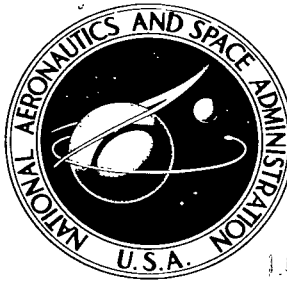


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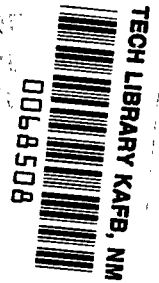
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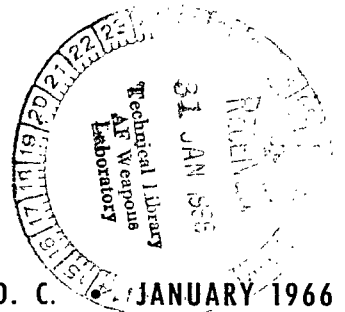
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ESTIMATION OF FLIGHT PERFORMANCE  
WITH CLOSED-FORM APPROXIMATIONS  
TO THE EQUATIONS OF MOTION

*by Charlie M. Jackson, Jr.*  
*Langley Research Center*  
*Langley Station, Hampton, Va.*





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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

An approximate method for calculating climb and acceleration performance of an air-breathing aircraft has been developed. Basically, the method involves linearization of the equations of motion to obtain closed-form expressions for the desired performance parameters. These expressions are applied over finite velocity intervals where the aerodynamic, propulsion, and flight-path parameters are assumed to be constant. The method is extended by a rapidly convergent iteration procedure to estimate the climb performance for a flight path limited by sonic-boom considerations.

Typical examples are presented where the performance is estimated for an air-breathing boost mission to hypersonic speeds and for a complete supersonic-transport mission. The examples illustrate the simplicity of the method and show that the estimated performance is in good agreement with the results of a numerical integration of the equations of motion. The application of the approximate method to these examples indicates the utility of the method for producing performance estimates suitable for preliminary airframe design, sizing studies, propulsion-system evaluation, and cursory heat-transfer and inlet-flow-field determinations along a flight path. In order to increase the utility of the method further, nomograms are presented which provide rapid solutions to the approximate performance equations.

INTRODUCTION

In many branches of the aerospace industry such as propulsion, structures, and aerodynamics, it is necessary to evaluate ideas and innovations with respect to mission performance in order to determine a profitable course of action. With the advent of high-performance aircraft, the steady-state methods of estimating climb mission performance have become inadequate even for preliminary design. Consequently, current methods of estimating climb performance involve the use of complicated high-speed digital-computer programs to integrate the equations of motion. These programs are not available to everyone and, even when they are available, their use requires considerable preliminary effort to assemble the input parameters in the required detail. A simplified computing

technique using a minimum of inputs, but capable of approximating the performance values obtained by the computer programs, is needed to fill the gap in performance estimation techniques.

An effort has been made in the present analysis to develop closed-form expressions which will estimate the variation of the performance parameters to an acceptable level of agreement with the digital-computing methods. A rigorous closed-form solution over a large range of Mach numbers and altitudes is not possible, because the variation of the aerodynamic and propulsion parameters is, in general, nonanalytic. With the present method, therefore, the total mission performance is calculated by using successive finite steps during which the aerodynamic and propulsion parameters are assumed to be constant.

### SYMBOLS

The units used for the physical quantities in this report are given both in U.S. Customary Units and in the International System of Units (SI). Factors relating the two systems are given in reference 1.

$C_{D,i}$	induced drag coefficient, $\frac{\text{Induced drag}}{qS}$
$C_{D,min}$	minimum drag coefficient, $\frac{\text{Minimum drag}}{qS}$
$C_L$	lift coefficient, $\frac{\text{Lift}}{qS}$
$C_{L,o}$	lift coefficient at minimum drag
$C_{L\alpha}$	lift-curve slope, per radian
D	total drag, lbf (N)
g	acceleration due to gravitational field of the earth, 32.174 ft/sec <sup>2</sup> (9.807 m/sec <sup>2</sup> )
h	geometric altitude above surface of the earth, ft (m)
$I_{sp}$	installed specific impulse, sec
$K_r$	reflection factor, 1.9
L	lift, lbf (N)
l	length of aircraft, ft (m)
M	Mach number

m	vehicle mass, slugs (kg)
p	reference pressure, lbf/ft <sup>2</sup> (N/m <sup>2</sup> ); atmospheric pressure at h/2
$\Delta p$	sonic-boom overpressure at ground level, lbf/ft <sup>2</sup> (N/m <sup>2</sup> )
q	dynamic pressure, lbf/ft <sup>2</sup> (N/m <sup>2</sup> )
R	range, ft (m) or nautical miles
$r_e$	radius of earth, $20.89 \times 10^6$ ft ( $6.3673 \times 10^6$ m)
S	reference area, ft <sup>2</sup> (m <sup>2</sup> )
sfc	specific fuel consumption, per hr
T	net installed thrust, lbf (N)
t	time, sec
V	velocity, ft/sec (m/sec) or $\frac{\text{nautical miles}}{\text{hr}}$
W	aircraft weight ( $W = mg$ ), lbf (N)
w	fuel flow rate, lbf/sec (N/sec)
$\alpha$	angle of attack, deg
$\beta$	Mach number parameter, $\sqrt{M^2 - 1}$
$\gamma$	flight-path angle measured positive up from the local horizontal, radians
$\delta$	thrust-vector angle measured up from the velocity vector, radians
$\psi$	range angle measured from center of earth, radians

Subscripts:

max	maximum
o	initial conditions
1	conditions at start of a specific computing interval
2	conditions at end of a specific computing interval

A dot over a symbol indicates the derivative with respect to time. A bar over a symbol indicates an average value.

## ANALYSIS

### FORMULATION OF APPROXIMATE PERFORMANCE EQUATIONS

An exact mathematical simulation of the flight of an aircraft within the atmosphere involves the solution of a set of nonlinear differential equations. It is the purpose of this analysis to linearize these equations (expressed in two-dimensional form) and produce useful closed-form expressions for the flight parameters of interest.

The complete differential equations which describe the motion of an aircraft through the atmosphere in the vertical plane are as follows:

$$\dot{V} = \frac{T \cos \delta - D}{m} - g \sin \gamma \quad (1a)$$

$$\dot{h} = V \sin \gamma \quad (1b)$$

$$\dot{\gamma} = \frac{T \sin \delta + L}{mV} - g \frac{\cos \gamma}{V} \left[ 1 - \frac{V^2}{g(r_e + h)} \right] \quad (1c)$$

$$\dot{\psi} = \frac{V \cos \gamma}{r_e + h} \quad (1d)$$

The assumptions that the earth is flat ( $r_e = \infty$ ) and that the thrust is aligned with the velocity vector ( $\delta = 0$ ) reduce the complexity of these expressions, but do not eliminate the nonlinearity of the system. The nonlinearity can be eliminated if the time derivative of the flight-path angle  $\gamma$  is assumed to be zero and the flight-path angle is assumed to be small enough for its cosine to be unity. With these assumptions equation (1c) is reduced to the simple concept that lift is equal to weight, and the system can be expressed in terms of a single differential equation as follows:

$$\frac{VdV}{g} + dh = \frac{TV}{mg} dt - \frac{DV}{mg} dt \quad (2)$$

where the drag  $D$  is now recognized as a function of weight since lift and weight are equal.

The basic differential energy balance expressed by equation (2) is identical to the expression obtained in reference 2. In reference 2 the energy equation is used to obtain minimum time-to-climb paths. In the present analysis this equation is used to develop a technique for rapid approximation of the weight, range, and time variation over arbitrary climb and descent paths.

In order to obtain a form of equation (2) in terms of the more commonly used aerodynamic, engine, and flight-path characteristics, a parabolic drag polar is assumed and the governing differential equation becomes

$$dV + \frac{g}{V} dh = \frac{dt}{m} \left( T - qS \left\{ C_{D,\min} + \left[ \frac{C_{D,i}}{(C_L - C_{L,o})^2} \right] \left( \frac{mg}{qS} - C_{L,o} \right)^2 \right\} \right) \quad (3)$$

If the thrust  $T$  is expressed in terms of fuel flow according to the relation

$$T = -wI_{sp} \quad (4)$$

and the parameter  $\frac{dh}{dV}$  is introduced to define the flight path, equation (3) can be arranged to give

$$\frac{dW}{W} = - \frac{dV \left[ 1 + \frac{g}{V} \left( \frac{dh}{dV} \right) \right]}{gI_{sp} \left\{ 1 - \frac{qSC_{D,\min}}{T} - \frac{\left[ C_{D,i} / (C_L - C_{L,o})^2 \right] (W - C_{L,o}qS)^2}{TqS} \right\}} \quad (5)$$

Equation (5) can be integrated if the weight in the lift term is assumed to be constant over the interval of integration. The result of this integration is a relatively simple closed-form expression for the weight change over the velocity interval from 1 to 2:

$$\ln \frac{W_1}{W_2} = \frac{V_2 - V_1 + g \left( \frac{\Delta h}{\Delta V} \right) \ln \frac{V_2}{V_1}}{g \frac{\bar{I}_{sp}}{\left( \frac{W_1}{S} \right)} \left\{ \frac{W_1}{S} - \frac{\bar{q} \bar{C}_{D,\min} + \frac{\left[ C_{D,i} / (C_L - C_{L,o})^2 \right] \left( \frac{W_1}{S} - \bar{C}_{L,o} \bar{q} \right)^2}{\bar{q}}}{\frac{\bar{T}}{W_1}} \right\}} \quad (6)$$

It is interesting to note that equation (6) can be expressed as

$$\ln \frac{W_1}{W_2} = \frac{\Delta V}{gI_{sp}} \left( \frac{1 + \frac{\Delta \text{ Potential energy}}{\Delta \text{ Kinetic energy}}}{1 - \frac{\text{Drag}}{\text{Thrust}}} \right) \quad (6a)$$

which is recognized as the classic field-free rocket equation with a modifier to correct for potential-energy gain and energy dissipation due to aerodynamic forces.

Once the weight variation is determined from equation (6), the time and range changes during the velocity interval can be estimated from the following expressions:

$$t_2 - t_1 = \frac{W_2 - W_1}{\bar{w}} \quad (7)$$

$$R_2 - R_1 = \frac{V_1 + V_2}{2}(t_2 - t_1) \quad (8)$$

where  $\bar{w}$  is the average fuel flow over the interval and is obtained from the average thrust and specific impulse by using equation (4).

