NASA TECHNICAL NOTE

NASA TN D-3244

c. 1

NASA TN D-3244

THEORETICAL RELATIONSHIPS FOR METEOROID PUNCTURE EXPERIMENTS

by Charles C. Dalton

George C. Marshall Space Flight Center
Huntsville, Ala.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1966

THEORETICAL RELATIONSHIPS FOR METEOROID PUNCTURE EXPERIMENTS

By Charles C. Dalton

George C. Marshall Space Flight Center
Huntsville, Ala.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – Price $2.00

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

Calculations for the results illustrated in Figure 1 were programmed by Mrs. Sylvia Bryant for a GE 225 digital computer. Discussions with Dr. O. K. Hudson, representing MSFC on the NASA Meteoroid Technology Advisory Working Group, were helpful in planning the analysis in relation to current and anticipated difficulties in the interpretation of experiments and in the analysis of hazards.

DEFINITION OF SYMBOLS

Symbol	Definition
$\bar{m}$	mass of a meteoroid which at mean density, mean velocity, and mean impact angle will just puncture a given sheet
β_5	logarithm of the puncture flux enhancement factor
v	meteoroid velocity in kilometers per second
ρ_p	meteoroid material density in grams per cubic centimeter
$\bar{v}$, $\bar{\rho}_p$	log $\bar{v}$ and log $\bar{\rho}_p$ represent the population means of log v and log ρ_p
m	meteoroid mass in grams
F_s	flux of incident meteoroids with mass equal to or greater than m
β_6	value of log F_s when m is one gram
β_2	slope of log F_s as a linear junction of log m
x	angle of impact in radians with respect to the normal for a puncturing meteoroid
p	effective thickness of a distributed puncture transducer in centimeters
t	target subscript
ρ_t	target material density in grams per cubic centimeter
H_t	target hardness in Brinell units
P_o	crater depth in centimeters
d	projectile diameter in centimeters
C_t	bulk velocity of sound in kilometers per second
E	10^{-6} x Young's modulus, kilograms per square centimeter
ϵ	ductility, percent elongation in 2-inch gauge length at fracture

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
ν	Poisson's ratio
k_2	a known function of target material parameters
k_3'	a parameter depending on material and dynamic properties of the projectile or meteoroid, thought to be 3 for dense projectiles
k_1	an unknown function of meteoroid material parameters
$\emptyset$	meteoroid puncture flux
β_n	celestial latitude of radiant of surface normal
λ_n	celestial longitude of radiant of surface normal
β_p	celestial latitude of earth's center seen from the satellite
λ_p	celestial longitude of earth's center seen from the satellite
θ	angular radius of the earth (meteoroid cross section) as seen from the satellite in radians
r	geocentric radial distance of the satellite in kilometers
r_e	radius of the earth in kilometers
i	inclination of meteoroid orbital plane to the ecliptic plane
β_e	celestial latitude of meteor radiant
f	cosmic weighting function for a photographic meteor
h_m	meteor height above sea level at the point of maximum brilliance
P	Öpik's probability that a meteoroid in a given orbit will encounter the earth during one revolution of the particle

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
f_f	reciprocal of the apparent fraction of the circle of celestial latitude through a meteor radiant
Z	meteor zenith angle in radians
s	constant scale factor to restore equality between the samle size and the sample sum of the weighting functions for photographic meteors
x_2	angle of impact in radians with respect to the normal for a meteoroid impacting onto a surface
$\bar{m}_x$	mass of a meteoroid which at mean density, mean velocity, and angle of impact x will just puncture a given sheet of material
P_{mx}	probability that the mass m of a meteoroid is sufficient for puncture when the angle of impact is x
R_1	the fraction of the potentially puncturing meteoroids from the exposed hemisphere which are intercepted by the apparent segment of the earth
R_2	the apparent fraction of the earth's disk
D	angular separation of the center of the earth's disk from the surface zenith in radians
γ	surface zenith angle (radians) of probable radiant
M	angle at the center of the earth's disk between the meridian of the exposed hemisphere and the celestial meridian
β_r	celestial latitude of the probable radiant of the puncturing meteoroid
λ_r	celestial longitude of the probable radiant of the puncturing meteoroid
α	half of the arc of the earth's disk (radians) subtended above the surface horizon
z	angle (radians) between the centroid of the apparent disk and the center of the earth
N	angle between celestial and surface meridians at the probable radiant

THEORETICAL RELATIONSHIPS FOR METEOROID PUNCTURE EXPERIMENTS

SUMMARY

In unshielded meteoroid measurement satellite experiments where vehicle attitude information is available without information about the meteoroid closing velocity vector, the probable radiant and the puncture flux enhancement factor for sporadic meteors should be based on the assumption that velocity vectors for ambient meteoroids are isotropically distributed. Then, although half of the meteoroids which impact onto a flat transducer are more than 45 degrees from the normal, those within 45 degrees puncture three times more effectively than the others, and half of the puncturing meteoroids are less than 33.5 degrees from the normal when the earth is below the surface horizon. This distribution also implies that the flux of puncturing meteoroids is five times greater than the flux of incident meteoroids with mass equal to or greater than the mass of the almost-puncturing meteoroid with the average values of density, velocity, and impact angle.

The formulas for the celestial coordinates of the probable radiant of a puncturing meteoroid have been corrected for earth-shielding. This correction is important near the earth even under nominal circumstances; e.g., when the orbit height is 500 kilometers, the radius of the earth's effective disk (as seen from the vehicle) is 70 degrees. In that case, when the elevations of the earth's center are 0, 15, 30, 45, and 60 degrees the probable radiants are deflected 5, 10, 20, 39, and 80 degrees, respectively, by the earth-shielding effect.

I. INTRODUCTION

Purposes

The main purpose of this analysis is to show how attitude information for an appropriate meteoroid puncture counting satellite can be used to infer the probable radiant of a puncturing meteoroid even when the earth partially obscures the celestial hemisphere to which the puncture transducer surface is exposed. A secondary purpose prerequisite to the main purpose is the derivation of the probability density function for the angle of impact of a puncturing meteoroid. A biproduct of the secondary purpose is a more accurate determination of the puncture flux enhancement factor; i.e., if a particular sheet of material is just puncturable by a meteoroid of mean density, mean velocity, mean impact angle, and mass $\bar{m}$, then the mean flux of puncturing meteoroids exceeds the mean flux of incident meteoroids with mass equal to or greater than $\bar{m}$ by β_5 order of magnitude [1]. The more accurate determination of β_5 is a secondary purpose of this analysis.

