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THE UNIVERSITY OF MICHIGAN

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TECHNICAL REPORT NO. 88

ELECTRON PHYSICS LABORATORY

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ABSTRACT

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On the basis of a one-dimensional, small-signal, single-velocity beam theory, the "forbidden" and "permitted" regions of wave amplification are determined in terms of the system parameter space (B,Ω,g) by examining the real roots of the determinantal equation for a beam wave system in which the wave is propagated along a lossless circuit. The system parameters are defined as $B = v_o/u_o$, $\Omega = B(\omega_p/\omega)$ and $g = B(Z_c I_o/2V_o)$, with v_o and u_o being the phase velocity of the unperturbed circuit wave and the initial average electron-beam velocity, respectively. ω_p and ω are the electron-beam plasma frequency and the angular frequency of the electromagnetic wave respectively. Z_c is the coupling impedance, I_o is the d-c beam current and V_o is the d-c beam voltage.

The necessary conditions for wave amplification are obtained for forward- and backward-wave interaction without imposing any restriction upon the system parameters. It is shown that wave amplification is possible under rather general circumstances, e.g., even if the value of (ω_p/ω) is greater than unity, provided that the value of B lies in the proper range for a given value of g. Furthermore, at synchronism (i.e., $B \simeq 1$), for $g < 1$ it is observed that there are two ranges of values of $\Omega_o = (\omega_p/\omega)$ over which wave amplification is possible. For example, for $g = 0.1$ these ranges are given by $0 < \Omega_o < 0.5$ and $1.82 < \Omega_o < 2.25$. In the first range it is the forward-propagating wave that is amplified, whereas in the second range it is the backward-propagating wave which is amplified. These results may be applied directly to the examination of whistler-mode propagation in ionospheric plasmas. Author

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THE NECESSARY CONDITIONS FOR AMPLIFICATION OF AN ELECTROMAGNETIC WAVE INTERACTING WITH A DRIFTING ELECTRON STREAM

I. INTRODUCTION

The amplification process arising in a coupled electromagnetic-wave electron-beam system characteristic of a laboratory TWT has been developed fully by Pierce[1], assuming a small-amplitude wave and a single-velocity electron beam. Since the ionospheric plasma in the presence of the earth's magnetic field can act as an effective slow-wave structure for a traveling wave and with the assumption that a corpuscular stream discharged from the sun provides the required beam, Pierce's theory on the traveling-wave-amplification process can be applied to the study of VLF emissions in the ionosphere[2,3]. For example, the TWT amplification process has been suggested as a possible mechanism for the generation of a certain type of VLF emission in the exosphere by Gallet and Helliwell[2] and has been investigated theoretically by Dowden[3] in great detail.

It is well known that for a laboratory TWT the small-signal theory predicts that the interaction between the propagating electromagnetic wave and the electron beam is strongest when u_o, the velocity of the linear electron beam, is so adjusted that it is in near synchronism with the phase velocity of the electromagnetic wave v_o. For strongly coupled systems, u_o must be greater than v_o for maximum amplification. It has been suggested in the literature[2] that the equality $v_o = u_o$ is the condition for VLF emission signal amplification. However, it should be pointed out that the equality $v_o = u_o$, as the condition of amplification, is valid only under a special assumption that the electron-beam plasma frequency ω_p is much smaller than the angular frequency of the electromagnetic wave, i.e., $\omega_p \ll \omega$, which is usually true in the case of a laboratory TWT but is not generally satisfied in the case of ionospheric phenomena, such as in a whistler-mode propagation[4]. Furthermore the fact that a forward-wave amplifier, known as the Crestatron[5], can be operated successfully without the condition of synchronism $v_o \simeq u_o$ tends to suggest the inadequacy of considering the condition of synchronism as the condition for VLF emission signal amplification.

In view of the fact that the observation of VLF noise bands with the Alouette I Satellite made by Belrose and Barrington[6] indicates that the TWT theory is perhaps the most hopeful among the various theories proposed for the generation mechanism of VLF emissions, it seems desirable to reexamine the above-mentioned condition for wave amplification. It is therefore the purpose of the present paper to obtain and discuss the necessary conditions for wave amplification under most general circumstances within the framework of a one-dimensional, small-signal, single-velocity beam theory.

II. DETERMINANTAL EQUATION

2.1 Single-Beam System

For a one-dimensional analysis, the quantities of interest associated with an electron beam, such as the space-charge density ρ, the electron velocity u and the convection current density J, can be written as follows:

$$\rho(z,t) = \rho_o(z) + \rho_1(z,t) \quad ,$$

$$u(z,t) = u_o(z) + u_1(z,t)$$

and

$$J(z,t) = J_o(z) + J_1(z,t) \quad . \tag{1}$$

where the subscripts o and 1 denote the time average value and the time-varying part of the quantity respectively. If the single-velocity assumption is made, the convection current density is given as the product of the velocity u and the space-charge density ρ

$$J = \rho u = (\rho_o + \rho_1)(u_o + u_1) \tag{2}$$

and under a small-signal assumption Eq. 2 yields

$$J_o = \rho_o u_o \tag{3}$$

and

$$J_1 = \rho_o u_1 + \rho_1 u_o \quad . \tag{4}$$

When the time-varying components of the quantities of interest are assumed to have the form $e^{(j\omega t - \Gamma z)}$, the continuity equation gives

$$\Gamma J_1 = j\omega\rho_1 \tag{5}$$

On the other hand, for a nonrelativistic analysis Newton's second law of motion gives

$$\frac{du}{dt} = \eta[E_z + E_s] \quad , \tag{6}$$

where η = (e/m), the ratio of electronic charge to mass for an electron taken as a negative quantity, E_z is the impressed field set up by the wave-propagating medium, while E_s is a local space-charge field caused by the bunches in the beam. Poisson's equation is written as

$$\frac{\partial E_s}{\partial z} = \frac{\rho_1}{\epsilon_o} \tag{7}$$

and with the aid of the Eq. 5 it yields

$$E_s = - \frac{J_1}{j\omega\epsilon_o} , \tag{8}$$

where ϵ_o is the dielectric constant of vacuum. It is not difficult to see that Eq. 6 can be written, with the aid of Eq. 8, as

$$u_1(j\omega - \Gamma u_o) = \eta \left(E_z - \frac{J_1}{j\omega\epsilon_o} \right) . \tag{9}$$

Upon elimination of ρ_1 and u_1 from Eqs. 4, 5 and 9, the following electronic equation is obtained:

$$J_1 = \frac{j\omega\rho_o\eta E_z}{\left[(j\omega - \Gamma u_o)^2 + \frac{\rho_o\eta}{\epsilon_o} \right]} , \tag{10}$$

which gives the value of the r-f current density in terms of the impressed field. Equation 10 can also be conveniently written in terms of the total r-f beam current i_1 instead of the current density such that

$$i_1 = \frac{j\beta_e \left(\frac{I_o}{2V_o}\right) E_z}{\left[(j\beta_e - \Gamma)^2 + \beta_p^2\right]} , \tag{11}$$

where

$$\beta_e = \left(\frac{\omega}{u_o}\right) , \quad \beta_p = \left(\frac{\omega_p}{u_o}\right) , \quad \omega_p^2 = \frac{\eta\rho_o}{\epsilon_o} ,$$

$$J_o = \rho_o u_o , \quad V_o = \frac{u_o^2}{-2\eta} , \quad I_o = -AJ_o$$

and

$$i_1 = AJ_1 , \tag{12}$$

where A is the cross-sectional area of the beam. It should be noted that if the β_p^2 term in Eq. 11 is neglected, then Eq. 11 is identical with Eq. 2.22 of Pierce[1]. Often, to take into account the actual field conditions, that is, the fact that the fields are far from being one-dimensional, a modified value $\omega_q^2 = R^2\omega_p^2$ is used instead of ω_p^2, where R is a space-charge reduction factor which is a function of the particular geometry.

On the other hand, the circuit equation can be given as follows[7]:

$$E_z = \frac{\Gamma^2 \Gamma_o Z_c}{(\Gamma_o^2 - \Gamma^2)} i_1 , \tag{13}$$

where Z_c is the coupling impedance between the drifting beam and the wave and Γ_o is the cold-circuit propagation constant.

It should be pointed out that Pierce treats the local space-charge fields as passive modes of the circuit and his space-charge term appears in the circuit equation. His result is

$$E_z = \left[\frac{\Gamma^2 \Gamma_o Z_c}{(\Gamma_o^2 - \Gamma^2)} - \frac{j\Gamma^2}{\omega C_1} \right] i_1 \quad , \tag{14}$$

where C_1 is a lumped capacitance representing the effect of the passive modes.

When Eq. 11 and Eq. 13 are combined, the determinantal equation for the propagation constant is obtained:

$$\frac{(\Gamma_o^2 - \Gamma^2)[(j\beta_e - \Gamma)^2 + \beta_q^2]}{j\beta_e \Gamma^2 \Gamma_o} = \frac{I_o Z_c}{2V_o} \quad . \tag{15}$$

Pierce obtains an equation equivalent to Eq. 15 by introducing a space-charge parameter Q and the gain parameter C defined by

$$C^3 = \frac{I_o Z_c}{4V_o} \qquad \text{and} \qquad Q = \frac{\beta_e}{2\omega C_1 Z_c} \quad , \tag{16}$$

the result being

$$(j\beta_e - \Gamma)^2 - 4\Gamma^2 Q C^3 = \frac{2C^3 \Gamma^2 \Gamma_o j\beta_e}{(\Gamma_o^2 - \Gamma^2)} \quad . \tag{17}$$

The numerical meaning attached to the parameters Q and R has been discussed in detail by Beck[7] and also by Chodorow and Susskind[8]. Several authors[1,9,10], starting with Pierce, have treated the solution of Eq. 15 or Eq. 17 in the general case of a nonsynchronous beam, a slightly lossy circuit and nonzero space charge. The method is straightforward; one attempts by suitable substitutions and combinations of the parameters to express the secular equation in the simplest form. Numerical values are then inserted and the roots are extracted by a standard technique. The merits consist of compactness, freedom from unnecessary approximation and the range of conditions studied. For example, Brewer and Birdsall[10] use Eq. 17 and Pierce's notation as the starting point in their calculation of a normalized propagation constant for a TWT.

It should be noted that in a BWA the electronic equation is precisely the same as it is in a TWA so that Eq. 11 is still valid. The circuit equation is, however, different. It differs from Eq. 13 in the sign of the right-hand side alone. The secular equation for a BWA can be given as

$$\frac{(\Gamma_o^2 - \Gamma^2)[(j\beta_e - \Gamma)^2 + \beta_q^2]}{j\beta_e\Gamma^2\Gamma_o} = -\frac{I_oZ_c}{2V_o}, \qquad (18)$$

or

$$(j\beta_e - \Gamma)^2 - 4\Gamma^2QC^3 = -\frac{2C^3\Gamma^2\Gamma_o j\beta_e}{(\Gamma_o^2 - \Gamma^2)}. \qquad (19)$$

2.2 Multibeam System

The determinantal equation for an N-beam system can be obtained in the following manner. For the νth beam, from Eqs. 4 and 5 respectively,

$$J_{1\nu} = \rho_{o\nu} u_{1\nu} + \rho_{1\nu} u_{o\nu} \tag{20}$$

and

$$\Gamma J_{1\nu} = j\omega\rho_{1\nu} \ . \tag{21}$$

Upon elimination of $\rho_{1\nu}$ from Eqs. 20 and 21,

$$J_{1\nu} = \frac{j\omega\rho_{o\nu} u_{1\nu}}{(j\omega - \Gamma u_{o\nu})} \ , \qquad \nu = 1, 2, \ldots N \ . \tag{22}$$

On the other hand, Eq. 6 gives

$$(j\omega - \Gamma u_{o\nu})u_{1\nu} = \eta[E_z + E_s] \tag{23}$$

and Eq. 7 becomes

$$\frac{\partial E_s}{\partial z} = \frac{1}{\epsilon_o} \frac{1}{A_T} \sum_{\nu=1}^{N} A_\nu \rho_{1\nu} \ , \tag{24}$$

where

$$A_T = \sum_{\nu=1}^{N} A_\nu \ ,$$

A_ν is the cross-sectional area of the νth beam and A_T is the total cross-sectional area of the system. With the aid of Eq. 21, Eq. 24 becomes

$$E_s = -\frac{1}{j\omega\epsilon_o} \frac{1}{A_T} \sum_{\nu=1}^{N} A_\nu J_{1\nu} \ . \tag{25}$$

When $u_{1\nu}$ and E_s are eliminated from Eqs. 22, 23 and 25,

$$J_{1\nu} = \frac{j\omega\rho_{o\nu}\eta}{(j\omega - \Gamma u_{o\nu})^2}\left[E_z - \frac{1}{j\omega\epsilon_o A_T}\sum_{\nu=1}^{N} A_\nu J_{1\nu}\right] \qquad (26)$$

With the total r-f beam current of the system, i_1, as

$$i_1 = \sum_{\nu=1}^{N} i_{1\nu} = \sum_{\nu=1}^{N} A_\nu J_{1\nu} \qquad (27)$$

Eq. 26 gives the following electronic equation for the N-beam system

$$i_1 = E_z \left[\frac{\displaystyle\sum_{\nu=1}^{N} \frac{j\beta_{e\nu}\left(\frac{I_{o\nu}}{2V_{o\nu}}\right)}{(j\beta_{e\nu} - \Gamma)^2}}{\displaystyle 1 + \frac{1}{A_T}\sum_{\nu=1}^{N} A_\nu \frac{\beta_{p\nu}^2}{(j\beta_{e\nu} - \Gamma)^2}}\right], \qquad (28)$$

where

$$\beta_{e\nu} = \left(\frac{\omega}{u_{o\nu}}\right), \quad \beta_{p\nu} = \left(\frac{\omega_{p\nu}}{u_{o\nu}}\right), \quad \omega_{p\nu}^2 = \frac{\rho_{o\nu}\eta}{\epsilon_o},$$

$$J_{o\nu} = \rho_{o\nu}u_{o\nu}, \quad V_{o\nu} = \frac{u_{o\nu}^2}{-2\eta} \text{ and } I_{o\nu} = -A_\nu J_{o\nu}. \qquad (29)$$

On the other hand, from Eq. 13 the r-f current of the νth beam can be written as

$$i_{1\nu} = \frac{(\Gamma_o^2 - \Gamma^2)}{\Gamma^2\Gamma_o Z_{c\nu}} E_z, \qquad (30)$$

where $Z_{c\nu}$ denotes the coupling impedance between the circuit and the νth beam.

Substituting Eq. 30 into Eq. 27 yields the circuit equation

$$i_1 = E_z \left[\left(\frac{\Gamma_o^2 - \Gamma^2}{\Gamma^2 \Gamma_o} \right) \frac{1}{Z_T} \right] , \tag{31}$$

where

$$\frac{1}{Z_T} = \sum_{\nu=1}^{N} \frac{1}{Z_{c\nu}} . \tag{32}$$

By combining Eqs. 28 and 31, the desired determinantal equation is obtained:

$$\frac{(\Gamma_o^2 - \Gamma^2)}{\Gamma^2 \Gamma_o} = \frac{\displaystyle\sum_{\nu=1}^{N} \frac{j\beta_{e\nu} \left(\frac{I_{o\nu}}{2V_{o\nu}} \right) Z_T}{(j\beta_{e\nu} - \Gamma)^2}}{1 + \displaystyle\frac{1}{A_T} \sum_{\nu=1}^{N} A_\nu \frac{\beta_{p\nu}^2}{(j\beta_{e\nu} - \Gamma)^2}} . \tag{33}$$

It is of interest to observe that for an intermingled N-beam system, in which all beams have the identical cross-sectional area A (i.e., $A_T = A_\nu = A$ for $\nu = 1, 2 \ldots N$), Eq. 24 becomes

$$\frac{\partial E_s}{\partial z} = \frac{1}{\epsilon_o} \sum_{\nu=1}^{N} \rho_{1\nu} \tag{34}$$

and Eq. 33 becomes

$$\frac{(\Gamma_o^2 - \Gamma^2)}{\Gamma^2\Gamma_o} = \frac{\sum_{\nu=1}^{N} \frac{j\beta_{e\nu}\left(\frac{I_{o\nu}}{2V_{o\nu}}\right) Z_T}{(j\beta_{e\nu} - \Gamma)^2}}{1 + \sum_{\nu=1}^{N} \frac{\beta_{p\nu}^2}{(j\beta_{e\nu} - \Gamma)^2}} \tag{35}$$

Furthermore, for a single-beam system (i.e., N = 1) Eq. 35 is reduced to Eq. 15.

