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SOME NOTES ON LIQUID SLOSHING

IN COMPARTMENTED CYLINDRICAL TANKS

by

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INTRODUCTION

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As one relatively simple means of avoiding dynamic coupling between sloshing of liquid propellants and automatic control system response, or elastic body response, it has been suggested that propellant tanks be compartmented so as to raise or otherwise modify the normal sloshing frequencies. There is also the possibility of an overall reduction in the total force response as a result of phasing of the liquid motions in different compartments. Frequencies are also increased in the case of clustered tank configurations (over that for a single tank of the same capacity), but the weight penalty is more severe and additional complications may be introduced through other dynamic coupling effects.

The present paper gives some results of experimental studies of frequencies and total force response in rigid cylindrical tanks compartmented into sectors by vertical walls and excited in translation. These data are correlated with theoretical values, where available. Some theoretical values for cylindrical tanks with annular cross-sections are also shown for comparative purposes. The experimental equipment and procedures are similar to those employed in (1)*.

Author

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* Numbers in parentheses refer to the References given at the end of the paper.

TANK CONFIGURATIONS

Several different compartmented tank configurations are considered in this paper, as shown in Figure 1. All of these are cylindrical with flat bottoms, filled with liquid to a depth h, and subjected to an axial acceleration a. In all cases, the vertical walls extend from above the liquid free surface to the tank bottom. The direction of the translational excitation is as shown in Figure 1.

LIQUID NATURAL FREQUENCIES

It is well-known that the liquid natural frequencies in a cylindrical tank are virtually independent of liquid depth for $h/d \geqslant 1$. Table 1 gives values of liquid natural frequencies, in terms of the dimensionless parameter $\omega_n^2 d/a$, for the various compartmented tank configurations for $h/d \geqslant 1$. The data, partly theoretical and partly experimental, is taken from (2-5) and the results of the present study. It is noted that the half and quarter tank configurations have their lowest natural frequencies considerably above that for the uncompartmented tank; but the eighth (45^o sector) tank has a very much higher value than any of the other configurations. Of course, as the liquid depth decreases ($h/d < 1$) the natural frequencies also decrease, but the relative values for the various configurations remain essentially unchanged (Fig. 2). As shown in Table 1, there is some difference between theoretical (4) and experimental values of the natural frequency parameter for the quarter tank.

The annular tank (4, 5) exhibits an unusual variation of natural frequencies of the outer ring compartment with diameter ratio k. The lowest natural frequency (for $h/d > 1$) remains substantially the same or slightly less than for the uncompartmented tank, reaching a minimum value of $\omega_n^2 d/a$ of approximately 2.0 at k = 0.8. On the other hand, the second resonant mode (again for $h/d > 1$) shows a very large increase in frequency, reaching a maximum value of $\omega_n^2 d/a$ of more than 30.0 at k = 0.8. The variation of these frequencies with depth for $h/d < 1$, for the single case of k = 0.5, is shown in Figure 2. The frequency variations for the inner cylindrical tank are, of course, the same as for an uncompartmented tank, based on the appropriate inner tank diameter.

The essential effect of sector compartmentation is to raise the lowest liquid frequency and to introduce new resonant frequencies, as shown in Table 1. The lower frequencies, corresponding to resonances of various sector combinations, are generally not widely separated.

Because of the weight penalty associated with increasing numbers of sectors, consideration has also been given to effects of wall perforation. For a quarter tank with perforated sector walls of 23% opening, it was found that the essential effect was to provide sector fluid exchange resulting in an effective geometrical configuration intermediate between the uncompartmented tank and the solid wall quarter tank. The two different sector resonances are blended into one, as a consequence of the liquid interchange. However, it is also required that the perforation hole size be maintained relatively small in order that the lowest resonant frequencies do not decrease to the order of those of the uncompartmented tank. This effect is shown in Figure 3 where the natural frequency parameter $\omega_n^2 d/a$ is plotted against an equivalent Reynolds Number (1), based on perforation hole diameter*. The value of the natural frequency parameter is seen to vary from between 5.0 and 6.0, corresponding to the solid wall quarter tank ($R_{NH} \longrightarrow 0$), to somewhat more than 3.0, approximately the value for the uncompartmented tank ($R_{NH} \longrightarrow \infty$), The transition zone is clearly dependent upon the excitation amplitude and probably on liquid depth (for $h/d < 1$) as well.

