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PROGRAMMER'S MANUAL for Interplanetary Error Propagation Program

CONTRACT NAS 5-3342

prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

PHILCO | WDL DIVISION

A SUBSIDIARY OF Ford Motor Company | PALO ALTO, CALIFORNIA

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Prepared by

PHILCO CORPORATION
A Subsidiary of Ford Motor Company
WDL Division
Palo Alto, California

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GODDARD SPACE FLIGHT CENTER
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ABSTRACT

WDL-TR2184 UNCLASSIFIED
PROGRAMMER'S MANUAL FOR
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This report discusses the subroutines that are used in the Interplanetary Error Propagation Program. A narrative description and card listing are provided for each subroutine; flow diagrams are included, if applicable. A listing of assigned locations in common is also presented.

Author

- -

FOREWORD

This report is submitted to the National Aeronautics and Space Administration, Goddard Space Flight Center, in fulfilling the requirements of Contract NAS 5-3342.

The documentation provided by Philco WDL in support of the Interplanetary Error Propagation Program consists of the following three volumes:

- WDL-TR2184, "Programmer's Manual for Interplanetary Error Propagation Program"
- WDL-TR2185, "User's Manual for Interplanetary Error Propagation Program"
- Guidance and Control System Engineering Department Technical Report No. 4," The application of State Space Methods to Navigation Problems," by Stanley F. Schmidt

These volumes discusse the theory of the Schmidt-Kalman filter used in the program for data smoothing, the manner in which the program is used, and subroutine description and listing.

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LIST OF TABLES

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PROGRAM SUBROUTINE LISTING

ARKTNS	Single Precision Arctangent
ARKTAN	Double Precision Arctangent
ASINH(X)	Function Evaluation
BODY	Calculates Accelerations Due to Perturbing Bodies
BVEC	Calculates B Vector
CHNGP	Determines when to Shift Body Center
COMPHQ	Computations for ONBTR and MONBTR Subroutines
CONST1	Array of Input Constants
CONVPI	Converts Input Covariance Matrix to 1950
CORRTP	Updates P Matrix
CROSS	Cross Product
(CSH)S	Fortran II Card Image Input Subroutine
DE6FN	Fap Integration Subroutine
DOT	Function Forming Dot Product
EARTR	Updates Covariance Matrix for Earth Based Tracking
ECLIP	Transforms Coordinates through Transformations
ENCKE	Calculates Perturbation due to Deviation from CONIC
ERP	Prints Out Ephemeris Error
ERPT	Prints Out Time of Ephemeris Error
FINP	Data Input Subroutine
FNORM	Norm of A Vector
GHA	Greenwich Hour Angle
GOTOB	Main Subroutine for Integration of Trajectories
GOTOR	Iterates to Solve Kepler's Equation
GUID	Performs Guidance Calculations
HOUR	Reads Printer Clock
HPHT	Performs Matrix Multiplication H*P*HT
INPUT	Converts Inputs to Equinox of 1950 Reference
INTR	FAP Ephemeris Subroutine
INV3	Inverts Up to a 6 By 6 Matrix
INVAO	Forms Inverse of Transistion Matrix
LOADO	Obtains Transition Matrix From T Array

LOADT	Puts Unit ICS On Perturbation Equations
MASS	Arranges Gravitational Constants of Bodies Considered
MATRX	Multiplies A*B=C or A*B*AT=C Max Dimension (10.10)
MATSUB	Error Propagation Logic Subroutine
MNA	Transformation to Selenocenic Coordinates
MNAND	Transformation for Selenocenic Velocities
MONBTR	Updates Covariance Matrix for Moon Beacons
MULT	Multiplies Two 3 by 3 Matrices
NUTAIT	Calculates Nutation Matrix
OBLN	Calculates Acceleration Due to Oblateness
ONBTR	Updates Covariance Matrix for Onboard Tracking
ORTC	Outputs Orbital Parameters
OUTC	Outputs Trajectory
OUTDAT	Outputs Calendar Date
OUTP	Outputs RMS Values of Orbital Parameters
PTRAN	Transforms P Matrix
RETRO	Performs Retro Fire
ROTATE	Calculates Transformation for Rotation About an Axis
ROTEQ	Calculates Matrix from Equinox 1950 to Mean Equinox of Date
RVIN	Transforms Coordinates From Spherical to Cartesian
RVOUT	Transforms Coordinates from Cartesian to Spherical
SDEC	Second Derivative Subroutine
SETN	Set Read and Write Tape Numbers
SHIFTP	Shifts Body Center
STEPC	Move Along Conic in Time
TIMEC	Converts Calendar Date to Days from 1950
TIMED	Converts Days Hours Min Sec to Seconds
TRAC	Tracking Station Coordinates
TRANSH	Transforms H Matrix From Date to 1950

PROGRAMMER'S MANUAL FOR INTERPLANETARY ERROR PROPAGATION PROGRAM

SECTION 1

INTRODUCTION

1.1 GENERAL

This manual contains subroutine descriptions, logic, and FORTRAN listings for the Interplanetary Error Propagation Program. Listings of the quantities which are in the four common storage arrays (T,S,C, and IC) are also included.

1.2 GENERAL DESCRIPTION OF METHOD OF ORBIT DETERMINATION USED IN PROGRAM

The classical approach to the determination of an orbit from tracking data is the maximum likelihood, least-squares technique. This program uses a different approach to this problem. The relationships between the observed tracking data and the initial conditions (or an equivalent set of parameters) which determine the orbit are highly nonlinear. Therefore, it is assumed that a preliminary estimate of the initial conditions and, therefore, the orbit, is known. The equations are then linearized about this "nominal" orbit and an adjustment to the nominal orbit found from the differences between the observed tracking data and the predicted tracking data based on the nominal orbit. In the least-squares technique, the data is processed in parallel in the sense that the normal equations are summed over all the available tracking data before a modification to the nominal orbit is determined.

The Schmidt-Kalman technique differs from the classical least-squares approach in two ways: first, the estimate used is based upon the minimization of a risk function different from that used in the least-squares approach and, second, the data is processed serially instead of in parallel. The Schmidt-Kalman technique depends upon a linearization of the relations between the orbit parameters and the observed tracking data. Therefore, it assumes some nominal trajectory and an associated error estimate (covariance matrix) for the nominal trajectory.

GENERAL FLOW DIAGRAM

OF OVERALL ERROR PROPAGATION PROGRAM

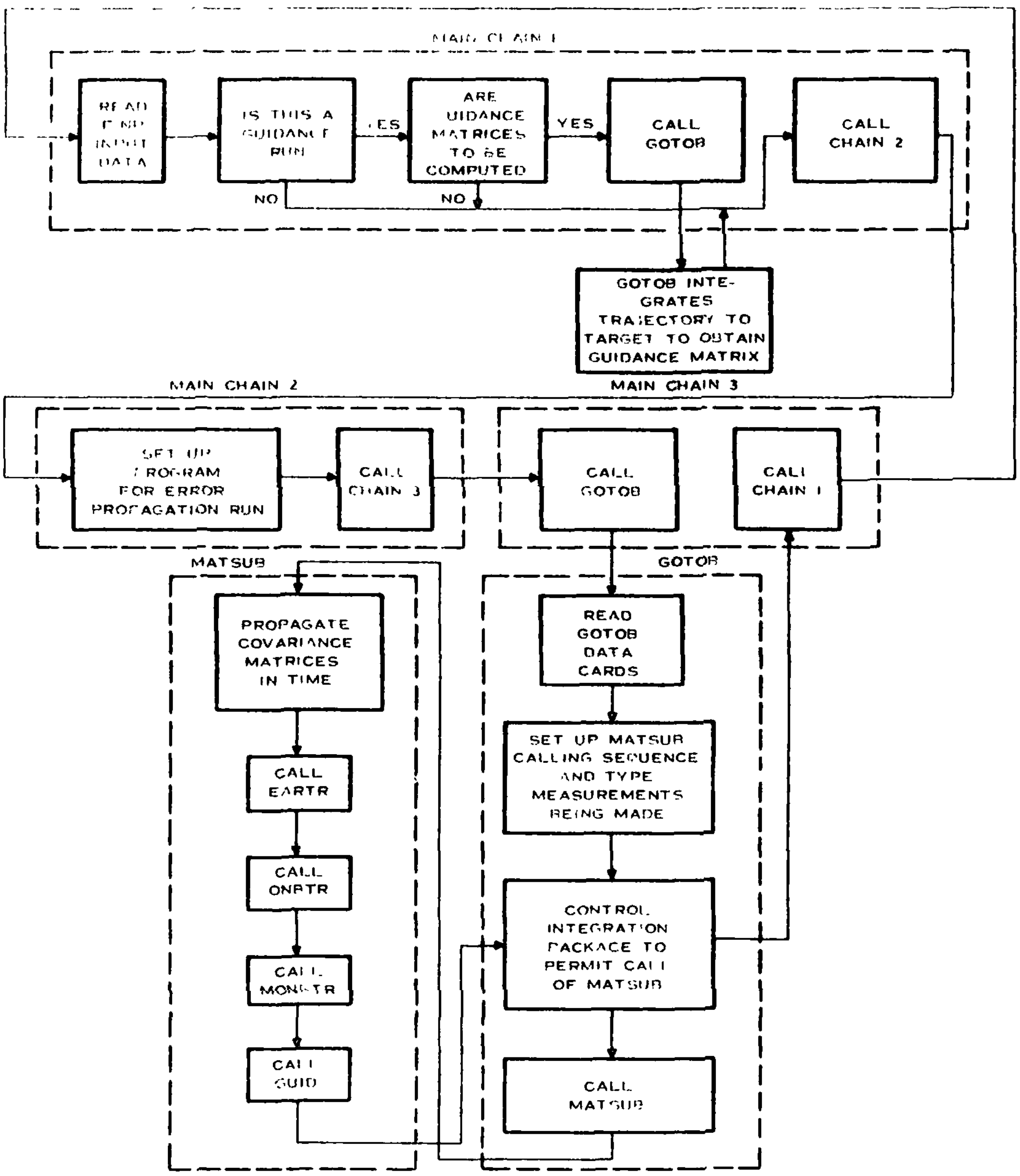

Figure 1-1

The serial processing of the tracking data and associated updating of the nominal orbit occurs as follows: At the start of the time period for which tracking data are to be processed and an improved estimate of the orbit is to be obtained, it is assumed that an estimate of the orbit (a set of state variables) and an error estimate exist. Both of these estimates are updated to an instant of time when tracking data are available. Based upon the available tracking data at this time, a revised estimate of the nominal trajectory is obtained together with a revised estimate of the associated covariance matrix. These two estimates are then propagated along the orbit until additional tracking data become available. In this way, the orbit is updated and tracking data are processed serially.

1.3 GENERAL PROGRAM DESCRIPTION

This program is capable of error studies and their propagation along lunar and planetary trajectories. The design of the program is such as to facilitate the accurate and rapid evaluation of tracking systems for a given lunar or interplanetary trajectory. The program is also able to analyze the errors based on ground tracking as well as on-board tracking independently and in combination. The program has the option to calculate the errors and their projection to the target planets for a probe or a spacecraft based on the following type of measurements.

a. <u>Earth-based tracking</u>
   1. Range
   2. Range rate
   3. Azimuth
   4. Elevation
   5. Minitrack data

b. <u>On-board tracking</u>
   1. Right ascension
   2. Declination
   3. Range
   4. Range rate
   5. Range and range rate from beacons on the target planets.

The program is also capable of propagating guidance covariance matrices along the nominal trajectory. The program has the facility of using two types of guidance laws: (1) Fixed time of arrival, and (2) Variable time of arrival with the constraint that the energy with respect to the target be held constant.

The program is composed of three chains. Each link is complete in itself; that is, each contains a main program and all necessary subroutines. Control is passed from one link to the next by the CALL CHAIN statement. The program requires an ephemeris tape for operation. Figure 1-1 shows a general block diagram of the flow through the program.

The following tables show the assigned locations in "common storage":

Table 1-1: "C" Array ----------- Floating Point Variables
Table 1-2: "S" Array ----------- Floating Point Constants
Table 1-3: "IC" Array ---------- Fixed Point Variables
Table 1-4: "T" Array ----------- Storage for Integration Routine

TABLE 1-1

LIST OF ASSIGNED QUANTITIES IN COMMON "C" ARRAY

DIMENSION 1000			
CELL	FORTRAN NAME	DIMENSION	DESCRIPTION
C(10)	TW		Whole days from 1950 at last rectification
C(11)	TF		Fractional days from 1950 at last rectification
C(12)	TSECO		Time in seconds at last rectification
C(13)	TP1		Time in whole days
C(14)	TP2		Time in fractional days
C(15)	XP	(3)	X, Y, Z, equinox of 1950
C(18)	VXP	(3)	$\dot{X}$, $\dot{Y}$, $\dot{Z}$ equinox of 1950
C(21)	AE	(3)	Acceleration due to deviation from conic
C(24)	AO	(3)	Acceleration due to oblateness
C(27)	CA	(3)	Acceleration due to perturbing bodies
C(30)	TSEC		Seconds from start - zero reference time
C(33)	X	(3)	Reference conic X, Y, Z
C(36)	VX	(3)	Reference conic $\dot{X}$, $\dot{Y}$, $\dot{Z}$
C(46)	XSO	(3)	Initial position for STEPC conic
C(49)	VXSO	(3)	Initial velocity for STEPC conic
C(55)	μ		Gravity constant of central body
C(56)	VKB	(6)	Gravitational constants of bodies used
C(62)	PO	(22)	Ephemeris positions of bodies
C(84)	VE	(22)	Ephemeris velocities of bodies
C(106)	RBO	(6)	Distances to bodies

DIMENSION 1000			
CELL	FORTRAN NAME	DIMENSION	DESCRIPTION
C(112)	RBOP	(6)	Distances to bodies
C(120)	EN	(3,3)	Nutation matrix
C(129)	EA	(3,3)	Transformation from 1950 to mean equinox of date
C(138)	AN	(3,3)	Transformation from 1950 to true equator of date
C(147)	ECL	(3,3)	Transformation from 1950 to ecliptic of date
C(156)	ECX	(3)	Position: ecliptic coordinates
C(159)	ECV	(3)	Velocity: ecliptic coordinates
C(162)	BET		Angle from line of nodes to radius vector
C(163)	THT		True anomaly
C(171)	BP	(3,3)	Acceleration partials for use in variational equations
C(180)	SMA		Semi-Major Axis
C(181)	ECC		Eccentricity
C(182)	RCA		Radius of closest approach
C(183)	OINC		Orbital inclination
C(184)	OMG		Longitude of ascending node
C(185)	BEP		Argument of periapsis
C(186)	XED	(3)	Position: equator date
C(189)	VED	(3)	Velocity: equator date
C(192)	RCV	(3)	$\vec{R} \times \vec{v}$
C(195)	RCV2		$\lvert \vec{R} \times \vec{v} \rvert^2$
C(196)	R2		R^2

DIMENSION 1000			
CELL	FORTRAN NAME	DIMENSION	DESCRIPTION
C(197)	V^2		V^2
C(198)	RDV		$\vec{R} \cdot \vec{V}$
C(199)	C3		$C_3 = V^2 - \frac{2\mu}{R}$
C(460)	PFTA	(6,6)	Deviation From Nominal Covariance Matrix
C(568)	TRANS	(3,6)	Guidance partial matrix being used on runs
C(622)	TLAST	(4,6)	Delta time since last observation in ONBTR
C(647)	TOUT		Time to output for MATSUB
C(648)	TGUIDE		Time to make guidance correction
C(649)	TSECP		LAST TIME through MATSUB
C(650)	TPRINT		Output print interval
C(651)	TSTART		Start time
C(652)	P	(6,6)	Knowledge of state covariance matrix
C(752)	AOS	(6,6)	Transition matrix from start to present time
C(788)	STPARS	(3,3)	Station partials MONBTR
C(797)	CIOMP		Compute guidance matrices
C(800)	STPARD	(3,3)	Station partials MONBTR
C(888) C(900)			Constants used in COMPHQ
C(973)	OUTPUT	(6)	Data for output purposes

TABLE 1-2

LIST OF ASSIGNED QUANTITIES IN COMMON "S" ARRAY

DIMENSION 1000			
CELL	FORTRAN NAME	DIMENSION	DESCRIPTION
S(1)			Earth Planetary Mass
S(2)			Moon Planetary Mass
S(3)			Sun Planetary Mass
S(4)			Venus Planetary Mass
S(5)			Mars Planetary Mass
S(6)			Jupiter Planetary Mass
S(7)			SPARE
S(8)			SPARE
S(9)			SPARE
S(10) } S(19) }			Earth Constants
S(20) } S(29) }			Moon Constants
S(30) } S(39) }			Sun Constants
S(40) } S(49) }			Venus Constants
S(50) } S(59) }			Mars Constants
S(60) } S(69) }			Jupiter Constants
S(70)			86400. Conversion factor days to seconds
S(71)			1/86400. Conversion factor seconds to days

DIMENSION 1000			
CELL	FORTRAN NAME	DIMENSION	DESCRIPTION
S(72)			.017453296 Conversion factor for degrees to radians
S(73)			57.29578 Conversion factor for radians to degrees
S(80)	AT		Launch latitude - degrees
S(81)	ON		Launch longitude - degrees
S(82)	AL		Launch altitude - km.
S(83)	TLTI		Time from launch to injection, days hours. min sec
S(84)	FAZ		Firing azimuth
S(110)	TSTOP		Time to stop calling MATSUB
S(111)	TIM		Type of input
S(112)	DATE		Start date
S(113)	FDATE		Fractional start date
S(114)	CIBDY		Central body
S(115)	X	(3)	Initial position
S(118)	VX	(3)	Initial velocity
S(121)	TIBDY		Target Body
S(122)	OUTTP		Type of output
S(124)	SKTB		Type of stop

EARTH BASED TRACKING DATA

N = 0 - 19

CELL	DESCRIPTION	UNITS
S(125)+N*15	Variance of range	KM^2
S(126) +N*15	Variance of azimuth	RAD^2
S(127)+N*15	Variance of elevation	RAD^2
S(128)+N*15	Variance of range rate	KM^2/SEC^2
S(129)+N*15	Variance of latitude	RAD^2
S(130)+N*15	Variance of longitude	RAD^2
S(131)+N*15	Variance of altitude	KM^2
S(132)+N*15	Variance of azimuth biases	RAD^2
S(133)+N*15	Variance of elevation biases	RAD^2
S(134)+N*15	Latitude of station	DEG
S(135)+N*15	Longitude of station	DEC
S(136)+N*15	Altitude of station	KM
S(137)+N*15	Station name	
S(138)+N*15	Period of observation	SEC
S(139)+N*15	Variance of time bias	SEC^2
S(425)	Velocity of light error	KM^2/SEC^2

ON-BOARD TRACKING DATA

CELL	DESCRIPTION	UNITS
S(426)	Variance of range	KM^2
S(427)	Variance of right ascension	RAD^2
S(428)	Variance of declination	RAD^2
S(429)	Variance of range rate	KM^2/SEC^2
S(430)	Variance of range bias	KM^2
S(431)	Variance of right ascension bias	RAD^2

CELL	DESCRIPTION	UNITS
S(432)	Variance of declination bias	RAD^2
S(433)	Variance of range rate bias	KM^2/SEC^2
S(434)	Variance of clock time bias	SEC^2
S(435)	Spare	
S(436)	Spare	

N = 0 - 5 EARTH, MOON, SUN, VENUS, MARS, JUPITER

CELL	FORTRAN NAME	DIMENSION	DESCRIPTION	UNITS
S(437)+4*N			Period of range observation	SEC
S(438)+4*N			Period of right ascension Observation	SEC
S(439)+4*N			Period of declination Observation	SEC
S(440)+4*N			Period of range rate Observation	SEC
S(471)			Shutoff Error (Rocket)	$(\%)^2$
S(473)			Pointing Error (Motor)	$(RAD)^2$
S(475)			Type of guidance decision 0-Time + Ratio	
S(476)	TGUID	(6)	Times for guidance correction	
S(483)	GDCOR		Ratio for making guidance correction	
S(484)	DMON		Error in monitoring guidance correction	$(\%)^2$
S(485)	ENKE		Correction criteria for deivation from reference conic	

CELL	FORTRAN NAME	DIMENSION	DESCRIPTION	UNITS
			MOON BEACON TRACKING DATA N = 0 - 9	
S(500)+N*15			Variance of range	KM^2
S(501)+N*15			Variance of right ascension	RAD^2
S(502)+N*15			Variance of declination	RAD^2
S(503)+N*15			Variance of range rate	KM^2/SEC^2
S(504)+N*15			Variance of range bias	KM^2
S(505)+N*15			Variance of right ascension bias	RAD^2
S(506)+N*15			Variance of declination bias	RAD^2
S(507)+N*15			Variance range rate bias	KM^2/SEC^2
S(508)+N*15			Variance clock bias	SEC^2
S(509)+N*15			Latitude of station	DEG
S(510)+N*15			Longitude of station	DEG
S(511)+N*15			Altitude of station	KM
S(512)+N*15			Station name	
S(513)+N*15			Period of observation	SEC
S(514)+N*15			SPARE	
S(650)	PARI	(21)	Initial deviation from nominal covariance matrix	
S(671)	PI	(21)	Initial knowledge of state covariance matrix	
S(720)	PUPIN		Type of coordinates for PI	
S(721)	PARIN		Type of coordinates for PARI	
S(722)	FTA	(3,6)	Guidance transition matrix fixed time arrival	

CELL	FORTRAN NAME	DIMENSION	DESCRIPTION	UNITS
S(740)	CTE	(3,6)	Guidance transition matrix constant energy WRT* target	

MOON BEACON DATA

N = 0 - 9

S(758)+N*3			Variance of station latitude	RAD^2
S(759)+N*3			Variance of station longitude	RAD^2
S(760)+N*3			Variance of station altitude	KM^2

EARTH BASED TRACKING DATA

N = 0 - 19

S(788)+N*5			Tracking Station horizon	RAD
S(789)+N*5			Variance of M dircos	
S(790)+N*5			Variance of M dircos bias	
S(791)+N*5			Variance of L dircos	
S(792)+N*5			Variance of L dircos bias	
S(888)	RETR		Key to call subroutine RETRO	
S(900)	RAD	(6)	Radius to bodies	
			* with respect to	

TABLE 1-3

LIST OF ASSIGNED QUANTITIES IN COMMON "IC" ARRAY

DIMENSION 300			
CELL	FORTRAN NAME	DIMENSION	DESCRIPTION
IC(1)	N		# of sets of equations
IC(2)	IOR		body center
IC(3)	NOR		new body center
IC(4)	KSTP		type stop
IC(5)	KTYPE		switch indicating type stop
IC(6)	ITARG		target body
IC(7)	KOUT		type of output
IC(8)	IMMSUB		key to test if MATSUB is to be called
IC(10)			+ if tracking station 1 considered
IC(11)			+ if tracking station 2 considered
IC(12)			+ if tracking station 3 considered
. . .			
IC(29)			+ if station 20 considered
IC(30)			+ range, 0 no range } Station # 1
IC(31)			+ AZ-EL, 0 no AZ-EL } Station # 1
IC(32)			+ dircos, 0 no dircos } Station # 1
IC(33)			+ range rate, 0 no range rate } Station # 1
. . .			

CELL	FORTRAN NAME	DIMENSION	DESCRIPTION	
IC(106)			+ range, 0 no range	Station # 20
IC(107)			+ AZ-EL, 0 no AZ-EL	
IC(108)			+ dircos 0 no dircos	
IC(109)			+ range rate, 0 no range rate	
IC(110)			Earth observed + yes, 0 no	
IC(111)			Moon observed, + yes, 0 no	
IC(112)			Sun observed, + yes, 0 no	
IC(113)			Venus observed, + yes, 0 no	
IC(114)			Mars observed, + yes, 0 no	
IC(115)			Jupiter observed, + yes, 0 no	
IC(116)			range, + yes, 0 no	Earth on-board measurements
IC(117)			rt. ascension, + yes, 0 no	
IC(118)			declination, + yes, 0 no	
IC(119)			range rate, + yes, 0 no	
⋮				
IC(136)			range, + yes, 0 no	Jupiter on-board measurements
IC(137)			rt. ascension, + yes, 0 no	
IC(138)			declination, + yes, 0 no	
IC(139)			range rate, + yes, 0 no	
IC(140)			$^{+}_{o}$ Moon beacon station 1	10 Moon beacon stations
⋮				
IC(149)			$^{+}_{o}$ Moon beacon station 10	

CELL	FORTRAN NAME	DIMENSION	DESCRIPTION	
IC(150)			$\overset{+}{o}$ range	
IC(151)			$\overset{+}{o}$ right ascension	station
IC(152)			$\overset{+}{o}$ declination	one
IC(153)			$\overset{+}{o}$ range rate	
⋮				
IC(186)			$\overset{+}{o}$ range	
IC(187)			$\overset{+}{o}$ right ascension	station
IC(188)			$\overset{+}{o}$ declination	ten
IC(189)			$\overset{+}{o}$ range rate	
IC(190)	LSTAT		Key for earth based tracking	
IC(191)	LONB		Key for on-board tracking	
IC(192)	LMB		Key for moon beacon tracking	
IC(193)	IGUID		Positive if guidance run	
IC(194)	ISTAT	(20)	Keep account which of 20 trackers saw vehicle last time in MATSUB	
IC(214)	NOUT		Output indicator	
IC(215)	PUPIN		type of P matrix input	
IC(216)	PARIN		type of PAR matrix input	
IC(217)	IGDTP		type of guidance	
IC(218)	IGDKEY		guidance key set in main	
IC(219)	IGD		number of guidance corrections	
IC(220)	IMOONB	(20)	Keep account which of 10 moon beacons saw vehicle last time in MATSUB	
IC(231)	NSTEP		Key for STEPC	

TABLE 1-4

LIST OF ASSIGNED QUANTITIES IN COMMON "T" ARRAY

This is the common storage block for the integration package. See the subroutine writeup for DE6FN for definitions of the quantities in storage.

DIMENSION 1360				
CELL	DESCRIPTION			
T(1)	N	No. of Equations	initial conditions which must be supplied (T(1)–T(N+4))	
T(2)	X			
T(3)	H	Step Size		
T(4)	y_i	i = 1,N		
T(N+4)	y_i'	i = 1,N		
T(2N+4)	y_i''	i = 1,N	second derivatives stored by V subroutine	
T(3N+4)	y_i'	i = 1,N	most significant	
T(4N+4)	y_i'	i = 1,N	least significant	
T(5N+4)	$\Delta y_i'$	i = 1,N		
T(6N+4)	$\Delta y_i'$	i = 1,N		
T(7N+4)	y_i	i = 1,N	most significant	
T(8N+4)	y_i	i = 1,N	least significant	
T(9N+4)	y_{i3}	i = 1,N	saved for central difference equations (T(9N+4)–T(10N+4))	
T(10N+4)	y_{i3}'	i = 1,N		

DIMENSION 1360	
CELL	DESCRIPTION
T(11N+4)	y''_{i0} i = 1,N
T(12N+4)	y''_{i1} i = 1,N
T(13N+4)	y''_{i2} i = 1,N — saved from 6R, R-K steps. Used to form difference tables in an attempt to proceed to a cowell step.
T(14N+4)	y''_{i3} i = 1,N
T(15N+4)	y''_{i4} i = 1,N
T(16N+4)	y''_{i5} i = 1,N
T(17N+4)	y''_{i6} i = 1,N
T(18N+4)	used only if coordinates changed.
T(19N+4) . . . . . . T(30N-8)	11 N storages. N sets of differences saved for next cowell step.

MAIN CHAINS

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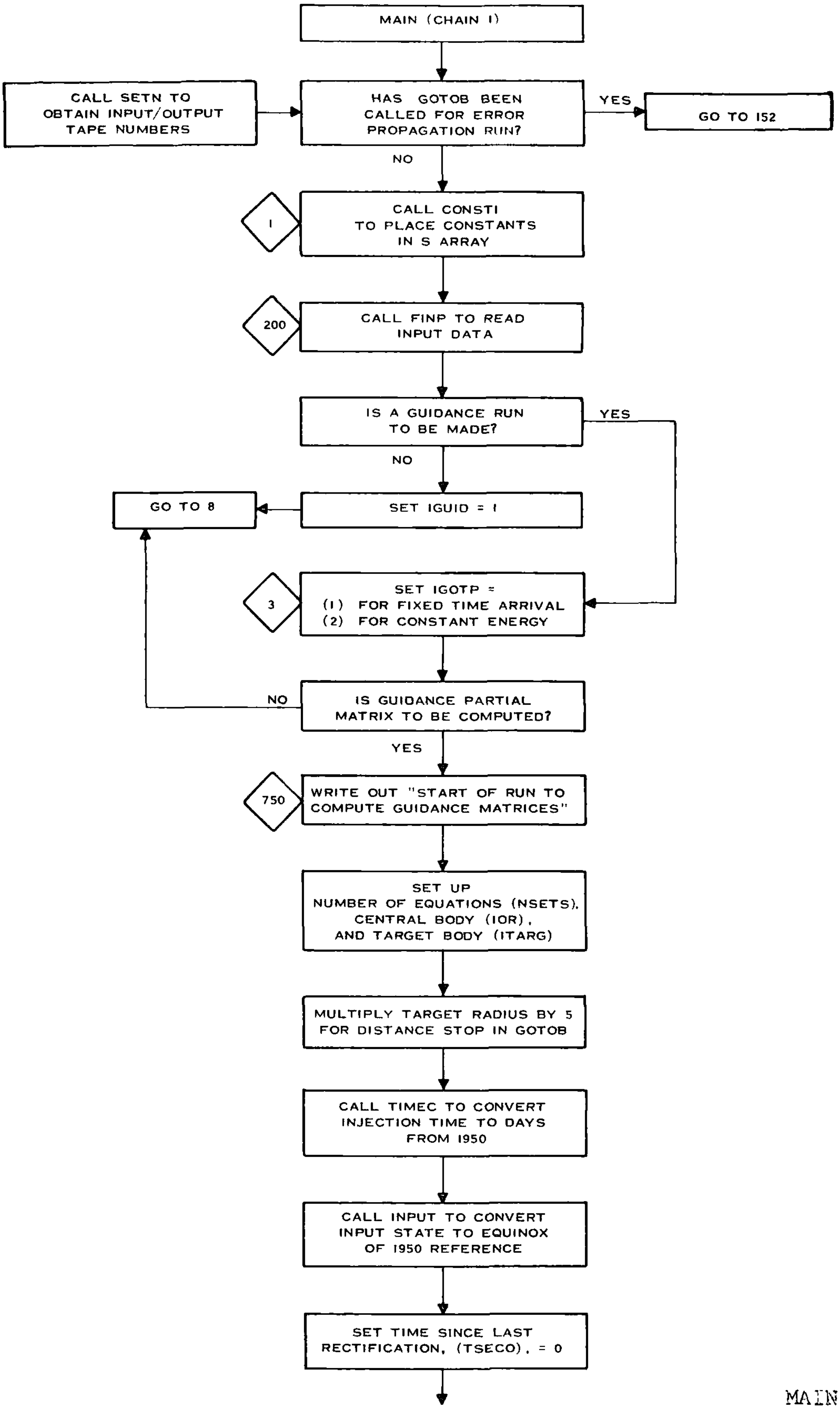
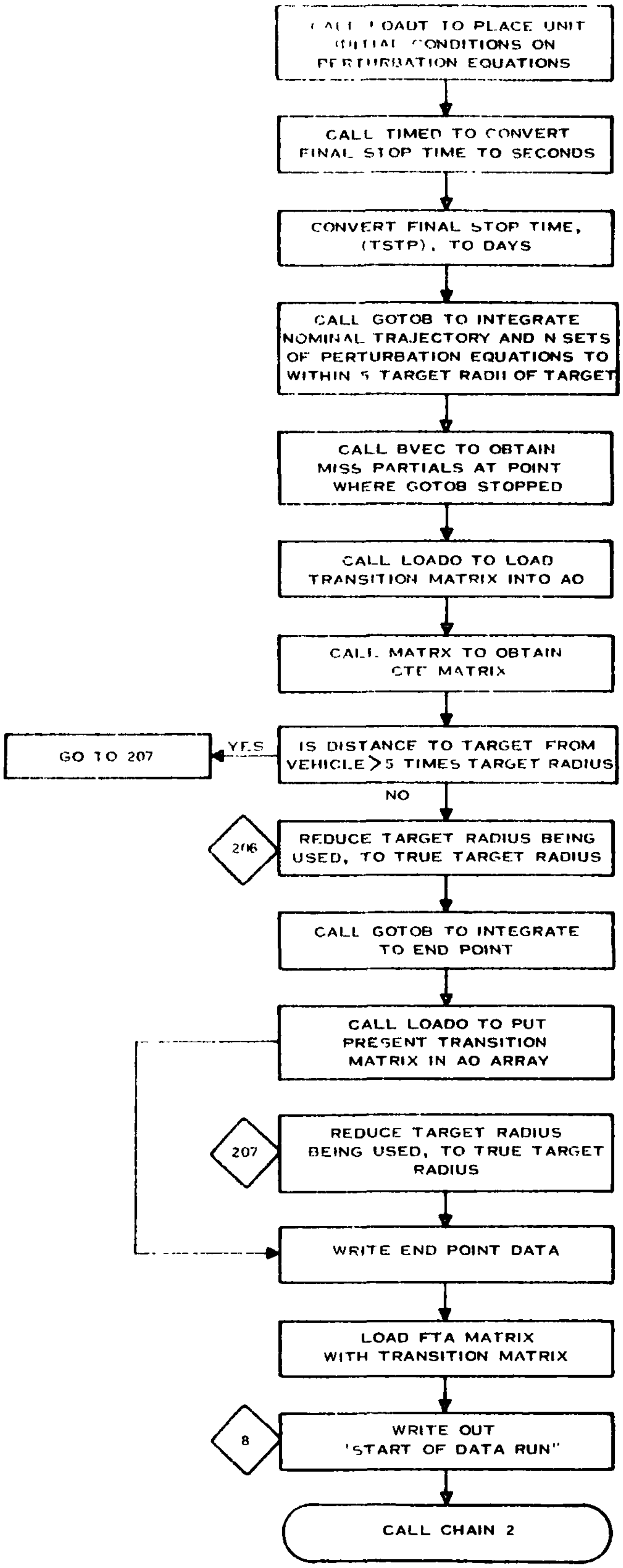
Subroutine: MAIN (Chain 1)

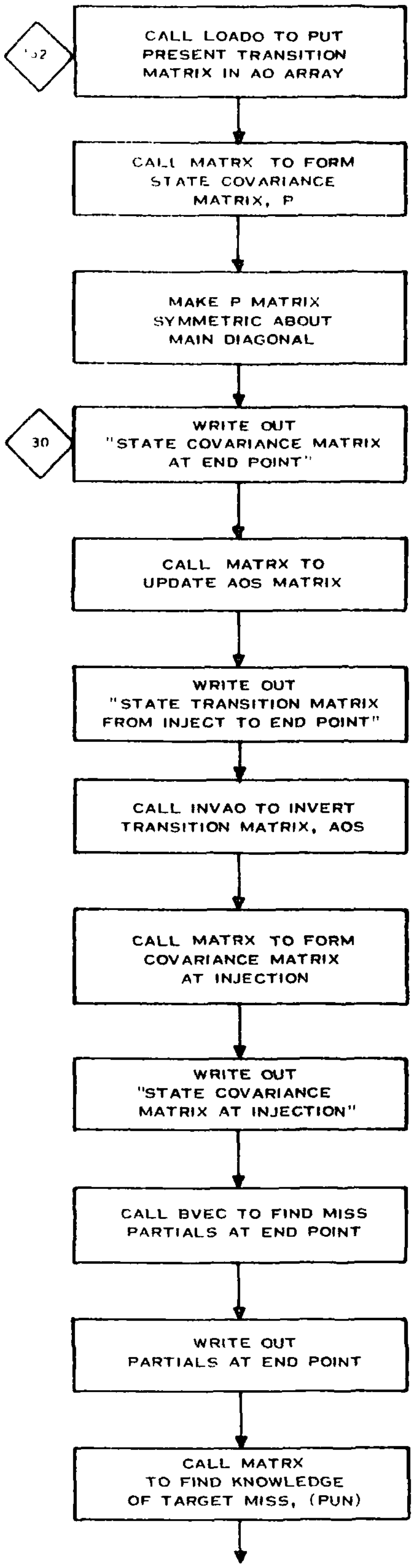
Purpose: Set up program to perform guidance runs to obtain transition and partial matrices. Call Chain 2 for error propagation runs and write out end point data after program returns from Chain 3.

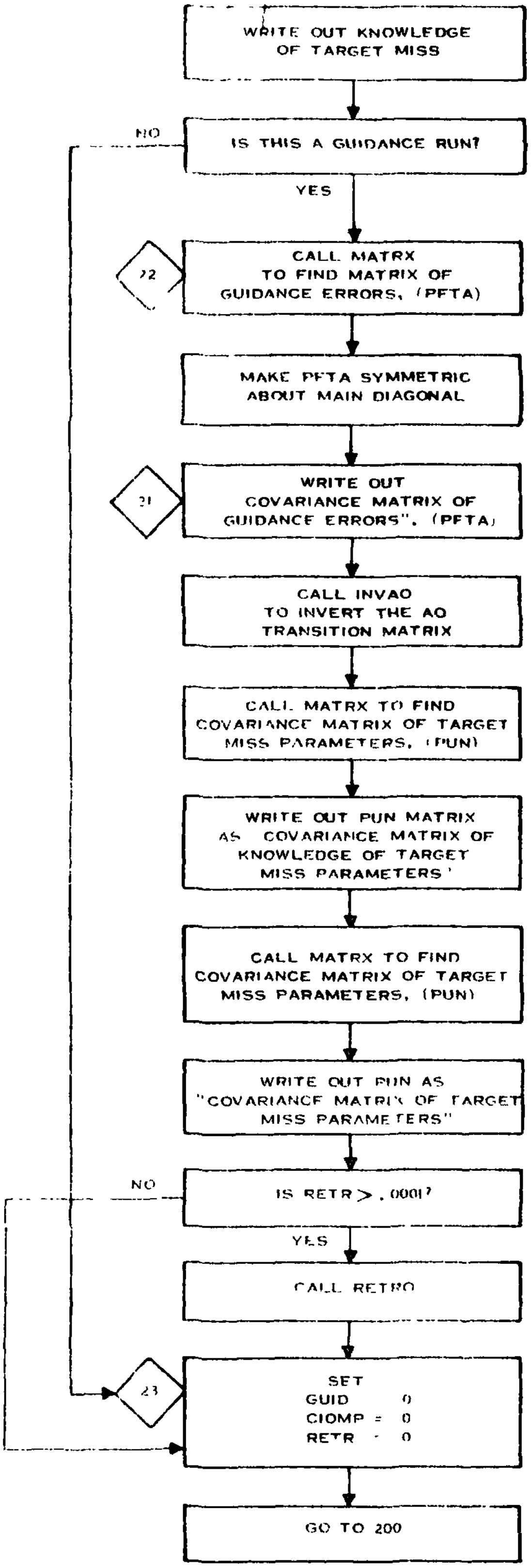
Common storages used or required:	T, S, C, IC
Subroutines required:	BVEC, CHAIN, CONST1, FINP, GOTOB, INPUT, INVAO, LOADO, LOADT, MATRX, RETRO, TIMEC, TIMED
Functions required:	None
Approximate number of storages required:	793 DEC



MAIN (CHAIN-1) -2


CALL LOADT TO PLACE UNIT INITIAL CONDITIONS ON PERTURBATION EQUATIONS
CALL TIMED TO CONVERT FINAL STOP TIME TO SECONDS
CONVERT FINAL STOP TIME, (TSTP), TO DAYS
CALL GOTOB TO INTEGRATE NOMINAL TRAJECTORY AND N SETS OF PERTURBATION EQUATIONS TO WITHIN 5 TARGET RADII OF TARGET
CALL BVEC TO OBTAIN MISS PARTIALS AT POINT WHERE GOTOB STOPPED
CALL LOADO TO LOAD TRANSITION MATRIX INTO AO
CALL MATRX TO OBTAIN CTF MATRIX
IS DISTANCE TO TARGET FROM VEHICLE > 5 TIMES TARGET RADIUS
YES
GO TO 207
NO
206
REDUCE TARGET RADIUS BEING USED, TO TRUE TARGET RADIUS
CALL GOTOB TO INTEGRATE TO END POINT
CALL LOADO TO PUT PRESENT TRANSITION MATRIX IN AO ARRAY
207
REDUCE TARGET RADIUS BEING USED, TO TRUE TARGET RADIUS
WRITE END POINT DATA
LOAD FTA MATRIX WITH TRANSITION MATRIX
8
WRITE OUT 'START OF DATA RUN"
CALL CHAIN 2


32
CALL LOADO TO PUT PRESENT TRANSITION MATRIX IN AO ARRAY
CALL MATRX TO FORM STATE COVARIANCE MATRIX, P
MAKE P MATRIX SYMMETRIC ABOUT MAIN DIAGONAL
30
WRITE OUT "STATE COVARIANCE MATRIX AT END POINT"
CALL MATRX TO UPDATE AOS MATRIX
WRITE OUT "STATE TRANSITION MATRIX FROM INJECT TO END POINT"
CALL INVAO TO INVERT TRANSITION MATRIX, AOS
CALL MATRX TO FORM COVARIANCE MATRIX AT INJECTION
WRITE OUT "STATE COVARIANCE MATRIX AT INJECTION"
CALL BVEC TO FIND MISS PARTIALS AT END POINT
WRITE OUT PARTIALS AT END POINT
CALL MATRX TO FIND KNOWLEDGE OF TARGET MISS, (PUN)


WRITE OUT KNOWLEDGE OF TARGET MISS
IS THIS A GUIDANCE RUN?
NO
YES
22
CALL MATRX TO FIND MATRIX OF GUIDANCE ERRORS, (PETA)
MAKE PETA SYMMETRIC ABOUT MAIN DIAGONAL
21
WRITE OUT COVARIANCE MATRIX OF GUIDANCE ERRORS". (PETA)
CALL INVAO TO INVERT THE AO TRANSITION MATRIX
CALL MATRX TO FIND COVARIANCE MATRIX OF TARGET MISS PARAMETERS, (PUN)
WRITE OUT PUN MATRIX AS COVARIANCE MATRIX OF KNOWLEDGE OF TARGET MISS PARAMETERS'
CALL MATRX TO FIND COVARIANCE MATRIX OF TARGET MISS PARAMETERS, (PUN)
WRITE OUT PUN AS "COVARIANCE MATRIX OF TARGET MISS PARAMETERS"
NO
IS RETR > .0001?
YES
CALL RETRO
23
SET GUID = 0 CIOMP = 0 RETR = 0
GO TO 200

MAIN (CHAIN 1)

```
*     LABEL                                                             MAI1
CEC20D2   MAIN PROGRAM                                                  MAI10000
C       ERROR PROPAGATION                                               MAI10010
C       ALONG INTERPLANETARY TRAJECTORIES                               MAI10015
C       REQUIRES FOLLOWING SUBROUTINES AND FUNCTIONS                    MAI10020
C       ARKTNS   SINGLE PRECISION ARCTANGENT                            MAI10030
C       ARKTAN   DOUBLE PRECISION ARCTANGENT                            MAI10040
C       ASINH(X) FUNCTION EVALUATION                                    MAI10050
C       BODY     CALCULATES ACCELERATIONS DUE TO PERTURBING BODIES      MAI10060
C       BVEC     CALCULATES B VECTOR                                    MAI10070
C       CHNGP    DETERMINES WHEN TO SHIFT BODY CENTER                   MAI10090
C       COMPHQ   COMPUTATIONS FOR ONBTR AND MONBTR SUBROUTINES          MAI10095
C       CONST1   ARRAY OF INPUT CONSTANTS                               MAI10100
C       CONVPI   CONVERTS INPUT COVARIENCE MATRIX TO 1950               MAI10110
C       CORRTP   UPDATES P MATRIX                                       MAI10020
C       CROSS    CROSS PRODUCT                                          MAI10130
C       DOT      FUNCTION FORMING DOT PRODUCT                           MAI10140
C       DE6FN    FAP  INTEGRATION SUBROUTINE                            MAI10160
C       EARTR    UPDATES COVARIANCE MATRIX FOR EARTH BASED TRACKING     MAI10170
C       ECLIP    TRANSFORMS COORDINATES THROUGH TRANSFORMATIONS         MAI10180
C       ENCKE    CALCULATES PERTURBATION DUE TO DEVIATION FROM CONIC    MAI10190
C       ERP      PRINTS OUT EPHEMERIS ERROR                             MAI10195
C       ERPT     PRINTS OUT TIME OF EPHEMERIS ERROR                     MAI10197
C       FINP     DATA INPUT SUBROUTINE                                  MAI10200
C       FNORM    NORM OF A VECTOR                                       MAI10210
C       GHA      GREENWICH HOUR ANGLE                                   MAI10220
C       GOTOB    MAIN SUBROUTINE FOR INTEGRATION OF TRAJECTORIES        MAI10230
C       GOTOR    ITERATES TO SOVE KEPLERS EQUATION                      MAI10235
C       GUID     PERFORMS GUIDANCE CALCULATIONS                         MAI10240
C       HPHT     PERFORMS MATRIX MULTIPLICATION H*P*HT                  MAI10245
C       INPUT    CONVERTS INPUTS TO EQUINOX OF 1950 REFERENCE           MAI10250
C       INTR     FAP EPHEMERIS SUBROUTINE                               MAI10260
C       INVAO    FORMS INVERSE OF TRANSITION MATRIX                     MAI10270
C       INV3     INVERTS UP TO A 6 BY 6 MATRIX                          MAI10280
C       LOADO    OBTAINS TRANSITION MATRIX FROM T ARRAY                 MAI10290
C       LOADT    PUTS UNIT ICS ON PERTURBATION EQUATIONS                MAI10300
C       MASS     ARRANGES GRAVITATIONAL CONSTANTS OF BODIES CONSIDERED  MAI10310
C       MATRX    MULTIPLIES A*B=C OR A*B*AT=C MAX DIMENSION(10,10)      MAI10320
C       MATSUB   ERROR PROPOGATION LOGIC SUBROUTINE                     MAI10330
C       MNA      TRANSFORMATION TO SELENOCENTIC COORDINATES             MAI10340
C       MNAND    TRANSFORMATION FOR SELENOCENTRIC VELOCITIES            MAI10345
C       MONBTR   UPDATES COVARIANCE MATRIX FOR MOON BEACONS             MAI10350
C       MULT     MULTIPLIES TWO 3 BY 3 MATRICES                         MAI10370
C       NUTAIT   CALCULATES NUTATION MATRIX                             MAI10380
C       OBLIN    CALCULATES ACCELERATION DUE TO OBLATENESS              MAI10390
C       ONBTR    UPDATES COVARIANCE MATRIX FOR ONBOARD TRACKING         MAI10400
C       ORTC     OUTPUTS ORBITAL PARAMETERS                             MAI10420
C       OUTC     OUTPUTS  TRAJECTORY                                    MAI10440
C       OUTDAT   OUTPUTS CALENDAR DATE                                  MAI10430
C       OUTP     OUTPUTS RMS VALUES OF ORBITAL PARAMETERS               MAI10450
C       PTRAN    TRANSFORMS P MATRIX                                    MAI10452
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AIN (CHAIN 1) (Cont'd)

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C       RETRO     PERFORMS RETRO FIRE                                          MAI10455
C       ROTATE    CALCULATES TRANSFORMATION FOR ROTATION ABOUT AN AXIS          MAI10460
C       ROTEQ     CALCULATES MATRIX FROM EQUINOX 1950 TO MEAN EQUINOX           MAI10465
C                 OF DATE                                                      MAI10468
C       RVIN      TRANSFORMS COORDINATES FROM SPHERICAL TO CARTESIAN            MAI10470
C       RVOUT     TRANSFORMS COORDINATES FROM CARTESIAN TO SPHERICAL            MAI10480
C       SDEC      SECOND DERIVATIVE SUBROUTINE                                 MAI10530
C       SETN      SET READ AND WRITE TAPE NUMBERS                              MAI10535
C       SHIFTP    SHIFTS BODY CENTER                                           MAI10540
C       STEPC     MOVE ALONG CONIC IN TIME                                     MAI10545
C       TIMEC     CONVERTS CALENDAR DATE TO DAYS FROM 1950                     MAI10550
C       TIMED     CONVERTS DAYS HOURS,MIN SEC TO SECONDS                       MAI10560
C       TRAC      TRACKING STATION COORDIATES                                  MAI10570
C       TRANSH    TRANSFORMS H MATRIX FROM DATE TO 1950                        MAI10580
      COMMON T,S,C,IC                                                          MAI10600
      DIMENSION T(1360),S(1000),C(1000),IC(300),X(3)                           MAI10610
     1,VX(3),XP(3),VXP(3),AN(3,3),UM(6),BV(4),PBV(3,6)                         MAI10620
     2,AO(6,6),PARI(21),PI(21),ISTAT(20),FTA(3,6)                              MAI10630
     3,TRANS(3,6)                                                              MAI10640
     4,PFTA(6,6),P(6,6),TGUID(6)                                               MAI10650
      DIMENSION AOS(6,6),RAD(6), RBOP(6),CTE(3,6),AOI(6,6),PUN(3,3)            MAI10660
      EQUIVALENCE(T,TDUM),(S,SDUM),(C,CDUM),(IC,IDUM),                         MAI10670
     1(IC(8),IMMSUB),(S(112),DATE),(S(113),FDATE),(S(115),X),(S(118),VX)MAI10680
     2,(IC(193),IGUID),(S(110),TSTOP),(IC(2),IOR),(S(111),TIM)                 MAI10690
     3,(IC(6),ITARG),(IC(4),KSTP),(IC(1),NSETS),(C(10),TW),(C(11),TF)          MAI10700
     4,(C(138),AN),(C(12),TSECO),(S(71),STD),(S(1),UM),(C(15),XP)              MAI10710
     5,(C(18),VXP),(C(13),TP1),(C(14),TP2),(S(671),PI),(S(650),PARI)           MAI10720
     6,(IC(194),ISTAT),(IC(7),KOUT),(C(649),TSECP),(S(122),OUTTP)              MAI10730
     7,(C(460),PFTA),(C(652),P)                                                MAI10740
     8,(S(114),CIBDY),(S(121),TIBDY),(S(722),FTA),(S(124),SKTB)                MAI10750
     9,(S(720),PUPIN),(S(721),PARIN)                                           MAI10760
      EQUIVALENCE (C(752),AOS),(S(900),RAD),(C(112),RBOP),(C(30),TSEC)         MAI10770
     1,(S(740),CTE),(C(648),TGUIDE),(C(850),AOI),(C(850),PUN)                  MAI10780
      EQUIVALENCE (S(476),TGUID),(IC(219),IGD)                                 MAI10790
     1,(IC(218),IGDKY),(IC(217),IGDTP),(C(568),TRANS)                          MAI10800
      EQUIVALENCE (T(1358),TSTP),(C(797),COMP),(S(888),RETR)                   MAI10810
      CALL SETN(NIN,NUTS)                                                      MAI10811
      IF(T(1359)-1359.)1,152,1                                                 MAI10820
  1   CALL CONST1                                                              MAI10830
C     THE CONSTS OF S ARRAY ARE READ IN                                        MAI10840
 200  CONTINUE                                                                 MAI10850
      WRITE OUTPUT TAPE NUTS,700                                               MAI10851
 700  FORMAT(26H1 START OF FINP INPUT DATA)                                    MAI10855
      CALL FINP(22,S,TIM,CIBDY,TIBDY,GUID,SKTB,CIOMP                           MAI10860
     1,TSTOP,X,VX,DATE,FDATE,FTA,PI,PARI,TGUID,GID,PUPIN,PARIN,CTE,            MAI10865
     2RETR,OUTTP,                                                         132HMAI10870
     2S      TIM    CIBDY TIBDY GUID  SKTB  CIOMP TSTOP X      VX    DATE  MAI10880
     3FDATE FTA    PI    PARI  TGUID GID   PUPIN PARIN CTE    RETR  OUTTP  MAI10885
     4)                                                                        MAI10890
C      GUID IS SET POSITIVE IF GUIDANCE IS CONSIDERED                          MAI10900
      IF(GUID) 2,2,3                                                           MAI10910
```

```
    2  CONTINUE
       IGUID=1
       GO TO 8
    3  CONTINUE
       IGD=GID
       IGDTP = GUID
       IF(IGDTP-3) 204,204,205
  205  CONTINUE
       IGDTP=2
  204  CONTINUE
       IGUID=2
C       CIOMP IS SET POSITIVE IF MATRICES ARE TO BE COMPUTED
       IF(CIOMP) 8,8,4
    4  CONTINUE
       COMP=CIOMP
       WRITE OUTPUT TAPE NUTS,750
  750  FORMAT(43H1START OF RUN TO COMPUTE GUIDANCE MATRICIES)
       NSETS=7
       IOR=S(114)
       ITARG=S(121)
       RAD(ITARG)=5.*RAD(ITARG)
       KIN=S(111)
       CALL TIMEC(DATE,FDATE,TW,TF)
       CALL INPUT(KIN,XP,VXP,TW,TF,AN,X,VX)
       TSECO =0.
       KSTP=1
       CALL LOADT
       CALL TIMED(TSTOP,TSTPS)
       TSTP=TSTPS*STD
       IMMSUB=1
       CALL GOTOB(TSTP)
       CALL BVEC(IOR,ITARG,XP,VXP,BV,PBV,TP1,TP2,UM)
       CALL LOADO(AO)
       CALL MATRX(PBV,AO,CTE,3,6,6,0)
       IF(RBOP(ITARG)-RAD(ITARG))206,206,207
  206  CONTINUE
       RAD(ITARG)=RAD(ITARG)/5.
       TSECO=TSEC
       CALL GOTOB(TSTP)
       CALL LOADO(AO)
       GO TO 208
  207  CONTINUE
       RAD(ITARG)=RAD(ITARG)/5.
  208  CONTINUE
       WRITE OUTPUT TAPE NUTS,706,((AO(I,J),J=1,6),I=1,6)
  706  FORMAT(31H0TRANSITION MATRIX TO END POINT//(6E17.8))
       WRITE OUTPUT TAPE NUTS,707,((CIE(I,J),J=1,6),I=1,3)
  707  FORMAT(49H0B VECTOR TRANSITION MATRIX. BY ROWS B.T,B.R,VINF//
      1(6E17.8))
       DO 6 I=1,3
       DO 6 J=1,6
```

```
      FTA(I,J)=AO(I,J)                                              MAI11430
    6 CONTINUE                                                      MAI11440
    8 CONTINUE                                                      MAI11450
      WRITE OUTPUT TAPE NUTS,751                                    MAI11460
  751 FORMAT(18H1START OF DATA RUN)                                 MAI11470
      CALL CHAIN(2,B3)                                              MAI11480
  152 CALL LOADO(AO)                                                MAI11490
C     AO IS TRANSITION MATRIX FROM LAST TIME IN MATSUB              MAI11500
      CALL MATRX(AO,P,P,6,6,6,1)                                    MAI11510
      DO 30 I=1,6                                                   MAI11511
      DO 30 J=I,6                                                   MAI11512
      P(I,J)=(P(I,J)+P(J,I))/2.                                     MAI11514
      P(J,I)=P(I,J)                                                 MAI11516
   30 CONTINUE                                                      MAI11518
C     P IS STATE COVARIACE MATRIX AT END POINT                      MAI11520
      WRITE OUTPUT TAPE NUTS,800,((P(I,J),J=1,6),I=1,6)             MAI11530
  800 FORMAT(38H0 STATE COVARIANCE MATRIX AT END POINT              MAI11540
     1/(6E17.8))                                                    MAI11550
      CALL MATRX(AO,AOS,AOS,6,6,6,0)                                MAI11560
C     AOS IS TRANSITION MATRIX FROM INJECTION TO END POINT          MAI11570
      WRITE OUTPUT TAPE NUTS,801,((AOS(I,J),J=1,6)I=1,6)            MAI11580
  801 FORMAT(50H0 STATE TRANSITION MATRIX FROM INJECT TO END POINT  MAI11590
     1/(6E17.8))                                                    MAI11600
      CALL INVAO(AOS,AOI)                                           MAI11610
C     FORMS INVERSE OF AOS                                          MAI11620
      CALL MATRX(AOI,P,AOS,6,6,6,1)                                 MAI11630
      WRITE OUTPUT TAPE NUTS,802,AOS                                MAI11640
  802 FORMAT(38H0 STATE COVARIANCE MATRIX AT INJECTION              MAI11650
     1/(6E17.8))                                                    MAI11660
      CALL BVEC(IOR,ITARG,XP,VXP,BV,PBV,TP1,TP2,UM)                 MAI11670
      WRITE OUTPUT TAPE NUTS,810,((PBV(J,I),I=1,6),J=1,3)           MAI11680
  810 FORMAT(40H0 B*T,B*R,VINF PARTIALS AT END POINT PBV            MAI11690
     1/(6E17.8))                                                    MAI11700
      CALL MATRX(PBV,P,PUN,3,6,6,1)                                 MAI11710
      WRITE OUTPUT TAPE NUTS,811,PUN                                MAI11720
  811 FORMAT(47H0 COVARIANCE MATRIX OF KNOWLEGE OF B*T,B*R,VINF     MAI11730
     1/(3E20.8))                                                    MAI11740
      IGUID = IGUID                                                 MAI11750
      GO TO (23,22),IGUID                                           MAI11760
   22 CONTINUE                                                      MAI11770
      CALL MATRX(AO,PFTA,PFTA,6,6,6,1)                              MAI11780
      DO 31 I=1,6                                                   MAI11781
      DO 31 J=I,6                                                   MAI11782
      PFTA(I,J)=(PFTA(I,J)+PFTA(J,I))/2.                            MAI11784
      PFTA(J,I)=PFTA(I,J)                                           MAI11786
   31 CONTINUE                                                      MAI11788
      WRITE OUTPUT TAPE NUTS,804,((PTFA(I,J),J=1,6),I=1,6)          MAI11790
  804 FORMAT(38H0 COVARIANCE MATRIX OF GUIDANCE ERRORS              MAI11800
     1/(6E17.8))                                                    MAI11810
      CALL INVAO(AO,AOI)                                            MAI11820
      CALL MATRX(TRANS,AOI,TRANS,3,6,6,0)                           MAI11830
```

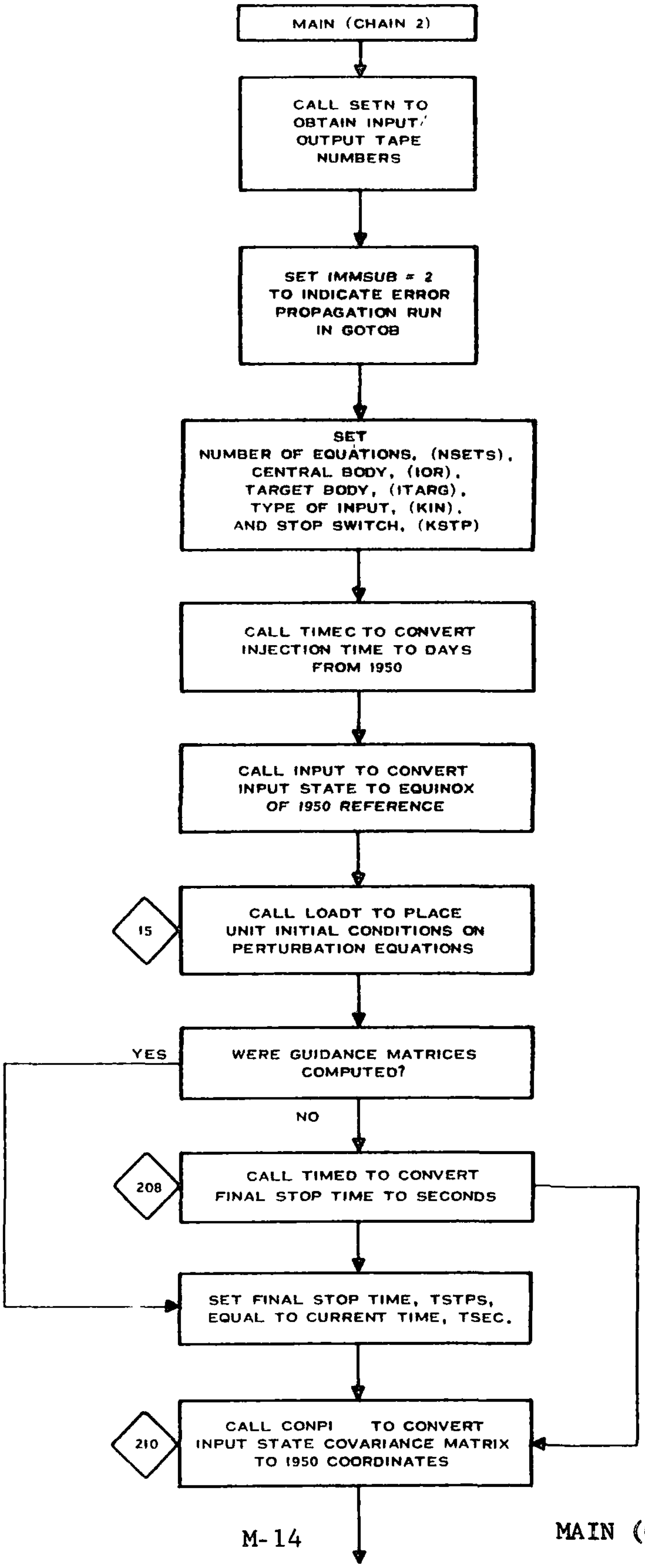
MAIN (CHAIN 1) (Cont'd)

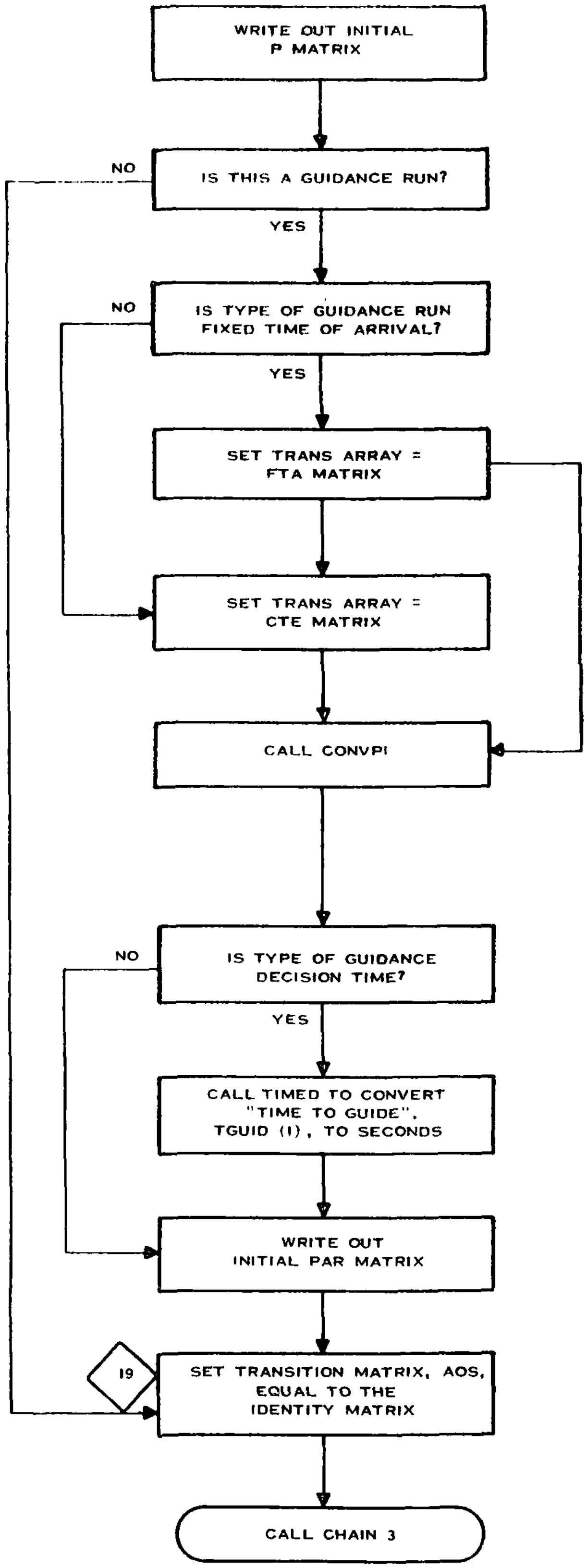
```
      CALL MATRX(TRANS,P,PUN,3,6,6,1)                                   MAI1184
      WRITE OUTPUT TAPE NUTS,803,PUN                                    MAI1185
  803 FORMAT(58H0 COVARIANCE MATRIX OF KNOWLEDGE OF TARGET MISS PARAMETEMAI1186
     1RS                                                                MAI1187
     2/(3E20.8))                                                        MAI1188
      CALL MATRX(TRANS,PFTA,PUN,3,6,6,1)                                MAI1189
      WRITE OUTPUT TAPE NUTS,807,PUN                                    MAI1190
  807 FORMAT(45H0 COVARIANCE MATRIX OF TARGET MISS PARAMETERS           MAI1191
     1/(3E20.8))                                                        MAI1192
      IF(RETR-.0001)23,23,10                                            MAI1192
   10 CONTINUE                                                          MAI1192
      CALL RETRO                                                        MAI1192
   23 CONTINUE                                                          MAI1193
      RETR=0.                                                           MAI1193
      GUID=0                                                            MAI1194
      CIOMP=0                                                           MAI1195
      GO TO 200                                                         MAI1196
      END                                                               MAI1
```

Subroutine: MAIN (Chain 2)

Purpose: Set up program to perform error propagation run. Call Chain 3 to make data run.

Common storages used or required:	T, S, C, IC
Subroutines required:	CHAIN, CONVPI, INPUT, TIMEC, TIMED
Functions required:	None
Approximate number of storages required:	317 DEC


MAIN (CHAIN 2)
CALL SETN TO OBTAIN INPUT/ OUTPUT TAPE NUMBERS
SET IMMSUB = 2 TO INDICATE ERROR PROPAGATION RUN IN GOTOB
SET NUMBER OF EQUATIONS, (NSETS), CENTRAL BODY, (IOR), TARGET BODY, (ITARG), TYPE OF INPUT, (KIN), AND STOP SWITCH, (KSTP)
CALL TIMEC TO CONVERT INJECTION TIME TO DAYS FROM 1950
CALL INPUT TO CONVERT INPUT STATE TO EQUINOX OF 1950 REFERENCE
15
CALL LOADT TO PLACE UNIT INITIAL CONDITIONS ON PERTURBATION EQUATIONS
WERE GUIDANCE MATRICES COMPUTED?
YES
NO
208
CALL TIMED TO CONVERT FINAL STOP TIME TO SECONDS
SET FINAL STOP TIME, TSTPS, EQUAL TO CURRENT TIME, TSEC.
210
CALL CONPI TO CONVERT INPUT STATE COVARIANCE MATRIX TO 1950 COORDINATES



MAIN (CHAIN 2) -3

MAIN (CHAIN 2)

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*      LABEL                                                                    MAI2
*      SYMBOL TABLE                                                             MAI2
CEC20E2      PART 2 MAIN CONTINUED                                              MAI20000
       COMMON T,S,C,IC                                                          MAI20010
       DIMENSION T(1360),S(1000),C(1000),IC(300),X(3)                           MAI20020
      1,VX(3),XP(3),VXP(3),AN(3,3),UM(6),BV(4),PBV(3,6)                         MAI20030
      2,AO(6,6),PARI(21),PI(21),ISTAT(20),FTA(3,6)                              MAI20040
      3,TRANS(3,6)                                                              MAI20050
      4,PFTA(6,6),P(6,6),TGUID(6)                                               MAI20060
       DIMENSION AOS(6,6),RAD(6), RBOP(6),CTE(3,6),AOI(6,6),PUN(3,3)            MAI20070
       EQUIVALENCE(T,TDUM),(S,SDUM),(C,CDUM),(IC,IDUM),                         MAI20080
      1(IC(8),IMMSUB),(S(112),DATE),(S(113),FDATE),(S(115),X),(S(118),VX)MAI20090
      2,(IC(193),IGUID),(S(110),TSTOP),(IC(2),IOR),(S(111),TIM)                 MAI20100
      3,(IC(6),ITARG),(IC(4),KSTP),(IC(1),NSETS),(C(10),TW),(C(11),TF)          MAI20110
      4,(C(138),AN),(C(12),TSECO),(S(71),STD),(S(1),UM),(C(15),XP)              MAI20120
      5,(C(18),VXP),(C(13),TP1),(C(14),TP2),(S(671),PI),(S(650),PARI)           MAI20130
      6,(IC(194),ISTAT),(IC(7),KOUT),(C(649),TSECP)                             MAI20140
      7,(C(460),PFTA),(C(652),P)                                                MAI20150
      8,(S(114),CIBDY),(S(121),TIBDY),(S(722),FTA),(S(124),SKTB)                MAI20160
      9,(S(720),PUPIN),(S(721),PARIN)                                           MAI20170
       EQUIVALENCE (C(752),AOS),(S(900),RAD),(C(112),RBOP),(C(30),TSEC)         MAI20180
      1,(S(740),CTE),(C(648),TGUIDE),(C(850),AOI),(C(850),PUN)                  MAI20190
       EQUIVALENCE (S(476),TGUID),(IC(219),IGD)                                 MAI20200
      1,(IC(218),IGDKY),(IC(217),IGDTP),(C(568),TRANS)                          MAI20210
       EQUIVALENCE (T(1358),TSTP),(C(797),COMP)                                 MAI20220
       CALL SETN(NIN,NOUT)                                                      MAI20225
       IMMSUB=2                                                                 MAI20230
       NSETS=7                                                                  MAI20240
       IOR  =S(114)                                                             MAI20250
       ITARG=S(121)                                                             MAI20260
       KIN  =S(111)                                                             MAI20270
       KSTP=S(124)                                                              MAI20280
       CALL TIMEC(DATE,FDATE,TW,TF)                                             MAI20290
       CALL INPUT(KIN,XP,VXP,TW,TF,AN,X,VX)                                     MAI20300
       TSECO =0.                                                                MAI20310
       DO 15  I=1,20                                                            MAI20320
       ISTAT(I)=0                                                               MAI20330
   15  CONTINUE                                                                 MAI20340
       KOUT=S(122)                                                              MAI20350
       CALL LOADT                                                               MAI20360
       TSECP=0.                                                                 MAI20370
       IF(COMP)208,208,209                                                      MAI20380
  208  CONTINUE                                                                 MAI20390
       CALL TIMED(TSTOP,TSTPS)                                                  MAI20400
       GO TO 210                                                                MAI20410
  209  TSTPS=TSEC                                                               MAI20420
  210  CONTINUE                                                                 MAI20430
       TSTP=TSTPS*STD                                                           MAI20440
       INPUP=PUPIN                                                              MAI20450
       CALL CONVPI(INPUP,PI,P)                                                  MAI20460
       WRITE OUTPUT TAPE NOUT,708                                               MAI20470
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AIN (CHAIN 2) (Cont'd)

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708   FORMAT(18H0INITIAL  P MATRIX)                                MAI20480
      WRITE OUTPUT TAPE NOUT,709,P                                 MAI20490
709   FORMAT(6E17.8)                                               MAI20500
      IGUID=IGUID                                                  MAI20510
      GO TO (19,18), IGUID                                         MAI20520
   18 CONTINUE                                                     MAI20530
      IGDTP=IGDTP                                                  MAI20540
      GO TO(211,212,212),IGDTP                                     MAI20550
211   CONTINUE                                                     MAI20560
      DO 202 I=1,3                                                 MAI20570
      DO 202 J=1,6                                                 MAI20580
      TRANS(I,J)=FTA(I,J)                                          MAI20590
202   CONTINUE                                                     MAI20600
      GO TO 214                                                    MAI20610
212   DO 213 I=1,3                                                 MAI20620
      DO 213 J=1,6                                                 MAI20630
213   TRANS(I,J)=CTE(I,J)                                          MAI20640
214   CONTINUE                                                     MAI20650
      IGDKY=1                                                      MAI20660
      INPAR=PARIN                                                  MAI20670
      CALL CONVPI(INPAR,PARI,PFTA)                                 MAI20680
      S(475) CONTAINS TYPE OF GUIDANCE DECISION 0 TIME + RATIO     MAI20690
      IF(S(475))17,17,20                                           MAI20700
 17   CONTINUE                                                     MAI20710
      CALL TIMED(TGUID(1),TGUIDE)                                  MAI20720
 20   CONTINUE                                                     MAI20730
      WRITE OUTPUT TAPE NOUT,710                                   MAI20740
710   FORMAT(20H0 INITIAL PAR MATRIX)                              MAI20750
      WRITE OUTPUT TAPE NOUT,709,PFTA                              MAI20760
 19   CONTINUE                                                     MAI20770
      DO 150 I=1,6                                                 MAI20780
      DO 151 J=1,6                                                 MAI20790
151   AOS (I,J) =0.                                                MAI20800
150   AOS (I,I)=1.                                                 MAI20810
      CALL CHAIN(3,B3)                                             MAI20820
      END                                                          MAI2
```

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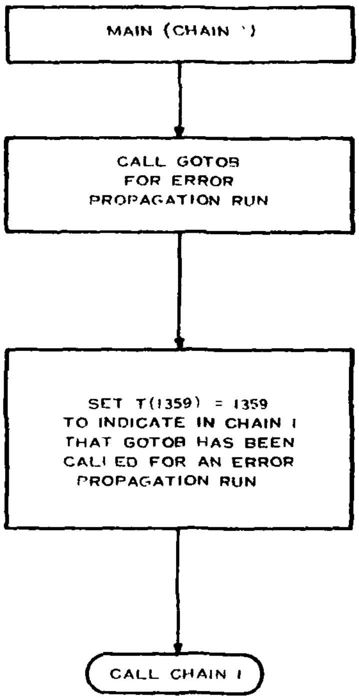
Subroutine: MAIN (Chain 3)

Purpose: Call subroutine GOTOB to perform integration of trajectory for an error propagation run. Set cell T(1359) = 1359 to indicate GOTOB has been called and call Chain 1.

Common storages used or required:	T
Subroutines required:	GOTOB
Functions required:	None
Approximate number of storages required:	


MAIN (CHAIN `)
CALL GOTOB
FOR ERROR
PROPAGATION RUN
SET T(1359) = 1359
TO INDICATE IN CHAIN I
THAT GOTOB HAS BEEN
CALLED FOR AN ERROR
PROPAGATION RUN
CALL CHAIN I

IN (CHAIN 3)

```
      LABEL                                                    MAI3
      SYMBOL TABLE                                             MAI3
C20C2        CHAIN 3 GOTOB ONLY                                MAI30000
      COMMON T                                                 MAI30010
      DIMENSION T(1360)                                        MAI30020
      EQUIVALENCE(T(1358),TSTP)                                MAI30030
      CALL GOTOB(TSTP)                                         MAI30040
      T(1359)=1359.                                            MAI30050
      CALL CHAIN(1,B3)                                         MAI30060
      END                                                      MAI3
```

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SUBROUTINES

(Subroutine discussion is presented in the following manner: (1) subroutine description, (2) subroutine flow diagrams, if applicable, (3) subroutine listings.)

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Function: ARKTAN

Purpose: To find the double precision arctangent θ of (Y/X) in the proper quadrant. $-\Pi \leq \theta \leq \Pi$ for N = 180; $0 \leq \theta \leq 360$ for N = 360.

Calling Sequence:

Z = ARKTAN (N, X, Y)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	N	1			N = 180 or 360, as above
I	X	1			
I	Y	1			
0	ARKTAN			radians	$\tan^{-1}$ (Y/X)

Common storages used or required: None

Subroutines required: None

Functions required: DATAN, other double precision routines

Approximate number of storages required:

```
*      LABEL                                                                ARKT
*      SYMBOL TABLE                                                         ARKT
CEC2021  FUNCTION ARKTAN                                                    ARKT0000
     CALL SETN(NIN,NOUT)                                                    ARKT0015
      FUNCTION ARKTAN(N,X,Y)                                                ARKT0010
D     P=3.14159265                                                          ARKT0020
      K=N/180                                                               ARKT0030
D     ARKTAN = ATANF(ABSF(Y/X))                                             ARKT0040
      IF(Y)1,2,3                                                            ARKT0050
    1 GO TO (4,5),K                                                         ARKT0060
    4 IF (X)10,11,12                                                        ARKT0070
D  10 ARKTAN = ARKTAN-P                                                     ARKT0080
      GO TO 9                                                               ARKT0090
D  11 ARKTAN = -P*.5                                                        ARKT0100
      GO TO 9                                                               ARKT0110
D  12 ARKTAN = -ARKTAN                                                      ARKT0120
      GO TO 9                                                               ARKT0130
    5 IF (X)13,14,15                                                        ARKT0140
D  13 ARKTAN = P+ARKTAN                                                     ARKT0150
      GO TO 9                                                               ARKT0160
D  14 ARKTAN = 3.*.5*P                                                      ARKT0170
      GO TO 9                                                               ARKT0180
D  15 ARKTAN = 2.*P-ARKTAN                                                  ARKT0190
      GO TO 9                                                               ARKT0200
    2 IF (X)16,17,9                                                         ARKT0210
D  16 ARKTAN = P                                                            ARKT0220
      GO TO 9                                                               ARKT0230
   17 WRITE OUTPUT TAPE NOUT,18                                             ARKT0240
   18 FORMAT (90H0                                 THE ARKTAN FUNCTION HAS ARKT0250
     1FAILED  -  ARKTAN(0/0) IS UNDEFINED/1H0)                              ARKT0260
      GO TO 9                                                               ARKT0270
    3 IF(X)8,19,9                                                           ARKT0280
D   8 ARKTAN = P-ARKTAN                                                     ARKT0290
      GO TO 9                                                               ARKT0300
D 19  ARKTAN = .5*P                                                         ARKT0310
    9 RETURN                                                                ARKT0320
      END                                                                   ARKT
```

Function: ARKTNS

Purpose: To find the arctangent θ of (Y/X) in the proper quadrant. $-\Pi \le \theta \le \Pi$ for N = 180; $0 \le \theta < 360$ for N = 360.

Calling Sequence:

Z = ARKTNS (N, X, Y)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	N	1			N = 180 or 360, as above
I	X	1			
I	Y	1			
O	ARKTNS			radians	$\tan^{-1}$ (Y/X)

Common storages used or required: None

Subroutines required: None

Functions required: ATANF

Approximate number of storages required: ________

```
*      LABEL                                                          ARKS
*      SYMBOL TABLE                                                   ARKS
CEC2022  FUNCTION ARKTNS                                              ARKS0000
       FUNCTION ARKTNS(N,X,Y)                                         ARKS0010
       CALL SETN(NIN,NOUT)                                            ARKS0015
       P=3.14159265                                                   ARKS0020
       K=N/180                                                        ARKS0030
       ARKTNS = ATANF(ABSF(Y/X))                                      ARKS0040
       IF(Y)1,2,3                                                     ARKS0050
     1 GO TO (4,5),K                                                  ARKS0060
     4 IF (X)10,11,12                                                 ARKS0070
    10 ARKTNS = ARKTNS-P                                              ARKS0080
       GO TO 9                                                        ARKS0090
    11 ARKTNS = -P*.5                                                 ARKS0100
       GO TO 9                                                        ARKS0110
    12 ARKTNS = -ARKTNS                                               ARKS0120
       GO TO 9                                                        ARKS0130
     5 IF (X)13,14,15                                                 ARKS0140
    13 ARKTNS = P+ARKTNS                                              ARKS0150
       GO TO 9                                                        ARKS0160
    14 ARKTNS = 3.*.5*P                                               ARKS0170
       GO TO 9                                                        ARKS0180
    15 ARKTNS = 2.*P-ARKTNS                                           ARKS0190
       GO TO 9                                                        ARKS0200
     2 IF (X)16,17,9                                                  ARKS0210
    16 ARKTNS = P                                                     ARKS0220
       GO TO 9                                                        ARKS0230
    17 WRITE OUTPUT TAPE NOUT,18                                      ARKS0240
    18 FORMAT (90H0                        THE ARKTAN FUNCTION HAS    ARKS0250
      1FAILED  -  ARKTAN(0/0) IS UNDEFINED/1H0)                       ARKS0260
       GO TO 9                                                        ARKS0270
     3 IF(X)8,19,9                                                    ARKS0280
     8 ARKTNS = P-ARKTNS                                              ARKS0290
       GO TO 9                                                        ARKS0300
    19 ARKTNS = .5*P                                                  ARKS0310
     9 RETURN                                                         ARKS0320
       END                                                            ARKS
```

Function: ASINH

Purpose: To compute $\theta = \text{arcsinh}(X) = \log_e (X + \sqrt{X^2 + 1})$

Calling Sequence:

Y = ASINH (X)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	X				
O	ASINH			radians	$\sinh^{-1}(X) = \log_e (X + \sqrt{X^2 + 1})$

Common storages used or required: None

Subroutines required: None

Functions required: LOGF

Approximate number of storages required:

```
*      LABEL                                              ASIN
*      SYMBOL TABLE                                       ASIN
       FUNCTION ASINH(X)                                  ASIN000
       ASINH=LOGF(X+SQRTF(X*X+1.0))                       ASIN001
       RETURN                                             ASIN002
       END                                                ASIN
```

Subroutine: BODY

Purpose: To compute the perturbing acceleration due to the perturbing bodies included in the ephemeris. It also computes the first variation of the perturbing acceleration for use in the integration of the variational equations.

Calling Sequence:

CALL BODY (XP, PO, VKB, RBO, RBOP, CA, NOR, BP, T, NEQ, U)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	XP	3	$\vec{R}$	Km	Position vector
I	PO	22	$\vec{R}_i$	Km	Positions of bodies in ephemeris
I	VKB	6	μ_i	Km^3/sec^2	Gravitational constants of bodies
					being considered
O	RBOP	6	$\lvert\vec{R}\rvert_{iv}$	Km	Magnitude of position of vehicle
					relative to i th body
O	RBO	6	$\lvert\vec{R}\rvert_i$	Km	Position of i^{th} body relative to
					central body

(Cont'd.)

Common storages used or required: None

Subroutines required: None

Functions required: FNORM

Approximate number of storages required: 445 DEC

(Cont'd.)

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	CA	3			Perturbation accelerations
I	NOR	1			Number of central body
O	BP	3,3	$\frac{\partial \ddot{x}_i}{\partial x_j}$		First variation of perturbation accelerations
I/O	T	1360			Common T Block
I	NEQ	1			Number of equations being integrated
I	U	1	μ	Km^3/sec^2	Central body gravitational constant

Equation Being Solved

The perturbation term due to the n bodies being considered is:

$$\vec{P} = -\sum_{j=1}^{n} \mu_j \left(\frac{\vec{R}_{jp}}{R_{jp}^3} + \frac{\vec{R}_j}{R_j^3} \right)$$

where $\vec{R}_{jp} = \vec{R} - \vec{R}_j$.

The vector $\vec{P}$ is placed in the cells called CA.

If the number of equations being solved is greater than three, the first variation of $\vec{P}$ with respect to position is determined and placed in the T block for the integration package

$$AP = \frac{\partial \ddot{X}_i}{\partial X_j} \qquad \begin{array}{l} i = 1,3 \\ j = 1,3 \end{array}$$

The form of AP is obtained by differentiating

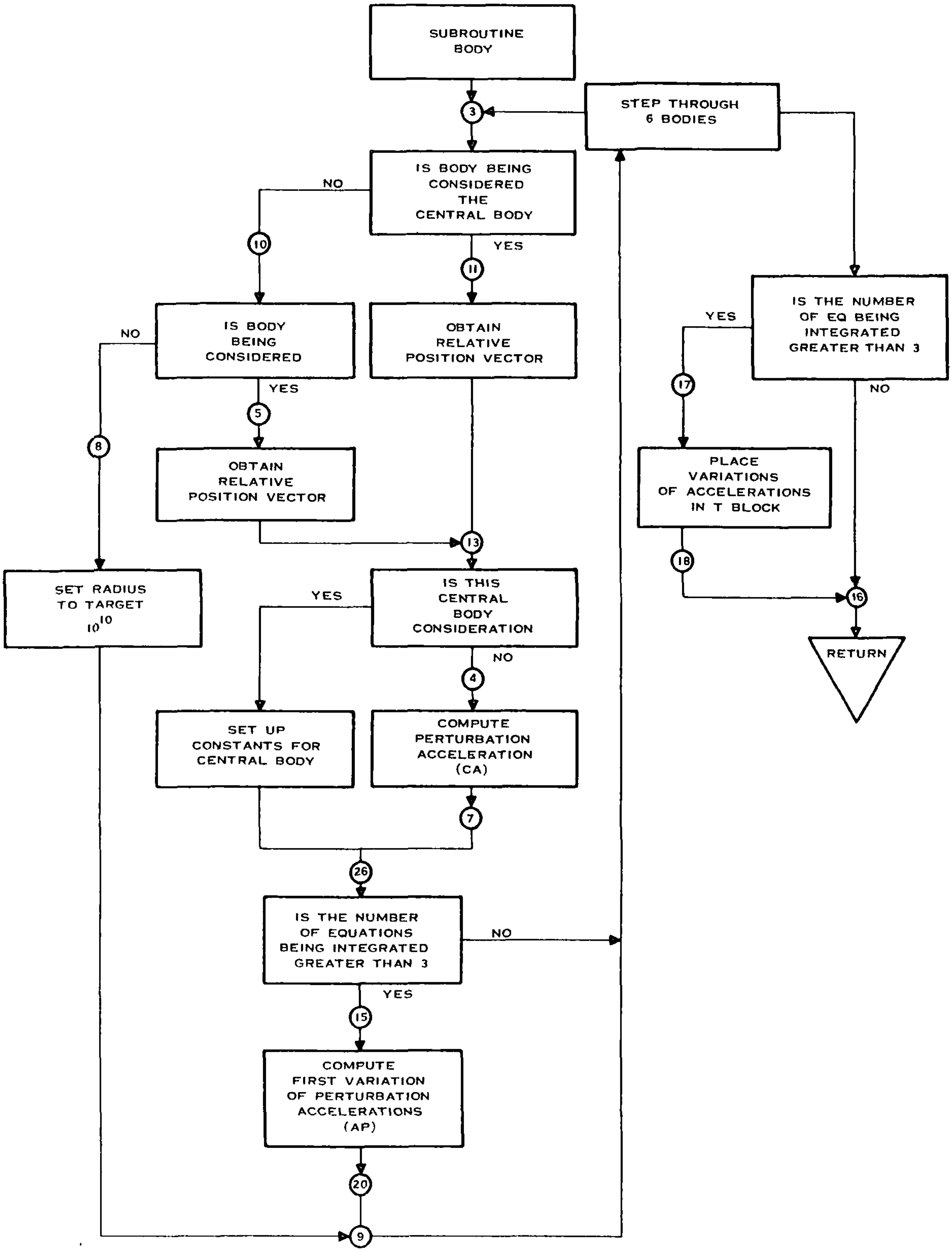
$$-\mu \frac{\vec{R}}{R^3} + \vec{P}$$

with respect to X_j. This yields:

$$AP_{ij} = -\sum_{k=0}^{n} \mu_k \left\{ \frac{1}{R_{kp}^3} \frac{\partial \vec{R}}{\partial X_j} - \frac{3}{R_{kp}^5} \left(\vec{R}_{kp} \cdot \frac{\partial \vec{R}}{\partial X_j} \right) \vec{R}_{kp} \right\}$$

with $\mu_o = \mu$ and $\vec{R}_{op} = \vec{R}$

$$\text{or } AP_{ij} = \sum_{k=0}^{n} \frac{\mu_k}{R_{kp}^3} \left\{ 3 \; \underline{\vec{R}_{kp} \vec{R}_{kp}^T} - I \right\}$$



SUBROUTINE BODY
STEP THROUGH 6 BODIES
IS BODY BEING CONSIDERED THE CENTRAL BODY
NO
YES
IS BODY BEING CONSIDERED
OBTAIN RELATIVE POSITION VECTOR
IS THE NUMBER OF EQ BEING INTEGRATED GREATER THAN 3
OBTAIN RELATIVE POSITION VECTOR
PLACE VARIATIONS OF ACCELERATIONS IN T BLOCK
SET RADIUS TO TARGET 10^10
IS THIS CENTRAL BODY CONSIDERATION
RETURN
SET UP CONSTANTS FOR CENTRAL BODY
COMPUTE PERTURBATION ACCELERATION (CA)
IS THE NUMBER OF EQUATIONS BEING INTEGRATED GREATER THAN 3
COMPUTE FIRST VARIATION OF PERTURBATION ACCELERATIONS (AP)
3
10
11
5
8
13
17
18
16
4
7
26
15
20
9

```
*      LABEL                                                              BODY
*      SYMBOL TABLE                                                       BODY
       SUBROUTINE BODY(X,PO,VKB,RBO,RBOP,CA,NOR,AP,T,NEQ,U)               BODY000
       DIMENSION X(3),PO(22),VKB(6),RBO(6),RBOP(6),CA(3),XN(18),RJP(3),   BODY001
      1AP(3,3),T(1360)                                                    BODY002
       N = NOR                                                            BODY003
       DO 1 I=1,15                                                        BODY004
       J = 23-I                                                           BODY005
     1 XN(I) = PO(J)                                                      BODY006
       DO 2 I=16,18                                                       BODY007
       J = 20-I                                                           BODY008
     2 XN(I) = PO(J)                                                      BODY009
       DO 3 I=1,3                                                         BODY010
       CA(I)=0.                                                           BODY011
     3 CONTINUE                                                           BODY012
       DO 9 I=1,6                                                         BODY013
       K1 = 3*I-3                                                         BODY014
       IF(I-N)10,11,10                                                    BODY015
    11 CONTINUE                                                           BODY016
       DO 12 J=1,3                                                        BODY017
       RJP(J)=X(J)                                                        BODY018
    12 CONTINUE                                                           BODY019
       GO TO 13                                                           BODY020
    10 CONTINUE                                                           BODY021
       IF (VKB(I)) 5,8,5                                                  BODY022
     5 DO 6 J=1,3                                                         BODY023
       K = K1+J                                                           BODY024
       RJP(J) = X(J) - XN(K)                                              BODY025
     6 CONTINUE                                                           BODY026
    13 CONTINUE                                                           BODY027
       RBOP(I) = FNORM(RJP)                                               BODY028
       RBO(I) = FNORM(XN(K1+1))                                           BODY029
       R3 = RBOP(I)**3                                                    BODY030
       X3 = RBO(I)**3                                                     BODY031
       FAC=VKB(I)/R3                                                      BODY032
       FAX=VKB(I)/X3                                                      BODY033
       IF(I-N)4,23,4                                                      BODY034
     4 CONTINUE                                                           BODY035
C      ACCELERATIONS (NEGATIVE) DUE TO PERTURBING BODIES                  BODY036
       DO 7 J=1,3                                                         BODY037
       K1J = K1 + J                                                       BODY038
       CA(J)=CA(J)+FAC*RJP(J)+FAX*XN(K1J)                                 BODY039
     7 CONTINUE                                                           BODY040
       GO TO 26                                                           BODY041
    23 CONTINUE                                                           BODY042
       FAC=U/R3                                                           BODY043
       FAX=U/X3                                                           BODY044
    26 CONTINUE                                                           BODY045
       DO 24 J=1,3                                                        BODY046
       RJP(J)=RJP(J)/RBOP(I)                                              BODY047
    24 CONTINUE                                                           BODY048
```

```
C       RJP IS NOW A UNIT VECTOR
        IF(NEQ-3)14,14,15
     15 CONTINUE
C       PARTIALS FOR PERTURBATION EQUATIONS
        DO 20 J=1,3
        J=J
        DO 20 K=J,3
        AP(J,K)=AP(J,K)+FAC*(3.*RJP(J)*RJP(K))
        IF(J-K)21,22,21
     21 CONTINUE
        AP(K,J)=AP(J,K)
        GO TO 20
     22 CONTINUE
        AP(J,J)=AP(J,J)-FAC
     20 CONTINUE
     25 FORMAT(3E16.8)
     14 CONTINUE
        GO TO 9
      8 RBOP(I) = 1.E10
      9 CONTINUE
        NS=NEQ/3-1
        IF(NS)16,16,17
     16 CONTINUE
        RETURN
     17 CONTINUE
        DO 18 I=1,NS
        NST=2*NEQ+3+3*I
        NSTP=3+3*I
        DO 19 J=1,3
        KK=NST+J
        T(KK)=0.
        DO 19 K=1,3
        JJ=NSTP+K
        T(KK)=T(KK)+AP(J,K)*T(JJ)
     19 CONTINUE
     18 CONTINUE
        GO TO 16
        END
```

Subroutine: BVEC

Purpose: To obtain the matrix of target miss partials; B·T, B·R, and V infinity with respect to the state.

Calling Sequence:

CALL BVEC (IOR, ITARG, X, V, BV, PBV, TW, TF, UM)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	IOR	1			Central body
I	ITARG	1			Target body
I	X	3	$\vec{R}$		Position vector
I	V	3	$\vec{\upsilon}$		Velocity vector
O	BV	4			Nominal Miss Parameters
O	PBV	(3,6)			Matrix of Miss Partials
I	TW	1	t	days	Whole days from 1950
I	TF	1	t	days	Fractional days from 1950
I	UM	1	μ	Km^3/sec^2	Gravitational constant

Common storages used or required: None

Subroutines required: DOT, INTRI, CROSS

Functions required: FNORM

Approximate number of storages required:

Derivation of the $\vec{B}$, $\hat{S}$, $\hat{T}$, and $\vec{R}$ vectors

The B, R, S, and T vectors are used to define a target miss (see Fig. 1 below)

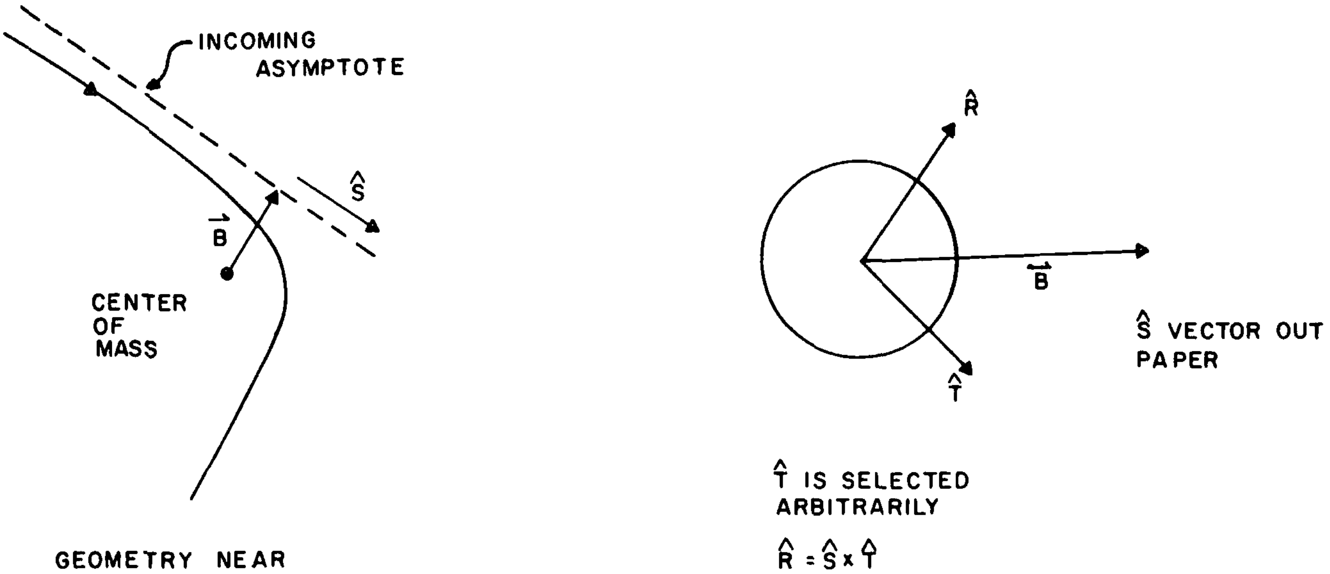


FIG. 1

$\vec{B}$, the position vector in the plane of the trajectory originating at the center of gravity of the target and directed perpendicularly to the incoming asymptote of the hyperbola, is approximately the vector miss which would occur if the target had no mass.

To determine the target miss direction relative to the target, a set of orthogonal vectors is computed. Let $\hat{S}$ be a unit vector in the direction of the incoming asymptote, $\hat{T}$ be a unit vector perpendicular to $\hat{S}$ that lies in a fixed plane of interest such as the equatorial plane, and $\hat{R}$ be a unit vector to form a right-handed system.

$$\hat{R} = \hat{S} \times \hat{T}$$

Since $\vec{B}$ is perpendicular to $\hat{S}$, it lies in the plane determined by $\hat{R}$ and $\hat{T}$. The variables which are used to describe the miss relative to a plane of interest are the projections of $\vec{B}$ on $\hat{T}$ and $\hat{R}$, i.e., $\vec{B}\cdot\hat{T}$ and $\vec{B}\cdot\hat{R}$.

The quantities used in the derivation of the vectors are the following:

$$h = |\vec{R} \times \vec{v}| \qquad C_3 = v^2 - \frac{2\mu}{R}$$

$$e = \sqrt{1 + \frac{h^2\, C3}{\mu^2}} \qquad P = \frac{h^2}{\mu}$$

True anamoly

$$\theta = \cos^{-1}\left[\frac{1}{\epsilon}\left(\frac{P}{R} - 1\right)\right]$$

Unit vectors to specify orientation of conic

$$\zeta = \frac{\vec{R} \times \vec{v}}{|\vec{R} \times \vec{v}|} \qquad \vec{M} = \vec{\zeta} \times \frac{\vec{R}}{|\vec{R}|}$$

$$\vec{\xi} = \frac{\vec{R}}{|\vec{R}|}\cos\theta - \vec{M}\sin\theta$$

$$\vec{h} = \vec{\zeta} \times \vec{\xi}$$

The unit vector in the direction of the asymptote is given by

$$\hat{S} = \frac{1}{e}\,\vec{\xi} + \sqrt{1 - \left(\frac{1}{e}\right)^2}\;\vec{n}$$

The impact parameter

$$\vec{B} = |\vec{B}| \left[\sqrt{1 - \left(\frac{1}{e}\right)^2} \ \vec{\xi} - \frac{1}{e} \ \vec{n} \right]$$

where $|\vec{B}| = \frac{\mu}{C3} \sqrt{e^2 - 1}$

Also
$$T_X = \frac{S_Y}{\sqrt{S_X^{\ 2} + S_Y^{\ 2}}}$$

$$T_Y = - \frac{S_X}{\sqrt{S_X^{\ 2} + S_Y^{\ 2}}}$$

$$T_Z = 0$$

and $\hat{R} = \hat{S} \ X \ \hat{T}$

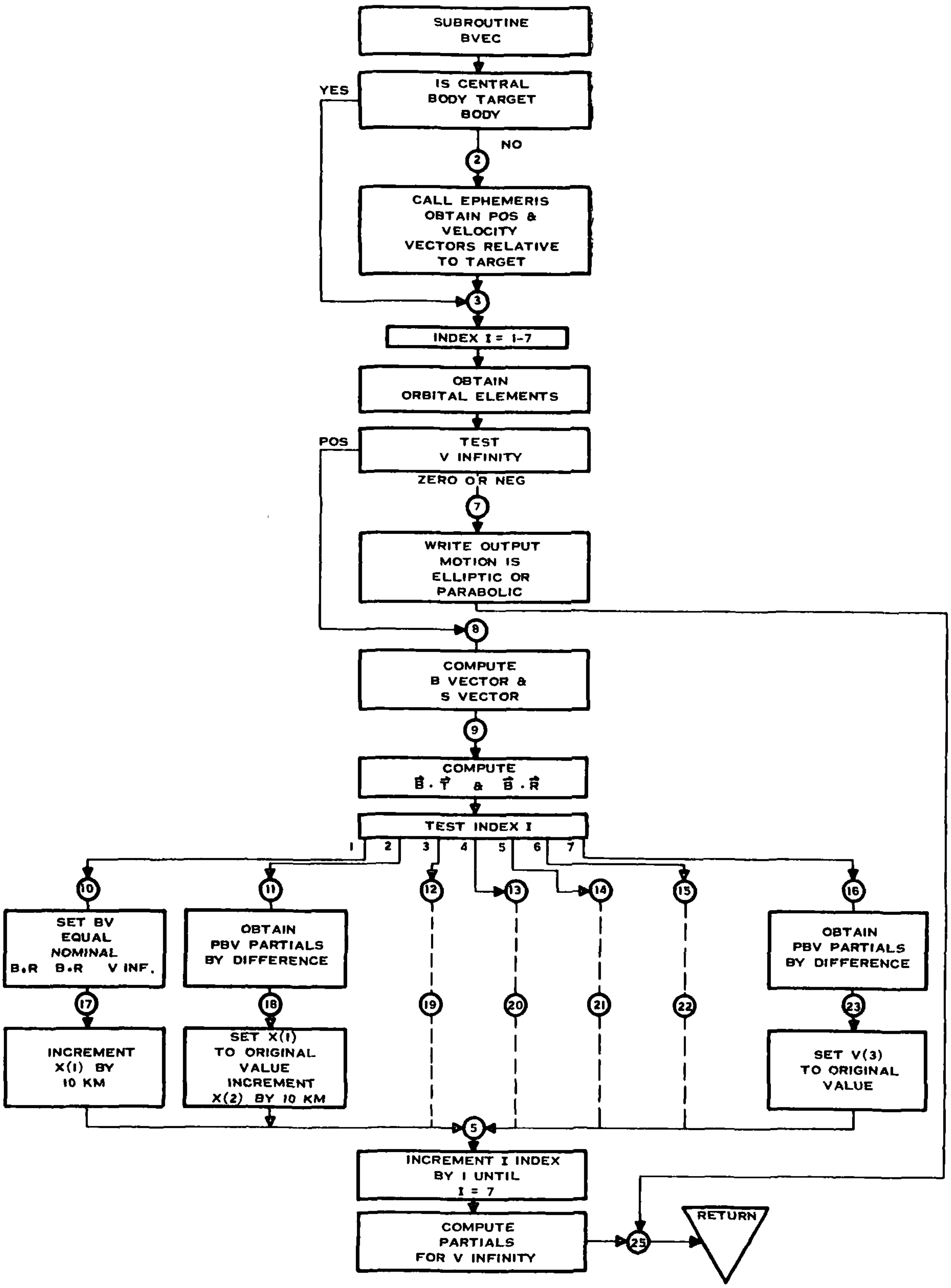
Derivation of partials of V infinity with respect to the state

$$VINF = \sqrt{v^2 - \frac{2\mu}{R}}$$

$$\frac{\partial VINF}{\partial X_i} = \frac{\mu \ X_i}{(VINF) \ R^3} \qquad i = 1,2,3$$

$$\frac{\partial VINF}{\partial X_i} = \frac{X_i}{(VINF)} \qquad i = 1,2,3$$

Reference: JPL EXTERNAL PUBLICATION NO. 674


SUBROUTINE BVEC
IS CENTRAL BODY TARGET BODY
YES
NO
CALL EPHEMERIS OBTAIN POS & VELOCITY VECTORS RELATIVE TO TARGET
INDEX I = 1-7
OBTAIN ORBITAL ELEMENTS
TEST V INFINITY
POS
ZERO OR NEG
WRITE OUTPUT MOTION IS ELLIPTIC OR PARABOLIC
COMPUTE B VECTOR & S VECTOR
COMPUTE B̂ · T̂ & B̂ · R̂
TEST INDEX I
SET BV EQUAL NOMINAL B•R B•R V INF.
OBTAIN PBV PARTIALS BY DIFFERENCE
OBTAIN PBV PARTIALS BY DIFFERENCE
INCREMENT X(I) BY 10 KM
SET X(I) TO ORIGINAL VALUE INCREMENT X(2) BY 10 KM
SET V(3) TO ORIGINAL VALUE
INCREMENT I INDEX BY I UNTIL I = 7
COMPUTE PARTIALS FOR V INFINITY
RETURN

```
*       LABEL                                                             BVEC
*       SYMBOL TABLE                                                      BVEC
CEC2001  SUBROUTINE BVEC                                                  BVEC0000
      SUBROUTINE BVEC (IOR,ITARG,X,V,BV,PBV,TW,TF,UM)                     BVEC0010
      DIMENSION X(3),V(3),XP(3),VP(3),BV(4),PBV(3,6),PO(22),VE(22)        BVEC0020
     1,BD(3),CP(3),RB(3),RM(3),UM(6),PP(3),QQ(3),S(3),B(3),RR(3),TT(3)    BVEC0030
      CALL SETN(NIN,NOUT)                                                 BVEC0035
      D=2.E6                                                              BVEC0040
      DO 1 I=1,3                                                          BVEC0050
      XP(I)=X(I)                                                          BVEC0060
      VP(I)=V(I)                                                          BVEC0070
    1 CONTINUE                                                            BVEC0080
      IF(IOR-ITARG) 2,3,2                                                 BVEC0090
    2 CONTINUE                                                            BVEC0100
      N=IOR-1                                                             BVEC0110
      CALL INTR1(TW,TF,N,PO,1,VE,D)                                       BVEC0120
      IC=26-3*ITARG                                                       BVEC0130
      DO 4 I=1,3                                                          BVEC0140
      IB=IC-I                                                             BVEC0150
      XP(I)=XP(I)-PO(IB)                                                  BVEC0160
      VP(I)=VP(I)-VE(IB)                                                  BVEC0170
    4 CONTINUE                                                            BVEC0180
    3 CONTINUE                                                            BVEC0190
      DO 5 I=1,7                                                          BVEC0200
      CALL CROSS(XP,VP,CP)                                                BVEC0210
      D=FNORM(CP)                                                         BVEC0220
      R=FNORM(XP)                                                         BVEC0230
      DO 6 J=1,3                                                          BVEC0240
      CP(J)=CP(J)/D                                                       BVEC0250
      RB(J)=XP(J)/R                                                       BVEC0260
    6 CONTINUE                                                            BVEC0270
      CALL CROSS(CP,RB,RM)                                                BVEC0280
      U=UM(ITARG)                                                         BVEC0290
      C3=DOT(VP,VP)-2.*U/R                                                BVEC0300
      IF(C3)7,7,8                                                         BVEC0310
    7 CONTINUE                                                            BVEC0320
      WRITE OUTPUT TAPE NOUT,100                                          BVEC0330
  100 FORMAT(32H MOTION IS ELLIPTIC OR PARABOLIC)                         BVEC0340
      GO TO 25                                                            BVEC0350
    8 CONTINUE                                                            BVEC0360
      P=D**2/U                                                            BVEC0370
      E2=1.+P*C3/U                                                        BVEC0380
      E = SQRTF(E2)                                                       BVEC0390
      A=U/C3                                                              BVEC0400
      SEM2=E2-1.                                                          BVEC0410
      SEM1=SQRTF(SEM2)                                                    BVEC0420
      ASE=A*SEM2/E                                                        BVEC0430
      SE =SEM1/E                                                          BVEC0440
      RDOT=DOT(XP,VP)/R                                                   BVEC0450
      CTHET=(P-R)/(E*R)                                                   BVEC0460
      STHET=(RDOT*SQRTF(P/U))/E                                           BVEC0470
```

```
      DO 9 J=1,3                                                        BVEC0480
      PP(J)=CTHET*RB(J)-STHET*RM(J)                                     BVEC0490
      QQ(J)=STHET*RB(J)+CTHET*RM(J)                                     BVEC0500
      S(J)=PP(J)/E+SE*QQ(J)                                             BVEC0510
      B(J)=ASE*PP(J)-A*SE*QQ(J)                                         BVEC0520
    9 CONTINUE                                                          BVEC0530
      S2=SQRTF(S(1)**2+S(2)**2)                                         BVEC0540
      TT(1)=S(2)/S2                                                     BVEC0550
      TT(2)=-S(1)/S2                                                    BVEC0560
      TT(3)=0.                                                          BVEC0570
      CALL CROSS(S,TT,RR)                                               BVEC0580
      BD(1)=DOT(B,TT)                                                   BVEC0590
      BD(2)=DOT(B,RR)                                                   BVEC0600
      I=I                                                               BVEC0610
      GO TO (10,11,12,13,14,15,16),I                                    BVEC0620
   10 CONTINUE                                                          BVEC0630
      DO 17 J=1,2                                                       BVEC0640
      BV(J)=BD(J)                                                       BVEC0650
   17 CONTINUE                                                          BVEC0660
      BV(3)=SQRTF(C3)                                                   BVEC0670
      XP(1)=XP(1)+10.                                                   BVEC0680
      GO TO 5                                                           BVEC0690
   11 CONTINUE                                                          BVEC0700
      DO 18 J=1,2                                                       BVEC0710
      PBV(J,1)=(BD(J)-BV(J))/10.                                        BVEC0720
   18 CONTINUE                                                          BVEC0730
      XP(1)=XP(1)-10.                                                   BVEC0740
      XP(2)=XP(2)+10.                                                   BVEC0750
      GO TO 5                                                           BVEC0760
   12 CONTINUE                                                          BVEC0770
      DO 19 J=1,2                                                       BVEC0780
      PBV(J,2)=(BD(J)-BV(J))/10.                                        BVEC0790
   19 CONTINUE                                                          BVEC0800
      XP(2)=XP(2)-10.                                                   BVEC0810
      XP(3)=XP(3)+10.                                                   BVEC0820
      GO TO 5                                                           BVEC0830
   13 CONTINUE                                                          BVEC0840
      DO 20 J=1,2                                                       BVEC0850
      PBV(J,3)=(BD(J)-BV(J))/10.                                        BVEC0860
   20 CONTINUE                                                          BVEC0870
      XP(3)=XP(3)-10.                                                   BVEC0880
      VP(1)=VP(1)+.01                                                   BVEC0890
      GO TO 5                                                           BVEC0900
   14 CONTINUE                                                          BVEC0910
      DO 21 J=1,2                                                       BVEC0920
      PBV(J,4)=(BD(J)-BV(J))*100.                                       BVEC0930
   21 CONTINUE                                                          BVEC0940
      VP(1)=VP(1)-.01                                                   BVEC0950
      VP(2)=VP(2)+.01                                                   BVEC0960
      GO TO 5                                                           BVEC0970
   15 CONTINUE                                                          BVEC0980
```

```
      DO 22 J=1,2                                                       BVEC0990
      PBV(J,5)=(BD(J)-BV(J))*100.                                       BVEC1000
   22 CONTINUE                                                          BVEC1010
      VP(2)=VP(2)-.01                                                   BVEC1020
      VP(3)=VP(3)+.01                                                   BVEC1030
      GO TO 5                                                           BVEC1040
   16 CONTINUE                                                          BVEC1050
      DO 23 J=1,2                                                       BVEC1060
      PBV(J,6)=(BD(J)-BV(J))*100.                                       BVEC1070
   23 CONTINUE                                                          BVEC1080
      VP(3)=VP(3)-.01                                                   BVEC1090
    5 CONTINUE                                                          BVEC1100
      VIN=SQRTF(C3)                                                     BVEC1110
      UR2=1.*U/R**3                                                     BVEC1120
      UR3=UR2/VIN                                                       BVEC1130
      DO 24 I=1,3                                                       BVEC1140
      J=I+3                                                             BVEC1150
      PBV(3,J)=VP(I)/VIN                                                BVEC1160
      PBV(3,I)=UR3*XP(I)                                                BVEC1170
   24 CONTINUE                                                          BVEC1180
   25 CONTINUE                                                          BVEC1190
      RETURN                                                            BVEC1200
      END                                                               BVEC
```

Subroutine: CHNGP

Purpose: To choose the appropriate body center as a function of distance from the present central body.

Calling Sequence:

CALL CHNGP (NOR, IOR, RAD)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	NOR	1			New body center
I	IOR	1			Present body center
I	RAD	6		Km.	RAD(I) = distance from body I
					to vehicle

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

Method of Selecting Body Center

Present Central Body	New Central Body	
1) Earth	Sun	$R_1 \geq 25 \times 10^5$
	Earth	$R_2 \geq 3 \times 10^4$
	Moon	$R_2 \leq 3 \times 10^4$
2) Moon	Moon	$R_2 < 3 \times 10^4$
	Earth	$R_2 \geq 3 \times 10^4$
3) Sun	Earth	$R_1 \leq 25 \times 10^5$
	Venus	$R_4 < 25 \times 10^5$
	Mars	$R_5 < 25 \times 10^5$
	Jupiter	$R_6 < 25 \times 10^5$
	Sun	(if none of above)
4) Venus	Venus	$R_4 < 25 \times 10^5$
	Sun	$R_4 \geq 25 \times 10^5$
5) Mars	Mars	$R_5 < 25 \times 10^5$
	Sun	$R_5 \geq 25 \times 10^5$
6) Jupiter	Jupiter	$R_6 \geq 25 \times 10^5$
	Sun	$R_6 < 25 \times 10^5$

where R_i = RAD(I) = distance from body(i) to the vehicle.

