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MASS TRANSFER COOLING IN LAMINAR BOUNDARY LAYERS WITH HYDROGEN INJECTED INTO NITROGEN AND CARBON DIOXIDE STREAMS

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ABSTRACT

Two component, laminar boundary layers on a surface with zero pressure gradient, including the effects of variable properties, are studied. Hydrogen is injected from the stationary, porous flat plate into a free stream of nitrogen or carbon dioxide. The partial differential equations describing the velocity, concentration and temperature distributions are first transformed into total differential equations then into integral forms which can be solved numerically on an electronic digital computer. The two situations treated are the constant wall temperature and the adiabatic wall. The Mach numbers considered are 0, 4, 8 and 12, the free stream temperature t_∞ ranges from 218°K to 1110°K and the wall temperature to free stream temperature ratio $T_w = t_w/t_\infty$ ranges from 0.5 to 6. In addition to velocity, concentration and temperature profiles, the heat transfer and skin friction are also obtained. The final results are presented in simplified curves which are useful for design calculations.

Author

INTRODUCTION

The effectiveness of mass transfer cooling for protecting surfaces moving at high velocity through the earth's atmosphere is well established. A considerable number of analytical and experimental studies of this process have been undertaken and accordingly it is now possible to predict with reasonable accuracy the heat transfer and skin friction associated with the mass transfer cooling of a body moving through the air environment. If the main stream is not air but some other gas such as nitrogen or carbon dioxide, there is some doubt as to the direct applicability of the available results. Since we expect to encounter such non-air environments when entering the atmospheres of other planets such as Mars or Venus it is important that we carry out pertinent analytical and experimental studies. With this goal in mind, the present analytical study assumes the free stream to be either nitrogen or carbon dioxide and the injected gas to be hydrogen; the geometry is taken to be a flat plate. Such a flat plate is a simple meaningful geometry from which we can get considerable physical insight into the details of the mass transfer cooling process.

ANALYTICAL FORMULATION

The formulation of the problem is straightforward. The well known boundary layer equations expressing conservation of mass, momentum, species and energy are transformed from partial differential equations into ordinary differential equations. The following restrictions have been imposed: (1) the flow is steady and laminar (2) radiation, thermal diffusion and diffusion thermo effects are neglected (3) no dissociation occurs. The physical properties of the binary gas mixture are assumed to be functions of the local temperature and concentration; the density and specific heat of the mixture are assumed to be given by the ideal gas laws while the transport terms (i.e. viscosity, thermal conductivity and ordinary diffusion coefficient) are calculated by the mixture laws of Ref.1. Complete details on the properties are given in Ref.2.

The transformed equations take the following form:

Momentum: $$\left[\left(\frac{f'}{\varphi_\rho \varphi_\mu}\right)'\right]' + f\left(\frac{f'}{\varphi_\rho \varphi_\mu}\right)' = 0$$

Diffusion: $$\left(\frac{W'}{S_c}\right)' + Scf\left(\frac{W'}{S_c}\right) = 0$$

Energy: $$\left(\frac{\varphi_k T'}{\varphi_\mu}\right)' + Pr_\infty \frac{\varphi_\mu}{\varphi_k} \left(\varphi_c f + \varphi_{c12} \frac{W'}{Sc}\right)\left(\frac{\varphi_k T'}{\varphi_\mu}\right)$$

$$+ \frac{Pr_\infty}{4} (\gamma_\infty - 1) M_\infty^2 \left[\left(\frac{f'}{\varphi_\rho \varphi_\mu}\right)'\right]^2 = 0$$

The boundary conditions become:

For $\eta = 0$:

$$f' = 0$$

$$f = f_w = -\frac{2\rho_w v_w}{\rho_\infty u_\infty}\sqrt{Re_{x,\infty}} = \text{constant}$$

$$W = W_w = \text{constant}$$

$$T = T_w = \text{constant or } \left.\frac{\partial T}{\partial \eta}\right)_w = 0$$

For η ∞

$$\lim f' = 2$$

$$\lim W = 0$$

$$\lim T = 1$$

The first boundary condition at the wall ($f' = o$) expresses the no-slip condition; the second expression brings out the fact that the normal velocity at the surface must either be zero or it must vary $1/\sqrt{Re_x}$ if we are to have a system of ordinary differential equations (i.e. if f is to be a function of η only). The actual magnitude of the dimensionless mass transfer rate f_w may be arbitrarily selected (within the limitations of boundary layer theory) or alternatively we can select the magnitude of the wall mass fraction. These two quantities - the mass flow of the injected coolant and the wall mass fraction - are not independent but are related by the expression:

$$f_w = W_w'/Sc_w\ (1-W_w)$$

This relationship results from the condition that there is no net flow of the main stream gas through the plate surface. In view of this restriction we can select either the wall mass fraction or the blowing rate but not both.

Finally the thermal boundary conditions correspond to the two cases studied: (1) The constant wall temperature (2) The recovery temperature case*. The governing equations were transformed into integral equations for the unknown velocity, mass fraction and temperature distribution taking into account the boundary conditions. The resulting equations were solved by successive approximation. Initial velocity, concentration and temperature profiles were used to obtain new profiles and the procedure was repeated until the equations were satisfied

The calculations were carried out for both the hydrogen-nitrogen and the hydrogen-carbon dioxide binary boundary layers. The range of Mach number and wall and free stream temperatures covered are shown in Tables I and II. In all cases the wall mass fraction was allowed to take on values of 0, 0.1, 0.2, 0.4 and 0.6; in the constant wall temperature cases of Table I a wall concentration of 0.8 was also studied

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*As used herein the recovery temperature is defined as that temperature corresponding to a zero temperature gradient at the wall

RESULTS

Only representative results will be presented here; readers interested in complete details are referred to Ref.2.

Typical boundary layer, temperature, mass fraction and velocity profiles are shown in Figs.1 and 2. Fig.1-A represents the boundary layer profiles for a hydrogen-nitrogen mixture at $M_\infty = 0$ (no dissipation), $T_w = 6$, $t_\infty = 218^\circ$ K. The temperature within the boundary layer is always lower than the wall temperature, so that the boundary layer is cooling the wall. The temperature gradient is reduced with increasing wall mass fraction W_w. This result is to be expected as a gas with finite heat capacity is being injected at the wall temperature. The thermal boundary layer thickness is obviously increased by increasing W_w. Fig.1-B shows a similar result for a hydrogen-carbon dioxide mixture under the same condition as in Fig.1-A; the thickness of the boundary layer of hydrogen-nitrogen mixture is less than that of hydrogen-carbon dioxide.

Fig.2-A represents the boundary layer profiles for hydrogen-nitrogen mixture at $M_\infty = 12$, $T_w = 6$, $t_\infty = 218^\circ$K. For each blowing rate, there is a maximum temperature within the boundary layer. This maximum temperature is reduced and moved away from the wall with increasing wall mass fraction W_w (i.e. increasing blowing rate), so that less and less heat is flowing to the wall from the fluid. Fig.2-B shows a similar result for a hydrogen-carbon

dioxide mixture under the same conditions as in Fig.2-A. It is found that at the same blowing rate, the maximum temperature within the boundary layer of the hydrogen-nitrogen mixture is always higher than that of the hydrogen-carbon dioxide mixture. As in the zero Mach number case, it was found that the boundary layer is thicker for the hydrogen-carbon dioxide mixture at $M_\infty=12$; this conclusion applies to all other constant wall temperature cases as well. From Figs.1 and 2, one can see that the maximum temperature and the thickness of the boundary layer are increased with increased free stream Mach number.

Normalized* skin friction coefficients, C_f/C_{fo} heat transfer q/q_o and recovery temperatures were obtained as a function of either the wall concentration or alternatively the blowing rate.

The dimensionless skin friction coefficient C_f/C_{fo} was found to be relatively insensitive to all parameters except the wall mass fraction, in all cases decreasing with increasing mass fraction. Figure 3 shows a summary of all the skin friction results for both mixtures and it can be seen that the dashed line represents all results with an accuracy of better than $\pm$ 15%.

The dimensionless heat transfer, q/q_o is not as well behaved as the skin friction coefficient. For example at low mass transfer

*The subscript o corresponds to the solid wall case under the same free stream conditions with the solid wall at the same temperature as the mass transfer cooled wall.

rates (i.e. at low values of the wall mass fraction) the heat transfer for the carbon dioxide may actually increase with the injection of hydrogen; this results from the fact that the thermal conductivity of the mixture increases with the injection of hydrogen and at low injection rates this can be the dominant factor. The normalized heat transfer, q/q_o, may not only take on values greater than unity, but may actually go to zero or change sign. This occurs if the selected value of the wall temperature is close to the recovery temperature. If the wall temperature is greater than the recovery temperature, heat flows from the wall to the fluid and correspondingly if the wall temperature is less than the recovery value, heat will flow to the wall. Figure 4 shows an extreme example of this behavior; here it may be seen that the heat flow goes to zero for the hydrogen-carbon dioxide binary system at $W_w \doteq 0.07$ for a Mach number of 4 when the free stream is 555° and the wall is at 1110° K. This corresponds to the recovery condition as is brought in Figure 5-B below.

