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JUNCTURE STRESS FIELDS IN MULTICELLULAR SHELL STRUCTURES

Final Report

Nine Volumes

Vol. VII Buckling Analysis of Segmental Orthotropic Cylinders Under Uniform Stress Distribution

by

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FOREWORD

This report is the result of a study on the buckling of segmental orthotropic cylinders under uniform axial compression. Work on this study was performed by staff members of Lockheed Missiles & Space Company in cooperation with the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration under Contract NAS 8-11480. Contract technical representative was H. Coldwater.

This volume is the the seventh of a nine-volume final report of studies conducted by the department of Solid Mechanics, Aerospace Sciences Laboratory, Lockheed Missiles & Space Company. Project Manager was K. J. Forsberg; E. Y. W. Tsui was Technical Director and Professor D. O. Brush of the University of California at Davis was a Consultant for the work.

The nine volumes of the final report have the following titles:

Vol. I	Numerical Methods of Solving Large Matrices
Vol. II	Stresses and Deformations of Fixed-Edge Segmental Cylindrical Shells
Vol. III	Stresses and Deformations of Fixed-Edge Segmental Conical Shells
Vol. IV	Stresses and Deformations of Fixed-Edge Segmental Spherical Shells
Vol. V	Influence Coefficients of Segmental Shells
Vol. VI	Analysis of Multicellular Propellant Pressure Vessels by the Stiffness Method
Vol. VII	Buckling Analysis of Segmental Orthotropic Cylinders under Uniform Stress Distribution
Vol. VIII	Buckling Analysis of Segmental Orthotropic Cylinders under Non-uniform Stress Distribution
Vol. IX	Summary of Results and Recommendations

SUMMARY

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This volume presents a theoretical solution to the buckling problem of orthotropic cylindrical panels under uniform axial compression. The panels are simply supported along the curved edges and elastically supported along the straight edges. There is included a digital computer program that was used to obtain design curves which give non-dimensional buckling loads for a wide range of geometric parameters for the cylindrical panels. Derivation of the expressions for the bending influence coefficients of orthotropic rectangular plates is also included.

Author

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NOTATION

$A_1 = d_1 h_1; A_2 = d_2 h_2$	cross-sectional areas of stringer and ring
$D = Et^3/12(1 - \nu^2)$	flexural rigidity of shell
$\hat{e}_1, \hat{e}_2$	eccentricities of stringer and ring
E, E_1, E_2	moduli of elasticity of the shell, stringer, and ring
$G = E/2(1 + \nu)$	shear modulus of shell
I_1, I_2	moments of inertia about the centroidal axis of stringer and shell; ring and skin
$\bar{I}_1, \bar{I}_2$	moments of inertia about the neutral axis of stringer and ring
J_1, J_2	torsional constants for the cross section of stringer and ring
K	buckling coefficient
L	length of panel
$\hat{M}_{( )}, \hat{N}_{( )}$	moments and stress resultants
$\hat{N}_{cr}$	critical buckling load
n	number of half-waves in the axial direction
P_x	total axial buckling load of the panel
R	radius of cylinder
r, s	spacings of rings and stringers
t	thickness of shell
$\hat{U}$	strain energy
$\hat{u}, \hat{v}, \hat{w}$	middle-surface displacement components in $\hat{x}$, $\hat{y}$, and $\hat{z}$ directions

$\hat{x}, \hat{y}, \hat{z}$	curvilinear coordinates for the middle-surface of shell
$\hat{V}$	total potential energy
$\hat{W}$	potential energy of external load
ν, ν_1, ν_2	Poisson's ratios of shell, stringer and ring
$\epsilon_{(\ )}, \gamma_{(\ )}$	direct and shear strains
$\chi_{(\ )}$	change of curvature or twist of middle-surface of shell
$(\ )_{,x}$	$\partial(\ )/\partial x$
Ω	angle subtending the width of panel (see Fig. 1)
$\theta = \hat{y}/R$	angular coordinate

$$\rho = 1 + (1 - \nu^2)\left(\frac{E_1}{E}\right)\left(\frac{d_1}{s}\right)\left(\frac{h_1}{t}\right)\left[\left(\frac{h_1}{t}\right)^2 + \frac{3(h_1/t + 1)^2}{(1 - \nu^2)(E_1/E)(d_1/s)(h_1/t) + 1}\right]$$

$\gamma^2 = \frac{1}{12}\left(\frac{t}{R}\right)^2$	constant

Section 1
INTRODUCTION

When multiceullular configurations are employed as fuel tanks or interstages of space vehicles, buckling of the cylindrical panels under axial compression may be critical as far as the load-carrying capacity of the structure is concerned. In general, the loads on such panels are distributed in a nonuniform manner. However, the critical uniform loading may serve as a valuable guide in the design of multicellular shell structures. Besides local buckling (between stiffeners) two different buckling modes are possible: a panel buckling mode and a general instability mode. We define as panel buckling here a mode in which the displacement pattern is either symmetrical or antimetrical about each of the lines of intersection between adjacent panels. In the general instability mode, on the other hand, the wavelength is larger than the width of one panel. For structures having twenty cells or less, it is evident that this deformation pattern produces very large axial membrane stresses in the cylindrical panels. Therefore, general instability will not be critical and because satisfactory methods are available for the local buckling mode, only the panel buckling mode will be considered here.

It is believed that the buckling of monocoque simply supported cylindrical panels was first studied by S. Timoshenko (Ref. 1). S. C. Redshaw (Ref. 2) extended the analysis to include a few cases of different boundary conditions along the generators. Results for the simply supported case agreed fairly well with tests made by J. S. Newell and W. H. Gale (Ref. 3). However, for the cases of fixed straight edges, the correlation was poor. A general expression for the critical compressive stress for slightly curved sheets with equal elastic restraints against rotation along the unloaded edges was obtained by E. Z. Stowell (Ref. 4).

Using a simplified version of the buckling determinant derived by W. Flügge (Ref. 5) for orthotropic cylinders, D. D. Dschou (Ref. 6) investigated the buckling of axially compressed segmental stiffened cylindrical shells. The general expression for the critical stress, however, is for the simple support boundary condition only, and eccentricity effects were not considered. Since then, the problem has received little attention (Refs. 7 and 8).

It is apparent that the available solutions cannot be directly applied to determine the critical compressive stresses of cylindrical segments subjected to the boundary conditions along the generators, which are applicable to the cylindrical panels in multicellular structures. The purpose of the present work is, therefore, to perform the necessary theoretical investigations required to formulate theory and to develop and analytical approach for predicting the axial buckling loads for both isotropic and orthotropic segmental cylindrical shells with displacements satisfying appropriate boundary conditions along the straight edges. Effects of eccentricities of stiffeners and bending of the web plates will also be examined.

Section 2
THEORETICAL FORMULATION

2.1 BASIC EQUATIONS

The analysis is based on the following assumptions:

- Stiffeners are located along coordinate lines.
- Strains in the stiffeners vary linearly with the radial coordinate and are compatible with those in the skin at the surfaces of contact.
- Stiffeners are under a uniaxial state of stress.
- In-plane shear forces are entirely carried by the skin.
- Stiffeners are so closely spaced that the effective width of the skin equals the spacing of the stringers or rings.

The strain-energy expression given in Sec. 2.2 for an orthotropic segmental cylinder as shown in Fig. 1 is based on the following elastic and kinematic relations (Refs. 9 and 10):

- Elastic relations

  For the skin:

$$\sigma_x = \frac{E}{1-\nu^2}\left[\epsilon_x + \nu\epsilon_\theta + z(\chi_x + \nu\chi_\theta)\right] \tag{2.1a}$$

$$\sigma_\theta = \frac{E}{1-\nu^2}\left[\epsilon_\theta + \nu\epsilon_x + z(\chi_\theta + \nu\chi_x)\right] \tag{2.1b}$$

  For the stiffeners:

$$\sigma_x = E_1(\epsilon_x + z\chi_x) \tag{2.2a}$$

$$\sigma_\theta = E_2(\epsilon_\theta + z\chi_\theta) \tag{2.2b}$$

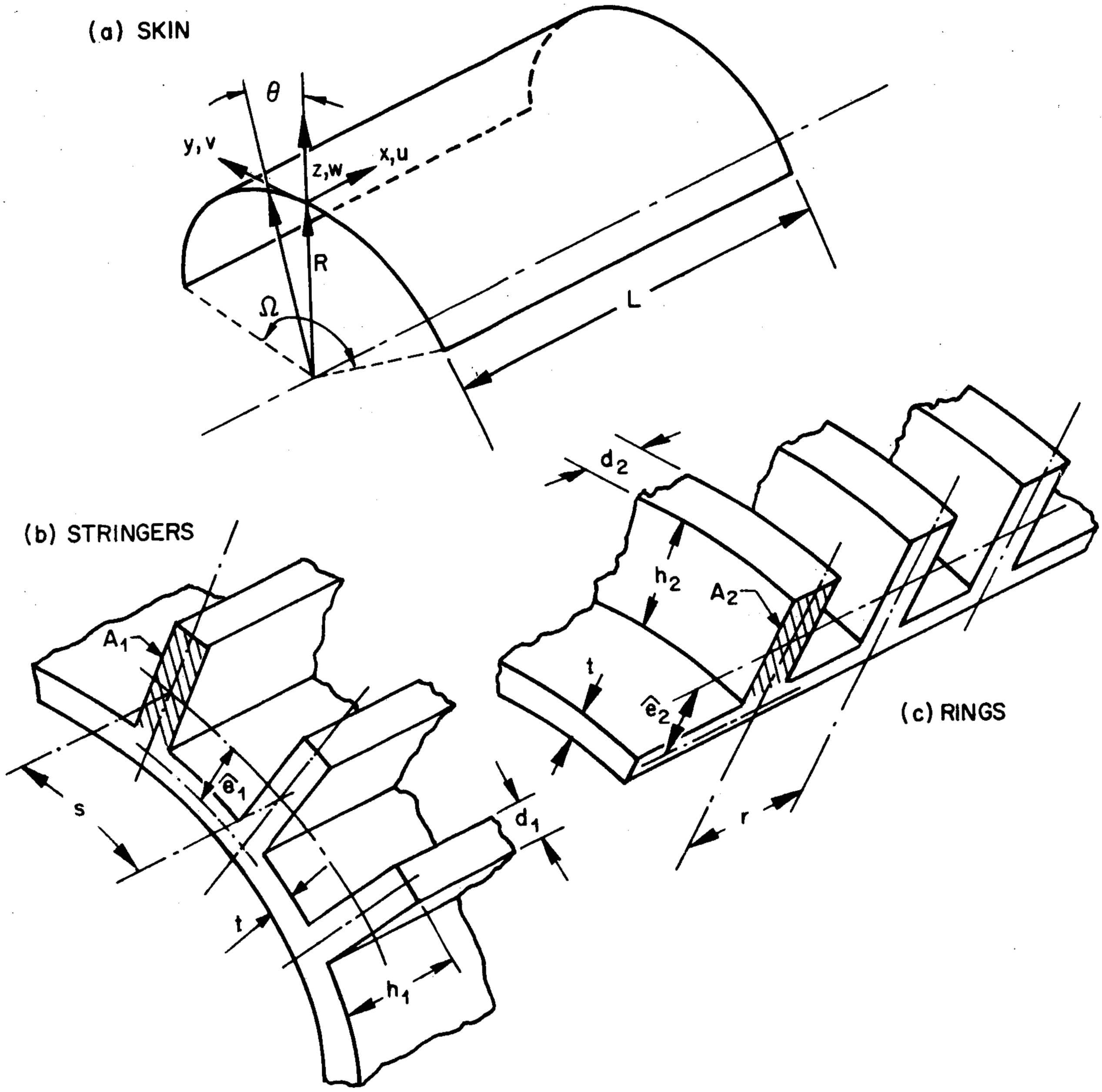

Fig. 1 Geometry of an Orthotropic Cylindrical Panel

- Middle-surface kinematic relations

Strain displacement:

$$\epsilon_x = u_{,x} + (w_{,x})^2/2 \tag{2.3a}$$

$$\epsilon_\theta = v_{,\theta} + w + (w_{,\theta})^2/2 \tag{2.3b}$$

$$\gamma_{x\theta} = u_{,\theta} + v_{,x} + (w_{,x})(w_{,\theta}) \tag{2.3c}$$

Curvature displacement:

$$\chi_x = -w_{,xx} \tag{2.4a}$$

$$\chi_\theta = -w_{,\theta\theta} \tag{2.4b}$$

$$\chi_{x\theta} = -w_{,x\theta} \tag{2.4c}$$

The nondimensional variables are given by

$$x = \hat{x}/R \quad , \quad \theta = \hat{y}/R \quad , \quad z = \hat{z}/R \tag{2.5a}$$

$$u = \hat{u}/R \quad , \quad v = \hat{v}/R \quad , \quad w = \hat{w}/R \tag{2.5b}$$

In terms of these relationships the corresponding stress resultants and moments are as follows:

$$N_x = \alpha_3\epsilon_x + \nu\epsilon_\theta + \alpha_1\chi_x \tag{2.6a}$$

$$N_\theta = \alpha_4\epsilon_\theta + \nu\epsilon_x + \alpha_2\chi_\theta \tag{2.6b}$$

$$N_{x\theta} = N_{\theta x} = \frac{1 - \nu}{2} \gamma_{x\theta} \tag{2.6c}$$

$$M_x = \alpha_5 \chi_x + \nu\gamma^2 \chi_\theta + \alpha_1 \epsilon_x \tag{2.6d}$$

$$M_\theta = \alpha_6 \chi_\theta + \nu\gamma^2 \chi_x + \alpha_2 \epsilon_\theta \tag{2.6e}$$

$$M_{x\theta} = \left[\gamma^2 + \frac{1 + \nu}{2tR^2} \frac{J_1}{(1 + \nu_1)s} \left(\frac{E_1}{E}\right)\right] (1 - \nu)\chi_{x\theta} \tag{2.6f}$$

$$M_{\theta x} = \left[\gamma^2 + \frac{1 + \nu}{2tR^2} \frac{J_2}{(1 + \nu_2)r} \left(\frac{E_2}{E}\right)\right] (1 - \nu)\chi_{\theta x} \tag{2.6g}$$

The nondimensional stress resultants and moments are defined as follows:

$$N_{(\ )} = \frac{1 - \nu^2}{Et} \hat{N}_{(\ )} \tag{2.7a}$$

$$M_{(\ )} = \frac{1 - \nu^2}{EtR} \hat{M}_{(\ )} \tag{2.7b}$$

and

$$\alpha_1 = \left(\frac{\hat{e}_1}{R}\right)\left(\frac{E_1}{E}\right) \frac{A_1(1 - \nu^2)}{st} \tag{2.8a}$$

$$\alpha_2 = \left(\frac{\hat{e}_2}{R}\right)\left(\frac{E_2}{E}\right) \frac{A_2(1 - \nu^2)}{rt} \tag{2.8b}$$

$$\alpha_3 = 1 + \left(\frac{E_1}{E}\right)\frac{A_1(1 - \nu^2)}{st} \tag{2.8c}$$

$$\alpha_4 = 1 + \left(\frac{E_2}{E}\right)\frac{A_2(1 - \nu^2)}{rt} \tag{2.8d}$$

$$\alpha_5 = \gamma^2 + (1 - \nu^2)\left(\frac{E_1}{E}\right)\frac{\bar{I}_1 + A_1\hat{e}_1^2}{stR^2} \tag{2.8e}$$

$$\alpha_6 = \gamma^2 + (1 - \nu^2)\left(\frac{E_2}{E}\right)\frac{\bar{I}_2 + A_2\hat{e}_2^2}{rtR^2} \tag{2.8f}$$

$$\alpha_7 = 2\gamma^2 + \frac{1 - \nu^2}{2tR^2}\left[\frac{J_1}{(1 + \nu_1)s}\left(\frac{E_1}{E}\right) + \frac{J_2}{(1 + \nu_2)r}\left(\frac{E_2}{E}\right)\right] \tag{2.8g}$$

$$\gamma^2 = \frac{1}{12}\left(\frac{t}{R}\right)^2 \tag{2.8h}$$

2.2 STRAIN-ENERGY EXPRESSIONS

The total potential energy of a stiffened cylindrical panel under axial load may be expressed as

$$\hat{V} = \hat{U} + \hat{W} \tag{2.9}$$

where $\hat{U}$ is the strain energy of the skin, stringers, and rings and $\hat{W}$ is the potential energy of the external load. From the relations in Sec. 2.1, it can be shown the strain energy of a cylindrical panel (Fig. 1) is given by the following equation.

$$\hat{U} = \int_0^{L/R}\int_{-\Omega/2}^{\Omega/2} \left(\epsilon_x^2 + \epsilon_x^2 + 2\nu\epsilon_x\epsilon_\theta + \frac{1-\nu}{2}\gamma_{x\theta}\right) dx\,d\theta$$

$$+ \frac{1}{12}\left(\frac{t}{r}\right)^2 \left[\chi_x^2 + \chi_\theta^2 + 2\nu\chi_x\chi_\theta + 2(1-\nu)\chi_{x\theta}^2\right] dx\,d\theta$$

$$+ \frac{1-\nu^2}{st}\frac{E_1}{E}\left\{A_1\left[\epsilon_x^2 + \frac{2\hat{e}_1}{R}\epsilon_x\chi_x + \left(\frac{\hat{e}_1}{R}\right)^2\chi_x^2\right] + \bar{I}_1\chi_x^2 + \frac{J_1}{2(1+\nu_1)}\chi_{x\theta}^2\right\} dx\,d\theta$$

$$+ \frac{1-\nu^2}{rt}\frac{E_2}{E}\left\{A_2\left[\epsilon_\theta^2 + \frac{2\hat{e}_2}{R}\epsilon_\theta\chi_\theta + \left(\frac{\hat{e}_2}{R}\right)^2\chi_\theta^2\right] + \bar{I}_2\chi_\theta^2 + \frac{J_2}{2(1+\nu_2)}\chi_{x\theta}^2\right\} dx\,d\theta \qquad (2.10)$$

where

$$U = \frac{2(1-\nu^2)}{EtR^2}\hat{U}$$

The potential energy of the external load is

$$\hat{W} = -\frac{\hat{N}_x}{2}\int_0^{L/R}\int_{-\Omega/2}^{\Omega/2} u_{,x}\,dx\,d\theta \qquad (2.11)$$

Equation (2.10) becomes in terms of the nondimensional parameter α_j

$$U = \int_0^{L/R}\int_{-\Omega/2}^{\Omega/2} \left[\alpha_3\epsilon_x^2 + \alpha_4\epsilon_\theta^2 + 2\alpha_1\epsilon_x\chi_x + 2\alpha_2\epsilon_\theta\chi_\theta + 2\nu\epsilon_x\epsilon_\theta + \frac{1-\nu}{2}\gamma_{x\theta}^2\right.$$

$$\left. + \alpha_5\chi_x^2 + \alpha_6\chi_\theta^2 + 2\nu\gamma^2\chi_x\chi_\theta + \left(\alpha_7 - 2\nu\gamma^2\right)\chi_{x\theta}^2\right] dx\,d\theta \qquad (2.12)$$

2.3 BUCKLING EQUATIONS

The equations for the state of neutral equilibrium and the associated boundary conditions can be obtained by equating to zero the first variation of the potential energy [Eq. (2.10] with respect to each of the displacement components u, v, w (i.e., $\delta\hat{V} = 0$) and by linearizing the resulting equations in accordance with the adjacent-equilibrium theory of stability. The linearized equations can then be expressed in terms of prebuckling stress resultants, incremental displacement components, and other parameters. Suitable mathematical manipulation and substitution lead to the following stability equations for axial compression:

$$\alpha_3 u_{,xx} + \frac{1-\nu}{2} u_{,\theta\theta} + \frac{1+\nu}{2} v_{,x\theta} + \nu w_{,x} - \alpha_1 w_{,xxx} = 0 \qquad (2.13a)$$

$$\frac{1+\nu}{2} u_{,x\theta} + \frac{1-\nu}{2} v_{,xx} + \alpha_4 v_{,\theta\theta} + \alpha_4 w_{,\theta} - \alpha_2 w_{,\theta\theta\theta} = 0 \qquad (2.13b)$$

$$N_x w_{,xx} + \nu u_{,x} - \alpha_4 (v_{,\theta} + w) - \alpha_2 w_{,\theta\theta} - \alpha_1 u_{,xxx}$$

$$- \alpha_2 (v_{,\theta\theta\theta} + w_{,\theta\theta}) + \alpha_5 w_{,xxxx} + \alpha_6 w_{,\theta\theta\theta\theta} + \alpha_7 w_{,xx\theta\theta} = 0 \qquad (2.13c)$$

Specializing these equations for a homogeneous isotropic shell yields the well-known Donnell equations.

2.4 BOUNDARY CONDITIONS

The natural or dynamic boundary conditions of the problem under consideration can be obtained from the expression for the first variation of the strain energy. These are as follows:

- At the curved edges

$$\alpha_3 u_{,x} + \nu (v_{,\theta} + w) - \alpha_1 w_{,xx} = 0 \qquad (2.14a)$$

$$u_{,\theta} + v_{,x} = 0 \tag{2.14b}$$

$$\alpha_5 w_{,xxx} + \nu\gamma^2 w_{,x\theta\theta} - \alpha_1 u_{,xx} + \left(\alpha_7 - 2\nu\gamma^2\right) w_{,x\theta} = 0 \tag{2.14c}$$

$$\alpha_5 w_{,xx} + \nu\gamma^2 w_{,\theta\theta} - \alpha_1 u_{,x} = 0 \tag{2.14d}$$

- At the straight edges

$$u_{\theta} + v_{,x} = 0 \tag{2.15a}$$

$$\alpha_4 (v_{,\theta} + w) + \nu u_{,x} - \alpha_2 w_{,\theta\theta} = 0 \tag{2.15b}$$

$$\alpha_6 w_{,\theta\theta\theta} + \nu\gamma^2 w_{,xx} - \alpha_2 (v_{,\theta\theta} + w_{,\theta}) + \left(\alpha_7 - 2\nu\gamma^2\right) w_{,xx\theta} = 0 \tag{2.15c}$$

$$\alpha_6 w_{,\theta\theta} + \nu\gamma^2 w_{,xx} - \alpha_2 (v_{,\theta} + w) = 0 \tag{2.15d}$$

For simplicity in the analysis, it is assumed here that the panels are simply supported at the curved edges.

$$\alpha_3 u_{,x} + \nu (v_{,\theta} + w) - \alpha_1 w_{,xx} = N_x = 0 \tag{2.16a}$$

$$v = 0 \tag{2.16b}$$

$$w = 0 \tag{2.16c}$$

$$\alpha_5 w_{,xx} + \nu\gamma^2 w_{,\theta\theta} - \alpha_1 u_{,x} = M_x = 0 \tag{2.16d}$$

Since the radial webs in multicellular shell structure elastically restrain the deformations along the line of intersection between two cylindrical panels, boundary conditions for both antimetric and symmetric buckling can be expressed in terms of the influence coefficients of the web plates. The displacement components parallel and perpendicular to the plane of the web plates (Fig. 2) are respectively given by

$$\bar{w} = w \cos \phi - v \sin \phi \tag{2.17a}$$

$$\bar{v} = w \sin \phi + v \cos \phi \tag{2.17b}$$

where

$$\phi = \begin{Bmatrix} \dfrac{\Omega - \mathfrak{D}}{2} \\ \\ \dfrac{-\Omega + \mathfrak{D}}{2} \end{Bmatrix} \quad \text{at} \quad \theta = \begin{Bmatrix} \Omega/2 \\ -\Omega/2 \end{Bmatrix}$$

The boundary conditions for the antimetric mode are

$$u = 0 \tag{2.18a}$$

$$\bar{w} = 0 \tag{2.18b}$$

$$\bar{v} - c_{11}(N_\theta \cos \phi + V_\theta \sin \phi)/2 - c_{12}M_\theta/2 = 0 \tag{2.18c}$$

$$w_{,\theta} - c_{21}(N_\theta \cos \phi + V_\theta \sin \phi)/2 - c_{22}M_\theta/2 = 0 \tag{2.18d}$$

In the above equations, N_θ and M_θ are given by Eqs. (2.6b) and (2.6e), and V_θ is the shear stress resultant.