III. THE NECESSARY CONDITIONS FOR WAVE AMPLIFICATION

3.1 The Determination of the Region of Possible Amplification

It is convenient to define the following propagation constants

$$\Gamma = jk \quad \text{and} \quad \Gamma_o = jk_o \tag{36}$$

so that Eqs. 15 and 18 can be combined into the following equation:

$$(k^2 - k_o^2)[\beta_q^2 - (k - \beta_e)^2] = \pm \frac{I_o Z_c}{2\,V_o} [\beta_e k_o k^2] \quad , \tag{37}$$

where the upper sign refers to a TWA and the lower sign refers to a BWA. It should be observed that Eq. 37 is a fourth-degree algebraic equation in k and thus has four roots. Thus this fact indicates the possibility of four modes of propagation for the system. For example, in the case of a negligibly small coupling (i.e., $Z_c \simeq 0$) the roots of Eq. 37 are $k = \pm k_o$ and $k = (\beta_e \pm \beta_q)$ which are the propagation constants of the two circuit waves and those of the two space-charge waves associated with the electron beam, respectively. Furthermore Eq. 37 can be written in the following normalized form:

$$(X^2 - 1)[(X - B)^2 - (\Omega^2 + g)] = \mp g \quad , \tag{38}$$

where

$$X = \left(\frac{k}{k_o}\right), \quad B = \left(\frac{\beta_e}{k_o}\right), \quad \Omega = \left(\frac{\beta_q}{k_o}\right)$$

and

$$\frac{g}{B} = \frac{I_o Z_c}{2V_o} \quad . \tag{39}$$

Similarly Eq. 35 can be expressed in terms of k and k_o. Then Eq. 35, being a (2^N+2)th-degree algebraic equation in k, has (2^N+2) roots. Once the various parameters of the system are specified, these roots can be obtained in principle. However, it is easily visualized that the solution of Eq. 35 becomes more complex as N increases, and it is not considered in the present investigation. Attention is directed toward the study of a single-beam system in the present discussion. When the losses of the circuit are negligible, k_o is real and can be written as $\beta_c = (\omega/v_o)$ with v_o being the phase velocity of the unperturbed circuit wave. Consequently Eq. 39 becomes

$$X = \left(\frac{k}{\beta_c}\right), \quad B = \left(\frac{v_o}{u_o}\right), \quad \Omega = \Omega_o B = (\omega_q/\omega)B \quad ,$$

and

$$g = \frac{I_o Z_c}{2V_o}\left(\frac{v_o}{u_o}\right) = 2C^3 B \tag{40}$$

in which the normalized propagation parameter X is generally complex, while the velocity parameter B, the frequency parameter Ω and the coupling parameter g are real. Furthermore, these parameters are related to the conveniently defined parameters (C,QC,b) in the TWT theory[1] as follows:

$$\pm \Omega_o = \frac{\pm C\sqrt{4QC}}{1 + C\sqrt{4QC}}$$

and

$$B = \frac{1}{1 + Cb} \quad . \tag{41}$$

For a given set of parameter values, Eq. 38 can be solved algebraically for the propagation parameter X, which in general may be complex. It should be noted that when X is complex, say $\tilde{X} = (p+ja)$, the wave function $e^{(j\omega t-\Gamma z)}$ takes the form,

$$e^{(j\omega t-\Gamma z)} = e^{j(\omega t-\tilde{X}\beta_c z)} = e^{a\beta_c z}\, e^{j(\omega t-p\beta_c z)} \quad , \tag{42}$$

which represents a nonuniform propagating wave provided that a and p are different from zero. Furthermore, since Eq. 38 is a quartic equation in X, if it has a complex root, $\tilde{X} = (p+ja)$, then $\tilde{X}^{*} = (p-ja)$ must also be a root. In other words, the complex roots of Eq. 38 must appear as a complex conjugate pair. Consequently the condition that the propagation parameter X becomes complex can be regarded as the necessary condition for wave amplification. Thus the problem becomes the determination of the proper combination of values of the parameters B, Ω and g for which Eq. 38 has <u>at least one pair of complex conjugate roots</u>. This region is referred to as the "permitted" region for wave amplification, defined in the system parameter space (B,Ω,g). The problem can also be considered equivalently as that of the determination of the proper combination of the parameters B, Ω and g for which Eq. 38 has four real roots, which is referred to as the "forbidden" region for wave amplification. It is of interest to note that the real roots of Eq. 38 can be determined rather easily by the following graphical method. Consider the real functions $H(x)$, $F(x)$ and $G(x)$, as defined by

$$H(x) = \frac{g}{(x^2 - 1)} , \tag{43}$$

$$F(x) = -(x - B)^2 + (\Omega^2 - g) \tag{44}$$

and

$$G(x) = (x - B)^2 - (\Omega^2 + g) , \tag{45}$$

where B, Ω, g and x are real. When these functions are plotted against x in a real x-y plane, they represent a composite curve with four branches (see Fig. 1), a parabola opening downwardly with its vertex located at the point $[B,(\Omega^2-g)]$ and a parabola opening upwardly with its vertex at the point $[B,-(\Omega^2+g)]$, respectively (see Fig. 2). Since the real roots of Eq. 38 satisfy the relationships

$$H(x) = F(x) \quad \text{for TWA} \tag{46}$$

and

$$H(x) = G(x) \quad \text{for BWA} , \tag{47}$$

when these curves are plotted on the same x-y plane, the x-coordinates of the intersection points of the H-curve with the F-curve and with the G-curve give the real roots of Eq. 38 for a TWA and for a BWA respectively. Moreover the number of these intersection points provides some useful information. For example, for a given set of values of the parameters g, B and Ω, if the number of intersections of the H-curve with the F-curve, n, or with the G-curve, ℓ, is four, then it indicates that there are four real distinct roots for Eq. 38. If $n = 3$ or $\ell = 3$ is observed, this implies that there are four real roots, two of which are equal, i.e.,

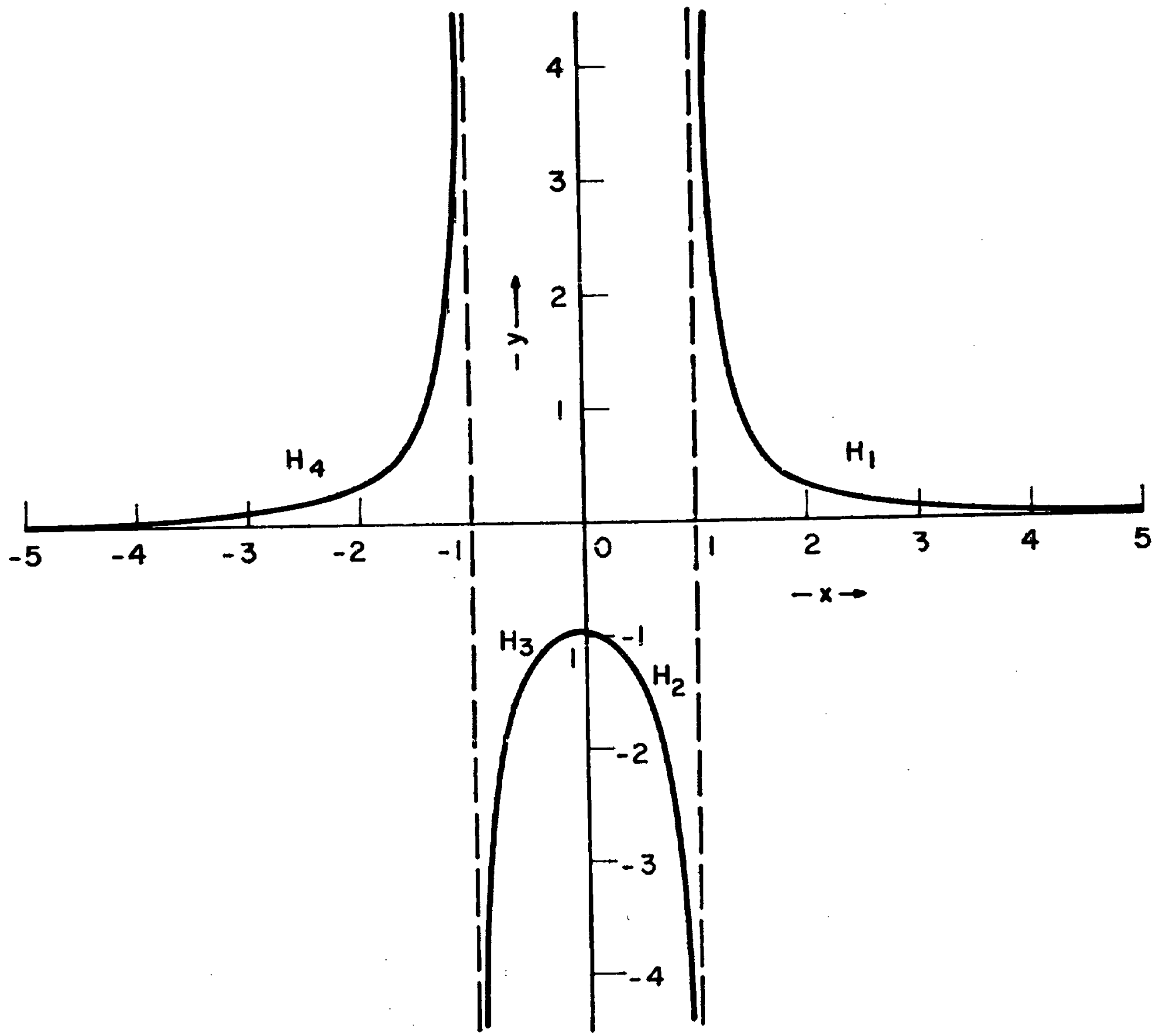

FIG. 1 PLOT OF THE FUNCTION H(x) VS. x.

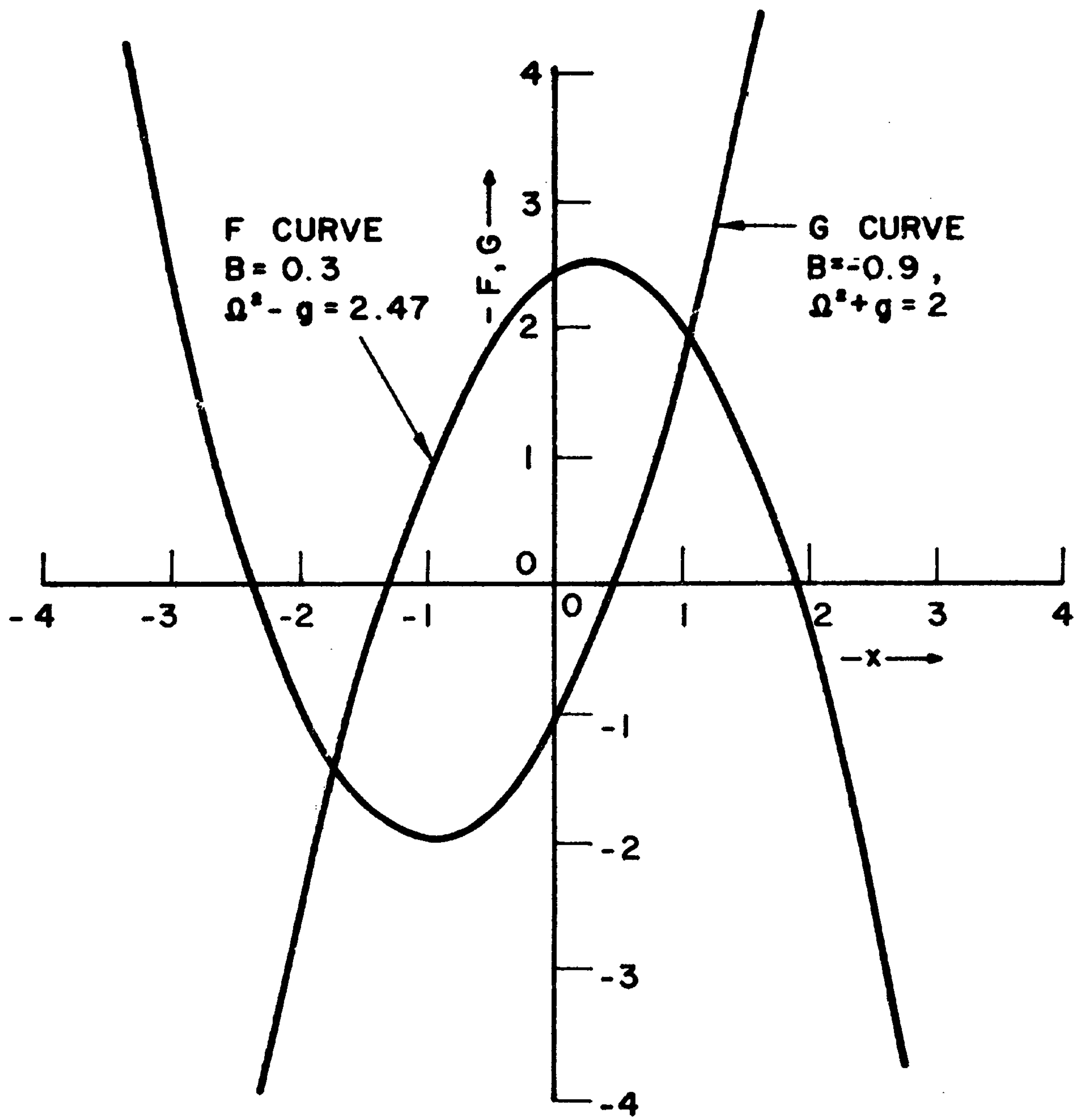

FIG. 2 PLOT OF THE FUNCTIONS F(x) AND G(x) VS. x.

the equal roots are represented by the tangent points of the H-curve with the F-curve or with the G-curve (see Fig. 3). On the other hand if $n = 2$ is observed, then there are two possibilities, namely that Eq. 38 has a pair of complex conjugate roots and two real distinct roots or it has two pairs of real equal roots, which is a special case of $n = 3$. However, there is no difficulty in distinguishing them by the graphical method. When $\ell = 2$ is observed it implies that Eq. 38 for a BWA has a pair of complex conjugate roots and two real distinct roots. $n = 0$ or $n = 1$ corresponds to the cases when Eq. 38 has no real root or a pair of real equal roots with a pair of complex conjugate roots, respectively. Consequently it can be said that Eq. 38 has at least one pair of complex conjugate roots if $\ell < 3$ for the case of a BWA or if $n < 3$ for the case of a TWA, provided that $n = 2$ does not correspond to the case of two tangent points between the H-curve and the F-curve.

It should be observed that if g is specified, then the H-curve is fixed and the parabolas can be shifted in position to any place in the x-y plane by varying the value of B or Ω without changing the shape or orientation. Therefore a change in location and in the number of intersection points of these curves can be easily observed. For example, for a given set of values of g and B, a change in n or ℓ with a change in the value of Ω can be observed. Since the F-curve or the G-curve move vertically (i.e., parallel to the y-axis) as Ω varies, the distribution of the intersection points changes accordingly (see Fig. 4). On the other hand, for a given set of values of g and Ω, the change in n or ℓ with the change in B can be investigated since the F-curve or the G-curve move horizontally (i.e., parallel to the x-axis) as B varies (see Fig. 5). Therefore the ranges of values of Ω or B for a given value of g, for which Eq. 38 has at least one pair of complex conjugate roots, can be quickly

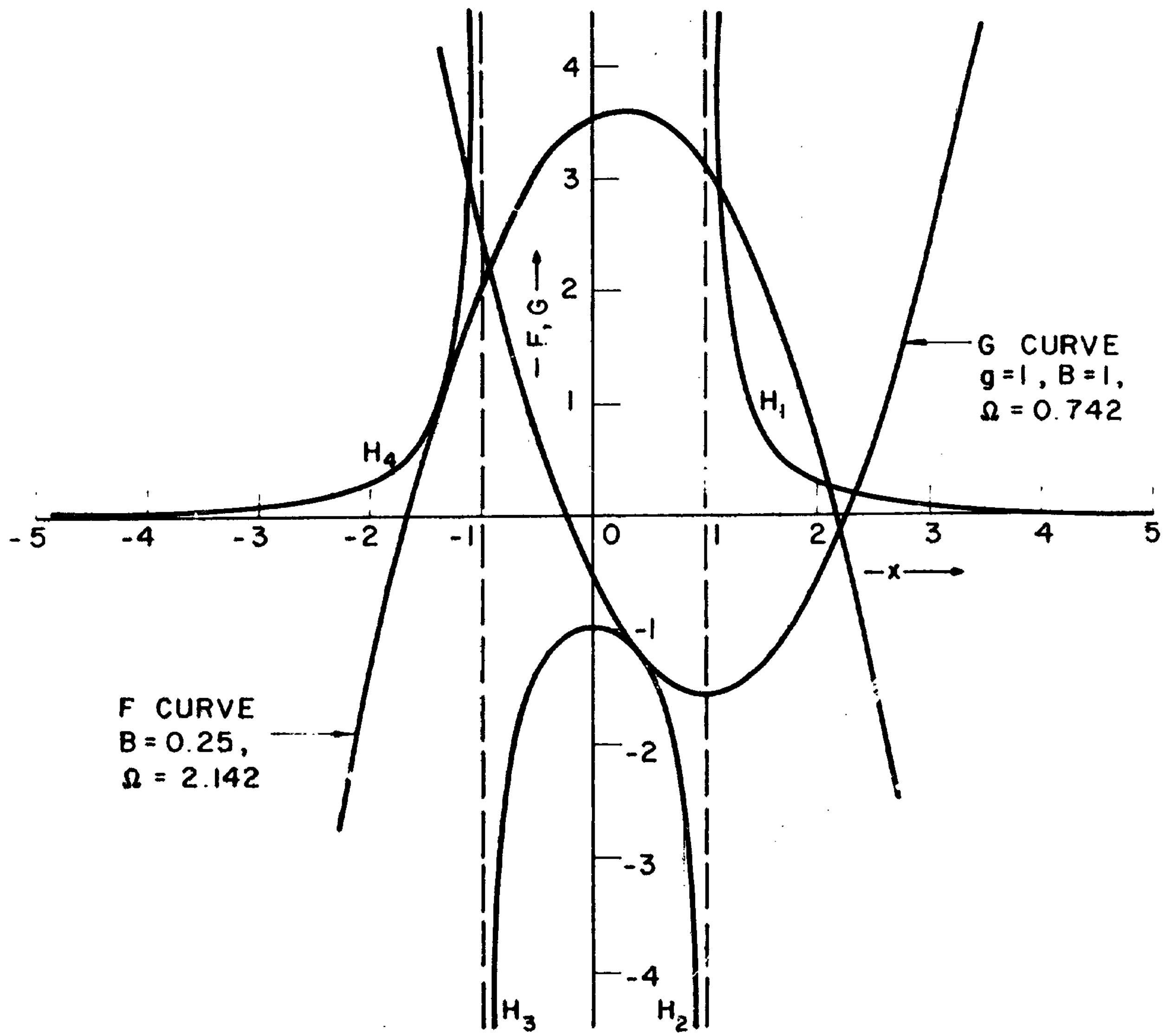

FIG. 3 ILLUSTRATION OF THE H-CURVE INTERSECTIONS AT THREE POINTS WITH THE F-CURVE ($n = 3$) AND WITH THE G-CURVE ($\ell = 3$).