Equations (6), (7), and (8) provide the means of estimating the performance of a climb path as long as there is an acceleration during the climb. If the flight plan requires a climb at constant velocity, then equation (6) becomes indeterminate, and it is necessary to develop an expression for the weight variation in terms of altitude. The differential equation for the weight change during a climb at constant velocity is obtained from a combination of equations (3) and (4) with  $dV$  set equal to zero. Integration of these combined equations yields

$$\ln \frac{W_1}{W_2} = \frac{h_2 - h_1}{\frac{V_1 \bar{I}_{sp}}{\left(\frac{W_1}{S}\right)} \left\{ \frac{W_1}{S} \frac{\bar{q} \bar{C}_{D,\min} + \frac{C_{D,i}/(C_L - C_{L,o})^2}{\bar{q}}}{\bar{T}} \left(\frac{W_1}{S} - \bar{C}_{L,o} \bar{q}\right)^2 \right\}} \quad (9)$$

The performance during a descent flight plan can be obtained from equations (6), (7), and (8). However, since the weight change during a descent is usually small (propulsion system in low fuel-flow condition), a better estimate of the descent performance can be obtained if the governing equation is solved for the flight-time variation over the velocity interval. This variation then can be used to obtain the range and weight variations. Assuming no weight change during a descent and deceleration calculation interval, equation (3) can be integrated to give

$$t_2 - t_1 = \frac{V_2 - V_1 + g \left(\frac{\Delta h}{\Delta V}\right) \ln \frac{V_2}{V_1}}{g \left\{ \frac{\bar{T}}{W_1} \frac{\bar{q} \bar{C}_{D,\min} + \frac{C_{D,i}/(C_L - C_{L,o})^2}{\bar{q}}}{\left(\frac{W_1}{S}\right)} \left(\frac{W_1}{S} - \bar{C}_{L,o} \bar{q}\right)^2 \right\}} \quad (10)$$



With the time variation predicted by equation (10) and the range and weight variations obtained from equations (7) and (8), the performance of a descent and deceleration flight mode is defined for the condition of low fuel flow.

## METHOD OF APPLICATION AND COMPARISON OF RESULTS

Since the method of applying the simplified performance equations to complete missions involves performance calculations over successive finite intervals of velocity or altitude, it would be helpful to establish the definition of proper interval size and criteria to define an average value of the input parameters for each interval. The methods for defining these quantities can best be illustrated by considering examples. At the same time, the utility of the present approximate method can be determined by comparisons of the results obtained for these examples with the numerical solutions of the exact equations.

### Basic Calculating Method Applied to Climb and Acceleration on a Specified Altitude-Velocity Schedule

As previously stated, the equation to be used to obtain the weight variation for a climb and acceleration flight (eq. (6)) was derived with the assumption that, within the velocity interval considered, the aerodynamic, engine, and flight-path parameters must be constant. The criteria for choosing the proper velocity intervals for calculation depend on maintaining a fairly accurate representation of the input parameters with respect to the preceding assumption and on accurate representation of the thrust and drag inputs in critical regions. For example, as the transonic acceleration becomes critical, it is necessary to take small intervals in the transonic speed range to insure accurate representation of the thrust margin.

In the first example, the flight path is specified in the altitude-velocity plane for a climb and acceleration mission of a typical air-breathing hypersonic aircraft. The Mach number - altitude variation used is presented in figure 1(a) along with the propulsion parameters for the mission for Mach numbers from 1 to 7. Aerodynamic characteristics are presented in figure 1(b). Note that although the variation of the slope of the lift curve  $C_{L\alpha}$  is presented, it is not used in the basic performance calculation but is used later in an auxiliary calculation to determine the variation of angle of attack along the flight path. At the initial condition ( $M = 1.0$ ), the wing loading  $\left(\frac{W}{S}\right)_0$  and thrust loading  $\left(\frac{T}{W}\right)_0$  used in the example are 61.2 lbf/ft<sup>2</sup> (2930.27 N/m<sup>2</sup>) and 0.442, respectively.

As a first step in obtaining the proper velocity-interval size for computing the weight variation for the example flight path, the total velocity range is divided into four equal intervals for the first calculation. Therefore, as shown in figure 1, variation of the

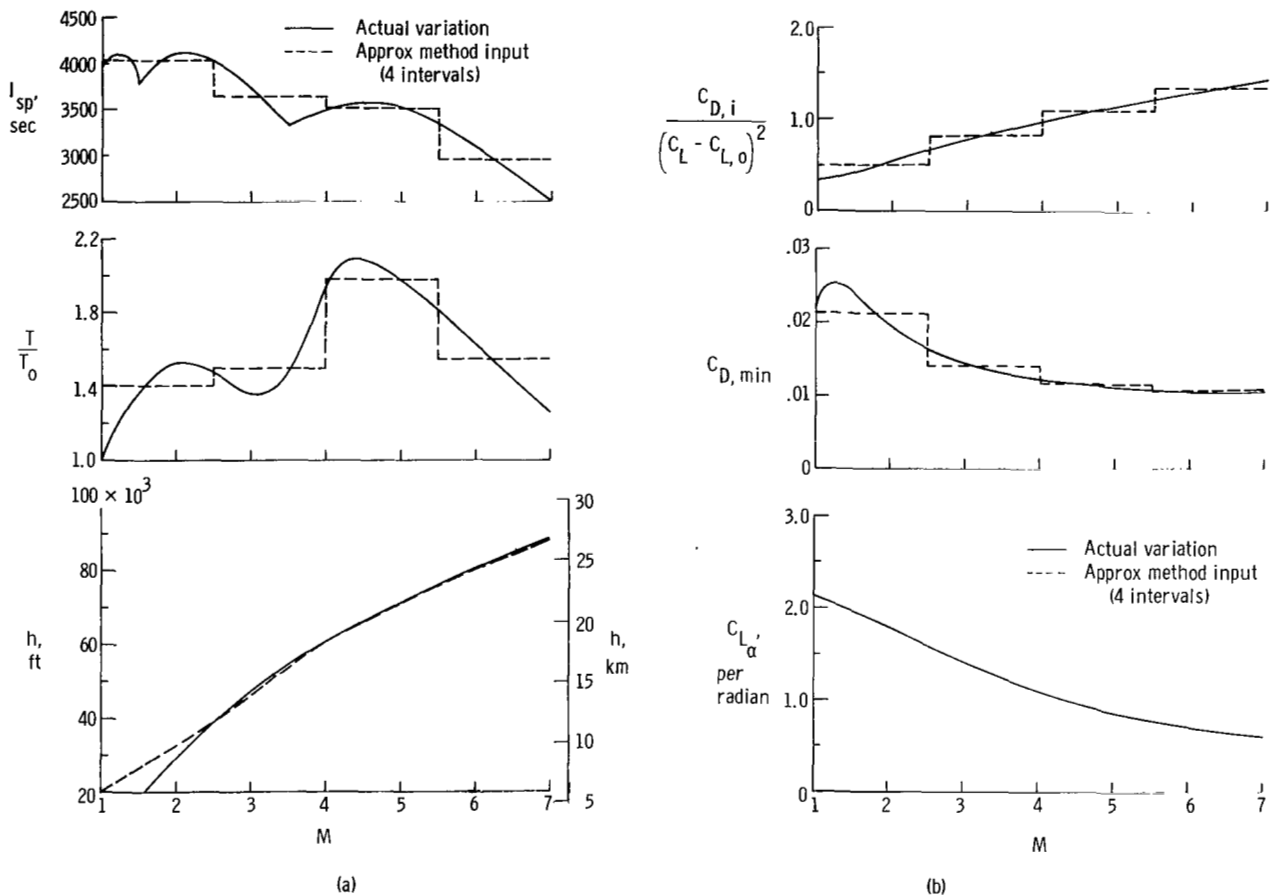


Figure 1.- Performance input parameters for typical air-breathing hypersonic aircraft.

representative input quantities is approximated by four steps, where the magnitude of the input parameters with the exception of  $\Delta h/\Delta V$  is assumed to be an "area average" for each step, that is, the magnitude is taken so that the area under the actual curve is equal to the area under the average value for each step. The flight path is represented by the input  $\Delta h/\Delta V$  which is obtained from the altitude - Mach number variation as shown in figure 1(a). It should be noted that, for the example presented, the aerodynamic inputs represent a symmetrical drag polar (i.e.,  $C_{L,0} = 0$ ).

With the "area average" values of the input parameters as indicated in figure 1, the weight change during each of the four intervals was calculated by using equation (6) and the results are shown as circular symbols in figure 2. In order to determine a sufficiently small interval size, additional calculations must be made where the existing intervals are halved successively until the resulting weight variation is not significantly affected. For example, in figure 2, the weight-ratio change calculated by using one interval for Mach numbers from 2.5 to 4.0 was about 3 percent (0.936 to 0.905); when two calculation intervals were used in this Mach number range, the weight-ratio change was

essentially unaffected (still about 3 percent, 0.925 to 0.893). Therefore, no advantage of further reduction of interval size in this Mach number range is indicated.

The procedure outlined in the previous paragraph was applied to the example and indicated, as shown in figure 2, that a total number of 12 intervals was sufficient; the majority of intervals was taken in the critical transonic range. Figure 2 indicates that these results are in good agreement with the weight variation obtained by the numerical integration of the exact two-dimensional equations of motion represented by the solid line. Although 12 calculations to determine the fuel burned for a climb path may seem excessive for a rapid method, it must be remembered that the calculations are simple, that a high degree of accuracy was required, and that a critical transonic-acceleration condition was used as an example.

Several thrust loadings were considered for the example of the hypersonic aircraft in order to give an indication of the effects of thrust margin on the utility of the approximate method. The calculated weight variation over the Mach number range from 1 to 7 is presented in figure 3 for the approximate method and the numerical integration of the exact equations of motion. As expected, the number of computing intervals necessary for the approximate method (indicated by the symbols in fig. 3) decreases as the thrust loading  $(\frac{T}{W})_0$  increases because the transonic acceleration is no longer critical and the results become relatively insensitive to minor variations in input parameters. In figure 3 the agreement of the approximate method with the

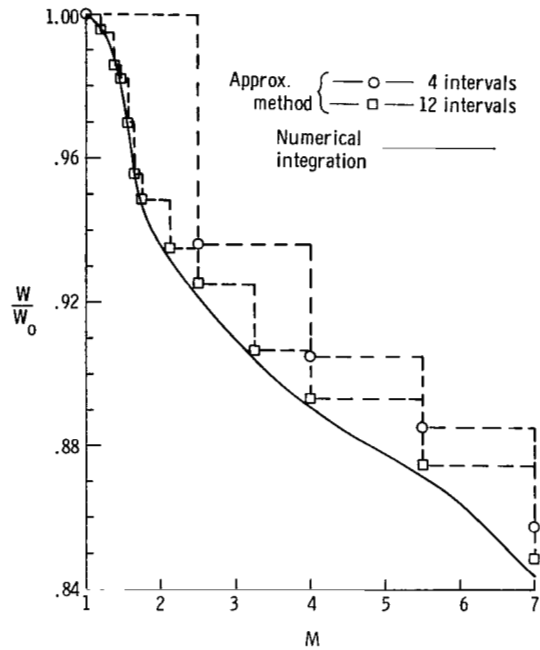


Figure 2.- Effects of computing-interval size.