The recovery temperatures are given in Figures 5-A and 5-B As anticipated, the influence of mass transfer (i.e. increasing the wall mass fraction) is to decrease the recovery temperature. It is also clear that the recovery temperature is much higher in the hydrogen-nitrogen boundary layer than in the hydrogen-carbon dioxide case. This can be explained by noting that the recovery temperature is given by the following formulation:

$$t_r = t_\infty + r(\gamma_\infty - 1)\frac{M_\infty^2}{2} t_\infty$$

The recovery factor is approximately the same for both mixtures; however, the value of γ_∞ for the low molecular weight gas nitrogen is higher than that of the carbon monoxide. Therefore we expect the recovery temperature of the hydrogen-nitrogen system to be higher.

GENERALIZED PRESENTATION OF RESULTS

It has proven possible to represent all of the analytical results on six curves. Figs.6, 7, 8. For each binary gas system C_f/C_{fo}, St/St_o and r/r_o are shown as functions of the dimensionless blowing parameter $f_w/\sqrt{C^*}$. The Chapman-Rubesin parameter C* is evaluated at the Eckert reference temperature. This same representation had earlier proven successful when the free stream gas was air (Ref.3). To apply these curves it is clearly necessary to know the solid wall values (i.e C_{fo}, St_o and r_o). Detailed inspection of the zero blowing situation has demonstrated that the Eckert reference temperature method is applicable when the free stream is either carbon dioxide or nitrogen.(Ref.4). To apply this method we evaluate all properties at the reference temperature

$$t_o^* = t_\infty + 0.50\,(t_w - t_\infty) + 0.22\,(t_{ro} - t_\infty).$$

The skin friction coefficient C_{fo}, the Stanton number, St_o and recovery factor r_o are then determined from the following relations:

$$C_{fo} = 0.664/\sqrt{u_\infty x/\nu^*}$$

$$C_{fo}/2\, St_o = (Pr^*)^{2/3}$$

$$r_o = \sqrt{Pr^*}$$

$$t_{ro} = t_\infty + r_o\, u_\infty^{\,2}/2\, C_{p\infty}$$

The use of the reference temperature method combined with Figures 6, 7 and 8 will yield values of skin friction coefficient, Stanton number and recovery factor which are sufficiently accurate for most engineering purposes.

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3. J. F. Gross, J. P. Hartnett, D. J. Masson and Carl Gazley, A Review of Binary Laminar Boundary Layer Characteristics, Int. Journ. of Heat Transfer, Vol.3, No.3, 1961.

4. H. A. Simon, C. S. Liu and J. P. Hartnett, The Eckert Reference Formulation Applied to High-Speed Laminar Boundary Layers of Nitrogen and Carbon Dioxide, University of Delaware, Department of Mechanical Engineering, Technical Note No.5, April 1965.

NOMENCLATURE

C^*	Chapman-Rubesin parameter, $\rho_2^*\mu_2^*/\rho_\infty\mu_\infty$
C_f	local skin friction coefficient
C_p	specific heat at constant pressure, cal/gm-°K
f	dimensionless stream function
q	heat flow per unit area and time, cal/cm^2-sec
r	recovery factor
t	temperature, °K
u	velocity component parallel to surface, cm/sec
v	velocity component normal to surface, cm/sec
x	coordinate along the body
y	coordinate normal to the body
D	diffusion coefficient, cm^2/sec
T	temperature ratio, t/t_∞
W	mass fraction of foreign gas
M	Mach number
Pr	Prandtl number
Sc	Schmidt number
t^*	reference temperature (see text)
γ	specific heat ratio
η	dimensionless wall distance
μ	dynamic viscosity, gm/cm-sec
ν	kinematic viscosity, cm^2/sec
ρ	density, gm/cm^3
ϖ_c	normalized mixture thermal capacity $(C_p/C_{p\infty})$

φ_{c12} normalized pure-component thermal capacity difference $(C_{p1}-C_{p2})/C_{p\infty}$

φ_k normalized mixture thermal conductivity (k/k_∞)

φ_ρ normalized mixture density (ρ/ρ_∞)

φ_μ normalized mixture viscosity (μ/μ_∞)

$Re_{x,\infty}$ Reynolds number $(\rho_\infty u_\infty x/\mu_\infty)$

h heat transfer coefficient

Subscripts

w evaluated at wall conditions

r refers to recovery conditions outside the boundary layer

o at solid surface

∞ Refers to free stream conditions

Superscript

* Evaluated at reference temperature

TABLE I CONSTANT WALL TEMPERATURE

M_∞	t_∞ °K	$T_w = \frac{t_w}{t_\infty}$
0	218	2
0	218	6
0	1110	0.5
4	218	2
4	218	6
4	555	1
4	555	2
4	555	3
4	1110	0.5
8	218	2
8	218	6
8	555	1
8	555	2
8	555	3
12	218	2
12	218	6

