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The Effect of Adiabatic Deceleration on the Cosmic Ray Spectrum in the Solar System*

E. N. Parker
Enrico Fermi Institute for Nuclear Studies and
Department of Physics
University of Chicago
Chicago, Illinois

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ABSTRACT

Cosmic rays in the magnetic fields carried in the solar wind are continually expanded and decelerated, in addition to being convected out of the solar system. The effect of the deceleration on the cosmic ray spectrum is calculated for the case that the outward convection of cosmic rays is small and for the case that the cosmic ray spectrum is a simple power law and the diffusion coefficient is independent of energy. The calculations are carried out for both particle momentum and energy. It is shown that a power law spectrum is preserved by the deceleration. It is shown that the deceleration contributes a decrease at high energies of about one third of the total modulation. Where the spectrum flattens out at lower energies, the deceleration produces an increase which partially cancels the reduction by convection. author

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I Introduction

The primary effect of the solar wind on cosmic rays is to convect the cosmic rays out of the solar system, thereby reducing the cosmic ray density below the level in interstellar space (Parker, 1958). It was shown more recently (Parker, 1965) that there is also a continual adiabatic deceleration of the cosmic rays in the quiet-day solar wind. The adiabatic deceleration follows from the fact that, in the frame of reference moving with the average wind velocity, the cosmic ray motion is a random walk through expanding magnetic fields. The rate of expansion of the fields carried in a steady radial wind with constant velocity $\mathbf{v}$ is $\nabla \cdot \mathbf{v} = 2v/r$. The formal calculations show that cosmic ray particles typically lose 5 - 20 percent of their original energy while penetrating into the solar system. The energy loss to the individual particles does not directly affect the total particle density, but the energy loss slides the cosmic ray spectrum down the energy scale, thereby affecting the density observed in a fixed energy interval. The density in a fixed energy interval may be increased where the spectrum is flat or is rising with increasing energy and may be decreased where the spectrum is falling. Several individuals have pointed out that the cosmic ray observations are becoming sufficiently detailed and quantitative that the energy loss should be included in the analysis.

The density reduction by convection and the energy loss associated with the Forbush decrease were worked out earlier (Parker, 1958, 1961, 1963, 1965). The purpose of the present paper is to illustrate the effect of the quiet-day energy loss and to compare the effect with the general density reduction caused by the outward convection. Unfortunately the effect of the energy loss, though simple in principle, is a little more complicate to compute than the outright removal by convection, because the energy loss depends upon the time spent at each radial distance r. Hence the simple examples given here are illustrative, showing the properties and relative magnitude of the deceleration effect, without representing all the possible complications for large, energy dependent convection and diffusion. But the examples contain the basic physical effects and should be useful for comparing and discussing the observations. The illustration will be carried out with the elementary example of a uniform radial solar wind v extending with spherical symmetry out to a distance $r = R$. The cosmic ray diffusion coefficient κ is taken to be uniform and isotropic in $r < R$ and infinite in $r > R$.

Consider briefly how a change ΔT of the kinetic energy T of each particle affects the observed spectrum $F(T)$. Let $F_0(T)$ represent the interstellar cosmic ray spectrum. Suppose that a particle observed with energy T had an energy $T' = T - \Delta T$ in interstellar space. Then an energy interval dT is related to dT' by

$$dT' = dT\left(1 - \partial \Delta T/\partial T\right).$$

Since $F(T)\,dT = F_0(T')\,dT'$, it follows that

$$F(T) = F_0(T - \Delta T)\left(1 - \partial \Delta T/\partial T\right).$$

But of course this omits the depression of the cosmic ray intensity by the factor $\exp(-Rv/\kappa)$ as a consequence of the convection. If Rv/κ is independent of energy, then including convection gives

$$F(T) = F_0(T-\Delta T)(1 - \partial\Delta T/\partial T)\exp(-Rv/\kappa).$$

If, however, Rv/κ is energy dependent, this simple form is not correct because the effective value of Rv/κ changes throughout deceleration. The complete Fokker-Planck equation must be used in this case with an energy dependent diffusion coefficient. We do not attempt the <u>general</u> calculation with an energy dependent diffusion coefficient here. The energy dependence of Rv/κ leads in practice to a somewhat smaller correction than the deceleration alone. Instead we shall illustrate the effects with the simple case that $Rv/\kappa \ll 1$. Then $\Delta T \ll T$ and

$$F(T) \cong F_0(T)\left[1 - \frac{Rv}{\kappa} - \Delta T\,\frac{d\ln F}{dT} - \frac{\partial\Delta T}{\partial T}\right] \tag{1}$$

to first order in Rv/κ, where Rv/κ may be a function of T. Equation (1) illustrates the effect on the spectrum of both convective reduction and the sliding of particles down the energy scale by the adiabatic deceleration.