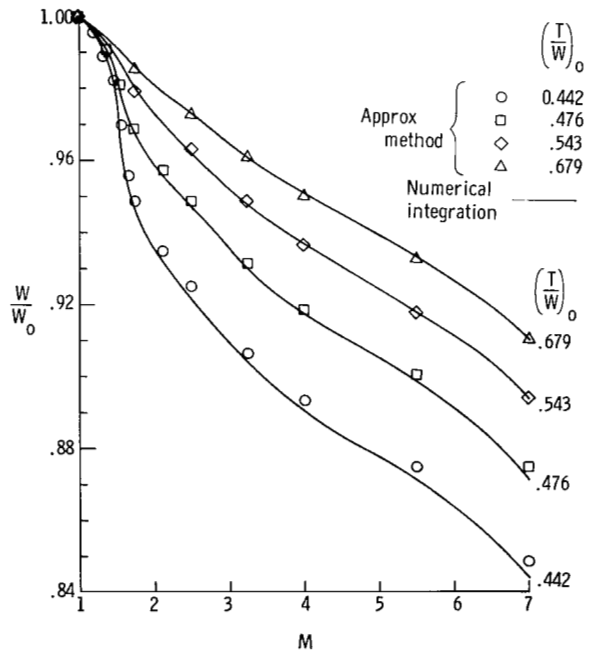
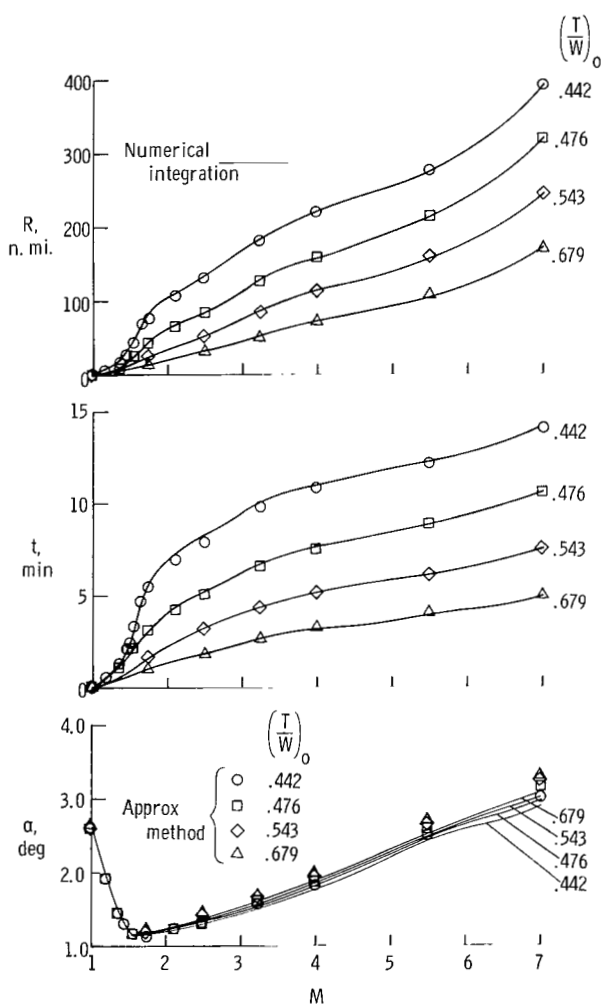


Figure 3.- Comparison of calculation techniques for determining weight variation for typical hypersonic aircraft.

integration of the equation of motion is within 3 percent for the most critical case and improves with an increase in thrust margin. The increased computing-interval size and improved accuracy suggests an overall increase in the utility of the approximate method as the aircraft-acceleration performance improves. The method is still quite useful, however, when performance is marginal. In the marginal case, care must be exercised in choosing the interval size and the average input parameters.

While the primary performance parameter for a climb and acceleration mission is usually fuel burned, the range and time are of equal importance, and for heat-transfer and inlet analyses, the angle-of-attack schedule is also necessary. Once the weight variation is obtained, these parameters can be easily estimated by the present analysis. Estimates of the range, time, and angle of attack are presented in figure 4 for the hypersonic aircraft with four engine sizes. Equations (7) and (8) were used to calculate range and time



variations. The angle of attack was obtained by applying the lift-curve slope ( $C_{L\alpha}$ , from fig. 1(b)) to the lift coefficient which was computed with the assumption that lift plus earth-centered centrifugal force is equal to weight. Figure 4 indicates that the results of approximate methods are in good agreement with the results of the integration of the equations of motion, with the exception of angle of attack which is acceptable for cursory inlet and heating analyses.

The simplicity of the approximate method, as outlined, allows rapid computation of mission performance with a slide rule or desk calculator and, in fact, graphical solutions to the performance equations have been developed. Graphical solutions to the weight, range, and time equations of this analysis are presented in the appendix in the form of nomograms. An example of the use of these nomograms is presented also.

Figure 4.- Comparison of calculation techniques for determining range, time, and angle-of-attack variations for typical hypersonic aircraft.

## Approximate Methods Applied to a Complete Supersonic-Cruise Mission

The utility of the approximate method of performance calculation has been demonstrated for a basic climb and acceleration flight mode where the climb path was specified in the altitude-velocity plane. In order to demonstrate the flexibility of the present approximate method, the performance of a complete supersonic-transport mission is calculated as an example. The mission includes a climb and acceleration to cruise Mach number subject to sonic-boom ground-overpressure limits, a climb at constant Mach number and "search" for the best cruise altitude, a cruise at constant Mach number and lift coefficient, and finally a descent along a specified flight path.

The aerodynamic parameters to be used in the example are presented in figure 5. For the climb phase of the mission, the propulsion-system performance parameters are presented in figure 6 as a function of altitude and Mach number. The thrust is presented as total propulsion-system thrust and corresponds to a thrust-to-weight ratio at take-off of 0.25. The take-off wing loading is specified to be 108.5 lb/ft<sup>2</sup> (5195.0 N/m<sup>2</sup>). For calculation purposes the climb mission is initiated at  $M = 0.3$  and  $h = 1500$  ft (457.2 m). The take-off weight of the aircraft (380,000 lbf (1.69 MN)) is reduced to 368,600 lbf (1.64 MN) to allow for take-off fuel.

Climb and acceleration to cruise Mach number. - The altitude - Mach number schedule for the climb phase of the mission is shown in figure 7. Initially the aircraft follows a specified subsonic profile. Above a Mach number of 1.0 the sonic-boom overpressure defines the flight profile until a structural limit is reached. The subsonic profile and structural limit are well defined in the Mach number - altitude plane; however, the sonic-boom overpressure path is unknown since it is a function of aircraft operating  $C_L$  as well as Mach number and altitude. The determination of the path which meets the sonic-boom-overpressure

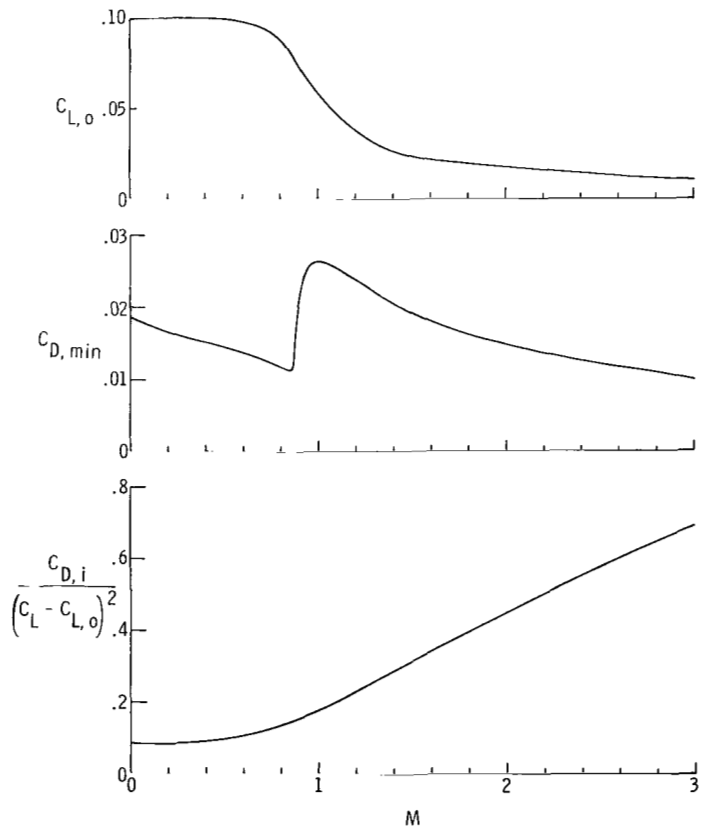


Figure 5.- Aerodynamic characteristics of supersonic-cruise aircraft.

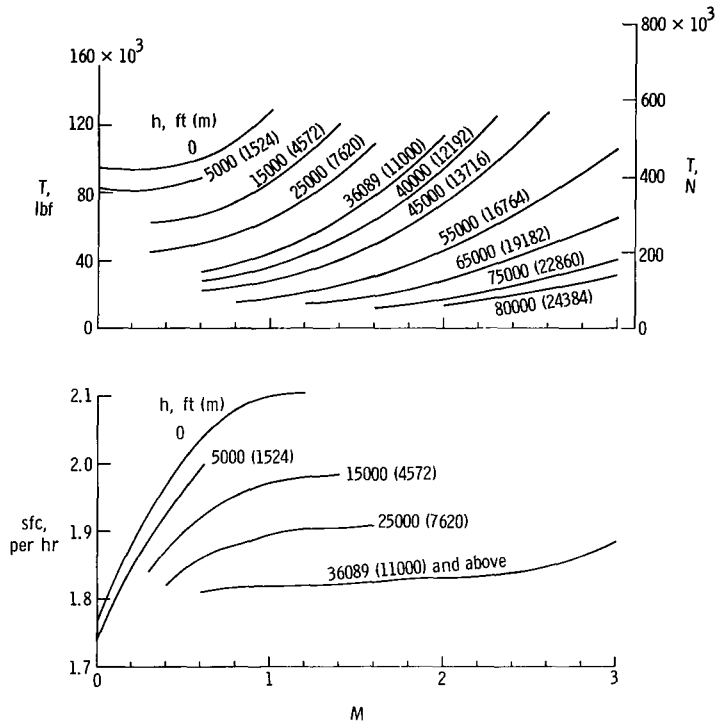


Figure 6.- Propulsion system maximum-power performance for supersonic-cruise aircraft.