Methods and Assumptions

In most of this analysis, it is not necessary to know the true mean value of parameters such as the logarithms of the closing velocity and the material density ρ of the meteoroids, but only the variation with respect to the unknown true mean. All indicated logarithms are to base ten. The following assumptions seem appropriate:

1. All puncturing meteoroids are considered to enter cislunar space from outside the earth-moon system with appreciable velocity. Meteoroid geocentric velocity at any point in cislunar space is sufficiently higher than vehicle orbital velocity that it is essentially the closing velocity. The logarithm of velocity, v, when v is in kilometers per second, for the population of incident meteoroids, is essentially normally distributed with mean and standard deviation given by equations (1) and (2), respectively [1].

$$\log \bar{v} = \log 26.7 = 1.43, \tag{1}$$

$$\sigma_{\log v} = 0.18. \tag{2}$$

2. Most of the puncturing meteoroids in all present experiments are dust balls instead of having the more substantial structure of meteorites. Although the mean density, ρ_p, for the population of dustball meteoroids is known only to within a factor two probable error, the logarithm of density, ρ_p, where the density is in grams per cubic centimeter, for the population of incident meteoroids, can be considered to be normally distributed with mean and standard deviation given by equations (3) and (4), respectively, [1].

$$\log \bar{\rho}_p = \log 0.44 \pm 0.30 = - 0.35 \pm 0.30, \tag{3}$$

$$\sigma_{\log \rho_p} = 0.44. \tag{4}$$

3. It is assumed that the puncture transducers are exposed to a celestial hemisphere, and that the probable radiant of the velocity vector of a puncturing meteoroid is determined with sufficient accuracy by neglecting any systematic variations in meteoroid physical or statistical parameters over that part of the apparent celestial hemisphere which is not subtended by the earth. Most of the punctures are expected to be from sporadic meteors rather than from stream meteoroids [1]; but

the known occurrence of a major meteor stream is an exception to the isotropic flux which is otherwise assumed. If a puncture is concurrent with a well-defined meteor stream with unobstructed radiant within the median angle of deviation from the surface normal radiant, then the probable radiant should be considered to be that of the concurrent meteor stream.

4. At any particular distance from the surface of the earth, the mean flux, F_s, of meteoroids per unit area per unit time with mass equal to or greater than m grams is considered to be an exponential function of mass m; i.e., [1],

$$F_s = 10^{\beta_6} m^{\beta_2} \tag{5}$$

where β_6 is a constant. In a model for cislunar meteoroid impact recently suggested as a criterion for current purposes by Dalton [1], β_2 is given as a function of distance from the surface of the earth with range $-1.34 \leq \beta_2 \leq -1$ in cislunar space and with an effective value of -1.23 for the orbit of the Pegasus-A meteoroid detection satellite. To determine the distribution of the angle of impact, x, for the population of puncturing meteoroids, the numerical value of β_2 for the Pegasus-A mission will be assumed throughout cislunar space; i.e.,

$$\beta_2 = -1.23. \tag{6}$$

5. In Dalton's [2] model for the puncture of a homogeneous metallic wall by hypervelocity impact of a spherical projectile, the relation between the minimum sufficient mass, m, of the projectile, its velocity, v, and the angle of impact, x, can be expressed as

$$\log m = 1.9 + \log(p^3 \rho_t H_t) - (3/2)(\log v + \log \cos x) - \log \rho_p, \tag{7}$$

where ρ_p is the density of the projectile and where ρ_t, H_t, and p are the density, hardness in Brinell units, and thickness in centimeters, respectively, for the punctured wall. Equation (7) is used in the present analysis only in the derivation of the distribution of the angle of impact, x, for the population of puncturing meteoroids.

6. Under contract number NAS7-219 to the Martin-Marietta Corporation for a survey and multivariate analysis of available laboratory hypervelocity impact data, Reismann, Donahue and Burkitt [3] found that in the highest of three velocity regimes the multiple correlation coefficient is nearly maximized (to 0.989) by representing the relation between projectile and target parameters for impact at normal incidence as follows:

$$p_o/d = 2.5(v/C_t)^{0.50}\ (E/E_t)^{1.31}\ (\nu/\nu_t)^{8.0}\ (\epsilon/\epsilon_t)^{0.43}, \qquad (8)$$

where

p_o = crater depth

d = projectile diameter

v = closing velocity in kilometers per second

t = target subscript

C_t = bulk velocity of sound in kilometers per second

E = 10^{-6} x Young's modulus, kilograms per square centimeter

ϵ = ductility, percent elongation in 2-inch gauge length at fracture

ν = Poisson's ratio.

The exponents in equation (8) are the result of some rounding off of decimals in the published [3] values. The relation between the thickness, p, of a puncturable sheet and crater depth, p_o, in a thick target has been treated analytically by Andriankin and Stepanov [4] and by Herrmann and Jones [5] and stochastically by Dalton [2, 6]. The results are problematical; but for meteoroid impact, the following relation would seem to be the most appropriate assumption:

$$p = 1.59\ p_o. \qquad (9)$$

Dalton [1] showed that when equations (1), (3) and (9) are assumed for meteoroids, the compatibility of equations (7) and (8) for aluminum 2024-T3 implies that the mass, $\bar{m}$, - of a meteoroid with mean log density

($\log \bar{\rho}_p = \log 0.44$), mean log velocity ($\log \bar{v} = \log 26.7$), and mean impact angle ($\bar{x}_2 = \pi/4$) - which will just puncture a particular metallic wall is

$$\bar{m} = 10^{10.985}(\rho C_t^{0.50} E_t^{1.31} \nu_t^{8.0} \epsilon_t^{0.43})^3. \tag{10}$$

It is assumed that equation (10) is superior to equation (7) for those purposes where it can be used; however, the exponent "3" and consequently also the exponent "10.985" in equation (10) are problematical for meteoroid impact, as well as the corresponding values in equation (7), and should be treated as unknown constants, say k_3 and k_1, respectively, when incident flux parameters (β_6 and β_2 in equation (5)) are to be inferred from puncture flux parameters (see Section II below).

II. RELATION BETWEEN PARAMETERS FOR IMPACT AND PUNCTURE FLUXES

Corresponding to the meteoroid impact flux, F_s, in equation (5), there will be a puncture flux, $\emptyset$, per unit area per unit time, through a particular exposed wall or distributed puncture transducer, which can be represented by

$$\emptyset = 10^{\beta_6+\beta_5} \bar{m}^{\beta_2}, \tag{11}$$

where β_6 and β_2 are the same constants used in equation (5) to express the incident (or impact) flux, F_s, the constant, β_5, is the logarithm of the puncture flux enhancement factor to be derived in Section IV, and $\bar{m}$ is the nominally puncturable meteoroid mass which is now predicted by the improvised equation (10).