FORCED VIBRATION RESPONSE

Experimental data for total force response in uncompartmented, half, and quarter tanks with solid walls are shown in Figure 4 in terms of force magnitude and phase versus excitation frequency. Similar data is

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* The experimental data of Figure 3 was obtained for a tank with $d = 14.4$ in., perforation hole sizes of 0.020 and 0.078 in., and with water and methylene chloride as test liquids.

shown in Figure 5 for an uncompartmented tank and for a quarter tank with both solid and perforated walls. It can readily be seen that a slightly greater amount of total force damping results from using perforated walls, compared to solid sector walls, and that all of these provide some amplitude damping over the non-compartmented tank.

Results similar to those shown in Figures 4 and 5 have been obtained for various other liquid depths ($h/d < 1$) and excitation amplitudes, but are not presented in this paper in the interests of brevity.

Calculated values of total force response in annular tanks are given in (5). For quarter tanks, the appropriate equations for total force response are given in (4), but calculated results are not yet available.

DISCUSSION

In general, one may come to several conclusions regarding the effects of tank sectorization, although these are not clear-cut and lead at times to conflicting opinions regarding the advantages of such compartmentation. First, compartmentation by sectors leads to higher resonant frequencies (the increase becomes very large for sector angles less than 90°), and in the case of solid sector walls at least the lowest frequencies are not very widely separated (Fig. 4). Perforation of the sector walls by holes of proper size may serve the purpose of weight reduction and at the same time provide for liquid interchange between sectors which blends the two lowest liquid resonant frequencies into one; however, the effects of perforation on the creation of turbulence in the liquid may be undesirable for certain types of propellants. Finally, the amount of force amplitude damping provided by compartmentation, while significant, is certainly much less than can be obtained through more customary ring baffle arrangements and probably involves greater weight penalties.

The attractiveness of the annular tank resides in the wide separation between freqeuncies of the inner and outer compartments and in the fact that phasing of the liquid motions between the two compartments results in a lower total force. Values of the diameter ratio $0.5 < k < 0.9$ give not only the widest separation of frequencies but also almost equal fluid masses in the two compartments so that force cancellation is maximized.

Thus, it appears that final selection of a tank configuration must depend on the overall vehicle dynamics; in some cases it may be sufficient to provide adequate force damping by means of ring baffles, while in others it may be more desirable to modify the propellant sloshing frequencies by compartmentation. But, in either case, an analysis of the overall vehicle dynamics must be undertaken which includes some representation of the effects of the sloshing liquid. Equivalent mechanical models are often employed in such dynamic analyses (6); appropriate models for quarter and annular tanks are given in (4, 5).

REFERENCES

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2. Abramson, H. N., and Ransleben, G. E., Jr., "Total Force Response Resulting From Liquid Sloshing in a Rigid Cylindrical Tank With a Vertical Center Wall Baffle," Tech. Rept. No. 9, Contract No. DA-23-072-ORD-1251, Southwest Research Institute, May 1961.

3. Unpublished data, The Martin Company, Baltimore, 1960.

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5. Bauer, H., "Theory of the Fluid Oscillations in a Circular Cylindrical Ring Tank Partially Filled With Liquid," NASA Tech. Note D-557 (December 1960).

6. Abramson, H. N., Chu, W. H., and Ransleben, G. E., Jr., "Representation of Fuel Sloshing in Cylindrical Tanks by an Equivalent Mechanical Model," ARS Journal, 31, 12, pp. 1697-1705, December 1961.

TABLE 1. VALUES OF LIQUID NATURAL FREQUENCY PARAMETER $\omega_n^2 d/a$ FOR VARIOUS TANK CONFIGURATIONS FOR $h/d \geqslant 1$

