vertical incidence due to the sources occupying a unit volume, located in the neighborhood of x'_α. By taking V_s to be a unit volume in Eq. 64, with the aid of Eq. 65, p' can be given as

$$p' = \frac{k_o}{2\lambda^2}\left[\frac{kT_oXZ}{1+Z^2}\right]\left(\frac{\ell_{22}+\ell_{33}}{R^2}\right) . \tag{80}$$

On the other hand, let p" be the power flow density which would be expected at x_α if an equivalent point source, radiating isotropically with the power $w_o(f, x'_\alpha)$, were placed at x'_α. With the aid of Eq. 50, p" can be expressed as

$$p'' = \frac{w_o(f,x'_\alpha)}{4\pi R^2} = \frac{1}{4\pi}\left[\frac{kT_oZ^2}{1+Z^2}\right]\left(\frac{\ell_{11}+\ell_{22}+\ell_{33}}{R^2}\right) . \tag{81}$$

Then by examining the ratio $\eta = (p'/p'')$, given by

$$\eta = \frac{p'}{p''} = \frac{1}{2\pi}\left(\frac{\omega}{c}\right)^3\left(\frac{X}{Z}\right)\left[\frac{\ell_{22}+\ell_{33}}{\ell_{11}+\ell_{22}+\ell_{33}}\right] , \tag{82}$$

where c is the speed of light in vacuum, the desired information should be obtainable.

It should be pointed out that η can be considered as the efficiency of conversion as long as it has a value less than unity. However, for a certain frequency range, particularly in the microwave range, it is possible that η may be greater than unity, in which case η must be looked upon as the directivity of a nonisotropic radiating system located in the ionosphere rather than as the efficiency of conversion.

V. PARTICULAR CASES

It is not difficult to see that when $Z^2 \ll 1$, say $Z \leq 1/10$, considerable simplification results in the evaluations of the various quantities discussed in the previous sections, such as $w_o(f,x'_\alpha)$, $W(f,V_s)$, $\overline{p}_r(\omega,x_\alpha)$, $P_o(f)$ and η. As an illustration, the following special cases are considered here:

$$\underline{\text{Case I}}: \quad Z^2 \ll 1 \ , \quad 1 \ll G^2 \ ,$$

$$\underline{\text{Case II}}: \quad Z^2 \ll 1 \ , \quad 3\,G^2 \ll 1 \ ,$$

$$\underline{\text{Case III}}: \quad 4\,Z^2 \ll 1 \ , \quad 3 \ll G^2 \ ,$$

$$\underline{\text{Case IV}}: \quad 4\,Z^2 \ll 1 \ , \quad 4\,G^2 \ll 1 \ . \tag{83}$$

For convenience, the subscripts 1, 2, 3 and 4 are introduced in the quantities W, P_o, w_o and η to indicate the fact that Cases I, II, III and IV are being considered, e.g., W_1 denotes that W for Case I is being considered, etc.

When $Z^2 \ll 1$, the factor $\Theta(Z,u)$ given in Eq. 53 is reduced to a much simpler form so that the integration of Θ appearing in Eq. 52 can be carried out without the aid of any numerical method as shown in the appendix. Consequently, Eq. 51 becomes

$$W(f,V_s) = (\varphi_1 - \varphi_o) \int_{r_o}^{r_1} kT_o Z^2 \Omega^2 (\cos\theta_o - \cos\theta_1 + 2I_o)\, dr \ , \tag{84}$$

where

$$I_o(\theta_o,\theta_1) = \int_{u_1}^{u_o} \Theta(Z^2 \ll 1, u)\, du \tag{85}$$

with

$$u_o = \cos\theta_o \quad \text{and} \quad u_1 = \cos\theta_1 \ .$$

Suppose that the amount of radiation from a layer, i.e., a region consisting of a spherical shell of thickness Δh, is to be determined;

then $\varphi_o = 0$, $\varphi_1 = 2\pi$, $\theta_o = 0$, $\theta_1 = \pi$, $r_1 = r_o + \Delta h$ and $r_o = a + h$ can be taken. If Δh is sufficiently small so that T_o and Z can be considered invariant with respect to h over Δh, then from Eq. 84,

$$W_1(f,h) = V_o k T_o(h) \; Z_1^2(h,f) \tag{86}$$

and

$$W_2(f,h) = 3 V_o k T_o(h) \; Z_2^2(h,f) \quad , \tag{87}$$

where

$$V_o = \frac{4\pi}{3} (r_1^3 - r_o^3) \simeq 4\pi \; r_o^2 \, \Delta h \quad . \tag{88}$$

