December 1965 M-77-65-9

Technical Report

JUNCTURE STRESS FIELDS IN MULTICELLULAR SHELL STRUCTURES

Final Report

Nine Volumes

Vol. IX Summary of Results and Recommendations

by

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FOREWORD

This report summarizes the results of the second phase of studies on juncture stress fields in multicellular shell structures and provides recommendations for possible future studies on the same subject. The work described in this report was performed by staff members of Lockheed Missiles and Space Company in cooperation with the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration under Contract NAS 8-11480. Contract technical representative was H. Coldwater.

This volume is the last of a nine-volume final report of studies conducted by the department of Solid Mechanics, Aerospace Sciences Laboratory, Lockheed Missiles and Space Company, under the supervision of K. J. Forsberg. The project was under the technical direction of E. Y. W. Tsui with associates B. O. Almroth, F. A. Brogan, C. T. Chen, J. M. Massard, E. V. Pittner, L. H. Sobel and P. Stern.

The titles of the nine volumes of the final report are:

- Vol. I — Numerical Methods of Solving Large Matrices, M-77-65-5, August 1965.
- Vol. II — Stresses and Deformations of Fixed-Edge Segmental Cylindrical Shells, M-77-65-1, June 1965.
- Vol. III — Stresses and Deformations of Fixed-Edge Segmental Conical Shells, M-77-65-2, June 1965.
- Vol. IV — Stresses and Deformations of Fixed-Edge Segmental Spherical Shells, M-77-65-4, July 1965.
- Vol. V — Influence Coefficients of Segmental Shells, M-77-65-5, August 1965.
- Vol. VI — Analysis of Multicellular Propellant Pressure Vessels by the Stiffness Method, M-77-65-7, November 1965.
- Vol. VII — Buckling Analysis of Segmental Orthotropic Cylinders under Uniform Stress Distribution, M-77-65-3, July 1965

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Section 1
INTRODUCTION

1.1 Background

In order to achieve the necessary reliability of multicellular propellant containers for application as large boosters, it is essential to fully clarify and reduce to theory the associated juncture stress fields peculiar to such shell structures. As a result of the first phase of research studies on this subject carried out by staff members of LMSC under Contract NAS 8-11079, a linear theory has been established for the prediction of static response of the shell structures under consideration*. Since an analytic solution is not available for this type of problem, a numerical procedure was formulated for use with high-speed digital computers. The feasibility of the selected numerical technique was demonstrated after a direct method of solving large systems of simultaneous equations was satisfactorily developed during the previous investigation*.

In spite of these achievements, however, two basic problems still existed insofar as the capability of performing the optimization of such structures is concerned. The first problem was the development of digital programs to generate stresses and deformations under loads and thermal gradients, and the second was to develop analyses and associated digital programs to predict buckling loads of orthotropic cylindrical panels under specific loadings and appropriate boundary conditions.

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*Tsui, E. Y. W., et al., "Investigation of Juncture Stress Fields in Multicellular Shell Structures," LMSC M-03-63-1 (NASA CR-61050), Feb. 1964.

1.2 Scope of work

In essence, the work involved in the present phase of the research studies on juncture stress fields in multicellular shell structures was designed to yield solutions to both the bending and buckling problems just mentioned. Specifically, the objectives of work reported here can be stated, as follows:

I The Bending Problem

Develop digital computer programs for the specific analytical approach formulated under NAS 8-11079 predicting membrane and discontinuity fields as well as deflections throughout a specific type of shell with junctures peculiar to multicellular structures. These programs should be capable of predicting stresses and deformations of the specific structures using isotropic bulkheads and orthotropic radial webs and cylindrical segments. The developed programs should include:

(1) Detail description of the programs.

(2) Numerical examples of sample problems, using the analytical procedure and the method of solving large sets of simultaneous equations developed by Lockheed, especially in the process of evaluating the influence coefficients of shell segments as well as satisfying the compatibility conditions along shell junctures. This work should also include the optimization of mesh size and total computer time. The resulting method of analysis will be limited to the elastic theory as applied to thin shell structures, and accounting for variations in physical properties of materials and specific range of snell geometry. Thermal stresses due to temperature gradients should also be included.

II The Buckling Problem

Perform the necessary theoretical investigations required to formulate theory and develop an analytical approach for prediction of the buckling loads of both isotropic and orthotropic cylindrical panels with displacements satisfying appropriate boundary conditions, and develop digital programs to generate design curves, preferably presented in a form of merit indices readily adaptable to optimum strength/weight design. This work will include literature search on the buckling analysis of cylindrical panels prior to the initiation of the theoretical investigation.

Section 2
SUMMARY OF RESULTS

2.1 The Bending Analysis

Results of the work carried out in the present studies which are directly related to the bending of the elements as well as the entire multicellular shell structure are described in Vols. I through VI of the final report. These results are summarized briefly below.

Volume I presents two basic numerical techniques for solving large systems of algebraic simultaneous equations resulting from the finite-difference approximation of the partial differential equations of thin elastic shells. Of the methods available for solving such systems of equations, the matrix factorization and two-line successive over-relaxation methods are discussed in detail. It is found that the direct methods generally require more computer running time than the iterative methods, especially when the size of the matrix is large. However, the direct methods permit rapidly varying mesh spacing which is desirable for the accurate determination of the boundary-layer bending behavior of shell elements involved in multicellular pressure vessels. Besides, the direct method as developed is capable of solving up to six thousand equations using the IBM 7094 and Fortran II Version II language.

Volume II presents a set of basic equations for both isotropic and orthotropic thin elastic cylindrical shells and a digital program which provides solutions for fixed-edge stiffened and unstiffened cylindrical panels under loads and change of temperature. The method of solution consists basically of obtaining the deformations (u, v, w) at various discrete stations of the structure by finite-difference approximation. The corresponding stress resultants, strains and stresses may then be computed. The following options are available in the program:

1. Construction
    (a) Isotropic
    (b) Orthotropic
2. Finite-Difference Mesh
    (a) Uniform spacing
    (b) Graded spacing in the x-direction
    (c) Symmetry in the x-direction
3. Loading Conditions
    (a) Uniform normal pressure
    (b) Hydro-static pressure
    (c) Linear temperature gradient through the shell thickness

The program is designed to compute not only the fixed-edge forces due to loads or thermal gradients, but also the displacements, stress resultants and strains in the loaded region of the panel. The finite-difference mesh network is specified completely by prescribing the number of rows (n) and columns (m) exclusive of the boundaries. The maximum number of rows and columns is 24 and 80 respectively. Thus, the maximum of $3 \times n \times m = 5760$ unknowns can be solved since there are three displacement components in each station. Output can be tabulated and plotted simultaneously. A numerical example (Fig. 12) showing the deformations and stress resultants of an orthotropic panel under uniform pressure is also presented (see Table 4 and Fig. 13). Figure 1 shows the variations of the fixed-edge forces along the boundaries of the same panel under uniform pressure.