Tank Configuration	Lowest Natural Frequency theory	Lowest Natural Frequency experiment	Second Natural Frequency theory	Second Natural Frequency experiment
Uncompartmented	3.68^{*}	3.6^{*}	10.66^{**}	10.6^{**}
Half	----	$5.1^{\square}$	----	$6.1^{\square\square}$
Quarter (90° sector)	$6.1^{+\nabla}$	5.0^{+}	$7.6^{++\nabla}$	6.5^{++}
Eighth (45° sector)	----	$7.7^{\triangle}$	----	$10.6^{\triangle\triangle}$
Annular ($k = 0.2$)	$3.4^{\Diamond}$	----	$9.9^{\Diamond}$	----
Annular ($k = 0.5$)	$2.7^{\Diamond}$	----	$13.1^{\Diamond}$	----
Annular ($k = 0.8$)	$2.2^{\Diamond}$	----	$31.6^{\Diamond}$	----

Notes:

$*$	first mode normal sloshing resonance
$**$	second mode normal sloshing resonance
$\square$	in-phase resonance (exp., Ref. 2)
$\square\square$	out-of-phase resonance (exp., Ref. 2)
$+$	sector 2, 4 resonance
$++$	sector 1, 3 resonance
$\triangle$	sector 1, 5 resonance (exp., Ref. 3)
$\triangle\triangle$	sector 3, 7 resonance (exp., Ref. 3)
∇	theoretical values (Ref. 4)
$\Diamond$	outer (ring) tank resonance (theory, Ref. 5)