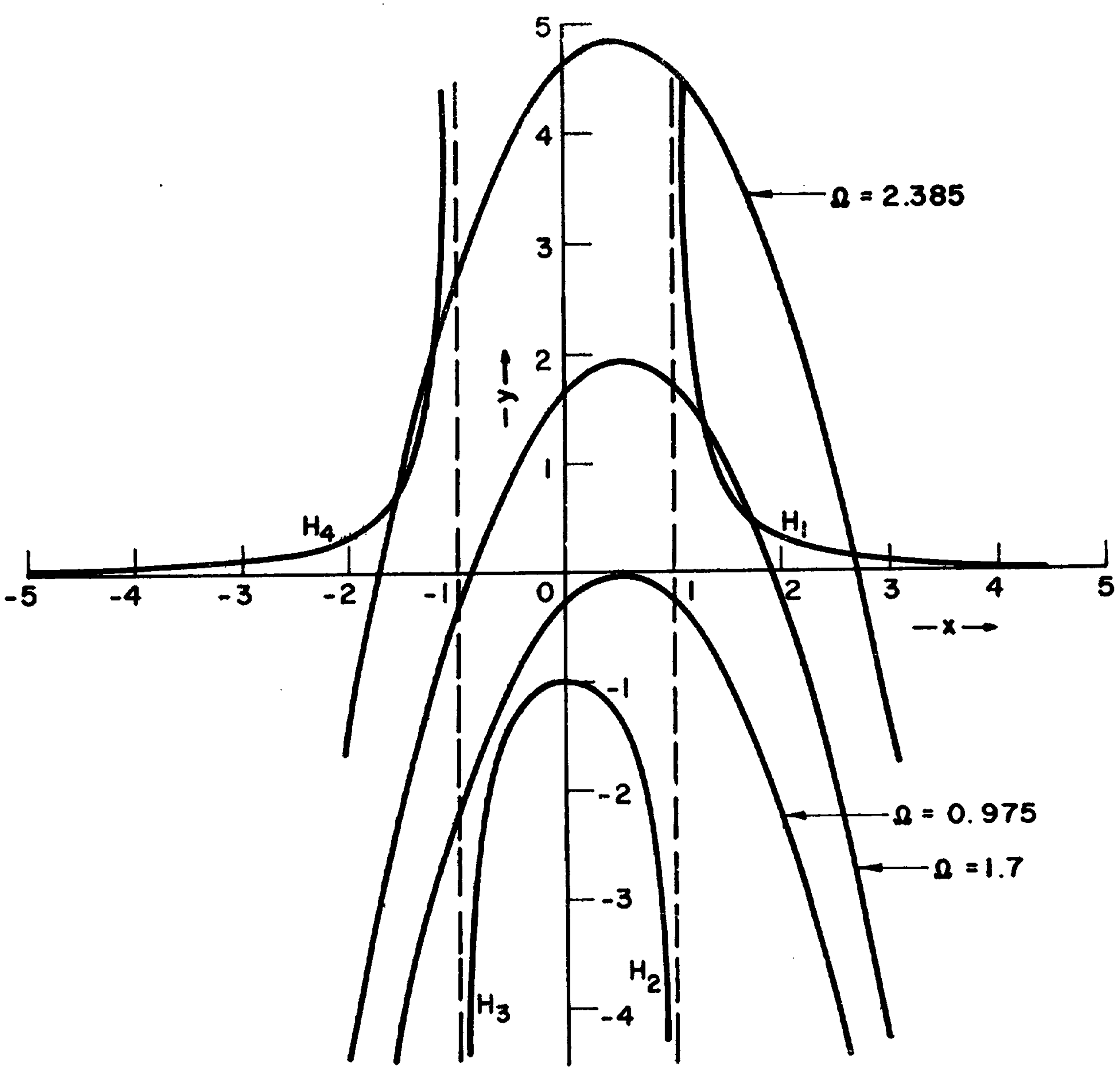

FIG. 4 ILLUSTRATION OF THE MOVEMENT OF THE F-CURVE WITH A VARIATION OF Ω. (g = 1.0, B = 0.5)

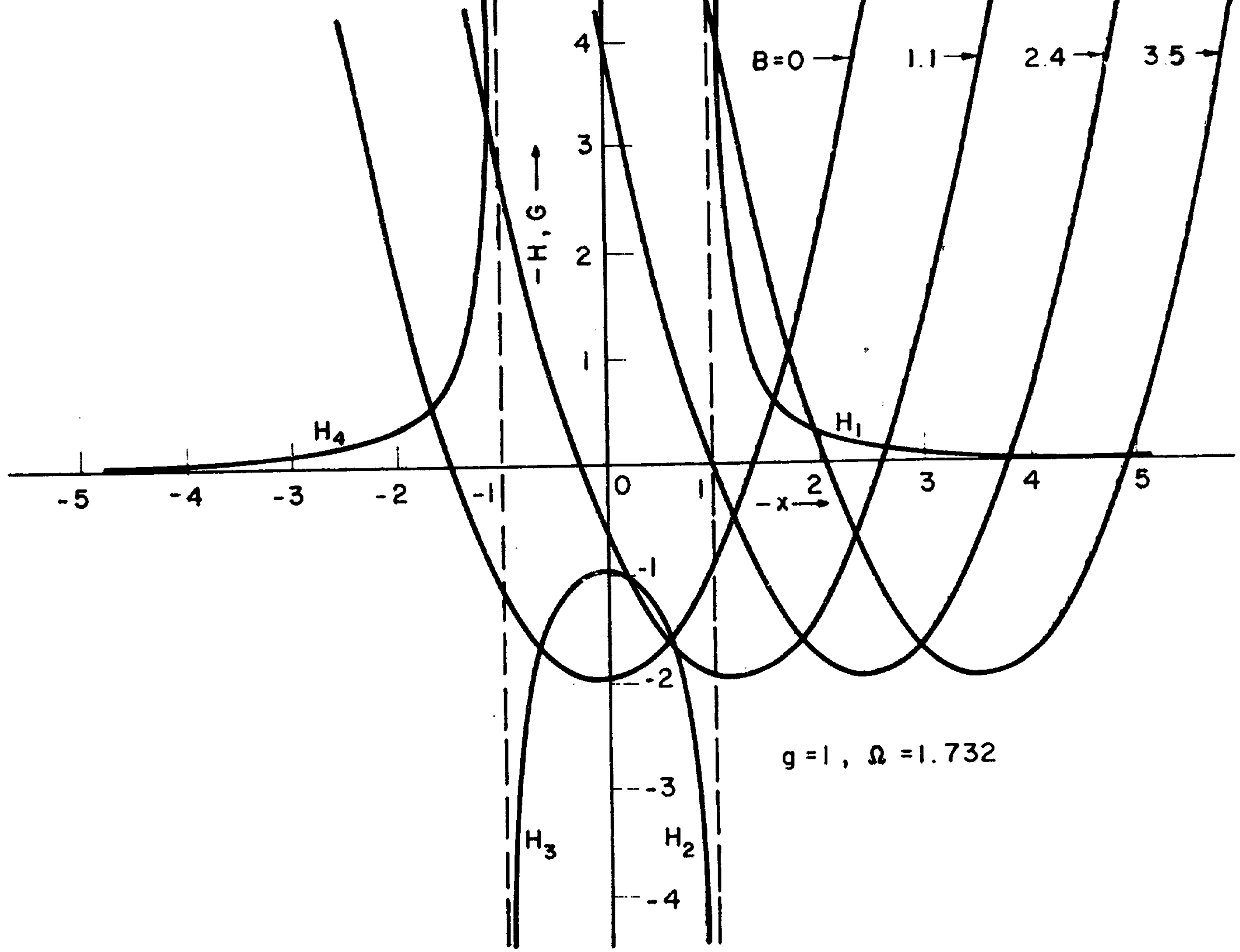

FIG. 5 ILLUSTRATION OF THE MOVEMENT OF THE G-CURVE WITH A VARIATION OF B FOR A GIVEN VALUE OF g AND Ω.

determined. The results thus obtained are illustrated and shown in Figs 6 and 7 for the cases of a TWA and a BWA respectively. In these figures, the shaded region represents a "forbidden" region for wave amplification, while the unshaded region represents the "permitted" region for wave amplification. The boundaries between these two regions correspond to the situations $n = 3$ or $\ell = 3$ which represent the cases where the F-curve or the G-curve is tangent to one of the branches of the H-curve. The F_m-curve, with $m = 1, 2, 3$ and 4, represents the proper combination of B, Ω and g for which the F-curve is tangent to the H_m-branch, whereas the G_m-curve, with $m = 1$ and 2, represents that for which the G-curve is tangent to the H_m-branch. It should be noted that the points labeled with K, L and M in Fig. 6 represent the conditions of two tangent points between the F-curve and the H-curve which correspond to the case where Eq. 38 for a TWA has two pairs of real equal roots (see Fig. 8).

When the coordinates of the points K, L, M, P and Q in the B-Ω plane (i.e., g = constant) are denoted by (B_k, Ω_k), $(0, \Omega_L)$, $(0, \Omega_M)$, $(B_p, 0)$ and $(B_Q, 0)$ respectively, the "permitted" region for wave amplification can be described in the following manner.

<u>For a TWA</u>.

If g and B are given and if $B_p < B_k$, then

$$F_2 < \Omega < F_4 \quad \text{for} \quad 0 \leq B < B_p ,$$

$$0 < \Omega < F_4 \quad \text{for} \quad B_p \leq B < B_k ,$$

$$0 < \Omega < F_1 \quad \text{and} \quad F_3 < \Omega < F_4 \quad \text{for} \quad B_k \leq B ,$$

and if $B_k < B_p$, then

$$F_2 < \Omega < F_4 \quad \text{for} \quad 0 \leq B < B_k ,$$

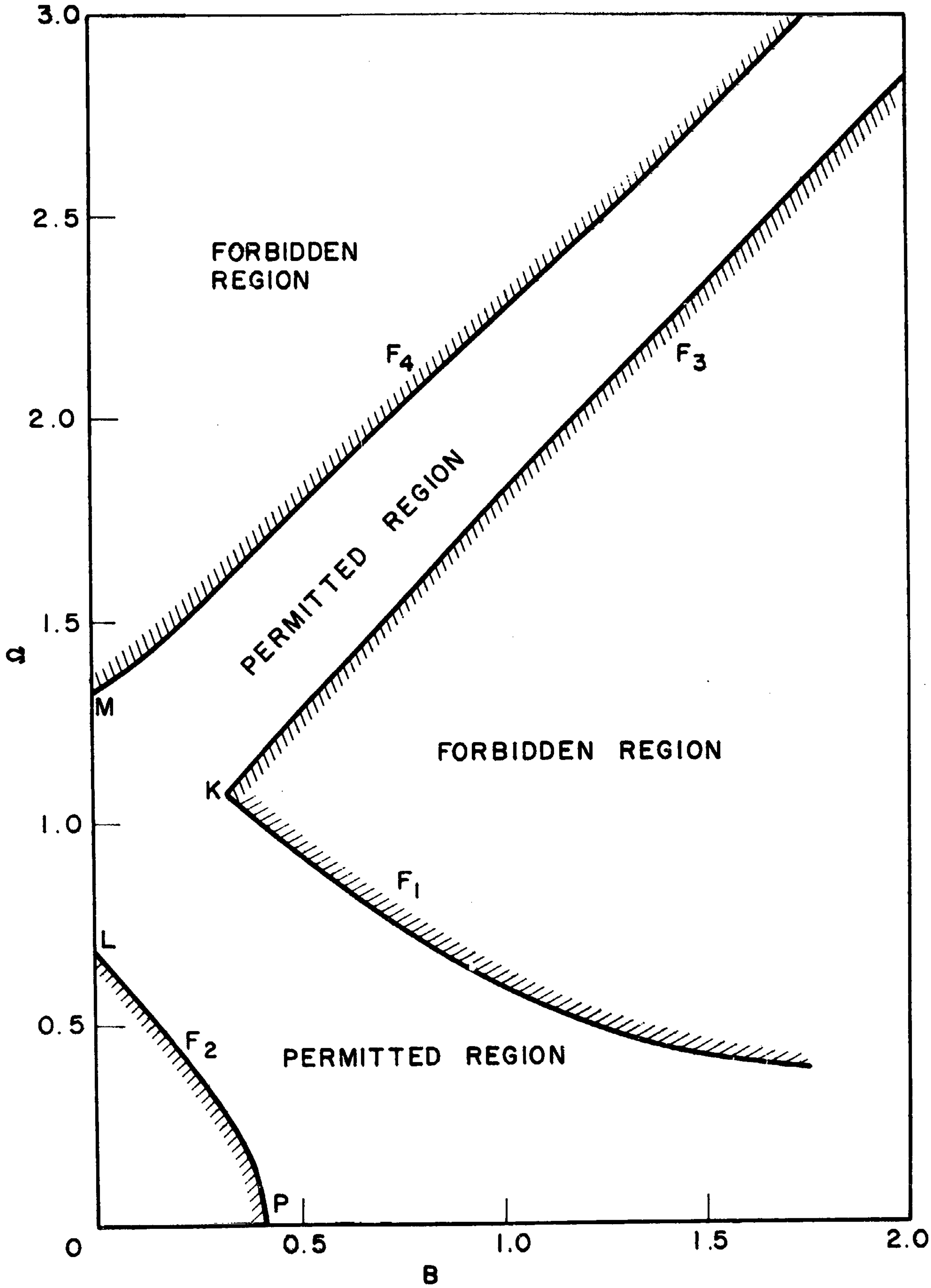

FIG. 6a "FORBIDDEN" AND "PERMITTED" REGIONS OF WAVE AMPLIFICATION FOR A TWA WITH g = 0.1 IN THE B-Ω PLANE.

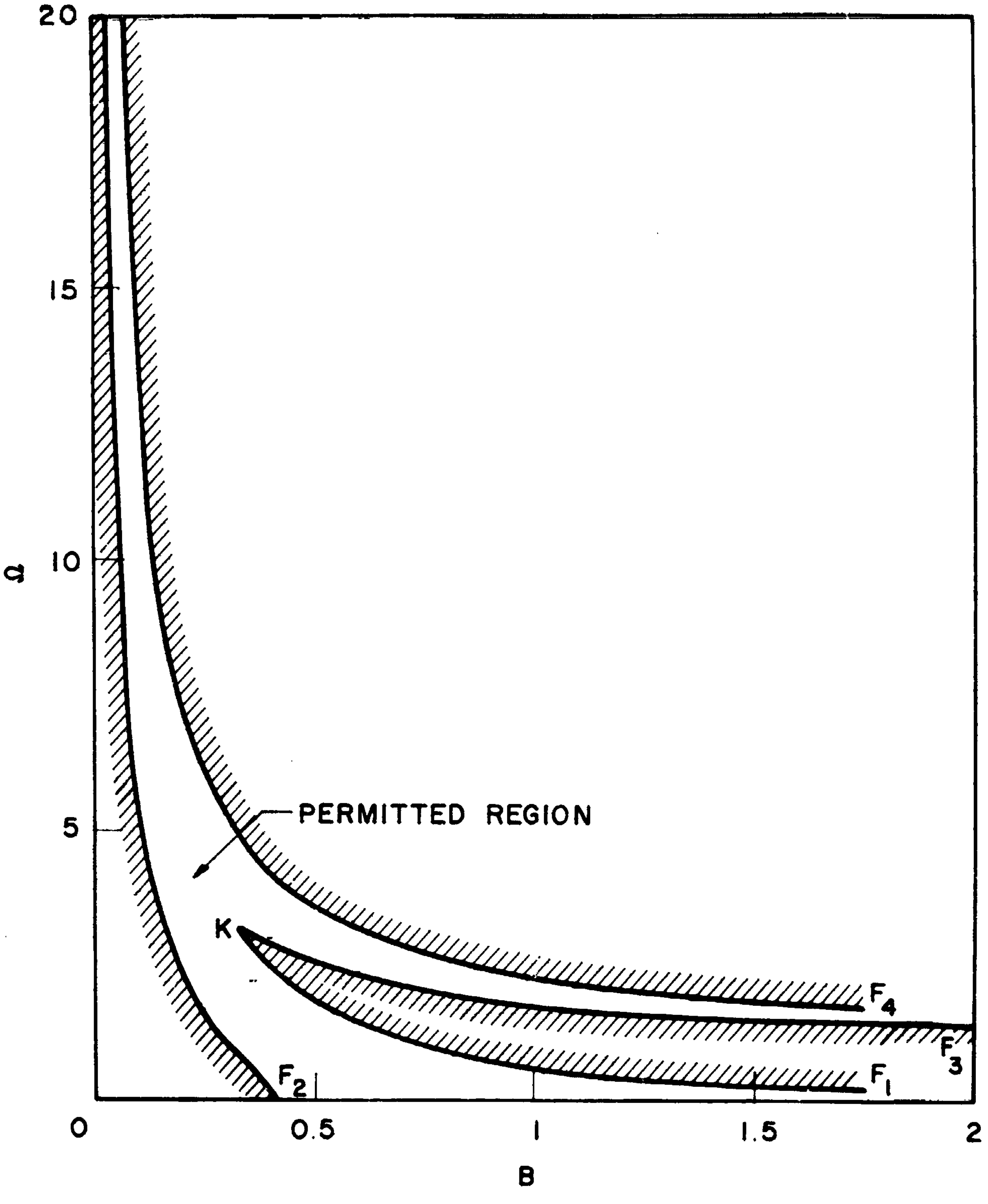

FIG. 6b "FORBIDDEN" AND "PERMITTED" REGIONS OF WAVE AMPLIFICATION IN THE B-Ω_o PLANE WITH g = 0.1 AND $\Omega_o = (\omega_p/\omega)$.

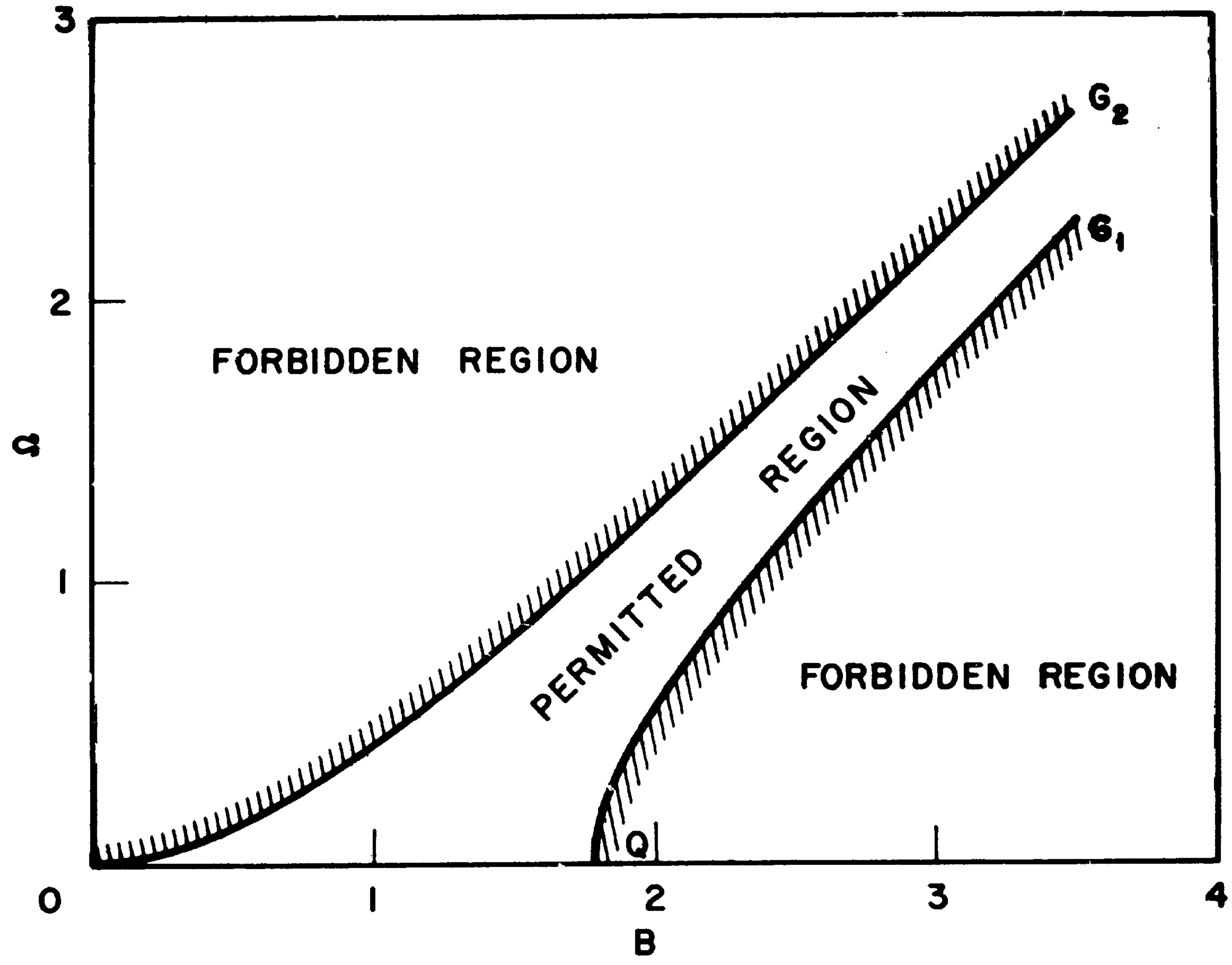

FIG. 7 "FORBIDDEN" AND "PERMITTED" REGIONS OF WAVE AMPLIFICATION FOR A BWA WITH $g = 0.1$.