II The Basic Equations and Earlier Calculations

For the simple case of a steady uniform wind with spherical symmetry and isotropic diffusion, the general Fokker-Planck equation (Parker, 1965) reduces to

$$\frac{V}{r^2}\frac{\partial}{\partial r}(r^2 U) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(\kappa r^2 \frac{\partial U}{\partial r}\right) + \frac{\partial}{\partial T}\left(\frac{dT}{dt}U\right) = 0 \qquad (2)$$

for the cosmic ray distribution $U(r, T)$ over radial distance and kinetic energy T. It is readily shown that adiabatic expansion of an isotropic distribution of cosmic rays at the rate $\nabla \cdot \underset{\sim}{v}$ decreases the particle energy T at the rate

$$\frac{1}{T}\frac{dT}{dt} = -\frac{n(T)}{3}\nabla \cdot \underset{\sim}{v}$$

where

$$n(T) = \frac{T + 2Mc^2}{T + Mc^2}$$

and M is the particle mass in grams. Obviously $n = 2$ for non-relativistic particles $(T \ll Mc^2)$ and $n = 1$ for extreme relativistic particles $(T \gg Mc^2)$. To make the problem tractable, the calculations which follow avoid the algebraic complexity of $n(T)$ for intermediate particle energies, and treat only n = constant. With

$\nabla \cdot \underset{\sim}{v} = + 2v/r$, it follows that

$$\frac{dT}{dt} = - \frac{2nv}{3} \frac{T}{r}$$

¶ The particle momentum is, in fact, simpler to treat, and is used in many contemporary analyses of cosmic ray variations. The distribution $\Upsilon(r, p)$ over momentum p satisfies the same Fokker-Planck equation as $U(r, T)$, viz (2), with T replaced by p in the equation. The simplification results from the fact that

$$\frac{1}{p} \frac{dp}{dt} = - \frac{1}{3} \nabla \cdot \underset{\sim}{v}$$

for all values of p . Since momentum and energy are both used in different analyses of the observations, we shall carry through the analysis for the more comlicated case of $U(r, T)$, treating n as a constant. The final analytical results then apply to momentum merely by replacing T by p and putting $n = 1$.

Solution of (2) was carried out (Parker, 1965) for the special case that the diffusion coefficient κ is independent of particle energy. The variables may then be separated by putting $U(r, T) = f(T) R(\zeta)$, where $\zeta = vr/\kappa$ and $\zeta_0 = vR/\kappa$, yielding the two equations

$$\frac{T}{f}\frac{df}{dT} = -1 - i\alpha \tag{3}$$

$$\frac{d^2R}{d\zeta^2} + \left(\frac{2}{\zeta} - 1\right)\frac{dR}{d\zeta} - 2\left(1 + i\frac{n\alpha}{3}\right)\frac{R}{\zeta} = 0 \tag{4}$$

where α is the separation constant. It was shown that the general solution in the absence of sources and sinks in $r < R$ is

$$U(r,T) = \frac{T_0}{T}\int_{-\infty}^{+\infty} d\alpha\, g(\alpha) \exp\left(-i\alpha \ln\frac{T}{T_0}\right) {}_1F_1\left[2\left(1+\frac{in\alpha}{3}\right); 2; \zeta\right] \tag{5}$$

where ${}_1F_1$ is the usual confluent hypergeometric function.

The Green's function is readily obtained from the general expression (5), and is particularly instructive because it traces the deceleration of individual particles. Put $U(R,T) = \delta(T - T_0)$, representing unit density of monoenergetic particles $T = T_0$ in interstellar space, with free escape of all decelerated particles back into interstellar space from $r = R$. The integral in (5) is a Fourier transform between α and $\ln T/T_0$. Inverting the transform gives $g(\alpha)$, which upon substitution into (5) gives the Green's function

$$G(r, T, T_0) \tag{6}$$

$$= \frac{1}{2\pi T} \int_{-\infty}^{+\infty} d\alpha \exp\left(-i\alpha \ln \frac{T}{T_0}\right) \frac{{}_1F_1\left[2(1+in\alpha/3); 2; \zeta\right]}{{}_1F_1\left[2(1+in\alpha/3); 2; \zeta_0\right]} .$$

The cosmic ray distribution $U(r, T)$ for any arbitrary interstellar spectrum $F_0(T)$ is then

$$U(r,T) = \int_0^{\infty} dT_0 \, F_0(T_0) \, G(r, T, T_0) . \tag{7}$$

The main computational problem is the integration in (6). This was done (Parker, 1965) for $\zeta_0 >> 1$, yielding

$$G(0, T, T_0) \sim \frac{3}{2n T_0} \zeta_0^2 \left(\frac{T}{T_0}\right)^{3/n-1} \exp\left\{-\zeta_0\left[1 + \left(\frac{T}{T_0}\right)^{3/2n}\right]\right\} \tag{8}$$

valid for $\zeta_0 > 5$. A plot of $G(0, T, T_0)$ is given in the eariler paper, along with a plot of the resulting mean particle energy $\langle T \rangle$ as a function of ζ_0 . No application of the general Green's function to a specific cosmic ray spectrum was given in the earlier paper.