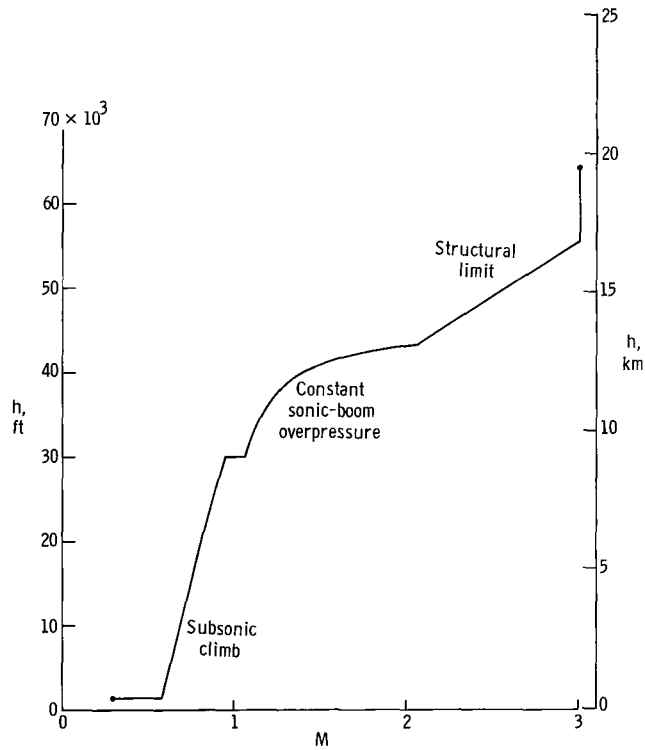


Figure 7.- Typical climb profile for supersonic-cruise mission.

requirement involves the calculation of the sonic-boom characteristics for the aircraft configuration. A method for determining these characteristics is presented in detail in reference 3. For the present analysis, the sonic-boom characteristics of the example aircraft have been incorporated into the following expression for sonic-boom overpressure as a function of Mach number, altitude, and operating lift coefficient:

$$\left(\frac{\Delta p}{P}\right)_{\max} = \frac{K_r \beta^{1/4}}{\left(\frac{h}{l}\right)^{3/4}} \left(0.048 + 2.4 \frac{\beta}{2} C_L \frac{S}{l^2}\right) \quad (11)$$

Since the operating  $C_L$  is dependent on the weight variation, which is indeed the primary unknown in the performance calculation, an iteration process is necessary to define the portion of the flight profile which is limited by the sonic boom.

The results of the iteration procedure, made by assuming an overpressure limit of 2.25 lbf/ft<sup>2</sup> (107.73 N/m<sup>2</sup>), are presented in figures 8 and 9. The process is started by making an initial, arbitrary estimate (fig. 8) of the weight variation with Mach number in

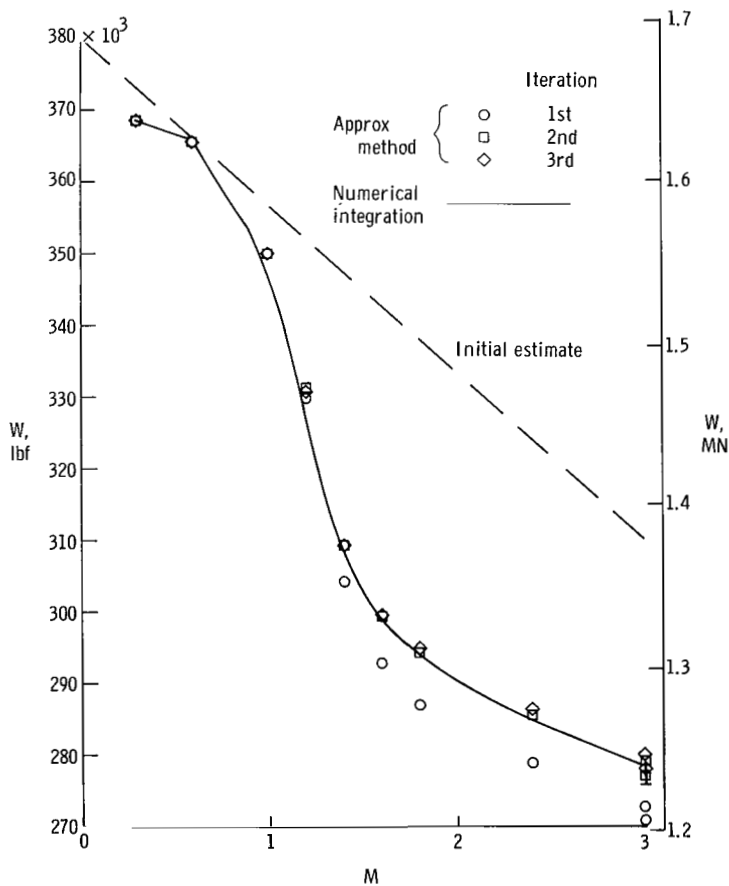


Figure 8.- Climb-weight relation for supersonic-cruise mission.

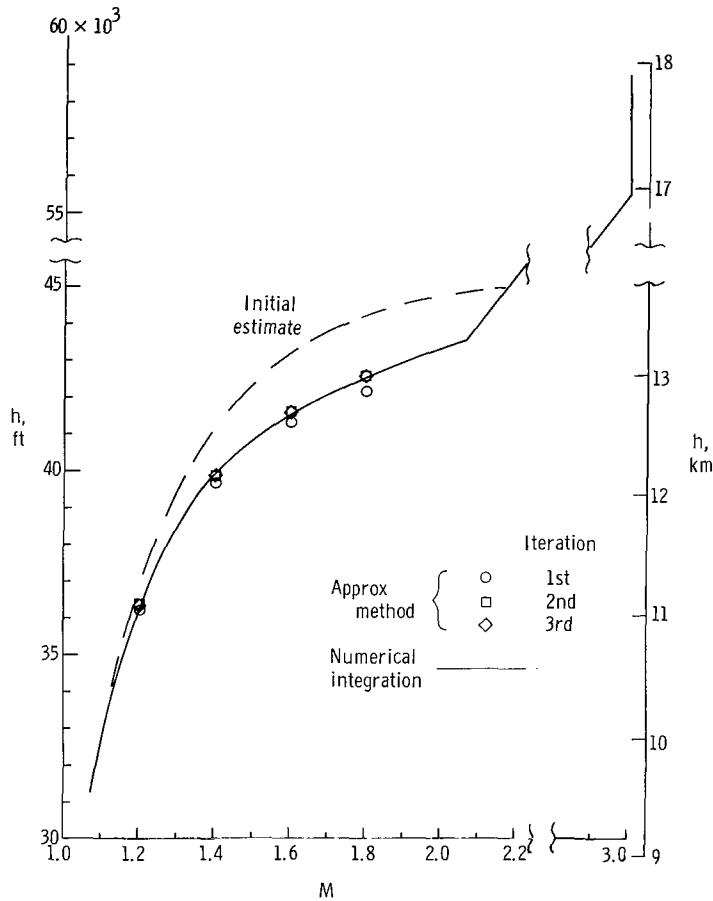


Figure 9.- Determination of constant sonic-boom overpressure flight path.  
 $\Delta p = 2.25 \text{ lb/ft}^2 (107.7 \text{ N/m}^2)$ .

order that an initial flight profile (fig. 9) which satisfies the sonic-boom limit can be determined. With this flight profile, the aerodynamic and propulsion inputs are defined; the first-iteration weight variation is shown in figure 8. This weight variation produces the flight profile described by the circular symbols in figure 9. The iteration procedure is continued until, after three iterations, the flight profile is defined to the desired accuracy. The rapid convergence of the iteration process and the agreement of the results with those of the integration of the equations of motion indicate that the approximate method can be easily extended to estimate the performance for a climb profile limited by the sonic boom.

The weight variation shown in figure 8 was used to determine the range and time variations for the climb and acceleration phase of the mission according to equations (7) and (8). The results of the range and time calculations shown in figure 10 again indicate



good agreement with the variations obtained by numerical integration of the equations of motion.

Climb at cruise Mach number and search for best cruise conditions.- Once the cruise Mach number is attained, a climb at constant Mach number is initiated to obtain the best cruise altitude. At this point in the analysis, the best cruise altitude is not known so the climb is arbitrarily taken to 64 000 feet (19 507 m) (fig. 7). The weight variation for the climb was calculated by using equation (9) with altitude-calculating intervals of about 2000 feet (609.6 m). The weight change from 55 500 feet (16 916.4 m) to 64 000 feet (19 507.2 m) was about 2000 lbf (8896.0 N) and is illustrated in figure 8 at the cruise Mach number of 3.0.

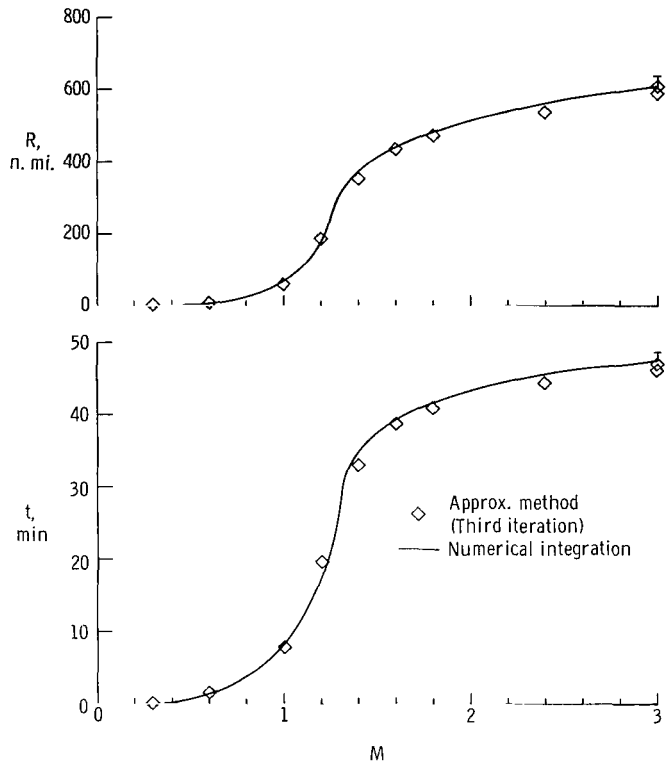


Figure 10.- Comparison of calculating techniques for determining range and time variations for climb phase of supersonic-cruise mission.