TABLE II RECOVERY CASE

M_∞	t_∞ °K
4	218
4	555
4	1110
8	218
8	555
12	218

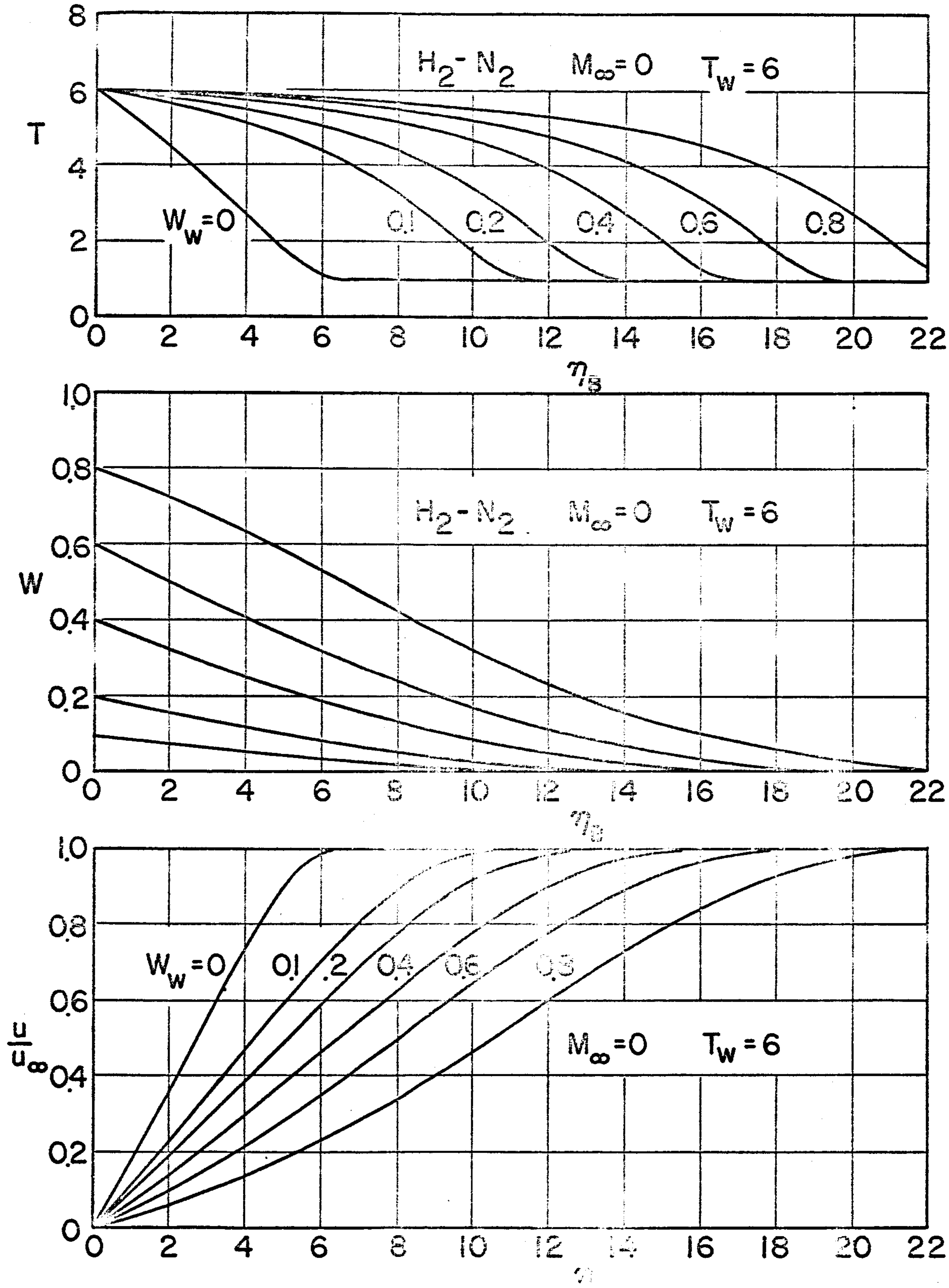


FIG. 1-A

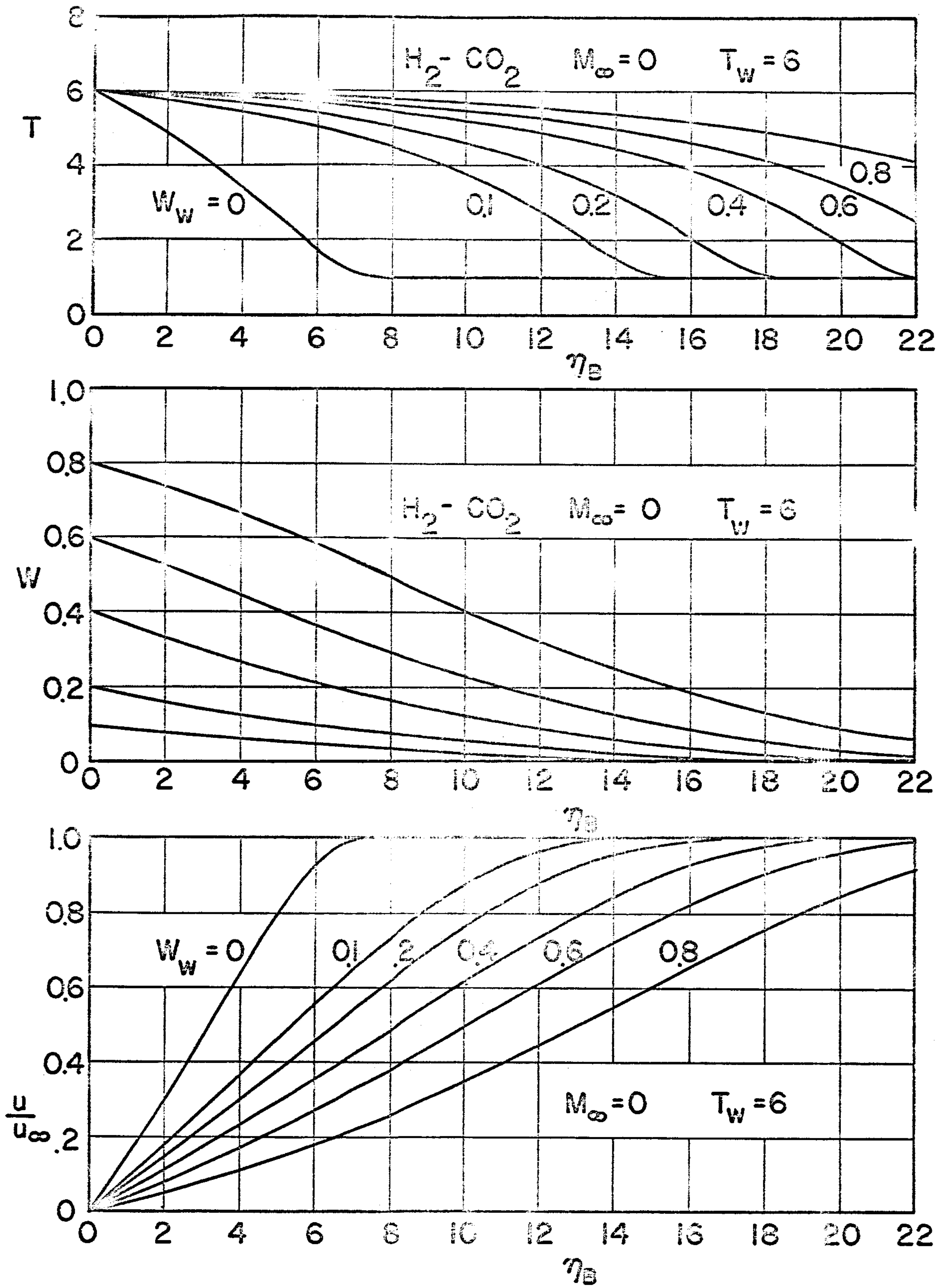
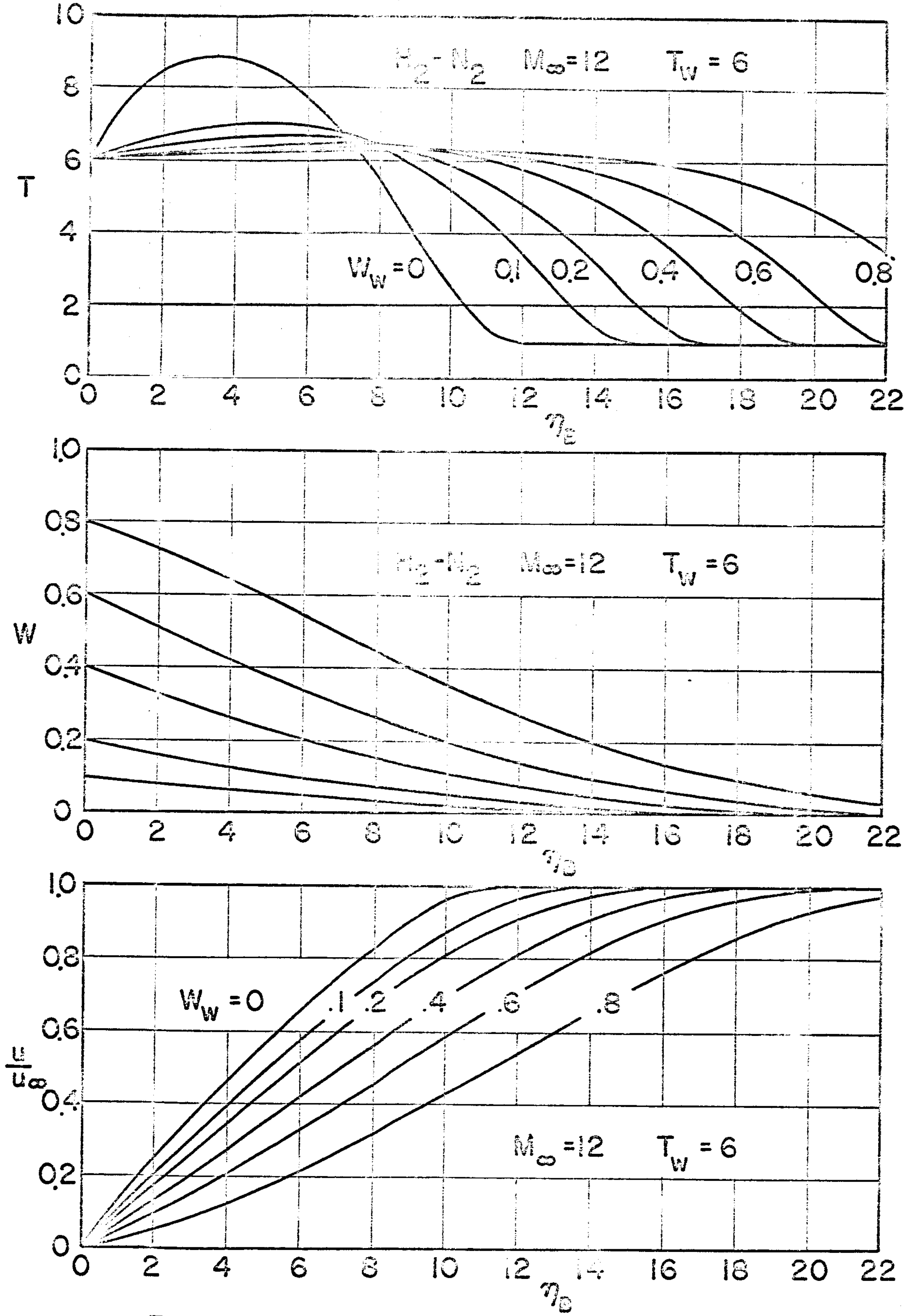

8
6
T
4
2
0
$H_2 - CO_2$
$M_\infty = 0$
$T_w = 6$
$W_w = 0$
0.1
0.2
0.4
0.6
0.8
0 2 4 6 8 10 12 14 16 18 20 22
η_B
1.0
0.8
0.6
W
0.4
0.2
0
$H_2 - CO_2$
$M_\infty = 0$
$T_w = 6$
0 2 4 6 8 10 12 14 16 18 20 22
η_B
1.0
0.8
0.6
0.4
$\frac{u}{u_\infty}$ .2
0
$W_w = 0$
0.1
.2
0.4
0.6
0.8
$M_\infty = 0$
$T_w = 6$
0 2 4 6 8 10 12 14 16 18 20 22
η_B

