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SUMMARY

A boundary-value-collocation procedure was applied in conjunction with the Williams stress function to determine values of the stress-intensity factor K_I for single-edge cracks in plate specimens subject to splitting forces applied close to the crack and acting transversely to it. The results are presented in terms of the dimensionless quantity $Y = K_I BH^{3/2}/Pa$, where B and H are the specimen thickness and half-depth, a is the effective crack length, and P is the applied load. The results are practically independent of the ratio of effective specimen length to specimen half-depth q/H, when this ratio is not less than $a/H + 2$, and are then in excellent agreement with those derived by other investigators from compliance measurements. In the limit, as H/a approaches zero, the value of Y approaches that obtained in an elementary analysis which treats the specimen as a pair of built-in cantilever beams.

INTRODUCTION

Plate specimens of the general configuration shown in figure 1 are used for plane strain fracture toughness measurements (ref. 1) and for fatigue crack propagation rate measurements. This type of specimen is essentially a narrow plate with a crack extending from one of the short edges along the longitudinal centerline. The forces P are applied close to the crack and act transversely to it, thus tending to split the specimen by extending the crack. Such specimens are referred to in this report as single-edge-notch specimens subject to splitting forces. They are sometimes referred to elsewhere as crackline loaded specimens, or as double cantilever specimens. The length of the crack a is taken as the distance from the end of the crack to the line of action of the load P. (All symbols used in this report are defined in the appendix.)

A first approximation to the strain energy release rate with crack extension for such a specimen is readily calculated from simple beam theory by regarding the specimen as a pair of built-in cantilever beams. This approach was used by Gilman in a study of the mechanics of cleavage (ref. 2). In terms of the stress-intensity factor K_I, the result obtained is that K_I is equal to

$2\sqrt{3}\ Pa/H^{3/2}$ for small values of H/a, where H is the depth of each cantilever arm (ref. 3). A better approximation was obtained by Ripling, Mostovoy, and Patrick (ref. 4), who modified the cantilever beam treatment and used experimental compliance measurements to establish the best value for a correction term.

Because of the potential usefulness of this type of specimen, it was considered desirable to conduct an independent study of the configuration by the boundary-collocation procedure used in earlier studies of other types of single-edge-notch specimens by Gross, Srawley, and Brown (refs. 5, 6, and 7). It was decided that this study should also include an evaluation of the effect of a variable not considered in reference 4, namely, the ratio of the effective specimen length to the effective crack length q/a. It is apparent that the stress-intensity factor will be independent of q/a for sufficiently large values of that ratio, but it is of practical interest to know how large a ratio is sufficient.

In general, the stress-intensity factor K_I will depend on the five variables P, a, q, H, and B (fig. 1). It is convenient to express the relation between K_I and four of these variables in dimensionless form in terms of the stress-intensity coefficient $Y = K_I BH^{3/2}/Pa$, which is a function of the ratios a/H and q/H. For practical purposes, the dependence of Y on q/H is negligible when q/H is sufficiently larger than a/H. The relation between Y and a/H for sufficiently large q/H will be referred to as the K-calibration for the single-edge-notch specimen subject to splitting forces. The K-calibration results discussed in this report cover a range of values of a/H from 2 to 10, which is sufficient for practical purposes. For each value of a/H, results were computed for values of the parameter q/H of $a/H + 1$, $a/H + 2$, and $a/H + 3$ sufficient to define the practical limit of influence of this parameter. For a/H equal to 4, results were also computed for q/H equal to $a/H + 6$.

ANALYTICAL AND COMPUTATIONAL PROCEDURE

The method of analysis consists in finding a stress function X that satisfies the biharmonic equation $\nabla^4 X = 0$ and also the boundary conditions at a finite number of stations along the boundary of a single-edge-notched specimen, such as shown in figure 2. The biharmonic equation and the boundary conditions along the crack are satisfied by the Williams stress function (refs. 8 and 9). Because of symmetry (fig. 1), the coefficient of the sine terms in the general stress function must be zero, hence

$$X(r,\theta) = \sum_{n=1,2\ldots}^{\infty} \left\{ (-1)^{n-1} d_{2n-1} r^{n+(1/2)} \left[-\cos\left(n - \frac{3}{2}\right)\theta + \frac{2n-3}{2n+1}\cos\left(n + \frac{1}{2}\right)\theta \right] + (-1)^n d_{2n} r^{n+1} \left[-\cos(n-1)\theta + \cos(n+1)\theta \right] \right\} \quad (1)$$