By the suggestion at the end of Section I about the problematical exponents in equation (10), an expression for $\bar{m}$ alternative to equation (10) is

$$\bar{m} = 10^{k_1+k_2k_3} \rho^{k_3}, \tag{12}$$

where k_1 and k_3 are unknown functions of the projectile material and dynamic characteristics, to be determined for meteoroids by satellite experiments, and where k_2 is a function of the target material parameters as follows:

$$k_2 = 0.50 \log C_t + 1.31 \log E_t + 8.0 \log \nu_t + 0.43 \log \epsilon_t. \tag{13}$$

By equations (11) and (12), one can write the following relation between log $\emptyset$ and (k_2 + log p), both of which (by equation (13)) have known values for each of the puncture transducers of different material and effective thickness in the flight experiments:

$$\log \emptyset = (\beta_6 + \beta_5 + k_1\beta_2) + k_3\beta_2 \, (k_2 + \log p). \qquad (14)$$

Dalton's [1] calculations for the Explorer XVI satellite imply -5.417 and -5.697 for log $\emptyset$ for the 1-mil and 2-mil puncture sensors, respectively, and -6.175 and -5.904 for (k_2 + log p) for the same respective sensors. The 2-point solution to equation (14) for the Explorer XVI orbit would be -1.20.

$$\log \emptyset = -11.80 - 1.033 \, (k_2 + \log p). \qquad (15)$$

The resulting slope in equation (15) would imply by equation (14) that, if 3 were the correct value of k_3, then the value for β_2 would be -0.344. The value of β_2 which Hawkins and Upton [8] found for photographic meteors is -1.34; and by Dalton's [1] model the effective value of β_2 for the Explorer XVI orbit would be -1.20.

Extrapolation of the puncture results for Explorer XVI by equation (15) would indicate a higher puncture rate for thicker structures than one would want to anticipate for design purposes without verification by further puncture experiments. The results in equation (15) are not established with much confidence because the numbers of punctures (44 and 11 for the 1-mil and 2-mil sensors, respectively) are statistically small samples, especially when the extreme diversity of the population of incident meteoroids is considered. More pertinently, one should anticipate that the relation between log $\emptyset$ and (k_2 + log p) in equation (14) may not be linear.

A sufficiently accurate value for β_5 will be derived in Section IV; but this leaves more unknowns (4) in equation (14) than conditions (2) which can be applied toward their determination from Pegasus-type data. If values for β_6 and β_2 can be assumed from the statistics and physical theory of meteors, then the meteoroid impact parameters k_1 and k_3 can be found as constants (if equation (14) is linear) or as functions of (k_2 + log p) (if equation (14) is nonlinear). But, because the impact parameters k_1 and k_3 are not known, it is not even possible to determine the value of β_6 in equation (14) by assuming a value for β_2, or vice versa. Therefore, by equations (5), (11), (12), and (14), it follows that the nominally minimum sufficient mass for puncturing meteoroids cannot be determined from meteoroid puncture data alone, unless the incident flux, cumulative with respect to mass, is known as a function of the mass.

III. DISTRIBUTION OF ANGLE OF IMPACT FOR INCIDENT METEOROIDS ISOTROPICALLY FROM AN APPARENT HEMISPHERE ONTO A PLAIN OR CONVEX SURFACE

Appropriate puncture experiments should give satellite attitude and punctured sensor identification at each meteoroid puncture event (see Reference 9 for a discussion of the Pegasus-A experiment). This information can be used to determine the celestial latitude, β_n, and celestial longitude, λ_n, of the radiant of the normal to the punctured surface, the corresponding celestial coordinates (β_p, λ_p) for the center of the earth (as seen from the punctured satellite), and the angular radius θ of the earth's effective meteoroid encounter cross section seen from the satellite; i.e., approximately,

$$\theta = \sin^{-1} [(r_e + 100)/r], \tag{16}$$

where r is the geocentric radial distance of the satellite in kilometers and r_e is the radius of the earth in kilometers. Therefore, when the probability distribution of the radiants of incident meteoroids is assumed, and when the relation between the probability distributions of the angles of impact of incident meteoroids and of puncturing meteoroids is established (see Section V), the celestial coordinates (β_m,λ_m) of the probable radiant of the puncturing meteoroid can be determined as functions of the values of β_n, λ_n, β_p, λ_p, and θ at the time of puncture. In this manner, any large sample of punctures will give a distribution of probable radiants, although the resulting distribution may be somewhat more widely dispersed than the unknown population of true radiants.

The dispersion of the probable radiants of the puncturing meteoroids in appropriate experiments will have two spurious components because of some uncertainty in the satellite attitude information and because the meteoroid velocity vector in on-board coordinates is not measured but must be inferred probabilistically. Nevertheless, the resulting information is of considerable interest because the other three sources of such information are somewhat problematical and they involve different ranges of meteoroid mass than that for which puncture experiments are designed. One such source of information is the interpretation of isophotes of zodiacal light from small dust particles in interplanetary space. Other inferences can be attempted from the distribution of the orbits of comets and from the effect of the combined action of several physical processes on the debris from disintegrating comets. More intensive studies of the distribution of meteoroid radiants have been based on the visual, photographic, and radio meteor data; but several sources of observational bias complicate these analyses and may prejudice the results.

Without specific contrary information, one could assume that the distribution of the closing velocity vectors is isotropic and therefore that the probability density for meteor radiants is uniformly distributed over the exposed celestial hemisphere. Then the probability density function $f(\beta_e)$ for the celestial latitude β_e of the radiants in the interval $0 \leqq \beta_e \leqq \pi/2$ would be $\cos \beta_e$; and, by integration, the mean (expected value) and median for β_e would be $(\pi/2 - 1)$ and $\pi/6$ radians (32.7 and 30.0 degrees), respectively.

Lovell's [10] results for the distribution of sporadic visual meteors, corrected for horizon zenith attraction and twilight effects, have been interpreted by Davison and Winslow [11] to indicate that approximately half of the sporadic meteoroids have orbital inclinations, i, within ±15 degrees from the ecliptic. (Orbit inclination, i, in the interval $0 \leqq i \leqq \pi/2$, and radiant celestial latitude, β_e, are practically interchangable here.) But the masses of the meteoroids had to be sufficient for duplicate visual observations (see Lovell's [10] description of Prentice's technique for visual meteors), and therefore several orders of magnitude higher than the largest mass one can practically expect to detect in present experiments. Then, by Hawkins [12] models for the distributions of meteoroids of cometary origin (most of those encountered in present experiments) and of asteroidal origin, the 2-observer visual meteor sample might be expected to have angles of inclination, i, distributed somewhat more like the asteroids. Allen [13] gives the arithmetic mean for the inclination of the orbits of the asteroids as approximately 10 degrees.

Assuming an isotropic distribution of the radiants of meteoroids (encountered by an observer in orbit about the sun) is not equivalent to assuming completely randomly distributed meteoroid orbits (with inclination, i, in the interval $0 \leqq i \leqq \pi/2$). Briggs [14] indicates that sin i would be the probability density function for the inclination, i, of completely random orbits (with all meteoroids considered, i.e., not just those detected by a moving observer). Then, by integration, the mean (expected value) of i would be one radian (57.3 degrees).