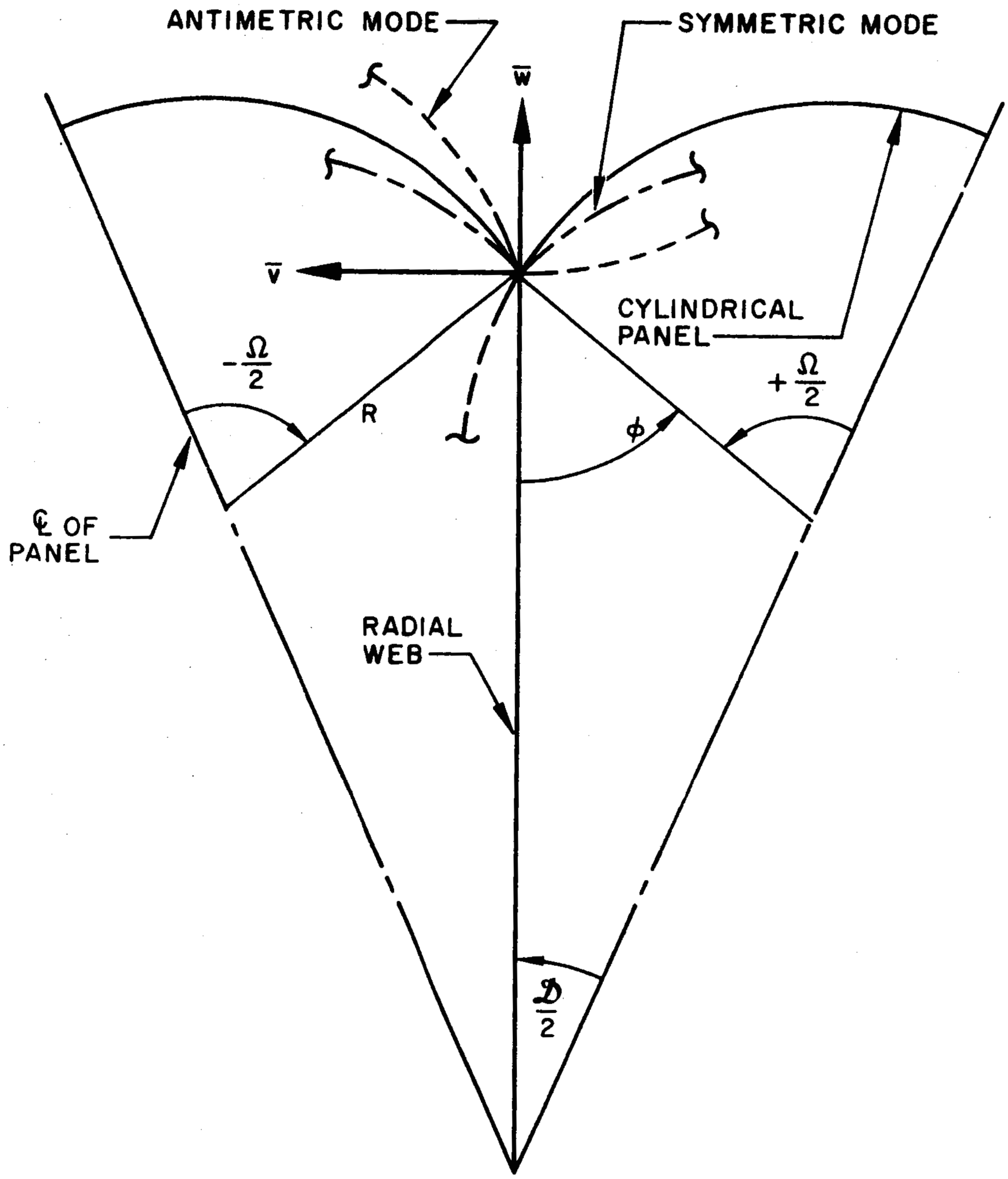

Fig. 2 Buckling Modes of Panel

which may be expressed in terms of the displacement components as follows:

$$V_\theta = \left(\alpha_7 - \nu\gamma^2\right)w_{,\theta xx} + \alpha_6 w_{,\theta\theta\theta} - \alpha_2(v_{,\theta\theta} + w_{,\theta}) \tag{2.19}$$

The coefficients c_{11}, c_{12}, c_{21}, and c_{22} are the bending influence coefficients of the web plates. A method to determine these coefficients is given in Appendix A.

For the symmetrical mode of panel buckling the boundary conditions at the straight edges are

$$u - c_{33}N_{x\theta}/2 = 0 \tag{2.20a}$$

$$\bar{v} = 0 \tag{2.20b}$$

$$\bar{w} - c_{44}(N_\theta \sin\phi + V_\theta \cos\phi)/2 = 0 \tag{2.20c}$$

$$w_{,\theta} = 0 \tag{2.20d}$$

where c_{33} and c_{44} are the stretching influence coefficients of the web plates. For the multicellular shell structures under consideration, it has been found numerically that the stretching flexibility of the web plates, especially when they are stiffened, is very small.

2.5 SOLUTION

For the case of uniform axial compression, Eqs. (2.13) are linear differential equations with constant coefficients. The simple support boundary conditions for the curved edges as given by Eqs. (2.16) are satisfied by the following displacement functions:

$$u = \cos anx \sum_{j=1}^{8} A_j \exp\left[\lambda_j(\theta/\Omega)\right] \tag{2.21a}$$

$$v = \sin anx \sum_{j=1}^{8} B_j \exp\left[\lambda_j(\theta/\Omega)\right] \tag{2.21b}$$

$$w = \sin anx \sum_{j=1}^{8} C_j \exp\left[\lambda_j(\theta/\Omega)\right] \tag{2.21c}$$

where

$$a = \pi(R/L)$$

Substitution of the displacement functions into the equations of equilibrium [Eqs. (2.13)] yields the following characteristic equation:

$$\begin{aligned}
&\left\{\left(\frac{1-\nu}{2}\right)\left[(\alpha_1)^2 - \alpha_3\alpha_5\right]a^8\right\}n^8 + \left[(1-\nu)\nu\alpha_1 a^6\right]n^6 + \left[\left(\frac{1-\nu}{2}\right)\left(\nu^2 - \alpha_3\alpha_4\right)a^4\right]n^4 \\
&+ \left\{\left[\alpha_5(\alpha_3\alpha_4 - \nu) + \left(\frac{1-\nu}{2}\right)\alpha_3\alpha_7 - (\alpha_1)^2\alpha_4\right]a^6n^6 + (1-\nu)\left[\alpha_2\alpha_3 + \alpha_1\alpha_4\right]a^4n^4\right\}(\tilde{\lambda})^2 \\
&+ \left\{\nu(1-\nu)\alpha_2 a^2n^2 + \left[(\nu - \alpha_3\alpha_4)\alpha_7 - \left(\frac{1-\nu}{2}\right)(\alpha_3\alpha_6 + \alpha_4\alpha_5) - (1+\nu)\alpha_1\alpha_2\right]a^4n^4\right\}(\tilde{\lambda})^4 \\
&+ \left\{\left[\alpha_6(\alpha_3\alpha_4 - \nu) + \left(\frac{1-\nu}{2}\right)\alpha_4\alpha_7 - (\alpha_2)^2\alpha_3\right]a^2n^2\right\}(\tilde{\lambda})^6 + \left\{\left(\frac{1-\nu}{2}\right)\left[(\alpha_2)^2 - \alpha_4\alpha_6\right]\right\}(\tilde{\lambda})^8 \\
&\qquad - N_{x_o}\left\{\left(\frac{1-\gamma}{2}\right)\alpha_3 a^6n^6 + (\nu - \alpha_3\alpha_4)a^4n^4(\tilde{\lambda})^2 + \left(\frac{1-\nu}{2}\right)\alpha_4 a^2n^2(\tilde{\lambda})^4\right\} = 0
\end{aligned} \tag{2.22}$$

where

$$N_{x_o} = \frac{1 - \nu^2}{Et} \frac{P_x}{\Omega R \left[1 + \dfrac{\hat{e}_1/R}{1 + (E/E_1)st/(1 - \nu^2)A_1} \right]}$$

and

$$\tilde{\lambda}_j = \frac{1}{\Omega} \lambda_j$$

Equations (2. 13) are identically satisfied if $\lambda_j (j = 1, 2 \ldots 8)$ in Eqs. (2. 22) are chosen to be the eight roots of this characteristic equation. The A_j, B_j and C_j in Eqs. (2. 22) are constants of integration. For each given value of λ_j, A_j and B_j may be expressed in terms of C_j by use of the in-plane equilibrium equations [Eqs. (2. 13a, b)]. These constants are governed by the homogeneous equation system which is obtained if Eqs. (2. 22) are substituted into the eight boundary conditions for the straight edges (four at each edge). The critical value of the load is found as the lowest value of N_{x_o} for which the determinant of the coefficients of this equation system vanishes.

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Section 3
NUMERICAL ANALYSIS

The critical load of an axially compressed stiffened cylindrical panel may be obtained by use of the computer program which is discussed in Appendix B. This program is based on the theoretical formulation as described in the previous section. It should be noted, particularly in the preliminary design stage, that the inverse problem is frequently posed. That is, the axial load and sometimes material or certain geometrical parameters are given and it is the task of the stress analyst to determine the panel geometry. It is certainly possible that this problem may be solved by the application of the computer program to a number of cases in which one or more parameters have been varied. In many cases it may also be possible to devise an optimizing computer program for the inverse problem. However, because of the many alternatives that are open, the usefulness of such a program will necessarily be somewhat restricted. In view of these considerations, it is obvious that the computer program would be a very valuable tool if a set of design curves were provided. Unfortunately, the large number of independent parameters for stiffened panels makes a complete parametric study impossible. Therefore, the numerical analysis presented here is primarily intended to help the designer to obtain a reasonably close estimate of the critical loads. With such an estimate available, the determination of more accurate dimensions by use of the computer program is expedited.

Table 1 shows some results from a study of the influence of the angle ϕ on the buckling load. It was found that within the range of values applicable to multicellular design, ϕ has little effect on the critical load. Consequently, for the case in which the stiffness of the web plate can be omitted, the usual simple support boundary conditions give a good approximation for the panels in the shell structures under consideration.

Table 1

INFLUENCE OF ANGLE ϕ ON BUCKLING LOAD

ϕ (deg)	Buckling Load $N_{cr}\left(\frac{E}{10^7}\right)$	Remarks
0	11, 170	$R/t = 242$
7.5	11, 265	$L/R = 1.5$
		$d_1/R = d_2/R = 0.006$
		$h_1/R = h_2/R = 0.017$
11.25	11, 313	$E_1/E = E_2/E = 1$
		$\nu = 0.3$
		$\Omega = 30$ deg

Figure 3 shows critical loads as a function of the panel angle, Ω , for an external stringer stiffened panel. The nondimensional buckling curves are for the following four different boundary conditions along the straight edges:

$$
\begin{array}{llll}
\text{PS1} & u = 0 \; ; \; v = 0 \; ; \; w = 0 \; ; \; M_\theta = 0 \\
\text{PC1} & u = 0 \; ; \; v = 0 \; ; \; w = 0 \; ; \; w_{,\theta} = 0 \\
\text{PS2} & u = 0 \; ; \; N_\theta = 0 \; ; \; w = 0 \; ; \; M_\theta = 0 \\
\text{PC2} & u = 0 \; ; \; N_\theta = 0 \; ; \; w = 0 \; ; \; w_{,\theta} = 0
\end{array}
$$

In view of Fig. 3, the curve PS2 gives the critical load for the case in which the web plate stiffness is omitted, and curve PC1 corresponds to the case of rigid web plate. It is evident that for sufficiently narrow panels the influence of the web plate stiffness may be significant. It can be observed that to restrain tangential displacements is more effective than to restrain rotations at the edge (PS1-curve is well above the PC2-curve).

The critical load corresponding to a series of values of the web plate stiffness has been computed for a panel which is stiffened by an internal waffle pattern. The results

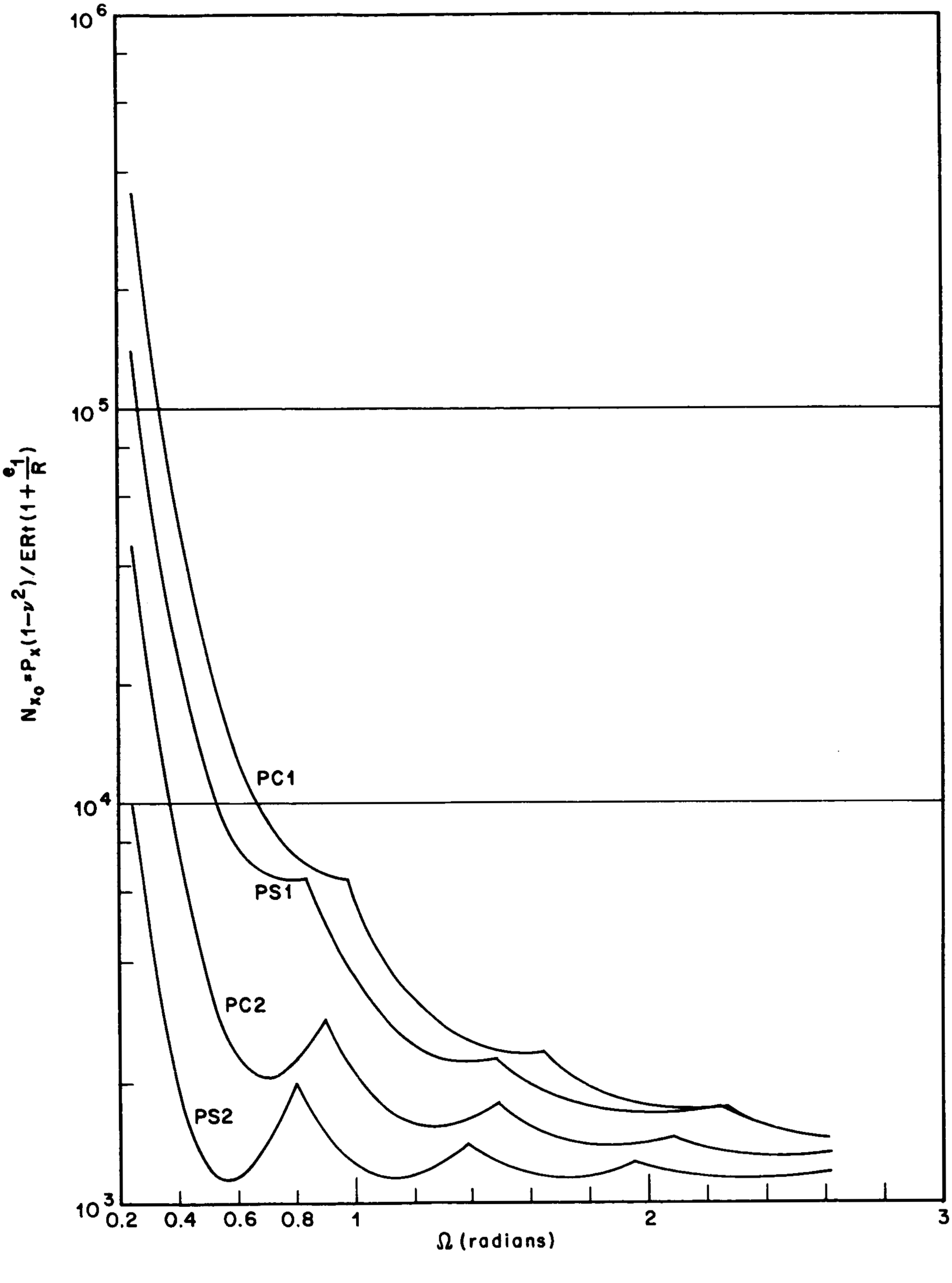


Fig. 3 Effects of Ω on Buckling Stress

are shown in Fig. 4. The web plate here is stiffened with the same waffle pattern as that for the cylinder but the stiffener height was varied. It is seen that the critical load does not increase significantly unless the plate is very stiff in comparison to that of the cylindrical panel. It appears that in most cases the simple support conditions should lead to a good approximation.

If the simply supported panel is entirely or primarily stiffened in the circumferential direction, the critical load would be the same as that obtained from a complete cylinder. For panels which are essentially stringer stiffened or with approximately the same stiffening in both directions, an estimate of the buckling load can be obtained by use of Figs. 5 through 16. For the case of simple support. Figs. 5 through 10 give buckling coefficients as a function of the angle Ω for various values of the parameters ρ, L/R, and R/t. Figures 11 through 16 give corresponding results for waffle stiffened cylindrical panels. Results are presented for external as well as internal stiffening. The graphs apply strictly only for rectangular stiffeners but they will yield reasonable buckling loads for other stiffener shapes.

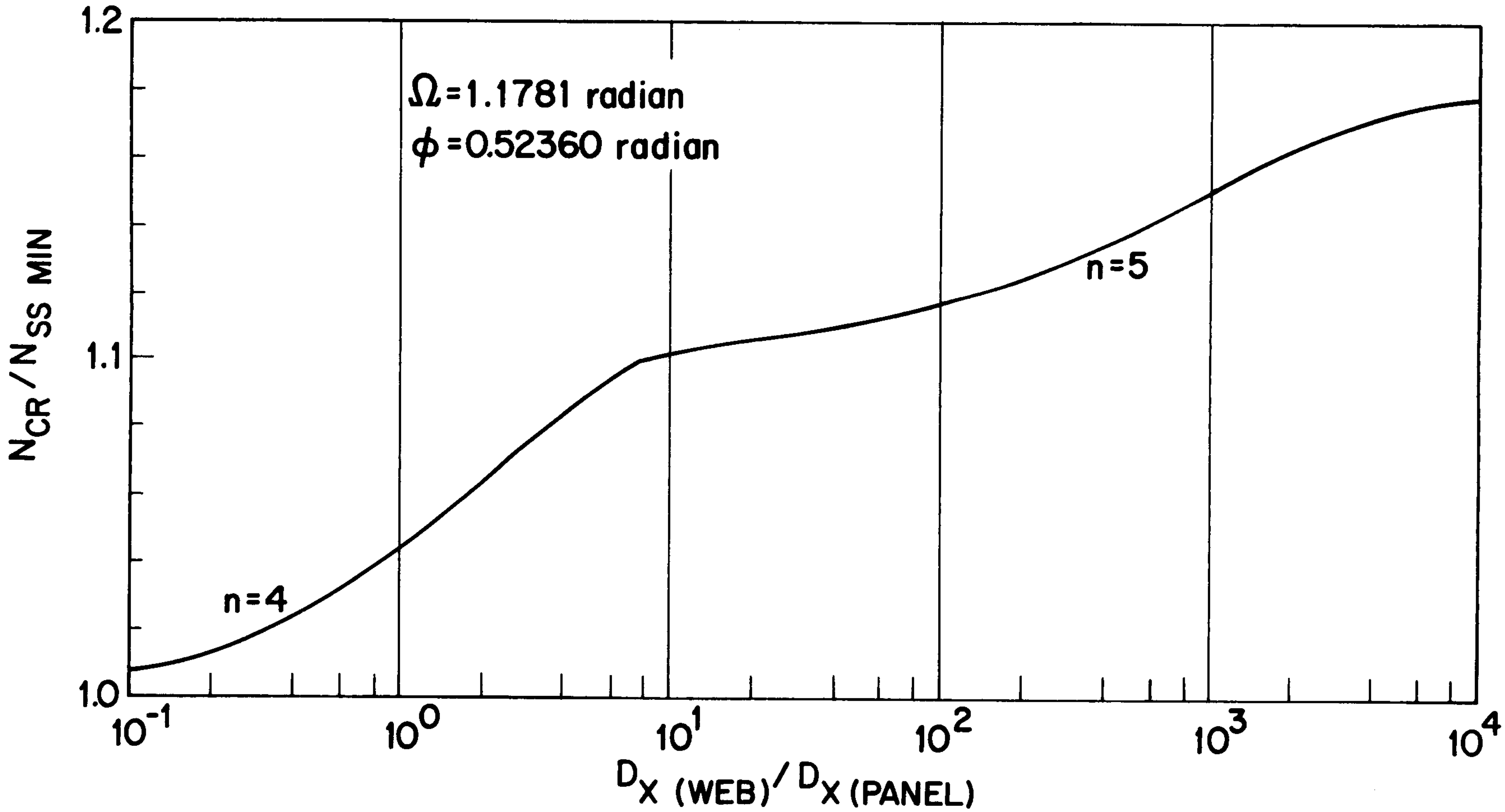


Fig. 4 Effect of Web Plate Stiffness on Panel Buckling (Antimetric Mode)

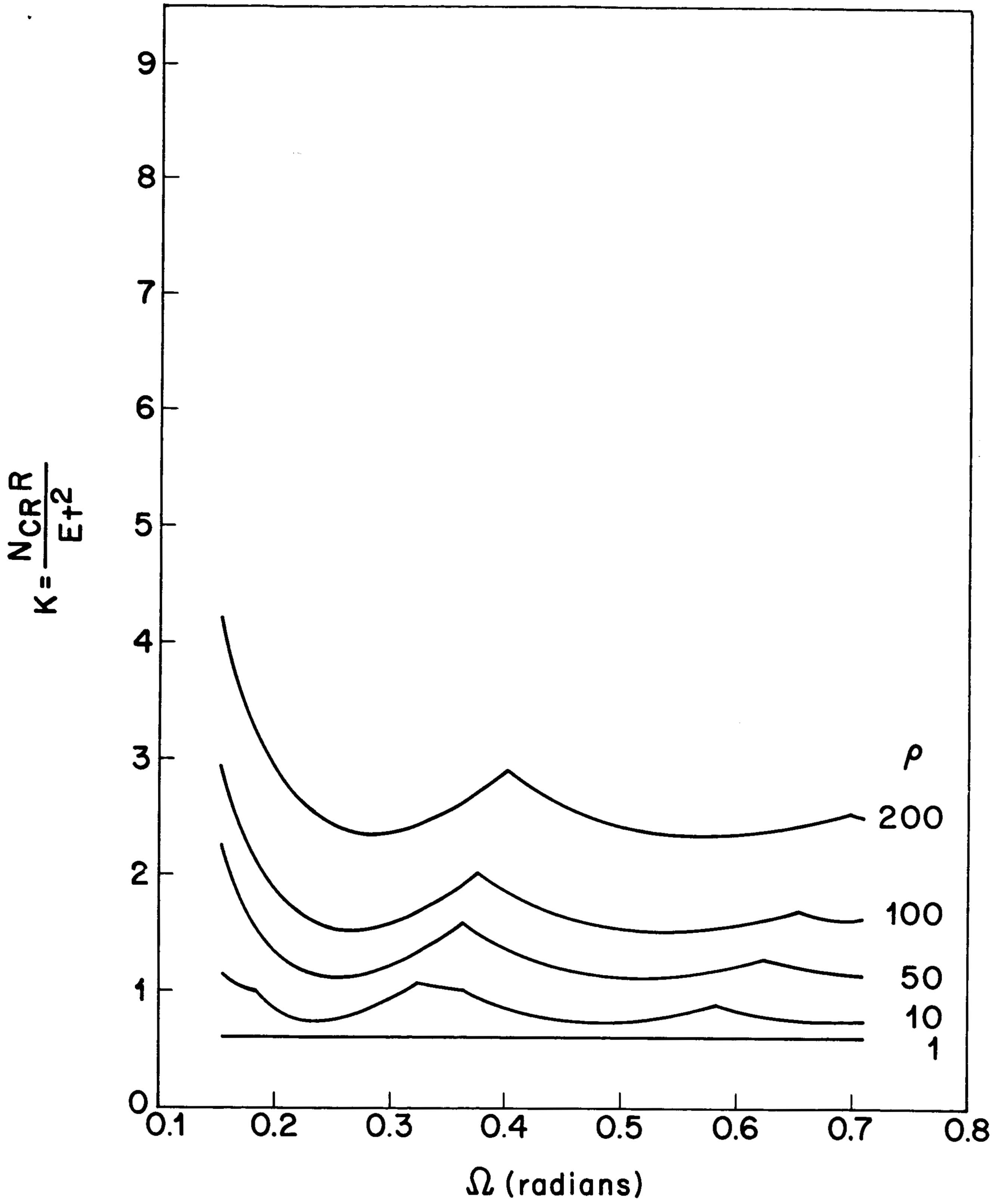


Fig. 5a Buckling Coefficients for Panels With Internal Stringers – L/R = 0.5; R/t = 400