```
      LABEL                                                            CHNG
      SYMBOL TABLE                                                     CHNG
C2002                                                                  CHNG
      SUBROUTINE CHNGP(NOR,IOR,RAD)                                    CHNG0000
      DIMENSION RAD(6)                                                 CHNG0010
      IO = IOR                                                         CHNG0020
      NOR = IOR                                                        CHNG0025
      GO TO (1,2,3,4,4,4), IO                                          CHNG0030
    1 IF (RAD(1)-2500000.) 11,12,12                                    CHNG0040
   11 IF (RAD(2)-30000.) 13,14,14                                      CHNG0050
   12 NOR = 3                                                          CHNG0060
      GO TO 14                                                         CHNG0070
   13 NOR = 2                                                          CHNG0080
   14 RETURN                                                           CHNG0090
    2 IF (RAD(2)-30000.) 14,21,21                                      CHNG0100
   21 NOR = 1                                                          CHNG0110
      GO TO 14                                                         CHNG0120
    3 IF (RAD(1)-2500000.) 31,31,32                                    CHNG0130
   31 NOR = 1                                                          CHNG0140
      GO TO 14                                                         CHNG0150
   32 DO 34 I=4,6                                                      CHNG0160
      IF (RAD(I)-2500000.) 33,34,34                                    CHNG0170
   33 NOR = I                                                          CHNG0180
      GO TO 14                                                         CHNG0190
   34 CONTINUE                                                         CHNG0200
      GO TO 14                                                         CHNG0210
    4 IF (RAD(IO)-2500000.) 14,41,41                                   CHNG0220
   41 NOR = 3                                                          CHNG0230
      GO TO 14                                                         CHNG0240
      END                                                              CHNG
```

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Subroutine: COMPHQ

Purpose: To compute the H matrix of measurement partials with respect to the vehicle state and the variance of the measurements. The subroutine is called by ONBTR, on-board tracking subroutine, and MONBTR, the moon beacon tracking subroutine. The types of measurements which are evaluated are range, right ascension, declination, and range rate. After computation of the H matrix and the measurement error, subroutine CORRTP is called to perform the filtering of the measurement and updating of the state covariance matrix, P.

Calling Sequence:

CALL COMPHQ (JJJ, KK, NN)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	JJJ	1			Type of measurement
I	KK	1			On-board or moon beacon
I	NN	1			Key for locating constants in
					storage

Common storages used or required: T, S, C, IC

Subroutines required: CORRTP, DOT

Functions required: SQRT

Approximate number of storages required: 388 DEC

Calling Sequence Elaboration

The calling sequence of COMPHQ consists of CALL COMPHQ (JJJ, KK, NN) where:

JJJ is a key which indicates the type of measurement being performed.

1. Range
2. Right ascension
3. Declination
4. Range rate

KK is a key which indicates if the measurements being made are: (1) on-board measurements or (2) moon beacons.

NN is a key which is used to obtain constants out of the common S array.

COMPHQ Partial Derivatives

To perform the covariance matrix updating for on-board observations and moon based beacons, a number of partial derivatives are required. The partials are derived in the following paragraphs.

The following quantities and relationships are used in the derivations. The vehicle's postion, $\vec{X}$, and velocity, $\dot{\vec{X}}$, relative to the celestial body or moon beacon are obtained from common. These vectors may be written as:

$$\vec{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = -\vec{X}_V + \vec{X}_B = \begin{pmatrix} -X_V & +X_B \\ -Y_V & +Y_B \\ -Z_V & +Z_B \end{pmatrix}$$

$$\dot{\vec{X}} = \begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{pmatrix} = -\dot{\vec{X}}_V + \dot{\vec{X}}_B$$

Two relationships which will be used in the derivation are:

$$\frac{\partial PAR}{\partial \vec{X}} = - \frac{\partial PAR}{\partial \vec{X}_V}$$

$$\frac{\partial PAR}{\partial \dot{\vec{X}}} = - \frac{\partial PAR}{\partial \dot{\vec{X}}_V}$$

The FORTRAN program names for the above vector are as follows:

FORTRAN NAME	DIMENSION	DERIVATION NAME	DIMENSION
XREL	(3)	$\vec{X}$	(3)
VREL	(3)	$\dot{\vec{X}}$	(3)

The quantities being measured are the following:

$$R = \text{RANGE} = |\vec{X}| = (\vec{X}\cdot\vec{X})^{1/2}$$

$$R = \text{Range Rate} = \frac{d}{dt}|\vec{X}| = \frac{\vec{X}\cdot\dot{\vec{X}}}{R}$$

$$RA = \tan^{-1}\left\{\frac{X_2}{X_1}\right\}$$

$$DEC = \sin^{-1}\left\{\frac{X_3}{R}\right\}$$

A. The following is a derivation of the partials relating the types of measurements and the vehicle state. In the program, these partials are used as a row vector. The FORTRAN name of the vector is H. H may be written as follows:

$$H = \left(\frac{\partial MEAS}{\partial X_V}; \frac{\partial MEAS}{\partial Y_V}; \frac{\partial MEAS}{\partial Z_V}; \frac{\partial MEAS}{\partial \dot{X}_V}; \frac{\partial MEAS}{\partial \dot{Y}_V}; \frac{\partial MEAS}{\partial \dot{Z}_V}\right)$$

(1) Derivation of partials for range measurement.

$$R = (\vec{X}\cdot\vec{X})^{1/2}$$

$$H_R = \left(\frac{\partial R}{\partial \vec{X}_V} ; \frac{\partial R}{\partial \dot{\vec{X}}_V} \right) = \left(-\frac{X_1}{R}; -\frac{X_2}{R}; -\frac{X_3}{R}; 0; 0; 0 \right)$$

(2) Derivation of partials for range rate measurement.

$$\dot{R} = \frac{\vec{X}\cdot\dot{\vec{X}}}{R}$$

$$H_{RR} = \left(\frac{\partial \dot{R}}{\partial \vec{X}_V}; \frac{\partial \dot{R}}{\partial \dot{\vec{X}}_V} \right) = \left(-\frac{\dot{X}_1}{R} + \frac{\dot{R}}{R^2} X_1; -\frac{\dot{X}_2}{R} + \frac{\dot{R}}{R^2} X_2; -\frac{\dot{X}_3}{R} + \frac{\dot{R}}{R^2} X_3; \right.$$

$$\left. \frac{X_1}{R}; \frac{X_2}{R}; \frac{X_3}{R} \right)$$

(3) Derivation of partials for right ascension measurement.

$$RA = \mathrm{Tan}^{-1} \left(\frac{X_2}{X_1} \right) \equiv \mathrm{Tan}^{-1} \gamma$$

$$\frac{\partial RA}{\partial PAR} = (1 + \gamma^2)^{-1} \frac{\partial \gamma}{\partial PAR}$$

$$(1 + \gamma^2)^{-1} = \frac{X_1^2}{X_1^2 + X_2^2}$$

$$H_{RA} = \left(\frac{\partial RA}{\partial \vec{X}_V}; \frac{\partial RA}{\partial \dot{\vec{X}}_V} \right) = \left(\frac{X_2}{X_1^2 + X_2^2}; \frac{X_1}{X_1^2 + X_2^2}; 0; 0; 0; 0 \right)$$

(4) Derivation of partials for declination measurement.

$$DEC = \sin^{-1}\left(\frac{X_3}{R}\right) \equiv \sin^{-1}\gamma$$

$$\frac{\partial DEC}{\partial PAR} = [1 - \gamma^2]^{-1/2} \quad \frac{\partial \gamma}{\partial PAR}$$

$$[1 - \gamma^2]^{-1/2} = \frac{R}{\sqrt{X_1^2 + X_2^2}}$$

$$H_{DEC} = \left(\frac{\partial DEC}{\partial \vec{X}_V}; \frac{\partial DEC}{\partial \dot{\vec{X}}_V}\right) = \left(\frac{X_1 X_3}{\sqrt{X_1^2 + X_2^2}\,R^2}; \quad \frac{X_2 X_3}{R^2\sqrt{X_1^2 + X_2^2}}; \quad \frac{\sqrt{X_1^2 + X_2^2}}{R^2}; \right.$$

$$\left. 0;\ 0;\ 0\right)$$

B. In order to include errors in the measurements being made due to station location errors and time bias, the partials of the measurements with respect to latitude, longitude, altitude, and time are required. In the program, the station location errors are used as a row vector with FORTRAN name DUM. DUM may be written as follows:

$$DUM = \left(\frac{\partial MEAS}{\partial LAT}; \frac{\partial MEAS}{\partial LON}; \frac{\partial MEAS}{\partial ALT}\right)$$

The following matrices are obtained from MONBTR and used in the computation of DUM.

$$STPARS = \begin{pmatrix} \frac{\partial X_1}{\partial LAT} & \frac{\partial X_1}{\partial LON} & \frac{\partial X_1}{\partial ALT} \\ \frac{\partial X_2}{\partial LAT} & \frac{\partial X_2}{\partial LON} & \frac{\partial X_2}{\partial ALT} \\ \frac{\partial X_3}{\partial LAT} & \frac{\partial X_3}{\partial LON} & \frac{\partial X_3}{\partial ALT} \end{pmatrix} \qquad STPARD = \begin{pmatrix} \frac{\partial \dot{X}_1}{\partial LAT} & \frac{\partial \dot{X}_1}{\partial LON} & \frac{\partial \dot{X}_1}{\partial ALT} \\ \frac{\partial \dot{X}_2}{\partial LAT} & \frac{\partial \dot{X}_2}{\partial LON} & \frac{\partial \dot{X}_2}{\partial ALT} \\ \frac{\partial \dot{X}_3}{\partial LAT} & \frac{\partial \dot{X}_3}{\partial LON} & \frac{\partial \dot{X}_3}{\partial ALT} \end{pmatrix}$$

(1) Derivation of range measurement error partials.

$$\frac{\partial R}{\partial LAT} = \sum_{i=1}^{3} \frac{\partial R}{\partial X_i} \frac{\partial X_i}{\partial LAT}$$

$$\frac{\partial R}{\partial LAT} = \sum_{i=1}^{3} -H_R(1,i)\ STPARS(i,1)$$

Similarly

$$\frac{\partial R}{\partial LON} = \sum_{i=1}^{3} -H_R(1,i)\ STPARS(i,2)$$

$$\frac{\partial R}{\partial ALT} = \sum_{i=1}^{3} -H_R(1,i)\ STPARS(i,3)$$

(2) Derivation of range rate measurement error partials.

$$\frac{\partial \dot{R}}{\partial LAT} = \sum_{i=1}^{3} \frac{\partial \dot{R}}{\partial X_i} \frac{\partial X_i}{\partial LAT} + \frac{\partial R}{\partial \dot{X}_i} \frac{\partial \dot{X}_i}{\partial LAT}$$

$$\frac{\partial \dot{R}}{\partial LAT} = \sum_{i=1}^{3} -H_{RR}(1,i)\ STPARS(i,1) - H_{RR}(1,i+3)\ STPARD(i,1)$$

Similarly

$$\frac{\partial \dot{R}}{\partial LON} = \sum_{i=1}^{3} -H_{RR}(1,i)\ STPARS(i,2) - H_{RR}(1,i+3)\ STPARD(i,2)$$

$$\frac{\partial \dot{R}}{\partial ALT} = \sum_{i=1}^{3} -H_{RR}(1,i)\ STPARS(i,3) - H_{RR}(1,i+3)\ STPARD(i,3)$$

(3) Derivation of right ascension measurement error partials.

$$\frac{\partial RA}{\partial LAT} = \sum_{i=1}^{3} \frac{\partial RA}{\partial x_i} \frac{\partial x_i}{\partial LAT}$$

$$\frac{\partial RA}{\partial LAT} = \sum_{i=1}^{3} -H_{RA}(1,i)\ STPARS(i,1)$$

Similarly

$$\frac{\partial RA}{\partial LON} = \sum_{i=1}^{3} -H_{RA}(1,i)\ STPARS(i,2)$$

$$\frac{\partial RA}{\partial ALT} = \sum_{i=1}^{3} -H_{RA}(1,i)\ STPARS(i,3)$$

(4) Derivation of declination measurement error partials.

$$\frac{\partial DEC}{\partial LAT} = \sum_{i=1}^{3} \frac{\partial DEC}{\partial x_i} \frac{\partial x_i}{\partial LAT}$$

$$\frac{\partial DEC}{\partial LAT} = \sum_{i=1}^{3} -H_{DEC}(1,i)\ STPARS(i,1)$$

Similarly

$$\frac{\partial DEC}{\partial LON} = \sum_{i=1}^{3} -H_{DEC}(1,i)\ STPARS(i,2)$$

$$\frac{\partial DEC}{\partial ALT} = \sum_{i=1}^{3} -H_{DEC}(1,i)\ STPARS(i,3)$$

(5) Derivation of time derivatives of measurement quantities to permit inclusion of time bias errors.

$$\frac{\partial R}{\partial t} = \dot{R} = \frac{\vec{X}\cdot\dot{\vec{X}}}{R}$$

$$\frac{\partial RA}{\partial t} = \sum_{i=1}^{3} \frac{\partial RA}{\partial X_i}\ \frac{\partial X_i}{\partial t}$$

$$\frac{\partial RA}{\partial t} = \sum_{i=1}^{3} -H_{RA}(1,i)\ \dot{X}_i$$

Similarly

$$\frac{\partial DEC}{\partial t} = \sum_{i=1}^{3} -H_{DEC}(1,i)\ \dot{X}_i$$

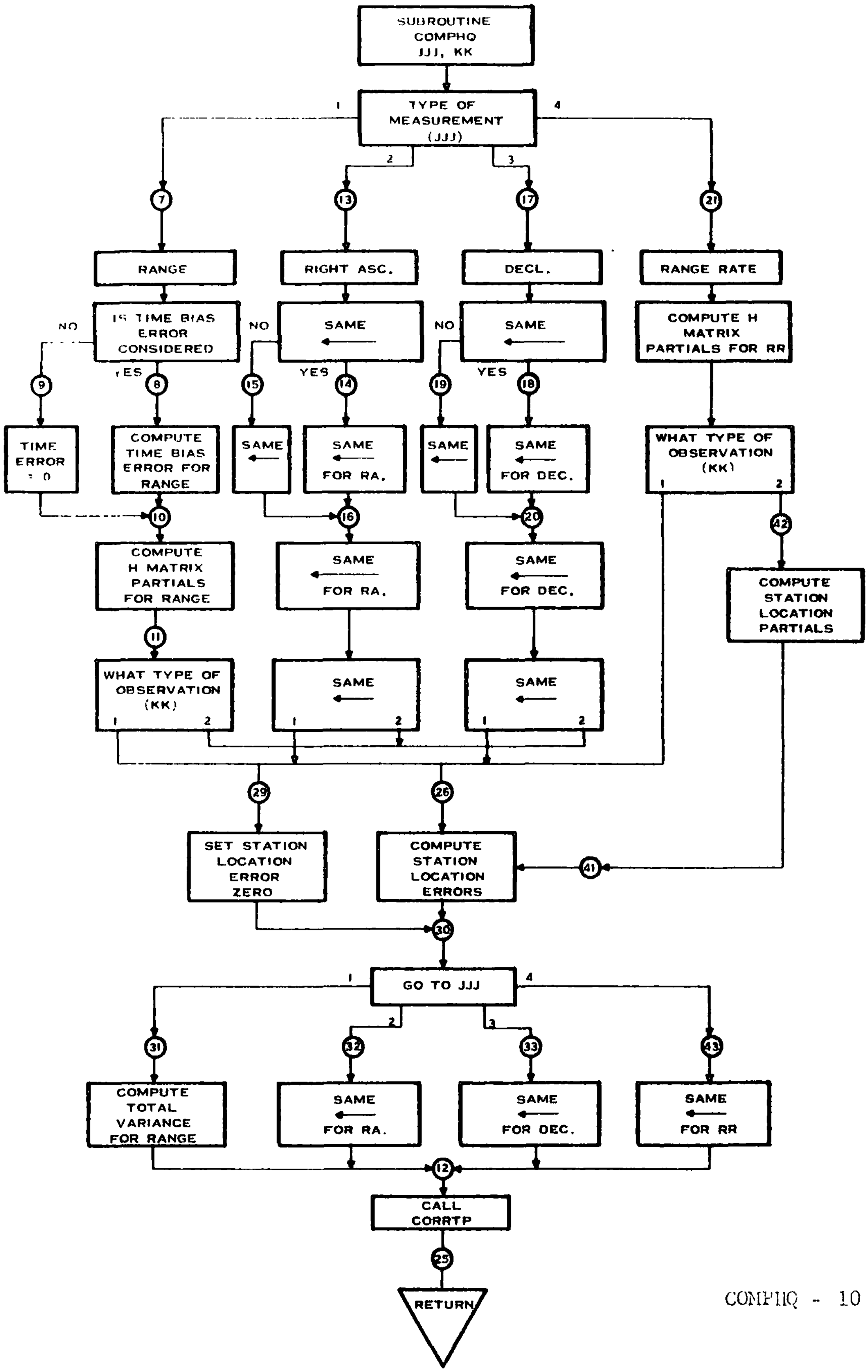
The errors in the measurements are computed in the following manner for station location errors and time bias.

$$QQSTAT = \left(DUM_{MEAS}\right)^2 \begin{pmatrix} \sigma^2 LAT \\ \sigma^2 LON \\ \sigma^2 ALT \end{pmatrix} \begin{matrix} \text{Station} \\ \text{Location} \\ \text{Error} \end{matrix}$$

$$1x1 \qquad 1x3 \qquad 3x1$$

$$QQDOT = \left(\frac{\partial MEAS}{\partial t} \cdot \sigma_{TIME} \right)^2 \qquad \text{Time Bias Error}$$

The computation of the H matrices and the measurement errors are the primary computations in COMPHQ. COMPHQ calls CORRTP which uses quantities to update the state covariance matrix.



SUBROUTINE COMPHQ JJJ, KK
TYPE OF MEASUREMENT (JJJ)
1
2
3
4
RANGE
RIGHT ASC.
DECL.
RANGE RATE
IS TIME BIAS ERROR CONSIDERED
NO
YES
SAME
COMPUTE H MATRIX PARTIALS FOR RR
TIME ERROR = 0
COMPUTE TIME BIAS ERROR FOR RANGE
SAME FOR RA.
SAME FOR DEC.
WHAT TYPE OF OBSERVATION (KK)
COMPUTE H MATRIX PARTIALS FOR RANGE
COMPUTE STATION LOCATION PARTIALS
SET STATION LOCATION ERROR ZERO
COMPUTE STATION LOCATION ERRORS
GO TO JJJ
COMPUTE TOTAL VARIANCE FOR RANGE
SAME FOR RR
CALL CORRTP
RETURN

COMPHQ - 10

```
*       LABEL                                                           COMP
*       SYMBOL TABLE                                                    COMP
CEC2008  SUBROUTINE COMPHQ                                              COMP
        SUBROUTINE COMPHQ(JJJ,KK,NN)                                    COMP0000
        COMMON T,S,C,IC                                                 COMP0010
        DIMENSION T(1360),S(1000),C(1000),IC(1)                         COMP0020
       1,XREL(3),VREL(3),H(6),P(6,6),STPARS(3,3),DUM(3),OUTPUT(6)       COMP0025
        DIMENSION STPARD(3,3)                                           COMP0030
        EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM),(IC,ICDUM)               COMP0040
       1,(C(895),XREL),(C(898),VREL),(C(894), OBNO),(C(893),XMAG),      COMP0050
       2(C(892),DEX),(C(891),DEN1),(C(890),DEN2),(C(788),STPARS)        COMP0060
       3,(C(889),DEX2),(C(888),DEX3),(C(652),P),(C(973),OUTPUT)         COMP0065
       4,(S(73),RTD)                                                    COMP0070
        EQUIVALENCE (C(800),STPARD)                                     COMP0075
C       KK  1=ONBOARD MEASUREMENTS   2=MOON BEACONS                     COMP0080
C       JJJ  1=RANGE  2=RIGHT ASC.  3=DECL.  4=RANGE RATE               COMP0090
        NN=NN                                                           COMP0100
        JJJ=JJJ                                                         COMP0110
        KK=KK                                                           COMP0120
        NNN=NN/5+657                                                    COMP0130
        GO TO (7,13,17,21),JJJ                                          COMP0140
      7 CONTINUE                                                        COMP0150
        IF(S(NN+8))9,9,8                                                COMP0160
      8 CONTINUE                                                        COMP0170
C       GET HERE TO COMPUTE ERROR IN RANGE OBS. DUE TO TIME BIAS        COMP0180
        RDOT=DOT(XREL,VREL)*DEX3                                        COMP0190
C       RDOT=RANGE RATE                                                 COMP0200
        OUTPUT(2)=RDOT                                                  COMP0205
        RDOT2=RDOT*RDOT                                                 COMP0210
        QQRDOT=RDOT2*S(NN+8)                                            COMP0220
C       QQRDOT=RANGE VARIANCE DUE TO TIME BIAS                          COMP0230
        GO TO 10                                                        COMP0240
      9 CONTINUE                                                        COMP0250
        QQRDOT=0.                                                       COMP0260
     10 CONTINUE                                                        COMP0270
C       THE FOLLOWING ARE H MATRIX CALCULATIONS FOR RANGE               COMP0280
        DO 11 I=1,3                                                     COMP0290
        K=I+3                                                           COMP0300
        H(I)=-XREL(I)*DEX3                                              COMP0310
        H(K)=0.                                                         COMP0320
     11 CONTINUE                                                        COMP0330
        GO TO (29,26),KK                                                COMP0340
     31 CONTINUE                                                        COMP0350
        QQ=S(NN)/OBNO+S(NN+4)+QQRDOT+S(425)*XMAG**2+QQSTAT              COMP0360
C       QQ=TOTAL RANGE VARIANCE-INSTR.+BIAS+TIME BIAS ERROR+SPEED LIGHT COMP0370
C       +STATION LOCATION ERRORS                                        COMP0380
     12 CONTINUE                                                        COMP0390
        CALL CORRTP (QQ,H,P)                                            COMP0400
C       CORRTP UPDATES COVARIANCE MATRIX FOR MEASUREMENT                COMP0410
        GO TO 25                                                        COMP0420
     13 CONTINUE                                                        COMP0430
```

```
      IF(S(NN+8))15,15,14
   14 CONTINUE
C     GET HERE TO COMPUTE ERROR IN RA MEASUREMENT DUE TO CLOCK BIAS
      RADOT=(VREL(2)*XREL(1)-VREL(1)*XREL(2))*DEN2
      CUTPUT(4)=RADOT*RTD
C     RADOT=RIGHT ASCENSION RATE
      RADOT2=RADOT*RADOT
      QQRADT=RADOT2*S(NN+8)
C     QQRADT=RIGHT ASCENSION VARIANCE DUE TO TIME BIAS
      GO TO 16
   15 CONTINUE
      QQRADT=0.
   16 CONTINUE
C     THE FOLLOWING ARE H MATRIX CALCULATIONS FOR RA
      H(6)=0.
      H(5)=0.
      H(4)=0.
      H(3)=0.
      H(1)= XREL(2)*DEN2
      H(2)=-XREL(1)*DEN2
      GO TO (29,26),KK
  32  CONTINUE
      QQ=S(NN+1)/OBNO+S(NN+5)+QQRADT+QQSTAT
C     QQ=TOTAL RIGHT ASCENSION VARIANCE
      GO TO 12
   17 CONTINUE
      IF(S(NN+8))19,19,18
   18 CONTINUE
C     GET HERE TO COMPUTE ERROR IN DEC.MEAS.DUE TO CLOCK BIAS
      CRUD=XREL(1)*VREL(1)+XREL(2)*VREL(2)
      CRUD2=VREL(3)*DEX-XREL(3)*CRUD
      DEDOT=CRUD2/(SQRTF(DEX)*DEX2)
      OUTPUT(6)=DEDOT*RTD
      DEDOT2=DEDOT*DEDOT
      QQDEDT=DEDOT2*S(NN+8)
C     QQDEDT=DECLINATION VARIANCE DUE TO TIME BIAS
      GO TO 20
   19 CONTINUE
      QQDEDT=0.
   20 CONTINUE
C     THE FOLLOWING ARE H MATRIX COMP. FOR DECLINATION
      H(4)=0.
      H(5)=0.
      H(6)=0.
      H(1)= XREL(1)*XREL(3)*DEN1
      H(2)= XREL(2)*XREL(3)*DEN1
      H(3)=-DEX*DEN1
      GO TO (29,26),KK
  33  CONTINUE
      QQ=S(NN+2)/OBNO+S(NN+6)+QQDEDT+QQSTAT
C     QQ=TOTAL DECLINATION VARIANCE
```

```
      GO TO 12                                                          COMP093
   21 CONTINUE                                                          COMP094
C     THE FOLLOWING ARE H MATRIX COMP. FOR RANGE RATE                   COMP095
      RDOT=DOT(XREL,VREL)*DEX3                                          COMP096
      OUTPUT(2)=RDOT                                                    COMP096
      DO 22 J=1,3                                                       COMP097
      K=J+3                                                             COMP098
      H(K)=-XREL(J)*DEX3                                                COMP099
      H(J)=-DEX3*(VREL(J)+H(K)*RDOT)                                    COMP100
   22 CONTINUE                                                          COMP100
      GO TO (29,42),KK                                                  COMP100
   42 CONTINUE                                                          COMP100
      DO 40 I=1,3                                                       COMP100
      DUM(I)=0.                                                         COMP101
      DO 40 J=1,3                                                       COMP101
      JJ=J+3                                                            COMP101
   40 DUM(I)=DUM(I)+H(J)*STPARS(J,I)+H(JJ)*STPARD(J,I)                  COMP101
      GO TO 41                                                          COMP102
   43 CONTINUE                                                          COMP102
      QQ=S(NN+3)/OBNO+S(NN+7)+QQSTAT                                    COMP102
C     QQ=RANGE RATE VARIANCE                                            COMP103
      GO TO 12                                                          COMP104
   26 CONTINUE                                                          COMP105
C     THE FOLLOWING COMPUTATIONS ARE FOR STATION LOCATION ERRORS        COMP106
      DO 27 I=1,3                                                       COMP107
      DUM(I)=0.                                                         COMP108
      DO 27 J=1,3                                                       COMP109
      DUM(I)=DUM(I)+H(J)*STPARS(J,I)                                    COMP110
   27 CONTINUE                                                          COMP111
   41 CONTINUE                                                          COMP111
      QQSTAT=0.                                                         COMP112
      DO 28 I=1,3                                                       COMP113
      L=NNN+I                                                           COMP114
      QQSTAT=QQSTAT+(DUM(I)**2)*S(L)                                    COMP115
   28 CONTINUE                                                          COMP116
      GO TO 30                                                          COMP117
   29 CONTINUE                                                          COMP118
      QQSTAT=0.                                                         COMP119
   30 CONTINUE                                                          COMP120
      GO TO(31,32,33,43),JJJ                                            COMP121
   25 CONTINUE                                                          COMP122
      RETURN                                                            COMP123
      END                                                               COMP
```

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Subroutine: CONST1

Purpose: To read permanent or semipermanent constants into the common S array.

Calling Sequence:

CALL CONST1

Input and **Output**

Common storages used or required:	T, S
Subroutines required:	None
Functions required:	None
Approximate number of storages required:	669 DEC

```
*      LABEL                                                          CONS
*      SYMBOL TABLE                                                   CONS
CEC2026  SUBROUTINE CONST1                                            CONS
       SUBROUTINE  CONST1                                             CONS0000
       COMMON T,S                                                     CONS0010
       DIMENSION T(1360),S(1000)                                      CONS0020
       EQUIVALENCE (T,TDUM),(S,SDUM)                                  CONS0030
C      PLANETARY MASSES 1-6,                                          CONS0040
C      PLANETARY MASSES 1-6 ,ORDER EARTH,MOON,SUN,VENUS,MARS,JUPITER  CONS0050
C      -KM**3/SEC2                                                    CONS0060
       S(1)=398603,2                                                  CONS0070
       S(2)=4900.7588                                                 CONS0080
       S(3)=.13271545E12                                              CONS0090
       S(4)=324769,50                                                 CONS0100
       S(5)=42915.515                                                 CONS0110
       S(6)=.12671059E9                                               CONS0120
C      6-9 ARE SPARES                                                 CONS0130
C      10-19 ARE EARTH CONSTANTS,RATE,J,H,D,EQUAT RAD,POLAR RAD       CONS0140
C      RAD/SEC                                                        CONS0150
       S(10)=7.2921152E-5                                             CONS0160
       S(11)=.162345E-2                                               CONS0170
       S(12)=-.575E-5                                                 CONS0180
       S(13)=.7875E-5                                                 CONS0190
       S(14)=6378.2064                                                CONS0200
       S(15)=6356.5838                                                CONS0210
C      20-29 ARE MOON CONSTANTS ,G,A,B,C,RAD                          CONS0220
       S(20)=.6671E-19                                                CONS0230
       S(21)=.88746E29                                                CONS0240
       S(22)=.88764E29                                                CONS0250
       S(23)=.88801E29                                                CONS0260
       S(24)=1738.                                                    CONS0270
C      30-59 SUN CONSTANTS                                            CONS0280
C      40-49 VENUS CONSTANTS                                          CONS0290
C      50-59 MARS CONSTANTS                                           CONS0300
C      60-69 JUPITER CONSTANTS                                        CONS0310
C CONVERSION FACTOR DAYS TO SECONDS                                   CONS0320
       S(70)=86400.                                                   CONS0330
C CONVERSION FACTOR SECONDS TO DAYS                                   CONS0340
       S(71)=1./S(70)                                                 CONS0350
C CONVERSION FACTOR FOR DEGREES TO RADIANS                            CONS0360
       S(72)=.017453296                                               CONS0370
C CONVERSION FACTOR FOR RADIANS TO DEGREES                            CONS0380
       S(73)=57.29578                                                 CONS0390
       S(80)=28.48713                                                 CONS0400
       S(81)=279.42315                                                CONS0410
       S(82)=.00257                                                   CONS0420
       S(84)=90.                                                      CONS0430
C      111 IS TYPE OF INPUT(1)EQUATOR 1950 (2)EQUATOR OF DATE(3)      CONS0440
C      EARTH-FIXED                                                    CONS0450
C      (4),(5),(6) ARE SPHERICAL INPUT IN, RESPECTIVELY, (1),(2),(3)  CONS0460
C      112 AND 113 ARE(YEAR MONTH.DAY)(HOUR MIN.SEC) OF INJECTION DATE CONS0470
```

```
C      114 IS CENTRAL BODY NUMBER,(1)EARTH,(2)MOON(3)SUN(4)VENUS(5)    CONS04
C      MARS(6)JUPITER                                                  CONS04
C      115-120 ARE INPUT STATE CORRESPONDING TO S(111) THRU 114.      CONS05
C      121  TARGET BODY                                                CONS05
C 122 TYPE OF OUTPUT;KOUT                                              CONS05
       S(121)=2.                                                       CONS05
C 123 TSECO                                                            CONS05
C N GOES FROM 0 TO 19                                                  CONS05
C(125)+N*15  VARIANCE OF RANGE              KM**2                      CONS05
C(126)+N*15  VARIANCE OF AZIMUTH            RAD**2                     CONS05
C(127)+N*15  VARIANCE OF ELEVATION          RAD**2                     CONS05
C(128)+N*15  VARIANCE OF RANGE RATE         KM**2/SEC**2               CONS05
C(129)+N*15  VARIANCE OF LATITUDE           RAD**2                     CONS06
C(130)+N*15  VARIANCE OF LONGITUDE          RAD**2                     CONS06
C(131)+N*15  VARIANCE OF ALTITUDE           KM**2                      CONS06
C(132)+N*15  VARIANCE OF AZIMUTH BIASES     RAD**2                     CONS06
C(133)+N*15  VARIANCE OF ELEVATION BIASES   RAD**2                     CONS06
C(134)+N*15  LATITUDE OF STATION            DEG                        CONS06
C(135)+N*15  LONGTUDE OF STATION            DEG                        CONS06
C(136)+N*15  ALTITUDE OF STATION            KM                         CONS06
C(137)+N*15  STATION NAME                                              CONS06
C(138)+N*15  PERIOD OF OBSERVATION          SEC                        CONS06
C(139)+N*15  VARIANCE OF TIME BIAS          SEC**2                     CONS07
C      STATION 1 ANTIGUA RADAR 91.1                                    CONS07
       S(125)=.000225                                                  CONS07
       S(126)=.000004                                                  CONS07
       S(127)=.000004                                                  CONS07
       S(128)=.00000001                                                CONS07
       S(129)=.0000000001                                              CONS07
       S(130)=.0000000001                                              CONS07
       S(131)=.0001                                                    CONS07
       S(134)=17.0267                                                  CONS07
       S(135)=298.2247                                                 CONS08
       S(136)=0.                                                       CONS08
       S(137)=6HANTIG                                                  CONS08
       S(138)=1.                                                       CONS08
C      STATION 2 ASCENSION RADAR 12.16                                 CONS08
       S(140)=.000225                                                  CONS08
       S(141)=.000004                                                  CONS08
       S(142)=.000004                                                  CONS08
       S(143)=.00000001                                                CONS08
       S(144)=.0000000001                                              CONS08
       S(145)=.0000000001                                              CONS09
       S(146)=.0001                                                    CONS09
       S(149)=-7.898                                                   CONS09
       S(150)=345.587393                                               CONS09
       S(151)=0.                                                       CONS09
       S(152)=6HASCENS                                                 CONS09
       S(153)=1.                                                       CONS09
C      STATION 3 MILLSTONE HILL RADAR                                  CONS09
       S(155)=.000225                                                  CONS09
```

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      S(156)=.000004
      S(157)=.000004
      S(158)=.00000001
      S(159)=.0000000001
      S(160)=.0000000001
      S(161)=.0001
      S(164)=42.4232
      S(165)=288.5080
      S(166)=0.
      S(167)=6HMILHIL
      S(168)=1.
C     STATION 4 MOBILE STATION MTS
      S(170)= 2500.E-6
      S(171)= .04 E-6
      S(172)= .04 E-6
      S(173)= .04 E-6
      S(174)= 1. E-10
      S(175)= 1. E-10
      S(176)= 9. E-6
      S(177)= .0016E-6
      S(178)= .0016E-6
      S(179)=-25.88472
      S(180)= 27.70528
      S(181)= 1.54302
      S(182)=6HMOBMTS
      S(183)= 30.
      S(184)= 9.E-6
C     STATION 5 AMR TRACKER GE
      S(185)=.000225
      S(186)=.000004
      S(187)=.000004
      S(188)=.00000001
      S(189)=.0000000001
      S(190)=.0000000001
      S(191)=.0001
      S(194)=28.278103
      S(195)=279.418297
      S(196)=0.
      S(197)=6HGEAMR
      S(198)=1.
C     STATION 6 BERMUDA
      S(200)=.000225
      S(201)=.000004
      S(202)=.000004
      S(203)=.00000001
      S(204)=.0000000001
      S(205)=.0000000001
      S(206)=.0001
      S(209)=32.160973
      S(210)=295.299179
      S(211)=0.
```

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      S(212)=6HBERMUD                                              CONS15
      S(213)=1.                                                    CONS15
C     STATION 7 GOLDSTONE RECEIVER                                 CONS15
C     50M SIG                                                      CONS15
      S(215) =2500.E-6                                             CONS15
C     .2MIL                                                        CONS15
      S(216) =.04 E-6                                              CONS15
C     .2MIL                                                        CONS15
      S(217) =.04 E-6                                              CONS15
C     .2M/SEC                                                      CONS15
      S(218) =.04 E-6                                              CONS16
C      63.78M                                                      CONS16
      S(219) =1.E-10                                               CONS16
C      63.78M                                                      CONS16
      S(220) =1.E-10                                               CONS16
C     3M                                                           CONS16
      S(221) =9.E-6                                                CONS16
C     .04MR                                                        CONS16
      S(222) =.0016E-6                                             CONS16
C     .04MR                                                        CONS16
      S(223) =.0016E-6                                             CONS17
      S(224) =35.3895                                              CONS17
      S(225) =243.15175                                            CONS17
      S(226) =1.03754                                              CONS17
      S(227)=6HGOSTRC                                              CONS17
      S(228) =30.                                                  CONS17
      S(229) =9.E-6                                                CONS17
C     STATION 8 GOLDSTONE TRANSMITTER                              CONS17
      S(230)=.000225                                               CONS17
      S(231)=.000004                                               CONS17
      S(232)=.000004                                               CONS18
      S(233)=.00000001                                             CONS18
      S(234)=.0000000001                                           CONS18
      S(235)=.0000000001                                           CONS18
      S(236)=.0001                                                 CONS18
      S(239) =35.29985                                             CONS18
      S(240) =243.19539                                            CONS18
      S(241) =.98949                                               CONS18
      S(242)=6HGOSTTR                                              CONS18
      S(243)=1.0                                                   CONS18
C     STATION 9 GBI RADAR XN-2                                     CONS19
      S(245)=.000225                                               CONS19
      S(246)=.000004                                               CONS19
      S(247)=.000004                                               CONS19
      S(248)=.00000001                                             CONS19
      S(249)=.0000000001                                           CONS19
      S(250)=.0000000001                                           CONS19
      S(251)=.0001                                                 CONS19
      S(254)=26.459736                                             CONS19
      S(255)=281.651892                                            CONS19
      S(256)=0.                                                    CONS20
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      S(257)=6HGBIRAD                                           CONS2010
      S(258)=1.0                                                CONS2020
C     STATION 10 JOHANNESBURG                                   CONS2030
C     50M SIG                                                   CONS2040
      S(260) =2500.E-6                                          CONS2050
C     .2MIL                                                     CONS2060
      S(261) =.04 E-6                                           CONS2070
C     .2MIL                                                     CONS2080
      S(262) =.04 E-6                                           CONS2090
C     .2M/SEC                                                   CONS2100
      S(263) =.04 E-6                                           CONS2110
C      63.78M                                                   CONS2120
      S(264) =1.E-10                                            CONS2130
C      63.78M                                                   CONS2140
      S(265) =1.E-10                                            CONS2150
C     3M SIG                                                    CONS2160
      S(266) = 9.E-6                                            CONS2170
C     .04MR                                                     CONS2180
      S(267) = .0016E-6                                         CONS2190
C     .04MR                                                     CONS2200
      S(268) = .0016E-6                                         CONS2210
C      DEG                                                      CONS2220
      S(269) =-25.88735                                         CONS2230
C      DEG                                                      CONS2240
      S(270) = 27.68478                                         CONS2250
C      KM                                                       CONS2260
      S(271) = 1.38192                                          CONS2270
      S(272)=6HJOHABG                                           CONS2280
C      SEC                                                      CONS2290
      S(273) = 30.                                              CONS2300
C     3MS SIG                                                   CONS2310
      S(274) = 9.E-6                                            CONS2320
C     STATION IN HAWAII                                         CONS2330
      S(275)=.000225                                            CONS2340
      S(276)=.000004                                            CONS2350
      S(277)=.000004                                            CONS2360
      S(278)=.00000001                                          CONS2370
      S(279)=.000000001                                         CONS2380
      S(280)=.000000001                                         CONS2390
      S(281)=.0001                                              CONS2400
      S(284)=18.823066                                          CONS2410
      S(285)=204.314722                                         CONS2420
      S(286)=0.                                                 CONS2430
      S(287)=6HHAWAII                                           CONS2440
      S(288)=1.0                                                CONS2450
C     STATION 12 JODRELL BANK                                   CONS2460
      S(290)=.000225                                            CONS2470
      S(291)=.000004                                            CONS2480
      S(292)=.000004                                            CONS2490
      S(293)=.00000001                                          CONS2500
      S(294)=.000000001                                         CONS2510
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      S(295)=.0000000001                                     CONS25
      S(296)=.0001                                           CONS25
      S(299)=53.049636                                       CONS25
      S(300)=357.693889                                      CONS25
      S(301)=0.                                              CONS25
      S(302)=6HJODBNK                                        CONS25
      S(303)=1.0                                             CONS25
C     STATION 13 PUERTO RICO RADAR 9.1                       CONS25
      S(305)=.000225                                         CONS26
      S(306)=.000004                                         CONS26
      S(307)=.000004                                         CONS26
      S(308)=.00000001                                       CONS26
      S(309)=.0000000001                                     CONS26
      S(310)=.0000000001                                     CONS26
      S(311)=.0001                                           CONS26
      S(314)=18.060396                                       CONS26
      S(315)=292.911805                                      CONS26
      S(316)=0.                                              CONS26
      S(317)=6HPURICO                                        CONS27
      S(318)=1.0                                             CONS27
C     STATION 14 SAN SALVADORE                               CONS27
      S(320)=.000225                                         CONS27
      S(321)=.000004                                         CONS27
      S(322)=.000004                                         CONS27
      S(323)=.00000001                                       CONS27
      S(324)=.0000000001                                     CONS27
      S(325)=.0000000001                                     CONS27
      S(326)=.0001                                           CONS27
      S(329)=23.173566                                       CONS28
      S(330)=285.4956                                        CONS28
      S(331)=0.                                              CONS28
      S(332)=6HSANSAL                                        CONS28
      S(333)=1.0                                             CONS28
C     STATION 15 WOOMERA                                     CONS28
C     50W SIG                                                CONS28
      S(335) = 2500.E-6                                      CONS28
C     .2MIL                                                  CONS28
      S(336) = .04E-6                                        CONS28
C     .2MIL                                                  CONS29
      S(337) = .04E-6                                        CONS29
C     .2M/SEC                                                CONS29
      S(338) = .04E-6                                        CONS29
C     63.78M                                                 CONS29
      S(339) = 1.E-10                                        CONS29
C     63.78M                                                 CONS29
      S(340) = 1.E-10                                        CONS29
C      3M                                                    CONS29
      S(341) = 9.E-6                                         CONS29
C     .04MR                                                  CONS30
      S(342) = .0016E-6                                      CONS30
C     .04MR                                                  CONS30
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      S(343) = .0016E-6                                       CONS3030
      S(344) =-31.38287                                       CONS3040
      S(345) =136.88502                                       CONS3050
      S(346) = .15079                                         CONS3060
      S(347)=6HWOMERA                                         CONS3070
C     SEC                                                     CONS3080
      S(348) = 30.                                            CONS3090
C     3MS                                                     CONS3100
      S(349) = 9.E-6                                          CONS3110
C   STATION 16   JPL CAPE CANAVERAL                           CONS3120
      S(350)=.000225                                          CONS3130
      S(351)=.000004                                          CONS3140
      S(352)=.000004                                          CONS3150
      S(353)=.00000001                                        CONS3160
      S(354)=.0000000001                                      CONS3170
      S(355)=.0000000001                                      CONS3180
      S(356)=.0001                                            CONS3190
C     DEG                                                     CONS3200
      S(359) = 28.48713                                       CONS3210
C     DEG                                                     CONS3220
      S(360) = 279.42315                                      CONS3230
C     KM                                                      CONS3240
      S(361) = .00257                                         CONS3250
      S(362) = 6HCAPJET                                       CONS3260
      S(363)=1.0                                              CONS3265
C    425 =SPEED OF LIGHT ACCURACY                             CONS3270
C     STATION 17 MAJUNGA                                      CONS3271
      S(369)=.0000000001                                      CONS3272
      S(370)=.0000000001                                      CONS3273
      S(371)=.0001                                            CONS3274
      S(374)=-15.216666                                       CONS3275
      S(375)=46.38333                                         CONS3276
      S(377)=6HMAJUNG                                         CONS3277
      S(378)=30.                                              CONS3278
C     STATION 18 CARNARVON                                    CONS3279
      S(384)=.0000000001                                      CONS3280
      S(385)=.0000000001                                      CONS3281
      S(386)=.0001                                            CONS3282
      S(389)=-24.8666                                         CONS3283
      S(390)=113.63333                                        CONS3284
      S(392)=6HCARNVN                                         CONS3285
      S(393)=30.                                              CONS3286
C     STATION 19 ROSMAN                                       CONS3287
      S(399)=.0000000001                                      CONS3288
      S(400)=.0000000001                                      CONS3289
      S(401)=.0001                                            CONS3290
      S(404)=35.20000                                         CONS3291
      S(405)=277.1333                                         CONS3292
      S(407)=6HROSMAN                                         CONS3293
      S(408)=30.                                              CONS3294
      S(425)=1.E-12                                           CONS3295
```

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C   FROM S(426)  TO  S(470) IS ONBOARD ERROR DATA                       CONS330
      S(475)=1.0                                                        CONS331
      S(476)=100.                                                       CONS332
      S(477)=200.                                                       CONS333
      S(478)=300.                                                       CONS334
      S(479)=400.                                                       CONS335
      S(480)=500.                                                       CONS336
      S(481)=600.                                                       CONS337
      S(485)=.03                                                        CONS337
C  N GOES FROM 0 TO 9  LUNAR BASED BEACONS                              CONS338
C (500)+N*15  VARIANCE OF RANGE                        KM2              CONS339
C (501)+N*15  VARIANCE OF AZIMUTH                      RAD2             CONS340
C (502)+N*15  VARIANCE OF ELEVATION                    RAD2             CONS341
C (503)+N*15  VARIANCE OF RANGE RATE                   KM2/SEC2         CONS342
C (504)+N*15  VARIANCE OF LATITUDE                     RAD2             CONS343
C (505)+N*15  VARIANCE OF LONGITUDE                    RAD2             CONS344
C (506)+N*15  VARIANCE OF ALTITUDE                     KM2              CONS345
C (507)+N*15  VARIANCE OF AZIMUTH BIAS                 RAD2             CONS346
C (508)+N*15  VARIANCE OF ELEVATION BIAS               RAD2             CONS347
C (509)+N*15  LATITUDE OF STATION                      DEG              CONS348
C (510)+N*15  LONGITUDE OF STATION                     DEG              CONS349
C (511)+N*15  ALTITUDE OF STATION                      KM               CONS350
C (512)+N*15  STATION NAME                                              CONS351
C (513)+N*15  PERIOD OF OBSERVATION                    SEC              CONS352
C (514)+N*15  SPARE                                                     CONS353
C (650) TO (685)  IS INITIAL P MATRIX                                   CONS354
C (686) TO (721)      INITIAL RAR MATRIX                                CONS355
C (722) TO (739)      TRANSITION MATRIX  FTA GUIDANCE                   CONS356
      S(900)=6378.165                                                   CONS357
      S(901)=1738.                                                      CONS358
      RETURN                                                            CONS362
      END                                                               CONS
```

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Subroutine: CONVPI

Purpose: Subroutine CONVPI transforms the input covariance matrices to the mean equinox of 1950 coordinate system. The input covariance matrices may be input in four coordinate systems:

1. Instantaneous launch pad tangent plane
2. Instantaneous injection tangent plane
3. True equinox of date
4. Mean equinox of 1950.

Calling Sequence:

CALL CONVPI (ITYPE, PIN, POUT)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	ITYPE	1			Logic key which indicates the
					type of input coordinate frame
I	PIN	21		Km^2, $(Km/Sec)^2$	Upper half of input covariance
					matrix in vector form
0	POUT	6, 6	$E(\vec{X}_{50}\ \vec{X}_{50}^{\ T})$	Km^2, $(Km/sec)^2$	Output covariance matrix (1950)

Common storages used or required: T, S, C, IC

Subroutines required: CROSS, GHA, MULT, NUTAIT, PTRAN, ROTEQ, TIMED, TRAC

Functions required: COS SIN FNORM

Approximate number of storages required: 476 DEC

Calling Sequence Elaboration

The calling sequence consists of CALL CONVPI (INTYPE, PIN, POUT) where:

INTYPE is a logic key which indicates the type of coordinate system in which the input covariance matrix is expressed. The range of the key is one to five. One through four indicate the four coordinate systems described above. A five in the call sequence indicates that the input matrix is in the same coordinate frame as the input matrix of the preceding call of the subroutine.

PIN is in the input covariance matrix. The matrix must be in the form of a column matrix with dimension (21). The (21) elements represent the upper half of the input covariance matrix.

$$PIN = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ & 7 & 8 & 9 & 10 & 11 \\ & & 12 & 13 & 14 & 15 \\ & & & 16 & 17 & 18 \\ & & & & 19 & 20 \\ & & & & & 21 \end{bmatrix}$$

Input Elements

The required order of the input covariance matrix is the following:
For INTYPE = 1 or 2

$$PIN = E \begin{bmatrix} AV \\ NOR \\ ALT \\ \dot{AV} \\ \dot{NOR} \\ \dot{ALT} \end{bmatrix}_{6x1} \begin{bmatrix} AV \\ NOR \\ ALT \\ \dot{AV} \\ \dot{NOR} \\ \dot{ALT} \end{bmatrix}^T_{1x6}$$

where

$$\hat{AV} \equiv \frac{\vec{R} \times \vec{v}}{|\vec{R} \times \vec{v}|} \times \frac{\vec{R}}{|\vec{R}|} = \text{coordinate along velocity}$$

$$\hat{NOR} \equiv \frac{\vec{R} \times \vec{v}}{|\vec{R} \times \vec{v}|} = \text{coordinate normal to orbit plane}$$

$$\hat{ALT} = \frac{\vec{R}}{|\vec{R}|} = \text{coordinate in direction of altitude}$$

Dots represent corresponding rates.

For INTYPE = 3 or 4

$$PIN = E \begin{bmatrix} X \\ Y \\ Z \\ \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix}_{6x1} \begin{bmatrix} X \\ Y \\ Z \\ \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix}^T_{1x6}$$

POUT is the output covariance matrix in cartesian coordinates mean equator 1950.

CONVPI Transformations

INTYPE = 1 Transformation from launch pad tangent plane to 1950.

Required input constants for operation:

LAUNCH PAD COORDINATES

S(80)	Altitude	(KM)
S(81)	Longitude	(DEG)
S(82)	Latitude	(DEG)

LAUNCH AZIMUTH

S(84)	Azimuth	(DEG)

TIME FROM LAUNCH TO INJECTION

S(83)	Time	DAYS HOURS. MIN SEC

The transformation from launch pad azimuth coordinates, (AV, NOR, ALT) to mean equinox of 1950 (X_{50}, Y_{50}, Z_{50}) is obtained as follows. Subroutine TRAC is called with launch time and launch pad coordinates to obtain a set of orthogonal unit vectors. The unit vectors, in equator of date, are: $\hat{U}_D$, unit up vector through the launch pad, $\hat{E}_D$, unit east vector, and $\hat{N}_D$, unit north vector. These unit vectors written in matrix form represent the transformation from equator of date coordinates to launch pad coordinates of date

$$\underset{3\times1}{\vec{X}_{LP}} = \underset{3\times3}{\begin{bmatrix} \hat{N}_D^T \\ \hat{E}_D^T \\ \hat{U}_D^T \end{bmatrix}} \underset{3\times1}{\vec{X}_{DATE}} \tag{1}$$

The transformation from launch pad coordinates, (N, E, U), to the desired launch azimuth coordinates, (AV, NOR, ALT), is a rotation about the $\hat{U}$ vector by an angle equal to the launch azimuth.

$$\underset{3x1}{\vec{X}_{LAZ}} = \underset{3x3}{\begin{bmatrix} \cos LAZ & \sin LAZ & 0 \\ -\sin LAZ & \cos LAZ & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3x1}{\vec{X}_{LP}} = \underset{3x3}{(ROT)} \underset{3x1}{\vec{X}_{LP}} \tag{2}$$

substituting for $\vec{X}_{LP}$ from equation (1) yields

$$\underset{3x1}{\vec{X}_{LAZ}} = \underset{3x3}{(ROT)} \underset{3x3}{\begin{bmatrix} \hat{N}_D^T \\ \hat{E}_D^T \\ \hat{U}_D^T \end{bmatrix}} \underset{3x1}{\vec{X}_{DATE}} \tag{3}$$

Subroutines ROTEQ and NUTAIT are called to obtain transformation matrix (AN) which is the transformation from mean equinox of 1950 to true equator of date.

$$\underset{3x1}{\vec{X}_{DATE}} = \underset{3x3}{(AN)} \underset{3x1}{\vec{X}_{1950}} \tag{4}$$

Substituting equation (4) into (3) yields

$$\underset{3x1}{\vec{X}_{LAZ}} = \underset{3x3}{(ROT)} \underset{3x3}{\begin{bmatrix} \hat{N}_D^T \\ \hat{E}_D^T \\ \hat{U}_D^T \end{bmatrix}} \underset{3x3}{(AN)} \underset{3x1}{\vec{X}_{1950}} \tag{5}$$

or

$$\vec{X}_{1950} = \left\{ (AN)^T \begin{bmatrix} \hat{N}_D^T \\ \hat{E}_D^T \\ \hat{U}_D^T \end{bmatrix}^T (ROT)^T \right\} \vec{X}_{LAZ} = (TRANS)\ \vec{X}_{LAZ} \qquad (6)$$

$$\begin{matrix} 3x1 & & 3x3 & 3x3 & 3x3 & 3x1 & & 3x3 & 3x1 \end{matrix}$$

The corresponding velocity transformation is the following:

$$\dot{\vec{X}}_{1950} = (TRANS)\ \dot{\vec{X}}_{LAZ} \qquad (7)$$

under the assumption that $(\dot{AN}) = 0$ which implies $(TR\dot{A}NS) = 0$.

The 3x3 matrix, (TRANS), is the desired transformation from launch azimuth coordinates to mean equator of 1950. Subroutine PTRAN is called with a 2 in the call list to perform the desired covariance matrix transformation.

$$E\left\{\vec{X}_{1950}\vec{X}^T_{1950}\right\} = \left\{\begin{matrix} \underset{3x3}{(TRANS)} & \underset{3x3}{0} \\ \underset{3x3}{0} & \underset{3x3}{(TRANS)} \end{matrix}\right\} E\left\{\vec{X}_{LAZ}\vec{X}^T_{LAZ}\right\} \left\{\begin{matrix} (TRANS) & 0 \\ 0 & (TRANS) \end{matrix}\right\}^T \qquad (8)$$

$$\begin{matrix} 6x6 & & 6x6 & & 6x6 & & 6x6 \end{matrix}$$

where $\vec{X}_{1950}$ and $\vec{X}_{LAZ}$ now represent the total state vector (position and velocity).

<u>INTYPE</u> = 2 Transformation from injection tangent plane to 1950.

The transformation from injection tangent plane coordinates, (AV, NOR, ALT), to mean equinox 1950 is obtained as follows. The unit vectors for the injection tangent plane coordinates are obtained in equator of 1950.

$$AV_{50} = \frac{\vec{R} \times \vec{v}}{|\vec{R} \times \vec{v}|} \times \frac{\vec{R}}{|\vec{R}|}$$

$$NOR_{50} = \frac{\vec{R} \times \vec{v}}{|\vec{R} \times \vec{v}|} \quad (9)$$

$$ALT_{50} = \frac{\vec{R}}{|\vec{R}|}$$

These vectors written in matrix form represent the transformation from mean equinox of 1950 to injection tangent plane

$$\underset{3x1}{\vec{X}_{ITP}} = \underset{3x3}{\begin{bmatrix} \hat{A}V_{50}^{T} \\ \hat{N}OR_{50}^{T} \\ \hat{A}LT_{50}^{T} \end{bmatrix}} \quad \underset{3x1}{\vec{X}_{1950}} \quad (10)$$

or

$$\underset{3x1}{\vec{X}_{1950}} = \underset{3x3}{\begin{bmatrix} \hat{A}V_{50}^{T} \\ \hat{N}OR_{50}^{T} \\ \hat{A}LT_{50}^{T} \end{bmatrix}^{T}} \quad \underset{3x1}{\vec{X}_{ITP}} = \underset{3x3}{(TRANS)} \ \underset{3x1}{\vec{X}_{ITP}} \quad (11)$$

The corresponding velocity transformation is

$$\underset{3x1}{\dot{\vec{X}}_{1950}} = \underset{3x3}{(TRANS)} \ \underset{3x1}{\dot{\vec{X}}_{ITP}} \quad (12)$$

Subroutine PTRAN is called with a 2 in the call list and the operation presented in equation (8) is performed with the 3x3 matrix, (TRANS), defined in equation (11).

<u>INTYPE</u> = 3 Transformation from equator of date to equator of 1950.

The transformation from equator of date to equator of 1950 is obtained as follows. Subroutines ROTEQ and NUTAIT are called to obtain (AN), the required transformation matrix.

$$\underset{3x1}{\vec{X}_{DATE}} = \underset{3x3}{(AN)} \underset{3x1}{\vec{X}_{1950}} \quad (13)$$

or

$$\underset{3x1}{\vec{X}_{1950}} = \underset{3x3}{(AN)^T} \underset{3x1}{\vec{X}_{DATE}} = \underset{3x3}{(TRANS)} \underset{3x1}{\vec{X}_{DATE}} \quad (14)$$

The corresponding velocity transformation is

$$\dot{\vec{X}}_{1950} = (TRANS) \dot{\vec{X}}_{DATE} \quad (15)$$

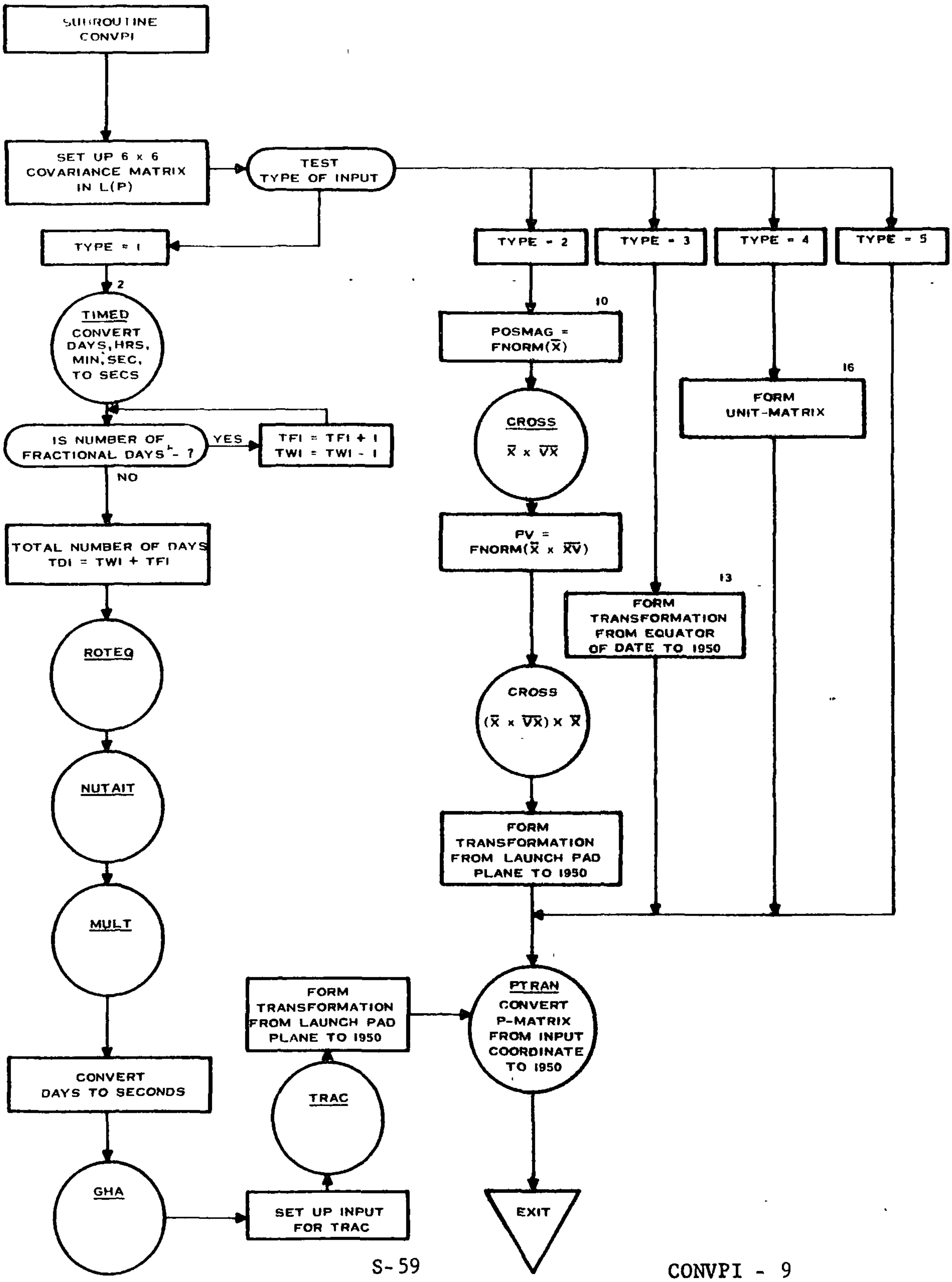
Subroutine PTRAN is called with a 2 in the call list and the operation presented in equation (8) is performed with the 3x3 matrix, (TRANS), defined in equation (14).

<u>INTYPE</u> = 4 Transformation from equator of 1950 to equator of 1950.

The transformation is the unit matrix.

$$\underset{3x1}{\vec{X}_{1950}} = \underset{3x3}{(I)} \underset{3x1}{\vec{X}_{1950}} = (TRANS) \vec{X}_{1950} \quad (16)$$

<u>INTYPE</u> = 5 The subroutine calls PTRAN to perform the operation presented in equation (8) with the transformation matrix, (TRANS), which is in storage from the previous call of CONVPI.


SUBROUTINE CONVPI
SET UP 6 x 6 COVARIANCE MATRIX IN L(P)
TEST TYPE OF INPUT
TYPE = 1
2
TIMED
CONVERT DAYS, HRS, MIN, SEC, TO SECS
IS NUMBER OF FRACTIONAL DAYS - ?
YES
TFI = TFI + 1
TWI = TWI - 1
NO
TOTAL NUMBER OF DAYS
TDI = TWI + TFI
ROTEQ
NUTAIT
MULT
CONVERT DAYS TO SECONDS
GHA
SET UP INPUT FOR TRAC
TRAC
FORM TRANSFORMATION FROM LAUNCH PAD PLANE TO 1950
TYPE = 2
TYPE = 3
TYPE = 4
TYPE = 5
10
POSMAG = FNORM(X̄)
CROSS
X̄ x V̄X̄
PV = FNORM(X̄ x X̄V̄)
CROSS
(X̄ x V̄X̄) x X̄
FORM TRANSFORMATION FROM LAUNCH PAD PLANE TO 1950
13
FORM TRANSFORMATION FROM EQUATOR OF DATE TO 1950
16
FORM UNIT-MATRIX
PTRAN
CONVERT P-MATRIX FROM INPUT COORDINATE TO 1950
EXIT

```
*      LABEL
*      SYMBOL TABLE
CEC2010  SUBROUTINE CONVPI
       SUBROUTINE CONVPI(INPUTP,PI,PS)
       COMMON T,S,C,IC
       DIMENSION T(1360),S(1000),C(1000),IC(1)
      1,PI(21),P(6,6),A(3,3),EM(3,3),AN(3,3),U(3)
      2,E(3),EN(3),PDUMP(3,3),DUM(3,3),PS(6,6)
      3,X(3),VX(3), ORORB(3),ALV(3),RT(3)
       EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM),(IC,ICDUM)
      1,(S(70),DTS),(S(72),DR),(S(83),TLTI)
      2,(C(138),AN), (S(115),X),(S(118),VX),(C(10),TW)
      3,(C(11),TF),(C(129),A),(C(120),EM)
C      INPUTP DETERMINES TYPE OF INPUT 1.LAUNCH PAD TANGENT PLANE
C      2.INJECTION TANGENT PLANE 3.EQUATOR DATE 4.EQ1950 5.USE MATRIX
C      THE REQUIRED ORDER OF INPUT VECTOR OF WHICH PI IS THE COVARIANCE
C      MATRIX FOR TANGENT PLANE CASES IS FOLLOWING:
C      1.ALONG VELOCITY,2.NORMAL TO ORBIT,3.ALTITUDE,4,5,6 CORRES.RATES
       K=0
       DO 1 I=1,6
       I=I
       DO 1 J=I,6
       K=K+1
       P(I,J)=PI(K)
       P(J,I)=P(I,J)
    1  CONTINUE
       INPUTP=INPUTP
       GO TO (2,10,13,16,8),INPUTP
    2  CONTINUE
       CALL TIMED(TLTI,TT)
       TFI=TF-(TT/86400.)
       TWI=TW
    3  CONTINUE
       IF(TFI)4,5,5
    4  CONTINUE
       TFI=TFI+1.
       TWI=TWI-1.
       GO TO 3
    5  CONTINUE
       TDI=TWI+TFI
       CALL ROTEQ(TDI,A)
       CALL NUTAIT(TDI,OM,CR,DDC,EM,EPSIL)
       CALL MULT(EM,A,AN,N)
       FSEC=DTS*TFI
       CALL GHA(FSEC,TWI,GH,EM(2,1),OMEGA)
       GH=GH*DR
       AT=S(80)*DR
       ON=S(81)*DR
       AL=S(82)
       CALL TRAC(ON,AT,AL,GH,U,E,EN,RT,AC,SL,CL,ST,CT)
       FAZ=S(84)*DR
```

```
      SAZ=SINF(FAZ)                                                    CONV05
      CAZ=COSF(FAZ)                                                    CONV05
      DO 6 I=1,3                                                       CONV05
      PDUMP(I,1)=CAZ*EN(I)+SAZ*E(I)                                    CONV05
      PDUMP(I,2)=-SAZ*EN(I)+CAZ*E(I)                                   CONV05
      PDUMP(I,3)=U(I)                                                  CONV05
    6 CONTINUE                                                         CONV05
      DO 7 I=1,3                                                       CONV05
      DO 7 J=1,3                                                       CONV05
      DUM(I,J)=0.                                                      CONV06
      DO 7 K=1,3                                                       CONV06
      DUM(I,J)=DUM(I,J)+AN(K,I)*PDUMP(K,J)                             CONV06
    7 CONTINUE                                                         CONV06
C     DUM IS TRANSFORMATION FROM LAUNCH PAD PLANE TO 1950              CONV06
    8 CONTINUE                                                         CONV06
      CALL PTRAN(P,DUM,PS,2)                                           CONV06
C     PTRAN CONVERTS P MATRIX FROM INPUT COORDINATES TO EQ 1950        CONV06
      GO TO 15                                                         CONV06
   10 CONTINUE                                                         CONV06
      POSMAG=FNORM(X)                                                  CONV07
      CALL CROSS(X,VX, ORORB)                                          CONV07
      PV=FNORM(ORORB)                                                  CONV07
      CALL CROSS (ORORB,X,ALV)                                         CONV07
      AL=POSMAG*PV                                                     CONV07
      DO 11 I=1,3                                                      CONV07
      PDUMP(I,1)=ALV(I)/AL                                             CONV07
      PDUMP(I,2)= ORORB(I)/PV                                          CONV07
      PDUMP(I,3)=X(I)/POSMAG                                           CONV07
   11 CONTINUE                                                         CONV07
      DO 12 I=1,3                                                      CONV08
      DO 12 J=1,3                                                      CONV08
      DUM(I,J)=0.                                                      CONV08
      DO 12 K=1,3                                                      CONV08
      DUM(I,J)=DUM(I,J)+AN (K,I)*PDUMP(K,J)                            CONV08
   12 CONTINUE                                                         CONV08
C     DUM IS TRANSFORMATION FROM INJECTION TAN.PLANE TO 1950           CONV08
      GO TO 8                                                          CONV08
   13 CONTINUE                                                         CONV08
      DO 14 I=1,3                                                      CONV08
      DO 14 J=1,3                                                      CONV09
      DUM(I,J)=AN (J,I)                                                CONV09
C     DUM IS TRANSFORMATION FROM EQUATOR OF DATE TO 1950               CONV09
   14 CONTINUE                                                         CONV09
      GO TO 8                                                          CONV09
   16 CONTINUE                                                         CONV09
      DO 17 I=1,3                                                      CONV09
      DO 18 J=I,3                                                      CONV09
      DUM(I,J)=0.                                                      CONV09
      DUM(J,I)=DUM(I,J)                                                CONV09
   18 CONTINUE                                                         CONV10
      DUM(I,I)=1.                                                      CONV10
```

```
 17   CONTINUE                                                   CONV1020
      GO TO 8                                                    CONV1030
 15   CONTINUE                                                   CONV1040
      RETURN                                                     CONV1050
      END                                                        CONV
```

Subroutine: CORRTP

Purpose: To perform the filtering or weighting of the observation being made and updating of the state covariance matrix for the inclusion of the observation.

Calling Sequence:

CALL CORRTP (QQ, H, P)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	QQ	1	σ^2_{MEAS}		Measurement variance
I	H	(1,6)	$\frac{\partial \text{ MEAS}}{\partial \text{ STATE}}$		Measurement partials
I,O	P	(6,6)	$E(\vec{x}\,\vec{x}^T)$		State covariance matrix

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required: 124 DEC

Equation Being Solved

The updating of the state covariance matrix for an observation assuming the Schmidt-Kahlman optimum filter is used on the data is the following:

$$P_{NEW} = P_{OLD} - P_o H^T (H P_o H^T + QQ)^{-1} H P_o$$

```
*       LABEL                                                    CORR
*       SYMBOL TABLE                                             CORR
CEC2025  SUBROUTINE CORRTP                                       CORR
        SUBROUTINE CORRTP(QQ,H,P)                                CORR00
        DIMENSION P(6,6),H(6),PMT(6)                             CORR00
        DIMENSION PCOR(6,6)                                      CORR00
        BB=0.                                                    CORR00
        DO 43 J=1,6                                              CORR00
        PMT(J)=0.                                                CORR00
        DO 44 K=1,6                                              CORR00
        PMT(J)=PMT(J)+P(J,K)*H(K)                                CORR00
   44   CONTINUE                                                 CORR00
        BB=BB+PMT(J)*H(J)                                        CORR00
   43   CONTINUE                                                 CORR01
        BB = BB+QQ                                               CORR01
        DO 45 J=1,6                                              CORR01
        DO 45 K=1,6                                              CORR01
        PCOR(J,K)=(PMT(J)*PMT(K))/BB                             CORR01
        P(J,K)=P(J,K)-PCOR(J,K)                                  CORR01
   45   CONTINUE                                                 CORR01
        RETURN                                                   CORR01
        END                                                      CORR
```

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Subroutine: CROSS

Purpose: To find the cross product of two 3-dimensional vectors.

Calling Sequence:

CALL CROSS (A, B, C)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	A	3			First input vector
I	B	3			Second input vector
O	C	3			Result = AXB

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

```
*      LABEL                                                    CROS
*      SYMBOL TABLE                                             CROS
       SUBROUTINE CROSS(A,B,C)                                  CROS0000
       DIMENSION A(3),B(3),C(3)                                 CROS0010
       C(1)=A(2)*B(3)-A(3)*B(2)                                 CROS0020
       C(2)=A(3)*B(1)-A(1)*B(3)                                 CROS0030
       C(3)=A(1)*B(2)-A(2)*B(1)                                 CROS0040
       RETURN                                                   CROS0050
       END                                                      CROS
```

Subroutine: (CSH)S

Purpose: This is the standard Fortran II card image input subroutine modified to output the card image, accept logical input and output tape numbers from SETN, and call subroutine HOUR to read the clock if desired.

Calling Sequence:

CALL (CSH)S (BUFF)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	BUFF	12			Buffer for contents of card read

Common storages used or required: None.

Subroutines required: SETN, HOUR, System read routines.

Functions required: None.

Approximate number of storages required:

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Subroutine: DE6FN

Identification

RW DE6FN
704 - FORTRAN SAP Language Subroutine
STL

Purpose

This FØRTRAN subprogram integrates a set of N simultaneous second-order ordinary differential equations in which first derivatives may or may not appear. It is the FØRTRAN version of the standard subroutine RW DE6F.

Restrictions

Same as explained in the write-up for RW DE6F.

Method

A fourth-order Runge-Kutta method (RW DE5F) is used to start the integration and to change the step-size during integration. A Cowell "second-sum" method based on sixth differences is used to continue the integration. While input to this routine is single precision, double precision is used internally to control round-off errors. Truncation error can be controlled by choosing an appropriate step-size, or by using the variable step-size mode of operation. The set-up entry uses the auxiliary subroutine to evaluate the second-order derivatives. The values of the variables and derivatives are consistent at all times. A detailed description of the method used is available in Appendix A.

Usage

a. Calling LIST for set-up entry (performed prior to initiating the integration).

```
CALL DE6FN(I0,I1,T(1),N,V,J,K,A3,A4,A5,A6)
```

I0 is the same as P0 of RW DE6F write-up.

+1 1st derivatives are present in the evaluation of the second derivatives.
-1 1st derivatives are missing in the evaluation of the second derivatives.

I1 is the same as P1 of RW DE6F write-up.

+1 Variable step-size mode of operation is used.
-1 Fixed step-size mode of operation is used.

T is the name of the floating point array which is of dimension 30N+3 and is reserved by the user. This region should be located in COMMON, since it must be referred to by the main program and by the subroutine V. T(1) need not be set equal to N; however, all other requirements that concern the usage of area T apply. The value of N is not available to the programmer in T(1) and if it is required by the auxiliary subroutine, it should appear in COMMON also.

N is the number of equations (an integer).

V is the name of a FØRTRAN subroutine for evaluating the derivatives y_i. This subroutine must be named in the main program by the use of an "F" card.

J equates to B in RW DE6F write-up (an integer).

K equates to R in RW DE6F write-up (an integer).

A3 equates to $\alpha+3$ in RW DE6F write-up (floating point).

A4 equates to $\alpha+4$ in RW DE6F write-up (floating point).

A5 equates to $\alpha+5$ in RW DE6F write-up (floating point).

A6 equates to $\alpha+6$ in RW DE6F write-up (floating point).

For meaning of N,V,J,K,A3,A4,A5,A6 refer to RW DE6F write-up.

Region T contains the following information prior to Set-up Entry.

T(2)	x	value of independent variable
T(3)	h	value of step-size
T(4) thru	y_i thru	values of the independent variables y_i
T(N+3)	y_N	
T(N+4) thru	y_i' thru	values of the first derivatives y_i'
T(2N+3)	y_N'	
T(2N+4) thru	y_i'' thru	values of the second derivatives y_i''
T(3N+3)	y_N''	
T(3N+4) - T(30N+4)		temporary storage

More detailed description of this storage may be found in the RW DE6F write-up.

b. RW DE6FN entry to integrate one step.

CALL DE6FPI (ACCUM)

Upon return ACCUM will have the results which formerly appeared in the accumulator.

c. RW DE6FN entry for a running start.

CALL DE6FP2

d. RW DE6FN entry for a change of step-size in Cowell/Runge-Kutta System.

CALL DE6FP3(H)
H is the new value of h which was formerly placed in the accumulator prior to entry.

e. RW DE6FN entry for a change of step-size for a final integration.

CALL DE6FP4
T(3) must be set to the correct value before the CALL.

f. RW DE6FN entry to initiate a running change in coordinates and to set a list call to a non-zero value.

CALL DE6FNZ (VALUE)

g. RW DE6FN entry to change from variable mode to fixed mode.

CALL DE6FNG

h. RW DE6FN entry to change from fixed mode to variable mode.

CALL DE6FPS

i. RW DE6FN entry to change the value of h_{min}.

CALL DE6FMN (VALUE)
VALUE is the new h_{min} in floating point.

j. RW DE6FN entry to change the value of h_{max}.

CALL DE6FMA (VALUE)
VALUE is the new h_{max} value in floating point.

k. RW DE6FN entry to change the value of y_{min}.

CALL DE6FCM (VALUE)
VALUE is the new y_{min} value in floating point.

l. RW DE6FN entry to change the value of R after a Cowell integration step.

CALL DE6FCH(NR,R)
NR is the integer value of R.
R is the value in floating point.

Identification

RWDE6F Floating Point Cowell (Second Sum), Runge-Kutta
Integration of Second-Order Equations
704 - SAP
STL

Purpose

To solve a set of N simultaneous second-order ordinary differential equations in which first derivatives may or may not appear.

Restrictions

No internal checks are made for overflow or underflow. The user must provide an auxiliary subroutine which evaluates the second-order derivatives. The initial conditions must be set up prior to the first set up entry.

Method

A fourth-order Runge-Kutta method* (RWDE5F) is used to start the integration and to change the step-size during integration. A Cowell "second-sum" method based on sixth differences is used to continue the integration. While input to this routine is single precision, double precision is used internally to control round-off errors. Truncation error can be controlled by choosing an appropriate step-size, or by using the variable step-size mode of operation. The set-up entry uses the auxiliary subroutine to evaluate the second-order derivatives. The values of the variables and derivatives are consistent at all times. A detailed description of the method used is available in Appendix A of this Subroutine.

Usage

Calling sequence to set up a problem:

Loc.	Instruction	Comments
γ	TSX DE6F,4	Set up entry.
α+1	PO T,O,V	Option, addresses of storage and auxiliary subroutines.
α+2	P1 B,O,R	Option and Parameters
α+3	DEC 1E-S	S is the number of significant figures desired.
α+4	DEC h_{min}	Minimum step-size. Floating point.

* Scarborough, T.B., Numerical Mathematical Analysis, Third Edition, John Hopkins Press, Baltimore, 1955 (pp. 301-302)

Loc.	Instruction	Comments
α+5	DEC h_{max}	Maximum step-size. Floating point.
α+6	DEC y_{min}	Minimum y_i value allowed for testing. Floating point. (See Appendix A for details of y_{min}.)

Calling sequence to integrate all variables one step:

α	TSX DE6F+1,4	Integration entry.
α+1	Return	Sign of AC will be plus if the integration was Runge-Kutta and minus if Cowell.

The address T is the first of 30N+3 calls arranged as follows:

T	PZE N,0,0	N is the number of equations. Fixed point.
T+1	x	Value of independent variable. Floating point.
T+2	h	Value of step-size. Floating point.
T+23 thru T+N+2	y_i thru y_N	Values of the dependent variables y_i. Floating point
T+N+3 thru T+2N+2	y'_i thru y'_N	Values of the first derivatives y'_i Floating point
T+3N+3 thru T+3N+3	y''_i thru Y''_N	Locations where the user's auxiliary subroutine must store the second derivatives y''_i.
T+3N+3 thru T+30N+2		27N cells of temporary storage.

T+3N+3 thru T+9N+2 (6N) are used by the Runge-Kutta subroutine (RWDE5F). T+4N+3 thru T+5N+2 and T+8N+3 thru T+9N+2 contain the least significant parts (except when a change of coordinates is in progress) of y'_i and y_i respectively, and must be preserved throught the entire integration procedure. The final 21N cells of the T storage are used by the Cowell subroutine and must also be preserved. (See Appendix E of this subroutine for a detailed description of these 27N cells.)

The address V is the entry point of the auxiliary subroutine which evaluates the derivatives y''_i and is entered by the calling sequence:

α	TSX V,4	Index registers need not be saved in V.
α+1	Return	Return must be made via a TRA 1,4.

The first B ($\leq$ N) equations are tested to determine whether it is necessary to halve or possibly to double the step-size or to proceed with a Cowell integration step.

For a given step-size h, the Cowell integration step is h and the Runge-Kutta integration step is h/R.

Options

PO - PZE 1st derivatives are present in the evaluation of the second derivatives
- MZE 1st derivatives are missing in the evaluation of the second derivatives.
P1 = PZE Variable step-size mode of operation is used.
= MZE Fixed step-size mode of operation is used.

If 1E-S, h_{min}, h_{max}, and y_{min} are not specified (0 in first calling sequence), the subroutine will set them to 1E-9, 0, 1E18, and 1., respectively.

Special Usage (See Appendices A and B for complete details.)

The following special usages are possible:

1. Running change of coordinates.
2. Running start.
3. Change of step-size by user in the Cowell/Runge-Kutta system.
4. Change of step-size for a final integration or at some prescribed value of x.

Space Required (In addition to T and V).

955 calls of program and constants. (Includes DE5F.)

44 calls of COMMON thru COMMON + 43.

Timing

Set-up time. (V=time for 1 entry to the auxiliary subroutine.)

$$.012 \left[12N + 512\right] \text{ ms.} + 1V.$$

To integrate one step:

1. Runge-Kutta (AC=+ after an integration return.)

A. With 1st derivatives.

$$.012 \left[476N + (8N+18)/R + 240\right] \text{ ms.} + 4V.$$

B. Without 1st derivatives.

$$.012 \left[383N + (8N + 18)/R + 240\right] \text{ ms.} + 3V.$$

Timing (Continued)

2. Cowell (AC= - after an integration return.)

A. With 1st derivatives.

.012 [2901N + 92B + 194] ms. + 2V.

+ .012 [2124N + 34] ms. if previous integration was Runge-Kutta.

B. Without 1st derivatives.

.012 2334N + 92B + 194 ms. + 2V.

+ .012 2124N + 88B + 34 ms. if previous integration was Runge-Kutta.

3. Change of Coordinates. (In addition to first part of 2A or 2B.)

.012 [2286N] ms. + 8V.

APPENDIX A

METHOD

This routine is prepared to solve the following system:

$$1)\quad \left.\begin{array}{l} y_i'' = f_i\,(x, y_i, \ldots, y_N, y_i', \ldots, y_N') \\ \\ y_i(x_0) = y_{i0}, \quad y_i(x_0) = y'_{i0} \end{array}\right\}\quad i = 1, 2, \ldots, N$$

In case none of the f_i involve the first derivatives y_i', time is saved by indicating this in the set-up. The Runge-Kutta routine RWDE5F is used to start the integration and also to change the step-size h. The user must ask for each integration step, and the routine will follow this sequence:

1. R Runge-Kutta steps of size $\frac{h\ start}{R}$ are taken to obtain y_{i1}, y_{i1}', y_{i1}''. This is continued until we reach y_{i6}, y_{i6}', y_{i6}'', after a total of 6R Runge-Kutta steps. The integer R (=4 if unspecified) simply allows Runge-Kutta to operate at a smaller step than the main program.

2. For each of the N equations, that part of the difference table above the diagonal line is constructed in three steps:

		y''_{i0}						
			Δ^{I}_{i0}					
		y''_{i1}		Δ^{II}_{i0}				
			Δ^{I}_{i1}		Δ^{III}_{i0}			
		y''_{i2}		Δ^{II}_{i1}		Δ^{IV}_{i0}		
			Δ^{I}_{i2}		Δ^{III}_{i1}		Δ^{V}_{i0}	
$''F_{i4}$		y''_{i3}		Δ^{II}_{i2}		Δ^{IV}_{i1}		Δ^{VI}_{i0}
	$'F_{i4}$		Δ^{I}_{i3}		Δ^{III}_{i2}		Δ^{V}_{i1}	
$''F_{i5}$		y''_{i4}		Δ^{II}_{i3}		Δ^{IV}_{i2}		Δ^{VI}_{i1}
	$'F_{i5}$		Δ^{I}_{i4}		Δ^{III}_{i3}		Δ^{V}_{i2}	
$''F_{i6}$		y''_{i5}		Δ^{II}_{i4}		Δ^{IV}_{i3}		
	$'F_{i6}$		Δ^{I}_{i5}		Δ^{III}_{i4}			
$''F_{i7}$		y''_{i6}		Δ^{II}_{i5}				
	$'F_{i7}$		Δ^{I}_{i6}					
$''F_{i8}$		y''_{i7}						
	$'F_{i8}$							
$''F_{i9}$								

First the known y''_{i0} through y''_{i6} are differenced to give the right half of the table. Next are calculated in extra precision:

$$2)\quad ''F_{i4} = \frac{y_{i3}}{h^2} - C_0\, y''_{i3} - C_2\, \Delta^{II}_{i2} - C_4\, \Delta^{IV}_{i1} - C_6\, \Delta^{VI}_{i0}$$

$$3) \quad 'F_{i4} = \frac{y'_{i3}}{h} - D_0\, y''_{i3} - D_1\, \Delta^{I}_{i3} - D_2\, \Delta^{II}_{i2} - D_3\, \Delta^{III}_{i2}$$

$$- D_4\, \Delta^{IV}_{i1} - D_5\, \Delta^{V}_{i1} - D_6\, \Delta^{VI}_{i0}$$

The table is then completed down to the diagonal line, by requiring the difference between any entry and the entry above to equal the entry to the right. The constants used in equations (2) - (7) are given in the description of the Livermore Cowell routine.

3. Before going to a Cowell step, the step-size h is tested. The tests are omitted, however, if the user so indicates in the initial calling sequence, in which case h is fixed. Only the first B equations are used to test, where $1 \le B \le N$ and $B = N$ if unspecified. We determine--

$$V = \frac{\max}{1 \le i \le B} \left| \frac{\Delta^{V}_{i1}}{\max(y_{i6},\, y_{min})} \right| . \quad \text{If} \quad V \ge \frac{10^{3-S}}{h^2} \text{, then the}$$

ratio of 5th difference to function is too large -- if S decimal places are to be retained at that step. Therefore, h is reduced to h/2 and Runge-Kutta re-entered for another sequence of 6R steps. These begin with the latest calculated values (y_{i6}, y'_{i6}) and no ground is retraced. The constant y_{min} (=1 if unspecified) prevents division by y near a zero; for example, in sine calculation y_{min}=.1 avoids difficulty near 180^{o}. The integer S, taken as 9 if unspecified, allows a large h if chosen smaller, say S = 7.

$$\text{If} \quad \frac{10^{-1-S}}{h^2} < V < \frac{10^{3-S}}{h^2} \text{, we proceed to a Cowell step.}$$

If $V \le \frac{10^{-1-S}}{h^2}$, we may be able to double h. We test further to see that

$$W = \frac{\max}{1 \le i \le B} \left| \frac{\Delta^{VI}_{i0}}{\max (y_{i6}, y_{min})} \right| \le \frac{10^{-1-S}}{h^2},$$

and if so, we re-enter Runge-Kutta after replacing h by 2h, since the step-size h has led to needlessly small difference to function ratios. Of course, h is not halved or doubled if this would violate h_{min} or h_{max}, which are 0 and 10^{18} if unspecified.

4. The Cowell integration begins with predictions:

$$4) \quad y_{i7} = h^2 \left({}''F_{i8} + N_0 y''_{i6} + N_1 \Delta^{I}_{i5} + \ldots + N_6 \Delta^{VI}_{i0} \right)$$

$$5) \quad y'_{i7} = h \left({}'F_{i7} + \dot{N}_0 y''_{i6} + \dot{N}_1 \Delta^{I}_{i5} + \ldots + \dot{N}_6 \Delta^{VI}_{i0} \right)$$

These use the row of the difference table above the diagonal line; only this row is needed for a Cowell step and is kept up to date as the integration proceeds. (We mention that the above prediction for y'_{i7} is omitted in the missing first derivative option.) Now from y_{i7} and y'_{i7}, we obtain y''_{i7} and then complete the row of differences out to Δ^{VI}_{i1} under the diagonal line in the table. With this row, we calculate final corrected values --

$$6) \quad y_{i7} = h^2 \left({}''F_{i8} + B_0 y''_{i7} + \ldots + B_6 \Delta^{VI}_{i1} \right)$$

$$7) \quad y'_{i7} = h \left({}'F_{i7} + \dot{B}_0 y''_{i7} + \ldots + \dot{B}_6 \Delta^{VI}_{i1} \right)$$

From these we get corrected values for y''_{i7}, and recalculate the entire row under the diagonal line. This completes the integration step. Using the new row of differences, the next step begins by testing the step-size (i.e., at 3.).

Further Properties of the Program

In some problems, information about the first derivative (velocity) may be less reliable than information about the function (position). The user may then choose a "running change of coordinates" or a "running start;" these depend on the fact that with 8 consecutive values of the y_i (and the y_i' in case first derivatives are present in the f_i) the Cowell part of the program can be self-starting. The mathematics is simple: step No. 1 is omitted, and No. 2 modified to calculate $''F_{i4}$ and $''F_{i5}$ from Eq. (2) (instead of $''F_{i4}$ and $'F_{i4}$). The difference table may again be completed, and Cowell integration begins. The user, having tested the AC to establish that the previous step was a Cowell step, begins a running change of coordinates by setting cell DE6F + 500 to non-zero. He then sets up a counter and begins immediately to store 8 consecutive values of the y_i starting at T+3+11N (and y_i' starting at T+3+3N, if they appear in the f_i). After changing the coordinates the 8th time, the user may change the second derivative evaluation routines; if x and h are to be in new units this should also be done. When another step is asked for, the routine will form a difference table in the new coordinates and proceed to a Cowell step.

The mechanics of a running start are similar; after going through the set-up routine, the user loads his values of y_{i0} through y_{i7} (and the y_i' if needed in the f_i) into the same locations as above and makes the required transfer.

There may also be occasions on which the user will wish to modify h himself; e.g., if he wishes to produce the numerical solution at some prescribed value of x, or if he wishes to approach a running change of coordinates at a step-size smaller than that being used by the routine. The technique of modifying h is described in Appendix B.

APPENDIX B

USAGE AND CODING INFORMATION

There are essentially two entries to the subroutine. The first entry must be made once at the beginning to set up the addresses, options, parameters, etc., of the routine for integration of N simultaneous second-order, ordinary differential equations, in which first derivatives may or may not appear. The first entry utilizes the auxiliary subroutine to evaluate the second-order derivatives at the initial conditions. Thus, the initial conditions must be set up prior to the first entry. The second entry may be used any number of times after the first to integrate all y_i from x to x+h by a Cowell step; or x to x+h/R by a Runge-Kutta step.

The first entry has the following calling sequence:

Loc.	Instruction	Comments
α	TSX DE6F,4	Set-up entry.
$\alpha+1$	PO T,O,V	Option, addresses of storage and auxiliary subroutine.
$\alpha+2$	P1 B,O,R	Option and parameters.
$\alpha+3$	DEC 1E-S	S is the number of significant figures desired.
$\alpha+4$	DEC h_{min}	Minimum step-size. Floating point.
$\alpha+5$	DEC h_{max}	Maximum step-size. Floating point.
$\alpha+6$	DEC y_{min}	Minimum y_i value allowed for testing. Floating point.
$\alpha+7$	Return	

(α+1): T is the address of the first word of a block of 30N+3 cells of temporary storage arranged as follows:

Loc.	Contents	Comments
T	PZE N,0,0	N is the number of equations. Fixed point.
T+1	x	Value of independent variable. Floating point.
T+2	h	Value of step-size. Floating point.
T+3 thru T+N+2	y_i thru y_N	Values of the dependent variables y_i. Floating point.
T+N+3 thru T+2N+2	y_i' thru y_N'	Values of the first derivatives y_i'. Floating point.
T+2N+3 thru T+3N+2	y_i'' thru y_N''	Locations where the user's auxiliary subroutine must store the second derivatives y_i''. Floating point.
T+3N+3 thru	T+30N+2	27N cells of temporary storage.

The next 27N storages of T are temporary storages. The Runge-Kutta subroutine uses the first 6N cells (T+3N+2 thru T+9N+3) and the Cowell routine uses the final 21N cells (T+9N+3 thru T+30N+2). However, if a change of coordinates (see Special Usage) is made, the Cowell routine will also use the first 6N cells. The attached T Storage Chart shows the set-up of the entire T region. The N cells starting at T+5N+3 and the N cells starting at T+6N+3 are used by the Runge-Kutta subroutine to compute $\Delta' y_{iN}$ and Δy_{iN}. However, $\Delta' y_{iN}$ and Δy_{iN} are destroyed before final exit, and these cells contain intermediate values of no significance to the user. The left side of the chart shows the storage for the normal case where 6R Runge-Kutta steps are taken (using the first 9N cells of T for each integration) before an attempt is made to proceed to a Cowell step. At the beginning of each set of R steps, the Cowell subroutine saves the values of the second derivatives (7 sets starting at T+11N+3). In addition, the values of y_{i3} and y_{i3}' are saved, starting at T+9N+3 and T+10N+3 respectively, for use in the central difference equations where $''F_{i4}$ and $'F_{i4}$ are calculated. Care must be exercised in using certain values in the temporary region.

For instance, after a Runge-Kutta integration step, the most significant values of y_i and y_i', starting at T+7N+3 and T+3N+3 respectively, will be the values of the previous integration; while the least significant values of y_i and y_i', starting at T+8N+3 and T+4N+3, respectively, will be the values of the present integration. The Cowell routine also saves the least significant values of y_i and y_i' (unless a change of coordinates is in progress) in these same storages at the end of each integration. The 11N storages starting at T+19N+3 contain the right half of the N difference tables, an example of which is shown in Appendix A. The right side of the chart shows other values which are stored in the T region during a change of coordinates and will be explained later under Special Usage. Even through only one symbol is given (y_{i0}', etc.), it should be understood that N values are stored as in the left side of the chart. Thus, y_{i0}' signifies y_{10}', y_{20}', y_{30}', ... , y_{NO}'.

The address V is the entry point of an auxiliary subroutine which the user must provide to evaluate the second derivatives y_i''. This subroutine must store y_i'' in T+2N+3 through T+3N+2 as shown above. The subroutine is entered by the calling sequence:

Loc.	Instruction	Comments
α	TSX V,4	Index registers need not be saved.
α+1	Return	Return via a TRA 1,4.

The derivatives y_i'' are evaluated during the set-up and at the end of each integration step. Thus, the values of the variables and the derivatives are consistent at all times. Extra precision is recommended for the evaluation of the second derivatives y_i''.

PO should be set to PZE if the first derivatives are present in the evaluation of the second derivatives. If first derivatives are not present, PO should be set to MZE.

(α+2): P1 should be set to PZE if a variable step-size is wanted. For a fixed step-size; P1 should be set to MZE. The former allows doubling and halving while the latter restricts the routine to a fixed r. The user may change the mode of operation externally at any time by setting cell DE6F+501 to plus for a variable step-size and minus for a fixed step-size.

Only the first B ($1 \leq B \leq N$) equations are tested to determine doubling or halving of h. Thus, the user should arrange the N equations in descending order of importance, and specify B accordingly. If B = 0 in the calling sequence, it will be set to N.

R is the ratio of the Cowell step-size to the Runge-Kutta step-size. Thus, smaller integration steps can be taken in the Runge-Kutta subroutine by setting R greater than 1. If R = 0 in the calling sequence, it will be set to 4. R is saved in the decrement of cell DE6F+516 and in floating point in cell DE6F+517. After any Cowell integration step (AC= -), the user chould change R by changing both of these cells.

(α+3): 1E-S is a floating point number where S is the number of significant figures of accuracy desired at each step. The user should experiment with S to fit his own particular problem. The 1 of 1E-S may also be varied from 1 to 9 (1E-S thru 9E-S) for a final degree of control over the accuracy testing. If 1E-S=0 in the calling sequence, it will be set to 1E-9.

(α+4): h_{min} is a floating point number giving a lower bound for h. h_{min} is saved in cell DE6F+509 and can be changed at any time.

(α+5): h_{max} is a floating point number giving an upper bound for h. If h_{max}=0 in the calling sequence, the upper bound will be set to 1E18. h_{max} is saved in cell DE6F+510 and can be changed at any time.

(α+6): y_{min} is a positive floating point number which is used in testing the step-size. If y_{min}=0 in the calling sequence, it will be set to 1. y_{min} is saved in cell DE6F+511 and can be changed at any time.

If the fixed step-size mode of operation is selected (P1=MZE), then B, 1E-S, h_{min}, h_{max}, and y_{min} are all ignored by the subroutine. (If P1=MZE, set B = 1 for maximum efficiency.)

The integration entry has the following calling sequence:

Loc.	Instruction	Comments
α	TSX DE6F+1,4	Integrates all variables one step.
α+1	Return	

Upon return from the integration entry, the accumulator will be plus if the integration was a Runge-Kutta step and minus if the integration was a Cowell step. Ordinarily, x will have been advanced to x +h/R for a Runge-Kutta step and x + h for a Cowell step. However, in the variable h mode, it is possible that the value of h in T+2 prior to the integration entry has been changed to h/2 or 2h. In this case, the integration step will be a Runge-Kutta step, and the value of x will be either x + h/2R or x + 2h/R. All values of y_i, y_i', and y_i'' will be consistent with the new value of x. The user must never change the value of the step-size h except as described under Special Usage.

Special Usage (See Appendix A. Further Properties of the Program.)

1. Running Change of Coordinates. (Normal Entries.)

After any Cowell integration step (AC= -), the user may initialize the beginning of a change of coordinates by setting cell DE6F+500 to non-zero. Starting with the present values, he begins to save eight consecutive sets of y_i starting at T+11N+3 (and y_i' starting at T+3N+3 if they appear in the y_i''). He continues to use the integration entry above. The routine will detect the non-zero value stored in cell DE6F+500 and will begin a count-down in cell DE6F+502 from 8 (-1) 0. The next seven integrations will be Cowell steps and all testing will be discontinued during this period. After the eighth set of y_i (y_{i7}), and y_i' (y_{i7}') if necessary, have been stored (DE6F+502 will have a fixed point 1 in the address), the user may change the second derivative evaluation routine V and the units of x and h. The units of the eight sets of y_i and y_i' may be changed while storing each set, or after all eight sets have been stored. When another integration step is asked for, the routine will perform the change of coordinates and proceed to a Cowell integration step. Cell DE6F+500 will be restored to zero, and an 8 will be restored to the address of cell DE6F+502. Thus, the routine will be ready for another change of coordinates and will operate under standard conditions.

2. Running Start. (Special Entry.)

A running start is similar to a running change of coordinates except that the user must supply all eight sets of y_i (and y_i' if necessary) at one time. The following sequence of operations must be followed:

A. Set up the initial conditions in the T storage. Only N, x, and h are needed, although the eight sets of y_i (but not y_i') can also be stored at this time. x must correspond to y_{i7} and h must be the interval at which the y_i have been obtained. Thus, $x = x_0 + 7h$ where x_0 corresponds to y_0 (and y_0').

B. Use the first entry calling sequence to set up all parameters and options. The V subroutine will be used but will have no effect on the problem. Also, cells T+4N+3 thru T+5N+2 and T+8N+3 thru T+9N+2 will be set to zero. Thus, the eight sets of y_i' must be stored after the setup entry.

c. Store eight consecutive sets (equal intervals) of y_i starting at cell T+11N+3 (and eight consecutive sets of y_i' starting at cell T+3N+3, if needed).

D. Execute the following calling sequence one time:

Loc.	Instruction	Comments
α	TSX DE6F+16,4	Enter only once.
α+1	Return	Integrates 1 step.

E. Continue with the regular integration entry (TSX DE6F+1,4) to integrate further. Step D integrates all variables one Cowell step, and $x(x_0 + 7h)$ is advanced to $x + h(x_0 + 8h)$. From this point, the routine will operate under normal conditions.

3. Change of Step-Size in the Cowell/Runge-Kutta System. (Special Entry.)

During the integration procedure, the user may wish to output for a specific value of x without interrupting the Cowell/Runge-Kutta integration procedure. Or, he may wish to change the value of the step size h prior to a running change of coordinates. He can do this after any integration entry with the following procedure:

Loc.	Instruction	Comments
α	TSX DE6F+22,4	Changes h and starts new series of 6R Runge-Kutta steps.
α+1	Return	

Thus, the integration step will be h/R. Continue with the regular integration entry (TSX DE6F+1,4) to integrate further.

If the above procedure is being used to reduce the step-size prior to a change of coordinates, the user must prevent the routine from doubling again until after the change of coordinates. Doubling can be prevented either by storing zero in cell DE6F+510 (h_{max}), or by setting cell DE6F+501 (fixed step size) negative prior to the above entry. After the change of coordinates, the user may restore the above cells.

4. Change of Step Size for a Final Integration. (Special Entry.)

This is simply a direct transfer to the Runge-Kutta integration subroutine and should be used to end exactly at a specific value of x.

After changing the value of h in T+2,

Loc.	Instruction	Comment
α	TSX DE6F+588,4	Integrates one step with the Runge-Kutta subroutine.
$\alpha+1$	Return	

The integration step will be the value of h in T+2. This procedure could be used in the middle of the integration procedure if 3. above (TSX DE6F+22,4) is used immediately afterwards to restart in the Cowell/ Runge-Kutta system.

In addition to the user's auxiliary subroutine and the 30N+3 cells for the T storage, the storage requirements are 955 words for RWDE6F plus 44 words of COMMON.

The value of the independent variable x is accumulated in double precision when incremented by h. The least significant part of x is saved in cell DE6F+718.

APPENDIX B - T STORAGE CHART

	N	x	h					
T+3	y_1	y_2	.	.	y_N	Initial conditions which user must supply.		
T+N+3	y_1'	y_2'	.	.	y_N'			
T+2N+3	y_1''	y_2''	.	.	y_N''	2nd derivatives stored by V subroutine.		
T+3N+3	y_1'	y_2'	.	.	y_N'	Most significant	y_{10}'	y_{10}''
T+4N+3	y_1'	y_2'	.	.	y_N'	Least significant	y_{11}'	y_{11}''
T+5N+3	$\Delta y_1'$	$\Delta y_2'$	.	.	$\Delta y_N'$	Destroyed by R.K.	y_{12}'	y_{12}''
T+6N+3	$\Delta y_1'$	$\Delta y_2'$	.	.	$\Delta y_N'$	Destroyed by R.K.	y_{k3}'	y_{k3}''
T+7N+3	y_1	y_2	.	.	y_N	Most significant	y_{14}'	y_{14}''
T+8N+3	y_1	y_2	.	.	y_N	Least significant	y_{15}'	y_{15}''
T+9N+3	y_{13}	y_{23}	.	.	y_{N3}	Saved for central	y_{16}'	y_{16}''
T+10N+3	y_{13}'	y_{23}'	.	.	y_{N3}'	difference equations	y_{17}'	y_{17}''
T+11N+3	y_{10}''	y_{20}''	.	.	y_{N0}''	Normal Case.	y_{10}	
T+12N+3	y_{11}''	y_{21}''	.	.	y_{N1}''	Saved from 6R R-K	y_{11}	
T+13N+3	y_{12}''	y_{22}''	.	.	y_{N2}''	steps. Used to form	y_{12}	
T+14N+3	y_{13}''	y_{23}''	.	.	y_{N3}''	difference tables in	y_{13}	
T+15N+3	y_{14}''	y_{24}''	.	.	y_{N4}''	an attempt to proceed	y_{14}	
T+16N+3	y_{15}''	y_{25}''	.	.	y_{N5}''	to a Cowell step.	y_{15}	
T+17N+3	y_{16}''	y_{26}''	.	.	y_{N6}''		y_{16}	
T+18N+3	Used only if coordinates changed.						y_{17}	

Coordinate Change (T+3N+3 to T+10N+3): 8 sets of consecutive y_i' saved by user if $y'' = f(x,y,y')$. Cowell stores y_i'' over y_i'.

Coordinate Change (T+11N+3 to T+18N+3): 8 sets of consecutive y_i saved by user. Cowell uses (with V sub) to form y_i'' above.