On the other hand, if $4 Z^2 \ll 1$, say $2 Z \leq 1/10$, then Eq. 79 becomes

$$P_o(f) = \frac{\pi}{\lambda} \int_{h_o}^{h_o+\Delta h} k T_o X Z \; \xi(\theta,h) \; dh \quad , \tag{89}$$

where

$$\xi(\theta,h) = \overline{\Gamma}_o \; (4 Z^2 \ll 1,h) \quad . \tag{90}$$

Once again, if Δh is sufficiently small, Eq. 89 gives

$$P_{o3}(f) = \xi_o(\theta,h_o) \; k T_o X_3 Z_3 \left(\frac{\Delta h}{\lambda_3}\right) \pi \tag{91}$$

and

$$P_{o4}(f) = 2 k T_o X_4 Z_4 \left(\frac{\Delta h}{\lambda_4}\right) \pi \quad , \tag{92}$$

where

$$\xi_o(\theta,h_o) = \frac{2 + G^2 \sin^2\theta}{G^2 \, (1 + 3 \cos^2\theta)} \quad ; \quad 3 \ll G^2 \quad . \tag{93}$$

Similarly, it is not difficult to see that Eq. 50 gives the following expressions, with the aid of Eq. 40:

$$w_{o3}(f,h) = kT_o Z_3^2 \tag{94}$$

and

$$w_{o4}(f,h) = 3kT_o Z_4^2 , \tag{95}$$

while Eq. 82 gives

$$\eta_3 = \frac{1}{2\pi}\left(\frac{\omega}{c}\right)^2\left(\frac{\omega_p}{c}\right)\left(\frac{\omega_p}{\nu}\right) \xi_o(\theta) \tag{96}$$

and

$$\eta_4 = \frac{1}{2\pi}\left(\frac{\omega}{c}\right)^2\left(\frac{\omega_p}{c}\right)\left(\frac{\omega_p}{\nu}\right) \frac{2}{3} . \tag{97}$$

Before discussing the significance of the above obtained simplified results, it should be noted that the factor $G(\omega,r)$ given in Eq. 19 can be approximated in the following manner. Since the radius of the earth a is approximately 6370 km and the magnetization[8] M is $0.935/4\pi$ amp/m^2 for 1945 for an altitude h (with $r = a + h$) up to about 200 km, the ratio h/a is much smaller than unity so that

$$G(\omega,r) \simeq \frac{6.95 \times 10^8}{f}\left[1 - 3\left(\frac{h}{a}\right)\right] . \tag{98}$$

This suggests that for a height up to about 200 km, $G = Y(\theta = \pi/2)$ is practically invariant with respect to h. On the other hand, the collision frequency $\nu(h)$ varies considerably with the height h in this same range of height[15]. For example, when h equals 50, 70, 80, 85, 90, 100 and 150 km, ν takes on values of 100, 20, 2, 0.65, 0.2, 0.05 and 0.001 mc per second respectively, and above 150 kv ν varies very little. Consequently, the condition $(\nu/\omega_b)^2 \ll 1$ occurs in the range of height up to about 200 km. As for the electron density $N_o(h)$ and the plasma frequency $\omega_p(h)$, at h = 80 km, $N_o = 3 \times 10^8$ per m^3 and $\omega_p = 1$ mc; at h = 90 km, $N_o = 3 \times 10^{10}$ per m^3 and $\omega_p = 10$ mc.

Furthermore it should be noted that since $Z(h,f)$ is defined as $\nu(h)/\omega$, the range of radiation frequency in which the condition $Z^2(h,f) \ll 1$, say $Z(h) \leq 1/10$, is satisfied depends upon the height h. For example, it is 3.14 mc $\leq f$ at $h = 80$ km and 0.314 mc $\leq f$ at $h = 90$ km. Whereas the condition $1 \ll G^2$, say $10 \leq G$, is satisfied when $f \leq 70$ mc and the condition $3\, G^2 \ll 1$ is satisfied when 12 kmc $\leq f$ for the range of height up to about 200 km, since G is practically constant.

Since Cases III and IV are subcases of I and II respectively, the attention is focused here on Cases III and IV. That is to say the remarks made on Case III apply to Case I as well, and similarly, the remarks on Case IV can be applied to Case II.

In the region of the ionosphere between 85 km and 200 km the frequency ranges specified by Cases III and IV now can be given as follows:

<u>Case III (and Case I)</u>: 1.6 mc $\leq f \leq 42$ mc

[r-f noise] ,

<u>Case IV (and Case II)</u>: 14 kmc $\leq f$

[microwave noise] .

Having established the correspondence between the ranges of parameters Z and G, and the ranges of height and radiation frequency, some meaningful interpretations can be given to the simplified results obtained earlier.