In order to determine the altitude for the most efficient cruise, it is necessary to examine the various parameters used to determine cruise performance as the climb at cruise Mach number progresses. These parameters are easily identified by considering the Bréguet range equation which is used to determine the cruise performance

$$\Delta R = \frac{V \left( \frac{L}{D} \right)}{\text{sfc}} \ln \frac{W_1}{W_2} \quad (12)$$

where the velocity  $V$  is expressed in nautical miles per hour and the value of  $\frac{V \left( \frac{L}{D} \right)}{\text{sfc}}$  is assumed to be constant during the cruise.

Since equation (12) indicates that the altitude for best cruise at some specified Mach number is that for a maximum value of  $\frac{V \left( \frac{L}{D} \right)}{\text{sfc}}$ , the variation of cruise  $L/D$  and  $\text{sfc}$  must be investigated during the climb at cruise Mach number. The aerodynamic characteristics from figure 5 and the weight during the climb are used to obtain cruise  $L/D$  since lift is assumed to be equal to weight for the cruise condition. The engine  $\text{sfc}$  is

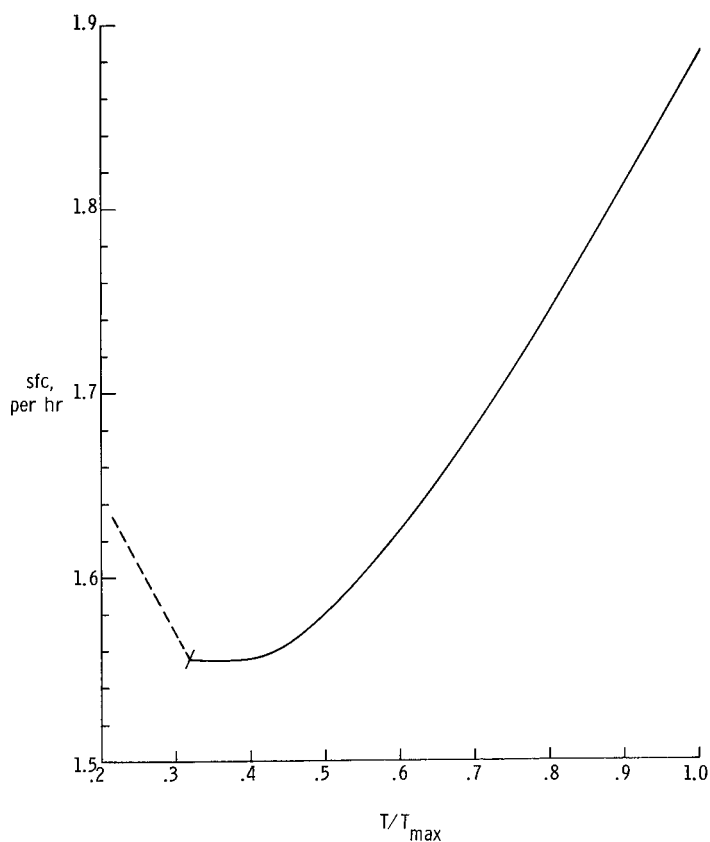


Figure 11.- Propulsion-system performance for reduced power at  $M = 3$ .

obtained by assuming the required thrust equal to drag and using the reduced-power characteristics of the engine at cruise Mach number as shown in figure 11. The variation of the aerodynamic and propulsion cruise parameters with altitude at the start of cruise are presented in figure 12. From figure 12 it can be seen that, as altitude increases, engine performance decreases and aerodynamic cruise efficiency  $\frac{v(\frac{L}{D})}{sfc}$  reaches a maximum at 63 500 feet (19 354.8 m). At this altitude cruise is initiated.

Cruise.- The cruise range is calculated from equation (12) where the initial weight  $W_1$  is the end-of-climb weight and the final weight  $W_2$  is arbitrarily chosen to be 228 000 pounds (1.014 MN) for this example. One way in which cruise-flight equilibrium can be main-

tained is by holding constant throttle setting and angle of attack to allow the aircraft to gain altitude as weight decreases. A detailed analysis of this approach is presented in reference 4. As the altitude increases, the value of the cruise efficiency may increase or decrease depending on the engine-airframe match. In order to obtain a more realistic value of the efficiency parameter for range calculation, an average value of  $\frac{v(\frac{L}{D})}{sfc}$  is used in equation (12). For the present example, the end-of-cruise altitude is 67 500 feet (20 574 m) (calculated by assuming constant  $C_L$  during cruise) and the average value of the efficiency parameter is 6815 nautical miles; this value yields a cruise range of 1365 nautical miles.

Descent.- The descent and deceleration flight path is shown in figure 13 from a Mach number of 3.0 and an altitude of 67 500 feet (20 574 m) to the end conditions for the mission. The time for descent was calculated from equation (10) by using five velocity intervals as indicated in figure 13. Propulsion characteristics for the descent and deceleration phase of the mission were assumed to be representative of the engine performance

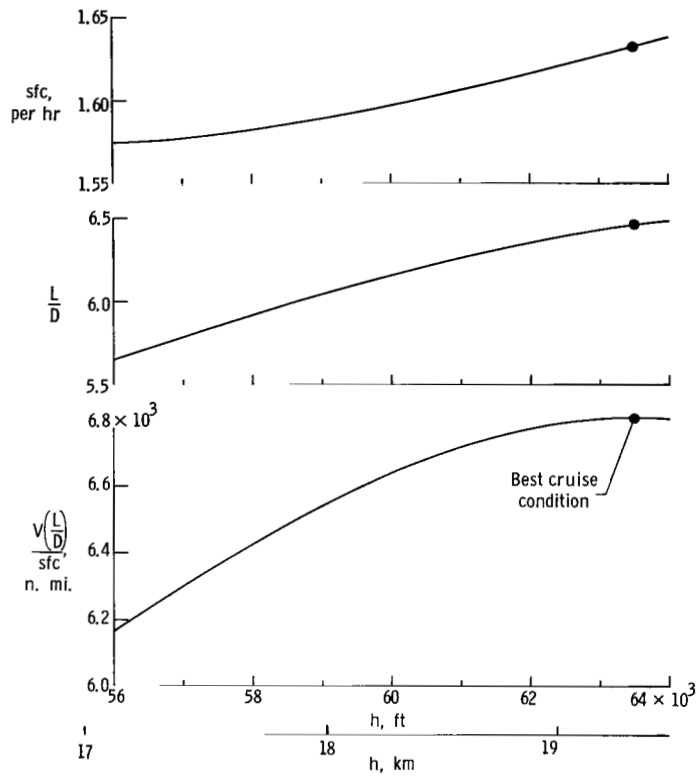


Figure 12.- Variation of initial cruise characteristics with cruise altitude.

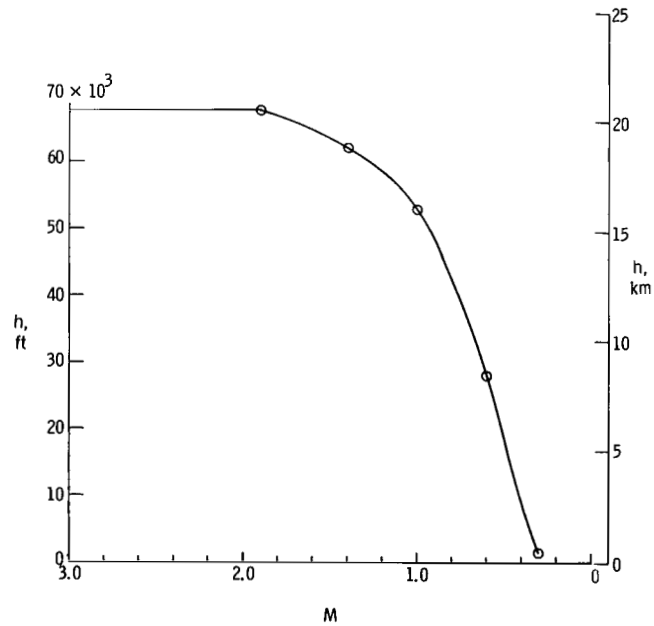


Figure 13.- Descent flight profile for supersonic-cruise mission.

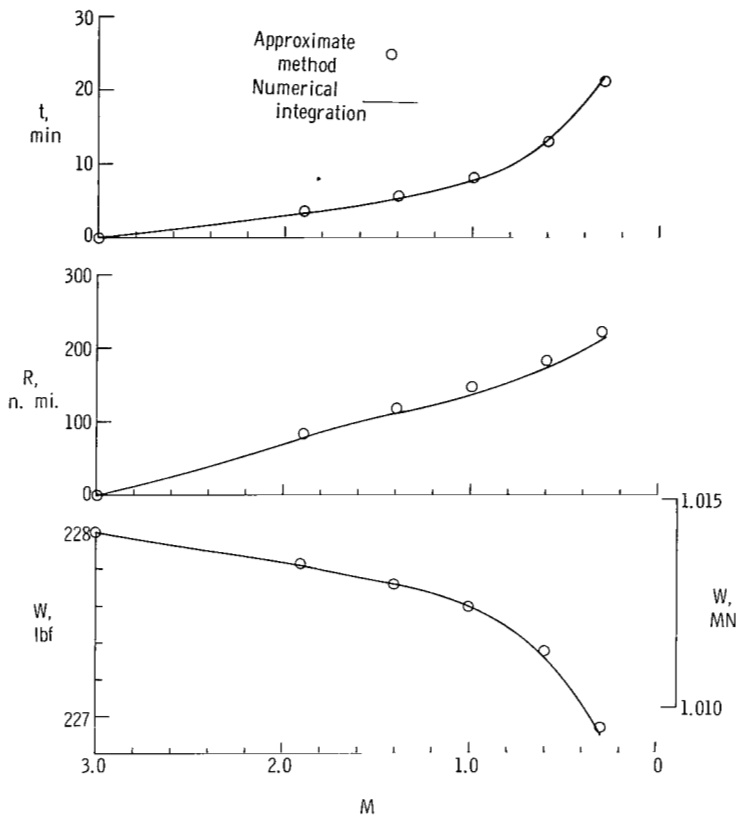


Figure 14.- Comparison of calculating techniques for determining descent performance of supersonic-cruise mission.

at idle condition. The net thrust was assumed to be zero and the total fuel flow was assumed to be 0.83 lbf/sec (3.692 N/sec) for the entire descent phase. The aerodynamic characteristics were identical to those used in the climb with the exception that  $C_{D,min}$  was increased by 0.03 from a Mach number of 1.0 down to 0.3 to account for spoiler and flap drag. Range and weight variations were obtained from equations (7) and (8) by using the time variation calculated from equation (10). These results compare well with the numerical integration of the equations of motion as is shown in figure 14.