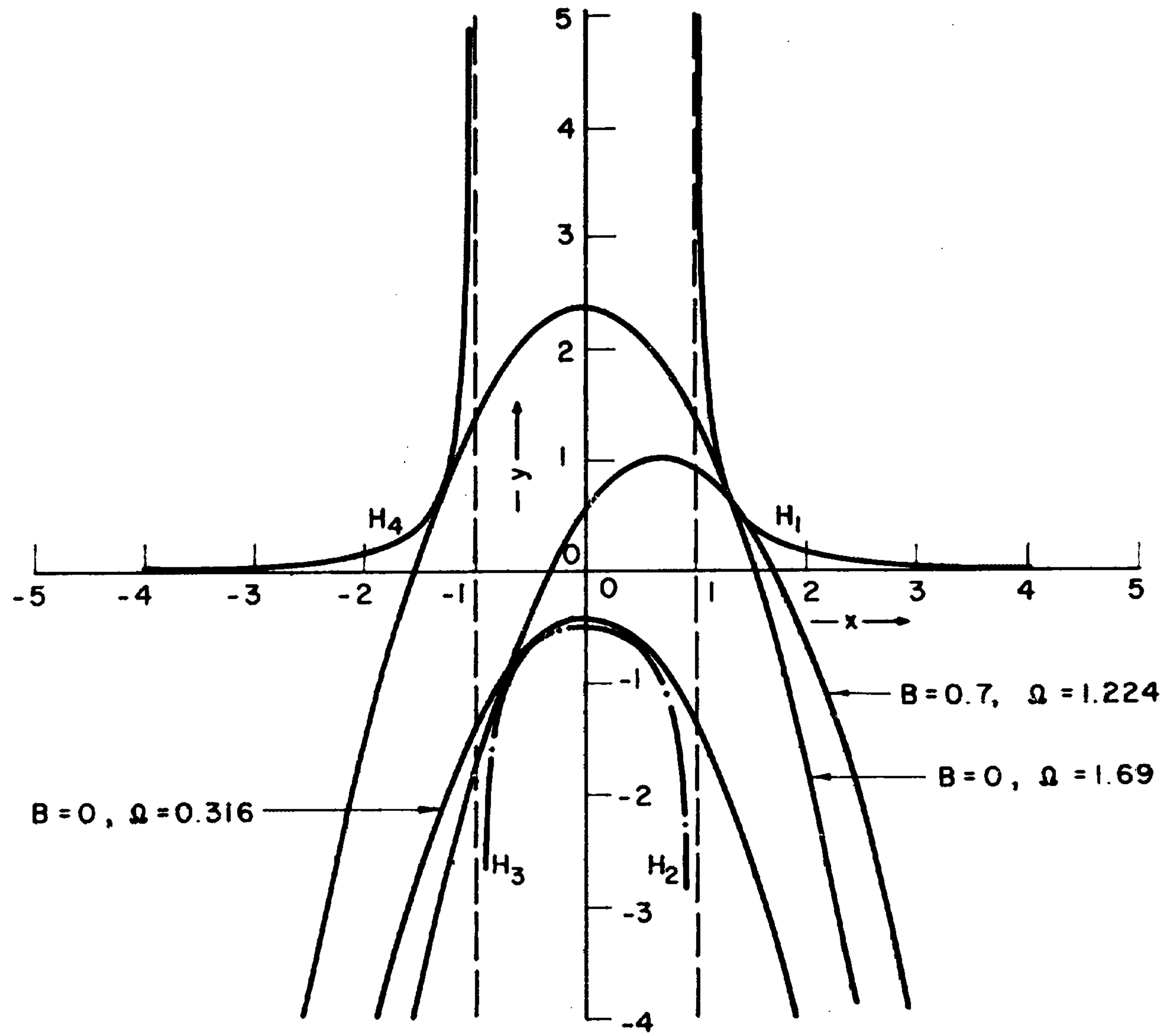

FIG. 8 ILLUSTRATION OF THE H-CURVE TANGENT TO THE F-CURVE AT TWO PLACES ($g = 0.5$).

$$F_2 < \Omega < F_1 \quad \text{and} \quad F_3 < \Omega < F_4 \quad \text{for} \quad B_k \leq B < B_p$$

and

$$0 < \Omega < F_1 \quad \text{and} \quad F_3 < \Omega < F_4 \quad \text{for} \quad B_p \leq B . \tag{48}$$

On the other hand, if g and Ω are given, then

$$F_2 < B < F_1 \quad \text{for} \quad 0 < \Omega < \Omega_L ,$$

$$0 < B < F_1 \quad \text{for} \quad \Omega_L \leq \Omega < \Omega_k ,$$

$$0 < B < F_3 \quad \text{for} \quad \Omega_k \leq \Omega < \Omega_M$$

and

$$F_4 < B < F_3 \quad \text{for} \quad \Omega_M \leq \Omega . \tag{49}$$

For a BWA.

If g and B are given, then

$$0 < \Omega < G_2 \quad \text{for} \quad 0 < B < B_Q$$

and

$$G_1 < \Omega < G_2 \quad \text{for} \quad B_Q \leq B . \tag{50}$$

On the other hand, if g and Ω are given, then

$$G_2 < B < G_1 \quad \text{for} \quad 0 \leq \Omega . \tag{51}$$

3.2 The Boundaries of the "Forbidden" Region in the B-Ω-g Space

When $n = 3$ or $\ell = 3$, Eq. 38 can be expressed in the following form:

$$(x - c)^2 (x - a) (x - b) = 0 , \tag{52}$$

where a, b and c are real. On the other hand, Eq. 38 can also be written as

$$F(x) \equiv x^4 - 2Bx^3 - [1 + g_o + (\Omega^2 - B^2)]\, x^2 + 2Bx + (\Omega^2 - B^2) = 0 \tag{53}$$

where $g_o = -g$ for a TWA and $g_o = +g$ for a BWA.

Expanding Eq. 52 and then comparing it with Eq. 53 gives

$$(a + b) + 2c = 2B \;, \tag{54}$$

$$c^2 + 2c(a + b) + ab = -[1 + g_o + (\Omega^2 - B^2)] \;, \tag{55}$$

$$c^2(a + b) + 2abc = -2B \tag{56}$$

and

$$abc^2 = (\Omega^2 - B^2) \tag{57}$$

From Eqs. 54 and 56 it follows that

$$(1 + c^2)\,(a + b) + 2c(1 + ab) = 0 \tag{58}$$

and from Eqs. 55 and 57,

$$2c(a + b) + (1 + c^2)\,(1 + ab) = -g_o \;. \tag{59}$$

If Eqs. 58 and 59 are solved simultaneously, a and b can be expressed in terms of c as follows.

$$a = \frac{cg_o}{\Delta} \pm \sqrt{D} \;, \tag{60}$$

$$b = \frac{cg_o}{\Delta} \mp \sqrt{D} \;, \tag{61}$$

where

$$D = \left[\left(\frac{cg_o}{\Delta}\right)^2 + 1 + \frac{g_o}{\Delta}\,(1 + c^2)\right] \tag{62}$$

and

$$\Delta = (1 - c^2)^2 \;. \tag{63}$$

It should be observed that for a BWA, $D > 0$ since $g_o > 0$, and a and b have an opposite algebraic sign, i.e., if $a > 0$ then $b < 0$ or if $a < 0$ then $b > 0$. On the other hand, for a TWA, since $g_o < 0$, it is possible that D becomes zero, in which case a becomes equal to b, which leads to $n = 2$, a special case of $n = 3$ which has been discussed previously.

Since c is a root of Eq. 53 [i.e., $P(c) = 0$],

$$c^4 - 2Bc^3 - [1 + g_o + (\Omega^2 - B^2)]c^2 + 2Bc + (\Omega^2 - B^2) = 0 \qquad (64)$$

and at the same time since c is also the equal roots of Eq. 53, $(dP/dx)_{x=c} = 0$, i.e.,

$$2c^3 - 3Bc^2 - [1 + g_o + (\Omega^2 - B^2)]c + B = 0 \ . \qquad (65)$$

Thus c must satisfy Eqs. 64 and 65 simultaneously. When Eq. 65 is multiplied by a factor (c/2) and is then subtracted from Eq. 64, the result is

$$Bc^3 + [1 + g_o + (\Omega^2 - B^2)]c^2 - 3Bc - 2(\Omega^2 - B^2) = 0 \ . \qquad (66)$$

It should be noted that the set of Eqs. 65 and 66 is equivalent to the set of Eqs. 64 and 65. Furthermore it is not difficult to show that Eq. 66 can also be derived from Eqs. 54, 55, 56 and 57. When the terms containing c^3 are eliminated from Eqs. 65 and 66,

$$p_2 c_o^2 - p_1 c_o + p_o = 0 \ , \qquad (67)$$

which in turn gives

$$c_o = \frac{p_1 \pm \sqrt{p_1^2 - 4p_o p_2}}{2p_2} \ , \qquad (68)$$

where

$$p_o = [3B^2 - 4\Omega^2] \quad ,$$

$$p_1 = [(B^2 + 5) - (g_o + \Omega^2)]B$$

and

$$p_2 = [(B^2 + 2) + 2(g_o + \Omega^2)] \qquad (69)$$

The double signs appearing in Eq. 68 must be chosen so that for a given set of values of B, Ω and g, c_o, obtained from Eq. 68, must satisfy Eq. 64.

Substituting c_o, given by Eq. 68, into Eq. 65 yields an equation relating the parameters g, B and Ω in such a way that $n = 3$ or $\ell = 3$. Thus Eq. 65 with the aid of Eqs. 68 and 69 can be regarded as the equation of the boundary surfaces of the "forbidden" region for wave amplification in the B-Ω-g space. It is not difficult to verify that if a given set of values of the parameters B, Ω and g can be represented by a point on the F_m-curve or on the G_m-curve, as shown respectively in Fig. 6 or Fig. 7, then the value of c_o, given by Eq. 68 with the aid of Eq. 69, does satisfy Eq. 65. Furthermore it should be observed that for a given value of g and B there are three values of Ω^2 which satisfy Eq. 65. When a positive angular frequency ω is considered, Ω must be taken as a positive quantity. The number of real positive Ω which satisfy Eq. 65 depends upon the ranges in which the given B lies. For example, as shown in Fig. 6 for the case of $B_p < B_k < B$, there are three positive Ω which correspond to those values of Ω given by the F_1-, F_3- and F_4-curves.

IV. DISCUSSION OF RESULTS

The "forbidden" region for wave amplification in the B-Ω plane for different values of the parameter g is shown in Fig. 9 for a TWA and in Fig. 10 for a BWA respectively. It should be observed that as the value of g decreases the "permitted" region for wave amplification decreases accordingly. As $g \rightarrow 0$, the F_4-curve approaches the F_3-curve and the F_2-curve approaches the F_1-curve. Thus the "permitted" region disappears. Similarly the G_2-curve approaches the G_1-curve as $g \rightarrow 0$. This is obvious from Eq. 38, because when g = 0 there are always four real distinct roots since a lossless circuit is being considered.

Figure 9 shows that in the case of a TWA, for given values of g and B, there is a range or ranges of values of Ω over which wave amplification is possible. For $B < B_k$, there is only one such range, while if $B \geq B_k$, two such ranges of Ω are observed. It should be noted that with Ω equal to $B(\omega_p/\omega)$, if the average electron-charge density ρ_o is assumed to be uniform over the electron beam, then ω_p is fixed and since B is given, for a range of Ω there corresponds a band of angular frequency ω. Consequently the above observation suggests that for $B < B_k$ there is only one frequency band, while for $B \geq B_k$ there are two frequency bands over which the wave can be amplified. On the other hand, in the case of a BWA Fig. 10 shows that there is only one frequency band over which wave amplification is possible. Furthermore the widths of these amplification bands decrease with a decrease in the value of g. It is of interest to note that the "permitted" and "forbidden" regions for wave amplification can also be given in the B-Ω_o-g space, as shown in Fig. 6b.

For a typical laboratory TWT, under normal operating conditions the value of B is near unity and the values of $\Omega_o = (\omega_p/\omega)$ and C are both much

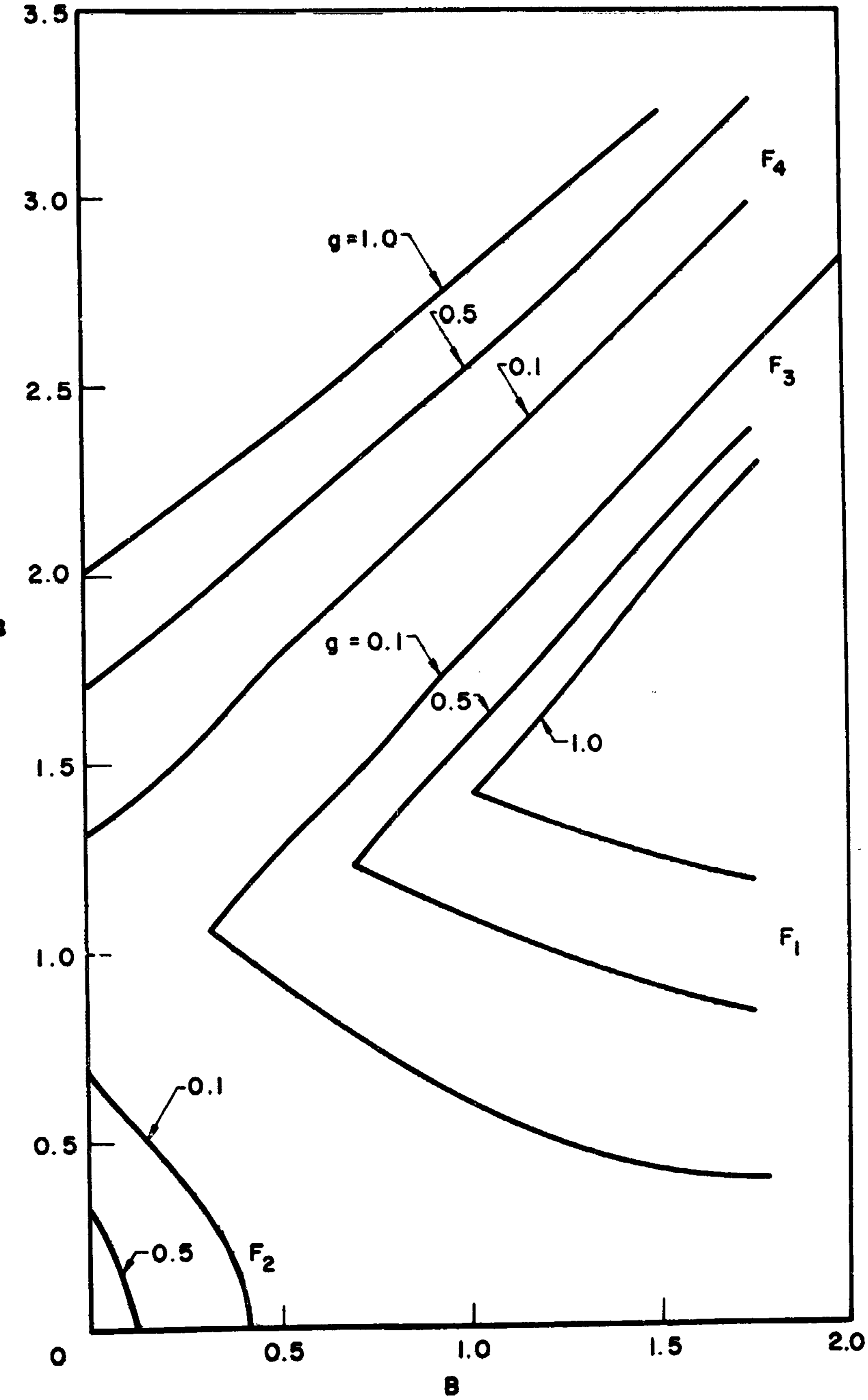

FIG. 9 "FORBIDDEN" AND "PERMITTED" REGIONS OF WAVE AMPLIFICATION FOR A TWA WITH g AS A PARAMETER.