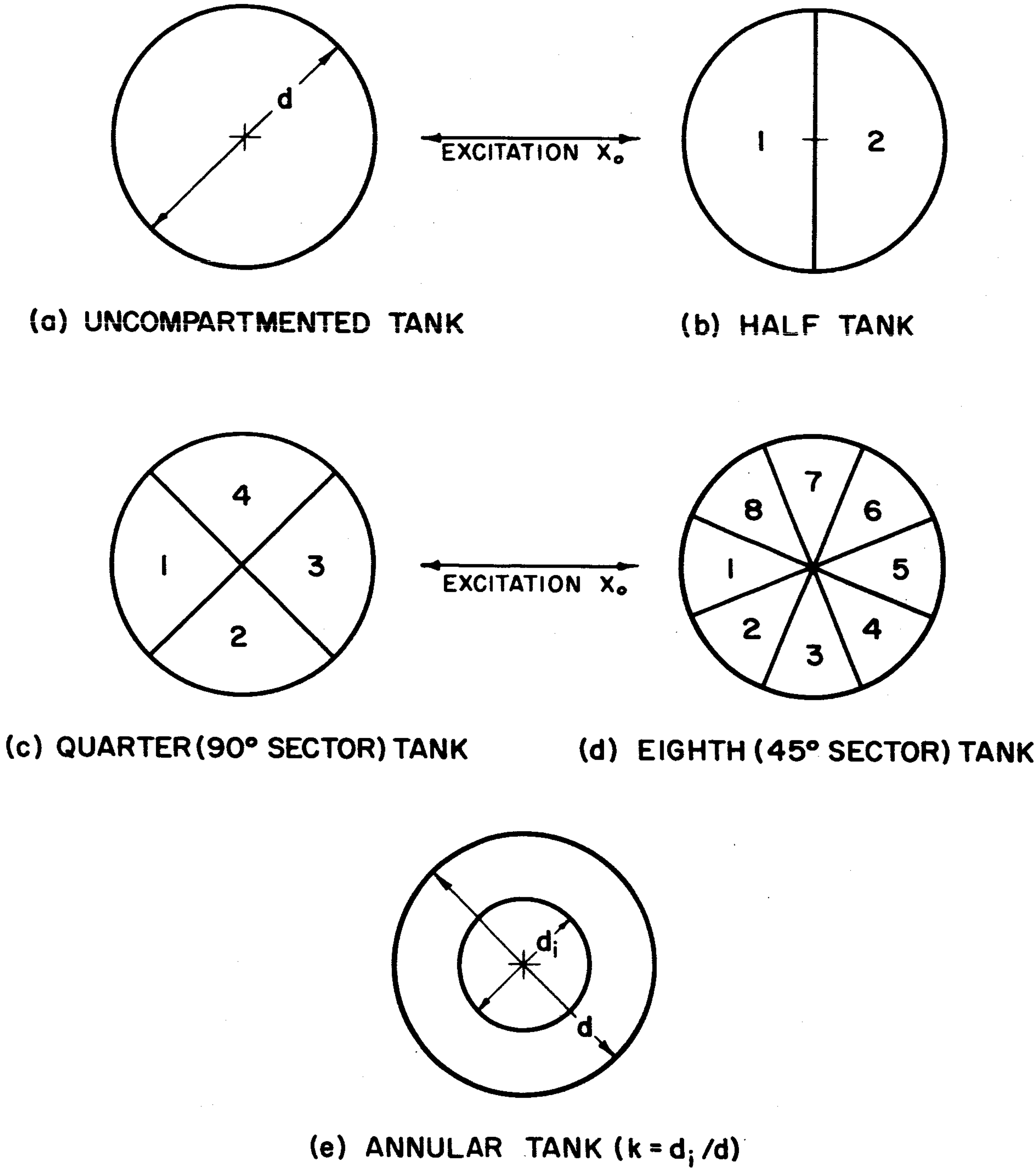

FIGURE I. TANK CONFIGURATIONS

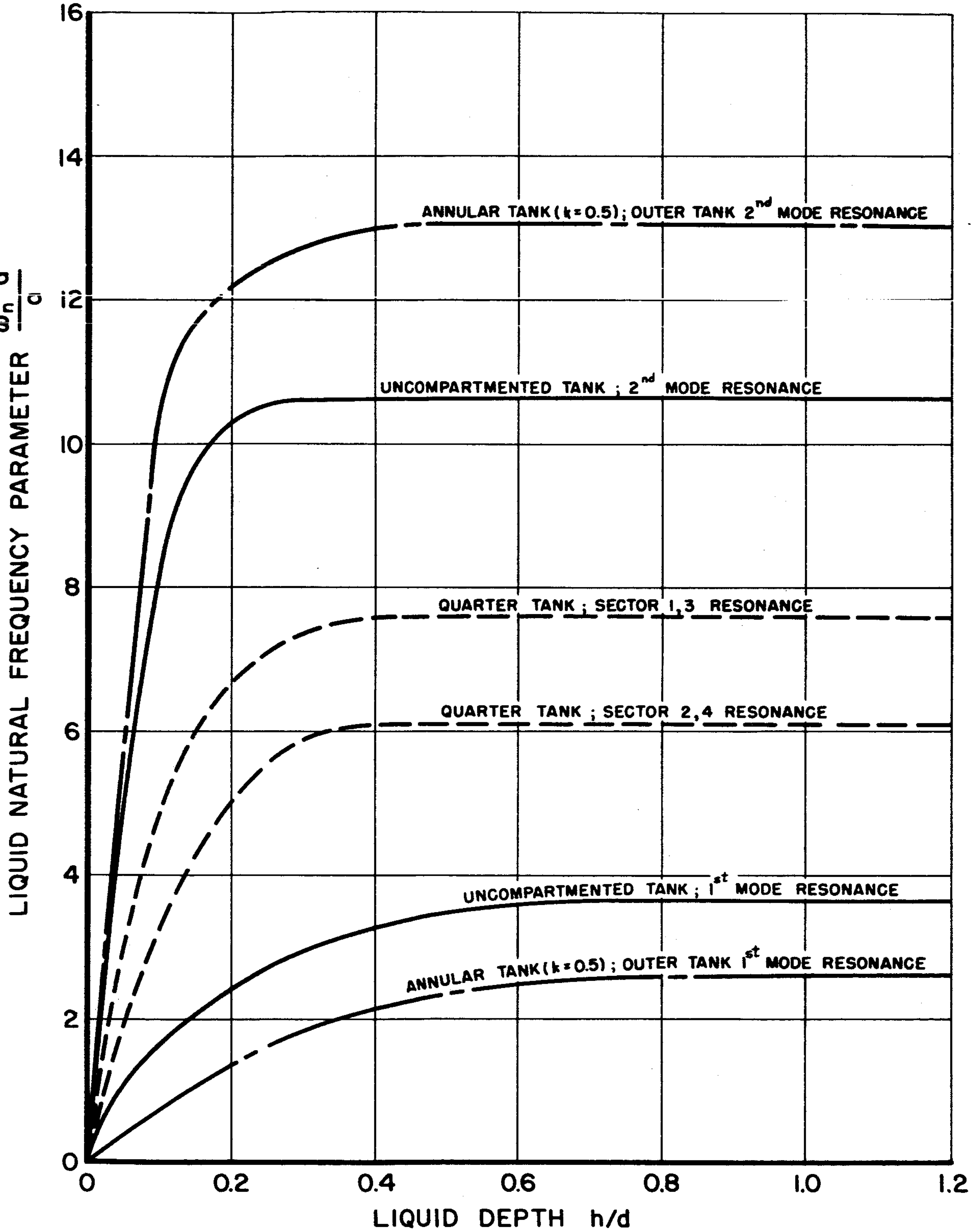

FIGURE 2. VARIATION IN LIQUID NATURAL FREQUENCIES WITH LIQUID DEPTH FOR VARIOUS TANK CONFIGURATIONS (THEORETICAL VALUES)