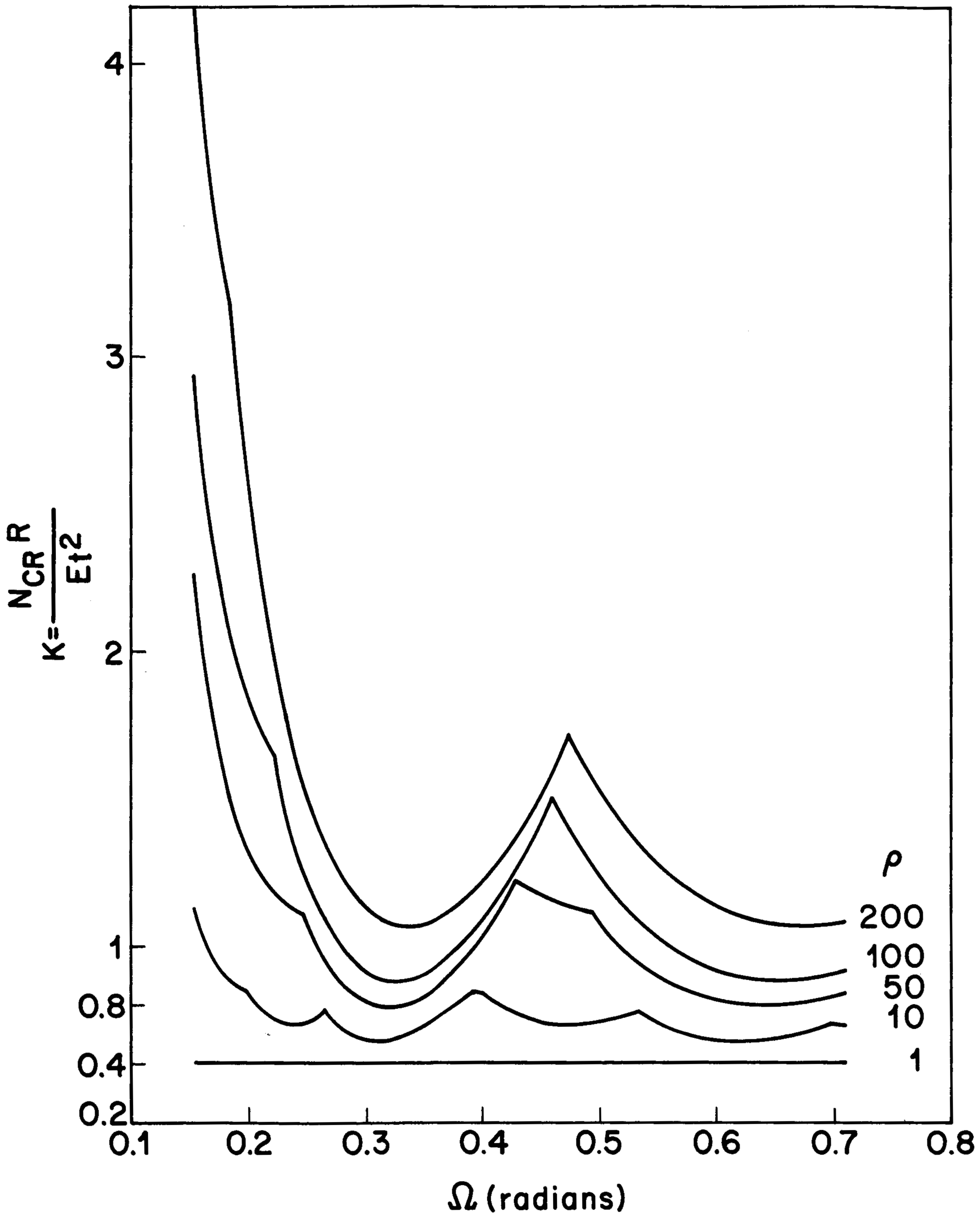


Fig. 5b Buckling Coefficients for Panels With Internal Stringers – L/R = 1.0; R/t = 400

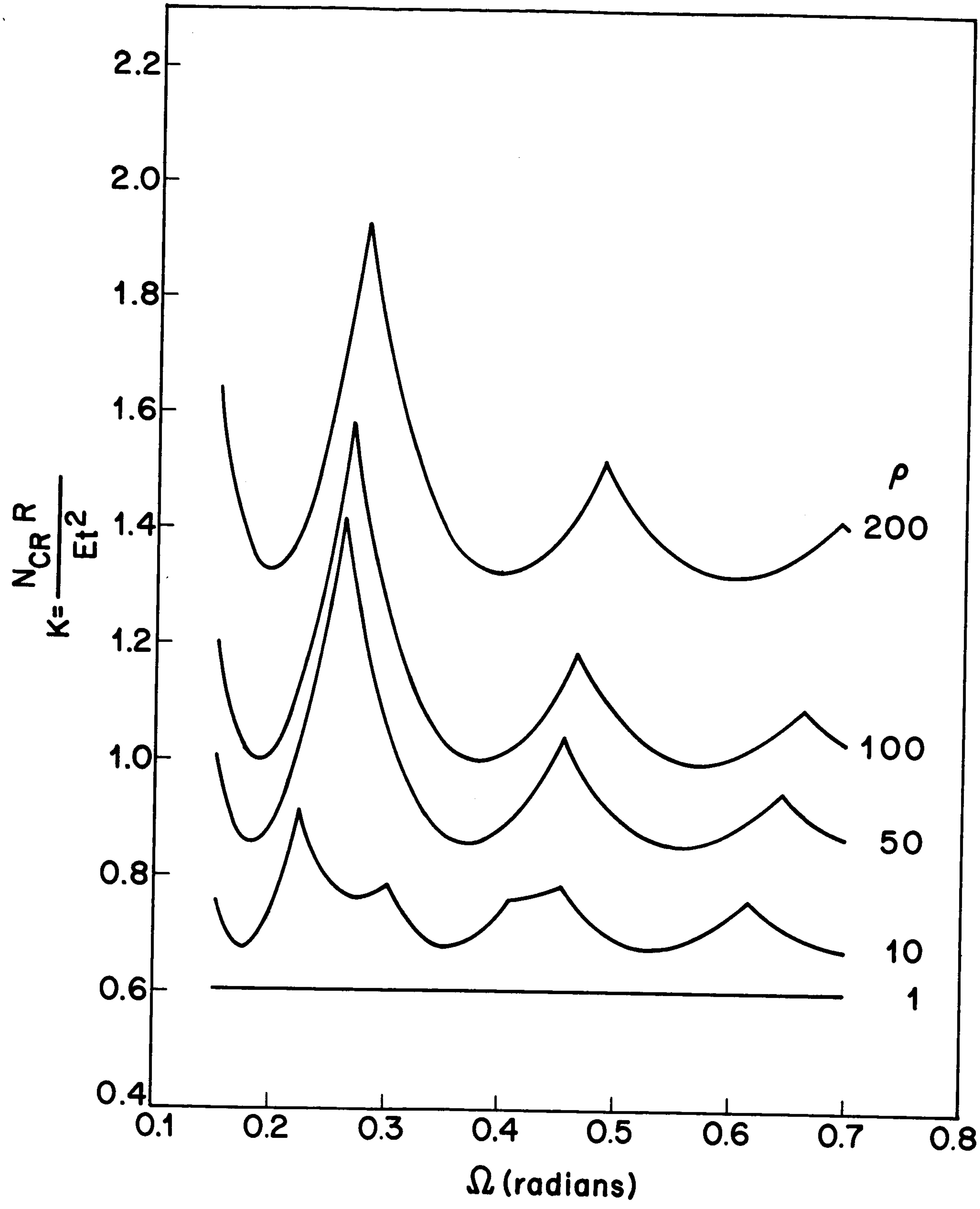


Fig. 6a Buckling Coefficients for Panels With Internal Stringers – $L/R = 0.5$; $R/t = 1000$

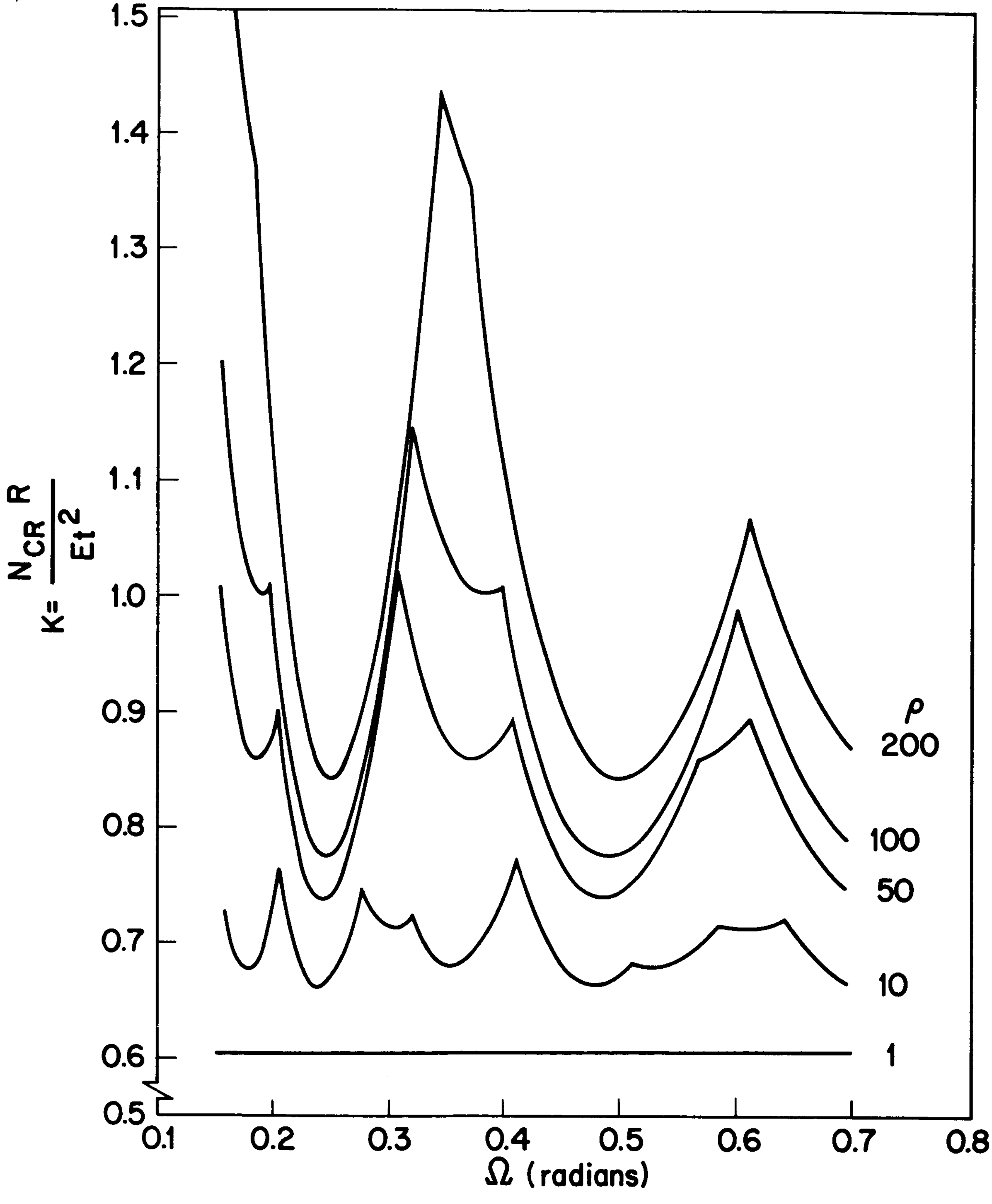


Fig. 6b Buckling Coefficients for Panels With Internal Stringers – L/R = 1.0; R/t = 1000

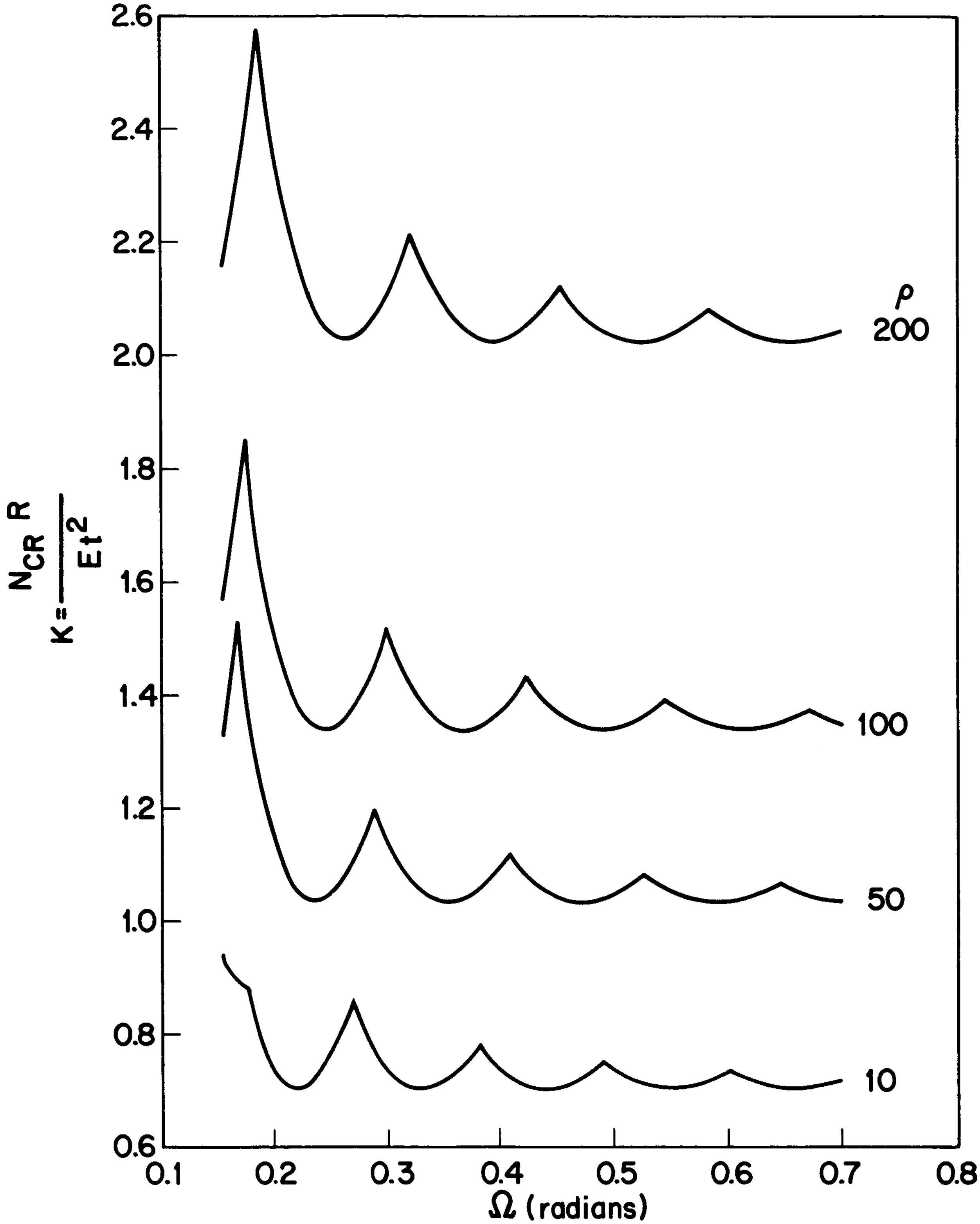


Fig. 7a Buckling Coefficients for Panels With Internal Stringers – L/R = 0.25; R/t = 2000

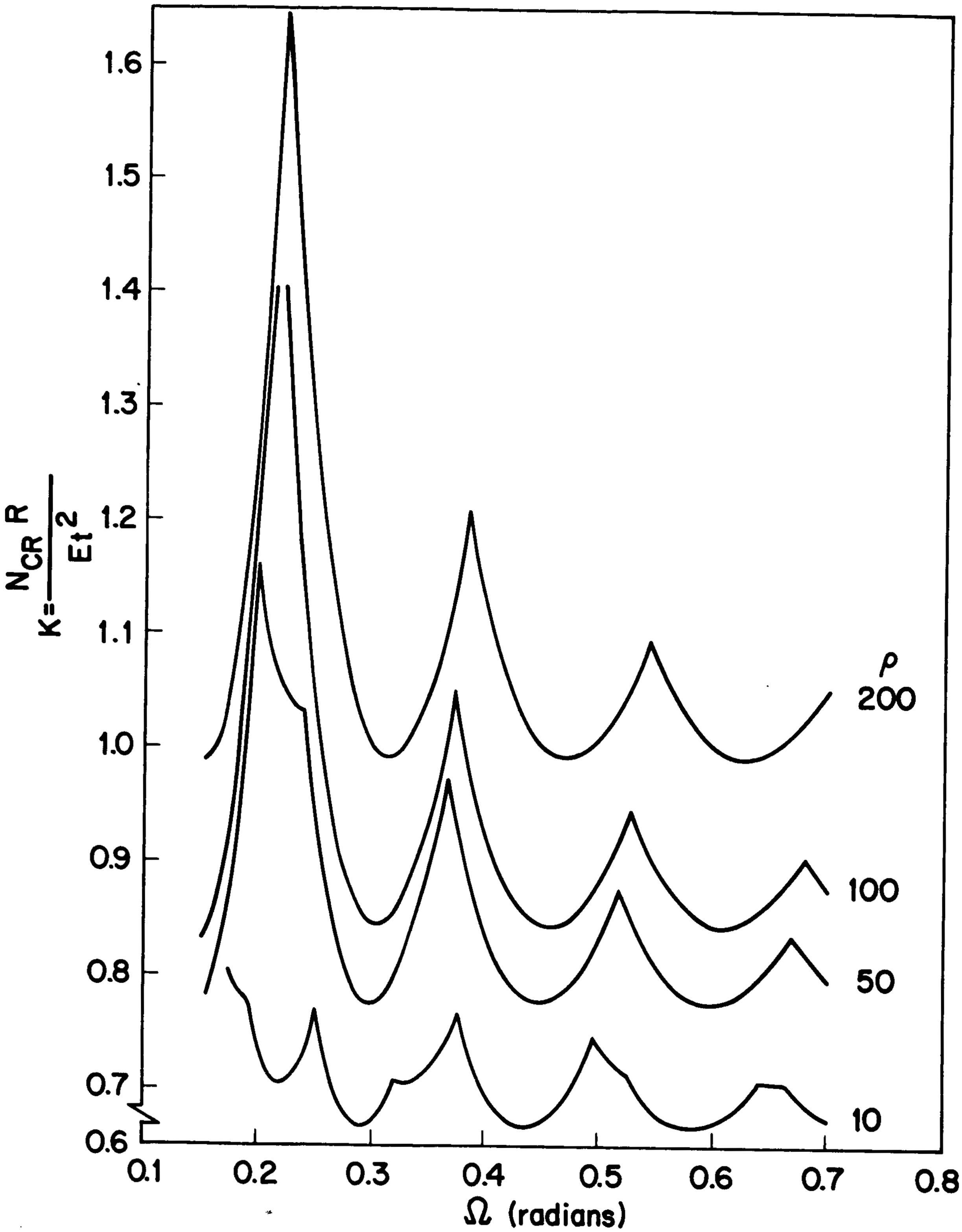


Fig. 7b Buckling Coefficients for Panels With Internal Stringers – $L/R = 0.5$; $R/t = 2000$

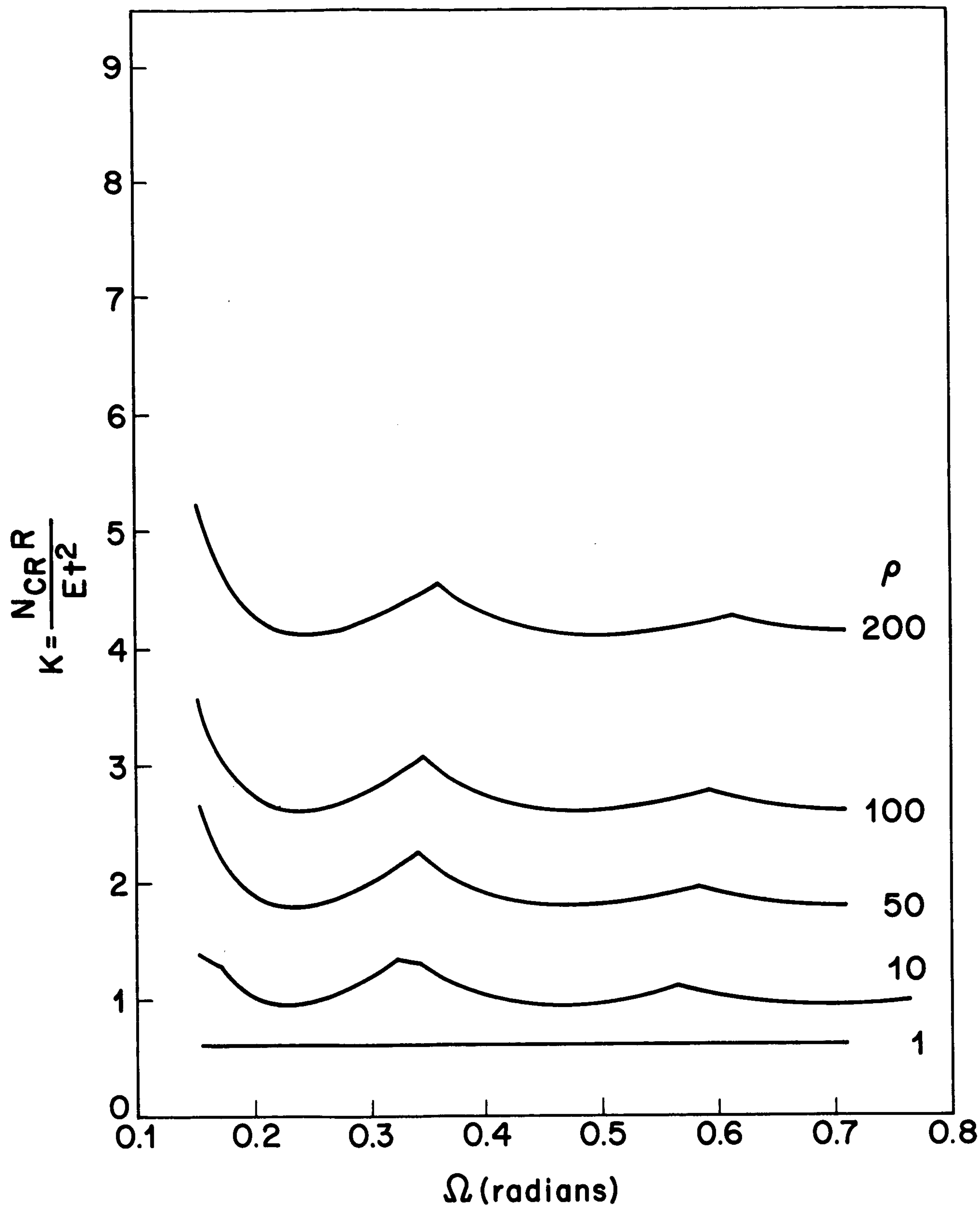


Fig. 8a Buckling Coefficients for Panels With External Stringers – $L/R = 0.5$; $R/t = 400$