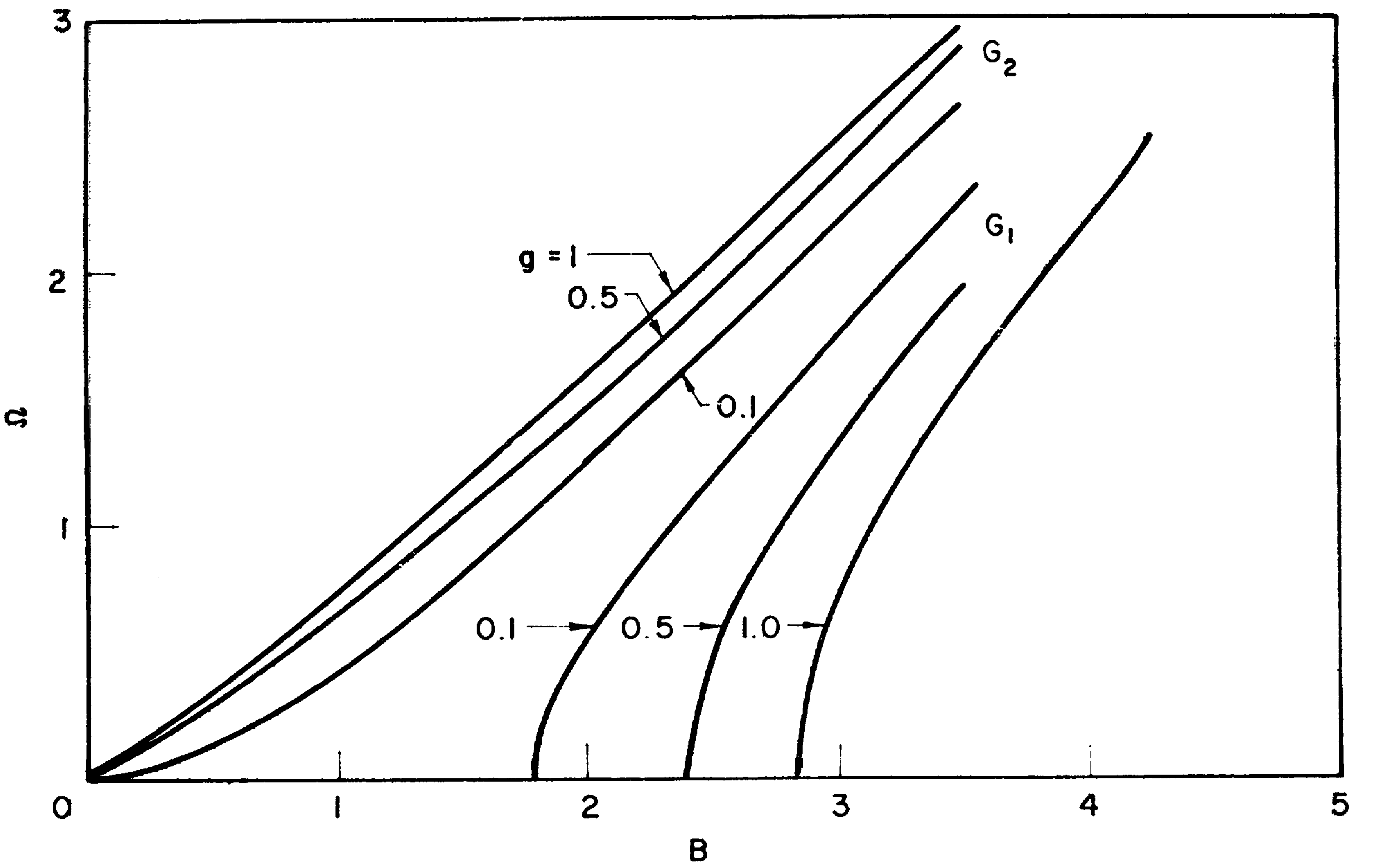

FIG. 10 "FORBIDDEN" AND "PERMITTED" REGIONS OF WAVE AMPLIFICATION FOR A BWA WITH g AS A PARAMETER.

smaller than unity so that [illegible] that the point representing this operating condition can be located in the "permitted" region in the B-Ω-g space (e.g., see Fig. 6a) or in the B-Ω_o-g space (e.g., see Fig. 6b). Thus for the analysis of a laboratory TWA, that portion of the "permitted" region which is below the F_1-curve, as shown in Figs. 6a or 6b is of primary interest. On the other hand, for the analysis of the traveling-wave-amplification process as a natural phenomenon one will, in general, be interested in that portion of the "permitted" region between the F_3-curve and the F_4-curve which is referred to as the R_U-region, as well as in the portion below the F_1-curve which is referred to as the R_L-region, since the physical environment in nature can not be easily controlled and the values of Ω_o and B may be arbitrary. In view of the fact that various experimental ionospheric observations tend to indicate that the value of (ω_p/ω) is usually of the order of unity or much greater, for example, in the case of VLF emission, the R_U-region should be of more interest than the R_L-region.

It is not difficult also to compare the effectiveness of the amplification process which can take place in the R_L- and R_U-regions by comparing the values of the imaginary part of the complex normalized propagation parameter $\tilde{X} = (p+ja)$ where p and a are the phase and amplitude factors respectively. As an illustration, plots of the real part p_m and the imaginary part a_m of the complex propagation parameter $\tilde{X}_m$ for the system, with m = 1,2,3 and 4, against Ω with g = 0.1 are shown in Figs. 11a, 11b and 11c for the cases of B = 0.5, 1.0 and 1.5 respectively. It should be noted that there are two ranges of Ω; one lies below and the other lies above Ω = 1.0 over which the amplification of a wave is possible. Furthermore it is of interest to observe that for the range Ω < B (i.e., $\omega_p/\omega < 1$) Eq. 38 has three roots with a positive real part and one root

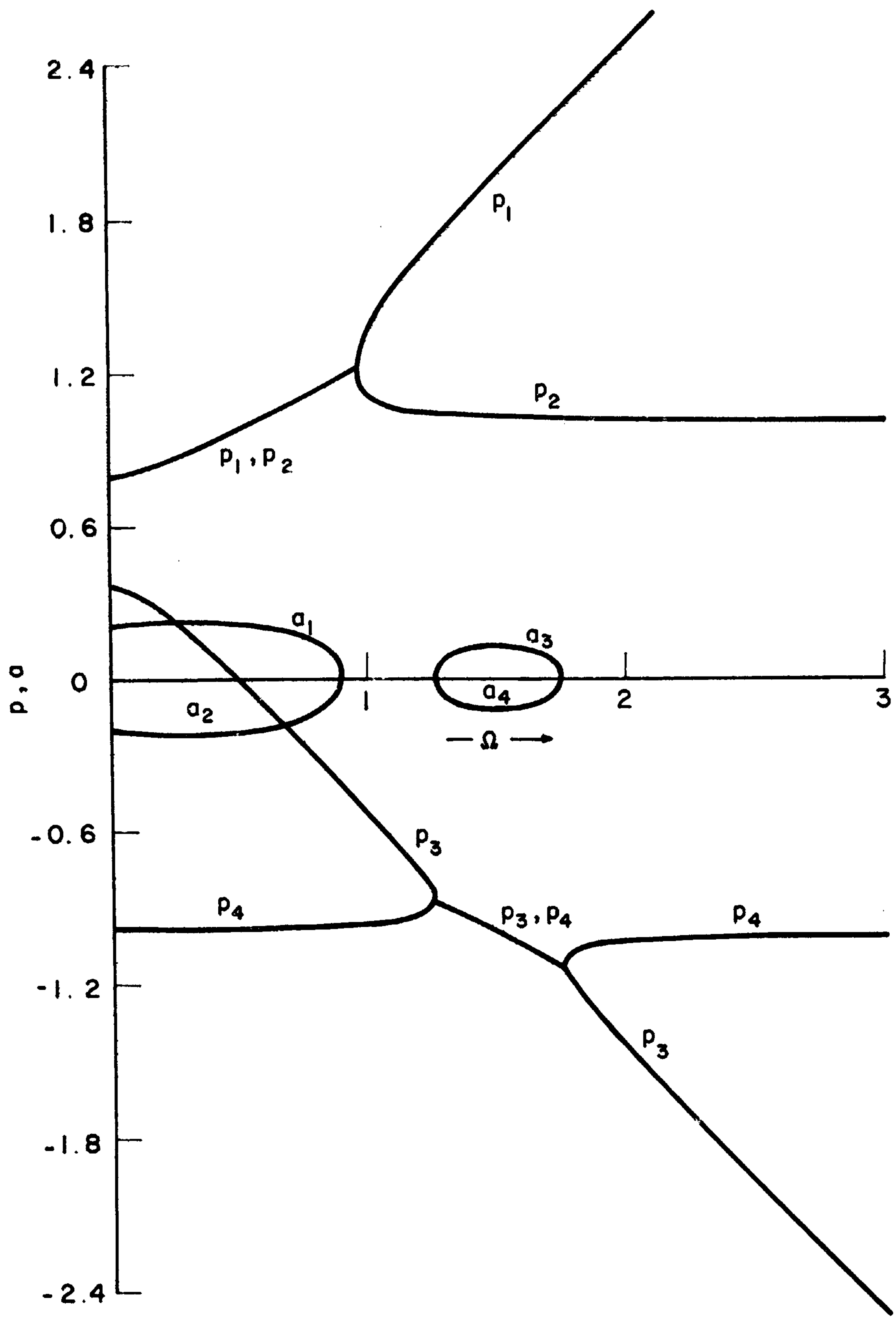

FIG. 11a PLOT OF p's AND a's VS. Ω AT B = 0.5 WITH g = 0.1.

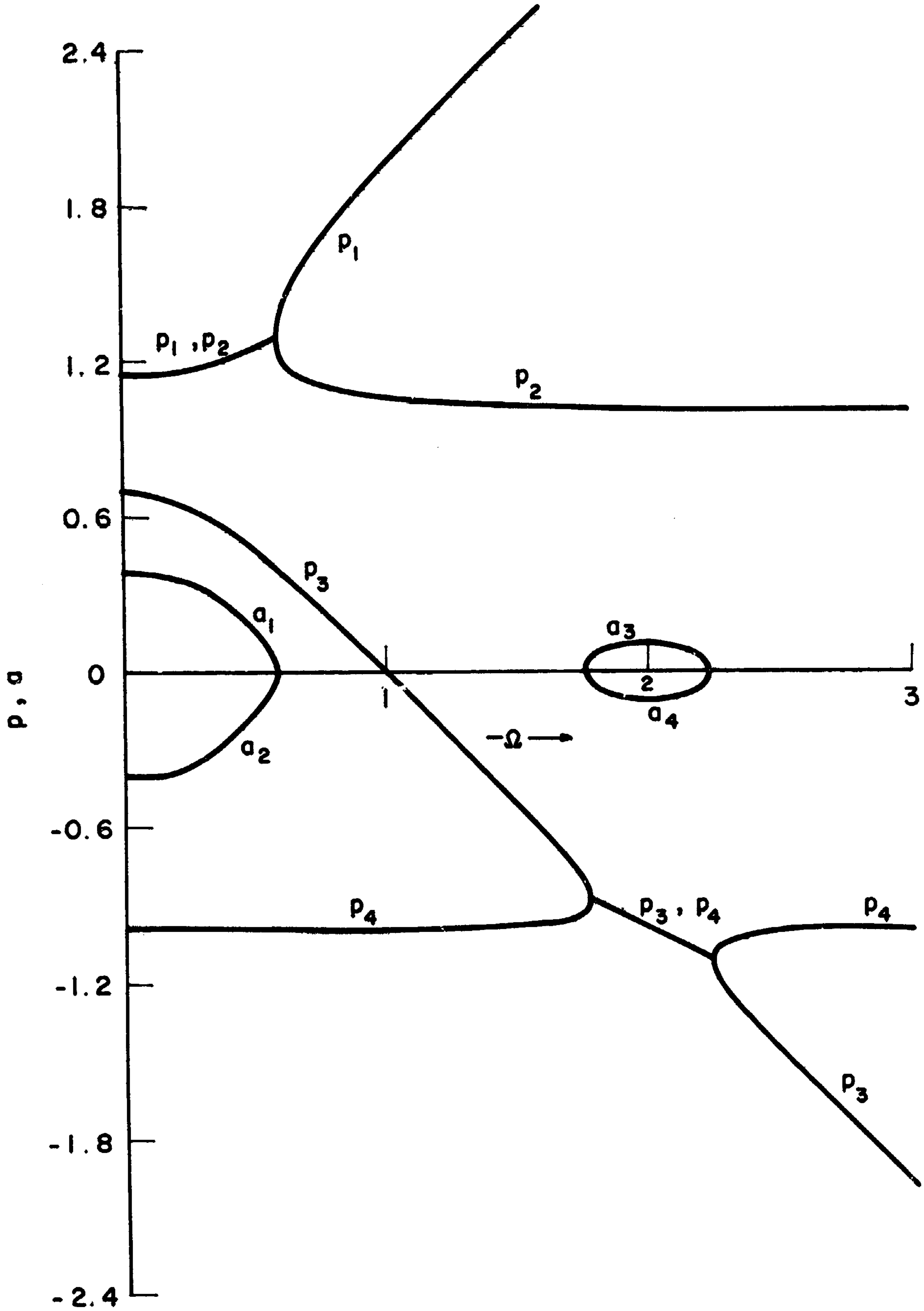

FIG. 11b PLOT OF p's AND a's VS. Ω AT B = 1.0 WITH g = 0.1.

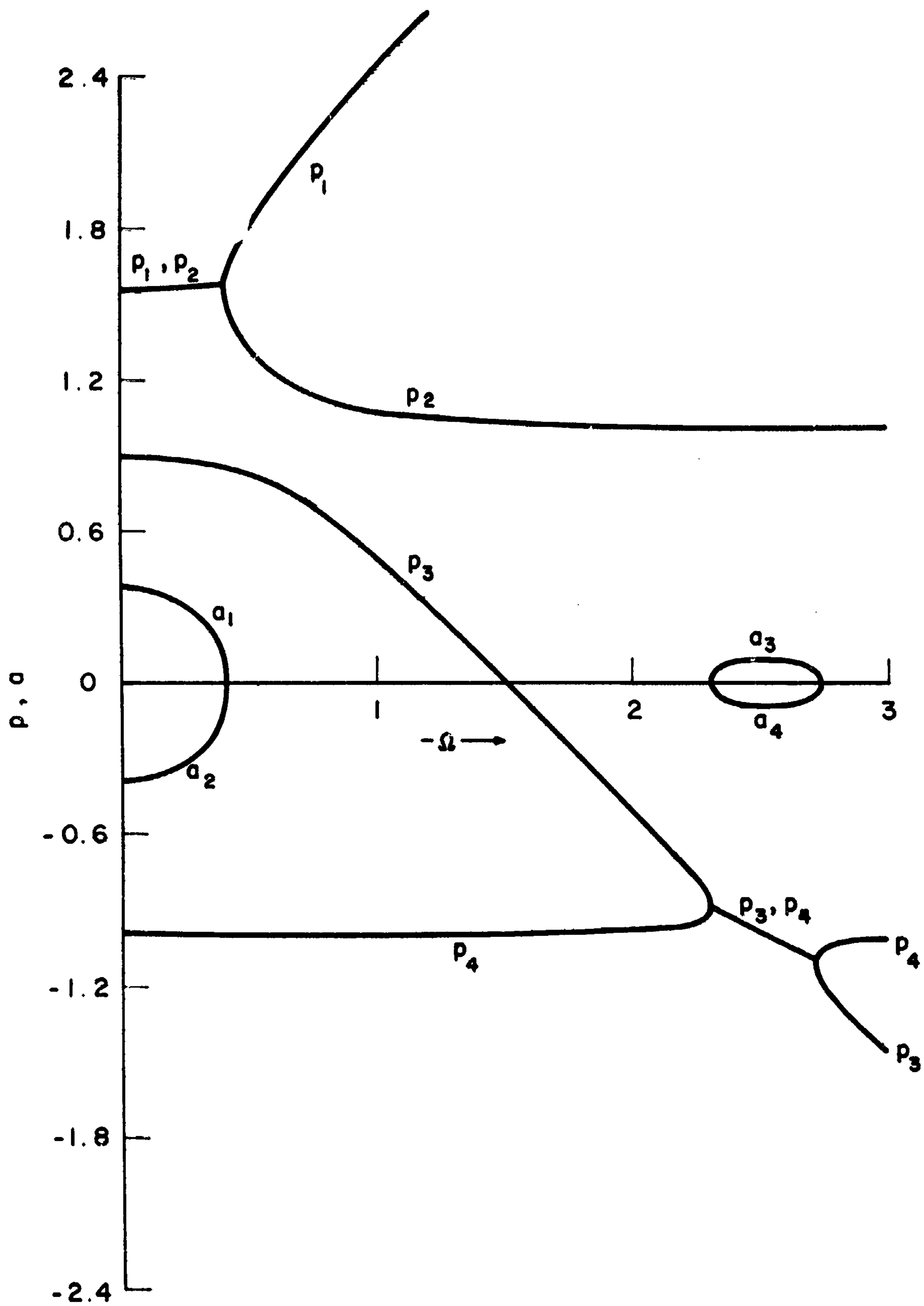

FIG. 11c PLOT OF p's AND a's VS. Ω AT B = 1.5 WITH g = 0.1.

with a negative real part, which implies that the system may support one backward- and three forward-propagating waves. On the other hand, for the range $\Omega > B$ (i.e., $\omega_p > \omega$) Eq. 38 has two roots with positive real parts and two roots with negative real parts, which suggests that the system can support two forward- and two backward-propagating waves. However when $\Omega = B$ (i.e., $\omega_p = \omega$), regardless of the value of g, $X = 0$ is one of the roots of Eq. 38. Consequently there are only three propagating waves in the system. The maximum values of the amplitude factor a vs. B with g as a parameter are shown in Figs. 12a and 12b for the R_L- and R_U-regions of the "permitted" region respectively. The values of the phase factor p corresponding to the maximum a vs. B with g as a parameter are shown in Figs. 13a and 13b for the "permitted" R_L- and R_U-regions respectively. The maximum value of a vs. the coupling parameter g with B as a parameter is shown in Figs. 14a and 14b for the R_L- and R_U-regions respectively. The comparison of Figs. 12a and 12b shows that for a given value of g the amplification process is more effective in the R_L-region than in the R_U-region. The comparison of Figs. 14a and 14b suggests the same. To show this fact more clearly the ratio of the maximum value of the amplitude factor a_3 in the R_U-region to the maximum value of the amplitude factor a_1 in the R_L-region (i.e., max a_3/max a_1) is plotted against B for a given value of g and is shown in Fig. 12c. It should be observed that this ratio is less than unity and it decreases as B increases. For example, for g = 0.01 at B = 1, max a_1 is approximately five times greater than max a_3, while at B = 0.5 max a_1 is approximately twice as great as max a_3. On the other hand, Fig. 14c, which is obtained by combining Figs. 14a and 14b, shows the variation of the ratio (max a_3/max a_1) for a fixed value of B. This figure suggests again that wave amplification should be more effective in the R_L-region than in the R_U-region. It should also be noted