A completely random distribution of orbits may be indicated for long period comets; but the debris from the disintegration of comets may have orbits which are distributed less randomly. Davison and Winslow [15] describe this situation as follows: "The zodiacal dust particles are thought to be more highly concentrated in the plane of the ecliptic. They have two components of drift as they orbit the sun, one toward the plane of the ecliptic and one toward the sun [16]. The drift toward the plane of the ecliptic results from the attraction of the planets. The drift or spiraling in toward the sun is a result of ... the Poynting-Robertson effect (Lovell [10] and Wyatt and Whipple [17])."

By establishing isophotes of the zodiacal light and interpreting them in relation to the Poynting-Robertson effect and data from observed meteors, Briggs [14] found that the probability density function f(i) for meteoroid orbit inclination, i, $(0 \leqq i \leqq \pi/2)$ is

$$
\begin{aligned}
f(i) &= (1.66 - 0.66i) \sin i, && \text{for aphelion} > 6.6 \text{ astronomical units} \\
&= 37e^{-6i} \sin i, && \text{for aphelion} \leqq 6.6 \text{ astronomical units.}
\end{aligned}
$$

Then, by integration, the mean (expected value) of the inclination, i, of the orbit of any high-aphelion particles (whether or not encountered by a moving observer) would be $1.66 - 0.66 (\pi - 2) = 0.907$ radians (52 degrees). The corresponding mean i for low-aphelion particles would be 12/37 radians (18.6 degrees). In this analysis, Briggs indicated that particles in the 1 to 50 micron range play the dominant role in the zodiacal light; and Ehricke [18] says that, according to Takakubo, the radius of the particles most effective is about 20 microns. Also, by the relation which Briggs indicated between particle radius and material density, a spherical particle with a radius of 20 microns would have a mass of $10^{-8.2}$ grams.

The author has completed only the first part [26] of his planned four-part analysis of the 285 photographic meteors common to the two tables of data for random samples of sporadic meteors published in 1958 and 1961 by Hawkins and Southworth [19, 20]. Some preliminary results were given in Reference 1; e.g., the sample range for meteoroid mass, m, in grams (above the atmosphere) is $10^{-2.2} \leqq m \leqq 10^{1.0}$, with a mean for log m of -1.0. Figure 1 shows the cosmically weighted cumulative distribution of celestial latitude, β_e, with an arithmetic median β_e of 25 degrees for the positive values of β_e. Asterisk and circle points in Figure 1 represent positive and negative values of β_e, respectively. Figure 1 was obtained by weighting each meteor datum by

$$f = s\, h_m^{-2}\, v^{-3/2}\, P^{-1}\, f_f\, e^{-0.18Z} \tag{17}$$

where h_m is the meteor height above sea level at the point of maximum brilliance, v is the closing velocity, P is Öpik's (see Whipple [21]) probability that a meteoroid in a given orbit will encounter the earth during one revolution of the particle; f_f is the reciprocal of the apparent fraction of the circle of celestial latitude through the

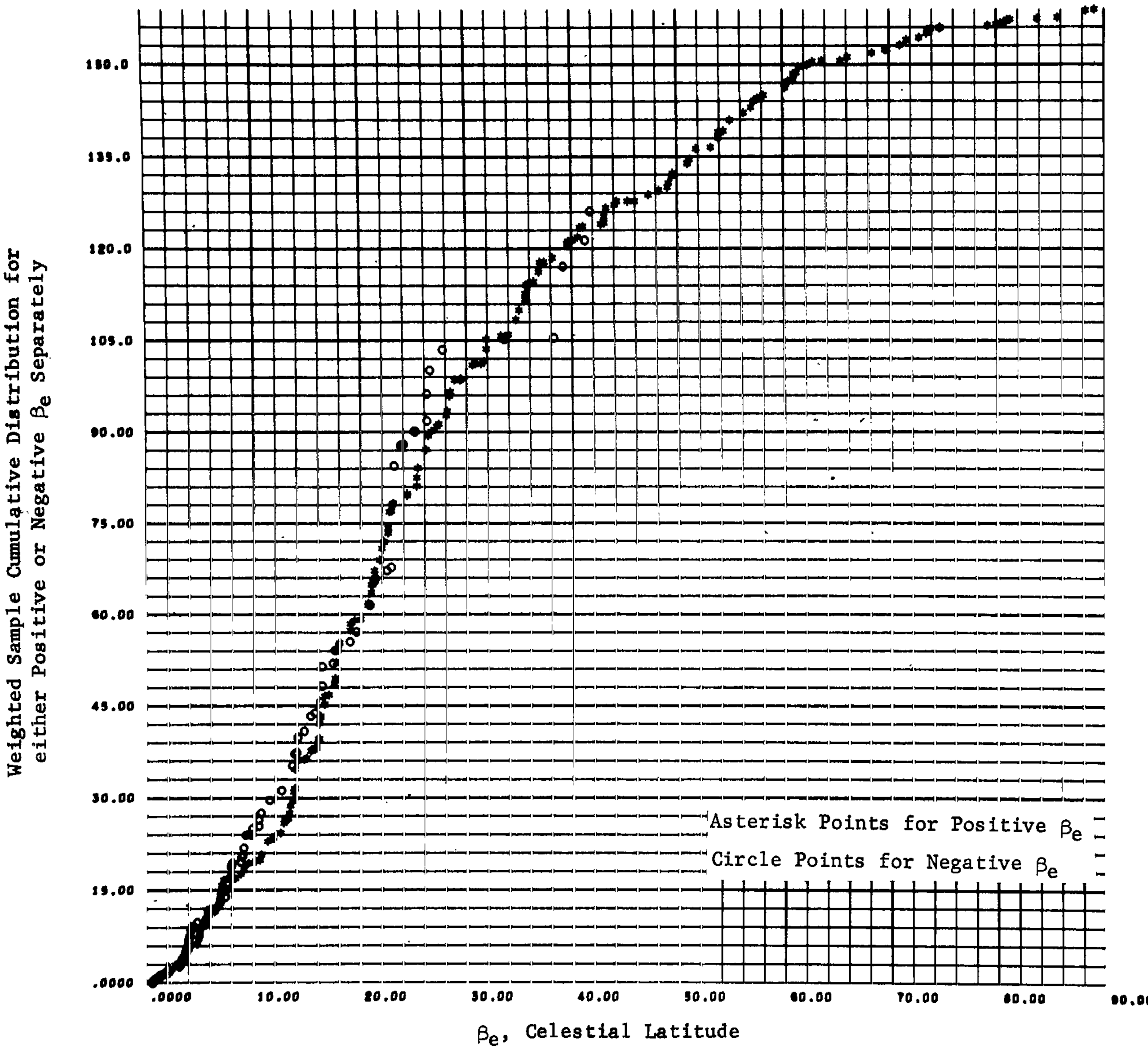

FIGURE 1. COSMICALLY WEIGHTED CUMULATIVE DISTRIBUTION OF CELESTIAL LATITUDE OF RADIANT FOR HAWKINS AND SOUTHWORTH'S [19,20] RANDOM SAMPLE OF SPORADIC PHOTOGRAPHIC METEORS

meteor radiant, e is the base of natural logarithms, Z is the meteor zenith angle in radians, and s is a constant scale factor which restores equality between the sample size and the sample sum of the weighting functions. The constant, -0.18, in the zenith angle weighting factor in equation (17) was chosen to minimize the sum of the squares of the differences between the cumulative distributions for positive and negative β_e below 42 degrees in Figure 1. (The zenith angle weighting factor is an approximate alternative to weighting with respect to meteor celestial longitude.) It is not yet known to what extent the results in Figure 1 may be changed by deleting Öpik's earth encounter probability, P, from the weighting function, f (equation (17)); but the corresponding results would be more appropriate for the present consideration (observer subject to the earth's orbital motion).