The stresses in terms of χ obtained by partial differentiation are as follows:

$$\left.\begin{aligned}
\sigma_y &= \frac{\partial^2\chi}{\partial x^2} = \frac{\partial^2\chi}{\partial r^2}\cos^2\theta - 2\frac{\partial^2\chi}{\partial\theta\,\partial r}\frac{\sin\theta\cos\theta}{r} + \frac{\partial\chi}{\partial r}\frac{\sin^2\theta}{r} \\
&\qquad + 2\frac{\partial\chi}{\partial\theta}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial^2\chi}{\partial\theta^2}\frac{\sin^2\theta}{r^2} \\
\sigma_x &= \frac{\partial^2\chi}{\partial y^2} = \frac{\partial^2\chi}{\partial r^2}\sin^2\theta + 2\frac{\partial^2\chi}{\partial\theta\,\partial r}\frac{\sin\theta\cos\theta}{r} + \frac{\partial\chi}{\partial r}\frac{\cos^2\theta}{r} \\
&\qquad - 2\frac{\partial\chi}{\partial\theta}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial^2\chi}{\partial\theta^2}\frac{\cos^2\theta}{r^2} \\
-\tau_{xy} &= \frac{\partial^2\chi}{\partial x\,\partial y} = \sin\theta\cos\theta\frac{\partial^2\chi}{\partial r^2} + \frac{\cos 2\theta}{r}\frac{\partial^2\chi}{\partial r\,\partial\theta} - \frac{\sin\theta\cos\theta}{r^2}\frac{\partial^2\chi}{\partial\theta^2} \\
&\qquad - \frac{\sin\theta\cos\theta}{r}\frac{\partial\chi}{\partial r} - \frac{\cos 2\theta}{r^2}\frac{\partial\chi}{\partial\theta}
\end{aligned}\right\} \tag{2}$$

The boundary-collocation procedure (ref. 5) consists in solving 2m simultaneous algebraic equations corresponding to the known values of χ and either $\partial\chi/\partial x$ or $\partial\chi/\partial y$ at m selected stations along the boundary ABCD of figure 2; thus, values for the first 2m coefficients of the Williams stress function are obtained when the remaining terms are neglected. Only the value of the first coefficient d_1 is used for the present purpose since the stress-intensity factor K_I is equal to $-\sqrt{2\pi}\,d_1$, as shown in reference 5.

The required values of χ and its first derivatives at the m selected boundary stations were obtained from distributions of bending moment and parabolic shear stress distribution (fig. 2) equivalent to the actual loads shown in figure 1. The equations for these boundary values in dimensionless form are as follows:

Along A-B:

$$\frac{\chi}{P} = -\frac{6(a - V)}{Bh^3}\left(\frac{y^3}{3} - \frac{hy^2}{2}\right)$$

$$B\frac{\partial\left(\frac{\chi}{P}\right)}{\partial x} = -\frac{6}{h^3}\left(\frac{y^3}{3} - \frac{hy^2}{2}\right)$$

Along B-C:

$$\frac{\chi}{P} = \frac{x + a}{B}$$

$$B\frac{\partial}{\partial y}\left(\frac{\chi}{P}\right) = 0$$

Along C-D:

$$\frac{\chi}{P} = \frac{q}{B}$$

$$B\frac{\partial}{\partial x}\left(\frac{\chi}{P}\right) = 1 \tag{3}$$

The distance V of the end boundary from the crack tip (fig. 2) was chosen to be 0.8 a. From physical considerations, it is clear that the boundary should be chosen neither close to the crack nor close to the load point because of the stress-field disturbances near these positions. Preliminary studies similar to those described in reference 5 established that V/a equal to 0.8 was about optimum, and that minor variations from 0.8 had a negligible effect on the K-calibration.

For each set of selected values of the primary variable a/H and the parameter q/H, the collocation computation was carried successively over the range of m equal to 21 to 48 stations in increasing increments of three. Figure 3 is fairly representative of the resultant variation of the K-calibration against the number of boundary stations. Except for one case, the variation of the dimensionless stress-intensity factor Y obtained for m equal to 42, 45, and 48 was less than 1 percent and in most cases it was a small fraction of a percent. For a/H equal to 2 and q/a equal to 5/2, the variation was approximately $1\frac{1}{4}$ percent. From this observation taken together with the nature of the trends of the Y values, the K-calibration results shown in table I were based on the average value of Y for m equal to 42, 45, and 48.