Equation 91 suggests that the r-f noise power available at the antenna $P_{o3}(f)$ depends upon the polar angle θ through the function $\xi_o(\theta)$ given in Eq. 93, which has been plotted and shown in Fig. 2. Since $\xi_o(\theta)$ increases with θ from $\xi_o(0) = 1/(2G^2) \ll 1$ to $\xi_o(\pi/2) = 1$,

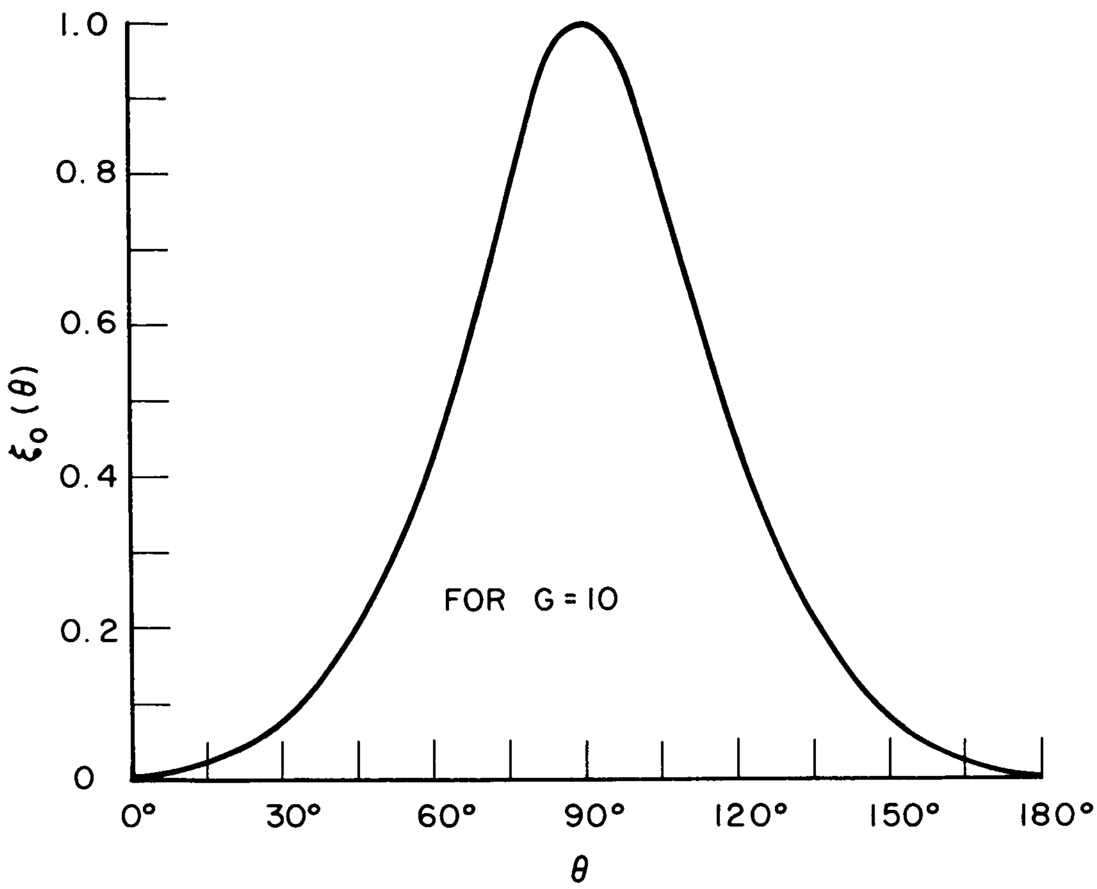

FIG. 2 THE FUNCTION $\xi_0(\theta)$, DEFINED BY EQ. 93, VERSUS THE THE POLAR ANGLE θ.

it can be said that the power level of r-f noise signal received at the antenna is lower in the polar cap zone ($\theta \simeq 0$) and is higher in the equatorial zone ($\theta \simeq \pi/2$). On the other hand, Eq. 92 suggests that the microwave noise power received does not depend upon the polar angle θ.

Equation 96 suggests that the efficiency of conversion, defined in Section IV, for the r-f noise does depend upon the polar angle θ and also upon the function $\xi_o(\theta)$. It implies that the efficiency is lower in the region near the polar cap ($\theta \simeq 0$) and is higher in the equatorial zone ($\theta \simeq \pi/2$). This, in turn, can be interpreted as follows: there is a larger fraction of the generated r-f noise power in the ionosphere at a given height, stored in the quasi-stationary fields in the region near the sources, in the polar cap region than in the equatorial zone.