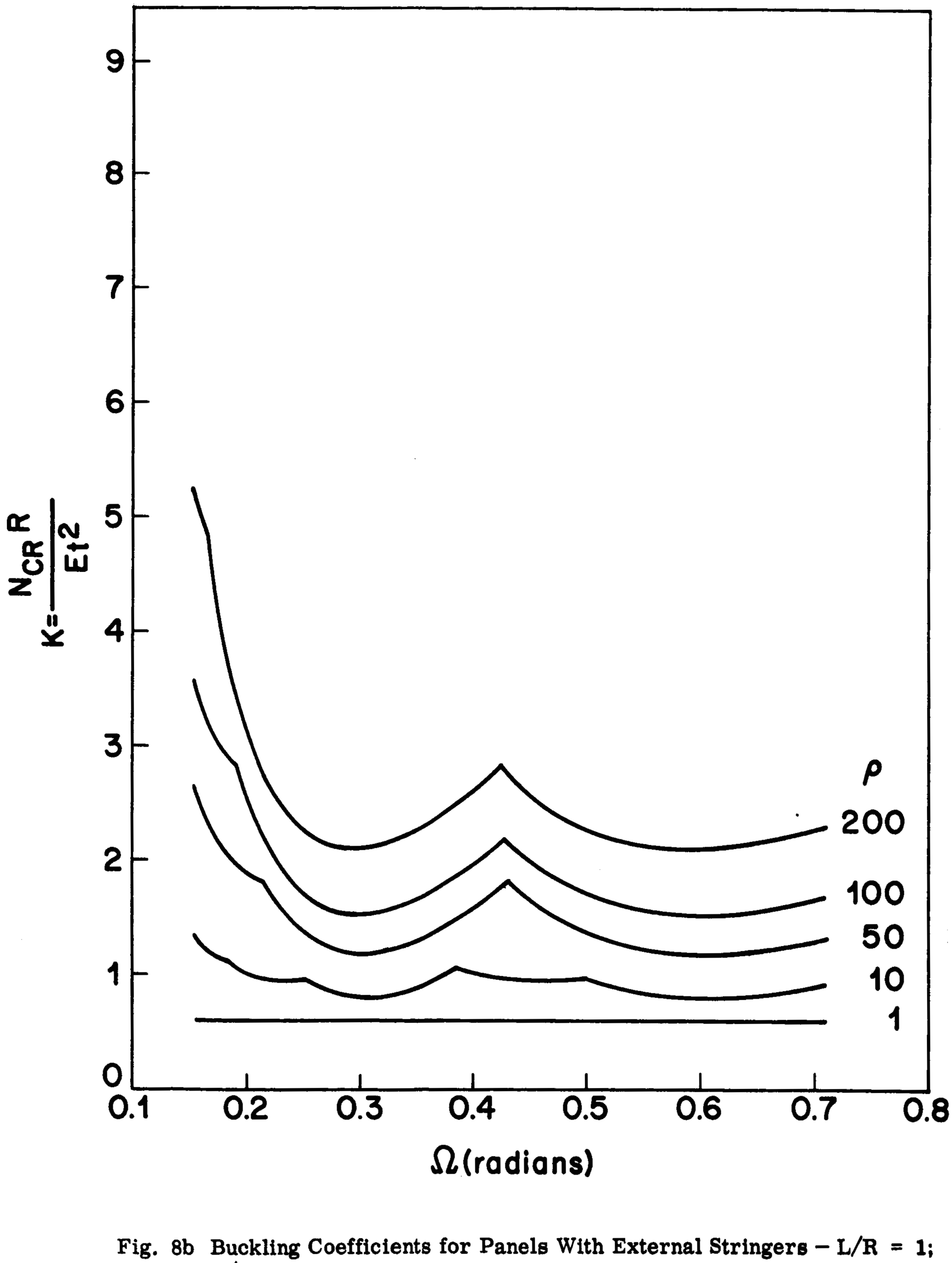


Fig. 8b Buckling Coefficients for Panels With External Stringers – L/R = 1; R/t = 400

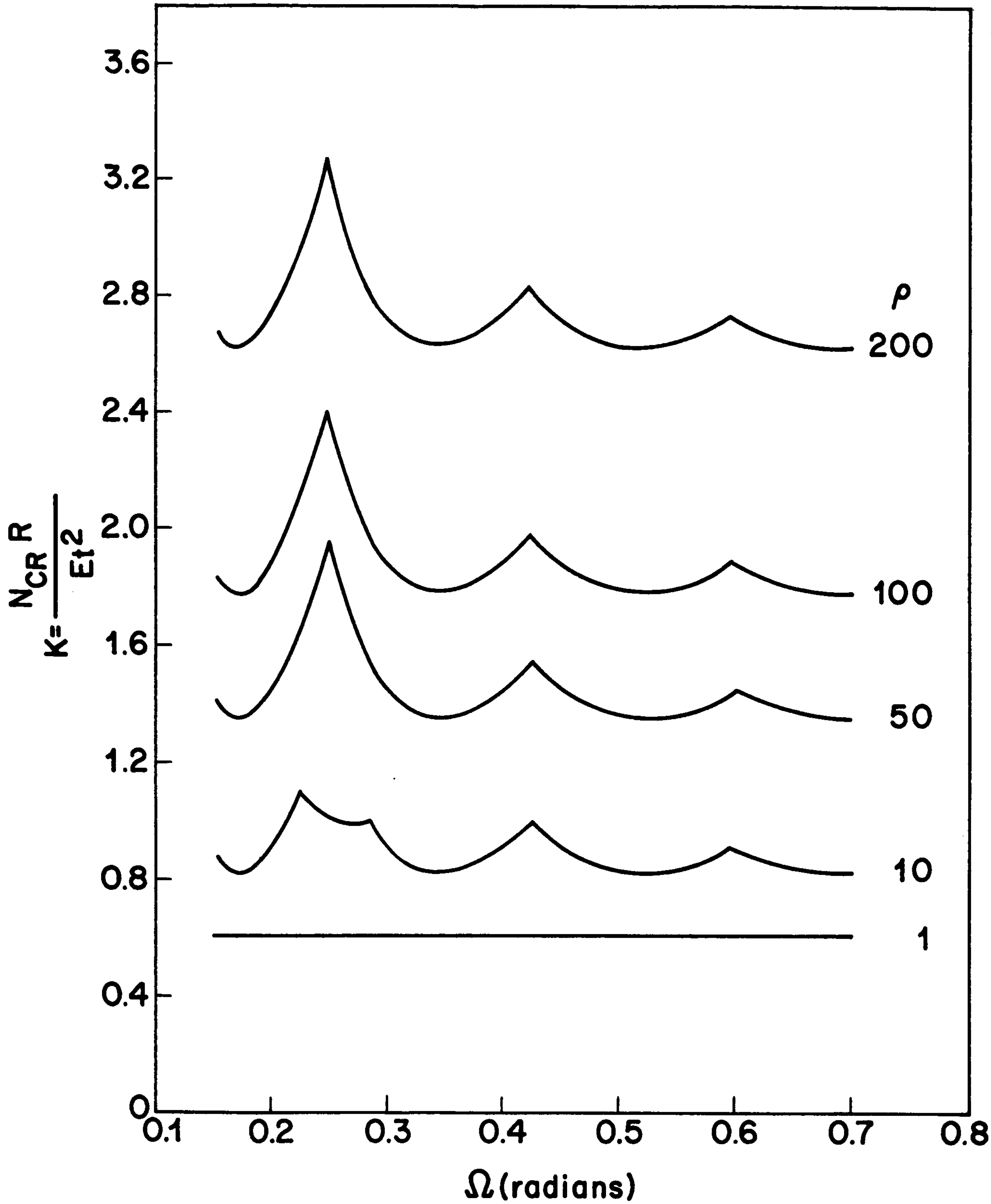


Fig. 9a Buckling Coefficients for Panels With External Stringers – L/R = 0.5; R/t = 1000

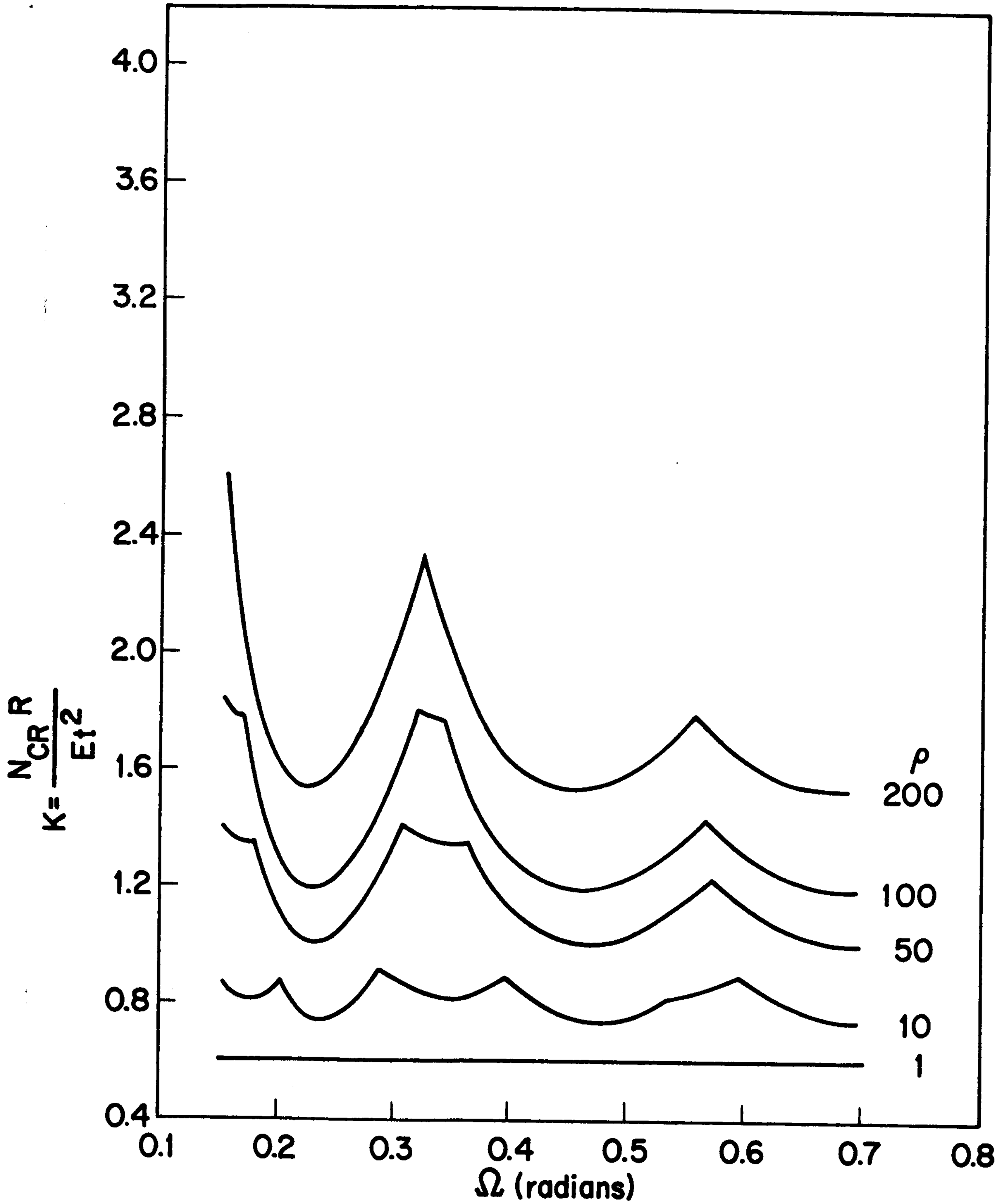


Fig. 9b Buckling Coefficients for Panels With External Stringers – L/R = 1.0; R/t = 1000

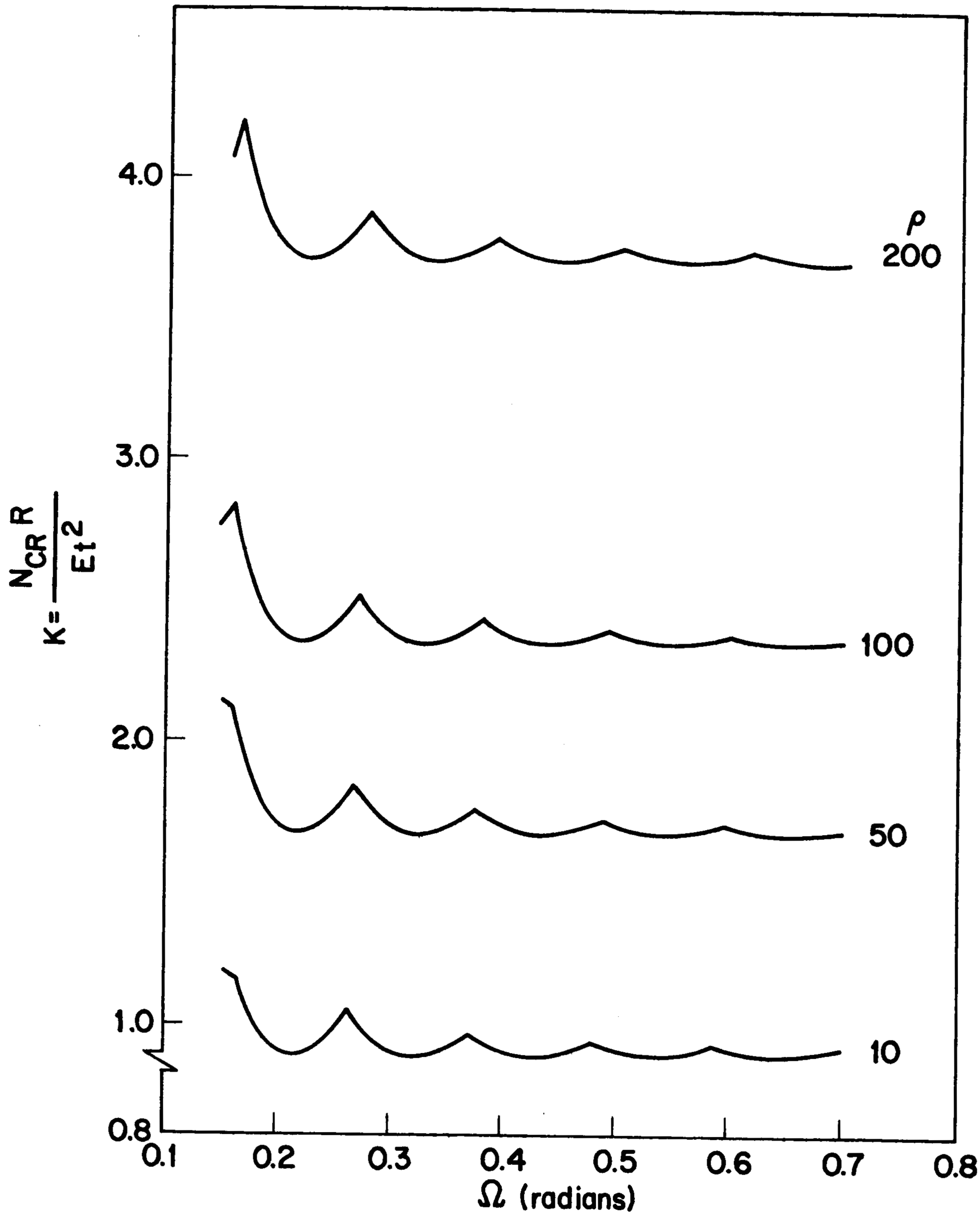


Fig. 10a Buckling Coefficients for Panels With External Stringers – L/R = 0.25; R/t = 2000

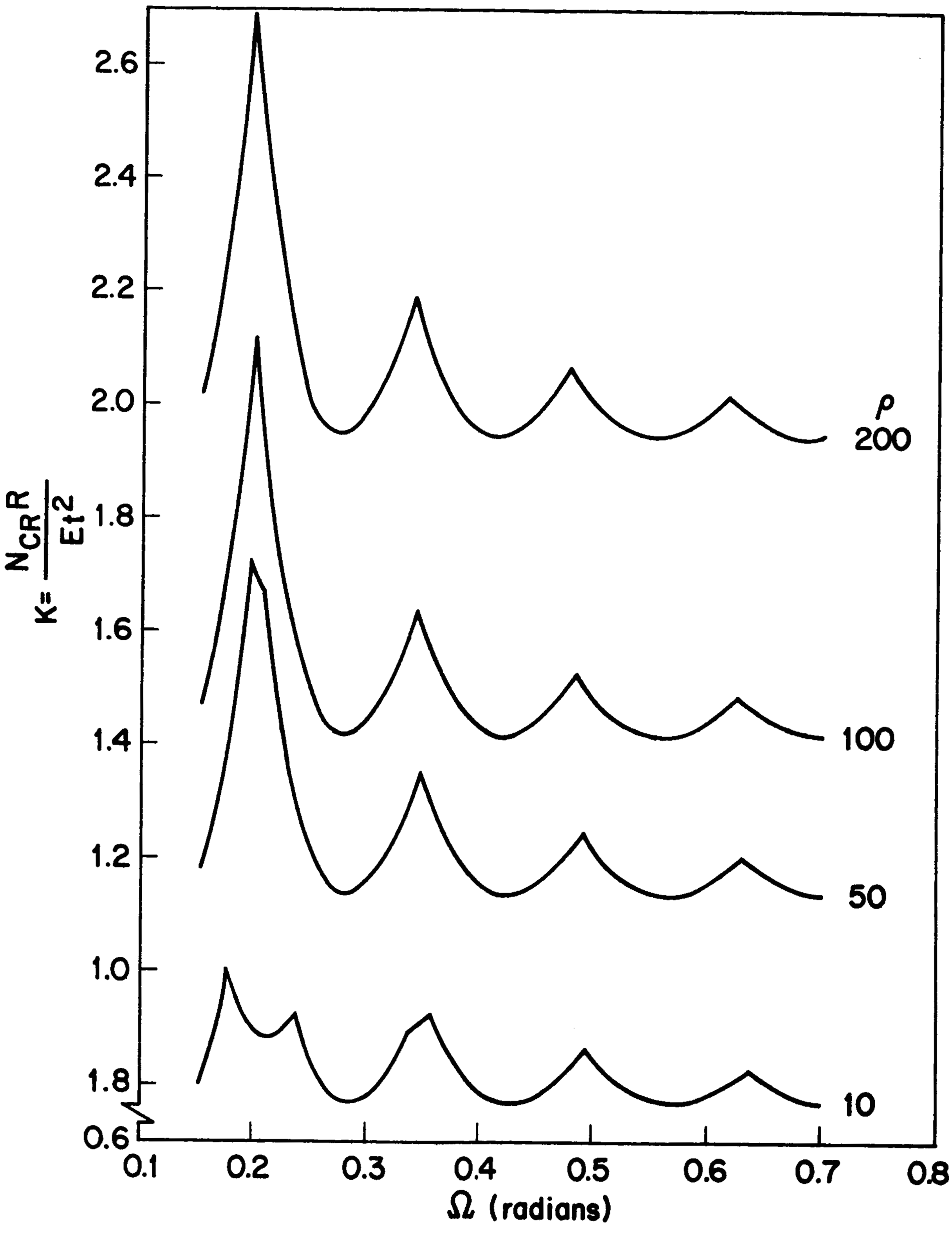


Fig. 10b Buckling Coefficients for Panels With External Stringers – L/R = 0.5; R/t = 2000

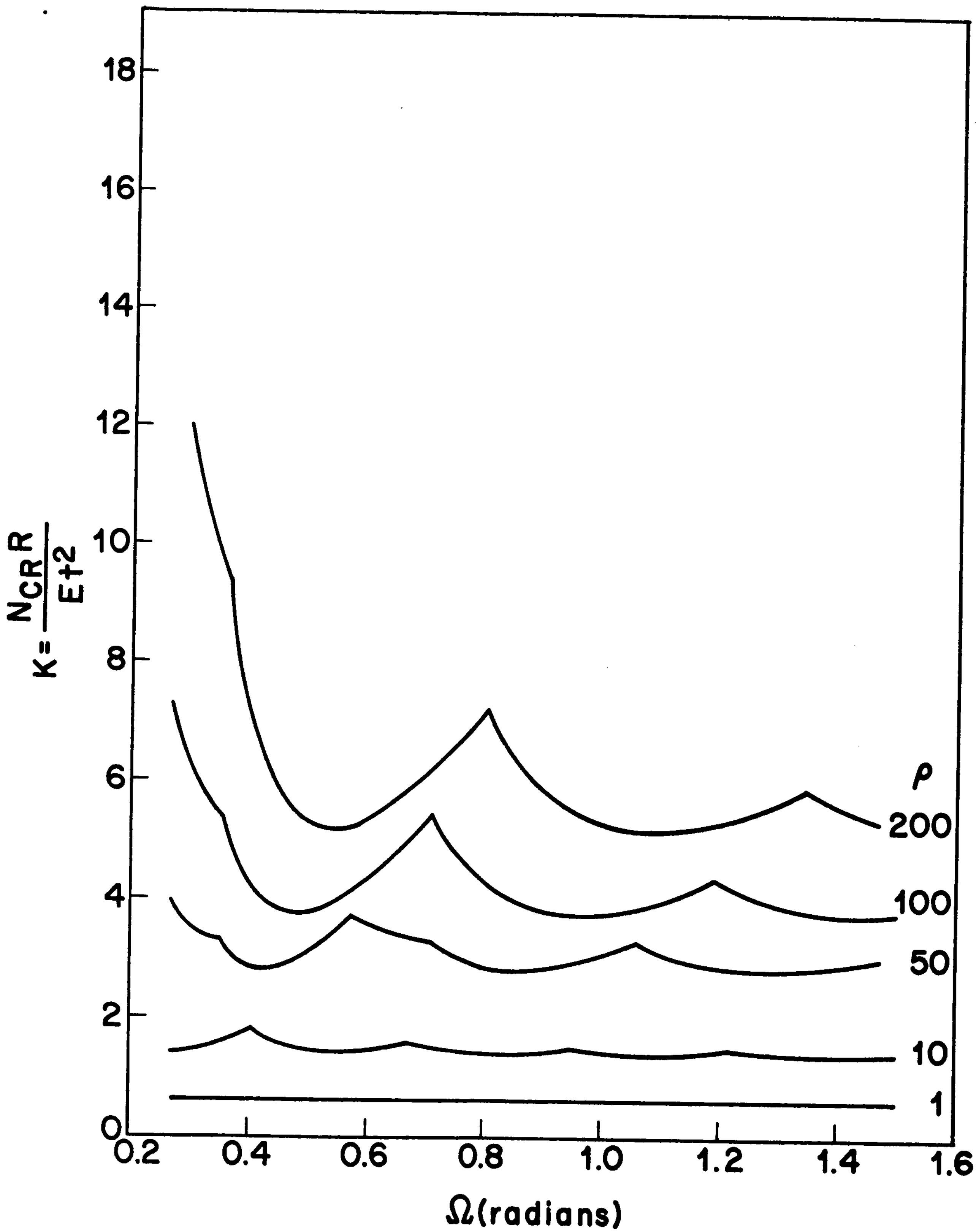


Fig. 11a Buckling Coefficients for Panels With Internal Waffles – L/R = 0.5; R/t = 400