III Effect on the Energy Spectrum

The present paper carries on from the previous calculations of the deceleration of a monoenergetic bunch of particles. The principle aim is to illustrate the effect on the cosmic ray spectrum. Two complementary illustrations are given: One for a simple power law cosmic ray spectrum and the other for a general cosmic ray spectrum with $Rv/\kappa << 1$.

A. Power Law: If the interstellar cosmic ray energy spectrum can be written as a simple power law $F_0 = CT^{-\gamma}$, then it is readily shown from equation (2) that the form of the spectrum is preserved throughout the solar wind. The solution is then

$$U(r,T) = CT^{-\gamma} \frac{{}_1F_1\{2[1+n(\gamma-1)/3]; 2; \zeta\}}{{}_1F_1\{2[1+n(\gamma-1)/3]; 2; \zeta_0\}} . \tag{9}$$

If $\zeta_0 << 1$, then

$$U(r,T) = CT^{-\gamma}\{1 - [1+n(\gamma-1)/3](\zeta_0 - \zeta) + O(\zeta_0^2)\} \tag{10}$$

If $\zeta_0 >> 1$, the asympotic expansion for the confluent hypergeometric function gives

$$U(0,T) \sim C T^{-\gamma} \Gamma\left\{2\left[1+n(\gamma-1)/3\right]\right\} \frac{\exp(-\zeta_0)}{\zeta_0^{2n(\gamma-1)/3}} \qquad (11)$$

at the origin $r = 0$.

It is of interest to compare these results at a fixed energy T with the value $C T^{-\gamma} \exp(\zeta - \zeta_0)$ that is derived from the diffusion equation if deceleration is ignored. If $\gamma < 1$, it is readily seen from both (10) and (11) that the deceleration increases the particle density at any given energy T. If $\gamma < 1 - 3/n$ the increase is so large as to more than compensate the decrease $\exp(-\zeta_0)$ caused by convection. If $\gamma > 1$ the deceleration decreases the particle density at any given T. As an example, above 10^{10}ev the cosmic ray spectrum has $\gamma \cong 2.5$. With $n = 1$ it is readily seen from (10) that when $\zeta_0 << 1$ the deceleration contributes a 50 percent additional decrease beyond the $1 - (\zeta_0 - \zeta)$ produced by convection alone. It is readily seen from (11) that when $\zeta_0 >> 1$ deceleration contributes the additional factor $2/\zeta_0$ to the decrease. Since presumably ζ_0 is not more than the order of one, the deceleration produces about one third of the total depression of the cosmic ray density at any given energy above 10^{10} ev. At lower energies the deceleration has less effect and may in fact produce an increase at low energies where $\gamma < 1$, as is perhaps better illustrated by the next example.

B. Small Rv/κ : When $Rv/\kappa \equiv \zeta_0 << 1$, a more general treatment of the problem is possible. For then the energy change ΔT of the individual particle is small and to a first approximation the energy dependence of the diffusion coefficient is a second order effect which can be neglected. The analysis is no longer restricted to the special case that $d\kappa/dT = 0$. Further, the calculation is simplified enough to permit the treatment of a general cosmic ray spectrum $F_0(T)$.

The main task when $\zeta_0 << 1$ is to evaluate the integral in (6). This computation is carried out in the appendix by noting that the confluent hypergeometric function reduces to a Bessel function, and then evaluating the integral as the sum of the residues of the poles enclosed by the contour. The resulting energy distribution of an initially monoenergetic group of particles is plotted in Fig. 1. The mean energy change of the particles at the origin is

$$\langle \Delta T(0) \rangle = \langle T(0) \rangle - T_0$$

$$= -\frac{n}{3} T_0 \zeta_0 \tag{12}$$

where $\langle T(r) \rangle$ is the mean energy change of the particles at $r\,(<R)$.

It is a straight forward matter to compute the effect of ΔT on the energy spectrum. Substituting (12) into (1) gives

$$F(T) = F_0(T)\left\{1 - \zeta_0\left[1 - \frac{n}{3}\left(\frac{T}{F}\frac{dF}{dT} + \frac{1}{n\zeta_0}\frac{d}{dT}(n\zeta_0 T)\right)\right]\right\}$$

to the order considered. The first term in the square brackets represents the reduction by convection. The remaining terms represent the energy loss. Suppose that in the extreme relativistic range $F_0(T) \propto T^{-\gamma}$ and $\kappa \propto T^{+\beta}$. Then $n = 1$ and $\zeta_0 \propto T^{-\beta}$,

$$F(T) = F_0(T)\left\{1 - \zeta_0(T)\left[1 + \tfrac{1}{3}(\gamma + \beta - 1)\right]\right\}$$

which agrees with (10) if we make κ independent of energy $(\beta = 0)$.