It should be noted that the tabulated or plotted output quantities are non-dimensional. These quantities can be related to the applied loads or thermal gradients, physical and geometrical properties of the panel in the following way:

a. For panels under Uniform Pressure (p_z):

$$\begin{aligned}
\hat{u} &= u(p_z\ R^2/Eh) \\
\hat{v} &= v(p_z\ R^2/Eh) \\
\hat{w} &= w(p_z\ R^2/Eh) \\
\hat{N}_\eta &= NTAN(p_z R) \\
\hat{N}_\zeta &= NNORM(p_z R) \\
\hat{Q} &= Q(p_z R) \\
\hat{M} &= M(p_z R^2)
\end{aligned} \qquad (1)$$

b. For panels under Hydrostatic Pressure ($\bar{p}_o + px$):

$$\begin{aligned}
\hat{u} &= u(R^2/Eh) \\
\hat{v} &= v(R^2/Eh) \\
\hat{w} &= w(R^2/Eh) \\
\hat{N}_\eta &= NTAN(R) \\
\hat{N}_\zeta &= NNORM(R) \\
\hat{Q} &= Q(R) \\
\hat{M} &= M(R^2)
\end{aligned} \qquad (2)$$

c. For panels under Linear Thermal Gradient through the thickness of shell:

$$\begin{aligned}
\hat{u} &= u(R) \\
\hat{v} &= v(R) \\
\hat{w} &= w(R) \\
\hat{N}_\eta &= NTAN(Eh) \\
\hat{N}_\zeta &= NNORM(Eh) \\
\hat{Q} &= Q(Eh) \\
\hat{M} &= M(EhR)
\end{aligned} \qquad (3)$$