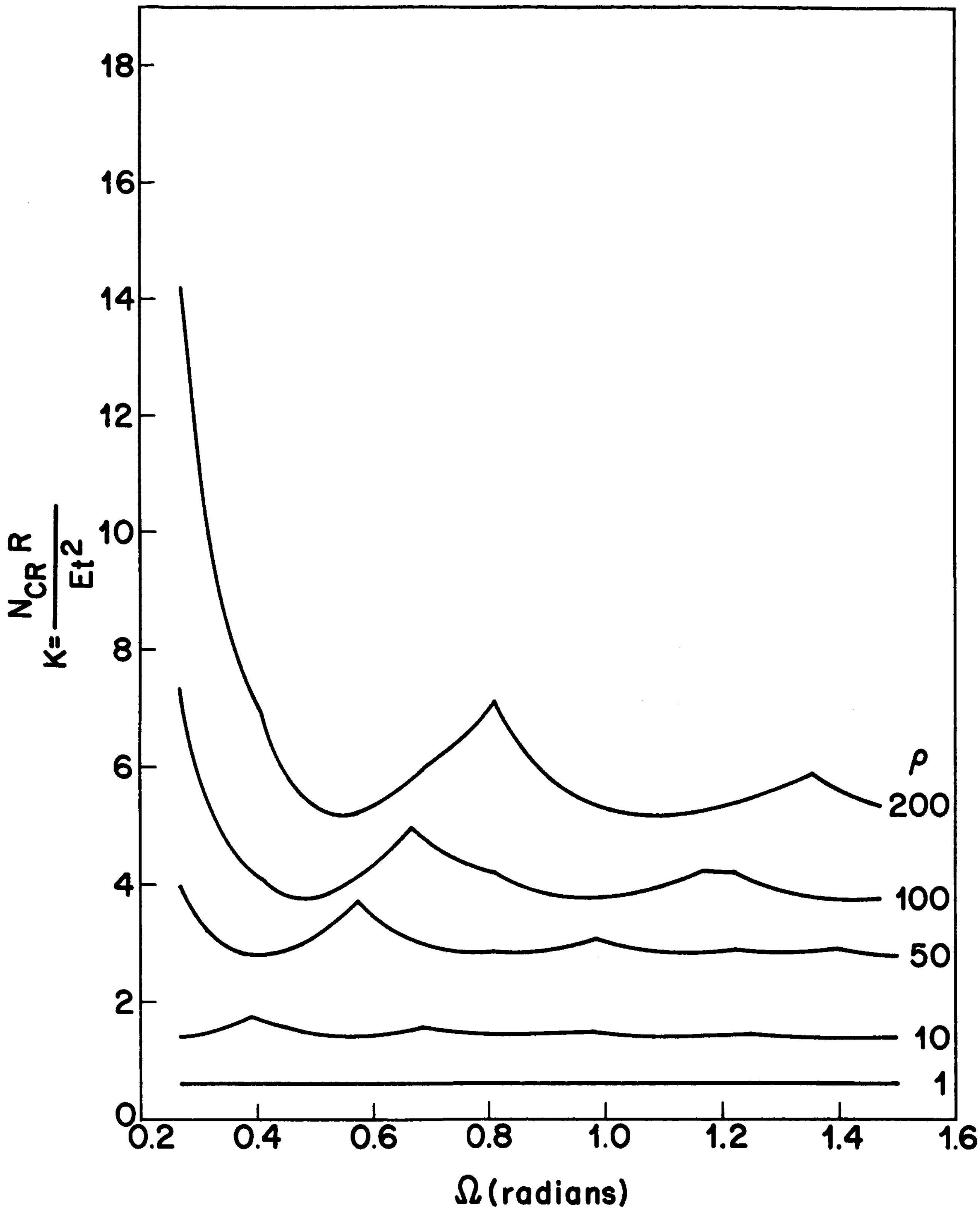


Fig. 11b Buckling Coefficients for Panels With Internal Waffles – L/R = 1.0; R/t = 400

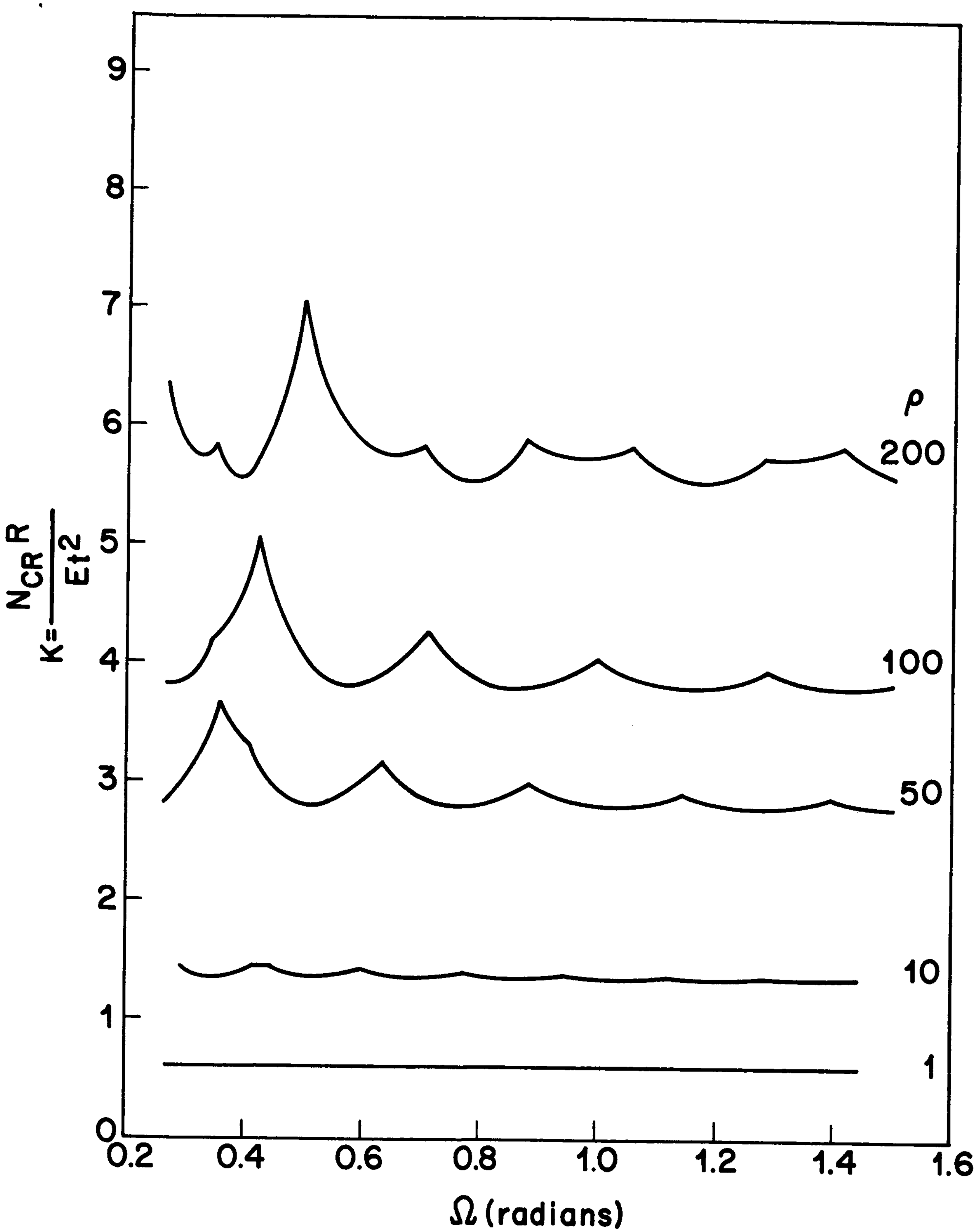


Fig. 12a Buckling Coefficients for Panels With Internal Waffles – L/R = 0.5; R/t = 1000

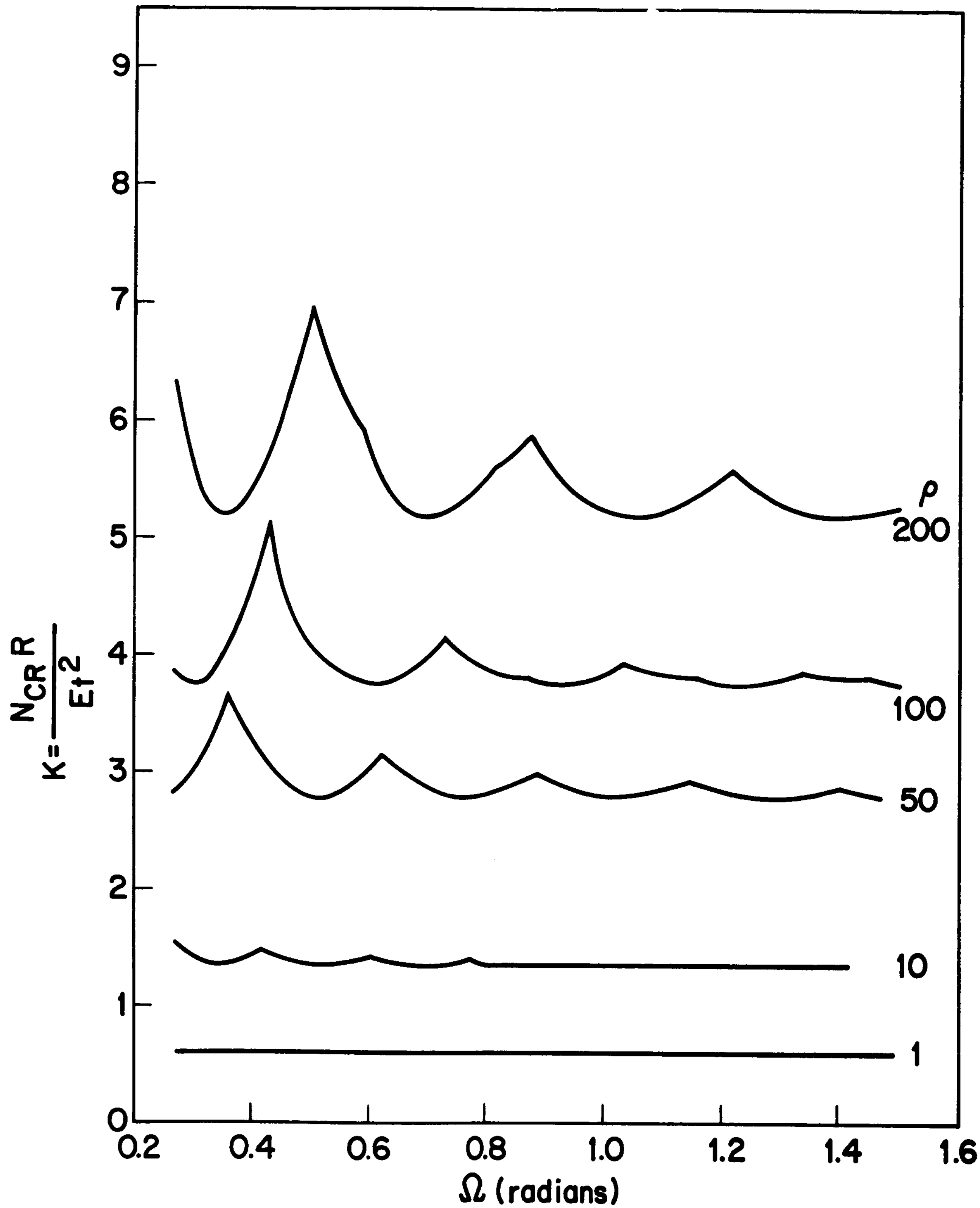


Fig. 12b Buckling Coefficients for Panels With Internal Waffles – L/R = 1; R/t = 1000

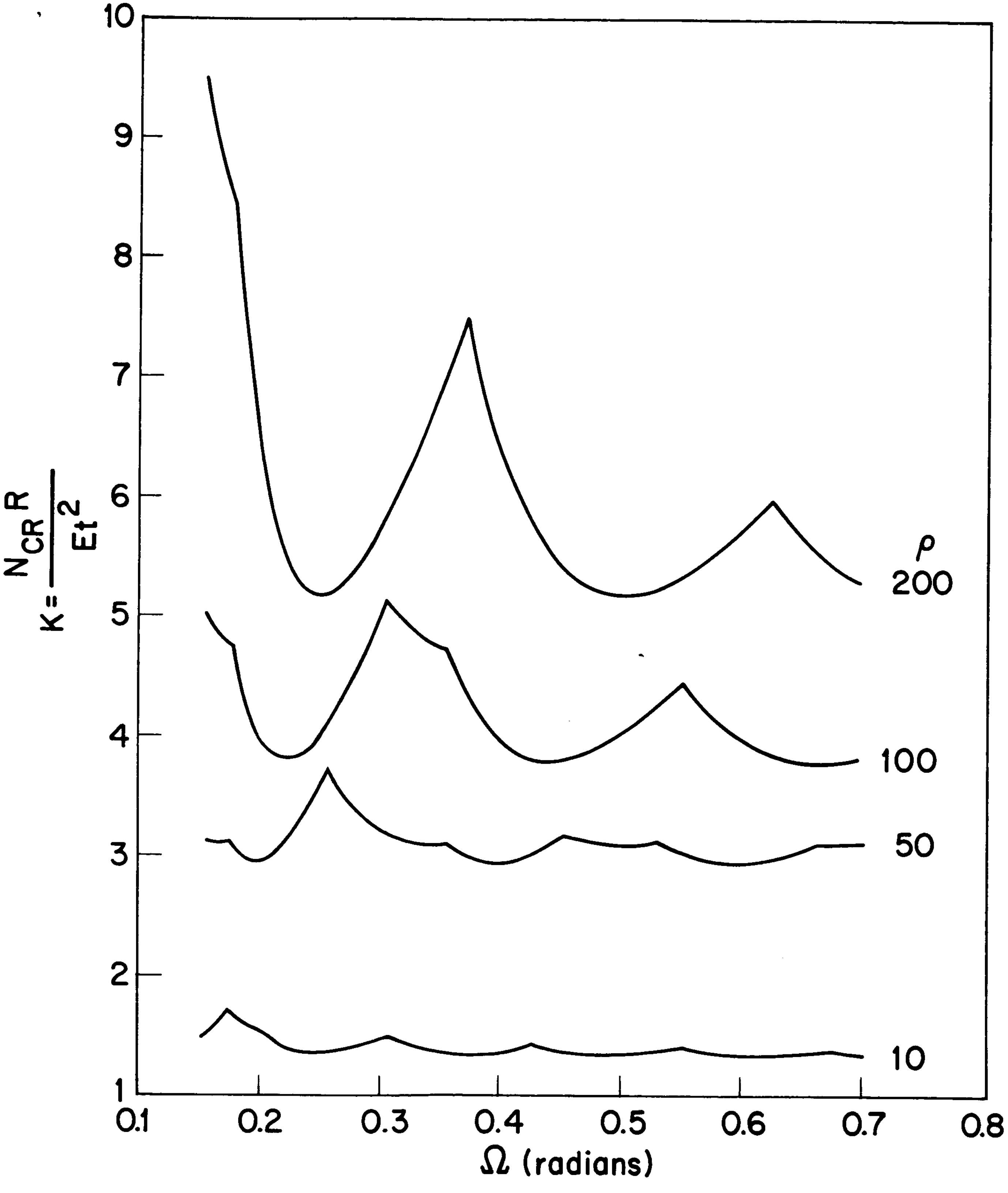


Fig. 13a Buckling Coefficients for Panels With Internal Waffles – L/R = 0.25; R/t = 2000

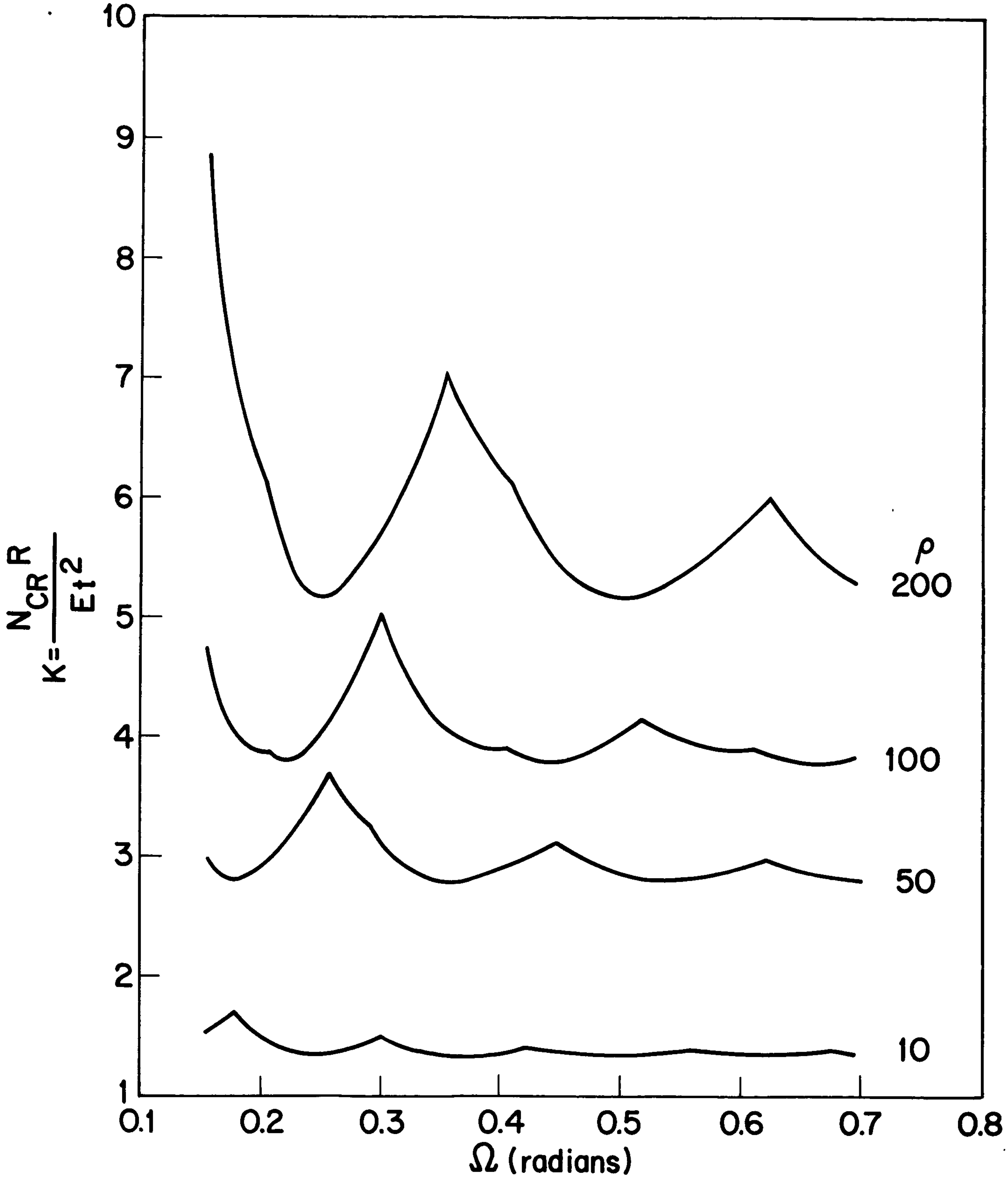


Fig. 13b Buckling Coefficients for Panels With Internal Waffles – L/R = 0.5; R/t = 2000

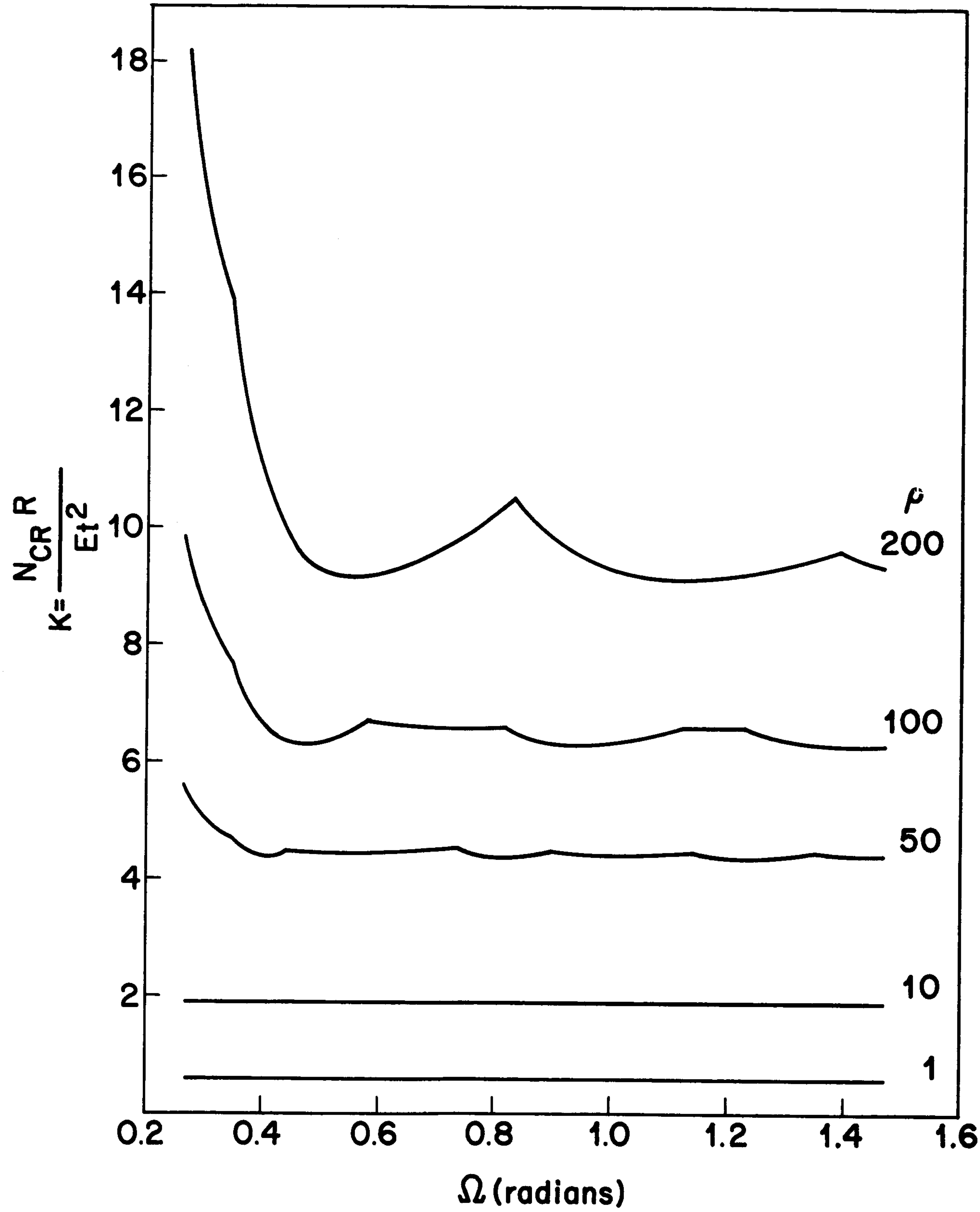


Fig. 14a Buckling Coefficients for Panels With External Waffles – L/R = 0.5; R/t = 400