The term $\frac{1}{3}(\gamma + \beta - 1)$ represents the effect of energy loss. As pointed out in the previous example, the energy loss contributes about half as much as the convection to the decrease of the cosmic ray spectrum at any given extreme relativistic particle energy. If β is large at very high energies, say $\beta = 2$, then the deceleration might there contribute more than the convection, but $\zeta_0(T)$, and the resulting decrease of the spectrum, is then small anyway.

At nonrelativistic energies γ may be zero, or negative, and β in some cases (Bryant, et al, 1965, and Gloeckler, 1965) of the order of 0.5 or less. The deceleration then contributes an increase.

It is interesting to see what happens at a maximum in the cosmic ray spectrum. Suppose that the interstellar cosmic ray spectrum has a maximum at $T = T_1$ with the simple parabolic form

$$F_0(T) = C\left[1 - \frac{(T-T_1)^2}{T_2^2}\right]$$

in some near neighborhood of the maximum. Then it is readily shown that

$$F(T) = F_0(T)\left\{1 - \zeta_0\left[1 - \frac{n}{3}\,\frac{(3T-T_1)(T-T_1) - T_2^2}{(T-T_1)^2 - T_2^2}\right]\right\}$$

if the energy dependence of ζ_0 is neglected. The maximum of $F(T)$ shifts down with the individual particles from T_1 to

$T_{max} = T_1\left(1 - n\zeta_0/3\right)$. The maximum intensity is reduced from C to

$$F_{max} = C\left\{1 - \zeta_0\left(1 - \frac{n}{3}\right)\right\}$$

to the order considered. Note that the deceleration contributes an increase of $J_0 n/3$ to the cosmic ray density at the maximum.

The general example for $Rv/\kappa << 1$ is useful up to about $Rv/\kappa = 0.5$. For Rv/κ larger than one, the form (8) can be used. Fig. 2. shows the mean particle energies at the origin computed from (8) and (12). The broken lines represent an arbitrary interpolation between the calculated curves for large and small Rv/κ.

V Discussion

We have presented the foregoing calculations to illustrate the effect of adiabatic deceleration in combination with outward convection on the mean cosmic ray spectrum in the solar system. The simple examples given here do not provide a general formula for the cosmic ray spectrum in terms of the spectrum in interstellar space, as can be written down for convection alone. But it is hoped that between the example of the power law spectrum with Rv/κ independent of particle energy, and the example of the general spectrum with $Rv/\kappa << 1$ but energy dependent, the important physical effects have been illustrated. The examples show that outward convection contributes about two thirds of the total cosmic ray depression at a given particle energy above a few times 10^9ev, with adiabatic deceleration contributing the rest. At lower energies the adiabatic deceleration contributes less, in some cases actually cancelling part of the decrease caused by convection. It must be remembered that adiabatic deceleration contributes by itself no net decrease to the total cosmic ray particle density, so that the mean contribution over the entire spectrum is zero.

The convection leads to a reduction of cosmic ray density at all energies.

It may prove desirable eventually to work out more complicated examples than given in the present paper. Numerical methods will have to be used in most cases because the variables in the Fokker-Planck equation do not separate. The actual anisotropy and inhomogeneity of the diffusion coefficient may be included too. It was pointed out (Parker, 1965) that the anisotropy has the same effect on the depression of the cosmic ray density in the inner solar system as a radial dependence of κ. It will be interesting to see if the two effects can be separated short of an extended survey through space. Singer, et al. (1962) have studied deceleration in some detail omitting the convection term from the Fokker-Planck equation.

Appendix

In order to carry out the integration indicated in (6) to obtain the Green's function $G(r, T, T_0)$ it is necessary to determine the properties of the confluent hypergeometric function as a function of the parameter α. This is most easily done by noting that when $\zeta < \zeta_0 \ll 1$, the coefficient of $dR/d\zeta$ in (4) may be approximated by $2/\zeta$ over the entire interval $(0, \zeta_0)$. The solution of the approximate form of the differential equation (4) is exactly $\zeta^{-1/2} J_1(\beta \zeta^{1/2})$ where $\beta^2 = -8(1 + in\alpha/3)$.