EXAMPLE PROBLEM ORTHOTROPIC CYLINDER NORMAL PRESSURE

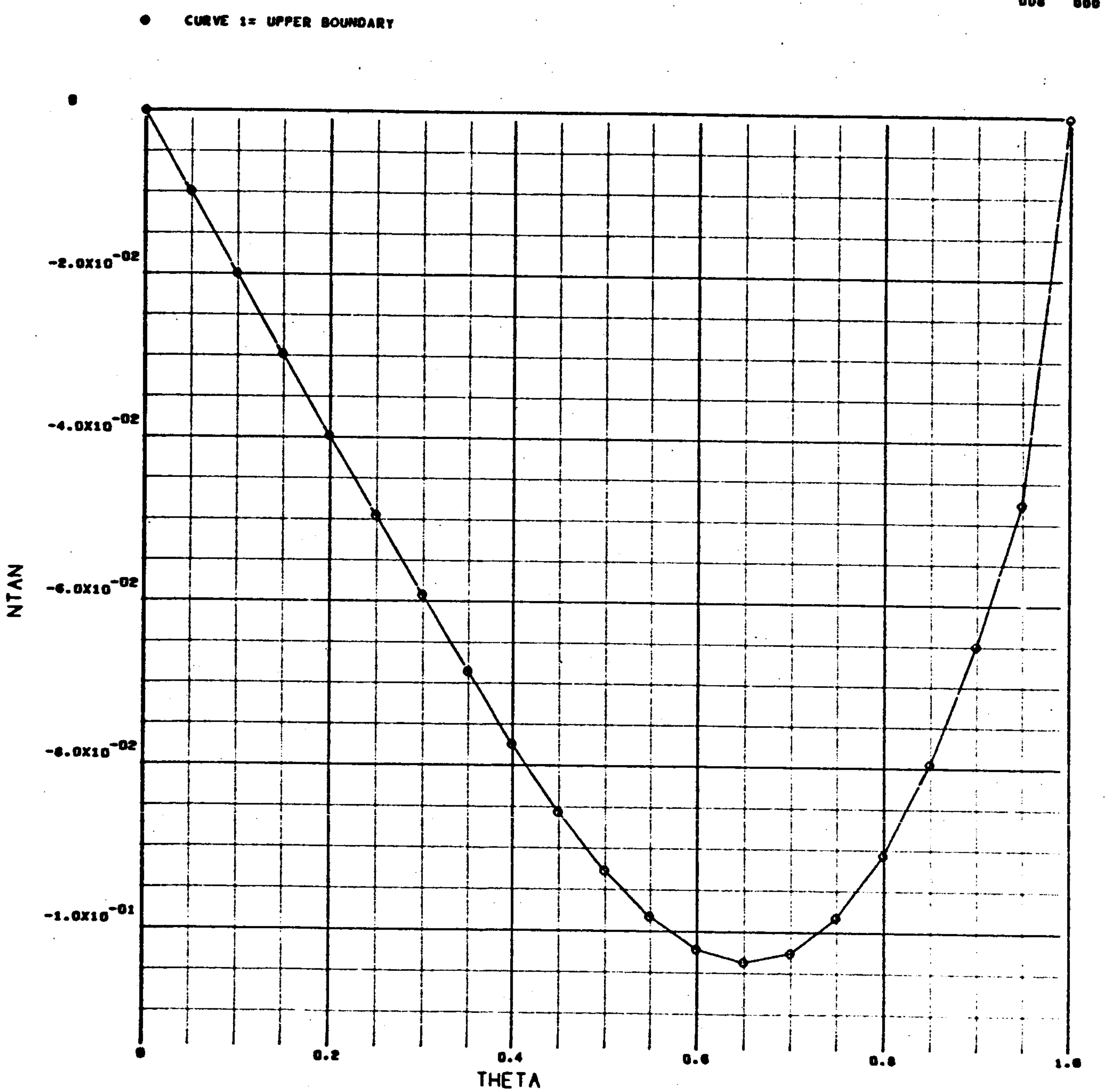

FIG. 1*a* BOUNDARY STRESS RESULTANTS

EXAMPLE PROBLEM ORTHOTROPIC CYLINDER NORMAL PRESSURE

SAN300 009 000

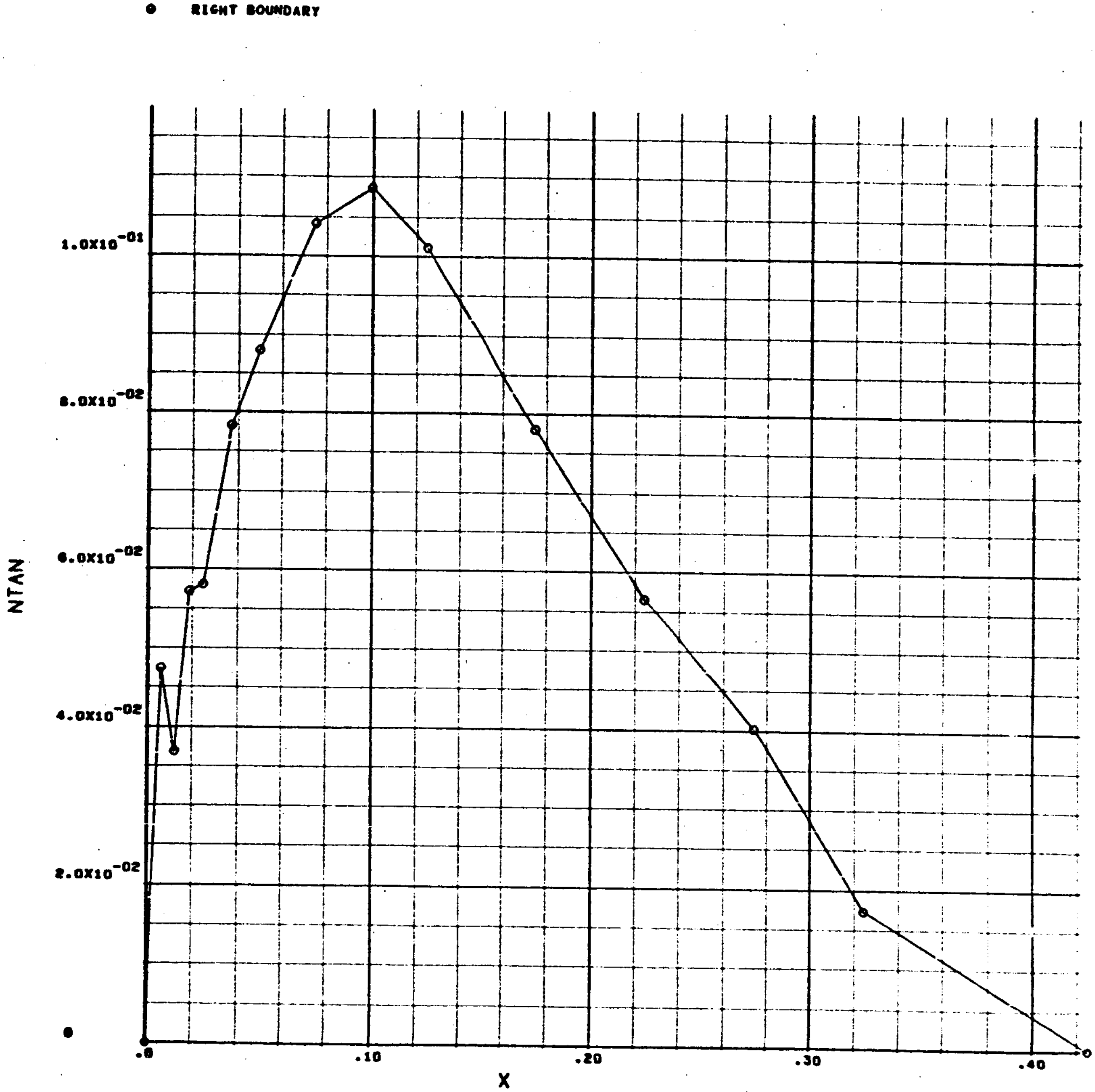

FIG. 1b BOUNDARY STRESS RESULTANTS

EXAMPLE PROBLEM ORTHOTROPIC CYLINDER NORMAL PRESSURE

BAM300
010 000

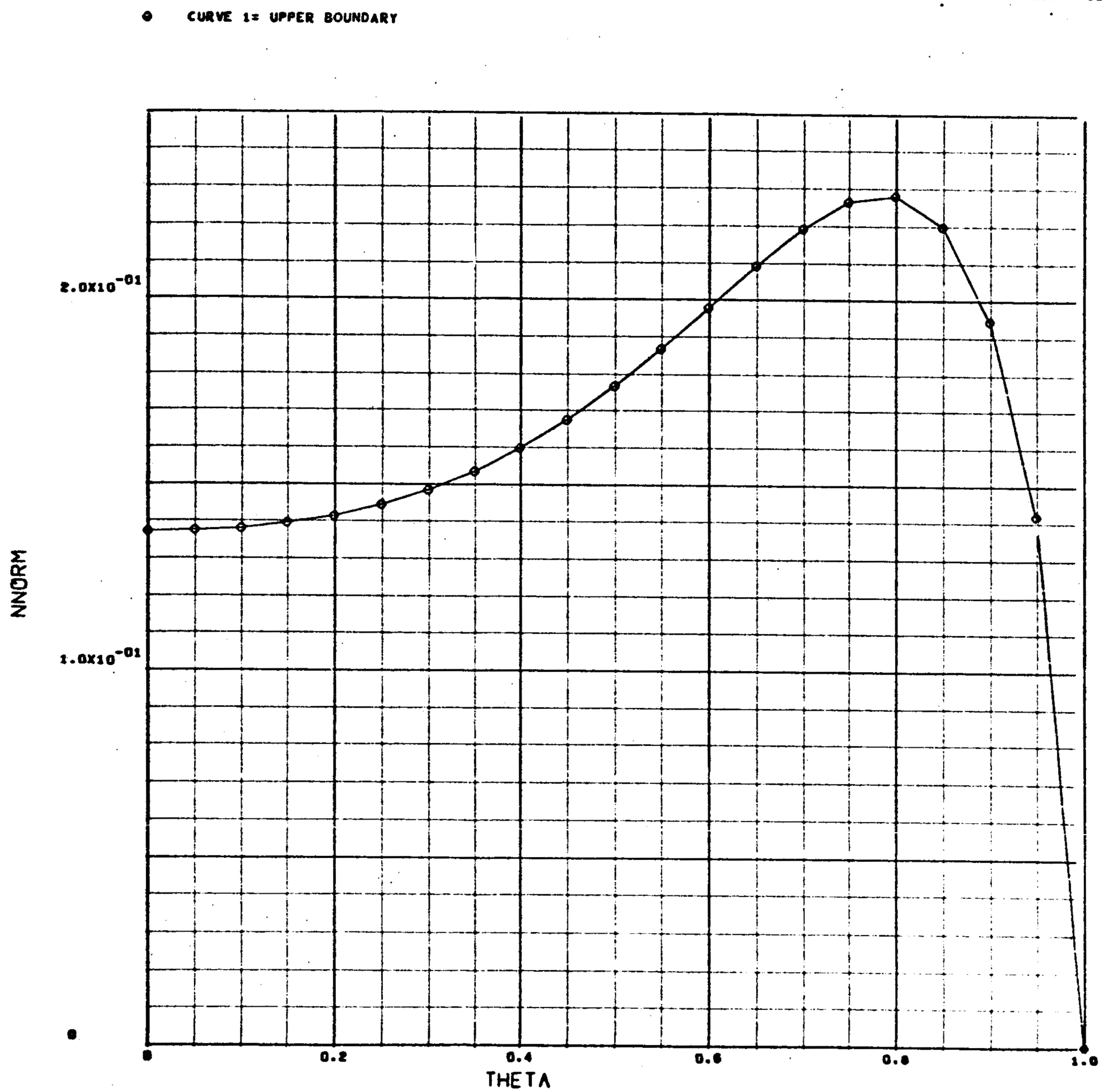

FIG. 1c BOUNDARY STRESS RESULTANTS