RESULTS AND DISCUSSION

Table I shows a comparison of results of the present work with corresponding results from reference 4 in terms of the dimensionless quantity $K_I BH^{3/2}/Pa$ as a function of relative crack length a/H. Over the range of relative crack lengths a/H varying from 2 to 10, the effect of relative specimen length q/H becomes negligible when not less than $a/H + 2$. Figure 3 shows a typical result in evaluating the effects of a large q/H ratio, $a/H + 6$, on the dimensionless quantity Y. The agreement between the analytical results for q/H greater than or equal to $a/H + 2$ and the experimental results of reference 4 is within 1.3 percent for all cases.

For a/H less than 2, from physical considerations one can conclude that an interaction between the stress field of the applied load and the crack tip is likely to occur. Thus, use of simple beam theory to obtain the stress function and its associated derivative at location V, would be an oversimplification of a complex condition.

The plot of Y against the reciprocal of relative crack length (fig. 4) is well fitted by a straight line passing through $Y = 3.46$ at $H/a = 0$. The value 3.46 is equal to the numerical coefficient $2\sqrt{3}$ obtained as a first approximation from simple beam theory, as discussed in the introduction. Thus, extrapolation of the collocation results to the limiting case of a very long, slender specimen leads to a value which is in good agreement with the elementary treatment.

K_{Ic} MEASUREMENT CAPACITY IN RELATION TO SPECIMEN DEPTH

The currently accepted criterion for evaluating K_{Ic}, the critical value of K at point of instability of crack extension in first or open mode, is that the nominal stress at the crack tip should not exceed the yield strength of the material σ_{ys} in a valid K_{Ic} test. Thus, the maximum nominal stress $6Pa/BH^2$ should not exceed σ_{ys}, which, on substituting in expression $Y^2 = K^2B^2H^3/P^2a^2$ gives $C_{IK}^2/\sigma_{ys}^2(2H) = Y^2/72$, where C_{IK} is used to denote the maximum acceptable value K_{Ic}. Figure 5 shows the dependence of dimensionless K_{Ic} measurement capacity on the relative crack length a/H for a sufficiently large effective specimen length. The specimen measurement capacity decreases with increasing relative crack length. For relative crack lengths greater than 6, the drop in capacity becomes exceedingly small.

The dependency of K_{Ic} measurement capacity on the specimen thickness is discussed at some length in reference 1 and is unrelated to the present work.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 9, 1965.

APPENDIX - SYMBOLS

a	effective crack length
B	specimen thickness
C_{IK}	estimate of maximum value of K_{Ic} that can be measured with specimen of given dimensions and yield strength
d_{2n}, d_{2n-1}	coefficients of Williams stress function
H	depth of arm of split beam
K_I	stress-intensity factor of elastic stress field in vicinity of crack tip
K_{Ic}	critical value of K at point of instability of crack extension in first or open mode, a measure of plane strain crack toughness of material
m	number of selected boundary stations for collocation solution
P	load applied to both ends of split beam
q	effective specimen length
r	polar coordinate referred to crack tip
V	distance from crack tip to boundary for collocation analysis
x,y	Cartesian coordinates referred to crack tip
Y	dimensionless stress-intensity coefficient, $KBH^{3/2}/Pa$
θ	polar coordinate referred to crack tip
σ_x	stress component in x-direction
σ_y	stress component in y-direction
σ_{ys}	0.2 percent offset tensile yield strength
τ_{xy}	shearing stress component
χ	stress function

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TABLE I. - COMPARISON OF RESULTS OF PRESENT WORK WITH CORRESPONDING RESULTS FROM REFERENCE 4 IN TERMS OF THE DIMENSIONLESS QUANTITY $K_I B H^{3/2}/Pa$ AS FUNCTION OF RELATIVE CRACK LENGTH

Relative crack length, a/H	Results of -			
	Boundary-collocation procedure			Reference 4 (experimental results)
	$\frac{q}{H} = \frac{a}{H} + 1$	$\frac{q}{H} = \frac{a}{H} + 2$	$\frac{q}{H} = \frac{a}{H} + 3$	
	$Y = K_I B H^{3/2}/Pa$			
2	5.88	4.62	4.65	4.61
3	5.44	4.25	4.24	4.21
4	5.22	4.06	4.05	4.01
5	5.09	3.94	3.93	3.90
6	5.00	3.86	3.85	3.83
7	4.93	3.81	3.80	3.78
8	4.90	3.77	3.76	3.73
9	4.87	3.73	3.72	3.70
10	4.82	3.71	3.72	3.67

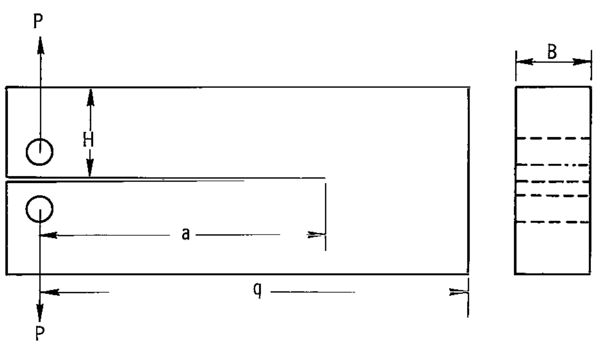

Figure 1. - Edge-cracked plate specimen subject to splitting forces.