However, Eq. 97 suggests that no such polar angle dependence exists for the microwave noise. For example, at h $\simeq$ 90 km, since $(\omega_p/c) \simeq 0.03$ and $(\omega_p/\nu) \simeq 50$, $\eta_3 \simeq 0.057\ \xi_o(\theta)$ and $\eta_4 \simeq 0.38$, which implies that about 38 percent of the microwave noise generated at 90 km in height reaches the receiving antenna whereas, at most, only about 6 percent of the r-f noise generated at the same height reaches the antenna.

VI. CONCLUSIONS

The attention has been focused in the present study on the effect of colliding electrons under the assumption that the effect of the motion of ions in the region of the ionosphere of interest is negligible.

The general expressions derived for $W(f,V_s)$, the thermal noise power generated per unit bandwidth from any given source region V_s of the ionosphere, and for $P_o(f)$, the available thermal power per unit bandwidth received at the detecting antenna due to the radiation from V_s, are valid for all frequency ranges and for most regions of interest in the ionosphere, i.e., where the electron collision process plays a major role. Once V_s is specified, the profiles of $T_o(h)$, $N_o(h)$ and $\nu(h)$ obtained from the experimental observations[16,17,18,19,20,21] can be used for the evaluation of $W(f,V_s)$ and $P_o(f)$. Thus the detailed information with regard to the spectral distribution of the thermal energy radiated from the ionosphere can be obtained with the aid of a numerical integration of the expressions derived in the present report. However, it has been demonstrated in Section V that several interesting conclusions can be drawn from even a simple consideration of some special cases.

It is suggested by Eq. 96 that a larger fraction of the r-f thermal noise power generated at a given height in the ionosphere can be converted into r-f thermal noise signals which can reach the receiving antenna in the equatorial zone ($\theta \simeq \pi/2$) than in the polar cap zone ($\theta \simeq 0$). In other words, a smaller fraction of the generated energy can be stored in the region near the source in the equatorial zone than in the polar cap zone. It should be noted that this theoretical observation does not contradict the experimentally[13,14] observed fact, in the ionospheric radio wave absorption measurement, that absorption is higher in the polar cap zone than in the equatorial zone.

It is of interest to note that, as suggested by Eq. 91, the r-f noise power available at the receiving antenna does depend upon the geographical location of the detecting antenna and that the power level is higher in the equatorial zone than in the polar cap zone.

It is indeed desirable that the present theory be tested and verified with some sort of experimental observation, e.g., a laboratory experiment. In other words, if the ionospheric plasma condition can be realistically represented with a laboratory experiment, then it will permit a study of the characteristics of thermal radiation in great detail and a test of the soundness of the present theory.

It should be pointed out that the present analysis may not be as rigorous as a microscopic treatment using the Boltzmann transport equation with the proper collision integral. However, this method of analysis does offer a simple and direct way of analyzing the thermal radiation from an anisotropic ionized medium and it radiation characteristics.

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APPENDIX A

DERIVATION OF VARIOUS EQUATIONS

1. Derivation of Eq. 40

Since $\tilde{L}_{\alpha\beta}$ is defined as

$$\tilde{L}_{\alpha\beta} = 2\left[\tilde{\sigma}_{\alpha\beta} + \tilde{\sigma}^*_{\beta\alpha}\right] = 2\left[\tilde{D}\,\tilde{C}_{\alpha\beta} + \tilde{D}^*\,\tilde{C}^*_{\beta\alpha}\right] , \qquad (A.1)$$

where the symbol ~ denotes the complex quantity, by letting $\tilde{D} = c_o + j\,d_o$ and $\tilde{C}_{\alpha\beta} = a_{\alpha\beta} + jb_{\alpha\beta}$ in Eq. A.1

$$\tilde{L}_{\alpha\beta} = 2\,[\{c_o(a_{\alpha\beta} + a_{\beta\alpha}) - d_o(b_{\alpha\beta} + b_{\beta\alpha})\}$$

$$+ j\,\{d_o(a_{\alpha\beta} - a_{\beta\alpha}) + c_o(b_{\alpha\beta} - b_{\beta\alpha})\}] \; . \qquad (A.2)$$

On the other hand, $\tilde{D}$ is given by Eq. 22, and its real part c_o and imaginary part d_o are given by

$$c_o = \frac{\omega\epsilon_o XZ\,(Z^2 + Y^2 - 3)}{(1 + Z^2)\,Q}$$

and

$$d_o = \frac{\omega\epsilon_o X\,(1 - 3Z^2 - Y^2)}{(1 + Z^2)\,Q} , \qquad (A.3)$$

where

$$Q = (Y^2 + Z^2 - 1)^2 + 4\,Z^2 \; .$$

$\tilde{L}_{\alpha\beta}$ can be arranged into the following convenient form:

$$\tilde{L}_{\alpha\beta} = 4\omega\epsilon_o(XZ/1 + Z^2)\,[\ell_{\alpha\beta} + jm_{\alpha\beta}] \; , \quad \alpha,\beta = 1, 2, 3 \; , \qquad (A.4)$$