T+19N+3	Δ^{VI}_{10}	Δ^{V}_{11}	Δ^{IV}_{12}	Δ^{III}_{13}	Δ^{II}_{14}	Δ^{I}_{15}	y_{16}''	$''F_{18}$	$''F_{18}$	$'F_{17}$	$'F_{17}$
T+19N+14	Δ^{VI}_{20}	Δ^{V}_{21}	Δ^{IV}_{22}	Δ^{III}_{23}	Δ^{II}_{24}	Δ^{I}_{25}	y_{26}''	$''F_{28}$	$''F_{28}$	$'F_{27}$	$'F_{27}$
T+19N+25	Δ^{VI}_{30}	Δ^{V}_{32}	Δ^{IV}_{32}	Δ^{III}_{33}	Δ^{II}_{34}	Δ^{I}_{35}	y_{36}''	$''F_{38}$ ↑	$''F_{38}$ ↑	$'F_{37}$ ↑	$'F_{37}$ ↑
.	.	.	.	.	.	.	.				
.	.	.	.	.	.	.	.	Most	Lst.	Most	Lst.
.	.	.	.	.	.	.	.	Sig. ↓	Sig. ↓	Sig. ↓	Sig. ↓
T+30N-8	Δ^{VI}_{N0}	Δ^{V}_{N1}	Δ^{IV}_{N2}	Δ^{III}_{N3}	Δ^{II}_{N4}	Δ^{I}_{N5}	y_{N6}''	$'F_{N8}$	$''F_{N8}$	$'F_{N7}$	$'F_{N7}$

11N storages. N sets of differences saved for next Cowell step. (See Appendix A)

```
*       FAP
         TTL DE6FN  COWELL INTEGRATION ROUTINE  CORRECTED 6/63
         LBL DE6FN
         COUNT   1100
         ENTRY DE6FN
         ENTRY DE6FP1
         ENTRY DE6FP2
         ENTRY DE6FP3
         ENTRY DE6FNZ
         ENTRY DE6FPS
         ENTRY DE6FNG
         ENTRY DE6FCH
         ENTRY DE6FCM
         ENTRY DE6FP4
         ENTRY DE6FMN
         ENTRY DE6FMA
         PZE 0
         PZE 0
         PZE 0
DE6FN    SXD DE6FN-1,1          SAVE 1R
         SXD DE6FN-2,2
         SXD DE6FN-3,4
         CLA 5,4
         STA DUX1
         CLA 6,4
         STA BCOM               LOC B
         CLA 7,4
         STA RCOM               LOC R
BCOM     CLA *
         STA CALL+1
         ARS 18
         STO CELM1
RCOM     CLA *
         ADD CELM1
         STO CALL+2             2ND WORD
         CLA 8,4
         STA *+1
         CLA *
         STO CALL+3             3RD WORD
         CLA 9,4
         STA *+1
         CLA *
         STO CALL+4             4TH WORD
         CLA 10,4
         STA *+1
         CLA *
         STO CALL+5             5TH WORD
         CLA 11,4
         STA *+1
         CLA *
         STO CALL+6             6TH WORD
```

```
        CLA 2,4                   SET SIGN                      DE6FN050
        STA *+1                                                 DE6FN051
        CLA *                                                   DE6FN052
        TPL *+3                   2ND WORD                      DE6FN053
        CLS CALL+2                                              DE6FN054
        STO CALL+2                                              DE6FN055
        CLA 4,4                   LOC ENDT                      DE6FN056
        STA NTOTA                                               DE6FN057
        CLA 3,4                                                 DE6FN058
        STA REVER+3                                             DE6FN059
        STA REVER+5                                             DE6FN060
        STA NTOT                                                DE6FN061
        STA COMN                                                DE6FN062
        STA COMCEL                                              DE6FN063
        STA SVTD                                                DE6FN064
NTOTA   CLA *                                                   DE6FN065
        ARS 18                                                  DE6FN066
NTOT    STO *                                                   DE6FN067
COMCEL  LDQ *                     FIND TOP T                    DE6FN068
        MPY TRTY                                                DE6FN069
        STQ CELM1                                               DE6FN070
        CLA SVTD                                                DE6FN071
        SUB CELM1                                               DE6FN072
        SUB TWO                                                 DE6FN073
        ADM CALL+1                                              DE6FN074
        STA START                 SAP T                         DE6FN075
        STO CALL+1                                              DE6FN076
        CLA 1,4                                                 DE6FN077
        STA *+1                                                 DE6FN078
        CLA *                                                   DE6FN079
        TPL *+3                   SET SIGN PO                   DE6FN080
        CLS CALL+1                                              DE6FN081
        STO CALL+1                                              DE6FN082
COMN    LDQ *                     COMPUTE                       DE6FN083
        MPY FIFTN                 MID POINT                     DE6FN084
        STQ CELM1                                               DE6FN085
        CLA CELM1                                               DE6FN086
        ADD ONE                                                 DE6FN087
        STO HN                                                  DE6FN088
        CLA START                                               DE6FN089
        ADD HN                                                  DE6FN090
        STA REVER+2                                             DE6FN091
        STA REVER+4                                             DE6FN092
        TSX REVER,4                                             DE6FN093
CALL    TSX DE6F,4                ENTER                         DE6FN094
        PZE ,,DUX                                               DE6FN095
        PZE *                                                   DE6FN096
        PZE *                                                   DE6FN097
        PZE *                                                   DE6FN098
        PZE *                                                   DE6FN099
        PZE *                                                   DE6FN100
```

```
       TSX  REVER,4                                    DE6FN1010
       LXD  DE6FN-1,1                                  DE6FN1020
       LXD  DE6FN-2,2                                  DE6FN1030
       LXD  DE6FN-3,4                                  DE6FN1040
       TRA  12,4                                       DE6FN1050
REVER  LXA  HN,1          REVERSE T BLOCK              DE6FN1060
       LXD  ZERO,2                                     DE6FN1070
IN     LDQ  *,1                                        DE6FN1080
       CLA  *,2                                        DE6FN1090
       STO  *,1                                        DE6FN1100
       STQ  *,2                                        DE6FN1110
       TXI  *+1,2,1                                    DE6FN1120
       TIX  IN,1,1                                     DE6FN1130
       TRA  1,4                                        DE6FN1140
DE6FP1 SXD  DE6FN-1,1                                  DE6FN1150
       SXD  DE6FN-2,2                                  DE6FN1160
       SXD  DE6FN-3,4                                  DE6FN1170
       TSX  REVER,4                                    DE6FN1180
       TSX  DE6F+1,4                                   DE6FN1190
       STO  CELM1                                      DE6FN1200
       TSX  REVER,4                                    DE6FN1210
       LXD  DE6FN-1,1                                  DE6FN1220
       LXD  DE6FN-2,2                                  DE6FN1230
       LXD  DE6FN-3,4                                  DE6FN1240
       CLA  1,4                                        DE6FN1250
       STA  *+2                                        DE6FN1260
       CLA  CELM1                                      DE6FN1270
       STO  *                                          DE6FN1280
       TRA  2,4                                        DE6FN1290
DE6FP2 SXD  DE6FN-1,1                                  DE6FN1300
       SXD  DE6FN-2,2                                  DE6FN1310
       SXD  DE6FN-3,4                                  DE6FN1320
       TSX  REVER,4                                    DE6FN1330
       TSX  DE6F+16,4                                  DE6FN1340
       TSX  REVER,4                                    DE6FN1350
       LXD  DE6FN-1,1                                  DE6FN1360
       LXD  DE6FN-2,2                                  DE6FN1370
       LXD  DE6FN-3,4                                  DE6FN1380
       TRA  1,4                                        DE6FN1390
DE6FP3 SXD  DE6FN-1,1                                  DE6FN1400
       SXD  DE6FN-2,2                                  DE6FN1410
       SXD  DE6FN-3,4                                  DE6FN1420
       TSX  REVER,4                                    DE6FN1430
       LXD  DE6FN-3,4                                  DE6FN1440
       CLA  1,4                                        DE6FN1450
       STA  *+1                                        DE6FN1460
       CLA  *                                          DE6FN1470
       TSX  DE6F+22,4                                  DE6FN1480
       TSX  REVER,4                                    DE6FN1490
       LXD  DE6FN-1,1                                  DE6FN1500
       LXD  DE6FN-2,2                                  DE6FN1510
```

```
        LXD DE6FN-3,4
        TRA 2,4
DE6FP4  SXD DE6FN-1,1
        SXD DE6FN-2,2
        SXD DE6FN-3,4
        TSX REVER,4
        TSX DE6F+588,4
        TSX REVER,4
        LXD DE6FN-1,1
        LXD DE6FN-2,2
        LXD DE6FN-3,4
        TRA 1,4
DE6FNZ  CLA 1,4
        STA *+1
        CLA *
        STO DE6F+500
        TRA 2,4
DE6FPS  CLA DE6F+501
        SSP
        STO DE6F+501
        TRA 1,4
DE6FNG  CLA DE6F+501
        SSM
        STO DE6F+501
        TRA 1,4
DE6FMN  CLA 1,4
        STA *+1
        CLA *
        STO DE6F+509
        TRA 2,4
DE6FMA  CLA 1,4
        STA *+1
        CLA *
        STO DE6F+510
        TRA 2,4
 DUX    SXD DUX2,4
        TSX REVER,4
 DUX1   TSX 0,4
        TSX REVER,4
        LXD DUX2,4
        TRA 1,4
 DUX2   PZE
DE6FCM  CLA 1,4
        STA *+1
        CLA *
        STO DE6F+511
        TRA 2,4
DE6FCH  CLA 1,4
        STA LINKA
        CLA 2,4
        STA LINKB
```

```
LINKA   CLA *                                                                  DE6FN2030
        STO DE6F+516                                                           DE6FN2040
LINKB   CLA *                                                                  DE6FN2050
        STO DE6F+517                                                           DE6FN2060
        TRA 3,4                                                                DE6FN2070
ZERO    PZE 0                                                                  DE6FN2080
ONE     DEC 1                                                                  DE6FN2090
TWO     DEC 2                                                                  DE6FN2100
 FIFTN  DEC 15                                                                 DE6FN2110
 HN     PZE 0                                                                  DE6FN2140
 SVTD   PZE 0                                                                  DE6FN2160
COMMON  BSS 44                                                                 DE6FN2170
DE6F    TRA DE6F+0799     COWELL METHOD INTEGRATION SUB (TR SETUP)             DE6FN2180
        SXD DE6F+0504,1             2ND ENTRY, BEGIN                           DE6FN2190
        SXD DE6F+0505,2        INTEGRATION                                     DE6FN2200
        SXD DE6F+0506,4                                                        DE6FN2210
        CLA DE6F+0500     TEST SWITCH FOR COORD. CHANGE                        DE6FN2220
        TZE DE6F+0011   0=NORMAL CASE                                          DE6FN2230
        CLA DE6F+0502      8(-1)0                                              DE6FN2240
        SUB DE6F+0531     1                                                    DE6FN2250
        STO DE6F+0502                                                          DE6FN2260
        TNZ DE6F+0011   CONTINUE IF NOT ZERO                                   DE6FN2270
        TRA DE6F+0091     TR FOR COORD. CHANGE                                 DE6FN2280
        CLA DE6F+0503     TEST R.K. OR COWELL                                  DE6FN2290
        TMI DE6F+0185    -=COWELL                                              DE6FN2300
        CLA DE6F+0709        TEST 1ST TIME FOR R.K.=+                          DE6FN2310
        TMI DE6F+0043    =2N=3RD,...,RTH R.K. INTEGR.                          DE6FN2320
        TRA DE6F+0030   TR TO BEGIN RUNGE KUTTA SERIES (R)                     DE6FN2330
        SXD DE6F+0504,1              PROGRAMMER MAY ENTER HERE FOR             DE6FN2340
        SXD DE6F+0505,2              SPECIAL COORD. CHANGE, 8 SETS             DE6FN2350
        SXD DE6F+0506,4              OF Y,S MUST BE IN T+11N+3                 DE6FN2360
        STZ DE6F+0502      AND T MUST BE AT 8TH Y.                             DE6FN2370
        SXD DE6F+0500,4             8 SETS OF Y PRIME MUST BE                  DE6FN2380
        TRA DE6F+0091     AT T+3N+3 IF F(X,Y,Y PRIME) USED.                    DE6FN2390
        SXD DE6F+0504,1        PROGRAMMER MAY ENTER HERE WITH H IN AC          DE6FN2400
        SXD DE6F+0505,2             AND START NEW RUNGE KUTTA SERIES           DE6FN2410
        SXD DE6F+0506,4             H/R WILL BE INTEGRATION STEP.              DE6FN2420
        STO COMMON+000         NEW H                                           DE6FN2430
        CLA DE6F+0500     CANNOT CHANGE H IF COORD.                            DE6FN2440
        TNZ DE6F+0006   CHANGE IS IN PROGRESS                                  DE6FN2450
        LDQ COMMON+000         NEW H                                           DE6FN2460
        TSX DE6F+0077,2             CHANGE VMIN, VMAX, WMAX FOR H2             DE6FN2470
        CLA DE6F+0519     H/R=R.K. H FOR EACH STEP                             DE6FN2480
        FDP DE6F+0517     R=TOTAL R.K. STEPS                                   DE6FN2490
        STQ 0                 (T+2)                                            DE6FN2500
        LDQ DE6F+0519     DELTA (T) = H                                        DE6FN2510
        FMP DE6F+0519                                                          DE6FN2520
        STO DE6F+0520     DELTA (T) SQUARED                                    DE6FN2530
        STZ DE6F+0518      SET K=0,(K=0(1)6)                                   DE6FN2540
        SSM                                                                    DE6FN2550
        STO DE6F+0709         FIX SWITCH FOR 2ND, ETC ENTRIES                  DE6FN2560
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LXA DE6F+0527,1              7N(FIX TO SAVE 2ND DERIV.)              DE6FN25
SXD DE6F+0527,1              7N(-N)N                                 DE6FN25
CLA DE6F+0516       R=R.K. STEPS PER DELTA(T)                        DE6FN25
STO DE6F+0515       R IN DECR.                                       DE6FN26
LXD DE6F+0515,1           R(-1)1                                     DE6FN26
CLA DE6F+0516       R=TOTAL R.K. STEPS PER D(T)                      DE6FN26
SUB DE6F+0515       R(I)=CURRENT RI=R(-1)1 IN DECR.                  DE6FN26
TNZ DE6F+0054     TR IF NOT 1ST R                                    DE6FN26
LXA DE6F+0710,4           N                                          DE6FN26
LXD DE6F+0527,2              7N(-N)N                                 DE6FN26
CLA 0,4                 T+3N+3(2ND DERIV.)                           DE6FN26
STO 0,2                 T+11N+3+(7N),(7N 2ND DERIVS)                 DE6FN26
TXI DE6F+0052,2,-1                                                   DE6FN26
TIX DE6F+0049,4,1                                                    DE6FN27
SXD DE6F+0527,2              7N(-N)N                                 DE6FN27
LXA DE6F+0518,2                                                      DE6FN27
TXH DE6F+0122,2,5            IF K=6, TR TO COWELL                    DE6FN27
TSX DE6F+0588,4         TR TO RUNGE KUTTA INTEGRATION                DE6FN27
TIX DE6F+0070,1,1            REDUCE R TIL R=RN                       DE6FN27
LXD DE6F+0516,1                  R=RN (RESTORE R FOR NEXT CYCLE)     DE6FN27
CLA DE6F+0518         K=0(1)6                                        DE6FN27
ADD DE6F+0531        1                                               DE6FN27
STO DE6F+0518                                                        DE6FN27
SUB DE6F+0533         3                                              DE6FN28
TNZ DE6F+0070       SAVE Y(3) AND Y(3) PRIME IF 0                    DE6FN28
LXA DE6F+0710,2           N                                          DE6FN28
CLA 0,2                T+N+3=Y(I)=DEPENDENT VAR. (YI)                DE6FN28
STO 0,2                T+10N+3=Y(3)                                  DE6FN28
CLA 0,2                T+2N+3=Y(I) PRIME=1ST DERIV.                  DE6FN28
STO 0,2                T+11N+3=Y(3)PRIME                             DE6FN28
TIX DE6F+0065,2,1                                                    DE6FN28
SXD DE6F+0515,1              SAVE R                                  DE6FN28
SSP                    +=RUNGE KUTTA                                 DE6FN28
STO DE6F+0503       -=COWELL METHOD                                  DE6FN29
LXD DE6F+0506,4                                                      DE6FN29
LXD DE6F+0504,1             1 R.K. STEP COMPLETED                    DE6FN29
LXD DE6F+0505,2                                                      DE6FN29
TRA 1,4                RETURN TO PROGRAMMER                          DE6FN29
CLA DE6F+0519         OLD H (SUB. FOR NEW VMIN, VMAX, WMAX)          DE6FN29
STO COMMON+001           H1=OLD H (SAVE FOR CHANGE OF HMAX)          DE6FN29
CLA DE6F+0520         OLD H SQ.=H1 SQ.                               DE6FN29
STO COMMON+000           H1 SQ.                                      DE6FN29
STQ DE6F+0519         NEW H=H2                                       DE6FN29
FMP DE6F+0519                                                        DE6FN30
STO DE6F+0520         NEW H SQ.=H2 SQ.                               DE6FN30
LXA DE6F+0533,5          3 IN IR1, IR4                               DE6FN30
CLA COMMON+000           H1 SQ.                                      DE6FN30
FDP DE6F+0520         H2 SQ.                                         DE6FN30
FMP DE6F+0515,4          CHANGE VMIN, VMAX, VMAX                     DE6FN30
STO DE6F+0515,4          (V/H1 SQ.)X(H1 SQ)/(H2 SQ.)                 DE6FN30
TIX DE6F+0085,4,1                                                    DE6FN30
```

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TRA 1,2                                                          DE6FN3080
LDQ 0                T+2=H (COORD. CHANGE BEGINS)                DE6FN3090
FMP DE6F+0544     =8.                                            DE6FN3100
FAD 0                T+1=T                                       DE6FN3110
STO 0                T+1                                         DE6FN3120
LXA DE6F+0528,1               8N                                 DE6FN3130
SXD DE6F+0528,1               8N(=N)0                            DE6FN3140
CLA 0                T+1=(T)                                     DE6FN3150
FAD 0                T+2=H                                       DE6FN3160
STO 0                T+1=(T+H)                                   DE6FN3170
LXA DE6F+0710,2          N                                       DE6FN3180
CLA 0,1               T+11N+3+(8N)=(Y0,Y1,....,Y7)               DE6FN3190
STO 0,2              T+N+3                                       DE6FN3200
CLA 0,1               T+3N+3+(8N)=1ST DERIV.                     DE6FN3210
STO 0,2               T+2N+3(IF SAVED)                           DE6FN3220
TXI DE6F+0106,1,-1        MOVE DEPENDENT VAR. TO T+3 THRU T+N+2  DE6FN3230
TIX DE6F+0101,2,1                                                DE6FN3240
TSX 0,4              V (FORM2ND DERIV.)                          DE6FN3250
LXD DE6F+0528,1               8N                                 DE6FN3260
LXA DE6F+0710,2          N                                       DE6FN3270
CLA 0,2              T+3N+3 (2ND DERIV.)                         DE6FN3280
STO 0,1               T+3N+3+(8N)                                DE6FN3290
TXI DE6F+0113,1,-1          STORE 2ND DERIV. (8 SETS)            DE6FN3300
TIX DE6F+0110,2,1                                                DE6FN3310
TXH DE6F+0096,1,0           LOOP FOR 8 SETS OF Y                 DE6FN3320
LDQ 0                 (T+2)=H=DELTA(T)                           DE6FN3330
TSX DE6F+0077,2             CHANGE VMIN, VMAX, WMAX              DE6FN3340
CLA DE6F+0519        H2=NEW H                                    DE6FN3350
FDP COMMON+001            H1=OLD H                               DE6FN3360
FMP DE6F+0512,1           CHANGE UNITS OF HMIN,HMAX,             DE6FN3370
STO DE6F+0512,1           AND XMIN                               DE6FN3380
TIX DE6F+0117,1,1                                                DE6FN3390
STZ DE6F+0521        CLEAR W, W=MAX (DELTA VII)/Y                DE6FN3400
STZ DE6F+0522        CLEAR V, V=MAX (DELTA VI)/Y                 DE6FN3410
LXA DE6F+0529,4               11N                                DE6FN3420
SXD DE6F+0529,4                                                  DE6FN3430
LXA DE6F+0710,4          N                                       DE6FN3440
LXA DE6F+0527,2               7N                                 DE6FN3450
SXD DE6F+0527,2               7N(=1)6N                           DE6FN3460
CLA 0,4                   T+N+3=Y                                DE6FN3470
STO DE6F+0711           SEE V AND W CALC.                        DE6FN3480
CLA DE6F+0502        NORMALLY=8 (NO CHANGE IN COORD.)            DE6FN3490
TNZ DE6F+0316        0=COORD. CHANGE                             DE6FN3500
TSX DE6F+0380,2               FORM DIFF.                         DE6FN3510
PZE 0                     T+3N+3+(7N)                            DE6FN3520
CLA DE6F+0520        H SQ.                                       DE6FN3530
TSX DE6F+0344,2             FORM SUM OF PRODUCTS                 DE6FN3540
PZE DE6F+0552,0,COMMON+21     C(I)XDELTAS                        DE6FN3550
CLA 0,4                   T+15N+3(Y3,COORD. CHANGE)              DE6FN3560
TSX DE6F+0482,2               ADD Y3 TO C+3,C+4                  DE6FN3570
TSX DE6F+0373,2               F4(II) IN A1 AND A2                DE6FN3580
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```
      CLS DE6F+0507
      STO DE6F+0714          -F4(II)
      CLS DE6F+0508
      STO DE6F+0715
      TSX DE6F+0380,2              FORM DIFF.
      PZE 0                  T+4N+3+(7N)
      TSX DE6F+0345,2         H SQ. STILL IN COM+2
      PZE DE6F+0552,0,COMMON+21
      CLA 0,4                T+16N+3,(Y4, COORD. CHANGE)
      TSX DE6F+0482,2              ADD Y4 TO C+3,C+4
      TSX DE6F+0373,2              F5(II) IN A1 AND A2
      CLA DE6F+0714
      STO COMMON+003         -F4(II)
      CLA DE6F+0715
      STO COMMON+004
      TSX DE6F+0491,2            F4(I)=F5(II)-F4(II)
      STO COMMON+003         F4(I)
      STQ COMMON+004
      CLA COMMON+008         Y4 (2ND DERIV.)
      TSX DE6F+0482,2              F5(I) IN COMMON+3,4
      LXA DE6F+0533,1              3
      TSX DE6F+0491,2            F5(II) OR F6(II) IF COORD. CHG.
      STO DE6F+0507
      STQ DE6F+0508
      CLA COMMON+012,1       Y(4),Y(5),Y(6) 2ND DERIV.
      TSX DE6F+0482,2              OR Y(5),Y(6),Y(7) IF COORD. CHANGE
      TSX DE6F+0491,2            FORM F5(II),F6(II),F7(II),F8(II)
      TIX DE6F+0163,1,1           OR F6(II),F7(II),F8(II),F9(II)

      STO COMMON+027         F8(II) OR F9(II)
      STQ COMMON+028
      CLA COMMON+003         F7(I) OR F8(I)
      STO COMMON+029
      CLA COMMON+004
      STO COMMON+030
      LXD DE6F+0529,1              11N(-11)11
      LXA DE6F+0541,2              11
      CLA COMMON+031,2       SAVE DIFF. AND F8(II),F7(I)
      STO 0,1                T+19N+3+(11N)
      TXI DE6F+0180,1,-1
      TIX DE6F+0177,2,1
      SXD DE6F+0529,1              SAVE FOR NEXT SET
      LXD DE6F+0527,2
      TXI DE6F+0184,2,-1
      TIX DE6F+0128,4,1           COMPLETE N EQNS.
      TSX DE6F+0414,4              TEST H (MAY TR R.K. AGAIN)
      CLA DE6F+0502         8(-1)0   IF COORD CHG. (NORMALLY=8)
      TNZ DE6F+0191       IF NOT=0, STILL IN COORD. CHG. OR NORMAL
      CLA DE6F+0538         8 RESTORE COUNTER FOR
      STO DE6F+0502         NEXT CHANGE IN COORD.
      STZ DE6F+0500         RESTORE SWCH TO NORMAL
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DE6FN (Cont'd)

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CLA DE6F+0519       H                                                DE6FN4100
STO                       T+2=(H) RESTORE AFTER R.K.                 DE6FN4110
FAD                       T+1=T=INDEPENDENT VAR.                     DE6FN4120
STO COMMON+000            T+H/2 (MOST SIG.)                          DE6FN4130
STQ COMMON+001            LST. SIG.                                  DE6FN4140
CLA COMMON+001                                                       DE6FN4150
FAD DE6F+0718             CUM. LST. SIG.                             DE6FN4160
FAD COMMON+000                                                       DE6FN4170
STQ DE6F+0718             SAVE LST. SIG. FOR NEXT INTEGRATION        DE6FN4180
STO                       T+1=(T+H)                                  DE6FN4190
LXA DE6F+0710,4              N                                       DE6FN4200
LXA DE6F+0529,1                11N                                   DE6FN4210
CLA DE6F+0530       T+19N+3+(7)                                      DE6FN4220
STA DE6F+0208                                                        DE6FN4230
STA DE6F+0218      T+19N+3+(7)                                       DE6FN4240
CLA DE6F+0520                                                        DE6FN4250
TSX DE6F+0344,2                                                      DE6FN4260
PZE 0,0,DE6F+0566         N+3+(7) STORED ABOVE (+11N)                DE6FN4270
LDQ DE6F+0520       H SQ                                             DE6FN4280
FMP 0,1                   T+19N+3+(11N+7)=F8(II)                     DE6FN4290
TSX DE6F+0473,2                                                      DE6FN4300
FAD COMMON+003                                                       DE6FN4310
STO 0,4                   T+N+3=NEW Y                                DE6FN4320
CLA DE6F+0717              TEST MISSING 1ST DERIV. IN F              DE6FN4330
TMI DE6F+0224      -=F(X,Y),+=F(X,Y,Y PRIME)                         DE6FN4340
CLA DE6F+0519       H                                                DE6FN4350
TSX DE6F+0344,2                                                      DE6FN4360
PZE 0,0,DE6F+0587         N+3+(7) (+11N)   INCR. BY 11               DE6FN4370
LDQ DE6F+0519       H                                                DE6FN4380
FMP 0,1                   T+19N+3+(11N+9)=F7(I)                      DE6FN4390
TSX DE6F+0473,2                                                      DE6FN4400
FAD COMMON+003                                                       DE6FN4410
STO 0,4                   T+2N+3=NEW Y PRIME                         DE6FN4420
CAL DE6F+0208      T+19N+3+(7)                                       DE6FN4430
ADM DE6F+0541       11                                               DE6FN4440
STA DE6F+0208                                                        DE6FN4450
STA DE6F+0218                                                        DE6FN4460
TXI DE6F+0229,1,-11            SET UP NEXT F8(II),F7(I)              DE6FN4470
TIX DE6F+0206,4,1           LOOP FOR N EQNS.                         DE6FN4480
TSX 0,4                   V (TR TO FORM 2ND DERIV.)                  DE6FN4490
LXA DE6F+0529,1                11N                                   DE6FN4500
LXA DE6F+0710,4              N                                       DE6FN4510
LXA DE6F+0541,2                11                                    DE6FN4520
CLA 0,1                   T+19N+3+(11N)                              DE6FN4530
STO COMMON+031,2          READ DIFF. TO COM+20                       DE6FN4540
TXI DE6F+0237,1,-1                                                   DE6FN4550
TIX DE6F+0234,2,1                                                    DE6FN4560
SXD DE6F+0529,1                                                      DE6FN4570
CLA 0,4                   T+3N+3=Y(7) 2ND DERIV.                     DE6FN4580
STO COMMON+039            Y(7) 2ND DERIV.                            DE6FN4590
LXA DE6F+0536,1                6                                     DE6FN4600
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DE6FN (Cont'd)

```
       LXA DE6F+0571,2         ZERO TO IR2                                    DE6FN461
       FSB COMMON+026,2        Y(6) 2ND DERIV. D5(I)...D0(VI)                 DE6FN462
       STO COMMON+038,2        D6(I),D5(II),D4(III),...D1(VI)                 DE6FN463
       TXI DE6F+0246,2,1           WE ARE GOING BACKWARDS HERE                DE6FN464
       TIX DE6F+0243,1,1                                                      DE6FN465
       CLA DE6F+0520       H SQ.                                              DE6FN466
       TSX DE6F+0344,2             B(I) X DELTAS                              DE6FN467
       PZE DE6F+0573,0,COMMON+40                                              DE6FN468
       LDQ DE6F+0520       H SQ.                                              DE6FN469
       FMP COMMON+027          F8(II)                                         DE6FN470
       TSX DE6F+0473,2                                                        DE6FN471
       FAD COMMON+003                                                         DE6FN472
       STO 0,4                 T+N+3=NEW Y                                    DE6FN473
       CLA DE6F+0500       TEST COORD. CHANGE                                 DE6FN474
       TNZ DE6F+0258      CANNOT SAVE IF COORD. CHANGE                        DE6FN475
       STQ 0,4                 T+9N+3=LST. SIG. (FOR R.K.)                    DE6FN476
       CLA DE6F+0519       H                                                  DE6FN477
       TSX DE6F+0344,2             B(I) DOT Y DELTAS                          DE6FN478
       PZE DE6F+0580,0,COMMON+40                                              DE6FN479
       LDQ DE6F+0519       H                                                  DE6FN480
       FMP COMMON+029          F7(I)                                          DE6FN481
       TSX DE6F+0473,2                                                        DE6FN482
       FAD COMMON+003                                                         DE6FN483
       STO 0,4                 T+2N+3=NEW Y PRIME                             DE6FN484
       CLA DE6F+0500                                                          DE6FN485
       TNZ DE6F+0269                                                          DE6FN486
       STQ 0,4                 T+5N+3=LST. SIG. (FOR R.K.)                    DE6FN487
       LXD DE6F+0529,1                                                        DE6FN488
       TIX DE6F+0233,4,1           COMPLETE N SETS                            DE6FN489
       TSX 0,4                 V (FORM Y(7) 2ND DERIV.                        DE6FN490
       STZ DE6F+0522       CLEAR V                                            DE6FN491
       STZ DE6F+0521       CLEAR W                                            DE6FN492
       LXA DE6F+0529,4             11N                                        DE6FN493
       LXA DE6F+0710,1           N                                            DE6FN494
       SXD DE6F+0529,4             11N(-11)11                                 DE6FN495
       LXA DE6F+0541,2             11                                         DE6FN496
       CLA 0,4                 T+19N+3+(11N)                                  DE6FN497
       STO COMMON+031,2        READ DIFF. TO COM+20                           DE6FN498
       TXI DE6F+0281,4,-1                                                     DE6FN499
       TIX DE6F+0278,2,1                                                      DE6FN500
       CLA 0,1                 T+3N+3=Y(7) 2ND DERIV.                         DE6FN501
       STO COMMON+039          Y(7) 2ND DERIV.                                DE6FN502
       CLA COMMON+029          F7(I) MOST SIG.                                DE6FN503
       STO COMMON+003                                                         DE6FN504
       CLA COMMON+030          F7(I) LST. SIG.                                DE6FN505
       STO COMMON+004                                                         DE6FN506
       CLA COMMON+039          Y(7) 2ND DERIV.                                DE6FN507
       TSX DE6F+0482,2                                                        DE6FN508
       STO COMMON+042          F8(I) MOST SIG.                                DE6FN509
       STQ COMMON+043          F8(I) LST. SIG.                                DE6FN510
       CLA COMMON+027          F8(II) MOST. SIG.                              DE6FN511
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DE6FN (Cont'd)

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 STO DE6F+0507
 CLA COMMON+028          F8(II) LST. SIG.
 STO DE6F+0508
 TSX DE6F+0491,2              DOUBLE PREC. FAD
 STO COMMON+040          F9(II) MOST. SIG.
 STQ COMMON+041          F9(II) LST. SIG.
 LXA DE6F+0536,4                6
 LXA DE6F+0571,2
 CLA COMMON+039,2        Y(7) 2ND DERIV.,D6(I),...,D1(VI)
 FSB COMMON+026,2        Y(6) 2ND DERIV.,D5(I),...,D0(VI)
 STO COMMON+038,2        D6(I),D5(II),...D1(VI)
 TXI DE6F+0305,2,1            WE ARE GOING BACKWARDS HERE
 TIX DE6F+0301,4,1
 LXD DE6F+0529,4                11N(-11)11
 LXA DE6F+0541,2                11
 CLA COMMON+044,2        SAVE FOR NEXT INTEGRATION
 STO 0,4                 T+19N+3+(11N)
 TXI DE6F+0311,4,-1
 TIX DE6F+0308,2,1
 TXH DE6F+0340,1,*            N-B IN DECR.
 TIX DE6F+0276,1,1            COMPLETE N SETS
 SSM                     --COWELL STEP
 TRA DE6F+0072       RETURN TO PROGRAMMER
 TSX DE6F+0380,2                 NORMAL CASE (NO COORD. CHANGE)
 PZE 0                   T+11N+3+(7N) (FORM DIFF.)
 CLA DE6F+0519       H
 TSX DE6F+0344,2              FORM SUM OF PRODUCTS
 PZE DE6F+0559,0,COMMON+21      D(I)XDELTAS
 CLA 0,4                 T+11N+3 (Y3 DOT)
 TSX DE6F+0482,2                ADD Y3 DOT TO C+3,C+4
 TSX DE6F+0373,2                F4(I) IN A1 AND A2
 CLA DE6F+0507
 STO DE6F+0714          F4(I)
 CLA DE6F+0508
 STO DE6F+0715
 CLA DE6F+0520       H SQ.
 TSX DE6F+0344,2              FORM SUM OF PRODUCTS
 PZE DE6F+0552,0,COMMON+21      D(I) X DELTAS
 CLA 0,4                 T+10N+3 (Y3)
 TSX DE6F+0482,2                ADD Y3 TO C+3,C+4
 TSX DE6F+0373,2                F4(II) IN A1 AND A2
 CLA DE6F+0714          F4(I)
 STO COMMON+003
 CLA DE6F+0715
 STO COMMON+004
 TXH DE6F+0453,4,*              N-B IN DECR. (TEST 1ST B EQNS)
 TRA DE6F+0161
 CLA 0,1                T+N+3=Y
 STO DE6F+0711          Y
 TSX DE6F+0455,2            FORM MAX V, W
 TRA DE6F+0313     FOR THIS Y
```

DE6FN (Cont'd)

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        STO COMMON+002          H SQ. OR H IN AC                                DE6FN563
        SXD DE6F+0707,1               SUB. TO FORM (CONSTANTS X DIFF.)          DE6FN564
        SXD DE6F+0708,2                                                         DE6FN565
        CAL 1,2                 A,0,B (A=CONSTANTS, B=DIFF.)                    DE6FN566
        STA DE6F+0356       CONSTANT ADDR. +7                                   DE6FN567
        ARS 18                                                                  DE6FN568
        STA DE6F+0357       DIFF. ADDR. +7                                      DE6FN569
        CLA COMMON+008          Y3 2ND DERIV.                                   DE6FN570
        STO COMMON+014          (OR Y4 IF COORD. CHANGE)                        DE6FN571
        STZ COMMON+003                                                          DE6FN572
        STZ COMMON+004                                                          DE6FN573
        LXA DE6F+0537,1               7                                         DE6FN574
        LDQ 0,1                 CO+7, OR DO+7, OR N6+7, ETC.                    DE6FN575
        FMP 0,1                 COMMON+21, OR COM+27, ETC.                      DE6FN576
        TSX DE6F+0473,2               ACCUM. PRODUCT IN C+3,C+4                 DE6FN577
        TIX DE6F+0356,1,1                                                       DE6FN578
        LDQ COMMON+002          H SQ. OR H                                      DE6FN579
        FMP COMMON+003          MOST SIG.                                       DE6FN580
        STO COMMON+003                                                          DE6FN581
        STQ COMMON+000                                                          DE6FN582
        LDQ COMMON+002          H SQ. OR H                                      DE6FN583
        FMP COMMON+004                                                          DE6FN584
        FAD COMMON+000                                                          DE6FN585
        FAD COMMON+003                                                          DE6FN586
        STO COMMON+003          MOST SIG. IN COMMON+3                           DE6FN587
        STQ COMMON+004          LST. SIG. IN COMMON+4                           DE6FN588
        LXD DE6F+0707,1                                                         DE6FN589
        LXD DE6F+0708,2                                                         DE6FN590
        TRA 2,2                                                                 DE6FN591
        CLA COMMON+003          DIVIDES C+3,C+4 BY C+2                          DE6FN592
        FDP COMMON+002          H SQ OR H                                       DE6FN593
        STQ DE6F+0507        MOST SIG. IN A1                                    DE6FN594
        FAD COMMON+004          REMAINDER + LST. SIG.                           DE6FN595
        FDP COMMON+002          H SQ. OR H                                      DE6FN596
        STQ DE6F+0508        LST. SIG. IN A2                                    DE6FN597
        TRA 1,2                                                                 DE6FN598
        SXD DE6F+0716,2             MOVE 2ND DERIV. AND FORM DIFF.              DE6FN599
        CAL 1,2                 STARTING ADDR. OF 2ND DERIV.                    DE6FN600
        STA DE6F+0385                                                           DE6FN601
        LXD DE6F+0527,2               7N(-1)6N                                  DE6FN602
        LXA DE6F+0571,1                                                         DE6FN603
        CLA 0,2                 START ADDR. (T+11N+3+(7N), ETC.)                DE6FN604
        STO COMMON+005,1        MOVE 2ND DERIV. TO COM+5                        DE6FN605
        STO COMMON+020,1        AND COM+20                                      DE6FN606
        TXI DE6F+0389,1,-1                                                      DE6FN607
        TIX DE6F+0385,2,*             N IN DECR.                                DE6FN608
        LXA DE6F+0536,2               6 (FORM DIFFERENCES)                      DE6FN609
        SXD COMMON+000,2                                                        DE6FN610
        LXA DE6F+0571,1         ZERO (2ND DERIV. STORED AT                      DE6FN611
        CLA COMMON+021,1       AT COMMON+5, AGAIN AT COM +20                    DE6FN612
        FSB COMMON+020,1       FORM DIFFERENCES IN                              DE6FN613
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DE6FN (Cont'd)

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 STO COMMON+020,1       COM+20 THRU COM+25              DE6FN6140
 TXI DE6F+0397,1,-1                                     DE6FN6150
 TIX DE6F+0393,2,1           LOOP 6,5,....,1 TIMES      DE6FN6160
 LXD COMMON+000,2                                       DE6FN6170
 TXH DE6F+0405,2,5           TR IF 1ST DIFF.            DE6FN6180
 TXH DE6F+0408,2,3           TR IF 2ND OR 3RD DIFF.     DE6FN6190
 TXH DE6F+0411,2,1           TR IF 4TH OR 5TH DIFF.     DE6FN6200
 TIX DE6F+0391,2,1                                      DE6FN6210
 LXD DE6F+0716,2                                        DE6FN6220
 TRA 2,2                                                DE6FN6230
 CLA COMMON+023         DELTA (I)3                      DE6FN6240
 STO COMMON+015         IN COM+15                       DE6FN6250
 TRA DE6F+0402                                          DE6FN6260
 CLA COMMON+022         DELTA (II)2 OR DELTA (III)2     DE6FN6270
 STO COMMON+021,2       IN COM+16 OR COM+17             DE6FN6280
 TRA DE6F+0402                                          DE6FN6290
 CLA COMMON+021         DELTA(IV)1 OR DELTA(V)1         DE6FN6300
 STO COMMON+021,2       IN COM+18 OR COM+19             DE6FN6310
 TRA DE6F+0402                                          DE6FN6320
 CLA DE6F+0500      SWITCH (0=NORMAL CASE)              DE6FN6330
 TNZ 1,4              STOP TEST IF NON/ZERO             DE6FN6340
 CLA DE6F+0501       TEST FIXED H                       DE6FN6350
 TMI 1,4                 ==FIXED H INTERGRATION         DE6FN6360
 CLA DE6F+0519      H=DELTA(T) COWELL TEST              DE6FN6370
 FDP DE6F+0719       2.                                 DE6FN6380
 STQ COMMON+000         H/2                             DE6FN6390
 CLA DE6F+0523      002.0 (WILL MULT BY 2 SQUARED       DE6FN6400
 STO COMMON+001         WILL MULT VMAX,VMIN, AND WMAX   DE6FN6410
 CLA DE6F+0509      MINIMUM H ALLOWED (CAN=0)           DE6FN6420
 LRS 0                  SET SIGNS PLUS                  DE6FN6430
 TLQ DE6F+0438      H/2 LESS THAN H MIN.                DE6FN6440
 CLA DE6F+0513      10 TO (3-S)/(H SQUARED) =MAX V      DE6FN6450
 LDQ DE6F+0522      V=MAX (DELTA VI)/Y                  DE6FN6460
 TLQ DE6F+0438      V LESS THAN MAX. V                  DE6FN6470
 CLA COMMON+000         H/2 OR 2H                       DE6FN6480
 STO DE6F+0519      SET H=H/2 (OR 2H) AND               DE6FN6490
 LXA DE6F+0533,4              3 (SCALE VMIN, VMAX, AND WMAX)   DE6FN6500
 CLA DE6F+0515,4            VMIN, VMAX, AND WMAX        DE6FN6510
 TZE DE6F+0436      SKIP IF ZERO                        DE6FN6520
 ADD COMMON+001         + OR - 002.0                    DE6FN6530
 STO DE6F+0515,4            NEW VALUES DUE TO CHANGE IN H   DE6FN6540
 TIX DE6F+0432,4,1                                      DE6FN6550
 TRA DE6F+0030    RETURN TO RUNGE KUTTA                 DE6FN6560
 CLA DE6F+0519      H=DELTA(T)                          DE6FN6570
 FAD DE6F+0519                                          DE6FN6580
 STO COMMON+000         2H                              DE6FN6590
 SSP                                                    DE6FN6600
 LDQ DE6F+0510      MAXIMUM H ALLOWED                   DE6FN6610
 TLQ DE6F+0452      2H GREATER THAN H MAX.              DE6FN6620
 LDQ DE6F+0512      .10 TO (-1-S)/(H SQUARED)= VMIN     DE6FN6630
 CLA DE6F+0522      V=MAX(DELTA VII)/Y                  DE6FN6640
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DE6FN- (Cont'd)

```
TLQ DE6F+0452     V GR. THAN V MIN
CLS DE6F+0523     002.0 DIVIDES BY 2 SQUARED
STO COMMON+001        DECREASES VMIN, WMAX, VMAX
CLA DE6F+0514    .10 TO (-1-S)/(H SQUARED)= HMAX
LDQ DE6F+0521     W=MAX (DELTA VII)/Y
TLQ DE6F+0429     W LESS THAN W MAX. (SET H=2H)
TRA 1,4               RETURN TO PROGRAM
TSX DE6F+0455,2           FORM LARGEST V AND W
TRA DE6F+0339
CLA DE6F+0501      SUB TO FORM MAX V AND W FOR 1ST N-B EQNS.
TMI 1,2                --FIXED H INTEGRATION
CLA DE6F+0711        Y
SSP                   SET Y PLUS
LDQ DE6F+0511     A=MIN. Y ALLOWED
TLQ DE6F+0462      TR IF Y GREATER THAN Y MIN.
STQ DE6F+0711        OTHERWISE, SET Y=Y MIN.=A
SXD COMMON+002,1
LXA DE6F+0532,1                :2
CLA COMMON+022,1        DELTA VI (OR DELTA V)
FDP DE6F+0711.       Y (OR Y MIN)
CLA DE6F+0523,1              MAX, W(OR V) THIS FAR
LRS 0                 SET SIGNS PLUS
TLQ DE6F+0470     NEW W (OR V) IS SMALLER
STQ DE6F+0523,1              NEW MAX. W (OR V)
TIX DE6F+0464,1,1          LOOP FOR V
LXD COMMON+002,1
TRA 1,2
STQ COMMON+000        LST SIG. (PREC. AND 1/2)
FAD COMMON+003        MOST SIG.
STO COMMON+003
STQ COMMON+001
CLA COMMON+000
FAD COMMON+001
FAD COMMON+004
STO COMMON+004        LST. SIG.
TRA 1,2
UFA COMMON+003         ADDS AC TO COM+3, COM+4
STO COMMON+003         AND STORES ANSWER IN C+3,C+4
STQ COMMON+000
CLA COMMON+000
UFA COMMON+004
FAD COMMON+003
STO COMMON+003         MOST SIG. IN COMMON+3
STQ COMMON+004         LST. SIG. IN COMMON+4
TRA 1,2
CLA DE6F+0507     DOUBLE PRECISION FAD
UFA COMMON+003
STO COMMON+001
STQ COMMON+000
CLA COMMON+000
UFA DE6F+0508
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DE6FN (Cont'd)

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UFA COMMON+004                                                            DE6FN7160
FAD COMMON+001                                                            DE6FN7170
TRA 1,2                                                                   DE6FN7180
BSS 1                  SWITCH (PROGRAMMER CHANGES) (0=NORMAL)             DE6FN7190
BSS 1                  FIXED H STORAGE (-=FIXED STEP SIZE)                DE6FN7200
BSS 1                  COUNTER FOR CHANGE OF COORD.                       DE6FN7210
BSS 1                  +=R.K. STEP,-=COWELL STEP                          DE6FN7220
BSS 1                  SAVE INDEX REG. FOR (IR1)                          DE6FN7230
BSS 1                  COWELL METHOD       (IR2)                          DE6FN7240
BSS 1                                      (IR4)                          DE6FN7250
BSS 1                  1ST NO.(MOST SIG.)                                 DE6FN7260
BSS 1                  1ST NO. (LST. SIG.)                                DE6FN7270
BSS 1                  MIN. H (ABS. VALUE) PROG. SPEC.                    DE6FN7280
BSS 1                  HMAX (IF H=0, SET HMAX= 1E18                       DE6FN7290
BSS 1                  A=MIN. Y ALLOWED FOR TEST                          DE6FN7300
BSS 1                  10 TO (-1-S)/(H SQUARED)=VMIN                      DE6FN7310
BSS 1                  10 TO (3-S)/(H SQUARED)=VMAX                       DE6FN7320
BSS 1                  10 TO (-1-S)/(H SQUARED)=WMAX                      DE6FN7330
BSS 1                  R(I)=CURRENT RI=R(-1)1 IN DECR.                    DE6FN7340
BSS 1                  R=TOTAL ITERATIONS PER D(T)                        DE6FN7350
BSS 1                  R (FLOATING PT.) (ABOVE=FIXED PT.)                 DE6FN7360
BSS 1                  0(1)8 (COUNTER FOR R.K. LOOPS)                     DE6FN7370
BSS 1                  H=DELTA(T)=CURRENT D(T)                            DE6FN7380
BSS 1                  DELTA (T) SQUARED                                  DE6FN7390
BSS 1                  W=MAX (DELTA VII)/Y                                DE6FN7400
BSS 1                  V=MAX(DELTA VI)/Y                                  DE6FN7410
OCT 002000000000       MULT. BY 2 SQUARED (OR DIVIDE)                     DE6FN7420
OCT 233000000000       NO TO FLOAT FIXED PT NO.                           DE6FN7430
DEC 1E18               IF H=0, SET HMAX = 1E18                            DE6FN7440
DEC 0                  2N                                                 DE6FN7450
DEC 0                  7N                                                 DE6FN7460
DEC 0                  8N                                                 DE6FN7470
DEC 0                  11N                                                DE6FN7480
DEC 0                  DUMMY=T+19N+3+(7)                                  DE6FN7490
DEC 1                  1                                                  DE6FN7500
DEC 2                  2                                                  DE6FN7510
DEC 3                  3                                                  DE6FN7520
DEC 4                  4                                                  DE6FN7530
DEC 5                  5                                                  DE6FN7540
DEC 6                  6                                                  DE6FN7550
DEC 7                  7                                                  DE6FN7560
DEC 8                  8                                                  DE6FN7570
DEC 9                  9                                                  DE6FN7580
DEC 10                 10                                                 DE6FN7590
DEC 11                 11                                                 DE6FN7600
DEC 13                 13                                                 DE6FN7610
DEC 1.                                                                    DE6FN7620
DEC -8.                -8.                                                DE6FN7630
DEC -8.333333333E-2    (SIGNS HAVE BEEN REVERSED)  (C0)                   DE6FN7640
DEC 0                  C1=0                                               DE6FN7650
DEC +4.166666667E-3    C2                                                 DE6FN7660
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DE6FN (Cont'd)

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DEC 0                   C3=0                                        DE6FN76
DEC -5,125661376E-4     C4                                          DE6FN76
DEC 0                   C5=0                                        DE6FN76
DEC +7,964065256E-5     C6                                          DE6FN77
DEC +0,5               (SIGNS HAVE BEEN REVERSED)  (D0)             DE6FN77
DEC +8,333333333E-2     D1                                          DE6FN77
DEC -4,166666667E-2     D2                                          DE6FN77
DEC -1,527777778E-2     D3                                          DE6FN77
DEC +7,638888889E-3     D4                                          DE6FN77
DEC +3,1580688E-3       D5                                          DE6FN77
DEC -1,5790344E-3       D6                                          DE6FN77
DEC +6,549575617E-2    (NO CHANGE IN SIGNS)    (N6)                 DE6FN77
DEC +6,820436508E-2     N5                                          DE6FN77
DEC +7,134589948E-2     N4                                          DE6FN78
DEC +0,075              N3                                          DE6FN78
DEC +7,916666667E-2     N2                                          DE6FN78
DEC +8,333333333E-2     N1                                          DE6FN78
DEC +8,333333333E-2     N0                                          DE6FN78
DEC -2,708608907E-3     B6                                          DE6FN78
DEC -3,141534392E-3     B5                                          DE6FN78
DEC -3,654100529E-3     B4                                          DE6FN78
DEC -4,166666667E-3     B3                                          DE6FN78
DEC -4,166666667E-3     B2                                          DE6FN78
DEC 0                   B1=0                                        DE6FN79
DEC +8,333333333E-2     B0                                          DE6FN79
DEC -1,136739418E-2     B6 DOT                                      DE6FN79
DEC -1,426917989E-2     B5 DOT                                      DE6FN79
DEC -1,875E-2           B4 DOT                                      DE6FN79
DEC -2,638888889E-2     B3 DOT                                      DE6FN79
DEC -4,166666667E-2     B2 DOT                                      DE6FN79
DEC -8,333333333E-2     B1 DOT                                      DE6FN79
DEC +0,5                B0 DOT                                      DE6FN79
DEC ,304224537          (NO CHANGE IN SIGNS)  (N6 DOT)              DE6FN79
DEC ,315591931          N5 DOT                                      DE6FN80
DEC ,329861111          N4 DOT                                      DE6FN80
DEC ,348611111          N3 DOT                                      DE6FN80
DEC ,375000000          N2 DOT                                      DE6FN80
DEC ,416666666          N1 DOT                                      DE6FN80
DEC ,500000000          N0 DOT                                      DE6FN80
TRA DE6F+0725      TO SET UP ROUTINE  (RUNGE KUTTA SUB)             DE6FN80
SXD DE6F+0707,1            SAVE INDEX REG.                          DE6FN80
SXD DE6F+0708,2          FROM                                       DE6FN80
SXD DE6F+0709,4          MAIN,PROGRAM (NEW R.K. STARTS)             DE6FN80
CLA 0                 (T+2)=H                                       DE6FN81
STO DE6F+0713          H                                            DE6FN81
FDP DE6F+0719      2.                                               DE6FN81
STQ DE6F+0714        H/2                                            DE6FN81
STQ DE6F+0715        H/2.                                           DE6FN81
FMP DE6F+0713        H                                              DE6FN81
STO DE6F+0711        (H SQ)/2                                       DE6FN81
FDP DE6F+0720      4.                                               DE6FN81
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DE6FN (Cont'd)

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      STQ DE6F+0712        (H SQ)/8                                    DE6FN8180
      CLA DE6F+0713        H                                           DE6FN8190
      FDP DE6F+0722      6.                                            DE6FN8200
      STQ DE6F+0716        H/6                                         DE6FN8210
      LXA DE6F+0710,1           N                                      DE6FN8220
      CLA 0,1                SAVE YI (T+N+3)=YI (M.S.)                 DE6FN8230
      STO 0,1                (T+8N+3)=YI MOST SIG.                     DE6FN8240
      CLA 0,1                SAVE YI PRIME (T+2N+3)                    DE6FN8250
      STO 0,1                (T+4N+3)=YI PRIME MOST SIG.               DE6FN8260
      TIX DE6F+0604,1,1                                                DE6FN8270
      LXA DE6F+0723,1              1                                   DE6FN8280
      SXD DE6F+0724,1              K=1(+1)4                            DE6FN8290
      LXA DE6F+0710,6           N TO IR2 AND IR4                       DE6FN8300
      TXL DE6F+0629,1,1            TR IF K=1                           DE6FN8310
      TXL DE6F+0649,1,2            TR IF K=2                           DE6FN8320
      TXL DE6F+0629,1,3            TR IF K=3                           DE6FN8330
      LXA DE6F+0710,2           K=4 (N TO IR4)                         DE6FN8340
      CLA 0,2                T+3N+3=F(X,Y,Y PRIME)                     DE6FN8350
      FAD 0,2                T+6N+3=CUM. D(YN) PRIME                   DE6FN8360
      STO 0,2                T+6N+3=NEW D(YN) PRIME                    DE6FN8370
      TIX DE6F+0616,2,1                                                DE6FN8380
      TXH DE6F+0669,1,3         OUT IF K=4                             DE6FN8390
      TIX DE6F+0615,1,1            ADD 2F(X,Y,PRIME) IF K=2,3          DE6FN8400
      LXD DE6F+0724,1              K                                   DE6FN8410
      TXH DE6F+0626,1,2            TR IF K=3                           DE6FN8420
      CLA DE6F+0717         TEST MISSING 1ST DERIV. IN F               DE6FN8430
      TMI DE6F+0628        *=F(X,Y,Y PRIME)                            DE6FN8440
      TSX 0,4                 V (TR OUT FOR F(X,Y,Y PRIME)             DE6FN8450
      LXD DE6F+0724,1                                                  DE6FN8460
      TXI DE6F+0610,1,1           INCREASE K BY 1                      DE6FN8470
      CLA 0                   T+1=(T)                                  DE6FN8480
      FAD DE6F+0714        H/2                                         DE6FN8490
      STO COMMON+000            T+H/2 (MOST SIG.)                      DE6FN8500
      STQ COMMON+001            LST. SIG.                              DE6FN8510
      CLA COMMON+001                                                   DE6FN8520
      FAD DE6F+0718             CUM. LST. SIG.                         DE6FN8530
      FAD COMMON+000                                                   DE6FN8540
      STQ DE6F+0718             SAVE LST. SIG. FOR NEXT INTEGRATION    DE6FN8550
      STO 0                   T+1=(T+H/2)                              DE6FN8560
      TIX DE6F+0639,1,1           IR1=1 IF K=1, IR1=2 IF K=3           DE6FN8570
      LDQ DE6F+0713,1          (H SQ)/8 OR (H SQ)/2 (K=1,3)            DE6FN8580
      FMP 0,2                 T+3N+3=F(X,Y,Y PRIME)                    DE6FN8590
      STO COMMON+000                                                   DE6FN8600
      LDQ DE6F+0715,1           (H/2) OR (H) K=1 OR 3                  DE6FN8610
      FMP 0,2                 T+4N+3=YN PRIME (M.S.)                   DE6FN8620
      FAD COMMON+000          (H SQ)/8 X F(X,Y,YP)                     DE6FN8630
      FAD 0,2                 T+8N+3=YN (M.S.)                         DE6FN8640
      STO 0,2                 T+N+3=YN AT (T+H/2) OR (T+H)             DE6FN8650
      TIX DE6F+0639,2,1           N WAS IN IR2                         DE6FN8660
      LXD DE6F+0724,1              RESTORE K TO 1 OR 3                 DE6FN8670
      CLA DE6F+0717         TEST MISSING 1ST DERIV. IN F               DE6FN8680
```

DE6FN (Cont'd)

```
TMI DE6F+0656       *=F(X,Y), +=F(X,Y,Y PRIME)                DE6FN86
LDQ DE6F+0716,1            H/2 OR H                           DE6FN87
FMP 0,4                 T+3N+3=F(X,Y,Y PRIME)                 DE6FN87
FAD 0,4                 T+4N+3=YN PRIME (M.S.)                DE6FN87
STO 0,4                 T+2N+3=NEW Y PRIME                    DE6FN87
TIX DE6F+0651,4,1                                             DE6FN87
LXA DE6F+0710,2            N                                  DE6FN87
TXL DE6F+0664,1,1            TR TO STORE IF K=1               DE6FN87
CLA 0,2                 T+3N+3=F(X,Y,Y PRIME)                 DE6FN87
FAD 0,2                 T+7N+3=CUM.DELTA(YN)                  DE6FN87
STO 0,2                 T+7N+3                                DE6FN87
TIX DE6F+0658,2,1                                             DE6FN88
TXL DE6F+0615,1,2                  TR IF K=2 (CONNOT=1)       DE6FN88
TXI DE6F+0615,1,-1                 REDUCE K FROM 3 TO 2       DE6FN88
CLA 0,2                 T+3N+3=F(X,Y,Y PRIME)                 DE6FN88
STO 0,2                 T+6N+3=STORE FOR D(YN)PRIME           DE6FN88
STO 0,2                 T+7N+3=STORE FOR D(YN)                DE6FN88
TIX DE6F+0664,2,1            N WAS IN IR2                     DE6FN88
TRA DE6F+0626       TR FOR K=2                                DE6FN88
LXA DE6F+0717,4           2N                                  DE6FN88
LXA DE6F+0710,2            N                                  DE6FN88
LDQ DE6F+0716          H/6                                    DE6FN89
FMP 0,4                 T+7N+3=D(YN)P AND D(YN)               DE6FN89
STQ COMMON+002                                                DE6FN89
UFA 0,2                 T+4N+3=YN PRIME (M.S.)                DE6FN89
STO COMMON+000                                                DE6FN89
STQ COMMON+001                                                DE6FN89
CLA COMMON+001                                                DE6FN89
UFA 0,2                 T+5N+3=YN PRIME (LST. SIG.)           DE6FN89
UFA COMMON+002                                                DE6FN89
FAD COMMON+000                                                DE6FN89
STO 0,4                 T+3N+3=(YN)P AND (YN) (M.S.)          DE6FN90
STQ 0,4                 T+7N+3=(YN)P AND (YN) (L.S.)          DE6FN90
TNX DE6F+0686,4,1          (YN)  DESTROYS F(X,Y,Y PRIME)      DE6FN90
TIX DE6F+0671,2,1         (YN)P AT T+N+3 (COMPLETED)          DE6FN90
TRA DE6F+0670       (YN)  AT T+2N+3 (NOT COMPLETED)           DE6FN90
LXA DE6F+0710,2            N                                  DE6FN90
LDQ DE6F+0713          H                                      DE6FN90
FMP 0,2                 T+3N+3=(YN PRIME + H/6(DY)            DE6FN90
STQ COMMON+002                                                DE6FN90
UFA 0,2                 T+8N+3=YN (M.S.)                      DE6FN90
STO COMMON+000                                                DE6FN91
STQ COMMON+001                                                DE6FN91
CLA COMMON+001                                                DE6FN91
UFA 0,2                 T+9N+3=YN (L.S.)                      DE6FN91
UFA COMMON+002                                                DE6FN91
FAD COMMON+000                                                DE6FN91
STO 0,2                 T+N+3=Y(N+1) (M.S.)                   DE6FN91
STQ 0,2                 T+9N+3=Y(N+1) (L.S.)                  DE6FN91
CLA 0,2                 T+6N+3=Y(N+1)P (L.S.)                 DE6FN91
STO 0,2                 T+5N+3 (SAVE LST. SIG.)               DE6FN91
```

```
TIX DE6F+0687,2,1                                                  DE6FN9200
TSX 0,4                  V (FORM 2ND DERIV.)                       DE6FN9210
LXD DE6F+0709,4                                                    DE6FN9220
LXD DE6F+0707,1                                                    DE6FN9230
LXD DE6F+0708,2                                                    DE6FN9240
TRA 1,4                 LEAVE SUB. FROM 2ND ENTRY                  DE6FN9250
PZE 2,0,0               IR1 SAVED IN DECR.                         DE6FN9260
BSS 11                                                             DE6FN9270
DEC 2.                  2.                                         DE6FN9280
DEC 4.                  4.                                         DE6FN9290
DEC 8.                  8.                                         DE6FN9300
DEC 6.                  6.                                         DE6FN9310
DEC 1                   1                                          DE6FN9320
BSS 1                   K=1(+1)4                                   DE6FN9330
SXD DE6F+0707,1              R.K. SET UP ENTRY                     DE6FN9340
SXD DE6F+0708,2                                                    DE6FN9350
SXD DE6F+0709,4                                                    DE6FN9360
STZ DE6F+0718            CLEAR LST. SIG. OF T                      DE6FN9370
CLA 1,4                 T,0,V                                      DE6FN9380
STA DE6F+0708       T=ADDRESS                                      DE6FN9390
STO DE6F+0717          -=F(X,Y), +=F(X,Y,Y PRIME)                  DE6FN9400
STA DE6F+0737                                                      DE6FN9410
ARS 18                   V TO ADDR.                                DE6FN9420
STA DE6F+0796       V                                              DE6FN9430
STA DE6F+0702        V                                             DE6FN9440
STA DE6F+0626                                                      DE6FN9450
CLA                     (T)=N                                      DE6FN9460
STO DE6F+0710         N                                            DE6FN9470
CAL DE6F+0708       T                                              DE6FN9480
ADD DE6F+0723       1                                              DE6FN9490
                                                                   DE6FN9500
STA DE6F+0629       T+1                                            DE6FN9510
STA DE6F+0637                                                      DE6FN9520
ADD DE6F+0723       1                                              DE6FN9530
STA DE6F+0591       T+2                                            DE6FN9540
ADD DE6F+0723       1                                              DE6FN9550
ADM DE6F+0710         N                                            DE6FN9560
STA DE6F+0604       T+N+3=YI (STARTS AT T+3)                       DE6FN9570
STA DE6F+0697       T+N+3=YI                                       DE6FN9580
STA DE6F+0646       T+N+3                                          DE6FN9590
ADM DE6F+0710         N                                            DE6FN9600
STA DE6F+0606       T+2N+3=YI PRIME (STARTS T+N+3)                 DE6FN9610
STA DE6F+0654                                                      DE6FN9620
ADM DE6F+0710         N                                            DE6FN9630
STA DE6F+0616       T+3N+3=2ND DERIV.                              DE6FN9640
STA DE6F+0681                                                      DE6FN9650
STA DE6F+0640       T+3N+3                                         DE6FN9660
STA DE6F+0652       T+3N+3                                         DE6FN9670
STA DE6F+0658       T+3N+3                                         DE6FN9680
STA DE6F+0664                                                      DE6FN9690
STA DE6F+0688       T+3N+3                                         DE6FN9700
```

DE6FN (Cont'd)

```
 ADM DE6F+0710         N                                          DE6FN9710
 STA DE6F+0653       T+4N+3                                       DE6FN9720
                                                                  DE6FN9730
 STA DE6F+0607                                                    DE6FN9740
 STA DE6F+0643       T+4N+3                                       DE6FN9750
 STA DE6F+0674       T+4N+3                                       DE6FN9760
 ADM DE6F+0710         N                                          DE6FN9770
 STA DE6F+0793       T+5N+3                                       DE6FN9780
 STA DE6F+0700                                                    DE6FN9790
 STA DE6F+0678                                                    DE6FN9800
 ADM DE6F+0710         N                                          DE6FN9810
 STA DE6F+0617       T+6N+3                                       DE6FN9820
 STA DE6F+0618                                                    DE6FN9830
 STA DE6F+0665                                                    DE6FN9840
 STA DE6F+0699       T+6N+3                                       DE6FN9850
 ADM DE6F+0710         N                                          DE6FN9860
 STA DE6F+0660       T+7N+3                                       DE6FN9870
 STA DE6F+0672                                                    DE6FN9880
 STA DE6F+0659                                                    DE6FN9890
 STA DE6F+0666                                                    DE6FN9900
 STA DE6F+0682       T+7N+3                                       DE6FN9910
 ADM DE6F+0710         N                                          DE6FN9920
 STA DE6F+0605       T+8N+3                                       DE6FN9930
 STA DE6F+0645                                                    DE6FN9940
 STA DE6F+0690                                                    DE6FN9950
 ADM DE6F+0710         N                                          DE6FN9960
 STA DE6F+0694       T+9N+3                                       DE6FN9970
 STA DE6F+0698       T+9N+3                                       DE6FN9980
 STA DE6F+0794                                                    DE6FN9990
```

DE6FN (Cont'd)

```
CAL  DE6F+0710          N                                          DE6FN10000
PAX  0,1                  N TO IR1                                 DE6FN10010
ADM  DE6F+0710          N                                          DE6FN10020
STA  DE6F+0717          2N TO ADDR. OF SAVE+10                     DE6FN10030
STZ  0,1                  T+5N+3 (CLEAR Y PRIME L.S.)              DE6FN10040
STZ  0,1                  T+9N+3 (CLEAR Y LST. SIG.)               DE6FN10050
TIX  DE6F+0793,1,1                                                 DE6FN10060
TSX  0,4                   V (FORM 2ND DERIV.)                     DE6FN10070
LXD  DE6F+0709,4              RESTORE IRC                          DE6FN10080
TXI  DE6F+0704,4,-1            TRA 2,4 FROM HERE                   DE6FN10090
SXD  DE6F+0504,1           COWELL SETUP                            DE6FN10100
SXD  DE6F+0505,2                                                   DE6FN10110
SXD  DE6F+0506,4                                                   DE6FN10120
CLA  1,4                  T,0,V                                    DE6FN10130
STO  DE6F+0805          FIX RUNGE KUTTA (1ST ENTRY)                DE6FN10140
TSX  DE6F+0587,4          TR TO SET UP RUNGE KUTTA                 DE6FN10150
PZE                       T,0,V                                    DE6FN10160
LXD  DE6F+0506,4                RESTORE IR4 AFTER R.K.             DE6FN10170
CAL  DE6F+0702         V                                           DE6FN10180
STA  DE6F+0107       V                                             DE6FN10190
STA  DE6F+0230        V                                            DE6FN10200
STA  DE6F+0271       V                                             DE6FN10210
CAL  DE6F+0717          2N                                         DE6FN10220
STA  DE6F+0526                                                     DE6FN10230
ADM  DE6F+0645         T+8N+3                                      DE6FN10240
STA  DE6F+0134        T+10N+3=T+3N+3+( 7N)                         DE6FN10250
STA  DE6F+0066                                                     DE6FN10260
STA  DE6F+0331                                                     DE6FN10270
ADM  DE6F+0710          N                                          DE6FN10280
STA  DE6F+0103        T+11N+3=T+3N+3+(8N)                          DE6FN10290
STA  DE6F+0111        T+3N+3+(8N)                                  DE6FN10300
STA  DE6F+0146        T+4N+3+(7N)                                  DE6FN10310
STA  DE6F+0068        T+11N+3                                      DE6FN10320
STA  DE6F+0321                                                     DE6FN10330
CAL  DE6F+0794         T+9N+3                                      DE6FN10340
STA  DE6F+0257                                                     DE6FN10350
CAL  DE6F+0700         T+5N+3                                      DE6FN10360
STA  DE6F+0268                                                     DE6FN10370
CAL  DE6F+0681         T+3N+3                                      DE6FN10380
STA  DE6F+0049                                                     DE6FN10390
STA  DE6F+0110                                                     DE6FN10400
STA  DE6F+0239                                                     DE6FN10410
STA  DE6F+0282        T+3N+3                                       DE6FN10420
CAL  DE6F+0606         T+2N+3                                      DE6FN10430
STA  DE6F+0067                                                     DE6FN10440
STA  DE6F+0104                                                     DE6FN10450
STA  DE6F+0223                                                     DE6FN10460
STA  DE6F+0265        T+2N+3                                       DE6FN10470
CAL  DE6F+0697         T+N+3                                       DE6FN10480
STA  DE6F+0065                                                     DE6FN10490
STA  DE6F+0102                                                     DE6FN10500
```

DE6FN (Cont'd)

```
STA DE6F+0129                                        DE6FN105
STA DE6F+0213                                        DE6FN105
STA DE6F+0254                                        DE6FN105
STA DE6F+0340      T+N+3                             DE6FN105
CAL DE6F+0629       T+1                              DE6FN105
STA DE6F+0093                                        DE6FN105
STA DE6F+0094                                        DE6FN105
STA DE6F+0097                                        DE6FN105
STA DE6F+0099                                        DE6FN105
STA DE6F+0193                                        DE6FN106
STA DE6F+0200      T+1                               DE6FN106
CAL DE6F+0591       T+2                              DE6FN106
STA DE6F+0032                                        DE6FN106
STA DE6F+0091                                        DE6FN106
STA DE6F+0923       T+2                              DE6FN106
STA DE6F+0098                                        DE6FN106
STA DE6F+0115                                        DE6FN106
STA DE6F+0192      T+2                               DE6FN106
LDQ DE6F+0538       8                                DE6FN106
MPY DE6F+0710         N                              DE6FN107
STQ DE6F+0528       8N                               DE6FN107
CAL DE6F+0528       8N                               DE6FN107
SBM DE6F+0710         N                              DE6FN107
STA DE6F+0527       7N                               DE6FN107
ADM DE6F+0526       2N                               DE6FN107
ADM DE6F+0526       2N                               DE6FN107
STA DE6F+0529       11N                              DE6FN107
ADM DE6F+0672       T+7N+3                           DE6FN107
STA DE6F+0050      T+18N+3=T+11N+3+(7N)              DE6FN107
STA DE6F+0317      T+18N+3=T+11N+3+(7N)              DE6FN108
ADM DE6F+0710         N                              DE6FN108
STA DE6F+0101      T+19N+3=T+11N+3+(8N)              DE6FN108
ADM DE6F+0537       7                                DE6FN108
STA DE6F+0530       T+19N+10=T+19N+3+(7)             DE6FN108
CAL DE6F+0101      T+19N+3                           DE6FN108
ADM DE6F+0529       11N                              DE6FN108
STA DE6F+0178      T+30N+3=T+19N+3+(11N)             DE6FN108
STA DE6F+0234                                        DE6FN108
STA DE6F+0278                                        DE6FN108
STA DE6F+0309      T+30N+3                           DE6FN109
ADM DE6F+0537       7                                DE6FN109
STA DE6F+0210      T+30N+10=T+19N+3+(11N+7)          DE6FN109
ADM DE6F+0532       2                                DE6FN109
STA DE6F+0220      T+30N+12=T+19N+3+(11N+9)          DE6FN109
CAL DE6F+0317      7+18N+3                           DE6FN109
SBM DE6F+0526       2N                               DE6FN109
STA DE6F+0149      T+16N+3                           DE6FN109
SBM DE6F+0710         N                              DE6FN109
STA DE6F+0138      T+15N+3                           DE6FN109
LXA DE6F+0710,1             N                        DE6FN110
SXD DE6F+0389,1                N TO DECR.            DE6FN110
```

DE6FN (Cont'd)

```
CLA 2,4                 PZE=VAR. STEP SIZE (B,0,R)                 DE6FN11020
STO DE6F+0501         MZE=FIXED STEP SIZE                          DE6FN11030
STZ COMMON+000                                                     DE6FN11040
STA COMMON+000            B (B LESS THAN OR =N)                    DE6FN11050
STZ DE6F+0516                                                      DE6FN11060
STD DE6F+0516         R=TOTAL R.K. LOOPS PER D(T)                  DE6FN11070
CLA DE6F+0516                                                      DE6FN11080
ARS 18                                                             DE6FN11090
TNZ DE6F+0903                                                      DE6FN11100
LXA DE6F+0534,1                 4                                  DE6FN11110
SXD DE6F+0516,1                   IF R=0, SET R=4                  DE6FN11120
CLA DE6F+0534         4                                            DE6FN11130
ADD DE6F+0524         233000000000                                 DE6FN11140
FAD DE6F+0571         FLOAT FIXED PT. NO.                          DE6FN11150
STO DE6F+0517         R IN FLOATING PT.                            DE6FN11160
CLA COMMON+000            TEST B=0                                 DE6FN11170
TNZ DE6F+0910                                                      DE6FN11180
CLA DE6F+0710           IF B=0, SET B=N                            DE6FN11190
STA COMMON+000             N                                       DE6FN11200
CAL DE6F+0710            N                                         DE6FN11210
SBM COMMON+000             B (OR N, IF B=0)                        DE6FN11220
TPL DE6F+0914         PROGRAMMER GOOFED, IF MINUS                  DE6FN11230
CLM                       SET B=N IF B TOO LARGE (N-B=0)           DE6FN11240
ALS 18                    N-B TO DECR.                             DE6FN11250
STD DE6F+0338        1ST B EQNS. TESTED                            DE6FN11260
STD DE6F+0312        N-B                                           DE6FN11270
CLA 3,4                   S                                        DE6FN11280
TNZ DE6F+0920         IF S=0, SET S=9                              DE6FN11290
CLA DE6F+0954          1E-9                                        DE6FN11300
STO COMMON+000             S (SEE VMIN,VMAX,WMAX.                  DE6FN11310
STZ DE6F+0503        +=R.K., -=COWELL                              DE6FN11320
STZ DE6F+0709           +=1ST R.K. INTEGRATION, -=2ND, ETC.        DE6FN11330
CLA                       T+2=DELTA(T)=H                           DE6FN11340
STO DE6F+0519                                                      DE6FN11350
CLA 6,4                    A=MIN. Y ALLOWED                        DE6FN11360
TNZ DE6F+0928                                                      DE6FN11370
CLA DE6F+0543         1. (IF A=0, SET A=1.)                        DE6FN11380
STO DE6F+0511                                                      DE6FN11390
CLA 4,4                   MINIMUM H ALLOWED  (FLOATING PT.)        DE6FN11400
SSP                                                                DE6FN11410
STO DE6F+0509                                                      DE6FN11420
CLA 5,4                   MAXIMUM H ALLOWED (FLOATING PT.)         DE6FN11430
SSP                                                                DE6FN11440
TNZ DE6F+0936          IF H=0, SET HMAX= 1E18                      DE6FN11450
CLA DE6F+0525          1E18                                        DE6FN11460
STO DE6F+0510                                                      DE6FN11470
LDQ DE6F+0519         DELTA(T)=H                                   DE6FN11480
FMP DE6F+0519                                                      DE6FN11490
STO DE6F+0520         H SQ.                                        DE6FN11500
CLA COMMON+000             1E-S                                    DE6FN11510
FDP DE6F+0520          H SQ.                                       DE6FN11520
```

DE6FN (Cont'd)

```
         FMP DE6F+0952       .1                                        DE6FN115
         STO DE6F+0512       VMIN=(10)**(-1-S)/H SQ.                   DE6FN115
         STO DE6F+0514       WMAX                                      DE6FN115
         FDP DE6F+0953       1E-4                                      DE6FN115
         STQ DE6F+0513       VMAX=(10)**(3-S)/H SQ.                    DE6FN115
         STZ DE6F+0500       SWITCH (PROGRAMMER SETS NON/ZERO)         DE6FN115
         CLA DE6F+0538       8 (SET UP COUNTER IN CASE                 DE6FN115
         STO DE6F+0502       OF CHANGE IN COORD.)                      DE6FN116
         LXD DE6F+0506,4         RETURN TO PROGRAMMER. (1ST ENTRY)     DE6FN116
         TXI DE6F+0074,4,-6      TRA 7,4                               DE6FN116
         DEC .1                  .1                                    DE6FN116
         DEC 1E-4                .0001=1E-4                            DE6FN116
         DEC 1E-9                1E-9 (IF S=0)                         DE6FN116
BREAK    EQU *                                                         DE6FN116
END      EQU *                                                         DE6FN116
```

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Function: DOT

Purpose: To find the dot product of two three-dimensional vectors.

Calling Sequence:

Z = DOT (X, Y)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	X	3			First input vector
I	Y	3			Second input vector
O	Z				Z = X(1)Y(1)+X(2)Y(2)+X(3)Y(3)

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

DOT

```
*       LABEL                                                  DOT0
*       SYMBOL TABLE                                           DOT0
        FUNCTION DOT(X,Y)                                      DOT0000
        DIMENSION X(3),Y(3)                                    DOT0010
        DOT = X(1)*Y(1) + X(2)*Y(2) + X(3)*Y(3)                DOT0020
        RETURN                                                 DOT0030
        END                                                    DOT0
```

Subroutine: EARTR

Purpose: Subroutine EARTR determines which tracking stations can observe the vehicle at the time EARTR is called. The subroutine then updates the state covariance matrix for the types of observations being performed by the tracking station. The possible types of observations are: range, range rate, azimuth-elevation, and direction cosines. The measurements may be corrupted by the following types of errors: random, bias, time bias, and station location errors. The subroutine input and output quantities are in common storage.

Calling Sequence:

CALL EARTR

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities placed in common

Common storages used or required: T, S, C, IC

Subroutines required: CORRTP, DOT, GHA, INTRI, MATRX, NUTAIT, SETN, TRAC, TRANSH

Functions required: ARKTNS, ATAN, COS, FNORM, SQRT, SIN

Approximate number of storages required: 1530

EARTR Partial Derivatives

To perform the covariance matrix updating for earth based tracking observations, a number of partial derivatives are required. The updating of the covaniance matrix for observations requires the partial derivatives relating the types of measurements and the vehicle state. The evaluation of the errors in the measurements due to station location errors and time bias requires the partials which relate the measurements and the station latitude, longitude, altitude, and time.

The following quantities and relationships are used in the derivations of the various partials. Four important vectors are obtained by calling subroutine TRAC. Three unit vectors, $\hat{U}$, $\hat{E}$, $\hat{N}$, which form a topocentric orthognal coordinate system and the vector X_T are obtained from the center of the earth to tracker.

The unit vectors are the up, east, and north vectors. These vectors may be written as follows:

$U_1 = \cos(LAT)\cos(LON)$

$U_2 = \cos(LAT)\sin(LON)$

$U_3 = \sin(LAT)$

$E_1 = -\sin(LON)$

$E_2 = \cos(LON)$

$E_3 = 0$

$N_1 = -\sin(LAT)\cos(LON)$

$N_2 = -\sin(LAT)\sin(LON)$

$N_3 = \cos(LAT)$

$LON = \omega(t + t_o) + LON'$

LON' = STATION LONGITUDE

ω = EARTH'S ROTATION RATE

ωt_o = GREENWICH'S LONGITUDE AT EPOCH

$$X_T = \left(\frac{R_E}{C} + H \right) U_1 \qquad H = \text{ALTITUDE}$$

$$Y_T = \left(\frac{R_E}{C} + H \right) U_2 \qquad C = \left[\cos^2(\text{LAT}) + \frac{R_P^2}{R_E^2} \sin^2(\text{LAT}) \right]^{1/2}$$

$$Z_T = \left(\frac{R_P^2}{R_E C} + H \right) U_3 \qquad R_E = \text{EQUATORIAL RADIUS}$$

$$R_P = \text{POLAR RADIUS}$$

The vehicle's position, $\vec{X}_v$, and velocity, $\dot{\vec{X}}_v$, are obtained from common storage.

$$\vec{X}_v = \begin{pmatrix} X_v \\ Y_v \\ Z_v \end{pmatrix} \qquad \dot{\vec{X}}_v = \begin{pmatrix} \dot{X}_v \\ \dot{Y}_v \\ \dot{Z}_v \end{pmatrix}$$

The tracking station velocity is obtained by differentiating $\vec{X}_T$.

$$\frac{d}{dt} (\vec{X}_T) = \dot{\vec{X}}_T = \begin{pmatrix} \dot{X}_T \\ \dot{Y}_T \\ \dot{Z}_T \end{pmatrix}$$

$$\dot{X}_T = -\omega \left(\frac{R_E}{C} + H \right) \cos(\text{LAT}) \sin(\text{LON})$$

$$\dot{Y}_T = \omega \left(\frac{R_E}{C} + H \right) \cos(\text{LAT}) \cos(\text{LON})$$

$$\dot{Z}_T = 0$$

The vector from the tracking station to vehicle may then be written as

$$\underset{3\times1}{\vec{X}} = \underset{3\times1}{\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}} = \underset{3\times1}{\vec{X}_v - \vec{X}_T} = \underset{3\times1}{\begin{pmatrix} X_v - X_T \\ Y_v - Y_T \\ Z_v - Z_T \end{pmatrix}}$$

A relationship which will be used in the derivations is

$$\frac{\partial PAR}{\partial \vec{X}_v} = \frac{\partial PAR}{\partial \vec{X}}$$

The FORTRAN program names for the above vectors are as follows:

FORTRAN NAME	DIMENSION	DERIVATION NAME	DIMENSION
U	(3)	U	(3)
E	(3)	E	(3)
EN	(3)	N	(3)
X	(3)	X_v	(3)
RT	(3)	X_T	(3)
Y	(3)	$X = X_v - X_T$	(3)
A	(1)	R_E	(1)
B	(1)	R_P	(1)
RAT	(1)	$\vert X \vert$	(1)
XD	(3)	$\dot{X} = \dot{X}_v - \dot{X}_T$	(3)

The quantities being measured are the following:

$$R = \text{RANGE} = |\vec{X}| = (\vec{X}\cdot\vec{X})^{1/2}$$

$$\dot{R} = \text{RANGE RATE} = \frac{d}{dt}|\vec{X}| = \frac{\vec{X}\cdot\dot{\vec{X}}}{R}$$

$$AZ = \tan^{-1}\left\{\frac{\vec{X}\cdot\hat{E}}{\vec{X}\cdot\hat{N}}\right\} \qquad EL = \sin^{-1}\frac{(\hat{U}\cdot\vec{X})}{R}$$

Below is a list of partial derivatives which will be used in the application of the chain rule for partial derivatives.

$$\frac{\partial U_1}{\partial LAT} = N_1 \qquad \frac{\partial E_1}{\partial LAT} = 0$$

$$\frac{\partial U_2}{\partial LAT} = N_2 \qquad \frac{\partial E_2}{\partial LAT} = 0$$

$$\frac{\partial U_3}{\partial LAT} = N_3 \qquad \frac{\partial E_e}{\partial LAT} = 0$$

$$\frac{\partial U_1}{\partial LON} = \cos(LAT)\ \sin(LON) \qquad \frac{\partial E_1}{\partial LON} = -\cos(LON)$$

$$\frac{\partial U_2}{\partial LON} = \cos(LAT)\ \cos(LON) \qquad \frac{\partial E_2}{\partial LON} = -\sin(LON)$$

$$\frac{\partial U_3}{\partial LON} = 0 \qquad \frac{\partial E_3}{\partial LON} = 0$$

$$\frac{\partial U_1}{\partial ALT} = 0 \qquad \frac{\partial E_1}{\partial ALT} = 0$$

$$\frac{\partial U_2}{\partial ALT} = 0 \qquad \frac{\partial E_2}{\partial ALT} = 0$$

$$\frac{\partial U_3}{\partial ALT} = 0 \qquad \frac{\partial E_3}{\partial ALT} = 0$$

$$\frac{\partial N_1}{\partial LAT} = -U_1 \qquad \frac{\partial U_1}{\partial t} = -\omega \sin(LON)\cos(LAT)$$

$$\frac{\partial N_2}{\partial LAT} = -U_2 \qquad \frac{\partial U_2}{\partial t} = \omega \cos(LON)\cos(LAT)$$

$$\frac{\partial N_3}{\partial LAT} = -U_3 \qquad \frac{\partial U_3}{\partial t} = 0$$

$$\frac{\partial N_1}{\partial LON} = \sin(LAT)\sin(LON) \qquad \frac{\partial E_1}{\partial t} = -\omega \cos(LON)$$

$$\frac{\partial N_2}{\partial LON} = -\sin(LAT)\cos(LON) \qquad \frac{\partial E_2}{\partial t} = -\omega \sin(LON)$$

$$\frac{\partial N_3}{\partial LON} = 0 \qquad \frac{\partial E_3}{\partial t} = 0$$

$$\frac{\partial N_1}{\partial ALT} = 0 \qquad \frac{\partial N_1}{\partial t} = \omega \sin(LAT)\sin(LON)$$

$$\frac{\partial N_2}{\partial ALT} = 0 \qquad \frac{\partial N_2}{\partial t} = -\omega \sin(LAT)\cos(LON)$$

$$\frac{\partial N_3}{\partial ALT} = 0 \qquad \frac{\partial N_3}{\partial t} = 0$$

FORTRAN 3x3 matrix called DRT

$$\underset{3\times3}{DRT} = \underset{3\times3}{\begin{pmatrix} \frac{\partial X_1}{\partial LAT} & \frac{\partial X_1}{\partial LON} & \frac{\partial X_1}{\partial ALT} \\ \frac{\partial X_2}{\partial LAT} & \frac{\partial X_2}{\partial LON} & \frac{\partial X_2}{\partial ALT} \\ \frac{\partial X_3}{\partial LAT} & \frac{\partial X_3}{\partial LON} & \frac{\partial X_3}{\partial ALT} \end{pmatrix}}$$

$$DRT(1,1) = -\frac{\partial X_T}{\partial LAT} = \left(\frac{R_E}{C} + H\right) \sin(LAT)\ \cos(LON) + U_1 \frac{R_E}{C^2} \frac{\partial C}{\partial LAT}$$

$$\frac{\partial C}{\partial LAT} = \frac{\sin(LAT)\ \cos(LAT)}{C} \left(1 - \frac{R_P^2}{R_E^2}\right)$$

$$DRT(2,1) = -\frac{\partial Y_T}{\partial LAT} = U_2 \frac{R_E}{C^2} \frac{\partial C}{\partial LAT} + \left(\frac{R_E}{C} + H\right) \sin(LAT)\ \sin(LON)$$

$$DRT(3,1) = -\frac{\partial Z_T}{\partial LAT} = -\left(\frac{R_P^2}{R_E C} + H\right) \cos(LAT) + U_3 \frac{R_P^2}{R_E C^2} \frac{\partial C}{\partial LAT}$$

$$DRT(1,2) = -\frac{\partial X_T}{\partial LON} = -E_1 \left(\frac{R_E}{C} + H\right) \cos(LAT)$$

$$DRT(2,2) = -\frac{\partial Y_T}{\partial LON} = -E_2 \left(\frac{R_E}{C} + H\right) \cos(LAT)$$

$$DRT(3,2) = 0$$

$$DRT(1,3) = -U_1$$

$$DRT(2,3) = -U_2$$

$$DRT(3,3) = -U_3$$

$$\frac{\partial \dot{X}_1}{\partial LAT} = -\frac{\partial \dot{X}_T}{\partial LAT} = -\omega \left(\frac{R_E}{C} + H\right) \sin(LAT)\ \sin(LON)$$

$$\frac{\partial \dot{X}_2}{\partial LAT} = -\frac{\partial \dot{Y}_T}{\partial LAT} = \omega \left(\frac{R_E}{C} + H\right) \sin(LAT)\ \cos(LON)$$

$$\frac{\partial \dot{X}_3}{\partial LAT} = -\frac{\partial \dot{Z}_T}{\partial LAT} = 0$$

$$\frac{\partial \dot{X}_1}{\partial LON} = - \frac{\partial \dot{X}_T}{\partial LON} = \omega \left(\frac{R_E}{C} + H \right) \cos(LAT) \cos(LON)$$

$$\frac{\partial \dot{X}_2}{\partial LON} = - \frac{\partial \dot{Y}_T}{\partial LON} = \omega \left(\frac{R_E}{C} + H \right) \cos(LAT) \sin(LON)$$

$$\frac{\partial \dot{X}_3}{\partial LON} = - \frac{\partial \dot{Z}_T}{\partial LON} = 0$$

$$\frac{\partial \dot{X}_1}{\partial ALT} = - \frac{\partial \dot{X}_T}{\partial ALT} = \omega \cos(LAT) \sin(LON)$$

$$\frac{\partial \dot{X}_2}{\partial ALT} = - \frac{\partial \dot{Y}_T}{\partial ALT} = -\omega \cos(LAT) \cos(LON)$$

$$\frac{\partial X_3}{\partial ALT} = - \frac{\partial Z_T}{\partial ALT} = 0$$

A. The following is a derivation of the partials relating the types of measurements and the vehicle state. In the program, these partials are used as a row vector. The FORTRAN name of the vector is H. H may be written as follows:

$$\underset{1x6}{H} = \underset{1x6}{\left(\frac{\partial MEAS}{\partial X_v}; \frac{\partial MEAS}{\partial Y_v}; \frac{\partial MEAS}{\partial Z_v}; \frac{\partial MEAS}{\partial \dot{X}_v}; \frac{\partial MEAS}{\partial \dot{Y}_v}; \frac{\partial MEAS}{\partial \dot{Z}_v} \right)}$$

(1) Derivation of partials for range measurement

$$R = (\vec{X} \cdot \vec{X})^{1/2}$$

$$\underset{1x6}{H_R} = \underset{1x6}{\left(\frac{\partial R}{\partial \vec{X}_v}; \frac{\partial R}{\partial \dot{\vec{X}}_v} \right)} = \underset{1x6}{\left(\frac{X_1}{R}; \frac{X_2}{R}; \frac{X_3}{R}; 0; 0; 0 \right)}$$

(2) Derivation of partials for range rate measurement.

$$\dot{R} = \frac{\vec{X}\cdot\dot{\vec{X}}}{R}$$

$$H_{RR} = \left(\frac{\partial \dot{R}}{\partial \vec{X}_v}; \frac{\partial \dot{R}}{\partial \dot{\vec{X}}_v}\right) = \left(\frac{\dot{X}_1}{R} - \frac{\dot{R}}{R^2}(X_1); \frac{\dot{X}_2}{R} - \frac{\dot{R}}{R^2}(X_2); \frac{\dot{X}_3}{R} - \frac{\dot{R}}{R^2}(X_3); \frac{X_1}{R}; \frac{X_2}{R}; \frac{X_3}{R}\right)$$

(3) Derivation of partials for azimuth measurement.

$$AZ = \mathrm{Tan}^{-1}\left(\frac{\vec{X}\cdot\hat{E}}{\vec{X}\cdot\hat{N}}\right) \equiv \mathrm{Tan}^{-1}\,\gamma$$

$$\frac{\partial AZ}{\partial PAR} = \left[1 + \gamma^2\right]^{-1} \frac{\partial\gamma}{\partial PAR}$$

$$\left[1 + \gamma^2\right]^{-1} = \frac{(\vec{X}\cdot\hat{N})^2}{S^2} \qquad S^2 = (\vec{X}\cdot\hat{E}) + (\vec{X}\cdot\hat{N})^2$$

$$H_{AZ} = \left(\frac{\partial AZ}{\partial \vec{X}_v}; \frac{\partial AZ}{\partial \dot{\vec{X}}_v}\right) = \left(\frac{E_1(\hat{N}\cdot\vec{X})}{S^2} - \frac{\hat{E}\cdot\vec{X}(N_1)}{S^2}; \frac{E_2(\hat{N}\cdot\vec{X})}{S^2} - \frac{\hat{E}\cdot\vec{X}(N_2)}{S^2}; \frac{E_3(\hat{N}\cdot\vec{X})}{S^2} - \frac{\hat{E}\cdot\vec{X}(N_3)}{S^2}; 0; 0; 0\right)$$

(4) Derivation of partials for elevation measurement.

$$EL = \mathrm{Sin}^{-1}\left(\frac{\hat{U}\cdot\vec{X}}{R}\right) \equiv \mathrm{Sin}^{-1}\,\gamma$$

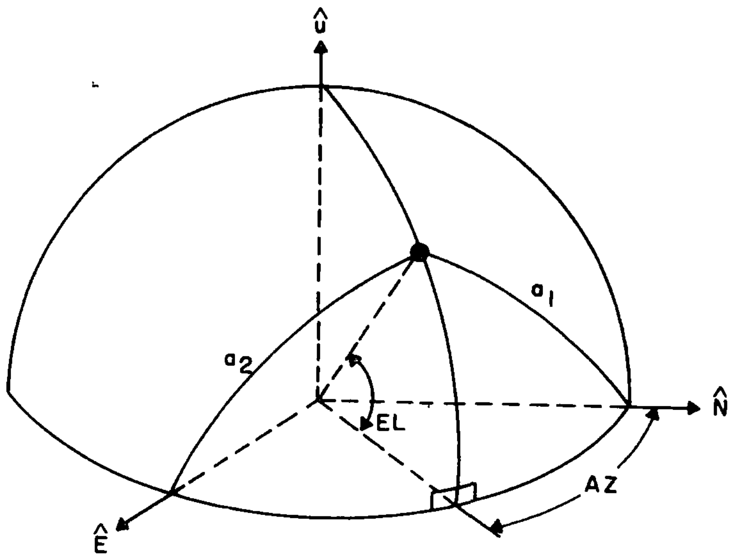
$$\frac{\partial EL}{\partial PAR} = \left[1 - \gamma^2\right]^{-1/2} \frac{\partial\gamma}{\partial PAR}$$

$$(1 - \gamma^2)^{-1/2} = \frac{R}{S}$$

$$H_{EL} = \left(\frac{\partial EL}{\partial \vec{X}} ; \frac{\partial EL}{\partial \dot{\vec{X}}}\right) = \left(\frac{U_1}{S} - \frac{(\hat{U}\cdot\vec{X})}{R^2 S} X_1 ; \frac{U_2}{S} - \frac{(\hat{U}\cdot\vec{X})}{R^2 S} X_2 ;\right.$$

$$\left.\frac{U_3}{S} - \frac{(\hat{U}\cdot\vec{X})}{R^2 S} ; 0; 0; 0\right)$$

(5) Derivation of partials for direction cosines. The partials for the ℓ and m direction cosines are obtained by combining the azimuth and elevation partials as follows:



$$m = \cos(a_1) = \cos AZ \cos EL$$

$$\ell = \cos(a_2) = \sin AZ \cos EL$$

$$\frac{\partial m}{\partial PAR} = - \cos AZ \sin EL \frac{\partial EL}{\partial PAR} - \sin AZ \cos EL \frac{\partial AZ}{\partial PAR}$$

$$\frac{\partial \ell}{\partial PAR} = - \sin AZ \sin EL \frac{\partial EL}{\partial PAR} + \cos AZ \cos EL \frac{\partial AZ}{\partial PAR}$$

$$H_m = \left(\frac{\partial m}{\partial \bar{x}} \; ; \; \frac{\partial m}{\partial \dot{\bar{x}}}\right) = \left(-\cos AZ \; \sin EL \;\; -\sin AZ \; \cos EL\right) \begin{pmatrix} H_{AZ} \\ H_{EL} \end{pmatrix}$$

1x6 1x6 1x2 2x6

$$H_\ell = \left(\frac{\partial \ell}{\partial \bar{x}} \; ; \; \frac{\partial \ell}{\partial \dot{\bar{x}}}\right) = \left(-\sin AZ \; \sin EL \;\; \cos AZ \; \cos EL\right) \begin{pmatrix} H_{AZ} \\ H_{EL} \end{pmatrix}$$

1x6 1x6 1x2 2x6

B. In order to include errors in the measurements being made due to station location errors and time bias, the partials of the measurements with respect to latitude, longitude, altitude, and time are required. In the program, the station location partials are used as a row vector with FORTRAN name SPART. SPART may be written as follows:

$$SPART = \left(\frac{\partial MEAS}{\partial LAT} \; ; \; \frac{\partial MEAS}{\partial LON} \; ; \; \frac{\partial MEAS}{\partial ALT}\right)$$

(1) Derivations of range measurement error partials.

$$\frac{\partial R}{\partial LAT} = \sum_{i=1}^{3} \frac{\partial R}{\partial x_i} \frac{\partial x_i}{\partial LAT}$$

$$\frac{\partial R}{\partial LAT} = \sum_{i=1}^{3} H_R(1,i) \; DRT(i,1)$$

Similarily

$$\frac{\partial R}{\partial LON} = \sum_{i=1}^{3} H_R(1,i) \; DRT(i,2)$$

$$\frac{\partial R}{\partial ALT} = \sum_{i=1}^{3} H_R(1,i)\ DRT(i,3)$$

$$SPART_R = \left(\frac{\partial R}{\partial LAT};\ \frac{\partial R}{\partial LON};\ \frac{\partial R}{\partial ALT}\right)$$

(2) Derivation of range rate measurement error partials.

$$\frac{\partial \dot{R}}{\partial LAT} = \sum_{i=1}^{3} \frac{\partial \dot{R}}{\partial X_i}\frac{\partial X_i}{\partial LAT} + \frac{\partial \dot{R}}{\partial \dot{X}_i}\frac{\partial \dot{X}_i}{\partial LAT}$$

$$\frac{\partial \dot{R}}{\partial LAT} = \sum_{i=1}^{3} H_{RR}(1,i)\ DRT(i,1) + H_{RR}(1,i+3)\ \frac{\partial \dot{X}_i}{\partial LAT}$$

Similarly

$$\frac{\partial \dot{R}}{\partial LON} = \sum_{i=1}^{3} H_{RR}(1,i)\ DRT(i,2) + H_{RR}(1,i+3)\ \frac{\partial \dot{X}_i}{\partial LON}$$

$$\frac{\partial \dot{R}}{\partial ALT} = \sum_{i=1}^{3} H_{RR}(1,i)\ DRT(i,3) + H_{RR}(1,i+3)\ \frac{\partial \dot{X}_i}{\partial ALT}$$

$$SPART_{RR} = \left(\frac{\partial \dot{R}}{\partial LAT};\ \frac{\partial \dot{R}}{\partial LON};\ \frac{\partial \dot{R}}{\partial ALT}\right)$$

(3) Derivation of azimuth measurement error partials.

$$\frac{\partial AZ}{\partial LAT} = \sum_{i=1}^{3} \frac{\partial AZ}{\partial X_i}\frac{\partial X_i}{\partial LAT} + \underbrace{\frac{\partial AZ}{\partial E_i}\frac{\partial E_i}{\partial LAT}}_{(=0)} + \frac{\partial AZ}{\partial N_i}\frac{\partial N_i}{\partial LAT}$$

$$\frac{\partial AZ}{\partial N_i} = -\frac{X_i(\vec{X}\cdot\hat{E})}{S^2} \qquad i = 1,2,3$$

$$\frac{\partial AZ}{\partial LAT} = \sum_{i=1}^{3} H_{AZ}(1,i)\ DRT(i,1) - \frac{X_i(\vec{X}\cdot\hat{E})}{S^2}\ \frac{\partial N_i}{\partial LAT}$$

$$\frac{\partial AZ}{\partial LON} = \sum_{i=1}^{3} \frac{\partial AZ}{\partial X_i}\ \frac{\partial X_i}{\partial LON} + \frac{\partial AZ}{\partial E_i}\ \frac{\partial E_i}{\partial LON} + \frac{\partial AZ}{\partial N_i}\ \frac{\partial N_i}{\partial LON}$$

$$\frac{\partial AZ}{\partial E_i} = \frac{X_i(\vec{X}\cdot\hat{N})}{S^2} \qquad i = 1,2,3$$

$$\frac{\partial AZ}{\partial LON} = \sum_{i=1}^{3} H_{AZ}(1,i)\ DRT(i,2) + \frac{X_i}{S^2}(\vec{X}\cdot\hat{N})\ \frac{\partial E_i}{\partial LON} - \frac{X_i}{S^2}(\vec{X}\cdot\hat{E})\ \frac{\partial N_i}{\partial LON}$$

$$\frac{\partial AZ}{\partial ALT} = \sum_{i=1}^{3} \frac{\partial AZ}{\partial X_i}\ \frac{\partial X_i}{\partial ALT} + \underbrace{\frac{\partial AZ}{\partial E_i}\ \frac{\partial E_i}{\partial ALT}}_{(=0)} + \underbrace{\frac{\partial AZ}{\partial N_i}\ \frac{\partial N_i}{\partial ALT}}_{(=0)}$$

$$\frac{\partial AZ}{\partial ALT} = \sum_{i=1}^{3} H_{AZ}(1,i)\ DRT(i,3)$$

$$SPART_{AZ} = \left(\frac{\partial AZ}{\partial LAT} ; \frac{\partial AZ}{\partial LON} ; \frac{\partial AZ}{\partial ALT} \right)$$

(4) Derivation of elevation measurement error partials.

$$\frac{\partial EL}{\partial LAT} = \sum_{i=1}^{3} \frac{\partial EL}{\partial X_i}\ \frac{\partial X_i}{\partial LAT} + \frac{\partial EL}{\partial U_i}\ \frac{\partial U_i}{\partial LAT}$$

$$\frac{\partial EL}{\partial U_i} = \frac{X_i}{S} \qquad i = 1,2,3$$

$$\frac{\partial EL}{\partial LAT} = \sum_{i=1}^{3} H_{EL}(1,i)\ DRT(i,1) + \frac{X_i}{S}\ \frac{\partial U_i}{\partial LAT}$$

$$\frac{\partial EL}{\partial LON} = \sum_{i=1}^{3} \frac{\partial EL}{\partial X_i}\ \frac{\partial X_i}{\partial LON} + \frac{\partial EL}{\partial U_i}\ \frac{\partial U_i}{\partial LON}$$

$$\frac{\partial EL}{\partial LON} = \sum_{i=1}^{3} H_{EL}(1,i)\ DRT(i,2) + \frac{X_i}{S}\ \frac{\partial U_i}{\partial LON}$$

$$\frac{\partial EL}{\partial ALT} = \sum_{i=1}^{3} \frac{\partial EL}{\partial X_i}\ \frac{\partial X_i}{\partial ALT} + \overbrace{\frac{\partial EL}{\partial U_i}\ \frac{\partial U_i}{\partial ALT}}^{(=0)}$$

$$\frac{\partial EL}{\partial ALT} = \sum_{i=1}^{3} H_{ALT}(1,i)\ DRT(i,3)$$

$$SPART_{EL} = \left(\frac{\partial EL}{\partial LAT} ; \frac{\partial EL}{\partial LON} ; \frac{\partial EL}{\partial ALT} \right)$$

(5) Derivation of direction cosine measurement error partials.

$$\underset{1x3}{SPART_m} = \underset{1x2}{\left(-\cos AZ\ \sin EL \quad -\ \sin AZ\ \cos EL\right)} \underset{2x3}{\begin{pmatrix} SPART_{AZ} \\ SPART_{EL} \end{pmatrix}}$$

$$\underset{1x3}{SPART_\ell} = \underset{1x2}{\left(-\sin AZ\ \sin EL \quad \cos AZ\ \cos EL\right)} \underset{2x3}{\begin{pmatrix} SPART_{AZ} \\ SPART_{EL} \end{pmatrix}}$$

(6) Derivations of time derivatives of measurement quantities to permit inclusion of time bias errors.

$$\frac{\partial R}{\partial t} = \dot{R} = \frac{\vec{X} \cdot \dot{\vec{X}}}{R}$$

$$\frac{\partial AZ}{\partial t} = \sum_{i=1}^{3} \frac{\partial AZ}{\partial X_i} \frac{\partial X_i}{\partial t} + \frac{\partial AZ}{\partial E_i} \frac{\partial E_i}{\partial t} + \frac{\partial AZ}{\partial N_i} \frac{\partial N_i}{\partial t}$$

$$\frac{\partial AZ}{\partial t} = \sum_{i=1}^{3} H_{AZ}(1,i)\ \dot{X}_i + \frac{\partial AZ}{\partial E_i}\ \dot{E}_i + \frac{\partial AZ}{\partial N_i}\ \dot{N}_i$$

$$\frac{\partial EL}{\partial t} = \sum_{i=1}^{3} H_{EL}(1,i)\ \dot{X}_i + \frac{\partial EL}{\partial E_i}\ \dot{E}_i + \frac{\partial EL}{\partial N_i}\ \dot{N}_i$$

(7) The time partials for the ℓ and m direction cosines are obtained by the following combination of azimuth and elevation partials.

$$\frac{\partial m}{\partial t} = \begin{pmatrix} -\cos AZ \sin EL & -\sin AZ \cos EL \end{pmatrix} \begin{pmatrix} \frac{\partial AZ}{\partial t} \\ \frac{\partial EL}{\partial t} \end{pmatrix}$$

1x1 1x2 2x1

$$\frac{\partial}{\partial t} = \begin{pmatrix} -\sin AZ \sin EL & \cos AZ \cos EL \end{pmatrix} \begin{pmatrix} \frac{\partial AZ}{\partial t} \\ \frac{\partial EL}{\partial t} \end{pmatrix}$$

1x1 1x2 2x1

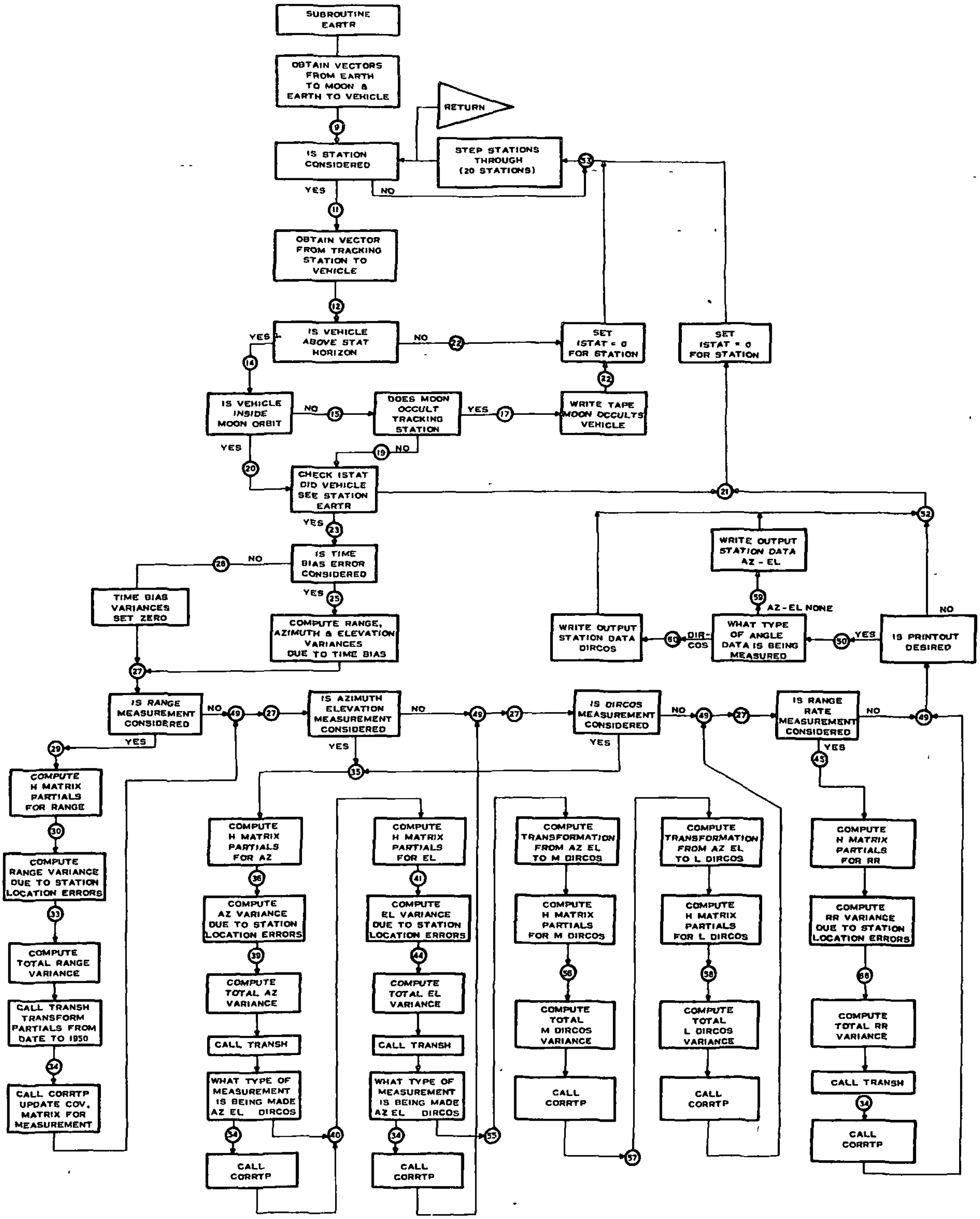
The errors in the measurements are computed in the following manner for station location errors and time bias.

$$QQS = \left(SPART_{MEAS}\right)^2 \begin{pmatrix} \sigma^2 LAT \\ \sigma^2 LON \\ \sigma^2 ALT \end{pmatrix} \qquad \text{Station Location Error}$$

1x1 1x3 3x1

$$QQDOT = \left\{\frac{\partial MEAS}{\partial t} \cdot \sigma_{TIME}\right\}^2 \qquad \text{Time Bias Error}$$

The computation of the H matrices and the measurement errors are the primary computations in EARTR. These quantities are input to subroutine CORRTP which does the weighting of the measurement and updating of the state covariance matrix.