FIG. 1 - B



H_2 - N_2 M_∞=12 T_W = 6
T
W_W =0
0.1
0.2
0.4
0.6
0.8
η_B
W
H_2 - N_2 M_∞=12 T_W = 6
η_B
$\frac{u}{u_\infty}$
W_W =0
.1
.2
.4
.6
.8
M_∞ =12 T_W = 6
η_B

FIG. 2-A

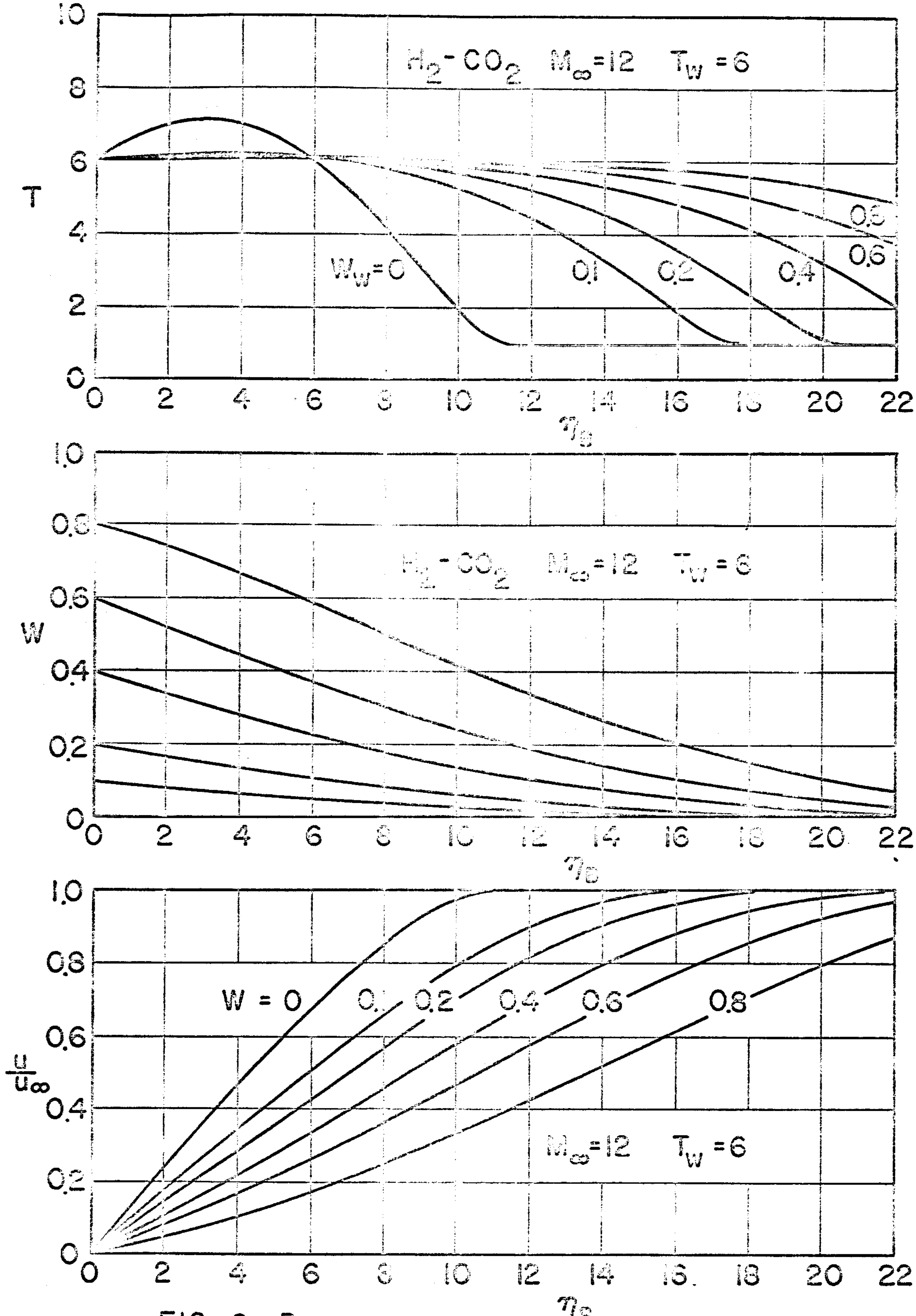


FIG. 2 - B

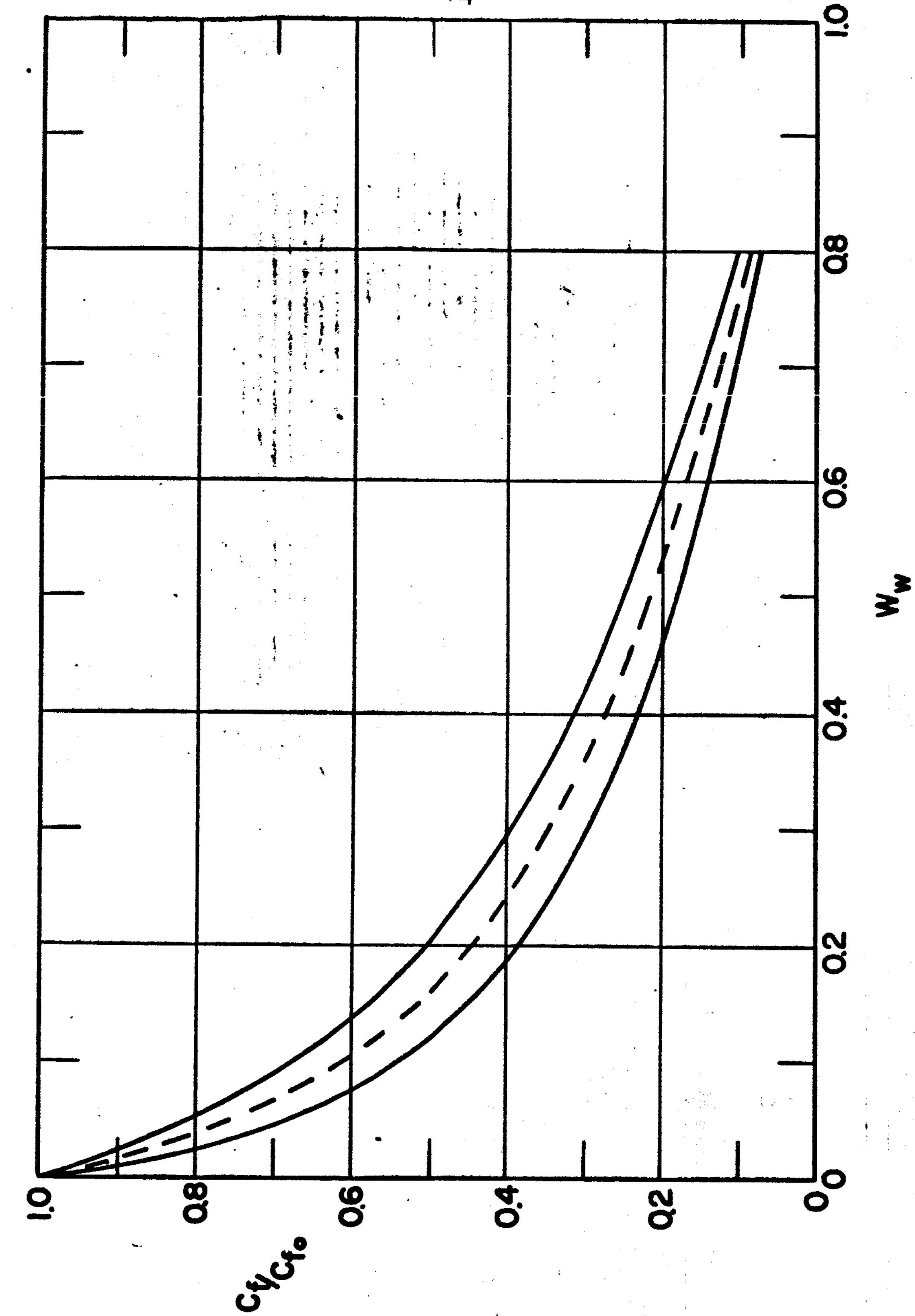