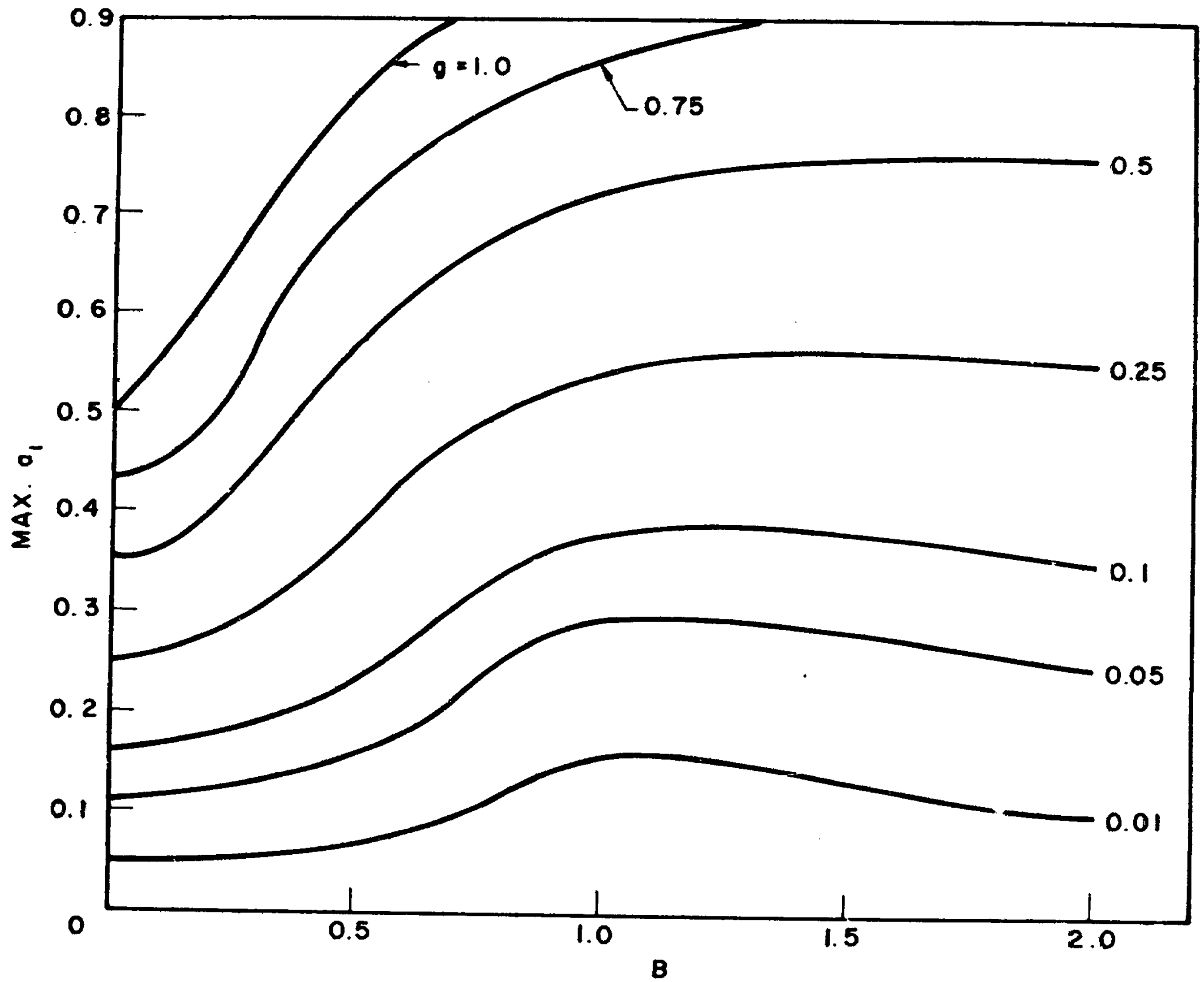

FIG. 12a PLOT OF THE MAXIMUM VALUE OF THE AMPLITUDE FACTOR a VS. B IN THE "PERMITTED" R_L-REGION WITH g AS A PARAMETER.

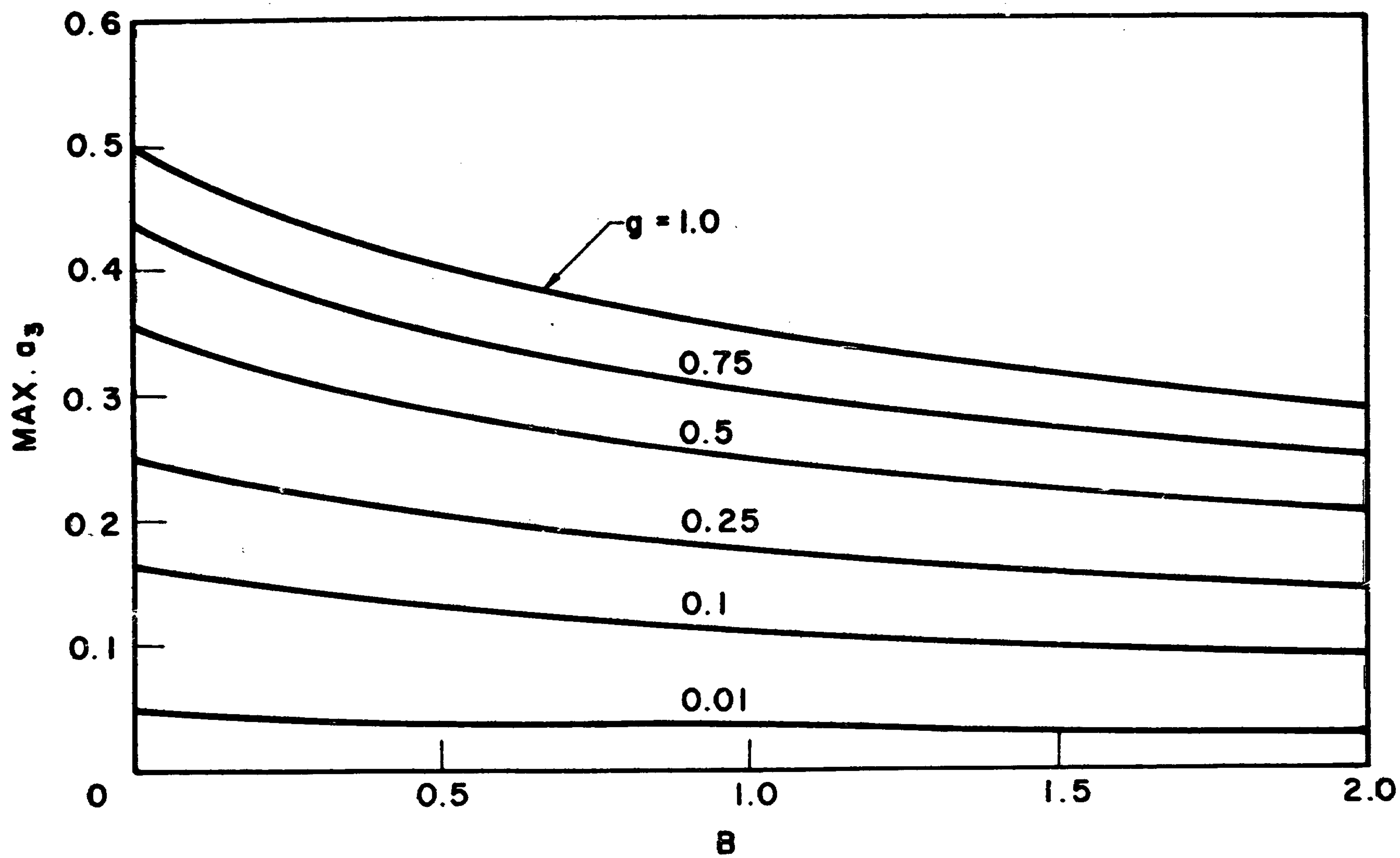

FIG. 12b PLOT OF THE MAXIMUM VALUE OF THE AMPLITUDE FACTOR a VS. B IN THE "PERMITTED" R_U-REGION WITH g AS A PARAMETER.

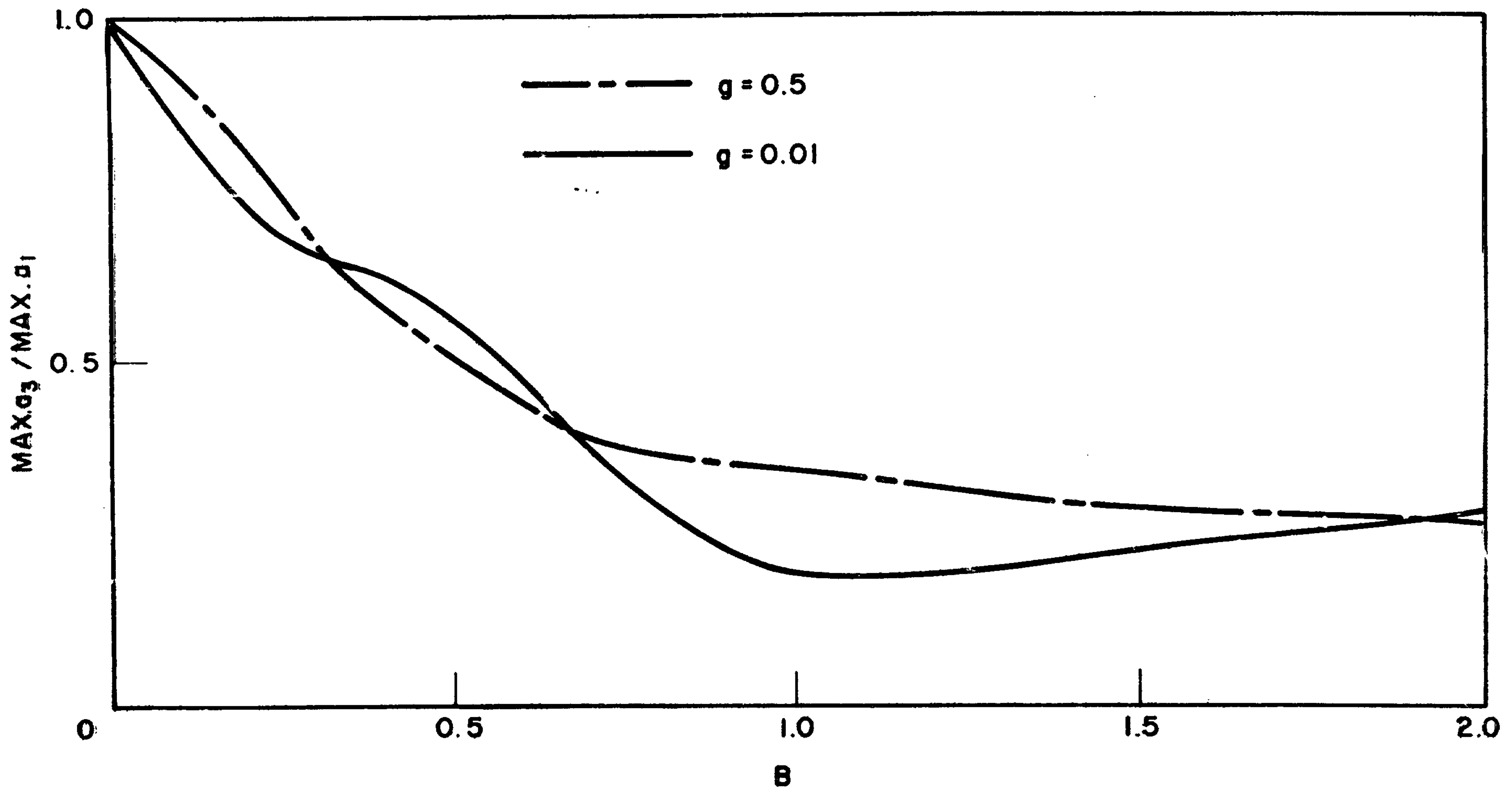

FIG. 12c PLOT OF (max a_s/max a_1) VS. B WITH g AS A PARAMETER. max a_1 AND max a_s DENOTE THE MAXIMUM VALUES OF THE AMPLITUDE FACTOR IN THE "PERMITTED" R_L- AND R_U-REGIONS RESPECTIVELY.

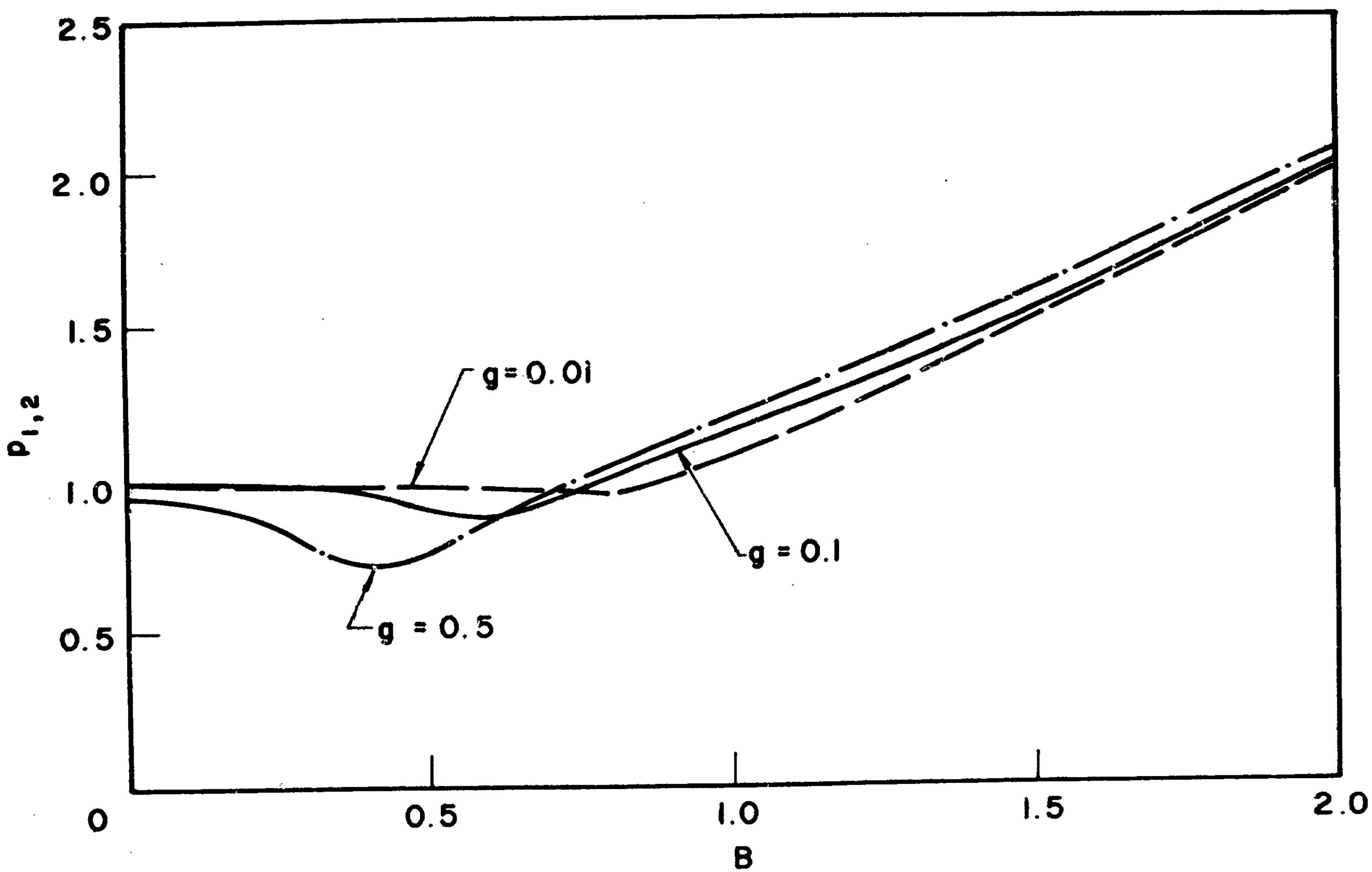

FIG. 13a PLOT OF THE PHASE FACTOR p CORRESPONDING TO THE MAXIMUM a VS. B IN THE "PERMITTED" R_L-REGION WITH g AS A PARAMETER.

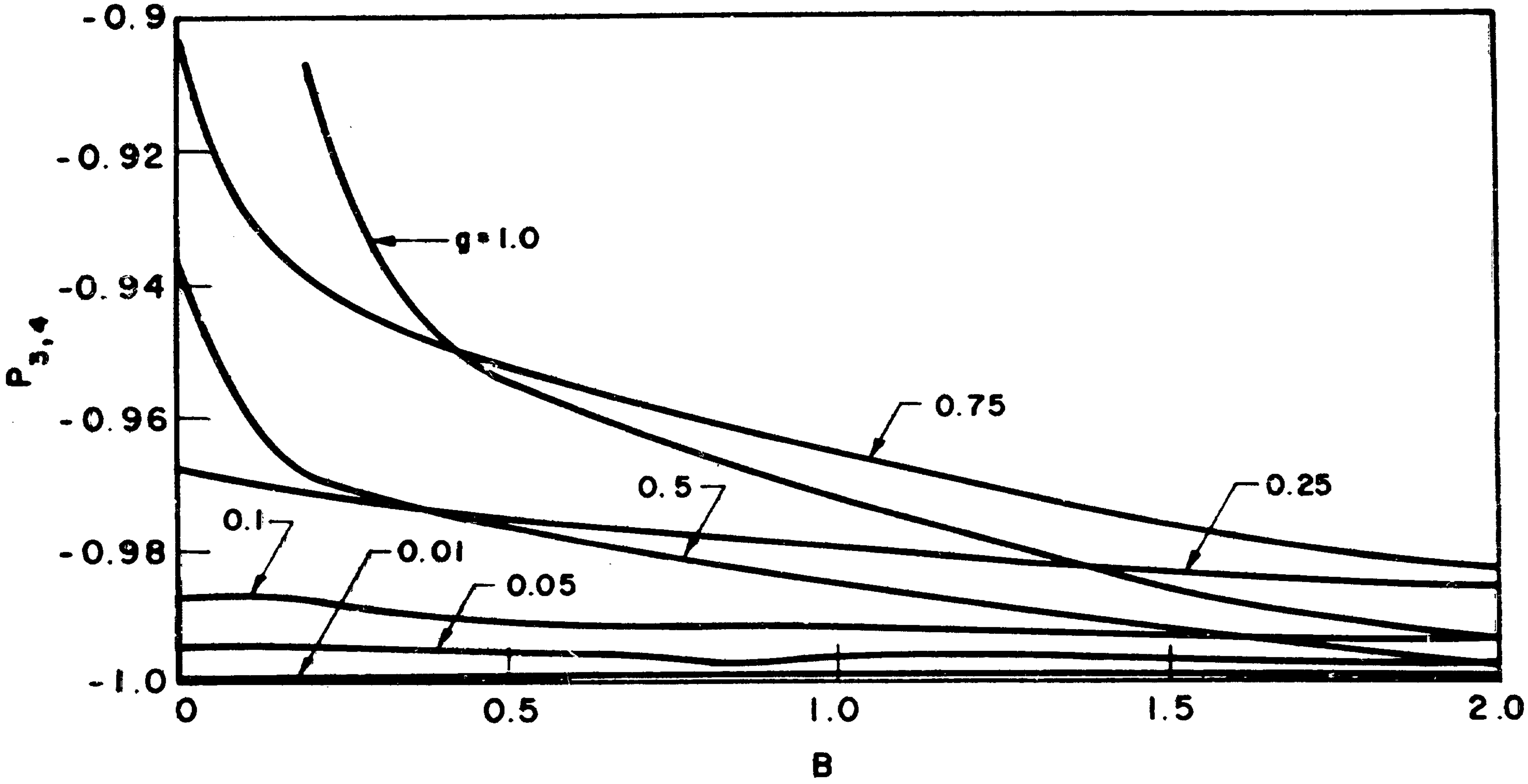

FIG. 13b PLOT OF THE PHASE FACTOR p_i CORRESPONDING TO THE MAXIMUM a VS. B IN THE "PERMITTED" R_U-REGION WITH g AS A PARAMETER.