EXAMPLE PROBLEM ORTHOTROPIC CYLINDER NORMAL PRESSURE

SAN300
011 000

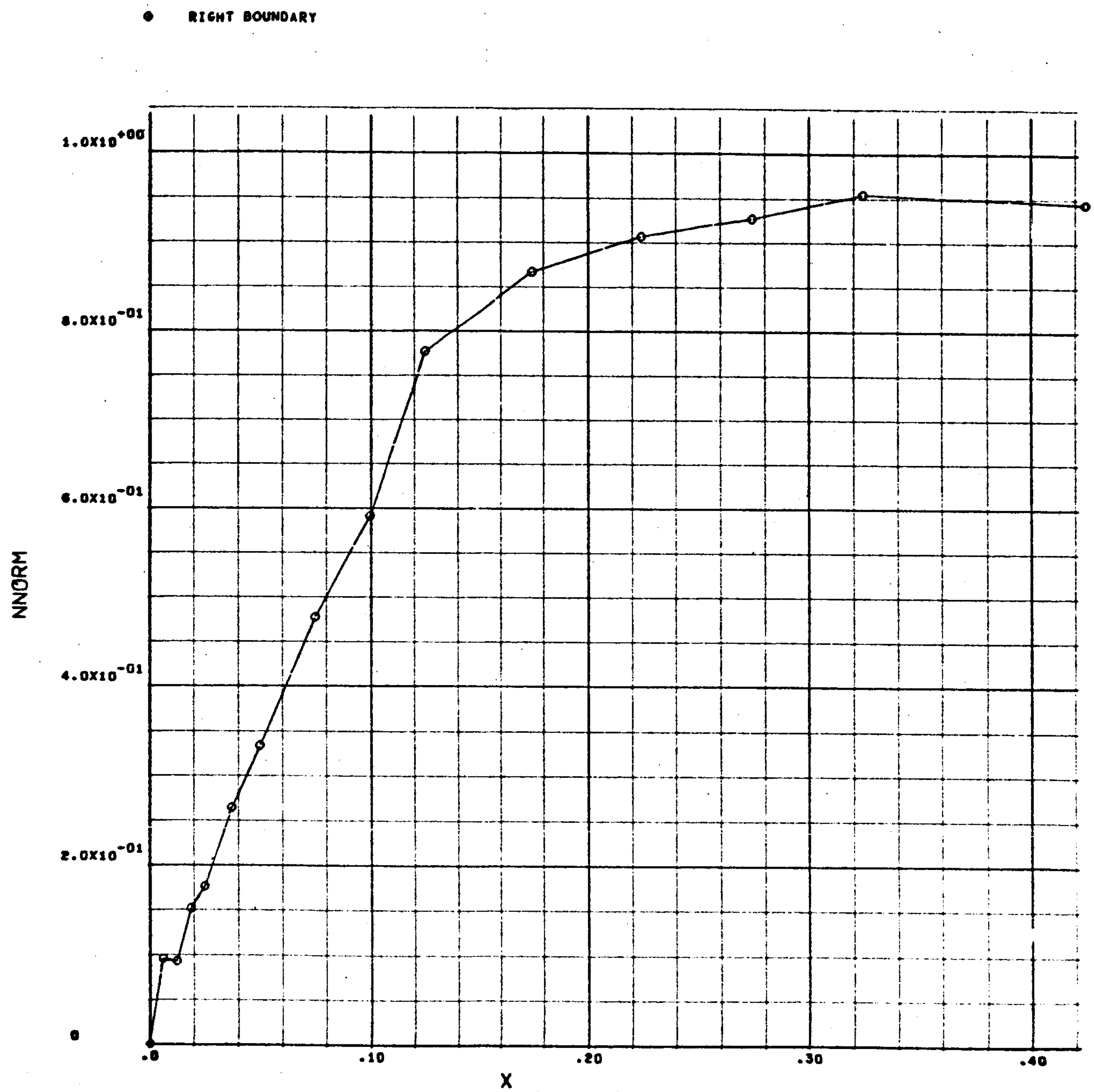

FIG. 1d BOUNDARY STRESS RESULTANTS

EXAMPLE PROBLEM ORTHOTROPIC CYLINDER NORMAL PRESSURE

BAH300
014 000

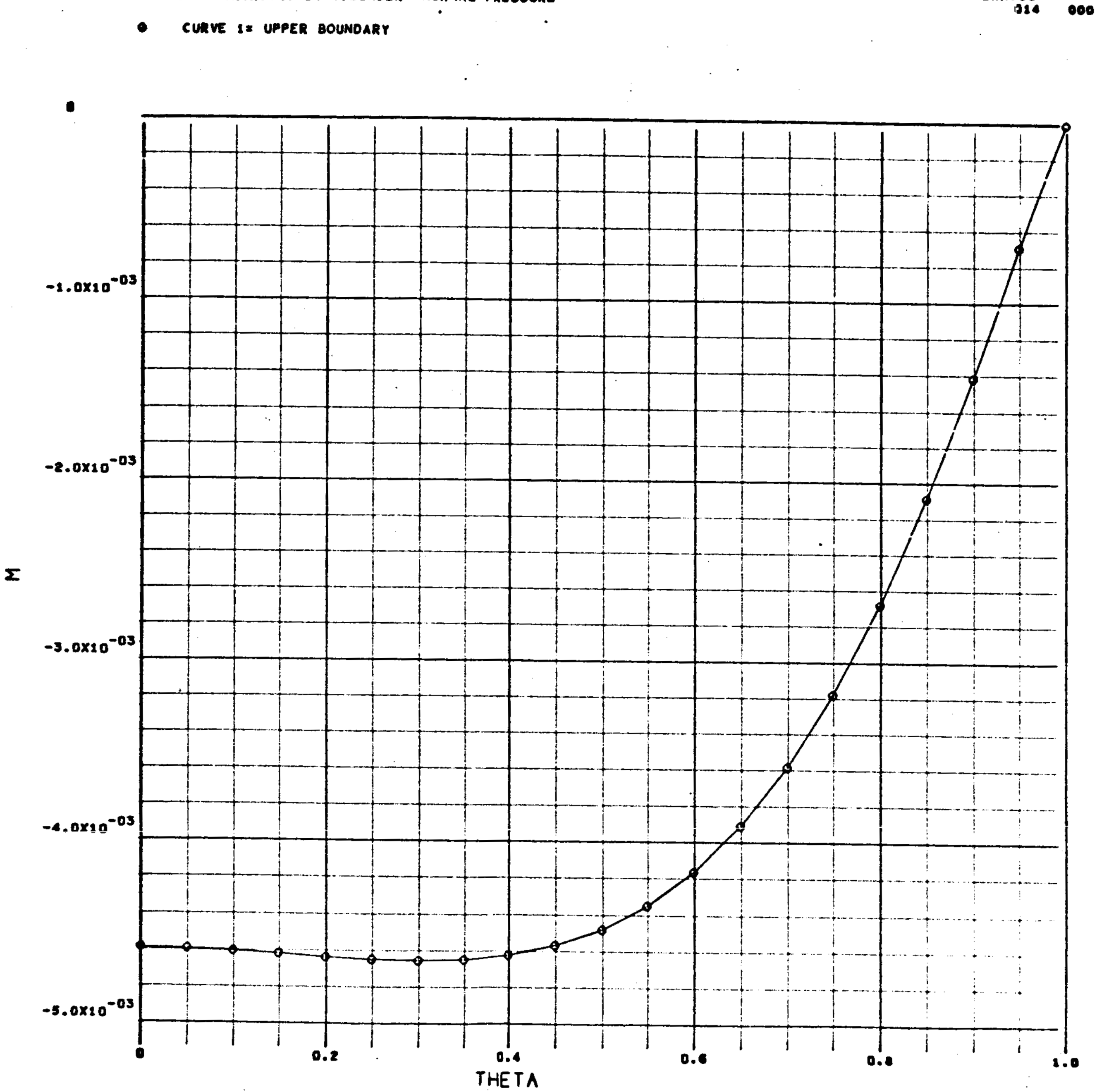

FIG. 1e BOUNDARY STRESS RESULTANTS

EXAMPLE PROBLEM ORTHOTROPIC CYLINDER NORMAL PRESSURE

● RIGHT BOUNDARY

BAN300
013 000

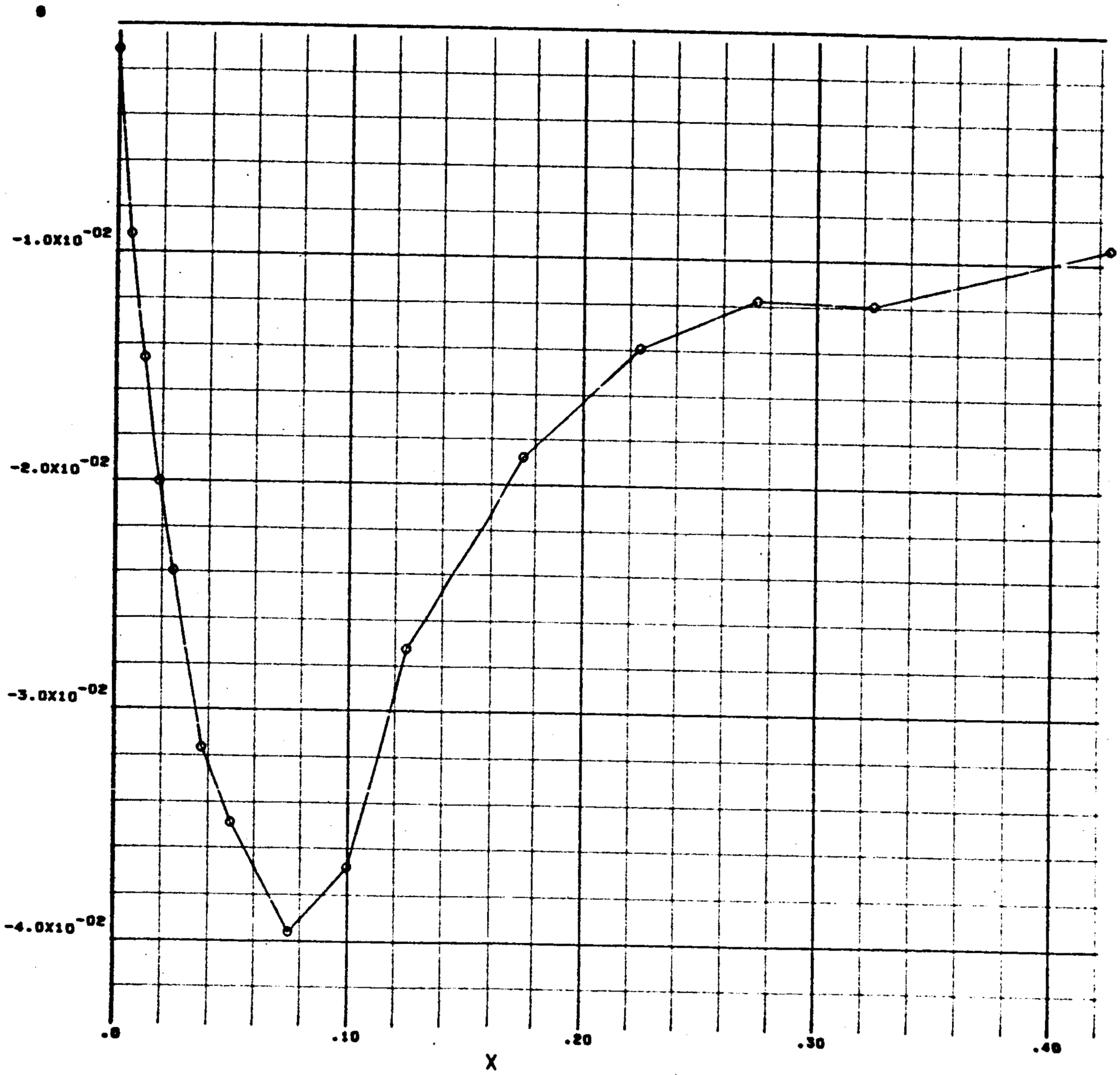

FIG. 1f BOUNDARY STRESS RESULTANTS