where

$$\ell_{\alpha\beta} = \frac{1}{2Q}\left[\{(Z^2 + Y^2 - 3)\ (a_{\alpha\beta} + a_{\beta\alpha})\} - \frac{1}{Z}\{(1 - 3Z^2 - Y^2)(b_{\alpha\beta} + b_{\beta\alpha})\}\right] \tag{A.5}$$

and

$$m_{\alpha\beta} = \frac{1}{2Q}\left[(1/Z)\{(1 - 3Z^2 - Y^2)(a_{\alpha\beta} - a_{\beta\alpha})\}\right.$$

$$\left. + \{(Z^2 + Y^2 - 3)(b_{\alpha\beta} - b_{\beta\alpha})\}\right] \ , \tag{A.6}$$

where $a_{\alpha\beta}$ and $b_{\alpha\beta}$ for α and β = 1, 2, 3 are given in Eq. 24. Upon substituting $a_{\alpha\beta}$ and $b_{\alpha\beta}$ into Eqs. A.5 and A.6, Eq. 40 is obtained.

2. Derivation of Eq. 49

Since the tensor (or matrix) $\{\gamma_{\alpha\beta}\}$, given by Eq. 20, can be expressed as the sum of a symmetric tensor (or matrix) $\{S_{\alpha\beta}\}$ and an antisymmetric tensor (or matrix) $\{T_{\alpha\beta}\}$

$$\{\gamma_{\alpha\beta}\} = \{S_{\alpha\beta}\} + \{T_{\alpha\beta}\} \ , \quad \alpha,\beta = 1, 2, 3 \ , \tag{A.7}$$

where

$$\{S_{\alpha\beta}\} = \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix} \text{ and } \{T_{\alpha\beta}\} = \begin{pmatrix} 0 & 0 & \gamma_{13} \\ 0 & 0 & \gamma_{23} \\ -\gamma_{13} & -\gamma_{23} & 0 \end{pmatrix}$$

with

$$\gamma = \frac{Z + j}{\omega\epsilon_o X} \ , \quad \gamma_{13} = \frac{G \sin\theta}{\omega\epsilon_o X} \text{ and } \gamma_{23} = \frac{-\ 2\ G\cos\theta}{\omega\epsilon_o\ X} \ , \tag{A.8}$$

it follows that

$$\underline{K}^+ \underline{\underline{\gamma}}\, \underline{K} = \underline{K}^+ \underline{\underline{S}}\, \underline{K} + \underline{K}^+ \underline{\underline{T}}\, \underline{K} = \frac{Z + j}{\omega\epsilon_o X} [K_1 K_1^* + K_2 K_2^* + K_3 K_3^*]$$

$$+ \{\gamma_{13} (K_1^* K_3 - K_1 K_3^*) + \gamma_{23} (K_2^* K_3 - K_2 K_3^*)\} \quad . \tag{A.9}$$

By noting that the quantity inside the curly bracket appearing on the right-hand side of Eq. A.9 is purely imaginary, since γ_{13} and γ_{23} are real,

$$\frac{1}{2} \text{Re} [\underline{K}^+ \underline{\underline{\gamma}}\, \underline{K}] = \frac{Z}{2\omega\epsilon_o X} [K_1 K_1^* + K_2 K_2^* + K_3 K_3^*] \quad . \tag{A.10}$$

3. Derivation of Eq. 51

For convenience, let L be defined as

$$L = \ell_{11} + \ell_{22} + \ell_{33} \quad ; \tag{A.11}$$

then it is not difficult to show with the aid of Eq. 40 that

$$L = 1 + 2\ell_{33} \quad , \tag{A.12}$$

where $\ell_{33}(Y,Z)$ is given in Eq. 40. Since only the parameter Y depends upon θ, in order to facilitate the integration with respect to θ, ℓ_{33} can be expressed conveniently as

$$\ell_{33}(Y,Z) = (Z^2 + 1)\, \Theta\, (Y,Z) \quad , \tag{A.13}$$

where

$$\Theta(Y,Z) = \frac{(1 + Z^2 + Y^2)}{Y^4 + 2(Z^2 - 1)\, Y^2 + (Z^2 + 1)^2} \quad . \tag{A.14}$$

The volume integration of $w_o(f,\bar{r})$, given by Eq. 50 with the aid of Eqs. A.12 and A.13, yields Eq. 51. The substitution of Eq. 23 into A.14, with $u = \cos\theta$, gives Eqs. 53 and 54.