In an analysis of the distribution of sporadic radio meteor radiants corrected for antenna selectivity, Elford, Hawkins, and Southworth [22] show contour diagrams of radiant density over the celestial sphere, for each of the first eight months of 1962. Also they show corresponding results for the entire period both before and after correction for velocity selection effects. The monthly mean results, not corrected for velocity selection effects, show much shifting of the contour diagrams of radiant density from month to month. The mean results for the entire period undergo major changes when corrected for velocity selection effects; e.g., the concentration of radiants around the apex of the earth's way disappears, and the previously supposed scarcity of radiants near the antipex becomes questionable. Correction for velocity selection effects is important for the present purpose because the incident flux of radio meteors is proportional to the $4\beta_2$ power of the velocity [22]; whereas, puncture flux through a given structure is thought to be proportional to the $1.5\beta_2$ power of the velocity [1]. The parameter β_2 is the slope of log F_s as a function of log m, and is thought to have an effective value between -1.34 [8] and -1.0 [22] for meteor data.

The main features of the preliminary results for the mean radiant distribution of sporadic radio meteors, corrected for velocity and other selection effects, are that radiants are more densely concentrated near the sun and antisun directions (about ±80 degrees latitude from the apex of the earth's way) and that radiant density decreases by a factor of about three within about 25 degrees from the center of each of the two main concentrations. Elford, Hawkins, and Southworth [22] indicate that these results are for meteoroids with mass not less than 1.3×10^{-6} grams.

The dispersion in the sample of probable radiants from puncture experiments will have a spurious component due to any error in the distribution of radiants which must be assumed in determining each of the probable radiants. It would be prohibitively tedious to apply a distribution with intricate variations with respect to celestial coordinates and season of the year. For any distribution with respect only to celestial latitude, it is not apparent from the discussion so far in this section that any improvement over the isotropic distribution could be found. Therefore, for simplicity and because the radiant activity within ±60 degrees from the antisun is not yet established with corrections for velocity selection effects for radio meteors, any variations in the distribution of radiants with respect to celestial longitude will be ignored also in the rest of this analysis.

Particles incident isotropically from an exposed hemisphere onto an element of a plane surface give a sin $2x_2$ probability density function for the angle of impact, x_2, with respect to the normal; i.e.,

$$f(x_2) = \sin 2x_2. \tag{18}$$

This result (equation (18)) is evident intuitively because that fraction of the celestial hemisphere which contributes radiants within the angular increment, dx_2, varies as sin x_2, and the projection of the element of area onto the plane normal to any radiant direction varies as cos x_2. Then, by integration, both the mean and the median for the angle of impact, x_2, for all meteoroids incident isotropically onto a plane surface, are equal to $\pi/4$ radians.

IV. PUNCTURE FLUX ENHANCEMENT FACTOR

Equation (7) gives the logarithm of meteoroid mass, m, as a linear function of the logarithms of impact velocity, v, and meteoroid density, ρ_p. But, in Section I, log v and log ρ_p are described as essentially normally distributed and statistically independent random variables. Then log m in equation (7) is a normally distributed random variable with mean and variance, by equations (1) through (4), as follows:

$$\log \bar{m}_x = -1.9 + \log(p^3 \rho_t H_t) - (3/2) \log \bar{v} - \log \bar{\rho}_p - (3/2) \log \cos x \tag{19}$$

$$\sigma^2_{\log m} = (3/2)^2\, \sigma^2_{\log v} + \sigma^2_{\log \rho_p} = (0.52)^2. \tag{20}$$

By taking the differential of equation (5), the number of meteoroids, $-dF_s$, impacting per square meter per second with masses between m and m + dm is

$$-dF_s = -10^{\beta_6} \beta_2 m^{\beta_2 - 1} dm. \tag{21}$$

Then, by equations (18) and (21), the flux of impacting meteoroids, within both an increment of mass dm and an increment of impact angle dx_2, is

$$-dF_s (\sin 2x_2) dx_2 = -10^{\beta_6} \beta_2 m^{\beta_2 - 1} \sin 2x_2 \, dx_2 dm. \tag{22}$$

By equation (22), the incremental number of punctures $d\emptyset$, per square meter per second, due to meteoroids in the increment of impact angle dx can be expressed by

$$d\emptyset = \left[\int_0^\infty -10^{\beta_6} \beta_2 P_{mx} m^{\beta_2 - 1} dm \right] \sin 2x \, dx, \tag{23}$$

where P_{mx} is the probability that the mass, m, of a meteoroid, which is incident with an angle of impact, x, is sufficient for puncture (i.e., P_{mx} is the cumulative distribution of the normally distributed variable log m as defined by equations (7), (19), and (20)). The subscript has been dropped from x_2 in equation (23) to indicate that the population of puncturing meteoroids is now considered rather than the population of impacting meteoroids.

The tangent line at the point $P_{mx} = 1/2$ for the curve relating P_{mx} and log m is a very close approximation to the curve in the interval $0.25 \leqq P_{mx} \leqq 0.75$, and its extension throughout the interval $0 \leqq P_{mx} \leqq 1$ is close enough for the present purpose; i.e.,

$$\left.\begin{aligned} P_{mx} &= 0 \quad \text{for} \quad m < \exp_{10}\left(\log \bar{m}_x - \frac{\sqrt{2\pi}}{2} \sigma_{\log m}\right) \\ &= 1 \quad \text{for} \quad m > \exp_{10}\left(\log \bar{m}_x + \frac{\sqrt{2\pi}}{2} \sigma_{\log m}\right) \\ &= (2\pi)^{-1/2} \left(\frac{\log m - \log \bar{m}_x}{\sigma_{\log m}}\right) + \frac{1}{2}, \quad \text{otherwise} \end{aligned}\right\}. \tag{24}$$

Therefore, by using the expressions from equation (24) in evaluating the ingegral in equation (23), it follows that the differential puncture flux, $d\emptyset$, corresponding to the differential impact angle, dx, is

$$d\emptyset = 10^{\beta_6}\left(\frac{1}{\sqrt{2\pi}\,\beta_2\,\sigma_{\log m}}\right)\left(10^{\frac{\sqrt{2\pi}}{2}\beta_2\sigma_{\log m}} - 10^{-\frac{\sqrt{2\pi}}{2}\beta_2\sigma_{\log m}}\right)\bar{m}_x^{\beta_2}\sin 2x dx. \tag{25}$$