FIG. 3

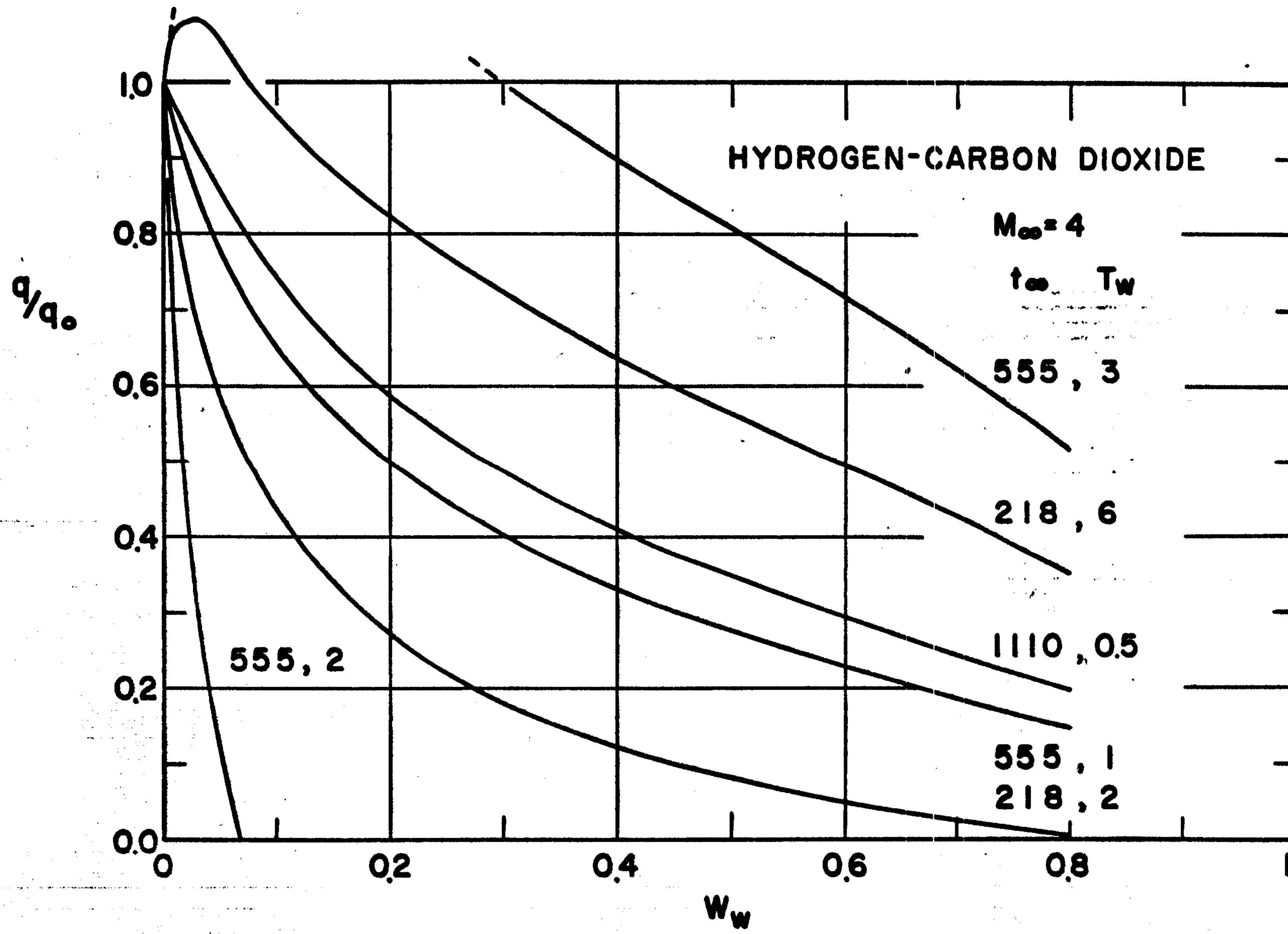


FIG. 4 T_W = CONSTANT

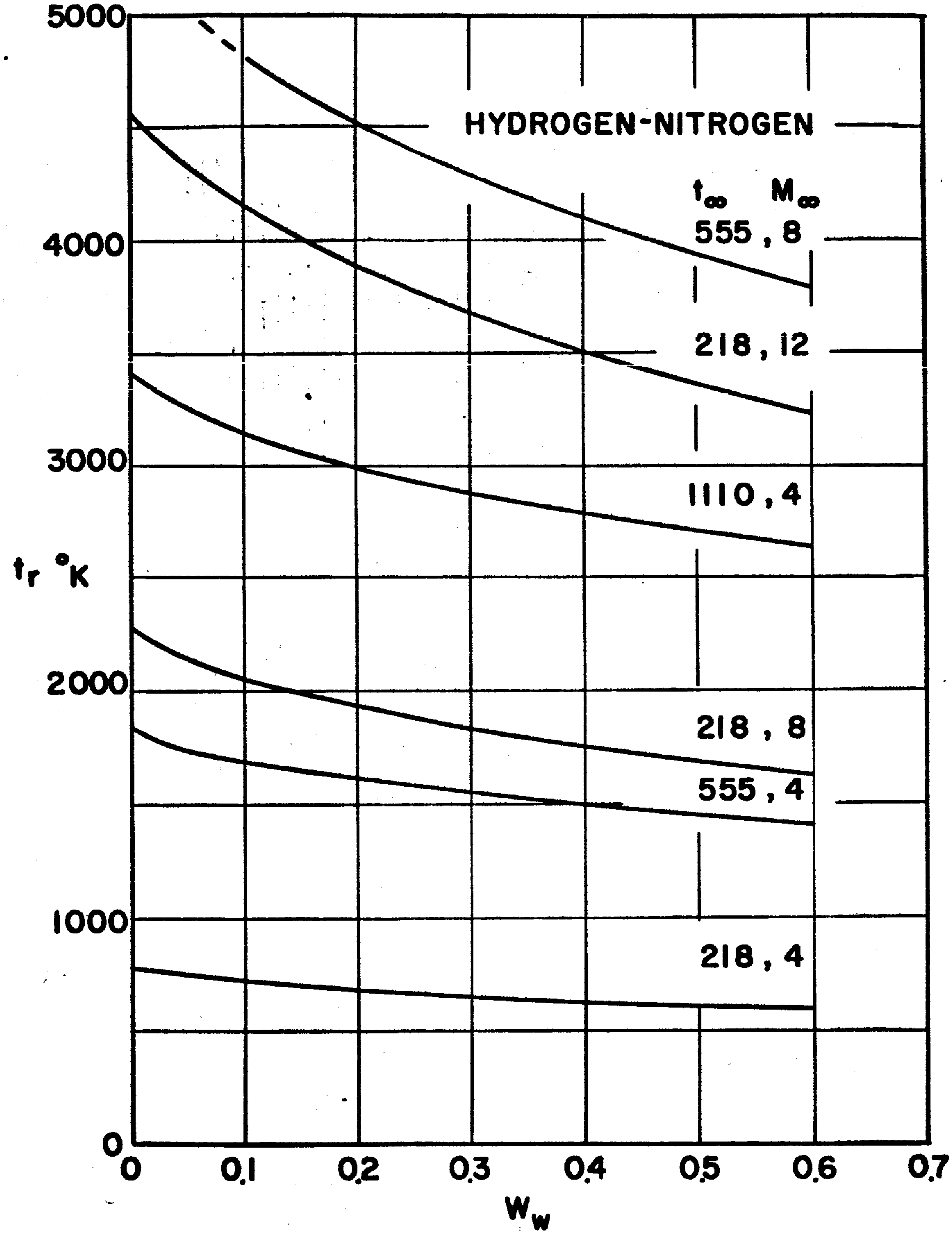


FIG. 5-A RECOVERY TEMPERATURE

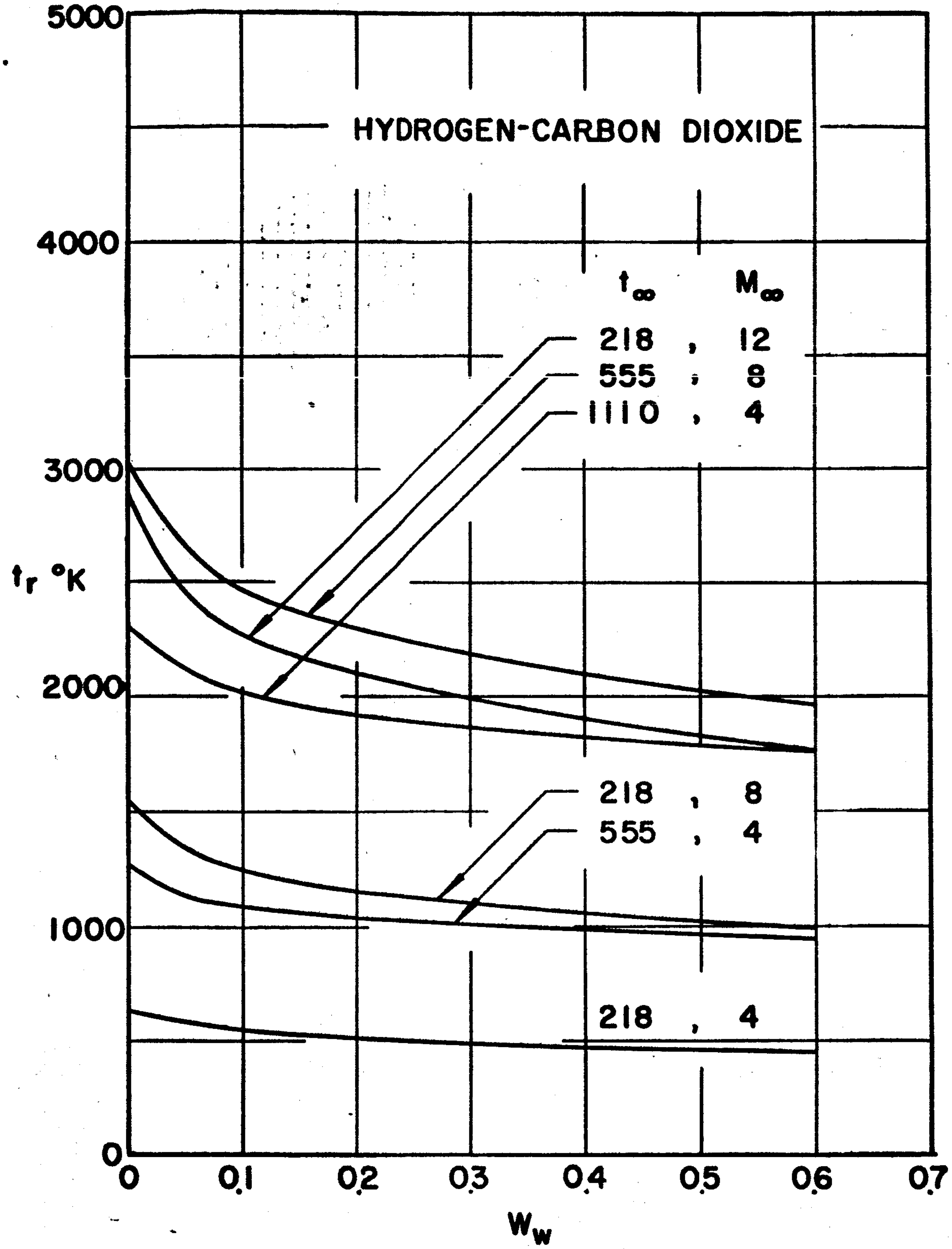


FIG. 5-B RECOVERY TEMPERATURE

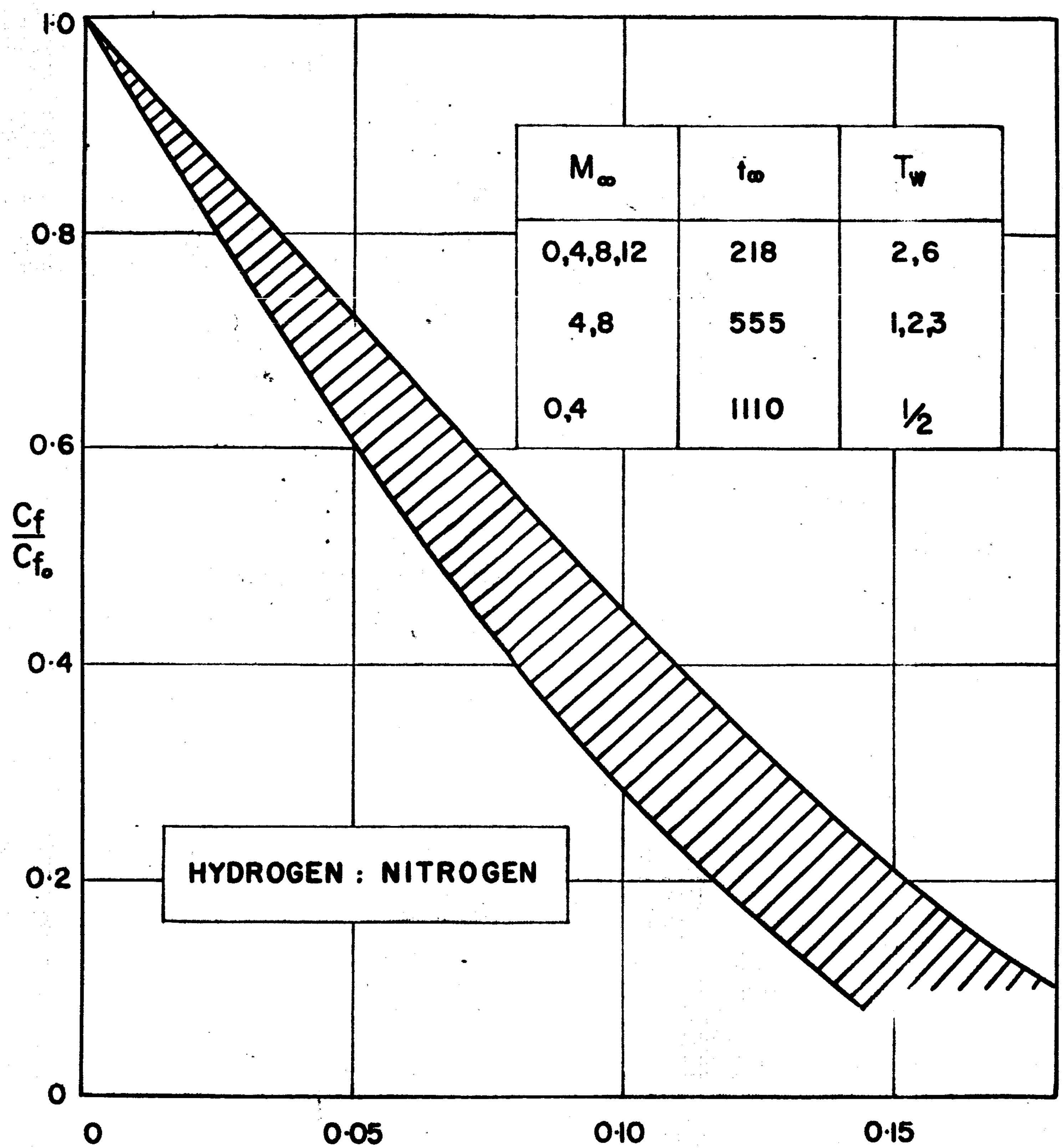