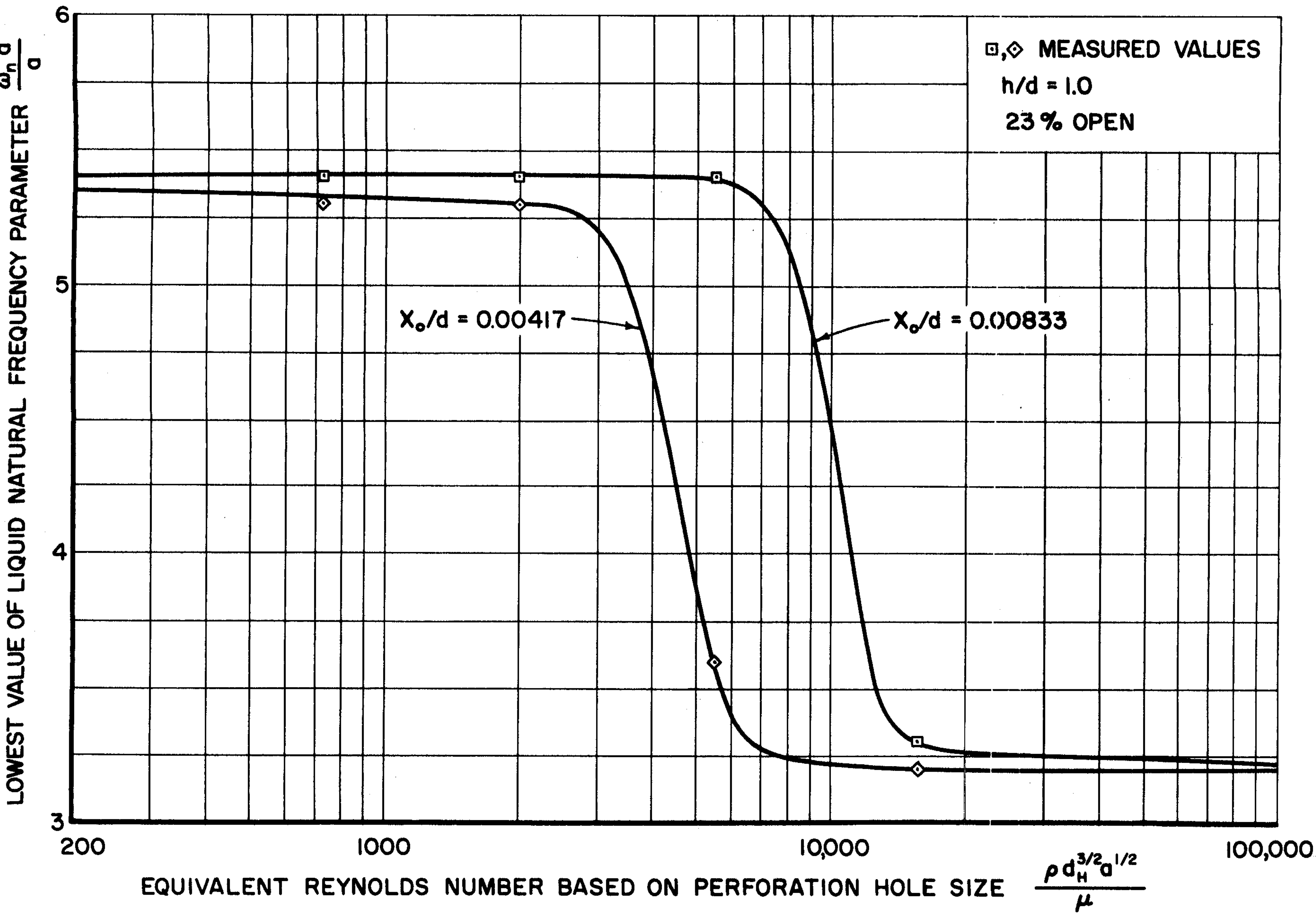

FIGURE 3. LIQUID NATURAL FREQUENCY VARIATION FOR A QUARTER TANK WITH PERFORATED SECTOR WALLS

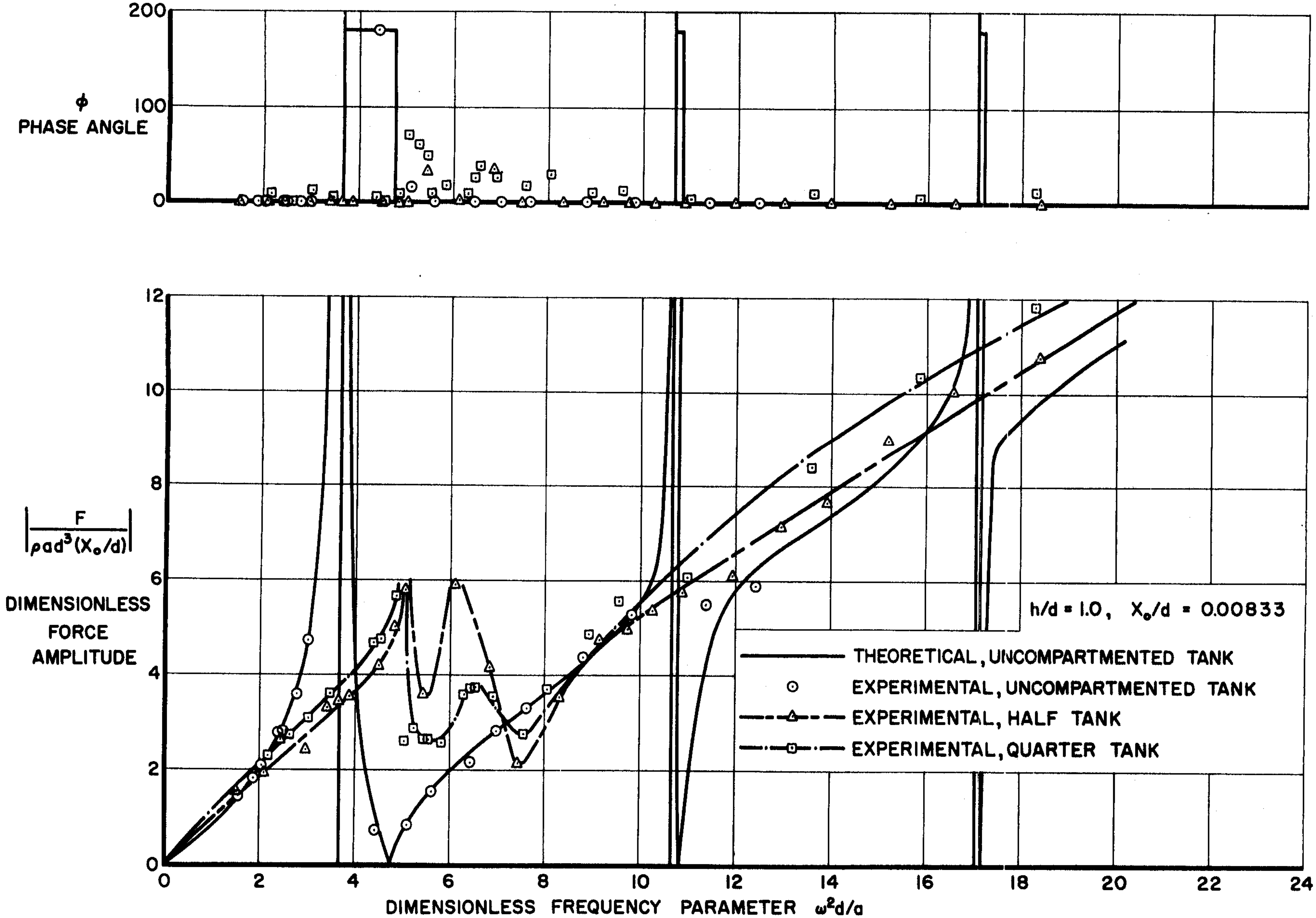