In equation (19), by replacing log cos x with the mean of log cos x_2, say log cos $\bar{x}_2$, one also replaces $\bar{m}_x$ (the mass of the nominally puncturing meteoroid at impact angle x) with $\bar{m}$ (the mass of the nominally puncturing meteoroid in equation (11)). Then, by equation (18),

$$\log\cos\bar{x}_2 = \int_0^{\pi/2} (\log\cos x_2)\sin 2x_2\, dx_2 = -(1/2)\log e; \tag{26}$$

and, by equations (19) and (26),

$$\log \bar{m}_x = \log \bar{m} - (3/4)\log e - (3/2)\cos x. \tag{27}$$

One finds, by substituting the antilogarithm of the right side of equation (27) for $\bar{m}_x$ in equation (25), and by using the numerical values of β_2 and $\sigma_{\log m}$ from equations (6) and (20), respectively, that

$$d\emptyset = 10^{\beta_6}\,\bar{m}^{\beta_2}(9.69 \sin 2x)(\cos x)^{1.84}\, dx. \tag{28}$$

Then, by integrating the right side of equation (28) over the interval $0 \leqq x \leqq \pi/2$,

$$\emptyset = (5.05)\, 10^{\beta_6}\,\bar{m}^{\beta_2} = 10^{\beta_6 + 0.70}\,\bar{m}^{\beta_2}. \tag{29}$$

Therefore, by equations (5) and (29), the puncture flux enhancement factor is five, instead of four [1], and β_5 in equation (11) is 0.70 instead of 0.60.

V. DISTRIBUTION OF ANGLE OF IMPACT FOR PUNCTURING METEOROIDS ISOTROPICALLY FROM AN APPARENT HEMISPHERE ONTO A PLAIN OR CONVEX SURFACE

The probability density function f(x) of the angle of impact, x, for the population of meteoroids which actually puncture a given sheet of material exposed to an isotropic flux, is, by equations (28) and (29),

$$f(x) = (d\phi/dx)/\phi = 1.92(\cos x)^{1.84} \sin 2x. \tag{30}$$

Then, by integrating equation (30), the probability that a puncturing meteoroid had an angle of impact of not more than x radians from the normal is $1 - (\cos x)^{3.84}$. Therefore, the median and third quartile of impact angle for puncturing meteoroids are 33.5 and 45.8 degrees, respectively, from the normal.

VI. PROBABLE RADIANT FOR A PUNCTURING METEOROID FROM A HEMISPHERE PARTIALLY OBSCURED BY THE EARTH

The fraction of the exposed celestial hemisphere which has the meteoroid radiants corresponding to a differential increment of impact angle, dx, varies as sin x. Then, by equation (30), the number of meteoroids which puncture a hemispherically exposed flat transducer and which have radiants within a constant differential solid angle in the vicinity of a point on a meridian of the exposed hemisphere is proportional to $(\cos x)^{2.84}$.

The hemispherical mean radiant density for the puncturing meteoroids is not expected to differ appreciably from the radiant density corresponding to the median impact angle for puncturing meteoroids (for which $(\cos x)^{2.84}$ equals $2^{-(2.84/3.84)}$; i.e., $(1.67)^{-1}$).

The apparent disk of the earth with angular radius, θ, in equation (16) subtends a solid angle of $2\pi(1 - \cos \theta)$ steradians, (see Reference 23). Then, of the total number of meteoroids which would puncture, the fraction R_1 obscured by the apparent fraction R_2 of the earth's disk is sufficiently approximated by

$$R_1 = 1.67(\cos x)^{2.84} (1 - \cos \theta) R_2, \tag{31}$$

where x is the separation of the centroid of the apparent segment from the surface zenith.

The separation D of the center of the earth's disk (β_p, λ_p) from the surface zenith (β_n, λ_n) is related to the given coordinates through the formula for the haversine of D (see Reference 25):

$$\text{hav } D = \text{hav}(\beta_p - \beta_n) + \cos \beta_n \cos \beta_p \text{ hav } (\lambda_n - \lambda_p). \tag{32}$$

By definition, the haversine and cosine of any angle A are related by

$$\text{hav } A = (1 - \cos A)/2. \tag{33}$$

Then, by equations (32) and (33), D is the smallest positive value in radians satisfying

$$\cos D = \cos(\beta_p - \beta_n) - [1 - \cos (\lambda_n - \lambda_p)] \cos \beta_n \cos \beta_p, \tag{34}$$

where D is in the interval $0 \leqq D \leqq \pi$.

When $D \geqq \pi/2 + \theta$, the earth is below the surface horizon, both R_2 and R_1 in equation (31) vanish, and the probable radiant of a puncturing meteoroid is the surface zenith (β_n, λ_n). But when $D < \pi/2 + \theta$, the probable radiant is on the earth's meridian (of the exposed hemisphere) and at an angular increment, γ, beyond the surface zenith, sufficiently approximated by

$$\left.\begin{aligned} \gamma &= \frac{xR_1}{1 - R_1} && \text{when } \frac{xR_1}{1 - R_1} \leqq 4/3 \\ &= \theta + D/3 && \text{when } \frac{xR_1}{1 - R_1} > 4/3 \end{aligned}\right\} . \tag{35}$$

When $D \leqq \pi/2 - \theta$, the earth is entirely above the surface horizon, R_2 becomes unity in equation (31), and D replaces x in both equations (31) and (35). But when D is in the interval $\pi/2 - \theta < D < \pi/2 + \theta$, only a segment of the earth's disk is above the surface horizon, the radius of the earth's disk to either horizon point of the periphery makes an angle α with the meridian of the exposed hemisphere, and the centroid of the apparent segment of the earth's disk is displaced by an angle z from the earth's center. Then α is a function of D and θ, z is a function of α and θ, R_2 in equation (31) is a function of α, and x in equations (31) and (35) is a function of D and z. These relations are approximated sufficiently from the plane geometry for circular segments (see Reference 24) as follows:

$$\cos \alpha = (D - \pi/2)/\theta, \quad 0 < \alpha < \pi, \tag{36}$$

$$R_2 = (\alpha - \sin \alpha \cos \alpha)/\pi, \tag{37}$$

$$z = (2\theta/3)(\sin^3\alpha)/(\alpha - \sin \alpha \cos \alpha), \tag{38}$$

$$x = D - z. \tag{39}$$

Let M represent the angle at the center of the earth's disk between the meridian of the exposed hemisphere and the celestial meridian; and let the celestial coordinates of the probable radiant of the puncturing meteoroid be β_r and λ_r. Then the spherical triangles which are formed on the celestial sphere by the celestial pole and the great-circle arcs D and $D + \gamma$ have the following solution (see Reference 25):

$$\text{hav } M = [\text{hav } (\pi/2 - \beta_n) - \text{hav } (\pi/2 - \beta_p - D)] \csc (\pi/2 - \beta_p) \csc D, \tag{40}$$

$$\text{hav } (\pi/2 - \beta_r) = \text{hav } (\pi/2 - \beta_p - D - \gamma) + \sin (\pi/2 - \beta_p) \sin (D + \gamma) \text{ hav } M, \tag{41}$$

$$\text{hav } (\lambda_r - \lambda_n) \cos \beta_r \cos \beta_n = \text{hav } \gamma - \text{hav } (\beta_n - \beta_r). \tag{42}$$