EXAMPLE PROBLEM ORTHOTROPIC CYLINDER NORMAL PRESSURE

BAM300 012 000

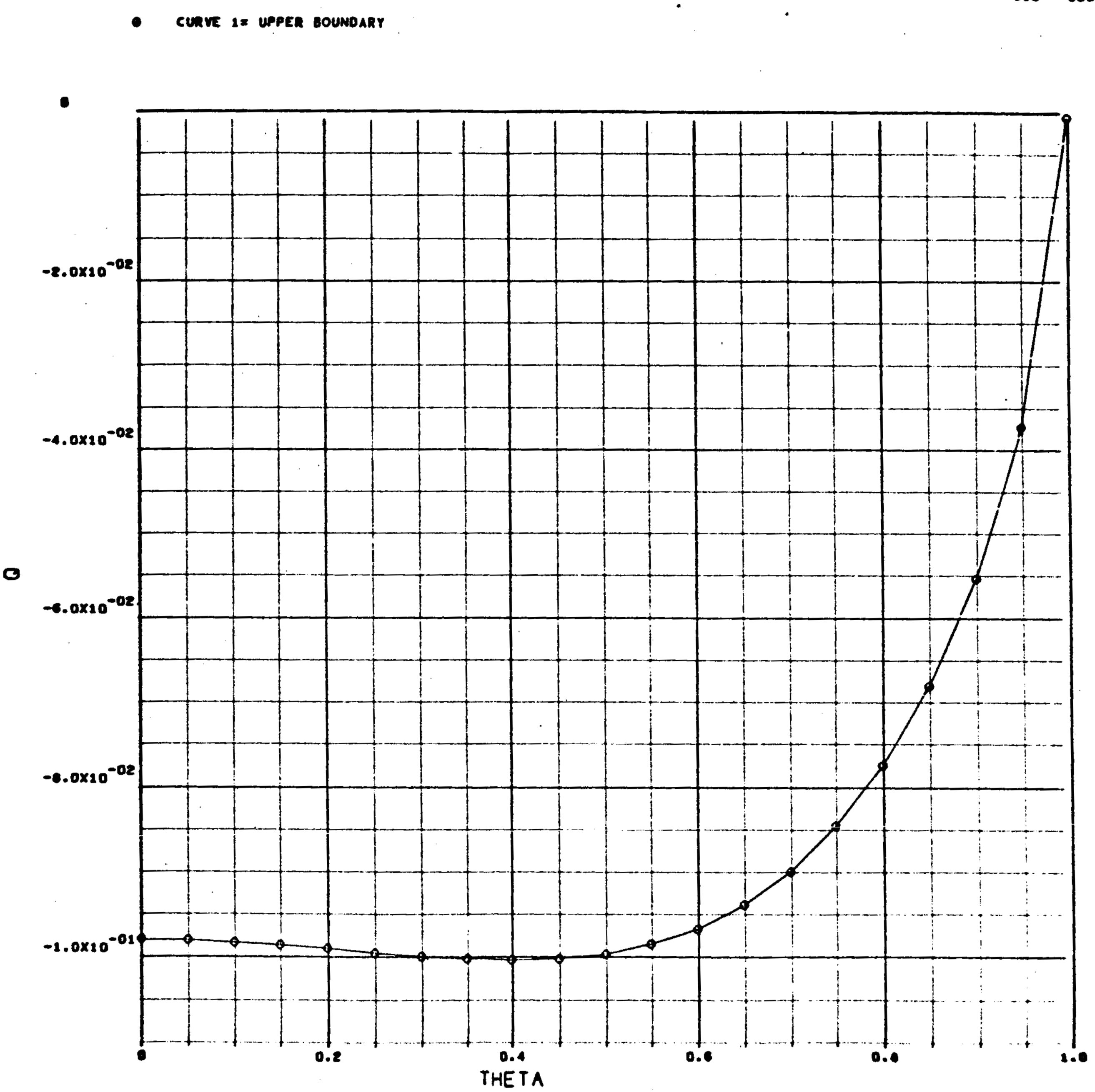

FIG. 1g BOUNDARY STRESS RESULTANTS

EXAMPLE PROBLEM ORTHOTROPIC CYLINDER NORMAL PRESSURE

BAN 00 J15 000

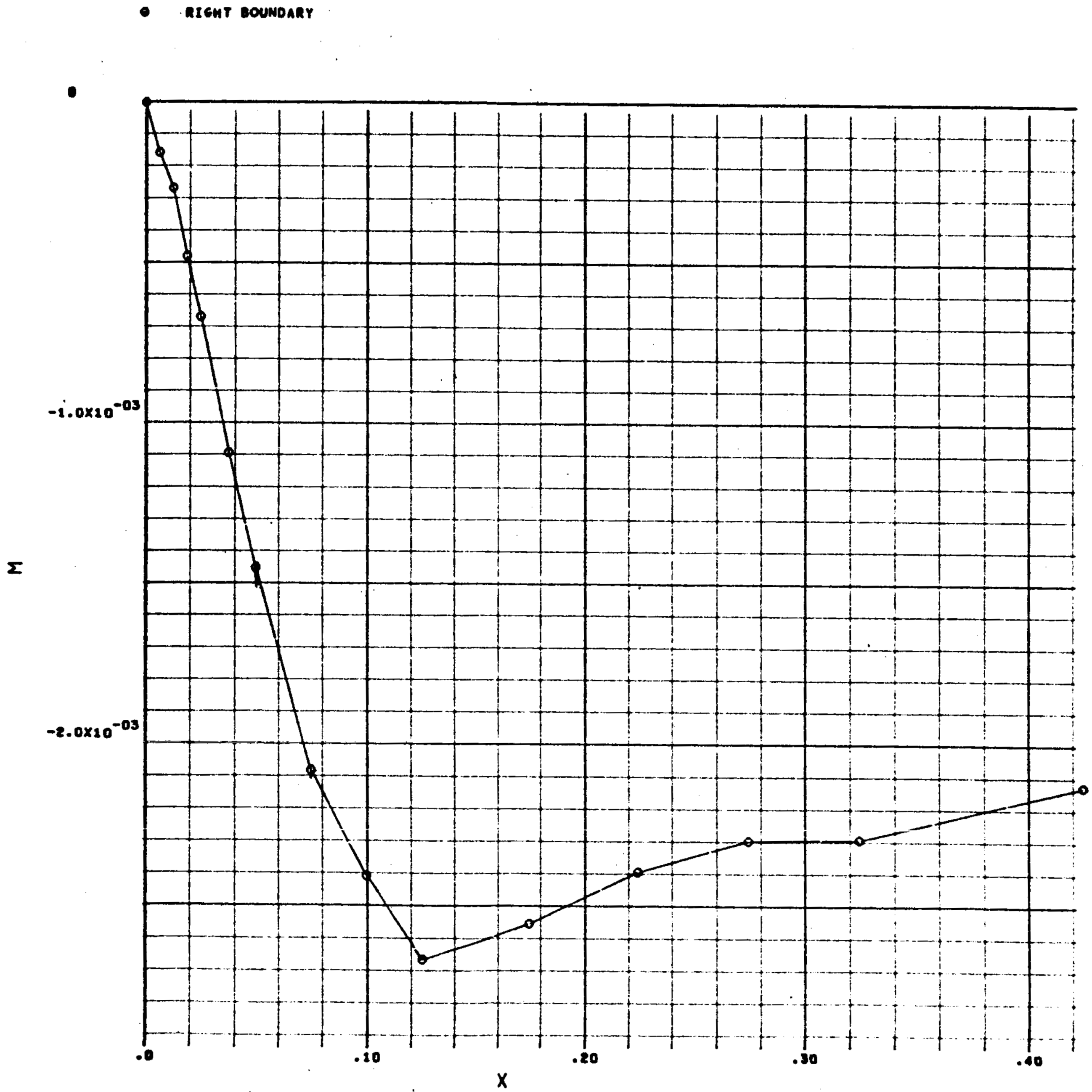

FIG. 1h BOUNDARY STRESS RESULTANTS

Volume III presents a set of basic equations for the isotropic shells with the wall thickness either uniform or varying linearly along the generator. A digital program is presented for the computation of the fixed-edge forces as well as displacements and strain in the loaded region due to intermediate loads or thermal gradients. The following program options are available:

1. Finite-difference mesh
   (a) Uniform spacing
   (b) Graded spacing along the generator
2. Loading Conditions
   (a) Uniform normal pressure
   (b) Gravitational loading
   (c) Linear temperature gradient through the thickness of shell.