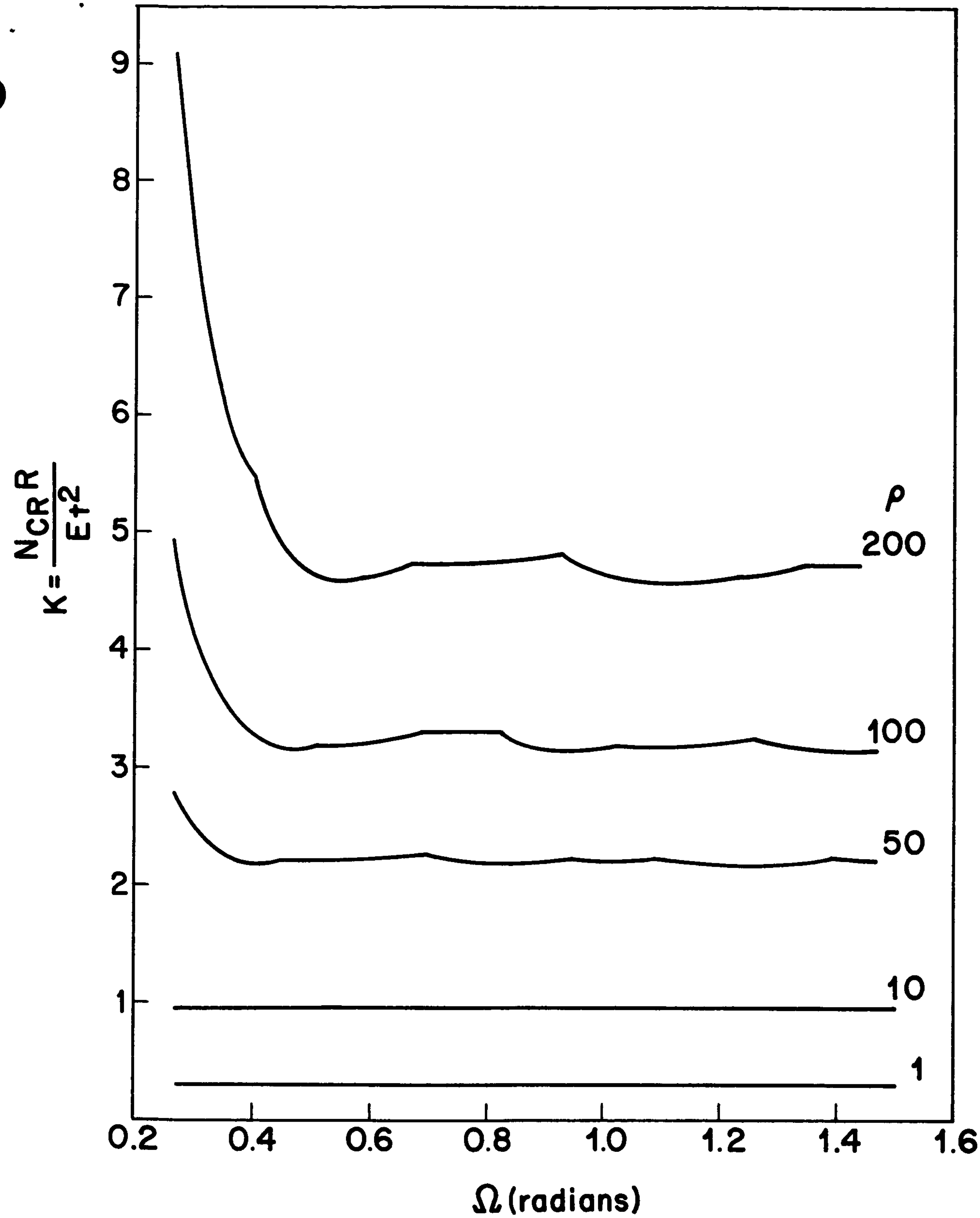


Fig. 14b Buckling Coefficients for Panels With External Waffles – L/R = 1.0; R/t = 400

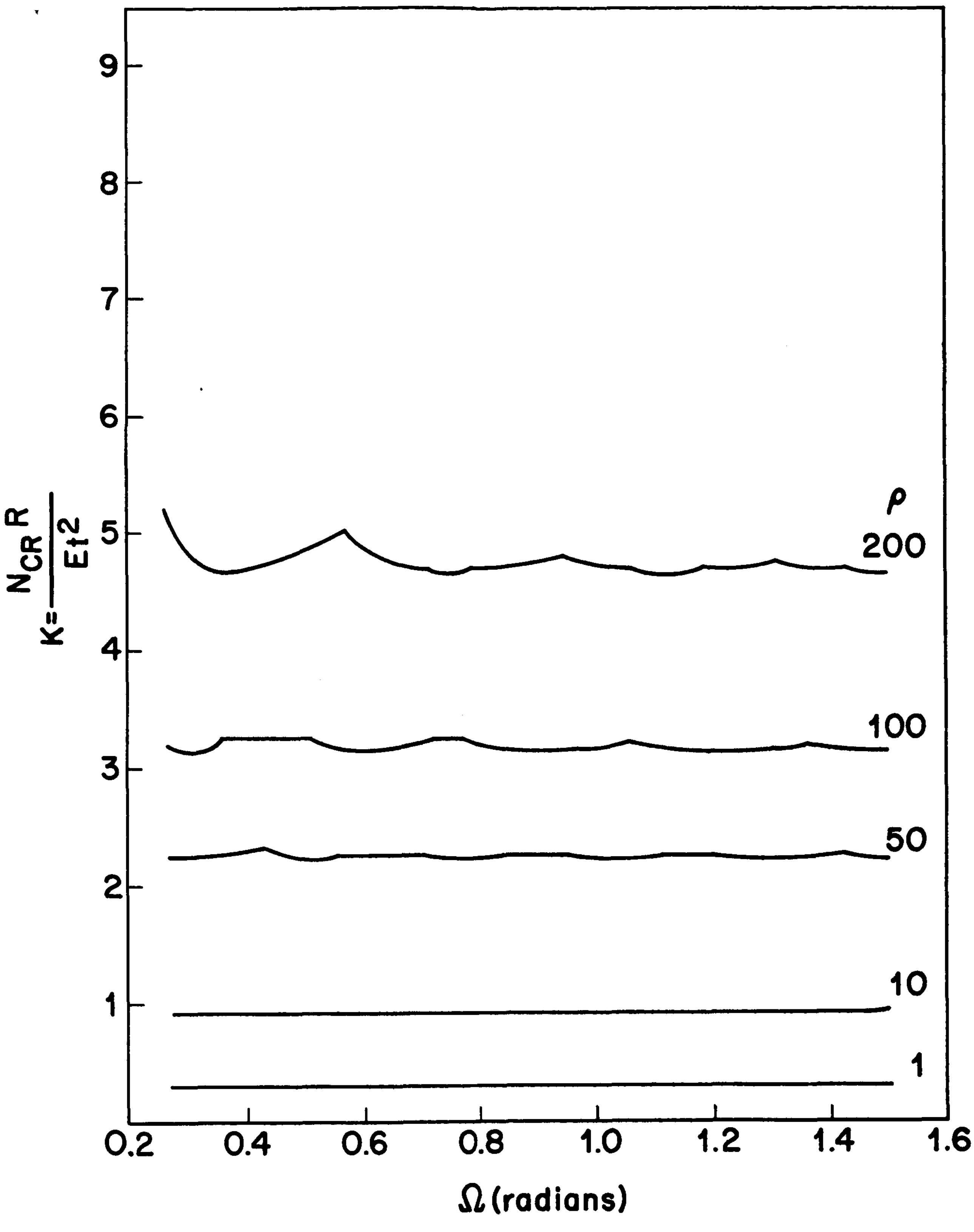


Fig. 15a Buckling Coefficients for Panels With External Waffles – L/R = 1.0; R/t = 1000

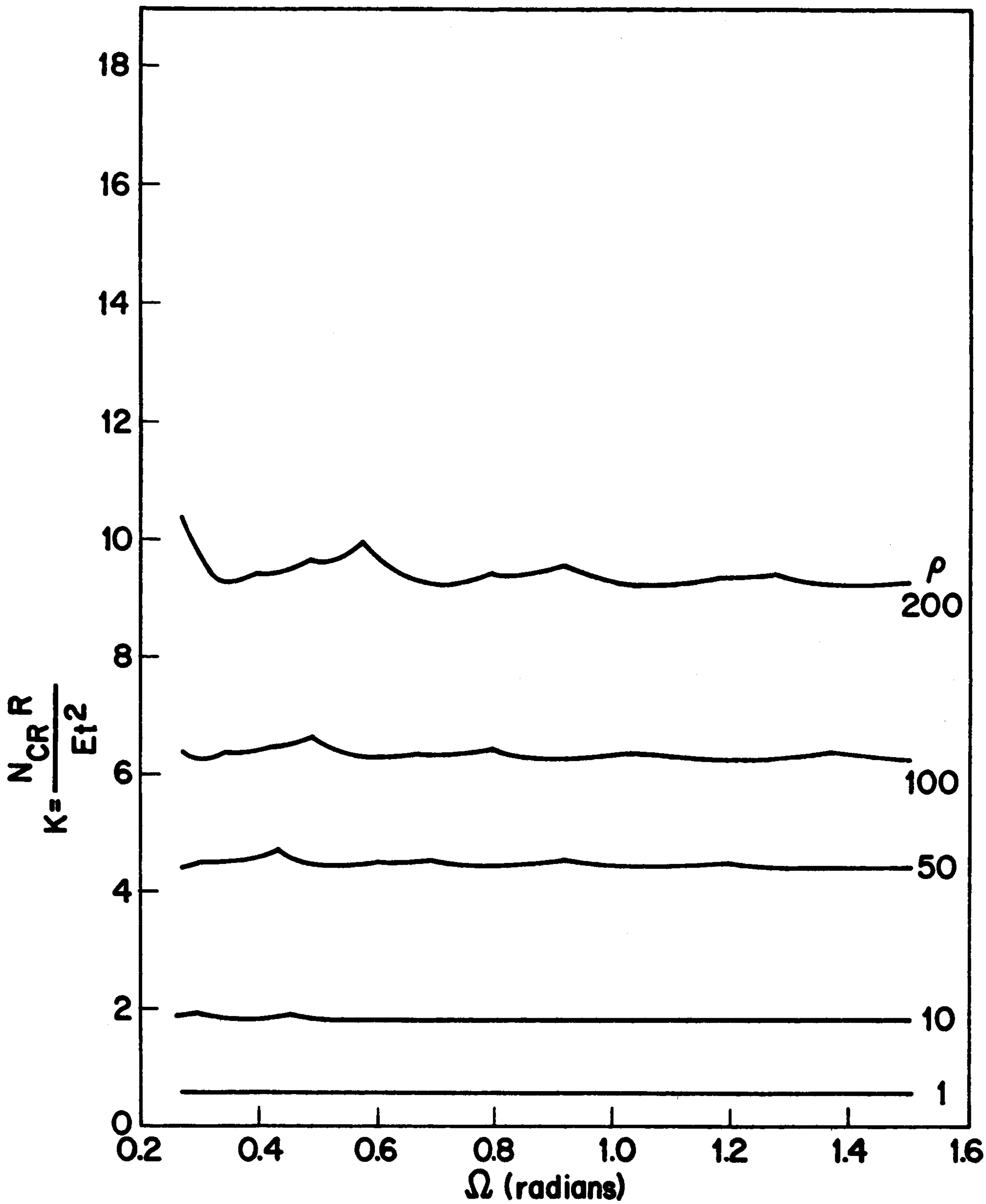


Fig. 15b Buckling Coefficients for Panels With External Waffles – L/R = 1.0; R/t = 1000

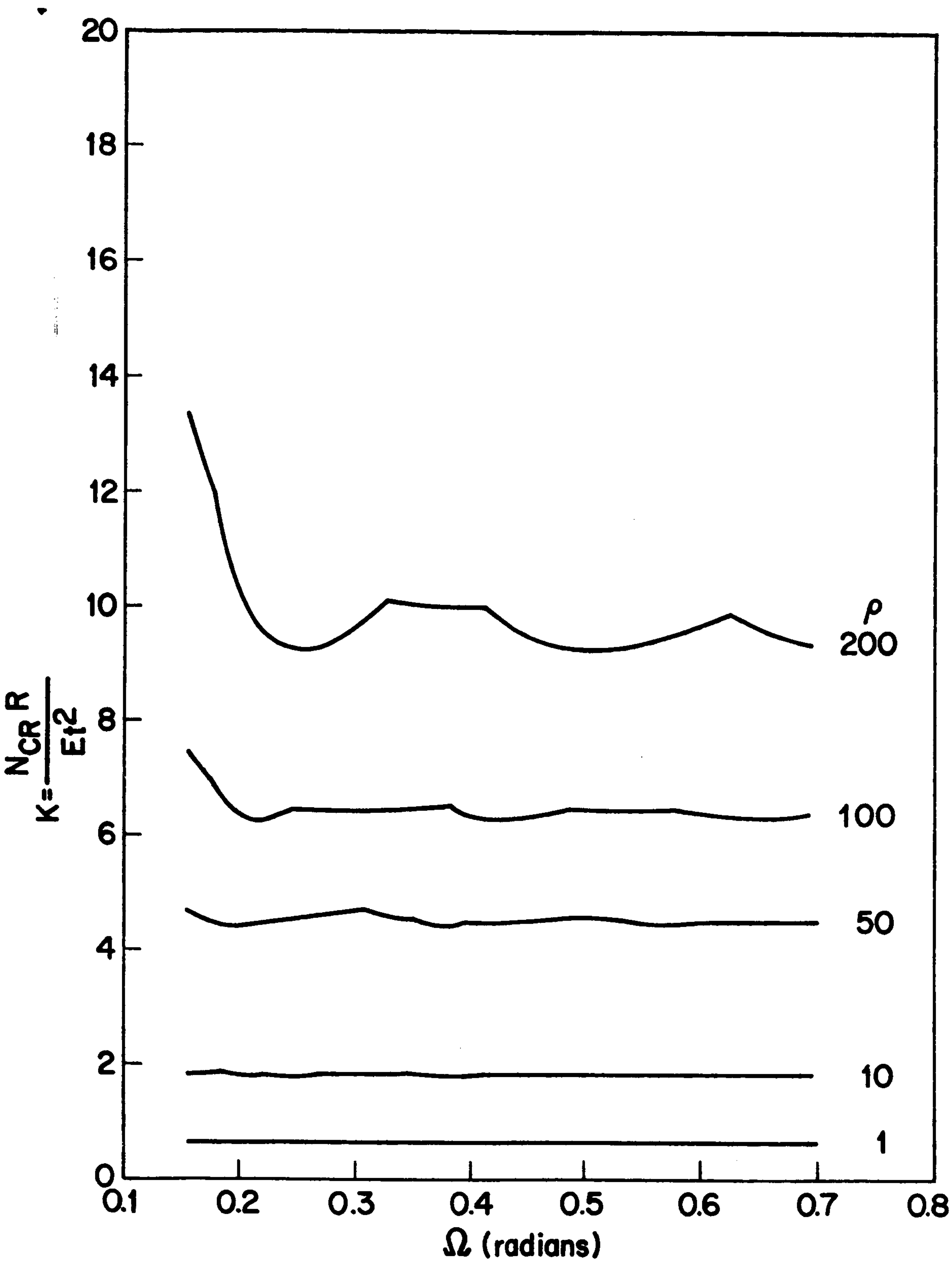


Fig. 16a Buckling Coefficients for Panels With External Waffles – L/R = 0.25; R/t = 2000

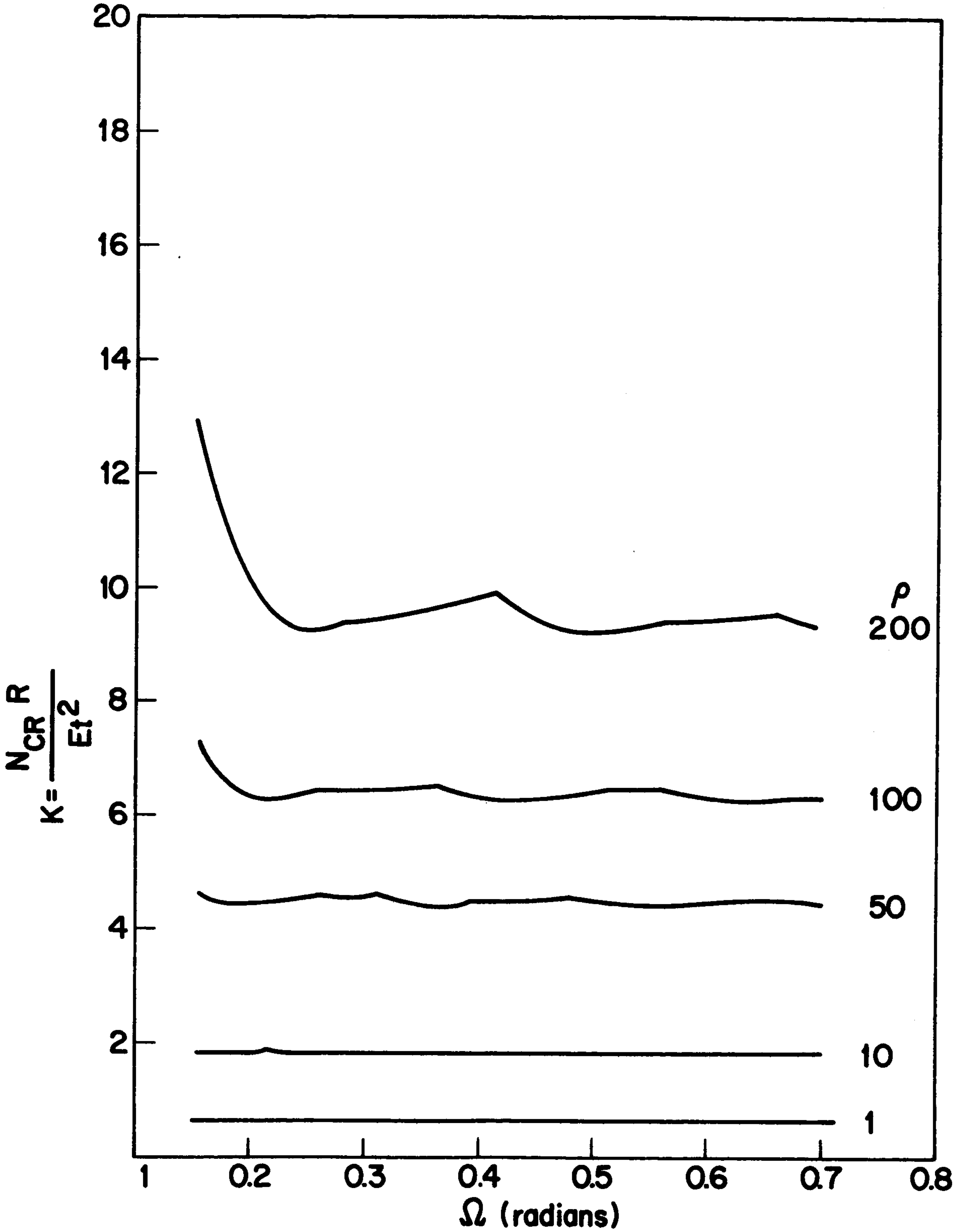


Fig. 16b Buckling Coefficients for Panels With External Waffles – L/R = 0.5; R/t = 2000

Appendix A

INFLUENCE COEFFICIENTS FOR ORTHOTROPIC PLATES

An analysis of influence coefficients for orthotropic rectangular plates subjected to edge moments and shears is presented in this appendix. The influence coefficients are used to determine the elastic restraint provided to cylindrical panels supported along the straight edges by stiffened web plates (see Fig. 2, Vol. II).

The following equation which governs the bending of an orthotropic flat plate may be obtained through specialization of the equations for an orthotropic cylinder (see Sec. 2, Vol. II):

$$D_x w_{,xxxx} + 2H w_{,xxyy} + D_y w_{,yyyy} = 0 \qquad (A.1)$$

where

$$H = D_1 + 2D_{xy}$$

$$D_x = \frac{Et^3}{12(1-\nu^2)} \left\{ 1 + \frac{E_1}{E}\left(\frac{d_1}{S_1}\right)\left(\frac{h_1}{t}\right)\left[\left(\frac{h_1}{t}\right)^2 + \frac{3[(h_1/t)+1]^2}{(1-\nu^2)(E_1/E)(d_1/S_1)(h_1/t)+1}\right](1-\nu^2) \right\}$$

$$D_y = \frac{Et^3}{12(1-\nu^2)} \left\{ 1 + \frac{E_2}{E}\left(\frac{d_2}{S_2}\right)\left(\frac{h_2}{t}\right)\left[\left(\frac{h_2}{t}\right)^2 + \frac{3[(h_2/t)+1]^2}{(1-\nu^2)(E_2/E)(d_2/S_2)(h_2/t)+1}\right](1-\nu^2) \right\}$$

$$D_1 = \frac{\nu E t^3}{12(1+\nu)}$$

$$D_{xy} = \frac{Et^3}{24(1+\nu)} \left\{ 1 + \frac{1}{2}\left[\frac{(E_1/E)J_1}{(1+\nu_1)S_1} + \frac{(E_2/E)J_2}{(1+\nu_2)S_2}\right](1+\nu) \right\}$$

and S_i is the spacing of the stringers; subscripts 1 and 2 refer to the x- and y-directions; w is the lateral displacement of the plate.

The x and y axes are taken so that the edges of the plate are at $x = 0$, $x = a$, $y = 0$, and $y = b$. The plate is assumed to be simply supported along the edges $y = 0$, $y = b$, and $x = a$. A moment M and a transverse shear force V are specified along the edge $x = 0$. These edge loadings may be represented by the following Fourier series:

$$M = \sum_{m=1}^{\infty} M_m \sin \frac{m\pi y}{b} \tag{A.2a}$$

$$V = \sum_{m=1}^{\infty} V_m \sin \frac{m\pi y}{b} \tag{A.2b}$$

Thus, the boundary conditions for the plate are

$\underline{y = 0}$ $$w = w_{,yy} = 0 \tag{A.3a}$$

$\underline{y = b}$ $$w = w_{,yy} = 0 \tag{A.3b}$$

$\underline{x = a}$ $$w = w_{,xx} = 0 \tag{A.3c}$$

$\underline{x = 0}$ $$M_x = M = \sum_{m=1}^{\infty} M_m \sin \frac{m\pi}{b} y = -D_x w_{,xx} - D_1 w_{,yy} \tag{A.3d}$$

$$\begin{aligned} V_x = V &= \sum_{m=1}^{\infty} V_m \sin \frac{m\pi}{b} y \\ &= -D_x w_{,xxx} - (H + 2D_{xy}) w_{,xyy} \end{aligned} \tag{A.3e}$$

By use of the following Fourier series representation of the displacement function, the space variables in Eq. (A. 1) can be separated and the boundary conditions along the edges $y = 0$ and $y = b$ are satisfied:

$$w(x, y) = \sum_{m=1}^{\infty} w_m(x) \sin \frac{m\pi y}{b} \tag{A.4}$$

Insertion of Eq. (A. 4) into Eq. (A. 1) results in the following ordinary differential equation for the Fourier function w_m :

$$D_x \frac{d^4 w_m}{dx^4} - 2\alpha_m^2 H \frac{d^2 w_m}{dx^2} + \alpha_m^4 D_y w_m = 0 \tag{A.5}$$

where

$$\alpha_m = \frac{m\pi}{b} \quad , \quad m = 1, 2, 3, \ldots$$

Since Eq. (A. 5) is linear and has constant coefficients, we substitute

$$w_m = \sum_{i=1}^{4} A_{im} e^{\lambda_i x} \tag{A.6}$$

into Eq. (A. 5) and obtain a characteristic equation of the fourth order. The roots of this characteristic equation are

$$\lambda_{1,2,3,4} = \pm \alpha_m \left(\frac{H}{D_x} \pm \sqrt{\frac{H^2}{D_x^2} - \frac{D_y}{D_x}} \right)^{1/2} \tag{A.7}$$

The form of the solution of Eq. (A.5) depends on the value of $H^2 - D_x D_y$, and thus the following three cases are possible:

Case 1 $\quad H^2 > D_x D_y$

$$w_m = A_{1m} \cosh(\lambda_1 x) + A_{2m} \sinh(\lambda_1 x) + A_{3m} \cosh(\lambda_2 x) + A_{4m} \sinh(\lambda_2 x) \qquad (A.8)$$

where

$$\lambda_1 = \alpha_m \left(\frac{H}{D_x} + \sqrt{\frac{H^2}{D_x^2} - \frac{D_y}{D_x}} \right)^{1/2}$$

$$\lambda_2 = \alpha_m \left(\frac{H}{D_x} - \sqrt{\frac{H^2}{D_x^2} - \frac{D_y}{D_x}} \right)^{1/2}$$

Case 2 $\quad H^2 = D_x D_y$

$$w_m = (A_{1m} + A_{2m} x) \cosh(\lambda x) + (A_{3m} + A_{4m} x) \sinh(\lambda x) \qquad (A.9)$$

where

$$\lambda = \alpha_m \sqrt{\frac{H}{D_x}}$$

Case 3 $\quad H^2 < D_x D_y$

$$w_m = [A_{1m} \cos(dx) + A_{2m} \sin(dx)] \cosh(cx) + [A_{3m} \cos(dx) + A_{4m} \sin(dx)] \sinh(cx)$$