4. Derivation of Eqs. 60 and 61

From Eqs. 57 and 58

$$\vec{E} = \frac{1}{j\,\omega\,\mu_o\,\epsilon_o}\nabla(\nabla\cdot\vec{A}) - j\omega\vec{A} \quad . \tag{A.15}$$

The fact that

$$\nabla\left(\frac{1}{R}e^{-jk_oR}\right) = -\vec{R}\left[\frac{1}{R^2} - \frac{jk_o}{R}\right]\frac{e^{-jk_oR}}{R} \quad ,$$

yields

$$\nabla\cdot\vec{A} = \frac{\mu_o}{4\pi}\int_{V_s}\vec{K}\cdot\nabla\left(\frac{1}{R}e^{-jk_oR}\right)dV'$$

$$= \frac{-\mu_o}{4\pi}\int_{V_s}(\vec{K}\cdot\vec{R})\left[\frac{1}{R^2} + \frac{jk_o}{R}\right]\frac{e^{-jk_oR}}{R}dV'$$

and

$$\nabla(\nabla\cdot\vec{A}) = \frac{\mu_o}{4\pi}\int_{V_s}\left[\vec{K}\left(\frac{-1}{R^2} - \frac{jk_o}{R}\right)\right.$$

$$\left. + (\vec{K}\cdot\vec{R})\,\vec{R}\left\{\frac{3}{R^4} + \frac{j3k_o}{R^3} - \frac{k_o^2}{R^2}\right\}\right]\frac{e^{-jk_oR}}{R}dV' \quad . \tag{A.16}$$

The substitution of Eqs. 55 and A.16 into Eq. A.5 gives

$$\vec{E} = \frac{1}{j\,\omega\,\epsilon_o}\,\frac{1}{4\pi}\int_{V_s}\left[\vec{K}\left\{k_o^2 - \frac{j\,k_o}{R} - \frac{1}{R^2}\right\}\right.$$

$$\left. - (\vec{K}\cdot\vec{R})\,\vec{R}\left\{\frac{k_o^2}{R^2} - \frac{j3k_o}{R^3} - \frac{3}{R^4}\right\}\right]\frac{e^{-jk_oR}}{R}dV' \quad . \tag{A.17}$$

On the other hand, from Eqs. 55 and 59

$$\vec{H} = \frac{1}{4\pi} \int_{V_s} \nabla \times \left(\vec{K} \frac{e^{-jk_oR}}{R} \right) dV'$$

$$= \frac{1}{4\pi} \int_{V_s} (\vec{K} \times \vec{R}) \left[\frac{j\, k_o}{R} + \frac{1}{R^2} \right] \frac{e^{-jk_oR}}{R} dV' \quad . \qquad (A.18)$$

5. Derivation of Eq. 64

If only the $1/R$ dependent radiation field is taken into account, from Eqs. A.17 and A.18 the following can be derived:

$$\vec{E}(x_\alpha) = \frac{1}{j\omega\epsilon} \frac{1}{4\pi} \int_{V_s} [\vec{k}' \times \{\vec{K}' (x'_\alpha) \times \vec{k}'\}] \frac{e^{-j\vec{k}' \cdot \vec{R}'}}{R'(x_\alpha, x'_\alpha)} dV' \qquad (A.19)$$

and

$$\vec{H}(x_\alpha) = \frac{j}{4\pi} \int_{V_s} \{\vec{K}''(x''_\alpha) \times \vec{k}''\} \frac{e^{-j\vec{k}'' \cdot \vec{R}''}}{R''(x_\alpha, x''_\alpha)} dV'' \quad , \qquad (A.20)$$

where $\vec{k}' = \vec{n}'\, k_o$ and $\vec{k}'' = \vec{n}''\, k_o$, with $\vec{n}'$ and $\vec{n}''$ being the unit vectors directed from the source points x'_α and x''_α to the point of observation x_α, respectively. The quantities with a prime denote those associated with a source located at the point x'_α and those with double primes denote association with a source at the point x''_α.

Then, from Eqs. 19 and 20, the complex Poynting vector can be given as follows:

$$\frac{1}{2}\,[\vec{E}(x_\alpha)\times\vec{H}^*(x_\alpha)]$$

$$= \frac{k_o^2}{(4\pi)^2}\,\frac{\sqrt{\mu_o/\epsilon_o}}{2}\int_{V_s}\int_{V_s}\vec{M}(x_\alpha,x_\alpha',x_\alpha'')\,\frac{e^{jk_o(\vec{n}''\cdot\vec{R}''-\vec{n}'\cdot\vec{R}')}}{R'\,R''}\,dV'\,dV'' \quad , \tag{A.21}$$

where

$$\vec{M}(x_\alpha,x_\alpha',x_\alpha'') = [\vec{K}''^*\times\vec{n}'']\times[\vec{n}'\times(\vec{K}'\times\vec{n}')] \quad . \tag{A.22}$$