Thus

$$ {}_1F_1\left[2(1+in\alpha/3); 2; \zeta\right] \cong \frac{2}{\beta \zeta^{1/2}} J_1(\beta \zeta^{1/2}) \tag{A1} $$

(the confluent hypergeometric function has the value one at $\zeta = 0$).
Note that this equality depends only upon $\zeta \ll 1$ and is valid for all values of α no matter how large. If follows that

$$ U(r, T) \cong \frac{1}{2\pi T}\left(\frac{\zeta_0}{\zeta}\right)^{1/2} \int_{-\infty}^{+\infty} d\alpha \exp\left(-i\alpha \ln\frac{T}{T_0}\right) \frac{J_1(\beta \zeta^{1/2})}{J_1(\beta \zeta_0^{1/2})} . \tag{A2} $$

This integral is readily evaluated using Cauchy's theorem.

It is evident from the definition of β that β has the asymptotic form $(8n\alpha/3)^{1/2} \exp i(3\pi/4)$ as $\alpha \to +\infty$ and $|8n\alpha/3|^{1/2} \exp(i\pi/4)$ as $\alpha \to -\infty$. At $\alpha = 0$ we have $\beta = i\, 2^{3/2}$. On the asymptotic branch $(8n\alpha/3)^{1/2} \exp(i\, 3\pi/4)$ of β the asymptotic form for the Bessel functions gives

$$\frac{\zeta^{1/2} J_1(\beta\zeta^{1/2})}{\zeta_0^{1/2} J_1(\beta\zeta_0^{1/2})} \sim \exp\left[(1+i)\left(\frac{4n\alpha}{3}\right)^{1/2}\left(\zeta^{1/2} - \zeta_0^{1/2}\right)\right]$$

and on $(8n\alpha/3)^{1/2} \exp(i\pi/4)$

$$\frac{\zeta^{1/2} J_1(\beta\zeta^{1/2})}{\zeta_0^{1/2} J_1(\beta\zeta_0^{1/2})} \sim \exp\left[(1-i)\left|\frac{4n\alpha}{3}\right|^{1/2}\left(\zeta^{1/2} - \zeta_0^{1/2}\right)\right].$$

Since $\zeta_0 > \zeta$, it is evident that these factors converge like $\exp(-|\alpha|^{1/2})$ as $\alpha \to \pm\infty$. Hence the integrand converges properly along the real axis.

The integrand has simple poles at the zeros of $J_1(\beta\zeta_0^{1/2})$, except for $\beta = 0$ which is a regular point. The zeros of $J_1(\beta\zeta_0^{1/2})$ occur only for the real values 3.829, 7.016, 10.173, 13.324, . . . of the argument $\beta \zeta_0^{1/2}$. If β_m denotes the value of β at the $m\underline{th}$ zero of $J_1(\beta \zeta_0^{1/2})$, it follows that the zeros lie at the values of α

$$\alpha_m = i \frac{3}{8n} \left(\beta_m^2 - 8\right).$$

For $\zeta_0 \ll 1$ is is readily shown that $\beta_m \gg 1$ for all m . Hence the poles lie along the positive imaginery α-axis.

Consider how the contour may be closed. If the imaginery part of α is large and positive, then

$$\beta \cong \left(\frac{8n}{3}\right)^{1/2} \left[|I_m \alpha|^{1/2} - i \frac{Re\, \alpha}{2|I_m \alpha|^{1/2}} \right].$$

The Bessel functions do not go to zero as $I_m \alpha \rightarrow +\infty$ so the integrand vanishes as $I_m \alpha \rightarrow +\infty$ if and only if $T < T_0$. If the imaginery part of α is large and negative, then β has again the above form. The Bessel functions both grow large as $exp\, |I_m \alpha|^{1/2}$, so that the integrand vanishes as $I_m \alpha \rightarrow -\infty$ if and only if $T > T_0$. It is evident, therefore, that when $T > T_0$ the contour

should be closed around the lower half plane. There are no poles in the lower half plane, with the result that $U(r, T) = 0$ for $T > T_0$.
This is to be expected since all the particles are decelerated. None are accelerated.

When $T < T_0$ the contour must be closed round the upper half plane, giving $U(r, T)$ as the sum of the residues of the poles. In the vicinity of the pole α_m write $\alpha = \alpha_m + \epsilon$. It is readily shown that

$$J_1(\beta \zeta_0^{1/2}) = -i \frac{4n\epsilon}{3\beta_m} \zeta_0^{1/2} J_0(\beta_m \zeta_0^{1/2})$$

in the neighborhood of the pole so that the sum of the residues gives

$$U(r,T) = -\frac{3}{4nT\zeta^{1/2}} \sum_{m=1}^{\infty} \left(\frac{T}{T_0}\right)^{3(\beta_m^2+8)/8n} \frac{\beta_m J_1(\beta_m \zeta^{1/2})}{J_0(\beta_m \zeta_0^{1/2})} . \tag{A3}$$

In particular, the energy distribution at the origin is

$$U(0,T) = -\frac{3}{8nT} \sum_{m=1}^{\infty} \left(\frac{T}{T_0}\right)^{3(\beta_m^2+8)/8n} \frac{\beta_m^2}{J_0(\beta_m \zeta_0^{1/2})} . \tag{A4}$$