(A.10)

where

$$c = \frac{\alpha_m}{\sqrt{2}} \sqrt{\beta^2 + \frac{H}{D_x}}$$

$$d = \frac{\alpha_m}{\sqrt{2}} \sqrt{\beta^2 - \frac{H}{D_x}}$$

$$\beta^2 = (D_y/D_x)^{1/2}$$

The integration constants A_{1m} to A_{4m} are to be determined such that the boundary condtiions [Eqs. (A.3c) through (A.3e)] at the edges $x = 0$ and $x = a$ are satisfied. This yields

$$(A_m) = [R_m]^{-1} (P_m) \tag{A.11}$$

where

$$(A_m) = \begin{pmatrix} A_{1m} \\ A_{2m} \\ A_{3m} \\ A_{4m} \end{pmatrix}$$

$$(P_m) = \begin{pmatrix} 0 \\ 0 \\ M_{xm} \\ V_{xm} \end{pmatrix}$$

$$
[R_m] = \begin{bmatrix}
\cosh(\lambda_1 a) & \sinh(\lambda_1 a) & \cosh(\lambda_2 a) & \sinh(\lambda_2 a) \\
\lambda_1^2 \cosh(\lambda_1 a) & \lambda_1^2 \sinh(\lambda_1 a) & \lambda_2^2 \cosh(\lambda_2 a) & \lambda_2^2 \sinh(\lambda_2 a) \\
-D_x \lambda_1^2 + D_1 \alpha_m^2 & 0 & -D_x \lambda_2^2 + D_1 \alpha_m^2 & 0 \\
0 & -D_x \lambda_1^3 + k \lambda_1 \alpha_m^2 & 0 & -D_x \lambda_2^3 + k \lambda_2 \alpha_m^2
\end{bmatrix}
$$

for $H^2 > D_x D_y$, or

$$
[R_m] = \begin{bmatrix}
\cosh(\lambda a) & a \cosh(\lambda a) & \sinh(\lambda a) & a \sinh(\lambda a) \\
\lambda^2 \cosh(\lambda a) & \begin{matrix} \lambda^2 a \cosh(\lambda a) \\ + 2\lambda \sinh(\lambda a) \end{matrix} & \lambda^2 \sinh(\lambda a) & \begin{matrix} \lambda^2 a \sinh(\lambda a) \\ + 2\lambda \cosh(\lambda a) \end{matrix} \\
-D_x \lambda^2 + D_1 \alpha_m^2 & 0 & 0 & -D_x (2\lambda) \\
0 & -D_x (3\lambda^2) + k \alpha_m^2 & -D_x \lambda^3 + k \lambda \alpha_m^2 & 0
\end{bmatrix}
$$

for $H^2 = D_x D_y$, or

$$
[R_m] = \begin{bmatrix}
\cos(da)\cosh(ca) & \sin(da)\cosh(ca) & \cos(da)\sinh(ca) & \sin(da)\sinh(ca) \\
\begin{matrix} (c^2 - d^2)\cos(da)\cosh(ca) \\ - 2cd \sin(da)\sinh(ca) \end{matrix} & \begin{matrix} (c^2 - d^2)\sin(da)\cosh(ca) \\ + 2cd \cos(da)\sinh(ca) \end{matrix} & \begin{matrix} (c^2 - d^2)\cos(da)\sinh(ca) \\ - 2cd \sin(da)\cosh(ca) \end{matrix} & \begin{matrix} (c^2 - d^2)\sin(da)\sinh(ca) \\ + 2cd \cos(da)\sinh(ca) \end{matrix} \\
-D_x (c^2 - d^2) + D_1 \alpha_m^2 & 0 & 0 & -D_x (2cd) \\
0 & -D_x d(3c^2 - d^2) + k d \alpha_m^2 & -D_x c(c^2 - 3d^2) + k c \alpha_m^2 & 0
\end{bmatrix}
$$

for $H_2 < D_x D_y$

and

$$
k = H + 2 D_{xy}
$$

Through use of Eqs. (A. 11), (A. 8), (A. 9), or (A. 10), the edge deflection $w_m(0)$ and edge rotation $(d/dx)w_m(0)$ are next computed for the following values of M_m and V_m:

Case I $M_m = 0$, $V_m = 1$

Case II $M_m = 1$, $V_m = 0$

Finally, for each harmonic m of edge loading, the influence coefficients c_{ij} are obtained as follows:

From Case I

$$c_{11} = w_m(0) \tag{A. 12a}$$

$$c_{21} = \frac{d}{dx} w_m(0) \tag{A. 12b}$$

From Case II

$$c_{12} = w_m(0) \tag{A. 13a}$$

$$c_{22} = \frac{d}{dx} w_m(0) \tag{A. 13b}$$

This procedure has been programmed for the IBM 7094 digital computer and the influence coefficients so obtained were used to arrive at the results presented in Sec. 3.

Appendix B
DIGITAL COMPUTER PROGRAM

The solution presented in Sec. 2 was programmed for the digital computer and a listing of the FORTRAN II program is given in this appendix. Use of the program requires input of geometrical and elastic properties of the cylindrical panel, estimated ranges of critical load, number of waves in the axial direction, and a set of control variables. In the general case, such properties as moments of inertia of stiffeners are required in the input, but a separate branch is provided in the program for use in the case of rectangular stiffeners. If this branch is used the stiffener dimensions are read in and stiffnesses are computed internally.

The program computes the determinant of the stability matrix and can give the output in two different ways. A series of values of the determinant may be printed out together with the corresponding load levels. Thus, the user can find the critical load as the lowest load for which the determinant vanishes. The computer can also determine the value of the critical load by use of an iterative procedure.

At certain values of the load a change may occur in the number of real roots of the characteristic equation. Multiple roots exist at such values and hence two columns in the stability matrix will be identical. As this leads to a zero determinant independently of the boundary conditions at the straight edges, the corresponding value of the load is a false solution to the buckling problem. When a series of determinant values are printed, such a solution may be recognized in the output since the number of real roots is included. In case the iterative method is used to find the critical load, the program will ignore zero crossings of the determinant when a change takes place in the number of real roots within 10^{-4} times the load step. Printing out the values of the roots of the characteristic equation is an additional option.

The buckling mode, which is symmetrical around the line of intersection between adjacent panels, is expected to be less critical than the antimetrical mode even for the case without web plates. In the presence of web plates, their stiffness will practically prevent the in-plane displacements and therefore the antimetric mode becomes critical. For completeness, the branch for the symmetrical mode is included in the computer program. However, to avoid unnecessary input, this program branch assumes a negligible effect for the web plate, and a lower bound to the symmetrical buckling mode can be computed. For the antimetric mode, the case of zero web plate stiffness corresponds to infinite flexibility constants (c_{11}, c_{12}, c_{21}, and c_{22}). For other cases the coefficients may be computed according to the solution given in Appendix A.

Figure B-1 shows a flow chart of the computer program. Geometric and elastic properties needed as input are listed in Table B-1. Load range and load step size are defined by the input parameters listed in Table B-2. In addition some control variables are input parameters. They are listed in Table B-3. The complete sequence of input parameters and the corresponding "read-in-formats" are shown in Table B-4.

A sample problem is demonstrated in Table B-5 (input) and Table B-6 (output). The coefficients c_{11}, c_{12}, c_{21}, and c_{22} are computed according to the solution in Appendix A.

The critical load is given under the heading NXO in the output (Table B-6). This buckling load is computed for an internally waffle stiffened panel as shown in Fig. B-2.

The complete program is given in Table B-7.

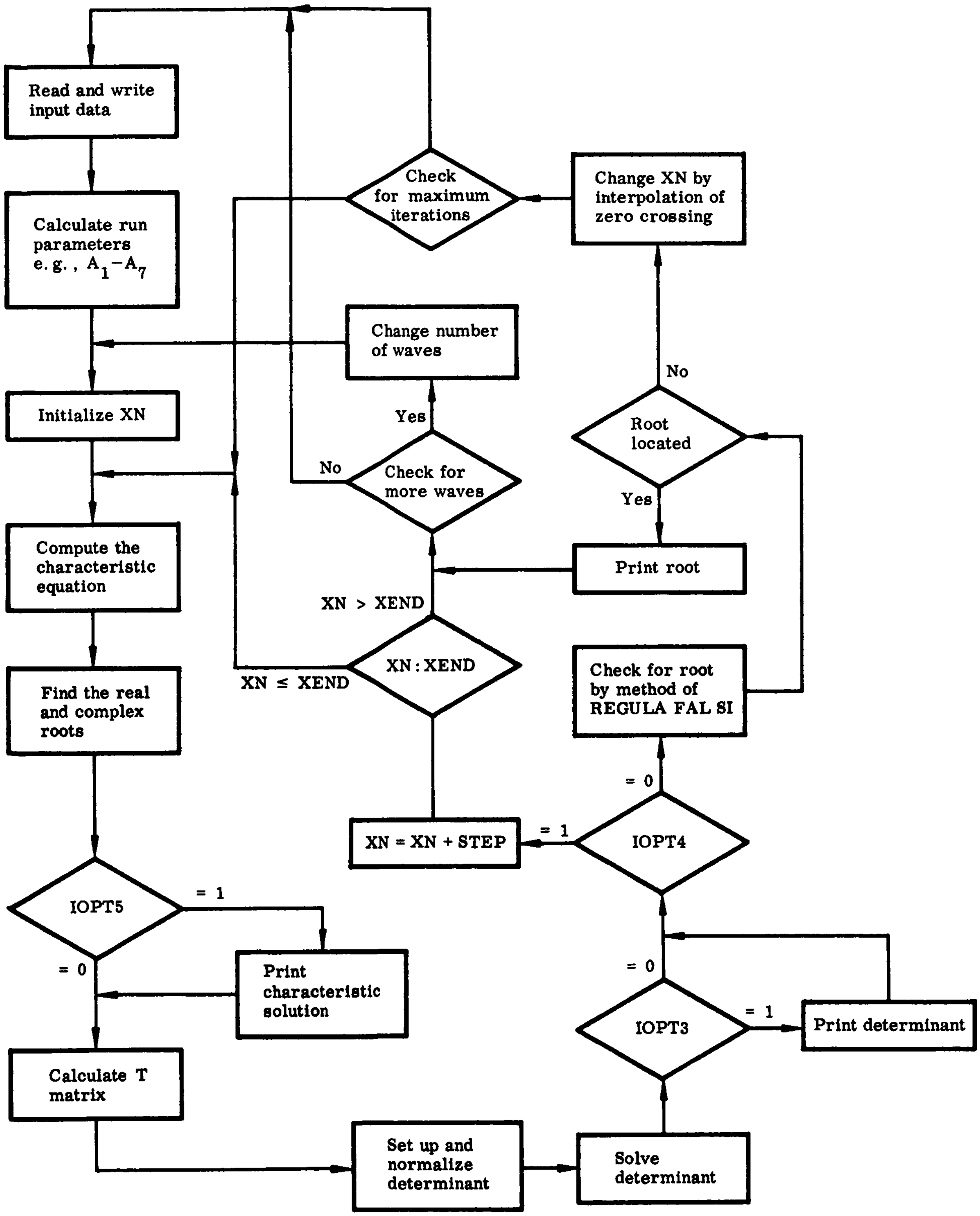


Fig. B-1 Flow Chart

Table B-1

GEOMETRIC AND ELASTIC PROPERTIES USED AS INPUT

	Computer Program Notation	Notation in Analysis (Sec. 2)	Item
	ZNU	ν	Poisson's ratio
	AL	L	Panel length
	AR	R	Panel radius
	AE	E	Young's modulus of skin
	AT	t	Thickness of skin
For Rectangular Stiffeners	DIS		Thickness of rectangular stringers
For Rectangular Stiffeners	D2		Thickness of rectangular rings
For Rectangular Stiffeners	H1		Height of rectangular stringers
For Rectangular Stiffeners	H2		Height of rectangular rings
For Rectangular Stiffeners	AAS		Spacing of rectangular stringers
For Rectangular Stiffeners	AB		Spacing of rectangular rings
	W	Ω	Angle subtending width of panel
	PHI	$\mathfrak{D}$	Cell angle
For Arbitrary Stiffeners	XA	s	Spacing of arbitrary stringers
For Arbitrary Stiffeners	XB	r	Spacing of arbitrary rings
For Arbitrary Stiffeners	XE1	$\hat{e}_1$	Eccentricity of arbitrary stringers
For Arbitrary Stiffeners	XE2	$\hat{e}_2$	Eccentricity of arbitrary rings
For Arbitrary Stiffeners	XA1	A_1	Area of arbitrary stringers
For Arbitrary Stiffeners	XA2	A_2	Area of arbitrary rings
For Arbitrary Stiffeners	XI1	$\bar{I}_1$	Moment of inertia of arbitrary stringers
For Arbitrary Stiffeners	XI2	$\bar{I}_2$	Moment of inertia of arbitrary rings
For Arbitrary Stiffeners	XIT1	J_1	Torsional constant for arbitrary stringers
For Arbitrary Stiffeners	XIT2	J_2	Torsional constant for arbitrary rings
	E1	E_1	Young's modulus of arbitrary stringers
	E2	E_2	Young's modulus of arbitrary rings
	$CC_{11}, CC_{12}, CC_{21}, CC_{22}$	$c_{11}, c_{12}, c_{21}, c_{22}$	Influence coefficients for web plate (included only if KC = 1)
	BN	n	Number of axial waves

Table B-2

LOAD RANGE AND STEP SIZE

Computer Program Notation	Item
XBEG	Starting value for axial load (compression corresponds to negative values). The numerical value should be chosen below the expected solution.
XEND	Maximum value of axial load. (The search for a solution will not be extended beyond this point.)
STEP	Increment of axial load used in search for solution.

Table B-3

CONTROL VARIABLES AND INPUT PARAMETERS

Computer Program Notation	Item
KC	= 0 for symmetrical mode = 1 for antisymmetrical mode
KN	Number of elements in the wave-number vector
LIM	Maximum number of iterations in the search for zero-crossing of determinant
ICARD1	= 0 The "option card" (Card No. 3) will not be included. Information for previous case applies = 1 Card No. 3 will be included
ICARD2	= 0 The "geometry card" (Card No. 4) not included = 1 (Card No. 4) will be included
ICARD3	= 0 The "α-card" (Card No. 5) is not included = 1 (Card No. 5) will be included
ICARD4	= 0 The "load-card" (Card No. 6) is not included = 1 (Card No. 6) will be included
IOPT3	= 0 The determinant is not printed = 1 The determinant is printed
IOPT4	= 0 Regular falsi; search is used = 1 Determinant computed for several points

Table B-3 (cont'd)

Computer Program Notation	Item
IOPT5	= 0 Roots of characteristic equation are printed
	= 1 Roots of characteristic equation are not printed
IOPT7	= 0 External stiffeners
	= 1 Internal stiffeners
IOPT11	= 0 Stiffeners with rectangular cross section
	= 1 Stiffeners with any cross section

Table B-4

INPUT PARAMETER SEQUENCE AND CORRESPONDING READ-INFORMATS

Card	FORTRAN Symbol	Format
1	Record	72H
2	ICARD1, ICARD2, ICARD3, ICARD4, KN, KC, LIM	4I1, 2I3, I4
3	BN(I)	6E12.8
4[(a)]	IOPT3, IOPT4, IOPT5, IOPT7, IOPT11	20I1
5[(b)]	ZNU, AL, AR, AE, AT	6E12.8
6a[(c)]	DIS, D2, H1, H2, AAS, AB, E1, E2, W	6E12.8
6b[(d)]	XA, XB, XE1, XE2, XA1, XA2, XI1, XI2, XIT1, XIT2, E1, E	6E12.8
7[(e)]	XBEG, XEND, STEP	6E12.8
8a[(f)]	PHI	6E12.8
8b[(g)]	PHI, C_{11}, C_{12}, C_{21}, C_{22}	6E12.8

(a) Omitted unless ICARD1 = 1.
(b) Omitted unless ICARD2 = 1.
(c) Omitted unless ICARD3 = 1, for rectangular stiffeners.
(d) Omitted unless ICARD3 = 1, for arbitrary stiffeners.
(e) Omitted unless ICARD4 = 1.
(f) For rigid webplate.
(g) For the case without webplate for symmetrical mode or with finite stiffness webplate for antimetrical mode.

Table B-5

INPUT DATA IN REQUIRED FORMAT

LOCKHEED MISSILES & SPACE COMPANY

80 COLUMN WORKSHEET

FORM LMSC 2874

PAGE ______ OF ______

MODULE | ORGN. | BLDG. | FAC. | JOB NUMBER | DATE OF REQUEST | DISPATCH NUMBER | PROGRAM | OPER. CONTROL NO. | DATE NEEDED | PRIORITY

PROGRAMMER | ORGN. | PHONE

```
1      OCTOBER   1   1965
1111  1  1 300
4        +01
00010
 3       +00 51477   +02 35598   +02 1       +08 147     +00
 221     +00 221     +00 603     +00 603     +00 6821    +01 6821    +01
 1       +08 1       +08 11781   +01
-8       +04-12       +05-25       +03
 5236    +00 25842557-02 43124422-03 43124422-03 15400285-03
```

Table B-7

EXAMPLE: CYLINDRICAL PANEL WITH INTERNAL WAFFLE

```
D1=   0.221000E-00, D2=   0.221000E-00, H1=   0.603000E 00, H2=   0.603000E 00,  A=   0.682100E 01,  B=   0.682100E 01

INTERNAL STIFFENERS.

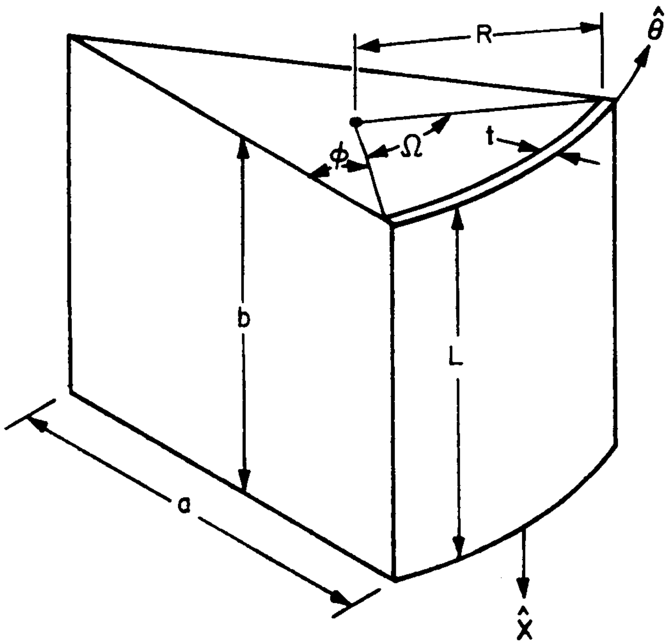
PARAMETER RANGE,   -0.80000E 04 TO    -0.12000E 05, STEP=  -0.25000E 03
ANTIMETRIC MODE  PHI = 5.2359999E-01  C11= 2.5842557E-03  C12= 4.3124421E-04  C21= 4.3124421E-04  C22= 1.5400285E-04

ZNU=   0.300000E-00,   L=   0.514770E 02,     R=   0.355980E 02, N=   0.400000E 01, E=   0.100000E 08, T=   0.147000E-00

OMEGA =   0.117810E 01   E1 =   0.100000E 08   E2 =   0.100000E 08

ALPHA1 - ALPHA7 ARE,
-0.127406E-02  -0.127406E-02   0.112094E 01   0.112094E 01   0.177343E-04   0.177343E-04   0.403729E-05

        N               NXO                                  DETR                         DETI
    4.0000000E 00     -8.9121157E 03      1.2453256E-26+ 60      -1.8650651E-32+  60
```



Panel	Web Plate
$L = 51.477$ in.	$a = 90.765759$ in.
$R = 35.598$ in.	$b \equiv L = 51.477$ in.
$\Omega = 3\pi/8$	$t = 0.147$ in.
$\phi = 0.52360$ radian	
$t = 0.147$ in.	
$d_1 = d_2 = 0.221$ in.	$d_1 = d_2 = 0.221$ in.
$h_1 = h_2 = 0.603$ in.	$h_1 = h_2 = 0.603$ in.
$s = r = 6.821$ in.	$s_1 = s_2 = 6.821$ in.

Fig. B-2 Cylindrical Panel (Stiffeners not shown)