Then, by equations (33), (40), and (41), the celestial latitude, β_r, of the probable radiant (in the interval $-\pi/2 \leqq \beta_r \leqq \pi/2$) can be calculated from:

$$\sin \beta_r = \sin(\beta_p + D + \gamma) + \frac{\sin(D + \gamma)}{\sin D} [\sin \beta_n - \sin(\beta_p + D)], \qquad (43)$$

where, by definition, $-\pi/2 \leqq \beta_r \leqq \pi/2$. Also, by equations (33) and (42), the celestial longitude, λ_r, of the probable radiant can be calculated from

$$\cos (\lambda_r - \lambda_n) = 1 + \frac{\cos \gamma - \cos(\beta_n - \beta_r)}{\cos \beta_r \cos \beta_n} . \qquad (44)$$

In addition to equation (44) for $\cos (\lambda_r - \lambda_n)$, one must have a formula for $\sin (\lambda_r - \lambda_n)$ to remove the quadrant ambiguity. Let N be the angle between the celestial and surface meridians at the probable radiant; i.e., $0 \leqq N \leqq \pi$,

$$\text{hav } N = [\text{hav } (\pi/2 - \beta_n) - \text{hav } (\pi/2 - \beta_r - \gamma)] \csc (\pi/2 - \beta_r) \csc \gamma. \qquad (45)$$

Then, by equations (33) and (45), N can be calculated from

$$\cos N = 1 + \frac{\sin \beta_n - \sin(\beta_r + \gamma)}{\cos \beta_r \sin \gamma} , \quad 0 \leqq N \leqq \pi. \qquad (46)$$

Therefore, by the law of sines,

$$\sin (\lambda_r - \lambda_n) = \frac{\sin \gamma \sin N}{\sin (\pi/2 - \beta_n)} = \frac{\sin \gamma \sin N}{\cos \beta_n} . \qquad (47)$$

VII. CONCLUSIONS

In appropriate meteoroid measurement satellite experiments where vehicle attitude information is available without information about the meteoroid closing velocity vector, the probable radiant and the puncture flux enhancement factor for sporadic meteors should be based on the assumption that velocity vectors for ambient meteoroids are isotropically distributed. Then, although half of the meteoroids which impact into a flat transducer are more than 45 degrees from the normal, those within 45 degrees puncture three times more effectively than the others, and half of the puncturing meteoroids are less than 33.5 degrees from the normal when the earth is below the surface horizon.

The probability density functions $f(x_2)$ and $f(x)$ and the corresponding cumulative distributions for angles of impact x_2 and x for incident and puncturing meteoroids, respectively, are as follows when the earth is below the surface horizon:

$$f(x_2) = \sin 2x_2 \tag{18}$$

$$\int_0^{x_2} f(x_2)\, dx_2 = (1 - \cos 2x_2)/2$$

$$f(x) = 1.92\,(\cos x)^{1.84} \sin 2x \tag{30}$$

$$\int_0^{x} f(x)\, dx = 1 - (\cos x)^{3.84}.$$

The probability density functions in equations (18) and (30) above also imply that the flux $\emptyset$ of puncturing meteoroids is five times greater than the flux F_s of incident meteoroids with mass m equal to or greater than the mass $\bar{m}$ of the almost-puncturing meteoroid with the average values of material density, velocity, and impact angle; i.e.,

$$F_s = 10^{\beta_6} m^{\beta_2} \tag{5}$$

$$\emptyset = 10^{\beta_6+0.70} \bar{m}^{\beta_2}. \tag{29}$$

The Pegasus-type experiments give puncture flux, $\emptyset$, as a function of target thickness, p, and target material parameter, k_2, rather than as a function of the parameters β_2 and β_6 relating flux and mass in equations (5) and (29) above; i.e.,

$$\bar{m} = 10^{k_1} (10^{k_2} p)^{k_3} \tag{12}$$

$$\log \emptyset = (\beta_6 + 0.70 + k_1\beta_2) + k_3\beta_2(k_2 + \log p) \tag{14}$$

where k_1 and k_3 depend on the unknown meteoroid material parameters, on the mechanics of impact, and possibly also on k_2 and p, and where k_2 is a known function of the known target material parameters as follows:

$$k_2 = 0.50 \log C_t + 1.31 \log E_t + 8.0 \log \nu_t + 0.43 \log \epsilon_t. \tag{13}$$

As observed from the site of puncture in a Pegasus-type experiment, the angular radius, θ, of the earth's effective meteoroid encounter cross section is sufficiently approximated from:

$$\sin \theta = (r_e + 100)/r \tag{16}$$

where r is the geocentric radial distance of the satellite in kilometers and r_e is the radius of the earth in kilometers.

The separation $D(0 \leqq D \leqq \pi$ radians) of the center of the earth's disk (at celestial coordinates β_p and λ_p) from the surface zenith (at celestial coordinates β_n and λ_n) is calculated from:

$$\cos D = \cos (\beta_p - \beta_n) - [1 - \cos (\lambda_n - \lambda_p)] \cos \beta_n \cos \beta_p. \tag{34}$$

When $D \geqq \pi/2 + \theta$ the earth is below the surface horizon and the probable radiant of a puncturing meteoroid is the surface zenith (β_n, λ_n). But when D is in the interval $\pi/2 - \theta < D < \pi/2 + \theta$, a fraction R_2 of the earth's disk with central angle 2α is apparent above the surface horizon and shields a fraction R_1 of the meteoroids from the exposed celestial hemisphere. The centroid of the apparent segment is separated z radians from the center of the earth's disk and x radians from the surface zenith. Then the probable radiant is on the earth's meridian (of the exposed hemisphere) and at an increment γ radians beyond the surface zenith. The relations between these parameters are sufficiently approximated by:

$$\cos \alpha = (D - \pi/2)/\theta, \qquad \theta < \alpha < \pi \tag{36}$$

$$R_2 = (\alpha - \sin \alpha \cos \alpha)/\pi \tag{37}$$

$$z = (2\theta \sin^3\alpha)/(3\pi R_2) \tag{38}$$

$$x = D - z \tag{39}$$

$$R_1 = 1.67(\cos x)^{2.84} (1 - \cos \theta) R_2 \tag{31}$$

$$\begin{aligned} \gamma &= \frac{xR_1}{1 - R_1}, \qquad \text{when } \frac{xR_1}{1 - R_1} \leqq 4/3 \\ &= \theta + D/3 \qquad \text{when } \frac{xR_1}{1 - R_1} > 4/3 \end{aligned} \tag{35}$$