Mission summary.- A summary of the basic performance of the example mission is shown in figure 15 where the results of the approximate techniques are again

compared with the results of numerical integration methods. This comparison also gives an indication of the effects of accumulative errors between the phases of the mission. For example, in figure 15, the aircraft at the initial cruise conditions is about 2000 pounds (8896 N) heavier and has about 40 nautical miles less range when the approximate method is used to estimate climb performance. Since the end-of-cruise weight is fixed, the climb fuel error results in more available cruise fuel and, therefore, more cruise range. The weight-range efficiency is considerably lower in the climb phase than in the cruise phase as evidenced by the relative slopes of the curves in figure 15. This difference in efficiency results in the rather high degree of sensitivity of the cruise range to the climb fuel errors for the example chosen. In spite of these discrepancies the basic mission performance as represented by this summary figure is in good agreement with the more exact methods involving numerical integration of the equations of motion.

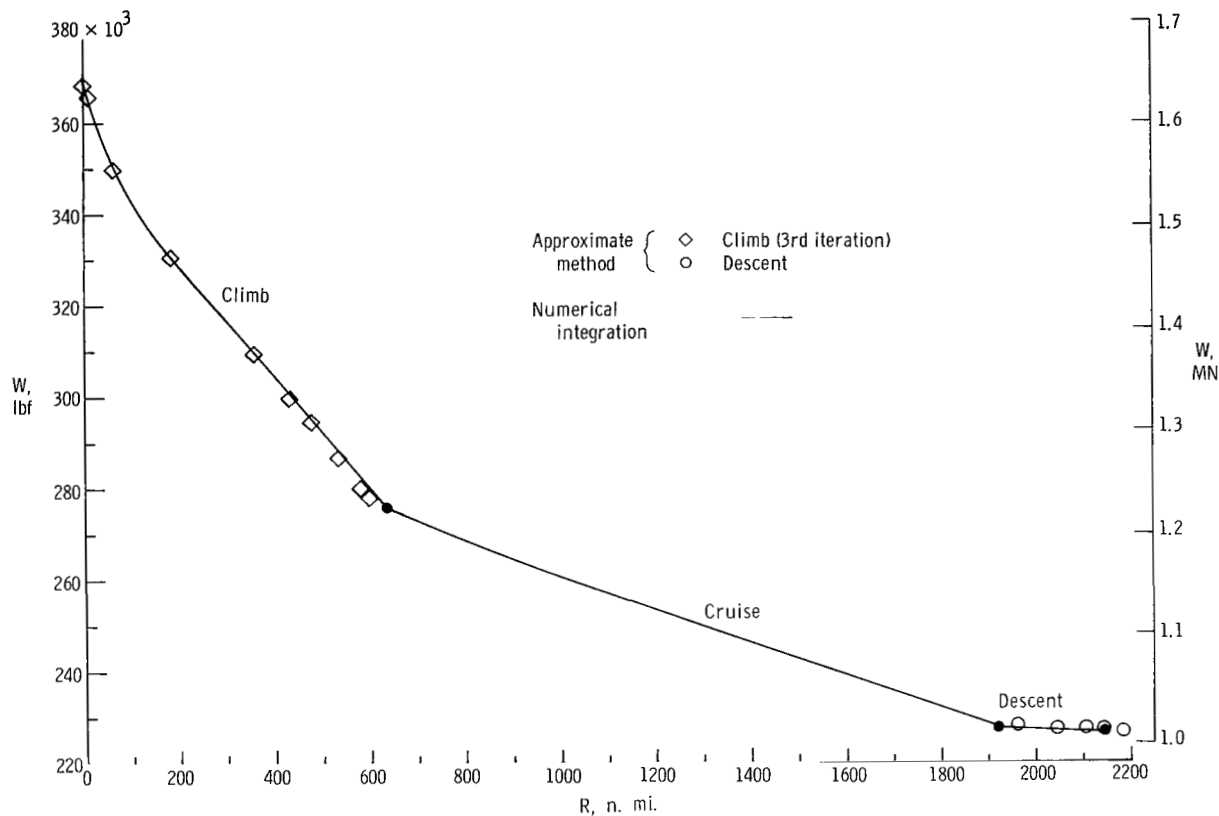


Figure 15.- Summary of supersonic-cruise-mission performance.

### CONCLUDING REMARKS

An approximate method for calculating climb and acceleration performance of an aircraft has been developed. Basically, the method involves linearization of the equations of motion to obtain closed-form expressions for the desired performance parameters. These expressions are applied over finite velocity intervals where the aerodynamic, propulsion, and flight-path parameters are assumed to be constant. The method is extended by a rapidly convergent iteration procedure to estimate the climb performance for a sonic-boom-limited flight path.

Typical examples are presented, where the performance is estimated for an air-breathing boost mission to hypersonic speeds and for a complete supersonic-transport mission. The examples illustrate the simplicity of the method and show that the estimated performance is in good agreement with the results of a numerical integration of the equations of motion. The application of the approximate method to these examples indicates the utility of the method for producing performance estimates suitable for preliminary airframe design, sizing studies, propulsion system evaluation, and cursory

heat-transfer and inlet-flow-field determination along a flight path. In order to increase the utility of the method further, nomograms are presented to provide rapid solutions to the approximate performance equations.

Langley Research Center,  
National Aeronautics and Space Administration,  
Langley Station, Hampton, Va., August 24, 1965.

## APPENDIX

### NOMOGRAPHIC SOLUTIONS TO THE APPROXIMATE PERFORMANCE EQUATIONS

In order to obtain rapid estimates of the performance of an aircraft, nomographic solutions have been developed for the equations of the approximate method presented in the text. The equation number and its corresponding nomogram are presented in the following table:

Equation	Nomogram	Figure
	Acceleration along flight path	16
6	Velocity-weight relation for climb and acceleration	17
7 and 8	Weight-range-time relation	18
9	Altitude-weight relation for climb at $\dot{V} = 0$	19
12	Weight-range relation during cruise	20
10	Velocity-time relation for descent and deceleration	21

The nomogram used to obtain the acceleration along the flight path (fig. 16) forms a basic part of equations (6), (9), and (10) and therefore provides an input to the climb and acceleration nomogram (fig. 17), the climb at constant velocity nomogram (fig. 19), and the descent and deceleration nomogram (fig. 21). The nomogram expressing the weight-range-time relation (fig. 18) is applicable to all modes of flight except the cruise (fig. 20) and when the thrust is assumed to be zero during the descent mode.

The use of the nomograms is illustrated for the example presented in the text, where the climb and acceleration performance is estimated for a typical air-breathing hypersonic aircraft with the initial thrust-to-weight ratio of 0.679. The values of the necessary input parameters are presented in table I for six velocity intervals from a Mach number of 1.0 to 7.0. With the use of the nomograms of figures 16 and 17, the acceleration along the flight path and the weight variation were obtained for each velocity interval. The resulting weight variation was then used as an input to the weight-range-time nomogram (fig. 18) to obtain the time and range variations. The necessary inputs and the results obtained from the nomograms are presented in table II. For the first calculating interval of the example ( $1.0 \leq M \leq 1.75$ ) the necessary operations are outlined on the nomograms of figures 16 to 18. In order to illustrate the use of the nomograms for climb at constant velocity, cruise, and descent and deceleration flight modes, the necessary operations are represented by the dashed lines in figures 19 to 21 for typical inputs.

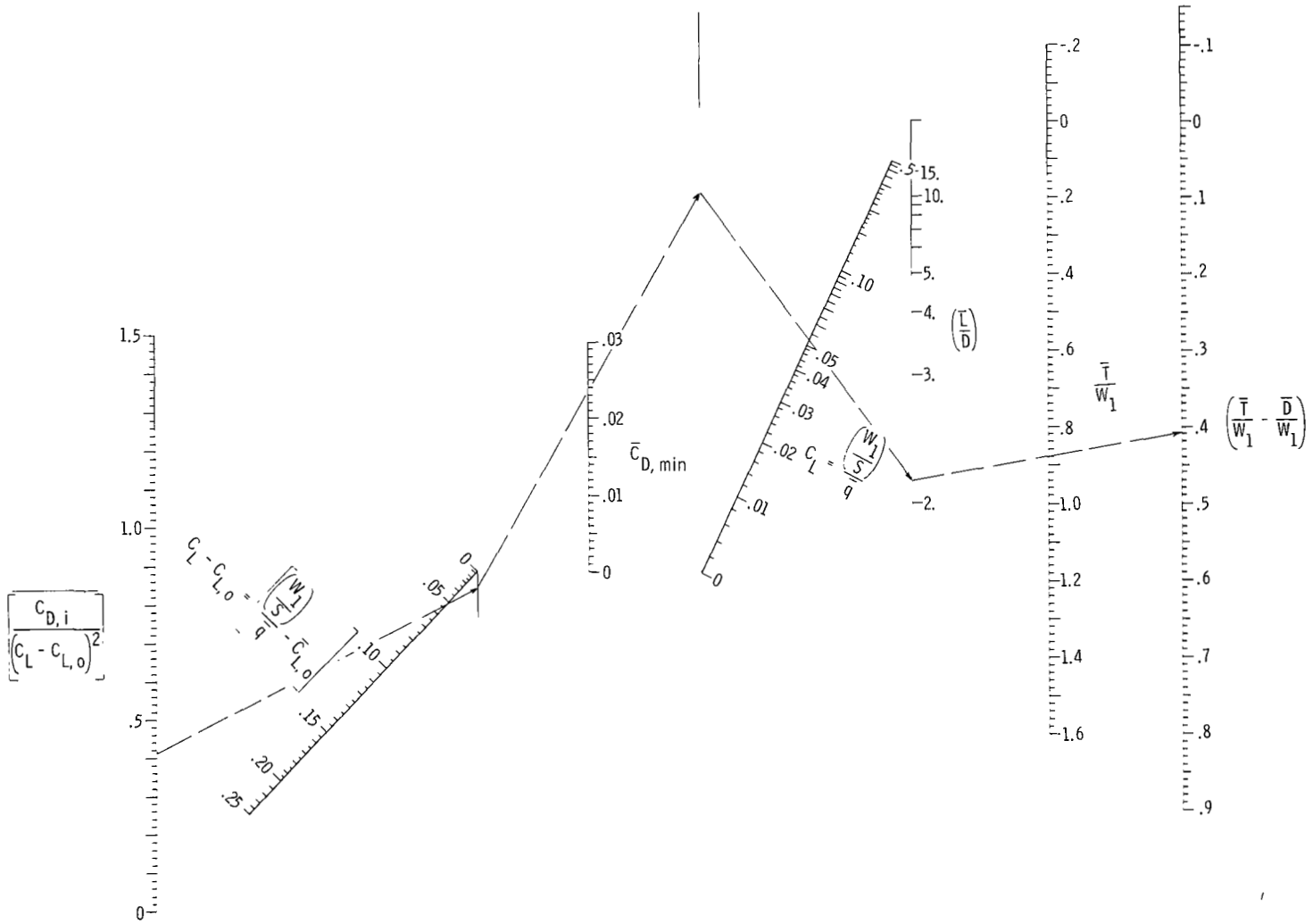


Figure 16.- Acceleration along flight path.