With the aid of the vector algebraic identities, it is not difficult to show that $\vec{M}$ is expressible as

$$\vec{M} = \{(\vec{n}'\cdot\vec{K}')(\vec{n}'\cdot\vec{n}'')\}\,\vec{K}''^* - (\vec{K}'\cdot\vec{n}'')\,\vec{K}''^*$$

$$- \{(\vec{n}'\cdot\vec{K}')(\vec{n}'\cdot\vec{K}''^*)\}\,\vec{n}'' + (\vec{K}'\cdot\vec{K}''^*)\,\vec{n}'' \quad . \tag{A.23}$$

It should be noted that each term on the right-hand side of Eq. A.23 contains a factor of the form $(K_\alpha'\,K_\beta''^*)$ with $\alpha,\beta = 1, 2, 3$. It has been pointed out in connection with Eq. 56 that since K_α' contains the Dirac delta function of the form $\delta(x_\alpha - x_\alpha')$ and K_α'' contains $\delta(x_\alpha - x_\alpha'')$, $(K_\alpha'\,K_\beta'')$ is proportional to $\delta(x_\alpha - x_\alpha')\,\delta(x_\alpha - x_\alpha'')$. This is consistent with the discussion given in Section III with respect to the spatial correlation of the source current. Consequently, $x_\alpha' = x_\alpha''$, $\vec{n}' = \vec{n}'' = \vec{n}$ and $R' = R'' = R(x_\alpha,x_\alpha')$ can be set in the integral of Eq. A.21, with the result

$$\frac{1}{2}\,[\vec{E}(x_\alpha)\times\vec{H}^*(x_\alpha)] = \frac{k_o^2}{(4\pi)^2}\,\frac{\sqrt{\mu_o/\epsilon_o}}{2}\int_{V_s}\vec{n}(x_\alpha,x_\alpha')\,\frac{M_o(x_\alpha,x_\alpha')}{R^2(x_\alpha,x_\alpha')}\,dV' \quad , \tag{A.24}$$

where

$$M_o(x_\alpha, x'_\alpha) = K(x'_\alpha)\ K^*(x'_\alpha) - [\vec{n}(x_\alpha, x'_\alpha) \cdot \vec{K}(x'_\alpha)][\vec{n}(x_\alpha, x'_\alpha) \cdot \vec{K}^*(x'_\alpha)] \quad . \tag{A.25}$$

By letting

$$\vec{n} = \vec{u}_1\, n_1 + \vec{u}_2\, n_2 + \vec{u}_3\, n_3$$

and

$$\vec{K} = \vec{u}_1\, K_1 + \vec{u}_2\, K_2 + \vec{u}_3\, K_3 \quad , \tag{A.26}$$

M_o becomes

$$M_o = (1 - n_1^2)[K_1\ K_1^*] + (1 - n_2^2)\ [K_2\ K_2^*] + (1 - n_3^2)[K_3\ K_3^*]$$

$$- n_1\, n_2\, [K_1\ K_2^* + K_1^*\ K_2] - n_1\, n_3\, [K_1\ K_3^* + K_1^*\ K_3] - n_2\, n_3 [K_2\ K_3^* + K_3^*\ K_2] \quad , \tag{A.27}$$

which can also be conveniently written in matrix notation as

$$M_o = \underline{K}^+\ \underline{\underline{\Omega}}\ \underline{K} \quad , \tag{A.28}$$

where $\underline{K}$ is a column matrix, $\underline{K}^+$ a row matrix and $\underline{\underline{\Omega}}$ is a square real symmetric matrix whose elements are given by

$$\Omega_{\alpha\beta} = (\delta_{\alpha\beta} - n_\alpha\, n_\beta) \quad , \quad \alpha, \beta = 1,\ 2,\ 3 \quad , \tag{A.29}$$

where $\delta_{\alpha\beta}$ is the usual Kronecker delta.

Now, with the aid of Eq. 38, from A.27,

$$< M_o >_{avg} = \frac{1}{2}\, \tau\, \Delta f \left[\left(1 - n_1^2\right) L_{11} + \left(1 - n_2^2\right) L_{22} + \left(1 - n_3^2\right) L_{33} \right.$$

$$\left. - n_1\, n_2 \left(L_{12} + L_{21}\right) - n_1\, n_3 \left(L_{13} + L_{31}\right) - n_2\, n_3 \left(L_{23} + L_{32}\right) \right] \quad , \tag{A.30}$$

which can be further written with the aid of Eqs. 39 and 40 as

$$< M_o >_{avg} = 2\omega\epsilon_o \left(\frac{XZ}{1 + Z^2}\right) \tau \, \Delta f \, \Gamma \quad , \qquad \text{(A.31)}$$

where

$$\Gamma(x_\alpha, x'_\alpha) = \left[\left(1 - n_1^2\right) \ell_{11} + \left(1 - n_2^2\right) \ell_{22} + \left(1 - n_3^2\right) \ell_{33} - 2n_1 \, n_2 \, \ell_{12}\right] \quad . \qquad \text{(A.32)}$$