There are no restrictions on the geometrical dimensions of panels. However, the accuracy with which the basic differential equations are approximated may vary for different configurations of the conical shell. A numerical solution showing a fixed-edge uniform thickness conical panel under uniform pressure is also included in this volume. The nondimensional quantities tabulated and plotted in Table 5 and Fig. 10 respectively can be related to the applied loads and the properties of the conical panel, as follows:

a. For panels under Uniform Pressure (p_z):

$$
\begin{aligned}
\hat{u} &= u[p_z X_L(1 - \nu^2)/E] \\
\hat{v} &= v[p_z X_L(1 - \nu^2)/E] \\
\hat{w} &= w[p_z X_L(1 - \nu^2)/E] \\
\hat{N}_\eta &= \text{NTAN}\ [h\ p_z(1 - \nu)/2] \\
\hat{N}_\zeta &= \text{NNORM}(p_z h) \\
\hat{Q} &= Q(h^3 p_z/12\ X_L^2) \\
\hat{M} &= M(h^3 p_z/12\ X_L)
\end{aligned}
\qquad (4)
$$

b. For panels under Gravitational Load:

$$\hat{u} = u[h_o\hat{\rho}\ X_L(1 - \nu^2)/E]$$

$$\hat{v} = v[h_o\hat{\rho}\ X_L(1 - \nu^2)/E]$$

$$\hat{w} = w[h_o\hat{\rho}\ X_L(1 - \nu^2)/E]$$

$$\hat{N}_\eta = \text{NTAN}\ [hh_o\hat{\rho}(1 - \nu)/2] \qquad (5)$$

$$\hat{N}_\zeta = \text{NNORM}(hh_o\hat{\rho})$$

$$\hat{Q} = Q(h_o\hat{\rho}h^3/12X_L^2)$$

$$\hat{M} = M(h_o\hat{\rho}h^3/12X_L)$$

where

$$\hat{\rho} = \text{material density}$$

c. For panels under Linear Thermal Gradient across the thickness of shell

$$\hat{u} = u(X_L)$$

$$\hat{v} = v(X_L)$$

$$\hat{w} = w(X_L)$$

$$\hat{N}_\eta = \text{NTAN}\ [Eh/2(1 + \nu)] \qquad (6)$$

$$\hat{N}_\zeta = \text{NNORM}\ [Eh/(1 - \nu^2)]$$

$$\hat{Q} = Q(D/X_L^2)$$

$$\hat{M} = M(D/X_L)$$

where

$$D = E\hat{h}^3/12(1 - \nu^2)$$

Volume IV of this final report presents a set of basic equations for an isotropic spherical shell and a digital program which provides solutions for spherical panels under loads and changes of temperature. The program is designed to compute the fixed-edge forces. However, displacements, strains and stress resultants in the loaded region are also generated simultaneously. The following program options are available:

1. Finite-difference mesh
   (a) Uniform spacing
   (b) Graded spacing in the φ-direction
   (c) Symmetry in the φ-direction
2. Loading conditions
   (a) Uniform normal pressure
   (b) Linear temperature distribution across the thickness of shell.

Since the boundaries are not along the coordinate lines for the panel under consideration, the appropriate boundary conditions can not be satisfied exactly by the established finite-difference scheme. In order to approximate the boundary conditions as closely as possible, two methods for orienting one of the coordinates were devised for the subdivided segments of the spherical panel. A numerical example, using the first orientation of coordinate "φ" for the spherical subsegment which adjoins the conical panel (under uniform pressure) is also given (see Table 5 and Figs. 6 and 11). The nondimensional output quantities are related to the applied loads and the physical properties of the panel in the following way:

a. For panels under Uniform Pressure (p_z):

$$\begin{aligned}
\hat{u} &= u(p_z R^2/E\hat{h}) \\
\hat{v} &= v(p_z R^2/E\hat{h}) \\
\hat{w} &= w(p_z R^2/E\hat{h}) \\
\hat{N}_\eta &= \mathrm{NTAN}\ [p_z R/2(1 + \nu)]
\end{aligned} \tag{7}$$

$$\hat{N}_\zeta = \text{NNORM } [p_z R/(1 - \nu^2)$$

$$\hat{Q} = Q[p_z RK/(1 - \nu^2)]$$

$$\hat{M} = M[p_z R^2 K/(1 - \nu^2)]$$

b. For panels under Linear Thermal Gradient across the thickness

$$\hat{u} = u(R)$$

$$\hat{v} = v(R)$$

$$\hat{w} = w(R)$$

$$\hat{N}_\eta = \text{NTAN } [E\hat{h}/2(1 + \nu)] \tag{8}$$

$$\hat{N}_\zeta = \text{NNORM } [E\hat{h}/(1 - \nu^2)]$$

$$\hat{Q} = Q(D/R^2)$$

$$\hat{M} = M(D/R)$$

where

$$D = E\hat{h}^3/12(1 - \nu^2)$$

It is noted that Volumes II, III and IV as a whole provide the necessary information for the first step toward the solution of the bending problem.

Volume V introduces a technique for determining stiffness influence coefficients of cylindrical, conical and spherical panels, using the modified digital programs developed for fixed-edge shells under applied loads. Numerical values of influence coefficients for sample cylindrical, conical and spherical panels are also listed. This volume describes the necessary numerical procedure for the second step toward the solution of the bending problem.

Volume VI presents a detailed description of the third or final step toward the solution of the bending problem. It provides a technique to set up and solve the pertinent boundary-value problem of a structure composed of shell elements. The technique, known as the direct matrix stiffness or displacement method, is applied to the multicellular shell structure to predict the stresses and deformations due to loads and thermal gradients. The problem is formulated from the standpoint of transformations of coordinate systems, the compatibility and equilibrium requirements at the junctures, and the solution of a large set of algebraic equations. It is shown that the ordering of equations plays an important role in forming a desirable overall banded matrix. In addition to the consideration of solving the overall matrix equations simultaneously, the method of relaxing the fixed boundaries of shell elements successively is also introduced. The practicability of the technique is demonstrated by numerical examples which show how different shell elements are matched along their common juncture lines. Solutions of these examples are generated by a digital program designed and developed for this particular purpose.