Table B-7

STUDY OF JUNCTURE STRESS FIELDS: STIFFENED PANEL UNDER UNIFORM AXIAL COMPRESSION

```
C CONTRACT NAS 8=11480  TO NASA G.C. MARSHALL SPACE FLIGHT CENTER       COMP0010
C                        HUNTSVILLE, ALABAMA                            COMP0020
C BY SOLID MECHANICS , AEROSPACE SCIENCES LABORATORY , 52-20            COMP0030
C LOCKHEED MISSILES AND SPACE COMPANY, PALO ALTO CALIF                  COMP0040
C      STIFFENED PANEL, UNIFORM AXIAL COMPRESSION , REGULA FALSI        COMP0050
      DIMENSION RN(100)                                                 COMP0060
D     DIMENSION C(9)                                                    COMP0070
I     DIMENSION T(8,8),X(8),RU(8),RV(8),D1(1),DET(1),CO(5),ANSW(4)      COMP0080
      COMMON KEXP                                                       COMP0090
      COMMON IOPT3,IOPT4,IOPT5                                          COMP0100
C     IOPT3=0   DO NOT PRINT DET                                        COMP0110
C     IOPT3=1   PRINT DET                                               COMP0120
C     IOPT4=0   REGULA FALSI SEARCH FOR DET=0                           COMP0130
C     IOPT4=1   COMPUTE  DET FOR SEVERAL POINTS                         COMP0140
C     IOPT5=0   DO NOT PRINT CHARACTERISTIC SOLUTION                    COMP0150
C     IOPT5=1   PRINT CHARACTERISTIC SOLUTION                           COMP0160
C     IOPT7=0   EXTERNAL                                                COMP0170
C     IOPT7=1   INTERNAL                                                COMP0180
C     IOPT11=0     RECTANGULAR CROSS-SECTION.                           COMP0190
C     IOPT11=1     ARBITRARY CROSS-SECTION                              COMP0200
C     KC=0  SYMMETRIC MODE                                              COMP0210
C     KC=1  ANTIMETRIC MODE                                             COMP0220
      DELT = .00005                                                     COMP0230
      READ INPUT TAPE 5,1020                                            COMP0240
   10 CONTINUE                                                          COMP0250
      READ INPUT TAPE 5,990,ICARD1,ICARD2,ICARD3,ICARD4,KN,KC,LIM       COMP0260
      WRITE OUTPUT TAPE 6,1020                                          COMP0270
C     ICARD1=0  SKIP OPTION CARD                                        COMP0280
C     ICARD2=0  SKIP GEOMETRY CARD                                      COMP0290
C     ICARD3=0  SKIP ALPHA CARD                                         COMP0300
C     ICARD4=0  SKIP STEP SIZE CONTROL                                  COMP0310
      READ INPUT TAPE 5,1000,(RN(I),I=1,KN)                             COMP0320
      KKK=1                                                             COMP0330
      IF (ICARD1) 30,30,20                                              COMP0340
```

```
20 CONTINUE                                                        COMP0350
   READ INPUT TAPE 5,1010,IOPT3,IOPT4,IOPT5,IOPT7,IOPT11            COMP0360
30 IF (ICARD2) 50,50,40                                            COMP0370
40 CONTINUE                                                        COMP0380
   READ INPUT TAPE 5,1000,ZNU,AL,AR,AE,AT                          COMP0390
   Y1=(1.-ZNU)/2.                                                  COMP0400
   Y2=(1.+ZNU)/2.                                                  COMP0410
   W1=(1.-ZNU**2)/2.                                               COMP0420
   W2=(AT/AR)**2/12.                                               COMP0430
   W4=(1.-ZNU**2)                                                  COMP0440
   ARL = 3.14159265*AR/AL                                          COMP0450
   XN = XN1S                                                       COMP0460
50 IF (ICARD3) 80,80,60                                            COMP0470
60 IF (IOPT11) 70,70,160                                           COMP0480
70 CONTINUE                                                        COMP0490
   READ INPUT TAPE 5,1000,D1S,D2,H1,H2,AAS,AB,E1,E2,W               COMP0500
   WRITE OUTPUT TAPE 6,1110,D1S,D2,H1,H2,AAS,AB                    COMP0510
80 IF (IOPT11) 90,90,170                                           COMP0520
90 CONTINUE                                                        COMP0530
   H1R=H1/AR                                                       COMP0540
   H2R=H2/AR                                                       COMP0550
   H1T=H1/AT                                                       COMP0560
   H2T=H2/AT                                                       COMP0570
   D1B = D1S*E1/(AB*AE)                                            COMP0580
   D2A = D2*E2/(AAS*AE)                                            COMP0590
   A1=W1*D1B*H1R*(H1T+1.)                                          COMP0600
   A2=W1*D2A*H2R*(H2T+1.)                                          COMP0610
   A3=1.+2.*W1*D1B*H1T                                             COMP0620
   A4=1.+2.*W1*D2A*H2T                                             COMP0630
   A5=W2+W1/6.*H1R**2*D1B*(4.*H1T+6.+3./H1T)                       COMP0640
   A6=W2+W1/6.*H2R**2*D2A*(4.*H2T+6.+3./H2T)                       COMP0650
   IF (D1S-H1) 100,110,110                                         COMP0660
100 A5=H1T*D1B*(D1S/AR)**2                                         COMP0670
   GO TO 120                                                       COMP0680
110 A5=H1T*D1B*H1R**2                                              COMP0690
120 IF (D2-H2) 130,140,140                                         COMP0700
130 A6=H2T*D2A*(D2/AR)**2                                          COMP0710
   GO TO 150                                                       COMP0720
140 A6=H2T*D2A*H2R**2                                              COMP0730
```

```
150 A7=2.*W2+Y1/3.*(W5+W6)                                                COMP0740
    GO TO 180                                                             COMP0750
160 CONTINUE                                                              COMP0760
    READ INPUT TAPE 5,1000,XA,XB,XE1,XE2,XA1,XA2,XI1,XI2,XIT1,XIT2,E1,COMP0770
   1E2,W                                                                  COMP0780
    WRITE OUTPUT TAPE 6,1120,XA,XB,XE1,XE2,XA1,XA2,XI1,XI2,XIT1,XIT2     COMP0790
170 CONTINUE                                                              COMP0800
    XAI=XA*AT                                                             COMP0810
    BT=XB*AT                                                              COMP0820
    E1E = E1/AE                                                           COMP0830
    E2E = E2/AE                                                           COMP0840
    A1E=XA1*XE1                                                           COMP0850
    A2E=XA2*XE2                                                           COMP0860
    A1=W4*A1E/(AR*BT) * E1E                                               COMP0870
    A2=W4*A2E/(AR*XAT) * E2E                                              COMP0880
    A3=1.+W4*XA1/BT * E1E                                                 COMP0890
    A4=1.+W4*XA2/XAT * E2E                                                COMP0900
    A5=W2+W4*(XI1+A1E*XE1)/(BT*AR**2) * E1E                               COMP0910
    A6=W2+W4*(XI2+A2E*XE2)/(XAT*AR**2) * E2E                              COMP0920
    A7=2.*W2+Y1/(AT*AR**2)*(XIT1/XB*E1E + XIT2/XA*E2E)                    COMP0930
    GO TO 200                                                             COMP0940
180 IF (IOPT7) 200,200,190                                                COMP0950
190 A1=-A1                                                                COMP0960
    A2=-A2                                                                COMP0970
    WRITE OUTPUT TAPE 6,1030                                              COMP0980
200 CONTINUE                                                              COMP0990
210 IF (ICARD4) 270,270,220                                               COMP1000
220 CONTINUE                                                              COMP1010
    READ INPUT TAPE 5,1000,XBEG,XEND,STEP                                 COMP1020
    WRITE OUTPUT TAPE 6,1100,XBEG,XEND,STEP                               COMP1030
    READ INPUT TAPE 5,1000,PHI,CC11,CC12,CC21,CC22                        COMP1040
    IF (KC) 230,230,240                                                   COMP1050
230 WRITE OUTPUT TAPE 6,1130,PHI                                          COMP1060
    GO TO 250                                                             COMP1070
240 WRITE OUTPUT TAPE 6,1140,PHI,CC11,CC12,CC21,CC22                      COMP1080
    XBOT = (CC11*CC22-CC12*CC21)*2.                                       COMP1090
    AXBOT = ABSF(XBOT)                                                    COMP1100
    C3 =  ABSF(CC22)/AXBOT                                                COMP1110
    C4 = -ABSF(CC21)/AXBOT                                                COMP1120
```

```
      C5 = C4                                                           COMP1130
      C6 =  ABSF(CC11)/AXBOT                                            COMP1140
      C7=0.                                                             COMP1150
      C8=0.                                                             COMP1160
  250 FUDGE=2.*W1/(AE*AT)                                               COMP1170
      XBEG=XBEG*FUDGE                                                   COMP1180
      XEND=XEND*FUDGE                                                   COMP1190
      STEP=STEP*FUDGE                                                   COMP1200
  260 CONTINUE                                                          COMP1210
      STEPS=STEP                                                        COMP1220
      XN1S=XN                                                           COMP1230
  270 CONTINUE                                                          COMP1240
      XN =XBEG                                                          COMP1250
      AN=BN(KKK)                                                        COMP1260
      WRITE OUTPUT TAPE 6,1070,ZNU,AL,AR,AN,AE,AT                       COMP1270
      WRITE OUTPUT TAPE 6,1080,W,E1,E2                                  COMP1280
      WRITE OUTPUT TAPE 6,1090,A1,A2,A3,A4,A5,A6,A7                     COMP1290
  280 CONTINUE                                                          COMP1300
      IOUT = 15                                                         COMP1310
  290 CONTINUE                                                          COMP1320
      IF (IOP15) 300,300,320                                            COMP1330
  300 CONTINUE                                                          COMP1340
  310 WRITE OUTPUT TAPE 6,1060                                          COMP1350
  320 CONTINUE                                                          COMP1360
      STEP=STEPS                                                        COMP1370
      Y4=A3*A4-ZNU                                                      COMP1380
      Y6=12.                                                            COMP1390
      ANN = AN                                                          COMP1400
      AN = AN*ARL                                                       COMP1410
      AN2=AN**2                                                         COMP1420
      AN3=AN**3                                                         COMP1430
      AN4=AN2**2                                                        COMP1440
      AN5=AN2*AN3                                                       COMP1450
      AN6=AN3*AN3                                                       COMP1460
      AN8=AN4*AN4                                                       COMP1470
      ALIM=XEND                                                         COMP1480
      DEI=0.                                                            COMP1490
      DEI(2)=0.                                                         COMP1500
      STEPS=STEP                                                        COMP1510
```

```
      K=0                                                               COMP1520
      K1=0                                                              COMP1530
      KOUNT=0                                                           COMP1540
      TH=(W-PHI)/2.                                                     COMP1550
      CTH=COSF(TH)                                                      COMP1560
      STH=SINF(TH)                                                      COMP1570
      AA = XN                                                           COMP1580
C     COMPUTE THE CHARACTERISTIC EQUATION                               COMP1590
  330 CONTINUE                                                          COMP1600
      C(5) = Y1 *(-A3*A5+A1**2)*AN8+ (1.-ZNU)*ZNU*A1*AN6                COMP1610
     1   +Y1*(ZNU**2-A3*A4)*AN4 - XN*Y1*A3*AN6                          COMP1620
      C(4) =(A5*Y4 + Y1*A3*A7 - A1**2*A4)*AN6 + ((1.-ZNU)*A2*A3         COMP1630
     1   +2.* Y1*A1*A4)*AN4                        + XN*Y4*AN4          COMP1640
      C(3)=Y1*2.*ZNU*A2*AN2 - (Y4*A7 + Y1*(A3*A6 + A4*A5))*AN4          COMP1650
     1   - XN*Y1*A4*AN2 - (1.+ZNU)* A1*A2*AN4                           COMP1660
      C(2) = (A6*Y4 + Y1*A4*A7 - A2**2*A3)*AN2                          COMP1670
      C(1) = Y1*(A2**2 - A4*A6)                                         COMP1680
      DO 340 I=10,14                                                    COMP1690
  340 C(I)=0.                                                           COMP1700
      CALL MULLER (C, 4, X, X(9))                                       COMP1710
      IF (IOPT5) 380,380,350                                            COMP1720
  350 WRITE OUTPUT TAPE 6,1040,(X(I),X(I+8),I=1,4)                      COMP1730
      WRITE OUTPUT TAPE 6,980,(C(I),I=1,5)                              COMP1740
      DO 360 I=1,5                                                      COMP1750
  360 CO(I) = C(I)                                                      COMP1760
      DO 370 I=1,4                                                      COMP1770
I     ANSW(I) = (0., 0.)                                                COMP1780
      DO 370 J=1,5                                                      COMP1790
I 370 ANSW(I) = X(I)*ANSW(I) + CO(J)                                    COMP1800
      WRITE OUTPUT TAPE 6,980,(ANSW(I),ANSW(I+4),I=1,4)                 COMP1810
  380 CONTINUE                                                          COMP1820
      NI2 = NI1                                                         COMP1830
      NI1 = 0                                                           COMP1840
      DO 410 I=1,4                                                      COMP1850
      I1=5-I                                                            COMP1860
      IF (ABSF(X(I1))-1.E6*ABSF(X(I1+8))) 400,390,390                   COMP1870
  390 NI1 = NI1+1                                                       COMP1880
I 400 Z = SQRTF(X(I1))                                                  COMP1890
I     X (2*I1-1) = Z                                                    COMP1900
```

```
I      X (2*I1) = -7                                                    COMP1910
  410 CONTINUE                                                          COMP1920
      DO 420 I=1,8                                                      COMP1930
I     Z=X(I)                                                            COMP1940
I     Z2=Z*Z                                                            COMP1950
I     Z3=Z*Z2                                                           COMP1960
I     Z4=Z2**2                                                          COMP1970
I     Z5=Z*Z4                                                           COMP1980
I     T20 = Y1*(A3*AN4 + A4*Z4) - Y4*AN2*Z2                             COMP1990
I     BU(I) = (Y1*(A1*AN5 + ZNU*AN3) + A4*AN*Z2*(Y1-A1*AN2)             COMP2000
     1   - Y2*A2*AN*Z4) / T20                                           COMP2010
I     BV(I) = -((Y2*((1.,0.)+A1*AN2) - A3*A4)*AN2*Z + (Y1*A4 + A2*A3    COMP2020
     1   *AN2)*Z3 - Y1*A2*Z5) / T20                                     COMP2030
  420 CONTINUE                                                          COMP2040
      GNA = - W2*ZNU+A7                                                 COMP2050
      DO 450 I=1,8                                                      COMP2060
I     ELI = EXPF((.5,0.) * X(I) * W)                                    COMP2070
I     RLI = (1.,0.)/ELI                                                 COMP2080
I     OLB = (1.,0.) + X(I)*BV(I)                                        COMP2090
I     C11=A4*OLB-ZNU*AN*BU(I)-A2*X(I)**2                                COMP2100
I     C22=A6*X(I)**2-AN2*ZNU*W2-A2*OLB                                  COMP2110
I     C11=C11*AE*AT    /W4                                              COMP2120
I     C22=C22*AE*AT*AR     /W4                                          COMP2130
I     C12=AE*AT*(A6*X(I)**3-A2*(BV(I)*X(I)**2+X(I))-(A7-ZNU*W2)*AN**2   COMP2140
     1*X(I)) /W4                                                        COMP2150
I     C21=C12*CTH-C11*STH                                               COMP2160
I     C11=C11*CTH+C12*STH                                               COMP2170
      IF (KC) 430,430,440                                               COMP2180
C     SYMMETRIC MODE                                                    COMP2190
I 430 T(1,I)=ELI*X(I)                                                   COMP2200
I     T(2,I)=RLI*X(I)                                                   COMP2210
I     T(3,I)=ELI*(BV(I)*CTH+STH)                                        COMP2220
I     T(4,I)=RLI*(BV(I)*CTH-STH)                                        COMP2230
I     C1S=BU(I)*AR*W4                                                   COMP2240
I     C2S=    AE*AT*(BV(I)*AN+BU(I)*X(I))                               COMP2250
I     T(5,I)=ELI*(C1S*C7+C2S)                                           COMP2260
I     T(6,I)=RLI*(C1S*C7-C2S)                                           COMP2270
I     T(7,I)=ELI*((CTH-BV(I)*STH)*C8-C21/AR)                            COMP2280
I     T(8,I)=RLI*((CTH+BV(I)*STH)*C8-C21/AR)                            COMP2290
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      GO TO 450                                                         COMP2300
C     ANTIMETRIC MODE                                                   COMP2310
I 440 T(3,I)=ELI*(CTH-RV(I)*STH)                                        COMP2320
I     T(4,I)=RLI*(CTH+RV(I)*STH)                                        COMP2330
I     T(1,I)=BU(I)*ELI                                                  COMP2340
I     T(2,I)=BU(I)*RLI                                                  COMP2350
I     T(5,I)=ELI*(C11+C3*(RV(I)*CTH+STH)*AR-C4*X(I))                    COMP2360
I     T(6,I)=RLI*(C11-C3*(RV(I)*CTH-STH)*AR+C4*X(I))                    COMP2370
I     T(7,I)=ELI*(C22-C5*(RV(I)*CTH+STH)*AR+C6*X(I))                    COMP2380
I     T(8,I)=RLI*(C22+C5*(RV(I)*CTH-STH)*AR-C6*X(I))                    COMP2390
  450 CONTINUE                                                          COMP2400
      DDET=1.                                                           COMP2410
      KEXP=0                                                            COMP2420
      DO 540 I=1,8                                                      COMP2430
      CALL NORM (T(1,I),T(1,I+8),DET2)                                  COMP2440
      DD=DET2                                                           COMP2450
  460 IF (DD-1.E+10) 480,470,470                                        COMP2460
  470 DD=DD*1.E-10                                                      COMP2470
      KEXP=KEXP+10                                                      COMP2480
      GO TO 460                                                         COMP2490
  480 IF (DD-1.E-10) 490,490,500                                        COMP2500
  490 DD=DD*1.E+10                                                      COMP2510
      KEXP=KEXP-10                                                      COMP2520
      GO TO 480                                                         COMP2530
  500 CONTINUE                                                          COMP2540
  510 DDET=DDET*DD                                                      COMP2550
  520 IF (DDET-1.E+10) 540,530,530                                      COMP2560
  530 DDET=DDET*1.E-10                                                  COMP2570
      KEXP=KEXP+10                                                      COMP2580
      GO TO 520                                                         COMP2590
  540 CONTINUE                                                          COMP2600
I     D1=DET                                                            COMP2610
      CALL PLXDET (T, T(1,9), 8, DET, DET(2))                           COMP2620
      DET=DET*DDET                                                      COMP2630
      DET(2)=DET(2)*DDET                                                COMP2640
      IF (IOPT3) 550,620,550                                            COMP2650
  550 IF (IOUT-50) 560,560,590                                          COMP2660
  560 IOUT=IOUT+1                                                       COMP2670
      IF (IOPT5) 580,580,570                                            COMP2680
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  570 CONTINUE                                                              COMP2690
      WRITE OUTPUT TAPE 6,1060                                              COMP2700
      IOUT=IOUT+3                                                           COMP2710
  580 CONTINUE                                                              COMP2720
      GO TO 610                                                             COMP2730
  590 IOUT=3                                                                COMP2740
      WRITE OUTPUT TAPE 6,1020                                              COMP2750
  600 WRITE OUTPUT TAPE 6,1060                                              COMP2760
  610 CONTINUE                                                              COMP2770
      XNX = XN/FUDGE                                                        COMP2780
      WRITE OUTPUT TAPE 6,1050,ANN,XNX,DET,KEXP,DET(2),KEXP,NI1             COMP2790
  620 IF (IOPT4) 630,640,630                                                COMP2800
  630 XN = XN + STEP                                                        COMP2810
      IF ((XN-XEND)*STEP) 330,330,960                                       COMP2820
  640 KK=KEXP1-KEXP                                                         COMP2830
      AAZ=AA                                                                COMP2840
      IF (KK) 650,670,650                                                   COMP2850
  650 IF (D1) 660,670,660                                                   COMP2860
  660 XD=10.**KK                                                            COMP2870
      D1=D1*XD                                                              COMP2880
      D1(2)=D1(2)*XD                                                        COMP2890
      FOLD=FOLD*XD                                                          COMP2900
      FTWO=FTWO*XD                                                          COMP2910
      KEXP1=KEXP                                                            COMP2920
  670 IF (K) 650,680,860                                                    COMP2930
  680 IF (ABSF(DET)-ABSF(DET(2))) 690,720,720                               COMP2940
  690 IF (D1(2)/DET(2)) 700,780,780                                         COMP2950
  700 IF (NI1-NI2) 740,710,740                                              COMP2960
C     IGNORE CROSSING  IF NUMBER OF COMPLEX ROOTS HAS CHANGED               COMP2970
C     ZERO CROSSING OF IMAGINARY PART                                       COMP2980
  710 K=-1                                                                  COMP2990
      FOLD=D1(2)                                                            COMP3000
      FNEW=DET(2)                                                           COMP3010
      GO TO 900                                                             COMP3020
  720 IF (D1/DET) 730,780,780                                               COMP3030
  730 IF (NI1-NI2) 740,770,740                                              COMP3040
C     CHANGE STEPSIZE                                                       COMP3050
  740 KOUNT=KOUNT+1                                                         COMP3060
      IF (KOUNT-3) 750,750,760                                              COMP3070
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  750 AA=AA1                                                       COMP3080
      STEP=STEP/10.                                                COMP3090
I     DET=D1                                                       COMP3100
      NI1=NI2                                                      COMP3110
      GO TO 780                                                    COMP3120
  760 KOUNT=0                                                      COMP3130
      STEP=STEPS                                                   COMP3140
      GO TO 780                                                    COMP3150
C     ZERO CROSSING OF REAL PART                                   COMP3160
  770 K=1                                                          COMP3170
      FOLD=D1                                                      COMP3180
      FNEW=DET                                                     COMP3190
      GO TO 900                                                    COMP3200
  780 AA1=AA                                                       COMP3210
      AA=AA+STEP                                                   COMP3220
      KEXP1=KEXP                                                   COMP3230
      K1=K1+1                                                      COMP3240
      IF (K1-LIM) 800,800,790                                      COMP3250
  790 WRITE OUTPUT TAPE 6,1150                                     COMP3260
      GO TO 960                                                    COMP3270
  800 IF ((AA-ALIM)*STEP) 820,820,810                              COMP3280
  810 WRITE OUTPUT TAPE 6,1160                                     COMP3290
      GO TO 960                                                    COMP3300
  820 CONTINUE                                                     COMP3310
  830 XN = AA                                                      COMP3320
      GO TO 330                                                    COMP3330
  840 WRITE OUTPUT TAPE 6,1170                                     COMP3340
      GO TO 960                                                    COMP3350
C     SEARCH FOR DET=0                                             COMP3360
  850 K=K-1                                                        COMP3370
      FNEW=DET(2)                                                  COMP3380
      GO TO 870                                                    COMP3390
  860 K=K+1                                                        COMP3400
      FNEW=DET                                                     COMP3410
  870 IF (FOLD*FNEW) 880,940,930                                   COMP3420
  880 CONTINUE                                                     COMP3430
  890 IF (XABSF(K)-LIM) 900,900,840                                COMP3440
  900 STEP1= (AA-AA1)*FOLD/(FOLD-FNEW)                             COMP3450
      AA2=AA                                                       COMP3460
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      AA=AA1+STEP1                                                      COMP3470
      FTWO=FNEW                                                         COMP3480
      IF (ABSF((AAZ-AA)/AA)-DELT) 940,940,910                           COMP3490
  910 IF (XMODF(ABSF(K),3)) 920,920,830                                 COMP3500
  920 AA=(AA1+AA2)*.5                                                   COMP3510
      GO TO 830                                                         COMP3520
  930 STEP1=AA2-AA                                                      COMP3530
C     CROSSING BETWEEN AA AND AA2                                       COMP3540
      AA1=AA                                                            COMP3550
      AA=AA2                                                            COMP3560
      FOLD=FNEW                                                         COMP3570
      FNEW=FTWO                                                         COMP3580
      KEXP1=KEXP                                                        COMP3590
      GO TO 880                                                         COMP3600
C     ROOT LOCATED                                                      COMP3610
  940 ROOT= AA1+(AA-AA1)*FOLD/(FOLD-FNEW)                               COMP3620
      XNX = ROOT/FUDGE                                                  COMP3630
  950 WRITE OUTPUT TAPE 6,1050,ANN,XNX,DET,KEXP,DET(2),KEXP             COMP3640
  960 CONTINUE                                                          COMP3650
      IF (KKK-KN) 970,10,10                                             COMP3660
  970 KKK=KKK+1                                                         COMP3670
      GO TO 270                                                         COMP3680
  980 FORMAT (1H0, 8E15.8)                                              COMP3690
  990 FORMAT (4I1,2I3,I4)                                               COMP3700
 1000 FORMAT (6E12.8)                                                   COMP3710
 1010 FORMAT (20I1)                                                     COMP3720
 1020 FORMAT (72H1                                                      COMP3730
     1                                                        )         COMP3740
 1030 FORMAT  (30H0 INTERNAL STIFFENERS,                            )   COMP3750
 1040 FORMAT (41H0 SOLUTION OF THE CHARASTERITIC EQUATION   /(1P8E15.7))COMP3760
 1050 FORMAT (1P3E20.7, 1H+, I3, 1PE20.7, 1H+, I4, I20)                 COMP3770
 1060 FORMAT ( 1H0,12X, 1HN, 18X, 3HNX0,17X, 16X,4HDETR,21X,            COMP3780
     1 4HDETI    )                                                      COMP3790
 1070 FORMAT (6H0 ZNU=E15.6, 6H,   L=E15.6, 9H,      R=E15.6,4H, N=E15.6COMP3800
     1 , 4H, E=E15.6, 4H, T=E15.6  )                                    COMP3810
 1080 FORMAT (9H0 OMEGA =, E15.6, 7H   E1 =, E15.6, 7H   E2 =, E15.6)   COMP3820
 1090 FORMAT (24H0 ALPHA1 - ALPHA7 ARE,       /7E15.6 )                 COMP3830
 1100 FORMAT (19H0 PARAMETER RANGE,   E14.5,5H TO   E14.5,7H, STEP=E14.5COMP3840
     1   )                                                              COMP3850
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1110 FORMAT (5H  D1=                      E15,6,5H, D2=E15.6,5H, H1=E15.6,  COMP3860
    1 5H, H2=E15.6, 5H,  A= E15.6, 5H,  B= E15,6  )                          COMP3870
1120 FORMAT (120H        A              R             E1           E2       ACOMP3880
    1A1          AA2           I1           I2           IT1          IT2    COMP3890
    2 / 10E12.5 )                                                            COMP3900
1130 FORMAT (15H SYMMETRIC MODE5X5HPHI =PE15,8)                              COMP3910
1140 FORMAT(16H ANTIMETRIC MODE2X5HPHI =1PE14,7,2X4HC11=E14.7,2X4HC12= COMP3920
    1 E14.7,2X4HC21=E14.7,2X4HC22=E14,7)                                     COMP3930
1150 FORMAT ( 50H0 MAXIMUM NUMBER OF STEPS EXCEEDED                         )COMP3940
1160 FORMAT (50H0  UPPER LOAD LIMIT EXCEEDED                               ) COMP3950
1170 FORMAT (50H0   REGULA FALSI DOES NOT CONVERGE                          ) COMP3960
     END                                                                     COMP3970
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