In view of the fact that $< M_o >_{avg} = (1/2)\ \mathrm{Re}\ (M_o)$, and $\vec{n}$ and R are real, the time average Poynting vector $\vec{p}(x_\alpha)$ can be given from Eqs. A.24 and A.31 as

$$\vec{p}(x_\alpha) = \frac{1}{2} \mathrm{Re}\ [\vec{E} \times \vec{H}^*]$$

$$= \frac{k_o \, \Delta f}{2\lambda^2} \int_{V_s} \vec{n}(x \ , x') \left[\frac{\tau \ X \ Z}{1 + Z^2}\right] \frac{\Gamma(x_\alpha, x'_\alpha)}{R^2(x_\alpha, x'_\alpha)} \, dV' \quad , \qquad \text{(A.33)}$$

where λ is the free space wavelength, Z, X and $\tau = k \, T_o$, with T_o being the electron temperature in the source region, are the functions of x'_α and $k_o = 2\pi/\lambda$ is used.

6. Evaluation of the Integral $I_o(\theta_o, \theta_1)$ Defined in Eq. 85

Note that $I_o(\theta_o, \theta_1)$ is defined as

$$I_o(\theta_o, \theta_1) = \int_{u_1}^{u_o} \theta(Z^2 \ll 1, u) \, du \ ; \quad u_o = \cos \theta_o \quad \text{and} \quad u_1 = \cos \theta_1 \qquad \text{(A.34)}$$

when $Z^2 \ll 1$, $\theta(Z,u)$ given by Eq. 53 is reduced to the following form:

$$\theta_o(u) = \theta(Z^2 \ll 1, u)$$

$$= \frac{1}{3\,G^2}\,\frac{1}{(u^2 + a_o^2)} + \frac{2}{(3\,G^2)^2}\,\frac{1}{(u^2 + a_o^2)^2} \quad ,$$

where

$$a_o^2 = \frac{(G^2 - 1)}{3G^2} \quad . \tag{A.35}$$

Then the integral I_o can be evaluated readily with the aid of a standard table of integrals[22].

Case 1

$1 < G \quad ; \quad \left(0 < a_o^2\right)$ [using 120.1 and 120.2 of Reference 22]

$$I_o(\theta_o,\theta_1) = \frac{1}{(3\,G^2)}\,\frac{1}{(G^2 - 1)}\left[\frac{G^2}{a_o}\left\{\tan^{-1}\left(\frac{u_o}{a_o}\right) - \tan^{-1}\left(\frac{u_1}{a_o}\right)\right\} + \left\{\frac{u_o}{u_o^2 + a_o^2} - \frac{u_1}{u_1^2 + a_o^2}\right\}\right] . \tag{A.36}$$

Case 2

$G < 1 \quad ; \quad \left(a_o^2 < 0\right) \quad$ or $\quad \left(b_o^2 = -a_o^2 > 0\right)$

[using 140.1 and 140.2 of Reference 22] ,

$$I_o(\theta_o,\theta_1) = \frac{1}{(3\,G^2)}\,\frac{1}{(1 - G^2)}\left[\frac{G^2}{2b_o}\log\left\{\left|\frac{b_o + u_o}{b_o - u_o}\right|\left|\frac{b_o - u_1}{b_o + u_1}\right|\right\} + \left\{\frac{u_o}{b_o^2 - u_o^2} - \frac{u_1}{b_o^2 - u_1^2}\right\}\right] ,$$

where

$$b_o^2 = \frac{(1 - G^2)}{3G^2} \quad . \tag{A.37}$$

Case 3

$$G = 1 \; ; \; a_o = 0$$

from A.35, so that

$$\theta_o(u) = \frac{1}{3}\frac{1}{u^2} + \frac{2}{9}\frac{1}{u^4}$$

and

$$I_o = \frac{1}{3}\left(\frac{1}{u_1} - \frac{1}{u_o}\right) + \frac{2}{27}\left(\frac{1}{u_1^3} - \frac{1}{u_o^3}\right) \tag{A.38}$$

provided that the interval (u_o, u_1) does not contain the point $u = 0$, i.e., $\theta = \pi/2$. It should be noted that when $1 \ll G^2$, Eq. A.36 can be simplified by letting $a_o^2 = 1/3$. Similarly, when $G^2 \ll 1$, $b_o^2 = 1/(3G^2)$ can be used in Eq. A.37.

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