FIG. 6-A $\frac{-2\,\rho_w\, v_w\sqrt{Re_x}}{\rho_\infty\, u_\infty\sqrt{C_f}}$

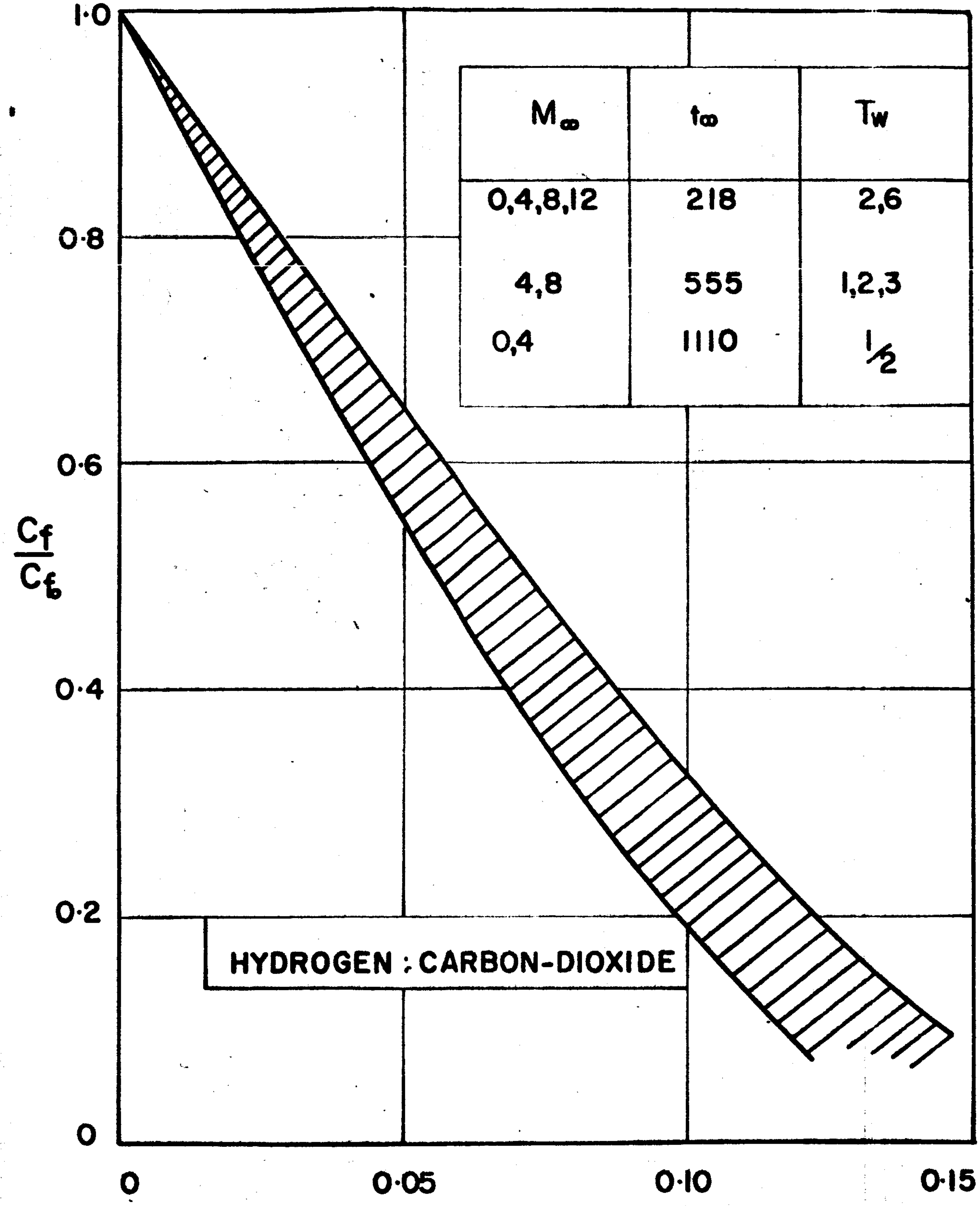


FIG. 6-B $\dfrac{-2\,\rho_w\, V_w\sqrt{Re_x}}{\rho_\infty\, u_\infty\sqrt{C^*}}$

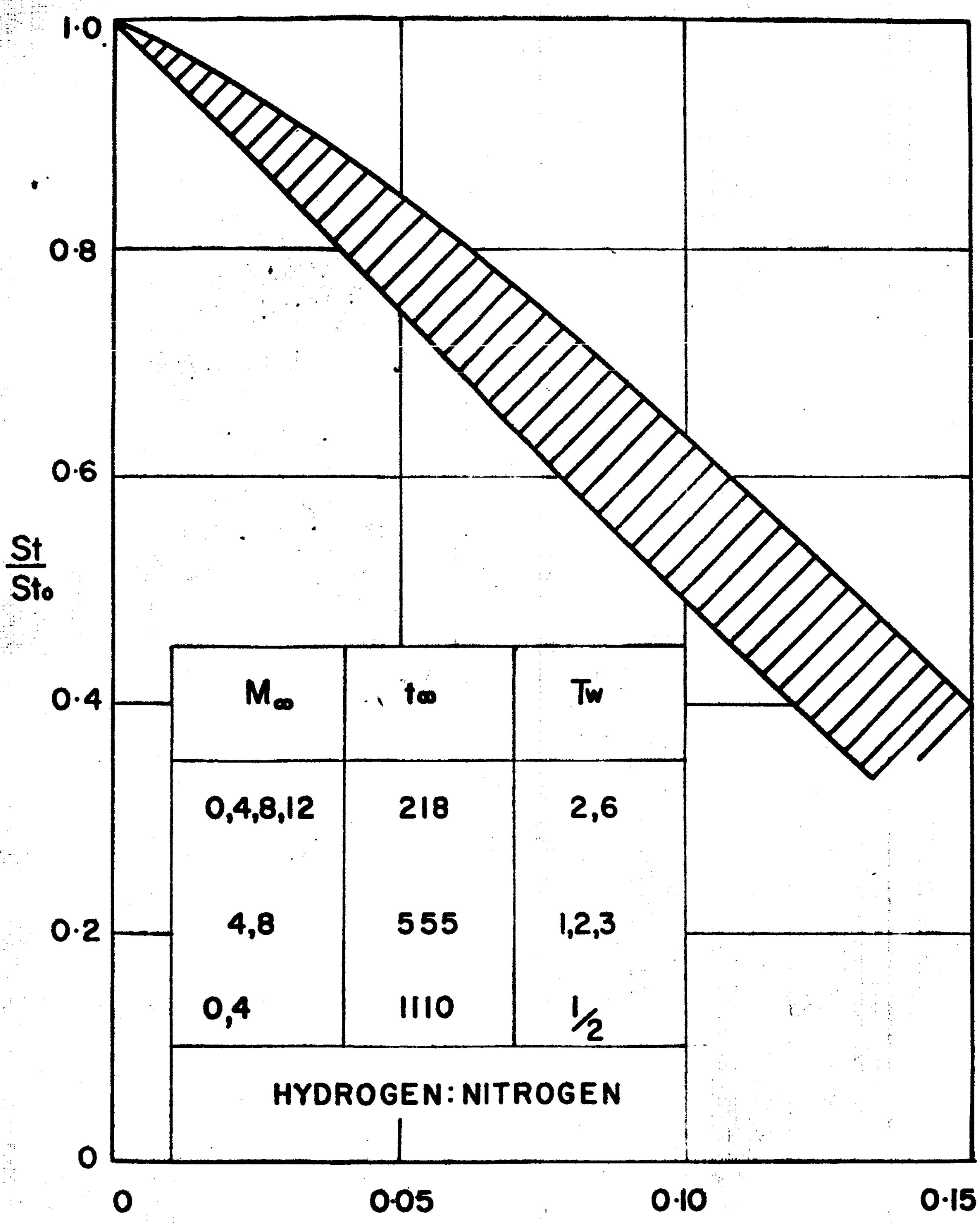


