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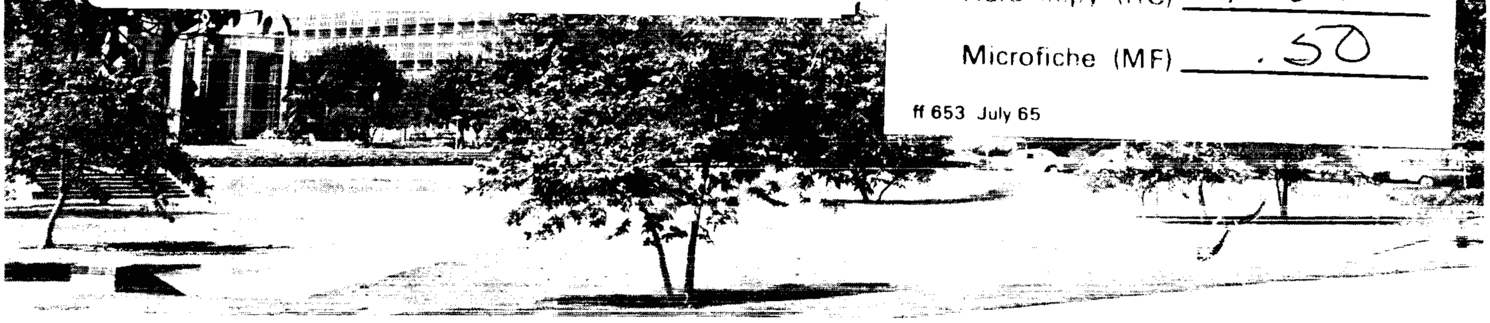

Propagation of Ion Acoustic Waves
Along Cylindrical Plasma Columns

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Propagation of Ion Acoustic Waves Along Cylindrical Plasma Columns

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It has been found by Wong, Motley and D'Angelo[1] that the propagation of ion acoustic waves along an external axial magnetic field between 4kg and 14 kg in highly ionized plasmas is not affected by the size and shape of the cylindrical plasma column. The experimental results indeed agree with a plane wave analysis even though the plasma radius R was of the order of the wavelength λ. Neither changing the plasma diameter by a half nor altering the density profile through varying the axial magnetic field (4-14kg) seemed to have any effect on the phase velocity and damping. On the other hand, Little[2] has observed the effects of boundaries on ion acoustic waves in a weakly ionized plasma in the presence of a weak magnetic field (less than 50 gauss) with $\lambda \approx R$. Malmberg and Wharton[3] also found that in the propagation of electrostatic electron oscillations the size of the plasma column must be taken into account when $kR \approx 1$. These experimental results led us to re-examine the theory of ion acoustic waves to include transverse boundary effects.

Making the electrostatic approximation for ion acoustic waves $\underline{E} = -\nabla\phi$ and $\nabla \cdot \underline{D} = 0$, we can derive a wave equation for the wave potential ϕ:

$$\varepsilon_{\perp} \nabla_{\perp}^2 \phi - \varepsilon_{zz} k^2 \phi = 0 \qquad (1)$$

where $\varepsilon_\perp$ and ε_{zz} are elements of the dielectric tensor $\underline{\underline{\varepsilon}}$

$$\underline{\underline{\varepsilon}} = \begin{vmatrix} \varepsilon_\perp & \varepsilon_2 & 0 \\ -\varepsilon_2 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_{zz} \end{vmatrix}$$

and k = the wave number in the direction of propagation.