The series (A3) and (A4) converge if, and only if $T < T_0$. There are however some difficulties which arise with the convergence when average values are computed. For instance, the particle density is

$$N(r) = \int_0^{T_0} dT\, U(r,T)$$

$$= -\frac{2}{\zeta^{1/2}} \sum_{m=1}^{\infty} \frac{\beta_m J_1(\beta_m \zeta^{1/2})}{(\beta_m^2 + 8)\, J_0(\beta_m \zeta_0^{1/2})} \quad . \tag{A5}$$

This series converges for $0 < \zeta < 1$, as is readily shown from the asymptotic form of the individual terms for large m $(\beta_m \zeta^{1/2} >> 1)$. But for $\zeta = 0$ the situation is a little more complicated. Write $\beta_m^2 \zeta_0 \equiv \mu_m^2$, so that μ_m is the m<u>th</u> root of $J_1(\mu_m)$. Then in the limit of small ζ_0

$$N(r) = -\sum_{m=1}^{\infty} \frac{\mu_m^2}{(\mu_m^2 + 8\zeta_0)\, J_0(\mu_m)}$$

(A6)

$$= -\sum_{m=1}^{\infty} \frac{1}{J_0(\mu_m)} + 8\zeta_0 \sum_{m=1}^{\infty} \frac{1}{\mu_m^2 J_0(\mu_m)} + \cdots \quad .$$

The first term is an infinite series whose sum does not exist. It represents the unit density which obtains in the absence of an interaction between the wind and the cosmic rays $(\zeta_0 = 0)$. The second series converges and represents the reduction of the cosmic ray intensity to first order in ζ_0. Numerically we find

$$\sum_{m=1}^{\infty} \frac{1}{\mu_m^2 J_1(\mu_m)} = -0.125, \tag{A7}$$

so that the reduction is

$$\Delta N(r) = -1.00\, \zeta_0 \tag{A8}$$

to this accuracy, in agreement with the well known result $\exp(-\zeta_0)$ obtained when the Fokker-Planck equation is summed over T before being integrated.

The mean particle energy is

$$\langle T(r) \rangle = \frac{1}{N(r)} \int_0^{T_0} dT\, T\, U(r,T)$$

(A9)

$$= -\frac{2T_0}{\zeta^{1/2}} \exp \zeta_0 \sum_{m=1}^{\infty} \frac{\beta_m J_1(\beta_m \zeta^{1/2})}{(\beta_m^2 + B + B_m/3)\, J_0(\beta_m \zeta^{1/2})}$$

so that the small mean energy loss is

$$\langle -\Delta T(r) \rangle = T_0 - T(r)$$

$$= -\frac{16 n T_0 \exp \zeta_0}{3 \zeta^{1/2}} \sum_{m=1}^{\infty} \frac{\beta_m J_1(\beta_m \zeta^{1/2})}{(\beta_m^2 + 8)(\beta_m^2 + 8 + 8n/3)\, J_0(\beta_m \zeta_0^{1/2})}$$

$$\cong -\frac{16 n T_0}{3 \zeta^{1/2}} \sum_{m=1}^{\infty} \frac{J_1(\beta_m \zeta^{1/2})}{\beta_m^3 J_0(\beta_m \zeta_0^{1/2})} \tag{A10}$$

$$= -\frac{16 n T_0}{3} \left(\frac{\zeta_0}{\zeta}\right)^{1/2} \zeta_0 \sum_{m=1}^{\infty} \frac{J_1(\mu_m \zeta^{1/2}/\zeta_0^{1/2})}{\mu_m^3 J_0(\mu_m)} \;. \tag{A11}$$

At the origin, this reduces to

$$\langle -\Delta T(0)\rangle = -\frac{8n T_0}{3}\zeta_0 \sum_{m=1}^{\infty}\frac{1}{\mu_m^2 J_1(\mu_m)}$$

$$= +\frac{n}{3} T_0 \zeta_0 \qquad \text{(A12)}$$

upon using (A7) for the numerical value of the series. It is evident from (A11) that for any fixed value of r/R, the energy reduction is directly proportional to ζ_0. At the origin the fractional energy change is simply $(n/3)\,\zeta_0$.