FIG. 7-A $\frac{-2\rho_w v_w\sqrt{Re_x}}{\rho_\infty u_\infty\sqrt{C^*}}$

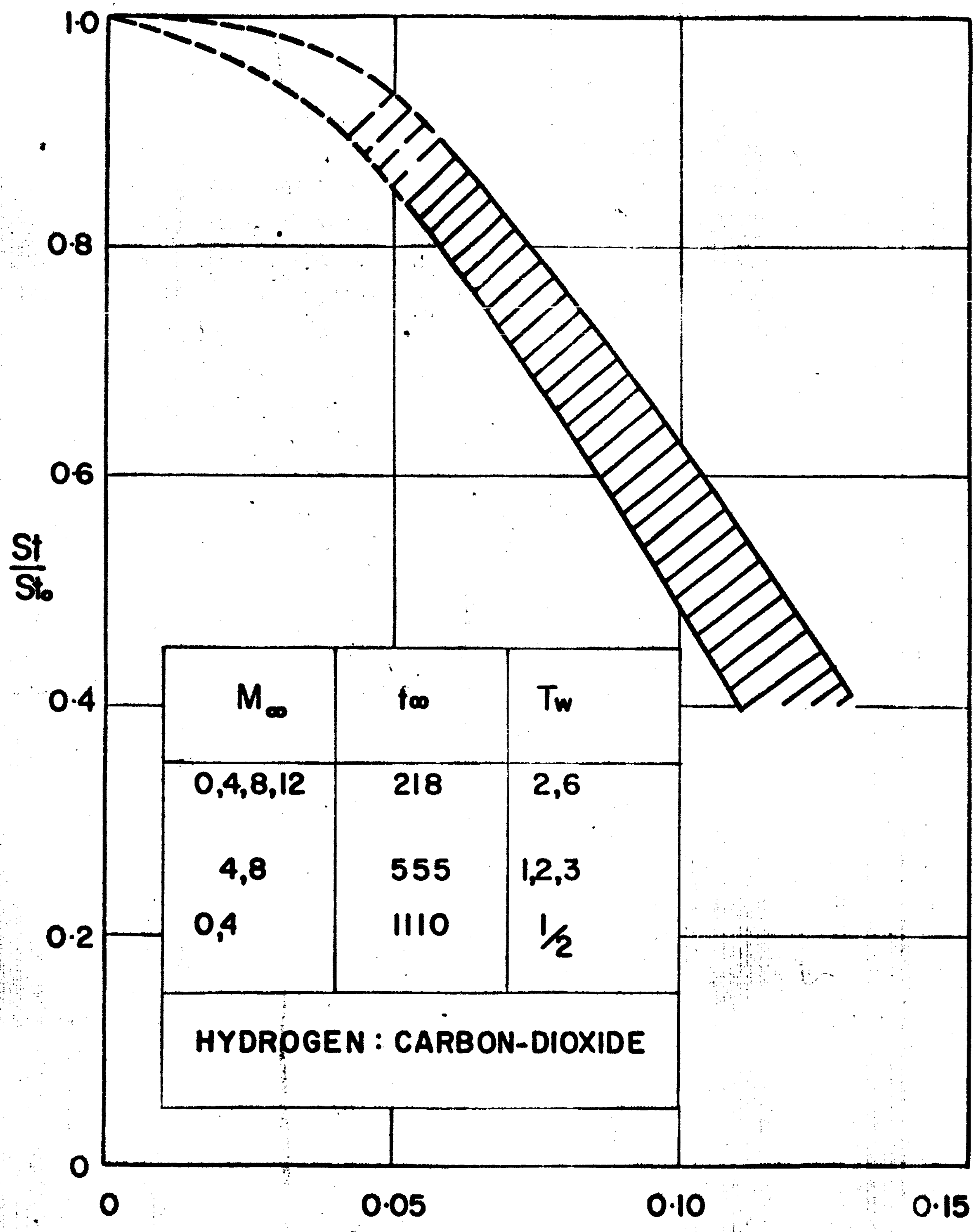


FIG. 7-B $\frac{-2\rho_w v_w \sqrt{Re_x}}{\rho_\infty u_\infty \sqrt{C^*}}$

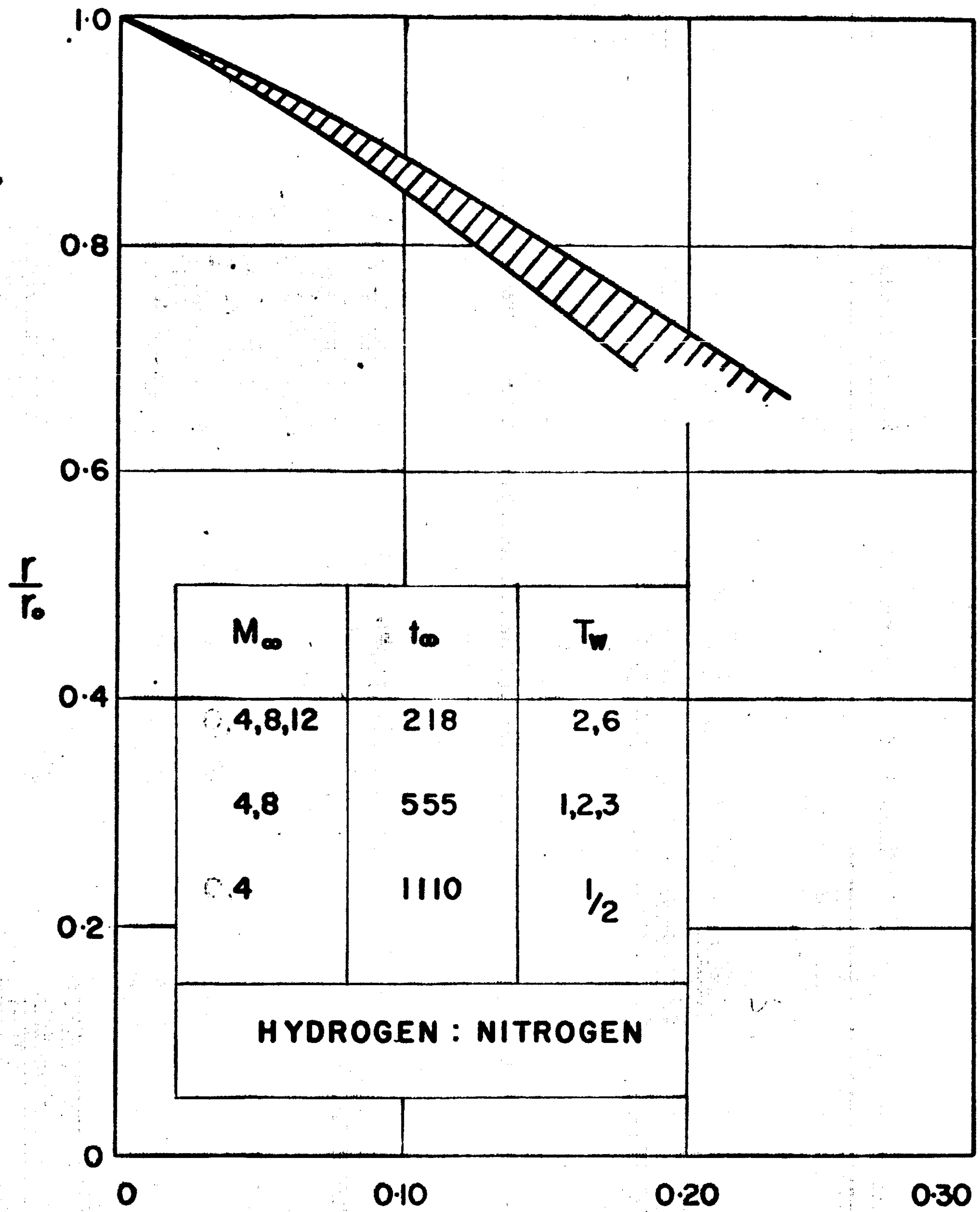
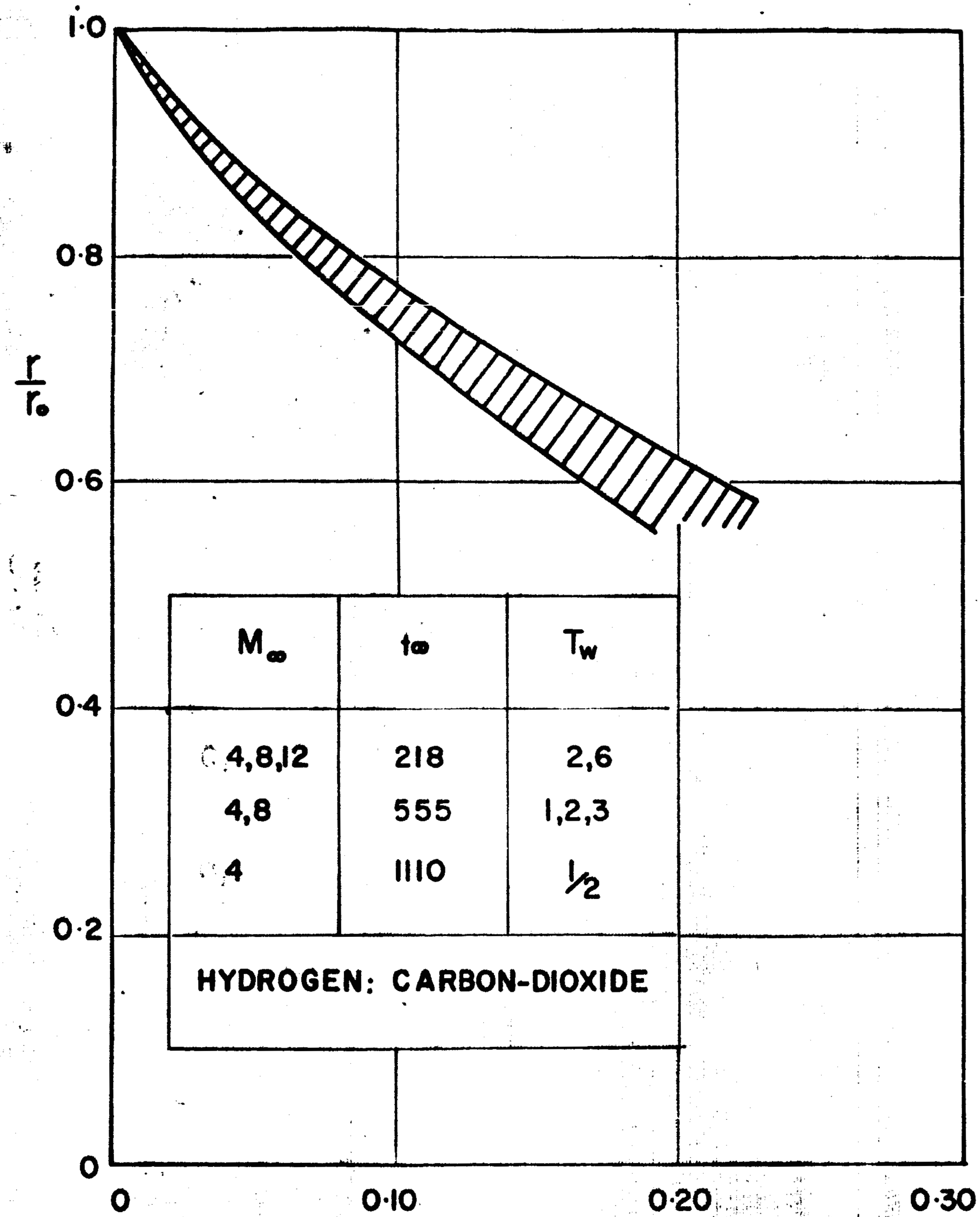


FIG. 8-A $\dfrac{-2\,\rho_w\,v_w\sqrt{Re_x}}{\rho_\infty\,u_\infty\sqrt{C^*}}$



1·0
0·8
0·6
0·4
0·2
0
$\frac{r}{r_o}$
0
0·10
0·20
0·30
M_∞
t_∞
T_w
4,8,12
218
2,6
4,8
555
1,2,3
4
1110
1/2
HYDROGEN: CARBON-DIOXIDE
$\frac{-2\,\rho_w\,v_w\sqrt{Re_x}}{\rho_\infty\,u_\infty\sqrt{C^*}}$

FIG. 8-B