From a theory taking the ions as collisionless and the electrons as a fluid (as appropriate to WMD's experiment[1]) we obtained

$$\varepsilon_{zz} = 1 - \frac{k_D^2}{k^2} Z' \left(\frac{\omega}{kv_i}\right) + 2 \frac{T_i}{T_e} \frac{k_D^2}{k^2}$$

$$\varepsilon_\perp \doteqdot 1 + k_D^2 \rho_i^2 \qquad \text{For} \quad \omega < \omega_{ci} \ll \omega_{ce} \quad \text{and} \quad \frac{B^2}{8\pi KT} \gg 1$$

where $k_D^2 = \dfrac{4\pi ne^2}{KT_i}$ $\qquad$ ρ_i = ion cyclotron radius

and ω_{ci} ω_{ce} the ion and electron cyclotron frequency respectively. Neglecting the electron inertia in the ion wave, it can be shown that

$$\frac{e\phi}{KT} = \frac{n}{n_o}$$

where n_o = the unperturbed plasma density

n = the perturbed plasma density

Equation (1) can thus be written as

$$\nabla_\perp^2 n - k^2 \frac{\varepsilon_{zz}}{\varepsilon_\perp} n = 0 \tag{2}$$

We shall make use of the following experimental observation in our selection of the appropriate solution for equation (2):

1) $n_o(r) \simeq J_o\left(\frac{\chi_1 r}{R}\right)$, where χ_1 = 1st root of the zeroth order Bessel function

R = plasma radius.

2) $\frac{n}{n_o} \simeq$ constant along the radial direction.

3) azimuthal symmetry.

The solution of equation (2) will then be $n(r) \propto J_o\left(\sqrt{\frac{-k^2\varepsilon_{zz}}{\varepsilon_\perp}}\, r\right)$

with the condition that $k^2\varepsilon_{zz} = -\frac{\varepsilon_\perp \chi_1{}^2}{R^2}$

This condition can be rewritten as

$$1 - \frac{k_D{}^2}{k^2} Z'\left(\frac{\omega}{kv_i}\right) + 2\frac{T_i}{T_e}\frac{k_D{}^2}{k^2} + \frac{\varepsilon_\perp \chi_1{}^2}{k^2R^2} = 0$$

Under the experimental conditions of WMD

$k << k_D$ and $T_i = T_e$

we have $Z'\left(\frac{\omega}{kv_i}\right) \simeq 2 + \frac{\varepsilon_\perp \chi_1{}^2}{k_D{}^2R^2}$

The effects of transverse boundary conditions are important only if

$$\left|\frac{\varepsilon_\perp \chi_1{}^2}{k_D{}^2R^2}\right| \simeq 2 \qquad (3)$$

A similar condition is also arrived at when both ions and electrons are treated as fluids.

In WMD's experiment

$T_e = T_i = 2500°K$

$n \simeq 10^{11}$ cm/sec

$R = 1.5$ cm,

$k_D \cong 6 \times 10^2\ \text{cm}^{-1}$

$4 \times 10^{-2}\ \text{cm} < \rho_i < 2 \times 10^{-1}\ \text{cm}$ for $14\ \text{kg} > B > 4\ \text{kg}$

we have $\varepsilon_\perp \simeq k_D^2 \rho_i^2$ since $k_D \rho_i >> 1$

Condition (3) reduces to $$\frac{\rho_i^2}{R^2} = \frac{2}{x_1^2} \qquad (4)$$

Thus we come to the conclusion that the transverse boundary conditions are important if the ion cyclotron radius (rather than the wavelength of the ion wave) is comparable to the plasma radius. This agrees with the experimental finding of WMD: the independence of propagation characteristics on the size of the plasma column in the presence of a magnetic field such that $\rho_i << R$. Examination of condition (3) shows that the transverse boundary effects on ion acoustic waves can be vitiated by the application of a sufficiently high magnetic field which reduces the boundary contribution with respect to the electron contribution in ion acoustic waves. In contrast, the longitudinal electron plasma oscillations are sensitively dependent upon the boundary conditions since the corresponding ion contribution to the electron oscillations is zero. The importance of such dependence has been shown experimentally[3] and theoretically.[4,5] In summary, the independence of ion acoustic waves on transverse boundary conditions is attainable under typical laboratory conditions and further simplifies its laboratory investigation and increases its role in plasma diagnosis.[6]

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