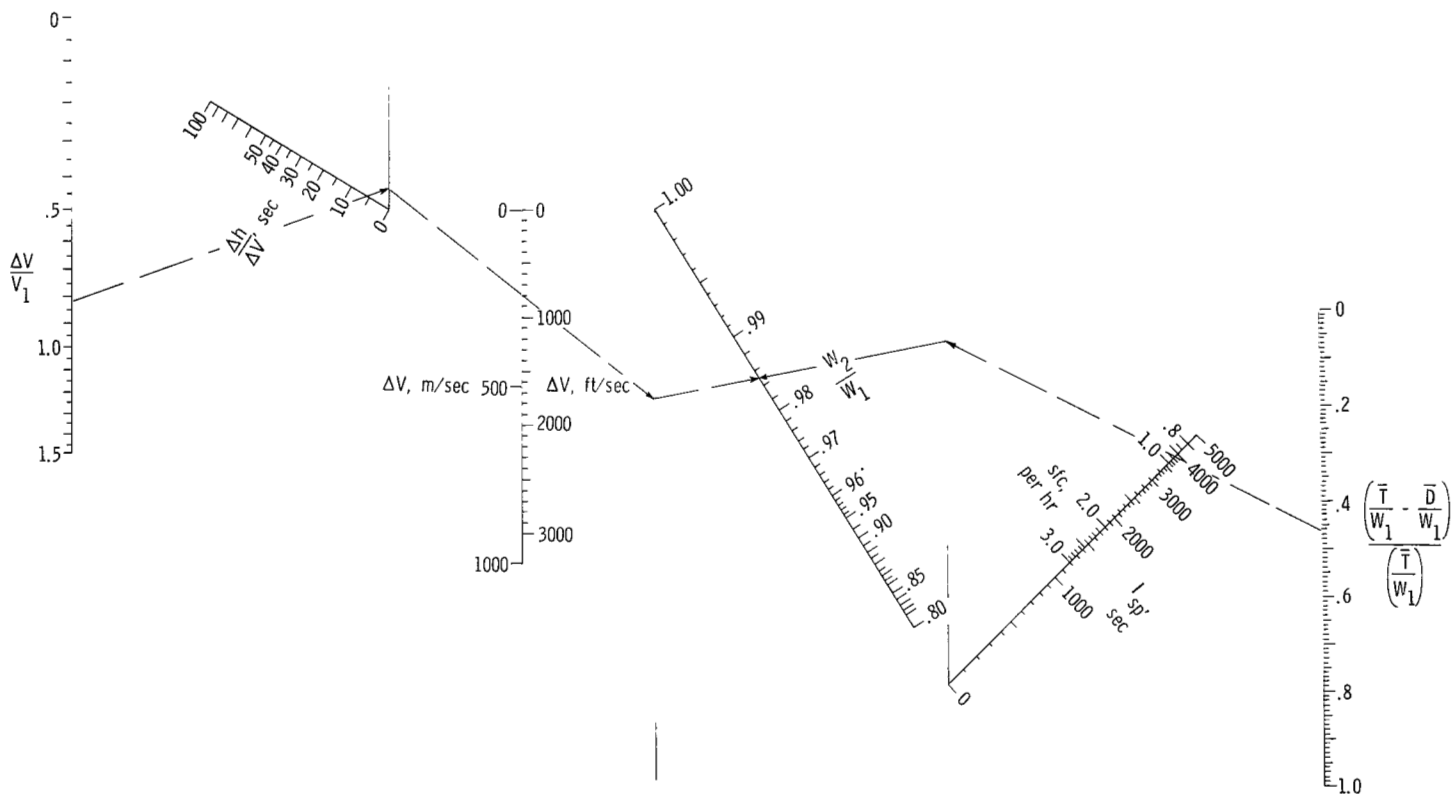


Figure 17.- Weight-velocity relation for climb and acceleration.

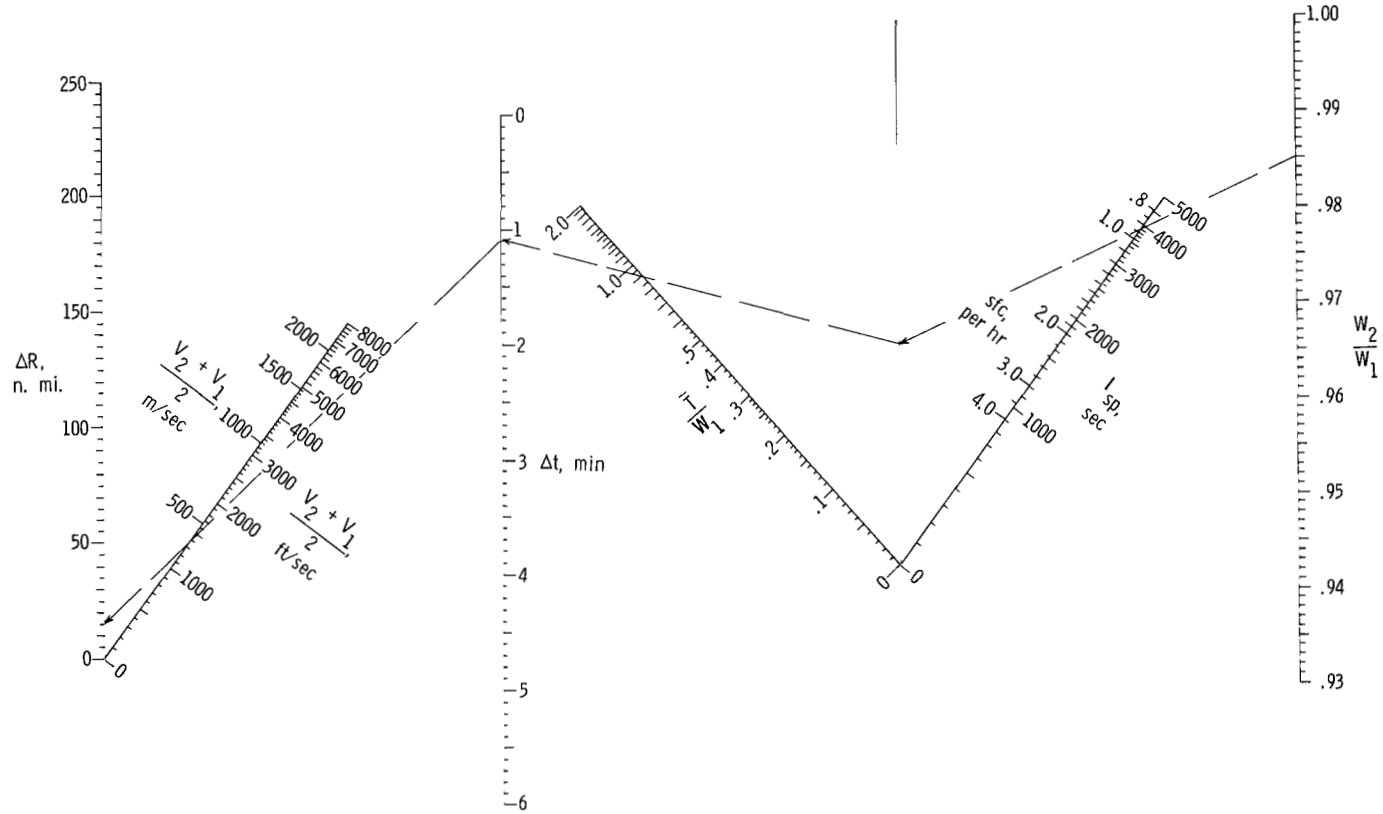


Figure 18.- Weight-range time relation.

# APPENDIX

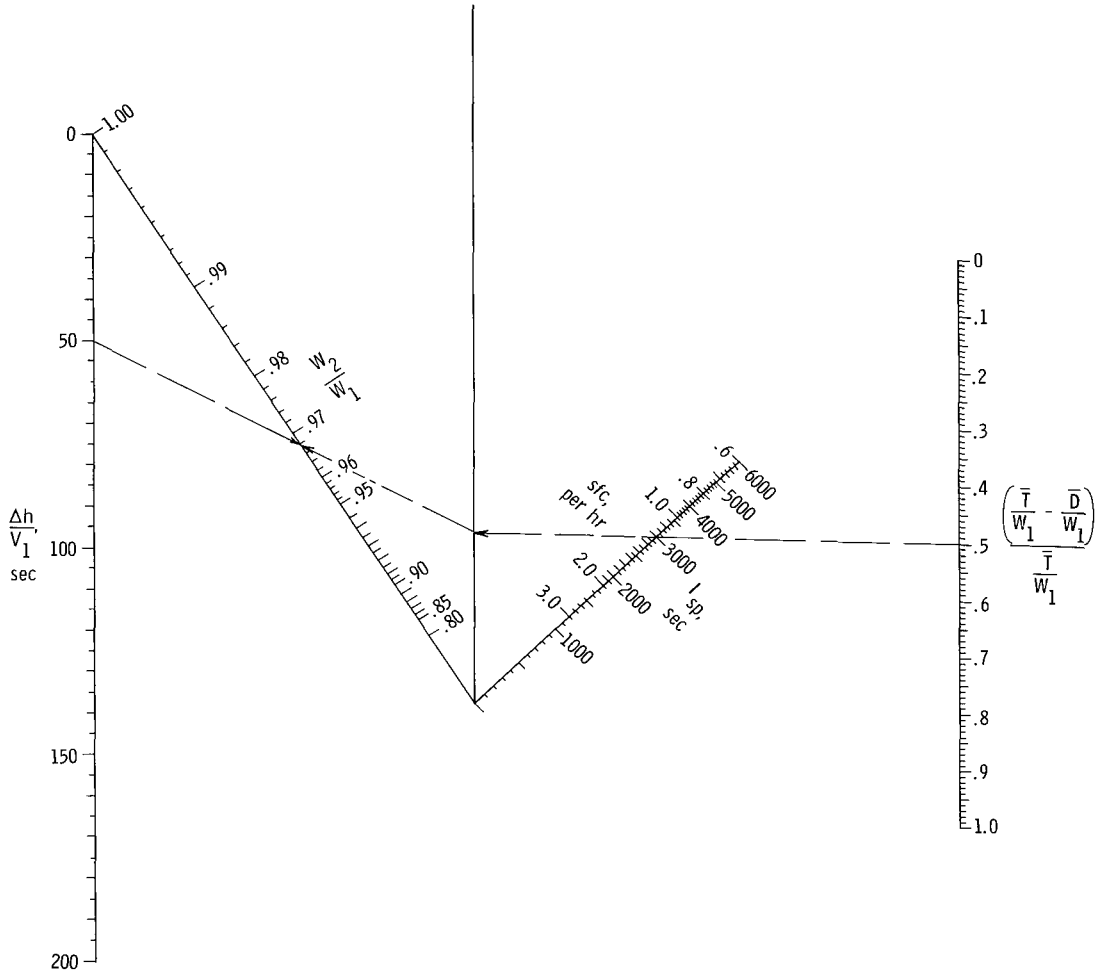


Figure 19.- Altitude-weight relation for climb at  $\dot{V} = 0$ .