Also, when $D \leqq \pi/2 - \theta$ the earth is entirely above the surface horizon, R_2 becomes unity in equation (31) and D replaces x in both equations (31) and (35). Then, when at least part of the earth is apparent above the surface horizon ($D < \pi/2 + \theta$), the celestial coordinates of the probable radiant (β_r, λ_r) can be calculated from:

$$\sin \beta_r = \sin (\beta_p + D + \gamma) + \frac{\sin(D + \gamma)}{\sin D} [\sin \beta_n - \sin (\beta_p + D)]$$

$$-\frac{\pi}{2} \leqq \beta_r \leqq \frac{\pi}{2}, \tag{43}$$

$$\cos (\lambda_r - \lambda_n) = 1 + \frac{\cos \gamma - \cos (\beta_n - \beta_r)}{\cos \beta_r \cos \beta_n} \tag{44}$$

$$\sin (\lambda_r - \lambda_n) = \frac{\sin \gamma \sin N}{\cos \beta_n} \tag{47}$$

where

$$\cos N = 1 + \frac{\sin \beta_n - \sin(\beta_r + \gamma)}{\cos \beta_r \sin \gamma}, \quad 0 \leqq N \leqq \pi. \tag{46}$$

The formulas for the celestial coordinates of the probable radiant (β_r, λ_r) of a puncturing meteoroid have been corrected for earth-shielding. This correction is important near the earth even under nominal circumstances; e.g., when the orbit height is 500 kilometers, the radius of the earth's disk (as seen from the vehicle) is 70 degrees. In that case, when the elevations of the earth's center are 0, 15, 30, 45, and 60 degrees, the probable radiants are deflected 5, 10, 20, 39, and 80 degrees, respectively, by the earth-shielding effect.

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama, November 25, 1965

REFERENCES

1. Dalton, C. C., "Cislunar Meteoroid Impact and Puncture Models with Predicted Pegasus Satellite Punctures," NASA Technical Memorandum X-53187, January 13, 1965, Unclassified.

2. Dalton, C. C., "Estimation of Tolerance Limits for Meteoroid Hazard to Space Vehicles ...," NASA Technical Note D-1996, February 1964.

3. Reismann, H., J. D. Donahue and W. C. Burkitt, "Multivariable Analysis of the Mechanics of Penetration of High Speed Particles," Martin-Marietta Corporation, Interim Technical Progress Report No. NASA-CR-64-5, Contract No. NAS7-219, April 1, 1963 through May 1964.

4. Andriankin, E. I. and Yu. S. Stepanov, "Impact Penetration Depth of Meteoric Particles," Artificial Earth Satellites, Vol. 15, pp. 45-53, January 1964.

5. Herrmann, W. and A. H. Jones, "Survey of Hypervelocity Impact Information," Aeroelastic and Structures Research Laboratory, Massachusetts Institute of Technology, Report No. 99-1, September 1961.

6. Dalton, C. C., "Meteoroid Hazard to Space Vehicles ...," NASA Technical Memorandum X-789, December 1962.

7. Hawkins, G. S., and E. K. L. Upton, "The Influx Rate of Meteors in the Earth's Atmosphere," Astronomical Journal, Vol. 128, pp. 727-735, 1958.

8. Pegasus Data Evaluation Group, "Pegasus-A Satellite Bulletin No. 1," Marshall Space Flight Center, February 23, 1965.

9. Lovell, A. C. B., "Meteor Astronomy," Oxford University Press, London, 1954.

10. Davison, E. H., and P. C. Winslow, Jr., "Direct Evaluation of Meteoroid Hazard," Aerospace Engineering, February 1962, pp. 24-33.

11. Hawkins, G. S., "Interplanetary Debris Near the Earth," Annual Review of Astronomy and Astrophysics, pp. 149-164, Annual Reviews, Inc., Palto Alto, California, 1964.

REFERENCES (Continued)

12. Allen, C. W., "Astronomical Quantities," The Athlone Press (University of London), 1955.

13. Briggs, R. E., "Steady-State Distribution of Meteoric Particles under the Operation of the Ponyting-Robertson Effect," Astronomical Journal, Vol. 67, No. 10, pp. 710-723, December 1962.

14. Davison, E. H., and P. C. Winslow, Jr., "Space Debris Hazard Evaluation," NASA Technical Note D-1105, December 1961.

15. Robertson, H. P., "Dynamical Effects of Radiation in the Solar System," Royal Astronomical Society Monthly Notices, Vol. 97, April 1937, pp. 423-438.

16. Wyatt, S. P., Jr. and F. L. Whipple, "The Ponyting-Robertson Effect on Meteor Orbits," Astrophysical Journal, Vol. 111, No. 1, p. 134, 1950.

17. Ehricke, K. A., "Space Flight, Vol. 1: Environment and Celestial Mechanics," Principles of Guided Missile Design, Ed.,G. M. Merrill, D. van Nostrand Co., 1960.

18. Hawkins, G. S. and R. B. Southworth, "The Statistics of Meteors in the Earth's Atmosphere," Smithsonian Contributions to Astrophysics, Vol. 2, No. 11, 1958, Smithsonian Institution, Washington, D. C.

19. Hawkins, G. S. and R. B. Southworth, "Orbital Elements of Meteors," Smithsonian Contributions to Astrophysics, Vol. 4, No. 3, 1961, Smithsonian Institution, Washington, D. C.

20. Whipple, F. L., "Photographic Meteor Orbits and Their Distribution in Space," The Astronomical Journal, Vol. 59, No. 6, pp. 201-217, July 1954.

21. Elford, W. G., G. S. Hawkins and R. B. Southworth, "The Distribution of Sporadic Meteor Radiants," NASA Research Report No. 11, Contract NASr-158, Harvard College Observatory and Smithsonian Astrophysical Observatory, Cambridge Mass., December 1964.

22. Manned Spacecraft Criteria and Standards Board, "Meteoroid Environment in Near-Earth, Cislunar, and Near-Lunar Space," Engineering Criteria Bulletin No. EC-1, November 8, 1963, NASA Manned Spacecraft Center, Houston, Texas.

REFERENCES (Concluded)

23. Hudson, R. G., "The Engineers' Manual," 2nd Ed., John Wiley & Sons, New York, 1939.

24. Kells, L. M., W. F. Kern and J. R. Bland, "Spherical Trigonometry with Naval and Military Applications," McGraw Hill Book Co., New York, 1942.

25. Dalton, C. C., "Statistical Analysis of Photographic Meteor Data - Part I - Öpik's Luminous Efficiency and Supplemented Whipple Weighting," NASA Technical Memorandum X-53325, September 2, 1965.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546