SUBROUTINE EARTR
OBTAIN VECTORS FROM EARTH TO MOON & EARTH TO VEHICLE
RETURN
IS STATION CONSIDERED
STEP STATIONS THROUGH (20 STATIONS)
YES
NO
OBTAIN VECTOR FROM TRACKING STATION TO VEHICLE
IS VEHICLE ABOVE STAT HORIZON
SET ISTAT = 0 FOR STATION
SET ISTAT = 0 FOR STATION
IS VEHICLE INSIDE MOON ORBIT
DOES MOON OCCULT TRACKING STATION
WRITE TAPE MOON OCCULTS VEHICLE
CHECK ISTAT DID VEHICLE SEE STATION EARTR
IS TIME BIAS ERROR CONSIDERED
TIME BIAS VARIANCES SET ZERO
COMPUTE RANGE, AZIMUTH & ELEVATION VARIANCES DUE TO TIME BIAS
WRITE OUTPUT STATION DATA AZ - EL
WRITE OUTPUT STATION DATA DIRCOS
WHAT TYPE OF ANGLE DATA IS BEING MEASURED
AZ-EL NONE
DIR-COS
IS PRINTOUT DESIRED
IS RANGE MEASUREMENT CONSIDERED
IS AZIMUTH ELEVATION MEASUREMENT CONSIDERED
IS DIRCOS MEASUREMENT CONSIDERED
IS RANGE RATE MEASUREMENT CONSIDERED
COMPUTE H MATRIX PARTIALS FOR RANGE
COMPUTE RANGE VARIANCE DUE TO STATION LOCATION ERRORS
COMPUTE TOTAL RANGE VARIANCE
CALL TRANSH TRANSFORM PARTIALS FROM DATE TO 1950
CALL CORRTP UPDATE COV. MATRIX FOR MEASUREMENT
COMPUTE H MATRIX PARTIALS FOR AZ
COMPUTE AZ VARIANCE DUE TO STATION LOCATION ERRORS
COMPUTE TOTAL AZ VARIANCE
CALL TRANSH
WHAT TYPE OF MEASUREMENT IS BEING MADE AZ EL DIRCOS
CALL CORRTP
COMPUTE H MATRIX PARTIALS FOR EL
COMPUTE EL VARIANCE DUE TO STATION LOCATION ERRORS
COMPUTE TOTAL EL VARIANCE
CALL TRANSH
WHAT TYPE OF MEASUREMENT IS BEING MADE AZ EL DIRCOS
CALL CORRTP
COMPUTE TRANSFORMATION FROM AZ EL TO M DIRCOS
COMPUTE H MATRIX PARTIALS FOR M DIRCOS
COMPUTE TOTAL M DIRCOS VARIANCE
CALL CORRTP
COMPUTE TRANSFORMATION FROM AZ EL TO L DIRCOS
COMPUTE H MATRIX PARTIALS FOR L DIRCOS
COMPUTE TOTAL L DIRCOS VARIANCE
CALL CORRTP
COMPUTE H MATRIX PARTIALS FOR RR
COMPUTE RR VARIANCE DUE TO STATION LOCATION ERRORS
COMPUTE TOTAL RR VARIANCE
CALL TRANSH
CALL CORRTP

EARTR

```
*       LABEL
*       SYMBOL TABLE
        SUBROUTINE EARTR
C       SUBROUTINE EARTR UPDATES THE COVARIANCE MATRIX USED
C       IN EARTH TRACKING
        COMMON T,S,C,IC
        DIMENSION  PO(22),VE(22),XED(3),VED(3),AN(3,3),RBOP(6),POSM(3),
      1VOSM(3),POVRM(3),EM(3,3),EN(3),U(3), E(3),RT(3),Y(3),TD(3),
      2ISTAT(20),H(6),X(3),VX(3),T(1360),S(1000),C(1000),IC(1),ISA(20),
      3IMSTA(4,20),P(6,6),SPART(3),DRT(3,3),XD(3),SPART2(3),HA(6),HCOS(6)
        EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM)
      1,(IC(3),NOR),(C(13),TW),(C(14),TF),(C(62),PO),(C(84),VE)
      2,(C(112),RBOP),(C(138),AN),(C(120),EM),(C(15),X),(C(18),VX),
      3(S(70),DTS),(S(71),STD),(S(72),DR),(S(73),RD),(IC(194),ISTAT)
      4,(IC(10),ISA),(IC(30),IMSTA),(IC(214),NOUT),(C(652),P)
      5,(C(649),TSECP),(C(30),TSEC),(S(10),W),(S(14),A),(S(15),B)
        CALL SETN(NIN,NUTS)
        NB=NOR-1
        DIS=1.E10
        CALL INTR1 (TW,TF,NB,PO,1,VE,DIS)
        CALL MATRX(AN,X,XED,3,3,1,0)
        CALL MATRX(AN,VX,VED,3,3,1,0)
C       XED=VECTOR FROM NB BODY TO VEHICLE, VED=VELOCITY EQUATOR OF DATE
        MS=1
        IF(RBOP(1)-330000.)10,2,2
C       RBOP= MAGNITUDE OF VECTOR FROM VEHICLE TO BODY
      2 CONTINUE
        IF (NOR-1)3,5,3
      3 CONTINUE
C       HERE IF OUTSIDE MOON AND NOT EARTH REFERENCED
        DO 4 I=1,3
        POSM(I)=0.
        VOSM(I)=0.
        DO 4 J=1,3
        K=23-J
        XED(I)=XED(I)-AN(I,J)*PO(K)
        VED(I)=VED(I)-AN(I,J)*VE(K)
        L=20-J
        POSM(I)=POSM(I)+AN(I,J)*(PO(L)-PO(K))
        VOSM(I)=VOSM(I)+AN(I,J)*(VE(L)-VE(K))
C       POSM IS A VECTOR FROM EARTH TO MOON
      4 CONTINUE
        GO TO 7
      5 CONTINUE
C       HERE IF OUTSIDE MOON AND EARTH REFERENCED
        DO 6 I=1,3
        POSM(I)=0
        VOSM(I)=0
        DO 6 J=1,3
        K=20-J
        POSM(I)=POSM(I)+AN(I,J)*PO(K)
```

EARTR (Cont'd)

```
      VOSM(I)=VOSM(I)+AN(I,J)*VE(K)
    6 CONTINUE
    7 CONTINUE
      MS=2
      DO 8 I=1,3
      POVRM(I)=XED(I)-POSM(I)
    8 CONTINUE
      RADM=FNORM(POVRM)
      DO 9 I=1,3
      POVRM(I)=POVRM(I)/RADM
    9 CONTINUE
   10 CONTINUE
      TIME=TW+TF
      CALL       NUTAIT (TIME,OM,CR,DT,EM,ERSIL)
      FSEC=DTS*TF
      CALL GHA(FSEC,TW,GH,EM(2,1),OMEGA)
      GH=GH*DR
      NDUM=783
      NN=110
      DO 53 III=1,20
      NDUM=NDUM+5
      NN=NN+15
      IF(ISA(III)) 53,53,11
   11 CONTINUE
      AT=S(NN+9)*DR
      ON=S(NN+10)*DR
      AL=S(NN+11)
      SNAME=S(NN+12)
      CALL TRAC(ON,AT,AL,GH,U,E,EN,RT,AC,SL,CL,ST,CT,A,B)
C     TRAC  OBTAINS UNIT VECTORS UP,EAST AND NORTH. RT= VECTOR
C     TO TRACK STATION
      DO 12 J=1,3
      Y(J)=XED(J)-RT(J)
C     Y=VECTOR FROM TRACK STATION TO VEHICLE. TD=UNIT VECTOR IN
C     DIRECTION OF Y
   12 CONTINUE
      RAT=FNORM(Y)
      DO 13 J=1,3
      TD(J)=Y(J)/RAT
   13 CONTINUE
      CE=DOT( U,TD)
C     DOT FORMS A DOT PRODUCT OF ELEMENTS IN PARENTHESES
      SEC=CE/SQRTF(1.-CE**2)
      ELEV=ATANF(SEC)
      HORCOR=S(NDUM)
C     S(NDUM) CONTAINS HORIZON CORRECTION FOR STATION
      IF(ELEV-HORCOR)22,14,14
   14 CONTINUE
C     HERE IF VEHICLE IS ABOVE STATION HORIZON
      GO TO (20,15),MS
   15 CONTINUE
```

EARTR (Cont'd)

```
C       TO DETERMINE IF MOON OCCULTS VEHICLE
        DOTTD=DOT(POVRM,TD)
        IF(DOTTD)19,19,16
     16 CONTINUE
        IF(DOTTD-(SQRTF(RADM**2-1739.**2))/RADM)19,19,17
     17 WRITE OUTPUT TAPE NUTS,18,SNAME
     18 FORMAT (27H0MOON OCCULTS VEHICLE FROM,A6,8H TRACKER)
        GO TO 22
     19 CONTINUE
     20 CONTINUE
        SEC=DOT(TD,EN)
        CE=DOT(TD,E)
        AZMTH =ARKTNS(360,SEC,CE)
        IF(ISTAT(III))21,21,23
C ISTAT IS SET ONE IF THE STATION SAW VEHICLE ON PREVIOUS TIME
C       THROUGH MATSUB
C       ZERO IF IT COULD NOT SEE IT
     21 CONTINUE
C       HERE IF STATION SEES THE VEHICLE
        ISTAT(III)=1
        GO TO 53
     22 CONTINUE
        ISTAT(III)=0
C       VEHICLE WAS NOT OBSERVED BY STATION
        GO TO 53
     23 CONTINUE
        OBNO=INTF((TSEC-TSECP)/S(NN+13))
C       OBNO IS THE NUMBER OF OBSERVATIONS SINCE LAST TIME IN MATSUB
        XTU=DOT(Y, U)
        XTE=DOT(Y,E)
        XTN=DOT(Y,EN)
        TPD2=XTE**2+XTN**2
        TPD=SQRTF(TPD2)
        ALLY=XTN/TPD2
        GAM=XTU*XTE
        BET=XTE/TPD2
        DEL=XTU*XTN
        D=1./RAT**2/TPD
        CA=AC/A
        BA=B*B/(A*A)
        GG=CA*(1.-BA)*SL*CL*CA*CA
        DRT(1,1)=A*GG*U(1)+(AC+AL)*EN(1)
        DRT(2,1)=A*GG*U(2)+(AC+AL)*EN(2)
        DRT(3,1)=(B*B/A)*GG*U(3)+BA*(AC+AL)*EN(3)
        DRT(1,2)=(AC+AL)*CL*E(1)
        DRT(2,2)=(AC+AL)*CL*E(2)
        DRT(3,2)=0.0
        DO 24 I=1,3
        DRT(I,3)=U(I)
C       THE ABOVE     QUATITIES ARE USED ONLY FOR H MATRIX CALCULATIONS
     24 CONTINUE
```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EARTR (Cont'd)

```
      AA=S(10)*CL*(AC+AL)                                              EART142
      XD(1)=VED(1)-AA*E(1)                                             EART143
      XD(2)=VED(2)-AA*E(2)                                             EART144
      XD(3)=VED(3)                                                     EART145
      XDTU=DOT(XD,U)                                                   EART146
      XDTE=DOT(XD,E)                                                   EART147
      XDTN=DOT(XD,EN)                                                  EART148
      RDOT=DOT(XD,Y)/RAT                                               EART149
C     RDOT=RANGE RATE                                                  EART150
      ADOT=W*SL-W*CL*ALLY*XTU+ALLY*XDTE-BET*XDTN                       EART151
C     ADOT=AZIMUTH RATE                                                EART152
      EDOT=(1./TPD)*(W*CL*XTE+XDTU-RDOT*XTU/RAT)                       EART153
C     EDOT=ELEVATION RATE                                              EART154
      IF(S(NN+14))26,26,25                                             EART155
   25 CONTINUE                                                         EART156
      RDOT2=RDOT*RDOT                                                  EART157
      QQRDOT=RDOT2*S(NN+14)                                            EART158
C     QQRDOT=RANGE VARIANCE DUE TO TIME BIAS                           EART159
      ADOT2=ADOT*ADOT                                                  EART160
      QQADOT=ADOT2*S(NN+14)                                            EART161
C     QQADOT=AZIMUTH VARIANCE DUE TO TIME BIAS                         EART162
      EDOT2=EDOT*EDOT                                                  EART163
      QQEDOT=EDOT2*S(NN+14)                                            EART164
C     QQEDOT=ELEVATION VARIANCE DUE TO TIME BIAS                       EART165
      GO TO 27                                                         EART166
   26 CONTINUE                                                         EART167
      QQRDOT=0.                                                        EART168
      QQADOT=0.                                                        EART169
      QQEDOT=0.                                                        EART170
   27 CONTINUE                                                         EART171
      DO 49 II=1,4                                                     EART172
      IF (IMSTA(II,III)) 49,49,28                                      EART173
   28 CONTINUE                                                         EART174
      II=II                                                            EART175
      GO TO (29,35,35,45),II                                           EART176
C     II DESIGNATES TYPE OF MEASUREMENT 1=RANGE 2=AZ+EL 3=DIRCOS       EART176
C     4=RANGE RATE                                                     EART176
   29 CONTINUE                                                         EART177
C     CALCULATES H MATRIX FOR RANGE                                    EART178
      DO 30 J=1,3                                                      EART179
      H(J)=TD(J)                                                       EART180
   30 CONTINUE                                                         EART181
      DO 32 I=1,3                                                      EART182
      SPART(I)=0.                                                      EART183
C     SPART=PARTIALS OF RANGE WRT STATION LOCATION                     EART184
      DO 31 K=1,3                                                      EART185
      SPART(I)=SPART(I)-H(K)*DRT(K,I)                                  EART186
   31 CONTINUE                                                         EART187
      SPART2(I)=SPART(I)*SPART(I)                                      EART188
   32 CONTINUE                                                         EART189
      QQS=0.                                                           EART190
```

EARTR (Cont'd)

```
      DO 33 I=1,3                                                       EART1910
C     QQS=RANGE VARIANCE DUE TO STATION LOCATION ERRORS                 EART1920
      KL=NN+3+I                                                         EART1930
      QQS=QQS+SPART2(I)*S(KL)                                           EART1940
   33 CONTINUE                                                          EART1950
      QQ=S(NN)/OBNO+RAT*S(425)*RAT+QQS+QQRDOT                           EART1960
C     QQ=TOTAL RANGE VARIANCE                                           EART1970
      CALL TRANSH(H,1)                                                  EART1980
C     TRANSH TRANSFORMS H MATRIX FROM EQUINOX OF DATE TO 1950           EART1990
   34 CONTINUE                                                          EART2000
      CALL CORRTP (QQ,H,P)                                              EART2010
C     CORRTP UPDATES COVARIANCE MATRIX FOR PARTICULAR TYPE              EART2015
C     OF MEASUREMENT                                                    EART2016
      GO TO 49                                                          EART2020
   35 CONTINUE                                                          EART2030
C     THE FOLLOWING CALCULATIONS GIVE H MATRIX FOR AZIMUTH              EART2040
      DO 36 J=1,3                                                       EART2050
      HA(J)=ALLY*E(J)-BET*EN(J)                                         EART2060
   36 CONTINUE                                                          EART2070
      DO 37 I=1,3                                                       EART2080
      SPART(I)=0.                                                       EART2090
C     SPART=PARTIALS OF AZIMUTH WRT STATION LOCATION                    EART2100
      DO 37 K=1,3                                                       EART2110
      SPART(I)=SPART(I)-HA(K)*DRT(K,I)                                  EART2120
   37 CONTINUE                                                          EART2130
      SPART(1)=SPART(1)+BET*XTU                                         EART2140
      PLUS=BET*SL*XTE-ALLY*(Y(1)*CT+Y(2)*ST)                            EART2150
      SPART(2)=SPART(2)+PLUS                                            EART2160
      DO 38 I=1,3                                                       EART2170
      SPART2(I)=SPART(I)*SPART(I)                                       EART2180
   38 CONTINUE                                                          EART2190
      QQSA=0.                                                           EART2200
      DO 39 I=1,3                                                       EART2210
C     QQSA=AZIMUTH VARIANCE DUE TO STATION LOCATION ERRORS              EART2220
      KL=NN+3+I                                                         EART2230
      QQSA=QQSA+SPART2(I)*S(KL)                                         EART2240
   39 CONTINUE                                                          EART2250
      QQ=S(NN+1)/OBNO+S(NN+7)+QQSA+QQADOT                               EART2260
C     QQ=TOTAL AZIMUTH VARIANCE                                         EART2270
      CALL TRANSH(HA,1)                                                 EART2280
      IF(II-2)54,54,40                                                  EART2290
   54 CONTINUE                                                          EART2300
      CALL CORRTP(QQ,HA,P)                                              EART2310
   40 CONTINUE                                                          EART2320
C     THE FOLLOWING CALCULATIONS GIVE H MATRIX FOR ELEVATION            EART2330
      DO 41 J=1,3                                                       EART2340
      H(J)= U(J)/TPD-Y(J)*XTU*D                                         EART2350
   41 CONTINUE                                                          EART2360
      DO 42 I=1,3                                                       EART2370
      SPART(I)=0.                                                       EART2380
C     SPART=PARTIALS OF ELEVATION WRT STATION LOCATION                  EART2390
```

EARTR (Cont'd)

```
      DO 42 K=1,3
      SPART(I)=SPART(I)-H(K)*DRT(K,I)
   42 CONTINUE
      SPART(1)=SPART(1)+XTN/TPD
      SPART(2)=SPART(2)+XTE*CL/TPD
      DO 43 I=1,3
      SPART2(I)=SPART(I)*SPART(I)
   43 CONTINUE
      QQS=0.
      DO 44 I=1,3
C     QQS=ELEVATION VARIANCE DUE TO STATION LOCATION ERRORS
      KL=NN+3+I
      QQS=QQS+SPART2(I)*S(KL)
   44 CONTINUE
      QQ=S(NN+2)/OBNO+S(NN+8)+QQS+QQEDOT
C     QQ=TOTAL ELEVATION VARIANCE
      CALL TRANSH(H,1)
      IF(II-2)34,34,55
   55 CONTINUE
      DIRCOS=DOT(EN,TD)
      SINANG=SQRTF(1.-DIRCOS**2)
      TANANG=SINANG/DIRCOS
      ANGM=ATANF(TANANG)
      ANGM=ANGM*RD
      CA=COSF(AZMTH)
      SA=SINF(AZMTH)
      SE=SINF(ELEV)
      CE=COSF(ELEV)
      C1=-SA*CE
      C2=-CA*SE
      ANGMDT=C1*EDOT+C2*ADOT
      ANGMDT=ANGMDT*RD
      DO 56 I=1,3
      HCOS(I)=C1*H(I)+C2*HA(I)
   56 CONTINUE
      QQSCOS=C1*QQS+C2*QQSA
C     QQSCOS IS THE VARIANCE IN THE M DIRECTION COSINE DUE TO STATION
C     LOCATION ERRORS
      QQCDOT=C1*QQEDOT+C2*QQADOT
      QQ=S(NDUM+1)/OBNO+S(NDUM+2)+QQSCOS+QQCDOT
C     QQ=TOTAL VARIANCE OF M DIRECTION COSINE
      CALL CORRTP(QQ,HCOS,P)
   57 CONTINUE
      DIRCOS=DOT(E,TD)
      SINANG=SQRTF(1.-DIRCOS**2)
      TANANG=SINANG/DIRCOS
      ANGL=ATANF(TANANG)
      ANGL=ANGL*RD
      C1=CA*CE
      C2=-SA*SE
      ANGLDT=C1*EDOT+C2*ADOT
```

EARTR (Cont'd)

```
      ANGLDT=ANGLDT*RD
      DO 58 I=1,3
      HCOS(I)=C1*H(I)+C2*HA(I)
   58 CONTINUE
      QQSCOS=C1*QQS+C2*QQSA
C     QQSCOS IS THE L DIRECTION COSINE ERROR DUE TO STATION LOCATION
C     ERROR
      QQCDOT=C1*QQEDOT+C2*QQADOT
      QQ=S(NDUM+3)/OBNO+S(NDUM+4)*QQSCOS+QQCDOT
C     QQ=TOTAL L DIRECTION COSINE VARIANCE
      CALL CORRTP(QQ,HCOS,P)
      GO TO 49
   45 CONTINUE
C     THE FOLLOWING CALCULATIONS GIVE H MATRIX FOR RANGE RATE
   47 DO 48 J=1,3
      K=J+3
      H(K)=TD(J)
      H(J)=(1./RAT)*(XD(J)-RDOT*TD(J))
   48 CONTINUE
      CNST=B*B/A*((1./CA)**3)
      DO 65 J=1,3
      JJ=3*J-2
      SPART(J)=-DOT(H,DRT(JJ))
   65 CONTINUE
      SPART(1)=SPART(1)+S(10)*U(3)*(CNST+AL)*XTE/RAT
      SPART(2)=SPART(2)-AA*(TD(2)*E(1)-TD(1)*E(2))
      SPART(3)=SPART(3)-S(10)*EN(3)*XTE/RAT
      QQS=0.
      DO 66 I=1,3
      KL=NN+3+I
      QQS=QQS+(SPART(I)**2)*S(KL)
   66 CONTINUE
      QQ=S(NN+3)/OBNO+QQS
      CALL TRANSH(H,2)
      GO TO 34
   49 CONTINUE
      NOUT=NOUT
      GO TO (52,50),NOUT
   50 CONTINUE
      IF(IMSTA(3,III))59,59,60
   59 CONTINUE
      EDOT=EDOT*RD
      ADOT=ADOT*RD
      AZMTH =AZMTH*RD
      ELEV=ELEV*RD
      WRITE OUTPUT TAPE NUTS,51,SNAME,RAT,RDOT,AZMTH,ADOT,ELEV,EDOT
   51 FORMAT (17H TRACKER STATION ,A6,/4H RNGE15.8,5H  RGRE15.8,
     15H  AZME15.8,5H  AZRE15.8,5H  ELEE15.8,5H  ELRE15.8)
      GO TO 52
   60 CONTINUE
      WRITE OUTPUT TAPE NUTS,61,SNAME,RAT,RDOT,ANGL,ANGLDT,ANGM,ANGMDT
```

EARTR (Cont'd)

```
61  FORMAT (17H TRACKER STATION ,A6,/4H RNGE15.8,5H  RGRE15.8,          EART32
   15H  LCSE15.8,5H  LCRE15.8,5H  MCSE15.8,5H  MCRE15.8)                 EART33
 52 CONTINUE                                                             EART33
    GO TO 21                                                             EART33
 53 CONTINUE                                                             EART33
    RETURN                                                               EART33
    END                                                                  EART
```

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Subroutine: ECLIP

Purpose: To replace each of two 3-dimensional vectors by its product with a given 3 x 3 matrix. This subroutine is used for transforming position and velocity vectors.

Calling Sequence:

CALL ECLIP (X, VX, ECL)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I-O	X	3			Xout = (ECL) Xin
I-O	VX	3			VXout = (ECL) VXin
I	ECL	3,3			TRANSFORMATION MATRIX

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

```
*       LABEL                                                    ECLP
*       SYMBOL TABLE                                             ECLP
        SUBROUTINE ECLIP(X,VX,ECL)                               ECLP0000
        DIMENSION X(3),VX(3),XP(3),VXP(3),ECL(3,3)               ECLP0010
        DO 1 I=1,3                                               ECLP0020
        XP(I) = 0.                                               ECLP0030
        VXP(I) = 0.                                              ECLP0040
        DO 1 J=1,3                                               ECLP0050
        XP(I) = XP(I) + ECL(I,J)*X(J)                            ECLP0060
      1 VXP(I) = VXP(I) + ECL(I,J)*VX(J)                         ECLP0070
        DO 2 I=1,3                                               ECLP0080
        X(I) = XP(I)                                             ECLP0090
      2 VX(I) = VXP(I)                                           ECLP0100
        RETURN                                                   ECLP0110
        END                                                      ECLP
```

Subroutine: ENCKE

Purpose: To perform the calculation of the ENCKE contribution to the acceleration. These are perturbations in acceleration due to deviation from the osculating conic.

Calling Sequence:

CALL ENCKE (U, X, D, AE)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	U	1	μ	Km^3/sec^2	Gravitational constant
I	X	3	X,Y,Z	Km	Position on the conic
I	D	3	X',Y',Z'	Km	Position deviation from the conic
O	AE	3		Km/sec^2	ENCKE acceleration terms

Common storages used or required: None

Subroutines required: None

Functions required: SQRT, DOT

Approximate number of storages required: 158 DEC

Equation Being Solved

The perturbation acceleration due to deviation from the reference conic is the following

$$\vec{AE} = \frac{\mu}{R_o^3}\left(\vec{R}\,F(Q) - \vec{D}\right)$$

where $\vec{R} = \vec{R}_o + \vec{D}$. The solution, R_o, for the position in the two body orbit is provided by STEPC, and is saved from step to step so that a new R_o is calculated only when the time has changed.

$F(Q) = 1 - (1 + 2Q)^{-3/2}$ is calculated from the series expansion

$$F(Q) = Q\sum_{j=0}^{6} a_j Q^j$$

where

$$a_0 = 3$$

$$a_1 = -7.5$$

$$a_2 = 17.5$$

$$a_3 = 39.375$$

$$a_4 = 86.625$$

$$a_5 = 187.6875$$

$$a_6 = 402.1875$$

and

$$Q = \frac{\vec{D}\cdot\left(\vec{R}_o + \frac{\vec{D}}{2}\right)}{R_o^2}$$

ENCKE

```
*       LABEL                                                          ENCK
*       SYMBOL TABLE                                                   ENCK
CEC2014  SUBROUTINE ENCKE                                              ENCK
        SUBROUTINE ENCKE (U,X,D,AE)                                    ENCK00
        DIMENSION X(3),D(3),AE(3)                                      ENCK00
        R = DOT(X,X)                                                   ENCK00
        DD = DOT(D,D)                                                  ENCK00
        RD = DOT(X,D)                                                  ENCK00
        Q = (RD+.5*DD)/R                                               ENCK00
        F = 1./SQRTF(2.*Q+1.)**3                                       ENCK01
      2 A = Q*(3.+Q*(-7.5+Q*(17.5+Q*(-39.375+Q*(86.625+Q*(-187.6875+Q* ENCK01
       1402.1875))))))                                                 ENCK01
      4 R3 = (SQRTF(R))*R                                              ENCK01
        UA = U*A/R3                                                    ENCK01
        UF = U*F/R3                                                    ENCK01
        DO 5 I=1,3                                                     ENCK01
      5 AE(I) = UA*X(I)-UF*D(I)                                        ENCK01
        RETURN                                                         ENCK01
        END                                                            ENCK
```

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Subroutine: ERP

Purpose: To write on the output tape: EPHEMERIS FAILED DUE TO TAPE REDUNDANCIES, and then to CALL EXIT. Should this message occur, and the tape was actually mounted on the correct unit, try again or try a different copy of the ephemeris tape.

Calling Sequence:

CALL ERP

Input and **Output**

Common storages used or required: None

Subroutines required: SETN, EXIT

Functions required:

Approximate number of storages required:

```
*     LABEL                                                             ERPO
CEC20GD                                                                 ERPO
      SUBROUTINE ERP                                                    ERP0000
      CALL SETN(NIN,NOUT)                                               ERP0010
  1   FORMAT(42H EPHEMERIS FAILED DUE TO TAPE REDUNDANCIES)             ERP0020
      WRITE OUTPUT TAPE NOUT,1                                          ERP0030
      CALL EXIT                                                         ERP0040
      RETURN                                                            ERP0050
      END                                                               ERPO
```

Subroutine: ERPT

Purpose: To write on the output tape: EPHEMERIS FAILED LOOKING FOR T = ( ) DAYS SINCE 1950.0, and to call EXIT.

The cause of this condition is either incorrect input or not having the ephemeris tape mounted correctly.

Calling Sequence:

CALL ERPT(T)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	T			days	Time (in days since 1950.0)
					for which INTR was searching
					the tape

Common storages used or required: None

Subroutines required: SETN, EXIT

Functions required: None

Approximate number of storages required:

```
*     LABEL                                                    ERPT
CEC20GE                                                        ERPT
      SUBROUTINE ERPT(T)                                       ERPT0000
      CALL SETN(NIN,NOUT)                                      ERPT0010
      WRITE OUTPUT TAPE NOUT,2,T                               ERPT0020
 2    FORMAT(33H EPHEMERIS FAILED LOOKING FOR T =,F20.0,       ERPT0030
     118H DAYS SINCE 1950,0)                                   ERPT0040
      CALL EXIT                                                ERPT0050
      RETURN                                                   ERPT0060
      END                                                      ERPT
```

Subroutine: FINP

Identification

RW FINP - Decimal, Octal, BCD, Variable Data Input
7090 FAP Subroutine
W.J. Stoner, August 24, 1961
Aerospace Corporation

Purpose

To read a set of Hollerith punched data and/or header cards into core with one FORTRAN CALL statement.

To convert the data fields to binary and store in core according to their associated conversion codes.

Restrictions

This routine uses (CSH)S to accomplish the BCD card image read. Tape troubles or other errors from this routine are indicated by the printout of HPR 1,4.

This routine uses (EXE) to print HPR 2,4 in case of errors such as non-Hollerith characters, data out of range, illegal format, subscripts too large for the array previously defined, etc. Upon detection of any error, control is immediately sent to (EXE) and no more cards are processed.

Method

Decimal numbers are converted to binary integers and then scaled to the indicated power of ten.

Octal numbers are converted to binary integers.

Hollerith words are stored directly.

Range: Decimal to floating binary conversion $10^{\pm 38}$
Decimal to fixed binary; 1 to 9 digits*
Decimal integer to binary integer; 1 to 5 digits
Octal integer to binary integer; 0 to 235 - 1

*the magnitude of the number depends upon the location of the decimal point.

Usage

Format:

1. The data card format, available on keypunch form M-1, consists of four subfields containing the conversion code, location, number, and exponent, respectively.

	Data Field	Data Field	Data Field	Data Field
Subfield	1	2	3	4
Conversion Code	1	19	37	55
Location	2-6	20-24	38-42	56-60
Value	7-16	25-34	43-52	61-70
Exponent	17-18	35-36	53-54	71-72

where conversion code is one of the alphabetic characters defined below which specifies the type of conversion to be used on the value field, the location specifies the cell into which the converted value field is to be stored, the value subfield contains the data to be converted, and the exponent contains the power of ten by which floating data is to be scaled, or the location of the binary point of fixed point data.

2. The header card format consists of a conversion code in column 1, a sequence number in columns 2-6, and any Hollerith information in columns 7-72.

Decimal Points:

Decimal points may be placed anywhere in the value field except that they may not occur in the same column as a minus sign (11 punch) since this results in a non-Hollerith character. If the decimal point would normally appear at the right of the number punched in the value field, then it is optional.

Minus Signs:

Minus signs are 11 punches over any digit of the field. If all of the available columns of the field are not used, minus signs may be punched as the left character of the field.

Values:

Values must always be written to the extreme left of a field. It is not necessary that the entire field be filled as the first blank denotes the end of value. Superfluous low order zeros should be omitted as they increase conversion error.

The only exception to partial fields is BCK data where the entire field, including blanks, is stored.

Location:

The location may be specified by either absolute octal, a variable or array name, or the element subscripts in a one or two dimensional array. If the locations contain five digits, it is interpreted as octal. All five columns must be punched for octal locations.

If the location contains at least one non-numeric character, it is interpreted as a variable or array name which must appear exactly as given in the CALL statement (see Calling Sequence below). The contents of the number and exponent fields, if they are numeric data, are stored in the cell for the variable or the first cell for the array. This location then becomes the origin for all subscript locations following until another variable or array name is encountered. Caution must be taken to load an array name prior to subscript locations.

If the location contains four or fewer digits, it is interpreted as a subscript except for conversion code H explained below. Single dimension array subscripts must be left-justified with leading zeros optional. Two dimension array subscripts must be denoted by two subfields of two columns each containing i and j, respectively. The i and j subfields must be separated by a comma and must contain two-digit integers.

If the location is left blank, then the location counter within the routine is decreased by 1 and the associated number is stored in the cell immediately preceding the cell where the last number was stored. Thus, an entire array may be read in by specifying the initial location only.

Conversion Codes:

Blank: Floating decimal

The number in the value field times the power of ten in the exponent field is converted to floating binary. Checks are made for overflow and format errors.

F: Fixed decimal

The number in the value field is converted to fixed point binary and stored with the binary point located at the position specified by the number in the exponent field. An overflow error check is made.

I: Decimal integer

The number in the value field is converted to a fixed point binary integer with the binary point following position 17. The exponent field is ignored. A decimal point is considered an error.

B: Octal

The value plus exponent fields are converted as a logical ocatal word.

It is not necessary to include leading zeros but the first octal digit must always occupy the leftmost position of the field.

D: BCI Data

The contents of the value plus exponent fields are interpreted as two BCI words and stored in two consecutive cells in descending order beginning at the location specified by the location field.

H: Heading Card

A card with an H in column 1 is considered a BCI heading card. If the location field is blank, the card is ignored. If the location field contains a left-justified one to four digit positive decimal integer V (octal, negative, or variable locations are not permitted), columns 7-72 of the card are stored directly in 11 consecutive words in descending order. The location of the first of these words is calculated by the routine as HEAD (1+11*(V-1)) where HEAD is defined as the last variable or array named in the CALL statement. Each card may be used as one record of output using FORMAT option A with column 7 of the card providing the code for printer spacing on output.

A: Variable names as data

The value plus exponent fields are interpreted in a pseudo FAP instruction format AAAAA T DDDDD P where the fields to replace are address, tag, decrement, and prefix, respectively. The address and decrement fields are defined normally to be 5 characters and the tag and prefix as one octal numeric character each. Any field containing less than the normal number of characters must end with a comma while fields of normal length must not. Any address or decrement field containing less than 5 numeric characters are converted as octal. Any address or decrement field containing at least one non-numeric character is interpreted as a variable or array name. Variable addresses cause the entire word from the compiler generated calling sequence to be loaded into the location word (i.e., the TSX X is stored in the location specified if X is the variable appearing in the address field). Variable decrements cause the right-most 18 bits from the compiler generated calling sequence to be loaded into the location word's prefix and decrement. Numeric tags and prefixes are loaded directly into the corresponding parts of the location words. Null fields are not loaded. Since the first blank indicates the end of the loading of a word, address only, adress-tag, address-tag-decrement, or entire word may be loaded as desired.

G: Temporary Origin

The value in the first location field on the card is used as a temporary origin for tables. The location is saved and if data cards follow with blank location fields the corresponding data is stored consecutively in descending order beginning with the cell specified in the location in the G card. Columns 7-72 are ignored and may be used to identify the table.

The first nonblank location starts a new origin. If this nonblank location is a subscript, it references the last variable or array named, which may or may not have been on the G card.

J: Transfer

The location specified with this prefix must be an octal address and is the only part of the data field that is interpreted. The subroutine causes a transfer to the octal location specified and does not interpret the remaining fields on the card.

L: Two dimension array i_{max}, j_{max} definition

The location field contains the name of the array to be loaded. The value field is defined to consist of 2 subfields, separated by a comma, of 2 columns each containing the two-digit decimal integers for i_{max} and j_{max} respectively where i_{max} and j_{max} generally appear in a DIMENSIØN statement. The i_{max} and j_{max} values are retained to compute the successive subscripted locations until redefined. Blank address fields may follow this array definition if successive elements of the array are to be loaded.

M: Two dimension array i_{max}, j_{max} definition

Conversion is identical to L except the entire array is preset to zero.

E: End Case

This defines an end-of-case and control is returned to the FORTRAN object program. The rest of this field and the remaining fields on the card are ignored.

Calling Sequence:

The following two types of CALL statements may be used:

I. CALL FINP (n,X,Y,ZETA,...,mHX(5)Y(5)ZETA(2)...) where

A. n is the number of variables and/or arrays in the list, excluding n itself.

B. X, Y, ZETA,... are the names of variables and/or arrays restricted to at most 5 characters each, one character of which is non-numeric.

C. m is 6 times n. Hence, mH allows for 6n Hollerith characters to follow.

D. X(5)Y(5)ZETA(2)... is a list of the items previously named in exactly the same order with (i) indicating the number, i, of blanks necessary to provide six Hollerith characters for each item. Since each item name is restricted to 5 characters, the minimum value of (i) is (1).

II. CALL FINP (0) where the number of items is given as zero. This CALL statement must be used only after a CALL statement of type I has been executed. When the subroutine encounters a zero for the number of items, it immediately refers to the last executed CALL FINP with a nonzero number of items for the names of the items to be loaded.

Space Requirements

613 cells.

Number of Pages

Writeup	6
Listing	12
Total	18

```
*       FAP                                                          FINP
        LBL FINP                                                     FINP
        TTL     FINP INPUT ROUTINE                                   FINP
        ENTRY   FINP                                                 FINP01
        REM     CALL FINP (N,A,B,DATA,...,6NHA     B     DATA   ...) FINP000
        REM          WHERE N IS NUM OF VARIABLES AND                 FINP000
        REM          6N IS 6 TIMES NUM OF VARIABLES                  FINP000
FINP    TXH     0,0,0                                                FINP000
        SXD     GIN00,4                                              FINP000
        SXD     GIN02,1                                              FINP000
        SXD     GIN05,2                                              FINP000
        SXD     STOP1,4                                              FINP000
        SXD     STOP2,4                                              FINP001
        CLA*    1,4                                                  FINP001
        TNZ     INPT1                                                FINP001
        STO     COMMON+28         FLAG TO USE PREVIOUS CALL          FINP001
        TXL     INPT2                                                FINP001
INPT1   STO     COMMON+28         FLAG TO USE PRESENT CALL           FINP001
        SXD     FINPT,4                                              FINP001
        NOP                                                          FINP001
        COM                                                          FINP001
        STD     *+1                                                  FINP001
        TXI     *+1,4,0                                              FINP002
        SXD     GIN00,4                                              FINP002
INPT2   LXD     FINPT,4                                              FINP002
        CLA*    1,4                                                  FINP002
        ARS     18                                                   FINP002
        STO     COMMON+29         N                                  FINP002
        ALS     18                                                   FINP002
        COM                                                          FINP002
        STD     TBL03                                                FINP002
        STD     *+1                                                  FINP002
        TXI     *+1,4,0                                              FINP003
        CLA     0,4                                                  FINP003
        STA     GIN01             ADDRESS OF HEADER                  FINP003
        CLA     1,4                                                  FINP003
        STA     TBL01             ADDRESS OF BCD TABLE               FINP003
        STO     ERROR                                                FINP003
GINA    LXD     GIN00,4                                              FINP003
        CALL    (CSH)S                                               FINP003
        PZE     COMMON,0,-1                                          FINP003
STOP1   PZE     1,4                                                  FINP003
        AXC     COMMON,1                                             FINP004
        TXI     *+1,1,-12                                            FINP004
        SXD     GINC,1                                               FINP004
        SXD     GIN11+1,1                                            FINP004
        AXC     COMMON,1                                             FINP004
        CLA     BLANK                                                FINP004
GINB    CAS     0,1                                                  FINP004
        TXL     GINC+2                                               FINP004
        TXI     *+2,1,-1                                             FINP004
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        TXL     GINC+2                                                FINP0049
GINC    TXH     GINB,1,**                                             FINP0050
        TXL     GINA                                                  FINP0051
        AXC     COMMON,1                                              FINP0052
        LDQ 0,1                                                       FINP0053
GIN01   PXD **                   SAVE PREFIX CODE                     FINP0054
        LGL 6                                                         FINP0055
        STO PREFIX                                                    FINP0056
        STQ 0,1                                                       FINP0057
        SUB     ME                  E                                 FINP0058
        TZE     GIN13               END                               FINP0059
        LGL 30                   SAVE LAST DIGIT OF ADDRESS           FINP0060
        ANA MASK1                                                     FINP0061
        STO ADDIND                                                    FINP0062
        STZ SUM                  TEST FOR BLANK ADDRESS               FINP0063
        STZ SIGN                                                      FINP0064
        CLA     0,1                                                   FINP0065
        CAS     MASK2                                                 FINP0066
        TXL     *+2                                                   FINP0067
        TXL     GIN06               BLANK                             FINP0068
        LRS     18                  TEST FOR MATRIX                   FINP0069
        ANA     MASK1                                                 FINP0070
        SUB     MASK4                                                 FINP0071
        TNZ     GIN17-2                                               FINP0072
        TSX     MTX00,4             MATRIX                            FINP0073
        TRA     GINER               OUTSIDE MATRIX                    FINP0074
        TRA     GIN20                                                 FINP0075
MASK4   OCT     73                                                    FINP0076
        AXT     5,4                                                   FINP0077
        LDQ     0,1                                                   FINP0078
GIN17   PXD                                                           FINP0079
        LGL     6                                                     FINP0080
        PAX     0,2                                                   FINP0081
        TXL     GIN18,2,9                                             FINP0082
        TXH     GIN19,2,48                                            FINP0083
        TXL     GIN19,2,47                                            FINP0084
GIN18   TIX     GIN17,4,1                                             FINP0085
        TXL     GIN16                                                 FINP0086
GIN19   CLA     0,1                 SYMBOLIC                          FINP0087
        ORA     MASK3                                                 FINP0088
        TSX     TBL00,4                                               FINP0089
        LLS     35                                                    FINP0090
        STA     GIN00                                                 FINP0091
GIN20   CLA     MASK3                                                 FINP0092
        STO     ADDIND                                                FINP0093
        TXL     GIN07                                                 FINP0094
GIN16   CLA     ADDIND              TEST FOR OCTAL                    FINP0095
        SUB MASK3                                                     FINP0096
        TZE GIN04                                                     FINP0097
        LXA 5B35,2               SET-UP OCTAL ADDRESS                 FINP0098
        TSX OCTAL,4                                                   FINP0099
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GIN02   TXH 5,,**                                                   FINP010
GIN03   TXL GIN07,,**                                               FINP010
GIN04   LXA 100B35,2          SET UP DECIMAL ADDRESS                FINP010
        CLA 4B35                                                    FINP010
        TSX DECIM,4                                                 FINP010
GIN05   TXH 6,,**                                                   FINP010
        XCA                                                         FINP010
        SUB     1B35                                                FINP010
        XCA                                                         FINP010
        TXL GINER,2,100                                             FINP010
        TXL     GIN07                                               FINP011
GIN06   LDQ MASK2                                                   FINP011
GIN07   STQ CRDADD                                                  FINP011
        LXA PREFIX,4          SET-UP TRA TO                         FINP011
        TXH GINER,4,48            SUB-PROGRAM                       FINP011
        TXL GIN08,4,47                                              FINP011
        LXD 100B35,4                                                FINP011
        TXL GIN09                                                   FINP011
GIN08   TXH     GINER,4,36                                          FINP011
        TXL     *+2,4,32                                            FINP011
        TXI     GIN09,4,-23                                         FINP012
        TXH     GINER,4,25                                          FINP012
        TNX GINER,4,16                                              FINP012
GIN09   TXI GIN09+1,1,-1                                            FINP012
        TRA GIN10,4                                                 FINP012
        TXL     MCD00            M MATRIX IJ(ZERO                   FINP012
        TXL     MCD00            L MATRIX IJ-NO ZERO                FINP012
        TXL     GINER            K                                  FINP012
        TRA*    CRDADD                                              FINP012
        TXL     IGIN00           I        INTEGER                   FINP012
        TXL     HGIN00           H        HEADER                    FINP013
        TXL     GGIN00           G        TABLE ORIGIN              FINP013
        TXL     FGIN00           F        FIXED FOINT               FINP013
        TXL GINER           E                                       FINP013
        TXL     DGIN00           D        BCD                       FINP013
        TXL GINER           C                                       FINP013
        TXL     BGIN00           B        OCTAL                     FINP013
        TXL     ACD0             A        SYMBOLIC                  FINP013
GIN10   TXL     LGIN00,,12       BLANK  FLOATINS                    FINP013
GIN11   TXI GIN11+1,1,-1           TEST END CARD                    FINP013
        TXH     GIN01-1,1,**                                        FINP014
GIN12   TXL     GINA             TO NEXT CARD                       FINP014
GIN13   LXD     GIN00,4                                             FINP014
        LXD     GIN02,1                                             FINP014
        LXD     GIN05,2                                             FINP014
        TRA     2,4              EXIT                               FINP014
GINER   PXD                                                         FINP014
        LDI     STOP2                                               FINP014
        CALL    (EXE)                                               FINP014
STOP2   PZE     2,4                                                 FINP014
 ME     BCI     1,00000E                                            FINP015
```

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BLANK   BCI      1,                                                FINP0151
        REM                                                        FINP0152
HGINO0  CLA      CRDADD                                            FINP0153
        CAS      MASK2                                             FINP0154
        TXL      *+2                                               FINP0155
        TXL      GIN12                 BLANK                       FINP0156
        LDQ      CRDADD                                            FINP0157
        MPY      11B35                                             FINP0158
        STQ      COMMON                                            FINP0159
        CAL      GINO1                                             FINP0160
        SBM      COMMON                                            FINP0161
        STA HGINO1                                                 FINP0162
        LXD      1B35,4                                            FINP0163
        LXA      11B35,1                                           FINP0164
        CLA      COMMON+12,1                                       FINP0165
HGINO1  STO      **,4                                              FINP0166
        TXI      *+1,4,1                                           FINP0167
        TIX HGINO1-1,1,1                                           FINP0168
        TXL GIN12                                                  FINP0169
GGINO0  CLA ADDIND                                                 FINP0170
        SUB MASK3                                                  FINP0171
        TZE GGINO1                                                 FINP0172
        PXD                                                        FINP0173
        TXL GGINO2                                                 FINP0174
GGINO1  CAL GINO0                                                  FINP0175
GGINO2  ADD      1B35                                              FINP0176
        ADD CRDADD                                                 FINP0177
        STA SGINO3                                                 FINP0178
        TXL GIN12                                                  FINP0179
BGINO0  STZ SUM                                                    FINP0180
        STZ SIGN                                                   FINP0181
        LXA M6B35,2            6                                   FINP0182
        TSX OCTAL,4                                                FINP0183
        TXI      BGINO4,1,-1                                       FINP0184
        TXI BGINO1,1,-1                                            FINP0185
BGINO1  LXA M6B35,2            6                                   FINP0186
        TSX OCTAL,4                                                FINP0187
        TXH 4                                                      FINP0188
BGINO2  TSX SGINO0,4                                               FINP0189
BGINO3  TXL GIN11,,**          EXIT FOR NEW PREFIX                 FINP0190
BGINO4  TXH      BGINO3,2,5       BLANK                            FINP0191
        TXL      BGINO2                                            FINP0192
IGINO0  TSX DECNO,4           INTERGER FORMAT                      FINP0193
        TXH GINER,2,0              ERROR FOR NON-INTEGER           FINP0194
        LLS      18                                                FINP0195
        TXL BGINO2                                                 FINP0196
DGINO0  LDQ 0,1                                                    FINP0197
        TSX SGINO0,4                                               FINP0198
        CLA MASK2                                                  FINP0199
        STO CRDADD                                                 FINP0200
        LDQ 1,1                                                    FINP0201
```

```
        TXI BGINO2,1,-1                                     FINP0202
DECNO   SXD BGINO3,4                                        FINP0203
        CLA 1,1                                             FINP0204
        ALS 24                                              FINP0205
        SLW COMMON+20                                       FINP0206
        STZ SUM                                             FINP0207
        STZ SIGN                                            FINP0208
        LXA 100B35,2                                        FINP0209
        CLS M6B35            6                              FINP0210
        TSX DECIM,4                                         FINP0211
        TXI DECN2+1,1,-1     END ON BLANK                   FINP0212
        TXI DECN1,1,-1                                      FINP0213
DECN1   CLA 4B35             4                              FINP0214
        TSX DECIM,4                                         FINP0215
DECN2   TXH 2,,**                                           FINP0216
        TXL DECN4,2,99                                      FINP0217
        TXL GIN11,2,100                                     FINP0218
DECN3   LXD 1B35,2                                          FINP0219
DECN4   LXD BGINO3,4                                        FINP0220
        LXD BGINO3,4                                        FINP0221
        TRA 1,4                                             FINP0222
DECEN   SXD GINO3,4                                         FINP0223
        TSX DECNO,4                                         FINP0224
        STQ COMMON+13                                       FINP0225
        SXD DECN2,2                                         FINP0226
        CLA DECN2                                           FINP0227
        STZ SUM                                             FINP0228
        STZ SIGN                                            FINP0229
        LDQ COMMON+20                                       FINP0230
        STQ 0,1                                             FINP0231
        TSX DECIM,4                                         FINP0232
        TXH                                                 FINP0233
        LXD GINO3,4                                         FINP0234
        TRA 1,4                                             FINP0235
FGINOO  TSX DECEN,4          FIXED PT, CONVERSION           FINP0236
        STQ COMMON                                          FINP0237
        CAL DECN2                                           FINP0238
        COM                                                 FINP0239
        ADD 1B17                                            FINP0240
        PDX 0,2                                             FINP0241
        CLA COMMON                                          FINP0242
        SUB BREF,2                                          FINP0243
        TMI GINER                                           FINP0244
        STA FGINO1                                          FINP0245
        LDQ COMMON+13                                       FINP0246
        MPY FREF,2                                          FINP0247
FGINO1  LRS **                                              FINP0248
        TNZ GINER                                           FINP0249
        TXL BGINO2,,**                                      FINP0250
LGINOO  TSX DECEN,4          FLTG. PT. CONVERSION           FINP0251
        STQ COMMON                                          FINP0252
```

```
        LXD DECN2,2                                   FINP0253
        PXD ,2                                        FINP0254
        ARS 18                                        FINP0255
        SSM                                           FINP0256
        ADD COMMON                                    FINP0257
        LRS 35                                        FINP0258
        DVP 10B35                                     FINP0259
        SUB M9B35                                     FINP0260
        PAX ,2                                        FINP0261
        CLM                                           FINP0262
        LLS 35                                        FINP0263
        ADD 5B35              HIGH DIGIT              FINP0264
        TMI GINER                                     FINP0265
        PAX ,4                                        FINP0266
        SXD FGINO1+2,1        SAVE ADD REFERENCE      FINP0267
        CLA XREF1,2                                   FINP0268
        ADD XREF2,4                                   FINP0269
        ADD 126B35                                    FINP0270
        TPL LGINOA                                    FINP0271
        STZ COMMON+13                                 FINP0272
LGINOA  PAX 0,1                                       FINP0273
        LDQ FREF1,2                                   FINP0274
        MPR FREF2,4                                   FINP0275
        LRS 35                                        FINP0276
        MPY COMMON+13                                 FINP0277
        LLS 2                                         FINP0278
        TZE LGINO3                                    FINP0279
LGINO1  TZE LGINO2                                    FINP0280
        LRS 1                                         FINP0281
        TXI LGINO1,1,1                                FINP0282
LGINO2  PXD ,1                                        FINP0283
        ARS 18                                        FINP0284
        LLS 0                                         FINP0285
        LRS 8                                         FINP0286
LGINO3  LXD FGINO1+2,1                                FINP0287
        TNZ GINER                                     FINP0288
        TXL BGINO2            EXIT TO STORE           FINP0289
SGINO0  CLA CRDADD            STORE ROUTINE           FINP0290
        SUB MASK2                                     FINP0291
        TZE SGINO1                                    FINP0292
        CLA ADDIND                                    FINP0293
        SUB MASK3                                     FINP0294
        TZE SGINOA                                    FINP0295
        CLA CRDADD                                    FINP0296
        TXL SGINO2                                    FINP0297
SGINOA  CAL      GINO0                                FINP0298
        SUB      CRDADD                               FINP0299
        TXL SGINO2                                    FINP0300
SGINO1  CAL SGINO3            ADD 1 TO OLD ADDRESS    FINP0301
        SUB      1B35                                 FINP0302
SGINO2  STA SGINO3                                    FINP0303
```

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SGINO3 STO **                                                        FINP0304
       TRA 1,4                                                       FINP0305
OCTAL  LDQ 0,1                SET-UP OCTAL INTEGER                   FINP0306
OCTAL1 PXD                                                           FINP0307
       LGL 3                                                         FINP0308
       TNZ OCTAL2                                                    FINP0309
       CAL SUM                                                       FINP0310
       LGL 3                                                         FINP0311
       SLW SUM                                                       FINP0312
       TIX OCTAL1,2,1                                                FINP0313
       TXI OCTAL3,4,-1                                               FINP0314
OCTAL2 LGL 3                                                         FINP0315
       SUB MASK3                                                     FINP0316
       TNZ GINER                                                     FINP0317
OCTAL3 LDQ SUM                                                       FINP0318
       TRA 1,4                                                       FINP0319
DECIM  SXD DECIM2,4           GENERATE DECIMAL INTEGER               FINP0320
       PAX ,4                                                        FINP0321
DECIM1 CLM                                                           FINP0322
       LDQ 0,1                                                       FINP0323
       LGL 6                                                         FINP0324
       STQ 0,1                                                       FINP0325
       SUB 10B35                                                     FINP0326
       TMI DECIM6                                                    FINP0327
       SUB 17B35                                                     FINP0328
       TNZ DECIM3                                                    FINP0329
       LXD 10B35,2                                                   FINP0330
DECIM2 TXL DECIM7,,**                                                FINP0331
DECIM3 SUB 5B35                                                      FINP0332
       TMI GINER                                                     FINP0333
       SUB 10B35                                                     FINP0334
       TNZ DECIM4                                                    FINP0335
       SUB 10B35                                                     FINP0336
DECIM4 TPL DECIM5                                                    FINP0337
       STO SIGN                                                      FINP0338
       TXL DECIM6                                                    FINP0339
DECIM5 ADD M6B35                                                     FINP0340
       TNZ GINER                                                     FINP0341
DECIM8 LXD DECIM2,4           END ON BLANK                           FINP0342
       LDQ SUM                                                       FINP0343
       CLA SIGN                                                      FINP0344
       LRS 0                                                         FINP0345
       TRA 1,4                                                       FINP0346
DECIM6 ADD 10B35                                                     FINP0347
       STO COMMON+14                                                 FINP0348
       LDQ SUM                                                       FINP0349
       MPY 10B35                                                     FINP0350
       LLS 35                                                        FINP0351
       ADD COMMON+14                                                 FINP0352
       STO SUM                                                       FINP0353
       TXI DECIM7,2,1                                                FINP0354
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DECIM7  TIX DECIM1,4,1                                                  FINP0355
        LXD DECIM2,4                                                    FINP0356
        TXI DECIM8+1,4,-1        END ON DIGIT COUNT                     FINP0357
        DEC 133                                                         FINP0358
100B35  DEC 100                                                         FINP0359
        DEC 67                                                          FINP0360
        DEC 34                                                          FINP0361
1B35    DEC 1                                                           FINP0362
        DEC -33                                                         FINP0363
        DEC -66                                                         FINP0364
        DEC -99                                                         FINP0365
        DEC -132                                                        FINP0366
XREF2   DEC .918354961680                                               FINP0367
        DEC .788860905380                                               FINP0368
        DEC .677626357980                                               FINP0369
        DEC .582076609280                                               FINP0370
        DEC .5B0                                                        FINP0371
        DEC .858993459280                                               FINP0372
        DEC .737869763080                                               FINP0373
        DEC .633825300280                                               FINP0374
        DEC .544451787180                                               FINP0375
FREF2   DEC 30                                                          FINP0376
        DEC 27                                                          FINP0377
        DEC 24                                                          FINP0378
        DEC 20                                                          FINP0379
17B35   DEC 17                                                          FINP0380
        DEC 14                                                          FINP0381
10B35   DEC 10                                                          FINP0382
        DEC 7                                                           FINP0383
4B35    DEC 4                                                           FINP0384
BREF    DEC 1                                                           FINP0385
        DEC -3                                                          FINP0386
M6B35   DEC -6                                                          FINP0387
M9B35   DEC -9                                                          FINP0388
        DEC -13                                                         FINP0389
        DEC -16                                                         FINP0390
        DEC -19                                                         FINP0391
        DEC -23                                                         FINP0392
        DEC -26                                                         FINP0393
XREF1   DEC -29                                                         FINP0394
        DEC .931322574780                                               FINP0395
        DEC .745058059780                                               FINP0396
        DEC .596046447880                                               FINP0397
        DEC .953674316580                                               FINP0398
        DEC .762939453180                                               FINP0399
        DEC .610351562580                                               FINP0400
        DEC .9765625B0                                                  FINP0401
        DEC .78125B0                                                    FINP0402
        DEC .625B0                                                      FINP0403
FREF    DEC .5B0                                                        FINP0404
        DEC .8B0                                                        FINP0405
```

```
         DEC  .64B0                                              FINP040
         DEC  .512B0                                             FINP040
         DEC  .8192B0                                            FINP040
         DEC  .65536B0                                           FINP040
         DEC  .524288B0                                          FINP041
         DEC  .8388608B0                                         FINP041
         DEC  .67108864B0                                        FINP041
FREF1    DEC  .536870912B0                                       FINP041
5B35     DEC  5                                                  FINP041
126B35   PZE  126                                                FINP041
MASK1    OCT  000000000077                                       FINP041
MASK2    OCT  606060606000                                       FINP041
MASK3    OCT  000000000060                                       FINP041
1B17     OCT  1000000                                            FINP041
 2B35    DEC  2B35                                               FINP042
 3B35    PZE  3                                                  FINP042
 8B35    DEC  8B35                                               FINP042
11B35    PZE       11                                            FINP042
12B35    PZE  12                                                 FINP042
MTX00    SXA       MTX10,4         TSX MTX00,4                   FINP042
         AXT       2,4             IJ RETURN                     FINP042
MTX01    SXA       MTX02,4         NORMAL RETURN                 FINP042
         STZ       SIGN                                          FINP042
         STZ       SUM                                           FINP042
         LXA       100B35,2                                      FINP043
         CLA       2B35                                          FINP043
         TSX       DECIM,4                                       FINP043
         TXH       0,0,0                                         FINP043
         TXL       GINER,2,100                                   FINP043
MTX02    AXT       **,4                                          FINP043
         STQ       I+2,4           I,J                           FINP043
         LDQ       0,1             REMOVE                        FINP043
         LGL       6                  COMMA                      FINP043
         STQ       0,1                                           FINP043
         TIX       MTX01,4,1                                     FINP044
MTX10    AXT       **,4                                          FINP044
         CLA       J                                             FINP044
         SUB       1B35                                          FINP044
         XCA                                                     FINP044
         MPY       IMAX                                          FINP044
         XCA                                                     FINP044
         ADD       I                                             FINP044
         LDQ       IJ                                            FINP044
         TLQ       1,4             LOC GTR MAX                   FINP044
         SUB       1B35                                          FINP045
         XCA                                                     FINP045
         TRA       2,4                                           FINP045
*        SET UP IJ WORD                                          FINP045
MCD00    SXA       MCD02,4                                       FINP045
         CAL       GIN00                                         FINP045
         ADD       1B35                                          FINP045
```

```
        STA     SGIN03                                          FINP0457
        TSX     MTX00,4                                         FINP0458
MCD02   NOP     **                                              FINP0459
        LDQ     I                                               FINP0460
        STQ     IMAX                                            FINP0461
        MPY     J                                               FINP0462
        STQ     IJ                                              FINP0463
        LXA     MCD02,4                                         FINP0464
        TXL     MCD04,4,12                                      FINP0465
        LXA     IJ,4                                            FINP0466
        TXI     *+1,4,-1                                        FINP0467
        SXD     *+6,4                                           FINP0468
        CLA     GIN00                                           FINP0469
        STA     *+2                                             FINP0470
        LXD     1B35,4                                          FINP0471
        STZ     **,4                                            FINP0472
        TXI     *+1,4,1                                         FINP0473
        TXL     *-2,4,**                                        FINP0474
MCD04   TXI     GIN11,1,-1                                      FINP0475
*                                                               FINP0476
ACD0    STZ     CTR                                             FINP0477
        STZ     PASS                                            FINP0478
ACD1    STZ     PF                                              FINP0479
        AXT     5,2                                             FINP0480
        TSX     SYM00,4                                         FINP0481
        AXT     0,4             BLANK                           FINP0482
        TXH     ACD6,2,1                                        FINP0483
        TXL     ACD12,4,0       NULL AND BLANK                  FINP0484
        TXL     ACD6+1          NULL                            FINP0485
ACD6    STQ     PF                                              FINP0486
        TXL     ACD10,4,0       BLANK                           FINP0487
        AXT     1,2                                             FINP0488
        TSX     SYM00,4                                         FINP0489
        AXT     0,4             BLANK                           FINP0490
        TXL     ACD10,2,1       NULL                            FINP0491
        XCL                                                     FINP0492
        ALS     15                                              FINP0493
        STT     PF                                              FINP0494
ACD10   LDQ     PF                                              FINP0495
        CLS     PASS                                            FINP0496
        STO     PASS                                            FINP0497
        TPL     ACD15                                           FINP0498
        STQ     WORD                                            FINP0499
        TXH     ACD1,4,0        NOT BLANK                       FINP0500
ACD12   LXA     CTR,4                                           FINP0501
        TXH     ACD14,4,6                                       FINP0502
        TXI     *+1,1,-1                                        FINP0503
        TXL     *+3,4,1         BLANK                           FINP0504
ACD14   LDQ     WORD                                            FINP0505
        TSX     SGIN00,4                                        FINP0506
        TXL     GIN11                                           FINP0507
```

```
ACD15  RQL     18                                               FINP05
       SLQ     WORD                                             FINP05
       TXL     ACD12                                            FINP05
*                                                               FINP05
SYM00  SXD     SYM02,2          LXA L(NO CHARACTERS),2          FINP05
       SXA     SYM14,4          TSX SYM00,4                     FINP05
       STZ     PSYM             BALNK                           FINP05
       AXT     1,2              NORMAL IR2=1 IF NULL            FINP05
SYM01  CLA     CTR                                              FINP05
       ADD     1B35                                             FINP05
       STO     CTR                                              FINP05
       SUB     10B35+1          7                               FINP05
       TNZ     *+2                                              FINP05
       TXI     *+1,1,-1                                         FINP05
       LDQ     0,1                                              FINP05
       CAL     PSYM                                             FINP05
       LGL     6                                                FINP05
       SLW     PSYM                                             FINP05
       STQ     0,1                                              FINP05
       ANA     MASK1                                            FINP05
       SUB     MASK3                                            FINP05
       TZE     SYM20            BLANK                           FINP05
       SUB     11B35                                            FINP05
       TZE     SYM21            COMMA                           FINP05
       TXI     *+1,2,1                                          FINP05
SYM02  TXL     SYM01,2,**                                       FINP05
       CAL     PSYM                                             FINP05
SYM03  TXI     *+1,2,-1                                         FINP05
       SXA     SYM06,2          NO CHARACTERS                   FINP05
       LDQ     MASK2                                            FINP05
SYM04  LGL     6                LEFT JUSTIFY                    FINP05
       TXI     *+1,2,1                                          FINP05
       TXL     SYM04,2,5                                        FINP05
       SLW     SYMBL                                            FINP05
       CAL     PSYM                                             FINP05
       ANA     BLANK                                            FINP05
       TNZ     SYM10            SYMBOLIC                        FINP05
SYM06  AXT     **,2             NO CHARACTERS                   FINP05
       LDQ     SYMBL                                            FINP05
       STZ     SUM                                              FINP05
       STZ     SIGN                                             FINP05
       TXL     SYM08,2,4        DECIMAL                         FINP05
       TSX     OCTAL1,4         OCTAL                           FINP05
       TXL     GINER                                            FINP05
       TSX     SYM14,2                                          FINP05
SYM08  CLA     0,1                                              FINP05
       STQ     0,1                                              FINP05
       STO     SYMBL                                            FINP05
       PXA     ,2                                               FINP05
       TSX     DECIM,4                                          FINP05
       TXL     GINER                                            FINP05
```

```
        CLA    SYMBL                                    FINP0559
        STO    0,1                                      FINP0560
        TSX    SYM14,2                                  FINP0561
SYM10   CLA    SYMBL                                    FINP0562
        TSX    TBL00,4                                  FINP0563
SYM14   AXT    **,4                                     FINP0564
        TRA    2,4              EXIT                    FINP0565
SYM20   TXI    *+1,4,1          BLANK                   FINP0566
        SXA    SYM14,4                                  FINP0567
SYM21   TXL    SYM14+1,2,1      NULL FIELD              FINP0568
SYM25   CAL    PSYM                                     FINP0569
        ARS    6                                        FINP0570
        SLW    PSYM                                     FINP0571
        TXL    SYM03                                    FINP0572
TBL00   LXD    TBL01+1,2                                FINP0573
TBL01   CAS    **,2                                     FINP0574
        TXL    *+2                                      FINP0575
        TXI    TBL02,2,-2                               FINP0576
        TXI    *+1,2,-1                                 FINP0577
TBL03   TXH    TBL01,2,**                               FINP0578
        TXL    GINER,0,-1                               FINP0579
TBL02   SXA    *+1,2                                    FINP0580
        AXC    0,2                                      FINP0581
        SXA    *+2,2                                    FINP0582
        LXD    FINPT,2                                  FINP0583
        LDQ    **,2                                     FINP0584
        LXD    TBL03+1,2                                FINP0585
        TRA    1,4                                      FINP0586
IMAX    PZE                                             FINP0587
IJ      PZE                                             FINP0588
COMMON  BSS    31                                       FINP0589
GIN00   SYN    COMMON+30                                FINP0590
PF      SYN    COMMON+15                                FINP0591
PASS    SYN    COMMON+16                                FINP0592
CTR     EQU    COMMON+17                                FINP0593
SYMBL   EQU    COMMON+18                                FINP0594
PSYM    EQU    COMMON+19                                FINP0595
CRDADD  EQU    COMMON+21                                FINP0596
SIGN    EQU    COMMON+22                                FINP0597
SUM     EQU    COMMON+23                                FINP0598
ADDIND  EQU    COMMON+24                                FINP0599
PREFIX  EQU    COMMON+25                                FINP0600
WORD    SYN    COMMON+26                                FINP0601
ERROR   SYN    COMMON+27                                FINP0602
I       SYN    PASS                                     FINP0603
J       SYN    CTR                                      FINP0604
FINPT   SYN    FINP                                     FINP0605
        END                                             FINP
```

FINP-18

Function: FNORM

Purpose: To find the magnitude of a 3-dimensional vector.

Calling Sequence:

Y = FNORM(X)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	X	3	X_1, X_2, X_3		
O	FNORM	1			$FNORM = (X_1^2 + X_2^2 + X_3^2)^{\frac{1}{2}}$

Common storages used or required: None

Subroutines required: None

Functions required: SQRTF

Approximate number of storages required:

FNORM

```
*      SYMBOL TABLE                                                      FNOR
CEC2018  FUNCTION FNORM                                                  FNOR
       FUNCTION FNORM(X)                                                 FNOR0000
       DIMENSION X(3)                                                    FNOR0010
     1 FNORM = SQRTF(X(1)**2+X(2)**2+X(3)**2)                            FNOR0020
     3 RETURN                                                            FNOR0030
       END                                                               FNOR
```

Subroutine: GHA

Purpose: To determine the Greenwich Hour Angle of the first point of Aries for a given date and time.

Calling Sequence:

CALL GHA (TSEC, D, GHAN, DA, OMEGA)

Input and Output

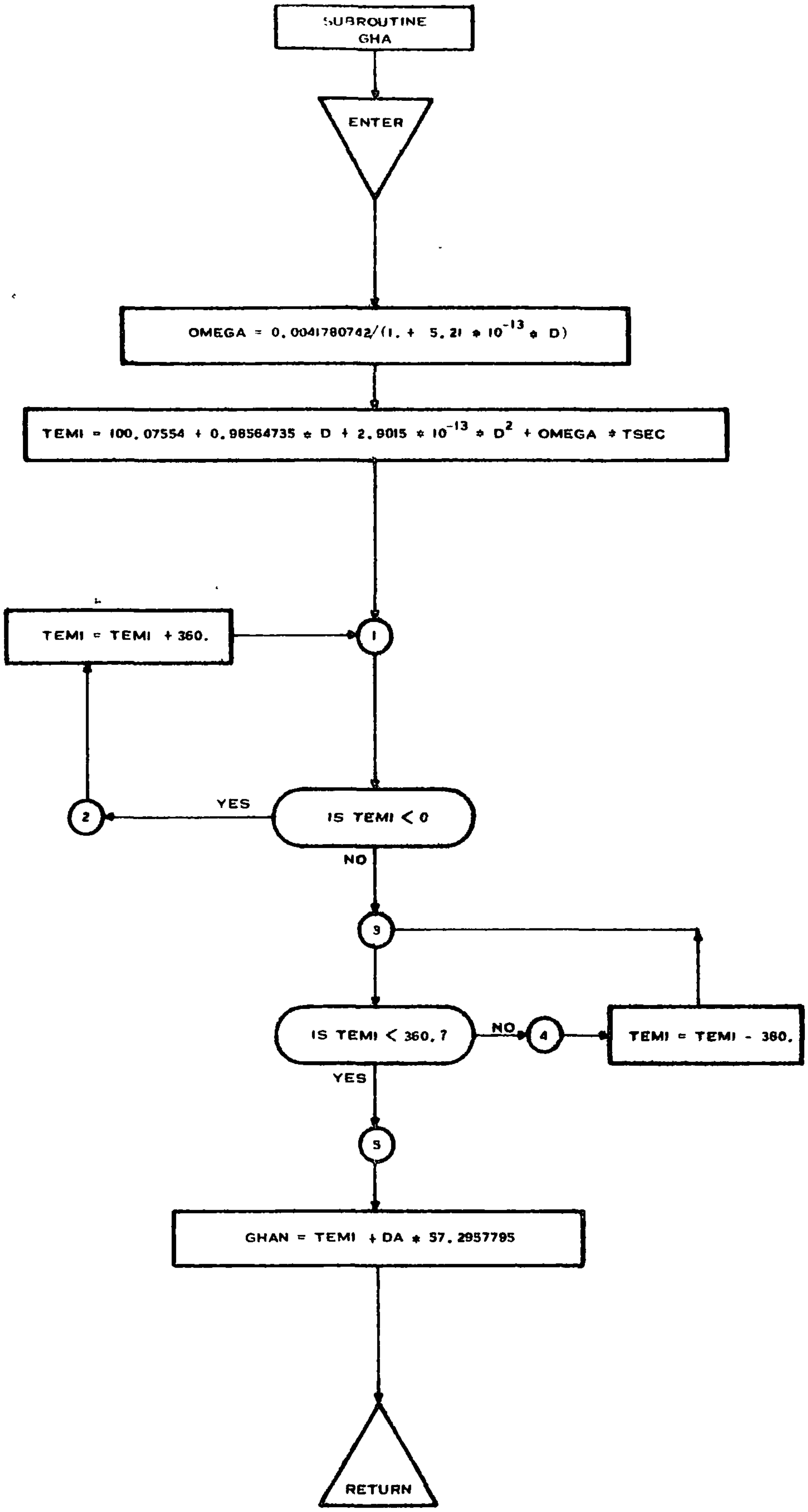
I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TSEC	1		seconds	Fractional part of day from D
I	D	1		days	Whole days from Jan. 1, 1950
O	GHAN	1		degrees	Greenwich Hour Angle,
					$0 \leq GHA < 360$
I	DA	1			Adjustment due to nutation*
O	OMEGA	1	ω_e	degrees/sec	Rate of rotation of the earth
					*DA = EN(2,1) of EN nutation
					matrix from NUTATE

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:


SUBROUTINE GHA
ENTER
OMEGA = 0.0041780742/(1. + 5.21 * 10^-13 * D)
TEMI = 100.07554 + 0.98564735 * D + 2.9015 * 10^-13 * D^2 + OMEGA * TSEC
TEMI = TEMI + 360.
1
IS TEMI < 0
YES
2
NO
3
IS TEMI < 360.?
NO
4
TEMI = TEMI - 360.
YES
5
GHAN = TEMI + DA * 57.2957795
RETURN

GHA

```
*       LABEL
CEC20AG
        SUBROUTINE GHA(TSEC,D,GHAN,DA,OMEGA)
        OMEGA = .0041780742/(1.+5.21E-13*D)
        DD=D
D       DD=DD*(.98564735/360.)
D       DD=DD-INTF(DD)
        DF=DD
        TEM1 = 100.07554+360.*DF     +2.9015E-13*D*D+OMEGA*TSEC
      1 IF (TEM1) 2,3,3
      2 TEM1 = TEM1+360.
        GO TO 1
      3 IF (TEM1-360.) 5,4,4
      4 TEM1 = TEM1-360.
        GO TO 3
      5 GHAN = TEM1+DA*57.2957795
        RETURN
        END
```

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Subroutine: GOTOB

Purpose: It is a logic type of subroutine to read input data cards, set up a sequence for calling subroutine MATSUB, interpret types of measurements being made, and control integration package to perform specific operations (read new data, call MATSUB, stop, etc.) at specified times.

Calling Sequence:

CALL GOTOB (TSTPD)

Input and **Output**

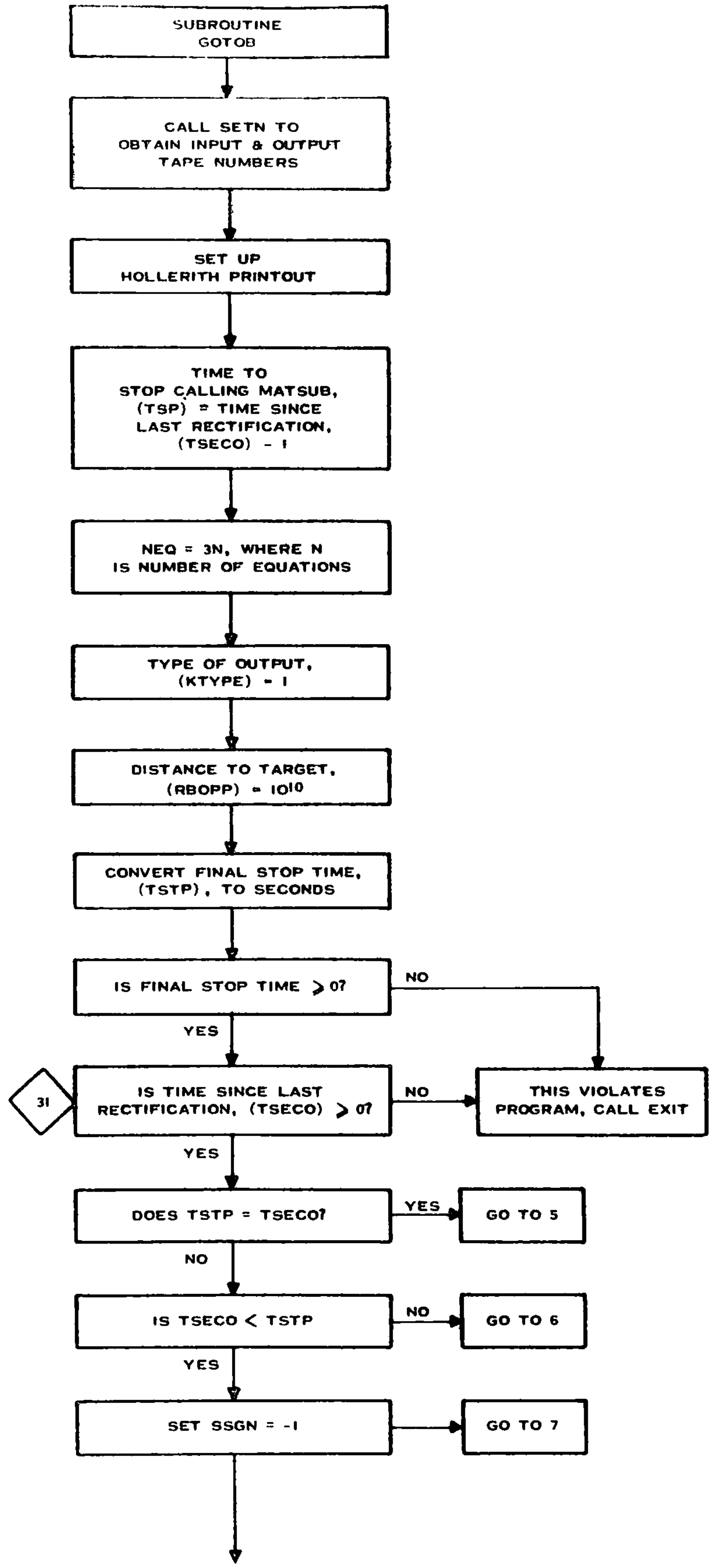
I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TSTPD	1	time	days	Desired stop time in days from
					start

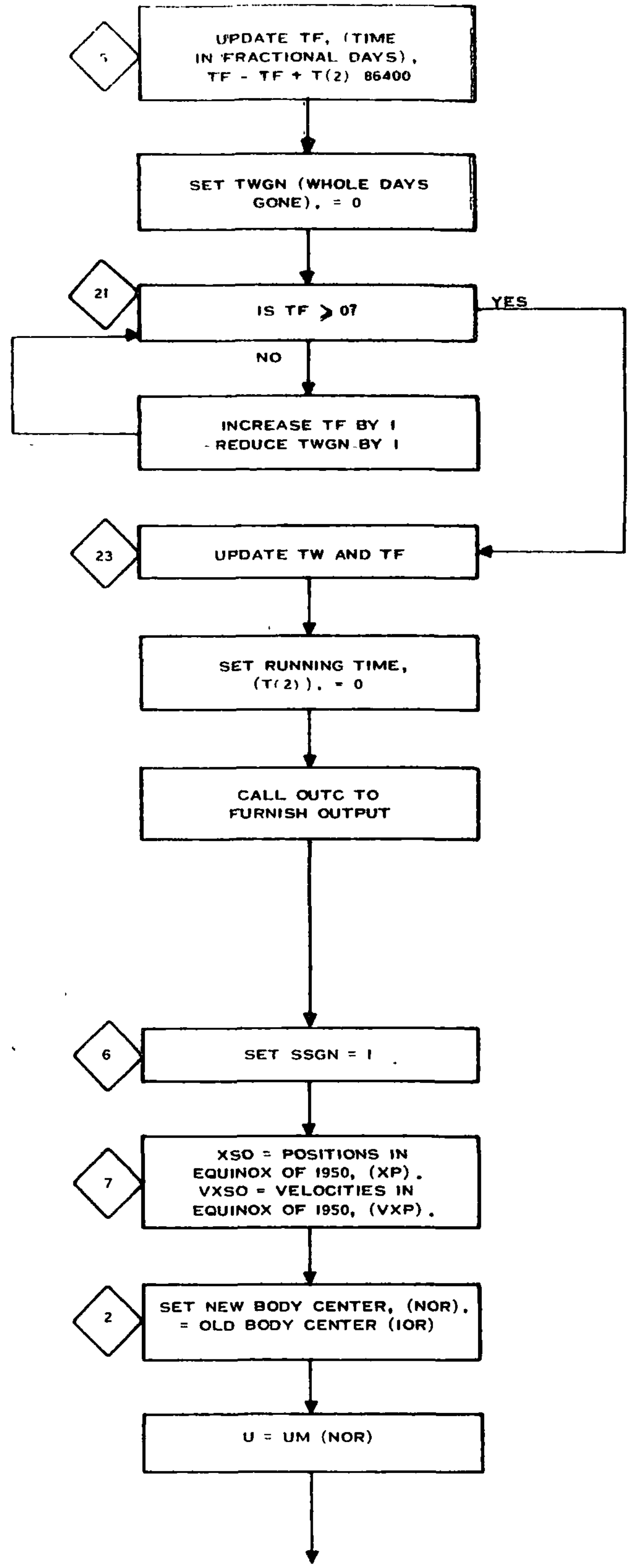
Common storages used or required: T, S, C, IC

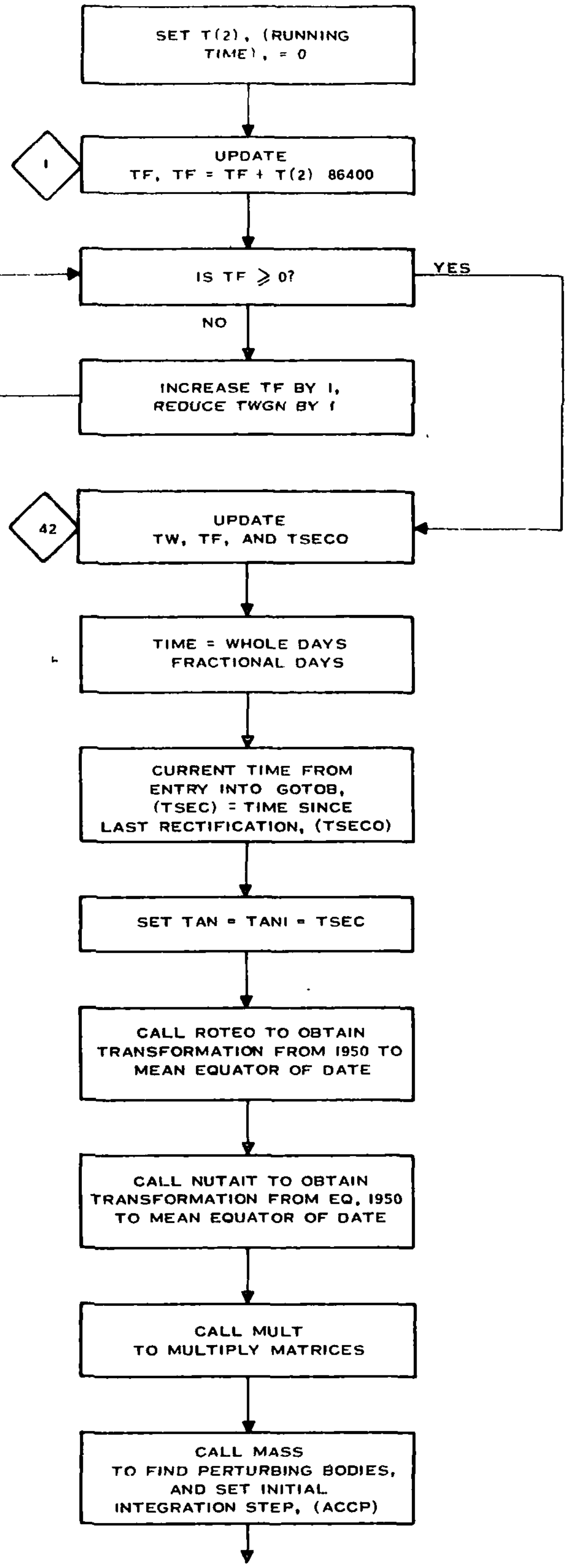
Subroutines required: CHNGP, DE6FN, DE6FP, DE6FP3, DE6FP4, EXIT, MASS, MATSUB, MULT, NUTAIT, OUTC, ROTEQ, SDEC, SETN, SHIFTP, TIMED

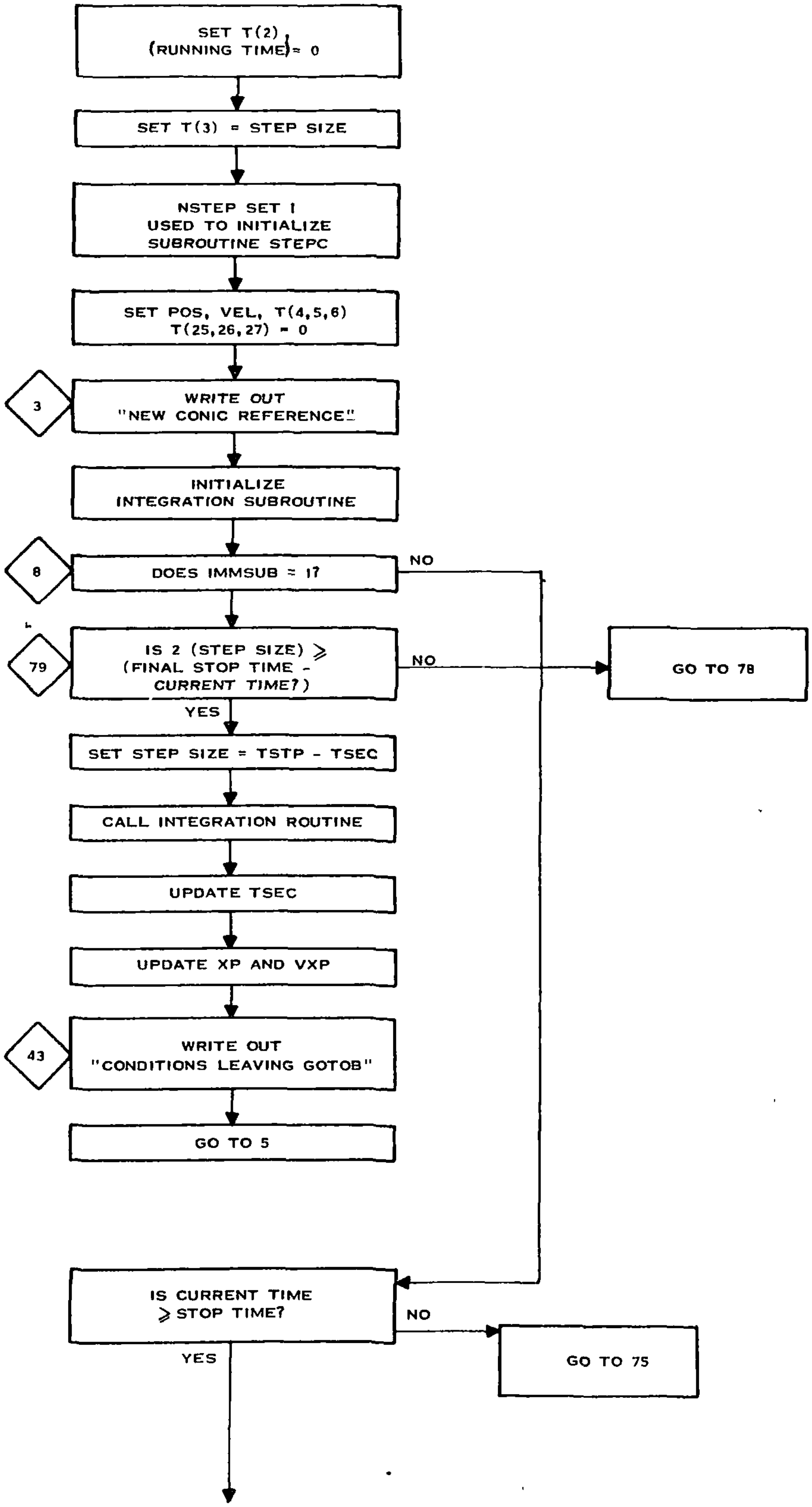
Functions required: FNORM (FIL)(RTN)(SLI)

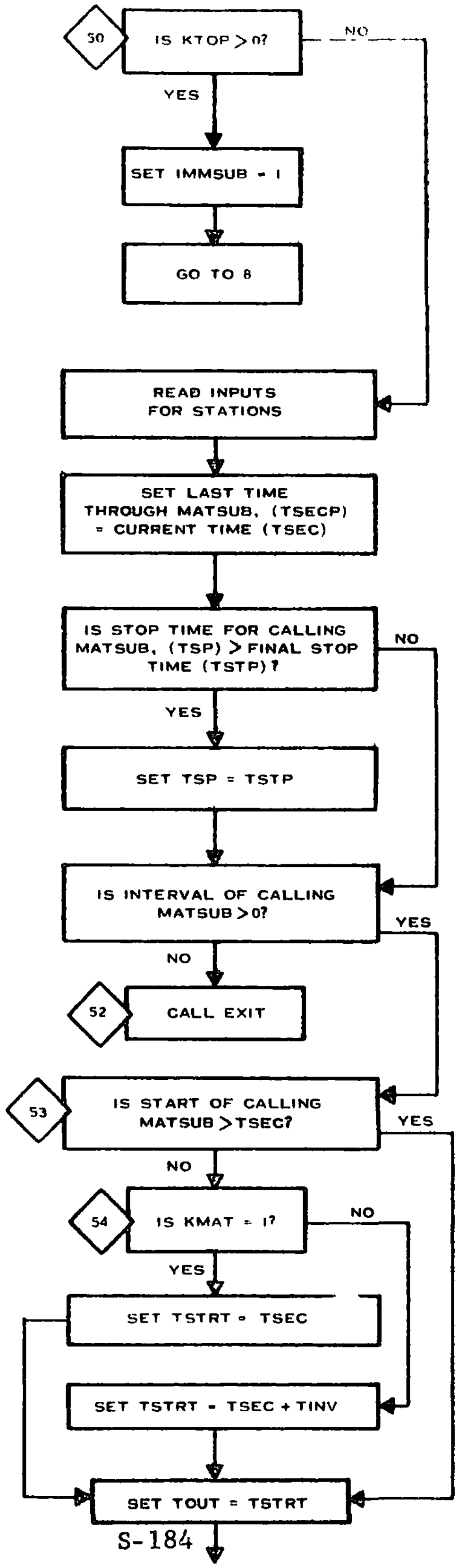
Approximate number of storages required: 1242 DEC

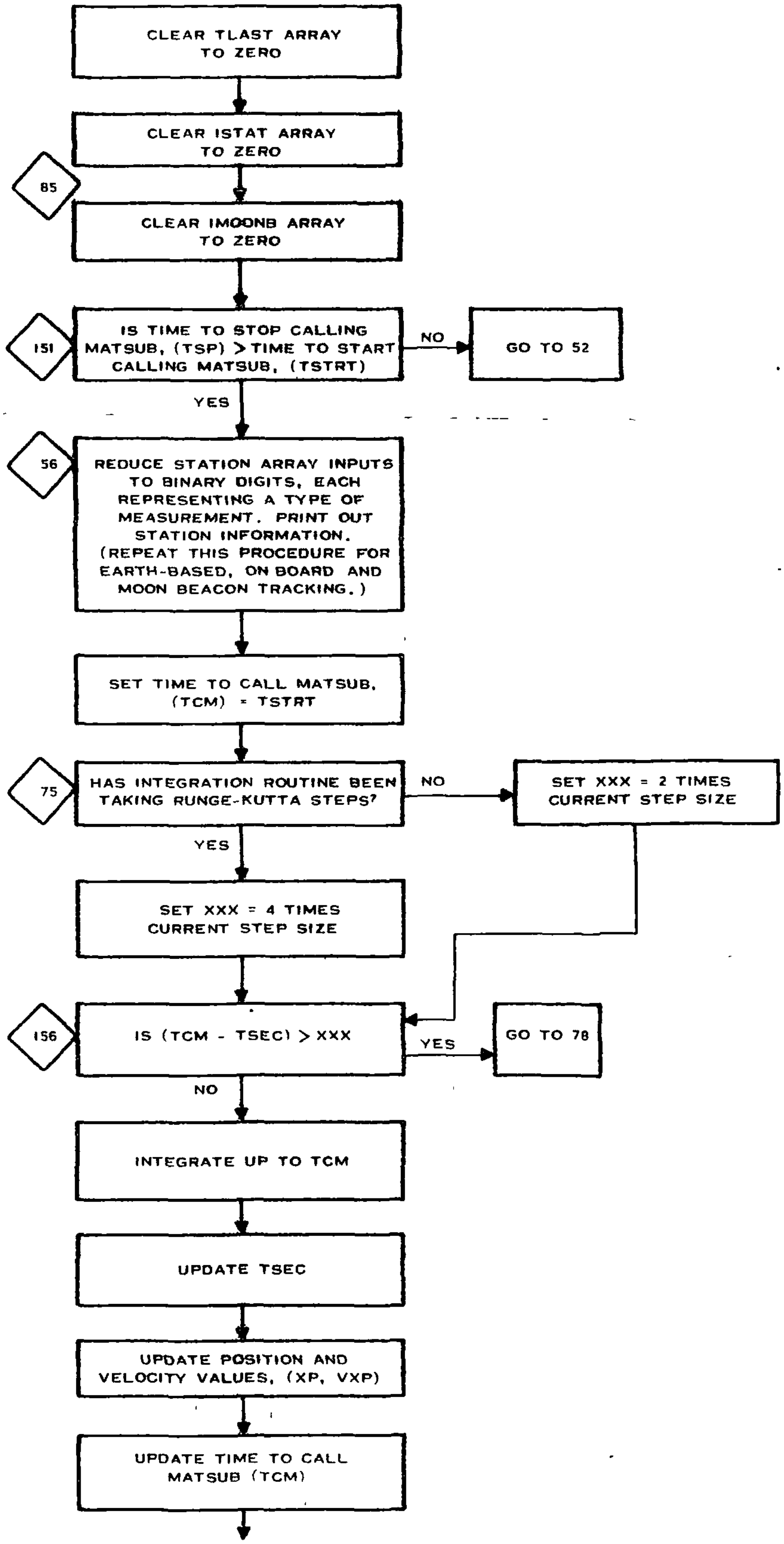

SUBROUTINE GOTOB
CALL SETN TO OBTAIN INPUT & OUTPUT TAPE NUMBERS
SET UP HOLLERITH PRINTOUT
TIME TO STOP CALLING MATSUB, (TSP) = TIME SINCE LAST RECTIFICATION, (TSECO) - 1
NEQ = 3N, WHERE N IS NUMBER OF EQUATIONS
TYPE OF OUTPUT, (KTYPE) = 1
DISTANCE TO TARGET, (RBOPP) = 10^10
CONVERT FINAL STOP TIME, (TSTP), TO SECONDS
IS FINAL STOP TIME > 0?
NO
YES
31
IS TIME SINCE LAST RECTIFICATION, (TSECO) > 0?
NO
THIS VIOLATES PROGRAM, CALL EXIT
YES
DOES TSTP = TSECO?
YES
GO TO 5
NO
IS TSECO < TSTP
NO
GO TO 6
YES
SET SSGN = -1
GO TO 7

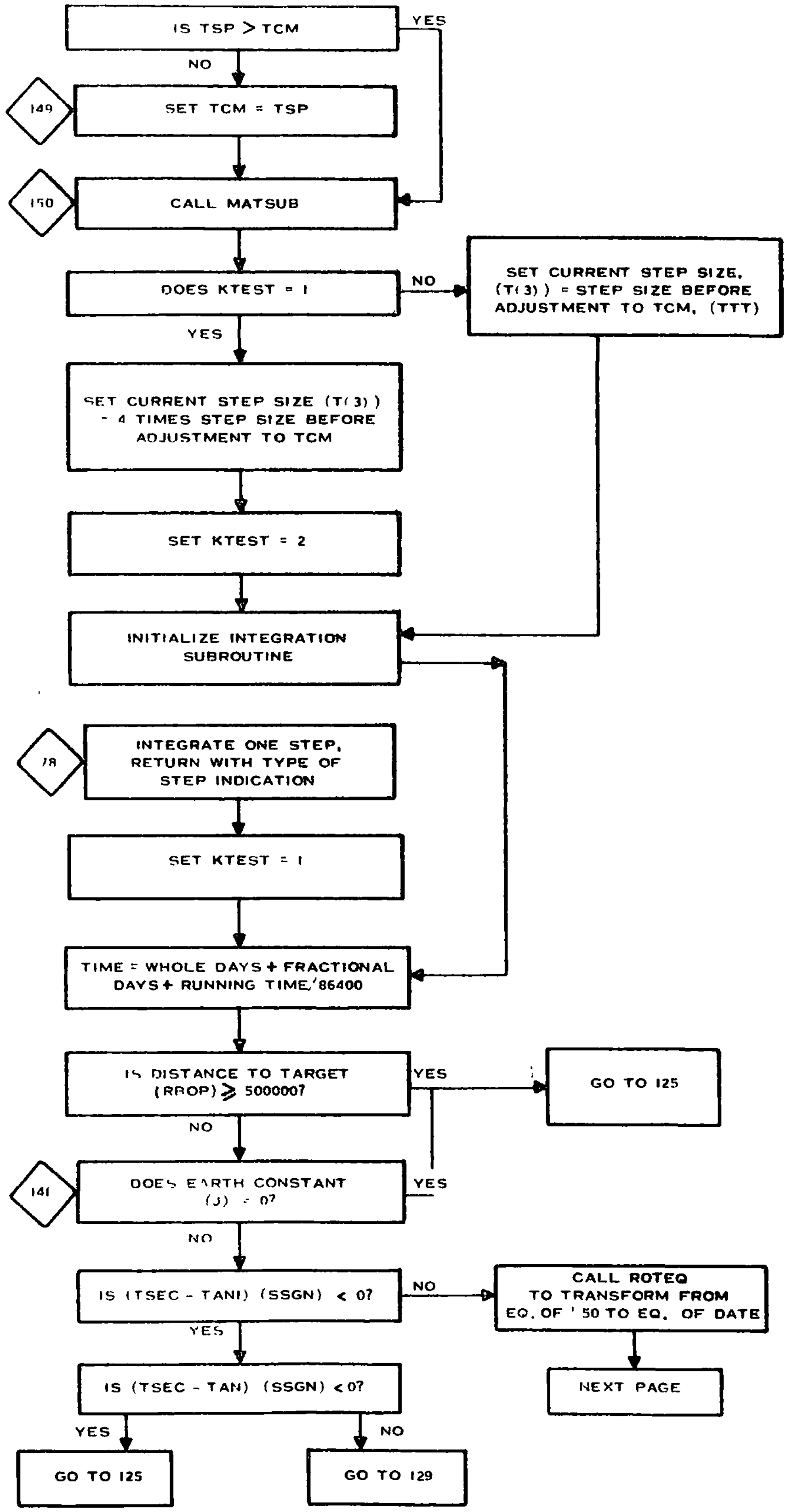

UPDATE TF, (TIME IN FRACTIONAL DAYS), TF - TF + T(2) 86400
SET TWGN (WHOLE DAYS GONE), = 0
21
IS TF > 0?
YES
NO
INCREASE TF BY 1 REDUCE TWGN BY 1
23
UPDATE TW AND TF
SET RUNNING TIME, (T(2)), = 0
CALL OUTC TO FURNISH OUTPUT
6
SET SSGN = 1
7
XSO = POSITIONS IN EQUINOX OF 1950, (XP). VXSO = VELOCITIES IN EQUINOX OF 1950, (VXP).
2
SET NEW BODY CENTER, (NOR), = OLD BODY CENTER (IOR)
U = UM (NOR)


SET T(2), (RUNNING TIME), = 0
1
UPDATE TF, TF = TF + T(2) 86400
IS TF ⩾ 0?
YES
NO
INCREASE TF BY 1, REDUCE TWGN BY 1
42
UPDATE TW, TF, AND TSECO
TIME = WHOLE DAYS FRACTIONAL DAYS
CURRENT TIME FROM ENTRY INTO GOTOB, (TSEC) = TIME SINCE LAST RECTIFICATION, (TSECO)
SET TAN = TANI = TSEC
CALL ROTEO TO OBTAIN TRANSFORMATION FROM 1950 TO MEAN EQUATOR OF DATE
CALL NUTAIT TO OBTAIN TRANSFORMATION FROM EQ. 1950 TO MEAN EQUATOR OF DATE
CALL MULT TO MULTIPLY MATRICES
CALL MASS TO FIND PERTURBING BODIES, AND SET INITIAL INTEGRATION STEP, (ACCP)

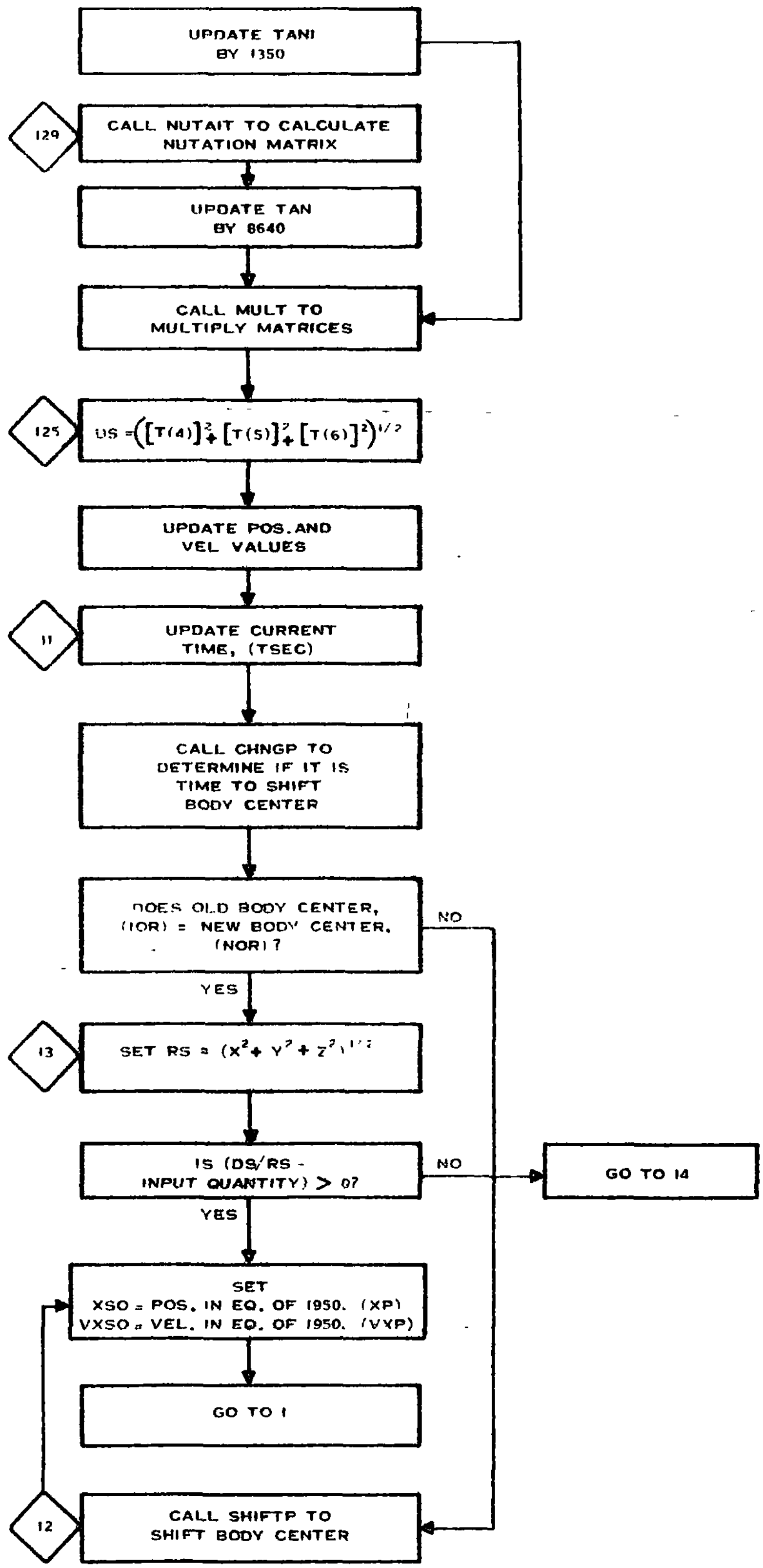

SET T(2) (RUNNING TIME) = 0
SET T(3) = STEP SIZE
NSTEP SET 1 USED TO INITIALIZE SUBROUTINE STEPC
SET POS, VEL, T(4,5,6) T(25,26,27) = 0
3
WRITE OUT "NEW CONIC REFERENCE"
INITIALIZE INTEGRATION SUBROUTINE
8
DOES IMMSUB = 1?
NO
79
IS 2 (STEP SIZE) ≥ (FINAL STOP TIME - CURRENT TIME?)
NO
GO TO 78
YES
SET STEP SIZE = TSTP - TSEC
CALL INTEGRATION ROUTINE
UPDATE TSEC
UPDATE XP AND VXP
43
WRITE OUT "CONDITIONS LEAVING GOTOB"
GO TO 5
IS CURRENT TIME ≥ STOP TIME?
NO
GO TO 75
YES

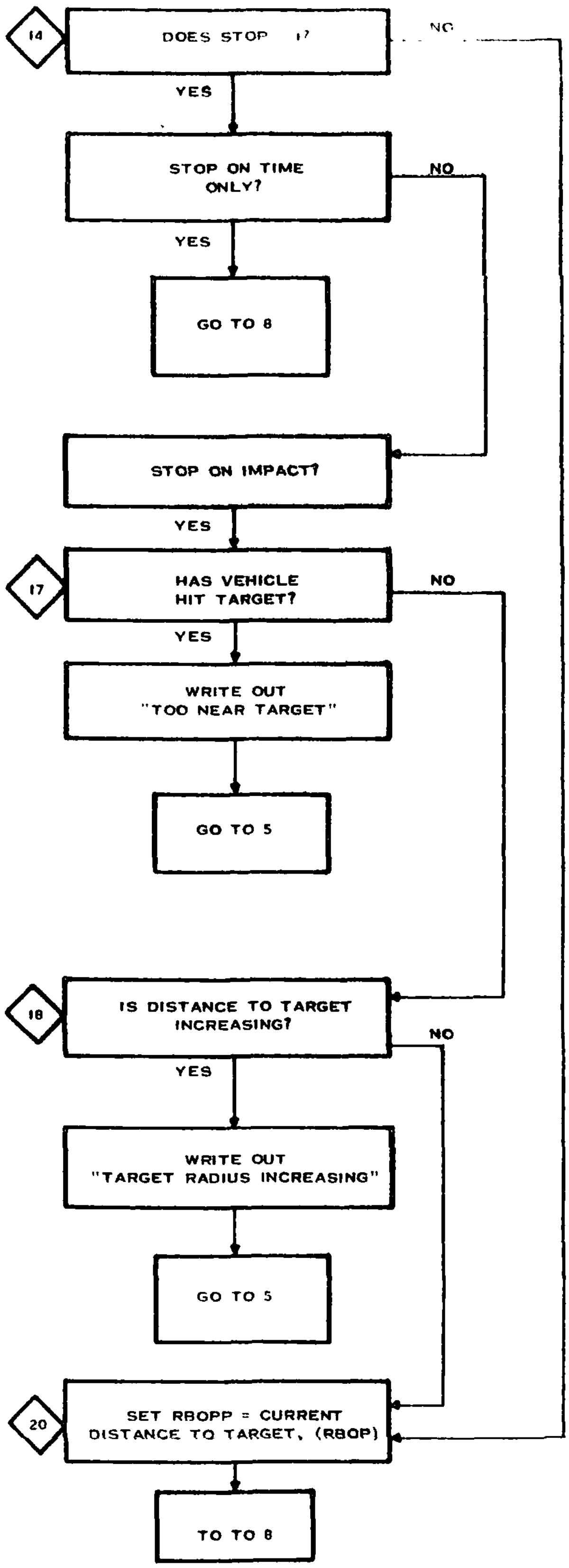

50
IS KTOP > 0?
NO
YES
SET IMMSUB = 1
GO TO 8
READ INPUTS FOR STATIONS
SET LAST TIME THROUGH MATSUB, (TSECP) = CURRENT TIME (TSEC)
IS STOP TIME FOR CALLING MATSUB, (TSP) > FINAL STOP TIME (TSTP)?
NO
YES
SET TSP = TSTP
IS INTERVAL OF CALLING MATSUB > 0?
YES
NO
52
CALL EXIT
53
IS START OF CALLING MATSUB > TSEC?
YES
NO
54
IS KMAT = 1?
NO
YES
SET TSTRT = TSEC
SET TSTRT = TSEC + TINV
SET TOUT = TSTRT


CLEAR TLAST ARRAY TO ZERO
CLEAR ISTAT ARRAY TO ZERO
85
CLEAR IMOONB ARRAY TO ZERO
151
IS TIME TO STOP CALLING MATSUB, (TSP) > TIME TO START CALLING MATSUB, (TSTRT)
NO
GO TO 52
YES
56
REDUCE STATION ARRAY INPUTS TO BINARY DIGITS, EACH REPRESENTING A TYPE OF MEASUREMENT. PRINT OUT STATION INFORMATION. (REPEAT THIS PROCEDURE FOR EARTH-BASED, ON BOARD AND MOON BEACON TRACKING.)
SET TIME TO CALL MATSUB, (TCM) = TSTRT
75
HAS INTEGRATION ROUTINE BEEN TAKING RUNGE-KUTTA STEPS?
NO
SET XXX = 2 TIMES CURRENT STEP SIZE
YES
SET XXX = 4 TIMES CURRENT STEP SIZE
156
IS (TCM - TSEC) > XXX
YES
GO TO 78
NO
INTEGRATE UP TO TCM
UPDATE TSEC
UPDATE POSITION AND VELOCITY VALUES, (XP, VXP)
UPDATE TIME TO CALL MATSUB (TCM)



IS TSP > TCM
YES
NO
149
SET TCM = TSP
150
CALL MATSUB
DOES KTEST = 1
NO
SET CURRENT STEP SIZE, (T(3)) = STEP SIZE BEFORE ADJUSTMENT TO TCM, (TTT)
YES
SET CURRENT STEP SIZE (T(3)) = 4 TIMES STEP SIZE BEFORE ADJUSTMENT TO TCM
SET KTEST = 2
INITIALIZE INTEGRATION SUBROUTINE
78
INTEGRATE ONE STEP, RETURN WITH TYPE OF STEP INDICATION
SET KTEST = 1
TIME = WHOLE DAYS + FRACTIONAL DAYS + RUNNING TIME/86400
IS DISTANCE TO TARGET (RROP) ≥ 5000000?
YES
GO TO 125
NO
141
DOES EARTH CONSTANT (J) = 0?
YES
NO
IS (TSEC - TANI) (SSGN) < 0?
NO
CALL ROTEQ TO TRANSFORM FROM EQ. OF '50 TO EQ. OF DATE
NEXT PAGE
YES
IS (TSEC - TAN) (SSGN) < 0?
YES
NO
GO TO 125
GO TO 129


UPDATE TANI BY 1350
129
CALL NUTAIT TO CALCULATE NUTATION MATRIX
UPDATE TAN BY 8640
CALL MULT TO MULTIPLY MATRICES
125
US = ([T(4)]² + [T(5)]² + [T(6)]²)^1/2
UPDATE POS. AND VEL VALUES
11
UPDATE CURRENT TIME, (TSEC)
CALL CHNGP TO DETERMINE IF IT IS TIME TO SHIFT BODY CENTER
DOES OLD BODY CENTER, (IOR) = NEW BODY CENTER, (NOR)?
NO
YES
13
SET RS = (X² + Y² + Z²)^1/2
IS (DS/RS - INPUT QUANTITY) > 0?
NO
GO TO 14
YES
SET
XSO = POS. IN EQ. OF 1950. (XP)
VXSO = VEL. IN EQ. OF 1950. (VXP)
GO TO 1
12
CALL SHIFTP TO SHIFT BODY CENTER