¶ It is convenient for purposes of plotting to write the series (A3) and (A4) in a more invariant form. It is readily seen from (A12) that the particles undergo very little deceleration when $\zeta_0 \ll 1$ so that $U(r, T)$ is negligible except in a small neighborhood, of the order of a few times $\langle \Delta T\rangle$, of T_0. So put $T = T_0 - s\langle -\Delta T\rangle = T_0(1 - sn\,\zeta_0/3)$, where s is a number of the order of unity in the region of physical interest. In this region write

$$\left(\frac{T}{T_0}\right)^{3\beta_m^2/8n} = \left(1 - sn\,\zeta_0/3\right)^{3\mu_m^2/8n\,\zeta_0}$$

$$= \left[\left(1 - sn\,\zeta_0/3\right)^{3/sn\,\zeta_0}\right]^{s\mu_m^2/8}$$

$$= \exp(-s\mu_m^2/8)$$

in the limit as $\zeta_0 \longrightarrow 0$. Then (A3) and (A4) may be written

$$U(r,T) \cong -\frac{3}{4nT_0\zeta_0}\left(\frac{R}{r}\right)^{1/2}\sum_{m=1}^{\infty}\frac{\mu_m J_1(\mu_m r^{1/2}/R^{1/2})\exp(-s\mu_m^2/8)}{J_0(\mu_m)} , \qquad \text{(A13)}$$

$$U(0,T) = -\frac{3}{8nT_0\zeta_0}\sum_{m=1}^{\infty}\frac{\mu_m^2 \exp(-s\mu_m^2/8)}{J_0(\mu_m)} \qquad \text{(A14)}$$

in terms of the energy displacement $s\langle -\Delta T\rangle$. Fig. 1 is plotted from (A14).

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Figures

Fig. 1. The energy distribution at the origin $r = 0$ is shown in terms of the parameter $s = \Delta T(0) / \langle \Delta T(0) \rangle$ for monoenergetic particles with $T = T_0$ introduced steadily at $r = R$.

Fig. 2. The mean particle energy at the origin is shown for particles introduced with energy $T = T_0$ at $r = R$. The straight lines represent the asymptotic form for $Rv/\kappa << 1$. The curved lines represent the asymptotic form for $Rv/\kappa >> 1$. The broken lines represent an arbitrary interpolation across intermediate values of Rv/κ.