FIGURE 4. TOTAL FORCE RESPONSE FOR UNCOMPARTMENTED, HALF, AND QUARTER TANKS WITH SOLID WALLS

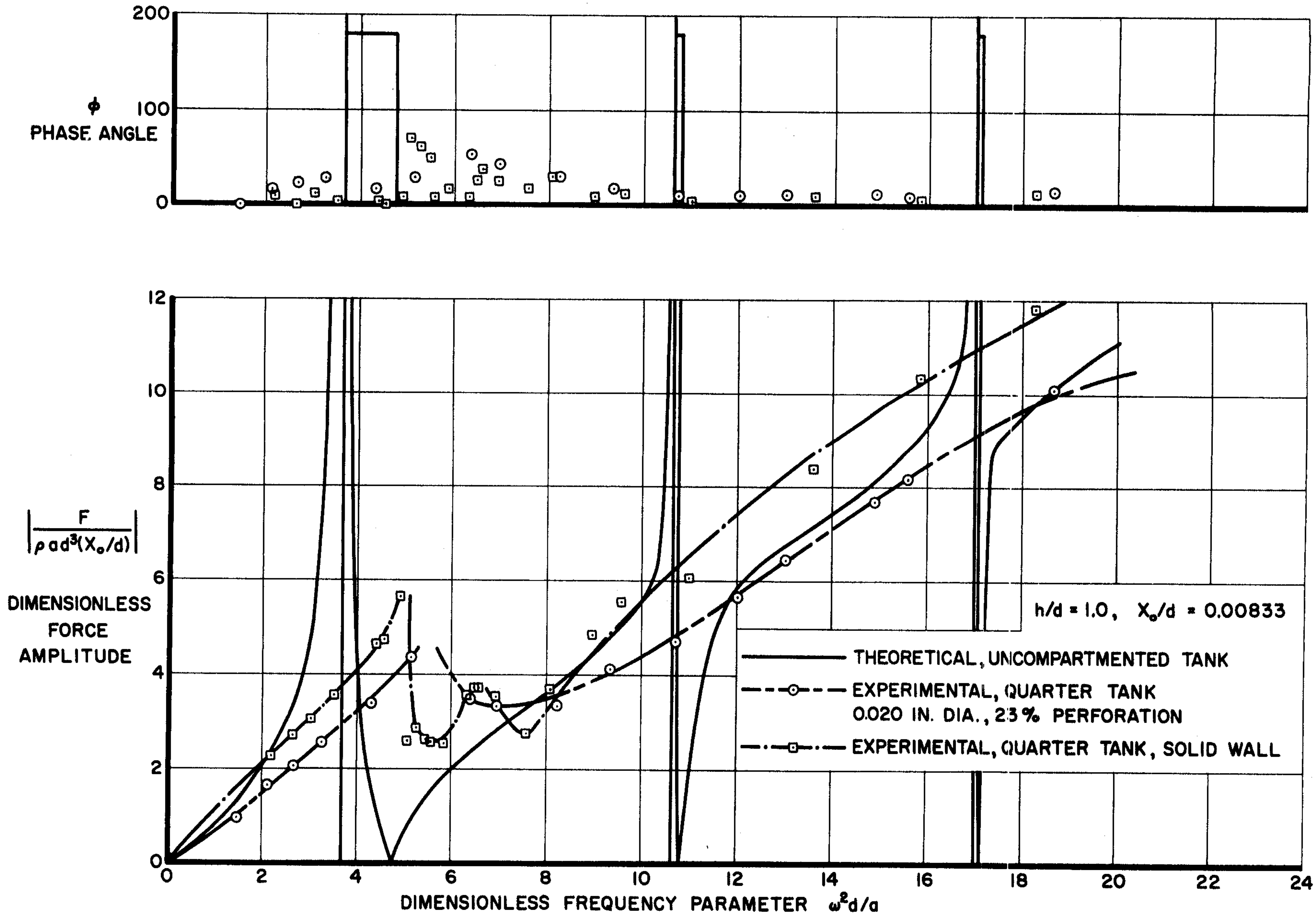

FIGURE 5. TOTAL FORCE RESPONSE FOR A QUARTER TANK WITH PERFORATED SECTOR WALLS