14
DOES STOP 1?
NO
YES
STOP ON TIME ONLY?
NO
YES
GO TO 8
STOP ON IMPACT?
YES
17
HAS VEHICLE HIT TARGET?
NO
YES
WRITE OUT "TOO NEAR TARGET"
GO TO 5
18
IS DISTANCE TO TARGET INCREASING?
NO
YES
WRITE OUT "TARGET RADIUS INCREASING"
GO TO 5
20
SET RBOPP = CURRENT DISTANCE TO TARGET, (RBOP)
TO TO 8

GOTOB-10

GOTOB

```
*       LABEL                                                                    GOTB
*       SYMBOL TABLE                                                             GOTB
CEC2004  SUBROUTINE GOTOB                                                        GOTB
      SUBROUTINE GOTOB(TSTPD)                                                    GOTB00
      DIMENSION T(1360),S(1000),C(1000),IC(300),VX(3)                            GOTB00
      DIMENSION VXP(3),XP(3),AE(3),AO(3),AP(3),PO(22),VE(22),CA(3)               GOTB00
     1,X(3),UM(6),RAD(6),RBOP(6),                      IMOONB(10)                GOTB00
     1,VKB(6),RBO(6),AN(3,3),EN(3,3),EA(3,3)                                     GOTB00
     1,XSO(3),VXSO(3)                                                            GOTB00
      DIMENSION BP(3,3),TLAST(4,6)                                               GOTB00
      DIMENSION ISTAT(20),IONB(6),IMB(10),KS(4,20),KONB(4,6),KMB(4,10)           GOTB00
     1,ISA(20)                                                                   GOTB00
      DIMENSION SMEAS(4),BNAME(6),BEACNO(10),TOBM(4)                             GOTB00
      COMMON T,S,C,IC                                                            GOTB00
      EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM)                                     GOTB01
      EQUIVALENCE (S(1),UM),(S(11),VJ),(S(12),H),(S(13),D),(S(14),RE)            GOTB01
     1,(S(900),RAD),(C(9),SSGN),(C(10),TW),(C(11),TF),(C(12),TSECO)              GOTB01
     2,(C(13),TP1),(C(14),TP2),(C(15),XP),(C(18),VXP),(C(21),AE)                 GOTB01
     3,(C(24),AO),(C(27),CA),(C(30),TSEC),(C(33),X),(C(36),VX)                   GOTB01
     5,(C(46),XSO),(C(49),VXSO),         (C(55),U),(C(56),VKB),(C(62),PO)        GOTB01
     6,(C(84),VE),(C(106),RBO),(C(112),RBOP),(C(120),EN),(C(129),EA)             GOTB01
     7,(C(138),AN),(C(171),BP)                                                   GOTB01
     8,(IC(1),N),(IC(2),IOR),(IC(3),NOR),(IC(4),KSTP),(IC(5),KTYPE)              GOTB01
     9,(IC(6),ITARG),(IC(7),KOUT),(IC(220),IMOONB)                               GOTB02
      EQUIVALENCE (IC(10),ISA),(IC(110),IONB),(IC(140),IMB),(IC(30),KS)          GOTB02
     1,(IC(116),KONB),(IC(150),KMB),(C(650),TPRINT),(IC(9),LPRINT)               GOTB02
     2,(C(651),TSTRT),(IC(190),LSTAT),(IC(191),LONB),(IC(192),LMB)               GOTB02
     3,(IC(8),IMMSUB),(C(622),TLAST),(C(649),TSECP),(IC(194),ISTAT)              GOTB02
     4,(C(647),TOUT)            ,(IC(231),NSTEP),(S(485),ENKE)                   GOTB02
      CALL SETN(NIN,NOUT)                                                        GOTB02
C SMEAS MEANS TYPE OF STATION MEASUREMENT                                         GOTB02
      SMEAS(1) = 6HRANGE.                                                        GOTB02
      SMEAS(2)= 6HAZ-ELE                                                         GOTB02
      SMEAS(3)= 6HDIRCOS                                                         GOTB03
      SMEAS(4) = 6HRNGRAT                                                        GOTB03
C BNAME MEANS BODY NAME                                                           GOTB03
      BNAME(1) = 6HEARTH.                                                        GOTB03
      BNAME(2) = 6HMOON..                                                        GOTB03
      BNAME(3) = 6HSUN...                                                        GOTB03
      BNAME(4) = 6HVENUS.                                                        GOTB03
      BNAME(5) = 6HMARS..                                                        GOTB03
      BNAME(6) = 6HJUPITR                                                        GOTB03
C TOBM MEANS TYPE OF ONBOARD MEASUREMENT                                          GOTB03
      TOBM(1) = 6HRANGE.                                                         GOTB04
      TOBM(2) = 6HRTASCN                                                         GOTB04
      TOBM(3) = 6HDECLIN                                                         GOTB04
      TOBM(4) = 6HRNGRAT                                                         GOTB04
      KTEST=2                                                                    GOTB04
      TSP =TSECO-1.                                                              GOTB04
      LPRINT=1                                                                   GOTB04
      KMAT=1                                                                     GOTB04
```

(Cont'd)

```
      K=I+NEQV                                                          GOTB1320
      XP(I)=X(I)+T(I+3)                                                 GOTB1330
      VXP(I)=VX(I)+T(K)                                                 GOTB1340
   43 CONTINUE                                                          GOTB1350
      WRITE OUTPUT TAPE NOUT,39                                         GOTB1360
   39 FORMAT(25H0CONDITIONS LEAVING GOTOB)                              GOTB1370
      WRITE OUTPUT TAPE NOUT,750                                        GOTB1380
 750  FORMAT(21H STOPPED ON TIME STOP)                                  GOTB1390
      GO TO 5                                                           GOTB1400
   10 CONTINUE                                                          GOTB1410
      IF(TSEC-TSP)75,50,50                                              GOTB1420
   50 CONTINUE                                                          GOTB1430
C     KTOP IS SET +1 ON LAST CARD OF SEQUENCE FOR NORMAL TIME STOP      GOTB1435
C     OF GOTOB                                                          GOTB1436
      IF(KTOP)51,51,80                                                  GOTB1440
   80 CONTINUE                                                          GOTB1450
      IMMSUB=1                                                          GOTB1460
      GO TO 8                                                           GOTB1470
   51 READ INPUT TAPE NIN,700,ISA,IONB,IMB                              GOTB1480
  700 FORMAT(20I2,4X,6I2,4X,10I2)                                       GOTB1490
      READ INPUT TAPE NIN,701,KTOP,TSTR,TSP1,TINT,TPRIN                 GOTB1500
  701 FORMAT(1X,I1,4E17.8)                                              GOTB1510
      WRITE OUTPUT TAPE NOUT,708,KTOP,TSTR,TSP1,TINT,TPRIN              GOTB1520
  708 FORMAT(/7H0 KTOP=,I1,7H  TSTR=,1E17.8,7H  TSP1=,1E17.8,7H  TINT=  GOTB1530
     1,1E17.8,8H  TPRIN=,1E17.8)                                        GOTB1540
      TSECP=TSEC                                                        GOTB1550
C     SUBROUTINE TIMED CONVERTS INPUT TIMES FROM FORMAT                 GOTB1555
C     (DAYS HOURS).(MIN SEC)                                            GOTB1556
      CALL TIMED(TSTR,TSTRT)                                            GOTB1560
C     TSTRT =START TIME FOR CALLING MATSUB                              GOTB1570
      CALL TIMED(TSP1,TSP)                                              GOTB1580
C     TSP   =STOP TIME AND INDICATOR FOR NEW DATA                       GOTB1590
      STOP=KTOP                                                         GOTB1600
      IF(TSP-TSTP)101,101,100                                           GOTB1610
  100 TSP=TSTP                                                          GOTB1620
  101 CONTINUE                                                          GOTB1630
      CALL TIMED(TINT,TINV)                                             GOTB1640
C     TINV  =INTERVAL OF CALLING MATSUB                                 GOTB1650
      CALL TIMED(TPRIN,TPRINT)                                          GOTB1660
C     TPRINT=PRINT INTERVAL FOR MATSUB                                  GOTB1670
      IF(TINV) 52,52,53                                                 GOTB1680
   52 WRITE OUTPUT TAPE NOUT,702                                        GOTB1690
  702 FORMAT(26H0YOU BOTCHED UP INPUT DATA)                             GOTB1700
      CALL EXIT                                                         GOTB1710
   53 CONTINUE                                                          GOTB1720
      IF(TSTRT-TSEC) 54,54,158                                          GOTB1730
   54 CONTINUE                                                          GOTB1740
      GO TO (152,153),KMAT                                              GOTB1750
  152 TSTRT=TSEC                                                        GOTB1760
      TOUT=TSEC                                                         GOTB1770
      GO TO 158                                                         GOTB1780
```

GOTOB-(Con't)

```
  153 TSTRT=TSEC+TINV
      TOUT=TSEC+TPRINT
      GO TO 151
  158 CONTINUE
      TOUT=TSTRT
      DO 84 I=1,4
      DO 84 J=1,6
      TLAST(I,J) =0.
   84 CONTINUE
      DO 85 I=1,20
      ISTAT(I) =0
   85 CONTINUE
      DO 86 I=1,10
      IMOONB(I)=0,
   86 CONTINUE
  151 CONTINUE
      IF(TSP-TSTRT)  52,52,55
   55 CONTINUE
      LSTAT=1
      LONB=1
      LMB=1
      L=122
      DO 61 I=1,20
      L=L+15
      IF (ISA(I)) 61,61,56
   56 CONTINUE
      WRITE OUTPUT TAPE NOUT,709,I,S(L)
  709 FORMAT (17H  STATION NUMBER ,I2,2H ,,A6,2H ,,10H  MEASURES)
      BBB = ISA(I)
      LSTAT=2
      DO 60 J=1,4
      CCC=BBB/2.
      DDD=INTF(CCC)
      IF(CCC-DDD) 57,57,58
   57 CONTINUE
C     THE  KS, KONB, AND KMB ARRAYS ARE  COLUMN WISE ARRANGED IN BINARY
C     FOR INDICATING TYPE MEASUREMENTS MADE .KS FOR EARTH BASED TRACKING
C     KONB FOR ONBOARD TRACKING, KMB FOR LUNAR BEACONS.
      KS(J,I)=0
      GO TO 59
   58 CONTINUE
      KS(J,I)=1
      WRITE OUTPUT TAPE NOUT,703,SMEAS(J)
  703 FORMAT (1H A6)
   59 CONTINUE
      BBB=DDD
   60 CONTINUE
   61 CONTINUE
      DO 62 I=1,6
      IF (IONB(I))  62,62,63
   63 CONTINUE
```

GOTOB (Cont'd)

```
      WRITE OUTPUT TAPE NOUT,704,BNAME(I)
  704 FORMAT (28H  VEHICLE TO CELESTIAL BODY ,A6,10H  MEASURES)
      LONB=2
      BBB=IONB(I)
      DO  68 J=1,4
      CCC=BBB/2.
      DDD=INTF(CCC)
      IF(CCC-DDD) 64,64,65
   64 CONTINUE
      KONB(J,I)=0
      GO TO 67
   65 CONTINUE
      KONB(J,I)=1
      WRITE OUTPUT TAPE NOUT,703,TOBM(J)
   67 CONTINUE
      BBB=DDD
   68 CONTINUE
   62 CONTINUE
      DO  74 I=1,10
      IF (IMB(I))  74,74,69
   69 CONTINUE
      WRITE OUTPUT TAPE NOUT,706,I
  706 FORMAT (33H  VEHICLE TO LUNAR BEACON NUMBER ,I2,10H  MEASURES)
      LMB=2
      BBB=IMB(I)
      DO  73 J=1,4
      CCC=BBB/2.
      DDD=INTF(CCC)
      IF(CCC-DDD) 70,70,71
   70 CONTINUE
      KMB(J,I)=0
      GO TO 72
   71 CONTINUE
      KMB(J,I)=1
      WRITE OUTPUT TAPE NOUT,703,TOBM(J)
   72 CONTINUE
      BBB=DDD
   73 CONTINUE
   74 CONTINUE
      TCM=TSTRT
   75 CONTINUE
      IF(ACCUM) 154,154,155
  155 XXX=4.*T(3)
      GO TO 156
  154 XXX=2.*T(3)
  156 CONTINUE
      IF(TCM-TSEC-XXX) 76,76,78
   76 CONTINUE
      TT=TCM-TSEC
      TTT=T(3)
      T(3)=TT
```

GOTOB (Cont'd)

```
      DO 77 I=1,4                                                       GOTB281
      CALL DE6FP3(TT)                                                   GOTB282
   77 CONTINUE                                                          GOTB283
      TSEC = TSECO + T(2)                                               GOTB284
      DO 157 I=1,3                                                      GOTB285
      K=I+NEQV                                                          GOTB286
      XP(I) = X(I)+T(I+3)                                               GOTB287
  157 VXP(I) = VX(I)+T(K)                                               GOTB288
      TCM=TCM+TINV                                                      GOTB289
      IF(TCM-TSP) 150,150,149                                           GOTB290
  149 TCM=TSP                                                           GOTB291
  150 CONTINUE                                                          GOTB292
      CALL MATSUB                                                       GOTB293
      KMAT=2                                                            GOTB294
      IF(KTEST-1)81,81,82                                               GOTB295
  81  T(3)=4.*TTT                                                       GOTB296
      KTEST=2                                                           GOTB297
      GO TO 87                                                          GOTB298
  82  T(3)=TTT                                                          GOTB298
  87  CONTINUE                                                          GOTB299
      CALL DE6FN(-1,1,T,NEQ,SDEC,3,4,1.E-9,3.456,172800.,86400.)        GOTB300
      GO TO 113                                                         GOTB301
   78 CONTINUE                                                          GOTB302
      CALL DE6FP1(ACCUM)                                                GOTB303
      KTEST=1                                                           GOTB303
  113 TIME = TW+TF + T(2)/86400.                                        GOTB304
      IF (RBOP-500000.) 141,125,125                                     GOTB305
  141 IF (VJ) 127,125,127                                               GOTB306
  127 IF((TSEC-TAN1)*SSGN)142,126,126                                   GOTB307
  142 IF((TSEC-TAN)*SSGN)125,129,129                                    GOTB308
  126 CALL ROTEQ(TIME,EA)                                               GOTB309
      TAN1=TAN1+1350.*SSGN                                              GOTB310
      GO TO 140                                                         GOTB311
  129 CALL NUTAIT (TIME,CM,CRUD,DDC,EN,EPSIL)                           GOTB312
      TAN=TAN+8640.*SSGN                                                GOTB313
  140 CALL MULT(EN,EA,AN,0)                                             GOTB314
  125 CONTINUE                                                          GOTB315
      DS=FNORM(T(4))                                                    GOTB316
      DO 11 I=1,3                                                       GOTB317
      K=I+NEQV                                                          GOTB318
      XP(I)=X(I)+T(I+3)                                                 GOTB319
      VXP(I)=VX(I)+T(K)                                                 GOTB320
   11 CONTINUE                                                          GOTB321
      TSEC = TSECO + T(2)                                               GOTB322
      CALL CHNGP(NOR,IOR,RBOP)                                          GOTB323
      IF(IOR-NOR)12,13,12                                               GOTB324
   13 CONTINUE                                                          GOTB325
      RS=FNORM(X)                                                       GOTB326
      IF(DS/RS-ENKE)14,15,15                                            GOTB327
   15 CONTINUE                                                          GOTB328
      DO 16 I=1,3                                                       GOTB329
```

GOTOB (Cont'd)

```
      XSO(I)=XP(I)
      VXSO(I)=VXP(I)
   16 CONTINUE
      GO TO 1
   12 CONTINUE
      CALL SHIFTP(IOR,NOR,U,UM,XP,VXP,PO,VE,TF,TW,T(2))
      GO TO 15
   14 CONTINUE
      ITARG = ITARG
      IF(STOP-1.)20,83,83
  83  CONTINUE
      KSTP=KSTP
      GO TO (17,18,8),KSTP
   17 CONTINUE
      IF(RBOP(ITARG)-RAD(ITARG))19,19,18
   19 CONTINUE
      WRITE OUTPUT TAPE NOUT,39
      WRITE OUTPUT TAPE NOUT,34,RBOP(ITARG),RAD(ITARG),ITARG
   34 FORMAT(21H TOO NEAR TARG  RBOP=,1E17.8,5H RAD=,1E17.8,6H TARG=,I1)
      KTYPE=2
      GO TO 5
   18 CONTINUE
      IF(RBOP(ITARG)-RBOPP)20,35,35
   35 CONTINUE
      WRITE OUTPUT TAPE NOUT,39
      WRITE OUTPUT TAPE NOUT,36,RBOP(ITARG),RBOPP,ITARG
   36 FORMAT(29H TARG RADIUS INCREASING RBOP=,1E17.8,7H RBOPP=,1E17.8,6H
     1 TARG=,I1)
      KTYPE=2
      GO TO 5
   20 CONTINUE
      RBOPP=RBOP(ITARG)
      GO TO 8
      END
```

Subroutine: GOTOR

Purpose: To solve Kepler's equation for incremental excentric anomaly on a conic section given the incremental mean anomaly (time).

Calling Sequence:

CALL GOTOR (K, VM, C, F, E1)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	K	1	K		Orbit type (1) Elliptic
					(2) Hyperbolic
I	VM	1	ΔM	RAD	Incremental mean anomaly
I	C	2	$C_1 C_2$		Kepler's equation coefficients
O	F	4	F		Trigonometric functions of
					incremental eccentric anomaly
I/O		1	ΔE	RAD	Incremental eccentric anomaly

Common storages used or required: None

Subroutines required: None

Functions required: SIN, COS, EXP

Approximate number of storages required: 476 DEC

Discussion (See subroutine STEPC)

The incremental eccentric anomaly, φ, is implicitly expressed as a function of incremental mean anomaly, ΔM, and conic coefficients, C_1 and C_2, by transcendental equations

$$\Delta M = (\varphi - \sin\varphi) + C_1 \sin\varphi - C_2(\cos\varphi - 1) \qquad \underline{\text{elliptical}}$$

$$\Delta M = (\sinh\varphi - \varphi) + C_1 \sinh\varphi + C_2(\cosh\varphi - 1) \qquad \underline{\text{hyperbolic}}$$

The two equations above may be written as a single equation

$$\Delta M = f_1(\varphi) + C_1 f_3(\varphi) + C_2 f_4(\varphi)$$

if the convention below is adopted.

For the elliptical case, let

$$f_1(\varphi) = \varphi - \sin\varphi$$

$$f_2(\varphi) = 1 - \cos\varphi$$

$$f_3(\varphi) = \sin\varphi$$

$$f_4(\varphi) = \cos\varphi$$

and for the hyperbolic case, let

$$f_1(\varphi) = \sinh\varphi - \varphi$$

$$f_2(\varphi) = \cosh\varphi - 1$$

$$f_3(\varphi) = \sinh\varphi$$

$$f_4(\varphi) = \cosh\varphi$$

When $|\varphi| \leq 1$, numerical accuracy requires that $f_1(\varphi)$ and $f_2(\varphi)$ be computed by truncated series expansions. Otherwise, the computer library functions SINF, COSF and EXPF are used in the computation.

The slope of the function

$$F(\varphi) = f_1(\varphi) + C_1 f_3(\varphi) + C_2 f_4(\varphi)$$

at φ is seen to be

$$F'(\varphi) = f_2(\varphi) + C_1 f_4(\varphi) + C_2 f_3(\varphi).$$

$F(\varphi)$ is a monotonically increasing function of φ so that any solution of the equation $F(\varphi) = \Delta M$ is obviously unique. Newton's method of iteration is used. That is, letting φ_n be the n^{th} estimate of φ, the (n+1)st estimate is calculated from

$$\varphi_{n+1} = \varphi_n + \frac{\Delta M - F(\varphi_n)}{F'(\varphi_n)}$$

The iteration is halted and φ is said to be φ_n when

$$\left| \frac{\varphi_{n+1} - \varphi_n}{\varphi_{n+1} + \varphi_n} \right| \leq 3. \times 10^{-8}$$

or when n = 20, whichever occurs first. The $f_i(\varphi)$ as well as φ are output from GOTOR to avoid their re-computation outside the subroutine.

GOTOR

```
*     LABEL
CEC2091  NEW GOTOR
      SUBROUTINE GOTOR(K,VM,C,F,E1)
      DIMENSION C(2),F(4)
      NMAX=20
      N=0
      GO TO (1,2),K
    1 CONTINUE
C     FIRST GUESS IS OBTAINED FOR ELLIPTICAL CASE
    8 CONTINUE
      IF(E1-1.)30,31,31
   30 CONTINUE
      D2=E1*E1
      F(1)=E1*D2*(0.16666667E+00-D2*(0.833333333E-02-D2*(0.198412698E-03
     1-D2*(0.275573192E-05-D2*0.250521083E-07))))
      F(2)= D2*(0.5-D2*(0.41666667E-1-D2*(0.13888889E-2-D2*(0.24801587E-
     14-D2*0.27557319E-6))))
      F(3)=E1-F(1)
      F(4)=1.-F(2)
      GO TO 3
   31 CONTINUE
      F(3)=SINF(E1)
      F(4)=COSF(E1)
      F(1)=E1-F(3)
      F(2)=1.-F(4)
      GO TO 3
    2 CONTINUE
C     FIRST GUESS IS OBTAINED FORHYPERBOLIC CASE
    9 CONTINUE
      IF(E1-1.)32,33,33
   32 CONTINUE
      D2=E1*E1
      F(1) =E1*D2*(0.16666667E+00+D2*(0.833333333E-02+D2*(0.198412698E-03
     1+D2*(0.275573192E-05+D2*0.250521083E-07))))
      F(2)  =D2*(0.5+D2*(0.41666667E-1+D2*(0.13888889E-2+D2*(0.24801587E
     1-4+D2*0.27557319E-6))))
      F(3)=E1+F(1)
      F(4)=1.+F(2)
      GO TO 3
   33 CONTINUE
      EX=.5*EXPF(E1)
      OX=.25/EX
      F(3)=EX-OX
      F(4)=EX+OX
      F(1)=F(3)-E1
      F(2)=F(4)-1.
    3 CONTINUE
      CM=F(1)+C(1)*F(3)+C(2)*F(2)
      DM=F(2)+C(1)*F(4)+C(2)*F(3)
      DE=(VM-CM)/DM
      ERROR=DE
```

GOTOR(Cont'd)

```
      AB=ABSF(DE)                                                       GOTR050
      IF(AB-1.)10,10,11                                                 GOTR051
   11 DE=DE/AB                                                          GOTR052
   10 E2=E1+DE                                                          GOTR053
      IF(ABSF((E2-E1)/(E2+E1))-3.E-8)4,4,5                              GOTR054
    5 CONTINUE                                                          GOTR055
      IF(N-NMAX)6,7,7                                                   GOTR056
    7 CONTINUE                                                          GOTR057
      GO TO 4                                                           GOTR058
    6 CONTINUE                                                          GOTR059
      N=N+1                                                             GOTR060
      E1=E2                                                             GOTR061
      GO TO (8,9),K                                                     GOTR062
    4 CONTINUE                                                          GOTR063
      RETURN                                                            GOTR064
      END                                                               GOTR
```

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Subroutine: GUID

Purpose: To determine the times when guidance corrections are to be made. Compute the correction to be made and determine the error in the correction being performed. Then the subroutine updates both the state and guidance covariance matricies for the correction.

Calling Sequence:

CALL GUID

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities returned to common

Common storages used or required: T, S, C, IC

Subroutines required: INV3, MARX, SETN, SQRTF, TIMED

Functions required: (FIL)(STH)

Approximate number of storages required: 1022 DEC

Theory for updating covariance matricies for a guidance correction.

The following quantities will be used in the derivation to follow.

PAR(to) = covariance matrix of errors at injection

$\hat{x}(t)$ = estimate of state at time t

$\tilde{x}(t)$ = error in estimate of state at time t

$x(t)$ = true state at time t

$E(\tilde{x}\,\tilde{x}^T) = P(t)$ = covariance matrix of error in estimate at time t

The quantities $\hat{x}$, $\tilde{x}$, and x are all considered as deviation quantities from the nominal trajectory which arrives at the target in the desired manner.

Using linear theory for the deviations

$$x(t) = \Phi\, X(to)$$

where Φ is the transition matrix relating unit deviations in injection errors to deviations at time t. Then

$$PAR(t) = E(x(t)\; x(t)^T) = \Phi\, PAR(to)\, \Phi^T \qquad (1)$$

where PAR(t) is the covariance matrix of deviations from the nominal trajectory prior to any midcourse corrections.

Assuming we have a transition matrix which relates the vehicle state at time (t) to some desired conditions at the target, a guidance law may be derived to meet the desired end conditions. The two laws which the program can solve are: (1) Fixed Time of Arrival, and (2) Constant Energy with Respect to the Target.

$$\underline{C}(T) = (A_1 \ A_2) \begin{pmatrix} \underline{x}(t) \\ \dot{\underline{x}}(t) \end{pmatrix} \qquad (2)$$

$$3 \times 1 \quad 3 \times 6 \quad 6 \times 1$$

where $\underline{C}(T)$ = target conditions at time T

$(A_1 \ A_2)$ = transition matrix

For Fixed Time of Arrival:

$$\underline{C}(T) = \begin{pmatrix} x(T) \\ y(T) \\ z(T) \end{pmatrix}$$

$$3 \times 1 \quad 3 \times 1$$

For Constant Energy with Respect to the Target:

$$\underline{C}(T) = \begin{pmatrix} \delta \ B \cdot T(T) \\ \delta \ B \cdot R(T) \\ \delta \ VINF \end{pmatrix}$$

The guidance law is derived such that the estimate of $\underline{C}(T)$, which represents deviations from the nominal trajectory at the target, is zero when the vehicle arrives at the target.

$$\underline{C}(T) = A_1 \ \hat{\underline{x}}(t) + A_2 \ \hat{\dot{\underline{x}}}(t)$$

$$3x1 \quad 3x3 \ 3x1 \quad 3x3 \ 3x1 \qquad (3)$$

The velocity correction is selected such that:

$$\underline{C}(T) + A_2 \dot{\underline{x}}_g = 0 = A_1 \hat{\underline{x}}(t) + A_2 \hat{\dot{\underline{x}}}(t) + A_2 \dot{\underline{x}}_g \qquad (4)$$

$\dot{\underline{x}}_g$ is multiplied by A_2 because as can be seen in equation (3), A_2 is a matrix of sensitivity partials relating $\underline{C}(T)$ to velocity deviations at time (t).

Solving equation 4 for $\dot{\underline{x}}_g$ yields

$$\dot{\underline{x}}_g = A_2^{-1} \underline{C}(T) = -A_2^{-1} A_1 \hat{\underline{x}}(t) - I \hat{\dot{\underline{x}}}(t) \qquad (5)$$

$$\dot{\underline{x}}_g = - \begin{bmatrix} A_2^{-1} A_1 & I \end{bmatrix} \begin{pmatrix} \hat{\underline{x}}(t) \\ \hat{\dot{\underline{x}}}(t) \end{pmatrix} \qquad (6)$$

We are interested in the covariance matrix PAR at (T), that is, at the time of arrival, since this is the covariance matrix of errors in guidance.

If we find $E\,(x(t)\,x(t)^T)$ after a correction, the use of equation (1) will propagate it to the target.

Let subscript (a) define the state after a correction, and subscript (b) define the state before the correction.

Then

$$\begin{pmatrix} \underline{x}_a \\ \dot{\underline{x}}_a \end{pmatrix} = \begin{pmatrix} \underline{x}_b \\ \dot{x}_b \end{pmatrix} + \begin{pmatrix} 0 \\ \dot{\underline{x}}_g \end{pmatrix} + \begin{pmatrix} 0 \\ q \end{pmatrix}$$

where $\dot{x}_g$ is the computed velocity correction and q is the error in making the correction. This is under the assumption of an impulsive correction. Utilizing equation (5), and since $x_a = \hat{x}_a + \tilde{x}_a$

$$\begin{pmatrix} \underline{x}_a \\ \dot{\underline{x}}_a \end{pmatrix} = \begin{pmatrix} I & 0 \\ -A_2A_1 & 0 \end{pmatrix} \begin{pmatrix} \hat{\underline{x}}_b \\ \hat{\dot{\underline{x}}}_b \end{pmatrix} + \begin{pmatrix} \tilde{\underline{x}}_b \\ \tilde{\dot{\underline{x}}}_b \end{pmatrix} + \begin{pmatrix} 0 \\ q \end{pmatrix} \tag{7}$$

and

$$\underset{6 \times 6}{E\,(x_a x_a^T)} = \underset{6 \times 6}{\begin{bmatrix} I & 0 \\ -A_2A_1 & 0 \end{bmatrix}} \underset{6 \times 6}{E(\hat{x}_b \hat{x}_b^T)} \underset{6 \times 6}{\begin{bmatrix} I & 0 \\ -A_2A_1 & 0 \end{bmatrix}^T} + \underset{6 \times 6}{E\,(\tilde{x}_b \tilde{x}_b^T)}$$

$$+ \underset{6 \times 6}{E\left[\begin{pmatrix} 0 \\ q \end{pmatrix} (0 \quad q)^T\right]} + \text{CROSS TERMS}$$

The cross terms are zero because:

1. $E\,(\hat{x}\,\tilde{x}^T) = 0$ because of using an optimum estimation procedure

2. $E\,(\hat{x}\,q_s^T) = E(\hat{x}\,\hat{x}^T)\,E(S) = 0$

   $E\,(\hat{x}\,q_p^T) = E(\hat{x}\,\hat{x}^T)\,E(U) = 0$

   for the two types of correction errors described below.

The two types of errors considered in making a guidance correction are shutoff error and pointing.

A. Shutoff Error

Let the correction by $\dot{\underline{x}}_g = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$

The shutoff error is

$$\vec{\epsilon}_s = S\, \dot{\underline{x}}_g$$

where S is a scalar random variable which is gaussian $(0, \sigma_s)$.

B. Pointing Error

The pointing error is $\vec{\epsilon}_p = \vec{U} \times \dot{\underline{x}}_g$

where $U = (U_x, U_y, U_z)$ is a three dimensional spherical gaussian random variable with sigmas equal to σ_p and mean 0.

The two types of expected values of interest in equation (8) for the error in correction are $E\,(qq^T)$ and $E\,(\hat{x}\, q^T)$

$$\vec{q} = \vec{\epsilon}_p + \vec{\epsilon}_s$$

$$E(q\, q^T) = (\sigma_s^2 - \sigma_p^2)\, E(\dot{\underline{x}}_g\, \dot{x}_g^T) + \sigma_p^2\, E(\dot{x}_g^T\, \dot{x}_g)\, I$$

$$E(\hat{x}\, q^T) = 0$$

The two remaining expected values in equation (8) may be written as follows:

$$E(\hat{x}\, \hat{x}^T) = E(x - \tilde{x})\,(x - \tilde{x})^T = E(x\, x^T) - E(x\, \tilde{x}^T) + E(\tilde{x}\, \tilde{x}^T) - E(\tilde{x}\, x^T)$$

but $E(x\, \tilde{x}^T) = E\left([\hat{x} + \tilde{x}]\, \tilde{x}^T\right) = E\,(\tilde{x}\, \tilde{x}^T)$

since $E(\hat{x}\, \tilde{x}) = 0$

$$\therefore \quad E(\hat{x}\, \hat{x}^T) = E(x\, x^T) - E(\tilde{x}\, \tilde{x}^T)$$

$$E(\hat{x}\, \hat{x}^T) = PAR_b - P_b$$

Finally

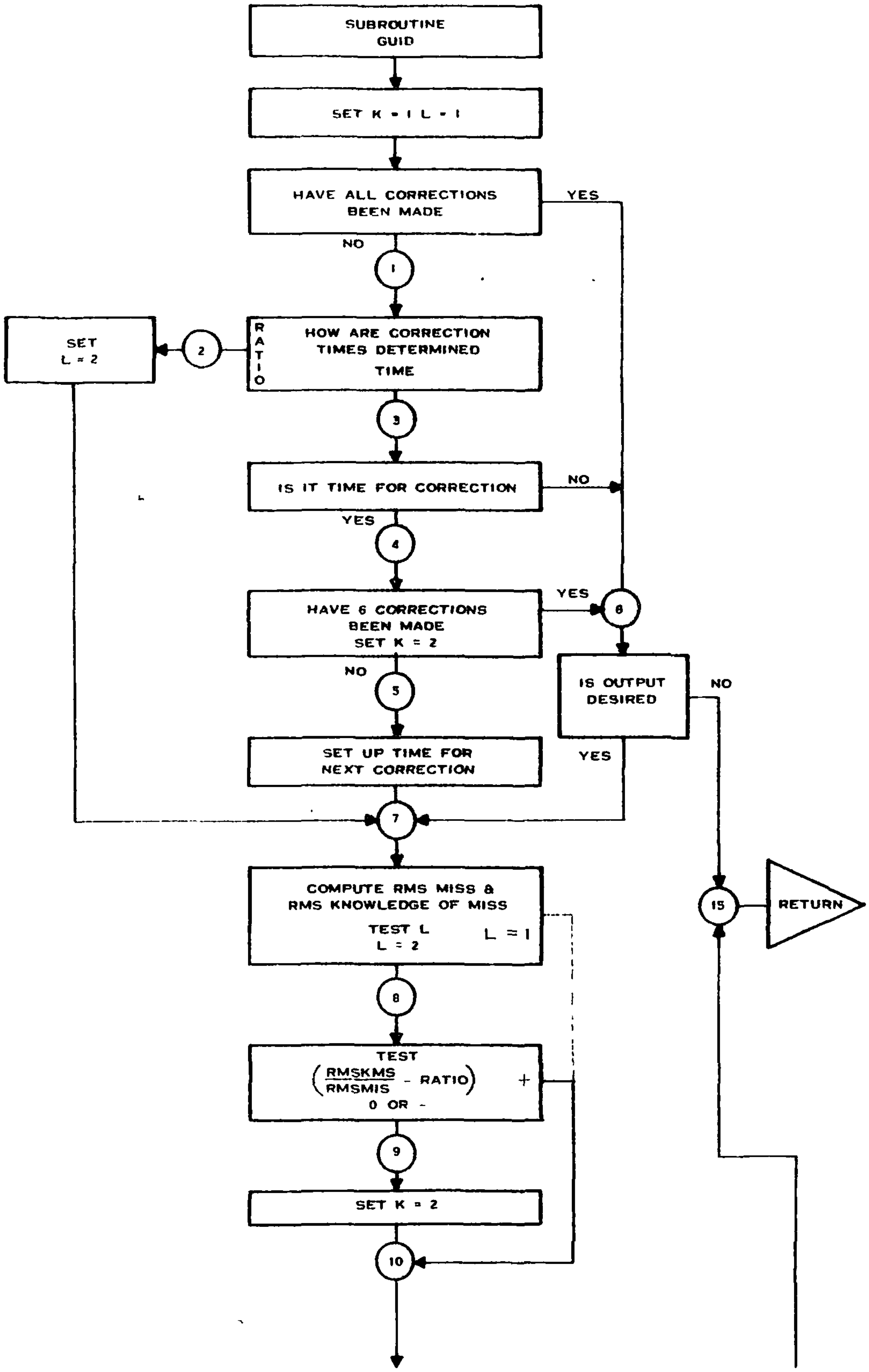
$$
\underset{6\times 6}{PAR_a} = \underset{6\times 6}{\begin{bmatrix} I & 0 \\ -A_2^{1} A_1 & 0 \end{bmatrix}} \underset{6\times 6}{\begin{bmatrix} PAR_b - P_b \end{bmatrix}} \underset{6\times 6}{\begin{bmatrix} I & 0 \\ -A_2^{1} A_1 & 0 \end{bmatrix}^T}
$$

$$
+ \underset{6\times 6}{P_b} + \underset{6\times 6}{\begin{bmatrix} 0 & 0 \\ 0 & E(qq^{1}) \end{bmatrix}}
$$

The covariance matrix of the knowledge of the state is updated to account for the lack of exact knowledge of the correction.

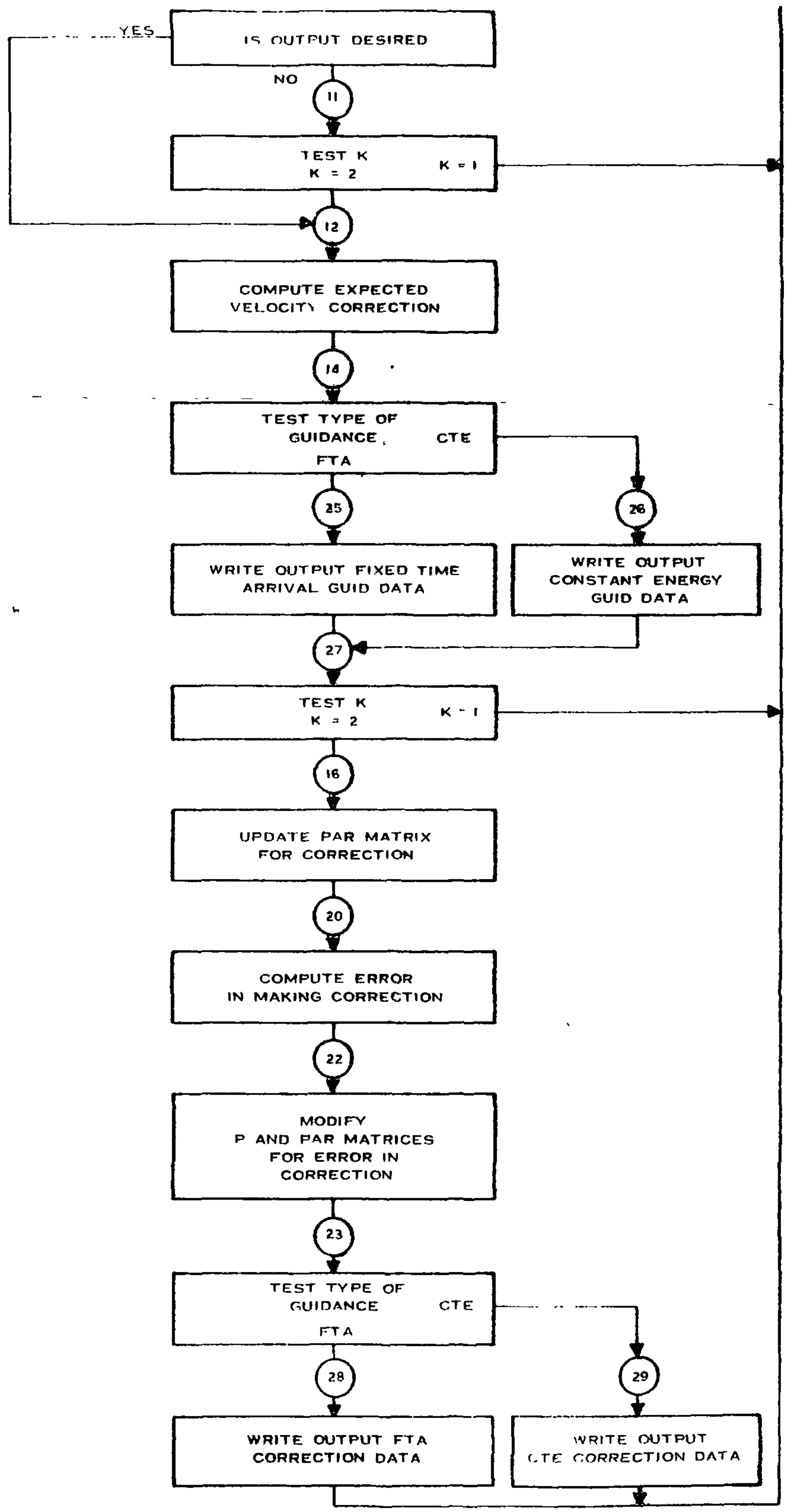
$$
\underset{6x6}{P_a} = \underset{6x6}{P_b} + \delta \underset{6x6}{\begin{bmatrix} 0 & 0 \\ 0 & E(qq^T) \end{bmatrix}}
$$

where δ indicates the ability to monitor (ON BOARD) the error in correction.


SUBROUTINE GUID
SET K = 1 L = 1
HAVE ALL CORRECTIONS BEEN MADE
YES
NO
1
SET L = 2
2
RATIO
HOW ARE CORRECTION TIMES DETERMINED
TIME
3
IS IT TIME FOR CORRECTION
NO
YES
4
HAVE 6 CORRECTIONS BEEN MADE SET K = 2
YES
6
IS OUTPUT DESIRED
NO
NO
5
SET UP TIME FOR NEXT CORRECTION
YES
7
COMPUTE RMS MISS & RMS KNOWLEDGE OF MISS
TEST L
L = 2
L = 1
15
RETURN
8
TEST
(RMSKMS / RMSMIS - RATIO)
0 OR -
+
9
SET K = 2
10

S-210

GUID-8


YES
IS OUTPUT DESIRED
NO
11
TEST K
K = 2
K = 1
12
COMPUTE EXPECTED VELOCITY CORRECTION
14
TEST TYPE OF GUIDANCE
CTE
FTA
25
26
WRITE OUTPUT FIXED TIME ARRIVAL GUID DATA
WRITE OUTPUT CONSTANT ENERGY GUID DATA
27
TEST K
K = 2
K = 1
16
UPDATE PAR MATRIX FOR CORRECTION
20
COMPUTE ERROR IN MAKING CORRECTION
22
MODIFY P AND PAR MATRICES FOR ERROR IN CORRECTION
23
TEST TYPE OF GUIDANCE
CTE
FTA
28
29
WRITE OUTPUT FTA CORRECTION DATA
WRITE OUTPUT CTE CORRECTION DATA

```
*     LABEL                                                              GUID
*     SYMBOL TABLE                                                       GUID
CEC2005  SUBROUTINE GUID                                                 GUID
      SUBROUTINE GUID                                                    GUID0000
      COMMON T,S,C,IC                                                    GUID0010
      DIMENSION T(1360),S(1000),C(1000),IC(1)                            GUID0020
     1,TPAR(3,6),PAR(6,6),P(6,6),DUP(3,3)                                GUID0030
     2,TGUID(6),DUM(3,3),DUN(3,3),DUMM(6,6)                              GUID0040
      EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM)                             GUID0050
     1,(IC,ICDUM),(C(568),TPAR),(C(460),PAR)                             GUID0060
     2,(C(652),P),(IC(218),N),(C(648),TGUIDE)                            GUID0070
     3,(IC(219),IGD),(IC(217),IGDTP),(IC(214),NOUT)                      GUID0080
     4,(C(30),TSEC),(S(476),TGUID),(S(483),RATIO)                        GUID0090
     5,(S(473),QQPT),(S(471),QQSO),(S(484),DMONIT)                       GUID0100
      CALL SETN(NIN,NUTS)                                                GUID0105
C     IGD=NUMBER OF GUIDANCE CORRECTIONS TO BE MADE                      GUID0110
C     N KEY USED TO TEST NUMBER OF GUIDANCE CORRECTIONS WHICH HAVE       GUID0120
C     BEEN MADE                                                          GUID0130
C     S(475)  0=GUIDE ON INPUT TIMES   + GUIDE ON A RATIO TEST           GUID0140
C     IGDTP 1.FTA  2.CTE  3.MINIMUM FUEL                                 GUID0150
C     DMONIT=ACCURACY OF MONITORING THE GUIDANCE CORRECTION              GUID0160
      NOUT=NOUT                                                          GUID0170
      K=1                                                                GUID0180
      L=1                                                                GUID0190
      IF(IGD-N)6,1,1                                                     GUID0200
 1    IF(S(475))3,3,2                                                    GUID0210
  2   L=2                                                                GUID0220
      GO TO 7                                                            GUID0230
 3    IF(TGUIDE-TSEC)4,4,6                                               GUID0240
 4    CONTINUE                                                           GUID0250
      K=2                                                                GUID0260
      N=N+1                                                              GUID0270
      IF(N-7)5,6,6                                                       GUID0280
    5 CALL TIMED(TGUID(N),TGUIDE)                                        GUID0290
      GO TO 7                                                            GUID0300
 6    CONTINUE                                                           GUID0310
      GO TO (15,7),NOUT                                                  GUID0320
 7    CALL MATRX(TPAR,P,DUM,3,6,6,1)                                     GUID0330
      RMSKMS=SQRTF(DUM(1,1)+DUM(2,2)+DUM(3,3))                           GUID0340
      CALL MATRX(TPAR,PAR,DUN,3,6,6,1)                                   GUID0350
      RMSMIS=SQRTF(DUN(1,1)+DUN(2,2)+DUN(3,3))                           GUID0360
      GO TO (10,8),L                                                     GUID0370
 8    IF((RMSKMS/RMSMIS)-RATIO)9,9,10                                    GUID0380
 9    K=2                                                                GUID0390
      N=N+1                                                              GUID0400
 10   GO TO (11,12),NOUT                                                 GUID0410
 11   GO TO (15,12),K                                                    GUID0420
 12   CONTINUE                                                           GUID0430
      DO 13 I=1,3                                                        GUID0440
      DO 13 J=1,3                                                        GUID0450
      JJ=J+3                                                             GUID0460
```

GUID (Cont'd)

```
      DUP(I,J)=-TPAR(I,JJ)                                            GUID047
   13 CONTINUE                                                        GUID048
      CALL INV3(DUP,3,DET)                                            GUID049
      DO 14 I=1,3                                                     GUID050
      DO 14 J=1,3                                                     GUID051
      DUM(I,J)=DUN(I,J)-DUM(I,J)                                      GUID052
   14 CONTINUE                                                        GUID053
      RMSPAP=SQRTF(PAR(1,1)+PAR(2,2)+PAR(3,3))                        GUID054
      RMSPAV=SQRTF(PAR(4,4)+PAR(5,5)+PAR(6,6))                        GUID055
      CALL MATRX(DUP,DUM,DUM,3,3,3,1)                                 GUID056
C     DUM IS THE COVARIANCE MATRIX OF INDICATED VELOCITY CORRECTION   GUID057
      RMSV2=DUM(1,1)+DUM(2,2)+DUM(3,3)                                GUID058
      RMSV=SQRTF(RMSV2)                                               GUID059
      IGDTP=IGDTP                                                     GUID059
      GO TO (25,26),IGDTP                                             GUID059
   25 CONTINUE                                                        GUID059
      WRITE OUTPUT TAPE NUTS,800,RMSMIS,RMSPAP,RMSKMS,RMSPAV,RMSV     GUID060
  800 FORMAT(18H0GUID DATA FOLLOWS,/                                  GUID061
     124H    RMS FTA TARGET MISS=,E15.8,                              GUID062
     225H    RMS POS DEV FROM NOM=,E15.8,/                            GUID063
     324H      RMS KNOW. OF MISS=,E15.8,                              GUID064
     425H    RMS VEL DEV FROM NOM=,E15.8,/                            GUID065
     524H            RMS VEL REQ=,E15.8)                              GUID065
      GO TO 27                                                        GUID065
   26 CONTINUE                                                        GUID065
      RMSMIS=SQRTF(DUN(1,1)+DUN(2,2))                                 GUID065
      RMSVIN=SQRTF(DUN(3,3))                                          GUID065
      WRITE OUTPUT TAPE NUTS,802,RMSMIS,RMSPAP,RMSVIN,RMSPAV,RMSKMS,RMSVGUID066
  802 FORMAT(18H0GUID DATA FOLLOWS,/                                  GUID066
     124H    RMS TARGET POS MISS=,E15.8,                              GUID066
     225H    RMS POS DEV FROM NOM=,E15.8,/                            GUID066
     324H     RMS VINFINITY MISS=,E15.8,                              GUID066
     425H    RMS VEL DEV FROM NOM=,E15.8,/                            GUID066
     324H      RMS KNOW. OF MISS=,E15.8,                              GUID066
     525H             RMS VEL REQ=,E15.8)                             GUID066
   27 CONTINUE                                                        GUID066
      GO TO (15,16),K                                                 GUID067
   15 RETURN                                                          GUID068
   16 CONTINUE                                                        GUID069
C     FOLLOWING IS UPDATING OF PAR MATRIX FOR GUIDANCE CORRECTION     GUID070
      DO 17 I=1,6                                                     GUID071
      DO 17 J=I,6                                                     GUID072
      PAR(I,J)=PAR(I,J)-P(I,J)                                        GUID073
      PAR(J,I)=PAR(I,J)                                               GUID074
   17 CONTINUE                                                        GUID075
      CALL MATRX(DUP,TPAR,DUP,3,3,3,0)                                GUID076
      DO 19 I=1,3                                                     GUID077
      II=I+3                                                          GUID078
      DO 18 J=1,3                                                     GUID079
      JJ=J+3                                                          GUID080
      DUMM(II,J)=DUP(I,J)                                             GUID081
```

```
      DUMM(I,JJ)=0.                                                     GUID0820
      DUMM(II,JJ)=0.                                                    GUID0830
 18   DUMM(I,J)=0.                                                      GUID0840
 19   DUMM(I,I)=1.                                                      GUID0850
      CALL MATRX(DUMM,PAR,PAR,6,6,6,1)                                  GUID0860
      DO 20I=1,6                                                        GUID0870
      DO 20 J=I,6                                                       GUID0880
      PAR(I,J)=PAR(I,J)+P(I,J)                                          GUID0890
      PAR(J,I)=PAR(I,J)                                                 GUID0900
 20   CONTINUE                                                          GUID0910
      CRUD=QQPT*RMSV2                                                   GUID0920
      QQ=QQSO-QQPT                                                      GUID0930
      DO 22 I=1,3                                                       GUID0940
      DO 21 J=I,3                                                       GUID0950
      DUN(I,J)=QQ*DUM(I,J)                                              GUID0960
      DUN(J,I)=DUN(I,J)                                                 GUID0965
 21   CONTINUE                                                          GUID0970
      DUN(I,I)=DUN(I,I)+CRUD                                            GUID0980
 22   CONTINUE                                                          GUID0990
      RMSERR=SQRTF(DUN(1,1)+DUN(2,2)+DUN(3,3))                          GUID1000
      DO 23 I=1,3                                                       GUID1010
      II=I+3                                                            GUID1020
      DO 23 J=I,3                                                       GUID1030
      JJ=J+3                                                            GUID1040
      PAR(II,JJ)=PAR(II,JJ)+DUN(I,J)                                    GUID1050
      PAR(JJ,II)=PAR(II,JJ)                                             GUID1060
      P(II,JJ)=P(II,JJ)+DMONIT*DUN(I,J)                                 GUID1070
      P(JJ,II)=P(II,JJ)                                                 GUID1080
   23 CONTINUE                                                          GUID1090
      CALL MATRX(TPAR,PAR,DUN,3,6,6,1)                                  GUID1100
      RMSPAP=SQRTF(PAR(1,1)+PAR(2,2)+PAR(3,3))                          GUID1105
      RMSPAP=SQRTF(PAR(1,1)+PAR(2,2)+PAR(3,3))                          GUID1110
      RMSPAV=SQRTF(PAR(4,4)+PAR(5,5)+PAR(6,6))                          GUID1115
      GO TO (28,29),IGDTP                                               GUID1120
 28   CONTINUE                                                          GUID1125
      RMSMIS=SQRTF(DUN(1,1)+DUN(2,2)+DUN(3,3))                          GUID1130
      WRITE OUTPUT TAPE NUTS,801,RMSERR,RMSPAP,RMSMIS,RMSPAV            GUID1140
801   FORMAT(32H  ***GUIDANCE CORRECTION MADE***,                       GUID1150
     1/24H      RMS ERROR IN CORR.=,E15.8,                              GUID1160
     225H    RMS POS DEV FROM NOM=,E15.8,                               GUID1170
     3/24H       RMS MIS AFT CORR.=,E15.8,                              GUID1180
     425H    RMS VEL DEV FROM NOM=,E15.8)                               GUID1190
      GO TO 15                                                          GUID1200
 29   CONTINUE                                                          GUID1210
      RMSMIS=SQRTF(DUN(1,1)+DUN(2,2))                                   GUID1220
      RMSVIN=SQRTF(DUN(3,3))                                            GUID1230
      WRITE OUTPUT TAPE NUTS,803,RMSERR,RMSPAP,RMSMIS,RMSPAV,RMSVIN     GUID1240
803   FORMAT(32H  ***GUIDANCE CORRECTION MADE***,                       GUID1250
     1/24H      RMS ERROR IN CORR.=,E15.8,                              GUID1260
     225H    RMS POS DEV FROM NOM=,E15.8,/                              GUID1270
     3/24H  RMS POS MIS AFT CORR.=,E15.8,                               GUID1280
```

```
     425H    RMS VEL DEV FROM NOM=,E15.8,                                GUID129
     5/24H     RMS VINF AFT CORR.=,E15.8)                                GUID130
  30  CONTINUE                                                           GUID131
      GO TO 15                                                           GUID132
      END                                                                GUID
```

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Subroutine: HOUR

Purpose: To provide a reading of the printer clock in hours since midnight. This is the STL subroutine RW HOUR.

Calling Sequence:

CALL HOUR(A)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	A			hours	Time

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required: 86 cells

<u>Restrictions</u>

Clock advance is suppressed during readout so that elapsed time will be off .4N seconds where N is the number of times the clock is selected.

HOUR will always provide a floating point number if the printer is available. Therefore, a faulty clock or the wrong board will be indicated.

<u>Use</u>

The main application of HOUR is to time cases through use of the FINP "CLOCK" option.

HOUR

```
*       FAP
*                HOUR SUBROUTINE 10-27-31
         ENTRY   HOUR
HOUR     CLA     1,4
         STA     SSAM
         SXA     SAMX,4
         SXA     SAMX+1,2
         TCOA    *                 DELAY

         RPRA                      ECHO CHECK
         SPRA    6                 READ CLOCK
         SPRA    9                 SUPPRESS SPACING
         SPRA    10                AND PRINTING
         AXT     24,4
         STZ     SAMZ+24,4
         TIX     *-1,4,1
         RCHA    SAMIO             TRANSMIT CLOCK READING
         TCOA    *
         STZ     SAMZ+18
         STZ     SAMZ+20
         AXT     18,4
SAM1     CAL     SAMZ+18,4         8-4 LEFT ADD ECHOS
         ORS     SAMZ+18           AND CONVERT TO
         LDQ     SAMZ+18           BCD
         PXD
         AXT     5,2
SAM3     LGL     1
         ALS     5
         TIX     SAM3,2,1
         LGL     1
         ACL     SAMZ+20
         SLW     SAMZ+20
         TIX     SAM1,4,2
         STZ     SAMZ+22
         AXT     6,4
         LDQ     SAMZ+20
SAM4     PXD
         LGL     6
         SLW     SAMZ+23
         CLA     SAMZ+22
         ALS     2
         ADD     SAMZ+22
         ALS     1
         ADD     SAMZ+23
         STO     SAMZ+22
         TIX     SAM4,4,1
         LDQ     SAMZZ
         DVP     SIX
         XCL
         ORA     SA
         FAD     SA
```

HOUR (Cont'd)

```
        CAS     M24
        FSB     M24                 MOD 24 HOURS
        NOP                         MIDNIGHT ONLY
SSAM    STO     **0
SAMX    AXT     **0,4
        AXT     **0,2
        TRA     2,4                 EXIT
SAMIO   IOCP    SAMZ,2+0,26         NON TRANSMIT 26 COPIES
        IOCP    SAMZ,0,1            27TH COPY 9L ECHO
        IOCP    SAMZ+1,2,3          NON TRANSMIT 9R,12L,12R
        IOCD    SAMZ+2,0,16         8L TO 1R ECHOS
SA      OCT     214000000000
M24     DEC     24,
SIX     DEC     6000B15
SAMZZ   PZE
SAMZ    BSS     24
        END
```

Subroutine: HPHT

Purpose: To perform the matrix multiplication $(RMS)^2 = H \cdot P \cdot H^T$. The square root of $(RMS)^2$ is taken and RMS, which is the standard deviation of the parameter being considered, is output from the subroutine.

Calling Sequence:

CALL HPHT (H, P, RMS)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	H	(1,6)	$\frac{\partial PAR}{\partial STATE}$		Parameter Partial Matrix
I	P	(6,6)	$E(\vec{X}\,\vec{X}^{T})$		State Covariance Matrix
O	RMS	1			Standard deviation of parameter
					due to state variance.

Common storages used or required: None

Subroutines required: None

Functions required: SQRT

Approximate number of storages required: ______

HPHT

```
*     LABEL                                                  HPHT
*     SYMBOL TABLE                                           HPHT
      SUBROUTINE HPHT(H,P,RMS)                               HPHT0000
      DIMENSION H(6),P(6,6),DUM(6)                           HPHT0010
      DO 1 I=1,6                                             HPHT0020
      DUM(I)=0.                                              HPHT0030
      DO 1 J=1,6                                             HPHT0040
      DUM(I)=DUM(I)+H(J)*P(J,I)                              HPHT0050
    1 CONTINUE                                               HPHT0060
      VAR2=0.                                                HPHT0070
      DO 2 I=1,6                                             HPHT0080
      VAR2=VAR2+DUM(I)*H(I)                                  HPHT0090
    2 CONTINUE                                               HPHT0100
      RMS=SQRTF(VAR2)                                        HPHT0110
      RETURN                                                 HPHT0120
      END                                                    HPHT
```

Subroutine: INPUT

Purpose: To transform input initial conditions of position and velocity in various coordinate systems to Cartesian coordinates with respect to earth's equator and equinox of 1950. Input may be Cartesian referred to (1) mean equator and equinox, 1950; (2) true equator and equinox of date; (3) earth-fixed or spherical; (4) mean equator and equinox of 1950; (5) true equator and equinox of date; (6) earth-fixed.

Calling Sequence:

CALL INPUT (K, XOUT, VOUT, TW, TF, AN, XIN, VIN)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	K	1			Type of input: K=1 for (1) above, etc.
O	XOUT	3		see following pgs	Position, equinox 1950.0
O	VOUT	3			Velocity, equinox 1950.0
I	TW	1		days	Whole number of days past 1950.
I	TF	1		days	Fractional number of days past TW
O	AN	3,3			Transformation matrix from system
					(1) to (2)
I	XIN	3		see following pgs	Input position vector
I	VIN	3		see following pgs	Input velocity vector

Common storages used or required: None

Subroutines required: ROTEQ, NUTAIT, MULT, RVIN, INV3, GHA, SETN

Functions required: SIN, COS

Approximate number of storages required:

Input format:

For Cartesian coordinates: [K = 1,2,3]

XIN(1) = X
XIN(2) = Y
XIN(3) = Z
VIN(1) = $\dot{X}$
VIN(2) = $\dot{Y}$
VIN(3) = $\dot{Z}$

For sperical coordinates: [K = 4,5,6]

XIN(1) = R = magnitude of radius vector
XIN(2) = O = declination (degrees)
XIN(3) = θ = right ascension (degrees)
VIN(1) = V = magnitude of velocity
VIN(2) = γ = flight path angle (degrees)
VIN(3) = σ = azimuth angle (degrees)

Input Transformations:

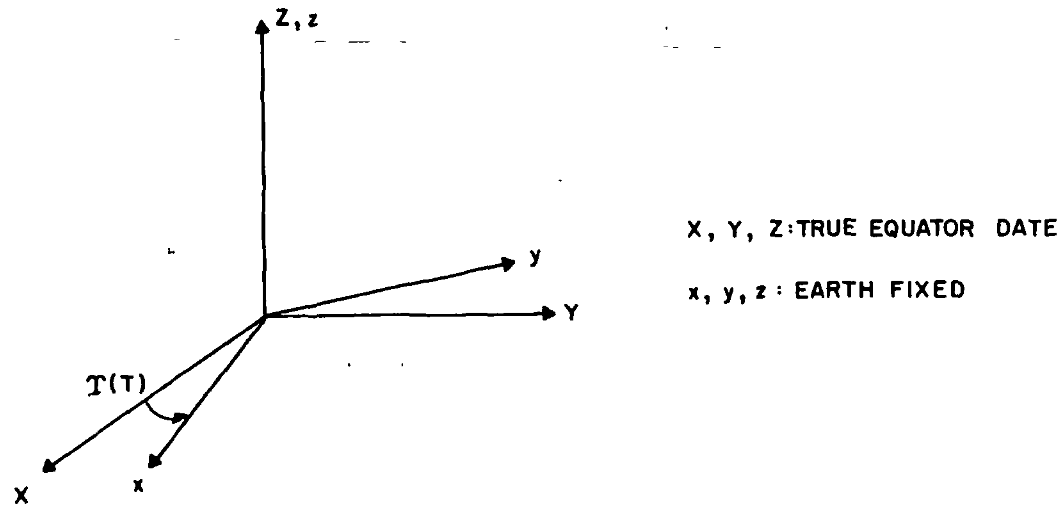
K = 1 The input is in mean equinox of 1950 and no transformation is required

$$\underset{3x1}{\vec{XOUT}_{50}} = \underset{3x1}{\vec{XIN}_{50}} \qquad \underset{3x1}{\vec{VOUT}_{50}} = \underset{3x1}{\vec{VIN}_{50}}$$

K = 2 Subroutines ROTEQ and NUTAIT are called to obtain the transformation matrix, AN, from mean equator 1950 to true equator of date

$$\underset{3x1}{\vec{XOUT}_{50}} = \underset{3x3}{(AN)^T} \underset{3x1}{XIN_{DATE}} \qquad \underset{3x1}{\vec{VOUT}_{50}} = \underset{3x3}{(AN)^T} \underset{3x1}{\vec{VIN}_{DATE}}$$

K = 3 The earth fixed Cantesian coordinate system is assumed to rotate with the earth. The X-Y plane is coincident with the earth's true equator of date, the X axis lying in the Greenwich meridian and the Z axis along the earth's spin axis. Subroutine GHA is called with time, T, to obtain the Greenwich hour angle, $\gamma^{(T)}$, of the true vernal equinox of date. The transformations from earth fixed coordinates to true equator of date are the following:



$$\underset{3\times1}{X_{DATE}} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{DATE} = \begin{pmatrix} \cos^{\gamma} & -\sin^{\gamma} & 0 \\ \sin^{\gamma} & \cos^{\gamma} & \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z \\ y \\ z \end{pmatrix}_{EF}$$

and

$$\underset{3x1}{V_{DATE}} = \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix}_{DATE} = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}_{EF} +$$

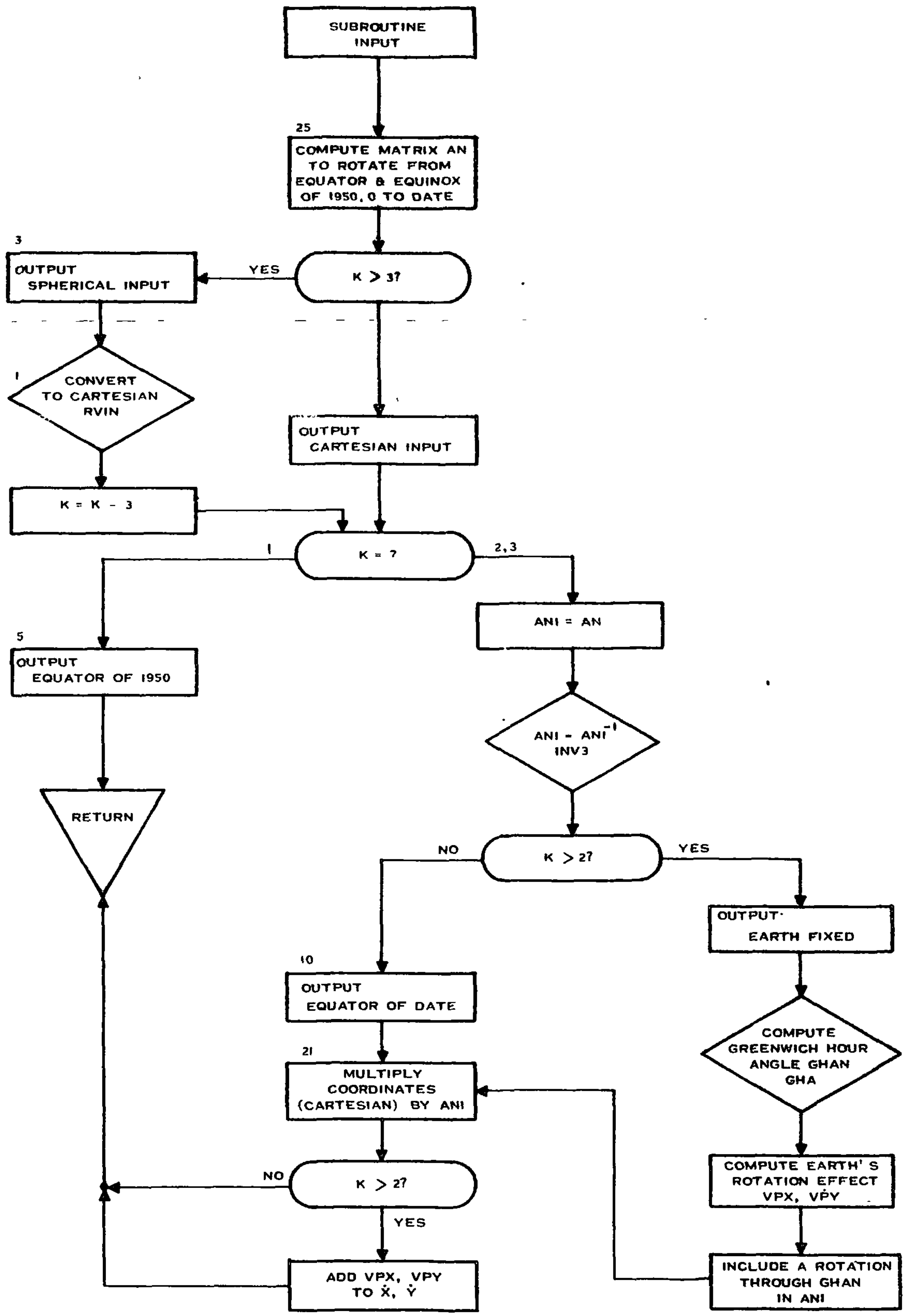
$$\omega \begin{pmatrix} -\sin\gamma & -\cos\gamma & 0 \\ \cos\gamma & -\sin\gamma & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{EF}$$

where $\omega = \frac{d}{dt}\{\gamma(t)\}$ = earth's rotation rate

Subroutines ROTEQ and NUTAIT are called to obtain the AN transformation which completes the transformation to equator 1950.

$$\underset{3x1}{\vec{X}OUT} = \underset{3x3}{(AN)^T} \underset{3x1}{\vec{X}_{DATE}} \qquad \underset{3x1}{\vec{V}OUT} = \underset{3x3}{(AN)^T} \underset{3x1}{\vec{V}_{DATE}}$$

<u>K = 4, 5, or 6</u> In each of these cases, subroutine RVIN is called to transform the spherical sets of coordinates to Cartesian coordinates of the same reference frame. Following this spherical to Cartesian conversion, the transformation procedures which are followed for the transformation to 1950 are identical to those described for the corresponding reference frames for K = 1, 2, or 3.


SUBROUTINE INPUT
25
COMPUTE MATRIX AN TO ROTATE FROM EQUATOR & EQUINOX OF 1950.0 TO DATE
3
OUTPUT SPHERICAL INPUT
YES
K > 3?
1
CONVERT TO CARTESIAN RVIN
OUTPUT CARTESIAN INPUT
K = K - 3
K = ?
1
2,3
5
OUTPUT EQUATOR OF 1950
ANI = AN
ANI - ANI⁻¹ INV3
RETURN
NO
K > 2?
YES
OUTPUT EARTH FIXED
10
OUTPUT EQUATOR OF DATE
COMPUTE GREENWICH HOUR ANGLE GHAN GHA
21
MULTIPLY COORDINATES (CARTESIAN) BY ANI
COMPUTE EARTH'S ROTATION EFFECT VPX, VPY
NO
K > 2?
YES
ADD VPX, VPY TO Ẋ, Ẏ
INCLUDE A ROTATION THROUGH GHAN IN ANI

INPUT

```
*      LABEL
*      SYMBOL TABLE
CEC2013  SUBROUTINE INPUT
       SUBROUTINE INPUT(K,X,V,TW,TF,AN,XIN,VIN)
C      XIN AND VIN ARE INPUTS, X AND V ARE OUTPUTS IN EQUINOX 1950
C      K IS TYPE INPUT (1) EQUATOR 1950, (2) EQUATOR OF DATE (3)
C      EARTH-FIXED
C      (4),(5),(6), ARE SPHERICAL INPUT RESP,,(1),(2),(3)
       DIMENSION X(3),V(3),SP(6),EN(3,3),A(3,3),AN(3,3),ANI(3,3),B(3,3)
       DIMENSION XIN(3),VIN(3)
       CALL SETN(NIN,NOUT)
       TD = TW+TF
       DO 25 I=1,3
       X(I)=XIN(I)
       V(I)=VIN(I)
   25  CONTINUE
       CALL ROTEQ(TD,A)
       CALL NUTAIT (TD,OM,CRUD,DDC,EN,EPSIL)
       CALL MULT(EN,A,AN,0)
       IF (K-3) 4,4,3
  4    WRITE OUTPUT TAPE NOUT,15,X,V
   15  FORMAT(17H CARTESIAN INPUT 6E17.8)
       GO TO 16
  3    WRITE OUTPUT TAPE NOUT,17,X,V
   17  FORMAT(17H SPHERICAL INPUT 6E17.8)
       DO 2 I=1,3
       J = I+3
       SP(I) = X(I)
    2  SP(J) = V(I)
    1  CALL RVIN(SP,X,V)
       K = K-3
   16  DO 12 I=1,3
       J = I+3
       SP(I) = X(I)
   12  SP(J) = V(I)
       GO TO (5,7,7), K
  5    WRITE OUTPUT TAPE NOUT,18
   18  FORMAT(16H EQUATOR OF 1950)
       GO TO 13
    7  CONTINUE
       DO 6 I=1,3
       DO 6 J=1,3
    6  ANI(I,J) = AN(I,J)
       CALL INV3(ANI,3,D)
       IF (K-2) 10,10,8
    8  DD = EN(2,1)
       WRITE OUTPUT TAPE NOUT,20
   20  FORMAT(12H EARTH FIXED)
       TS = TF*86400.
       CALL GHA(TS,TW,GHAN,DD,OMEGA)
       OMEGA = OMEGA*.017453296
```

INPUT (Cont'd

```
      GHAN = GHAN*.017453296                                            INPT04
      SP(4)=SP(4)-OMEGA*SP(2)                                           INPT04
      SP(5)=SP(5)+OMEGA*SP(1)                                           INPT04
      CALL ROTATE(3,GHAN,B,-1)                                          INPT04
      CALL MULT(ANI,B,EN,-1)                                            INPT04
      DO 9 I=1,3                                                        INPT05
      DO 9 J=1,3                                                        INPT05
    9 ANI(I,J) = EN(I,J)                                                INPT05
      GO TO 21                                                          INPT05
   10  WRITE OUTPUT TAPE NOUT,19                                        INPT05
   19 FORMAT(16H EQUATOR OF DATE)                                       INPT05
   21 DO 11 I=1,3                                                       INPT05
      X(I) = 0.                                                         INPT05
      V(I) = 0.                                                         INPT05
      DO 11 J=1,3                                                       INPT05
      X(I) = X(I) + ANI(I,J)*SP(J)                                      INPT06
   11 V(I) = V(I) + ANI(I,J)*SP(J+3)                                    INPT06
   13 RETURN                                                            INPT06
      END                                                               INPT
```

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Subroutine: INTR, INTRI

Purpose: The purpose of INTR is to interpolate as a function of the central body on the coordinates of the other bodies, to the given time, and return the interpolated position values. The purpose of INTRI is the same as INTR, but in addition it numerically differentiates the positions to obtain the velocities and also return them.

Calling Sequence:

CALL INTR, INTRI (TW, TF, NB, PO, NV, VE, DIS)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TW	1		whole days	Time in days since 1950.0
I	TF	1		fractional days	Time in fractional days since TW
I	NB	1			Central body
O	PO	22		Km	Array of position values
I	NV	1			Dummy variable
O	VE	22		Km/sec	Array of velocity values
I	DIS	1		Km	Jupiter included if DIS > 10^6

Common storages used or required: C

Subroutines required: FIX, FLOAT

Functions required:

Approximate number of storages required: 2063 OCTAL

For interpolation, the following formula is used:

$$y(t) = \left\{ uy_o + ty_1 \right\} + \left\{ \frac{u(u^2-1)}{3!} \delta^2 y_o + \frac{t(t^2-1)}{3!} \delta^2 y_1 \right\}$$

$$+ \left\{ \frac{u(u^2-1)(u^2-4)}{5!} \delta^4 y_o + \frac{t(t^2-1)(t^2-4)}{5!} \delta^4 y_1 \right\}$$

where $y_o = y(T_j)$

$y_1 = y(T_j + h)$

h = ephemeris interval

$t = \frac{T-T_j}{h}$

$u = 1-t$

$T_j \leqq T < T + h$

For the velocity values, the following is used:

$$\dot{y}(T) = \frac{1}{h} \left\{ -y_o + y_1 \right\} + \frac{1}{h} \left\{ - \frac{3u^2-1}{3!} \delta^2 y_o + \frac{3t^2-1}{3!} \delta^2 y_1 \right\}$$

$$+ \frac{1}{h} \left\{ - \frac{5u^4-15u^2+4}{5!} \delta^4 y_o + \frac{5t^4-15t^2+4}{5!} \delta^4 y_1 \right\}$$

The locations of variables in the position and velocity arrays are as follows:

PO(5)	Z value of Jupiter
PO(6)	Y value of Jupiter
PO(7)	X value of Jupiter

PO(8) - PO(10)	values of Mars
PO(11) - PO(13)	values of Venus
PO(14) - PO(16)	values of the Sun
PO(17) - PO(19)	values of the Moon
PO(20) - PO(22)	values of the Earth

The VE array is arranged the same as the PO array with velocity values in place of positions.

```
       FAP                                                                INTR
        TTL JPL EPHEMERIS LOOKUP   NEW ERP 6/63                          INTR0000
        LBL INTR                                                         INTR0010
        COUNT   530                                                      INTR0020
        ENTRY   INTR                                                     INTR0030
        ENTRY   INTR1                                                    INTR0040
        REM     CALL INTR A,B,C,D,E,F,G  POSITIONS AND VELOCITIES        INTR0050
        REM     A=ARG INTEGRAL DAYS SINCE 1950.0                         INTR0060
        REM     B=FRACTION OF DAY                                        INTR0070
        REM     C=CENTRAL BODY 0=EARTH 1=MOON 2=SUN 3=VEN 4=MARS         INTR0080
        REM     D=OUTPUT TABLE STARTING ADDRESS                          INTR0090
        REM     E=IGNORED (VELOCITY OPTION INDICATOR)                    INTR0100
        REM     F=ADDRESS OF VELOCITY TABLE                              INTR0110
        REM     G=RADIUS FROM CENTRAL BODY                               INTR0120
        REM     EPHEMERIS TAPE MUST BE MOUNTED ON A6                     INTR0130
INTR    TXL     *+3,,**                                                  INTR0140
INTR1   SXD     VEL,4                                                    INTR0150
        TXL     *+2,,**                                                  INTR0160
        STZ     VEL                                                      INTR0170
        CLA     1,4                                                      INTR0180
        STA     TAR                                                      INTR0190
        CLA     2,4                     FRA                              INTR0200
        STA     FRA                                                      INTR0210
        CLA     3,4                     CEN                              INTR0220
        STA     CEN                                                      INTR0230
        CLA     4,4                     OUP                              INTR0240
        STA     OUP                                                      INTR0250
        CLA     6,4                     VELOCITY                         INTR0260
        STA     XN.V                                                     INTR0270
        CLA     7,4                                                      INTR0280
        STA     RADS                                                     INTR0290
TAR     CLA     **                                                       INTR0300
        STO     TARG                                                     INTR0310
FRA     CLA     **                                                       INTR0320
        STO     TARG+1                                                   INTR0330
CEN     CLA     **                                                       INTR0340
        ARS     18                                                       INTR0350
        STO     CENTER                                                   INTR0360
RADS    CLA     **                                                       INTR0370
        STO     R                                                        INTR0380
        SXD     TRAP,4                                                   INTR0390
        SXD     TRAP+1,2                                                 INTR0400
        SXD     HELIO-1,1                                                INTR0410
        CLA     VEL                                                      INTR0420
        TNZ     NEU                     MUST INTERPOLATE TO              INTR0430
        REM                             OBTAIN VELOCITY                  INTR0440
        CLA     CENTER                                                   INTR0450
        SUB     KERNO                                                    INTR0460
        TNZ     NEU                     MUST INTERPOLATE FOR             INTR0470
        REM                             NEW CENTRAL BODY                 INTR0480
        CLA     TARG                                                     INTR0490
```

INTR, INTRI(Cont'd)

```
        SUB     TARGO                                     INTR050
        TNZ     NEU             TIME HAS CHANGED          INTR051
        CLA     TARG+1                                    INTR052
        SUB     TARGO+1                                   INTR053
        TZE     FLEE+6                                    INTR054
        REM                     ANALYZE TABLE             INTR055
        REM                     NEEDS AS A                INTR056
        REM                     FUNCTION OF               INTR057
        REM                     CENTRAL BODY              INTR058
NEU     CLA     TARG                                      INTR059
        FSB     TABLE                                     INTR060
        TMI     LOOKUP          POSITION EPHEMERIS TAPE   INTR061
        STO     COM+19          T-TO                      INTR062
        SUB     20.                                       INTR063
        TPL     LOOKUP                                    INTR064
        CLA     COM+19                                    INTR065
        FDH     4.                                        INTR066
        STQ     COM+18                                    INTR067
        CLA     COM+18          (T-TO)/4.                 INTR068
        CALL    FIX                                       INTR069
        CALL    FLOAT                                     INTR070
        STO     COM+17                                    INTR071
        CHS                                               INTR072
        FAD     COM+18                                    INTR073
        STO     COM+1                                     INTR074
        CLA     TARG+1                                    INTR075
        FDH     4.                                        INTR076
        STQ     COM                                       INTR077
        CLA     COM                                       INTR078
        FAD     COM+1                                     INTR079
        STO     TARG+2                                    INTR080
        LDQ     COM+19                                    INTR081
        FMP     =9.                                       INTR082
        CALL    FIX                                       INTR083
        ADD     KERNO+1                                   INTR084
        STA     GG                                        INTR085
        LDQ     COM+17                                    INTR086
        FMP     =54.                                      INTR087
        CALL    FIX                                       INTR088
        ADD     KERNO+2                                   INTR089
        STA     HH                                        INTR090
        AXT     2*SEP,4         MUST INTERPOLATE          INTR091
        STZ     XN+2*SEP,4      SO CLEAR STORAGE          INTR092
        TIX     *-1,4,1                                   INTR093
        AXT     BSEP,4          SET GRAVITATIONAL         INTR094
        STZ     KBO+BSEP,4      COEFFICIENTS              INTR095
        TIX     *-1,4,1         TO ZERO                   INTR096
        LXA     CENTER,4                                  INTR097
        PXD     ,4                                        INTR098
        COM                                               INTR099
        PDX     ,2                                        INTR100
```

NTR, INTRI (Cont'd)

```
        TXH     HELIO,4,1                                    INTR1010
GEO     AXT     3,1                                          INTR1020
        CLA     GRAV+3,1                                     INTR1030
        STO     KB0+3,1                                      INTR1040
        TIX     *-2,1,1                                      INTR1050
        STZ     KB0-1,2                                      INTR1060
        CLA     EWORTO                                       INTR1070
        STO     EWORT                                        INTR1080
        CLA     HWORTE                                       INTR1090
        STO     HWORT                                        INTR1100
        TXH     GG12,4,0                                     INTR1110
        CLA     RJ                  TEST FOR                 INTR1120
        SUB     R                   INCLUSION OF             INTR1130
        TPL     GG12                JUPITER IF               INTR1140
        CLA     HWORTJ              EARTH IS                 INTR1150
        STO     HWORT               CENTRAL BODY             INTR1160
        CLA     GRAV+6                                       INTR1170
        STO     KB6                                          INTR1180
        TXL     GG12,,**                                     INTR1190
HELIO   AXT     BSEP,1                                       INTR1200
        CLA     GRAV+BSEP,1                                  INTR1210
        STO     KB0+BSEP,1                                   INTR1220
        TIX     *-2,1,1                                      INTR1230
        STZ     KB2-3,2                                      INTR1240
ORTHO   CLA     EWORTO                                       INTR1250
        STO     EWORT                                        INTR1260
        CLA     HWORTO                                       INTR1270
        STO     HWORT                                        INTR1280
GG12    CLA     VEL                 CHECK                    INTR1290
        TZE     GG6                 FOR VELOCITY             INTR1300
        CLA     EWORTO              OPTIONS                  INTR1310
        STO     EWORT                                        INTR1320
        CLA     HWORTV                                       INTR1330
        STO     HWORT                                        INTR1340
GG6     CLA     EWORT                                        INTR1350
        TZE     GG+2                                         INTR1360
        CLA     EVEL                                         INTR1370
        STO     VEL+1                                        INTR1380
        CLA     TARG+1              GEOCENTRIC               INTR1390
        TSX     TAB,4               INTERPOLATION            INTR1400
GG      PZE     **,,9                                        INTR1410
        PZE     XN,,EWORT+1                                  INTR1420
        CLA     HWORT                                        INTR1430
        TZE     HH+2                                         INTR1440
        CLA     HVEL                                         INTR1450
        STO     VEL+1                                        INTR1460
        CLA     TARG+2              HELIOCENTRIC             INTR1470
        TSX     TAB,4               INTERPOLATION            INTR1480
HH      PZE     **,,54                                       INTR1490
        PZE     XN+3,,HWORT+1                                INTR1500
RESET   AXT     3,4                 PLACE                    INTR1510
```

INTR, INTRI (Cont'd)

```
        CLA    XN+6,4             COORDINATES                         INTR1520
        FDH    EVEL               IN OLD                              INTR1530
        STQ    XN,+18,4           FORMAT                              INTR1540
        CLA    XN+9,4                                                 INTR1550
        FDH    EVEL                                                   INTR1560
        STQ    XN,+12,4                                               INTR1570
        LDQ    XN+3,4                                                 INTR1580
        STQ    XN+6,4                                                 INTR1590
        FMP    MU                                                     INTR1600
        FSB    XN+18,4                                                INTR1610
        STO    XN+9,4             POSITION OF SUN                     INTR1620
        STZ    XN+3,4                                                 INTR1630
        LDQ    XN,+3,4                                                INTR1640
        STQ    XN,+6,4                                                INTR1650
        FMP    MU                                                     INTR1660
        FSB    XN,+18,4                                               INTR1670
        STO    XN,+9,4            VELOCITY OF SUN                     INTR1680
        STZ    XN,+3,4                                                INTR1690
        TIX    RESET+1,4,1                                            INTR1700
        LXA    CENTER,4                                               INTR1710
        LXD    VEL,2              IX2 NOT ZERO IF VELOCITY OPTION     INTR1720
        TXH    RVPRT,2,0          GET COORDINATES FOR PRINTING        INTR1730
        TXH    TESTM,4,0                                              INTR1740
        NZT    KB6                EARTH-CENTERED                      INTR1750
        TRA    FLEE                                                   INTR176
        AXT    3,1                                                    INTR177
        CLA    XN+21,1                                                INTR178
        FAD    XN+9,1                                                 INTR179
        STO    XN+21,1                                                INTR180
        TIX    *-3,1,1                                                INTR181
        TXL    FLEE                                                   INTR182
TESTM   TXH    RVPRT,4,1                                              INTR183
        AXT    3,1                MOON-CENTERED                       INTR184
        CLS    XN+6,1                                                 INTR185
        STO    XN+3,1                                                 INTR186
        CLA    XN+9,1                                                 INTR187
        FSB    XN+6,1                                                 INTR188
        STO    XN+9,1                                                 INTR189
        STZ    XN+6,1                                                 INTR190
        TIX    *-6,1,1                                                INTR191
FLEE    CLA    CENTER                                                 INTR192
        STO    KERNO                                                  INTR193
        CLA    TARG                                                   INTR194
        STO    TARGO                                                  INTR195
        CLA    TARG+1                                                 INTR196
        STO    TARGO+1                                                INTR197
        AXT    21,1               TRANSFER POSITIONS                  INTR198
        CLA    XN+21,1                                                INTR199
OUP     STO    **,1                                                   INTR200
        TIX    OUP-1,1,1                                              INTR201
        CLA    VEL                                                    INTR202
```

INTR, INTRI (Cont'd)

```
         TZE     OUT                 DO NOT TRANSFER VELOCITY        INTR2030
         AXT     21,1                                                INTR2040
         CLA     XN,+21,1            TRANSFER VELOCITIES             INTR2050
(N,V     STO     **,1                                                INTR2060
         TIX     XN,V-1,1,1                                          INTR2070
)UT      LXD     HELIO-1,1                                           INTR2080
         LXD     TRAP+1,2                                            INTR2090
         LXD     TRAP,4                                              INTR2100
         TRA     8,4                                                 INTR2110
         REM     TSX TAB,4           (AC)=INTERPOLATIVE ARGUMENT     INTR2120
         REM     PZE B,,K                                            INTR2130
         REM     PZE A,,C                                            INTR2140
         REM                         B=START OF DATA BLOCK           INTR2150
         REM                         K=WORDS PER SUB BLOCK           INTR2160
         REM                         A=START OF RESULT BLOCK         INTR2170
         REM                         C=SKIP CODE WORD LOCATION       INTR2180
'AB      SXD     COM+9,4                                             INTR2190
         SXD     COM+8,2                                             INTR2200
         SXD     COM+7,1                                             INTR2210
         STO     ARG                                                 INTR2220
         CLA     1,4                                                 INTR2230
         STA     TAB18                                               INTR2240
         LRS     18                                                  INTR2250
         ADD     1,4                                                 INTR2260
         STA     TAB21                                               INTR2270
         CLA     2,4                                                 INTR2280
         PAX     ,1                                                  INTR2290
         TXI     *+1,1,SEP                                           INTR2300
         SXA     TAB29,1                                             INTR2310
         ARS     18                                                  INTR2320
         STA     VELOP                                               INTR2330
         AXT     2,1                                                 INTR2340
         CLA     VEL                                                 INTR2350
         TNZ     VELOP                                               INTR2360
         TXI     VELOP,1,-1                                          INTR2370
'ELOP    CLA     **,1                PICK UP SKIP                    INTR2380
         STO     VEL+2               CODE WORD                       INTR2390
         TXL     POSOP,1,1                                           INTR2400
         REM                         VELOCITY OPTION                 INTR2410
         CLA     ARG                                                 INTR2420
         TSX     COEFF.,4            FORM THE                        INTR2430
         AXT     3,4                 E1(2J).                         INTR2440
         CLS     COM+13,4                                            INTR2450
         STO     COM+16,4                                            INTR2460
         TIX     *-2,4,1                                             INTR2470
         CLA     1.                  FORM THE                        INTR2480
         FSB     ARG                 E0(2J).                         INTR2490
         TSX     COEFF.,4                                            INTR2500
         TRA     TAB1-1                                              INTR2510
         REM                         POSITION OPTION                 INTR2520
OSOP     CLA     ARG                                                 INTR2530
```

INTR, INTRI (Cont'd)

```
         TSX    COEFF,4         FORM THE        INTR254
         AXT    3,4             E1(2J)          INTR255
         CLA    COM+13,4                        INTR256
         STO    COM+16,4                        INTR257
         TIX    *-2,4,1                         INTR258
         CLA    1.              FORM THE        INTR259
         FSB    ARG             E0(2J)          INTR260
         TSX    COEFF,4                         INTR261
         LXA    TAB29,4                         INTR262
         TXI    *+1,4,-SEP                      INTR263
         SXA    TAB29,4                         INTR264
         SXD    COM+4,1                         INTR265
TAB1     AXT    0,3                             INTR266
         LDQ    VEL+2                           INTR267
         PXD                                    INTR268
         LGL    2                               INTR269
         PAX    ,4                              INTR270
         STQ    VEL+2                           INTR271
         CAL    =03                             INTR272
         ORS    VEL+2                           INTR273
         TXH    END,4,2                         INTR274
         TXH    FORT,4,0                        INTR275
         TXI    *+1,1,-3                        INTR276
         TXI    TAB1+1,2,-1                     INTR277
FORT     AXT    3,4                             INTR278
TAB18    LDQ    **,1            X(0)            INTR279
         FMP    COM+13,4                        INTR280
         STO    COM                             INTR281
TAB21    LDQ    **,1            X(1)            INTR282
         FMP    COM+16,4                        INTR283
         FAD    COM                             INTR284
         STO    COM+4,4                         INTR285
         TXI    *+1,1,-1                        INTR286
         TIX    *-8,4,1                         INTR287
         FAD    COM+2                           INTR288
         FAD    COM+1                           INTR289
TAB29    STO    **,2            X(T)            INTR290
         TXI    TAB1+1,2,-1                     INTR291
END      LXD    COM+4,1                         INTR292
         TIX    VELOP,1,1                       INTR293
         LXD    COM+9,4                         INTR294
         LXD    COM+8,2                         INTR295
         LXD    COM+7,1                         INTR296
         TRA    3,4                             INTR297
COEFF    STO    COM+10          CALCULATE       INTR298
         LDQ    COM+10          POSITION        INTR299
         FMP    COM+10          COEFFICIENTS    INTR300
         STO    COM+12          FOR EVERETT,S   INTR301
         FSB    1.              INTERPOLATION   INTR302
         FDH    6.              FORMULA         INTR303
         FMP    COM+10                          INTR304
```

INTR, INTRI (Cont'd)

```
        STO    COM+11                                    INTR3050
        CLA    COM+12                                    INTR3060
        FSB    4.                                        INTR3070
        FDH    20.                                       INTR3080
        FMP    COM+11                                    INTR3090
        STO    COM+12                                    INTR3100
        TRA    1,4                                       INTR3110
COEFF,  STO    COM             CALCULATE                 INTR3120
        CLS    1.              VELOCITY                  INTR3130
        FDH    VEL+1           COEFFICIENTS              INTR3140
        STQ    COM+10          FOR EVERETT,S             INTR3150
        LDQ    COM             INTERPOLATION             INTR3160
        FMP    COM             FORMULA                   INTR3170
        STO    COM+12                                    INTR3180
        XCA                                              INTR3190
        FMP    3.                                        INTR3200
        FSB    1.                                        INTR3210
        FDH    6.                                        INTR3220
        FMP    COM+10                                    INTR3230
        STO    COM+11                                    INTR3240
        CLA    COM+12                                    INTR3250
        FSB    3.                                        INTR3260
        XCA                                              INTR3270
        FMP    5.                                        INTR3280
        XCA                                              INTR3290
        FMP    COM+12                                    INTR3300
        FAD    4.                                        INTR3310
        FDH    120.                                      INTR3320
        FMP    COM+10                                    INTR3330
        STO    COM+12                                    INTR3340
        TRA    1,4                                       INTR3350
RVPRT   AXT    0,2             EXPRESS ALL               INTR3360
        AXT    3,5             BODIES GEOCENTRICALLY     INTR3370
        CLA    XN+9,2                                    INTR3380
        FAD    XN+9,4                                    INTR3390
        STO    XN+9,2                                    INTR3400
        CLA    VEL                                       INTR3410
        TZE    *+4                                       INTR3420
        CLA    XN,+9,2                                   INTR3430
        FAD    XN,+9,4                                   INTR3440
        STO    XN,+9,2                                   INTR3450
        TXI    *+1,2,-1                                  INTR3460
        TIX    RVPRT+2,4,1                               INTR3470
        TXH    RVPRT+1,2,9-SEP                           INTR3480
        CLA    CENTER          BUFFER                    INTR3490
        ALS    1               CENTRAL                   INTR3500
        ADD    CENTER          BODY                      INTR3510
        PAC    ,4                                        INTR3520
        CLA    XN,4                                      INTR3530
        STO    COM+3,1                                   INTR3540
        CLA    XN,,4                                     INTR3550
```

INTR, INTRI (Cont'd)

```
        STO     COM+6,1                                     INTR356
        TXI     *+1,4,-1                                    INTR357
        TIX     *-5,1,1                                     INTR358
RVPRT1  AXT     0,2                 EXPRESS ALL             INTR359
        AXT     3,4                 BODIES IN TERMS         INTR360
        CLA     XN,2                OF THE CENTRAL          INTR361
        FSB     COM+3,4             BODY                    INTR362
        STO     XN,2                                        INTR363
        CLA     VEL                                         INTR364
        TZE     *+4                                         INTR365
        CLA     XN,,2                                       INTR366
        FSB     COM+6,4                                     INTR367
        STO     XN,,2                                       INTR368
        TXI     *+1,2,-1                                    INTR369
        TIX     RVPRT1+2,4,1                                INTR370
        TXH     RVPRT1+1,2,-SEP                             INTR371
        TXL     FLEE                                        INTR372
LOOKUP  CLA     TLAST                                       INTR373
        FSB     TARG                                        INTR374
        TNZ     *+2                                         INTR375
        TRA     ERR1                                        INTR376
        TMI     *-1                                         INTR377
        CLA     TARG                                        INTR378
        FSB     TFIRST                                      INTR379
        TPL     *+2                                         INTR380
        TRA     ERR1                                        INTR381
        FDH     20.                                         INTR382
        XCA                                                 INTR383
        CALL    FIX                                         INTR384
        CALL    FLOAT                                       INTR385
        XCA                                                 INTR386
        FMP     20.                                         INTR387
        FAD     TFIRST                                      INTR388
        STO     COM+1               TIME ON RECORD          INTR389
        CLA     TABLE                                       INTR390
        FSB     COM+1                                       INTR391
        TMI     FINDIT                                      INTR392
        FDH     20.                                         INTR393
        XCA                                                 INTR394
        CALL    FIX                                         INTR395
        ADD     1F                  RECORDS TO              INTR396
        PAX     ,4                  BE BACKED               INTR397
        TEFA    *+1                                         INTR398
        TXL     *+3,4,20            OVER                    INTR399
        REWA    6                                           INTR400
        TRA     FINDIT                                      INTR401
        BSRA    6                                           INTR402
        TIX     *-1,4,1                                     INTR403
FINDIT  AXT     10,2                                        INTR404
        SDHA    6                                           INTR405
        RTBA    6                                           INTR406
```

INTR, INTRI (Cont'd)

```
         RCHA    IO                                                    INTR4070
         TCOA    *                                                     INTR4080
         TEFA    ERR1                                                  INTR4090
         CLA     TABLE                                                 INTR4100
         SUB     COM+1                                                 INTR4110
         TMI     *+2                                                   INTR4120
         TNZ     LOOKUP                 TAPE NOT POSITIONED PROPERLY   INTR4130
         TNZ     FINDIT                                                INTR4140
         AXT     513,4                                                 INTR4150
         CAL     TABLE                  CHECK                          INTR4160
         ACL     A+513,4                SUM                            INTR4170
         TIX     *-1,4,1                                               INTR4180
         TRCA    BSRA                                                  INTR4190
         LAS     C                                                     INTR4200
         TRA     *+2                                                   INTR4210
         TRA     SCALE                                                 INTR4220
BSRA     BSRA    6                                                     INTR4230
         TIX     FINDIT+1,2,1                                          INTR4240
         CALL    ERP                                                   INTR4250
IO       IOCD    TABLE,,515                                            INTR4260
ERR1     CALL    ERPT                                                  INTR4270
         PZE     TARG                                                  INTR4280
         REM                            SCALING                        INTR4290
         REM                            LOOP                           INTR4300
SCALE    AXT     189,4                                                 INTR4310
         LDQ     A+189,4                SCALE                          INTR4320
         FMP     SCALE1                 GEOCENTRIC                     INTR4330
         STO     A+189,4                EPHEMERIS                      INTR4340
         TIX     *-3,4,1                                               INTR4350
         AXT     324,4                                                 INTR4360
         LDQ     B+324,4                SCALE                          INTR4370
         FMP     SCALE2                 HELIOCENTRIC                   INTR4380
         STO     B+324,4                EPHEMERIS                      INTR4390
         TIX     *-3,4,1                                               INTR4400
         CLA     GRAV                   COMPUTE                        INTR4410
         FAD     GRAV+1                 MASS RATIO                     INTR4420
         STO     COM                    OF MOON                        INTR4430
         CLA     GRAV+1                 TO EARTH-MOON                  INTR4440
         FDH     COM                    BARYCENTER                     INTR4450
         STQ     MU                     FOR POSITION OF EARTH          INTR4460
         TRA     NEU                                                   INTR4470
SCALE1   DEC     6378.150               EARTH RADIUS                   INTR4480
SCALE2   DEC     149598500.             ASTRONOMICAL UNIT (JPL, JULY 1961INTR4490
TFIRST   DEC     3892.0                 0 HR AUG 28, 1960 JD = 2437174.5 INTR4500
TLAST    DEC     7292.0                 0 HR DEC 19, 1969 JD = 2440554.5 INTR4510
TRAP     PZE                                                           INTR4520
         PZE                                                           INTR4530
RJ       DEC     1E6                    JUPITER TEST DISTANCE          INTR4540
TEMPDT   DEC     34.                    E.T.-U.T.                      INTR4550
         PZE                                                           INTR4560
GRAV     DEC     3.98602E5              EARTH                          INTR4570
```

INTR, INTRI (Cont'd)

```
        DEC     4.8983349E3         MOON                              INTR458
        DEC     1.3252312E11        SUN                               INTR459
        DEC     3.2429889E5         VENUS                             INTR460
        DEC     4.2915518E4         MARS                              INTR461
        DEC     0                   REMOVE BARYCENTER                 INTR462
        DEC     1.2652701E8         JUPITER                           INTR463
EVEL    DEC     86400.                                                INTR464
HVEL    DEC     345600.                                               INTR465
MU      PZE                         MASS RATIO OF MOON TO BARYCENTER  INTR466
        OCT     000000520052        MARS, JUPITER VELOCITY            INTR467
HWORT   OCT     0                                                     INTR468
        OCT     527777777777        MOON VELOCITY                     INTR469
EWORT   OCT     0                                                     INTR470
HWORTJ  OCT     000000005252        BARYCENTER, JUPITER               INTR471
HWORTE  OCT     000000005200        BARYCENTER                        INTR472
EWORTO  OCT     527777777777                                          INTR473
HWORTO  OCT     000052525252                                          INTR474
HWORTV  OCT     525252525252                                          INTR475
TARGO   DEC     20,,0               FORMER TIME                       INTR476
KERNO   PZE                         FORMER CENTER                     INTR477
        PZE     A                   GEOCENTRIC REFERENCE              INTR478
        PZE     B                   HELIOCENTRIC REFERENCE            INTR479
ARG     PZE     0                                                     INTR480
1F      DEC     1                                                     INTR481
1.      DEC     1.                                                    INTR482
3.      DEC     3.                                                    INTR483
4.      DEC     4.                                                    INTR484
5.      DEC     5.                                                    INTR485
6.      DEC     6.                                                    INTR486
20.     DEC     20.                                                   INTR487
120.    DEC     120.                                                  INTR488
86400.  DEC     86400.                                                INTR489
VEL     BSS     3                   VELOCITY OPTION,H,SKIP CODE       INTR490
*                                                                     INTR491
TABLE   DEC     0                   RESERVE                           INTR492
A       BSS     189                 FOR                               INTR493
B       BSS     324                 WORKING                           INTR494
C       BSS     1                   EPHEMERIS                         INTR495
*                                                                     INTR496
TARG    PZE                                                           INTR497
        PZE                                                           INTR498
        PZE                                                           INTR499
CENTER  PZE                                                           INTR500
COM     BSS     21                                                    INTR501
VAFLG   PZE                                                           INTR502
FQFLG   BSS     3                                                     INTR503
KBO     PZE                                                           INTR504
KB1     PZE                                                           INTR505
KB2     PZE                                                           INTR506
KB3     PZE                                                           INTR507
KB4     PZE                                                           INTR508
```

INTR, INTRI (Cont'd)

```
KB5     PZE                                                  INTR5090
KB6     PZE                                                  INTR5100
XN      BSS      21                                          INTR5110
XN.     BSS      21                                          INTR5120
T       PZE                                                  INTR5130
        PZE                                                  INTR5140
R       PZE                                                  INTR5150
BSEP    SYN      7                                           INTR5160
SEP     SYN      XN.-XN                                      INTR5170
        END                                                  INTR
```

Subroutine: INV3

Purpose: To invert a matrix of any dimension up to a 6 by 6.

Calling Sequence:

CALL INV3 (A, N, DETERM)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
IO	A	6,6			A(I) Matrix to be inverted
					A(O) Inverse of input matrix
I	N	1			Dimension of input matrix
O	DETERM	1			Matrix determinant

Common storages used or required: None

Subroutines required: None

Functions required: ABS

Approximate number of storages required:

INV3

```
*     LABEL
C     SUBROUTINE FOR INVERTING SQUARE MATRICES WHICH ARE 6 BY 6 OR LESS
C     (TO INVERT LARGER MATRICES,SAY M BY M, DIMENSION IPIVOT(M),A(M*M),
C     INDEX(M,2),PIVOT(M) AND RECOMPILE)
C     A IS THE SQUARE MATRIX TO BE INVERTED
C     N IS THE SIZE OF A (A IS AN N BY N MATRIX)
C     THE SUBROUTINE RETURNS A INVERSE IN PLACE OF A AND THE DETERMINANT
C     IN DETERM.
      SUBROUTINE INV3(A,N,DETERM)
      DIMENSION IPIVOT(6),A(36) ,INDEX(6,2),PIVOT(6)
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM),(AMAX,T,SWAP)
      DETERM=1.0
      DO 20  J=1,N
 20   IPIVOT(J)=0
      DO 550  I=1,N
      AMAX=0.0
      DO 105 J=1,N
      IF(IPIVOT(J)-1) 60,105,60
 60   DO 100 K=1,N
      IF(IPIVOT(K)-1) 80,100,740
  80  M = N*(K-1)+J
      IF  (ABSF(AMAX)-ABSF(A(M))) 85,100,100
 85   IROW=J
      ICOLUM=K
      AMAX = A(M)
 100  CONTINUE
 105  CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
      IF(IROW-ICOLUM) 140,260,140
 140  DETERM=-DETERM
      DO 200 L=1,N
      M = N*(L-1)
      M1 = M+ICOLUM
      M = M+IROW
      SWAP = A(M)
      A(M) = A(M1)
  200 A(M1) = SWAP
 260  INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      M = N*(ICOLUM-1)+ICOLUM
      PIVOT(I) = A(M)
      DETERM=DETERM*PIVOT(I)
      A(M) = 1.0
      DO 350 L=1,N
      M = N*(L-1)+ICOLUM
  350 A(M) = A(M)/PIVOT(I)
      DO 550 L1=1,N
      IF(L1-ICOLUM) 400,550,400
  400 M = N*(ICOLUM-1)+L1
      T = A(M)
      A(M) = 0.
```

INV3 (Cont'd)

```
      DO 450  L=1,N
      M = N*(L-1)
      M1 = M+ICOLUM
      M = M+L1
  450 A(M) = A(M)-A(M1)*T
  550 CONTINUE
      DO 710 I=1,N
      L=N+1-I
      IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      M = N*(JROW-1)
      M1 = N*(JOOLUM-1)
      DO 705 K=1,N
      M = M+1
      M1 = M1+1
      SWAP = A(M)
      A(M) = A(M1)
      A(M1) = SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
```

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Subroutine: INVAO

Purpose: To form the inverse of a transition matrix for a linear, conservative dynamic system. If Φ is a partioned matrix such as

$$\Phi = \begin{pmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{pmatrix} \quad \text{then INVAO computes } \Phi^{-1} = \begin{pmatrix} \Phi_4^T & -\Phi_2^T \\ -\Phi_3^T & \Phi_1^T \end{pmatrix}$$

Calling Sequence:

CALL INVAO (AO, AOI)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	AO	6,6	$\Phi(t_2,t_1)$		Transition Matrix
O	AO1	6,6	$\Phi^{-1}(t_2,t_1)$		Inverse Transition Matrix

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

INVAO

```
*      LABEL                                                    INVA
*      SYMBOL TABLE                                             INVA
       SUBROUTINE INVAO(AO,AOI)                                 INVA0000
       DIMENSION AO(6,6),AOI(6,6)                               INVA0010
       DO 1 I=1,3                                               INVA0020
       II=I+3                                                   INVA0030
       DO 1 J=1,3                                               INVA0040
       JJ=J+3                                                   INVA0050
       AOI(I,J)=AO(JJ,II)                                       INVA0060
       AOI(II,J)=-AO(JJ,I)                                      INVA0070
       AOI(II,JJ)=AO(J,I)                                       INVA0080
    1  AOI(I,JJ)=-AO(J,II)                                      INVA0090
       RETURN                                                   INVA0100
       END                                                      INVA
```

Subroutine: LOADO

Purpose: To transfer the transition matrix from its storage in the integration package's T block to the matrix in the call list.

Calling Sequence:

CALL LOADO (AO)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	AO	(6,6)	$\Phi(t_2 t_1)$		Transition Matrix

Common storages used or required: T

Subroutines required: None

Functions required: None

Approximate number of storages required:

LOADO

```
*      LABEL
*      SYMBOL TABLE
       SUBROUTINE LOADO(AC)
       COMMON T
       DIMENSION T(1360),AO(6,6)
       K=6
       JJ=27
       DO1 I=1,6
       DO 1 J=1,3
       K=K+1
       KK=J+3
       JJ=JJ+1
       AO(KK,I)=T(JJ)
       AO(J,I)=T(K)
    1  CONTINUE
       RETURN
       END
```

Subroutine: LOADT

Purpose: To place unit initial conditions in the T block for the variational equations which are being used to generate the transition matrix.

Calling Sequence:

CALL LOADT

Input and **Output**

Common storages used or required: T

Subroutines required: None

Functions required: None

Approximate number of storages required:

LOADT

```
*       LABEL                                                            LOAT
*       SYMBOL TABLE                                                     LOAT
        SUBROUTINE LOADT                                                 LOAT0000
C       SUBROUTINE LOADT PUTS UNIT ICS ON PERTURBATION EQUATIONS         LOAT0010
        COMMON T                                                         LOAT0020
        DIMENSION T(1360),X(3,6),V(3,6)                                  LOAT0030
        EQUIVALENCE (T(7), X),(T(28),V)                                  LOAT0040
        DO 2 I=1,3                                                       LOAT0050
        L=I+3                                                            LOAT0060
        DO 1 J=1,3                                                       LOAT0070
        K=J+3                                                            LOAT0080
        X(I,J)=0.                                                        LOAT0090
        V(I,K)=0.                                                        LOAT0100
        X(I,K)=0.                                                        LOAT0110
        V(I,J)=0.                                                        LOAT0120
      1 CONTINUE                                                         LOAT0130
        X(I,I)=1.                                                        LOAT0140
        V(I,L)=1.                                                        LOAT0150
      2 CONTINUE                                                         LOAT0160
        RETURN                                                           LOAT0170
        END                                                              LOAT
```

Subroutine: MASS

Purpose: To find the relevant gravitational constants to be used in computing planetary perturbations for a given central body. MASS also chooses the initial integration step size as a function of central body.

Calling Sequence:

CALL MASS (NOR, UM, VKB, X, ACCP)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	NOR	1			Central Body Indicator.
					1=Earth; 2, Moon; 3, Sun; 4,
					Venus; 5, Mars; 6, Jupiter
I	UM	6	μ_i	Km^3/sec^2	Gravitational constants, arranged
					as above
O	VKB	6	μ_i	Km^3/sec^2	μ_i for bodies
I	X	3	X, Y, Z	Km	Position coordinates
O	ACCP	1		seconds	Initial integration step size

Common storages used or required: None

Subroutines required: None

Functions required: FNORM

Approximate number of storages required:

Method of Establishing Integration Step Size and Perturbing Bodies

Central Body	Integration Step Size	Other Bodies to be Considered
Earth	60.0	Moon, Sun. If $\lvert X\rvert \geq 10^6$Km, Jupiter.
Moon	60.0	Earth, Sun.
Sun	43200.0	All
Venus, Mars, Jupiter	60.0	All

The bodies which are not to be used in calculation of pertubation accelerations are eliminated from consideration by placing zeros in array VKB(6). For the bodies which are being considered, the appropriate gravitational constant is placed in VKB(6).