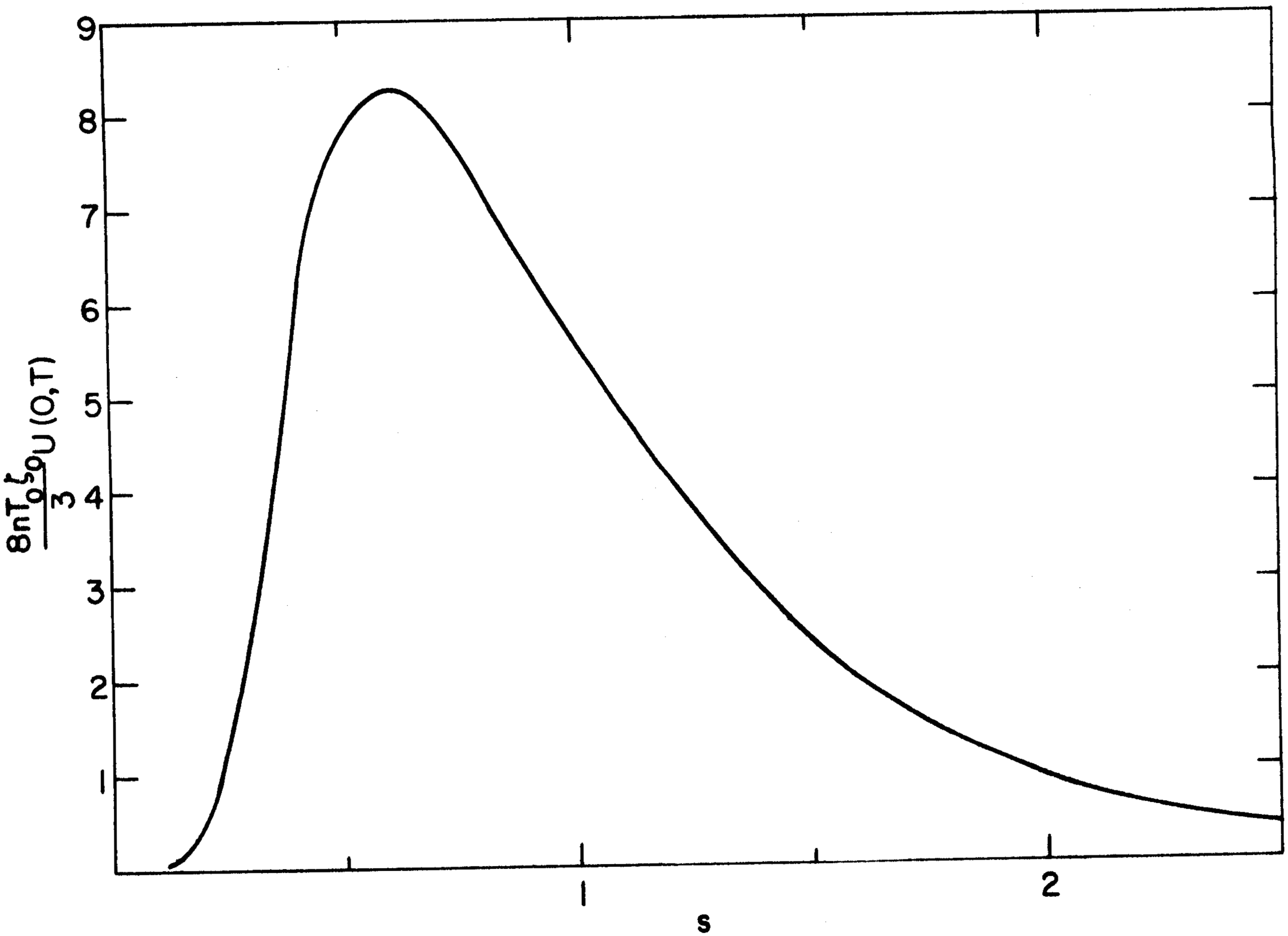

Fig. 1

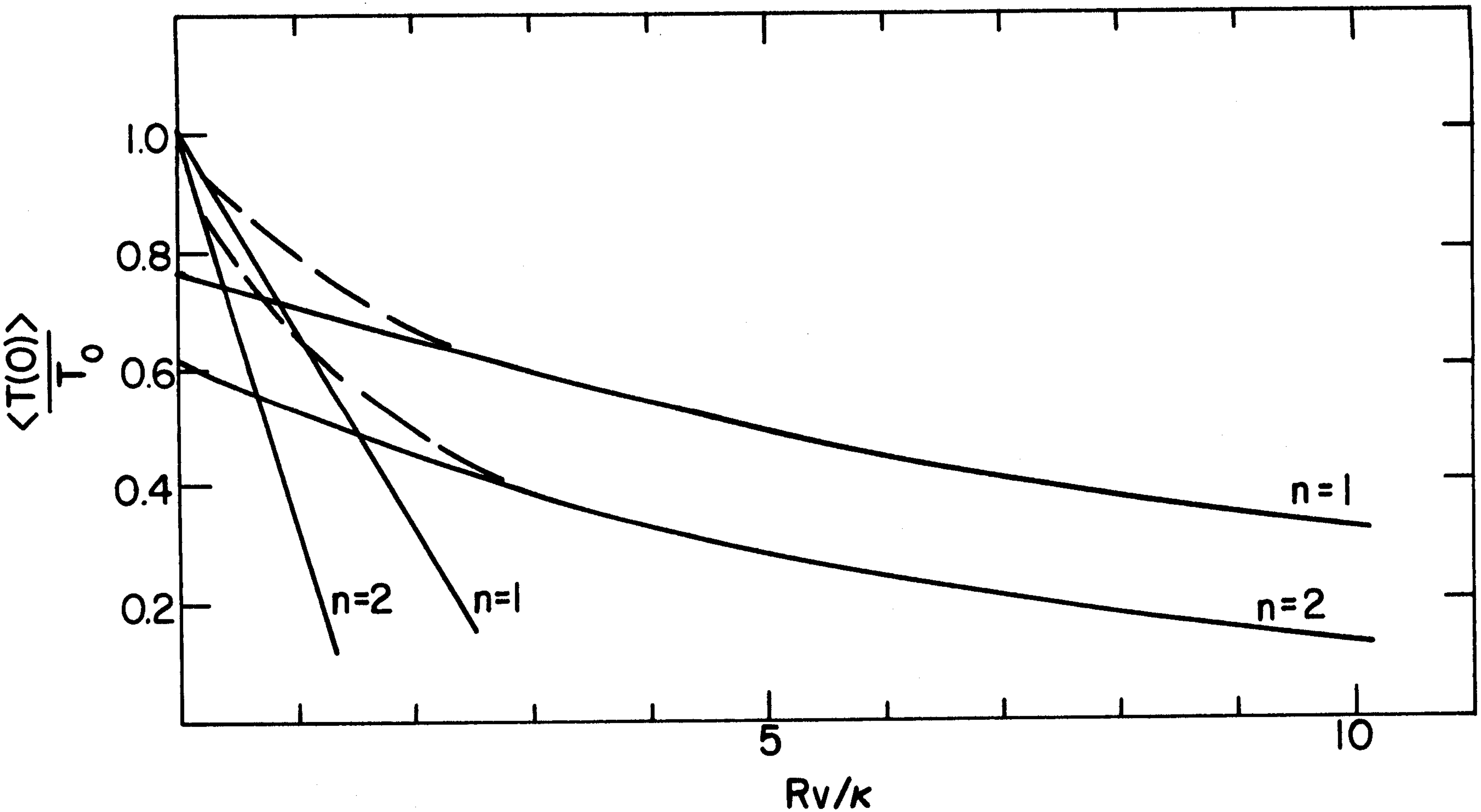

Fig. 2