State Univ. of New York

N66-15378

Progress Report: Product Form of Inverses of Sparse Matrices, NASA Grant NGR-33-015-013.

pages 8
CR 69270
code 1

This report summarizes the work performed to this date on the project "Product Form of Inverses of Sparse Matrices." It supplements a paper of the same title as the project. Copies of this paper have already been submitted to NASA.

In addition to the four methods that are mentioned in the paper, six others were considered. All the ten methods investigated so far will be referred to as M1, M2, --, M10. In all of the six methods, viz., M5-M10, the row count vector was updated in the same manner as in M2. The order in which the columns were selected was based on 'a priori' estimation of the column density measure. A brief description of the density measures associated with each of the methods M5 - M10 will now be given. The notation used in the following description will be that of the above-mentioned paper. In particular, we have the following notation:

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R_j = Initial number of non-zero elements in column j of the matrix A; j = 1, 2, ..., n.

D_j = Initial density measure associated with column j in M4; it is equal to the sum of the elements of row count vector corresponding to non-zero elements of the vector j. We recall that the element C_i in row i of the row count vector is equal to the number of non-zero elements in row i.

$C_r = \min_i C_i$, where C_i are the elements of the initial row count vector corresponding to non-zero elements of column j.

$C_i - 1 = \bar{C}_i$

$C_r - 1 = \bar{C}_r$

$D_j - R_j = \bar{D}_j = \Sigma' \bar{C}_i$, where prime denotes the sum corresponding to the non-zero elements of column j.

$P_{ij} = (C_i - 1)(R_j - 1)$, this is the maximum possible growth in the matrix if the i, j element was chosen as a pivot.

The density measures associated with each of the six methods can be described now as follows:

M5: The average value of P_{ij} for column j, $P_{aj} = (1 - 1/R_j)\,(D_j - R_j)$.

M6: Minimum P_{ij} for the column taking into account the non-zero elements present in the other rows and columns, $P^*_{rj} = \bar{C}_r\left(\bar{R}_j - \frac{\bar{D}_j - \bar{C}_r}{n-1}\right)$.

M7: Minimum P_{ij} for column j, $P_{rj} = (C_r - 1)\,(R_j - 1)$.

M8: The average value of P^*_{ij} (see M6 above), $P^*_{aj} = \frac{1}{R_j}\left\{\left(\bar{R}_j - \frac{\bar{D}_j}{n-1}\right)\bar{D}_j + \frac{\Sigma' C_i^2}{n-1}\right\} + \frac{\Sigma' C_i^2}{n-1}$

M9: Taking into consideration that the three elements of a column that correspond to the lowest C_i's have a higher probability of being selected as a pivot and combining M5 with it, we have

$$P_{amj} = (R_j - 1)\,\frac{9\bar{C}_{i_1} + 3\bar{C}_{i_2} + \bar{C}_{i_3}}{13} + \left(1 - \frac{1}{R_j}\right)(D_j - R_j).$$

M10: Column selection was done on the basis of $\min_j D_j^{(k)}$, however $D_j^{(k)}$ was recomputed at each step for 2, 3 and 4 vectors. This process is referred to as sub-optimization. This is in contrast with M4 where columns were sorted in order of ascending values of $D_j^{(o)}$ once for all and were not rearranged later on in the light of the behavior of $D_j^{(k)}$'s that remain to be pivoted into the basis.

On the basis of the computational experiments that were performed, the following unexpected results were obtained:

(a) Suboptimization in M10 did not do as well as M4 - M9. All of its three variations led to approximately 19% fewer non-zero entries than M1 whereas M4 - M9 give approximately 32% improvement.

(b) The relatively simple D_j criterion for the selection of vectors in M4 was as good as the more sophisticated criteria in M5- M9.

The reasons for the above-mentioned unexpected results are currently under investigation.

For the purposes of comparing the efficiencies of M1 - M10, a program was written in which at each step a column was selected on the basis of $\min_j P^*_{rj}$ (see M6). After the selected column was pivoted into the basis, the P^*_{rj}'s for all the other remaining columns were recomputed. It should be noted, this method will not be feasible in practice due to the time it takes to perform column sorts at each iteration. The densities obtained for the Product Form of Inverse by this control program were far less than those obtained by any of the feasible methods investigated so far. In comparison with M1, the control program gave 51% fewer non-zero entries; the corresponding figure for the best feasible method was 32%. Therefore, there is reason to believe that even further improvements in the techniques for the reduction in densities of Product Form of Inverses is possible by proper row and column choice.

The question of "a priori" estimation of the density of product Form of Inverse of a matrix of given density was also investigated. On the basis of probabilistic estimates a recursive formula has been derived for the methods currently in use in actual Linear Programming codes (M1 and M3). This formula gives a fairly good estimate for the density growth of the eta vectors. It agrees well with the results obtained for the practical production problems in Linear Programming codes. The formula is

$$d_k = d_{k-1} + \left(d_{k-1}^2 - d_{k-1}^3\right) \left(\frac{k}{n}\right)^2 .$$

where d_0 is the density of the original matrix A and d_k is the density of the eta vector formed at the $(k+1)^{th}$ iteration. This formula will be useful for the 'a priori' allocation of memory in the computers for the PFI of a given matrix. Further modifications of the above formula for M2, M4 - M9 and future methods are currently under investigation.

In addition to the current and future areas of investigation already mentioned, the following deserve to be mentioned in this project:

1. How the structure of the given matrix A affects the density of its PFI?

2. Further improvement in probabilistic updating of row count vector over the method given in the paper (method M2).

3. Experimentation with some sparse matrices from typical linear programming problems.

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NASA GRANT NGR-33-015-013

November, 1965

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