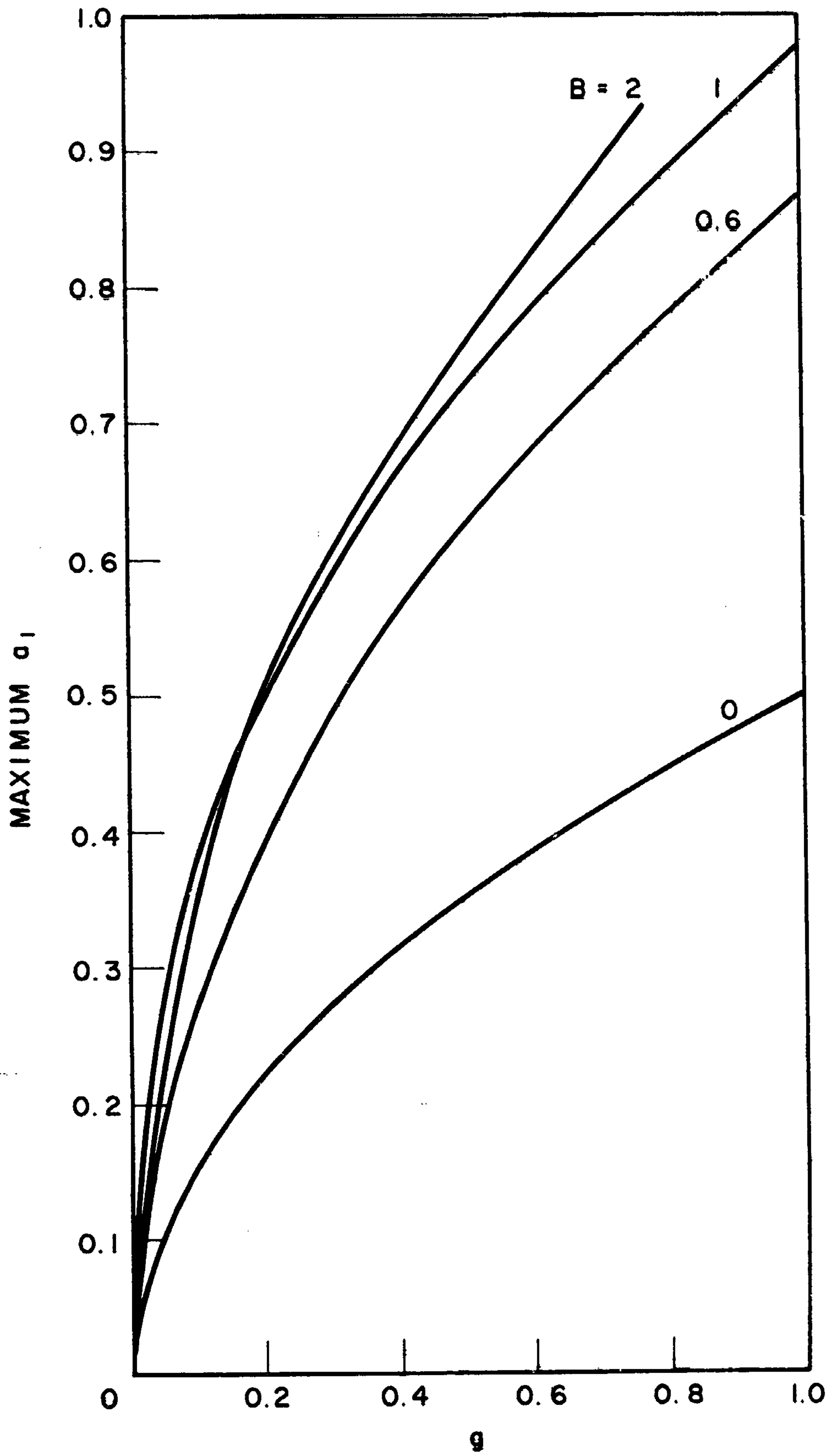

FIG. 14a PLOT OF THE MAXIMUM VALUE OF THE AMPLITUDE FACTOR a VS. g IN THE "PERMITTED" R_L-REGION WITH B AS A PARAMETER.

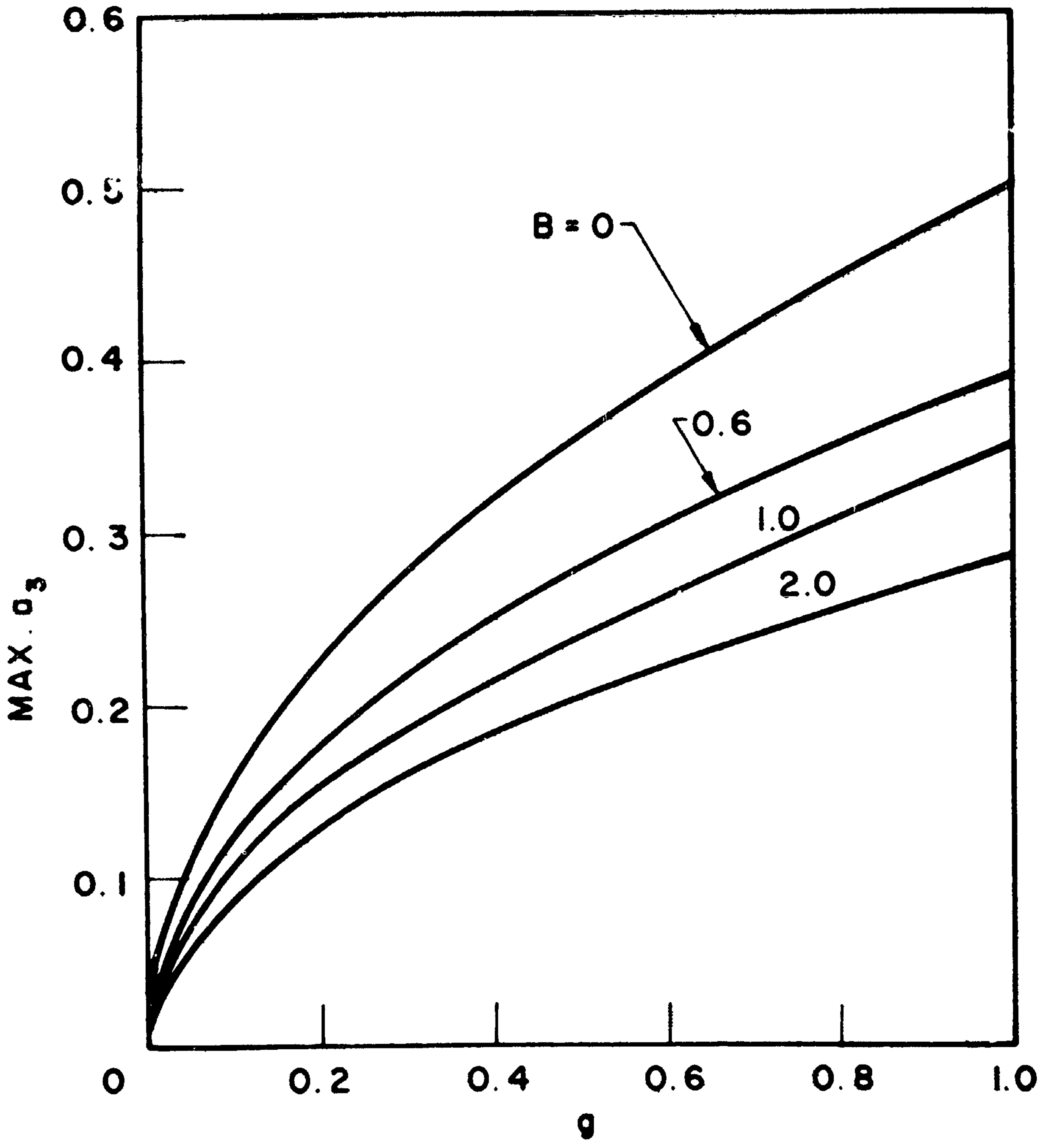

FIG. 14b PLOT OF THE MAXIMUM VALUE OF THE AMPLITUDE FACTOR a VS. g IN THE "PERMITTED" R_U-REGION WITH B AS A PARAMETER.

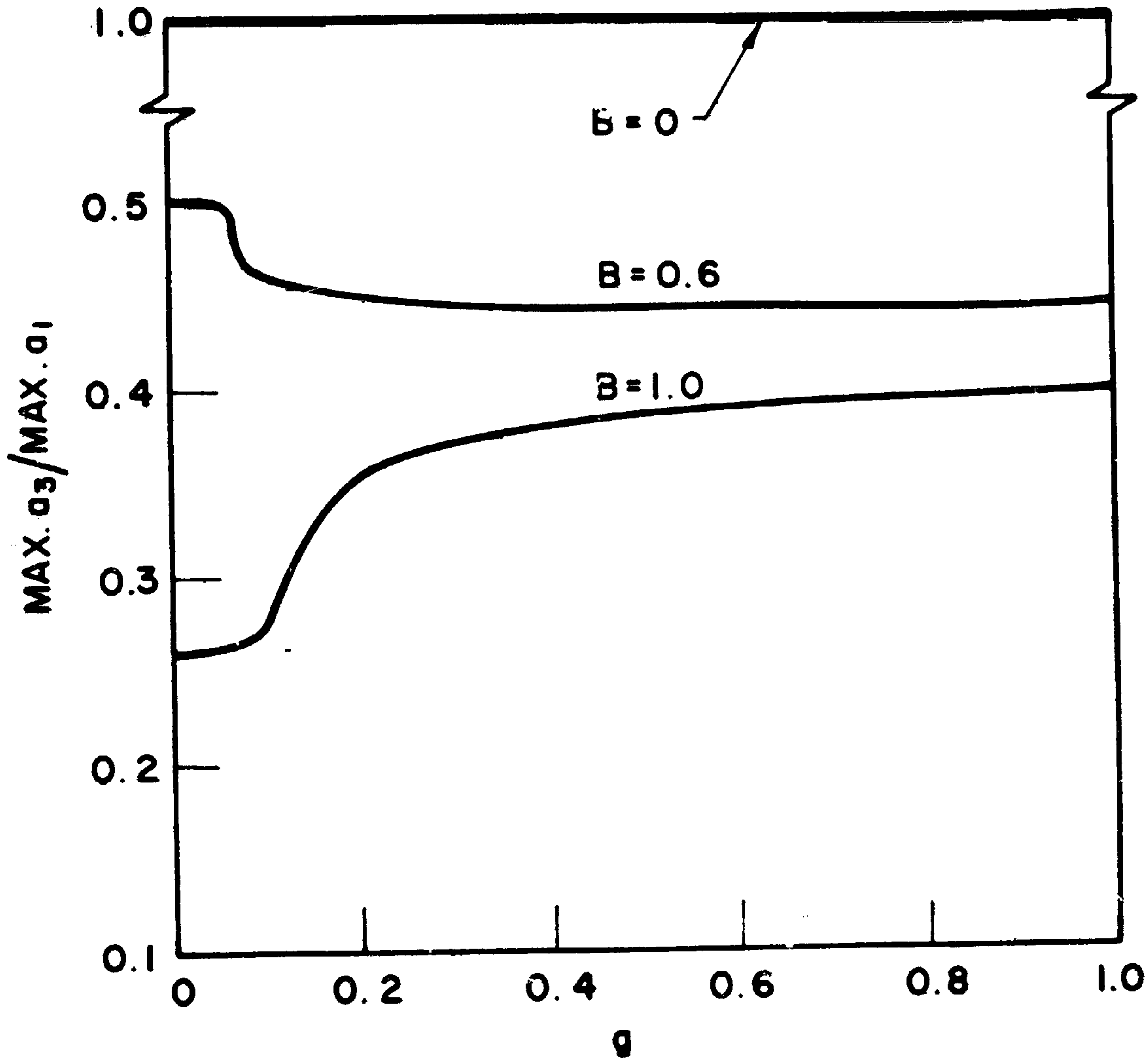

FIG. 14c PLOT OF (max a_3/max a_1) VS. g WITH B AS A PARAMETER. max a_1 AND max a_3 DENOTE THE MAXIMUM VALUES OF THE AMPLITUDE FACTOR IN THE "PERMITTED" R_L- AND R_U-REGIONS RESPECTIVELY.

that since the coupling parameter g is defined by a factor $(2C^3B)$ for a given value of B, Fig. 14c can easily be used to obtain the plot of the ratio (max a_3/max a_1) vs. C. It is of interest to observe that Figs. 13a and 13b suggest that in the "permitted" R_L-region the amplified wave is a forward-propagating one since $p > 0$, whereas in the "permitted" R_U-region the amplified wave is a backward-propagating one since $p < 0$. Furthermore since $p = (v_o/v_{ph})$ where v_{ph} is the phase velocity of the amplified wave and v_o is the phase velocity of the cold-circuit wave, Fig. 13b suggests that in the "permitted" R_U-region the amplified wave is propagating at a speed faster than that of the cold-circuit wave. Figure 15a shows the relationship between B and Ω for a given value of g which gives the maximum value of the amplitude factor a in the "permitted" R_U-region. It should be noted that for $B \simeq 0$, regardless of the value of g, the amplitude factor a has its maximum value at $\Omega \simeq 1$. Furthermore, for $B > 0.4$ with a given value of g, in order to obtain the maximum value for a, Ω must depend linearly upon B. For example, for $g = 0.1$ this linear relationship can be expressed approximately as $\Omega = (B+1)$. On the other hand, since $\Omega = \Omega_o B = (\omega_p/\omega)B$, this relationship can also be given by $[(\omega_p/\omega)-1]B = 1$. When it is plotted in the B-Ω_o plane it forms a hyperbola as shown in Fig. 15b. This figure suggests that for given values of g and B, in order to maintain the maximum amplitude factor a in the R_U-region, the value of (ω_p/ω) must be properly chosen, e.g., for $g = 0.1$ and $B = 1$, (ω_p/ω) must be equal to two. Furthermore for a given value of g, if B is decreased, then the gain parameter C is increased and to maintain the maximum amplification (ω_p/ω) must be increased accordingly, which is reasonable.

The plots of p and a against B of the complex conjugate pair propagation parameter for $\Omega = 0.025$ with g as parameter are shown in Fig. 16. This figure suggests that an increase in the value of g has a

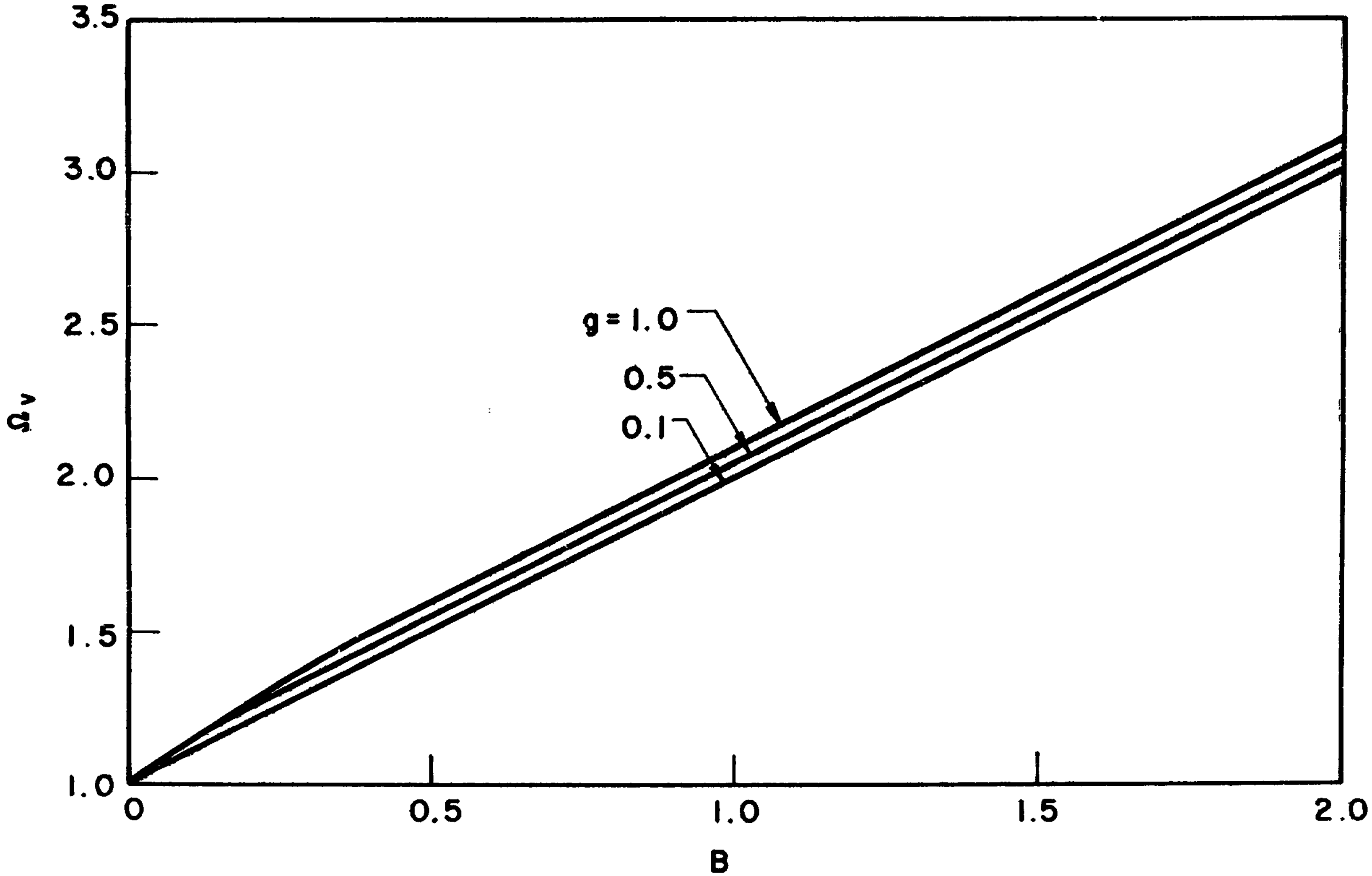

FIG. 15a PLOT OF Ω VS. B FOR THE MAXIMUM VALUE OF THE AMPLITUDE FACTOR a IN THE "PERMITTED" R_U-REGION WITH g AS A PARAMETER.