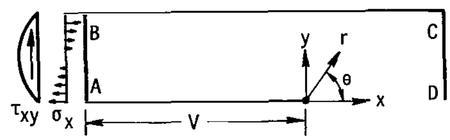

Figure 2. - Assumed equivalent distribution of boundary stresses at distance V from end of crack.

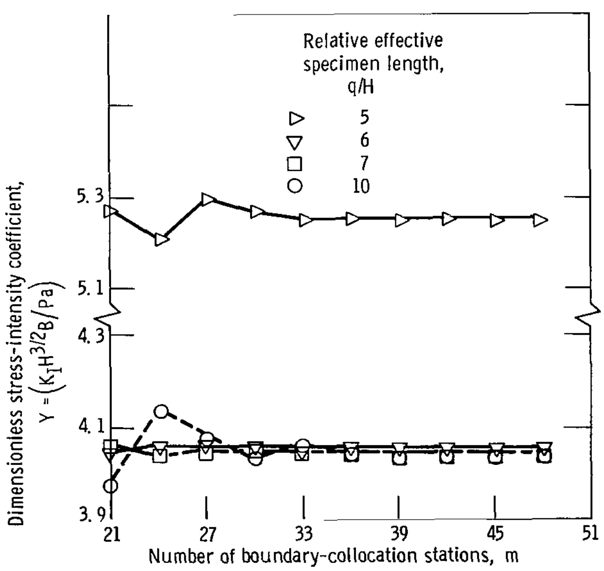

Figure 3. - Value of dimensionless stress-intensity coefficient Y against number of boundary-collocation stations for relative crack length a/H = 4.

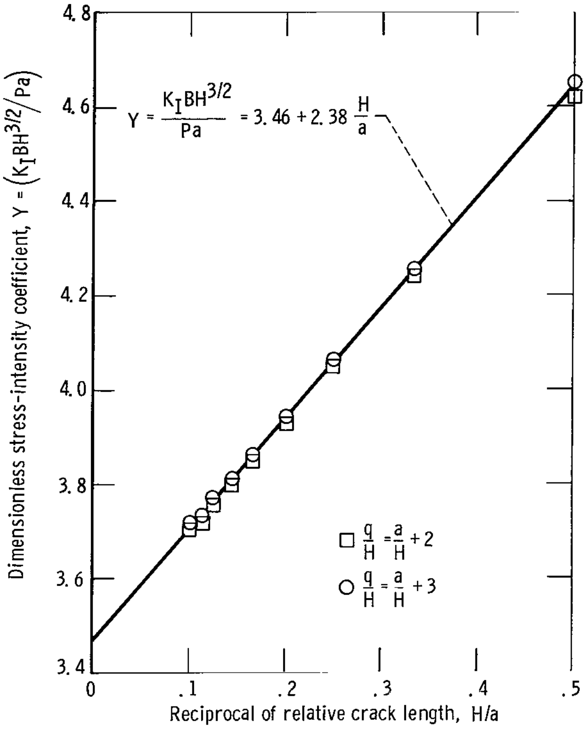

Figure 4. - Dimensionless stress-intensity coefficient as function of reciprocal of relative crack length.

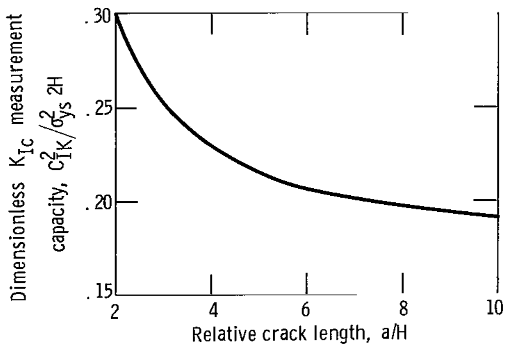

Figure 5. - Dependence of K_{Ic} measurement capacity C_{IK} of single-edge-notched specimens subjected to splitting forces on relative crack length.

NASA-Langley, 1966 E-3271

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