# APPENDIX

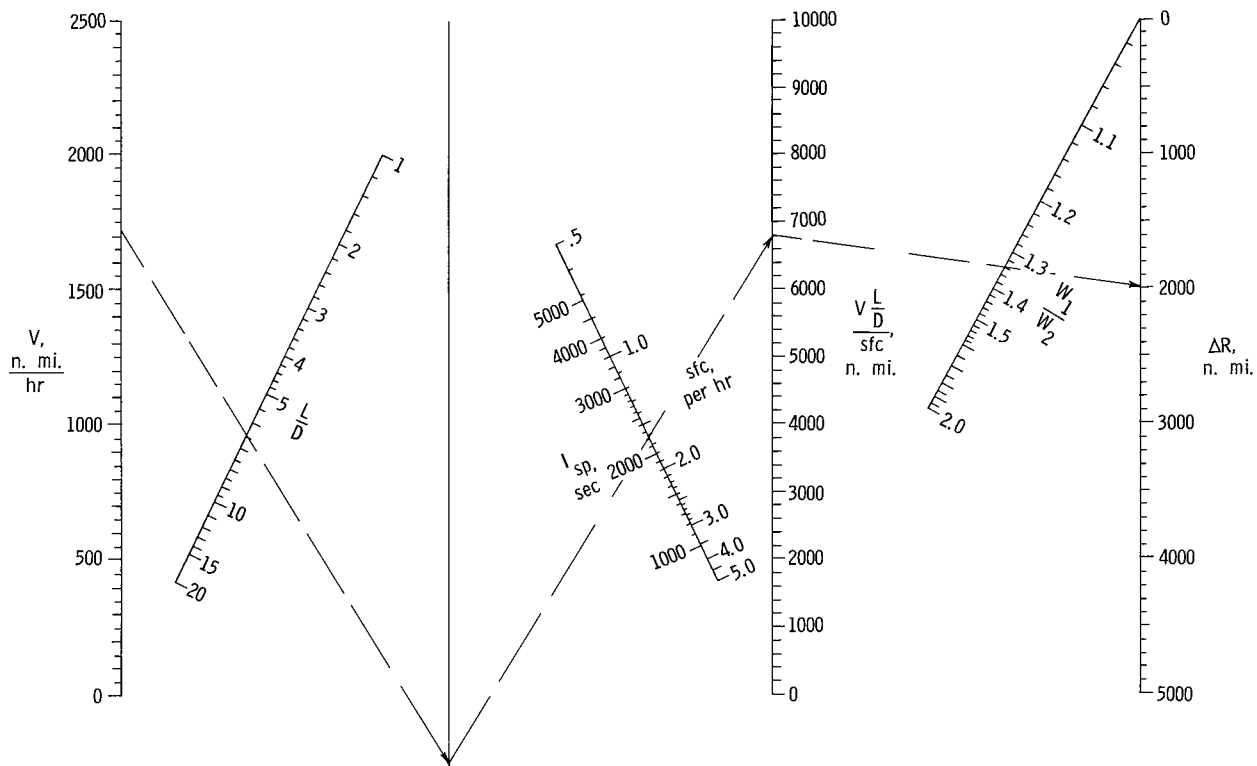


Figure 20.- Weight-range relation during cruise.

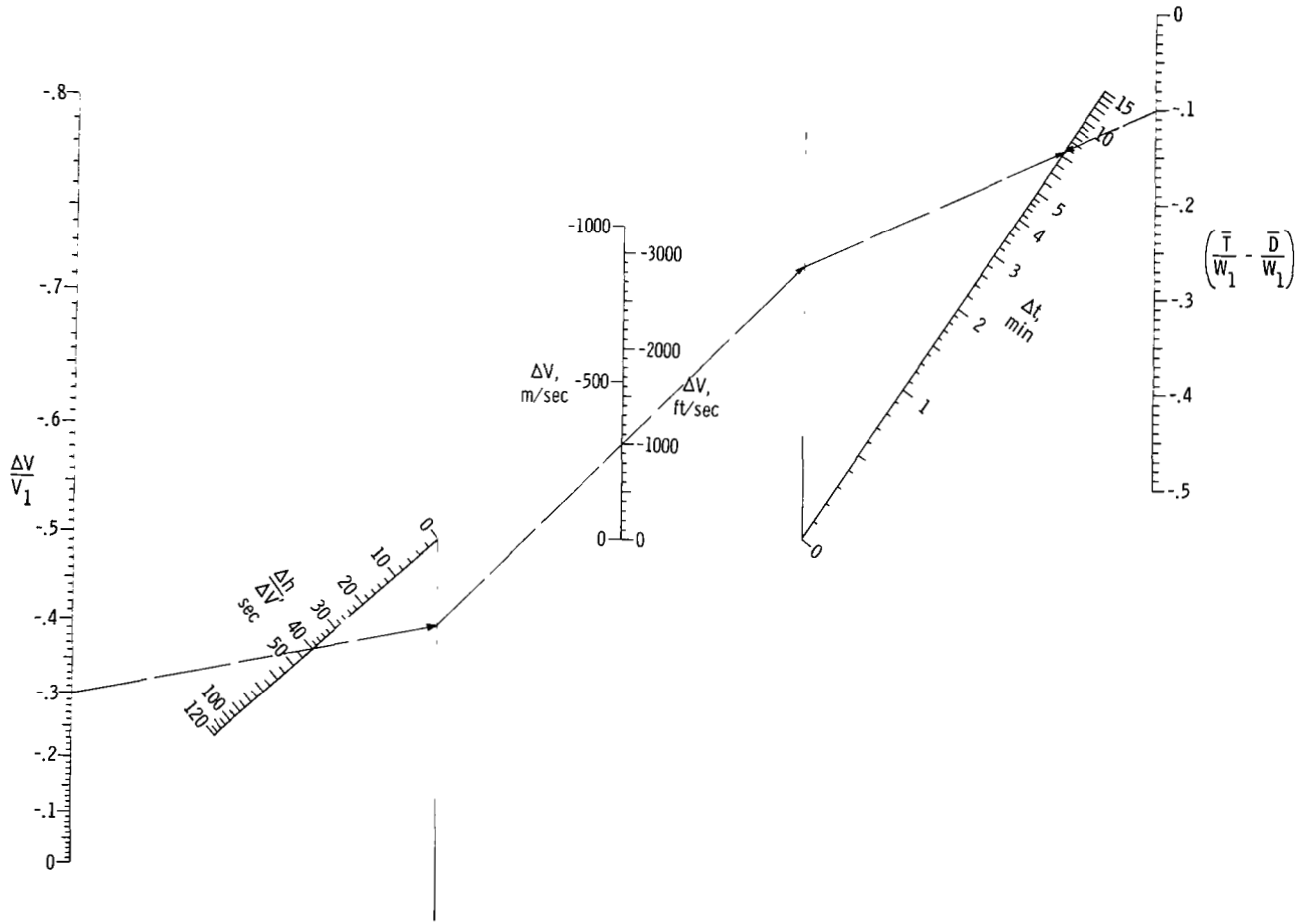


Figure 21.- Velocity-time relation for descent and deceleration.

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TABLE I.- INPUT PARAMETERS

M	h		$\bar{q}$		$\frac{\bar{T}}{W_0}$	$\bar{I}_{sp}$ , sec	$\bar{C}_{D,min}$	$\overline{C_{D,i}/(C_L - C_{L,o})^2}$	$\bar{C}_{L,o}$
	ft	m	lb/sq ft	N/m <sup>2</sup>					
1.00	20 000	6 096	1 175	56 259	0.880	3 870	0.02365	0.412	0
1.75	24 650	7 513	1 500	71 820	1.025	4 050	.01920	.566	0
2.50	39 100	11 917	1 500	71 820	.952	3 770	.01535	.737	0
3.25	51 400	15 666	1 500	71 820	1.069	3 414	.01294	.893	0
4.00	61 200	18 654	1 500	71 820	1.346	3 470	.01168	1.085	0
5.50	76 100	23 195	1 465	70 144	1.046	2 930	.01086	1.325	0
7.00	88 500	26 974							

TABLE II.- COMPUTED PARAMETERS AND PERFORMANCE RESULTS

M	V		$\Delta V$		$\frac{\Delta V}{V_1}$	$\frac{V_1 + V_2}{2}$		$\frac{\Delta h}{\Delta V}$ , sec	$\frac{W_1}{S}$		$C_L$	$C_L - C_{L,0}$	$\frac{\bar{T}}{W_1}$	$\frac{\bar{T}}{W_1} - \frac{\bar{D}}{W_1}$	$\frac{\frac{\bar{T}}{W_1} - \frac{\bar{D}}{W_1}}{\frac{\bar{T}}{W_1}}$	$\frac{W_2}{W_1}$	$\frac{W}{W_0}$	$\Delta t$ , min	$\Delta R$ , n. mi.
	ft/sec	m/sec	ft/sec	m/sec		ft/sec	m/sec		lbf/ft	N/m <sup>2</sup>									
1.00	973	297							61.2	2 930							1.00		
			797	243	0.819	1 372	418.2	5.83			0.0521	0.0521	0.880	0.410	0.466	0.9850		1.10	15.0
1.75	1 770	525							60.3	2 887							.9850		
			650	198	.367	2 095	638.5	22.23			.0402	.0402	1.040	.511	.491	.9868		.85	17.0
2.50	2 420	738							59.5	2 849							.9720		
			720	219	.297	2 780	847.3	17.08			.0397	.0397	.983	.572	.5825	.9881		.75	21.0
3.25	3 140	957							58.78	2 814							.9640		
			730	223	.232	3 505	1 068.3	13.56			.0392	.0392	1.113	.749	.673	.9892		.56	19.0
4.00	3 870	1 179							58.14	2 784							.9500		
			1 490	454	.385	4 615	1 406.6	10.00			.03876	.03876	1.4168	1.073	.759	.9815		.81	36.0
5.50	5 360	1 634							57.06	2 732							.9324		
			1 520	463	.284	6 120	1 865.4	8.16			.0389	.0389	1.122	.790	.704	.9763		1.03	62.0
7.00	6 880	2 097															.9103		



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