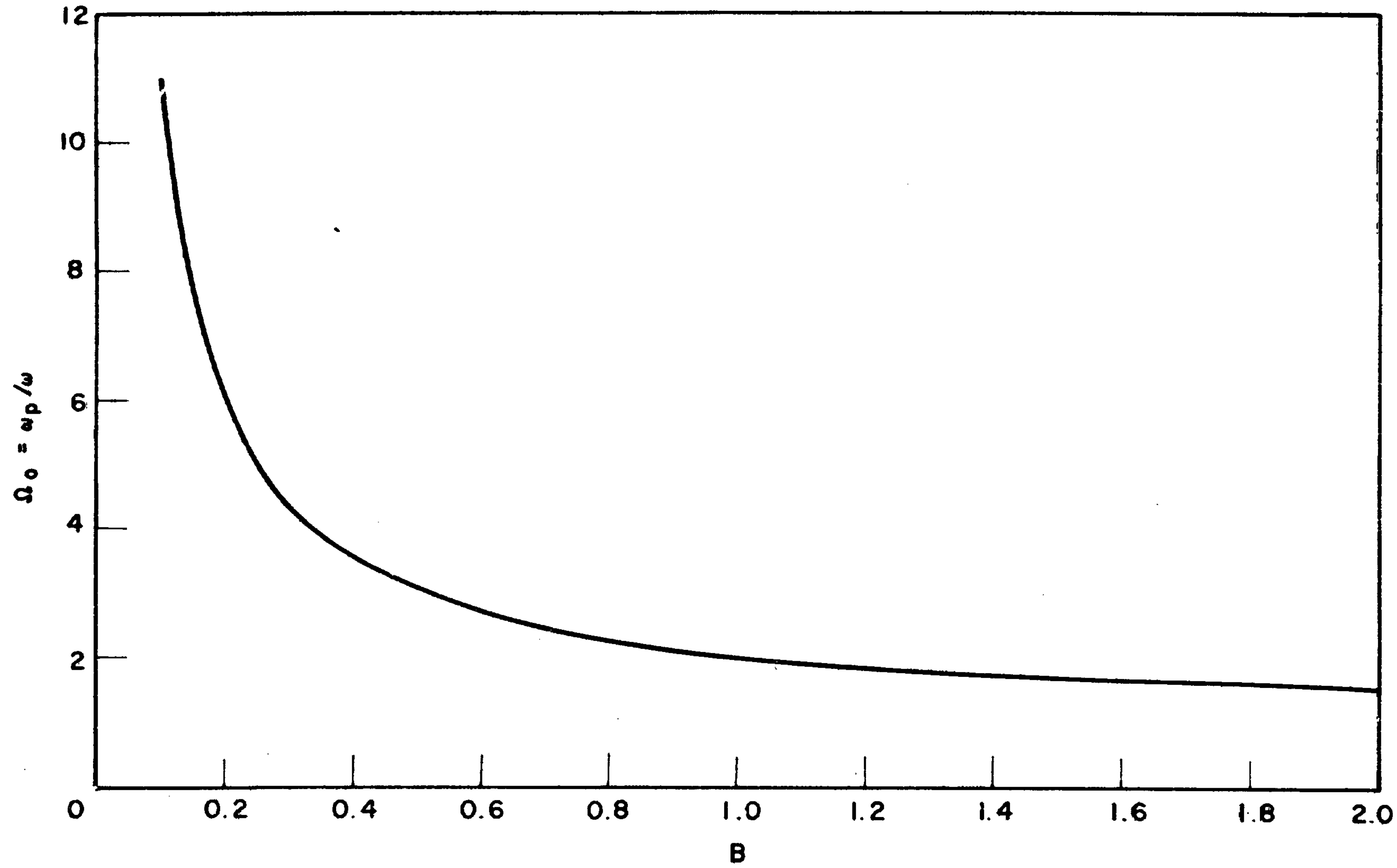

FIG. 15b PLOT OF $\Omega_o = (\omega_p/\omega)$ VS. B FOR THE MAXIMUM VALUE OF THE AMPLITUDE FACTOR a IN THE "PERMITTED" R_U-REGION WITH g = 0.1.

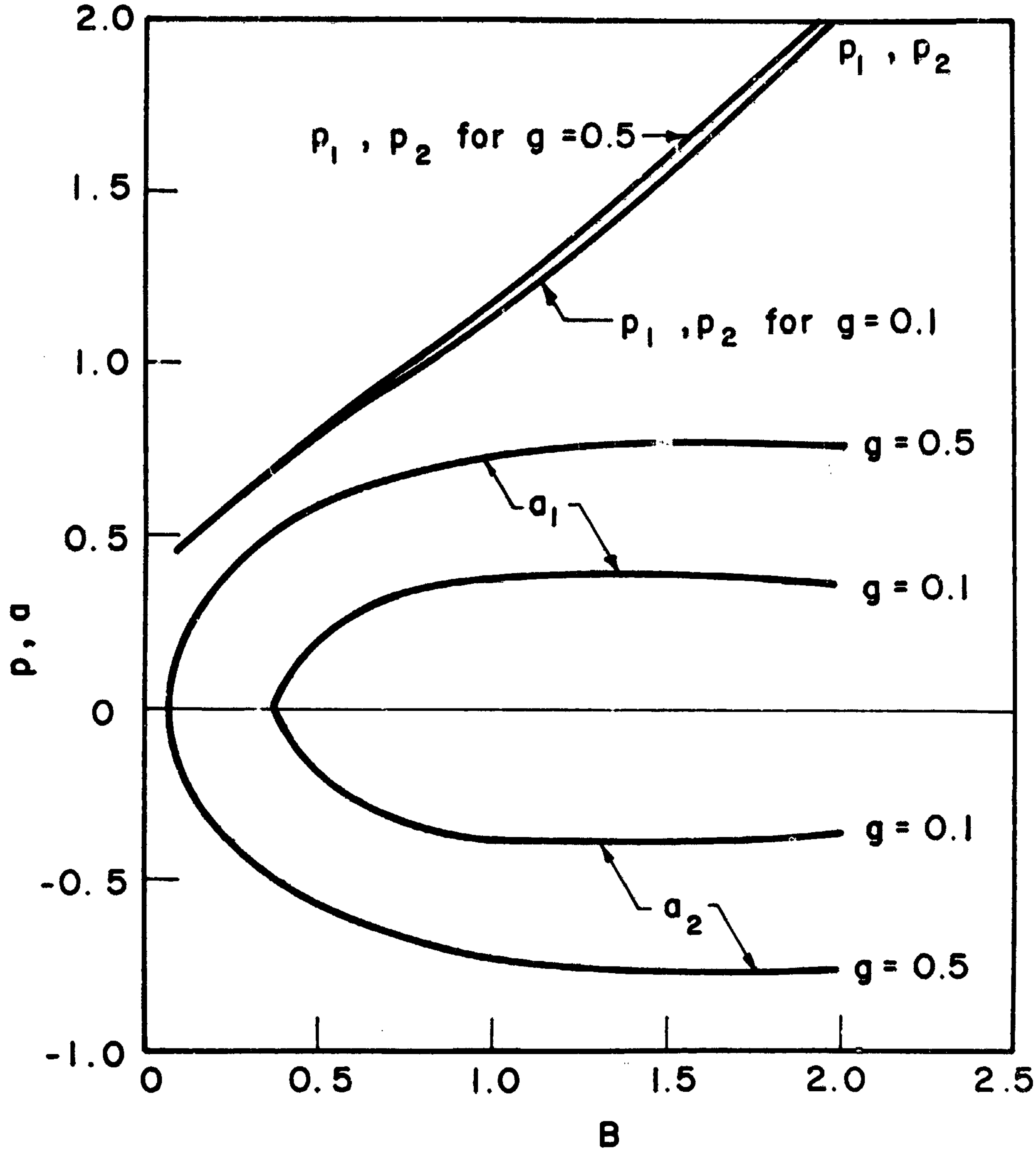

FIG. 16 PLOT OF p's AND a's VS. B AT Ω = 0.025 FOR THE COMPLEX CONJUGATE PAIR OF PROPAGATION PARAMETERS WITH g AS A PARAMETER.

considerable effect on the amplitude factor a, but only has a minor effect on the phase factor p of the amplified wave.

Finally the plots of p and a vs. B for g = 0.1 at Ω = 0.025 which is in the R_L-region, and at Ω = 2.0 which is in the R_U-region are shown in Figs. 17a and 17b respectively. The observation of these figures suggests that the amplifying range of value of B is much narrower in the R_U-region than in the R_L-region.

V. CONCLUDING REMARKS

In the present paper the "forbidden" and "permitted" regions in the B-Ω-g space for wave amplification are determined by examining the real roots of the determinantal equation of the system. The "permitted" region thus obtained represents all possible combinations of the system parameters B, Ω and g under which the amplification of a wave is possible. The conditions given by the inequalities (48) through (51), therefore, can be regarded as the necessary conditions for wave amplification. However it should be pointed out that these conditions are only the <u>necessary</u> conditions but not the <u>necessary</u> <u>and</u> <u>sufficient</u> conditions because the existence of a pair of complex conjugate roots of the determinantal equation merely implies the possibility of amplification and attenuation of the wave in the system. In order to know whether or not the resultant electromagnetic wave in the system will be amplified, the way in which the wave is excited and the boundary conditions which are to be imposed must be known.

The result of the present investigation indicates that wave amplification is possible even if the value of $\Omega_o = (\omega_p/\omega)$ is not small compared with unity, provided that the value of B is in the proper range, which

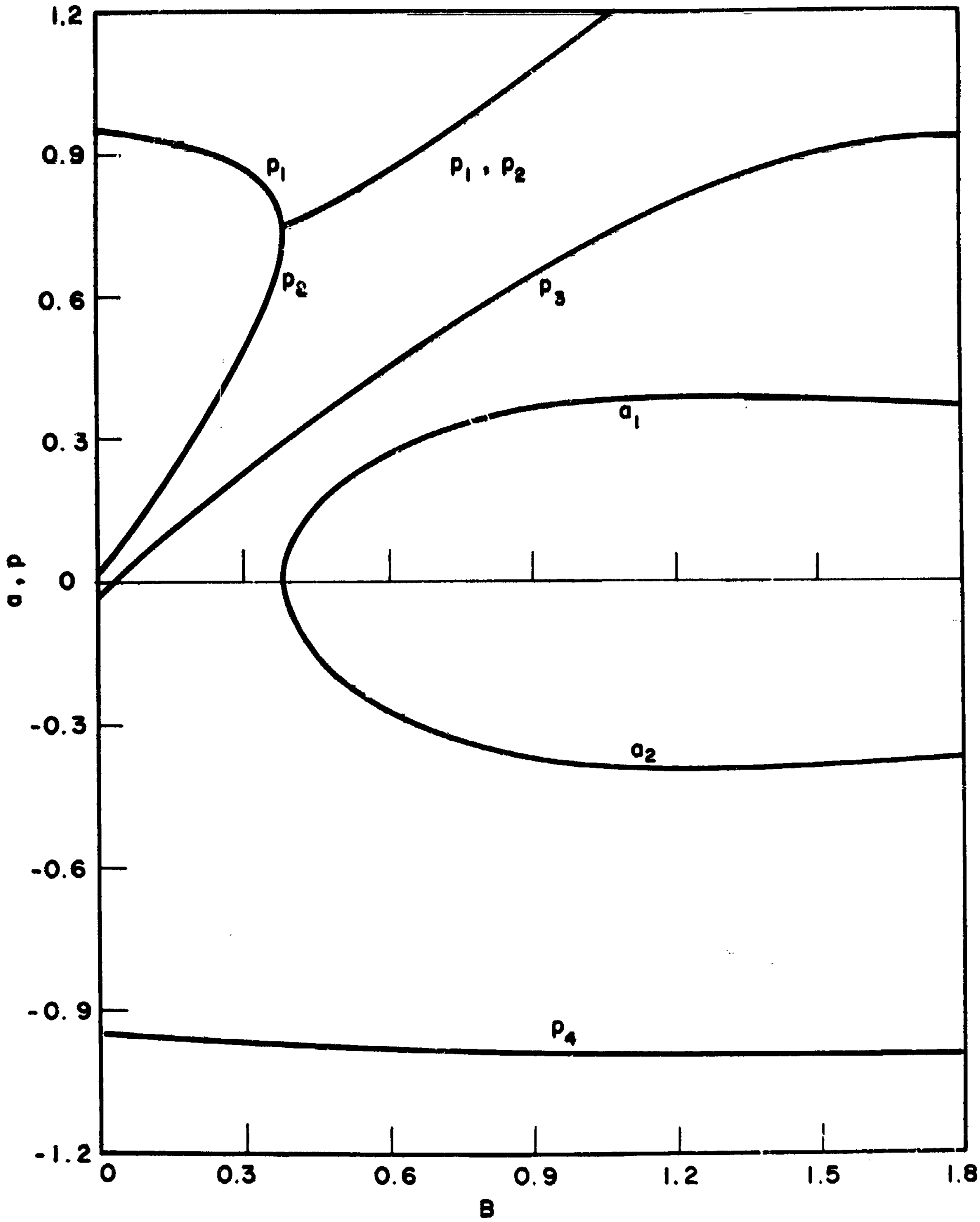

FIG. 17a PLOT OF p's AND a's VS. B AT $\Omega = 0.025$ FOR $g = 0.1$.

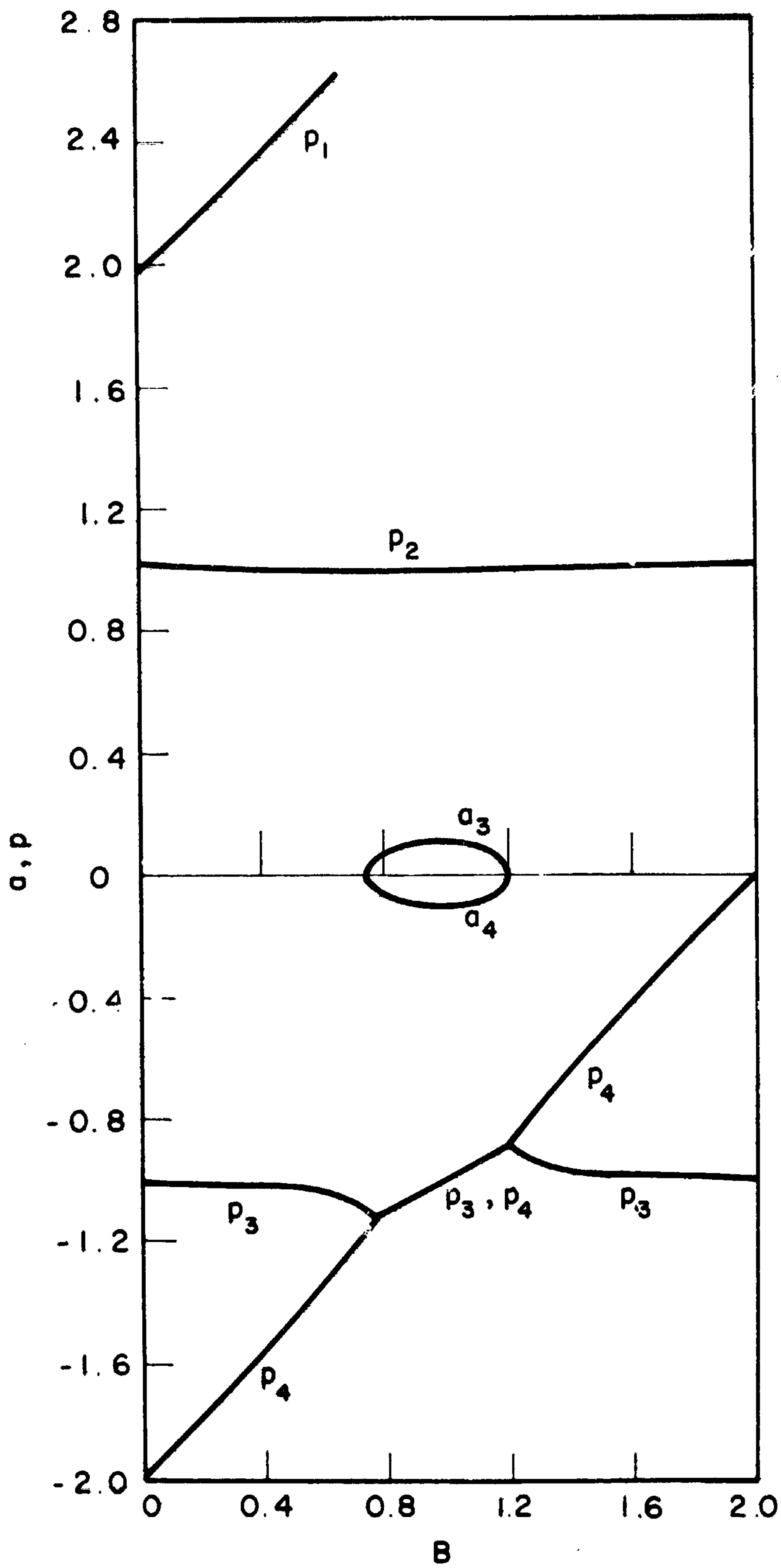

FIG. 17b PLOT OF p's AND a's VS. B AT Ω = 2.0 FOR g = 0.1.

depends upon the value of parameter g. Thus it tends to suggest the applicability of the TWT theory to the investigation of natural phenomena. It should, however, be pointed out that the synchronous condition $B \sim 1$ does not automatically imply wave amplification as is illustrated in Fig. 9. In order to have wave amplification the value of Ω must also lie in the proper range. For example, the point in the B-Ω-g space which represents the operating condition $B = 1$, $\Omega \ll 1$ and $g = 0.1$ lies within the "permitted" region, while that representing the condition $B = 1$, $\Omega \gg 1$ and $g = 0.1$ lies in the "forbidden" region where wave amplification is not possible.

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