2.2 The Buckling Analysis

Results of the studies concerning the buckling problem are described in Volumes VII and VIII of the final report. These results are summarized below:

Volume VII presents a theoretical solution for the buckling of orthotropic cylindrical panels with simply supported curved edges and elastically supported straight edges, and describes a digital computer program for the prediction of critical buckling loads under uniform compression applied in the axial direction only. A set of nondimensional buckling curves are provided for practical applications. These curves are based on the assumption that the straight edges are simply supported. Also, general expressions for the influence coefficients of rectangular orthotropic web plates are presented, and an investigation of the effects of flexibility of the web plate on the buckling of the panel is described.

Volume VIII presents a numerical analysis of the buckling of orthotropic cylindrical panels under both axial compression and shear loads applied at the generators. As a consequence of the introduction of shear loads, the stress distribution in the panel is no longer uniform and the inherent eigenvalue problem can be solved only through numerical analysis. A two-dimensional finite-difference scheme was used to solve the pertinent buckling equations. It is believed that this was the first attempt to solve the prescribed problem numerically. The objective of this work was to determine approximately the lower bound for buckling of cylindrical panels with the effect of web plates neglected.

Section 3
DISCUSSIONS AND RECOMMENDATIONS

3.1 Discussions

The scope of the present phase of studies on juncture stress fields involves three main topics, namely, (1) numerical methods for solving large matrices, (2) the bending analysis, and (3) the buckling analysis. In what follows these topics will be discussed in the same sequence.

Both direct Gaussian elimination and two-line iterative methods have been explored and successfully utilized to solve the related boundary-value problem. It was found that the latter method requires less computer time, especially when the size of matrix is large. However, Gaussian elimination has been used in most of the programs developed for the present work mainly because it was the first method developed. From the economics standpoint it would be desirable to replace the direct method used in some of these programs by the iterative technique especially if extensive parametric studies are to be made to generate design curves needed for practical applications. The two-dimensional finite-difference numerical procedure has been employed in an attempt to solve the buckling of cylindrical panels under non-uniform stress distribution. This technique seems quite promising but further work on programming is needed for obtaining meaningful solutions.

One of the remarkable achievements in the present phase of studies concerning the bending problem is the nondimensionalization of the basic equations for shell elements and the digital programs developed for the fixed-edge panels (see Volumes II, III and IV). If care is properly taken, these programs can be used in preliminary design for the approximate evaluation of the peak stresses at the interior as well as along the web plates of the panels of which the bulkheads are composed. However, as far as accurate stress distribution is concerned, the bending problem of the entire structure should be solved by the technique described in Volume VI, presumably using a package digital program. Unfortunately, the program for solving the overall matrix has not been fully developed, and additional programming work is needed for the complete mastery of the bending analysis of multicellular shell structures. In addition, it is conceivable that within the actual structure under consideration irregularities such as openings, manholes, etc. may exist. The determination of localized stresses within the neighborhood of these anisotropic or variable thickness junctures requires special techniques. Some of these techniques such as digital programs for the analysis of variable thickness axisymmetrical shells, are available in LMSC.

The accomplishment on the buckling of orthotropic cylindrical panels under uniform stress distribution as reported in Volume VII is by no means a complete solution to the problem. However, with the available non-dimensional design curves and the accompanying digital program, approximate buckling loads of the cylindrical panels within the multicellular shell structure as well as interstages can be predicted quite readily. From Figs. 5 through 16, one finds that within the range of R/t considered the buckling coefficients are slightly higher for an orthotropic panel when the stiffeners are placed outside of the skin rather than inside of the shell. For monocoque panels ($\rho = 1$), the buckling coefficient "K" is given by the classical value of $1/[3(1 - \nu^2)]^{1/2}$ if $\Omega > 0.2$. It is believed that the coefficients as presented in Figs. 5 to 16 are conservative and may well be used as a guide in preliminary design or in obtaining more accurate results using associated digital programs. Since the principle of minimization of energy has been employed, the buckling analysis of cylindrical panels under non-uniform stress distribution as formulated in Volume VIII is unique. However, difficulties have been encountered in developing the associated digital program which involves a two-dimensional finite-difference approximate of the buckling equations. As mentioned earlier, further work on the programming is desirable.

3.2 Recommendations for Further Studies

Based on the experience and the results obtained in the present studies (M-77-65-1 to 9) and those from the previous studies [M-03-63-1 (NASA CR-61050)] as well as the fact that a 200-in. model has been tested recently by NASA/MSFC with no attempt made to correlate the test results with those obtained from analytic solutions, it is recommended that the following additional studies be made:

1. Improvement of Numerical Techniques

Under the present contract, the use of iterative methods to solve large matrices has been explored. It is suggested that the most favorable iterative methods be incorporated in some of the programs that have been developed. Further development of iterative techniques is needed to introduce non-uniform spacing and to explore the "alternating direction" iterative methods. Also further improvement is needed on the numerical technique used to obtain eigenvalues of the determinants resulting from the finite-difference approximation of the buckling equations for non-uniform stress distribution.

2. Correlation of Test Results for the 200-In. Model with Analytic Solutions

Use the programs developed for bending analysis in conjunction with LMSC in-house digital programs to determine stresses and deformations at specific locations in the model and correlate these results with test data. This work will include the determination of stress distributions at the juncture of manholes and other irregularities in both isotropic and orthotropic elements.

3. Optimization of a Specific Prototype Multicellular Shell Structure

Utilize the bending and buckling programs developed under the present contract to optimize a specific complete multicellular structure assigned by NASA/MSFC. The optimum is here defined as the lightest weight structure wherein two or more buckling modes are critical under the applied loading, and in which the membrane as well as bending stresses do not exceed stated maxima. If the size of the specified structure is appreciable, the effect of dead weight of the structure should be taken into account. This implies that slight modification of the programs developed for the fixed-edge cylindrical and spherical panels is required. Two comparisons appear to be of interest: first, to determine the optimum type of stiffening and shell diameter/volume; second, to compare two or more specific types of stiffening given a shell with a fixed number of cells.

4. Evaluation of Residual Stresses and Joint Efficiency

As a result of previous investigation, it was noted that bi-axial residual stresses exist in the welded structure if it is not heat treated after welding. In order to incorporate these stresses in the numerical analysis, experimental determination of their magnitude and distribution is necessary. If these stresses are known, joint efficiency for both uni-axial and bi-axial states of stress can be evaluated, using an appropriate yield criterion for the bi-axial case. Results so obtained can be correlated by those obtained experimentally.

5. Determination of Model Characteristics and Dynamic Response of Bulkheads

In order to achieve overall structural integrity of the vehicle, knowledge of static response is not sufficient. In addition, capability to obtain the dynamic response of the shell structure during operational conditions is required. To achieve this capability, it is suggested that model characteristics of the shell elements for appropriate boundary conditions be determined and that a digital program for the prediction of dynamic response of the bulkhead under a simplified forcing function representing the launching and in-flight environment be developed.