MASS

```
*      LABEL
*      SYMBOL TABLE
       SUBROUTINE MASS(NOR,UM,VKB,X,ACCP)
       DIMENSION UM(6),VKB(6),X(3)
       N = NOR
       DO 1 I=1,6
    1  VKB(I) = 0.
       GO TO (2,3,6,4,4,4),N
    2  VKB(2) = UM(2)
       ACCP = 60.
       DIS = FNORM(X)
       VKB(3) = UM(3)
       IF (DIS-1000000.) 11,10,10
   10  VKB(6) = UM(6)
       GO TO 12
   11  VKB(6) = 0.
   12  RETURN
    3  VKB(1) = UM(1)
       VKB(3) = UM(3)
       ACCP = 60.
       RETURN
    6  ACCP = 43200.
       GO TO 7
    4  ACCP = 60.
    7  DO 5 I=1,6
    5  VKB(I) = UM(I)
       VKB(N) = 0.
       RETURN
       END
```

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Subroutine: MATRX

Purpose: To perform matrix multiplications of matricies with any dimensions up to maximum dimensions of 10 by 10. A zero in the call sequence (J) yields an output matrix $C = A \cdot B$. A one in the call sequence (J) yields an output matrix $C = A \cdot B \cdot A^T$. The matrix products are obtained in double precision.

Calling Sequence:

CALL MATRX (A, B, C, NRA, NCA, NCB, J)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	A	(10,10)			Input Matrix
I	B	(10,10)			Input Matrix
O	C	(10,10)			Output Matrix
I	NRA	1			Number of Rows of A
I	NCA	1			Number of Columns of A
I	NCB	1			Number of Columns of B
I	J	1			Type of multiplication desired

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

```
*      LABEL                                                   MATX
*      SYMBOL TABLE                                            MATX
CEC2000  SUBROUTINE MATRX                                      MATX
       SUBROUTINE MATRX(A,B,C,NRA,NCA,NCB,J)                   MATX0000
C      J=0 A*B=C                                               MATX0010
C      J=1 A*B*AT=C  T MEANS TRANSPOSE                         MATX0020
C      NRA=NUMBER OF ROWS OF A                                 MATX0030
C      NCA=COLUMNS OF A                                        MATX0040
C      NCB=COLUMNS OF B                                        MATX0050
C      ACCUMULATION OF PRODUCTS IN DOUBLE PRECISION            MATX0060
       DIMENSION A(100),B(100),C(100),D(100)                   MATX0080
D      DUD=0.                                                  MATX0090
D      DUM=0.                                                  MATX0100
       DO 1 I=1,NRA                                            MATX0110
       DO 1 K=1,NCB                                            MATX0120
       NC=I+(K-1)*NRA                                          MATX0130
D      CRUD=0.                                                 MATX0140
       DO 2 L=1,NCA                                            MATX0150
       NA=I+(L-1)*NRA                                          MATX0160
       NB=L+(K-1)*NCA                                          MATX0170
       DUD=A(NA)                                               MATX0180
       DUM=B(NB)                                               MATX0190
D      CRUD=CRUD+DUD*DUM                                       MATX0200
     2 CONTINUE                                                MATX0210
       D(NC)=CRUD                                              MATX0220
     1 CONTINUE                                                MATX0230
       IF(J)3,3,4                                              MATX0240
     3 CONTINUE                                                MATX0250
       KK=NCB*NRA                                              MATX0260
       DO 5 I=1,KK                                             MATX0270
     5 C(I)=D(I)                                               MATX0280
       GO TO 10                                                MATX0290
     4 CONTINUE                                                MATX0300
       DO 6 I=1,NRA                                            MATX0310
       DO 6 K=1,NRA                                            MATX0320
       NC=I+(K-1)*NRA                                          MATX0340
D      CRUD=0.                                                 MATX0350
       DO 7 L=1,NCA                                            MATX0360
       NA=I+(L-1)*NRA                                          MATX0370
       NB=K+(L-1)*NRA                                          MATX0380
       DUM=D(NA)                                               MATX0390
       DUD=A(NB)                                               MATX0400
D      CRUD=CRUD+DUM*DUD                                       MATX0410
     7 CONTINUE                                                MATX0420
       C(NC)=CRUD                                              MATX0430
     6 CONTINUE                                                MATX0440
    10 CONTINUE                                                MATX0510
       RETURN                                                  MATX0520
       END                                                     MATX
```

Subroutine: MATSUB

Purpose: The subroutine is primarily logic which controls (1) trajectory and data printout, (2) updating of the state covariance matrix for observations by calling subroutines EARTR, ONBTR, and MONBTR, and (3) updating of the guidance covariance matrix by calling subroutine GUID.

Calling Sequence:

CALL MATSUB

Input and Output

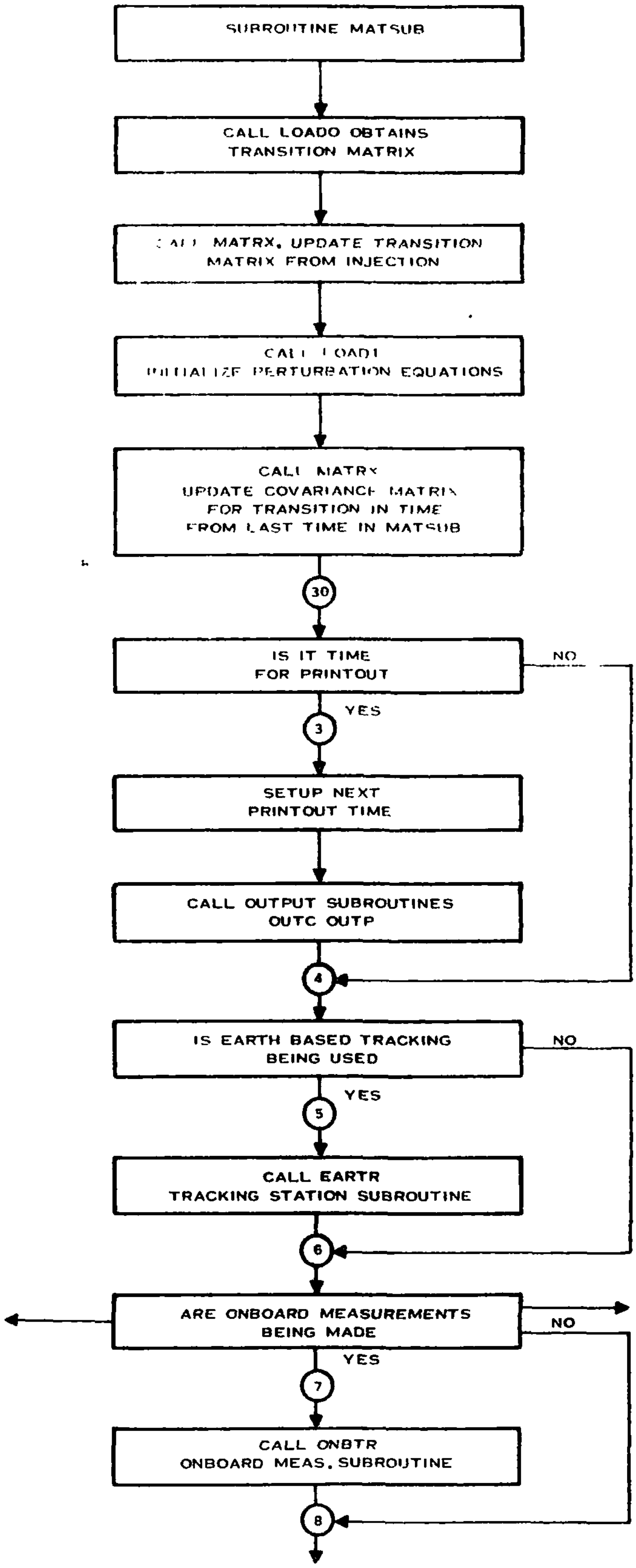
I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities returned to common

Common storages used or required: T, S, C, IC

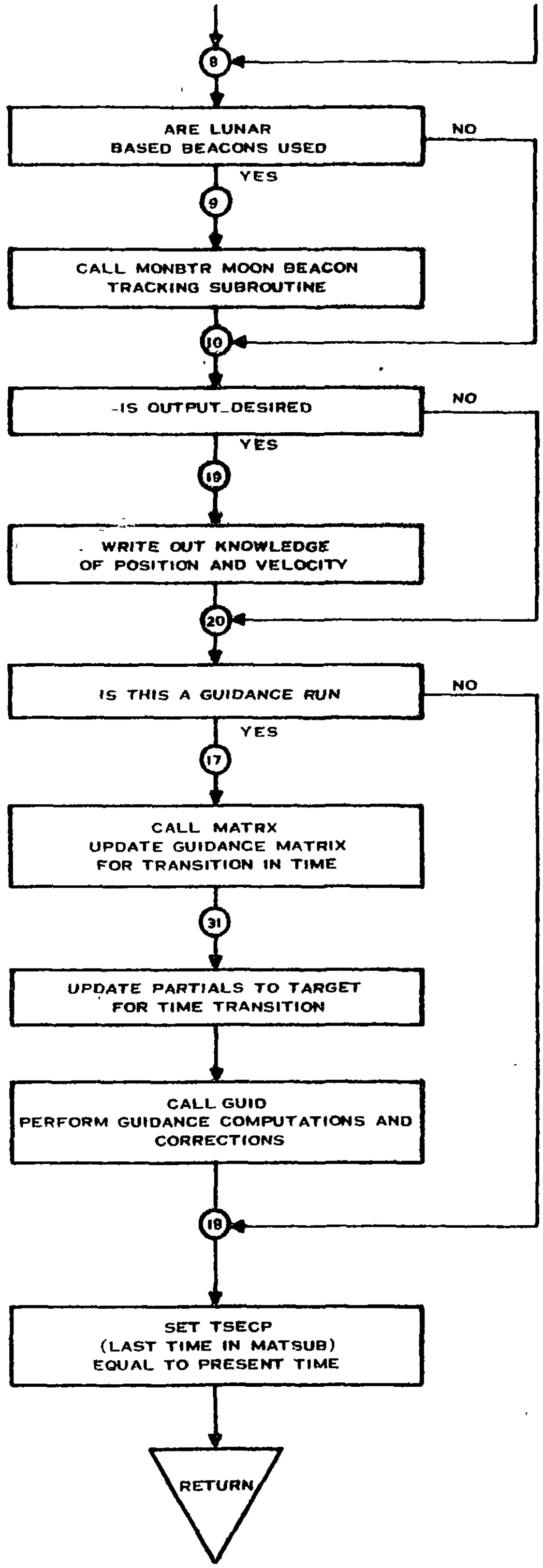
Subroutines required: EARTR, GUID, INVAO, LOADO, LOADT, MATRX, MONBTR, ONBTR, OUTC, OUTP

Functions required: SQRT (FIL)(STH)

Approximate number of storages required: 346 DEC


SUBROUTINE MATSUB
CALL LOADO OBTAINS TRANSITION MATRIX
CALL MATRX, UPDATE TRANSITION MATRIX FROM INJECTION
CALL LOAD1 INITIALIZE PERTURBATION EQUATIONS
CALL MATRX UPDATE COVARIANCE MATRIX FOR TRANSITION IN TIME FROM LAST TIME IN MATSUB
30
IS IT TIME FOR PRINTOUT
NO
YES
3
SETUP NEXT PRINTOUT TIME
CALL OUTPUT SUBROUTINES OUTC OUTP
4
IS EARTH BASED TRACKING BEING USED
NO
YES
5
CALL EARTR TRACKING STATION SUBROUTINE
6
ARE ONBOARD MEASUREMENTS BEING MADE
NO
YES
7
CALL ONBTR ONBOARD MEAS. SUBROUTINE
8

MATSUB -2


8
ARE LUNAR BASED BEACONS USED
NO
YES
9
CALL MONBTR MOON BEACON TRACKING SUBROUTINE
10
IS OUTPUT DESIRED
NO
YES
19
WRITE OUT KNOWLEDGE OF POSITION AND VELOCITY
20
IS THIS A GUIDANCE RUN
NO
YES
17
CALL MATRX UPDATE GUIDANCE MATRIX FOR TRANSITION IN TIME
31
UPDATE PARTIALS TO TARGET FOR TIME TRANSITION
CALL GUID PERFORM GUIDANCE COMPUTATIONS AND CORRECTIONS
18
SET TSECP (LAST TIME IN MATSUB) EQUAL TO PRESENT TIME
RETURN

```
*     LABEL                                                                 MATS
*     SYMBOL TABLE                                                          MATS
CEC2006  SUBROUTINE MATSUB                                                  MATS
      SUBROUTINE MATSUB                                                     MATS0000
      COMMON T,S,C,IC                                                       MATS0010
      DIMENSION T(1360),S(1000),C(1000),IC(1),AN(3,3),PO(22),VE(22),        MATS0020
     1EM(3,3),XP(3),VXP(3),EA(3,3),XED(3),VED(3),AO(6,6),P(6,6),            MATS0030
     2TFTA(3,6),PFTA(6,6),AOS(6,6),AOI(6,6)                                 MATS0040
      EQUIVALENCE (T ,TDUM),(S ,SDUM),(C ,CDUM),                            MATS0050
     1(C(138),AN),(C(13),TP1),(C(14),TP2),(C(62),PO),(C(84),VE),            MATS0060
     2(C(120),EM),(C(650),TPRINT),(C(651),TSTRT),(C(30),TSEC),              MATS0070
     3(C(15),XP),(C(18),VXP),(C(10),TW),(C(11),TF) ,(C(129),EA),            MATS0080
     4(C(652),P),(C(752),AOS),(C(850),AOI)                                  MATS0090
     5,(C(460),PFTA),(C(568),TFTA),(C(647),TOUT)                            MATS0100
      EQUIVALENCE (IC(2),IOR),(IC(7),KOUT),(IC(6),ITARG),(IC(9),LPRINT),MATS0110
     1(IC(190),LSTAT),(IC(191),LONB),(IC(192),LMB),(IC(193),IGUID),         MATS0120
     2(IC(214),NOUT),(C(649),TSECP)                                         MATS0130
      CALL SETN(NIN,NUTS)                                                   MATS0131
      NOUT=1                                                                MATS0140
      CALL LOADO(AO)                                                        MATS0145
      CALL MATRX(AO,AOS,AOS,6,6,6,0)                                        MATS0150
      CALL LOADT                                                            MATS0155
      CALL MATRX(AO,P,P,6,6,6,1)                                            MATS0160
      DO 30 I=1,6                                                           MATS0165
      DO 30 J=I,6                                                           MATS0170
      P(I,J)=(P(I,J)+P(J,I))/2.                                             MATS0175
      P(J,I)=P(I,J)                                                         MATS0180
   30 CONTINUE                                                              MATS0185
      IF(TOUT-TSEC) 3,3,4                                                   MATS0220
    3 CONTINUE                                                              MATS0230
      TOUT=TOUT+TPRINT                                                      MATS0240
      NOUT=2                                                                MATS0250
C     NOUT IS USED TO INDICATE OTHER OUTPUT WANTED WHEN = 2                 MATS0260
      CALL OUTC                                                             MATS0270
      CALL OUTP                                                             MATS0280
    4 CONTINUE                                                              MATS0290
C     LSTAT IS SET 2 BY INPUT CARDS IF ANY EARTH BASED TRACKING             MATS0300
      LSTAT=LSTAT                                                           MATS0310
      GO TO (6,5),LSTAT                                                     MATS0320
    5 CONTINUE                                                              MATS0330
      CALL EARTR                                                            MATS0340
C     EARTR UDDATS COV MATRIX P FOR EARTH BASED TRACKING                    MATS0350
    6 CONTINUE                                                              MATS0360
      LONB=LONB                                                             MATS0370
      GO TO (8,7),LONB                                                      MATS0380
C     LONB IS SET 2 BY INPUT CARDS IF ANY ONBOARD MEASUREMENTS MADE         MATS0390
    7 CONTINUE                                                              MATS0400
C     ONBTR  UPDATES COV MATRIX P FOR THE ONBOARD MEASUREMENTS              MATS0410
      CALL ONBTR                                                            MATS0420
    8 CONTINUE                                                              MATS0430
      LMB=LMB                                                               MATS0440
```

MATSUB (Cont'd)

```
      GO TO (10,9),LMB                                                  MATS045
C     LMB IS SET 2 BY INPUT CARDS IF MOON BASED BEACONS USED            MATS046
    9 CONTINUE                                                          MATS047
      CALL MONBTR                                                       MATS048
C    MONBTR UPDATS COV MATRIX P FOR BEACONS ON MOON                     MATS049
   10 CONTINUE                                                          MATS050
      GO TO (20,19), NOUT                                               MATS051
  19  CONTINUE                                                          MATS052
      RMSP=SQRTF (P (1,1)+P (2,2)+P (3,3))                              MATS053
      RMSVP=SQRTF(P (4,4)+P (5,5)+P (6,6))                              MATS054
      WRITE OUTPUT TAPE NUTS,700,RMSP,RMSVP                             MATS055
 700  FORMAT(42H KNOWLEDGE OF STATE AFTER ALL OBSERVATIONS,/            MATS056
     124H              RMS POSITION=,E15.8,25H              RMS VELOCITY=,  MATS057
     2E15.8)                                                            MATS058
  20  CONTINUE                                                          MATS059
C     NUMBERS 11-16 RESERVED FOR FUTURE MEASUREMENTS                    MATS060
      IGUID=IGUID                                                       MATS061
      GO TO (18,17),IGUID                                               MATS062
C     IGUID IS SET BY INPUT TO MAIN PROGRAM IF GUIDANCE CALCULATIONS AREMATS063
C     MADE                                                              MATS064
   17 CONTINUE                                                          MATS065
      CALL MATRX(AO,PFTA,PFTA,6,6,6,1)                                  MATS065
      DO 31 I=1,6                                                       MATS066
      DO 31 J=I,6                                                       MATS066
      PFTA(I,J)=(PFTA(I,J)+PFTA(J,I))/2.                                MATS067
      PFTA(J,I)=PFTA(I,J)                                               MATS067
  31  CONTINUE                                                          MATS068
      CALL INVAO(AO,AOI)                                                MATS068
      CALL MATRX(TFTA,AOI,TFTA,3,6,6,0)                                 MATS069
      CALL GUID                                                         MATS071
C     GUID PERFORMS THE GUIDANCE CALCULATIONS                           MATS072
   18 CONTINUE                                                          MATS073
      TSECP=TSEC                                                        MATS074
      RETURN                                                            MATS075
      END                                                               MATS
```

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Subroutine: MNA

Purpose: To provide the rotation matrix EMN which transforms moon centered coordinates in the earth's true equator and equinox to moon centered coordinates in the moon's true equator.

Calling Sequence:

CALL MNA (TIME, OM, CR, DT, EPSIL, RO, G, GP, WW, EMN)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TIME	1	d	days	Days since 1950.0
I	OM	1	Ω		Output from NUTAIT: (Arg. De-
					cending node)
I	CR	1	☾	radians	Output from NATAIT: (mean long.
					of the moon)
I	DT	1	$\Delta\Psi + d\Psi$	radians	Output from NUTAIT: (nutation in
					longitude)
I	EPSIL	1	$\bar{\epsilon} + \Delta\epsilon + d\epsilon$	radians	Output from NUTAIT: (mean obliquity
					and nutation in obliquity)

(Cont'd.)

Common storages used or required: None.

Subroutines required: System double precision routines.

Functions required: INTF, SQRT, SIN, COS, ACOS

Approximate number of storages required: __________

(Cont'd.)

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
0	RO	1	ρ		Libration term
0	G	1	g	radians	Mean anomaly of the moon
0	GP	1	g	radians	Mean anomaly of the sun
0	WW	1	ω	radians	Argument of the perigee of the
					moon
0	EMN	3,3			Rotation matrix

Transformation From Earth's True Equator to Moon's True Equator.

The two rectangular systems are related through Λ, Ω', and i by the rotation:

$$\underset{\text{MOON}}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \underset{\text{EARTH}}{\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}}$$

where

$$b_{11} = \cos \Lambda \cos \Omega' - \sin \Lambda \sin \Omega' \cos i$$

$$b_{12} = \cos \Lambda \sin \Omega' + \sin \Lambda \cos \Omega' \cos i$$

$$b_{13} = \cos \Lambda \sin i$$

$$b_{21} = -\sin \Lambda \cos \Omega' - \cos \Lambda \sin \Omega' \cos i$$

$$b_{22} = -\sin \Lambda \sin \Omega' + \cos \Lambda \cos \Omega' \cos i$$

$$b_{23} = \cos \Lambda \sin i$$

$$b_{31} = \sin \Omega' \sin i$$

$$b_{32} = -\cos \Omega' \sin i$$

$$b_{33} = \cos i$$

i is the inclination of the moon's true equator to the earth's equator

Ω' is the right ascension of the ascending node of the moon's true equator

Λ is the anomaly from the node to the X axis

$$\Lambda = \Delta + (☾ + \Upsilon) - (\Omega + \sigma)$$

Δ is the anomaly from the node to the ascending node of the moon's true equator on the ecliptic

Ω is the mean longitude of the descending node of the moon's mean equator on the ecliptic

☾ is the mean longitude of the moon

σ is the libration in the node

Υ is the libration in the mean longitude

ρ is the libration in the inclination.

$\delta\Psi$, ϵ, Ω, and ☾ are input quantities obtained from NUTAIT. The remainder are computed from the following equations.

I = inclination of moon's equator to ecliptic

$$I = 1.535^{\circ}$$

g = mean anomaly of moon

$$g = 215.54013 + 13.064992\ d$$

g' = mean anomaly of sun

$g' = 358.009067 + .9856005\ d$

ω = argument of perigee of moon

$\omega = 196.745632 + .1643586\ d$

where d = days from 1950.

$$\sigma \sin I = -.0302777 \sin g + .0102777 \sin (g + 2\omega) - .00305555 \sin (2g + 2\omega)$$

$$\tau = -.003333 \sin g + .0163888 \sin g' + .005 \sin 2\omega$$

$$\rho = -.0297222 \cos g + .0102777 \cos (g + 2\omega) - .00305555 \cos (2g + 2\omega)$$

$$\cos i = \cos (\Omega + \sigma + \delta\Psi) \sin \epsilon \sin (I + \rho) + \cos \epsilon \cos (I + \rho) \qquad 0 < i < 90^\circ$$

$$\sin \Omega' = -\sin (\Omega + \sigma + \delta\Psi) \sin (I + \rho) \csc i \qquad -90^\circ < \Omega' < 90^\circ$$

$$\sin \Delta = -\sin (\Omega + \sigma + \delta\Psi) \sin \epsilon \csc i$$

$$\cos \Delta = -\sin (\Omega + \sigma + \delta\Psi) \sin \Omega' \cos \epsilon - \cos (\Omega + \sigma + \delta\Psi) \cos \Omega' \qquad 0^\circ \le \Delta < 360^\circ$$

Reference: JPL Technical Report No. 32-223

```
*       LABEL
*       SYMBOL TABLE
 CEC20AH
        SUBROUTINE MNA(TIME,OM,CR,DT,EPSIL,RO,G,GP,WW,EM)
        DIMENSION EM(3,3),DF(3)
        D = TIME
        T = D/36525.
        T2 = T*T
        T3 = T2*T
        A=13.064992
        DO 6 I=1,3
        DD=D
 D      DD=DD*(A/360.)
 D      DD=DD-INTF(DD)
        DF(I)=DD
        GO TO (4,5,6),I
    4   A=.9856005
        GO TO 6
    5   A=.1643586
    6   CONTINUE
        G=215.54013+360.*DF(1)
        GP=358.009067+360.*DF(2)
        WW=196.745632+360.*DF(3)
        G = G*.017453296
        GP = GP*.017453296
        WW = WW*.017453296
        YN = 1.535*.017453296
        RO = -.0297222*COSF(G) + .01020777*COSF(G+2.*WW)
       1 -.00305555*COSF(2.*G+2.*WW)
        TA = -.003333*SINF(G) + .0163888*SINF(GP)
       1 +.005*SINF(2.*WW)
        SG = -.0302777*SINF(G) + .0102777*SINF(G+2.*WW)
       1 -.00305555*SINF(2.*G+2.*WW)
        SG = (SG*.017453296)/SINF(YN)
        RO = RO*.017453296
        TA = TA*.017453296
        YN = YN + RO
        RO = OM + SG + DT
        CI = COSF(RO)*SINF(EPSIL)*SINF(YN)
       1 +COSF(EPSIL)*COSF(YN)
        SI = 1. - CI**2
        SI = SQRTF(SI)
        SO = -SINF(RO)*SINF(YN)/SI
        CO = 1. - SO**2
        CO = SQRTF(CO)
        SD = -SINF(RO)*SINF(EPSIL)/SI
        CD = -SINF(RO)*SO*COSF(EPSIL) - COSF(RO)*CO
        DL = ACOSF(CD)
        IF(SD)1,3,3
  1     DL = 6.283185306 - DL
  3     CONTINUE
```

MNA (Cont'd)

```
CA = DL + (CR + TA) - (OM + SG)                     SMNA048
SA = SINF(CA)                                       SMNA049
CA = COSF(CA)                                       SMNA050
RO = COSF(RO)*SINF(EPSIL)/(SI*CD)                   SMNA051
EM(1,1) = CA*CO - SA*SO*CI                          SMNA052
EM(1,2) = CA*SO + SA*CO*CI                          SMNA053
EM(1,3) = SA*SI                                     SMNA054
EM(2,1) = -SA*CO - CA*SO*CI                         SMNA055
EM(2,2) = -SA*SO + CA*CO*CI                         SMNA056
EM(2,3) = CA*SI                                     SMNA057
EM(3,1) = SO*SI                                     SMNA058
EM(3,2) = -CO*SI                                    SMNA059
EM(3,3) = CI                                        SMNA060
RETURN                                              SMNA061
END                                                 SMNA
```

Subroutine: MNAND

Purpose: To compute the matrix M used to perform the velocity transformation corresponding to the position transformation described in MNA.

Calling Sequence:

CALL MNAND (TIME, RO, G, GP, WW, EM, DM)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TIME	1	d	days	Days since 1950.0
I	RO	1	ρ	radians	Output from MNA; see description
I	G	1	g	radians	Output from MNA; see description
I	GP	1	g'	radians	Output from MNA; see description
I	WW	1	ω	radians	Output from MNA; see description
I	EM	3,3	M		Output from MNA; matrix M
O	DM	3,3	$\dot{M}$		Matrix $\dot{M}$

Common storages used or required: None

Subroutines required: None

Functions required: COS

Approximate number of storages required:

Theory and Equations

In transforming lunacentric position coordinates relative to the earth's equator and equinox of 1950.0 to coordinates relative to the moon's true equator, three matrices are computed. Matrix A (from ROTEQ) rotates the coordinates to equinox of date, matrix N (from NUTAIT) accounts for the nutation of the earth about its precessing mean equator, and matrix M (from MNA) transforms to the position relative to the moon's true equator.

Then

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{moon}} = MNA \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{earth, 1950.0}}$$

Assuming that $\dot{N} = \dot{A} = 0$

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix}_{\text{moon}} = MNA \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix}_{\text{earth, 1950}} + \dot{M}NA \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{earth, 1950}}$$

The $\dot{M}$ matrix is computed from the equations:

$$\dot{M} = \begin{pmatrix} M_{21}\dot{\Lambda} & M_{22}\dot{\Lambda} & M_{23}\dot{\Lambda} \\ -M_{11}\dot{\Lambda} & -M_{12}\dot{\Lambda} & -M_{13}\dot{\Lambda} \\ 0 & 0 & 0 \end{pmatrix}$$

where M_{ij} are the elements of M, and

$$\dot{\Lambda} = \dot{\Delta} + \dot{☾} + \dot{t} - \dot{\Omega} - \dot{\sigma}$$

$$\dot{\Delta} = -\rho\ (\dot{\Omega} + \dot{\sigma})$$

$$\dot{☾} = 0.266170762 \times 10^{-5} - 0.12499171 \times 10^{-13}\ T \text{ rad/sec; } T = d/36525$$

$$\dot{\Omega} = -0.1069698435 \times 10^{-7} + 0.23015329 \times 10^{-3}\ T \text{ rad/sec}$$

$$\dot{t} = -0.1535272946 \times 10^{-9} \cos g$$
$$+0.569494067 \times 10^{-10} \cos g$$
$$+0.579473484 \times 10^{-11} \cos 2\omega \text{ rad/sec}$$

$$\dot{\sigma} = -0.520642191 \times 10^{-7} \cos g$$
$$+0.1811774451 \times 10^{-7} \cos (g+2\omega)$$
$$-0.1064057858 \times 10^{-7} \cos (2\omega+2g) \text{ rad/sec}$$

Reference: JPL Technical Report No. 32-223

```
*      LABEL                                                      MNAN
*      SYMBOL TABLE                                               MNAN
       SUBROUTINE MNAND(TIME,RO,G,GP,WW,EM,DM)                    MNAN0000
       DIMENSION DM(3,3),EM(3,3)                                  MNAN0010
       T = TIME/36525.                                            MNAN0020
       CRD = .266170762E-5 - .12499171E-13*T                      MNAN0030
       OMD = -.1069698435E-7 + .23015329E-13*T                    MNAN0040
       TAD = -1.535272946*COSF(G) + .569494067*COSF(GP)           MNAN0050
      1 + .0579473484*COSF(2.*WW)                                 MNAN0060
       TAD = .1E-9*TAD                                            MNAN0070
       SGD = -.520642191*COSF(G) + .1811774451*COSF(G+2.*WW)      MNAN0080
      1 -.1064057858*COSF(2.*WW+2.*G)                             MNAN0090
       SGD = .1E-6*SGD                                            MNAN0100
       DLD = -RO*(OMD + SGD)                                      MNAN0110
       CAD = DLD + CRD + TAD - OMD - SGD                          MNAN0120
       DM(1,1) = EM(2,1)*CAD                                      MNAN0130
       DM(1,2) = EM(2,2)*CAD                                      MNAN0140
       DM(1,3) = EM(2,3)*CAD                                      MNAN0150
       DM(2,1) = -EM(1,1)*CAD                                     MNAN0160
       DM(2,2) = -EM(1,2)*CAD                                     MNAN0170
       DM(2,3) = -EM(1,3)*CAD                                     MNAN0180
       DM(3,1) = 0.0                                              MNAN0190
       DM(3,2) = 0.0                                              MNAN0200
       DM(3,3) = 0.0                                              MNAN0210
       RETURN                                                     MNAN0220
       END                                                        MNAN
```

Subroutine: MONBTR

Purpose: To obtain the position and velocity vectors of the vehicle relative to the beacon and determine if the beacon is in view of the vehicle. The station location partials are computed and subroutine COMPHQ called to update the state covariance matrix for the measurements being made. The types of measurements possible are range, range rate, right ascension, and declination.

Calling Sequence:

CALL MONBTR

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities placed in common

Common storages used or required: T, S, C, IC

Subroutines required: COMPHQ, DOT, INTRI, MATRX, MNAND, MNA, NUTAIT, TRAC

Functions required: ASIN, ATAN, FNORM, SQRT (FIL)(SLO)(STH)

Approximate number of storages required: 495 DEC

MONBTR Partial Derivatives

MONBTR computes two partial matrices which are used in COMPHQ to evaluate station location errors. The matrices are the following:

$$\text{STPARS} = \begin{pmatrix} \frac{\partial x_1}{\partial LAT} & \frac{\partial x_1}{\partial LON} & \frac{\partial x_1}{\partial ALT} \\ \frac{\partial x_1}{\partial LAT} & \frac{\partial x_2}{\partial LON} & \frac{\partial x_2}{\partial ALT} \\ \frac{\partial x_3}{\partial LAT} & \frac{\partial x_3}{\partial LON} & \frac{\partial x_3}{\partial ALT} \end{pmatrix} .$$

$$\text{STPARD} = \begin{pmatrix} \frac{\partial \dot{x}_1}{\partial LAT} & \frac{\partial \dot{x}_1}{\partial LON} & \frac{\partial \dot{x}_1}{\partial ALT} \\ \frac{\partial \dot{x}_2}{\partial LAT} & \frac{\partial \dot{x}_2}{\partial LON} & \frac{\partial \dot{x}_2}{\partial ALT} \\ \frac{\partial \dot{x}_3}{\partial LAT} & \frac{\partial \dot{x}_3}{\partial LON} & \frac{\partial \dot{x}_3}{\partial ALT} \end{pmatrix}$$

where $\vec{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = -\vec{X}_V + \vec{X}_B \equiv$ XREL (FORTRAN)

$$\dot{\vec{X}} = \begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{pmatrix} = -\dot{\vec{X}}_v + \dot{\vec{X}}_B \equiv \text{VREL (FORTRAN)}$$

$$\frac{\partial x_i}{\partial LAT} = \frac{\partial x_{Bi}}{\partial LAT} \qquad \frac{\partial \dot{x}_i}{\partial LAT} = \frac{\partial \dot{x}_{Bi}}{\partial LAT} \qquad i = 1,2,3$$

$$\vec{X}_B = (MNA)^T \; \vec{X}_{BM}$$

$$\dot{\vec{X}}_B = (MNAND)^T \; \vec{X}_{BM} + (MNA)^T \dot{\vec{X}}_{BM} \quad (=0)$$

where subscripts BM indicate beacon moon-fixed coordinates.

The transformations MNA and MNAND are obtained by calling the subroutines MNA and MNAND.

$$\vec{X}_{BM} = (R + ALT) \begin{pmatrix} \cos LAT \cos LON \\ \cos LAT \sin LON \\ \sin LAT \end{pmatrix}$$

3 x 1   3 x 3   3 x 1

$$\begin{pmatrix} \frac{\partial X_1}{\partial PAR} \\ \frac{\partial X_2}{\partial PAR} \\ \frac{\partial X_3}{\partial PAR} \end{pmatrix} = (MNA)^T \begin{pmatrix} \frac{\partial X_{BM}}{\partial PAR} \\ \frac{\partial Y_{BM}}{\partial PAR} \\ \frac{\partial Z_{BM}}{\partial PAR} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial \dot{X}_1}{\partial PAR} \\ \frac{\partial \dot{X}_2}{\partial PAR} \\ \frac{\partial \dot{X}_3}{\partial PAR} \end{pmatrix} = (MNAND)^T \begin{pmatrix} \frac{\partial X_{BM}}{\partial PAR} \\ \frac{\partial Y_{BM}}{\partial PAR} \\ \frac{\partial Z_{BM}}{\partial PAR} \end{pmatrix}$$

$$\frac{\partial \vec{X}_{BM}}{\partial LAT} = (R + ALT) \begin{pmatrix} -\sin LAT \cos LON \\ -\sin LAT \sin LON \\ \cos LAT \end{pmatrix}$$

3 x 1 3 x 1

$$\frac{\partial \vec{X}_{BM}}{\partial LON} = (R + ALT) \begin{pmatrix} -\cos LAT \sin LON \\ \cos LAT \cos LON \\ 0 \end{pmatrix}$$

3 x 1 3 x 1

$$\frac{\partial \vec{X}_{BM}}{\partial ALT} = \begin{pmatrix} \cos LAT \cos LON \\ \cos LAT \sin LON \\ \sin LAT \end{pmatrix}$$

3 x 1 3 x 1

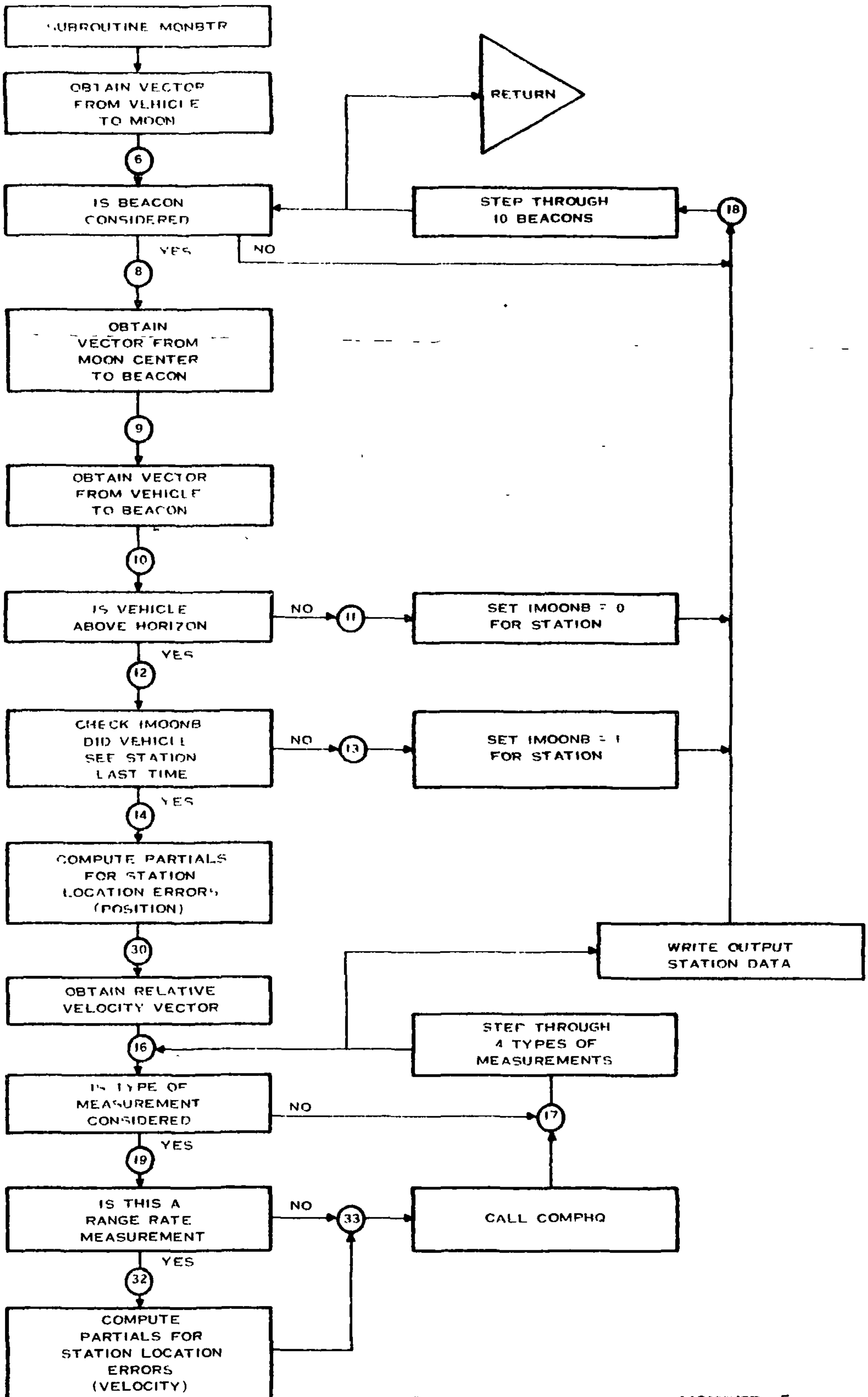
$$STPARS = (MNA)^T \left(\frac{\partial \vec{X}_{BM}}{\partial LAT} ; \frac{\partial \vec{X}_{BM}}{\partial LON} ; \frac{\partial \vec{X}_{BM}}{\partial ALT} \right)$$

3 x 3 3 x 3 3 x 3

$$STPARD = (MNAND)^T \left(\frac{\partial \vec{X}_{BM}}{\partial LAT} ; \frac{\partial \vec{X}_{BM}}{\partial LON} ; \frac{\partial \vec{X}_{BM}}{\partial ALT} \right)$$

3 x 3 3 x 3 3 x 3

Following computation of the matricies, subroutine COMPHQ is called to perform the updating of the state covariance matrix.


SUBROUTINE MONBTR
OBTAIN VECTOR FROM VEHICLE TO MOON
6
IS BEACON CONSIDERED
YES
NO
8
OBTAIN VECTOR FROM MOON CENTER TO BEACON
9
OBTAIN VECTOR FROM VEHICLE TO BEACON
10
IS VEHICLE ABOVE HORIZON
NO
11
SET IMOONB = 0 FOR STATION
YES
12
CHECK IMOONB DID VEHICLE SEE STATION LAST TIME
NO
13
SET IMOONB = 1 FOR STATION
YES
14
COMPUTE PARTIALS FOR STATION LOCATION ERRORS (POSITION)
30
OBTAIN RELATIVE VELOCITY VECTOR
16
IS TYPE OF MEASUREMENT CONSIDERED
NO
17
YES
19
IS THIS A RANGE RATE MEASUREMENT
NO
33
CALL COMPHQ
YES
32
COMPUTE PARTIALS FOR STATION LOCATION ERRORS (VELOCITY)
RETURN
STEP THROUGH 10 BEACONS
18
WRITE OUTPUT STATION DATA
STEP THROUGH 4 TYPES OF MEASUREMENTS

```
*       LABEL                                                            MONB
*       SYMBOL TABLE                                                     MONB
CEC2012  SUBROUTINE MONBTR                                               MONB
        SUBROUTINE MONBTR                                                MONB0000
        COMMON T,S,C,IC                                                  MONB0010
        DIMENSION                                                        MONB0020
       1T(1360),S(1000),C(1000),IC(1),XREL(3),VREL(3)                    MONB0030
       1,PO(22),VE(22),AN(3,3),EM(3,3),ISBCON(10),U(3),EN(3)             MONB0040
       2,RT(3),E(3),ETM(3,3),IMOONB(10),DM(3,3),DUM(3,3)                 MONB0050
       3,RMB(3),TYMEAS(4,10),X(3),VX(3),OUTPUT(6)                        MONB0060
        DIMENSION STPARS(3,3),DUD(3,3),STPARD(3,3)                       MONB0065
        EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM),(IC,ICDUM)                MONB0070
       1,(C(895),XREL),(C(898),VREL),(C(894), OBNO),(C(893),XMAG),       MONB0080
       2(C(892),DEX),(C(891),DEN1),(C(890),DEN2)                         MONB0090
       3,(C(889),DEX2),(C(888),DEX3),(C(652),P),(C(973),OUTPUT)          MONB0100
       1,(C(13),TW),(C(14),TF),(C(62),PO),(IC(3),NOR)                    MONB0110
       2,(IC(190),LSTAT),(C(138),AN),(S(24),A),(C(120),EM)               MONB0120
       3,(IC(140),ISBCON),(S(72),DR),(IC(220),IMOONB),(C(649),TSECP)     MONB0130
       4,(C(30),TSEC),(IC(140),TYMEAS),(C(15),X),(C(18),VX)              MONB0140
        EQUIVALENCE (C(800),STPARD),(C(788),STPARS)                      MONB0145
        CALL SETN(NIN,NUTS)                                              MONB0147
        DELTT=TSEC-TSECP                                                 MONB0150
        LSTAT=LSTAT                                                      MONB0160
        GO TO(1,2),LSTAT                                                 MONB0170
  1     CONTINUE                                                         MONB0180
        NB=NOR-1                                                         MONB0190
        DIS=1.E10                                                        MONB0200
        CALL INTR1(TW,TF,NB,PO,1,VE,DIS)                                 MONB0210
  2     CONTINUE                                                         MONB0220
        TIME=TW+TF                                                       MONB0230
        CALL NUTAIT(TIME,OM,CR,DT,EM,EPSIL)                              MONB0240
        CALL MNA(TIME,OM,CR,DT,EPSIL,RO,G,GP,WW,ETM)                     MONB0250
        CALL MNAND(TIME,RO,G,GP,WW,ETM,DM)                               MONB0260
        IF(NOR-2)3,5,3                                                   MONB0270
  3     CONTINUE                                                         MONB0280
        DO 4 I=1,3                                                       MONB0290
        J=20-I                                                           MONB0300
        XREL(I)=PO(J)-X(I)                                               MONB0310
        VREL(I)=VE(J)-VX(I)                                              MONB0320
  4     CONTINUE                                                         MONB0330
        GO TO 7                                                          MONB0340
  5     CONTINUE                                                         MONB0350
        DO 6 I=1,3                                                       MONB0360
        XREL(I)=-X(I)                                                    MONB0370
        VREL(I)=-VX(I)                                                   MONB0380
  6     CONTINUE                                                         MONB0390
C       XREL POSITION VECTOR FROM VEHICLE TO MOON CENTER 1950            MONB0400
C       VREL RELATIVE VELOCITY OF MOON CENTER 1950                       MONB0410
  7     CONTINUE                                                         MONB0420
        NN=485                                                           MONB0430
        DO 18 III=1,10                                                   MONB0440
```

INTR, INTRI(Cont'd)

```
         SUB      TARGO
         TNZ      NEU                    TIME HAS CHANGED
         CLA      TARG+1
         SUB      TARGO+1
         TZE      FLEE+6
         REM                             ANALYZE TABLE
         REM                             NEEDS AS A
         REM                             FUNCTION OF
         REM                             CENTRAL BODY
NEU      CLA      TARG
         FSB      TABLE
         TMI      LOOKUP                 POSITION EPHEMERIS TAPE
         STO      COM+19                 T-TO
         SUB      20.
         TPL      LOOKUP
         CLA      COM+19
         FDH      4.
         STQ      COM+18
         CLA      COM+18                 (T-TO)/4.
         CALL     FIX
         CALL     FLOAT
         STO      COM+17
         CHS
         FAD      COM+18
         STO      COM+1
         CLA      TARG+1
         FDH      4.
         STQ      COM
         CLA      COM
         FAD      COM+1
         STO      TARG+2
         LDQ      COM+19
         FMP      =9.
         CALL     FIX
         ADD      KERNO+1
         STA      GG
         LDQ      COM+17
         FMP      =54.
         CALL     FIX
         ADD      KERNO+2
         STA      HH
         AXT      2*SEP,4                MUST INTERPOLATE
         STZ      XN+2*SEP,4             SO CLEAR STORAGE
         TIX      *-1,4,1
         AXT      BSEP,4                 SET GRAVITATIONAL
         STZ      KBO+BSEP,4             COEFFICIENTS
         TIX      *-1,4,1                TO ZERO
         LXA      CENTER,4
         PXD      ,4
         COM
         PDX      ,2
```

INTR, INTRI (Cont'd)

```
        TXH    HELIO,4,1                                        INTR1010
GEO     AXT    3,1                                              INTR1020
        CLA    GRAV+3,1                                         INTR1030
        STO    KBO+3,1                                          INTR1040
        TIX    *-2,1,1                                          INTR1050
        STZ    KBO-1,2                                          INTR1060
        CLA    EWORTO                                           INTR1070
        STO    EWORT                                            INTR1080
        CLA    HWORTE                                           INTR1090
        STO    HWORT                                            INTR1100
        TXH    GG12,4,0                                         INTR1110
        CLA    RJ                  TEST FOR                     INTR1120
        SUB    R                   INCLUSION OF                 INTR1130
        TPL    GG12                JUPITER IF                   INTR1140
        CLA    HWORTJ              EARTH IS                     INTR1150
        STO    HWORT               CENTRAL BODY                 INTR1160
        CLA    GRAV+6                                           INTR1170
        STO    KB6                                              INTR1180
        TXL    GG12,,**                                         INTR1190
HELIO   AXT    BSEP,1                                           INTR1200
        CLA    GRAV+BSEP,1                                      INTR1210
        STO    KBO+BSEP,1                                       INTR1220
        TIX    *-2,1,1                                          INTR1230
        STZ    KB2-3,2                                          INTR1240
ORTHO   CLA    EWORTO                                           INTR1250
        STO    EWORT                                            INTR1260
        CLA    HWORTO                                           INTR1270
        STO    HWORT                                            INTR1280
GG12    CLA    VEL                 CHECK                        INTR1290
        TZE    GG6                 FOR VELOCITY                 INTR1300
        CLA    EWORTO              OPTIONS                      INTR1310
        STO    EWORT                                            INTR1320
        CLA    HWORTV                                           INTR1330
        STO    HWORT                                            INTR1340
GG6     CLA    EWORT                                            INTR1350
        TZE    GG+2                                             INTR1360
        CLA    EVEL                                             INTR1370
        STO    VEL+1                                            INTR1380
        CLA    TARG+1              GEOCENTRIC                   INTR1390
        TSX    TAB,4               INTERPOLATION                INTR1400
GG      PZE    **,,9                                            INTR1410
        PZE    XN,,EWORT+1                                      INTR1420
        CLA    HWORT                                            INTR1430
        TZE    HH+2                                             INTR1440
        CLA    HVEL                                             INTR1450
        STO    VEL+1                                            INTR1460
        CLA    TARG+2              HELIOCENTRIC                 INTR1470
        TSX    TAB,4               INTERPOLATION                INTR1480
HH      PZE    **,,54                                           INTR1490
        PZE    XN+3,,HWORT+1                                    INTR1500
RESET   AXT    3,4                 PLACE                        INTR1510
```

INTR, INTRI (Cont'd)

```
         CLA    XN+6,4                COORDINATES                     INTR152
         FDH    EVEL                  IN OLD                          INTR153
         STQ    XN,+18,4              FORMAT                          INTR154
         CLA    XN+9,4                                                INTR155
         FDH    EVEL                                                  INTR156
         STQ    XN,+12,4                                              INTR157
         LDQ    XN+3,4                                                INTR158
         STQ    XN+6,4                                                INTR159
         FMP    MU                                                    INTR160
         FSB    XN+18,4                                               INTR161
         STO    XN+9,4                POSITION OF SUN                 INTR162
         STZ    XN+3,4                                                INTR163
         LDQ    XN,+3,4                                               INTR164
         STQ    XN,+6,4                                               INTR165
         FMP    MU                                                    INTR166
         FSB    XN,+18,4                                              INTR167
         STO    XN,+9,4               VELOCITY OF SUN                 INTR168
         STZ    XN,+3,4                                               INTR169
         TIX    RESET+1,4,1                                           INTR170
         LXA    CENTER,4                                              INTR171
         LXD    VEL,2                 IX2 NOT ZERO IF VELOCITY OPTION INTR172
         TXH    RVPRT,2,0             GET COORDINATES FOR PRINTING    INTR173
         TXH    TESTM,4,0                                             INTR174
         NZT    KB6                   EARTH-CENTERED                  INTR175
         TRA    FLEE                                                  INTR176
         AXT    3,1                                                   INTR177
         CLA    XN+21,1                                               INTR178
         FAD    XN+9,1                                                INTR179
         STO    XN+21,1                                               INTR180
         TIX    *-3,1,1                                               INTR181
         TXL    FLEE                                                  INTR182
TESTM    TXH    RVPRT,4,1                                             INTR183
         AXT    3,1                   MOON-CENTERED                   INTR184
         CLS    XN+6,1                                                INTR185
         STO    XN+3,1                                                INTR186
         CLA    XN+9,1                                                INTR187
         FSB    XN+6,1                                                INTR188
         STO    XN+9,1                                                INTR189
         STZ    XN+6,1                                                INTR190
         TIX    *-6,1,1                                               INTR191
FLEE     CLA    CENTER                                                INTR192
         STO    KERNO                                                 INTR193
         CLA    TARG                                                  INTR194
         STO    TARGO                                                 INTR195
         CLA    TARG+1                                                INTR196
         STO    TARGO+1                                               INTR197
         AXT    21,1                  TRANSFER POSITIONS              INTR198
         CLA    XN+21,1                                               INTR199
OUP      STO    **,1                                                  INTR200
         TIX    OUP-1,1,1                                             INTR201
         CLA    VEL                                                   INTR202
```

INTR, INTRI (Cont'd)

```
        TZE     OUT                     DO NOT TRANSFER VELOCITY          INTR2030
        AXT     21,1                                                      INTR2040
        CLA     XN,+21,1                TRANSFER VELOCITIES               INTR2050
(N,V    STO     **,1                                                      INTR2060
        TIX     XN,V-1,1,1                                                INTR2070
)UT     LXD     HELIO-1,1                                                 INTR2080
        LXD     TRAP+1,2                                                  INTR2090
        LXD     TRAP,4                                                    INTR2100
        TRA     8,4                                                       INTR2110
        REM     TSX TAB,4               (AC)=INTERPOLATIVE ARGUMENT       INTR2120
        REM     PZE B,,K                                                  INTR2130
        REM     PZE A,,C                                                  INTR2140
        REM                             B=START OF DATA BLOCK             INTR2150
        REM                             K=WORDS PER SUB BLOCK             INTR2160
        REM                             A=START OF RESULT BLOCK           INTR2170
        REM                             C=SKIP CODE WORD LOCATION         INTR2180
'AB     SXD     COM+9,4                                                   INTR2190
        SXD     COM+8,2                                                   INTR2200
        SXD     COM+7,1                                                   INTR2210
        STO     ARG                                                       INTR2220
        CLA     1,4                                                       INTR2230
        STA     TAB18                                                     INTR2240
        LRS     18                                                        INTR2250
        ADD     1,4                                                       INTR2260
        STA     TAB21                                                     INTR2270
        CLA     2,4                                                       INTR2280
        PAX     ,1                                                        INTR2290
        TXI     *+1,1,SEP                                                 INTR2300
        SXA     TAB29,1                                                   INTR2310
        ARS     18                                                        INTR2320
        STA     VELOP                                                     INTR2330
        AXT     2,1                                                       INTR2340
        CLA     VEL                                                       INTR2350
        TNZ     VELOP                                                     INTR2360
        TXI     VELOP,1,-1                                                INTR2370
ELOP    CLA     **,1                    PICK UP SKIP                      INTR2380
        STO     VEL+2                   CODE WORD                         INTR2390
        TXL     POSOP,1,1                                                 INTR2400
        REM                             VELOCITY OPTION                   INTR2410
        CLA     ARG                                                       INTR2420
        TSX     COEFF.,4                FORM THE                          INTR2430
        AXT     3,4                     E1(2J).                           INTR2440
        CLS     COM+13,4                                                  INTR2450
        STO     COM+16,4                                                  INTR2460
        TIX     *-2,4,1                                                   INTR2470
        CLA     1.                      FORM THE                          INTR2480
        FSB     ARG                     E0(2J).                           INTR2490
        TSX     COEFF.,4                                                  INTR2500
        TRA     TAB1-1                                                    INTR2510
        REM                             POSITION OPTION                   INTR2520
OSOP    CLA     ARG                                                       INTR2530
```

INTR, INTRI (Cont'd)

```
        TSX     COEFF,4                FORM THE              INTR254
        AXT     3,4                    E1(2J)                INTR255
        CLA     COM+13,4                                     INTR256
        STO     COM+16,4                                     INTR257
        TIX     *-2,4,1                                      INTR258
        CLA     1.                     FORM THE              INTR259
        FSB     ARG                    E0(2J)                INTR260
        TSX     COEFF,4                                      INTR261
        LXA     TAB29,4                                      INTR262
        TXI     *+1,4,-SEP                                   INTR263
        SXA     TAB29,4                                      INTR264
        SXD     COM+4,1                                      INTR265
TAB1    AXT     0,3                                          INTR266
        LDQ     VEL+2                                        INTR267
        PXD                                                  INTR268
        LGL     2                                            INTR269
        PAX     ,4                                           INTR270
        STQ     VEL+2                                        INTR271
        CAL     =03                                          INTR272
        ORS     VEL+2                                        INTR273
        TXH     END,4,2                                      INTR274
        TXH     FORT,4,0                                     INTR275
        TXI     *+1,1,-3                                     INTR276
        TXI     TAB1+1,2,-1                                  INTR277
FORT    AXT     3,4                                          INTR278
TAB18   LDQ     **,1                   X(0)                  INTR279
        FMP     COM+13,4                                     INTR280
        STO     COM                                          INTR281
TAB21   LDQ     **,1                   X(1)                  INTR282
        FMP     COM+16,4                                     INTR283
        FAD     COM                                          INTR284
        STO     COM+4,4                                      INTR285
        TXI     *+1,1,-1                                     INTR286
        TIX     *-8,4,1                                      INTR287
        FAD     COM+2                                        INTR288
        FAD     COM+1                                        INTR289
TAB29   STO     **,2                   X(T)                  INTR290
        TXI     TAB1+1,2,-1                                  INTR291
END     LXD     COM+4,1                                      INTR292
        TIX     VELOP,1,1                                    INTR293
        LXD     COM+9,4                                      INTR294
        LXD     COM+8,2                                      INTR295
        LXD     COM+7,1                                      INTR296
        TRA     3,4                                          INTR297
COEFF   STO     COM+10                 CALCULATE             INTR298
        LDQ     COM+10                 POSITION              INTR299
        FMP     COM+10                 COEFFICIENTS          INTR300
        STO     COM+12                 FOR EVERETT,S         INTR301
        FSB     1.                     INTERPOLATION         INTR302
        FDH     6.                     FORMULA               INTR303
        FMP     COM+10                                       INTR304
```

INTR, INTRI (Cont'd)

```
        STO     COM+11                                  INTR3050
        CLA     COM+12                                  INTR3060
        FSB     4.                                      INTR3070
        FDH     20.                                     INTR3080
        FMP     COM+11                                  INTR3090
        STO     COM+12                                  INTR3100
        TRA     1,4                                     INTR3110
COEFF,  STO     COM             CALCULATE               INTR3120
        CLS     1.              VELOCITY                INTR3130
        FDH     VEL+1           COEFFICIENTS            INTR3140
        STQ     COM+10          FOR EVERETT,S           INTR3150
        LDQ     COM             INTERPOLATION           INTR3160
        FMP     COM             FORMULA                 INTR3170
        STO     COM+12                                  INTR3180
        XCA                                             INTR3190
        FMP     3.                                      INTR3200
        FSB     1.                                      INTR3210
        FDH     6.                                      INTR3220
        FMP     COM+10                                  INTR3230
        STO     COM+11                                  INTR3240
        CLA     COM+12                                  INTR3250
        FSB     3.                                      INTR3260
        XCA                                             INTR3270
        FMP     5.                                      INTR3280
        XCA                                             INTR3290
        FMP     COM+12                                  INTR3300
        FAD     4.                                      INTR3310
        FDH     120.                                    INTR3320
        FMP     COM+10                                  INTR3330
        STO     COM+12                                  INTR3340
        TRA     1,4                                     INTR3350
RVPRT   AXT     0,2             EXPRESS ALL             INTR3360
        AXT     3,5             BODIES GEOCENTRICALLY   INTR3370
        CLA     XN+9,2                                  INTR3380
        FAD     XN+9,4                                  INTR3390
        STO     XN+9,2                                  INTR3400
        CLA     VEL                                     INTR3410
        TZE     *+4                                     INTR3420
        CLA     XN.+9,2                                 INTR3430
        FAD     XN.+9,4                                 INTR3440
        STO     XN.+9,2                                 INTR3450
        TXI     *+1,2,-1                                INTR3460
        TIX     RVPRT+2,4,1                             INTR3470
        TXH     RVPRT+1,2,9-SEP                         INTR3480
        CLA     CENTER          BUFFER                  INTR3490
        ALS     1               CENTRAL                 INTR3500
        ADD     CENTER          BODY                    INTR3510
        PAC     ,4                                      INTR3520
        CLA     XN,4                                    INTR3530
        STO     COM+3,1                                 INTR3540
        CLA     XN.,4                                   INTR3550
```

INTR, INTRI (Cont'd)

```
       STO    COM+6,1                                    INTR3560
       TXI    *+1,4,-1                                   INTR3570
       TIX    *-5,1,1                                    INTR3580
RVPRT1 AXT    0,2             EXPRESS ALL                INTR3590
       AXT    3,4             BODIES IN TERMS            INTR3600
       CLA    XN,2            OF THE CENTRAL             INTR3610
       FSB    COM+3,4         BODY                       INTR3620
       STO    XN,2                                       INTR3630
       CLA    VEL                                        INTR3640
       TZE    *+4                                        INTR3650
       CLA    XN,,2                                      INTR3660
       FSB    COM+6,4                                    INTR3670
       STO    XN,,2                                      INTR3680
       TXI    *+1,2,-1                                   INTR3690
       TIX    RVPRT1+2,4,1                               INTR3700
       TXH    RVPRT1+1,2,-SEP                            INTR3710
       TXL    FLEE                                       INTR3720
LOOKUP CLA    TLAST                                      INTR3730
       FSB    TARG                                       INTR3740
       TNZ    *+2                                        INTR3750
       TRA    ERR1                                       INTR3760
       TMI    *-1                                        INTR3770
       CLA    TARG                                       INTR3780
       FSB    TFIRST                                     INTR3790
       TPL    *+2                                        INTR3800
       TRA    ERR1                                       INTR3810
       FDH    20.                                        INTR3820
       XCA                                               INTR3830
       CALL   FIX                                        INTR3840
       CALL   FLOAT                                      INTR3850
       XCA                                               INTR3860
       FMP    20.                                        INTR3870
       FAD    TFIRST                                     INTR3880
       STO    COM+1           TIME ON RECORD             INTR3890
       CLA    TABLE                                      INTR3900
       FSB    COM+1                                      INTR3910
       TMI    FINDIT                                     INTR3920
       FDH    20.                                        INTR3930
       XCA                                               INTR3940
       CALL   FIX                                        INTR3950
       ADD    1F              RECORDS TO                 INTR3960
       PAX    ,4              BE BACKED                  INTR3970
       TEFA   *+1                                        INTR3980
       TXL    *+3,4,20        OVER                       INTR3990
       REWA   6                                          INTR4000
       TRA    FINDIT                                     INTR4010
       BSRA   6                                          INTR4020
       TIX    *-1,4,1                                    INTR4030
FINDIT AXT    10,2                                       INTR4040
       SDHA   6                                          INTR4050
       RTBA   6                                          INTR4060
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INTR, INTRI (Cont'd)

```
        RCHA    IO                                                  INTR4070
        TCOA    *                                                   INTR4080
        TEFA    ERR1                                                INTR4090
        CLA     TABLE                                               INTR4100
        SUB     COM+1                                               INTR4110
        TMI     *+2                                                 INTR4120
        TNZ     LOOKUP                  TAPE NOT POSITIONED PROPERLY INTR4130
        TNZ     FINDIT                                              INTR4140
        AXT     513,4                                               INTR4150
        CAL     TABLE                   CHECK                       INTR4160
        ACL     A+513,4                 SUM                         INTR4170
        TIX     *-1,4,1                                             INTR4180
        TRCA    BSRA                                                INTR4190
        LAS     C                                                   INTR4200
        TRA     *+2                                                 INTR4210
        TRA     SCALE                                               INTR4220
BSRA    BSRA    6                                                   INTR4230
        TIX     FINDIT+1,2,1                                        INTR4240
        CALL    ERP                                                 INTR4250
IO      IOCD    TABLE,,515                                          INTR4260
ERR1    CALL    ERPT                                                INTR4270
        PZE     TARG                                                INTR4280
        REM                             SCALING                     INTR4290
        REM                             LOOP                        INTR4300
SCALE   AXT     189,4                                               INTR4310
        LDQ     A+189,4                 SCALE                       INTR4320
        FMP     SCALE1                  GEOCENTRIC                  INTR4330
        STO     A+189,4                 EPHEMERIS                   INTR4340
        TIX     *-3,4,1                                             INTR4350
        AXT     324,4                                               INTR4360
        LDQ     B+324,4                 SCALE                       INTR4370
        FMP     SCALE2                  HELIOCENTRIC                INTR4380
        STO     B+324,4                 EPHEMERIS                   INTR4390
        TIX     *-3,4,1                                             INTR4400
        CLA     GRAV                    COMPUTE                     INTR4410
        FAD     GRAV+1                  MASS RATIO                  INTR4420
        STO     COM                     OF MOON                     INTR4430
        CLA     GRAV+1                  TO EARTH-MOON               INTR4440
        FDH     COM                     BARYCENTER                  INTR4450
        STQ     MU                      FOR POSITION OF EARTH       INTR4460
        TRA     NEU                                                 INTR4470
SCALE1  DEC     6378.150                EARTH RADIUS                INTR4480
SCALE2  DEC     149598500.              ASTRONOMICAL UNIT (JPL, JULY 1961INTR4490
TFIRST  DEC     3892.0                  0 HR AUG 28, 1960 JD = 2437174.5 INTR4500
TLAST   DEC     7292.0                  0 HR DEC 19, 1969 JD = 2440554.5 INTR4510
TRAP    PZE                                                         INTR4520
        PZE                                                         INTR4530
RJ      DEC     1E6                     JUPITER TEST DISTANCE       INTR4540
TEMPDT  DEC     34.                     E.T.-U.T.                   INTR4550
        PZE                                                         INTR4560
GRAV    DEC     3.98602E5               EARTH                       INTR4570
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INTR, INTRI (Cont'd)

```
         DEC     4.8983349E3                MOON                             INTR458
         DEC     1.3252312E11               SUN                              INTR459
         DEC     3.2429889E5                VENUS                            INTR460
         DEC     4.2915518E4                MARS                             INTR461
         DEC     0                          REMOVE BARYCENTER                INTR462
         DEC     1.2652701E8                JUPITER                          INTR463
EVEL     DEC     86400.                                                      INTR464
HVEL     DEC     345600.                                                     INTR465
MU       PZE                                MASS RATIO OF MOON TO BARYCENTER INTR466
         OCT     000000520052               MARS, JUPITER VELOCITY           INTR467
HWORT    OCT     0                                                           INTR468
         OCT     527777777777               MOON VELOCITY                    INTR469
EWORT    OCT     0                                                           INTR470
HWORTJ   OCT     000000005252               BARYCENTER, JUPITER              INTR471
HWORTE   OCT     000000005200               BARYCENTER                       INTR472
EWORTO   OCT     527777777777                                                INTR473
HWORTO   OCT     000052525252                                                INTR474
HWORTV   OCT     525252525252                                                INTR475
TARGO    DEC     20,,0                      FORMER TIME                      INTR476
KERNO    PZE                                FORMER CENTER                    INTR477
         PZE     A                          GEOCENTRIC REFERENCE             INTR478
         PZE     B                          HELIOCENTRIC REFERENCE           INTR479
ARG      PZE     0                                                           INTR480
1F       DEC     1                                                           INTR481
1.       DEC     1.                                                          INTR482
3.       DEC     3.                                                          INTR483
4.       DEC     4.                                                          INTR484
5.       DEC     5.                                                          INTR485
6.       DEC     6.                                                          INTR486
20.      DEC     20.                                                         INTR487
120.     DEC     120.                                                        INTR488
86400.   DEC     86400.                                                      INTR489
VEL      BSS     3                          VELOCITY OPTION,H,SKIP CODE      INTR490
*                                                                            INTR491
TABLE    DEC     0                          RESERVE                          INTR492
A        BSS     189                        FOR                              INTR493
B        BSS     324                        WORKING                          INTR494
C        BSS     1                          EPHEMERIS                        INTR495
*                                                                            INTR496
TARG     PZE                                                                 INTR497
         PZE                                                                 INTR498
         PZE                                                                 INTR499
CENTER   PZE                                                                 INTR500
COM      BSS     21                                                          INTR501
VAFLG    PZE                                                                 INTR502
FQFLG    BSS     3                                                           INTR503
KB0      PZE                                                                 INTR504
KB1      PZE                                                                 INTR505
KB2      PZE                                                                 INTR506
KB3      PZE                                                                 INTR507
KB4      PZE                                                                 INTR508
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INTR, INTRI (Cont'd)

```
KB5     PZE                                  INTR5090
KB6     PZE                                  INTR5100
XN      BSS     21                           INTR5110
XN.     BSS     21                           INTR5120
T       PZE                                  INTR5130
        PZE                                  INTR5140
R       PZE                                  INTR5150
BSEP    SYN     7                            INTR5160
SEP     SYN     XN.-XN                       INTR5170
        END                                  INTR
```

Subroutine: INV3

Purpose: To invert a matrix of any dimension up to a 6 by 6.

Calling Sequence:

CALL INV3 (A, N, DETERM)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
IO	A	6,6			A(I) Matrix to be inverted
					A(O) Inverse of input matrix
I	N	1			Dimension of input matrix
O	DETERM	1			Matrix determinant

Common storages used or required: None

Subroutines required: None

Functions required: ABS

Approximate number of storages required: ________

INV3

```
*       LABEL
C       SUBROUTINE FOR INVERTING SQUARE MATRICES WHICH ARE 6 BY 6 OR LESS
C       (TO INVERT LARGER MATRICES,SAY M BY M, DIMENSION IPIVOT(M),A(M*M),
C       INDEX(M,2),PIVOT(M) AND RECOMPILE)
C       A IS THE SQUARE MATRIX TO BE INVERTED
C       N IS THE SIZE OF A (A IS AN N BY N MATRIX)
C       THE SUBROUTINE RETURNS A INVERSE IN PLACE OF A AND THE DETERMINANT
C       IN DETERM.
        SUBROUTINE INV3(A,N,DETERM)
        DIMENSION IPIVOT(6),A(36) ,INDEX(6,2),PIVOT(6)
        EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM),(AMAX,T,SWAP)
        DETERM=1.0
        DO 20  J=1,N
 20     IPIVOT(J)=0
        DO 550  I=1,N
        AMAX=0.0
        DO 105 J=1,N
        IF(IPIVOT(J)-1) 60,105,60
 60     DO 100 K=1,N
        IF(IPIVOT(K)-1) 80,100,740
  80    M = N*(K-1)+J
        IF (ABSF(AMAX)-ABSF(A(M))) 85,100,100
 85     IROW=J
        ICOLUM=K
        AMAX = A(M)
 100    CONTINUE
 105    CONTINUE
        IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
        IF(IROW-ICOLUM) 140,260,140
 140    DETERM=-DETERM
        DO 200 L=1,N
        M = N*(L-1)
        M1 = M+ICOLUM
        M = M+IROW
        SWAP = A(M)
        A(M) = A(M1)
  200   A(M1) = SWAP
 260    INDEX(I,1)=IROW
        INDEX(I,2)=ICOLUM
        M = N*(ICOLUM-1)+ICOLUM
        PIVOT(I) = A(M)
        DETERM=DETERM*PIVOT(I)
        A(M) = 1.0
        DO 350 L=1,N
        M = N*(L-1)+ICOLUM
  350   A(M) = A(M)/PIVOT(I)
        DO 550 L1=1,N
        IF(L1-ICOLUM) 400,550,400
  400   M = N*(ICOLUM-1)+L1
        T = A(M)
        A(M) = 0.
```

INV3 (Cont'd)

```
      DO 450  L=1,N
      M = N*(L-1)
      M1 = M+ICOLUM
      M = M+L1
  450 A(M) = A(M)-A(M1)*T
  550 CONTINUE
      DO 710 I=1,N
      L=N+1-I
      IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      M = N*(JROW-1)
      M1 = N*(JCOLUM-1)
      DO 705 K=1,N
      M = M+1
      M1 = M1+1
      SWAP = A(M)
      A(M) = A(M1)
      A(M1) = SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
```

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Subroutine: INVAO

Purpose: To form the inverse of a transition matrix for a linear, conservative dynamic system. If Φ is a partioned matrix such as

$$\Phi = \begin{pmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{pmatrix} \quad \text{then INVAO computes } \Phi^{-1} = \begin{pmatrix} \Phi_4^T & -\Phi_2^T \\ -\Phi_3^T & \Phi_1^T \end{pmatrix}$$

Calling Sequence:

CALL INVAO (AO, AOI)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	AO	6,6	$\Phi(t_2,t_1)$		Transition Matrix
O	AOI	6,6	$\Phi^{-1}(t_2,t_1)$		Inverse Transition Matrix

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required: ________

INVAO

```
*       LABEL                                                          INVA
*       SYMBOL TABLE                                                   INVA
        SUBROUTINE INVAO(AO,AOI)                                       INVA0000
        DIMENSION AO(6,6),AOI(6,6)                                     INVA0010
        DO 1 I=1,3                                                     INVA0020
        II=I+3                                                         INVA0030
        DO 1 J=1,3                                                     INVA0040
        JJ=J+3                                                         INVA0050
        AOI(I,J)=AO(JJ,II)                                             INVA0060
        AOI(II,J)=-AO(JJ,I)                                            INVA0070
        AOI(II,JJ)=AO(J,I)                                             INVA0080
    1   AOI(I,JJ)=-AO(J,II)                                            INVA0090
        RETURN                                                         INVA0100
        END                                                            INVA
```

Subroutine: LOADO

Purpose: To transfer the transition matrix from its storage in the integration package's T block to the matrix in the call list.

Calling Sequence:

CALL LOADO (AO)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	AO	(6,6)	$\Phi(t_2 t_1)$		Transition Matrix

Common storages used or required: T

Subroutines required: None

Functions required: None

Approximate number of storages required:

LOADO

```
*     LABEL
*     SYMBOL TABLE
      SUBROUTINE LOADO(AC)
      COMMON T
      DIMENSION T(1360),AO(6,6)
      K=6
      JJ=27
      DO1 I=1,6
      DO 1 J=1,3
      K=K+1
      KK=J+3
      JJ=JJ+1
      AO(KK,I)=T(JJ)
      AO(J,I)=T(K)
    1 CONTINUE
      RETURN
      END
```

Subroutine: LOADT

Purpose: To place unit initial conditions in the T block for the variational equations which are being used to generate the transition matrix.

Calling Sequence:

CALL LOADT

Input and Output

Common storages used or required: T

Subroutines required: None

Functions required: None

Approximate number of storages required:

LOADT

```
*       LABEL
*       SYMBOL TABLE
        SUBROUTINE LOADT
C       SUBROUTINE LOADT PLTS UNIT ICS ON PERTURBATION EQUATIONS
        COMMON T
        DIMENSION T(1360),X(3,6),V(3,6)
        EQUIVALENCE (T(7), X),(T(28),V)
        DO 2 I=1,3
        L=I+3
        DO 1 J=1,3
        K=J+3
        X(I,J)=0.
        V(I,K)=0.
        X(I,K)=0.
        V(I,J)=0.
      1 CONTINUE
        X(I,I)=1.
        V(I,L)=1.
      2 CONTINUE
        RETURN
        END
```

Subroutine: MASS

Purpose: To find the relevant gravitational constants to be used in computing planetary perturbations for a given central body. MASS also chooses the initial integration step size as a function of central body.

Calling Sequence:

CALL MASS (NOR, UM, VKB, X, ACCP)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	NOR	1			Central Body Indicator.
					1=Earth; 2, Moon; 3, Sun; 4,
					Venus; 5, Mars; 6, Jupiter
I	UM	6	μ_i	Km^3/sec^2	Gravitational constants, arranged
					as above
O	VKB	6	μ_i	Km^3/sec^2	μ_i for bodies
I	X	3	X, Y, Z	Km	Position coordinates
O	ACCP	1		seconds	Initial integration step size

Common storages used or required: None

Subroutines required: None

Functions required: FNORM

Approximate number of storages required:

Method of Establishing Integration Step Size and Perturbing Bodies

Central Body	Integration Step Size	Other Bodies to be Considered
Earth	60.0	Moon, Sun. If $\lvert X \rvert \geq 10^6$Km, Jupiter.
Moon	60.0	Earth, Sun.
Sun	43200.0	All
Venus, Mars, Jupiter	60.0	All

The bodies which are not to be used in calculation of pertubation accelerations are eliminated from consideration by placing zeros in array VKB(6). For the bodies which are being considered, the appropriate gravitational constant is placed in VKB(6).

```
*       LABEL
*       SYMBOL TABLE
        SUBROUTINE MASS(NOR,UM,VKB,X,ACCP)
        DIMENSION UM(6),VKB(6),X(3)
        N = NOR
        DO 1 I=1,6
    1   VKB(I) = 0.
        GO TO (2,3,6,4,4,4),N
    2   VKB(2) = UM(2)
        ACCP = 60.
        DIS = FNORM(X)
        VKB(3) = UM(3)
        IF (DIS-1000000.) 11,10,10
   10   VKB(6) = UM(6)
        GO TO 12
   11   VKB(6) = 0.
   12   RETURN
    3   VKB(1) = UM(1)
        VKB(3) = UM(3)
        ACCP = 60.
        RETURN
    6   ACCP = 43200.
        GO TO 7
    4   ACCP = 60.
    7   DO 5 I=1,6
    5   VKB(I) = UM(I)
        VKB(N) = 0.
        RETURN
        END
```

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Subroutine: MATRX

Purpose: To perform matrix multiplications of matricies with any dimensions up to maximum dimensions of 10 by 10. A zero in the call sequence (J) yields an output matrix C = A·B. A one in the call sequence (J) yields an output matrix $C = A \cdot B \cdot A^T$. The matrix products are obtained in double precision.

Calling Sequence:

CALL MATRX (A, B, C, NRA, NCA, NCB, J)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	A	(10,10)			Input Matrix
I	B	(10,10)			Input Matrix
O	C	(10,10)			Output Matrix
I	NRA	1			Number of Rows of A
I	NCA	1			Number of Columns of A
I	NCB	1			Number of Columns of B
I	J	1			Type of multiplication desired

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

```
*       LABEL                                                           MATX
*       SYMBOL TABLE                                                    MATX
CEC2000  SUBROUTINE MATRX                                               MATX
        SUBROUTINE MATRX(A,B,C,NRA,NCA,NCB,J)                           MATX0000
C       J=0 A*B=C                                                       MATX0010
C       J=1 A*B*AT=C  T MEANS TRANSPOSE                                 MATX0020
C       NRA=NUMBER OF ROWS OF A                                         MATX0030
C       NCA=COLUMNS OF A                                                MATX0040
C       NCB=COLUMNS OF B                                                MATX0050
C       ACCUMULATION OF PRODUCTS IN DOUBLE PRECISION                    MATX0060
        DIMENSION A(100),B(100),C(100),D(100)                           MATX0080
D       DUD=0.                                                          MATX0090
D       DUM=0.                                                          MATX0100
        DO 1 I=1,NRA                                                    MATX0110
        DO 1 K=1,NCB                                                    MATX0120
        NC=I+(K-1)*NRA                                                  MATX0130
D       CRUD=0.                                                         MATX0140
        DO 2 L=1,NCA                                                    MATX0150
        NA=I+(L-1)*NRA                                                  MATX0160
        NB=L+(K-1)*NCA                                                  MATX0170
        DUD=A(NA)                                                       MATX0180
        DUM=B(NB)                                                       MATX0190
D       CRUD=CRUD+DUD*DUM                                               MATX0200
      2 CONTINUE                                                        MATX0210
        D(NC)=CRUD                                                      MATX0220
      1 CONTINUE                                                        MATX0230
        IF(J)3,3,4                                                      MATX0240
      3 CONTINUE                                                        MATX0250
        KK=NCB*NRA                                                      MATX0260
        DO 5 I=1,KK                                                     MATX0270
      5 C(I)=D(I)                                                       MATX0280
        GO TO 10                                                        MATX0290
      4 CONTINUE                                                        MATX0300
        DO 6 I=1,NRA                                                    MATX0310
        DO 6 K=1,NRA                                                    MATX0320
        NC=I+(K-1)*NRA                                                  MATX0340
D       CRUD=0.                                                         MATX0350
        DO 7 L=1,NCA                                                    MATX0360
        NA=I+(L-1)*NRA                                                  MATX0370
        NB=K+(L-1)*NRA                                                  MATX0380
        DUM=D(NA)                                                       MATX0390
        DUD=A(NB)                                                       MATX0400
D       CRUD=CRUD+DUM*DUD                                               MATX0410
      7 CONTINUE                                                        MATX0420
        C(NC)=CRUD                                                      MATX0430
      6 CONTINUE                                                        MATX0440
     10 CONTINUE                                                        MATX0510
        RETURN                                                          MATX0520
        END                                                             MATX
```

Subroutine: MATSUB

Purpose: The subroutine is primarily logic which controls (1) trajectory and data printout, (2) updating of the state covariance matrix for observations by calling subroutines EARTR, ONBTR, and MONBTR, and (3) updating of the guidance covariance matrix by calling subroutine GUID.

Calling Sequence:

CALL MATSUB

Input and **Output**

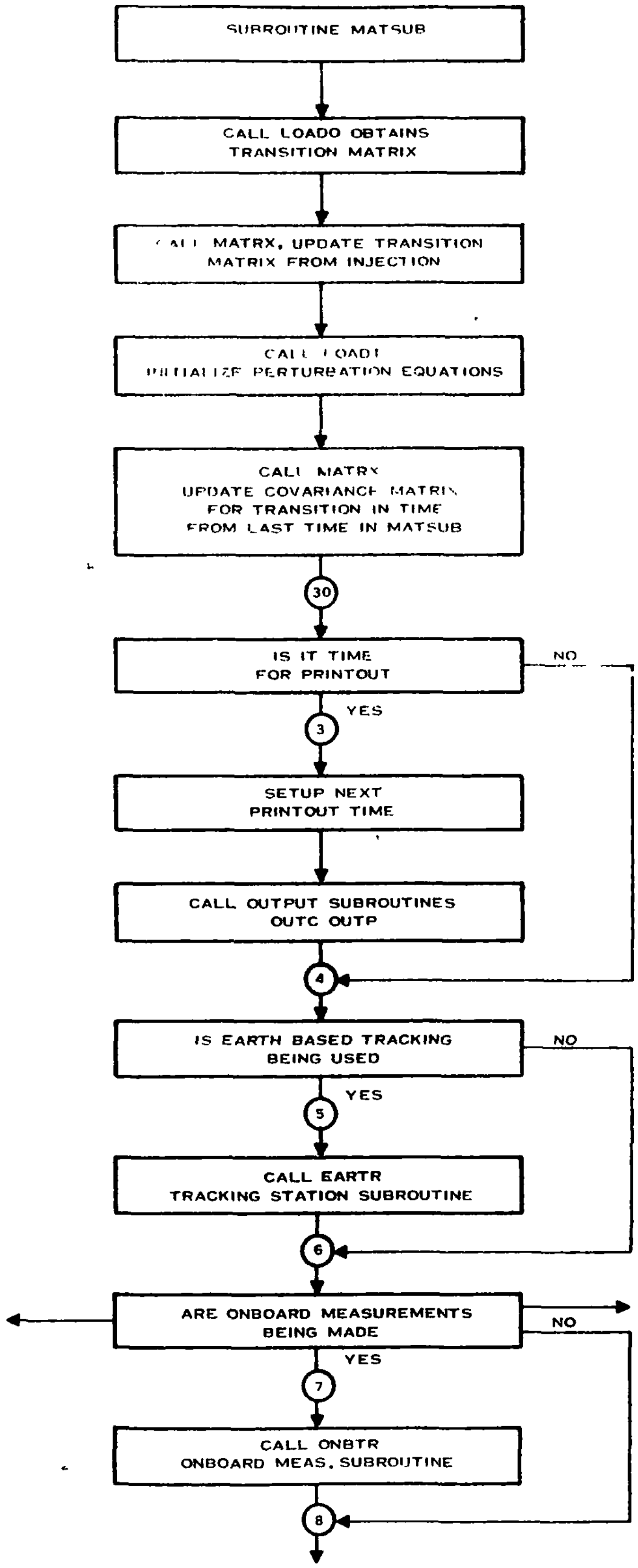
I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities returned to common

Common storages used or required: T, S, C, IC

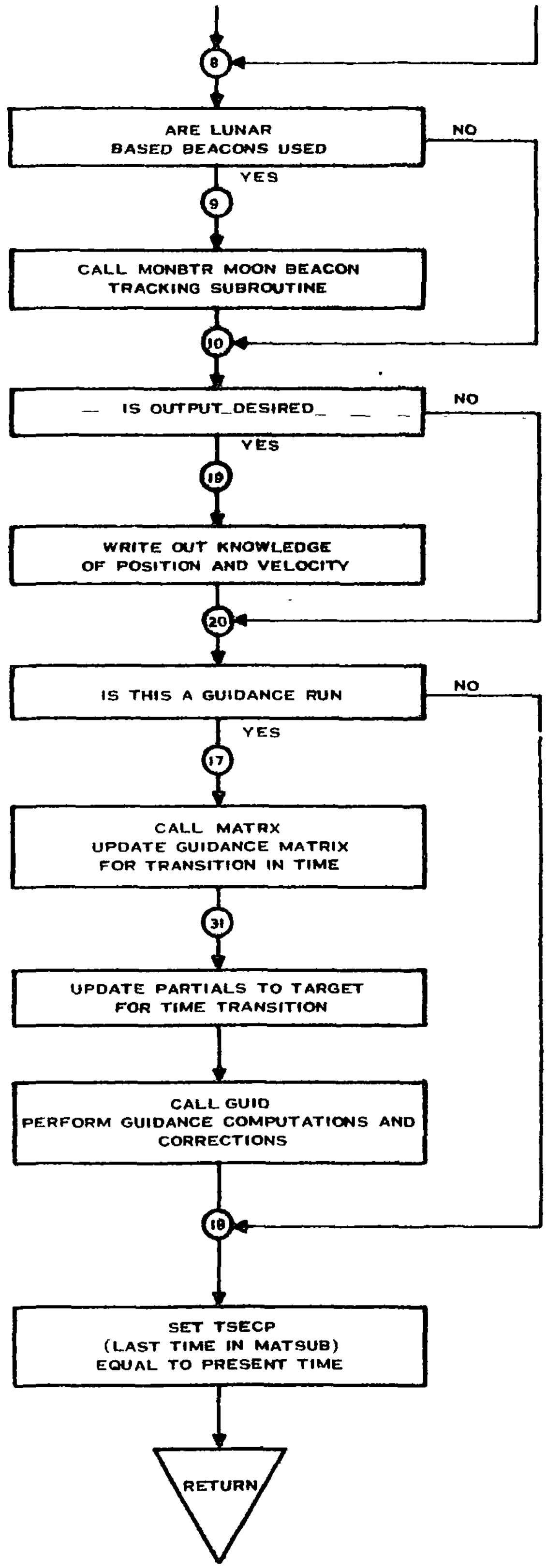
Subroutines required: EARTR, GUID, INVAO, LOADO, LOADT, MATRX, MONBTR, ONBTR, OUTC, OUTP

Functions required: SQRT (FIL)(STH)

Approximate number of storages required: 346 DEC


SUBROUTINE MATSUB
CALL LOADO OBTAINS TRANSITION MATRIX
CALL MATRX, UPDATE TRANSITION MATRIX FROM INJECTION
CALL LOADI INITIALIZE PERTURBATION EQUATIONS
CALL MATRX UPDATE COVARIANCE MATRIX FOR TRANSITION IN TIME FROM LAST TIME IN MATSUB
30
IS IT TIME FOR PRINTOUT
NO
YES
3
SETUP NEXT PRINTOUT TIME
CALL OUTPUT SUBROUTINES OUTC OUTP
4
IS EARTH BASED TRACKING BEING USED
NO
YES
5
CALL EARTR TRACKING STATION SUBROUTINE
6
ARE ONBOARD MEASUREMENTS BEING MADE
NO
YES
7
CALL ONBTR ONBOARD MEAS. SUBROUTINE
8

MATSUB -2


8
ARE LUNAR BASED BEACONS USED
NO
YES
9
CALL MONBTR MOON BEACON TRACKING SUBROUTINE
10
IS OUTPUT DESIRED
NO
YES
19
WRITE OUT KNOWLEDGE OF POSITION AND VELOCITY
20
IS THIS A GUIDANCE RUN
NO
YES
17
CALL MATRX UPDATE GUIDANCE MATRIX FOR TRANSITION IN TIME
31
UPDATE PARTIALS TO TARGET FOR TIME TRANSITION
CALL GUID PERFORM GUIDANCE COMPUTATIONS AND CORRECTIONS
18
SET TSECP (LAST TIME IN MATSUB) EQUAL TO PRESENT TIME
RETURN

```
*     LABEL                                                             MATS
*     SYMBOL TABLE                                                      MATS
CEC2006  SUBROUTINE MATSUB                                              MATS
      SUBROUTINE MATSUB                                                 MATS0000
      COMMON T,S,C,IC                                                   MATS0010
      DIMENSION T(1360),S(1000),C(1000),IC(1),AN(3,3),PO(22),VE(22),    MATS0020
     1EM(3,3),XP(3),VXP(3),EA(3,3),XED(3),VED(3),AO(6,6),P(6,6),        MATS0030
     2TFTA(3,6),PFTA(6,6),AOS(6,6),AOI(6,6)                             MATS0040
      EQUIVALENCE (T ,TDUM),(S ,SDUM),(C ,CDUM),                        MATS0050
     1(C(138),AN),(C(13),TP1),(C(14),TP2),(C(62),PO),(C(84),VE),        MATS0060
     2(C(120),EM),(C(650),TPRINT),(C(651),TSTRT),(C(30),TSEC),          MATS0070
     3(C(15),XP),(C(18),VXP),(C(10),TW),(C(11),TF) ,(C(129),EA),        MATS0080
     4(C(652),P),(C(752),AOS),(C(850),AOI)                              MATS0090
     5,(C(460),PFTA),(C(568),TFTA),(C(647),TOUT)                        MATS0100
      EQUIVALENCE (IC(2),IOR),(IC(7),KOUT),(IC(6),ITARG),(IC(9),LPRINT),MATS0110
     1(IC(190),LSTAT),(IC(191),LONB),(IC(192),LMB),(IC(193),IGUID),     MATS0120
     2(IC(214),NOUT),(C(649),TSECP)                                     MATS0130
      CALL SETN(NIN,NUTS)                                               MATS0131
      NOUT=1                                                            MATS0140
      CALL LOADO(AO)                                                    MATS0145
      CALL MATRX(AO,AOS,AOS,6,6,6,0)                                    MATS0150
      CALL LOADT                                                        MATS0155
      CALL MATRX(AO,P,P,6,6,6,1)                                        MATS0160
      DO 30 I=1,6                                                       MATS0165
      DO 30 J=I,6                                                       MATS0170
      P(I,J)=(P(I,J)+P(J,I))/2.                                         MATS0175
      P(J,I)=P(I,J)                                                     MATS0180
   30 CONTINUE                                                          MATS0185
      IF(TOUT-TSEC) 3,3,4                                               MATS0220
    3 CONTINUE                                                          MATS0230
      TOUT=TOUT+TPRINT                                                  MATS0240
      NOUT=2                                                            MATS0250
C     NOUT IS USED TO INDICATE OTHER OUTPUT WANTED WHEN = 2             MATS0260
      CALL OUTC                                                         MATS0270
      CALL OUTP                                                         MATS0280
    4 CONTINUE                                                          MATS0290
C     LSTAT IS SET 2 BY INPUT CARDS IF ANY EARTH BASED TRACKING         MATS0300
      LSTAT=LSTAT                                                       MATS0310
      GO TO (6,5),LSTAT                                                 MATS0320
    5 CONTINUE                                                          MATS0330
      CALL EARTR                                                        MATS0340
C     EARTR UDDATS COV MATRIX P FOR EARTH BASED TRACKING                MATS0350
    6 CONTINUE                                                          MATS0360
      LONB=LONB                                                         MATS0370
      GO TO (8,7),LONB                                                  MATS0380
C     LONB IS SET 2 BY INPUT CARDS IF ANY ONBOARD MEASUREMENTS MADE     MATS0390
    7 CONTINUE                                                          MATS0400
C     ONBTR  UPDATES COV MATRIX P FOR THE ONBOARD MEASUREMENTS          MATS0410
      CALL ONBTR                                                        MATS0420
    8 CONTINUE                                                          MATS0430
      LMB=LMB                                                           MATS0440
```

MATSUB (Cont'd)

```
      GO TO (10,9),LMB                                                  MATS0450
C     LMB IS SET 2 BY INPUT CARDS IF MOON BASED BEACONS USED            MATS0460
    9 CONTINUE                                                          MATS0470
      CALL MONBTR                                                       MATS0480
C    MONBTR UPDATS COV MATRIX P FOR BEACONS ON MOON                     MATS0490
   10 CONTINUE                                                          MATS0500
      GO TO (20,19), NOUT                                               MATS0510
  19  CONTINUE                                                          MATS0520
      RMSP=SQRTF (P (1,1)+P (2,2)+P (3,3))                              MATS0530
      RMSVP=SQRTF(P (4,4)+P (5,5)+P (6,6))                              MATS0540
      WRITE OUTPUT TAPE NUTS,700,RMSP,RMSVP                             MATS0550
 700  FORMAT(42H KNOWLEDGE OF STATE AFTER ALL OBSERVATIONS,/            MATS0560
     124H             RMS POSITION=,E15.8,25H            RMS VELOCITY=, MATS0570
     2E15.8)                                                            MATS0580
  20  CONTINUE                                                          MATS0590
C     NUMBERS 11-16 RESERVED FOR FUTURE MEASUREMENTS                    MATS0600
      IGUID=IGUID                                                       MATS0610
      GO TO (18,17),IGUID                                               MATS0620
C     IGUID IS SET BY INPUT TO MAIN PROGRAM IF GUIDANCE CALCULATIONS AREMATS0630
C     MADE                                                              MATS0640
   17 CONTINUE                                                          MATS0650
      CALL MATRX(AO,PFTA,PFTA,6,6,6,1)                                  MATS0655
      DO 31 I=1,6                                                       MATS0660
      DO 31 J=I,6                                                       MATS0665
      PFTA(I,J)=(PFTA(I,J)+PFTA(J,I))/2.                                MATS0670
      PFTA(J,I)=PFTA(I,J)                                               MATS0675
  31  CONTINUE                                                          MATS0680
      CALL INVAO(AO,AOI)                                                MATS0685
      CALL MATRX(TFTA,AOI,TFTA,3,6,6,0)                                 MATS0690
      CALL GUID                                                         MATS0710
C     GUID PERFORMS THE GUIDANCE CALCULATIONS                           MATS0720
   18 CONTINUE                                                          MATS0730
      TSECP=TSEC                                                        MATS0740
      RETURN                                                            MATS0750
      END                                                               MATS
```

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Subroutine: MNA

Purpose: To provide the rotation matrix EMN which transforms moon centered coordinates in the earth's true equator and equinox to moon centered coordinates in the moon's true equator.

Calling Sequence:

CALL MNA (TIME, OM, CR, DT, EPSIL, RO, G, GP, WW, EMN)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TIME	1	d	days	Days since 1950.0
I	OM	1	Ω		Output from NUTAIT: (Arg. De-
					cending node)
I	CR	1	☾	radians	Output from NATAIT: (mean long.
					of the moon)
I	DT	1	$\Delta\Psi + d\Psi$	radians	Output from NUTAIT: (nutation in
					longitude)
I	EPSIL	1	$\bar{\epsilon} + \Delta\epsilon + d\epsilon$	radians	Output from NUTAIT: (mean obliquity
					and nutation in obliquity)

(Cont'd)

Common storages used or required: None.

Subroutines required: System double precision routines.

Functions required: INTF, SQRT, SIN, COS, ACOS

Approximate number of storages required:

(Cont'd.)

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	RO	1	ρ		Libration term
O	G	1	g	radians	Mean anomaly of the moon
O	GP	1	g	radians	Mean anomaly of the sun
O	WW	1	ω	radians	Argument of the perigee of the
					moon
O	EMN	3,3			Rotation matrix

Transformation From Earth's True Equator to Moon's True Equator.

The two rectangular systems are related through Λ, Ω', and i by the rotation:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\text{MOON}} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}_{\text{EARTH}}$$

where

$$b_{11} = \cos \Lambda \cos \Omega' - \sin \Lambda \sin \Omega' \cos i$$

$$b_{12} = \cos \Lambda \sin \Omega' + \sin \Lambda \cos \Omega' \cos i$$

$$b_{13} = \cos \Lambda \sin i$$

$$b_{21} = -\sin \Lambda \cos \Omega' - \cos \Lambda \sin \Omega' \cos i$$

$$b_{22} = -\sin \Lambda \sin \Omega' + \cos \Lambda \cos \Omega' \cos i$$

$$b_{23} = \cos \Lambda \sin i$$

$$b_{31} = \sin \Omega' \sin i$$

$$b_{32} = -\cos \Omega' \sin i$$

$$b_{33} = \cos i$$

i is the inclination of the moon's true equator to the earth's equator

Ω' is the right ascension of the ascending node of the moon's true equator

Λ is the anomaly from the node to the X axis

$$\Lambda = \Delta + (☾ + \tau) - (\Omega + \sigma)$$

Δ is the anomaly from the node to the ascending node of the moon's true equator on the ecliptic

Ω is the mean longitude of the descending node of the moon's mean equator on the ecliptic

☾ is the mean longitude of the moon

σ is the libration in the node

τ is the libration in the mean longitude

ρ is the libration in the inclination.

$\delta\Psi$, ϵ, Ω, and ☾ are input quantities obtained from NUTAIT. The remainder are computed from the following equations.

I = inclination of moon's equator to ecliptic

$I = 1.535^{\circ}$

g = mean anomaly of moon

$g = 215.54013 + 13.064992\ d$

g' = mean anomaly of sun

g' = 358.009067 + .9856005 d

ω = argument of perigee of moon

ω = 196.745632 + .1643586 d

where d = days from 1950.

$$\sigma \sin I = -.0302777 \sin g + .0102777 \sin (g + 2\omega) - .00305555 \sin (2g + 2\omega)$$

$$\tau = -.003333 \sin g + .0163888 \sin g' + .005 \sin 2\omega$$

$$\rho = -.0297222 \cos g + .0102777 \cos (g + 2\omega) - .00305555 \cos (2g + 2\omega)$$

$$\cos i = \cos (\Omega + \sigma + \delta\Psi) \sin \epsilon \sin (I + \rho) + \cos \epsilon \cos (I + \rho) \qquad 0 < i < 90^\circ$$

$$\sin \Omega' = -\sin (\Omega + \sigma + \delta\Psi) \sin (I + \rho) \csc i \qquad -90^\circ < \Omega' < 90^\circ$$

$$\sin \Delta = -\sin (\Omega + \sigma + \delta\Psi) \sin \epsilon \csc i$$

$$\cos \Delta = -\sin (\Omega + \sigma + \delta\Psi) \sin \Omega' \cos \epsilon - \cos (\Omega + \sigma + \delta\Psi) \cos \Omega' \qquad 0^\circ \le \Delta < 360^\circ$$

Reference: JPL Technical Report No. 32-223

MNA

```
*       LABEL                                                           SMNA
*       SYMBOL TABLE                                                    SMNA
 CEC20AH                                                                SMNA
        SUBROUTINE MNA(TIME,OM,CR,DT,EPSIL,RO,G,GP,WW,EM)               SMNA0000
        DIMENSION EM(3,3),DF(3)                                         SMNA0001
        D = TIME                                                        SMNA0002
        T = D/36525.                                                    SMNA0003
        T2 = T*T                                                        SMNA0004
        T3 = T2*T                                                       SMNA0005
        A=13.064992                                                     SMNA0006
        DO 6 I=1,3                                                      SMNA0007
        DD=D                                                            SMNA0008
D       DD=DD*(A/360.)                                                  SMNA0009
D       DD=DD-INTF(DD)                                                  SMNA0010
        DF(I)=DD                                                        SMNA0011
        GO TO (4,5,6),I                                                 SMNA0012
    4   A=.9856005                                                      SMNA0013
        GO TO 6                                                         SMNA0014
    5   A=.1643586                                                      SMNA0015
    6   CONTINUE                                                        SMNA0016
        G=215.54013+360.*DF(1)                                          SMNA0017
        GP=358.009067+360.*DF(2)                                        SMNA0018
        WW=196.745632+360.*DF(3)                                        SMNA0019
        G = G*.017453296                                                SMNA0020
        GP = GP*.017453296                                              SMNA0021
        WW = WW*.017453296                                              SMNA0022
        YN = 1.535*.017453296                                           SMNA0023
        RO = -.0297222*COSF(G) + .01020777*COSF(G+2.*WW)                SMNA0024
       1 -.00305555*COSF(2.*G+2.*WW)                                    SMNA0025
        TA = -.003333*SINF(G) + .0163888*SINF(GP)                       SMNA0026
       1 +.005*SINF(2.*WW)                                              SMNA0027
        SG = -.0302777*SINF(G) + .0102777*SINF(G+2.*WW)                 SMNA0028
       1 -.00305555*SINF(2.*G+2.*WW)                                    SMNA0029
        SG = (SG*.017453296)/SINF(YN)                                   SMNA0030
        RO = RO*.017453296                                              SMNA0031
        TA = TA*.017453296                                              SMNA0032
        YN = YN + RO                                                    SMNA0033
        RO = OM + SG + DT                                               SMNA0034
        CI = COSF(RO)*SINF(EPSIL)*SINF(YN)                              SMNA0035
       1 +COSF(EPSIL)*COSF(YN)                                          SMNA0036
        SI = 1. - CI**2                                                 SMNA0037
        SI = SQRTF(SI)                                                  SMNA0038
        SO = -SINF(RO)*SINF(YN)/SI                                      SMNA0039
        CO = 1. - SO**2                                                 SMNA0040
        CO = SQRTF(CO)                                                  SMNA0041
        SD = -SINF(RO)*SINF(EPSIL)/SI                                   SMNA0042
        CD = -SINF(RO)*SO*COSF(EPSIL) - COSF(RO)*CO                     SMNA0043
        DL = ACOSF(CD)                                                  SMNA0044
        IF(SD)1,3,3                                                     SMNA0045
  1     DL = 6.283185306 - DL                                           SMNA0046
  3     CONTINUE                                                        SMNA0047
```

MNA (Cont'd)

```
CA = DL + (CR + TA) - (OM + SG)                SMNA048
SA = SINF(CA)                                  SMNA049
CA = COSF(CA)                                  SMNA050
RO = COSF(RO)*SINF(EPSIL)/(SI*CD)              SMNA051
EM(1,1) = CA*CO - SA*SO*CI                     SMNA052
EM(1,2) = CA*SO + SA*CO*CI                     SMNA053
EM(1,3) = SA*SI                                SMNA054
EM(2,1) = -SA*CO - CA*SO*CI                    SMNA055
EM(2,2) = -SA*SO + CA*CO*CI                    SMNA056
EM(2,3) = CA*SI                                SMNA057
EM(3,1) = SO*SI                                SMNA058
EM(3,2) = -CO*SI                               SMNA059
EM(3,3) = CI                                   SMNA060
RETURN                                         SMNA061
END                                            SMNA
```

Subroutine: MNAND

Purpose: To compute the matrix M used to perform the velocity transformation corresponding to the position transformation described in MNA.

Calling Sequence:

CALL MNAND (TIME, RO, G, GP, WW, EM, DM)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TIME	1	d	days	Days since 1950.0
I	RO	1	ρ	radians	Output from MNA; see description
I	G	1	g	radians	Output from MNA; see description
I	GP	1	g'	radians	Output from MNA; see description
I	WW	1	ω	radians	Output from MNA; see description
I	EM	3,3	M		Output from MNA; matrix M
O	DM	3,3	$\dot{M}$		Matrix $\dot{M}$

Common storages used or required: None

Subroutines required: None

Functions required: COS

Approximate number of storages required: ________

Theory and Equations

In transforming lunacentric position coordinates relative to the earth's equator and equinox of 1950.0 to coordinates relative to the moon's true equator, three matrices are computed. Matrix A (from ROTEQ) rotates the coordinates to equinox of date, matrix N (from NUTAIT) accounts for the nutation of the earth about its precessing mean equator, and matrix M (from MNA) transforms to the position relative to the moon's true equator.

Then
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{moon} = MNA \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{earth,\ 1950.0}$$

Assuming that $\dot{N} = \dot{A} = 0$

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix}_{moon} = MNA \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix}_{earth,\ 1950} + \dot{M}NA \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{earth,\ 1950}$$

The $\dot{M}$ matrix is computed from the equations:

$$\dot{M} = \begin{pmatrix} M_{21}\dot{\Lambda} & M_{22}\dot{\Lambda} & M_{23}\dot{\Lambda} \\ -M_{11}\dot{\Lambda} & -M_{12}\dot{\Lambda} & -M_{13}\dot{\Lambda} \\ 0 & 0 & 0 \end{pmatrix}$$

where M_{ij} are the elements of M, and

$$\dot{\Lambda} = \dot{\Delta} + \dot{☾} + \dot{t} - \dot{\Omega} - \dot{\sigma}$$

$$\dot{\Delta} = -\rho\ (\dot{\Omega} + \dot{\sigma})$$

$$\dot{☾} = 0.266170762 \times 10^{-5} - 0.12499171 \times 10^{-13}\ T \text{ rad/sec};\ T = d/36525$$

$$\dot{\Omega} = -0.1069698435 \times 10^{-7} + 0.23015329 \times 10^{-3}\ T \text{ rad/sec}$$

$$\dot{t} = -0.1535272946 \times 10^{-9} \cos g$$
$$+0.569494067 \times 10^{-10} \cos g$$
$$+0.579473484 \times 10^{-11} \cos 2\omega \text{ rad/sec}$$

$$\dot{\sigma} = -0.520642191 \times 10^{-7} \cos g$$
$$+0.1811774451 \times 10^{-7} \cos (g+2\omega)$$
$$-0.1064057858 \times 10^{-7} \cos (2\omega+2g) \text{ rad/sec}$$

Reference: JPL Technical Report No. 32-223

MNAND

```
*      LABEL
*      SYMBOL TABLE
       SUBROUTINE MNAND(TIME,RO,G,GP,WW,EM,DM)
       DIMENSION DM(3,3),EM(3,3)
       T = TIME/36525.
       CRD = .266170762E-5 - .12499171E-13*T
       OMD = -.1069698435E-7 + .23015329E-13*T
       TAD = -1.535272946*COSF(G) + .569494067*COSF(GP)
      1 + .0579473484*COSF(2.*WW)
       TAD = .1E-9*TAD
       SGD = -.520642191*COSF(G) + .1811774451*COSF(G+2.*WW)
      1 -.1064057858*COSF(2.*WW+2.*G)
       SGD = .1E-6*SGD
       DLD = -RO*(OMD + SGD)
       CAD = DLD + CRD + TAD - OMD - SGD
       DM(1,1) = EM(2,1)*CAD
       DM(1,2) = EM(2,2)*CAD
       DM(1,3) = EM(2,3)*CAD
       DM(2,1) = -EM(1,1)*CAD
       DM(2,2) = -EM(1,2)*CAD
       DM(2,3) = -EM(1,3)*CAD
       DM(3,1) = 0.0
       DM(3,2) = 0.0
       DM(3,3) = 0.0
       RETURN
       END
```

Subroutine: MONBTR

Purpose: To obtain the position and velocity vectors of the vehicle relative to the beacon and determine if the beacon is in view of the vehicle. The station location partials are computed and subroutine COMPHQ called to update the state covariance matrix for the measurements being made. The types of measurements possible are range, range rate, right ascension, and declination.

Calling Sequence:

CALL MONBTR

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities placed in common

Common storages used or required: T, S, C, IC

Subroutines required: COMPHO, DOT, INTRI, MATRX, MNAND, MNA, NUTAIT, TRAC

Functions required: ASIN, ATAN, FNORM, SQRT (FIL)(SLO)(STH)

Approximate number of storages required: 495 DEC

MONBTR Partial Derivatives

MONBTR computes two partial matrices which are used in COMPHQ to evaluate station location errors. The matrices are the following:

$$\text{STPARS} = \begin{bmatrix} \frac{\partial x_1}{\partial LAT} & \frac{\partial x_1}{\partial LON} & \frac{\partial x_1}{\partial ALT} \\ \frac{\partial x_1}{\partial LAT} & \frac{\partial x_2}{\partial LON} & \frac{\partial x_2}{\partial ALT} \\ \frac{\partial x_3}{\partial LAT} & \frac{\partial x_3}{\partial LON} & \frac{\partial x_3}{\partial ALT} \end{bmatrix}$$

$$\text{STPARD} = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial LAT} & \frac{\partial \dot{x}_1}{\partial LON} & \frac{\partial \dot{x}_1}{\partial ALT} \\ \frac{\partial \dot{x}_2}{\partial LAT} & \frac{\partial \dot{x}_2}{\partial LON} & \frac{\partial \dot{x}_2}{\partial ALT} \\ \frac{\partial \dot{x}_3}{\partial LAT} & \frac{\partial \dot{x}_3}{\partial LON} & \frac{\partial \dot{x}_3}{\partial ALT} \end{bmatrix}$$

where $\vec{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = -\vec{X}_V + \vec{X}_B \equiv$ XREL (FORTRAN)

$$\vec{\dot{X}} = \begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{pmatrix} = -\vec{\dot{X}}_v + \vec{\dot{X}}_B \equiv \text{VREL (FORTRAN)}$$

$$\frac{\partial x_i}{\partial LAT} = \frac{\partial x_{Bi}}{\partial LAT} \qquad \frac{\partial \dot{x}_i}{\partial LAT} = \frac{\partial \dot{x}_{Bi}}{\partial LAT} \qquad i = 1,2,3$$

$$\vec{X}_B = (MNA)^T \; \vec{X}_{BM}$$

$$\dot{\vec{X}}_B = (MNAND)^T \; \vec{X}_{BM} + (MNA)^T \dot{\vec{X}}_{BM} \quad (=0)$$

where subscripts BM indicate beacon moon-fixed coordinates.

The transformations MNA and MNAND are obtained by calling the subroutines MNA and MNAND.

$$\underset{3 \times 1}{\vec{X}_{BM}} = \underset{3 \times 3}{(R + ALT)} \underset{3 \times 1}{\begin{pmatrix} \cos LAT \cos LON \\ \cos LAT \sin LON \\ \sin LAT \end{pmatrix}}$$

$$\begin{bmatrix} \frac{\partial X_1}{\partial PAR} \\ \frac{\partial X_2}{\partial PAR} \\ \frac{\partial X_3}{\partial PAR} \end{bmatrix} = (MNA)^T \begin{bmatrix} \frac{\partial X_{BM}}{\partial PAR} \\ \frac{\partial Y_{BM}}{\partial PAR} \\ \frac{\partial Z_{BM}}{\partial PAR} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial \dot{X}_1}{\partial PAR} \\ \frac{\partial \dot{X}_2}{\partial PAR} \\ \frac{\partial \dot{X}_3}{\partial PAR} \end{bmatrix} = (MNAND)^T \begin{bmatrix} \frac{\partial X_{BM}}{\partial PAR} \\ \frac{\partial Y_{BM}}{\partial PAR} \\ \frac{\partial Z_{BM}}{\partial PAR} \end{bmatrix}$$

$$\frac{\partial \vec{X}_{BM}}{\partial LAT} = (R + ALT) \begin{pmatrix} -\sin LAT \cos LON \\ -\sin LAT \sin LON \\ \cos LAT \end{pmatrix}$$

3 x 1   3 x 1

$$\frac{\partial \vec{X}_{BM}}{\partial LON} = (R + ALT) \begin{pmatrix} -\cos LAT \sin LON \\ \cos LAT \cos LON \\ 0 \end{pmatrix}$$

3 x 1   3 x 1

$$\frac{\partial \vec{X}_{BM}}{\partial ALT} = \begin{pmatrix} \cos LAT \cos LON \\ \cos LAT \sin LON \\ \sin LAT \end{pmatrix}$$

3 x 1   3 x 1

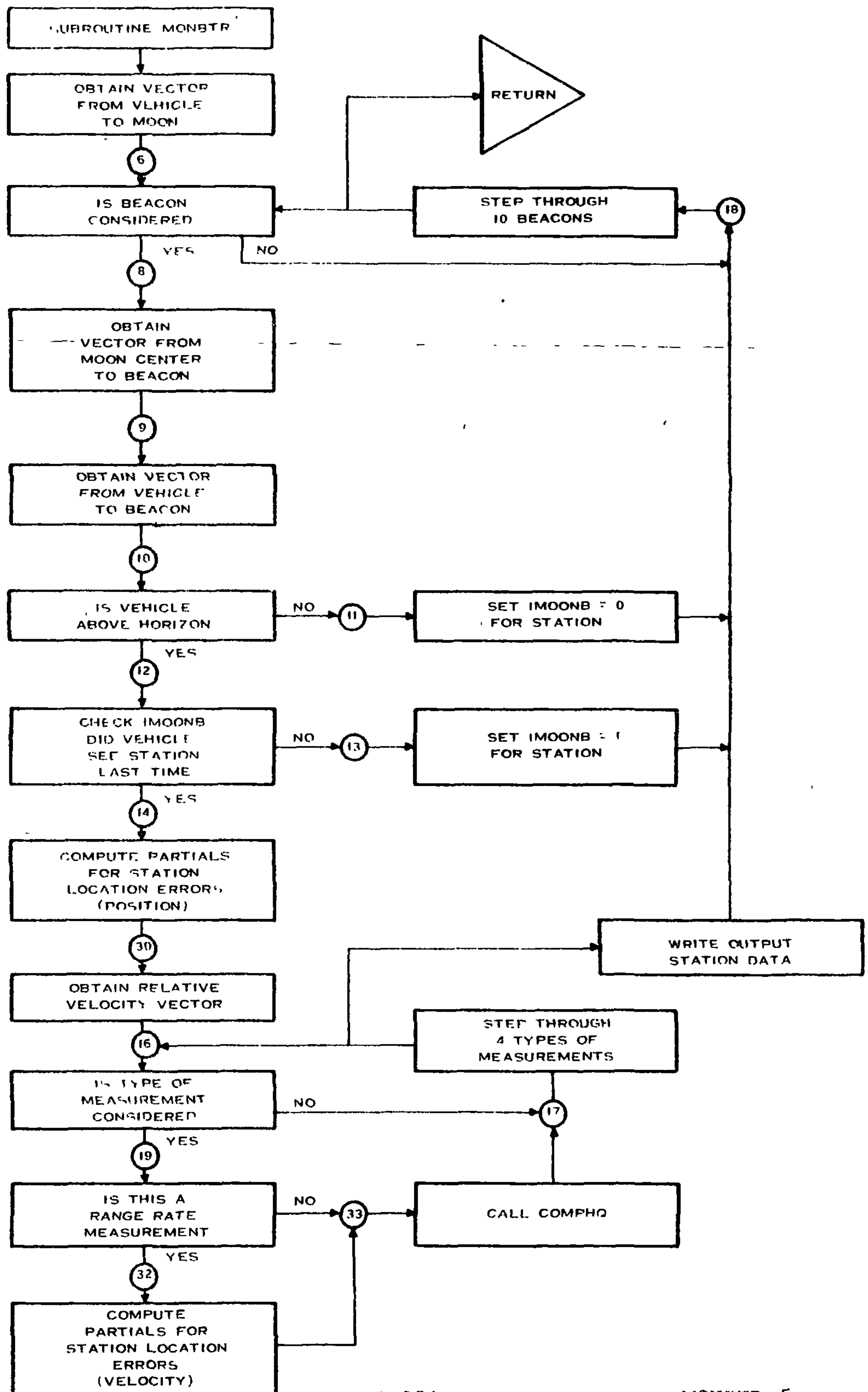
$$STPARS = (MNA)^T \left(\frac{\partial \vec{X}_{BM}}{\partial LAT} ; \frac{\partial \vec{X}_{BM}}{\partial LON} ; \frac{\partial \vec{X}_{BM}}{\partial ALT} \right)$$

3 x 3   3 x 3   3 x 3

$$STPARD = (MNAND)^T \left(\frac{\partial \vec{X}_{BM}}{\partial LAT} ; \frac{\partial \vec{X}_{BM}}{\partial LON} ; \frac{\partial \vec{X}_{BM}}{\partial ALT} \right)$$

3 x 3   3 x 3   3 x 3

Following computation of the matricies, subroutine COMPHQ is called to perform the updating of the state covariance matrix.


SUBROUTINE MONBTR
OBTAIN VECTOR FROM VEHICLE TO MOON
6
IS BEACON CONSIDERED
YES
NO
8
OBTAIN VECTOR FROM MOON CENTER TO BEACON
9
OBTAIN VECTOR FROM VEHICLE TO BEACON
10
IS VEHICLE ABOVE HORIZON
NO
11
SET IMOONB = 0 FOR STATION
YES
12
CHECK IMOONB DID VEHICLE SEE STATION LAST TIME
NO
13
SET IMOONB = 1 FOR STATION
YES
14
COMPUTE PARTIALS FOR STATION LOCATION ERRORS (POSITION)
30
OBTAIN RELATIVE VELOCITY VECTOR
16
IS TYPE OF MEASUREMENT CONSIDERED
NO
17
YES
19
IS THIS A RANGE RATE MEASUREMENT
NO
33
CALL COMPHQ
YES
32
COMPUTE PARTIALS FOR STATION LOCATION ERRORS (VELOCITY)
RETURN
STEP THROUGH 10 BEACONS
18
WRITE OUTPUT STATION DATA
STEP THROUGH 4 TYPES OF MEASUREMENTS

MONBTR

```
*       LABEL                                                                   MONB
*       SYMBOL TABLE                                                            MONB
CEC2012  SUBROUTINE MONBTR                                                      MONB
        SUBROUTINE MONBTR                                                       MONB0000
        COMMON T,S,C,IC                                                         MONB0010
        DIMENSION                                                               MONB0020
       1T(1360),S(1000),C(1000),IC(1),XREL(3),VREL(3)                           MONB0030
       1,PO(22),VE(22),AN(3,3),EM(3,3),ISBCON(10),U(3),EN(3)                    MONB0040
       2,RT(3),E(3),ETM(3,3),IMOONB(10),DM(3,3),DUM(3,3)                        MONB0050
       3,RMB(3),TYMEAS(4,10),X(3),VX(3),OUTPUT(6)                               MONB0060
        DIMENSION STPARS(3,3),DUD(3,3),STPARD(3,3)                              MONB0065
        EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM),(IC,ICDUM)                       MONB0070
       1,(C(895),XREL),(C(898),VREL),(C(894),IOBNO),(C(893),XMAG),              MONB0080
       2(C(892),DEX),(C(891),DEN1),(C(890),DEN2)                                MONB0090
       3,(C(889),DEX2),(C(888),DEX3),(C(652),P),(C(973),OUTPUT)                 MONB0100
       1,(C(13),TW),(C(14),TF),(C(62),PO),(IC(3),NOR)                           MONB0110
       2,(IC(190),LSTAT),(C(138),AN),(S(24),A),(C(120),EM)                      MONB0120
       3,(IC(140),ISBCON),(S(72),DR),(IC(220),IMOONB),(C(649),TSECP)            MONB0130
       4,(C(30),TSEC),(IC(140),TYMEAS),(C(15),X),(C(18),VX)                     MONB0140
        EQUIVALENCE (C(800),STPARD),(C(788),STPARS)                             MONB0145
        CALL SETN(NIN,NUTS)                                                     MONB0147
        DELTT=TSEC-TSECP                                                        MONB0150
        LSTAT=LSTAT                                                             MONB0160
        GO TO(1,2),LSTAT                                                        MONB0170
    1   CONTINUE                                                                MONB0180
        NB=NOR-1                                                                MONB0190
        DIS=1.E10                                                               MONB0200
        CALL INTR1(TW,TF,NB,PO,1,VE,DIS)                                        MONB0210
    2   CONTINUE                                                                MONB0220
        TIME=TW+TF                                                              MONB0230
        CALL NUTAIT(TIME,OM,CR,DT,EM,EPSIL)                                     MONB0240
        CALL MNA(TIME,OM,CR,DT,EPSIL,RO,G,GP,WW,ETM)                            MONB0250
        CALL MNAND(TIME,RO,G,GP,WW,ETM,DM)                                      MONB0260
        IF(NOR-2)3,5,3                                                          MONB0270
    3   CONTINUE                                                                MONB0280
        DO 4 I=1,3                                                              MONB0290
        J=20-I                                                                  MONB0300
        XREL(I)=PO(J)-X(I)                                                      MONB0310
        VREL(I)=VE(J)-VX(I)                                                     MONB0320
    4   CONTINUE                                                                MONB0330
        GO TO 7                                                                 MONB0340
    5   CONTINUE                                                                MONB0350
        DO 6 I=1,3                                                              MONB0360
        XREL(I)=-X(I)                                                           MONB0370
        VREL(I)=-VX(I)                                                          MONB0380
    6   CONTINUE                                                                MONB0390
C       XREL POSITION VECTOR FROM VEHICLE TO MOON CENTER 1950                   MONB0400
C       VREL RELATIVE VELOCITY OF MOON CENTER 1950                              MONB0410
    7   CONTINUE                                                                MONB0420
        NN=485                                                                  MONB0430
        DO 18 III=1,10                                                          MONB0440
```

```
      NN=NN+15
      IF(ISBCON(III))18,18,8
    8 CONTINUE
      AT=S(NN+9)*DR
      ON=S(NN+10)*DR
      AL=S(NN+11)
      SNAME=S(NN+12)
      GH=0.
      CALL TRAC(ON,AT,AL,GH,U,E,EN,RT,AC,SL,CL,ST,CT,A,A)
      CALL MATRX(ETM,AN,DUM,3,3,3,0)
      DO 9 I=1,3
      RMB(I)=0.
      DO 9 J=1,3
      RMB(I)=RMB(I)+DUM(J,I)*RT(J)
    9 CONTINUE
C     RMB RADIUS VECTOR TO MOON BEACON 1950
      DO 10 I=1,3
      XREL(I)=XREL(I)+RMB(I)
C     XREL VECTOR FROM VEHICLE TO MOON BEACON 1950
   10 CONTINUE
      TEST=DOT(RMB,XREL)
      IF(TEST)12,12,11
   11 IMOONB(III)=0
C     VEHICLE BELOW BEACON HORIZON
      GO TO 18
   12 CONTINUE
      IF(IMOONB(III))13,13,14
   13 CONTINUE
      IMOONB(III)=1
      GO TO 18
   14 CONTINUE
      RADIUS=A+AL
      DUD(1,1)=-RADIUS*SL*CT
      DUD(2,1)=-RADIUS*SL*ST
      DUD(3,1)=RADIUS*CL
      DUD(1,2)=-RADIUS*CL*ST
      DUD(2,2)=RADIUS*CL*CT
      DUD(3,2)=0.
      DUD(1,3)=CL*CT
      DUD(2,3)=CL*ST
      DUD(3,3)=SL
      DO 30 I=1,3
      DO 30 J=1,3
      STPARS(I,J)=0.
      DO 30 K=1,3
   30 STPARS(I,J)=STPARS(I,J)+DUM(K,I)*DUD(K,J)
      CALL MATRX(DM,AN,DUM,3,3,3,0)
      DO 15 I=1,3
      RMB(I)=0
      DO 15 J=1,3
      RMB(I)=RMB(I)+DUM(J,I)*RT(J)
```

MONBTR (Cont'd)

```
 15   CONTINUE                                                          MONB0810
C     RMB ROTATIONAL VELOCITY OF MOON BEACON 1950                       MONB0820
      DO 16 I=1,3                                                       MONB0830
      VREL(I)=VREL(I)+RMB(I)                                            MONB0840
C     VREL RELATIVE VELOCITY OF MOON BEACON WRT THE VEHICLE             MONB0850
 16   CONTINUE                                                          MONB0860
      OBNO=DELTT/S(NN+13)                                               MONB0870
      XMAG=FNORM(XREL)                                                  MONB0880
      DEX=XREL(1)**2+XREL(2)**2                                         MONB0890
      DEN1=1./SQRTF(DEX)/XMAG**2                                        MONB0900
      DEN2=1./DEX                                                       MONB0910
      DEX2=XMAG*XMAG                                                    MONB0920
      DEX3=1./XMAG                                                      MONB0930
C     THE ABOVE QUANTITIES ARE USED IN COMPUTATIONS OF H MATRIX         MONB0940
      DEC=ASINF(DEX3*XREL(3))                                           MONB0942
      OUTPUT(5)=DEC/DR                                                  MONB0944
      RA=ATANF(XREL(2)/XREL(1))                                         MONB0946
      OUTPUT(3)=RA/DR                                                   MONB0948
      OUTPUT(1)=XMAG                                                    MONB0949
      DO 17 JJJ=1,4                                                     MONB0950
      IF(TYMEAS(JJJ,III))17,17,19                                       MONB0960
  19  CONTINUE                                                          MONB0962
      IF(JJJ-4)33,32,33                                                 MONB0964
 32   CONTINUE                                                          MONB0966
      DO 34 I=1,3                                                       MONB0968
      DO 34 J=1,3                                                       MONB0970
      STPARD(I,J)=0.                                                    MONB0972
      DO 34 K=1,3                                                       MONB0974
 34   STPARD(I,J)=STPARD(I,J)+DUM(K,I)*DUD(K,J)                         MONB0975
 33   CONTINUE                                                          MONB0976
      CALL COMPHQ(JJJ,2,NN)                                             MONB0978
 17   CONTINUE                                                          MONB0980
      WRITE OUTPUT TAPE NUTS,700,III,OUTPUT                             MONB0982
700   FORMAT(13H MOON BEACON ,I2,                                       MONB0983
     1/4H RNGE15.8,5H  RGRE15.8,                                        MONB0984
     25H   RAE15.8,5H  RARE15.8,5H  DECE15.8,5H  DCRE15.8)              MONB0985
      DO 20 I=1,6                                                       MONB0986
 20   OUTPUT(I)=0.                                                      MONB0987
 18   CONTINUE                                                          MONB0990
      RETURN                                                            MONB1000
      END                                                               MONB
```

Subroutine: MULT

Purpose: To find the product matrix of two 3 x 3 matrices.

Calling Sequence:

CALL MULT (A, B, C)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	A	3,3			First input matrix
I	B	3,3			Second input matrix
O	C	3,3			C = AB

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

MULT

```
*       LABEL                                                        MULT
*       SYMBOL TABLE                                                 MULT
        SUBROUTINE MULT(A,B,C,N)                                     MULT0000
        DIMENSION A(3,3),B(3,3),C(3,3)                               MULT0010
        DO 1 I=1,3                                                   MULT0020
        DO 1 J=1,3                                                   MULT0030
        C(I,J) = 0.                                                  MULT0040
        DO 1 K=1,3                                                   MULT0050
      1 C(I,J) = C(I,J) + A(I,K)*B(K,J)                              MULT0060
      3 RETURN                                                       MULT0070
        END                                                          MULT
```

Subroutine: NUTAIT

Purpose: To evaluate the elements of the nutation matrix which relates the Cartesian coordinates expressed in the true equator and equinox to those in the mean equator and equinox.

Calling Sequence:

CALL NUTAIT (TIME, OM, CR, DT, EN, EPSIL)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TIME	1	T	days	Total number days from ref.epoch
O	OM	1	Ω	radians	Argument of moon's descending
					node
O	CR	1	CR	radians	Mean longitude of moon
O	DT	1	$\Delta\Psi + d\Psi$	radians	Nutation in longitude of equinox
O	EN	3,3	N	(-)	Nutation matrix
O	EPSIL	1	$\bar{\epsilon} + d\epsilon$	radians	True obliquity

Common storages used or required: None

Subroutines required: None

Functions required: INTF, SINF, COSF

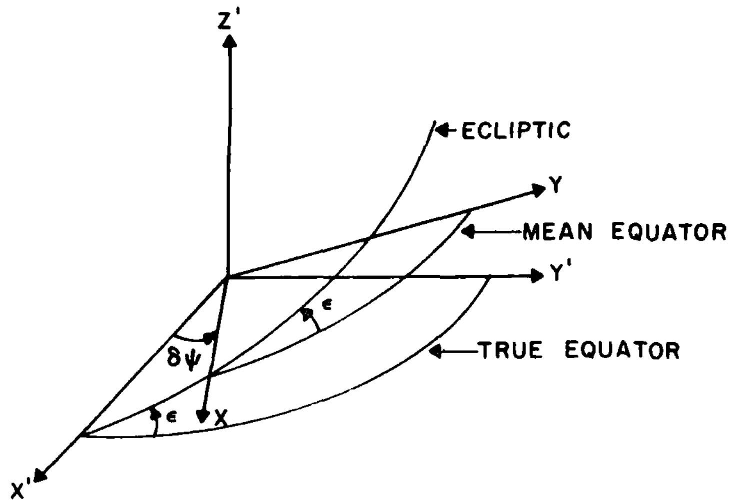
Approximate number of storages required:

NUTAIT

Method and Theory

The input time is converted to Julian centuries of 36525 days past the reference epoch (currently, 0^h January 1, 1950, E.T.). Where the computed angles are likely to be extremely large, the largest contributing term is reduced to the first revolution by the internal function DF(X). By using double precision arithmetic, this technique also helps to control round-off errors more closely as well as to keep the computed angles reasonable.

The relationship between the Cartesian coordinates expressed in the true equator and equinox and those expressed in the mean equator and equinox is shown in the following sketch:



The nutations are given by the following:

For obliquity $\delta\epsilon = \Delta\epsilon + d\epsilon$

For longitude $\delta\Psi = \Delta\Psi + d\Psi$

In the above equations, $\Delta\epsilon$, $\Delta\Psi$ express the long-period contributions and $d\epsilon$, $d\Psi$ the short-period contributions.

Both ϵ and Ψ are computed as functions of Ω, CR, Γ', L, Γ where

$$\Omega = 12.112790 - 360*DF(1) + .0020795T + .002081T^2 + .000002T^3$$

$$CR = 64.375452 + 360*DF(2) - .001131575T - .00113015T^2 - .0000019T^3$$

$$\Gamma' = 208.84399 + 360*DF(3) - .010334T - .010343T^2 - .000012T^3$$

$$\Gamma = 282.08053 + .0000470684D + .00045525T + .0004575T^2 + .000003T^3$$

$$L = 280.08121 + 360*DF(4) + .000303\ (T + T^2)$$

T in the above equations is the number of Julian years past the reference epoch and the function DF(X) is

$$DF(X) = d\ \frac{a}{360}$$

where a = .052953922, 13.176397, .11140408, .98564734
and d is the number of days past the reference epoch.

Then
$$\Delta\epsilon \times 10^4 = 25.5844 \cos\Omega - .2511 \cos 2\Omega + 1.5336 \cos 2L + .0666 \cos(3L - \Gamma) - .0258 \cos(L + \Gamma) - .0183 \cos(2L - \Omega) - .0067 \cos(2\Gamma' - \Omega)$$

$$d\epsilon \times 10^4 = .2456 \cos 2CR + .0508 \cos(2CR - \Omega) + .0369 \cos(3CR - \Gamma') - .0139 \cos(CR + \Gamma') - .0086 \cos(CR - \Gamma' + \Omega) + .0083 \cos(CR - \Gamma' - \Omega) + .0061 \cos(3CR + \Gamma' - 2L) + .0064 \cos(3CR - \Gamma' - \Omega)$$

$$\Delta\Psi \times 10^4 = -(47.8927 + .0482T) \sin\Omega + .5800 \sin 2\Omega - 3.5361 \sin 2L - .1378 \sin(3L - \Gamma) + .0594 \sin(L + \Gamma) + .0344 \sin(2L - \Omega) + .0125 \sin(2\Gamma' - \Omega) + .3500 \sin(L - \Gamma) + .0125 \sin(2L - 2\Gamma')$$

$$d\Psi \times 10^4 = -.5658 \sin 2CR - .0950 \sin (2CR - \Omega)$$
$$- .0725 \sin (3CR - \Gamma') + .0317 \sin (CR + \Gamma')$$
$$+ .0161 \sin (CR - \Gamma' + \Omega) + .0158 \sin (CR - \Gamma' - \Omega)$$
$$- .0144 \sin (3CR + \Gamma' - 2L) - .0122 \sin (3CR - \Gamma' - \Omega)$$
$$+ .1875 \sin (CR - \Gamma') + .0078 \sin (2CR - 2\Gamma')$$
$$+ .0414 \sin (CR + \Gamma' - 2L) + .0167 \sin (2CR - 2L)$$
$$- .0089 \sin (4CR - 2L).$$

The mean obliquity is calculated by

$$\bar{\epsilon} = 23.4457587 - .01309404T - .00000088T^2 + .00000050T^3$$

and the true obliquity by

$$\epsilon = \bar{\epsilon} + \delta\epsilon$$

The nutation matrix N relates the Cartesian coordinates expressed in the true equator and equinox to those in the mean equator and equinox by

$$\begin{vmatrix} X' \\ Y' \\ Z' \end{vmatrix} = N \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}$$

where the primed system is the true equator and equinox and the unprimed is the mean equator and equinox.

Thus

$$N_{11} = \cos \delta\Psi$$
$$N_{12} = -\sin \delta\Psi \cos \bar{\epsilon}$$
$$N_{13} = \sin \delta\Psi \sin \bar{\epsilon}$$
$$N_{21} = \sin \delta\Psi \cos \epsilon$$

$$N_{22} = \cos \delta\Psi \cos \epsilon \cos \bar{\epsilon} + \sin \epsilon \sin \bar{\epsilon}$$

$$N_{23} = \cos \delta\Psi \cos \epsilon \sin \bar{\epsilon} - \sin \epsilon \cos \bar{\epsilon}$$

$$N_{31} = \sin \delta\Psi \sin \epsilon$$

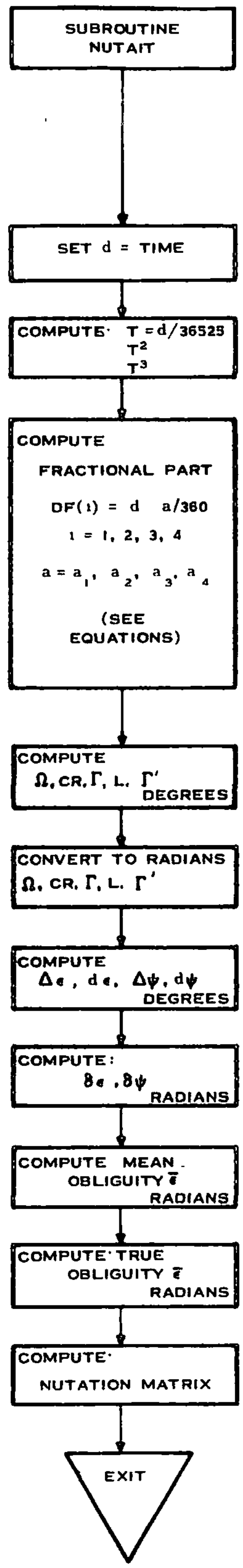
$$N_{32} = \cos \delta\Psi \sin \epsilon \cos \bar{\epsilon} - \cos \epsilon \sin \bar{\epsilon}$$

$$N_{33} = \cos \delta\Psi \sin \epsilon \sin \bar{\epsilon} + \cos \epsilon \cos \bar{\epsilon}$$

For numerical calculation, the above N matrix is expanded to the first order in $\delta\Psi$ and $\delta\epsilon$:

$$N = \begin{vmatrix} 1 & -\delta\Psi \cos \bar{\epsilon} & -\delta\Psi \sin \bar{\epsilon} \\ \delta\Psi \cos \bar{\epsilon} & 1 & -\delta\epsilon \\ \delta\Psi \sin \bar{\epsilon} & \delta\epsilon & 1 \end{vmatrix}$$

Reference: JPL Technical Report No. 32-223


SUBROUTINE NUTAIT
SET d = TIME
COMPUTE T = d/36525
T²
T³
COMPUTE
FRACTIONAL PART
DF(i) = d a/360
i = 1, 2, 3, 4
a = a1, a2, a3, a4
(SEE EQUATIONS)
COMPUTE
Ω, CR, Γ, L, Γ'
DEGREES
CONVERT TO RADIANS
Ω, CR, Γ, L, Γ'
COMPUTE
Δε, dε, Δψ, dψ
DEGREES
COMPUTE:
δε, δψ
RADIANS
COMPUTE MEAN OBLIGUITY ε̄
RADIANS
COMPUTE TRUE OBLIGUITY ε̃
RADIANS
COMPUTE
NUTATION MATRIX
EXIT

```
*         LABEL
*         SYMBOL TABLE
          SUBROUTINE NUTAIT (TIME,OM,CR,DT,EN,EPSIL)
          DIMENSION EN(3,3)
          D = TIME
          T = D/36525.
          T2 = T*T
          T3 = T2*T
          OM = 12.112790-.052953922*D+.0020795*T+.002081*T2+.000002*T3
          CR = 64.375452+13.176397*D-.001131575*T-.00113015*T2+.0000019*T3
          GP = 208.84399+.11140408*D-.010334*T-.010343*T2-.000012*T3
          VL = 280.08121+.98564734*D+.000303*(T+T2)
          G = 282.08053+.0000470684*D+.00045525*T+.0004575*T2+.000003*T3
          OM = OM*.017453296
          CR = CR*.017453296
          GP = GP*.017453296
          VL = VL*.017453296
          G = G*.017453296
          DE = 25.5844*COSF(OM)-.2511*COSF(2.*OM)+1.5336*COSF(2.*VL)
         1 +.0666*COSF(3.*VL-G)-.0258*COSF(VL+G)-.0183*COSF(2.*VL-OM)
         2 -.0067*COSF(2.*GP-OM)
          DD = .2456*COSF(2.*CR)+.0508*COSF(2.*CR-OM)+.0369*COSF(3.*CR-GP)
         1 -.0139*COSF(CR+GP)-.0086*COSF(CR-GP+OM)+.0083*COSF(CR-GP-OM)
         2 +.0061*COSF(3.*CR+GP-2.*VL)+.0064*COSF(3.*CR-GP-OM)
          DT = -(47.8927+.0482*T)*SINF(OM)+.58*SINF(2.*OM)
         1 -3.5361*SINF(2.*VL)-.1378*SINF(3.*VL-G)+.0594*SINF(VL+G)
         2 +.0344*SINF(2.*VL-OM)+.0125*SINF(2.*GP-OM)+.35*SINF(VL-G)
         3 +.0125*SINF(2.*VL-2.*GP)
          DS = -.5658*SINF(2.*CR)-.095*SINF(2.*CR-OM)-.0725*SINF(3.*CR-GP)
         1 +.0317*SINF(CR+GP)+.0161*SINF(CR-GP+OM)+.0158*SINF(CR-GP-OM)
         2 -.0144*SINF(3.*CR+GP-2.*VL)-.0122*SINF(3.*CR-GP-OM)
         3 +.1875*SINF(CR-GP)+.0078*SINF(2.*CR-2.*GP)
         4 +.0414*SINF(CR+GP-2.*VL)+.0167*SINF(2.*CR-2.*VL)
         5 -.0089*SINF(4.*CR-2.*VL)
          DE = .17453296E-5*(DE+DD)
          DT = .17453296E-5*(DT+DS)
          EB = 23.4457587-.01309404*T-.00000088*T2+.0000005*T3
          EB = EB*.017453296
          EPSIL = EB+DE
          EN(1,1) = 1.
          EN(1,2) = -DT*COSF(EB)
          EN(1,3) = -DT*SINF(EB)
          EN(2,1) = -EN(1,2)
          EN(2,2) = 1.
          EN(2,3) = -DE
          EN(3,1) = -EN(1,3)
          EN(3,2) = DE
          EN(3,3) = 1.
          RETURN
          END
```

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Subroutine: OBLN

Purpose: To compute the perturbing acceleration due to the earth's oblateness. The formula for the earth's potential includes the second, third and fourth spherical harmonics. Part of the input is a rotation matrix to equator and equinox of date, obtained by successive calls to ROTEQ, NUTAIT, and MULT. It also computes the first variation of the perturbing acceleration for the variational equations.

Calling Sequence:

CALL OBLN (XP, U, A, VJ, H, D, PO, AN, B, NEQ)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	XP	3	$\overline{R}$	Kilometers	Position vector
I	U	1	μ	Km^3/sec^2	Gravitational constant
I	A	1	R_E	Kilometers	Equatorial radius of the earth
I	VJ	1	J		Coefficient of second harmonic
I	H	1	H		Coefficient of third harmonic
I	D	1	D		Coefficient of fourth harmonic
O	PO	3	∇U		Perturbing acceleration
I	AN	3,3			Rotation matrix to equator and equinox of date

(Cont'd.)

Common storages used or required: None

Subroutines required: None

Functions required: SQRT

Approximate number of storages required: 480 DEC

(Cont'd.)

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	B	3,3	$\partial \ddot{X}_i / \partial X_j$		Derivatives of ∇U
I	NEQ	1			Number of differential equations
					being integrated

OBLN computes the perturbing acceleration due to the earth's oblateness. The oblate potential of the earth is assumed to contain the second, third, and fourth spherical harmonics:

$$U = \frac{\mu}{R} \left\{ \frac{JR_E^2}{3R^2} (1 - 3 \sin^2\phi) + \frac{HR_E^3}{5R^3} (3 \sin\phi - 5 \sin^3\phi) \right.$$

$$\left. + \frac{DR_E^4}{35R^4} (3 - 30 \sin^2\phi + 35 \sin^4\phi) \right\}$$

where R_E is the equatorial radius of the earth and ϕ is the geocentric latitude, $\overline{R} = (X_1, X_2, X_3)$ is expressed in the mean equator and equinox of 1950.

The position vector $\overline{r} = (X, Y, Z)$ expressed in the true equator and equinox of date must be obtained to determine $\sin\phi$. The proper rotation matrix AN has been computed. Then,

$$\overline{r} = AN\ \overline{R}$$

and

$$\sin\phi = \frac{Z}{R} .$$

The perturbing acceleration is equal to ∇U:

$$\frac{\partial U}{\partial X_j} = - J\mu \frac{R_E^2}{R^4} \left\{ \left(1 - \frac{5Z^2}{R^2}\right) \frac{X_j}{R} + 2 \frac{Z}{R} AN_{3j} \right\}$$

$$-H\mu \frac{R_E^3}{R^5} \left\{ \left(3 - \frac{7Z^2}{R^2}\right) \frac{Z}{R} \frac{X_j}{R} + \left(- \frac{3}{5} + \frac{3Z^2}{R^2}\right) AN_{3j} \right\}$$

$$-D\mu \frac{R_E^4}{R^6} \left\{ \left(\frac{3}{7} - \frac{6Z^2}{R^2} + \frac{9Z^4}{R^4}\right) \frac{X_j}{R} + \left(\frac{12}{7} - \frac{4Z^2}{R^2}\right) \frac{Z}{R} AN_{3j} \right\}$$

for $j = 1,2,3$.

If the number of second order differential equations is less than or equal to three, RETURN is called at this point.

OBLN also computes the derivatives of the perturbing acceleration for use in the integration package. For this purpose, only the term arising from the second harmonic is retained; also, the coordinates are regarded as being expressed in the reference system of the mean equator and equinox of 1950.

Starting with

$$\nabla U_a = \left(g_1 \frac{X_1}{R},\ g_1 \frac{X_2}{R},\ g_2 \frac{X_3}{R}\right)$$

where

$$g_1 = -J\mu \frac{R_E^2}{R^4} \left(1 - \frac{5Z^2}{R^2}\right)$$

$$g_2 = -J\mu \frac{R_E^2}{R^4} \left(3 - \frac{5Z^2}{R^2}\right)$$

yields

$$\frac{\partial \ddot{x}_1}{\partial x_j} = g_1 \frac{X_1}{R} \left(\frac{1}{X_1} \frac{\partial X_1}{\partial X_j} - \frac{3}{R^2} \bar{R} \cdot \frac{\partial \bar{R}}{\partial X_j} \right)$$

$$+ J\mu R_E^2 \frac{X_1}{R^7} \left\{ 10X_3 \frac{\partial X_3}{\partial X_j} + 2\left(1 - 10 \frac{X_3^2}{R^2}\right) \bar{R} \cdot \frac{\partial \bar{R}}{\partial X_j} \right\}$$

$$\frac{\partial \ddot{x}_2}{\partial x_j} = g_1 \frac{X_2}{R} \left(\frac{1}{X_2} \frac{\partial X_2}{\partial X_j} - \frac{3}{R^2} \bar{R} \cdot \frac{\partial \bar{R}}{\partial X_j} \right)$$

$$+ J\mu R_E^2 \frac{X_2}{R^7} \left\{ 10X_3 \frac{\partial X_3}{\partial X_j} + 2\left(1 - 10 \frac{X_3^2}{R^2}\right) R \cdot \frac{\partial R}{\partial X_j} \right\}$$

$$\frac{\partial \ddot{x}_3}{\partial x_j} = g_2 \frac{X_3}{R} \left(\frac{1}{X_3} \frac{\partial X_3}{\partial X_j} - \frac{3}{R^2} \bar{R} \cdot \frac{\partial \bar{R}}{\partial X_j} \right)$$

$$+ J\mu R_E^2 \frac{X_3}{R^7} \left\{ 10X_3 \frac{\partial X_3}{\partial X_j} + 2\left(3 - 10 \frac{X_3^2}{R^2}\right) R \cdot \frac{\partial \bar{R}}{\partial U_j} \right\}$$

Expanding the dot products and collecting terms gives

$$B_{11} = g_1 \left(\frac{1}{R} - \frac{3X_1^2}{R^3} \right) + 2J\mu R_E^2 \frac{X_1^2}{R^7} \left(1 - 10 \frac{X_3^2}{R^2}\right)$$

$$B_{12} = g_1 \left(- \frac{3X \cdot X_2}{R^3} \right) + 2 J\mu R_E^2 \frac{X_1 X_2}{R^7} \left(1 - 10 \frac{X_3^2}{R^2}\right)$$

$$B_{13} = g_1\left(-\frac{3X_1X_3}{R^3}\right) + 2\ J\mu\ R_E^2\ \frac{X_1X_3}{R^7}\left(6 - 10\frac{X_3^2}{R^2}\right)$$

$$B_{21} = B_{12}$$

$$B_{22} = g_1\left(\frac{1}{R} - \frac{3X_2^2}{R^3}\right) + 2\ J\mu\ R_E^2\ \frac{X_2^2}{R^7}\left(1 - 10\frac{X_3^2}{R^2}\right)$$

$$B_{23} = g_1\left(-\frac{3X_2X_3}{R^3}\right) + 2\ J\mu\ R_E^2\ \frac{X_2X_3}{R^7}\left(6 - 10\frac{X_3^2}{R^2}\right)$$

$$B_{31} = B_{13}$$

$$B_{32} = B_{23}$$

$$B_{33} = g_2\left(\frac{1}{R} - \frac{3X_3^2}{R^3}\right) + 2\ J\mu\ R_E^2\ \frac{X_3^2}{R^7}\left(8 - 10\frac{X_3^2}{R^2}\right)$$

where $B\lambda_j = \dfrac{\partial \ddot{X}_i}{\partial X_j}$

For $R > 3R_E$, B is set equal to zero.

```
*       LABEL
*       SYMBOL TABLE
CEC2031  SUBROUTINE OBLN
      SUBROUTINE OBLN(XP,U,A,VJ,H,D,PO,AN,B ,NEQ)
      DIMENSION X(3),PO(3),AN(3,3),XP(3),B (3,3)
      RE=6378.15
      DO 2 I=1,3
      X(I) = 0.
      DO 2 J=1,3
    2 X(I) = X(I) + AN(I,J)*XP(J)
      R2 = X(1)**2+X(2)**2+X(3)**2
      R = SQRTF(R2)
      UR2 = U/R2
      AR = A/R
      AR2 = AR*AR
      AR3 = AR*AR2
      AR4 = AR*AR3
      ZR = X(3)/R
      ZR2 = ZR*ZR
      ZR4 = ZR2*ZR2
      TM1 = (1.-5.*ZR2)/R
      TM2 = 2.*ZR
      TM3 = (3.-7.*ZR2)*ZR/R
      TM4 = -.6+3.*ZR2
      TM5 = (.42857143-6.*ZR2+9.*ZR4)/R
      TM6 = (1.7142857-4.*ZR2)*ZR
      TO1 = -VJ*UR2*AR2
      TO2 = -H*UR2*AR3
      TO3 = -D*UR2*AR4
      DO 1 I=1,3
    1 PO(I) = TO1*(TM1*XP(I)+TM2*AN(3,I))
     1        +TO2*(TM3*XP(I)+TM4*AN(3,I))
     2        +TO3*(TM5*XP(I)+TM6*AN(3,I))
      IF(NEQ-3)3,3,4
    3 CONTINUE
      RETURN
    4 CONTINUE
      IF(R-3.*RE)5,5,6
    6 CONTINUE
      DO 7 I=1,3
      DO 7 J=1,3
      B(I,J)=0.
    7 CONTINUE
      GO TO 3
    5 CONTINUE
      G5=-TO1/R
      G6=5.*ZR2
      G1=-G5*(1.-G6)
      G2=-G5*(3. -G6)
      G7=2.*G6
      G8=X(1)/R
```

```
      G9=X(2)/R
      G10=X(3)/R
      B(1,1)=G1*(1.-3.*G8**2)+2.*G5*G8**2*(1.-G7)
      B(1,2)=-3.*G1*G8*G9+2.*G5*G8*G9*(1.-G7)
      B(1,3)=-3.*G1*G8*G10+2.*G5*G8*G10*(6.-G7)
      B(2,1)=B(1,2)
      B(2,2)=G1*(1.-3.*G9**2)+2.*G5*G9**2*(1.-G7)
      B(2,3)=-3.*G1*G9*G10+2.*G5*G9*G10*(6.-G7)
      B(3,1)=B(1,3)
      B(3,2)=B(2,3)
      B(3,3)=G2*(1.-3.*G10**2)+2.*G5*G10**2*(8.-G7)
      GO TO 3
      END
```

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Subroutine: ONBTR

Purpose: The subroutine obtains the relative position and velocity vectors from the vehicle to any of the six following bodies; Earth, Moon, Sun, Venus, Mars, and Jupiter. The bodies to be observed are established by input data. The types of observation which can be made on the body are range, range rate, right ascension, and declination. The selection of types of measurements to be made is also by input data. ONBTR calls subroutine COMPHO to perform the updating of the covariance matrix for observations being made.

Calling Sequence:

CALL ONBTR

Input and Output

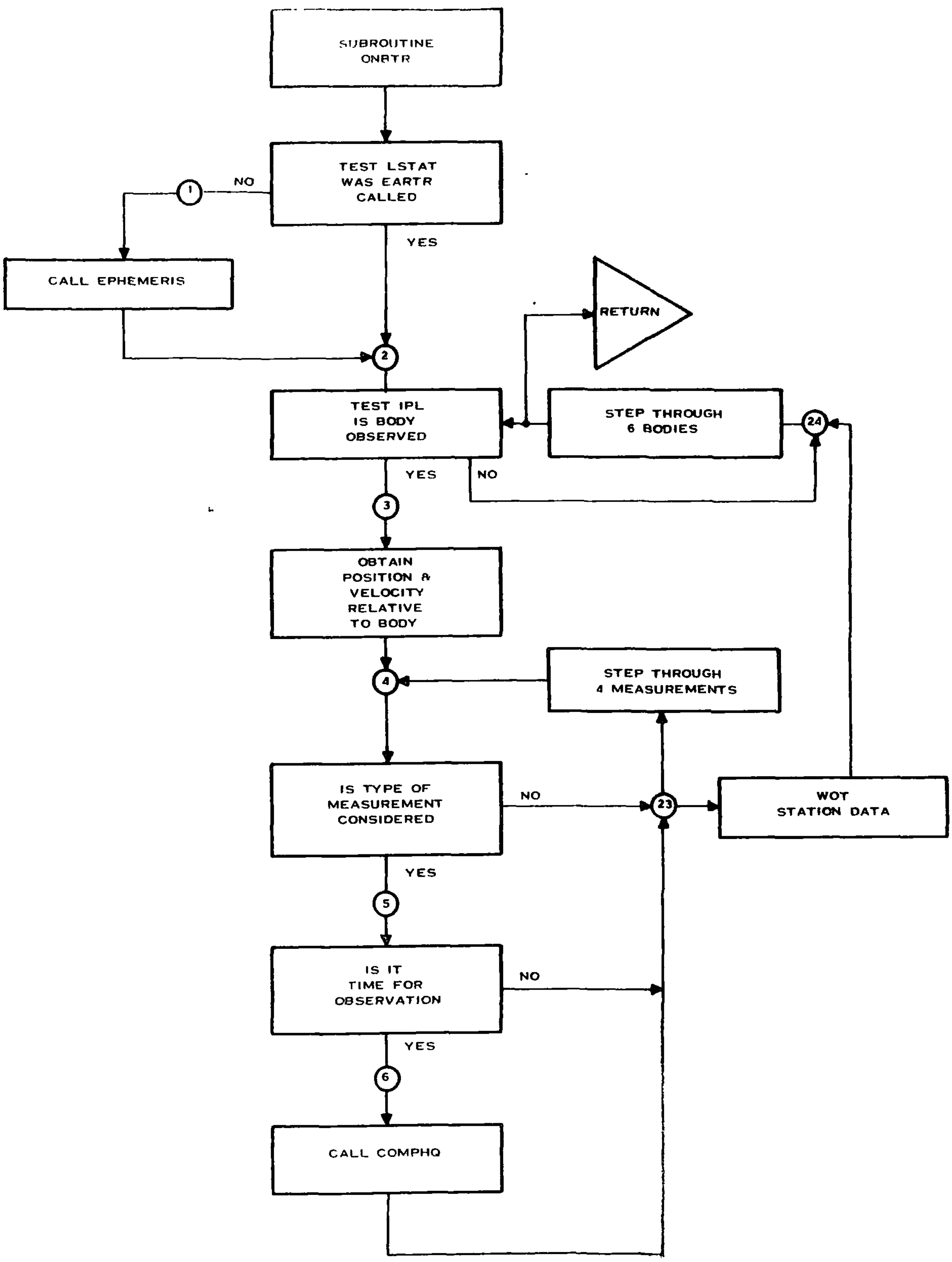
I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities placed in common

Common storages used or required: T, S, C, IC

Subroutines required: COMPHO, INTRI

Functions required: ASIN, ATAN, FNORM, SQRT, (FIL)(SLO)(STH)

Approximate number of storages required: 267 DEC


SUBROUTINE
ONBTR
TEST LSTAT
WAS EARTR
CALLED
NO
1
YES
CALL EPHEMERIS
RETURN
2
TEST IPL
IS BODY
OBSERVED
STEP THROUGH
6 BODIES
24
YES
NO
3
OBTAIN
POSITION &
VELOCITY
RELATIVE
TO BODY
STEP THROUGH
4 MEASUREMENTS
4
IS TYPE OF
MEASUREMENT
CONSIDERED
NO
23
WOT
STATION DATA
YES
5
IS IT
TIME FOR
OBSERVATION
NO
YES
6
CALL COMPHQ

```
*     LABEL                                                          ONBT
*     SYMBOL TABLE                                                   ONBT
CEC2007                                                              ONBT
      SUBROUTINE ONBTR                                               ONBT000
C     SUBROUTINE ONBTR UPDATES CONVARIANCE MATRIX FOR ONBOARD        ONBT001
C     MEASUREMENTS OF RANGE,RIGHT ASCENSION,DECLINATION,AND          ONBT002
C     RANGE RATE RELATIVE TO THE 6 CELESTIAL BODIES                  ONBT003
      COMMON T,S,C,IC                                                ONBT004
      DIMENSION PO(22),VE(22),IMPLAN(4,6),IPL(6),X(3),VX(3)          ONBT005
     1,T(1360),S(1000),C(1000),IC(1),XREL(3),VREL(3)                 ONBT006
     2,TLAST(4,6),OBRATE(4,6),H(6),P(6,6),OUTPUT(6)                  ONBT007
      EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM)                         ONBT008
     1,(IC(110),IPL),(IC(116),IMPLAN),(C(15),X),(C(18),VX)           ONBT009
     2,(IC(3),NOR),(C(13),TW),(C(14),TF),(IC(190),LSTAT)             ONBT010
     3,(C(649),TSECP),(C(30),TSEC),(S(437),OBRATE),(C(652),P)        ONBT011
     4,(C(622),TLAST),(C(62),PO),(C(84),VE)                          ONBT012
     1,(C(895),XREL),(C(898),VREL),(C(894), OBNO),(C(893),XMAG),     ONBT013
     2(C(892),DEX),(C(891),DEN1),(C(890),DEN2),(S(72),DR)            ONBT014
     3,(C(889),DEX2),(C(888),DEX3),(C(652),P),(C(973),OUTPUT)        ONBT015
      CALL SETN(NIN,NUTS)                                            ONBT015
      LSTAT=LSTAT                                                    ONBT016
      GO TO (1,2),LSTAT                                              ONBT017
    1 CONTINUE                                                       ONBT018
      NB=NOR-1                                                       ONBT019
      DIS=1.E10                                                      ONBT020
      CALL INTR1(TW,TF,NB,PO,1,VE,DIS)                               ONBT021
    2 CONTINUE                                                       ONBT022
      DO 24 JJ=1,6                                                   ONBT023
C     JJ DETERMINES BODY TO BE OBSERVED 1,EARTH 2,MOON 3,SUN         ONBT024
C     4,VENUS 5,MARS 6,JUPITER                                       ONBT025
      IF(IPL(JJ))24,24,3                                             ONBT026
    3 CONTINUE                                                       ONBT027
      LM=26-3*(JJ)                                                   ONBT028
      DO 4 I=1,3                                                     ONBT029
      MM=LM-I                                                        ONBT030
      XREL(I)=PO(MM)-X(I)                                            ONBT031
      VREL(I)=VE(MM)-VX(I)                                           ONBT032
    4 CONTINUE                                                       ONBT033
C     XREL=VECTOR FROM VEHICLE TO JJ-TH PLANET,EQUINOX 1950          ONBT034
C     VREL=RELATIVE VELOCITY VECTOR                                  ONBT035
      XMAG=FNORM(XREL)                                               ONBT036
      DEX=XREL(1)**2+XREL(2)**2                                      ONBT037
      DEN1=1./SQRTF(DEX)/XMAG**2                                     ONBT038
      DEN2=1./DEX                                                    ONBT039
      DEX2=XMAG*XMAG                                                 ONBT040
      DEX3=1./XMAG                                                   ONBT041
C     THE ABOVE QUANTITIES ARE USED IN COMPUTATIONS OF H MATRIX      ONBT042
      DELTT=TSEC-TSECP                                               ONBT043
      DEC=ASINF(DEX3*XREL(3))                                        ONBT043
      OUTPUT(5)=DEC/DR                                               ONBT043
      RA=ATANF(XREL(2)/XREL(1))                                      ONBT043
```

```
      OUTPUT(3)=RA/DR
      OUTPUT(1)=XMAG
      DO 23 JJJ=1,4
      IF(IMPLAN(JJJ,JJ))23,23,5
    5 CONTINUE
      TLAST(JJJ,JJ)=TLAST(JJJ,JJ)+DELTT
C     TLAST STORES TIME SINCE LAST JJJ TYPE OF OBSERVATION OF JJ BODY
C     JJJ DETERMINES TYPE OF OBS.1=RANGE,2=RIGHT ASCENSION,
C     3=DECLINATION,4=RANGE RATE
      NOB=TLAST(JJJ,JJ)/OBRATE(JJJ,JJ)
      OBNO=NOB
      IF(NOB)23,23,6
    6 CONTINUE
      JJJ=JJJ
      NN=426
      CALL COMPHQ(JJJ,1,NN)
      TLAST(JJJ,JJ)=TLAST(JJJ,JJ)-OBNO*OBRATE(JJJ,JJ)
   23 CONTINUE
      WRITE OUTPUT TAPE NUTS,700,JJ,OUTPUT
  700 FORMAT(16H CELESTIAL BODY ,I2,
     1/4H RNGE15.8,5H  RGRE15.8,
     25H   RAE15.8,5H  RARE15.8,5H  DECE15.8,5H  DCRE15.8)
      DO 20 I=1,6
   20 OUTPUT(I)=0.
   24 CONTINUE
      RETURN
      END
```

Subroutine: ORTC

Purpose: To convert Cartesian coordinates to conic elements for the output subroutines and to provide other auxiliary results used in computations in the output subroutine.

Calling Sequence:

CALL ORTC (X, DX, U, SMA, ECC, RCA, OINC, OMG, BEP, B, B2, R2, V2, A, C3, BET, THT)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	X	3	$\vec{R}$	Km	Position vector
I	VX	3	$\vec{V}$	Km/sec	Velocity vector
I	U	1	μ	Km^3/sec^2	Central body gravitational
					constant
O	SMA	1	a	Km	Semi-Major Axis
O	ECC	1	e		Eccentricity
O	RCA	1	V_p	Km	Radius of closest approach
O	OINC	1	i	RAD	Orbital inclination

(Cont'd.)

Common storages used or required: ________

Subroutines required: ________

Functions required: ________

Approximate number of storages required: ________

(Cont'd.)

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
0	OMG	1	Ω	RAD	Argument of ascending mode
0	BEP	1	ω	RAD	
0	B	3	$\vec{R} \times \vec{V}$		Quantities used in output
0	B2	1	$\lvert\vec{R} \times \vec{V}\rvert^2$		subroutine computations
0	R2	1	$\lvert\vec{R}\rvert^2$	Km^2	
0	V2	1	$\lvert\vec{V}\rvert^2$	$(Km/sec)^2$	
0	A	1	$\vec{R} \cdot \vec{V}$		
0	C3	1	$\upsilon^2 - \frac{2\mu}{R}$	$(Km/sec)^2$	
0	BET	1	η	RAD	
0	THT	1	θ	RAD	

The orbital elements determined by the subroutine are the following:

$$C3 = \upsilon^2 - \frac{2\mu}{R} \qquad \vec{R} \times \vec{\upsilon} = \vec{h}$$

Semi-Major Axis

$$a = SMA = \frac{\mu}{|C3|}$$

Eccentricity

$$e = ECC \sqrt{1 + \frac{C3\ |\vec{R} \times \vec{\upsilon}|^2}{\mu^2}} = \sqrt{1 + \frac{C_3\ |\vec{h}|^2}{\mu^2}}$$

Radius of Closest Approach

$$V_p = RCA = \frac{|\vec{R} \times \vec{U}|^2}{\mu(1 + e)} = \frac{|\vec{h}|^2}{\mu(1 + e)}$$

Orbital Inclination

$$i = OINC = Tan^{-1} \frac{\sqrt{h^2(1) + h^2(2)}}{h(3)}$$

Argument of the Ascending mode

$$\Omega = OMG = Tan^{-1} \frac{h(1)}{-h(2)}$$

True Anomaly

$$\theta = THT = Tan^{-1} \frac{(\vec{X} \cdot \vec{U}) \; |\vec{h}|}{|h|^2 - \mu R}$$

Argument of Periapsis plus True Anomaly

$$n = BET = \frac{X(3) \; |\vec{h}|}{X(2) \; h(1) - X(1) \; h(2)}$$

Argument of Periapsis

$$\omega = BEP = BET - THT$$

```
*      LABEL
*      SYMBOL TABLE
       SUBROUTINE ORTC(X,DX,U,SMA,ECC,RCA,OINC,OMG,BEP
      1,B,B2,R2,V2,A,C3,BET,THT)
       DIMENSION X(3),DX(3),B(3)
       CALL CROSS(X,DX,B)
       R2  = DOT(X,X)
       V2  = DOT(DX,DX)
       B2  = DOT(B,B)
       A   = DOT(X,DX)
       BB  = SQRTF(B2)
       R   = SQRTF(R2)
       C3  = V2-2.*U/R
       SMA= U/ABSF(C3)
       ECC= SQRTF(1.+C3*B2/U**2)
       OINC= ARKTNS(180,B(3),SQRTF(B(1)**2+B(2)**2))
       OMG= ARKTNS(360,-B(2),B(1))
       RCA= B2/(U*(1.+ECC))
       P   = B2/U
       THT= ARKTNS(360,U*(P-R),BB*A)
       BET= ARKTNS(360,X(2)*B(1)-X(1)*B(2),X(3)*BB)
       BEP= BET-THT
       IF(BEP) 2,3,3
     2 BEP=BEP+6.2831853
     3 CONTINUE
       RETURN
       END
```

Subroutine: OUTC

Purpose: OUTC writes out the trajectory elements in a coordinate system and reference system specified by an input quantity (KOUT). OUTC also writes out the orbital parameters found in ORTC, and writes out the selenographic latitude and longitude if the system is moon-centered.

Calling Sequence:

CALL OUTC

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
KOUT	IC(7)				0 for output in date
					1 for output in 1950
					2 for output in ecliptic coord.

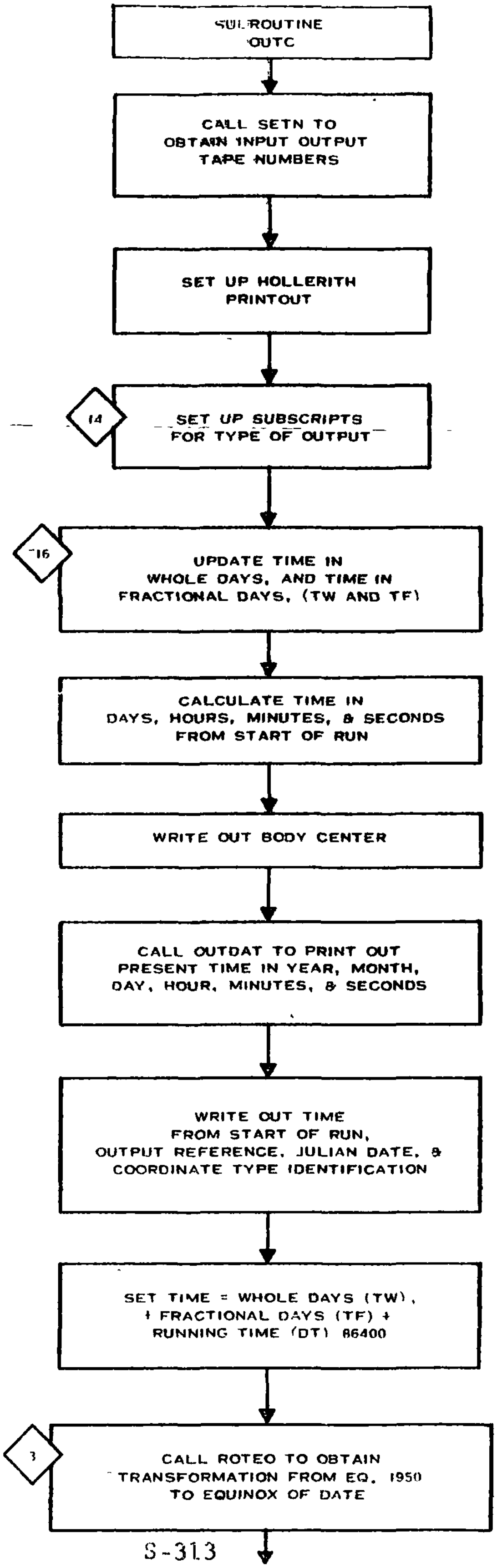
Common storages used or required: T, S, C, IC

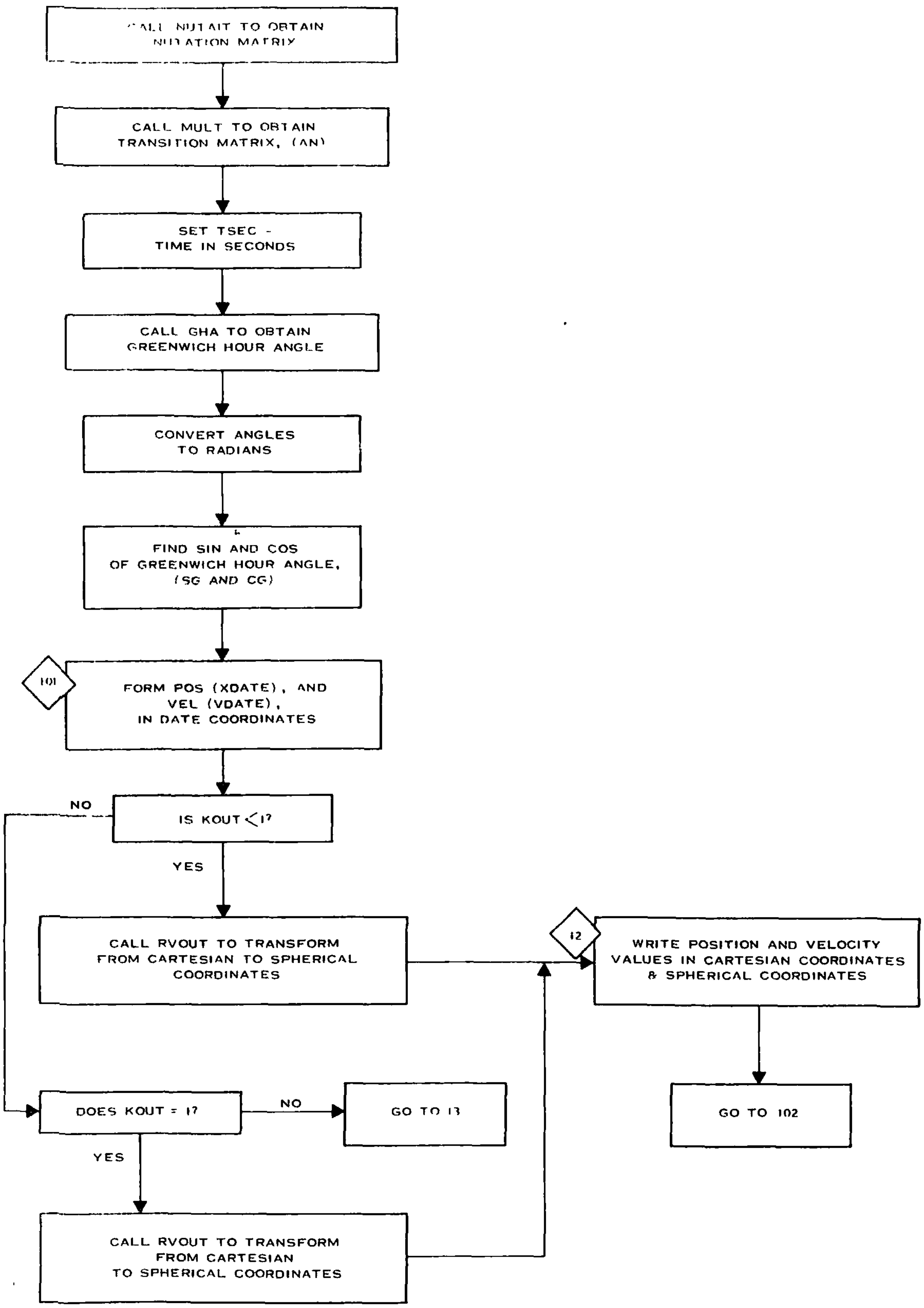
Subroutines required: see next page

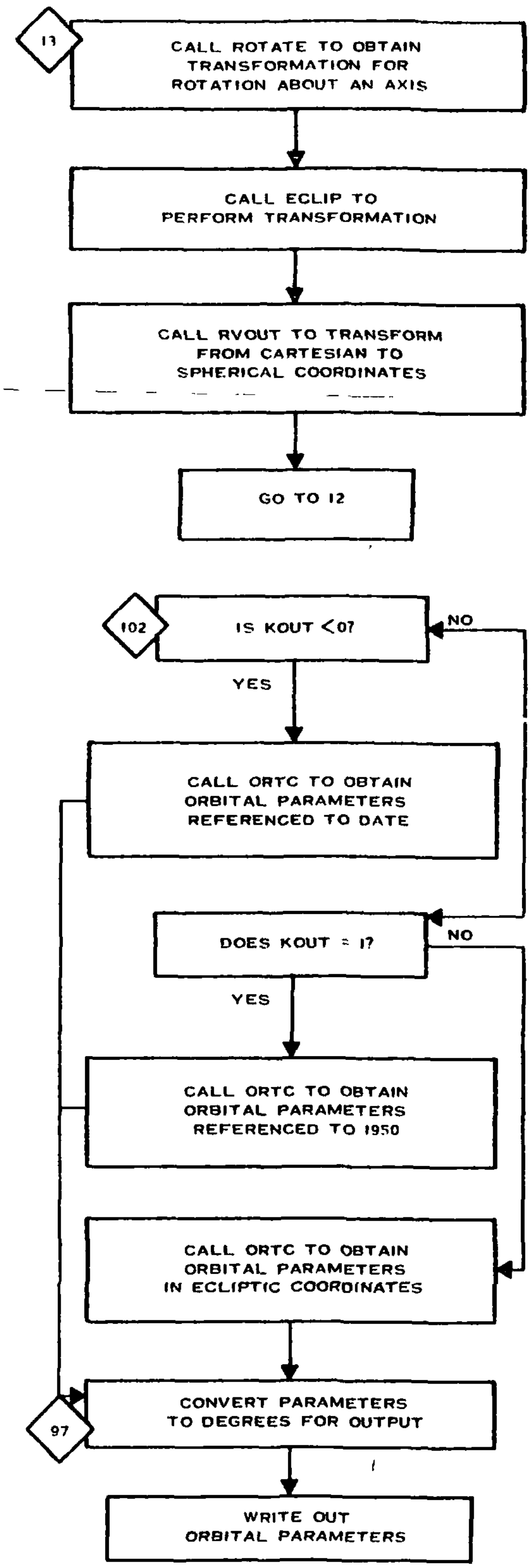
Functions required: see next page

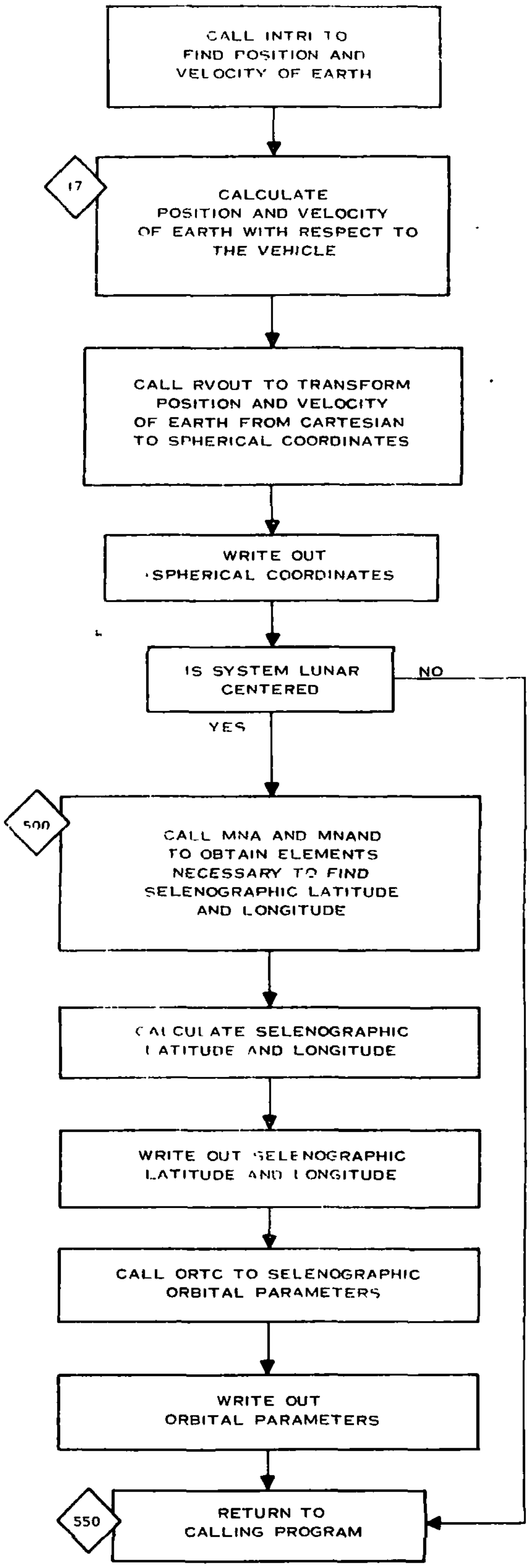
Approximate number of storages required: decimal 940; octal 1654

Subroutines Required	Functions Required
ECLIP	ARKTNS
GHA	ATAN
INTRI	COS
MNAND	SIN
NMA	SQRT
MULT	
NUTAIT	
ORTC	
OUTDAT	
ROTATE	
ROTEQ	
RVOUT	
SETN	


SUBROUTINE
OUTC
CALL SETN TO
OBTAIN INPUT OUTPUT
TAPE NUMBERS
SET UP HOLLERITH
PRINTOUT
14
SET UP SUBSCRIPTS
FOR TYPE OF OUTPUT
16
UPDATE TIME IN
WHOLE DAYS, AND TIME IN
FRACTIONAL DAYS, (TW AND TF)
CALCULATE TIME IN
DAYS, HOURS, MINUTES, & SECONDS
FROM START OF RUN
WRITE OUT BODY CENTER
CALL OUTDAT TO PRINT OUT
PRESENT TIME IN YEAR, MONTH,
DAY, HOUR, MINUTES, & SECONDS
WRITE OUT TIME
FROM START OF RUN,
OUTPUT REFERENCE, JULIAN DATE, &
COORDINATE TYPE IDENTIFICATION
SET TIME = WHOLE DAYS (TW),
+ FRACTIONAL DAYS (TF) +
RUNNING TIME (DT) 86400
3
CALL ROTEO TO OBTAIN
TRANSFORMATION FROM EQ. 1950
TO EQUINOX OF DATE


CALL NUTAIT TO OBTAIN NUTATION MATRIX
CALL MULT TO OBTAIN TRANSITION MATRIX, (AN)
SET TSEC - TIME IN SECONDS
CALL GHA TO OBTAIN GREENWICH HOUR ANGLE
CONVERT ANGLES TO RADIANS
FIND SIN AND COS OF GREENWICH HOUR ANGLE, (SG AND CG)
101
FORM POS (XDATE), AND VEL (VDATE), IN DATE COORDINATES
NO
IS KOUT < 1?
YES
CALL RVOUT TO TRANSFORM FROM CARTESIAN TO SPHERICAL COORDINATES
12
WRITE POSITION AND VELOCITY VALUES IN CARTESIAN COORDINATES & SPHERICAL COORDINATES
DOES KOUT = 1?
NO
GO TO 13
GO TO 102
YES
CALL RVOUT TO TRANSFORM FROM CARTESIAN TO SPHERICAL COORDINATES


13
CALL ROTATE TO OBTAIN TRANSFORMATION FOR ROTATION ABOUT AN AXIS
CALL ECLIP TO PERFORM TRANSFORMATION
CALL RVOUT TO TRANSFORM FROM CARTESIAN TO SPHERICAL COORDINATES
GO TO 12
102
IS KOUT <07
NO
YES
CALL ORTC TO OBTAIN ORBITAL PARAMETERS REFERENCED TO DATE
DOES KOUT = 1?
NO
YES
CALL ORTC TO OBTAIN ORBITAL PARAMETERS REFERENCED TO 1950
CALL ORTC TO OBTAIN ORBITAL PARAMETERS IN ECLIPTIC COORDINATES
CONVERT PARAMETERS TO DEGREES FOR OUTPUT
97
WRITE OUT ORBITAL PARAMETERS


CALL INTRI TO FIND POSITION AND VELOCITY OF EARTH
17
CALCULATE POSITION AND VELOCITY OF EARTH WITH RESPECT TO THE VEHICLE
CALL RVOUT TO TRANSFORM POSITION AND VELOCITY OF EARTH FROM CARTESIAN TO SPHERICAL COORDINATES
WRITE OUT SPHERICAL COORDINATES
IS SYSTEM LUNAR CENTERED
NO
YES
500
CALL MNA AND MNAND TO OBTAIN ELEMENTS NECESSARY TO FIND SELENOGRAPHIC LATITUDE AND LONGITUDE
CALCULATE SELENOGRAPHIC LATITUDE AND LONGITUDE
WRITE OUT SELENOGRAPHIC LATITUDE AND LONGITUDE
CALL ORTC TO SELENOGRAPHIC ORBITAL PARAMETERS
WRITE OUT ORBITAL PARAMETERS
550
RETURN TO CALLING PROGRAM

```
*       LABEL
*       SYMBOL TABLE
CEC2015  SUBROUTINE OUTC
        SUBROUTINE OUTC
        COMMON T,S,C,IC
        EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM),(IC,ICDUM)
        EQUIVALENCE (T(2),DT),(C(15),XP),(C(18),VXP),(C(129),EA)
       1,(C(120),EN),(C(138),AN),(C(62),PO),(C(84),VE),(C(10),TW)
       2,(C(11),TF),(IC(3),NOR),(IC(6),ITARG),(IC(7),KOUT),(C(55),U)
       3,(S(73),RTD),(C(180),SMA),(C(181),ECC),(C(182),RCA),(C(183),OINC)
       4,(C(184),OMG),(C(185),BEP),(C(186),XED),(C(189),VED)
       5,(C(156),ECX),(C(159),ECV),(C(147),ECL),(C(30),T2)
       6,(C(192),RCV),(C(195),RCV2),(C(196),R2),(C(197),V2),(C(198),RDV)
       7,(C(199),C3),(C(162),BET),(C(163),THT)
        DIMENSION T(1360),S(1000),C(1000),IC(1)
        DIMENSION XP(3),VXP(3),EA(3,3),EN(3,3),B(3,3),AN(3,3),
       1 BAN(3,3),PO(22),VE(22),XDATE(3),VDATE(3),XEARTH(3),VEARTH(3),
       2SPDATE(6),ECX(3),ECV(3),CNAME(6),VEV(3)
        DIMENSION XED(3),VED(3),PEV(3),RCV(3),RCVM(3)
        DIMENSION ANAME(2)
        DIMENSION BNAME(4)
        DIMENSION ECL(3,3)
        DIMENSION POS(3),VES(3),POM(3),VEM(3),POT(3),VET(3)
        CALL SETN(NIN,NOUT)
        ANAME(1) = 6H  DATE
        ANAME(2) = 6H1950.0
        BNAME(1) = 6H    EC
        BNAME(2) = 6HLIPTIC
        BNAME(3) = 6H  EQUA
        BNAME(4) = 6HTORIAL
        CNAME(1)=6H EARTH
        CNAME(2)=6H  MOON
        CNAME(3)=6H   SUN
        CNAME(4)=6H VENUS
        CNAME(5)=6H  MARS
        CNAME(6)=6HJUPITR
  202   CONTINUE
        KO=KOUT
        NOR = NOR
        IF (KOUT-1) 14,14,15
   14   KT = 3
        GO TO 16
   15   KT = 1
        KO=0
   16   CONTINUE
        TP2 = TF+DT/86400.
        TWGN=0.
   32   CONTINUE
        IF(TP2) 33,34,34
   33   CONTINUE
        TP2=TP2+1.
```

```
      TWGN=TWGN-1.
      GO TO 32
   34 CONTINUE
      TFP = INTF(TP2)
      TP2 = TP2-TFP
      TP1=TW+TFP+TWGN
      IT1=T2/86400.
      DT1=IT1
      DT1=T2-DT1*86400.
      IT2=DT1/3600.
      DT2=IT2
      DT2=DT1-DT2*3600.
      IT3=DT2/60.
      DT3=IT3
      DT3=DT2-DT3*60.
      TT = INTF(TP2+.5)
      TP2P = TP2+.5-TT
      TP1P = TP1 + TT + 2433282.
      WRITE OUTPUT TAPE NOUT,702,CNAME(NOR)
  702 FORMAT (//1H0A6,10H  CENTERED)
      CALL OUTDAT(TP1,TP2)
      WRITE OUTPUT TAPE NOUT,2,IT1,IT2,IT3,DT3,ANAME(KO  +1),TP1P,TP2P
     1 ,BNAME(KT),BNAME(KT+1)
    2 FORMAT( I4,5H DAYSI4,5H HRS,I4,5H MIN,F7.2,5H SEC.4XA6,3X12HJULIAN
     1 DATE F8.0,F8.8,10X2A6,12H COORDINATES)
      N = NOR
      TIME = TW+TF+DT/86400.
      DIS = 1.E10
      NR1 = N-1
      NV = 1
    3 CALL ROTEQ(TIME,EA)
      CALL NUTAIT(TIME,WM,CR,DA,EN,EPSIL)
      CALL MULT(EN,EA,AN,0)
      TSEC = 86400.*TP2
      CALL GHA(TSEC,TP1,GHAN,EN(2,1),OMEGA)
      GHAN = GHAN*.017453296
      OMEGA = OMEGA*.017453296
      SG = SINF(GHAN)
      CG = COSF(GHAN)
      DO 1 I=1,3
      XDATE(I) = 0.
      VDATE(I) = 0.
      DO 101 J=1,3
      XDATE(I) = XDATE(I) + AN(I,J)*XP(J)
      VDATE(I) = VDATE(I) + AN(I,J)*VXP(J)
  101 CONTINUE
      XED(I)=XDATE(I)
      VED(I)=VDATE(I)
    1 CONTINUE
      IF (KOUT-1) 11,10,13
   10 CALL RVOUT(XP,VXP,SPDATE)
```

```
      WRITE OUTPUT TAPE NOUT,5,XP,VXP,SPDAT
      GO TO 102
   11 CALL RVOUT(XDATE,VDATE,SPDATE)
   12 WRITE OUTPUT TAPE NOUT,5,XDATE,VDATE,SPDATE
      GO TO 102
   13 CALL ROTATE(1,EPSIL,ECL,1)
      CALL ECLIP(XDATE,VDATE,ECL)
      CALL RVOUT(XDATE,VDATE,SPDATE)
      DO 103 I=1,3
      ECX(I)=XDATE(I)
  103 ECV(I)=VDATE(I)
      GO TO 12
  102 CONTINUE
    5 FORMAT(4H   XE15.8,5H    YE15.8,5H    ZE15.8,5H   DXE15.8,
     1 5H   DYE15.8,5H   DZE15.8/4H   RE15.8,5H DECE15.8,5H   RAE15.8,
     2 5H    VE15.8,5H  PTHE15.8,5H   AZE15.8)
      IF (KOUT-1) 105,106,107
  105 CALL ORTC(XED,VED,U,SMA,ECC,RCA,OINC,OMG,BEP
     1,RCV,RCV2,R2,V2,RDV,C3,BET,THT)
      GO TO 97
  106 CALL ORTC(XP,VXP,U, SMA,ECC,RCA,OINC,OMG,BEP
     1,RCV,RCV2,R2,V2,RDV,C3,BET,THT)
      GO TO 97
  107 CALL ORTC(ECX,ECV,U,SMA,ECC,RCA,OINC,OMG,BEP
     1,RCV,RCV2,R2,V2,RDV,C3,BET,THT)
   97 CONTINUE
      ORIN =OINC*RTD
      OMEG =OMG*RTD
      APF  =BEP*RTD
      WRITE OUTPUT TAPE NOUT,4,SMA,ECC,ORIN,OMEG,APF,RCA
      CALL INTR1(TP1,TP2,NR1,PO,NV,VE,DIS)
      DO 17 I=1,3
      I4=23-I
      PEV(I)=PO(I4)-XP(I)
      VEV(I)=VXP(I)-VE(I4)
   17 CONTINUE
      DO 108 I=1,3
      XDATE(I)=0.
      VDATE(I)=0.
      DO 108 J=1,3
      XDATE(I)=XDATE(I)-AN(I,J)*PEV(J)
  108 VDATE(I)=VDATE(I)+AN(I,J)*VEV(J)
   22 XEARTH(3) = XDATE(3)
      VEARTH(3) = VDATE(3)
      XEARTH(1) = XDATE(1)*CG + XDATE(2)*SG
      XEARTH(2) = -XDATE(1)*SG + XDATE(2)*CG
      VEARTH(1)=VDATE(1)*CG+VDATE(2)*SG
      VEARTH(2)=-VDATE(1)*SG+VDATE(2)*CG
      CALL RVOUT(XEARTH,VEARTH,SPDATE)
      WRITE OUTPUT TAPE NOUT,31,SPDATE
   31 FORMAT(4H RTEE15.8,5H  LATE15.8,
```

```
     3 5H   LONE15.8,5H    VEE15.8,5H   PTEE15.8,5H   AZEE15.8)
  29  CONTINUE
      IF(N-2)550,500,550
 500  CALL MNA(TIME,WM,CR,DA,EPSIL,RR,G,GP,WW,BAN)
      DO 510 I=1,3
      XDATE(I)=0.
      VDATE(I)=0.
      DO 510 J=1,3
      XDATE(I)=XDATE(I)+BAN(I,J)*XED(J)
 510  VDATE(I)=VDATE(I)+BAN(I,J)*VED(J)
      SLAT=XDATE(3)/SQRTF(XDATE(1)**2+XDATE(2)**2)
      SLAT=ATANF(SLAT)*RTD
      SLNG=ARKTNS(360,XDATE(1),XDATE(2))*RTD
      WRITE OUTPUT TAPE NOUT,701,SLNG,SLAT
 701  FORMAT(19H SELENOGRAPHIC LON=,E17.8,5H LAT=,E17.8)
      CALL ORTC(XDATE,VDATE,U,AMS,EPS,RP,ORIN,OMEG,APF
     1,RCVM,RCVM2,R2M,V2M,RDVM,C3M,BETM,THTM)
      OMEG = OMEG*RTD
      ORIN = ORIN*RTD
C     GIVES CALENDER DATE FROM T1(WHOLE DAYS FROM 1950)
C     ANDT2(FRACT OF DAY)
      APF = APF*RTD
      WRITE OUTPUT TAPE NOUT,4,AMS,EPS,ORIN,OMEG,APF,RP
    4 FORMAT (4H SMAE15.8,5H  ECCE15.8,5H  INCE15.8,5H  LANE15.8,
     1 5H  APFE15.8,5H  RCAE15.8)
  550 RETURN
      END
```

Subroutine: OUTDAT

Purpose: OUTDAT finds and outputs the calendar date and Julian date, given the number of days since 1950.0.

Calling Sequence:

CALL OUTDAT (T1, T2)

Input and Output

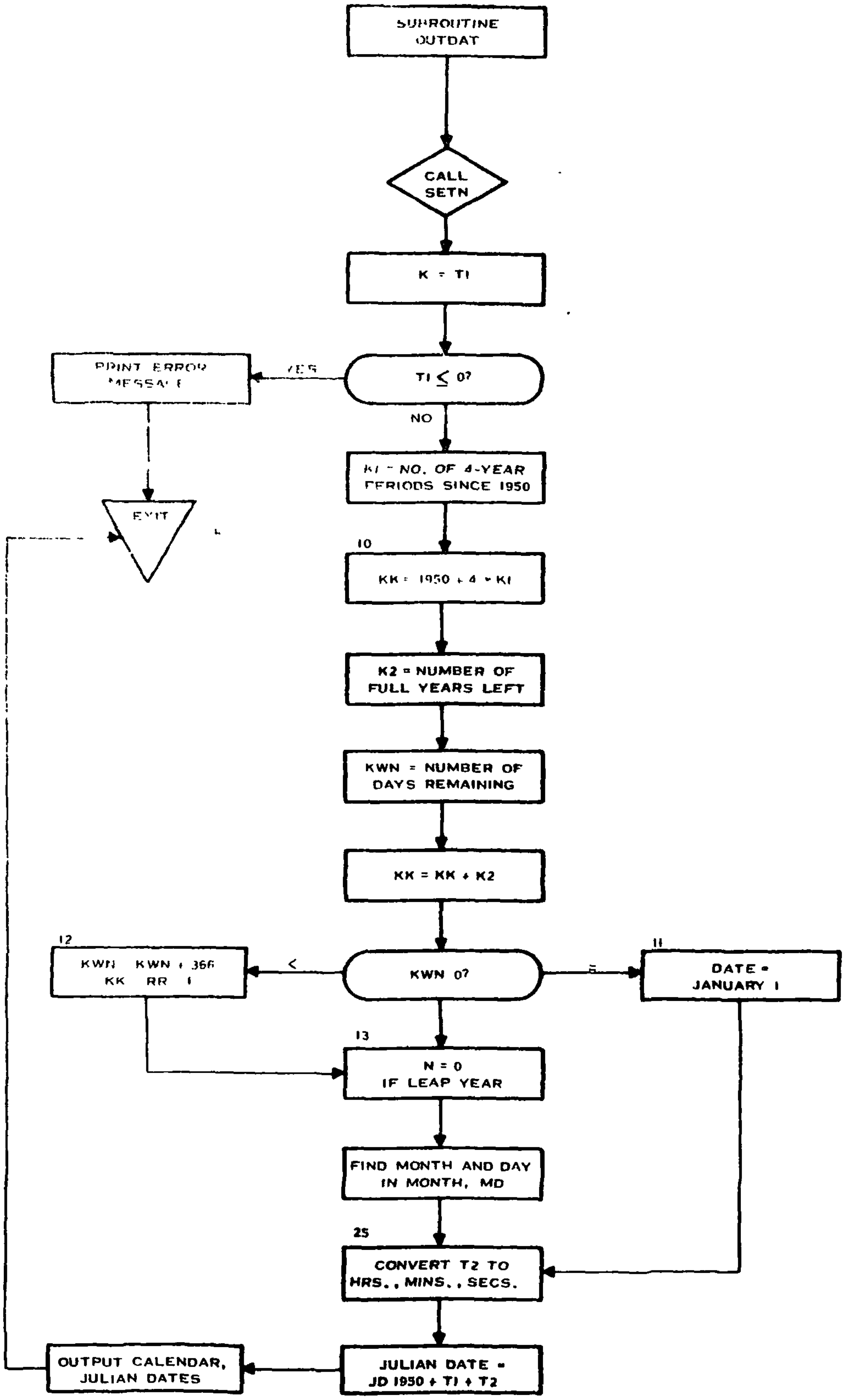
I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	T1	1		days	Integral number of days from
					1950.0.
I	T2	1		days	Fraction of day past T1

Common storages used or required: None

Subroutines required: SETN

Functions required: XMODF, INTF

Approximate number of storages required: 351 DEC


SUBROUTINE OUTDAT
CALL SETN
K = TI
PRINT ERROR MESSAGE
YES
TI ≤ 0?
NO
KI = NO. OF 4-YEAR PERIODS SINCE 1950
EXIT
10
KK = 1950 + 4 * KI
K2 = NUMBER OF FULL YEARS LEFT
KWN = NUMBER OF DAYS REMAINING
KK = KK + K2
12
KWN = KWN + 366
KK = KK - 1
<
KWN 0?
=
11
DATE = JANUARY 1
13
N = 0 IF LEAP YEAR
FIND MONTH AND DAY IN MONTH, MD
25
CONVERT T2 TO HRS., MINS., SECS.
OUTPUT CALENDAR, JULIAN DATES
JULIAN DATE = JD 1950 + TI + T2

```
*       LABEL
*       SYMBOL TABLE
CEC20DT
        SUBROUTINE OUTDAT (T1,T2)
C       GIVES CALENDER DATE FROM T1(WHOLE DAYS FROM 1950)
C       ANDT2(FRACT OF DAY)
        CALL SETN(NIN,NOUT)
        MD=0
        K=T1
        IF(T1)6,6,7
  6     WRITE OUTPUT TAPE NOUT,100
  8     RETURN
 100    FORMAT(23H DATE IS 1950 OR BEFORE)
  7     K1=K/1461
 10     KK=K1*4+1950
        K1=XMODF(K,1461)
        K2=K1/365
        KWN=XMODF(K1,365)-K2/3
        KK=KK+K2
        IF(KWN)12,11,13
 11     MONTH=1
        MD=1
        GO TO 25
 12     KWN=KWN+366
        KK=KK-1
 13     N=XMODF(KK,4)
        KNW=0
        JJ=0
 135    JJ=JJ+1
        IF(12-JJ)23,23,137
C       J,F,M,A,M,J,J,A,S,O,N,MONTHS FOR GO TO
 137    GO TO (14,15,14,16,14,16,14,14,16,14,16,14),JJ
 14     CONTINUE
        KNW=KNW+31
        GO TO 17
 15     CONTINUE
        IF(N)18,18,19
 18     KNW=KNW+1
 19     KNW=KNW+28
        GO TO 17
 16     KNW=KNW+30
 17     CONTINUE
        IF(KWN-KNW)20,21,22
 22     CONTINUE
        MD=KNW
        GO TO 135
     20 CONTINUE
        MONTH=JJ
        MD=KWN-MD+1
        GO TO 25
```

```
 21 CONTINUE
      MONTH=JJ+1
     MD=1
     GO TO 25
23   MONTH =12
     MD=KWN-MD+1
 25 CONTINUE
     TH=T2*24.
      THP=INTF(TH)
     NHOUR=THP
      THP=(TH-THP)*60.
     TH=INTF(THP)
      NMIN=TH
     THP=(THP-TH)*60.
     NSEC=THP
     TSEC=NSEC
     THP=(THP-TSEC)*1000.
     NFSEC=THP
     TT=INTF(T2+.5)
     TP2=T2+.5-TT
     TP1=T1+TT+2433282.
     WRITE OUTPUT TAPE NOUT,101,KK,MONTH,MD,NHOUR,NMIN,NSEC,NFSEC
    1,TP1,TP2
101 FORMAT(6H YEAR=,I4,8H  MONTH=,I2, 6H  DAY=,I2, 7H  HOUR=,I2,
    1 6H  MIN=,I2,6H  SEC=,I2,1H.,I3,10X,12HJULIAN DATE=,
    2F8.0,F8.8)
     GO TO 8
     END
```

Subroutine: OUTP

Purpose: Obtain the error data in the desired coordinate system for output purposes. The three coordinate systems available are: (1) Equator of Date, (2) Equator of 1950, and (3) Ecliptic. The coordinate system is determined by an input quantity KOUT.

Calling Sequence:

CALL OUTP

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities printed out

Common storages used or required: T, S, C, IC

Subroutines required: CROSS, HPHT, INV3, MATRX, PTRAN

Functions required: COS, FNORM, SIN, SQRT

Approximate number of storages required: 1179 DEC

A. <u>Derivation of Partials Used in Subroutine</u>

In order to determine the RMS error in knowledge of orbital elements, the partials which relate the elements and the vehicle state are required. The derivation of these partials follows. The angular elements are shown below in Fig. 1.

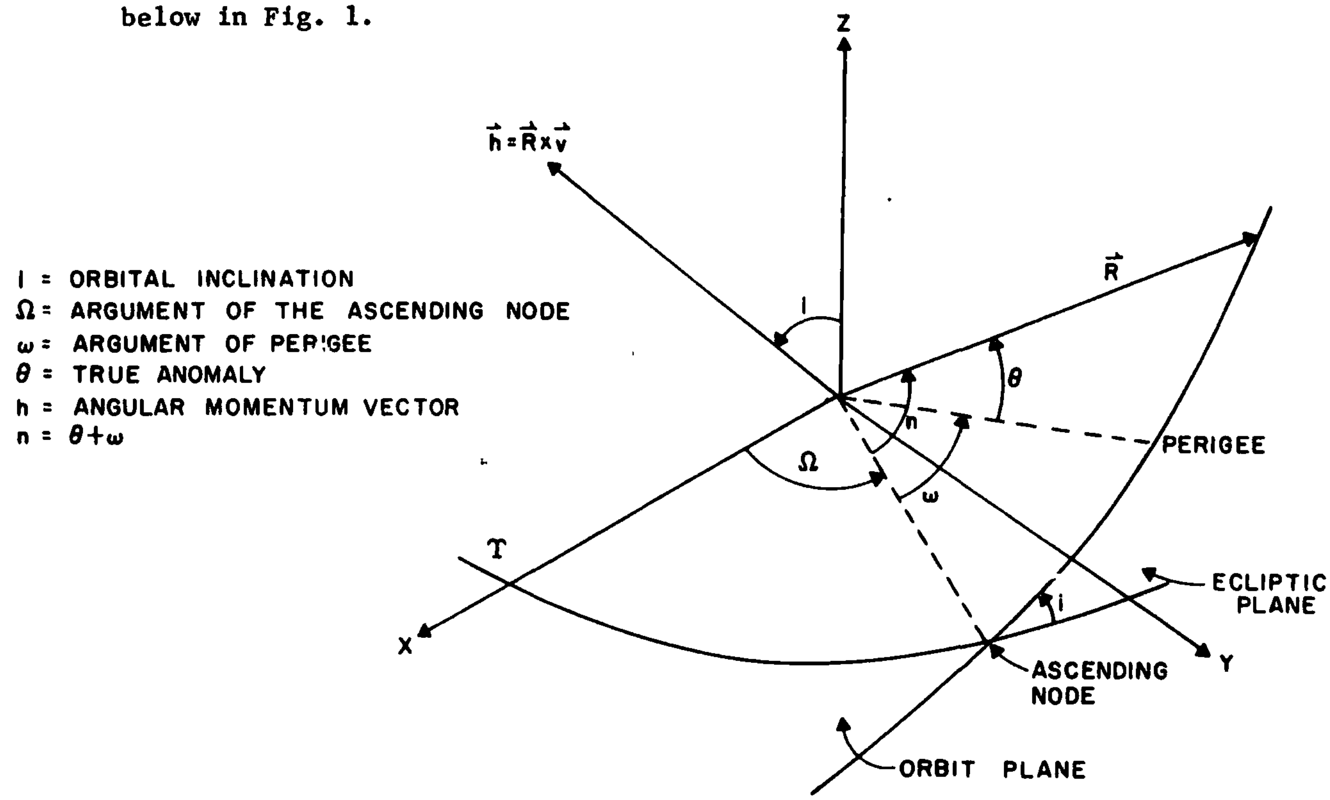


FIG. 1

1. Derivation of partial for eccentricity

$$\epsilon = \sqrt{1 + \frac{h^2 C3}{\mu^2}}$$

$$\nabla \epsilon = \frac{1}{\mu^2 \epsilon} \left\{ C3 \left[v^2 \vec{r} - (\vec{r}\cdot\vec{v}) \vec{v} \right] + h^2 \mu \frac{\vec{r}}{r^3} \right\}$$

$$\text{where } \nabla = \left(\frac{\partial}{\partial x} ; \frac{\partial}{\partial y} ; \frac{\partial}{\partial z} \right)$$

$$C3 = v^2 - \frac{2\mu}{r}$$

$$h^2 = |\vec{r} \times \vec{v}|^2$$

$$D\epsilon = \frac{1}{\mu^2 \epsilon} \left\{ h^2 \vec{v} + C3 \left[r^2 \vec{v} - (\vec{r} \cdot \vec{v}) \vec{r} \right] \right\}$$

where $D = \left(\frac{\partial}{\partial \dot{x}} \; ; \; \frac{\partial}{\partial \dot{y}} \; ; \; \frac{\partial}{\partial \dot{z}} \right)$

2. Derivation of partials for radius of closest approach

$$r_p = \frac{h^2}{\mu(1+\epsilon)}$$

$$\nabla r_p = \frac{1}{\mu\epsilon} \left\{ v^2 \vec{r} - (\vec{r} \cdot \vec{v}) \vec{v} - \mu \, r_p^2 \frac{\vec{r}}{r^3} \right\}$$

$$Dr_p = \frac{1}{\mu\epsilon} \left\{ \left(r^2 - r_p^2 \right) \vec{v} - (\vec{r} \cdot \vec{v}) \vec{r} \right\}$$

3. Derivation of partials for semi-major axis

$$a = - \frac{\mu}{|C3|}$$

$$\frac{\partial a}{\partial X_k} = 2a^2 \frac{X_k}{r^3}$$

$$\frac{\partial a}{\partial \dot{X}_k} = 2a^2 \frac{\dot{X}_k}{\mu}$$

for $k = 1,2,3$

$X_1 = X \quad X_2 = Y \quad X_3 = Z$

$\dot{X}_1 = \dot{X} \quad \dot{X}_2 = \dot{Y} \quad \dot{X}_3 = \dot{Z}$

4. Derivation of partials for longitude of the ascending node

$$\Omega = \text{Tan}^{-1} \frac{h(1)}{-h(2)}$$

$$\frac{\partial \Omega}{\partial x} = - \sin \Omega \frac{\dot{z}}{h \sin i} \qquad \frac{\partial \Omega}{\partial \dot{x}} = \sin \Omega \frac{z}{h \sin i}$$

$$\frac{\partial \Omega}{\partial y} = \cos \Omega \frac{\dot{z}}{h \sin i} \qquad \frac{\partial \Omega}{\partial \dot{y}} = - \cos \Omega \frac{z}{h \sin i}$$

$$\frac{\partial \Omega}{\partial z} = -\cos i \; \frac{\dot{z}}{h \sin i} \qquad \frac{\partial \Omega}{\partial \dot{z}} = \cos i \; \frac{z}{h \sin i}$$

5. Derivation of partials for inclination

$$i = \operatorname{Tan}^{-1} \frac{\sqrt{h(1)^2 + h(2)^2}}{h(3)}$$

$$\frac{\partial i}{\partial X_k} = \frac{\sin i \cos n}{\sin n} \; \frac{\partial \Omega}{\partial X_k} \qquad \text{for } k = 1,2,\dots.6$$

$$X_1 = X \qquad X_2 = Y \qquad X_3 = Z$$

$$X_4 = \dot{X} \qquad X_5 = \dot{Y} \qquad X_6 = \dot{Z}$$

6. Derivation of partials for argument of periapsis

$$\omega = n - \theta = \operatorname{Tan}^{-1} \frac{zh}{yh(1) - xh(2)} - \operatorname{Tan}^{-1} \frac{(\vec{r} \cdot \vec{v})\, h}{h^2 - \mu r}$$

$$\frac{\partial \omega}{\partial X_k} = -\frac{1}{\epsilon^2} (\alpha \cos n + \beta \sin n) \; \frac{\partial n}{\partial X_k} + \frac{1}{h\epsilon^2} \left\{ (\alpha \sin n - \beta \cos n) \right\}$$

$$\left\{ \frac{\partial h}{\partial X_k} - (\alpha + \cos n) \frac{\cos i}{\sin n} \; h \frac{\partial \Omega}{\partial X_k} \right\}$$

for k = 1,2,3

$$\frac{\partial \omega}{\partial \dot{x}} = -\frac{1}{\epsilon^2} (\alpha \cos n + \beta \sin n) \frac{\partial n}{\partial \dot{x}} + \frac{1}{h\epsilon^2} (\alpha \sin n - \beta \cos n) \left\{ \frac{\partial h}{\partial \dot{x}} \right.$$

$$\left. -(\alpha + \cos n) \frac{\cos i}{\sin n} \; h \frac{\partial \Omega}{\partial \dot{x}} \right\} - \frac{h}{\epsilon^2 \mu} \; \alpha \cos \Omega$$

$$\frac{\partial \omega}{\partial \dot{y}} = -\frac{1}{\epsilon^2}(\alpha \cos n + \beta \sin n)\frac{\partial n}{\partial \dot{y}} + \frac{1}{h\epsilon^2}(\alpha \sin n - \beta \cos n)\left\{\frac{\partial h}{\partial \dot{y}} - (\alpha + \cos n)\frac{\cos i}{\sin n}\, h\frac{\partial \Omega}{\partial \dot{y}}\right\} - \frac{1}{\mu\epsilon^2}\alpha \sin \Omega$$

$$\frac{\partial \omega}{\partial \dot{z}} = -\frac{1}{\epsilon^2}(\alpha \cos n + \beta \sin n)\frac{\partial n}{\partial \dot{z}} + \frac{1}{h\epsilon^2}(\alpha \sin n - \beta \cos n)\left\{\frac{\partial h}{\partial \dot{z}} - (\alpha + \cos n)\frac{\cos i}{\sin n}\, h\frac{\partial \Omega}{\partial \dot{z}}\right\} - \frac{h}{\mu\epsilon^2}\frac{\beta}{\sin i}$$

where

$$\alpha = \epsilon \cos \omega \qquad \beta = \epsilon \sin \omega$$

$$\frac{\partial h}{\partial X_k} = \frac{h \sin^2 i \cos n}{\cos i \sin n}\frac{\partial \Omega}{\partial X_k} \qquad k = 1,2,\ldots 6$$

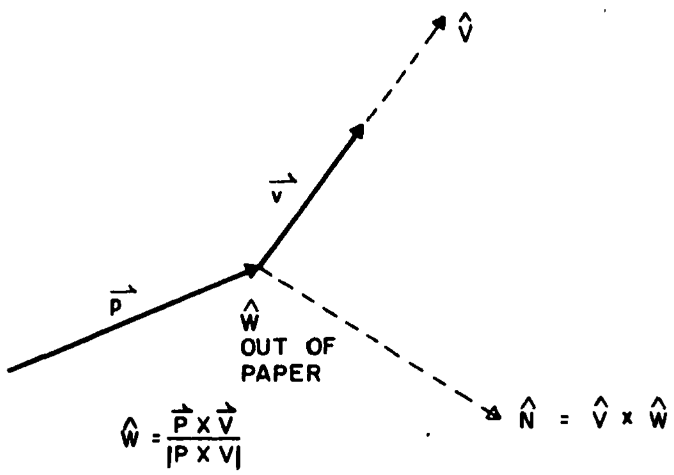
$$\frac{\partial n}{\partial x} = -\cos i\frac{\partial \Omega}{\partial x} - \frac{1}{r}\sin n \cos \Omega \qquad \frac{\partial n}{\partial \dot{x}} = -\cos i\frac{\partial \Omega}{\partial \dot{x}}$$

$$\frac{\partial n}{\partial y} = -\cos i\frac{\partial \Omega}{\partial y} - \frac{1}{r}\sin n \sin \Omega \qquad \frac{\partial n}{\partial \dot{y}} = -\cos i\frac{\partial \Omega}{\partial \dot{y}}$$

$$\frac{\partial n}{\partial z} = -\cos i\frac{\partial \Omega}{\partial z} + \frac{1}{r}\frac{\cos n}{\sin i} \qquad \frac{\partial n}{\partial \dot{z}} = -\cos i\frac{\partial \Omega}{\partial \dot{z}}$$

B. Derivation of Transformation from Cartesian x, y, z to Cartesian N, V, W coordinates

The coordinates N, V, and W represent unit vectors shown in Fig. 2 following.



(PAPER IS ORBIT PLANE)

$\vec{p}$ AND $\vec{v}$ ARE POSITION AND VELOCITY VECTORS

FIG. 2

$\hat{W}$ is in the direction of the angular momentum vector, $\hat{V}$ is along the velocity vector, and $\hat{N}$ forms an orthogonal coordinate system in the sense of $\hat{N} = \hat{V} \times \hat{W}$. The three unit vectors may be written as follows:

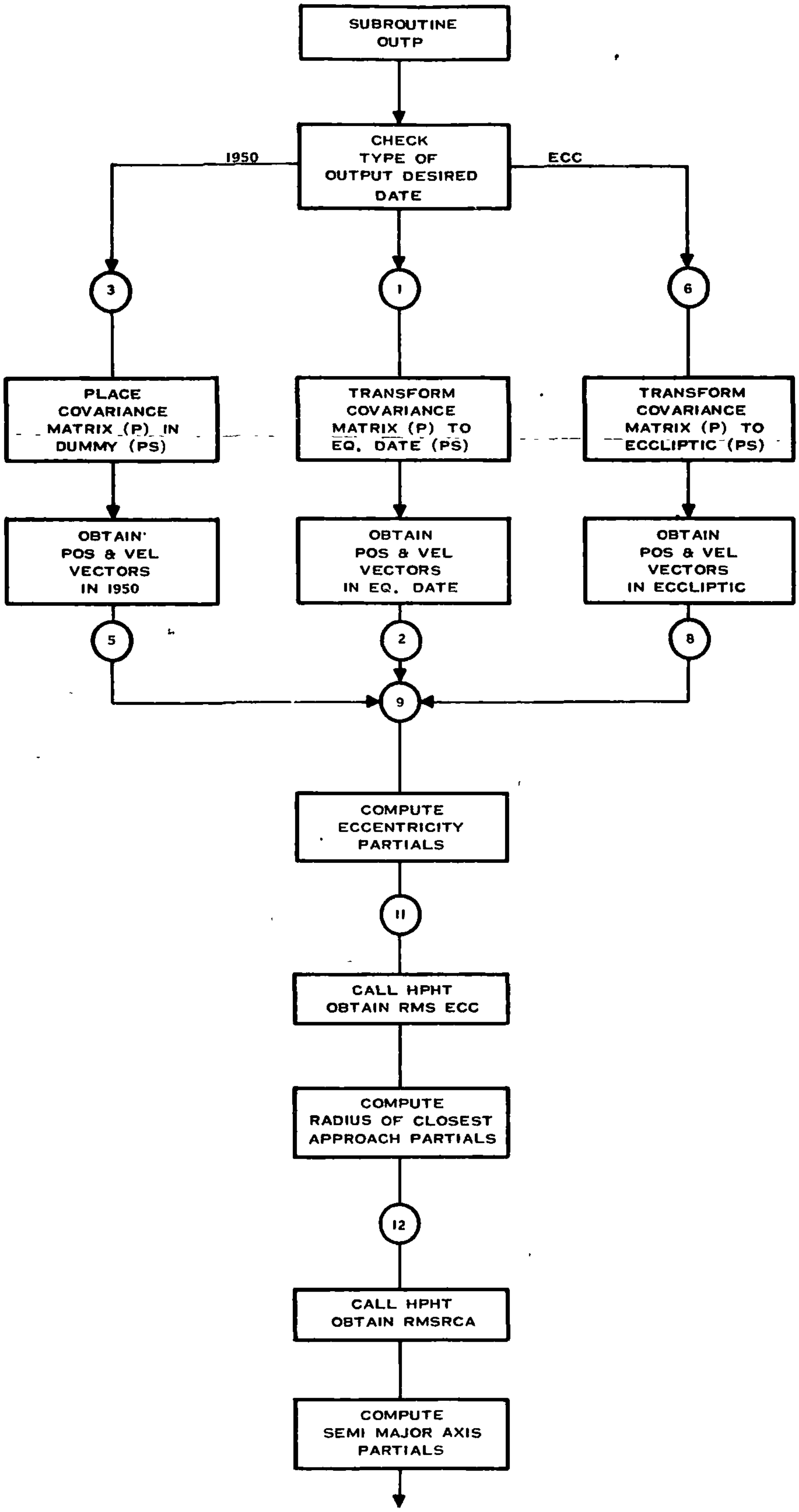
$$\hat{W} = \frac{\vec{R} \times \vec{v}}{|\vec{R} \times \vec{v}|}$$

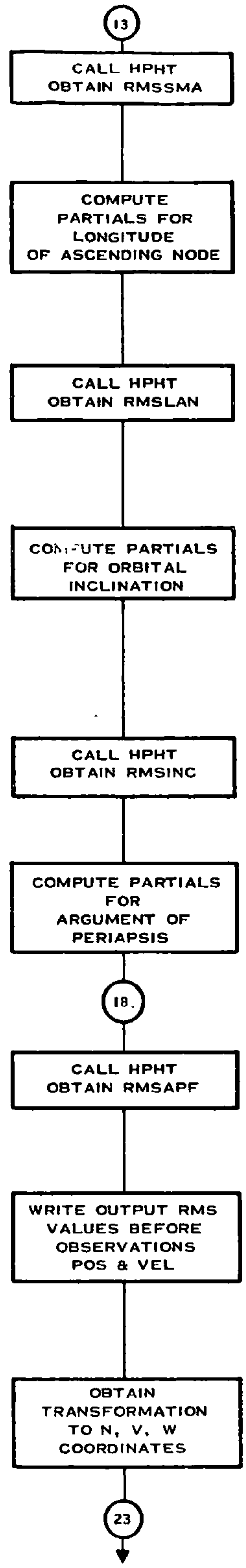
$$\hat{V} = \frac{\vec{v}}{|\vec{v}|}$$

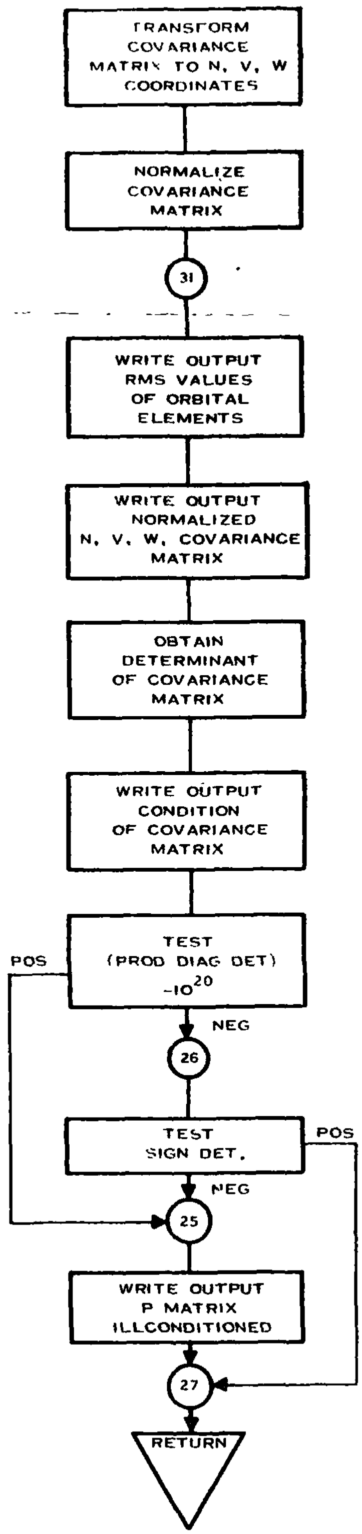
$$\hat{N} = \frac{\vec{v}}{|\vec{v}|} \times \frac{\vec{R} \times \vec{v}}{|\vec{R} \times \vec{v}|}$$

Writing the transposed vectors in matrix form yields the desired transformation from $\hat{i}$, $\hat{j}$, $\hat{k}$ to $\hat{N}$, $\hat{V}$, $\hat{W}$

$$\underset{3\times1}{\begin{pmatrix} N \\ V \\ W \end{pmatrix}} = \underset{3\times3}{\begin{pmatrix} \hat{N}^T \\ \hat{V}^T \\ \hat{W}^T \end{pmatrix}} \underset{3\times1}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}$$


SUBROUTINE OUTP
CHECK TYPE OF OUTPUT DESIRED DATE
1950
ECC
3
1
6
PLACE COVARIANCE MATRIX (P) IN DUMMY (PS)
TRANSFORM COVARIANCE MATRIX (P) TO EQ. DATE (PS)
TRANSFORM COVARIANCE MATRIX (P) TO ECCLIPTIC (PS)
OBTAIN POS & VEL VECTORS IN 1950
OBTAIN POS & VEL VECTORS IN EQ. DATE
OBTAIN POS & VEL VECTORS IN ECCLIPTIC
5
2
8
9
COMPUTE ECCENTRICITY PARTIALS
11
CALL HPHT OBTAIN RMS ECC
COMPUTE RADIUS OF CLOSEST APPROACH PARTIALS
12
CALL HPHT OBTAIN RMSRCA
COMPUTE SEMI MAJOR AXIS PARTIALS


13
CALL HPHT OBTAIN RMSSMA
COMPUTE PARTIALS FOR LONGITUDE OF ASCENDING NODE
CALL HPHT OBTAIN RMSLAN
COMPUTE PARTIALS FOR ORBITAL INCLINATION
CALL HPHT OBTAIN RMSINC
COMPUTE PARTIALS FOR ARGUMENT OF PERIAPSIS
18.
CALL HPHT OBTAIN RMSAPF
WRITE OUTPUT RMS VALUES BEFORE OBSERVATIONS POS & VEL
OBTAIN TRANSFORMATION TO N, V, W COORDINATES
23


TRANSFORM COVARIANCE MATRIX TO N, V, W COORDINATES
NORMALIZE COVARIANCE MATRIX
31
WRITE OUTPUT RMS VALUES OF ORBITAL ELEMENTS
WRITE OUTPUT NORMALIZED N, V, W, COVARIANCE MATRIX
OBTAIN DETERMINANT OF COVARIANCE MATRIX
WRITE OUTPUT CONDITION OF COVARIANCE MATRIX
TEST (PROD DIAG DET) -10^20
POS
NEG
26
TEST SIGN DET.
POS
NEG
25
WRITE OUTPUT P MATRIX ILLCONDITIONED
27
RETURN

```
*     LABEL                                                                  OUTP
CEC2016  SUBROUTINE OUTP                                                     OUTP
      SUBROUTINE OUTP                                                        OUTP0000
      COMMON T,S,C,IC                                                        OUTP0010
      DIMENSION T(1360),S(1000),C(1000),IC(1),P(6,6),AN(3,3),ECL(3,3)        OUTP0020
     1,ECX(3),ECV(3),XED(3),VED(3),XP(3),VXP(3),PS(6,6),X(3),VX(3)           OUTP0030
     2,DUM(3,3),  H (6),DUMC(3,3),DELH2(3),DELC3(3),DELECC(3),DELVH2(3)      OUTP0040
     3,DELVEC(3),DELRP(3),DELVRP(3),DELSMA(3),DELVSA(3),Q(6)                 OUTP0050
      DIMENSION Q1(6),RCV(3),DAG(6)                                          OUTP0060
      EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM),(IC,ICDUM)                      OUTP0070
     1,(C(652),P  ),(C(138),AN),(C(147),ECL),(C(156),ECX),(C(159),ECV)       OUTP0080
     2,(C(180),SMA),(C(181),ECC),(C(182),RCA),(C(183),OINC),(C(184),OMG)OUTP0090
     3,(C(185),APF),(C(186),XED),(C(189),VED),(C(192),RCV),(C(195),RCV2)OUTP0100
     4,(C(196),R2),(C(197),V2) ,(C(198),RDV),(C(199),C3),(C(15),XP)          OUTP0110
     5,(C(18),VXP),(IC(7),KOUT),(C(55) ,U),(C(162),BET),(C(163),THT)         OUTP0120
     6,(S(73),RTD)                                                           OUTP0130
      CALL SETN(NIN,NUTS)                                                    OUTP0138
C     KOUT=TYPE OUTPUT DESIRED  0=EQ OF DATE 1= EQ OF 1950 2=ECCLIPTIC       OUTP0140
      IF(KOUT-1) 1,3,6                                                       OUTP0150
    1 CONTINUE                                                               OUTP0160
      CALL PTRAN(P,AN,PS,2)                                                  OUTP0170
      DO 2 I=1,3                                                             OUTP0180
      X(I)=XED(I)                                                            OUTP0190
    2 VX(I)=VED(I)                                                           OUTP0200
      GO TO 9                                                                OUTP0210
    3 CONTINUE                                                               OUTP0220
      DO 4 I=1,6                                                             OUTP0230
      DO 4 J=1,6                                                             OUTP0240
    4 PS(I,J)=P(I,J)                                                         OUTP0250
      DO 5 I=1,3                                                             OUTP0260
      X(I)=XP(I)                                                             OUTP0270
    5 VX(I)=VXP(I)                                                           OUTP0280
      GO  TO 9                                                               OUTP0290
    6 CONTINUE                                                               OUTP0300
      DO 7 I=1,3                                                             OUTP0310
      DO 7 J=1,3                                                             OUTP0320
      DUM(I,J)=0.                                                            OUTP0330
      DO 7 K=1,3                                                             OUTP0340
    7 DUM(I,J)=DUM(I,J) + ECL(I,K)*AN(K,J)                                   OUTP0350
      CALL PTRAN( P,DUM,PS,2)                                                OUTP0360
      DO 8 I=1,3                                                             OUTP0370
      X(I)=ECX(I)                                                            OUTP0380
    8 VX(I)=ECV(I)                                                           OUTP0390
    9 CONTINUE                                                               OUTP0400
      R=SQRTF(R2)                                                            OUTP0410
      CALL CROSS(VX,RCV,DELH2)                                               OUTP0420
      CALL CROSS(RCV,X,DELVH2)                                               OUTP0430
      DO 10 I=1,3                                                            OUTP0440
   10 DELC3(I)=(U/R2/R)*X(I)                                                 OUTP0450
C     FOLLOWING IS COMPUTATION  OF  ECCENTRICITY PARTIALS                    OUTP0460
      CONST= 1./(ECC*U**2)                                                   OUTP0470
```

```
      DO 11 I=1,3
      II=I+3
      DELECC(I)=CONST*(C3*DELH2(I)+RCV2*DELC3(I))
      DELVEC(I)=CONST*(C3*DELVH2(I)+RCV2*VX(I))
      H(I)=DELECC(I)
      H(II)=DELVEC(I)
   11 CONTINUE
      CALL HPHT(H,PS,RMSECC)
C     FOLLOWING FOR PARTIALS OF RADIUS CLOSEST APPROACH =RCA
      CON=1.+ECC
      CONST=U*CON
      CONS =RCV2/(CONST*CON)
      CON=2./CONST
      DO 12 I=1,3
      II= I+3
      DELRP(I)=CON*DELH2(I)-CONS*DELECC(I)
      DELVRP(I)=CON*DELVH2(I)-CONS*DELVEC(I)
      H(I)=DELRP(I)
      H(II)=DELVRP(I)
   12 CONTINUE
      CALL HPHT(H,PS,RMSRCA)
C     FOLLOWING FOR PARTIALS OF SEMI MAJOR AXIS =SMA
      CONST= 1./( ECC-1.)
      CON= RCA*CONST**2
      DO 13 I=1,3
      II=I+3
      DELSMA(I)=CONST*DELRP(I)-CON*DELECC(I)
      DELVSA(I)=CONST*DELVRP(I)-CON*DELVEC(I)
      H(I)=DELSMA(I)
      H(II)=DELVSA(I)
   13 CONTINUE
      CALL HPHT(H,PS,RMSSMA)
      CI=COSF(OINC)
      SI=SINF(OINC)
      CW=COSF(OMG)
      SW=SINF(OMG)
C     FOLLOWING FOR LINE OF MODES =LAN  PARTIALS
      CON =SQRTF(RCV2)
      CONST=1./(CON*SI)
      CONS =VX(3)*CONST
      G(1) =-SW*CONS
      G(2) =CW *CONS
      G(3) =-CI*CONS/SI
      CONS =X(3)*CONST
      G(4) =SW*CONS
      G(5) =-CW*CONS
      G(6) = CI*CONS/SI
      CALL HPHT(G,PS,RMSLAN)
C     THE FOLLOWING FOR INCLINATION
      CONS=1./CON
      H(1)=CONS*(-VX(2)*SI + VX(3)*CW*CI)
```

```
      H(2)=CONS*(VX(1)*SI   + VX(3)*SW*CI)                                 OUTP1040
      H(3)=-CONS*CI*(VX(1)*CW+VX(2)*SW)                                     OUTP1050
      H(4)=CONS*(X(2)*SI-X(3)*CW*CI)                                        OUTP1060
      H(5)=-CONS*(X(1)*SI+X(3)*SW*CI)                                       OUTP1070
      H(6)= CONS*CI*(X(1)*CW + X(2)*SW)                                     OUTP1080
      CALL HPHT(H,PS,RMSINC)                                                OUTP1090
C    THE FOLLOWING ARE PARTIALS OF ARG. OF PERIAPSIS  = APF                 OUTP1100
      SN =SINF(BET)                                                         OUTP1110
      CN = COSF(BET)                                                        OUTP1120
      DO 17 I=1,6                                                           OUTP1130
      Q1(I) = -CI*Q(I)                                                      OUTP1140
   17 CONTINUE                                                              OUTP1150
      Q1(1) = Q1(1)-SN*CW/R                                                 OUTP1160
      Q1(2) = Q1(2)-SN*SW/R                                                 OUTP1170
      Q1(3) = Q1(3)+CN/(R*SI)                                               OUTP1180
      CP = COSF(APF)                                                        OUTP1190
      SP = SINF(APF)                                                        OUTP1200
      RAGHI = 1./SQRTF(RCV2)                                                OUTP1210
      CON = ECC*CP                                                          OUTP1220
      CONST = ECC*SP                                                        OUTP1230
      CONS = -(CON*CN+CONST*SN)/(ECC**2)                                    OUTP1240
      CONS1 = (CON*SN-CONST*CN)*RAGHI/(ECC**2)                              OUTP1250
      CONS2 = CON+CN                                                        OUTP1260
      CONS2 = CONS2*RAGHI*CI/SN                                             OUTP1270
      DO 18 I=1,3                                                           OUTP1280
      II = I+3                                                              OUTP1290
      H(I) = CONS*Q1(I)+CONS1*(RAGHI*DELH2(I)-CONS2*Q(I))                   OUTP1300
      H(II) = CONS*Q1(II)+CONS1*(RAGHI*DELVH2(I)-CONS2*Q(II))               OUTP1310
   18 CONTINUE                                                              OUTP1320
      CONS = -1./((RAGHI*U)*(ECC**2))                                       OUTP1330
      H(4)= H(4)+CONS*CW*CON                                                OUTP1340
      H(5)= H(5)+CONS*SW*CON                                                OUTP1350
      H(6) = H(6)+CONS*CONST/SI                                             OUTP1360
      CALL HPHT(H,PS,RMSAPF)                                                OUTP1370
      WRITE OUTPUT TAPE NUTS,700                                            OUTP1380
  700 FORMAT( 44X,31H RMS VALUES BEFORE OBSERVATIONS)                       OUTP1390
      DO 21 I=1,6                                                           OUTP1400
   21 H(I)    =SQRTF(PS(I,I))                                               OUTP1410
      WRITE OUTPUT TAPE NUTS,701,H                                          OUTP1420
  701 FORMAT(4H    XE15.8,5H    YE15.8,5H    ZE15.8,5H   DXE15.8,           OUTP1430
     15H   DYE15.8,5H   DZE15.8)                                            OUTP1440
      R50=FNORM(VXP)                                                        OUTP1580
      CALL CROSS(XP,VXP,DUM(1,3))                                           OUTP1590
      R51=FNORM(DUM(1,3))                                                   OUTP1600
      DO 22 I=1,3                                                           OUTP1610
      DUM(I,2)=VXP(I)/R50                                                   OUTP1620
   22 DUM(I,3)= DUM(I,3)/R51                                                OUTP1630
      CALL CROSS(DUM(1,2),DUM(1,3),DUM(1,1))                                OUTP1640
      DO 23 I=1,3                                                           OUTP1650
      DO 23 J=1,3                                                           OUTP1660
   23 DUMC(I,J)=DUM(J,I)                                                    OUTP1670
```

```
      CALL PTRAN(P,DUMC,PS,2)
      DO 30 I=1,6
      DAG(I)=SQRTF(PS(I,I))
 30   CONTINUE
      DO 31 I=1,6
      DO 32 J=I,6
      PS(I,J)=PS(I,J)/DAG(I)/DAG(J)
 32   CONTINUE
      PS(I,I)=1.
 31   CONTINUE
      RMSINC = RMSINC*RTD
      RMSAPF = RMSAPF*RTD
      RMSLAN = RMSLAN*RTD
     0WRITE OUTPUT TAPE NUTS,702,RMSSMA,RMSECC,RMSINC,RMSLAN,RMSAPF,
     1RMSRCA
702   FORMAT(4H SMAE15.8,5H  ECCE15.8,5H  INCE15.8,5H  LANE15.8,
     15H  APFE15.8,5H  RCAE15.8)
      WRITE OUTPUT TAPE NUTS,703,DAG
703   FORMAT(50X,22H RMS N,V,W COORDINATES,/21X,17H RMS POSITION, KM,
     142X,20H RMS VELOCITY KM/SEC,/7X,7H NORMAL, 12X,9H VELOCITY,11X,
     29H MOMENTUM,12X,7H NORVEL,14X,7H VELVEL,11X,9H MOMENVEL,/4H    E15
     3.8,(5E20.8))
      WRITE OUTPUT TAPE NUTS,704,((PS(I,J),J=1,6),I=1,6)
 704 FORMAT(30X,58H NORMALIZED STATE COVARIANCE MATRIX IN  N,V,W  COORD
     1INATES,/4H    E15.8,(5E20.8)/19X,(5E20.8)/39X,(4E20,8)/59X,
     2(3E20.8)/79X,(2E20.8)/99X,E20.8)
      CON = 1.
      CALL INV3(PS,6,DETER)
      CONST = CON/DETER
      WRITE OUTPUT TAPE NUTS,705,DETER,CON,CONST
 705 FORMAT(37H CONDITION OF STATE COVARIANCE MATRIX,/
     124H DETERMINANT OF MATRIX =,E15.8,3X,
     222H PRODUCT OF DIAGONAL =,E15.8,1X,
     324H RATIO PROD.DIAG,/DET. =,E15.8)
      IF (ABSF(CONST)-1.E20) 26,25,25
  26 IF (DETER) 25,25,27
      WRITE OUTPUT TAPE NUTS,706
 706 FORMAT(33H ***P MATRIX IS ILLCONDITIONED***)
  27 CONTINUE
      RETURN
      END
```

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Subroutine: PTRAN

Purpose: To compute the transformation of the state covariance matrix from one inertial coordinate system to another. The transformation which is obtained with KK = 2 is the following:

$$\underset{6x6}{PS} = \underset{6x6}{\begin{pmatrix} B & O \\ O & B \end{pmatrix}} \underset{6x6}{\begin{pmatrix} P \end{pmatrix}} \underset{6x6}{\begin{pmatrix} B^T & O \\ O & B^T \end{pmatrix}}$$

If KK is set 1 only, the diagonal 3x3's are computed.

Calling Sequence:

CALL PTRAN (P, B, PS, KK)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	P	(6,6)			Covariance Matrix
I	B	(3,3)			Transformation Matrix
O	PS	(6,6)			Transformed Matrix
I	KK	1			Logic Key

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

```
*      LABEL                                                            PTRN
*      SYMBOL TABLE                                                     PTRN
       SUBROUTINE PTRAN( P,B,PS,KK)                                     PTRN0000
       DIMENSION P(6,6),B(3,3),PS(6,6),DUM(6,6)                         PTRN0010
C      SUBROUTINE CALCULATES THE 6X6 MATRIX PS  FROM                    PTRN0020
C      P IS SYMMETRIC PS=( B   0 )(   P )( BT   0 )     T=TRANSPOSE     PTRN0030
C          ( 0   B )(      )( 0    BT)    K=1 OBTAINS ONLY DIAG 3X3     PTRN0035
       DO 1  I=1,6                                                      PTRN0040
       DO 1  J=1,6                                                      PTRN0050
       PS(I,J)=0.                                                       PTRN0060
    1  DUM(I,J)=0.                                                      PTRN0070
       DO 2 I=1,3                                                       PTRN0080
       I3=I+3                                                           PTRN0090
       DO 2 J=1,3                                                       PTRN0100
       J3=J+3                                                           PTRN0110
       DO 2 K=1,3                                                       PTRN0120
       K3 = K+3                                                         PTRN0130
       DUM(I,J)=DUM(I,J)+   B(I,K)*P(K,J)                               PTRN0140
       DUM(I3,J3)=DUM(I3,J3)+   B(I,K)*P(K3,J3)                         PTRN0150
       GO TO(2,3), KK                                                   PTRN0160
    3  DUM(I,J3) =DUM(I,J3) +  B(I,K)*P(K,J3)                           PTRN0170
    2  CONTINUE                                                         PTRN0180
       DO 5 I=1,3                                                       PTRN0190
       I3=I+3                                                           PTRN0200
       DO 5 J=1,3                                                       PTRN0210
       J3=J+3                                                           PTRN0220
       DO 4 K=1,3                                                       PTRN0230
       K3=K+3                                                           PTRN0240
       PS(I,J)=PS(I,J)  + DUM(I,K)*B(J,K)                               PTRN0250
       PS(I3,J3)=PS(I3,J3) +DUM(I3,K3)*B(J,K)                           PTRN0260
       GO TO(4,6), KK                                                   PTRN0270
    6  PS(I,J3) =PS(I,J3)  +DUM(I ,K3)*B(J,K)                           PTRN0280
    4  CONTINUE                                                         PTRN0290
       GO TO(5,7),KK                                                    PTRN0300
    7  PS(J3,I)=PS(I,J3)                                                PTRN0310
    5  CONTINUE                                                         PTRN0320
    8   CONTINUE                                                        PTRN0330
       RETURN                                                           PTRN0340
       END                                                              PTRN
```

Subroutine: RETRO

Purpose: To take the P and PAR covariance matricies through a retro maneuver into a circular orbit.

Calling Sequence:

CALL RETRO

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities returned to common

Common storages used or required: T, S, C, IC

Subroutines required: MATRX, SETN

Functions required: DOT, FNORM, SQRT

Approximate number of storages required: 779 DEC

Derivation of Guidance Law Through Retro Maneuver

The vehicle is assumed to be at periapsis at the time the subroutine is called. The maneuver is then carried out such that the vehicle is in circular orbit after retro fire.

The required velocity at periapsis for a circular orbit is:

$$\upsilon_p = \sqrt{\frac{\mu}{R_p}}$$

The required incremental velocity correction along the nominal trajectory is the following

$$\Delta \vec{V} = \left(\frac{\upsilon_p}{\upsilon_n} - 1\right) \vec{\upsilon}_n \qquad \upsilon_n = |\vec{\upsilon}_n| \tag{1}$$

where υ_n is the nominal periapsis velocity vector for the incoming trajectory.

The guidance law which governs the deviations from the nominal is obtained in the following manner. The deviation from nominal periapsis velocity (assuming linear theory) may be written as:

$$\delta \vec{\upsilon}_p = \dot{\underline{X}} + A\,\underline{X} \tag{2}$$

3x1 3x1(3x3)(3x1)

where

$$A = \begin{pmatrix} \frac{\partial \upsilon_p}{\partial X_1} & 0 & 0 \\ 0 & \frac{\partial \upsilon_p}{\partial X_2} & 0 \\ 0 & 0 & \frac{\partial \upsilon_p}{\partial X_3} \end{pmatrix}$$

and $\underline{X}$ and $\dot{\underline{X}}$ are the vehicle deviation state.

The partial derivatives in matrix A are the following:

$$\frac{\partial \upsilon_p}{\partial X_i} = - \frac{\mu X_i}{2 \upsilon_p R^3} \qquad i = 1,2,3$$

The estimate of periapsis velocity deviation from equation (2) is

$$\delta \hat{\dot{\upsilon}}_p = \hat{\dot{\underline{X}}} + A \hat{\underline{X}} \tag{3}$$

and the guidance law is obtained as follows:

$$\delta \hat{\dot{\upsilon}}_p + \dot{\underline{X}}_g = 0 = \hat{\dot{\underline{X}}} + A \hat{\underline{X}} + \dot{\underline{X}}_g$$

or

$$\underset{3x1}{\dot{\underline{X}}_g} = - \underset{3x6}{(A \;\; I)} \underset{6x1}{\begin{pmatrix} \hat{\underline{X}} \\ \hat{\dot{\underline{X}}} \end{pmatrix}} \tag{4}$$

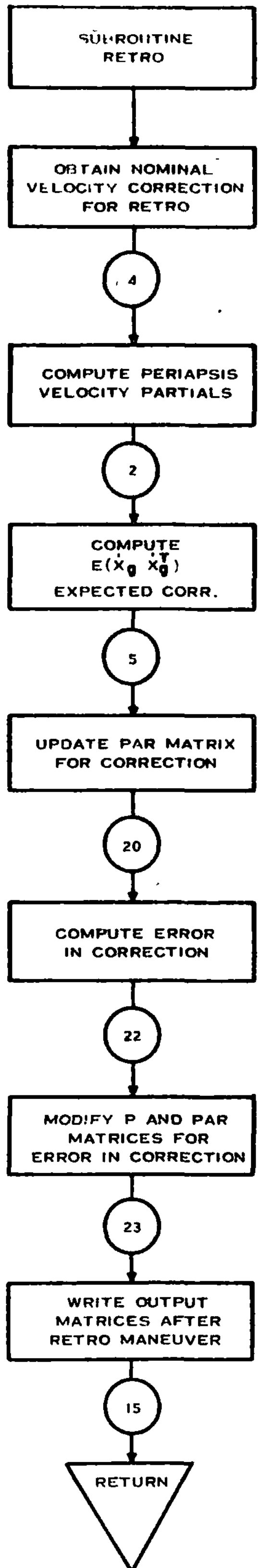
The total guidance correction is the sum of equations (1) and (4).

$$\dot{\underline{X}}_{gt} = - (A \;\; I) \begin{pmatrix} \hat{\underline{X}} \\ \hat{\dot{X}} \end{pmatrix} + \overrightarrow{\Delta V} \tag{5}$$

The guidance law expressed by equation (4) corresponds to equation (6) in the guidance correction derivation in subroutine GUID. The P and PAR covariance matricies are then updated as shown by the derivation in the GUID subroutine writeup. The additional term, $\Delta \vec{V}$, present in the total guidance correction required is treated as being independent of the deviation state. Therefore, the E $(\dot{\underline{X}}_{gt} \; \dot{X}^T_{gt})$ may be written as:

$$E (\dot{\underline{X}}_{gt} \; \dot{\underline{X}}^T_{gt}) = E (\dot{X}_g \; \dot{X}^T_g) + \overrightarrow{\Delta V} \; \overrightarrow{\Delta V}^T$$

$$\text{since } E(\dot{X}_g \; \overrightarrow{\Delta V}^T) = 0$$


SUBROUTINE RETRO
OBTAIN NOMINAL VELOCITY CORRECTION FOR RETRO
4
COMPUTE PERIAPSIS VELOCITY PARTIALS
2
COMPUTE $E(\dot{X}_g \dot{X}_g^T)$ EXPECTED CORR.
5
UPDATE PAR MATRIX FOR CORRECTION
20
COMPUTE ERROR IN CORRECTION
22
MODIFY P AND PAR MATRICES FOR ERROR IN CORRECTION
23
WRITE OUTPUT MATRICES AFTER RETRO MANEUVER
15
RETURN

S-344

```
*       LABEL                                                                  RETR
*       SYMBOL TABLE                                                           RETR
CEC2030  SUBROUTINE RETRO                                                      RETR
        SUBROUTINE RETRO                                                       RETR000
        COMMON T,S,C,IC                                                        RETR001
        DIMENSION T(1360),S(1000),C(1000),IC(1)                                RETR002
       1,TPAR(3,6),PAR(6,6),P(6,6),DUP(3,3)                                    RETR003
       2,TGUID(6),DUM(3,3),DUN(3,3),DUMM(6,6)                                  RETR004
        DIMENSION XP(3),VXP(3),CV(3)                                           RETR005
        EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM)                                 RETR006
       1,(IC,ICDUM),(C(568),TPAR),(C(460),PAR)                                 RETR007
       2,(C(652),P),(IC(218),N),(C(648),TGUIDE)                                RETR008
       3,(IC(219),IGD),(IC(217),IGDTP),(IC(214),NOUT)                          RETR009
       4,(C(30),TSEC),(S(476),TGUID),(S(483),RATIO)                            RETR010
       5,(S(473),QQPT),(S(471),QQSO),(S(484),DMONIT)                           RETR011
        EQUIVALENCE (C(15),XP),(C(18),VXP),(C(55),U)                           RETR012
        CALL SETN(NIN,NUTS)                                                    RETR013
        WRITE OUTPUT TAPE NUTS,701                                             RETR014
  701   FORMAT(94H THE DOLLOWING DATA ARE CONDITIONS AFTER RETRO INTO CIRCRETR015
       1ULAR ORBIT ASSUMING VEHICLE AT PERIGEE)                                 RETR016
        R2=DOT(XP,XP)                                                          RETR017
        R=FNORM(XP)                                                            RETR018
        VM=FNORM(VXP)                                                          RETR019
        VP=SQRTF(U/R)                                                          RETR020
        DELV=VP/VM                                                             RETR021
        DO 1 I=1,3                                                             RETR022
    1   CV(I)=DELV*VXP(I)                                                      RETR023
        WRITE OUTPUT TAPE NUTS,700,CV                                          RETR024
  700   FORMAT(35H CIRCULAR VELOCITY COMPONENTS X,Y,Z/(3E17.8))                RETR025
        DO 4 I=1,3                                                             RETR026
    4   CV(I)=(CV(I)-VXP(I))**2                                                RETR027
        CONS=-U/(2.*VP*R2*R)                                                   RETR028
        DO 2 I=1,3                                                             RETR029
        II=I+3                                                                 RETR030
        DO 3 J=1,3                                                             RETR031
        JJ=J+3                                                                 RETR032
        TPAR(I,J)=0.                                                           RETR033
    3   TPAR(I,JJ)=0.                                                          RETR034
        TPAR(I,I)=CONS*XP(I)                                                   RETR035
    2   TPAR(I,II)=-1.                                                         RETR036
        CALL MATRX(TPAR,P,DUM,3,6,6,1)                                         RETR037
        CALL MATRX(TPAR,PAR,DUN,3,6,6,1)                                       RETR038
        DO 5 I=1,3                                                             RETR039
        DO 6 J=1,3                                                             RETR040
    6   DUM(I,J)=DUN(I,J)-DUM(I,J)                                             RETR041
    5   DUM(I,I)=DUM(I,I)+CV(I)                                                RETR042
        RMSV2=DUM(1,1)+DUM(2,2)+DUM(3,3)                                       RETR043
        RMSV=SQRTF(RMSV2)                                                      RETR044
C       FOLLOWING IS UPDATING OF PAR MATRIX FOR GUIDANCE CORRECTION            RETR045
        DO 17 I=1,6                                                            RETR046
        DO 17 J=I,6                                                            RETR047
```

```
      PAR(I,J)=PAR(I,J)-P(I,J)
      PAR(J,I)=PAR(I,J)
 17   CONTINUE
      DO 19 I=1,3
      II=I+3
      DO 18 J=1,3
      JJ=J+3
      DUMM(II,J)=TPAR(I,J)
      DUMM(I,JJ)=0.
      DUMM(II,JJ)=0.
 18   DUMM(I,J)=0.
 19   DUMM(I,I)=1.
      CALL MATRX(DUMM,PAR,PAR,6,6,6,1)
      DO 20I=1,6
      DO 20 J=I,6
      PAR(I,J)=PAR(I,J)+P(I,J)
      PAR(J,I)=PAR(I,J)
 20   CONTINUE
      CRUD=QQPT*RMSV2
      QQ=QQSO-QQPT
      DO 22 I=1,3
      DO 21 J=I,3
      DUN(I,J)=QQ*DUM(I,J)
      DUN(J,I)=DUN(I,J)
 21   CONTINUE
      DUN(I,I)=DUN(I,I)+CRUD
 22   CONTINUE
      RMSERR=SQRTF(DUN(1,1)+DUN(2,2)+DUN(3,3))
      DO 23 I=1,3
      II=I+3
      DO 23 J=I,3
      JJ=J+3
      PAR(II,JJ)=PAR(II,JJ)+DUN(I,J)
      PAR(JJ,II)=PAR(II,JJ)
      P(II,JJ)=P(II,JJ)+DMONIT*DUN(I,J)
      P(JJ,II)=P(II,JJ)
   23 CONTINUE
      WRITE OUTPUT TAPE NUTS,702,RMSV,RMSERR
 702  FORMAT(24H RETRO GUID DATA FOLLOWS/
     115H   RMS VEL REQ=,E15.8,10X,18H   RMSERR IN CORR=,E15.8)
      WRITE OUTPUT TAPE NUTS,703,P
 703  FORMAT(9H P MATRIX/(6E17.8))
      WRITE OUTPUT TAPE NUTS,704,PAR
 704  FORMAT(11H PAR MATRIX/(6E17.8))
 15   RETURN
      END
```

Subroutine: ROTATE

Purpose: To compute a 3 x 3 rotation matrix for a transformation of coordinates about axis i (i = 1,2,3) through an angle $\pm\theta$.

[For example: rotation about axis 1, matrix = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$

Calling Sequence:

CALL ROTATE (NCOORD, ANG, RMTRIX, IROT)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	NCOORD	1	i, i = 1,2,3		Axis of rotation
I	ANG	1	θ	radians	Angle for rotation
O	RMTRIX	3,3			Resulting rotation matrix
I	IROT	1			If IROT < 0, use $-\theta$.

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required: ______

```
*       LABEL                                                            ROTA
*       SYMBOL TABLE                                                     ROTA
        SUBROUTINE ROTATE(NCOORD,ANG,RMTRIX,IROT)                        ROTA0000
        DIMENSION RMTRIX(3,3)                                            ROTA0010
        SANG = SINF(ANG)                                                 ROTA0020
        CANG = COSF(ANG)                                                 ROTA0030
        IF (IROT) 1,2,2                                                  ROTA0040
      1 SANG = -SANG                                                     ROTA0050
      2 DO 3 I=1,3                                                       ROTA0060
        DO 3 J=1,3                                                       ROTA0070
      3 RMTRIX(I,J) = 0.                                                 ROTA0080
        GO TO (4,5,6), NCOORD                                            ROTA0090
      4 I = 2                                                            ROTA0100
        J = 3                                                            ROTA0110
        GO TO 7                                                          ROTA0120
      5 I = 1                                                            ROTA0130
        J = 3                                                            ROTA0140
        GO TO 7                                                          ROTA0150
      6 I = 1                                                            ROTA0160
        J = 2                                                            ROTA0170
      7 RMTRIX(NCOORD,NCOORD) = 1.                                       ROTA0180
        RMTRIX(I,I) = CANG                                               ROTA0190
        RMTRIX(I,J) = SANG                                               ROTA0200
        RMTRIX(J,I) = -SANG                                              ROTA0210
        RMTRIX(J,J) = CANG                                               ROTA0220
        RETURN                                                           ROTA0230
        END                                                              ROTA
```

Subroutine: ROTEQ

Purpose: ROTEQ evaluates elements of the rotation matrix which relate the general precession of the earth's equator and the consequent retrograde motion of the equinox on the ecliptic. It is used to provide the transformation from mean equator and equinox of 1950.0 to mean equator and equinox of date.

Calling Sequence:

CALL ROTEQ (TIME, A)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TIME	1	T	days	Total number days from reference
					epoch (1950.0)
O	A	3,3	A		Rotation matrix

Common storages used or required: None

Subroutines required: None

Functions required: None

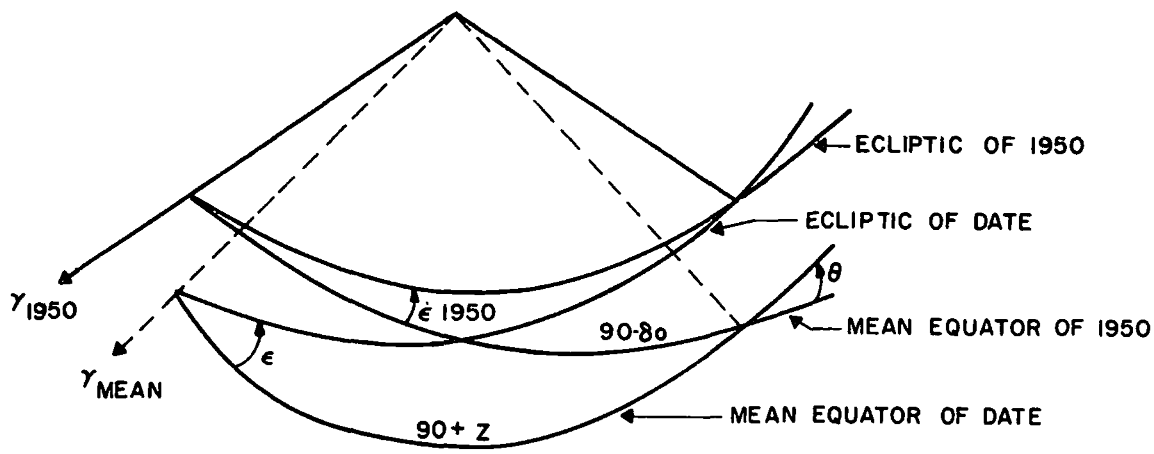
Approximate number of storages required:

Elements of Transformation

The rotation matrix may be represented as

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

where X, Y, Z are expressed in the mean equator and equinox of 1950.0 and x', y', z' are the coordinates in the mean equator and equinox of date.



The geometry of the precession has been represented by the three small parameters δ_o, z, and θ. Υ_{1950} is the mean equinox of 1950; $\bar{\epsilon}_{1950}$ is the mean obliquity of 1950; Υ_{mean} is the mean equinox of date; $\bar{\epsilon}$ is the mean obliquity of date. Measured in the mean equator of 1950 from the mean equinox of 1950, $90^{\circ} - \delta_o$ is the right ascension of the ascending node of the mean equator of date on the mean equator of 1950.

$90^{\circ} + z$ is the right ascension of the node measured in the mean equator of date from the mean equinox of date. θ is the inclination of the mean equator of date to the mean equator of 1950.

In terms of δ_o, z, and θ, (a_{ij}) is given by:

$$a_{11} = -\sin \delta_o \sin z + \cos \delta_o \cos z \cos \theta$$

$$a_{12} = -\cos \delta_o \sin z - \sin \delta_o \cos z \cos \theta$$

$$a_{13} = -\cos z \sin \theta$$

$$a_{21} = \sin \delta_o \cos z + \cos \delta_o \sin z \cos \theta$$

$$a_{22} = \cos \delta_o \cos z - \sin \delta_o \sin z \cos \theta$$

$$a_{23} = -\sin z \sin \theta$$

$$a_{31} = \cos \delta_o \sin \theta$$

$$a_{32} = -\sin \delta_o \sin \theta$$

$$a_{33} = \cos \theta$$

$$\delta_o = 2304''.997T + ''.302T^2 + ''.0179T^3$$

$$z = 2304''.997T + 1''.093T^2 + ''.0192T^3$$

$$\theta = 2004''.298T - ''.426T^2 - ''.0416T^3$$

with T the number of Julian centries of 36,525 days past the epoch 1950.0.

The actual computational form of (a_{ij}) is obtained by expanding the a_{ij} in power series in δ_o, z, θ and replacing the arguments by the above time series. The results are:

$$a_{11} = 1 - .00029697T^2 - .00000013T^3$$

$$a_{12} = -a_{21} = -.02234988T - .00000676T^2 + .00000221T^3$$

$$a_{13} = -a_{31} = -.00971711T + .00000207T^2 + .00000096T^3$$

$$a_{22} = 1 - .00024976T^2 - .00000015T^3$$

$$a_{23} = a_{32} = -.00010859T^2 - .00000003T^3$$

$$a_{33} = 1 - .00004721T^2 + .00000002T^3$$

```
*     LABEL                                                          ROTQ
*     SYMBOL TABLE                                                   ROTQ
      SUBROUTINE ROTEQ(TIME,A)                                       ROTQ000
      DIMENSION A(3,3)                                               ROTQ001
      T = TIME/36525.                                                ROTQ002
      T2 = T*T                                                       ROTQ003
      T3 = T2*T                                                      ROTQ004
      A(1,1) = 1. - .00029697*T2 - .00000013*T3                      ROTQ005
      A(1,2) = -.02234988*T - .00000676*T2 + .00000221*T3            ROTQ006
      A(2,1) = -A(1,2)                                               ROTQ007
      A(1,3) = -.00971711*T + .00000207*T2 + .00000096*T3            ROTQ008
      A(3,1) = -A(1,3)                                               ROTQ009
      A(2,2) = 1. - .00024976*T2 - .00000015*T3                      ROTQ010
      A(2,3) = -.00010859*T2 - .00000003*T3                          ROTQ011
      A(3,2) = A(2,3)                                                ROTQ012
      A(3,3) = 1. - .00004721*T2 + .00000002*T3                      ROTQ013
      RETURN                                                         ROTQ014
      END                                                            ROTQ
```

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Subroutine: RVIN

Purpose: To transform inertial equatorial spherical to Cartesian coordinates.

Calling Sequence:

CALL RVIN (A, B, C)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	A	5	A(1) = R		Magnitude of Position Vector
			A(2) = Φ	deg	Declination
			A(3) = θ	deg	Right Ascension
			A(4) = V		Magnitude of Velocity Vector
			A(5) = Γ	deg	Flight Path Angle
			A(6) = Σ	deg	Azimuth
0	B	3			X, Y, Z
0	C	3			$\dot{X}$, $\dot{Y}$, $\dot{Z}$

Common storages used or required: None

Subroutines required: None

Functions required: SIN, COS

Approximate number of storages required:

RVIN

The calling sequence consists of

CALL RVIN (XIN, XOUT, VOUT)

where: XIN is a vector with dimension six. The vector contains the spherical set in the following order

$$XIN = \begin{bmatrix} |R| \\ 0 \\ \theta \\ |V| \\ \gamma \\ \sigma \end{bmatrix}_{6x1}$$

|R| = Magnitude position vector
0 = Declination
θ = Right ascension
|V| = Magnitude velocity vector
γ = Flight path angle
σ = Azimuth

XOUT is a vector with dimension three. The vector contains the Cartesian components of position

$$XOUT = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3x1}$$

VOUT is a vector with dimension three. The vector contains the Cartesian components of velocity

$$VOUT = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}_{3x1}$$

RVIN TRANSFORMATIONS

The position transformation is obtained as follows (see Fig. 1 below).

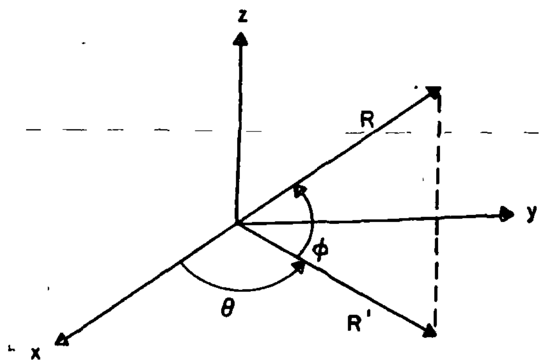

FIG 1

Projecting $\vec{R}$ on the x-y plane, θ is the angle from the x axis to the projection measured counterclockwise. The elevation of $\vec{R}$ above the x-y plane is the angle Φ. The formulas are

$$XOUT = \underset{3x1}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = |R| \underset{3x1}{\begin{pmatrix} \cos \Phi \cos \theta \\ \cos \Phi \sin \theta \\ \sin \Phi \end{pmatrix}} \quad (1)$$

The velocity transformation is obtained as follows. The azimuth and flight path angles refer to the tangent plane which is normal to the $\vec{R}$ vector (see Fig. 2).

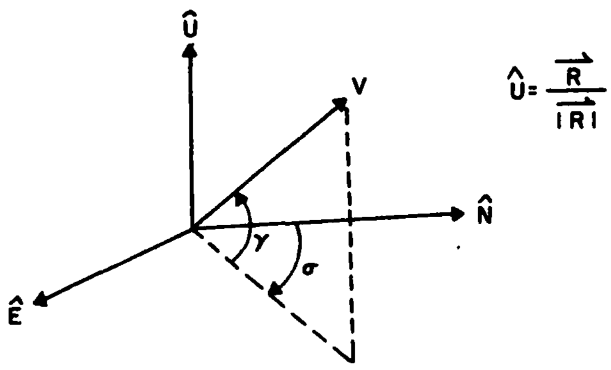

FIG. 2

The unit up, $\hat{U}$, east $\hat{E}$, and north, $\hat{N}$, vectors in terms of xyz are

$$\underset{3x1}{\hat{U}} = \underset{3x1}{\frac{\vec{R}}{|\vec{R}|}} = \underset{3x1}{\begin{pmatrix} \cos\Phi\cos\theta \\ \cos\Phi\sin\theta \\ \sin\Phi \end{pmatrix}} \qquad (2)$$

$$\underset{3x1}{\hat{E}} = \underset{3x1}{\frac{\hat{k} \times \hat{U}}{|k \times \hat{U}|}} = \underset{3x1}{\begin{pmatrix} -\sin\theta \\ \cos\theta \\ \Phi \end{pmatrix}} \qquad (3)$$

$$\underset{3x1}{\hat{N}} = \underset{3x1}{\hat{U} \times \hat{E}} = \underset{3x1}{\begin{pmatrix} -\sin\Phi\cos\theta \\ -\sin\Phi\sin\theta \\ \cos\Phi \end{pmatrix}} \qquad (4)$$

The above vectors may be written in matrix form to represent the transformation from the local tangent plane to the xyz coordinate system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \hat{U} & \hat{E} & \hat{N} \end{pmatrix} \begin{pmatrix} \dot{U} \\ \dot{E} \\ \dot{N} \end{pmatrix} = \begin{pmatrix} \cos\Phi\cos\theta & \sin\theta & -\sin\Phi\cos\theta \\ \cos\Phi\sin\theta & \cos\theta & -\sin\Phi\sin\theta \\ \sin\Phi & 0 & \cos\Phi \end{pmatrix} \begin{pmatrix} \dot{U} \\ \dot{E} \\ \dot{N} \end{pmatrix} \quad (5)$$

3x1 3x3 3x1 3x3 3x1

The velocity vector in the tangent plane coordinates (Fig. 2) is

$$\vec{V} = \begin{pmatrix} \dot{U} \\ \dot{E} \\ \dot{N} \end{pmatrix} = |V| \begin{pmatrix} \sin\gamma \\ \cos\gamma \, \sin\sigma \\ \cos\gamma \, \cos\sigma \end{pmatrix} \quad (6)$$

3x1 3x1

Substituting equation (6) into (5) yields the desired transformation

$$VOUT = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = |V| \begin{pmatrix} \hat{U} & \hat{E} & \hat{N} \end{pmatrix} \begin{pmatrix} \sin\gamma \\ \cos\gamma \, \sin\sigma \\ \cos\gamma \, \cos\sigma \end{pmatrix}$$

3x1 3x3 3x1

```
*     LABEL
*     SYMBOL TABLE
      SUBROUTINE RVIN(A,B,C)
      DIMENSION A(6),B(3),C(3),D(3,3),E(3)
      A(2) = A(2)*.017453293
      A(3) = A(3)*.017453293
      A(5) = A(5)*.017453293
      A(6) = A(6)*.017453293
      CP = COSF(A(2))
      SP = SINF(A(2))
      CT = COSF(A(3))
      ST = SINF(A(3))
      CG = COSF(A(5))
      SG = SINF(A(5))
      CS = COSF(A(6))
      SS = SINF(A(6))
      SPCT = SP*CT
      SPST = SP*ST
      CGSS = CG*SS
      CGCS = CG*CS
      CPCT = CP*CT
      CPST = CP*ST
      D(1,1) = CPCT
      D(1,2) = -ST
      D(1,3) = -SPCT
      D(2,1) = CPST
      D(2,2) = CT
      D(2,3) = -SPST
      D(3,1) = SP
      D(3,2) = 0.
      D(3,3) = CP
      B(1) = A(1)*CPCT
      B(2) = A(1)*CPST
      B(3) = A(1)*SP
      E(1) = A(4)*SG
      E(2) = A(4)*CGSS
      E(3) = A(4)*CGCS
      DO 1 I=1,3
      C(I) = 0.
      DO 1 J=1,3
    1 C(I) = C(I) + D(I,J)*E(J)
      RETURN
      END
```

Subroutine: RVOUT

Purpose: To convert Cartesian position and velocity coordinates to inertial equatorial spherical coordinates.

Calling Sequence:

CALL RVOUT (A, B, C)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	A	3	X,Y,Z		Position coordinates
I	B	3	$\dot{X},\dot{Y},\dot{Z}$		Velocity coordinates
O	C	6	C(1) = R		Magnitude of position vector
			C(2) = Φ	degrees	Declination
			C(3) = θ	degrees	Right ascension
			C(4) = V		Magnitude of velocity vector
			C(5) = Γ	degrees	Flight path angle
			C(6) = Σ	degrees	Azimuth

Common storages used or required: None

Subroutines required: CORSS

Functions required: ATANF, SQRTF, ARKTNS, DOT, ACOS, FNORM

Approximate number of storages required:

RVOUT

The transformations required to convert Cartesian coordinates to inertial equatorial spherical are the inverse of those required by, and described under, RVIN.

For the position coordinates:

$$R = \sqrt{X^2 + Y^2 + Z^2}$$

$$\Phi = \sin^{-1} \frac{Z}{R} \qquad -90^\circ \le \Phi \le 90^\circ$$

$$\theta = \tan^{-1} \frac{Y}{X} \quad \left.\begin{matrix} + 0 \text{ for } X > 0 \\ + 180^\circ \text{ for } X \le 0 \end{matrix}\right\}$$

For the velocity coordinates, $\dot{X}'$, $\dot{Y}'$, $\dot{Z}'$ are obtained by applying the inverse of the transformation matrix $[\hat{U}\ \hat{E}\ \hat{N}]_{3x3}$ given in RVIN. Then:

$$|V| = \sqrt{\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2}$$

$$\Gamma = \sin^{-1} \frac{\dot{X}'}{V} \qquad -90^\circ \le \Gamma \le 90^\circ$$

$$\Sigma = \tan^{-1} \frac{\dot{Y}'}{\dot{Z}'} \quad \left.\begin{matrix} + 0 \text{ for } \dot{Z}' > 0 \\ + 180 \text{ for } \dot{Z}' \le 0 \end{matrix}\right\}$$

```
*      LABEL                                                            RVOT
*      SYMBOL TABLE                                                     RVOT
CEC2017  SUBROUTINE RVOUT                                               RVOT
       SUBROUTINE RVOUT(A,B,C)                                          RVOT000
       DIMENSION A(3),B(3), C(6),E(3),G(3),H(3)                         RVOT001
       ACOSF(X) = ATANF(SQRTF(1.-X**2)/X)                               RVOT002
       C(1) = FNORM(A)                                                  RVOT003
       C(4) = FNORM(B)                                                  RVOT004
       C(2) = ATANF(A(3)/SQRTF(A(1)**2+A(2)**2))                        RVOT005
       C(3)=ARKTNS(360,A(1),A(2))                                       RVOT006
     4 F = DOT(A,B)                                                     RVOT007
       ARG = F/(C(1)*C(4))                                              RVOT008
       C(5) = ACOSF(ARG)                                                RVOT009
       IF (ARG) 5,6,6                                                   RVOT010
     5 C(5) = -1.5707963 - C(5)                                         RVOT011
       GO TO 7                                                          RVOT012
     6 C(5) = 1.5707963 - C(5)                                          RVOT013
     7 CALL CROSS(A,B,E)                                                RVOT014
       EN=FNORM(E)                                                      RVOT015
       DO 12 I=1,3                                                      RVOT016
   12  E(I)=E(I)/EN                                                     RVOT017
       CALL CROSS(E,A,G)                                                RVOT018
       GN = FNORM(G)                                                    RVOT019
       DO 8 I=1,3                                                       RVOT020
     8 G(I) = G(I)/GN                                                   RVOT021
       H(1) = 0.                                                        RVOT022
       H(2) = 0.                                                        RVOT023
       H(3) = 1.                                                        RVOT024
       CALL CROSS(H,A,E)                                                RVOT025
       ARG = DOT(E,G)                                                   RVOT026
       EN = FNORM(E)                                                    RVOT027
       ARG = ARG/EN                                                     RVOT028
       C(6) = ACOSF(ARG)                                                RVOT029
       IF (G(3)) 9,10,10                                                RVOT030
     9 C(6) = C(6) + 1.5707963                                          RVOT031
       GO TO 11                                                         RVOT032
    10 C(6) = -C(6) + 1.5707963                                         RVOT033
    11 CONTINUE                                                         RVOT034
       C(2) = C(2)*57.29578                                             RVOT035
       C(3) = C(3)*57.29578                                             RVOT036
       C(5) = C(5)*57.29578                                             RVOT037
       C(6) = C(6)*57.29578                                             RVOT038
       RETURN                                                           RVOT039
       END                                                              RVOT
```

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Subroutine: SDEC

Purpose: To compute the total perturbation accelerations due to earth's oblateness, perturbing bodies, and Encke perturbations due to deviations from reference conic. These perturbations are obtained by calling OBLN, BODY, and ENCKE and placed in the T block for use in the integration package.

Calling Sequence:

CALL ______

Input and **Output**

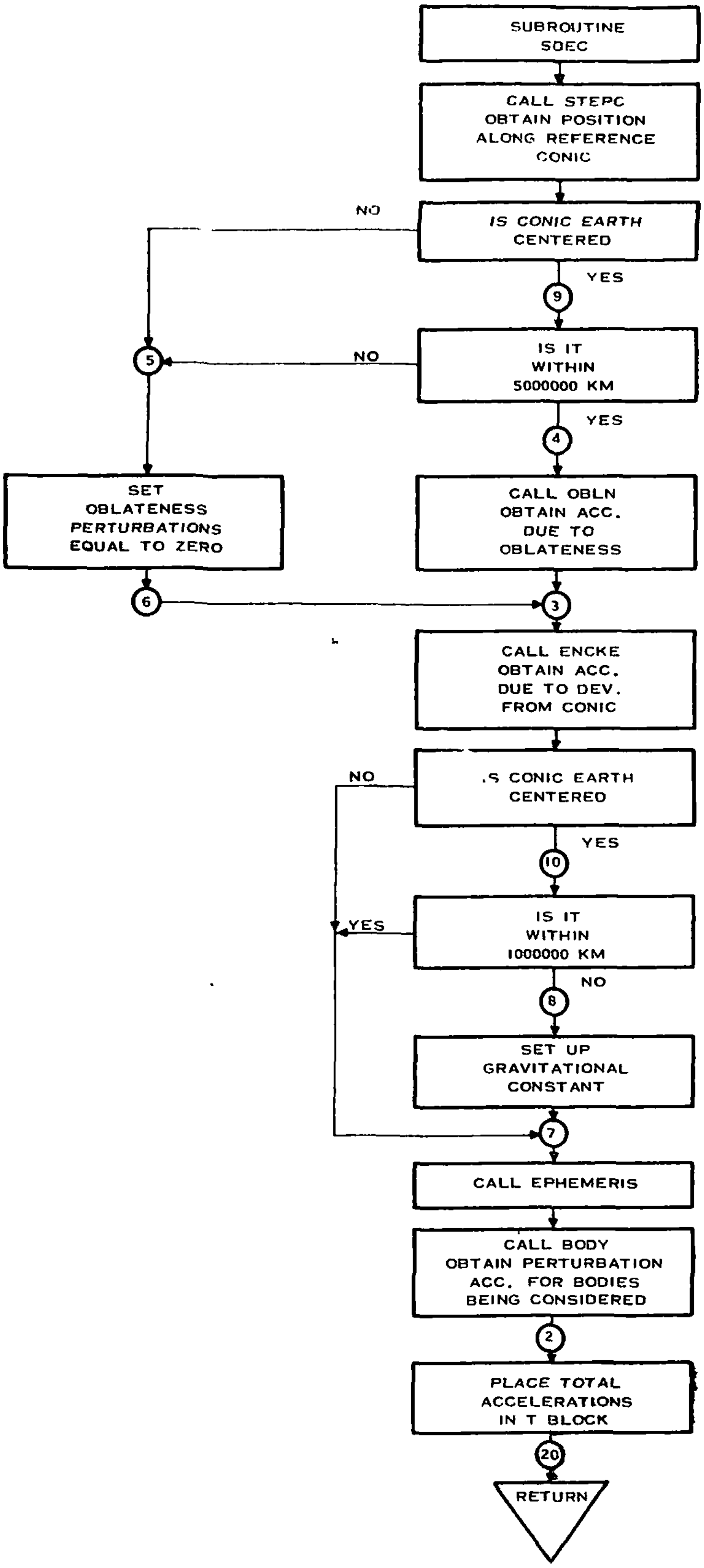
I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I					Quantities obtained from common
O					Quantities placed in common

Common storages used or required: T, S, C, IC

Subroutines required: BODY, ENCKE, INTR, OBLN, STEPC

Functions required: None

Approximate number of storages required: 195 DEC


SUBROUTINE SDEC
CALL STEPC OBTAIN POSITION ALONG REFERENCE CONIC
IS CONIC EARTH CENTERED
NO
YES
9
IS IT WITHIN 5000000 KM
NO
YES
5
4
SET OBLATENESS PERTURBATIONS EQUAL TO ZERO
CALL OBLN OBTAIN ACC. DUE TO OBLATENESS
6
3
CALL ENCKE OBTAIN ACC. DUE TO DEV. FROM CONIC
IS CONIC EARTH CENTERED
NO
YES
10
IS IT WITHIN 1000000 KM
YES
NO
8
SET UP GRAVITATIONAL CONSTANT
7
CALL EPHEMERIS
CALL BODY OBTAIN PERTURBATION ACC. FOR BODIES BEING CONSIDERED
2
PLACE TOTAL ACCELERATIONS IN T BLOCK
20
RETURN

```
*     LABEL                                                              SDEC
*     SYMBOL TABLE                                                       SDEC
CEC2024  SUBROUTINE SDEC                                                 SDEC
      SUBROUTINE SDEC                                                    SDEC000
      DIMENSION T(1360),S(1000),C(1000),IC(1)                            SDEC001
      DIMENSION RAD(3),                    X(3),VX(3),XP(3),             SDEC002
     1 AE(3),AO(3),AP(3),PO(22),VE(22),CA(3),VKB(6),                     SDEC003
     2 RBO(6),RBOP(6)          ,UM(6),VXP(3),AN(3,3),EN(3,3),EA(3,3)     SDEC004
     3,BP(3,3)    ,XSO(3),VXSO(3)                                        SDEC005
      COMMON T,S,C,IC                                                    SDEC006
      EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM)                             SDEC007
      EQUIVALENCE (S(11),VJ),(S(12),H),(S(13),D),(S(900),RAD)            SDEC008
      EQUIVALENCE (C(10),TW),(C(11),TF),(C(13),TP1),(C(14),TP2)          SDEC009
     1,(C(15),XP),(C(18),VXP),(C(21),AE),(C(24),AO),(C(27),CA)           SDEC010
     2,(C(33),X),(C(36),VX)                                              SDEC011
     4,(C(55),U),(C(56),VKB),(C(62),PO),(C(84),VE),(C(106),RBO)          SDEC012
     5,(C(112),RBOP),(C(120),EN),(C(129),EA),(C(138),AN),(C(171),BP)     SDEC013
     7,(IC(1),N),(IC(2),IOR),(IC(3),NOR),(IC(4),KSTP),(IC(5),KTYPE)      SDEC014
     8,(IC(6),ITARG),(S(14),RE),(C(46),XSO),(C(49),VXSO),(IC(231),NSTEP)SDEC015
      NEQ=3*N                                                            SDEC016
      CALL STEPC(NSTEP,T(2),XSO,VXSO,U,X,VX)                             SDEC017
      DO 1 I=1,3                                                         SDEC018
    1 XP(I) = X(I) + T(I+3)                                              SDEC019
      IF(NOR-1)5,9,5                                                     SDEC020
    9 IF (RBOP-500000.) 4,5,5                                            SDEC021
    4 CALL OBLN(XP,U,RE,VJ,H,D,AO,AN,BP,NEQ)                             SDEC022
      GO TO 3                                                            SDEC023
    5 DO 6 I=1,3                                                         SDEC024
      AO(I)=0.                                                           SDEC025
      DO 6 J=1,3                                                         SDEC026
      BP(I,J)=0.                                                         SDEC027
    6 CONTINUE                                                           SDEC028
    3 CALL ENCKE(U,X,T(4),AE)                                            SDEC029
      IF(NOR-1)7,10,7                                                    SDEC030
   10 IF (RBOP-1000000.) 7,8,8                                           SDEC031
    8 VKB(6) = UM(6)                                                     SDEC032
    7 NV = 0                                                             SDEC033
      NR1=NOR-1                                                          SDEC034
      TP2 = TF+T(2)/86400.                                               SDEC035
      TWGN=0.                                                            SDEC036
   21 CONTINUE                                                           SDEC037
      IF(TP2)22,23,23                                                    SDEC038
   22 CONTINUE                                                           SDEC039
      TP2=TP2+1.                                                         SDEC040
      TWGN=TWGN-1.                                                       SDEC041
      GO TO 21                                                           SDEC042
   23 CONTINUE                                                           SDEC043
      TFP = INTF(TP2)                                                    SDEC044
      TP2 = TP2-TFP                                                      SDEC045
      TP1=TW+TFP+TWGN                                                    SDEC046
      CALL INTR(TP1,TP2,NR1,PO,NV,VE,RBOP)                               SDEC047
```

```
      CALL BODY(XP,PO,VKB,RBO,RBOP,CA,NOR,BP,T,NEQ,U)               SDEC0480
      KJ=2*NEQ+3                                                     SDEC0490
      DO 2 I=1,3                                                     SDEC0500
      KI=KJ+I                                                        SDEC0510
    2 T(KI) = AO(I) + AE(I) -CA(I)                                   SDEC0520
   20 RETURN                                                         SDEC0530
      END                                                            SDEC
```

Subroutine: SETN

Purpose: SETN sets the logical tape numbers for input and output to conform with the system unit table.

Calling Sequence:

CALL SETN (NIN, NOUT)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	NIN	1			Logical input tape unit
O	NOUT	1			Logical output tape unit

Common storages used or required: None

Subroutines required: None

Functions required: None

Approximate number of storages required:

```
*       LABEL                                                                SETN
CEC2023                                                                      SETN0000
        SUBROUTINE SETN(NIN,NOUT)                                            SETN0010
C       CALLING THIS SUBROUTINE RESULTS IN THE USE OF                        SETN0020
C       INPUT TAPE 2 AND OUTPUT TAPE 3                                       SETN0030
C       USER MUST READ INPUT TAPE NIN,  AND WRITE OUTPUT NOUT,               SETN0040
        NIN=2                                                                SETN0050
        NOUT=3                                                               SETN0060
        RETURN                                                               SETN0070
        END                                                                  SETN
```

Subroutine: SHIFTP

Purpose: To change position and velocity coordinates from a coordinate system centered at body IOR to one centered at body NOR. SHIFTP also returns the new gravitational constant and the position and velocity of all bodies on the ephemeris tape referenced to the new center. (The latter is done by calling the ephemeris subroutine INTRI)

Calling Sequence:

CALL SHIFTP (IOR, NOR, U, UM, X, VX, PO, VE, TP1, TP2)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	IOR	1			Index of old central body
I	NOR	1			Index of new central body
O	U	1	μ	Km^3/sec^2	Gravitational constant, body NOR
I	UM	6			Array of gravitational constants
I-O	X	3	X, Y, Z	Km	Positions } input: * w.r.t. body IOR
I-O	VX	3	$\dot{X}, \dot{Y}, \dot{Z}$	Km/sec	Velocities } output: w.r.t. body NOR
O	PO	22		Km	Positions } of bodies on tape w.r.t.
O	VE	22		Km/sec	Velocities } body NOR
I	TP1	1		seconds	Double precision time in sec-
I	TP2	1		seconds	onds since 1950.

Common storages used or required: None

Subroutines required: INTRI

Functions required: None

Approximate number of storages required: ________

* with respect to

The sequence used in shifting centers is the following:

IOR (NOR)	CENTRAL BODY
1	Earth
2	Moon
3	Sun
4	Venus
5	Mars
6	Jupiter

```
*       LABEL                                                                SHTP
*       SYMBOL TABLE                                                         SHTP
        SUBROUTINE SHIFTP(IOR,NOR,U,UM,X,VX,PO,VE,TF,TW,T2)                  SHTP000
        COMMON T,S,C,IC                                                      SHTP001
        DIMENSION T(1360),S(1000),C(1000),IC(1)                              SHTP002
        DIMENSION UM(6),X(3),VX(3),PO(22),VE(22)                             SHTP003
        EQUIVALENCE (T,TDUM),(S,SDUM),(C,CDUM),(C(14),TP2),(C(13),TP1)       SHTP004
        N = NOR                                                              SHTP005
        IO = IOR                                                             SHTP006
        IOR = NOR                                                            SHTP007
        U = UM(N)                                                            SHTP008
        NV = 1                                                               SHTP009
        DIS = 1.E10                                                          SHTP010
        NR1 = NOR-1                                                          SHTP011
        CALL INTR1(TP1,TP2,NR1,PO,NV,VE,DIS)                                 SHTP012
        DO 1 I=1,3                                                           SHTP013
        J = 26-3*IO-I                                                        SHTP014
        X(I) = X(I)+PO(J)                                                    SHTP015
      1 VX(I) = VX(I)+VE(J)                                                  SHTP016
        RETURN                                                               SHTP017
        END                                                                  SHTP
```

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Subroutine: STEPC

Purpose: To obtain the state, i.e., radius vector, R, and velocity vector, V, of a probe which is separated by a time increment, t, from a specified state, R_o and V_o, assuming an inverse-square central force law with gravitational constant u.

Calling Sequence:

CALL STEPC (N, T, R, V, U, RR, VV)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	N	1			Initializing index
I	T	1	Δt	sec	Incremental time
I	R	3	$\vec{X}_o$	Km	Initial Conic position (t_o)
I	V	3	$\dot{\vec{X}}_o$	Km/sec	Initial Conic Velocity (t_o)
I	U	1	μ	Km^3/sec^2	Gravitational constant
O	RR	3	$\vec{X}$	Km	Position on Conic ($t_o + \Delta t$)
O	VV	3	$\dot{\vec{X}}$	Km/sec	Velocity on Conic ($t_o + \Delta t$)

Common storages used or required: None

Subroutines required: DOT, GOTOR

Functions required: ARKTNS, SQRT

Approximate number of storages required: 255 DEC

*Discussion

The inverse-square central force law is characterized by the equations

$$\dot{R} = \frac{dR}{dt} = V$$

$$\dot{V} = \frac{dV}{dt} = - \frac{u}{r^3} R \qquad (1)$$

where r is defined to be $+(R \cdot R)^{1/2}$.

The angular momentum vector, H, defined by the vector cross-product

$$H = R \times V \qquad (2)$$

is a constant with respect to time, as seen by

$$\dot{H} = \dot{R} x V + R x \dot{V} = V x V + R x \left(\frac{-u}{r^3} R \right) = 0, \text{ so that } H = H_o = R_o \times V_o.$$

Because the magnitude, h_o, of H_o is generally nonzero, the plane equations

$$R \cdot H = R \cdot H_o = 0$$

$$V \cdot H = V \cdot H_o = 0 \qquad (3)$$

which follow from the definition of H, lead to the conclusion that all motion occurs in the plane normal to H_o. That is, if R_o and V_o are nonzero and noncollinear vectors, any other vectors, such as R and V, must lie in the plane formed by R_o and V_o. Algebraically, one says that R and V may be expressed as linear combinations of R_o and V_o.

$$R = f R_o + g V_o$$

$$V = \dot{f} R_o + \dot{g} V_o \qquad (4)$$

* Vectors will be denoted by capital letters and scalars by lower-case letters with the single exception of the rather standard symbol, E, for eccentric anomaly.

The second of equations (4) follows from (1) and from the fact that R_o and V_o are not functions of the time increment, t. The scalars, f, $\dot{f}$, g, and $\dot{g}$, are functions of R_o, V_o, and t. The discussion which follows concerns the determination of these scalars.

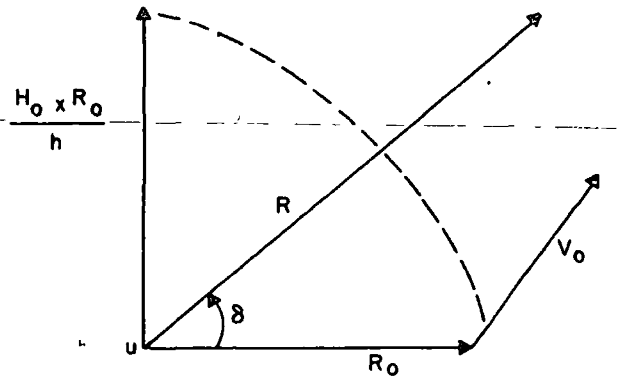

FIG 1

The vector $(H_o \times R_o)/h$ is seen to be perpendicular to R_o and to have the same magnitude, r_o, as R_o. Figure 1 shows that R may be written as the linear combination,

$$R = \frac{r}{r_o} \left[R_o \cos\delta + \frac{H_o \times R_o}{h} \sin\delta \right] \qquad (5)$$

Equation (5) may be rewritten by expanding the vector triple cross-product,

$$H_o \times R_o = (R_o \times V_o) \times R_o = V_o\,(R_o \cdot R_o) - R_o\,(R_o \cdot V_o).$$

$$R = \frac{r}{r_o} \left[R_o \left(\cos\delta - \frac{R_o \cdot V_o}{h} \sin\delta \right) + V_o \frac{r_o^2}{h} \sin\delta \right] \qquad (5a)$$

Comparing equation (5a) with equation (4),

$$f = \frac{r}{r_o}\left(\cos \delta - \frac{R_o \cdot V_o}{h} \sin \delta\right)$$

$$g = \frac{rr_o}{h} \sin \delta. \qquad (6)$$

At this point, the reader should note that while r_o, $R_o \cdot V_o$, and h are computable directly from R_o and V_o, the quantities r and δ have not been specifically defined in terms of R_o, V_o, and t. In order to do so, it is necessary to examine the conic solution to the inverse-square law equations of motion.

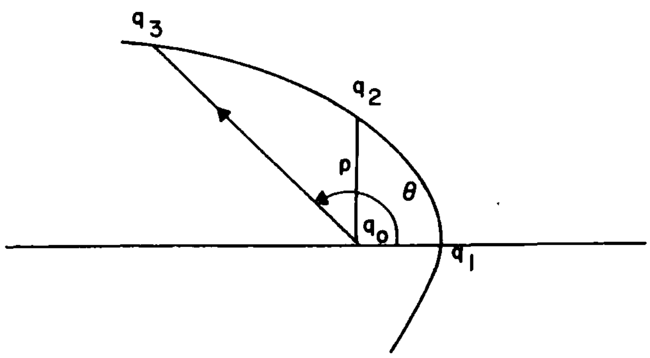

FIG 2

Figure 2 shows a conic section with focus at q_o and pericenter at q_1. The point q_3 on the conic is specified, in polar coordinates originating at q_o, by the radius, r, and true anomaly, θ, θ being measured positive counter-clockwise from the line $\overline{q_oq_1}$. The length $\overline{q_oq_2}$ is called the semi-latus rectum and is denoted by p. The parameter specifying the shape of the conic is the eccentricity, e, which may be calculated from the equation $e = \frac{\overline{q_oq_2}}{\overline{q_oq_1}} - 1$. The conic equation in an often-used form is,

$$r = \frac{p}{1+e\cos\theta} \qquad (7)$$

Conic sections are characterized by eccentricity as follows:

circular	if	$e = 0$
elliptical	if	$0 < e < 1$
parabolic	if	$e = 1$
hyperbolic	if	$e > 1$

The two cases of great interest here are the elliptical and hyperbolic cases. Figure 3 below shows an ellipse.

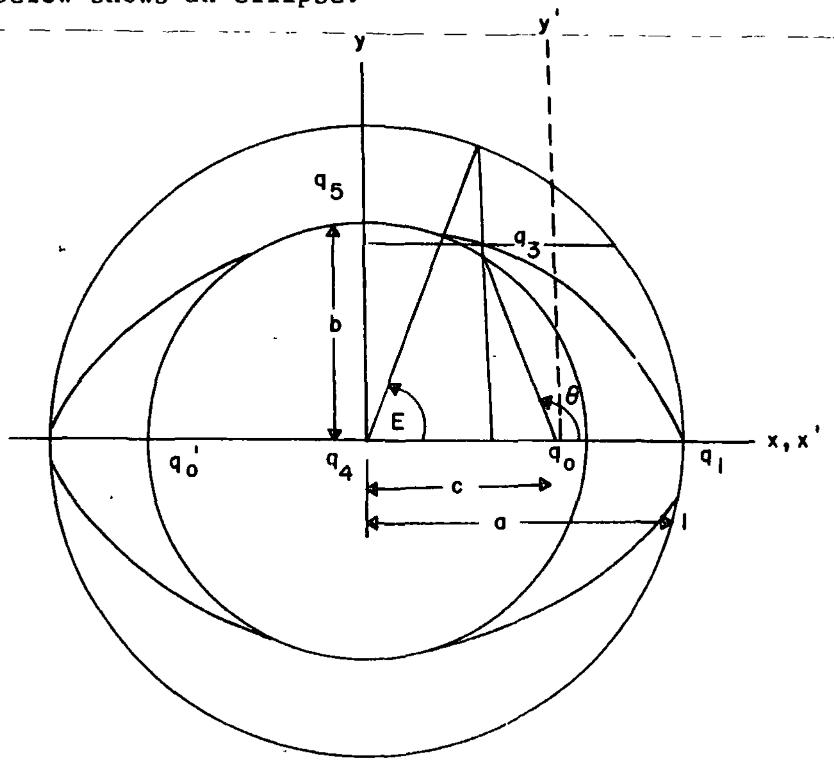

FIG. 3

The semi-major axis, a, is the distance from the point of symmetry, q_4, to the pericenter, q_1. The semi-minor axis, b, is the length $\overline{q_4 q_5}$. The eccentricity is equal to the ratio

$$c/a = \frac{\sqrt{a^2 - b^2}}{a}$$

The eccentric anomaly, E, is the argument of the projection of the point, q_3, from the line $\overline{q_4 q_1}$ onto a circle of radius, a, concentric with the ellipse. That is, the Cartesian coordinates of q_3 relative to q_4 are

$$x = a \cos E$$
$$y = b \sin E \tag{8}$$

Equations (8) give rise to the familiar equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2 E + \sin^2 E = 1.$$

Relative to coordinates centered at q_o, q_3 is described by

$$x' = r\cos\theta = a\cos E - ea$$
$$y' = y = r\sin\theta = b\sin E = a\sqrt{1-e^2}\sin E. \tag{8a}$$
$$r = x'^2 + y'^2 = a(1-e\cos E)$$

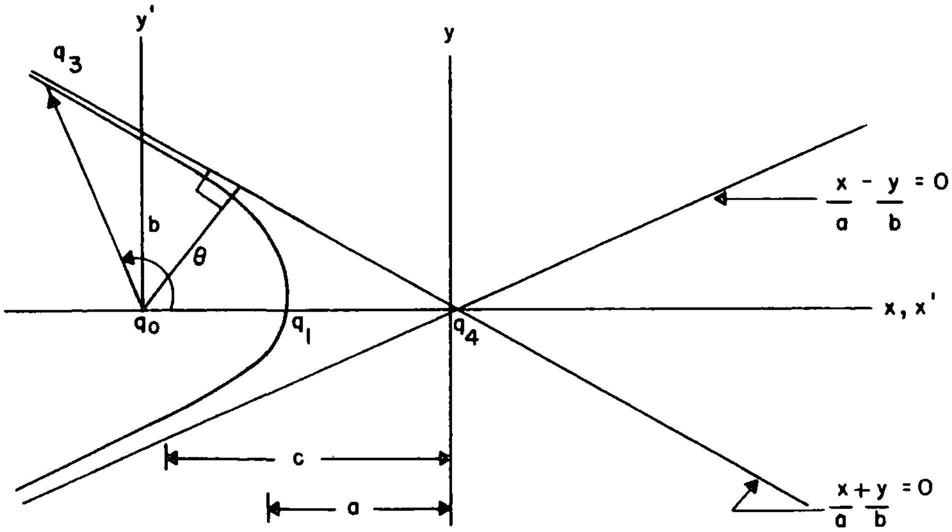

FIG 4

Figure 4 shows a conic section which is hyperbolic. The lines $\frac{x}{a} - \frac{y}{b} = 0$ and $\frac{x}{a} + \frac{y}{b} = 0$ are the asymptotes of the hyperbola. The semi-major axis, a, is again the distance from the point of symmetry, q_4, to the pericenter, q_1. The semi-minor axis, b, is defined as the distance from the focus, q_o, to one of the asymptotes. Eccentricity is given by the ratio

$$c/a = \sqrt{\frac{a^2 + b^2}{a^2}} .$$

The eccentric anomaly, E, (sometimes called F) is too difficult to picture to be included in Fig. 4. E is defined in such a way that q_3 is described in Cartesian coordinates by

$$x = -a \cosh E$$
$$y = b \sinh E, \qquad (9)$$

leading to the familiar hyperbolic equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \cosh^2 E - \sinh^2 E = 1$$

The Cartesian coordinates of q_3 relative to q_o are given by

$$x' = r\cos\theta = ea - a\cosh E$$
$$y' = r\sin\theta = b\sinh E = a\sqrt{e^2-1}\,\sinh E. \qquad (9a)$$
$$r = \sqrt{x'^2 + y'^2} = a(e\cosh E - 1)$$

Solution of the differential equations (1) yields p, a, and e as constants of the motion.

It is easily verified that

$$h = \sqrt{r_o^2 v_o^2 - (R_o \cdot V_o)^2} = r^2\dot{\theta}$$ angular momentum magnitude

$$p = h^2/u$$ semi-latus rectum

$$a = \frac{r_o}{\left| 2 - \frac{r_o v_o^2}{u} \right|}$$ semi-major axis

$$e^2 = 1 - p/a$$ eccentricity

$$n = \sqrt{u/a^3}$$ mean motion

It should be observed that p is non-negative so that a negative semi-major axis corresponds to an eccentricity greater than one; that is, to the hyperbolic case. Some useful identities are listed as equations (11).

$$\frac{R \cdot V}{r} = \frac{d}{dt} (R \cdot R)^{1/2} = \dot{r} = \frac{ep\dot{\theta} \sin\theta}{(1+e\cos\theta)^2} = \frac{ue}{h} \sin\theta$$

$$\cos\theta = \mp \frac{a}{r} (cE - e)$$

$$\sin\theta = \frac{a}{r} \sqrt{\left| e^2 - 1 \right|}\ sE = \sqrt{\frac{a}{u}}\ \frac{h}{r}\ sE$$

$$esE = \frac{R \cdot V}{\sqrt{ua}}$$

$$ecE = 1 \pm \frac{r}{a} \tag{11}$$

The convention has been adopted that when a double sense sign $\pm$ is used, the upper pertains to the hyperbolic case and the lower to the elliptical.

The symbols sE and cE mean sinE and cosE for the elliptical case, but sinhE and coshE for the hyperbolic case. Hence, if we let

$\Phi = E - E_o$, it follows that

$$s\Phi = sEcE_o - cEsE_o$$

$$c\Phi = cEcE_o \mp sEsE_o$$

and

$$sE = s\Phi cE_o + c\Phi sE$$

$$cE = c\Phi cE_o \pm s\Phi cE_o \qquad (12)$$

This angle, δ, of equation (5) is seen to represent the incremental true anomaly on the conic section containing R_o and R. Thus, we can

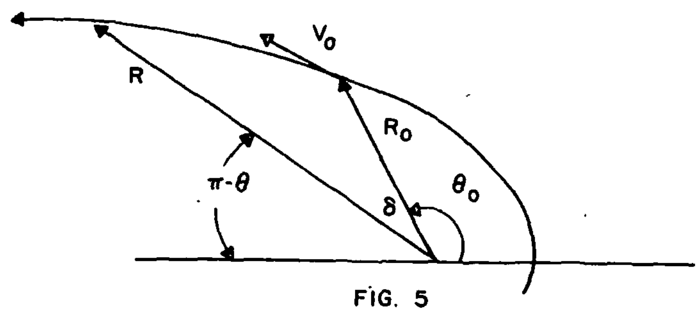

FIG. 5

find expressions for sin δ and cos δ in terms of θ and θ_o.

$$\sin\delta = \sin(\theta - \theta_o) = \sin\theta\cos\theta - \cos\theta\sin\theta_o$$

$$\cos\delta = \cos(\theta - \theta_o) = \cos\theta\cos\theta_o + \sin\theta\sin\theta_o. \qquad (13)$$

The incremental true anomaly may now be related to R_o and V_o through substitution of equations (11) and (12) into (13).

$$\sin\delta = \sin\theta\cos\theta_o - \cos\theta\sin\theta_o$$

$$= \sqrt{\frac{a^3}{u}}\ \frac{h}{rr_o}\ (\mp sEcE_o \pm esE \mp esE_o \pm cEsE_o)$$

$$= \frac{h}{nrr_o}\ (\mp s\Phi \pm esE \mp esE_o)$$

$$= \frac{h}{nrr_o}\ (\mp s\phi \pm ecE_o s\phi \pm esE_o c\phi \mp esE_o)$$

$$= \frac{h}{nrr_o}\left(\frac{r_o}{a}\ s\Phi \pm \frac{R_o \cdot V_o}{\sqrt{ua}}\ (c\Phi - 1)\right)$$

$$\cos\delta = \cos\theta\cos\theta_o + \sin\theta\sin\theta_o$$

$$= \frac{a^2}{rr_o}\left[cEcE_o - ecE - ecE_o + e^2 \pm (e^2-1)sEsE_o\right]$$

$$= \frac{a^2}{rr_o}\left[c\Phi - ecE - ecE_o + e^2 \pm e^2 sEsE_o\right]$$

$$= \frac{a^2}{rr_o}\left[c\Phi - ec\Phi cE_o \mp es\Phi sE_o - ecE_o + e^2 \pm e^2 sE_o cE_o s\Phi \pm e^2 s^2 E_o c\Phi\right]$$

$$= \frac{a^2}{rr_o}\left[\mp \frac{r_o}{a}\ c\Phi + \frac{r_o}{a}\ \frac{R_o \cdot V_o}{\sqrt{ua}}\ s\Phi + 1 \pm \frac{p}{a} - 1 \mp \frac{r_o}{a} \pm \frac{(R_o \cdot V_o)^2}{ua}\ c\Phi\right]$$

$$= \frac{a^2}{rr_o}\left[\mp \frac{r_o}{a}\ (c\Phi + 1) + \frac{r_o}{a}\ \frac{R_o \cdot V_o}{\sqrt{ua}}\ s\Phi \pm \frac{r_o^2 v_o^2}{ua} \pm \frac{(R_o \cdot V_o)^2}{ua}\ (c\Phi - 1)\right]$$

$$= \frac{a^2}{rr_o}\left[\mp \frac{r_o}{a}\ (c\Phi - 1) + \frac{r_o}{a}\ \frac{R_o \cdot V_o}{\sqrt{ua}}\ s\Phi + \frac{r_o}{a}\left(\frac{r_o v_o^2}{u} - 2\right)\right.$$

$$\left.\pm \frac{(R_o \cdot V_o)^2}{ua}\ (c\Phi - 1)\right]$$

$$= \frac{a^2}{rr_o}\left[\pm\left[\frac{(R_o \cdot V_o)^2}{ua} - \frac{r_o}{a}\right](c\Phi - 1) + \frac{r_o}{a}\ \frac{R_o \cdot V_o}{\sqrt{ua}}\ s\Phi + \frac{r_o^2}{a^2}\right]$$

The coefficients, f and g, of equation (6) are finally found to be:

$$f = \frac{r}{r_o}\left(\cos\delta - \frac{R_o \cdot V_o}{h}\sin\delta\right)$$

$$= \frac{a^2}{r_o^2}\left[\pm\left[\frac{(R_o \cdot V_o)^2}{ua} - \frac{r_o}{a} - \frac{(R_o \cdot V_o)^2}{ua}\right](c\Phi - 1) + \frac{r_o}{a}\left[\frac{R_o \cdot V_o}{\sqrt{ua}} - \frac{R_o \cdot V_o}{\sqrt{ua}}\right] s\Phi + \frac{r_o^2}{a^2}\right] = \mp \frac{a}{r_o}(c\Phi - 1) + 1 \tag{14}$$

$$g = \frac{rr_o}{h}\sin\delta$$

$$= \frac{1}{n}\frac{r_o}{a} s\Phi \pm \frac{R_o \cdot V_o}{\sqrt{ua}}(c\Phi - 1) \tag{15}$$

The velocity coefficients, $\dot{f}$ and $\dot{g}$, are found by noting that

$$\dot{\Phi} = \dot{E} = \sqrt{\frac{u}{a}}\,\frac{1}{r}$$

and therefore $\dot{f} = \mp \frac{a}{r_o}\frac{d}{dt}(c\Phi) = -\frac{a}{r_o}\dot{E}s\Phi = -\frac{\sqrt{ua}}{rr_o} s\Phi$ (16)

$$\dot{g} = \frac{\dot{E}}{n}\left(\frac{r_o}{a} c\Phi + \frac{R_o \cdot V_o}{\sqrt{ua}} s\Phi\right)$$

$$= \frac{a}{r}\left(\frac{r_o}{a} c\Phi + \frac{R_o \cdot V_o}{\sqrt{ua}} s\Phi\right) \tag{17}$$

where

$$\frac{r}{a} = \mp(1 - ecE) = \mp(1 - ec\Phi cE_o \mp es\Phi sE_o)$$

$$= \mp\left(1 - c\Phi\left(1 \pm \frac{r_o}{a}\right) \mp s\Phi \frac{R_o \cdot V_o}{\sqrt{ua}}\right)$$

$$= \pm(c\Phi - 1) + \frac{r_o}{a} c\Phi + \frac{R_o \cdot V_o}{\sqrt{ua}} s\Phi \tag{18}$$

Kepler's equation presents the incremental time, t, in terms of Φ

$$nt = \mp (E-esE) \pm (E_o-esE_o)$$

$$= \mp (\Phi-esE+esE_o)$$

$$= \mp (\Phi-es\Phi cE_o-ec\Phi sE_o+esE_o)$$

$$= \mp \left[\Phi-s\Phi\left(1 \pm \frac{r_o}{a}\right) - (c\Phi-1)\frac{R_o \cdot V_o}{\sqrt{ua}}\right]$$

$$= \mp (\Phi-s\Phi) + \frac{r_o}{a} s\Phi \pm \frac{R_o \cdot V_o}{\sqrt{ua}} (c\Phi-1) \qquad (19)$$

In order to solve equation (19) for Φ as a function of R_o, V_o and t, an iterative method is required. Any solution obtained is necessarily unique, since the right side of (19) is seen to be monotonically increasing for all Φ.

Equation (19) may be used to simplify equations (15) and (17).

$$g = \frac{1}{n}\left[\frac{r_o}{a} s\Phi+nt\pm (\Phi-s\Phi) - \frac{r_o}{a} s\Phi\right]$$

$$= t \pm \frac{1}{n} (\Phi-s\Phi) \qquad (15a)$$

$$\dot{g} = 1 \pm \frac{\dot{E}}{n} (1-c\Phi)$$

$$= 1 \mp \sqrt{\frac{u}{a}} \frac{1}{nr} (c\Phi-1)$$

$$= 1 \mp \frac{a}{r} (c\Phi-1) \qquad (17a)$$

Subroutine Description

The subroutine will calculate

(1) $R = R(R_o, V_o, t)$

$V = V(R_o, V_o, t)$

$\delta = \delta(R_o, V_o, t)$

The first time STEP is called with a given R_o and V_o (N=1), those coefficients which are dependent only on R_o and V_o are calculated. Thereafter, these coefficients are considered to be constants with respect to t.

Calculation	FORTAN Name
$r_o = \sqrt{R_o \cdot R_o}$	RM
$v_o = \sqrt{V_o \cdot V_o}$	VM
$a = \dfrac{r_o}{\left\|2 - \dfrac{r_o v_o^2}{u}\right\|}$	A
$n = \dfrac{1}{a}\sqrt{\dfrac{u}{a}}$	RAT
$R_o \cdot V_o = R_o \cdot V_o$	RV
$h = \sqrt{r_o^2 v_o^2 - (R_o \cdot V_o)^2}$	H

STEPC calls Subroutine GOTOR to solve equation (19),

$$nt = \mp (\Phi - s\Phi) + \frac{r_o}{a} s\Phi \pm \frac{R_o \cdot V_o}{\sqrt{ua}} (c\Phi - 1) \text{ for } \Phi.$$

GOTOR returns a vector F where

		Elliptical	Hyperbolic
F(1)	=	Φ-sin Φ	sinh Φ-Φ
F(2)	=	1-cos Φ	cosh Φ-1
F(3)	=	sin Φ	sinh Φ
F(4)	=	cos Φ	cosh Φ

The coefficients, f, g, $\dot{f}$ and $\dot{g}$, are then calculated.

Calculation	FORTRAN Name
$f = -\frac{r_o}{a} F(2) + 1$	EF
$g = -\frac{F(1)}{n} + t$	GE
$\frac{r}{a} = F(2) + \frac{r_o}{a} F(4) + \frac{R_o \cdot V_o}{\sqrt{ua}} F(3)$	ROA
$\dot{f} = - anF(3) / \frac{(r_o r)}{a}$	EFD
$\dot{g} = -\frac{a}{r} F(2) + 1$	GED
$\delta = \tan^{-1} \frac{hg}{f_o^2 f + R_o \cdot V_o g}$	TA
$R = fR_o + gV_o$	RR
$V = \dot{f}R_o + \dot{g}V_o$	VV

Reference: NASA X-640-63-71, ITEM Program Manual, Goddard Space Flight Center, Greenbelt, Maryland.

```
      LABEL                                                     STPC
*      SYMBOL TABLE                                             STPC
CEC2023  SUBROUTINE STEPC                                       STPC
       SUBROUTINE STEPC(N,T,R,V,U,RR,VV)                        STPC000
       DIMENSION R(3),V(3),RR(3),VV(3),C(2),F(4)                STPC001
       GO TO (1,2),N                                            STPC002
     1 CONTINUE                                                 STPC003
       PI=3.141592654                                           STPC004
       R2=DOT(R,R)                                              STPC005
       RM=SQRTF(R2)                                             STPC006
       V2=DOT(V,V)                                              STPC007
       RV=DOT(R,V)                                              STPC008
       VM=SQRTF(V2)                                             STPC009
       A=RM/(2.-RM*V2/U)                                        STPC010
       ABA=ABSF(A)                                              STPC011
       RAT=SQRTF(U/ABA)/ABA                                     STPC012
       C(1)=RM/ABA                                              STPC013
       C(2)=RV/SQRTF(U*ABA)                                     STPC014
       IF(A)3,3,4                                               STPC015
     4 CONTINUE                                                 STPC016
       K=1                                                      STPC017
       E1=RAT*T                                                 STPC018
       GO TO 15                                                 STPC019
     3 CONTINUE                                                 STPC020
       K=2                                                      STPC021
       E1=0.                                                    STPC022
    15 CONTINUE                                                 STPC023
     2 CONTINUE                                                 STPC024
C      T IS THE INCREMENTAL TIME IN ORBIT                       STPC025
       EMDT=RAT*T                                               STPC026
       CALL GOTOR(K,EMDT,C,F,E1)                                STPC027
       EF=-F(2)/C(1)+1.                                         STPC028
       GE=-F(1)/RAT+T                                           STPC029
       TA=ARKTNS(360,R2*EF+RV*GE,H*GE)                          STPC030
       ROA=F(2)+C(1)*F(4)+C(2)*F(3)                             STPC031
       EFD=-ABA*RAT*F(3)/(RM*ROA)                               STPC032
       GED=-F(2)/ROA+1.                                         STPC033
     8 CONTINUE                                                 STPC034
       DO 10 I=1,3                                              STPC035
       RR(I)=EF*R(I)+GE*V(I)                                    STPC036
       VV(I)=EFD*R(I)+GED*V(I)                                  STPC037
    10 CONTINUE                                                 STPC038
       RETURN                                                   STPC039
       END                                                      STPC
```

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Subroutine: TIMEC

Purpose: To compute whole days and fractional days since January 1, 1950, from calendar date input. This input is in the form (Years from 1900) x 10^2 + (Month of Year) + (Day of Month) x 10^{-2}, and (Hour of Day) x 10^2 + (Minutes of Hour) + (Seconds of Minute) x 10^{-2}. [See examples under input description]. The input date must be 1961 or later.

Calling Sequence:

CALL TIMEC (T1, T2, T3, T4)

Input and Output

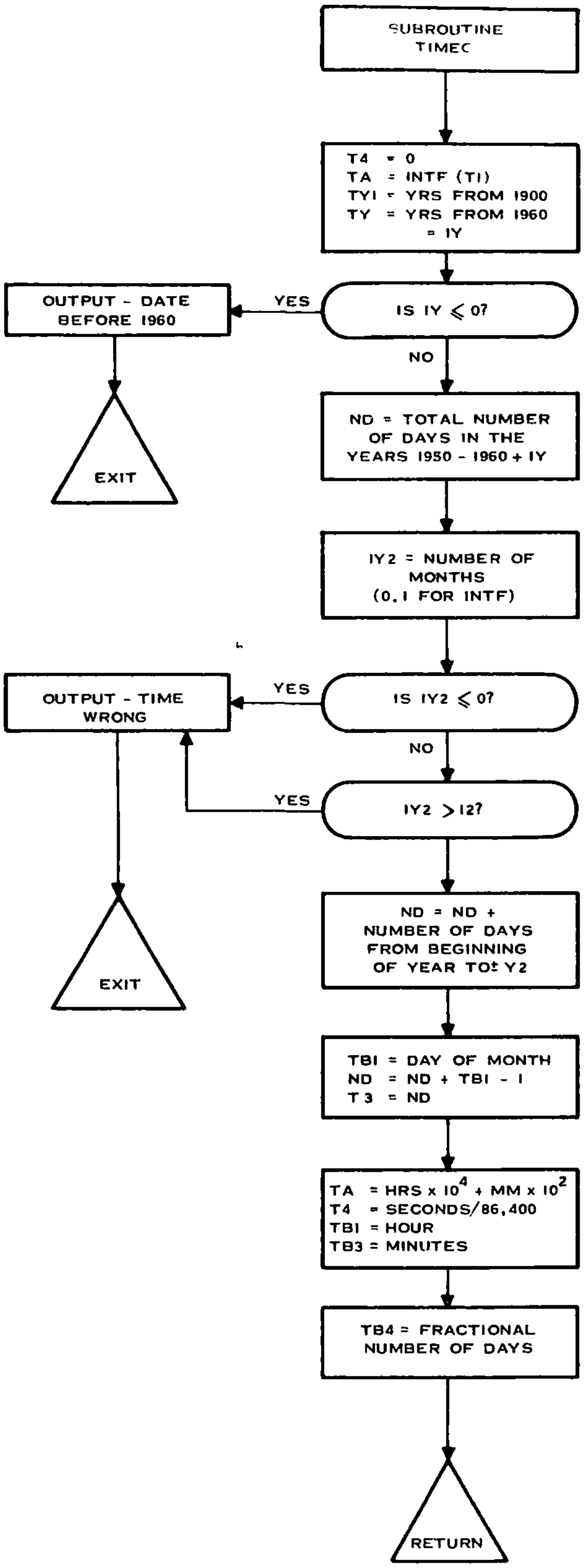
I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	T1	1			Year, month, day.
					Jan. 12, 1965 = 6501.12.
I	T2	1			Hour, minutes, seconds.
					1 PM, 1 min, 30.3 sec = 1301.303
0	T3	1		days	Whole number of days since 1950.
0	T4	1		days	Fractional number of days past
					T3.

Common storages used or required: None

Subroutines required: EXIT

Functions required: INTF

Approximate number of storages required: ____


SUBROUTINE TIMEC
T4 = 0
TA = INTF (TI)
TYI = YRS FROM 1900
TY = YRS FROM 1960
= IY
IS IY ⩽ 0?
YES
OUTPUT - DATE BEFORE 1960
EXIT
NO
ND = TOTAL NUMBER OF DAYS IN THE YEARS 1950 - 1960 + IY
IY2 = NUMBER OF MONTHS
(0.1 FOR INTF)
IS IY2 ⩽ 0?
YES
OUTPUT - TIME WRONG
EXIT
NO
IY2 > 12?
YES
ND = ND + NUMBER OF DAYS FROM BEGINNING OF YEAR TO IY2
TBI = DAY OF MONTH
ND = ND + TBI - I
T3 = ND
TA = HRS x 10^4 + MM x 10^2
T4 = SECONDS/86,400
TBI = HOUR
TB3 = MINUTES
TB4 = FRACTIONAL NUMBER OF DAYS
RETURN

```
*      LABEL
*      SYMBOL TABLE
       SUBROUTINE TIMEC(T1,T2,T3,T4)
C      T1 IS AN INPUT IN FORM (YEARS FROM 1900)(MONTH OF YEAR)
C      (DAY OF MONTH
C      WRITTEN AS 6501.12 FOR JAN. 12, 1965
C      T2 IS AN INPUT IN FORM (HOUR OF DAY)(MINUTE OF HOUR)
C      (SECOND OF MINUTE)
C      WRITTEN AS 1301.3032 FOR 1PM, 1 MINUTE, 30.32 SECOND
C      T3  IS WHOLE DAYS FROM 1950 OUTPUT
       CALL SETN(NIN,NOUT)
       T4=0.
       TA=INTF(T1)
       TA1=TA/100.+.01
       TY1= INTF(TA1)
       TY= TY1-60.
       IY=TY
       IF (IY) 15,15,16
 15    CONTINUE
       WRITE OUTPUT TAPE NOUT,17
 17    FORMAT (37H EXIT FROM TIMEC. DATE 1960 OR BEFORE)
       CALL EXIT
 16    CONTINUE
       ND=3652
       K=1
       DO 1 I=1, IY
       KK=K-I
       IF (KK) 2,2,3
     2 CONTINUE
       ND=ND + 366
       K=K+4
       GO TO 1
     3 CONTINUE
       ND=ND +365
     1 CONTINUE
       TY2=TA-TY1*100.+.1
       IY2=INTF(TY2)
       IF (IY2) 10,10,11
    11 CONTINUE
       IF (12-IY2) 10,12,12
    10 CONTINUE
       WRITE OUTPUT TAPE NOUT,13
 13    FORMAT(37H INPUT TIME IS WRONG. EXIT FROM TIMEC)
       CALL EXIT
    12 CONTINUE
       DO 4 I=1,IY2
       I=I
C            JAN FEB MAR APRIL MAY JUNE JULY AUG SEPT OCT NOV
       GO TO (4,6,7,6,8,6,8,6,6,8,6,8),I
     6 CONTINUE
       ND=ND + 31
```

```
      GO TO 4                                                           TIMC0480
    7 CONTINUE                                                          TIMC0490
      IF(KK-1)9,14,9                                                    TIMC0500
   14 CONTINUE                                                          TIMC0510
      ND=ND +29                                                         TIMC0520
      GO TO 4                                                           TIMC0530
    9 CONTINUE                                                          TIMC0540
      ND=ND +28                                                         TIMC0550
      GO TO 4                                                           TIMC0560
    8 CONTINUE                                                          TIMC0570
      ND=ND +30                                                         TIMC0580
    4 CONTINUE                                                          TIMC0590
      TB1=(T1-TA)*100.+.1                                               TIMC0600
      ND=ND +XINTF(TB1)                                                 TIMC0610
      ND=ND-1                                                           TIMC0620
      T3=ND                                                             TIMC0630
      TA=INTF(T2)                                                       TIMC0640
      T4=(T2-TA)/864.                                                   TIMC0650
      TB=TA/100.+.1                                                     TIMC0660
      TB1=INTF(TB)                                                      TIMC0670
      TB2=TA-TB1*100.+.1                                                TIMC0680
      TB3= INTF(TB2)                                                    TIMC0690
      T4=T4+TB1/24.+TB3/24./60.                                         TIMC0700
      RETURN                                                            TIMC0710
      END                                                               TIMC
```

Subroutine: TIMED

Purpose: To convert an input time from the format DAYS HOURS·MIN SEC to seconds. If the input is negative, it is assumed to be in seconds, and returned positive.

Calling Sequence:

CALL TIMED (T1, T2)

Input and **Output**

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	T1	1		-seconds	or (DAYS + 10^2 + HOURS + MIN
					x 10^{-2} + SEC x 10^{-4})
O	T2	1		seconds	

Common storages used or required: None

Subroutines required: None

Functions required: INTF

Approximate number of storages required: ________

```
*      LABEL                                                               TIMD
*      SYMBOL TABLE                                                        TIMD
       SUBROUTINE TIMED (T1,T2)                                            TIMD0000
       IF(T1)1,1,2                                                         TIMD0010
     1 CONTINUE                                                            TIMD0020
       T2=-T1                                                              TIMD0030
       RETURN                                                              TIMD0040
C      SUBROUTINE ACCEPTS T1 IN SEC IF THE SIGN IS INPUT NEGATIVE AND      TIMD0050
C      GIVES T2 IN + SEC IF T1 IS + AND ARRANGED AS (DAYS HOURS.MIN SEC)   TIMD0060
C      IT CONVERTS OUTPUT T2 TO SEC                                        TIMD0070
     2 CONTINUE                                                            TIMD0080
       TDH=INTF(T1)                                                        TIMD0090
        TMS=T1-TDH                                                         TIMD0100
       TM1=TMS*100.+.01                                                    TIMD0110
       TM=INTF(TM1)                                                        TIMD0120
       TS=(TMS*100.-TM)*100.+.01                                           TIMD0130
       TD=INTF(TDH/100.+.01)                                               TIMD0140
       TH=TDH-TD*100.+.01                                                  TIMD0150
       TH=INTF (TH)                                                        TIMD0160
       TS=INTF(TS)                                                         TIMD0170
       T2=TD*86400. +TH*3600. +TM*60.+TS                                   TIMD0180
       RETURN                                                              TIMD0190
       END                                                                 TIMD
```

Subroutine: TRAC

Purpose: To obtain the vector from a body center to a tracking station and the unit up, east, and north vectors. The coordinate system is a rotating body fixed system. The body may be treated as an oblate spheroid.

Calling Sequence:

CALL TRAC (TO, AT, H, GHA, U, E, EN, RT, AC, SL, CL, ST, CT, A, B)

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
I	TO	1		RAD	Tracker longitude
I	AT	1		RAD	Tracker latitude
I	H	1		Km	Tracker Altitude
I	GHA	1		RAD	Greenwich hour angle
O	U	3	$\hat{U}$		Up unit vector
O	E	3	$\hat{E}$		East unit vector
O	EN	3	$\hat{N}$		North unit vector
O	RT	3			Radius vector to tracker

(Cont'd.)

Common storages used or required: None

Subroutines required: None

Functions required: SIN, COS, SQRT

Approximate number of storages required: 194 DEC

(Cont'd.)

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
O	AC	1			Constant used in program
O	SL	1			Sin LAT
O	CL	1			Cos LAT
O	ST	1			Sin LONG
O	CT	1			Cos LONG
I	A	1	R_E	Km	Body's equatorial radius
I	B	1	R_P	Km	Body's polar radius

Derivation of Vectors obtained in TRAC

A. Derivation of Unit U, E, and N

The coordinate system of concern is shown in Fig. 1 below.

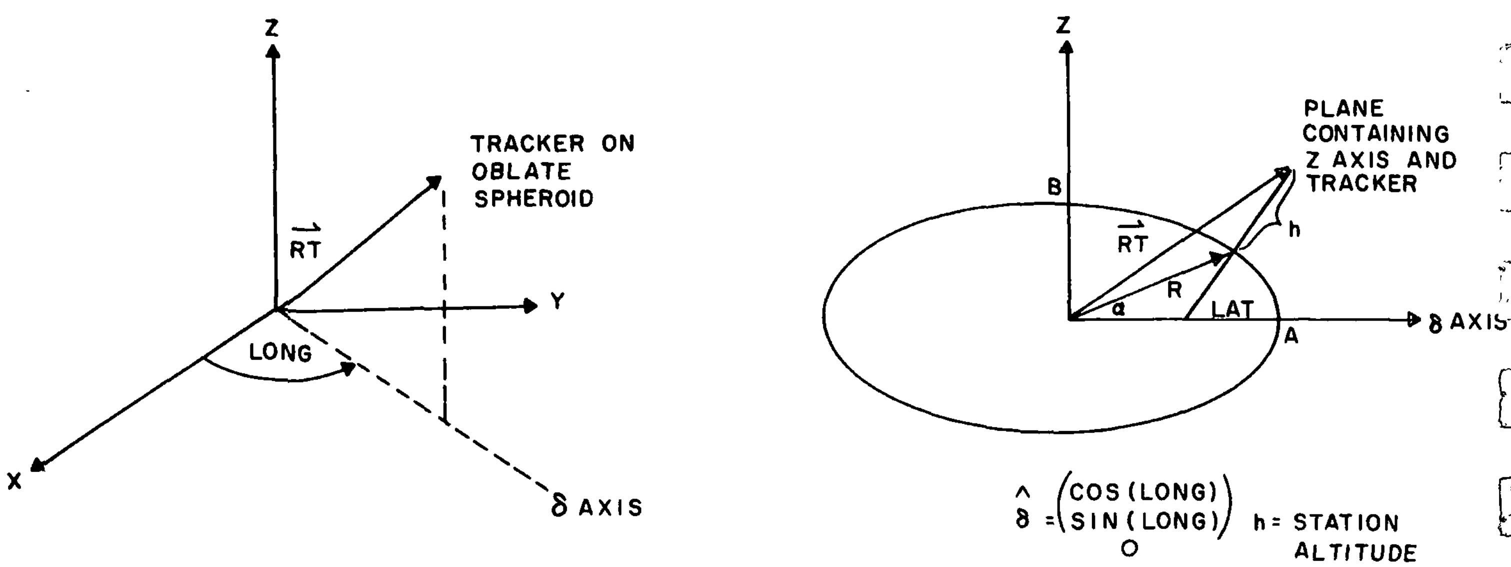


FIG 1

The unit up vector, U, may be written as:

$$\hat{U} = \begin{pmatrix} \cos(LAT)\ \cos(LONG) \\ \cos(LAT)\ \sin(LONG) \\ \sin(LAT) \end{pmatrix}$$

The unit east vector, E, is:

$$\hat{E} = \frac{\hat{k} \times \hat{u}}{|\hat{k} \times \hat{u}|} = \begin{pmatrix} -\sin(LONG) \\ \cos(LONG) \\ 0 \end{pmatrix}$$

The unit north vector, N, is:

$$\hat{N} = \hat{U} \times \hat{E} = \begin{pmatrix} -\sin(LAT)\cos(LONG) \\ -\sin(LAT)\ \sin(LONG) \\ \cos(LAT) \end{pmatrix}$$

B. Derivation of Radius Vector, RT, to Tracker.

$$\vec{RT} = \vec{R} + h\,\hat{U}$$

$$\vec{R} = |R|\cos\alpha \begin{pmatrix} \cos(LONG) \\ \sin(LONG) \\ 0 \end{pmatrix} + |R|\sin\alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Equation of ellipse in Z-δ plane

$$\frac{Z^2}{B^2} + \frac{\delta^2}{A^2} = 1$$

Substituting for Z, δ

$$R^2\left(\frac{\sin^2\alpha}{B^2} + \frac{\cos^2\alpha}{A^2}\right) = 1$$

$$\frac{dZ}{d\delta} = -\frac{B^2}{A^2}\ \frac{\delta}{Z}$$

$$\mathrm{Tan}(LAT) = \frac{A^2}{B^2}\ \frac{Z}{\delta}$$

$$\mathrm{Tan}\,\alpha = \frac{Z}{\delta}$$

$$R = \frac{AB}{\left(A^2 \sin^2\alpha + B^2 \cos^2\alpha\right)^{1/2}} \qquad \therefore \ \mathrm{Tan}\,\alpha = \frac{B^2}{A^2}\,\mathrm{Tan(LAT)}$$

$$R \cos\alpha = \frac{A}{\left(\frac{A^2}{B^2}\,\mathrm{Tan}^2\alpha + 1\right)^{1/2}} \qquad R \sin\alpha = \frac{B}{\left(1 + \frac{B^2}{A^2}\cot^2\alpha\right)^{1/2}}$$

Substituting for Tan α yields

$$R \cos\alpha = \frac{A}{\left(\frac{B^2}{A^2}\,\mathrm{Tan}^2(\mathrm{LAT}) + 1\right)^{1/2}} \qquad R \sin\alpha = \frac{B}{\left(1 + \frac{A^2}{B^2}\cot^2(\mathrm{LAT})\right)^{1/2}}$$

$$RT(1) = \left\{ \frac{A}{\left[\frac{B^2}{A^2}\sin^2(\mathrm{LAT}) + \cos^2(\mathrm{LAT})\right]^{1/2}} + h \right\} U(1)$$

$$RT(2) = \left\{ \frac{A}{\left[\frac{B^2}{A^2}\sin^2(\mathrm{LAT}) + \cos^2(\mathrm{LAT})\right]^{1/2}} + h \right\} U(2)$$

$$RT(3) = \left\{ \frac{B^2}{\left[B^2\sin^2(\mathrm{LAT}) + A^2\cos^2(\mathrm{LAT})\right]^{1/2}} + h \right\} U(3)$$

```
*       LABEL                                                          TRAC
*       SYMBOL TABLE                                                   TRAC
CEC2009                                                                TRAC
        SUBROUTINE TRAC (TO,AT,H,GHA,U,E,EN,RT,AC,SL,CL,ST,CT,A,B)     TRAC000
        DIMENSION RT(3),U(3),E(3),EN(3)                                TRAC001
C       TO AND LAT MUST BE INPUTED IN RAD,H IS ALTITUDE IN KM          TRAC002
        SL=SINF( AT)                                                   TRAC003
        CL=COSF( AT)                                                   TRAC004
        TA=TO+GHA                                                      TRAC005
C       GHA IS GREENWICH HOUR ANGLE,U IS UP,E IS EAST,                 TRAC006
C       EN IS NORTH UNIT VECTORS                                       TRAC007
        ST = SINF(TA)                                                  TRAC008
        CT = COSF(TA)                                                  TRAC009
        C = SQRTF(CL*CL+B*B*SL*SL/(A*A))                               TRAC010
        AC=A/C                                                         TRAC011
        U(1) = CL*CT                                                   TRAC012
        U(2) = CL*ST                                                   TRAC013
        U(3) = SL                                                      TRAC014
        RT(1) = (A/C+H)*U(1)                                           TRAC015
        RT(2) = (A/C+H)*U(2)                                           TRAC016
        RT(3) = (B*B/(A*C)+H)*U(3)                                     TRAC017
      4 E(1) = -ST                                                     TRAC018
        E(2) = CT                                                      TRAC019
        E(3) = 0.                                                      TRAC020
        EN(1) = -SL*CT                                                 TRAC021
        EN(2) = -SL*ST                                                 TRAC022
        EN(3) = CL                                                     TRAC023
        RETURN                                                         TRAC024
        END                                                            TRAC
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Subroutine: TRANSH

Purpose: To transform the partial matrix from equator and equinox of date to equator and equinox of 1950. The logic key is used to indicate if the last three partials are zero (LL = 1) or if all six partials have values (LL = 2).

Calling Sequence:

CALL TRANSH

Input and Output

I/O	Symbolic Name or Location	Program Dimensions	Mathematical Symbol	Data Dimensions or Units	Definition
IO	H	(1,6)	$\frac{\partial \text{ MEAS}}{\partial \text{ STATE}}$		Partials of measurements WRT*
					the state
I	LL	1			Logic key
					* With Respect To

Common storages used or required: T, S, C, IC

Subroutines required: None

Functions required: None

Approximate number of storages required:

```
*       LABEL                                                           TRAN
*       SYMBOL TABLE                                                    TRAN
        SUBROUTINE TRANSH(H,LL)                                         TRAN0000
C       SUBROUTINE TRANSFORMS H MATRIX TO EQUINOX 1950                  TRAN0010
        EQUIVALENCE(T,TDUM),(S,SDUM),(C,CDUM),(C(138),AN)               TRAN0020
        DIMENSION H(6),AN(3,3),DUM(1,6),T(1360),S(1000),C(1000),IC(1)   TRAN0030
        COMMON T,S,C,IC                                                 TRAN0040
        LL=LL                                                           TRAN0050
        DO 1 I=1,3                                                      TRAN0060
        K=I+3                                                           TRAN0070
        DUM(I)=0.                                                       TRAN0080
        DUM(K)=0.                                                       TRAN0090
        DO 1 J=1,3                                                      TRAN0100
        DUM(I)=DUM(I)+AN(J,I)*H(J)                                      TRAN0110
        GO TO(1,2),LL                                                   TRAN0120
    2   CONTINUE                                                        TRAN0130
        L=J+3                                                           TRAN0140
        DUM(K)=DUM(K)+AN(J,I)*H(L)                                      TRAN0150
    1   CONTINUE                                                        TRAN0160
        DO 3 I=1,6                                                      TRAN0170
        H(I)=DUM(I)                                                     TRAN0180
    3   CONTINUE                                                        TRAN0190
        RETURN                                                          TRAN0200
        END                                                             TRAN
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